Handling Correlation in Stacked Difference-in-Differences Estimates with Application to Medical Cannabis Policy

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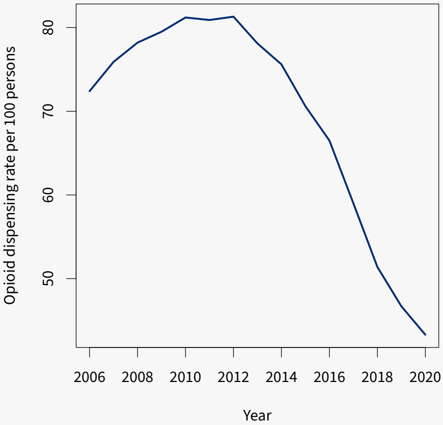
Slides are online!

slides.nickseewald.com/cmstatistics2023.pdf

Motivating Example: Medical Cannabis Laws and Opioid Prescribing

- **4x** increase in opioid prescribing in U.S. from 1999-2012
  - Opioid prescribing for chronic non-cancer pain has played a meaningful role
- Getting better: prescribing down since 2012, but still ~3x higher than 1999



Dart, R. C. et al. (2015). *New England Journal of Medicine*.
https://www.cdc.gov/drugoverdose/rxrate-maps/index.html

Federalism in the United States

"States are the laboratories of democracy."
(Louis Brandeis, *New State Ice Co. vs. Liebmann*)

States in the U.S. have wide latitude to implement or not implement policies and those policies can vary widely.

State laws permitting cannabis use are a great example of this.

Do Medical Cannabis Laws Change Opioid Prescribing?

- Cannabis industry & advocates argue medical cannabis for chronic pain could be a partial solution to opioid crisis via substitution
- Patients with chronic non-cancer pain are eligible to use cannabis under all existing state medical cannabis laws
- Some evidence of substitution among adults with chronic non-cancer pain

Question: What are the effects of state medical cannabis laws on receipt of opioid treatment among patients with chronic non-cancer pain?

Bicket, M. C., Stone, E. M., and McGinty, E. E. (2023). *JAMA Network Open*.

Motivating Example: Medical Cannabis Laws and Opioid Prescribing

Previous studies have found mixed results, but have key methodological limitations:

1. No individual-level data
2. General population samples lead to policy endogeneity

Motivating Example: Medical Cannabis Laws and Opioid Prescribing

Previous studies have found mixed results, but have key methodological limitations:

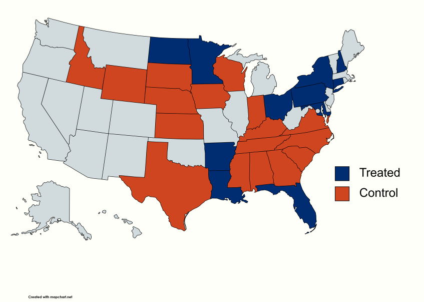
1. No individual-level data
2. General population samples lead to policy endogeneity

Individual-level data lets us identify the population, but adds methodological complexity.

Motivating Example: Medical Cannabis Laws and Opioid Prescribing

Our sample:

- 12 *treated* states that implemented a medical cannabis law between 2012 and 2019 and *do not also have recreational cannabis laws*
- 17 *comparison* states without medical or recreational cannabis laws

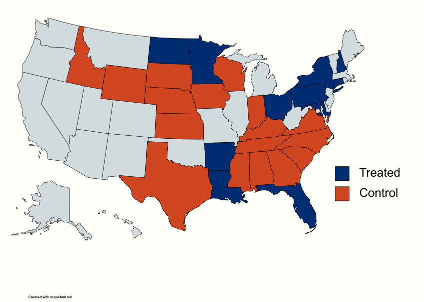


Motivating Example: Medical Cannabis Laws and Opioid Prescribing

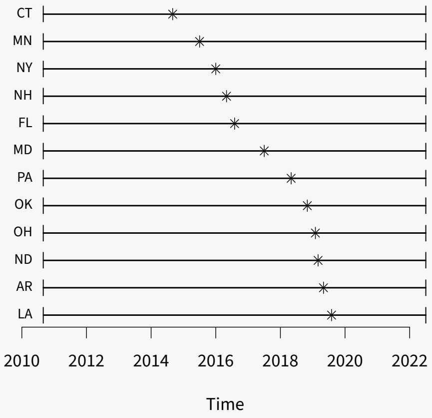
Our sample:

- 12 *treated* states that implemented a medical cannabis law between 2012 and 2019 and *do not also have recreational cannabis laws*
- 17 *comparison* states without medical or recreational cannabis laws

Goal: Estimate the effect of implementing a medical cannabis law on opioid prescribing outcomes, relative to what would have happened in the absence of treatment, among states that implemented such a law (an ATT).



Medical Cannabis Study: Study Periods



States implemented medical cannabis laws at different times

Difference-in-Differences with Multiple Time Periods

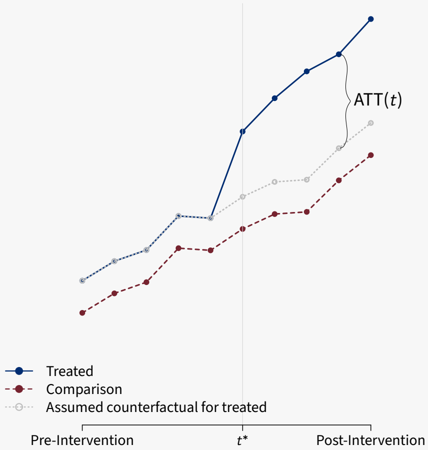
Now, times $t = \{1, \ldots, t^*, \ldots, T\}$; t^* first measurement after treatment.

