

F69 Laue X-Ray Diffraction

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9.1 Silicon Wafer

Varying X-ray tube current (heating current)

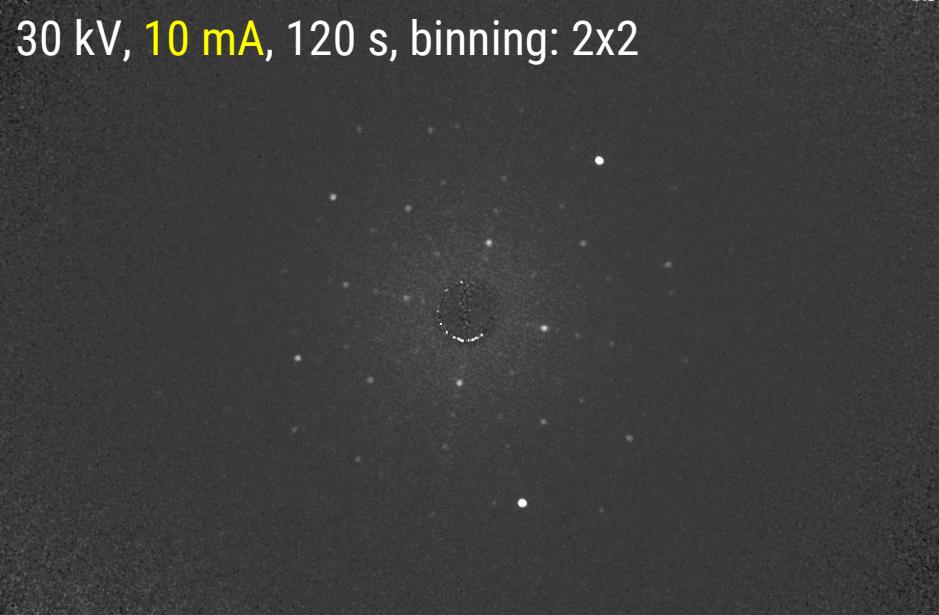
- **Observation:**

- Higher intensity
→ More reflexes visible

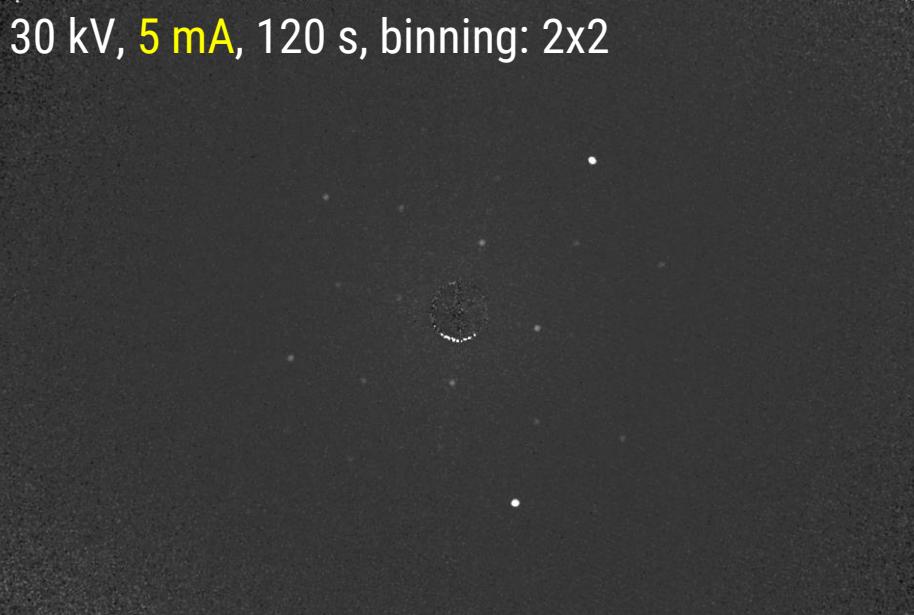
- **Explanation:**

- A higher current = more electrons
- More electrons = more X-ray photons
→ Higher intensity

30 kV, 10 mA, 120 s, binning: 2x2



30 kV, 5 mA, 120 s, binning: 2x2



9.1 Silicon Wafer

Varying X-ray tube voltage (acceleration voltage)

- **Observation:**

- The intensity of the diffraction reflexes is a little bit higher

- **Explanation:**

- The higher voltage leads to higher energies of the e^-
→ higher intensity of γ

22 kV, 5 mA, 120 s, binning: 2x2

A dark gray square image showing a faint, diffuse central spot with several small, sharp diffraction spots around it, indicating a polycrystalline silicon wafer.

30 kV, 5 mA, 120 s, binning: 2x2

A dark gray square image showing a more intense central spot and slightly more prominent diffraction spots compared to the 22 kV image, indicating higher X-ray intensity due to higher tube voltage.

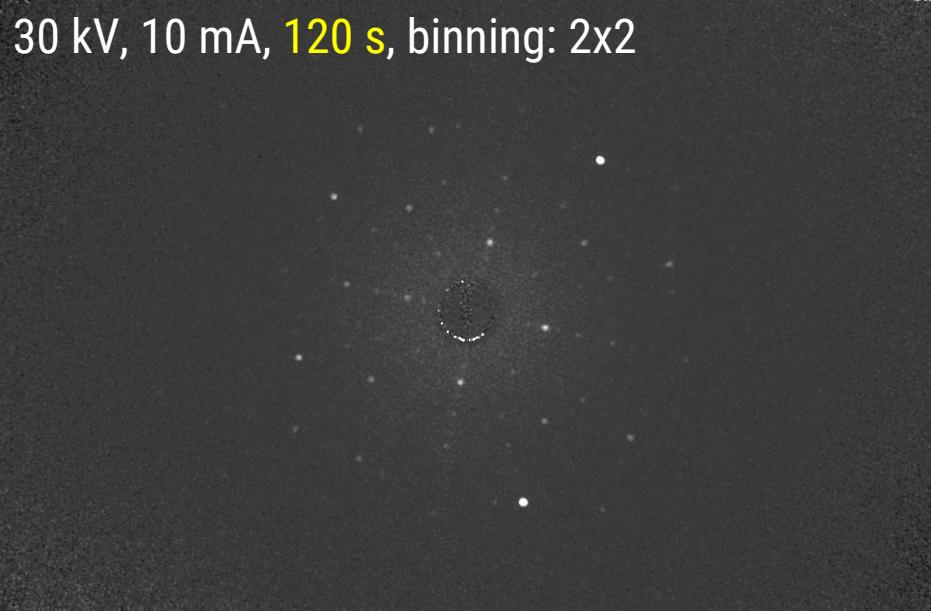
9.1 Silicon Wafer

Varying image-acquisition integration time

- **Observation:**

- The image gets brighter
- The intensity of the reflexes gets larger but at the same time the background gets more noisy

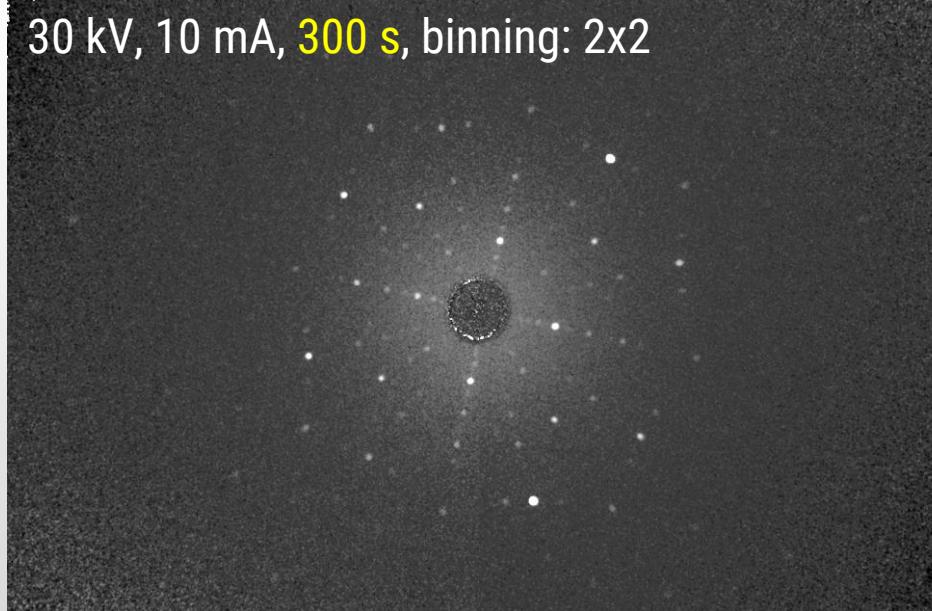
30 kV, 10 mA, 120 s, binning: 2x2



- **Explanation:**

- The larger the integration time, the brighter *all* pixels

30 kV, 10 mA, 300 s, binning: 2x2



9.1 Silicon Wafer

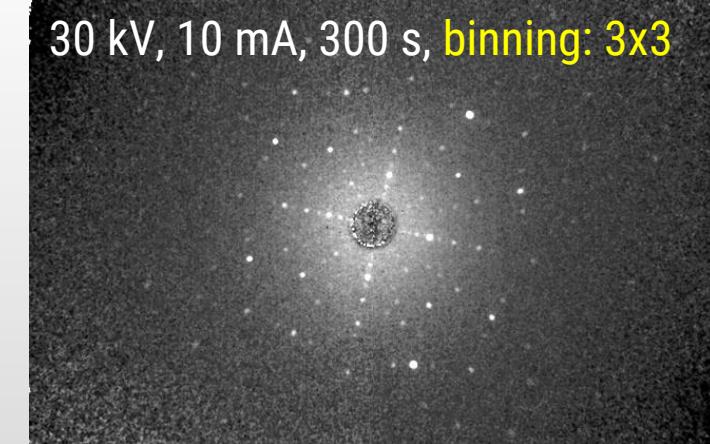
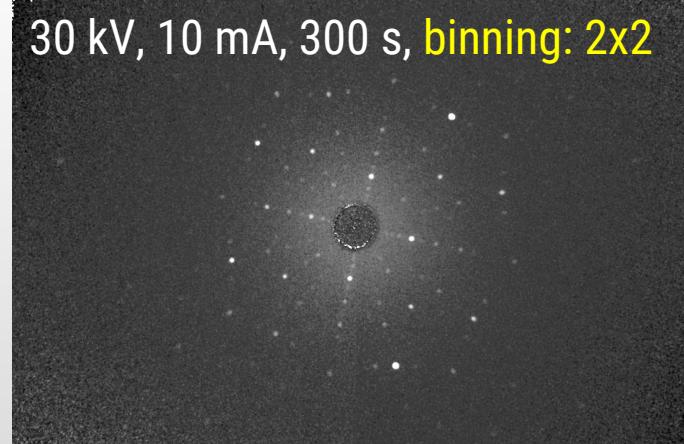
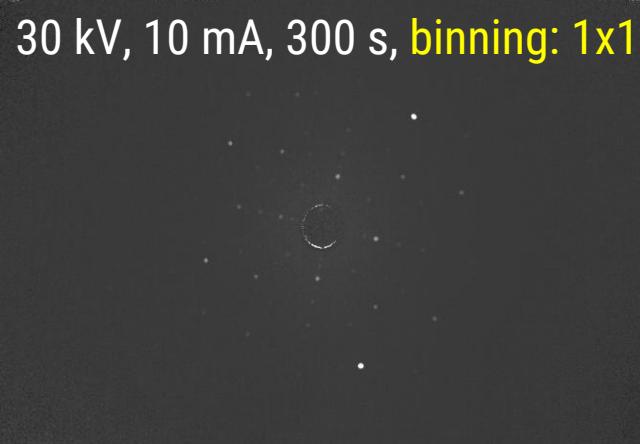
Varying software binning

- **Observation:**

- The image and the dots get brighter
- At the same time the background gets more and more noisy

