1. (10 points) Let $a_1 = \sqrt{2}, a_2 = \sqrt{2+\sqrt{2}}, \dots, a_n = \sqrt{2+\dots\sqrt{2+\sqrt{2+\sqrt{2}}}}$. Prove that $a_n < 2$ for all $n \geq 1$ using mathematical induction.

1. Base step. when $n=1$,

$a_1 = \sqrt{2}$, $\sqrt{2} < 2$ → therefore true when $n=1$

2. ~~Supp~~ Assume $a_k < 2$ and prove $a_{k+1} < 2$.
let $a_k = A$ then $a_{k+1} = \sqrt{2+A}$
$A < 2$ and we have to show $\sqrt{2+A} < 2$

$\sqrt{2+A} < 2$

square

$2+A < 4$

$A < 2$ since we assume $A<2$ the inequality is true.

thus. $a_n < 2$ for $n \geq 1$ is true.

2. (9 points) TRUE or FALSE. You don't have to justify.

False The set of all polynomials of degree 2 is a vector space with the usual operations.

True Let V be an inner product space. Then, any orthogonal set of non-zero vectors is linearly independent.

True The kernel of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ satisfies $3 \geq dim(Ker T) \geq 1$.