Alternative estimands:

$$\text{ATT}(t) = \text{E}\left[Y_t(1) - Y_t(0) \mid A = 1\right], \quad t \geq t^*$$

$$\text{ATT}_{\text{avg}} = \text{E}\left[\bar{Y}_{\{t \geq t^*\}}(1) - \bar{Y}_{\{t \geq t^*\}}(0) \mid A = 1\right]$$

Strength of counterfactual parallel trends assumpfbmtion varies with choice of estimand.



Two-Way Fixed Effects Estimation

A common "modeling" approach to estimate ATT:

$$Y_{sit} = \underbrace{\beta_{0,s}}_{\text{state fixed effects}} + \underbrace{\beta_{1,t}}_{\text{time fixed effects}} + \underbrace{\beta_2 A_{st}}_{\text{treatment}} + \varepsilon_{sit},$$

where

- $A_{st} = \mathbb{1}\{\text{state } s \text{ treated at time } t\}$
- β_0's are *state fixed effects*
- β_1's are *time fixed effects*

With 1 treated state or "simultaneous adoption",

$$\hat{\beta}_2 \equiv \left(\bar{Y}^{\text{tx}}_{\{t \geq t^*\}} - \bar{Y}^{\text{tx}}_{\{t < t^*\}}\right) - \left(\bar{Y}^{\text{ctrl}}_{\{t \geq t^*\}} - \bar{Y}^{\text{ctrl}}_{\{t < t^*\}}\right)$$

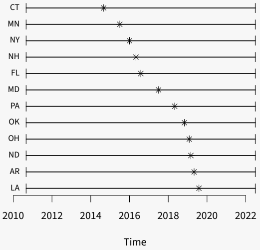
Two-Way Fixed Effects under Staggered Adoption

$$Y_{sit} = \beta_{0,s} + \beta_{1,t} + \boldsymbol{\beta_2} A_{st} + \varepsilon_{sit}$$

- Not all states implemented medical cannabis policy at the same time.
- Two-way fixed effects can yield a (very) biased overall effect estimate in this setting.
  - Problematic under time-varying treatment effects
  - Estimator inadvertently adjusts for post-treatment information

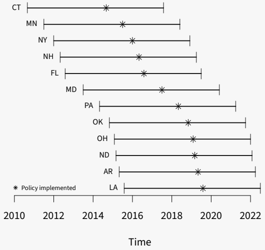
Goodman-Bacon, A. (2021). *Journal of Econometrics*.

Stacked Difference-in-Differences / Serial Trial Emulation



Start with full data

→



Anchor time

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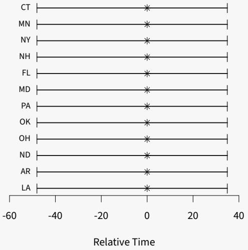


Estimate and aggregate

Hernán, M. A. and Robins, J. M. (2016). *American Journal of Epidemiology*; Ben-Michael, E., Feller, A., and Stuart, E. A. (2021). *Epidemiology*.

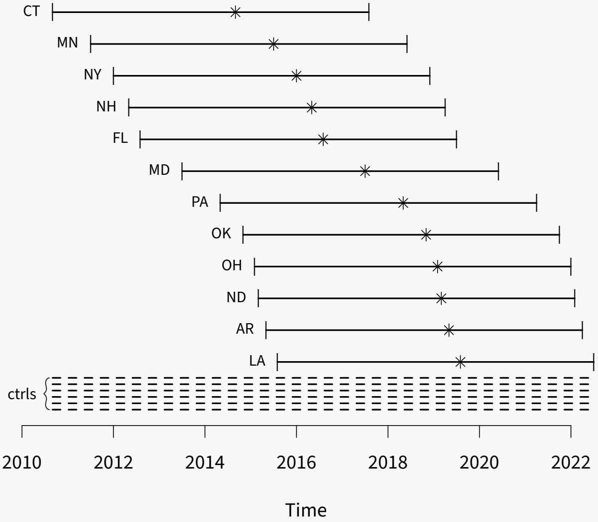
Medical Cannabis Study: State Cohorts

Data are individual-level commercial health insurance claims from $N = 583,820$ unique individuals in 29 states.

For each treatment state, we build a *cohort* of individuals in that state and the control states over the study period.

- Individuals included if they have a chronic non-cancer pain diagnosis in the pre-law period **and** are continuously enrolled in commercial health insurance for the full study period.

