

1. Derivative

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2. Gradient

$$\nabla = \begin{bmatrix} \frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y} \end{bmatrix}$$

$\nabla \perp \text{contour line}$

3. Directional Derivative

$$D_u f = \frac{\delta f}{\delta x} u_1 + \frac{\delta f}{\delta y} u_2 = \nabla \cdot u = |\nabla| |u| \cos(\theta)$$

$$D_u f = 0 \iff u // \text{contour line}$$

$$\max(D_u f) = |\nabla| \iff u // \nabla \iff u \perp \text{contour line}$$

(as $\max(\cos\theta) = 1$, achieved when $\theta = 0$ and \vec{u} points in the same direction as the gradient)