- **Explanation:**

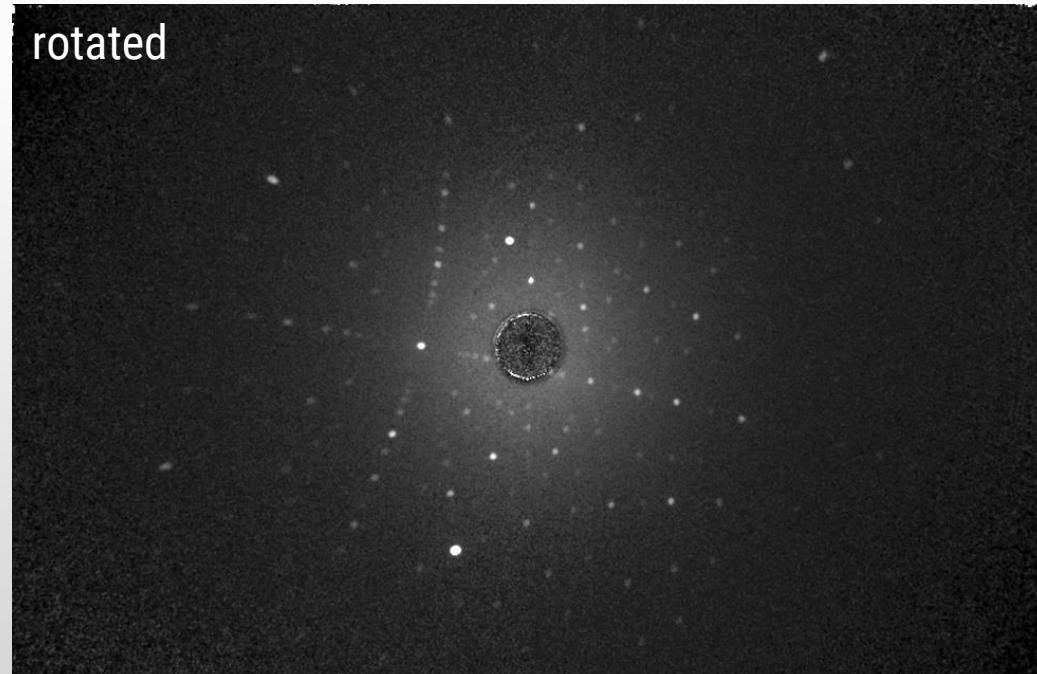
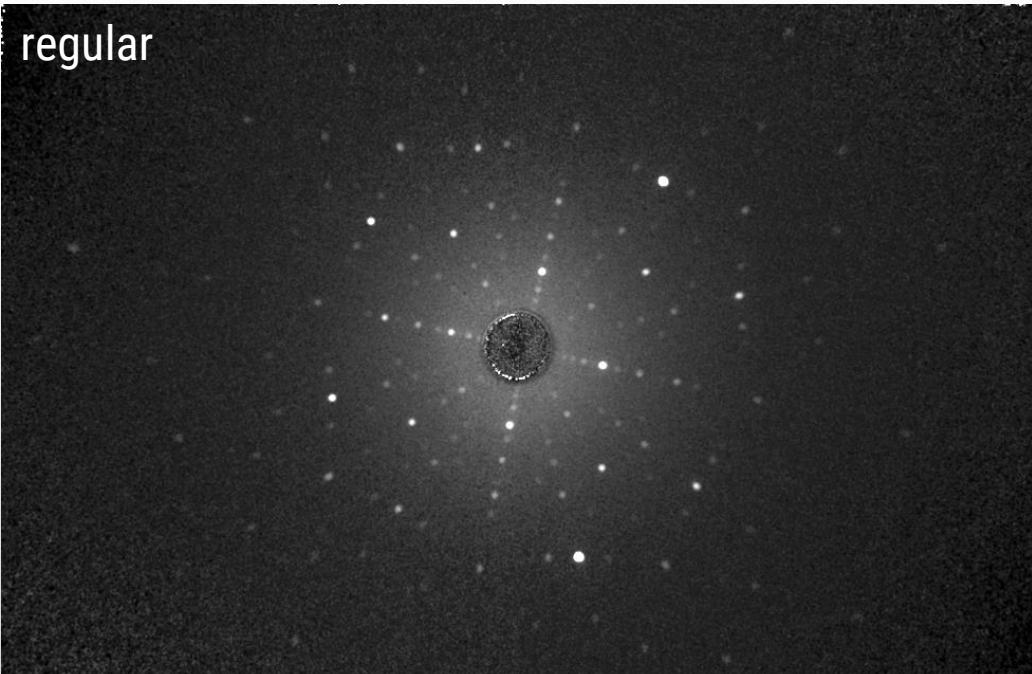
- The binning defines how many pixels get evaluated together, i.e. with a binning of 1x1 each pixel gets evaluated individually (integrated individually)



9.1 Silicon Wafer

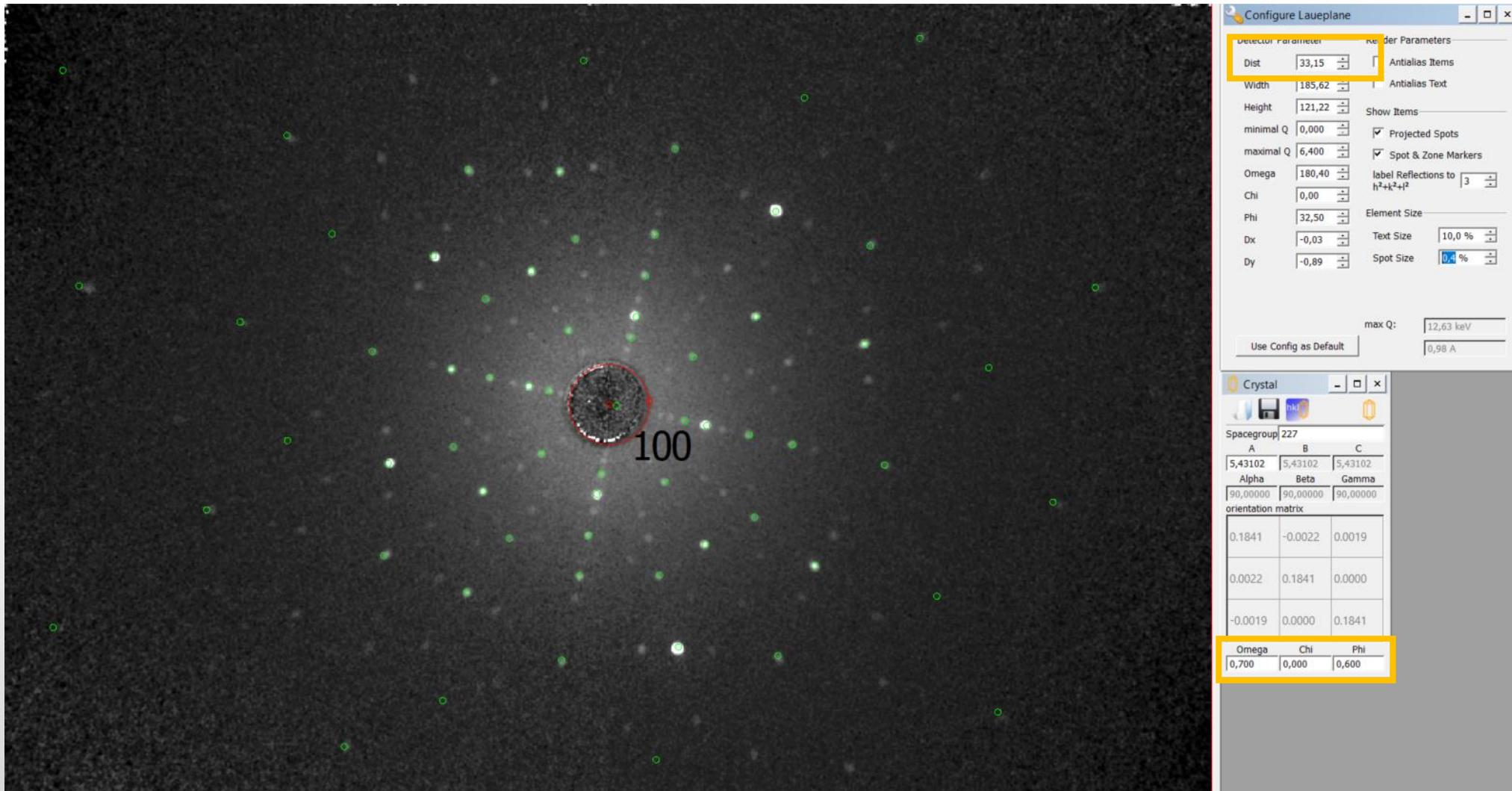
Results of adjusting the parameters

- The best recordings of the diffraction patterns that we received are done with the following parameters:
 - $U_a = 30kV$
 - $I_h = 15mA$
 - $\Delta t = 500s$
 - Binning: 2×2



