Math 375

Geonho Roh

(1) 3.6 Pr 1

$$\text{Compute each of the following determinants}$$

$$\text{a)} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & -4 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\text{Using upper trianglular form}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & -4 \\ 1 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & 4 & -6 \\ 1 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & 4 & -6 \\ 0 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & 4 & -6 \\ 0 & 0 & \frac{3}{4} \end{bmatrix}$$

$$det = 2 \times 6 \times \frac{3}{4} = 6$$

$$\text{b)} \begin{bmatrix} 3 & 0 & 8 \\ 5 & 0 & 7 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\text{using upper triangular form}$$

$$\begin{bmatrix} 3 & 0 & 8 \\ 5 & 0 & 7 \\ -1 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 & 8 \\ 0 & 20 & 17 \\ -1 & 4 & 2 \end{bmatrix} \rightarrow (\times 3 \text{ det}) \begin{bmatrix} 3 & 0 & 8 \\ 0 & 20 & 17 \\ -3 & 12 & 6 \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} 3 & 0 & 8 \\ 0 & 20 & 17 \\ 0 & 12 & 14 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 & 8 \\ 0 & 20 & 17 \\ 0 & 0 & 14 - \frac{3}{5}17 \end{bmatrix}$$

$$det = \frac{1}{3} \times 3 \times 20 \times (14 - \frac{51}{5}) = 280 - 204 = 76$$

$$\text{c)} \begin{bmatrix} a & 1 & 0 \\ 2 & a & 2 \\ 0 & 1 & a \end{bmatrix}$$

$$\text{using equation 3.2}$$

$$\text{det} = a \ \text{det} \begin{bmatrix} a & 2 \\ 1 & a \end{bmatrix} - 1 \ \text{det} \begin{bmatrix} 2 & 2 \\ 0 & a \end{bmatrix} + 0 \, \text{det} \begin{bmatrix} 2 & a \\ 0 & 1 \end{bmatrix}$$
$$= a(a^2 - 2) - 1(2a - 0) = a^3 - 4a$$

(2) 3.6 Pr 2

$$\text{If determinant of } \begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} = 1 \text{ compute the determinant of following matrices}$$

$$\begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} \det \begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} &= x \ \det \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} - y \det \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + z \det \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \\ &= x(-2) - y(3-2) + z(3-0) \\ &= -2x - y + 3z \end{aligned}$$

$$\text{a) } \det \begin{bmatrix} 2x & 2y & 2z \\ 3/2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \det 2 \begin{bmatrix} x & y & z \\ 3/2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \det \begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} = 1$$

$$\text{b) } \begin{bmatrix} x & y & z \\ 3x+3 & 3y & 3z+2 \\ x+1 & y+1 & z+1 \end{bmatrix}$$

$$\begin{aligned} \det \begin{bmatrix} x & y & z \\ 3x+3 & 3y & 3z+2 \\ x+1 & y+1 & z+1 \end{bmatrix} &= \det \begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ x+1 & y+1 & z+1 \end{bmatrix} = \det \begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ x+1 & y+1 & z+1 \end{bmatrix} \\ &= \det \begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} = 1 \end{aligned}$$

$$\text{c) } \begin{bmatrix} x-1 & y-1 & z-1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} \det \begin{bmatrix} x-1 & y-1 & z-1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} &= \det \begin{bmatrix} x & y & z \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \\ &= \det \begin{bmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} = 1 \end{aligned}$$

(3) 3.11 Pr 1

For each of the following statements about square matrices, give a proof or exhibit a counter example

$$\text{a) } \det A + \det B = \det(A+B)$$

$$\text{Not true}$$
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$
$$\det A = 1, \det B = 4$$
$$\det(A+B) = \det \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 9$$

$$\text{b) } \det\left((A+B)^2\right) = (\det(A+B))^2$$

$$\det\left((A+B)^2\right) = \det((A+B)(A+B))$$
$$\text{Since } \det AB = \det A \det B$$
$$\det((A+B)(A+B)) = \det(A+B)\det(A+B) = (\det(A+B))^2$$

$$\text{c) } \det\left((A+B)^2\right) = \det\left(A^2 + 2AB + B^2\right)$$

$$\text{For the equation to be true}$$
$$(A+B)(A+B) = A^2 + BA + AB + B^2 = A^2 + 2AB + B^2$$
$$\text{It has to satisfy } AB = BA$$

$$\text{However it is false since counter Ex exist}$$
$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$
$$AB = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$$
$$BA = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{d) } \det\left((A+B)^2\right) = \det\left(A^2 + B^2\right)$$

$$\text{If } \det\left((A+B)^2\right) = \det\left(A^2 + B^2\right)$$
$$AB + BA = 0$$
$$\text{Using the process from previous question we can easily check that}$$
$$AB + BA \neq 0$$

