

1. Vector

$$\vec{u} \cdot \vec{v} = u_1 * v_1 + u_2 * v_2 = |\vec{u}| |\vec{v}| \cos(\theta)$$

$$|\vec{u}|^2 = \vec{u} \cdot \vec{u}$$

Linear combination

$$a_1 \vec{v}_1 + \dots + a_m \vec{v}_m$$

Span

$$\text{span}(\vec{v}_1, \dots, \vec{v}_m) = \{a_1 \vec{v}_1 + \dots + a_m \vec{v}_m \mid a_1, \dots, a_m \in \mathbb{R}\}$$

Linear independent

A list $(\vec{v}_1, \dots, \vec{v}_m)$ is linear independent if the only choice of $a_1, \dots, a_m \in \mathbb{R}$ that makes $a_1 \vec{v}_1 + \dots + a_m \vec{v}_m = \vec{0}$ is $a_1 = \dots = a_m = 0$

Basic of V:

a list of vectors in V that

- linear independent
- span V

every $\vec{v} \in V$ can be written **uniquely** in form of a linear combination of the basic

Dim = len(basic)

Matrix

- Inverse matrix $A^{-1}A = I$

A is invertible if

- $\det(A) \neq 0$
- or we can not find a vector $\vec{x} \neq \vec{0}$ that: $A\vec{x} = \vec{0}$
proof: $A\vec{x} = \vec{0} \Leftrightarrow A^{-1}A\vec{x} = A^{-1}\vec{0} \Leftrightarrow \vec{x} = \vec{0}$

- Singular matrix = matrix that is not invertible $\Leftrightarrow \det(A)=0$
- Eigenvalue λ and eigenvector \vec{x} of a matrix A:

$$A\vec{x} = \lambda\vec{x}$$

How to find λ and \vec{x} of A:

$$A\vec{x} = \lambda\vec{x}$$

$$\Leftrightarrow (A - \lambda I)\vec{x} = \vec{0}$$

$$\Leftrightarrow (A - \lambda I) \text{ is singular}$$

$$\Leftrightarrow \det(A - \lambda I) = 0$$

find λ then find \vec{x}

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