MATH 375 HOMEWORK 9
DUE 5PM ON WEDNESDAY, DECEMBER 8 (NOTE UNUSUAL TIME AND DAY)

1. Show that if $f(x) = |x|$, then for any number m,

$$\lim_{h\to 0} (f(0+h) - f(0) - mh) = 0$$

but

$$\lim_{h\to 0} \frac{1}{h} ((f(0+h) - f(0) - mh) = 0$$

never holds. What does this say about f?

2. Find the Jacobian matrices of the following maps. Show your work.

(1) $f(x,y) = e^{x^2+y^3}$
(2) $f(x,y) = (xy, \sin(xy))$.

3. Consider the following function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{x^4+y^4}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Show that f is differentiable at $(0,0)$.

4. Consider the following function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Show that at $(0,0)$ the Jacobian of f exists, but f is not differentiable.

5. Apostol Section 8.9 Problem 13

6. Apostol Section 8.14 Problem 2

7. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ satisfy the conditions $f(\vec{0}) = (1, 2)$ and

$$Df(\vec{0}) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$$

Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be $g(x, y) = (x + 2y + 1, 3xy)$. Find $D(g \circ f)(\vec{0})$.

8. Let $f : \mathbb{R}^3 \to \mathbb{R}$, Let $g : \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Let $F : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$F(x, y) = f(x, y, g(x, y))$$

a) Find DF in terms of the partials of f and g.

b) If $F(x, y) = 0$ for all (x, y), find $D_1 g$ and $D_2 g$ in terms of the partials of f.