a) Show that $S = \{(1,1,-3),(2,1,1),(4,-7,-1)\}$ is a basis for $\mathbb{R}^3$. (*Hint*: Is there anything special about this set when you think about $\mathbb{R}^3$ with the dot product?)

$$\begin{matrix} 1 & 2 & 4 \\ 1 & 1 & -7 \\ -3 & 1 & -1 \end{matrix}$$

Any set of independent elements in V is a subset of basis
And every basis for V has same number of elements
therefore any set of n independent elements is basis. //

We need to show S is independent.
for S to be dependent there must be a, b, c which are not all zero.
that satisfy $a\begin{bmatrix}1\\1\\-3\end{bmatrix} + b\begin{bmatrix}2\\1\\1\end{bmatrix} + c\begin{bmatrix}4\\-7\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

8

$a+2b+4c=0$
$a+b-7c=0$) → $b+11c=0$
$-3a+b-c=0$) → $4b-22c=0$

$b=-11c$ and $b=\frac{11}{2}c$
~~there doest~~ there isn't solution

only $a=b=c=0$ satisfy the equation
so S is independent and also basis for $\mathbb{R}^3$

b) Find the components of the vector $(1,0,1) \in \mathbb{R}^3$ with respect to the basis S. ?

2/1

3 2 -2

2 3 -3

$a(1,1,-3)+b(2,1,1)+c(4,-7,-1) = (1,0,1)$

8

$a+2b+4c=1$
$a+b-7c=0$ > $b+11c=1$ > $b=\frac{1}{2}$, $c=\frac{1}{22}$ > $a=-\frac{4}{22}$
$-3a+b-c=1$ > $4b-22c=1$

$b+11c=1$
$2b+22c=2$
$4b-22c=1$

$6b=3$

$\frac{11}{22}-\frac{7}{22}$

$\frac{4}{22}$

$-\frac{4}{22}(1,1,-3)+\frac{1}{2}(2,1,1)+\frac{1}{22}(4,-7,-1) = (1,0,1) \in \mathbb{R}^3$

$(-\frac{4}{22}, \frac{1}{2}, \frac{1}{22})_S = (1,0,1)$

$\frac{4}{22}$ + 7 +