(4) 3.17 Pr 1

$$\text{Determine the cofactor of following matrices}$$

a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$cof(a_{11}) = (-1)^{1+1}4 = 4$$
$$cof(a_{12}) = (-1)^{1+2}3 = -3$$
$$cof(a_{21}) = (-1)^{2+1}2 = -2$$
$$cof(a_{22}) = (-1)^{2+2}1 = 1$$

$$\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

---

b) $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & 0 \end{bmatrix}$

$$cof(a_{11}) = (-1)^{1+1} \det \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} = 2$$
$$cof(a_{12}) = (-1)^{1+2} \det \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -1$$
$$cof(a_{13}) = (-1)^{1+3} \det \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} = 1$$
$$cof(a_{21}) = (-1)^{2+1} \det \begin{bmatrix} -1 & 3 \\ -2 & 0 \end{bmatrix} = -6$$
$$cof(a_{22}) = (-1)^{2+2} \det \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = 3$$
$$cof(a_{23}) = (-1)^{2+3} \det \begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix} = 5$$
$$cof(a_{31}) = (-1)^{3+1} \det \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix} = -4$$
$$cof(a_{32}) = (-1)^{3+2} \det \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} = -2$$
$$cof(a_{33}) = (-1)^{3+3} \det \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} = 2$$

$$\begin{bmatrix} 2 & -1 & 1 \\ -6 & 3 & 5 \\ -4 & -2 & 2 \end{bmatrix}$$

$$\text{c)} \begin{bmatrix} 3 & 1 & 2 & 4 \\ 2 & 0 & 5 & 1 \\ 1 & -1 & -2 & 6 \\ -2 & 3 & 2 & 3 \end{bmatrix}$$

$$cof(a_{11}) = (-1)^{1+1} \det \begin{bmatrix} 0 & 5 & 1 \\ -1 & -2 & 6 \\ 3 & 2 & 3 \end{bmatrix} = (-1)^{1+1}((-5)(-21) + 1(4)) = 109$$

$$cof(a_{12}) = (-1)^{1+2} \det \begin{bmatrix} 2 & 5 & 1 \\ 1 & -2 & 6 \\ -2 & 2 & 3 \end{bmatrix} = (-1)^{1+2}((2)(-18) + (-5)(15) + 1(-2)) = 113$$

$$cof(a_{13}) = (-1)^{1+3} \det \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 6 \\ -2 & 3 & 3 \end{bmatrix} = (-1)^{1+3}((2)(-21) + 1(1)) = -41$$

$$cof(a_{14}) = (-1)^{1+4} \det \begin{bmatrix} 2 & 0 & 5 \\ 1 & -1 & -2 \\ -2 & 3 & 2 \end{bmatrix} = (-1)^{1+4}((2)(4) + 5(1)) = -13$$

$$cof(a_{21}) = (-1)^{2+1} \det \begin{bmatrix} 1 & 2 & 4 \\ -1 & -2 & 6 \\ 3 & 2 & 3 \end{bmatrix} = (-1)^{2+1}((1)(-18) + (-2)(-21) + 4(4)) = -40$$

$$cof(a_{22}) = (-1)^{2+2} \det \begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 6 \\ -2 & 2 & 3 \end{bmatrix} = (-1)^{2+2}((3)(-18) + (-2)15 + 4(-2)) = -92$$

$$cof(a_{23}) = (-1)^{2+3} \det \begin{bmatrix} 3 & 1 & 4 \\ 1 & -1 & 6 \\ -2 & 3 & 3 \end{bmatrix} = (-1)^{2+3}((3)(-21) + (-1)15 + 4(1)) = 74$$

$$cof(a_{24}) = (-1)^{2+4} \det \begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & -2 \\ -2 & 3 & 2 \end{bmatrix} = (-1)^{2+4}((3)(4) + (-1)(-2) + 2(1)) = 16$$

$$\text{c)} \begin{bmatrix} 3 & 1 & 2 & 4 \\ 2 & 0 & 5 & 1 \\ 1 & -1 & -2 & 6 \\ -2 & 3 & 2 & 3 \end{bmatrix}$$

$$cof(a_{31}) = (-1)^{3+1} \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 1 \\ 3 & 2 & 3 \end{bmatrix} = (-1)^{3+1}((1)(13) + (-2)(-3) + 4(-15)) = -41$$

$$cof(a_{32}) = (-1)^{3+2} \det \begin{bmatrix} 3 & 2 & 4 \\ 2 & 5 & 1 \\ -2 & 2 & 3 \end{bmatrix} = (-1)^{3+2}((3)(13) + (-2)8 + 4(14)) = -79$$

