Geometry Exercises

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1 Chapter 1

1.1 I-1

Show that the function $f : \mathbb{E}^n \to \mathbb{E}^n$ defined by $f(\vec{v}) = 2\vec{v}$ is bijective but not an isometry.

We first show f is bijective. It is easy to see that f is invertible by defining $f^{-1}(\vec{v}) = \frac{1}{2}\vec{v}$ since:

$$f^{-1}(f(\vec{v})) = f^{-1}(2\vec{v}) = \frac{1}{2}(2\vec{v}) = \vec{v}.$$

Thus, f is bijective.

To show f is not an isometry, we proceed by counterexample. Let $(1,5),(2,3) \in \mathbb{E}^n$. For f to be an isometry we must have:

$$\begin{aligned}
||(1,5)-(2,3)|| &= ||f(1,5)-f(2,3)|| \\
||(-1,2)|| &= ||(-2,4)|| \\
\sqrt{(-1)^2+(2)^2} &= \sqrt{(-2)^2+(4)^2} \\
\sqrt{5} &= \sqrt{20} \\
\sqrt{5} &= 2\sqrt{5},
\end{aligned}$$

which is clearly false.

Therefore, f is bijective but not an isometry. $\square$

2 Chapter 6