Medical Cannabis Study: State Cohorts



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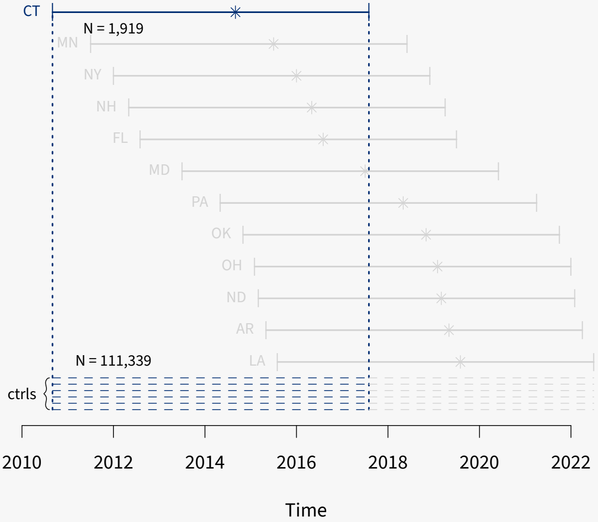
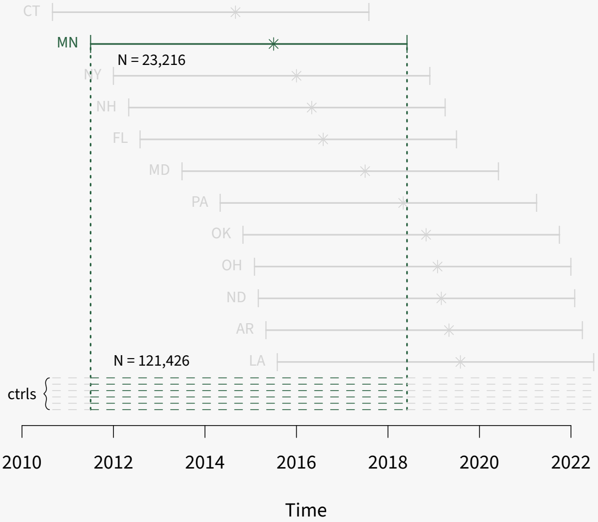
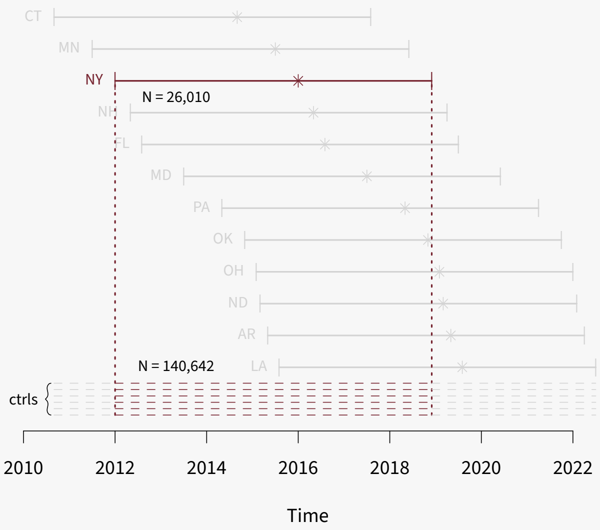


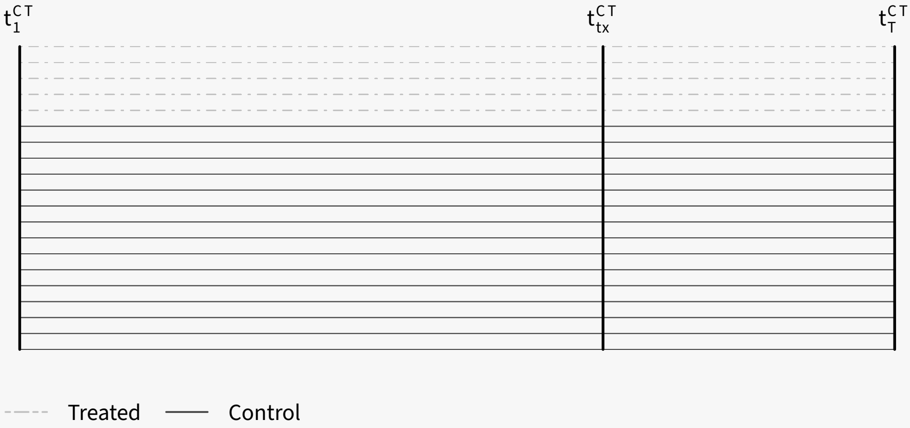
Medical Cannabis Study: State Cohorts



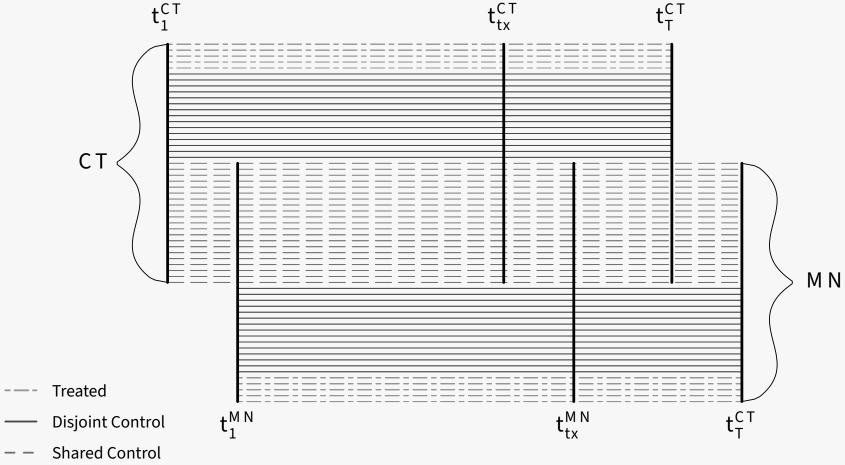
Medical Cannabis Study: State Cohorts



Cohort Schematic



Shared Control Individuals



Handling Correlation Induced by Shared Control Indviduals

Goal: Improved inference on overall ATT averaged across treated units.

