## Grade Table.

Question	Points	Score	
1	15	15	
2	10	10	
3	10	10	
4	12	12	
5	6	6	
6	10	8 -	2
7	12	6 -1	-
Total:	75	67	

1. (15 points) Solve the system

$$\begin{bmatrix} 1 & 0 & 1 \\ -4 & 1 & -1 \\ 6 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 1 \end{bmatrix}$$

by finding  $A^{-1}$  where  $A = \begin{bmatrix} 1 & 0 & 1 \\ -4 & 1 & -1 \\ 6 & -2 & 1 \end{bmatrix}$ .

AZ-C

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$$A^{1} = \begin{bmatrix} -1 & -2 & -1 \\ -2 & 4 & -3 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^{4}C = \begin{bmatrix} -1 & -2 & -1 \\ -2 & -6 & -8 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1/4 & -1 \\ -1/4 & -1 \\ -1/4 & -1 \end{bmatrix} \begin{bmatrix} -1/4 \\ -2 & -35 & -3 \\ 2 & +14 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1/4 \\ -2 & -35 & -3 \\ 2 & +14 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

101 -1 -2 -1 -4 1-1 -2 -4 -1 6 -2 1 2 2 1 8-4-2 2. (10 points) Let A and B be  $n \times n$  matrices. Prove that if either Ker A or Ker B is non-trivial, then Ker AB is also non-trivial.

Since der AB der Ader B and it der Ax her Ax more town

if one of dotA or dotBic zero that makes dotAB to @ zero.

The circle dotAB = dotAB = dotAde+B

if determinant is 0 then it is non-trivial.

(james)

3. (10 points) Let  $M_{2\times 2}$  be the vector space of all  $2\times 2$  matrices, with the basis

$$\{\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\}$$

Find the matrix representative with respect to the above basis for the linear transformation T transposition. i.e  $T: M_{2\times 2} \to M_{2\times 2}$  is given by  $T(A) = A^T$ . (Hint:  $M_{2\times 2}$  is a 4-dimensional vector space. So, this linear transformation should be represented by a 4 × 4 matrix.)

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to a balance

4. (12 points) If you know that det | Row 2 | = 4, what is the determinant of

$$B = \begin{bmatrix} 3\text{Row } 1 + 2\text{Row } 2 \\ \text{Row } 2 + 2\text{Row } 3 \\ \text{Row } 3 + \text{Row } 1 \end{bmatrix}$$

Di 100

0000

0.46

$$\begin{vmatrix} 3a+2b \\ b+2c \\ c+a \end{vmatrix} = \begin{vmatrix} 3a \\ b+2c \\ c+a \end{vmatrix} + \begin{vmatrix} 2b \\ b+2c \\ c+a \end{vmatrix} + \begin{vmatrix} 2a \\ b+2c \\ c+a \end{vmatrix} + \begin{vmatrix} 2a \\ b+2c \\ c+a \end{vmatrix} + \begin{vmatrix} 2b \\ b+2c \\ c+a \end{vmatrix} + \begin{vmatrix} 2b \\ 2c \\ c+a \end{vmatrix}$$

$$= \begin{vmatrix} 3a \\ b \\ c+a \end{vmatrix} + \begin{vmatrix} 3a \\ 2c \\ c+a \end{vmatrix} + \begin{vmatrix} 3a \\ 2c \\ c+a \end{vmatrix} + \begin{vmatrix} 2b \\ 2c \\ c+a \end{vmatrix}$$

$$= \begin{vmatrix} 3a \\ b \\ c+a \end{vmatrix} + \begin{vmatrix} 3a \\ 2c \\ c+a \end{vmatrix} + \begin{vmatrix} 3a \\ 2c \\ c+a \end{vmatrix} + \begin{vmatrix} 2b \\ 2c \\ c+a \end{vmatrix}$$

5. (6 points) Show that the set of vectors i(1, -3, 2), (2, 1, -3), (-3, 2, 1)) are linearly independent using determinants. Justify your answer.

1-3 2 By the property of characterists
= 1-3 When at a record are independent
-3 2 1 determinant is not zero, but when dependent missions

7+8 2 2 4.7

2-64

-1 1

since determinents o the rows are dependent

6. (10 points) TRUE or FALSE. You don't have to justify.

ab

we If AB and BA are defined then A and B are square matrices.

If A is a  $2 \times 2$  matrix, then det(2A) = 2det(A).

True If the rows of a square matrix A are linearly independent, so are the rows of  $A^2 = AA$ .

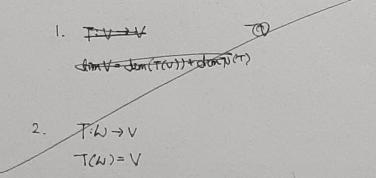
False Any system of linear equations has at most one solution.

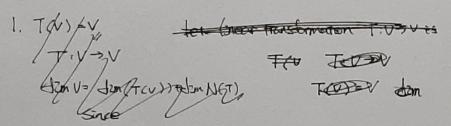
10 5

Take If the entries of both A and  $A^{-1}$  are integers, it is possible that det A = 3.

JerAJan'= 1

- 1. Show that V is isomorphic to itself.
- 2. Show that if V is isomorphic to W then W is isomorphic to V.
- 3. Show that if V is isomorphic to W and W is isomorphic to U then V is isomorphic to U.





by Consider linear transformation that images every elements of V to mess therefores then it is one-to-one and onta

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2. T(V) = W

dem V = dem(T(V)) + demN(T)

Since it is aneroone & onto dem V = dem(T(V)) \Rightarrow dem V = dem W

T(W) = V

dem W = dem(T(W) + demN(T))

Since it has to be ano-toone dem and dem W = dem V

dem W = dem(T(W)) = dem V it is one-to-one and however some demonstrant so W is isomorph
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3.  $T(u)=\omega$  T(u)=U T(T(v))=U TT(v)=USo there is a solution of the solution of