Exam 2: Topics and Practice Problems

Exam 2 will focus on the concepts below. This means that you should be able to reproduce any proofs we have done in class, and use theorems in these topics to solve problems or prove new results.

1 Topics

- Algebraic Operations on linear transformations, vector space of linear transformations
- Left and right inverse of a linear transformation
- One-to-one linear transformations
- Equivalent characterizations of 1-1'ness
- Invertible linear transformations
- Linear transformations with prescribed values
- Matrix representatives of linear transformations
- Choosing suitable bases to make matrix representative diagonal
- Vector space of matrices
- Matrix operations
- Finding a basis for the kernel of a matrix
- Finding a basis for the range of a matrix
- Solving systems of linear equations
- Row operations on matrices
- Reduced row echelon form
- Finding the inverse of a matrix
- Invertible matrices
- Axioms for determinants
- Computing determinants
- Applications of determinants

These topics are contained through Chapter 3 in the book.

2 Practice Problems

Once you are clear about all of the concepts above, solve all the homework problems. Not just look at their solutions, solve it again!

Here are some extra problems:

1. Professor Xavier throws a paper airplane out the window of his office. It catches a gust of wind and flies in a path given by $y = ax^2 + b\sqrt{x} + c$, for some constants over X-mansion. He is able to measure three points (x, y) that the paper airplane passes through: $(0,2), (1,2)$ and $(4,16)$.

(a) Write down an augmented matrix corresponding to the underlying linear system which must be solved.

(b) Put your matrix from part (a) into Echelon form.

(c) Use (b) to determine the exact expression for the path of the airplane.

2. Let $A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{bmatrix}$

a) Find a basis for the column space $C(A)$ of A.

b) Find a basis for the null space $N(A)$ of A.

3. Solve the system

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

by finding A^{-1} where $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$.

4. If you know that $det \begin{bmatrix} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \end{bmatrix} = 6$, what is the determinant of

$$B = \begin{bmatrix} \text{Row 1+2Row 2} \\ \text{Row 2+ Row 3} \\ \text{Row 3 +Row 1} \end{bmatrix}$$

5. Find the determinant of $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 3 & 0 \end{bmatrix}$

6. Section 2.8 pr 27

7. Section 2.12 pr 7

8. Section 2.16 pr 10

9. Section 2.20 pr 5

10. Section 2.21 pr 2

11. Prove that, if linear transformations A and B are invertible (and such that the product AB is defined), then the product AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

12. Prove, that if $A : V \to W$ is an isomorphism (i.e. a 1-1 and onto linear transformation) and $v_1, v_2, \ldots v_n$ is a basis in V, then $Av_1, Av_2, \ldots, Av_n$ is a basis in W.