

5. (6 points) Show that the set of vectors  $\{(1, -3, 2), (2, 1, -3), (-3, 2, 1)\}$  are linearly independent using determinants. Justify your answer.

$$\begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ -3 & 2 & 1 \end{pmatrix}$$

By the properties of determinants

$$\begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ -3 & 2 & 1 \end{pmatrix}$$

When all 3 rows are independent

$$\begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ -3 & 2 & 1 \end{pmatrix}$$

determinant is not zero, but when dependent it is zero

$$\det \begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ -3 & 2 & 1 \end{pmatrix} = 1 \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} - (-3) \begin{vmatrix} 2 & -3 \\ -3 & 1 \end{vmatrix} + 2 \begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix}$$

$$= 1(1+6) + 3(2-9) + 2(4+3)$$

$$= 7 + 3(-7) + 14$$

$$= 7 - 21 + 14$$

$$= 0$$

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$$= 1(1+6) + 3(2-9) + 2(4+3)$$

$$= 7 - 21 + 14$$

$$= 0$$

Since determinant is 0 the rows are dependent  
Using upper triangular form the det is product of diagonal elements

$$\begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ -3 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & 2 \\ 0 & 7 & -7 \\ 0 & -1 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & 2 \\ 0 & 7 & -7 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ -3 & 2 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & -3 & 2 \\ 0 & 7 & -7 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{determinant is } 1 \times 7 \times 0 = 0$$

Since determinant is 0 the rows are dependent

$$\begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ -3 & 2 & 1 \end{pmatrix}$$

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6. (10 points) TRUE or FALSE. You don't have to justify.

a b  
c d

True If  $AB$  and  $BA$  are defined then  $A$  and  $B$  are square matrices.

False If  $A$  is a  $2 \times 2$  matrix, then  $\det(2A) = 2\det(A)$ .

True If the rows of a square matrix  $A$  are linearly independent, so are the rows of  $A^2 = AA$ .

False Any system of linear equations has at most one solution.

False If the entries of both  $A$  and  $A^{-1}$  are integers, it is possible that  $\det A = 3$ .

$$\det A \det A^{-1} = 1$$

$$\det A^{-1} = \frac{1}{3}$$