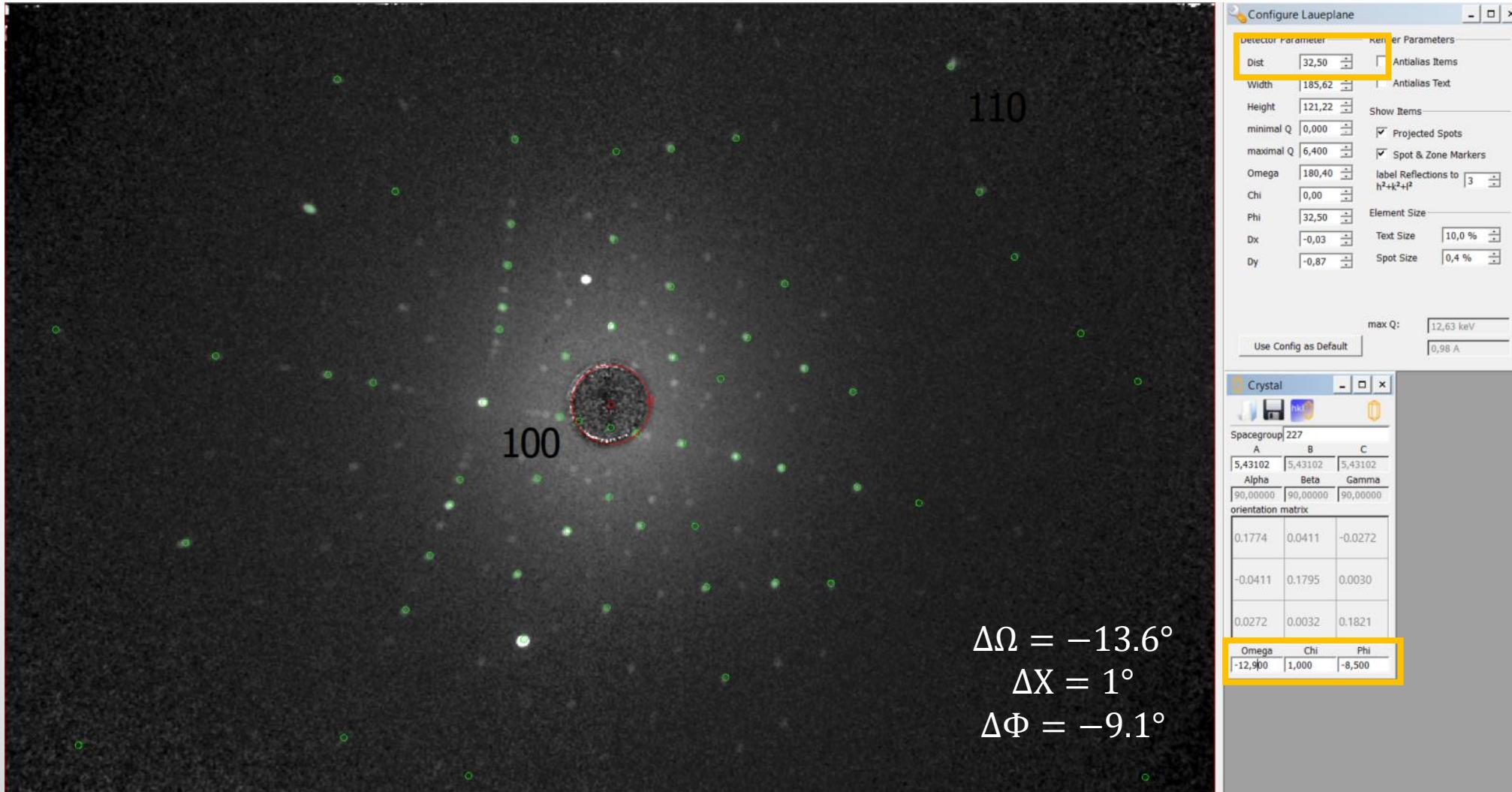
9.1 Silicon Wafer

30kV_15mA_500s.PNG



9.1 Silicon Wafer

30kV_15mA_500s_rotiert.PNG



9.1 Silicon Wafer

Effect of the rotation around Ω

$\Omega = -30^\circ$

111

100

111

110

$\Omega = 0^\circ$

100

$\Omega = +30^\circ$

111

100

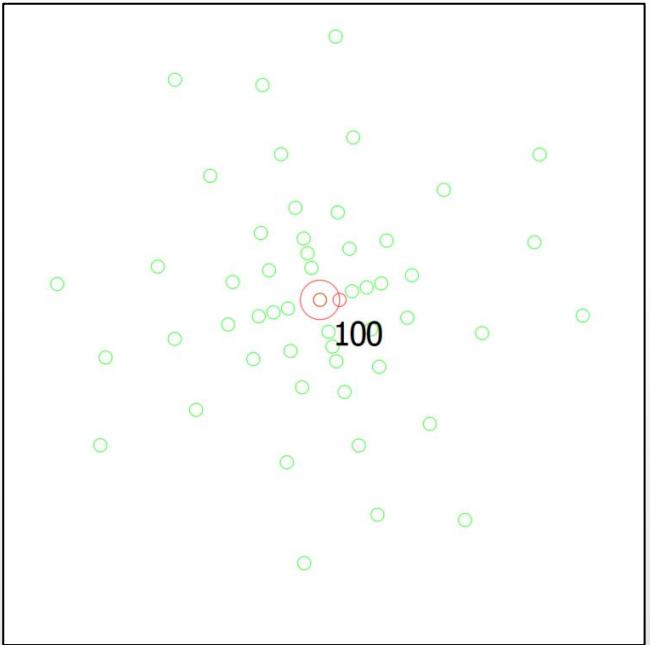
111

→ Rotation around z-axis

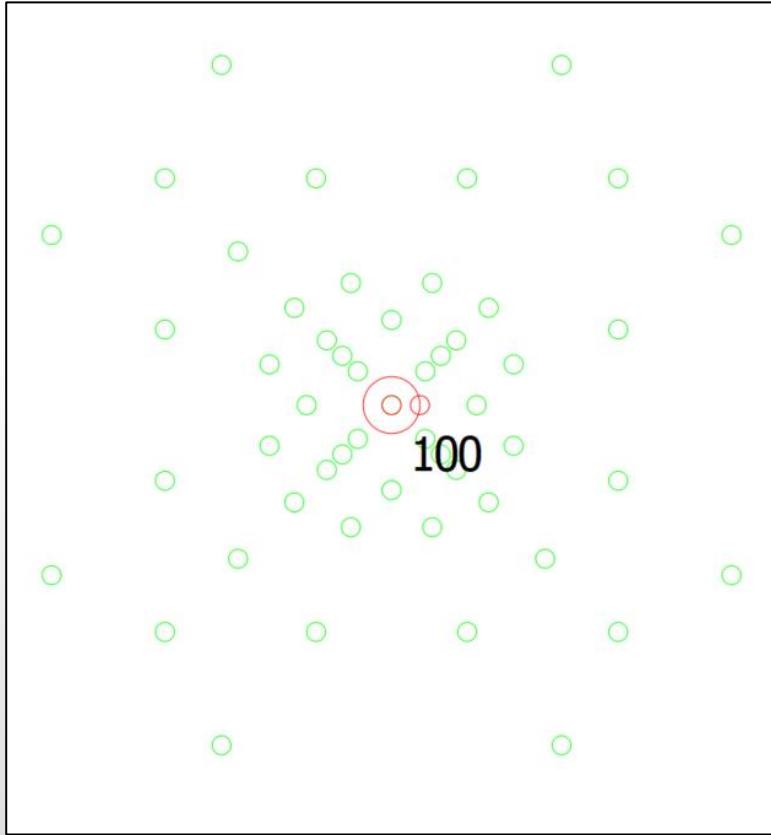
9.1 Silicon Wafer

Effect of the rotation around X

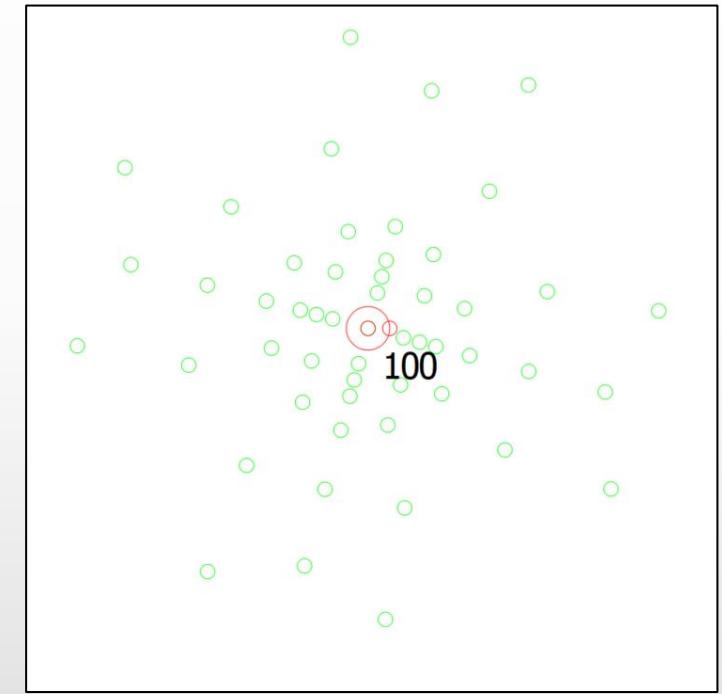
$X = -30^\circ$



$X = 0^\circ$



$X = +30^\circ$

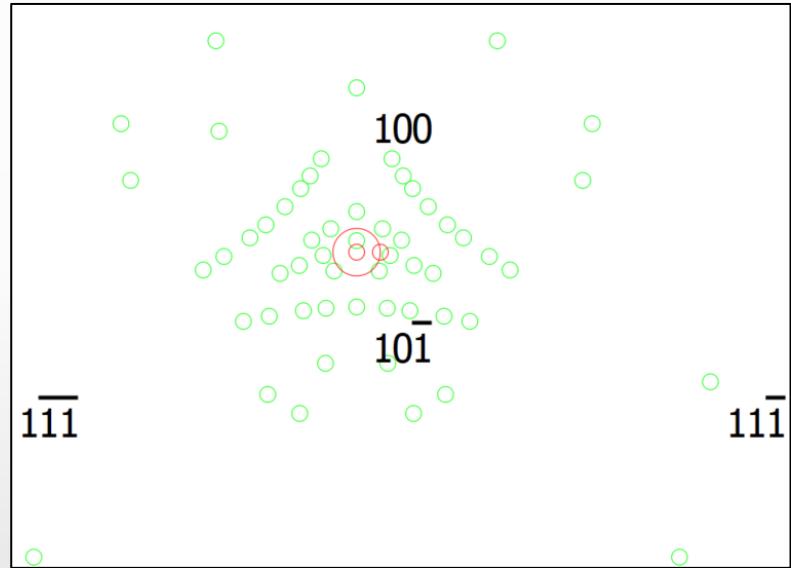


→ Rotation around x-axis (axis of view)

