NU1 Statements

A Collection of Unsubstantiated Claims

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Number Theory

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Divisibility

Notation. $a|b \iff \exists x : ax = b$

Proposition. -

1. $a|b \implies a|bc$
2. $a|b$ *and* $b|c \implies a|c$
3. $a|b$ *and* $a|c \implies a|bx + cy$
4. $a|b$ *and* $b \neq 0 \implies |a| \leq |b|$
5. $a|b$ *and* $b|a \implies a = \pm b$

Proposition. $a^n - b^n = (a - b) \sum_{i=0}^{n} a^{n-i} b^i$

Proposition. *If* n *is odd, then* $a^n + b^n = (a + b) \sum_{i=0}^{n} (-1)^i a^{n-i} b^i$

Definition. d, denoted (a, b), is the distingiushed common divisor of a and b iff

1. $d|a$ and $d|b$
2. $c|a$ and $c|b \implies c|d$

Proposition. (a, b) *exists, and is unique up to sign.*

Definition (Euclidean Algorithm)**.** Todo

Proposition. -

1. $(a, b) = (a, ak + b)$
2. $(a, b) = (ma, mb)$

Definition. a and b are relatively prime iff $(a, b) = 1$.

Lemma (Euclid)**.** $a|bc$ *and* $(a, b) = 1 \implies a|c$

Base 10 Divisibility

Proposition (Divisibility by 9)**.** $\overline{a_k \ldots a_2 a_1} \equiv a_k + \ldots + a_2 + a_1 \mod 9$

Proposition (Divisibility by 11)**.** $\overline{a_k \ldots a_2 a_1} \equiv \sum_{i=1}^{k} (-1)^{n-1} a_i \mod 11$

Proposition (Last n digit rule)**.** *If* $a|10^k$*, then* $\overline{\ldots a_k \ldots a_2 a_1} \equiv \overline{a_k \ldots a_2 a_1} \mod a$

Primes

Definition. p is irreducable iff $a|p \implies a = 1 \vee a = p$

Definition. p is prime iff $p|ab \implies p = a \vee p = b$

Theorem. *In $\mathbb{Z}$, irreducability and primality are equivalent.*

Theorem (Fundimental Theorem of Arithmetic)**.** *Every positive integer n has a unique canonical representation*

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k} = \prod_{i=1}^{k} p_i^{\alpha_i}$$

Where $p_1 < p_2 < \dots < p_k$ are primes.

Theorem. *There are infinitely many primes*

Congruences