$$cof(a_{33}) = (-1)^{3+3} \det \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & 1 \\ -2 & 3 & 3 \end{bmatrix} = (-1)^{3+3}((3)(-3) + (-1)8 + 4(6)) = 7$$

$$cof(a_{34}) = (-1)^{3+4} \det \begin{bmatrix} 3 & 1 & 2 \\ 2 & 0 & 5 \\ -2 & 3 & 2 \end{bmatrix} = (-1)^{3+4}((3)(-15) + (-1)14 + 2(6)) = 47$$

$$cof(a_{41}) = (-1)^{4+1} \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 1 \\ -1 & -2 & 6 \end{bmatrix} = (-1)^{4+1}((1)(32) + (-2)(1) + 4(5)) = -50$$

$$cof(a_{42}) = (-1)^{4+2} \det \begin{bmatrix} 3 & 2 & 4 \\ 2 & 5 & 1 \\ 1 & -2 & 6 \end{bmatrix} = (-1)^{4+2}((3)(32) + (-2)11 + 4(-9)) = 38$$

$$cof(a_{43}) = (-1)^{4+3} \det \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & 1 \\ 1 & -1 & 6 \end{bmatrix} = (-1)^{4+3}((3)(1) + (-1)11 + 4(-2)) = 16$$

$$cof(a_{44}) = (-1)^{4+4} \det \begin{bmatrix} 3 & 1 & 2 \\ 2 & 0 & 5 \\ 1 & -1 & -2 \end{bmatrix} = (-1)^{1+1}((3)(5) + (-1)(-9) + 2(-2)) = 20$$

$$\begin{bmatrix} 109 & 113 & -41 & -13 \\ -40 & -92 & 74 & 16 \\ -41 & -79 & 7 & 47 \\ -50 & 38 & 16 & 20 \end{bmatrix}$$

(5) 3.17 Pr 3

Find all values of the scalar λ for which the matrix $\lambda I - A$ is singular, if A is equal to

$$\text{a) } A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\begin{aligned} \lambda I - A &= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} \lambda & -3 \\ -2 & \lambda + 1 \end{bmatrix} \end{aligned}$$

$$\text{to be singular } \det \begin{bmatrix} \lambda & -3 \\ -2 & \lambda + 1 \end{bmatrix} = 0$$
$$\lambda^2 + \lambda - 6 = (\lambda + 3)(\lambda - 2)$$

$$-3, 2$$

$$\text{b) } A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix}$$

$$\begin{aligned} \lambda I - A &= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \lambda - 1 & 0 & -2 \\ 0 & \lambda + 1 & 2 \\ -2 & 2 & \lambda \end{bmatrix} \end{aligned}$$

$$\det \begin{bmatrix} \lambda - 1 & 0 & -2 \\ 0 & \lambda + 1 & 2 \\ -2 & 2 & \lambda \end{bmatrix} = 0$$
$$\begin{aligned} &\lambda - 1(\lambda^2 + \lambda - 4) - 2(2\lambda + 2) \\ &= \lambda^3 + \lambda^2 - 4\lambda - \lambda^2 - \lambda + 4 - 4\lambda - 4 \\ &= \lambda^3 - 9\lambda^2 \end{aligned}$$

$$0, 3, -3$$

$$\text{c) } A = \begin{bmatrix} 11 & -2 & 8 \\ 19 & -3 & 14 \\ -8 & 2 & -5 \end{bmatrix}$$

$$\begin{aligned} \lambda I - A &= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 11 & -2 & 8 \\ 19 & -3 & 14 \\ -8 & 2 & -5 \end{bmatrix} \\ &= \begin{bmatrix} \lambda - 11 & 2 & -8 \\ -19 & \lambda + 3 & -14 \\ 8 & -2 & \lambda + 5 \end{bmatrix} = 0 \end{aligned}$$

$$\det \begin{bmatrix} \lambda - 11 & 2 & -8 \\ -19 & \lambda + 3 & -14 \\ 8 & -2 & \lambda + 5 \end{bmatrix} = 0$$

$$\begin{gathered} \lambda - 11(\lambda^2 + 8\lambda + 15 - 28) - 2(-19\lambda - 95 + 112) - 8(38 - 8\lambda - 24) \\ = \lambda^3 + 8\lambda^2 + 15\lambda - 28\lambda - 11\lambda^2 - 88\lambda + 143 + 38\lambda - 34 - 112 + 64\lambda \\ = \lambda^3 - 3\lambda^2 + \lambda - 3 \end{gathered}$$

$$3, i, -i$$

(6) 4.4 Pr 1

$$\text{a) If } T \text{ has an eigenvalue } \lambda \text{, prove that } aT \text{ has the eigenvalue } a\lambda$$