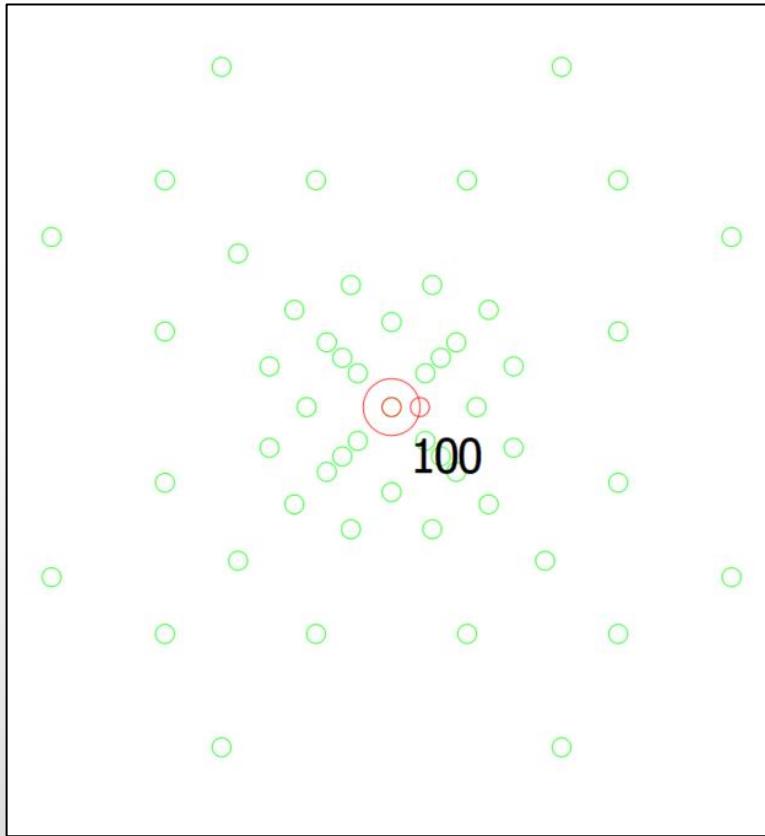
9.1 Silicon Wafer

Effect of the rotation around Φ

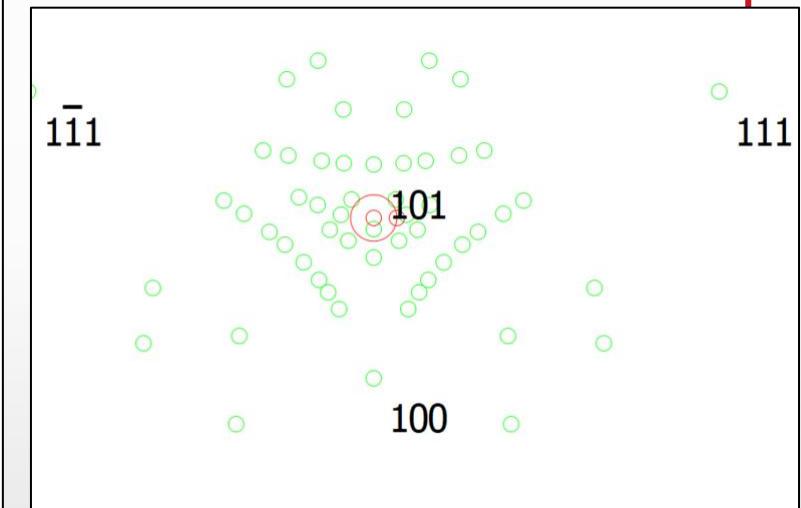
$\Phi = -30^\circ$



$\Phi = 0^\circ$



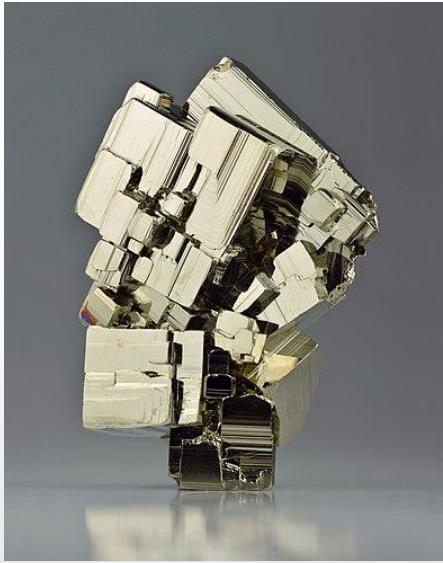
$\Phi = +30^\circ$



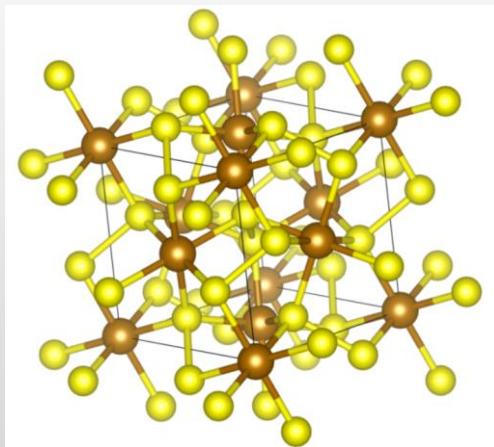
→ Rotation around y-axis

9.2 Crystal of your choice

Pyrite FeS_2



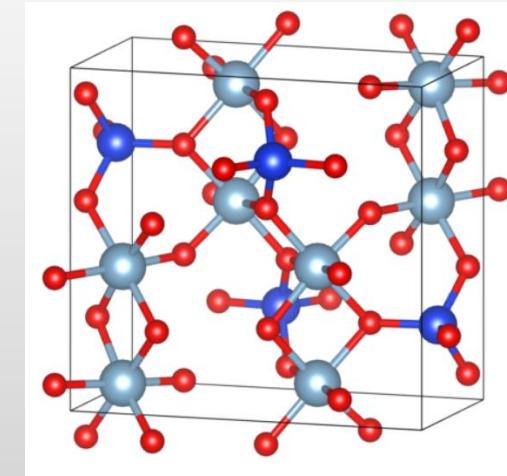
<https://en.wikipedia.org/wiki/Pyrite>



Topaz $Al_2SiO_4(F, OH)_2$



<https://en.wikipedia.org/wiki/Topaz>



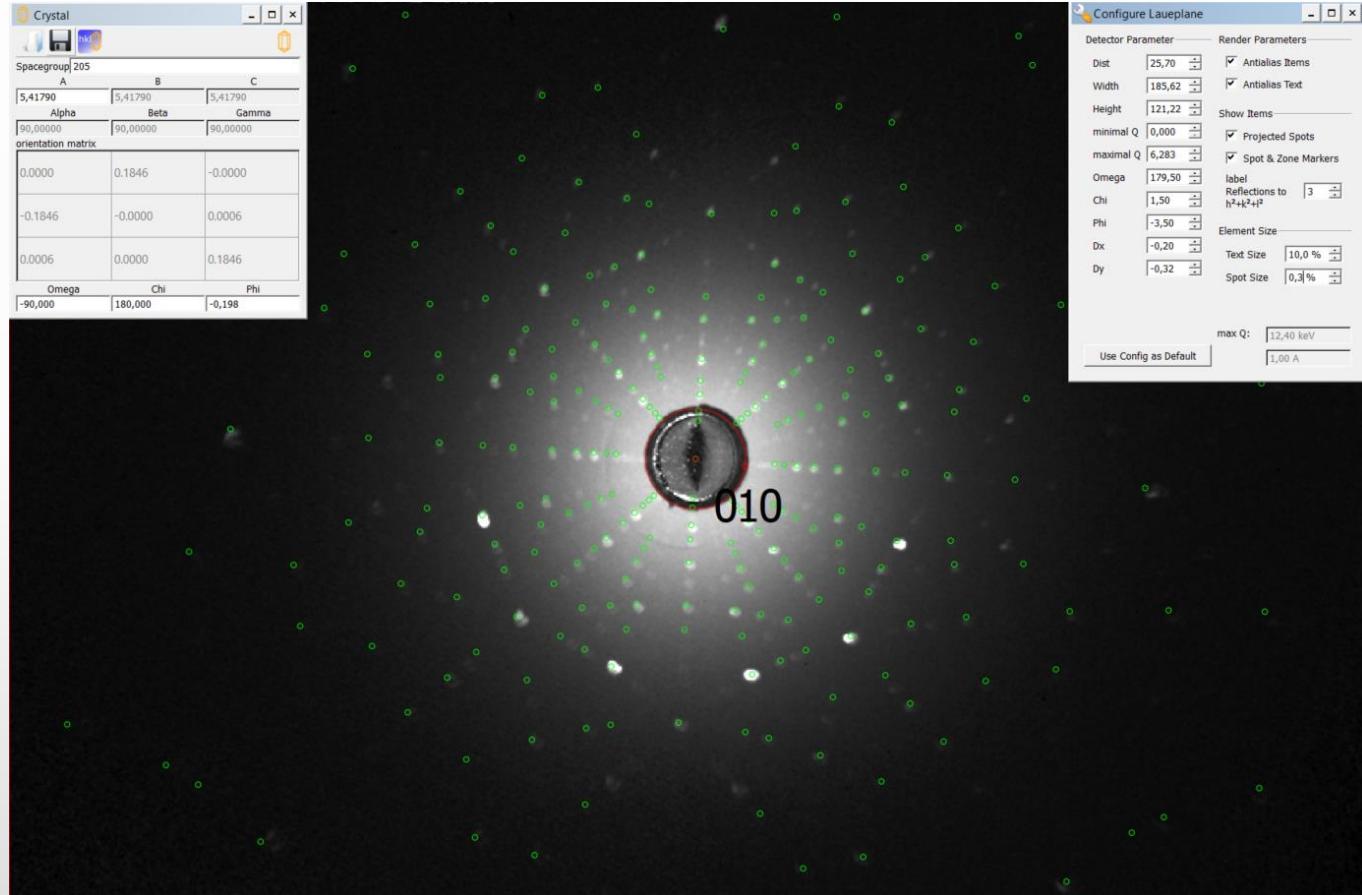
Crystal 1

Pyrite

9.2 Crystal of your choice

Pyrite 1

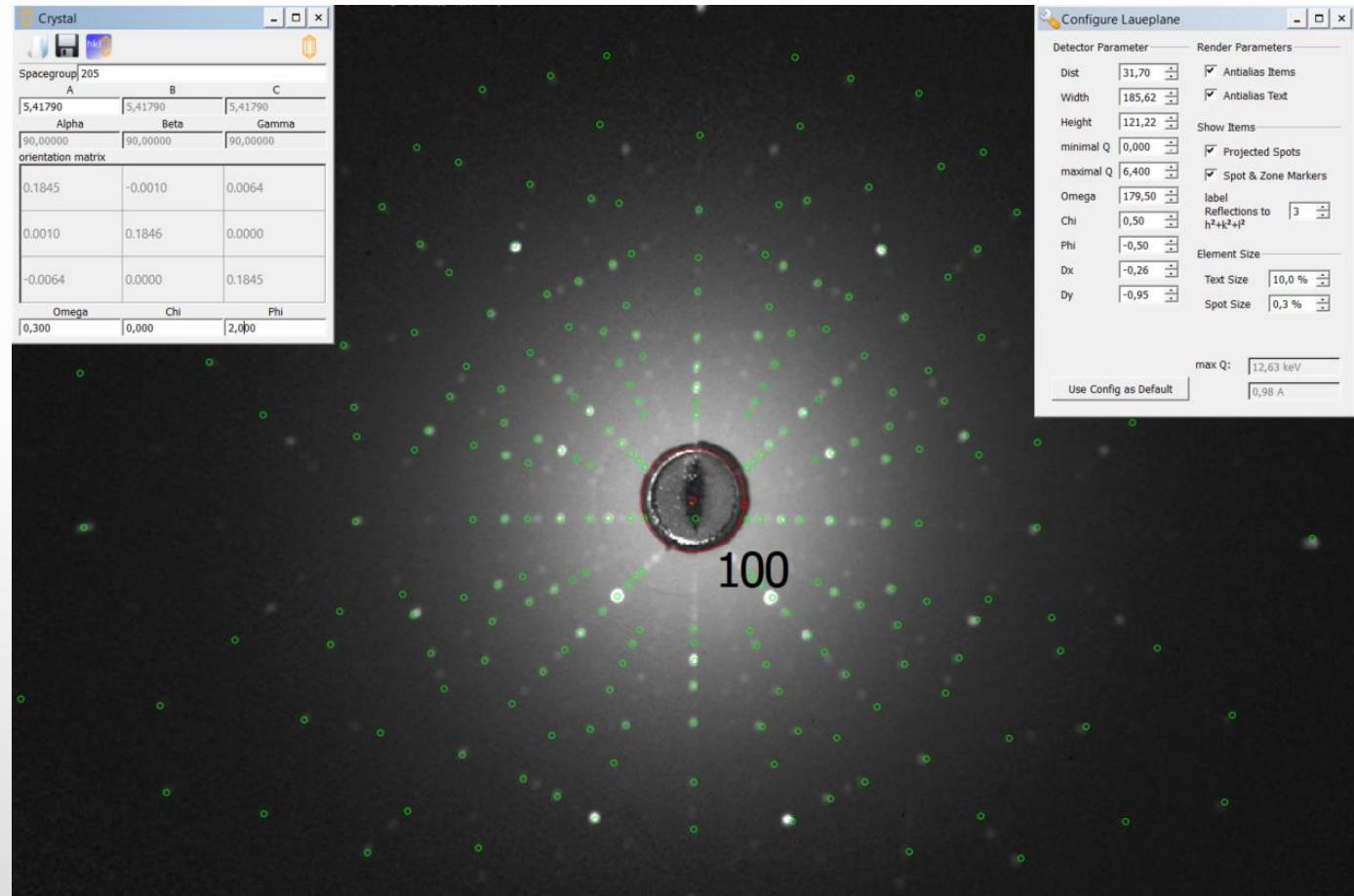
- Determined direction: 010
- Rotational symmetries: 4
- Mirror symmetries: 2
- Defects: not visible (a defect would disturb the symmetry)



9.2 Crystal of your choice

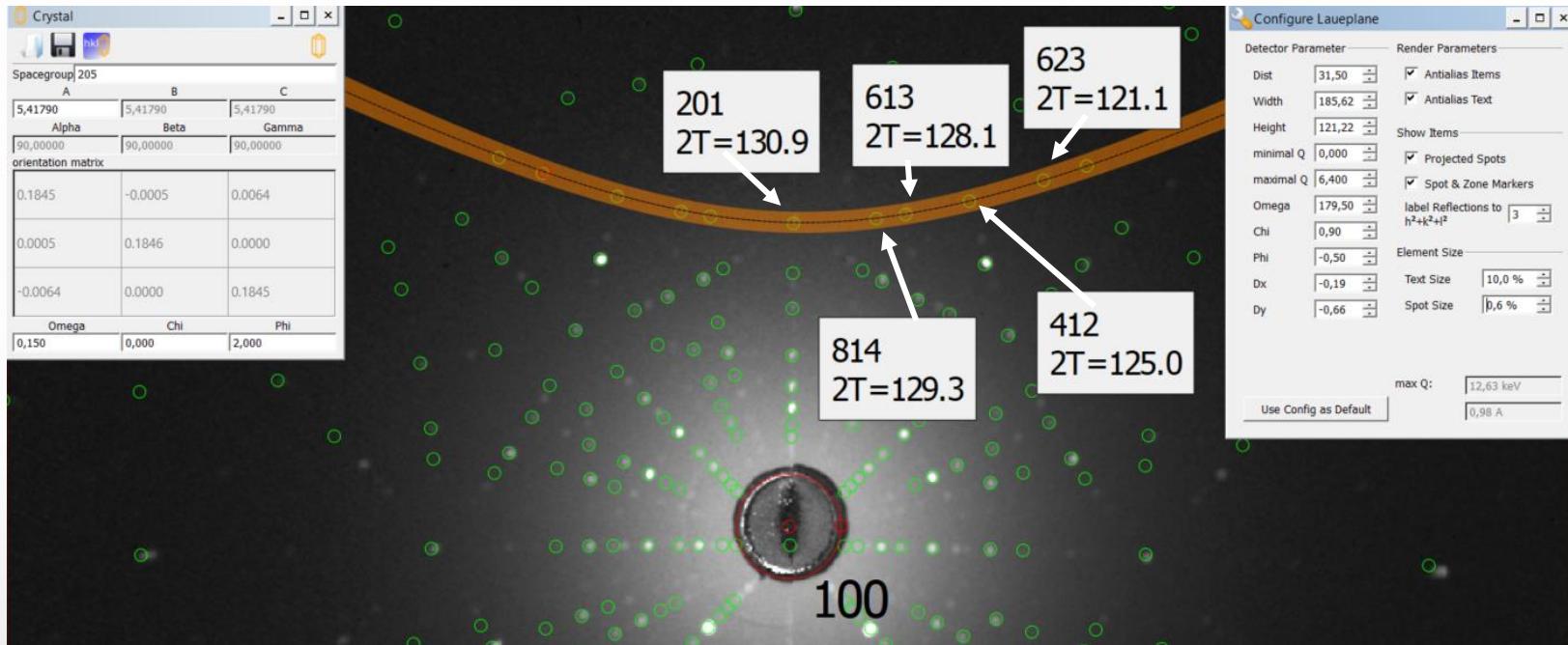
Pyrite 2

- Determined direction: 100
- Rotational symmetries: 4 (but skewed)
- Mirror symmetries: 2 (but skewed)