- ATT estimates remain unbiased under usual assumptions
- Failure to account for shared control individuals can lead to *incorrect inference*

Big Idea: Incorporate pairwise correlation between estimates into a generalized least squares-esque weighting procedure

Covariance between Diff-in-Diff Effect Estimates

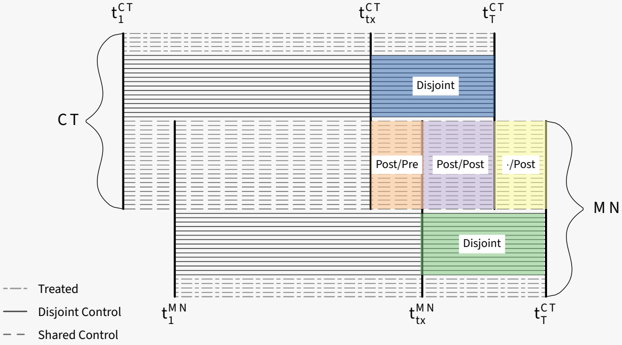
With only one treated unit, we could estimate ATT for cohort C as

$$\widehat{\mathrm{ATT}}_C = \bar{Y}^{\mathrm{tx}}_{s,\mathrm{post}} - \bar{Y}^{\mathrm{tx}}_{s,\mathrm{pre}} - \bar{Y}^{\mathrm{ctrl}}_{s,\mathrm{post}} - \bar{Y}^{\mathrm{ctrl}}_{s,\mathrm{pre}}$$

Assuming states are independent,

$$\begin{aligned}\mathrm{Cov}\left(\widehat{\mathrm{ATT}}_{C_1}, \widehat{\mathrm{ATT}}_{C_2}\right) = \mathrm{Cov}\left(\bar{Y}^{\mathrm{ctrl}}_{C_1,\mathrm{post}}, \bar{Y}^{\mathrm{ctrl}}_{C_2,\mathrm{post}}\right) + \mathrm{Cov}\left(\bar{Y}^{\mathrm{ctrl}}_{C_1,\mathrm{pre}}, \bar{Y}^{\mathrm{ctrl}}_{C_2,\mathrm{pre}}\right) \\ - \mathrm{Cov}\left(\bar{Y}^{\mathrm{ctrl}}_{C_1,\mathrm{post}}, \bar{Y}^{\mathrm{ctrl}}_{C_2,\mathrm{pre}}\right) - \mathrm{Cov}\left(\bar{Y}^{\mathrm{ctrl}}_{C_1,\mathrm{pre}}, \bar{Y}^{\mathrm{ctrl}}_{C_2,\mathrm{post}}\right)\end{aligned}$$

Covariances with Shared Control Individuals



$$\mathrm{Cov}\left(\bar{Y}^{\mathrm{ctrl}}_{\mathrm{CT,post}}, \bar{Y}^{\mathrm{ctrl}}_{\mathrm{MN,post}}\right) \text{``}=\text{''} \mathrm{Cov}\left(\bar{Y}_{\mathrm{CT\ Disjoint}} + \bar{Y}_{\mathrm{Post/Pre}} + \bar{Y}_{\mathrm{Post/Post}}, \bar{Y}_{\mathrm{MN\ Disjoint}} + \bar{Y}_{\mathrm{Post/Post}} + \bar{Y}_{\cdot\mathrm{/Post}}\right)$$

When Does This Matter?

- Setting / simplifying assumptions:
  - Exchangeable within-person correlation ρ
  - Within-period correlation ϕ, between-period correlation ψ
  - Interest is in ATT_{avg}
  - Individuals are independent of people who live in other states

  Depends on:
  - Number of measurement occasions in pre- and post-treatment periods
  - Number of measurement occasions *between* law implementations
  - Numbers of shared and unshared individuals in each control state

Between-Estimate Covariance in Stacked Diff-in-Diff

Here's some math:

$$\mathrm{Cov}\left(\widehat{\mathrm{ATT}}_\gamma, \widehat{\mathrm{ATT}}_\nu\right) = \frac{f\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta\right)}{N_\gamma^{\mathrm{ctrl}} N_\nu^{\mathrm{ctrl}}} \sum_{\zeta \in \text{ctrl states}} \sigma_\zeta^2 \Bigg[\underbrace{N_\gamma(\zeta) N_\nu(\zeta)}_{\#\text{ctrls per state } \zeta} \underbrace{\left(\phi_\zeta - \psi_\zeta\right)}_{\text{diff. in btwn-person corrs}}$$

$$+ \underbrace{N_{\gamma\cap\nu}(\zeta)}_{\#\text{shared ctrls}} \left(1 - \rho_\zeta - (\phi_\zeta - \psi_\zeta)\right) \Bigg],$$

