ECE 4750/6750 + CS 4501/6501: Digital Signal Processing **Due Tu 4/15**

Homework 5: Filters

Instructions: Submit a single Jupyter notebook (.ipynb) of your work to Canvas by 11:59pm on the due date. All code should be written in Python. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Do not look at another student's answers, do not use answers from the internet or other sources, and do not show your answers to anyone. **Cite any sources you used outside of the class material (webpages, etc.), and list any fellow students with whom you discussed the homework concepts.**

Important!!! Throughout this homework, you should not use any Python signal processing libraries to do the filter design or implementation. It is okay to use `numpy.convolve` when you have the impulse response function, $h[n]$. It is also okay to use any example code from class! You may want to cut-and-paste extensively from `WindowFunctions.ipynb`.

All-Pass Filters and the Phaser

1. Ohm's Acoustic Law [1] states that the human ear detects musical notes by their frequency content, but is insensitive to relative changes in phase. Let's test this!

   (a) Remember the all-pass system with real-valued coefficients from the lecture. This has system function:
   $$H(z) = \frac{(z^{-1} - c)(z^{-1} - \bar{c})}{(1 - \bar{c}z^{-1})(1 - cz^{-1})}.$$
   Write down a causal LCCDE for this system.

   (b) Write a Python function to implement this LCCDE. Your function should take as input the signal $x[n]$ and the complex number c, and it should return the output signal $y[n]$ after applying the all-pass filter.

   (c) Run your all-pass filter on the audio wave `synth.wav` [2] with $c = re^{-i\omega}$, where $r = 0.9$ and $\omega = \frac{\pi}{100}$. Play the resulting output. Do you hear any difference?

   (d) Plot the magnitude and phase response of this filter. Describe what it does to the signal.

   (e) Now take your filtered signal from part (c) and add it back to the original signal. (In other words, if $x[n]$ is the original signal, and $y[n]$ is the all-pass filtered signal from (c), you should simply produce $x[n] + y[n]$ in this part.) Now, does the resulting audio wave sound different from the original?

   (f) Plot the magnitude and phase response of the filter in part (e). Describe what it does to the signal and why it sounds different from the result in (c).

[1] `https://en.wikipedia.org/wiki/Ohm's_acoustic_law`

[2] This is a clip taken from the audio on this Wikipedia page: `https://en.wikipedia.org/wiki/Phaser_(effect)`. Also, read that page for more information on the DSP behind the phaser effect.

Filtering Electroencephalograhy (EEG) Data

2. In this part of the homework, you are going to design filters for processing EEG data. The EEG signals are measuring the electric activity of the brain (using electrodes on the scalp). Download the Jupyter notebook `EEG.ipynb` and the EEG dataset `chb01_18.mat` from the "Files" tab in Canvas. The notebook has some code you can use for loading and plotting the EEG signals. This data includes a segment where the subject is having a seizure, in which brain activity increases across multiple frequencies. Different frequency bands in EEG are associated with different electrophysiological phenomena. [3]

   (a) Create a low-pass filter to extract the δ band, which are frequencies < 4 Hz. First try designing a filter by hand by placing 21 zeros at even increments in the stop band. Plot the frequency response curve (in dB). Filter the EEG data and plot it. How does it compare with the original data?

   (b) Now build a low-pass filter with the same δ band cutoff, < 4 Hz, but this time use a 21 order Kaiser window ($M = 10$ in the slides). Again, plot the frequency response. Explain how this filter compares to the one in part (a) in terms of the pass-band, transition, and stop-band. Filter the EEG data and plot it. How is it different from the previous filter results?

   (c) Now increase your Kaiser low-pass filter to have order 201 ($M = 100$). Again, plot the frequency response, and explain how it compares to your order 21 Kaiser filter in terms of pass-band, transition, and stop-band. Filter the EEG data and plot it. How is it different from the previous Kaiser filter results?

   (d) Now let's look at the β band of frequencies, which are 12 - 30 Hz. Use an order 201 Kaiser window with the modulation transformation to create a band-pass filter in this range. Plot the frequency response. Filter the EEG data. Do you see activity in this band in the seizure period, and how does it compare to the activity in the δ band?

   (e) Finally, let's look at the very high frequency content above what is normally found in the brain. Design a high-pass filter for frequencies > 70 Hz using an order 201 Kaiser window and the modulation transformation. Plot the frequency response. Filter the EEG data. Do you see any difference in these frequencies in the seizure period compared to the normal period?

For Grads Only (or Extra Credit for Undergrads)

3. Returning to the all-pass filter from part 1, extend your filter to a full "phaser" with the following two steps: (1) add a cascade of multiple all-pass filters at different frequencies, try $\omega = \frac{k\pi}{100}$ for $k = 1, 2, 3, 4$; and (2) add a low-frequency oscillator (LFO) to the magnitude of the pole, c. The LFO should be of the form of a sine wave in time, so c varies in magnitude with time n like so:

   $$c[n] = \sin(\omega_{\mathrm{LFO}} n) c.$$

   In other words, your LCCDE will change at each time point n by plugging in $c[n]$ in place of c. **Note:** because of the varying $c[n]$, the final phaser system is *not* an LTI system! Set ω_{LFO} to correspond to 1 Hz. (Feel free to play with the c values and LFO frequencies to see what kind of effects you can get!)

[3] `https://en.wikipedia.org/wiki/Electroencephalography#Comparison_of_EEG_bands`