MAT 2141 Problems

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3.3.3: Prove that $F[x]$ is not finitely generated. Hint: Suppose $p_1, \ldots, p_n \in F[x]$. Find $p \in F[x]$ such that $p \notin Span\{p_1, \ldots, p_n\}$

Proof 3.3.3: Assume $F[x]$ is finitely generated. Given this assumption, it follows there exists some $p_1, \ldots, p_n \in F[x]$ such that $Span\{p_1, \ldots, p_n\} = F[x]$ where each p_i has degree i. Since $F[x]$ is the infinite set containing polynomials of all degrees, it follows there exists some polynomial of degree $n+1$. Since $Span\{p_1, \ldots, p_n\}$ can at most be of degree n, it follows $p_{n+1} \in F[x]$ is not an element of the span. This is a contradiction and therefore $F[x]$ cannot be finitely generated. QED.