Math 375

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(1) 4.4 Pr 4

$$\text{If } T : V \to V \text{ has the property that } T^2 \text{ has a nonnegative eigenvalue } \lambda^2$$
$$\text{Prove at least one of } \lambda \text{ or } -\lambda \text{ is an eigenvalue for } T.$$
$$T^2(x) = \lambda^2 x$$
$$T^2(x) - \lambda^2 x = 0$$
$$(T^2 - \lambda^2 I)x = 0$$

$$\det\left(T^2 - \lambda^2 I\right) = 0$$
$$\det\left(T^2 - \lambda^2 I\right) = \det\left(T - \lambda I\right)\det\left(T + \lambda I\right)$$

$$\det\left(T - \lambda I\right) \text{ is zero or } \det\left(T + \lambda I\right) \text{ is zero}$$
$$\text{Therefore at least one of } \lambda \text{ or } -\lambda \text{ is an eigenvalue for } T.$$

(2) 4.4 Pr 11

$$\text{Assume that a linear transformation } T \text{ has two eigenvectors } x \text{ and } y \text{ belonging to distinct}$$
$$\text{eigenvalues } i \text{ and } p. \text{ If } ax + by \text{ is an eigenvector of } T \text{ prove that } a = 0 \text{ or } b = 0$$
$$T(x) = ix \quad T(y) = py$$
$$\text{If } ax + by \text{ is an eigenvector } T(ax + by) = \lambda(ax + by)$$
$$aT(x) + bT(y) = \lambda ax + \lambda by$$

$$aT(x) + bT(y) = aix + bpy$$

$$\lambda ax + \lambda by = aix + bpy$$
$$(\lambda - i)ax + (\lambda - p)by = 0$$
$$\text{Since } x \text{ and } y \text{ are linearly independent}$$
$$(\lambda - i)a = 0 \quad (\lambda - p)b = 0$$
$$\text{if } a \neq 0 \text{ and } b \neq 0$$
$$\lambda = i = p$$
$$\text{However the problem said } i \text{ and } p \text{ are distinct eigenvalues}$$
$$\text{Therefore } a \neq 0 \text{ and } b \neq 0 \text{ is false}$$

$$\text{It proves that } a = 0 \text{ or } b = 0$$

(3) 4.4 Pr 12

$$\text{Let } T : S \to V \text{ be a llinear transformation such that every nonzero element of S is an eigenvector}$$
$$\text{Prove that there exist a scalar } c \text{ such that } T(x) = cx$$
$$\text{In other words the only transformation with this property is a scalar times the identity}$$

$$\text{let } x \text{ is an vector in } S$$
$$y \text{ is also a vector in } S \text{ which is dependent with } x$$
$$y = kx$$
$$T(y) = kT(x) = kcx = cy \text{ therefore } c \text{ exist}$$

$$\text{if } x \text{ and } y \text{ are independent}$$
$$\text{and they are belonging to distinct eigenvalues, the previous problem showed that}$$
$$x + y \text{ can't be eigenvector}$$
$$\text{It means for } ax + by \text{ which } a \neq 0 \text{ and } b \neq 0 \text{ to be eigenvector}$$
$$\text{belonging eigenvalues should be not distinct}$$

$$x \text{ and } y \text{ are independent and having eigenvalues that aren't distinct}$$
$$T(x) = \lambda x$$
$$T(y) = \lambda y$$

$$Txx^{-1} = \lambda xx^{-1}$$
$$T = \lambda I$$
$$Tyy^{-1} = \lambda yy^{-1}$$
$$T = \lambda I$$

$$\text{So } T(ax + by) = aT(x) + bT(y) = a\lambda x + b\lambda y = \lambda(ax + by)$$

$$\text{So } T \text{ is scalar times identity that has every nonzero elements as eigenvector}$$

(4) 4.8 Pr 1

$$\text{Determine the eigenvalues and eigenvectors of each of the matrices}$$
$$\text{for each eigenvalue } \lambda \text{ compute the dimension of the eigenspace } E(\lambda)$$
$$\text{a) } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{let } A$$

(5) 4.8 Pr 7

(6) 4.8 Pr 14

(7) 8.3 Pr 3

(8) 8.3 Pr 5