

## R Assignment 2

Alireza Sheikh-Zadeh, PhD

Document format: Follow the instructions given on the web page. Always review your solution word document before submission.

Plagiarism: You are not allowed to share your write-up with your peers. It's okay to advise your peers about how to solve a problem, but you never share your own write-up.

Problem 1: 28 points

Problem 2: 52 points

Format: 20 points

### Problem 1 (28 points)

Suppose you roll a die twice in succession, getting  $X_1$  and  $X_2$ . Then divide them, getting  $Y = \frac{X_1}{X_2}$ . Thus,  $Y$  is discrete, ranging from a minimum of  $1/6$  to a maximum of  $6$ . In R, we can generate all possible values for  $Y$  as follows.

```
# run this chunk of code:
X1 = c(1,2,3,4,5,6) # possible outcome of the first try
X2 = c(1,2,3,4,5,6) # possible outcome of the second try

Y = c() # defining an empty vector Y
# Then we divide each element of X1 by all value of X2 and record them in Y vector
for (i in X1) {
  for (j in X2) {
    Y = c(Y, i/j) # inserting X1/X2 into Y vector
  }
}
Y

## [1] 1.0000000 0.5000000 0.3333333 0.2500000 0.2000000 0.1666667 2.0000000
## [8] 1.0000000 0.6666667 0.5000000 0.4000000 0.3333333 3.0000000 1.5000000
## [15] 1.0000000 0.7500000 0.6000000 0.5000000 4.0000000 2.0000000 1.3333333
## [22] 1.0000000 0.8000000 0.6666667 5.0000000 2.5000000 1.6666667 1.2500000
## [29] 1.0000000 0.8333333 6.0000000 3.0000000 2.0000000 1.5000000 1.2000000
## [36] 1.0000000

# Now Y is ready to be used in part a
```

- Report the sorted list of  $Y$  values, then Report the mean of the  $Y$ . Present all values by 3 decimal places. (6 points)

```
# This is sorting of the Y values
```

```
sort(Y)
```

```
## [1] 0.1666667 0.2000000 0.2500000 0.3333333 0.3333333 0.4000000 0.5000000
## [8] 0.5000000 0.5000000 0.6000000 0.6666667 0.6666667 0.7500000 0.8000000
## [15] 0.8333333 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
## [22] 1.2000000 1.2500000 1.3333333 1.5000000 1.5000000 1.6666667 2.0000000
## [29] 2.0000000 2.0000000 2.5000000 3.0000000 3.0000000 4.0000000 5.0000000
## [36] 6.0000000
```

```
# This is the True Mean and expected value of Y values
```

```
mean(Y)
```

```
## [1] 1.429167
```

- b. Simulate 10000 (or more) i.i.d observations  $Ysim = (\frac{Xsim_1}{Xsim_2})$ . Don't print the outcome of Ysim (you can only print the first 6 value). (10 points)

```
# Simulation of 10000 outcomes for Ysim
```

```
set.seed(111)
```

```
Xsim1 <- sample(X1, 10000, replace = TRUE)
```

```
Xsim2 <- sample(X2, 10000, replace = TRUE)
```

```
Ysim <- Xsim1/Xsim2
```

```
# Did not print Ysim
```

- c. Draw the graph of successive average (cumulative mean) of Ysim. Discuss your observations based on what you see in the graph of successive average and the answer of part a for the mean(Y). (12 points)

```
# Graph illustrating the cumulative mean of Ysim
```

```
# The cumMean hovers around 1.429.
```

```
n = 10000
```

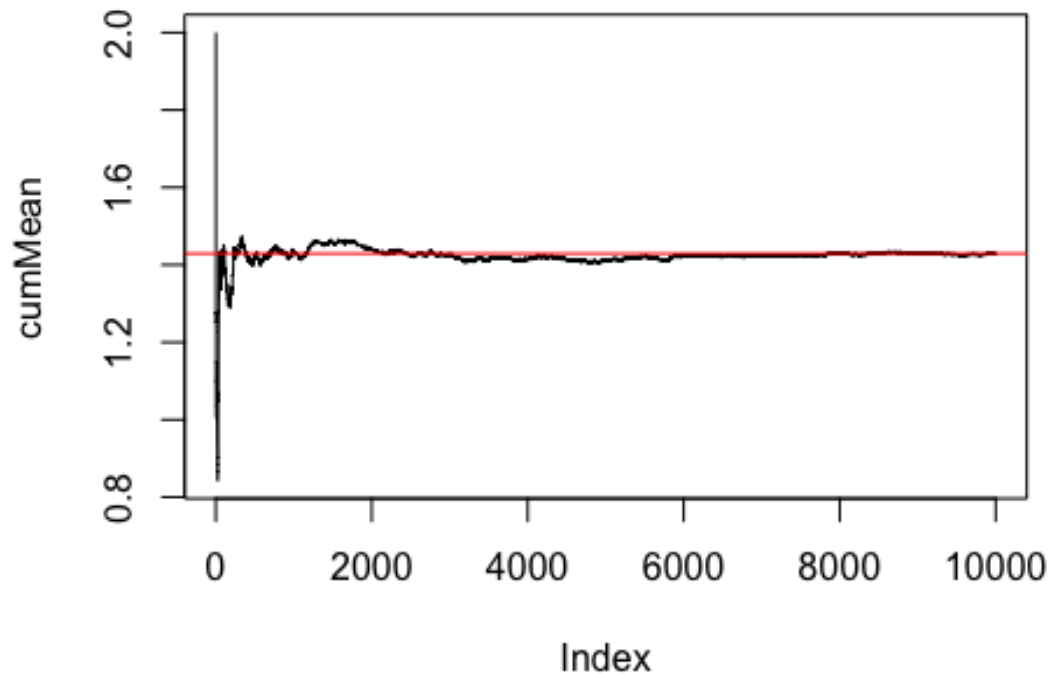
```
cumSumY <- cumsum(Ysim)
```

```
cumMean <- c()
```

```
for (i in 1:n) {
  cumMean[i] <- cumSumY[i]/i
}
```

```
plot(cumMean, type = "l")
```

```
abline(h = 1.429167, col = "red")
```



```
head(cumMean)
## [1] 2.000000 1.250000 1.277778 1.108333 1.086667 1.005556
tail(cumMean)
## [1] 1.429341 1.429248 1.429355 1.429263 1.429136 1.429073
#-----
# cumMean = cumsum(Ysim) / seq_along(Ysim)
```