9.2 Crystal of your choice

Pyrite 2 – zone axis for a particular set of five planes



- Five planes (201), (814), (613), (412), (623)

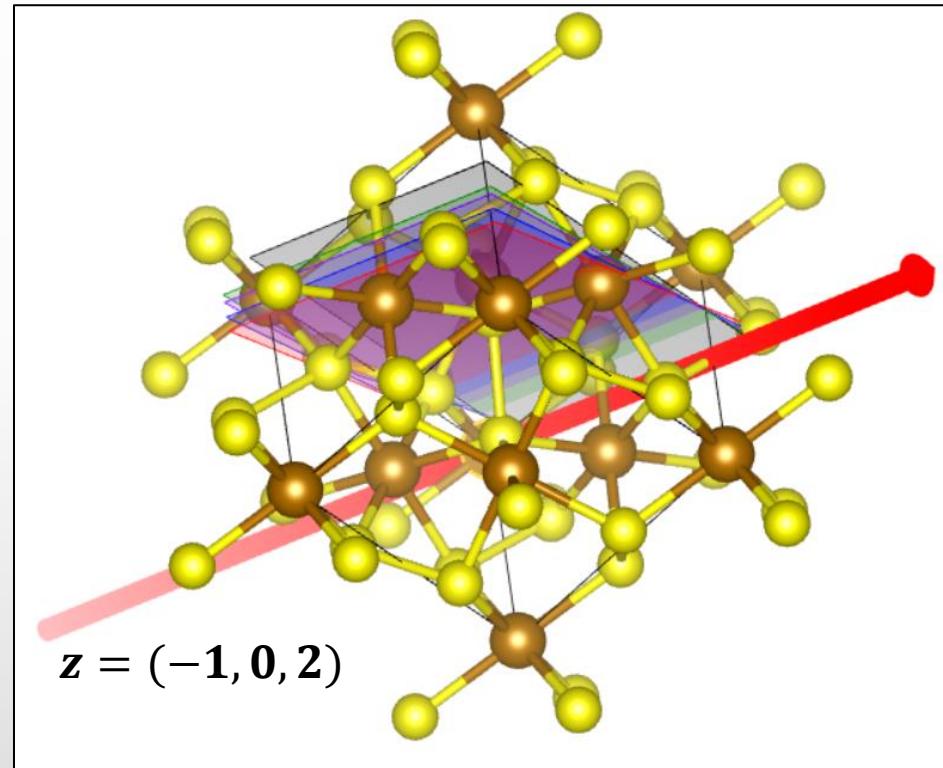
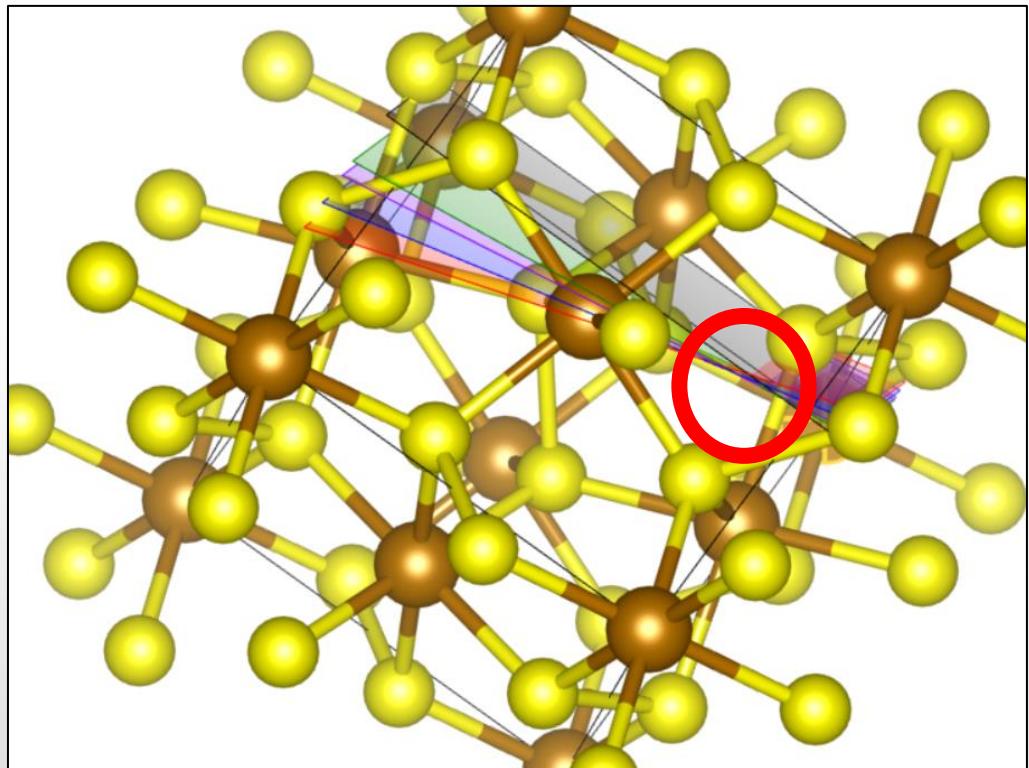
- The zone axis z can be calculated via

$$p_i \cdot z = 0 \text{ for all } i = 1, \dots, 5$$

- The result is: $z = (-1, 0, 2)$

9.2 Crystal of your choice

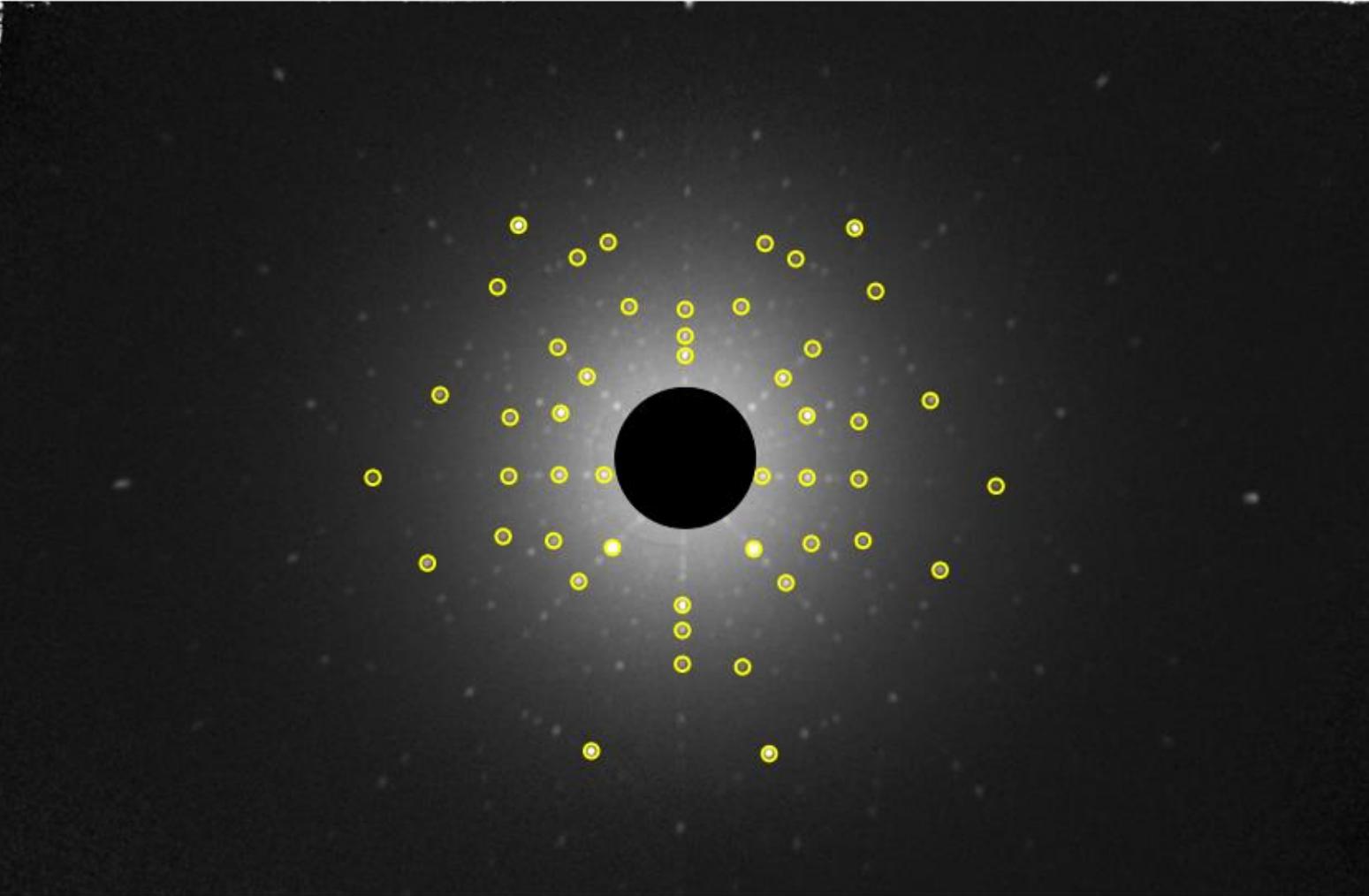
Pyrite 2 – zone axis for a particular set of five planes – visualization with VESTA



9.2 Crystal of your choice

Pyrite 2 – zone axis for a particular set of five planes – fit spots

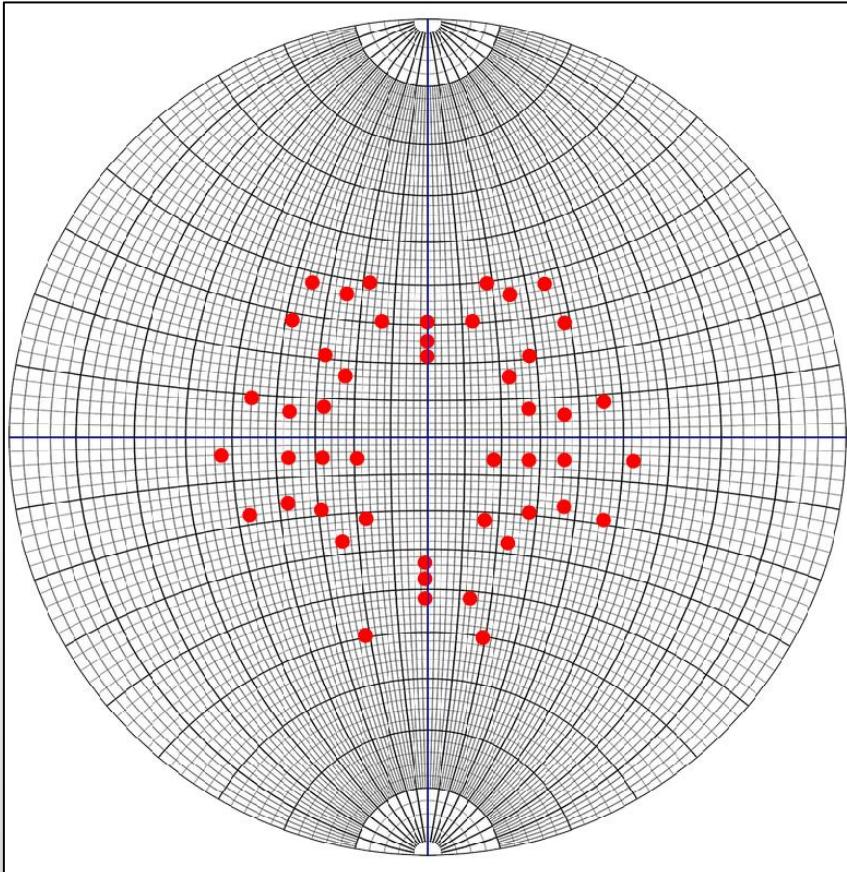
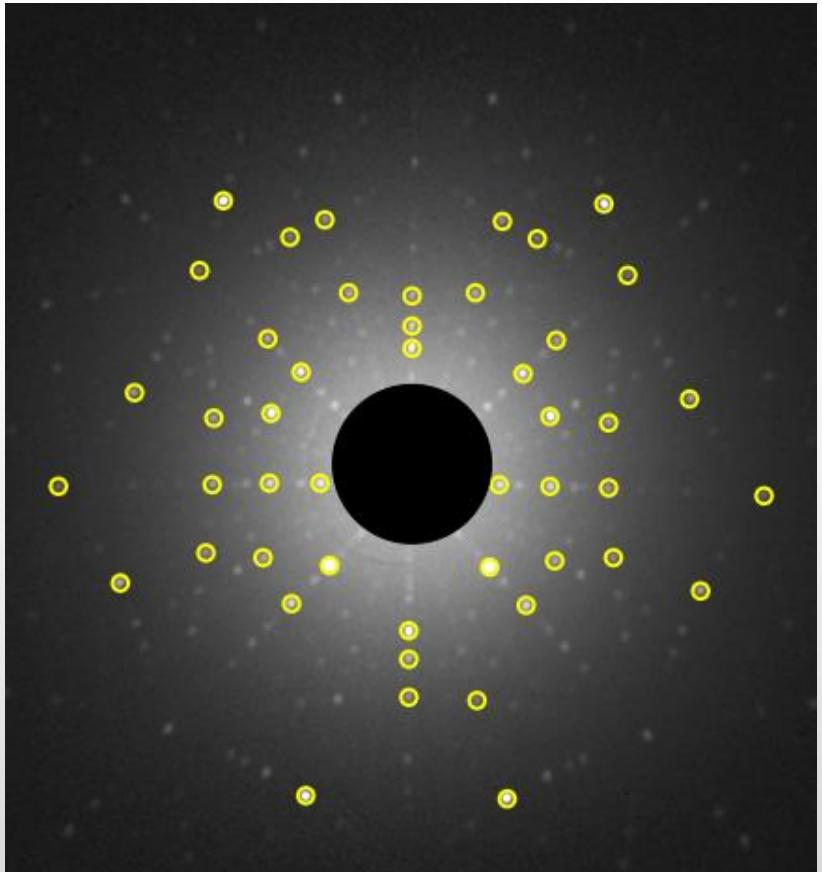
- We use a small python script to fit the spots (“less spots but no errors”)



