Signal Basics

Digital Signal Processing

January 24, 2023

Review: Discrete-Time Signals

A **discrete-time signal** is a function

$$x : \mathbb{Z} \to B,$$

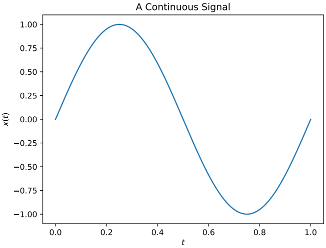
for some output set B (typically $B = \mathbb{R}$ or $B = \mathbb{C}$).

Equivalently, x is a sequence

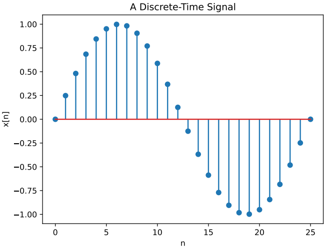
$$x[n] \in B, \quad -\infty \leq n \leq \infty.$$

Sampled Continuous Signals

Discrete-time signals often come from continuous signals:



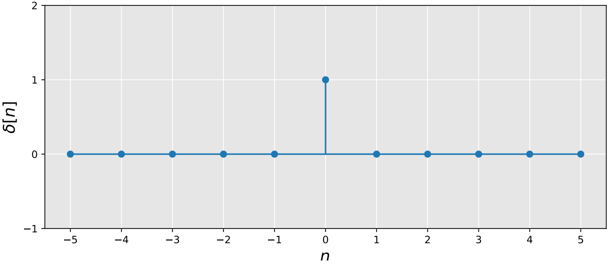
$$x_c(t) \quad \longrightarrow \quad x[n] = x_c(nT)$$



Here, $T \in \mathbb{R}$ is the **sampling period**, $\quad T = (1/25)\mathrm{s} = 0.04\mathrm{s}$

and $\frac{1}{T}$ is the **sampling frequency**. $\quad \frac{1}{T} = 25\mathrm{Hz}$

Unit Sample Function



$$\delta[n] = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases}$$

Also known as the **unit impulse function**.

Shifting the Unit Impulse

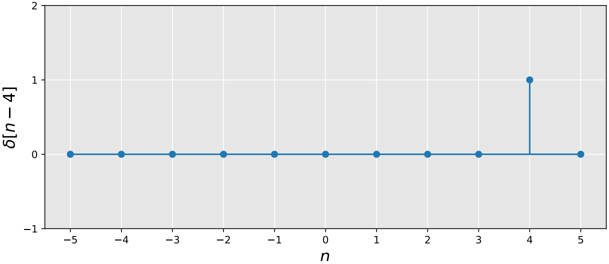
For any integer k,

$$\delta[n-k] = \begin{cases} 1, & n-k=0, \\ 0, & n-k \neq 0. \end{cases}$$

Or, in other words,

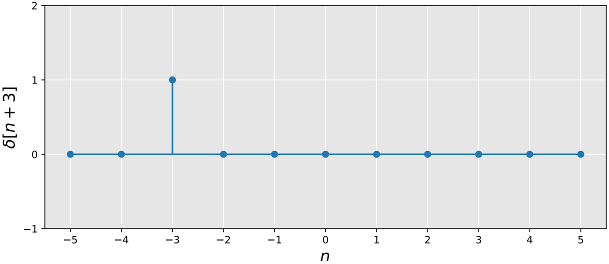
$$\delta[n-k] = \begin{cases} 1, & n=k, \\ 0, & n \neq k. \end{cases}$$

Shifting the Unit Impulse



$$\delta[n-4] = \begin{cases} 1, & n = 4, \\ 0, & n \neq 4. \end{cases}$$

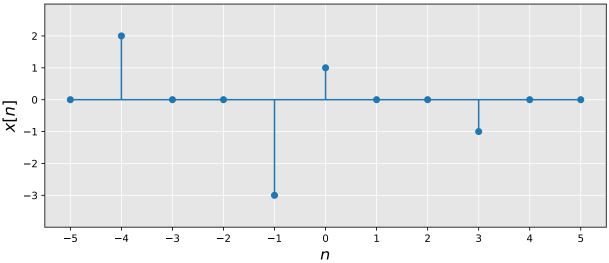
Shifting the Unit Impulse



$$\delta[n+2] = \begin{cases} 1, & n = -2, \\ 0, & n \neq -2. \end{cases}$$

Scaling and Adding Shifted Impulses

We can scale and add shifted impulses to construct signals:



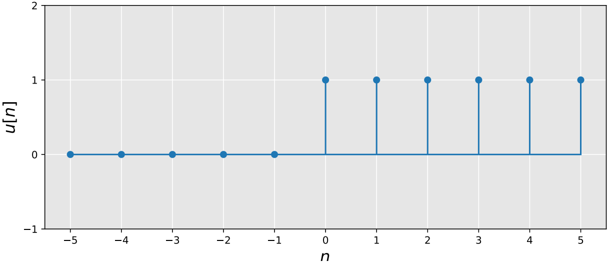
$$x[n] = 2\delta[n+4] - 3\delta[n+1] + \delta[n] - \delta[n-3]$$

Scaling and Adding Impulses

In fact, any sequence, $x[n]$, can be written as a sum of scaled, shifted impulses:

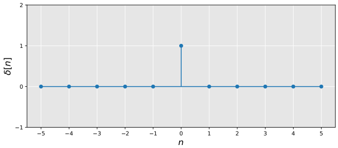
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