## Problem 2 (52 points)

A small hotel has 10 rooms. From experience they know that 20% of the time, people who make reservations do not show up, so as a result, they overbook by accepting 12 reservations for a given night. Let  $X$  be the number of no shows that night (people who don't show up).

- What is the expected number of “no shows” that night? What is the standard deviation of that number? (8 points)

```
# The expected number of no shows
```

```
Ex <- 12 * .2
Ex
## [1] 2.4
```

b. Is  $X$  continuous or discrete. (2 points)

# $X$  is discrete

c. What is the range of  $X$ ? (4 points)

## Range

# $x = 0-12$

d. Find the probability of all possible value of  $X$ . Round by 3 decimal places. (10 points)

*# Probability of all possible  $X$*

```
X <- 0:12
px <- dbinom(X, size = 12, prob = .2 )
```

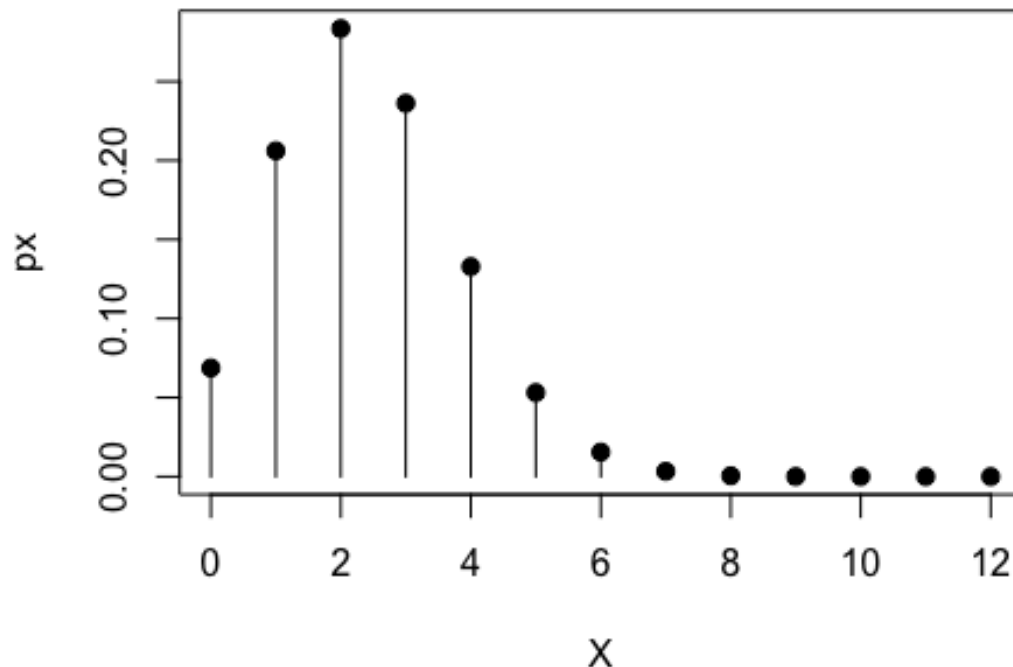
```
cbind(X, px)
```

```
##      X      px
## [1,] 0 6.871948e-02
## [2,] 1 2.061584e-01
## [3,] 2 2.834678e-01
## [4,] 3 2.362232e-01
## [5,] 4 1.328756e-01
## [6,] 5 5.315022e-02
## [7,] 6 1.550215e-02
## [8,] 7 3.321889e-03
## [9,] 8 5.190451e-04
## [10,] 9 5.767168e-05
## [11,] 10 4.325376e-06
## [12,] 11 1.966080e-07
## [13,] 12 4.096000e-09
```

e. Create a needle plot for all probability values of part d. (10 points)

*# This is a needle plot.*

```
plot(X, px, type = 'h')
points(X, px, pch=19)
```



- f. What is the probability that the hotel will end up with more customers than they can handle (that is, more people with reservations than available rooms will arrive)? (4 points)

```
#If x = 0 and 1 then they will have too many customers to handle
# P(x=0)
# 0 no show > 12 shows > 10 X
# 1 no show > 11 shows > 10 X
pbinom(1, 12, .2)

## [1] 0.2748779
```

- g. Simulate 10000 nights instances for the number of no-shows and call them *Xsim*; then estimate the expected value and standard deviation of the number of the simulated no-shows. What explains why the simulation results are different from the answers to a? (14 points)

```
# Simulate 10000 nights of no shows Xsim
# They are different because a is the expected number of returns while g is
# the simulation of different occurrences.
set.seed(123)
Xsim <- rbinom(n = 10, size = 12, prob = .2)
mean(Xsim)

## [1] 2.6
```

```
sd(Xsim)
```

```
## [1] 1.429841
```