9.2 Crystal of your choice

Pyrite 2 – zone axis for a particular set of five planes – stereographic projection

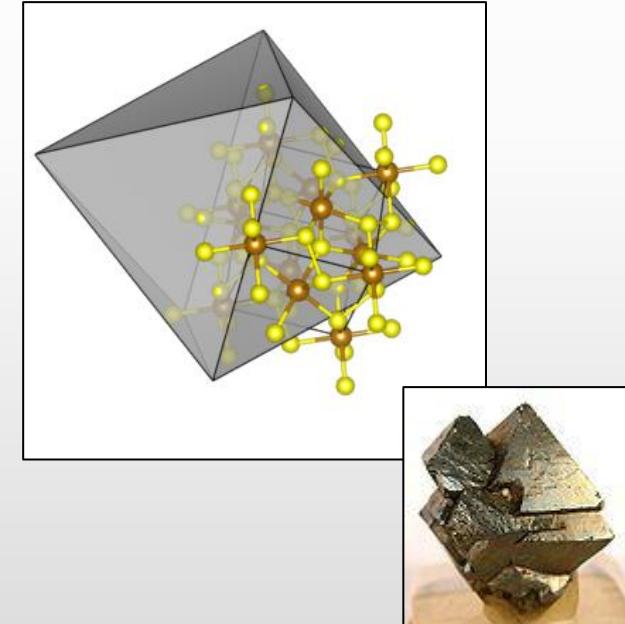
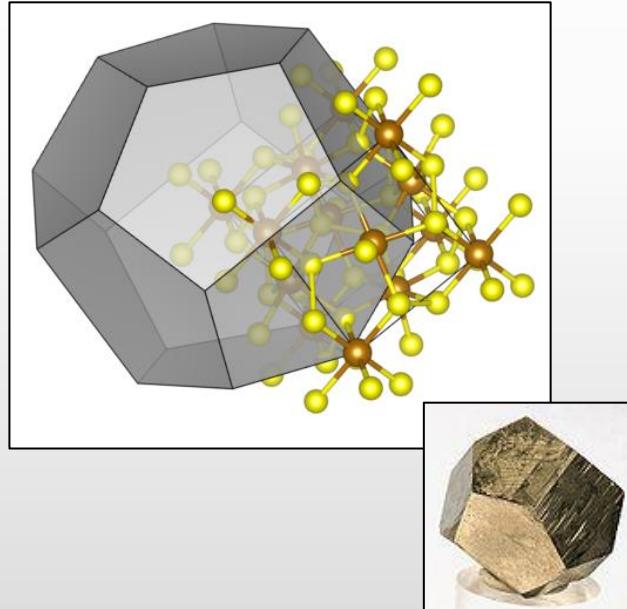
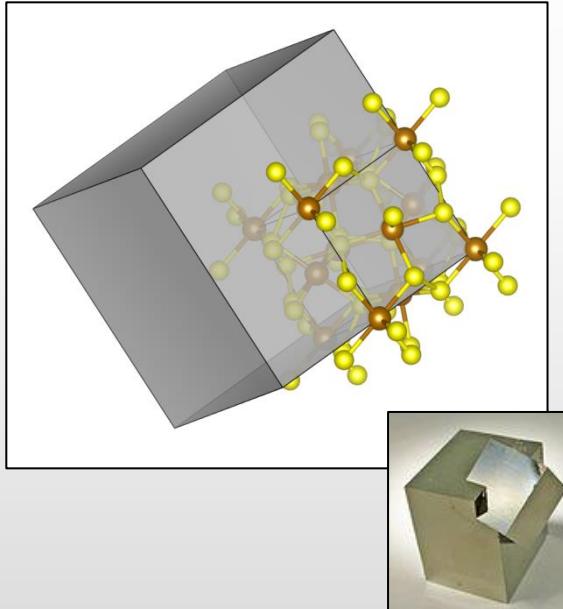
- We use a small python script to fit the spots (“less spots but no errors”)



9.2 Crystal of your choice

Pyrite – crystal shape

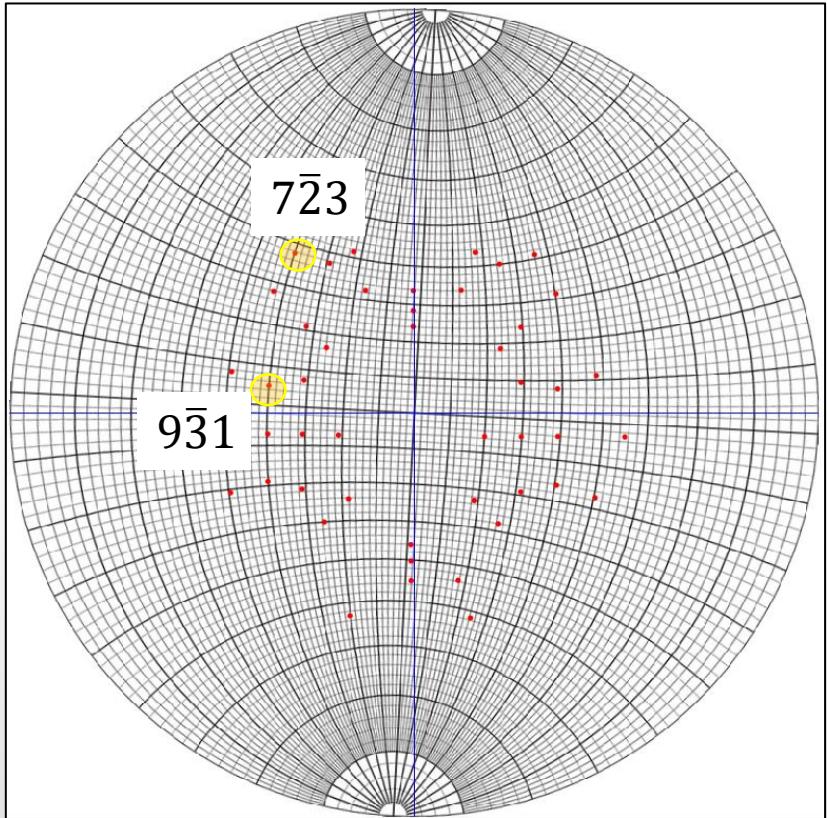
- Crystal-system: cubic
- Energetically favourable: Growing into directions of small (hkl) \rightarrow $(100), (010), (001)$
- Most common: $\{100\}, \{210\}, \{111\}$



