Homework 5: Filters

Instructions: Submit a single Jupyter notebook (.ipynb) of your work to Canvas by 11:59pm on the due date. All code should be written in Python. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Do not look at another student's answers, do not use answers from the internet or other sources, and do not show your answers to anyone. **Cite any sources you used outside of the class material (webpages, etc.), and list any fellow students with whom you discussed the homework concepts.**

All-Pass Filters and the Phaser

Ohm's Acoustic Law [1] states that the human ear detects musical notes by their frequency content, but is insensitive to relative changes in phase. Let's test this!

1. Remember the all-pass system with real-valued coefficients from the lecture. This has system function:
$$H(z) = \frac{(z^{-1} - c)(z^{-1} - \bar{c})}{(1 - \bar{c}z^{-1})(1 - cz^{-1})}.$$
Write down a causal LCCDE for this system.

2. Write a Python function to implement this LCCDE. Your function should take as input the signal $x[n]$ and the complex number c, and it should return the output signal $y[n]$ after applying the all-pass filter.

3. Run your all-pass filter on the audio wave `synth.wav` with $c = re^{-i\omega}$, where $r = 0.9$ and $\omega = \frac{\pi}{100}$. Play the resulting output. Do you hear any difference?

4. Plot the magnitude and phase response of this filter. Describe what it does to the signal.

5. Now take your filtered signal from part (c) and add it back to the original signal. (In other words, if $x[n]$ is the original signal, and $y[n]$ is the all-pass filtered signal from (c), you should simply produce $x[n] + y[n]$ in this part.) Now, does the resulting audio wave sound different from the original?

6. Plot the magnitude and phase response of the filter in part (d). Describe what it does to the signal and why it sounds different from the result in (c).

Filtering Electroencephalograhy (EEG) Data

Coming soon! Look for an announcement and download the PDF again when this part is released.

[1] `https://en.wikipedia.org/wiki/Ohm's_acoustic_law`