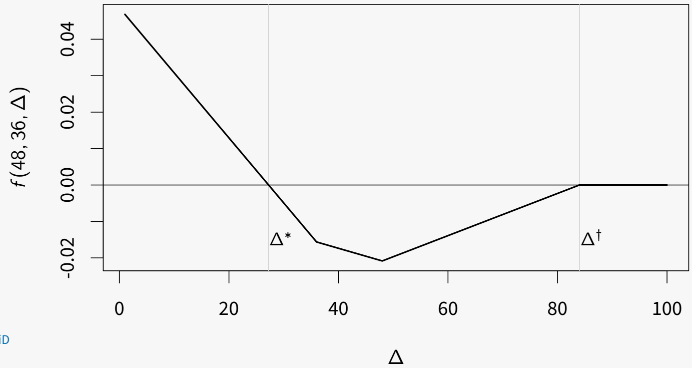
where

$$f\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta\right) = \frac{1}{T_{\mathrm{pre}}^2 T_{\mathrm{post}}^2} \cdot \Big[T_{\mathrm{pre}}^2 \max\left(T_{\mathrm{post}} - \Delta, 0\right) + T_{\mathrm{post}}^2 \max\left(T_{\mathrm{pre}} - \Delta, 0\right)$$

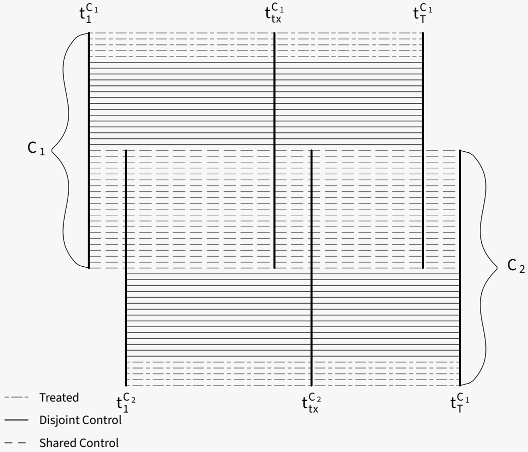
$$- T_{\mathrm{pre}} T_{\mathrm{post}} \min\left(T_{\mathrm{pre}}, T_{\mathrm{post}}, \Delta, \max\left(T_{\mathrm{pre}} + T_{\mathrm{post}} - \Delta, 0\right)\right) \Big].$$

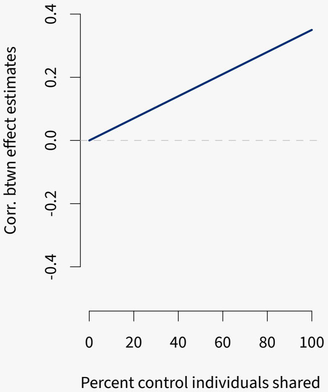
Sign of Between-Estimate Covariance Depends on Δ

$$f\left(T_{\text{pre}}, T_{\text{post}}, \Delta\right) = \frac{1}{T_{\text{pre}}^2 T_{\text{post}}^2} \cdot \Big[T_{\text{pre}}^2 \max\left(T_{\text{post}} - \Delta, 0\right) + T_{\text{post}}^2 \max\left(T_{\text{pre}} - \Delta, 0\right)$$
$$-T_{\text{pre}} T_{\text{post}} \min\Big(T_{\text{pre}}, T_{\text{post}}, \Delta, \max\left(T_{\text{pre}} + T_{\text{post}} - \Delta, 0\right)\Big)\Big].$$



Correlation Due to Shared Controls





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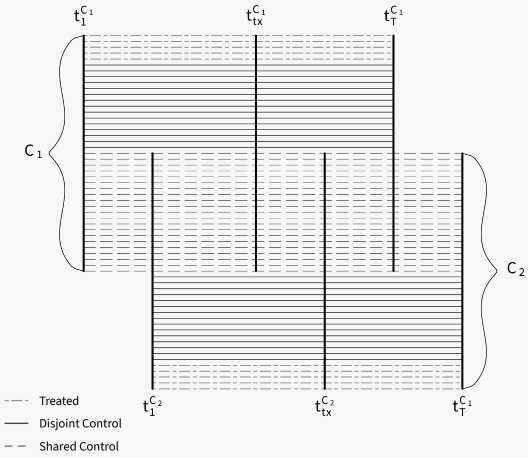

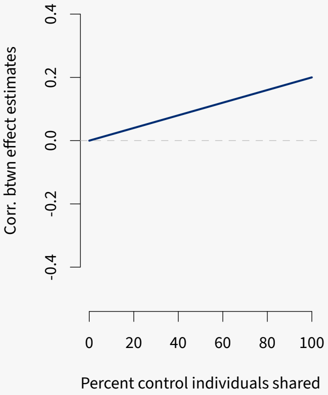

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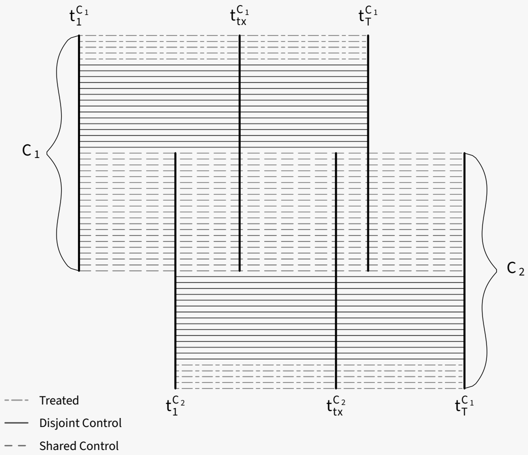