$$\begin{aligned} Tx &= \lambda x \\ aTx &= a\lambda x \\ (aT)x &= (a\lambda)x \end{aligned}$$

$$\begin{gathered} \text{b)) If } x \text{ is an eigenvector for both } T_1 \text{ and } T_2 \\ \text{prove that } x \text{ is also an eigenvector for } aT_1 + bT_2 \\ \text{How are the eigenvalues related?} \end{gathered}$$

$$\begin{aligned} T_1 x &= \lambda_1 x \\ T_2 x &= \lambda_2 x \end{aligned}$$

$$aT_1 x + bT_2 x = a\lambda_1 x + b\lambda_2 x = (a\lambda_1 + b\lambda_2)x$$

(7) 4.4 Pr 2 1

Assume $T : V \to V$ has an eigenvector x belonging to an eigenvalue λ
Prove that x is an eigenvector of T^2 belonging to λ^2 and more generally
x is an eigenvector of T^n belonging to λ^n
Then use the result from Exercise 1 to show that if P is a polynomial
then x is an eigenvector of $P(T)$ belonging to $P(\lambda)$

$$Tx = x\lambda$$
$$Txx^{-1} = x\lambda x^{-1}$$
$$T = x\lambda x^{-1}$$
$$T^2 = x\lambda x^{-1}x\lambda x^{-1} = x\lambda^2 x^{-1}$$
$$T^2 x = x\lambda^2 x^{-1}x = x\lambda^2$$

Using induction
if $T^n x = x\lambda^n$
$$T^n xx^{-1} = x\lambda^n x^{-1}$$
$$T^n = x\lambda^n x^{-1}$$
$$T^{n+1} = T^n T = x\lambda^n x^{-1}x\lambda x^{-1} = x\lambda^{n+1}x^{-1}$$
$$T^{n+1}x = x\lambda^{n+1}x^{-1}x = x\lambda^{n+1}$$

$$P(x) = a_n x^n + \cdots + a_0$$
$$P(T) = a_n T^n + \cdots + a_0$$
using part a from Exercise 1 if λ^n is eigenvalue of T^n then $a\lambda^n$
is eigenvalue of aT^n
using part b from Exercise 1 if T_1, T_2
are having same eigenvector then eigenvalue of their sum is equal to sum of each eigenvalue
$$P(T)x = (a_n T^n + \cdots + a_0)x = (a_n T^n x + \cdots + a_0 x)$$
For each $a_n T^n x = a_n \lambda^n x$
Using the 1-(b) eigenvalue of $P(T) = a_n\lambda^n x + \ldots a_0$

$$a_n\lambda^n + \cdots + a_0 = P(\lambda)$$

Therefore x is an eigenvector of $P(T)$ belonging to $P(\lambda)$

(8) 4.4 Pr 3

Assume rotating on counterclock wise

Consider the plane as a real linear space, $V = V_2(R)$ and let T be a rotation of V through an angle of $\frac{\pi}{2}$ radians. Although T has no eigenvectors,

prove that every nonzero vector is an eigenvector for T^2

let $T : V_2 \to V_2$ is a linear transformation that rotates (x, y) with degree of θ

basis is $(1,0)(0,1)$

$$T((1,0)) = (cos\theta, sin\theta) = cos\theta(1,0) + sin\theta(0,1)$$

$$T((0,1)) = (-sin\theta, cos\theta) = -sin\theta(1,0) + cos\theta(0,1)$$

Therefore rotation matrix $T = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

Since T is rotating with an angle of $\frac{\pi}{2}$ rad

$$T = \begin{bmatrix} cos\frac{\pi}{2} & -sin\frac{\pi}{2} \\ sin\frac{\pi}{2} & cos\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\vec{x} = (x, y)$$

$$T(\vec{x}) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$

$$\text{eigenvalue for T}$$

$$\det(\lambda I - T) = 0$$

$$\det\left(\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix} - \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\right)$$

$$= \det\left(\begin{bmatrix}\lambda & 1\\ -1 & \lambda\end{bmatrix}\right)$$

$$= \lambda^2 + 1 = 0$$

satisfying eigenvalues are $-i, i$ that means there is no real eigenvalue for T

$$T^2 = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

eigenvalue is $\det(\lambda I - T^2) = 0$

$$\det\left(\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix} - \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\right)$$

$$= \det\left(\begin{bmatrix}\lambda+1 & 0\\ 0 & \lambda+1\end{bmatrix}\right)$$

$$= (\lambda+1)^2 = 0$$

eigenvalue is -1

$$T^2(\vec{x}) = \begin{bmatrix}-x\\ -y\end{bmatrix} = (-1)\begin{bmatrix}x\\ y\end{bmatrix}$$

Therefore every nonzero vector is an eigenvector for T^2