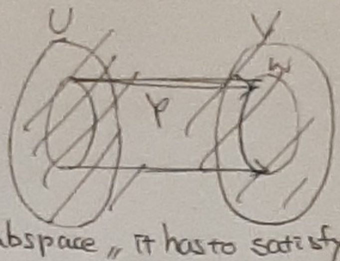
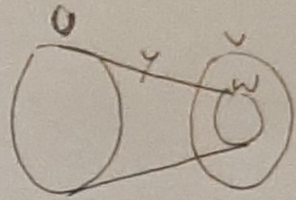
Let W be a subspace of V. Let Y be the subset of $\mathrm{Hom}(U, V)$ defined as follows:

$$Y = \{T : U \to V \mid T(U) \subset W\}.$$

Show that Y is a subspace.

$U \to V$





For a ~~subsp~~ subset to be subspace, it has to satisfy closure axioms.
Have to show Y satisfy ~~clous~~ closure axioms

$S, T \in Y$

so $S+T \in Y$
and
$cS \in Y$ $c = \mathbb{R}$

$x \in U$

using axiom of linear transformat

• first $\underbrace{cS(x) = S(cx)}_{\text{satisfied}}$ since $S \in Y$

need to show $cSx \in W$.

~~$S(a)+T(a)=$~~

• Second ~~$(S+T)(x) = S(T(x))$ $(ST)(x) = S(T(x))$~~
~~satisfied~~.

~~Therefore Y is a subspace.~~

for any S and T, $S(U)+T(U)$ since $S(U) \subset W$ $T(U) \subset W$ and W is a subspace
W is ~~closed~~ satisfying closure axioms, so it is closed under addition
therefore $S(U)+T(U) \subset W$ so $S+T$ also satisfy $S+T \in Y$

Therefore Y is a subspace

W