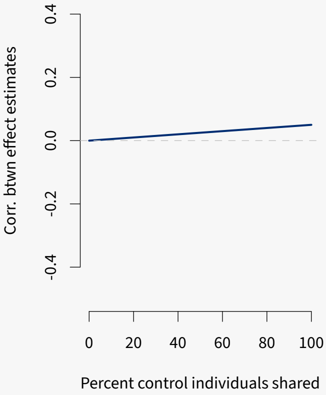

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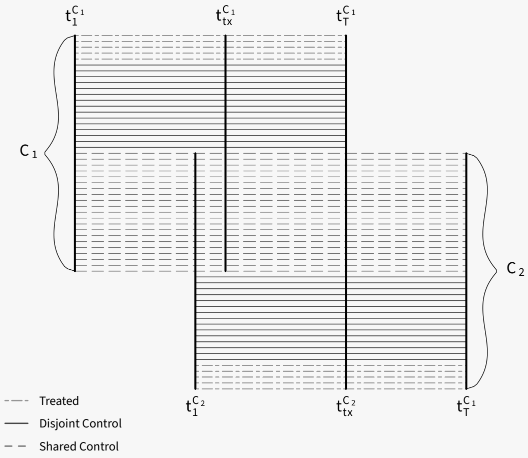

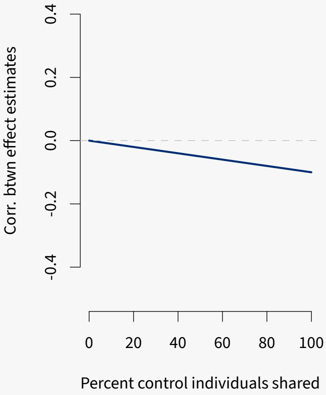

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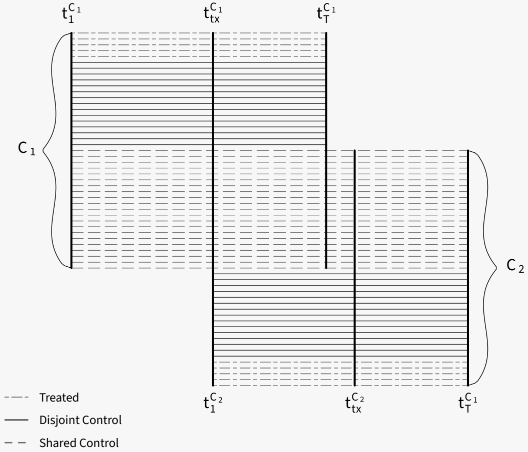

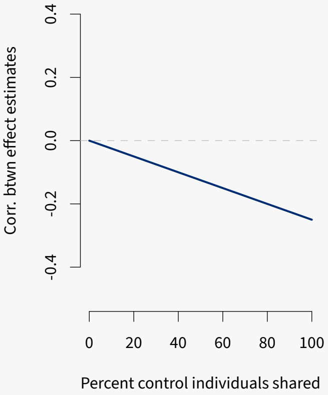

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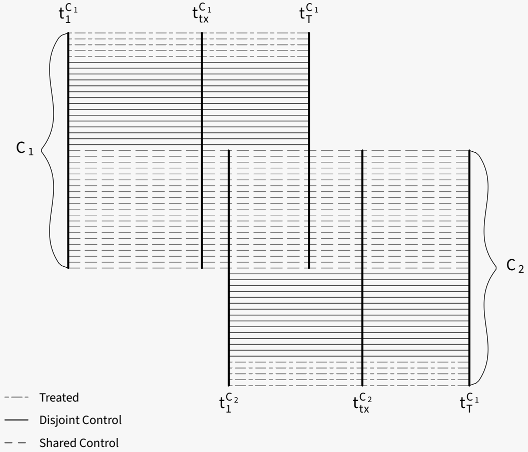

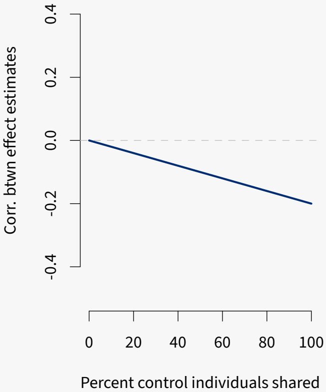

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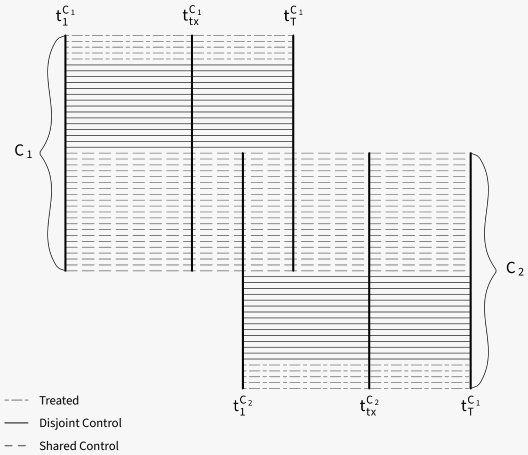

