(scrap)

Correction

4) $S = \{x_1, x_2, x_3\}$ $\Rightarrow$ independent subset of vector space V $y \in V$ but $y \notin L(S)$

prove $S \cup y$ is independent.

$S \cup y = \{x_1, x_2, x_3, y\}$ to prove it is independent $a_n \in \mathbb{R}$

have to show that $a_1x_1 + a_2x_2 + a_3x_3 + a_4y = 0$ and $a_1 = 0, a_2 = 0, a_3 = 0, a_4 = 0$

$a_1x_1 + a_2x_2 + a_3x_3 + a_4y = 0$

there can be two cases $a_4 = 0$ or $a_4 \neq 0$

i) $a_4 = 0$

$a_1x_1 + a_2x_2 + a_3x_3 = 0$ since S is independent.

a_1, a_2, a_3 must be zero

ii) $a_4 \neq 0$

let $a_4 = c$ which isn't zero

$a_1x_1 + a_2x_2 + a_3x_3 = -cy$

$(-\frac{a_1}{c})x_1 + (-\frac{a_2}{c})x_2 + (-\frac{a_3}{c})x_3 = y$ However the equation can't be true since $y \notin L(S)$

which means y can't be made by linear combination of S.

Therefore a_1, a_2, a_3, a_4 are all zero and $y \cup S$ is independent