1. Why Gauss elimination does not change the null space of matrix A?

let M be the product of elementary matrixes of A.

- suppose $\boldsymbol{x} \in \boldsymbol{N(A)}$, so Ax = 0. we have:

  $\boldsymbol{MAx = M(Ax) = M.0 = 0}$

  so $\boldsymbol{x} \in \boldsymbol{N(MA)}$ as well.
- suppose $\boldsymbol{y} \in \boldsymbol{N(MA)}$, so (MA).y=0. we have:

  $\boldsymbol{Ay = M^{-1}MAy = M^{-1}.0 = 0}$

  so $\boldsymbol{y} \in \boldsymbol{N(A)}$ as well.

That means when you apply Gauss elimination, you do change the solution space.

2. Why every elementary is invertible?

https://en.wikipedia.org/wiki/Elementary_matrix

https://math.vanderbilt.edu/sapirmv/msapir/prleelementary.html