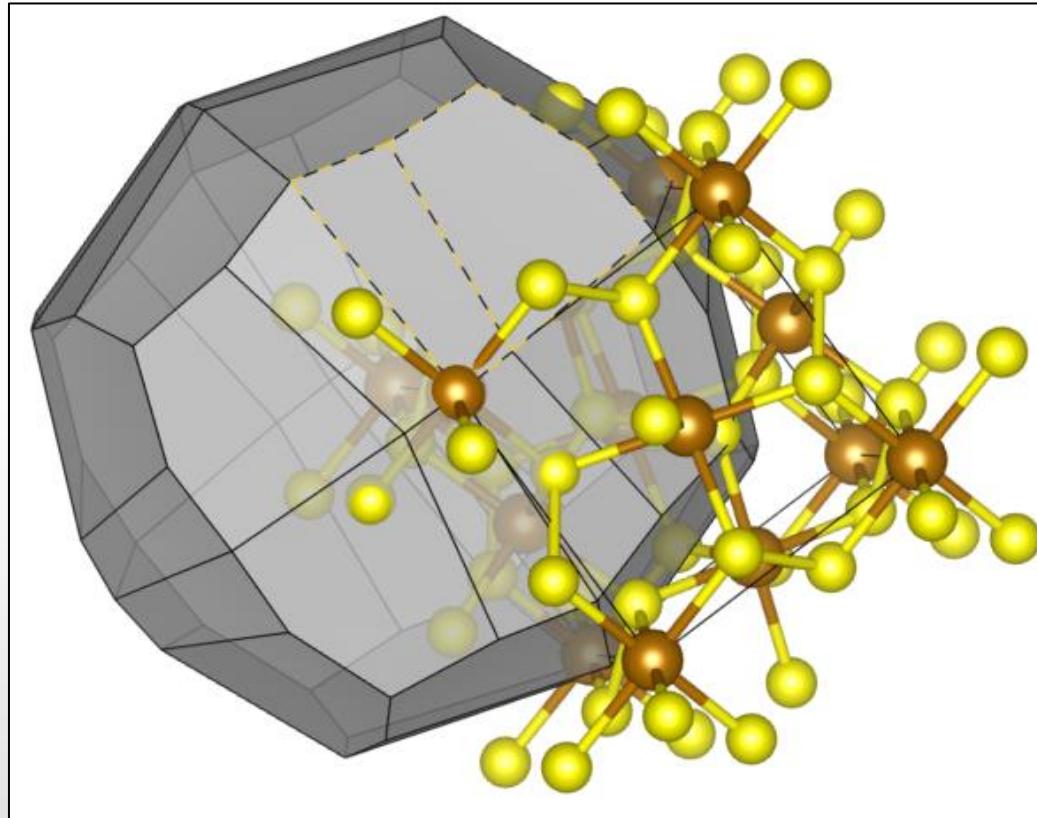
<https://de.wikipedia.org/wiki/Pyrit>

9.2 Crystal of your choice

Pyrite 2 – angle between planes



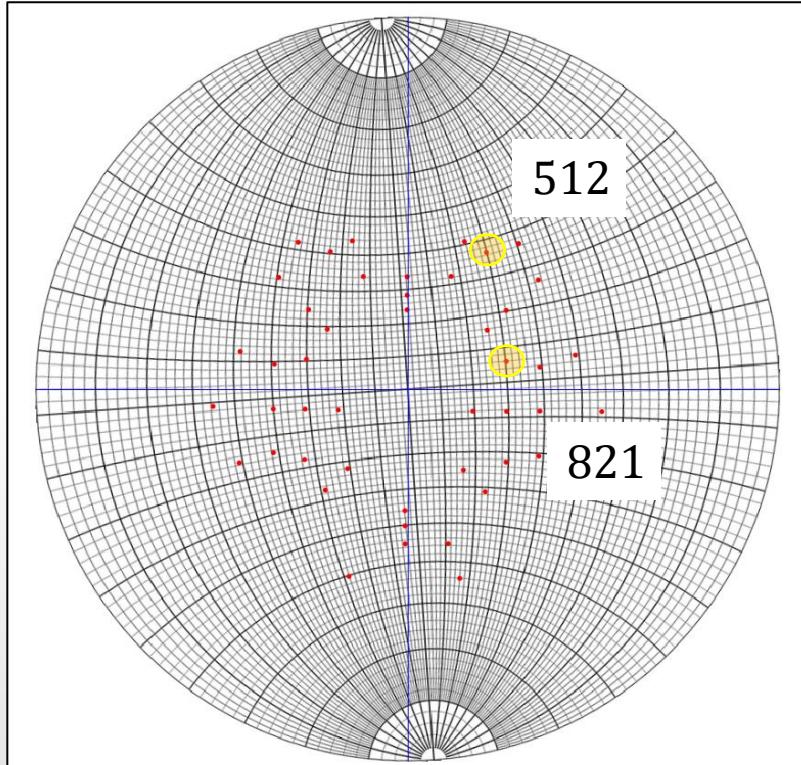
→ $(16.5 \pm 1.0)^\circ$



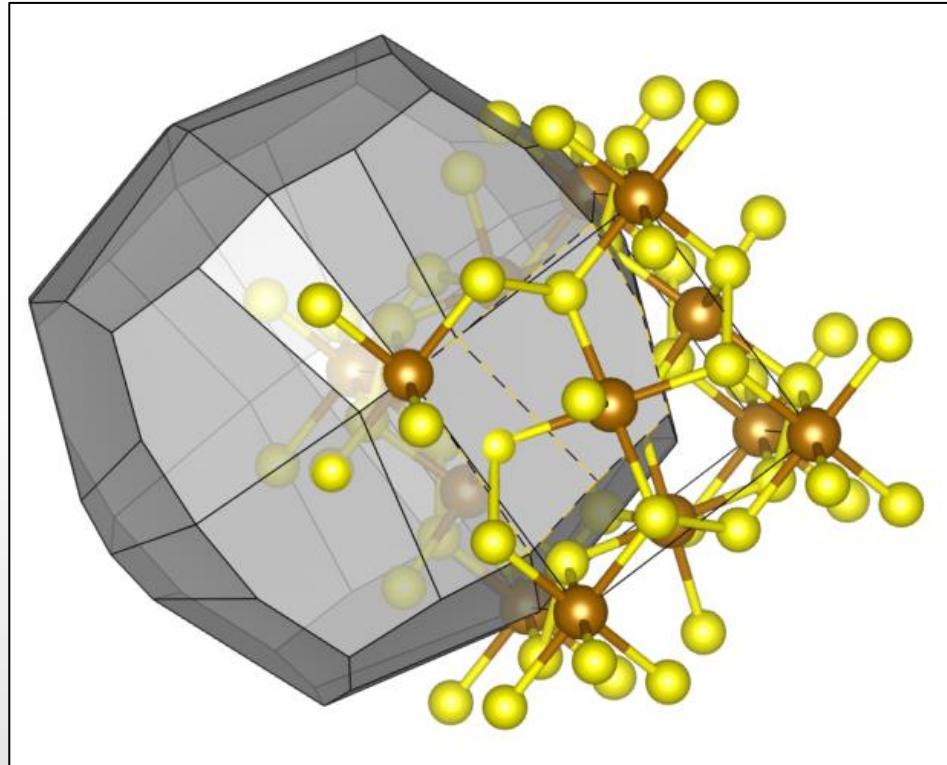
Angle between $(9 -3 1)$ and $(7 -2 3) = 16.5538^\circ$

9.2 Crystal of your choice

Pyrite 2 – angle between planes



→ $(15.0 \pm 1.0)^\circ$



Angle between (5 1 2) and (8 2 1) = 14.7398°

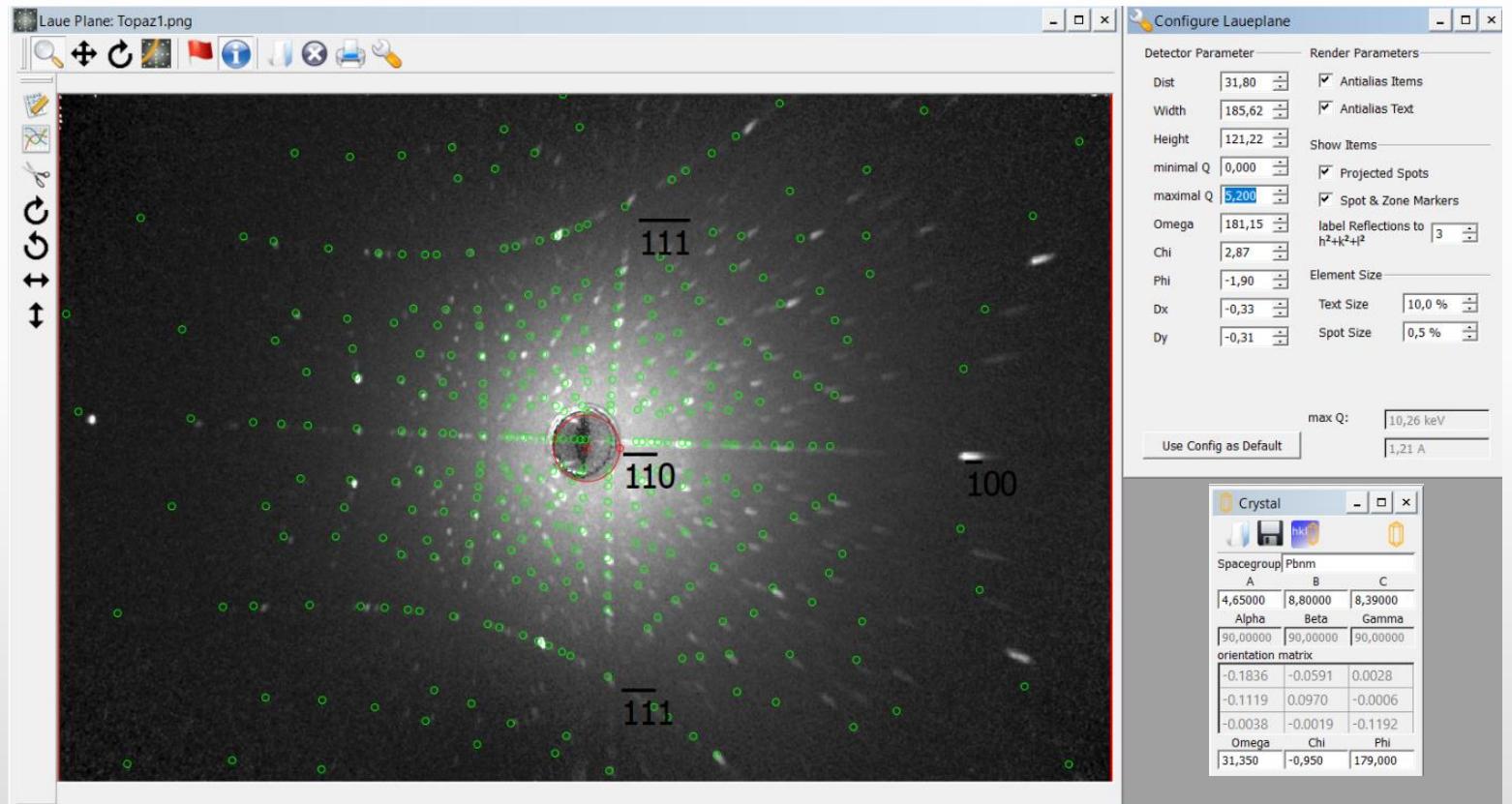
Crystal 2

Topaz

9.2 Crystal of your choice

Topaz 1

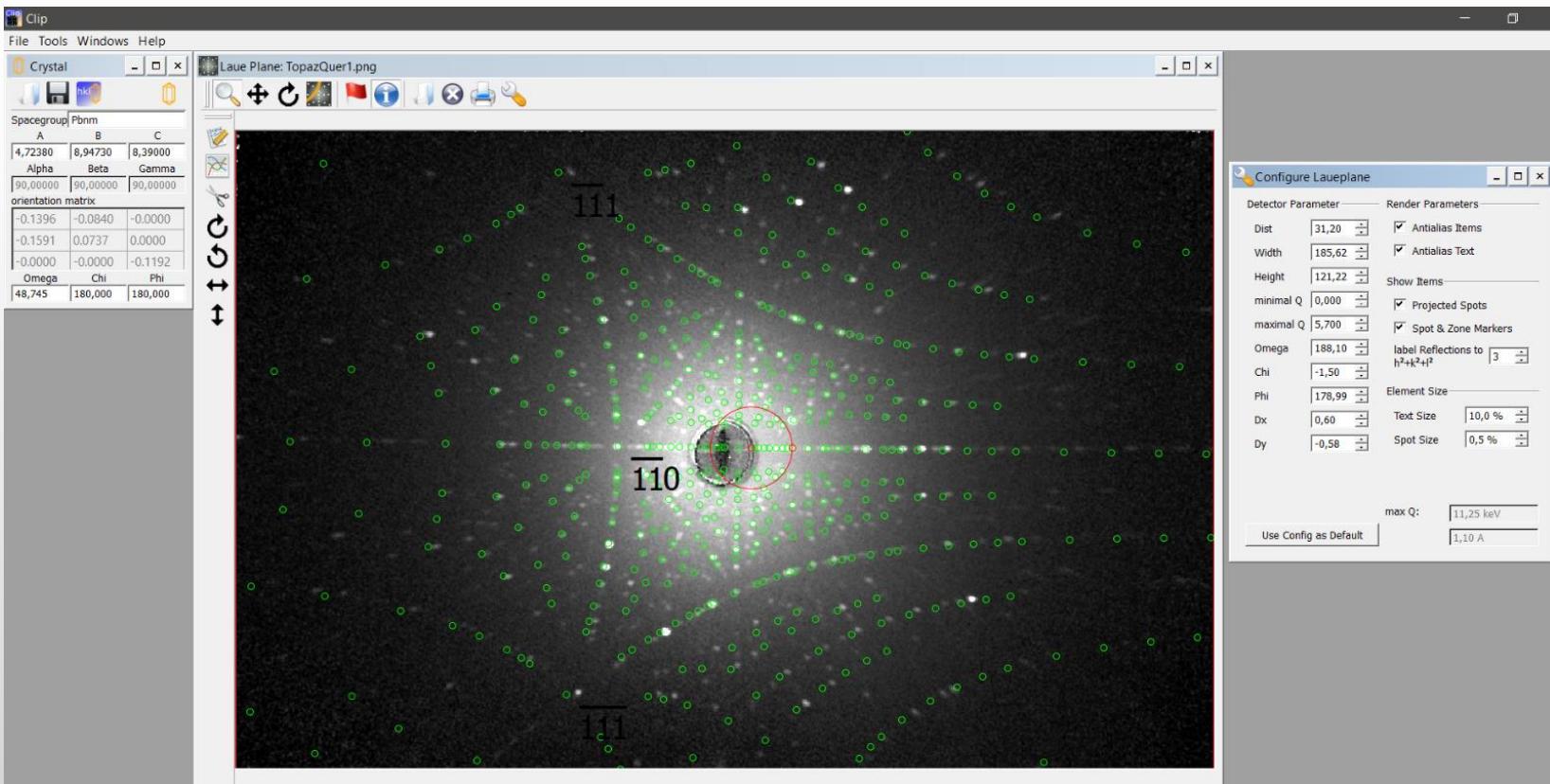
- Determined direction: $\bar{1}\bar{1}0$
- Rotational symmetries: <...>
- Mirror symmetries: <...>
- Defects: <...>