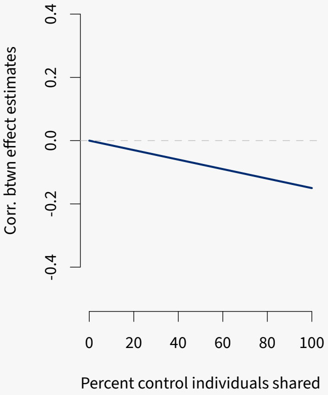

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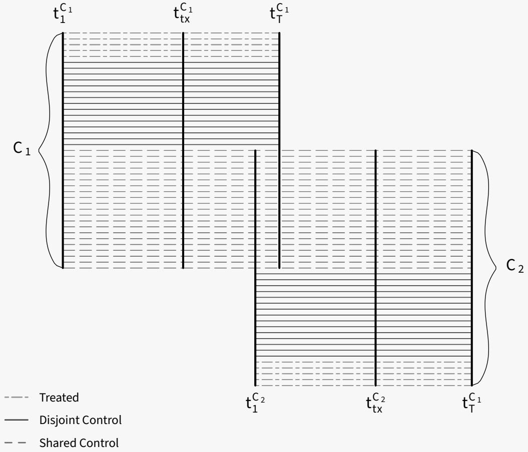

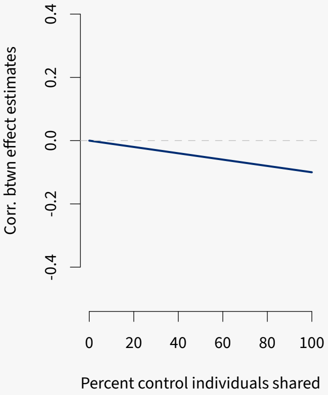

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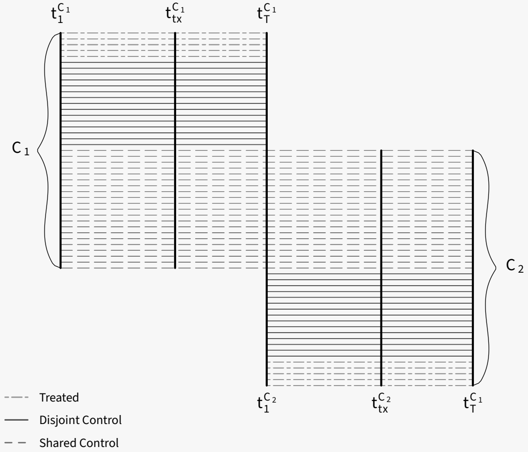

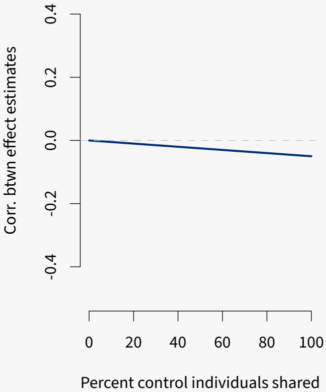

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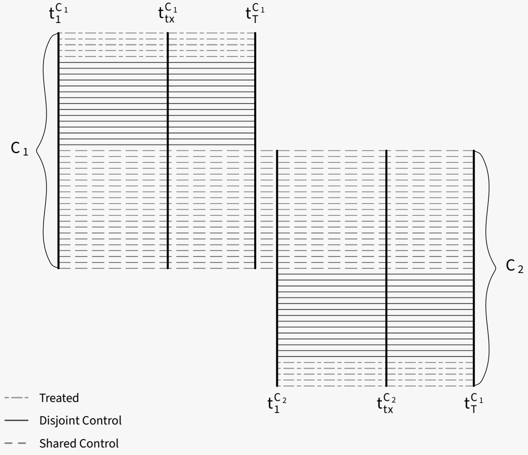
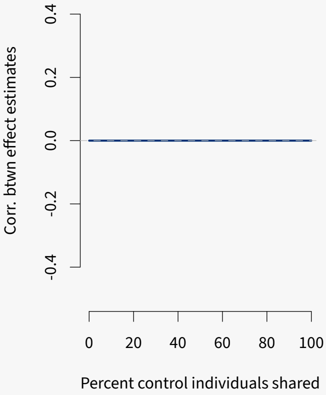




Aggregation: Inverse-Variance Weighting

If estimates are uncorrelated, could use inverse-variance weighted averaging to aggregate. Let V be a diagonal matrix with entries variances of the $\widehat{\mathrm{ATT}}$s. Then,

$$\widehat{\mathrm{ATT}}_{\text{ivw}} := \left(\mathbf{1}^\top \boldsymbol{V}^{-1}\mathbf{1}\right)^{-1} \mathbf{1}\boldsymbol{V}^{-1}\widehat{\mathbf{ATT}}_{\text{tx}} = \frac{1}{\sum_{s\in \text{tx states}} v_{ss}^{-1}} \sum_{s\in \text{tx states}} v_{ss}^{-1}\widehat{\mathrm{ATT}}_s,$$

This has variance

$$\mathrm{Var}\left(\widehat{\mathrm{ATT}}_{\text{ivw}}\right) = \left(\mathbf{1}^\top \boldsymbol{V}^{-1}\mathbf{1}\right)^{-1} = \frac{1}{\sum_{s\in \text{tx states}} 1/v_{ss}}.$$

This does **not** account for between-estimate correlation!

