7. (12 points) Recall that an *isomorphism* between two vector spaces V and W is a linear transformation $T : V \to W$ that is one to one and onto (i.e. $T(V) = W$). In this case we say that V is isomorphic to W. Prove any **TWO** of the following.

   1. Show that V is isomorphic to itself.
   2. Show that if V is isomorphic to W then W is isomorphic to V.
   3. Show that if V is isomorphic to W and W is isomorphic to U then V is isomorphic to U.

iso

1. ~~$T: V \to V$~~

~~$\dim V = \dim(T(V)) + \dim N(T)$~~

2. $T: W \to V$

$T(W) = V$

1. $T(V) \neq V$

$T: V \to V$

~~$\dim V = \dim(T(V)) + \dim N(T)$~~

~~Since~~

~~let linear transformation $T: V \to V$ is~~

Consider linear transformation that maps every elements of V to themselves then it is one-to-one and onto

2. $T(V) = W$

$\dim V = \dim(T(V)) + \dim N(T)$

Since it is onetoone & onto $\dim V = \dim(T(V)) \Rightarrow \dim V = \dim W$

$T(W) = V$

$\dim W = \dim(T(W)) + \dim N(T)$

Since it has to be one-toone and $\dim W = \dim V$

$\dim T(W) = \dim V$ it is one-to-one and having same dimension so W is isomorph

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3. $T(U) = W$

$T(W) = U$

$T(T(V)) = U$

$TT(V) = U$

So there exist linear tranformation that satisfy $T(V) = U$

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