Unit Step Function

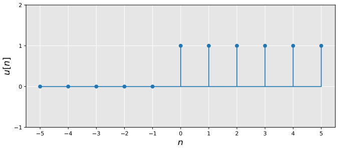


$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

Relationship Between Step and Impulse



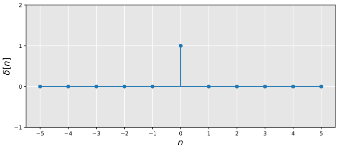
$\xrightarrow{\text{accumulate}}$



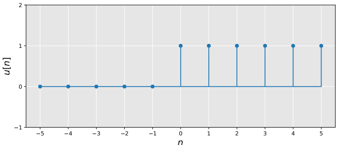
$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Discrete analogy to integration

Relationship Between Step and Impulse



difference $\longleftarrow$



$$\delta[n] = u[n] - u[n-1]$$

Discrete analogy to differentiation

Real Exponential Function

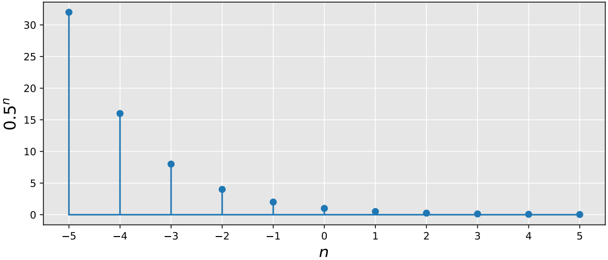
A **real exponential sequence** is of the form

$$x[n] = A\alpha^n,$$

for constants $A \in \mathbb{R}$ and $\alpha \in \mathbb{R}$.

Real Exponential Function

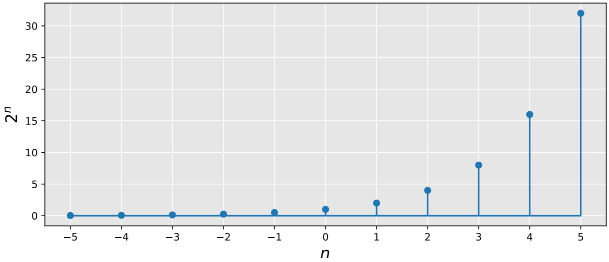
When $0 < \alpha < 1$, we get exponential **decay**:



$$x[n] = 0.5^n$$

Real Exponential Function

When $\alpha > 1$, we get exponential **growth**:



$$x[n] = 2^n$$

Real Exponential Function

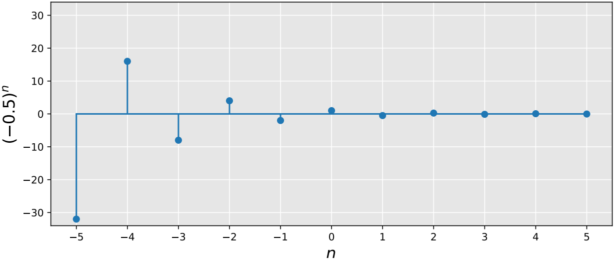
Note: taking reciprocal of α is equivalent to time-reversal:

Let $x[n] = \alpha^n$, then

$$x[-n] = \alpha^{-n} = \left(\alpha^{-1}\right)^n = \left(\frac{1}{\alpha}\right)^n$$

Real Exponential Function

When $\alpha < 0$, we get exponential **oscillation**:



$$x[n] = (-0.5)^n$$

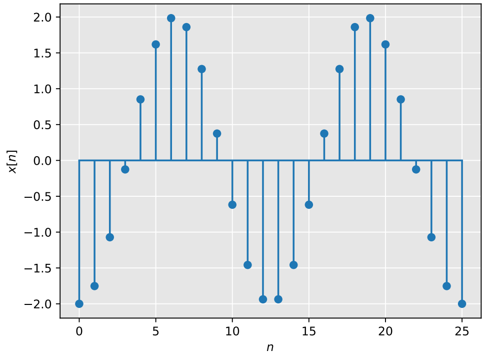
Real Exponential Function

Note: time shift is equivalent to multiplication:

Let $x[n] = \alpha^n$, then

$$x[n-k] = \alpha^{n-k} = \alpha^n \alpha^{-k} = \alpha^{-k} x[n]$$

Sinusoidal Function



$$A = 2, \quad \omega_0 = 4\pi, \quad \phi = \frac{1}{4}$$

$$x[n] = A\cos(\omega_0 n + \phi)$$

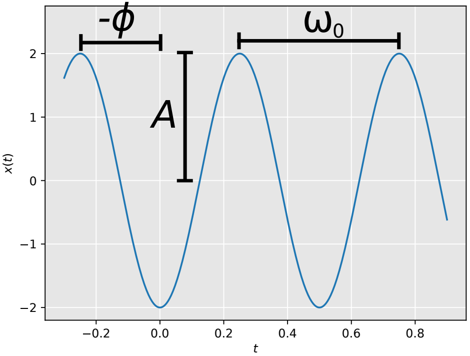
A : amplitude

ω_0 : frequency

ϕ : phase

Relation to Continuous Sinusoidal

Discrete-time sinusoidal is just a sampled continuous sinusoidal



$$A = 2, \quad \omega_0 = 4\pi, \quad \phi = \frac{1}{4}$$

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