Aggregation: GLS-Based Strategy

Now consider $\boldsymbol{W} = \text{Cov}\left(\textbf{ATT}\right)$.

Then,

$$\widehat{\text{ATT}}_{\text{gls}} = \left(\mathbf{1}^\top \boldsymbol{W}^{-1}\mathbf{1}\right)^{-1}\mathbf{1}\boldsymbol{W}^{-1}\widehat{\textbf{ATT}}_{\text{tx}}$$

and

$$\text{Var}\left(\widehat{\text{ATT}}_{\text{gls}}\right) = \left(\mathbf{1}^\top \boldsymbol{W}^{-1}\mathbf{1}\right)^{-1}.$$

Lin, D.-Y. and Sullivan, P. F. (2009). *Am J Hum Genet*.

Aggregation: GLS-Based Strategy

Now consider $\boldsymbol{W} = \mathrm{Cov}\left(\mathbf{ATT}\right)$.

Then,

$$\widehat{\mathrm{ATT}}_{\mathrm{gls}} = \left(\mathbf{1}^\top \boldsymbol{W}^{-1}\mathbf{1}\right)^{-1} \mathbf{1}\boldsymbol{W}^{-1}\widehat{\mathbf{ATT}}_{\mathrm{tx}}$$

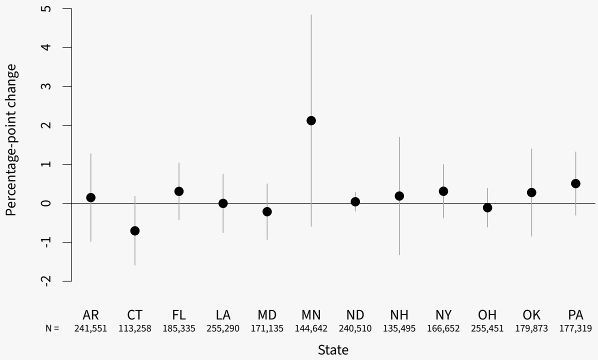
and

$$\mathrm{Var}\left(\widehat{\mathrm{ATT}}_{\mathrm{gls}}\right) = \left(\mathbf{1}^\top \boldsymbol{W}^{-1}\mathbf{1}\right)^{-1}.$$

For 2 treated states, $\mathrm{Var}\left(\widehat{\mathrm{ATT}}_{\mathrm{gls}}\right) > \mathrm{Var}\left(\widehat{\mathrm{ATT}}_{\mathrm{ivw}}\right)$, *unless* between-estimate correlation is positive and sufficiently small.

Lin, D.-Y. and Sullivan, P. F. (2009). *Am J Hum Genet*.

Medical Cannabis Study: Results



State-specific effects of medical cannabis laws on proportion of chronic noncancer pain patients receiving *any opioid prescription*, on average in a given month in first 3 years of law implementation

Between-Estimate Correlation

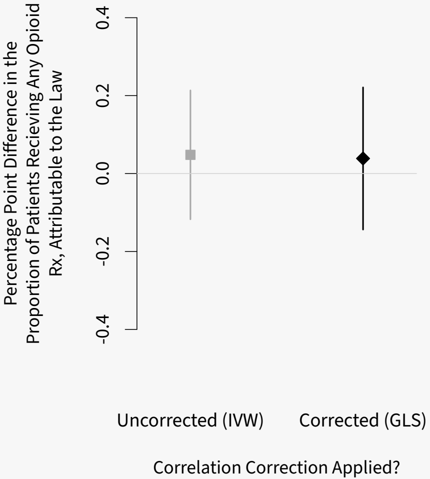
Correlation between state-specific estimates of the percentage point difference in proportion of patients prescribed any opioid, attributable to medical cannabis laws, in a given month in the first 3 years of law implementation.

- Correlations generally small in magnitude, but as high as 0.19.

	MN	NY	NH	FL	MD	PA	OK	OH	ND	AR	LA
CT	0.08	0.04	0.02	0.01	-0.02	-0.03	-0.03	-0.03	-0.05	-0.02	-0.02
MN		0.12	0.09	0.06	0.02	-0.02	-0.04	-0.05	-0.08	-0.04	-0.05
NY			0.1	0.08	0.04	0.01	-0.03	-0.04	-0.06	-0.03	-0.04
NH				0.08	0.05	0.02	-0.01	-0.02	-0.04	-0.03	-0.03
FL					0.05	0.03	0	-0.01	-0.02	-0.01	-0.03
MD						0.08	0.04	0.03	0.04	0.02	0.01
PA							0.07	0.06	0.09	0.04	0.03
OK								0.08	0.13	0.06	0.06
OH									0.19	0.09	0.09
ND										0.15	0.15
AR											0.09

Medical Cannabis Study: Results

- In this case, accounting for between-estimate correlation gives $\sim$10% *larger* SE



Conclusions

- Individual-level data is useful for identifying populations of interest in policy evaluation, but introduces methodological complexity.
- When using individual-level data that might be shared across cohorts in stacked diff-in-diff, it may be important to account for correlation between estimates
- A closed-form formula for induced correlation is available for select analyses

Acknowledgements

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