9.2 Crystal of your choice

Topaz cross

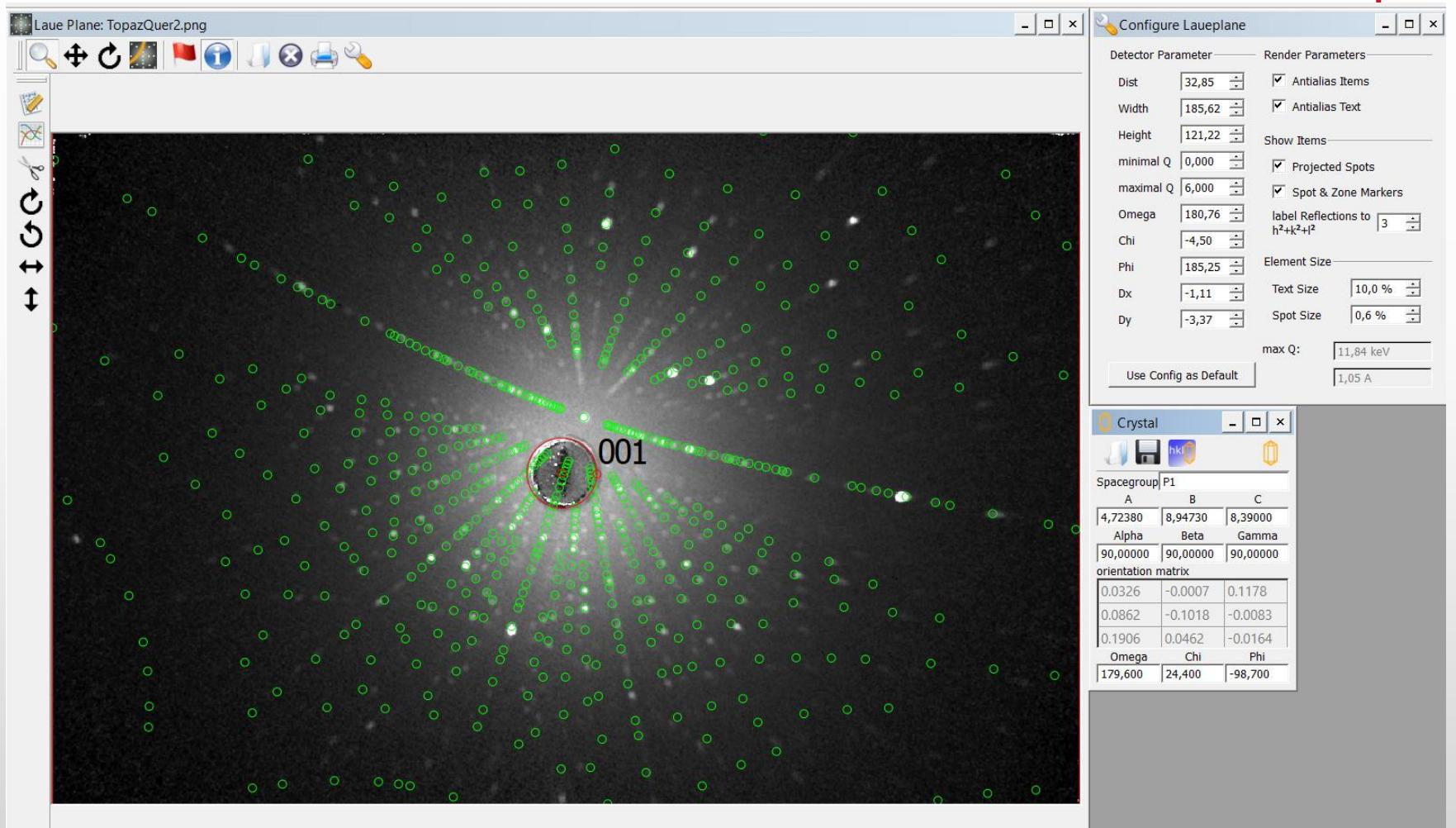
- Determined direction: $\bar{1}\bar{1}0$
- Rotational symmetries: <...>
- Mirror symmetries: <...>



9.2 Crystal of your choice

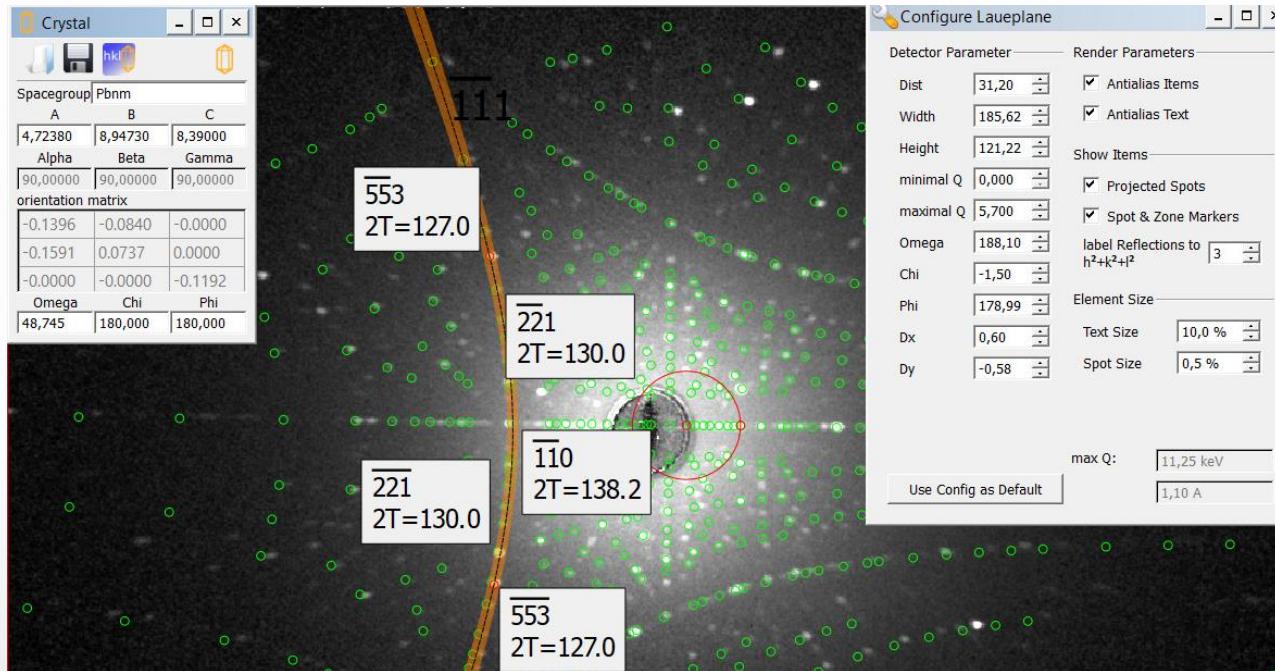
Topaz cross 2

- Determined direction: 100
- Rotational symmetries: $\langle \dots \rangle$
- Mirror symmetries: $\langle \dots \rangle$



9.2 Crystal of your choice

Topaz cross – zone axis for a particular set of five planes



- Five planes $(\bar{5}\bar{5}3), (\bar{2}\bar{2}1), (\bar{1}\bar{1}0), (\bar{2}\bar{2}\bar{1}), (\bar{5}\bar{5}\bar{3})$

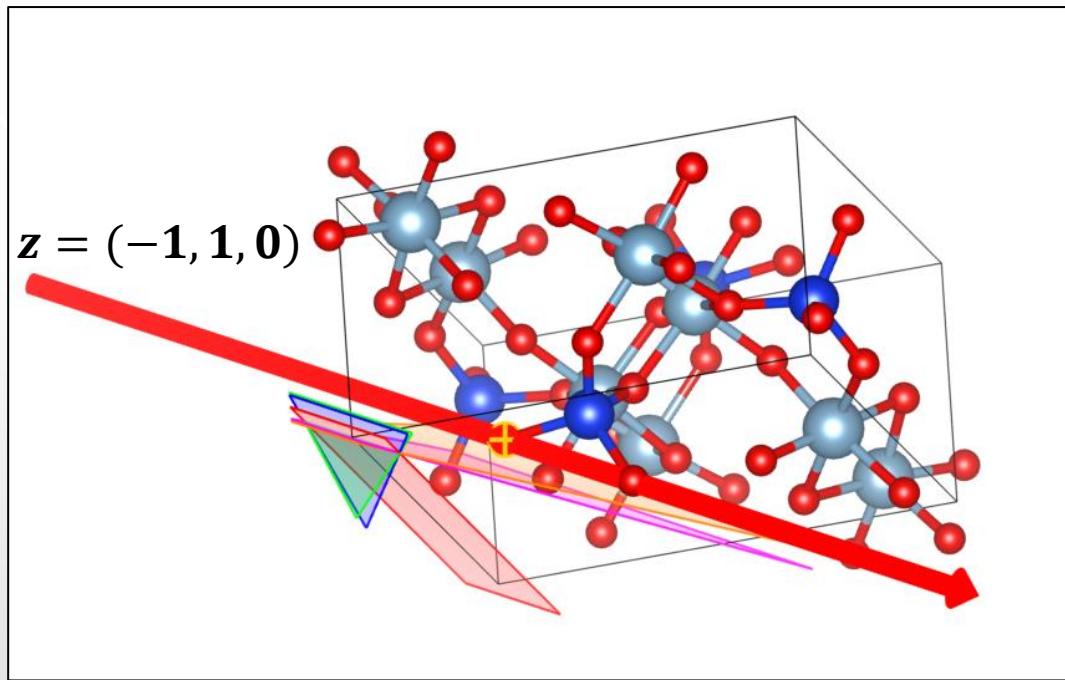
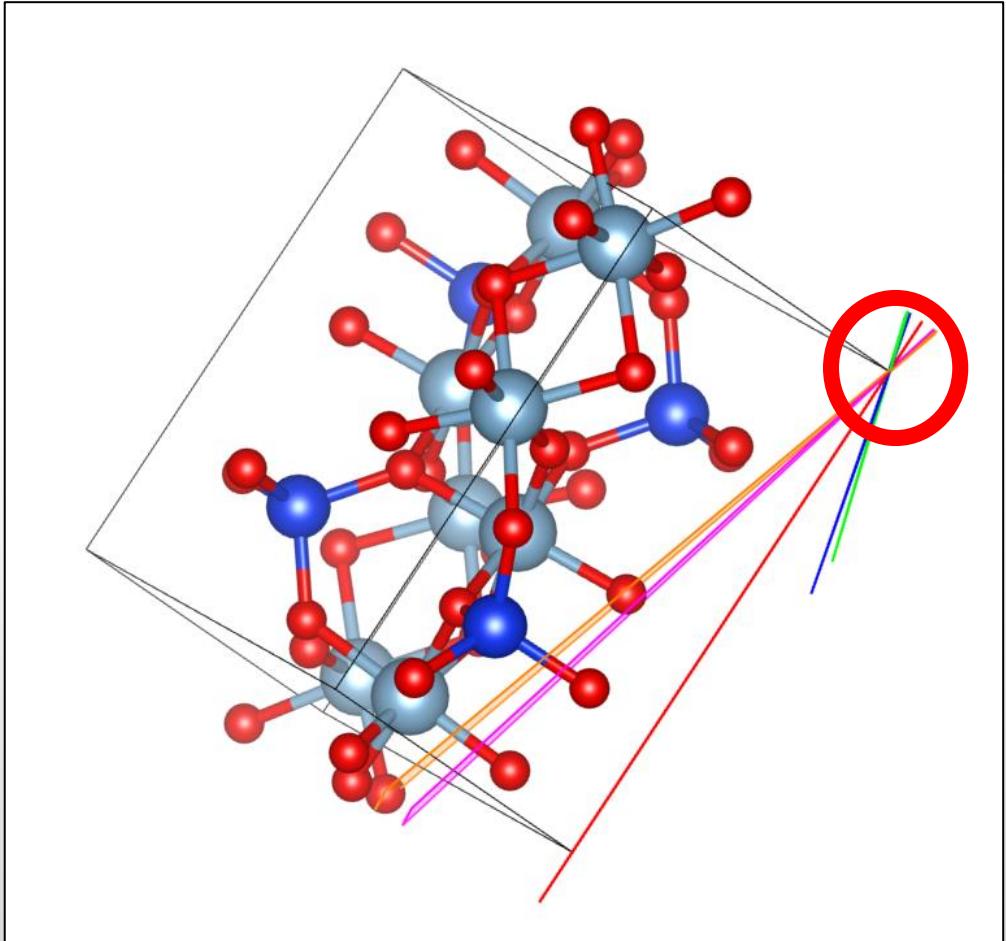
- The zone axis z can be calculated via

$$p_i \cdot z = 0 \text{ for all } i = 1, \dots, 5$$

- The result is: $z = (-1, 1, 0)$

9.2 Crystal of your choice

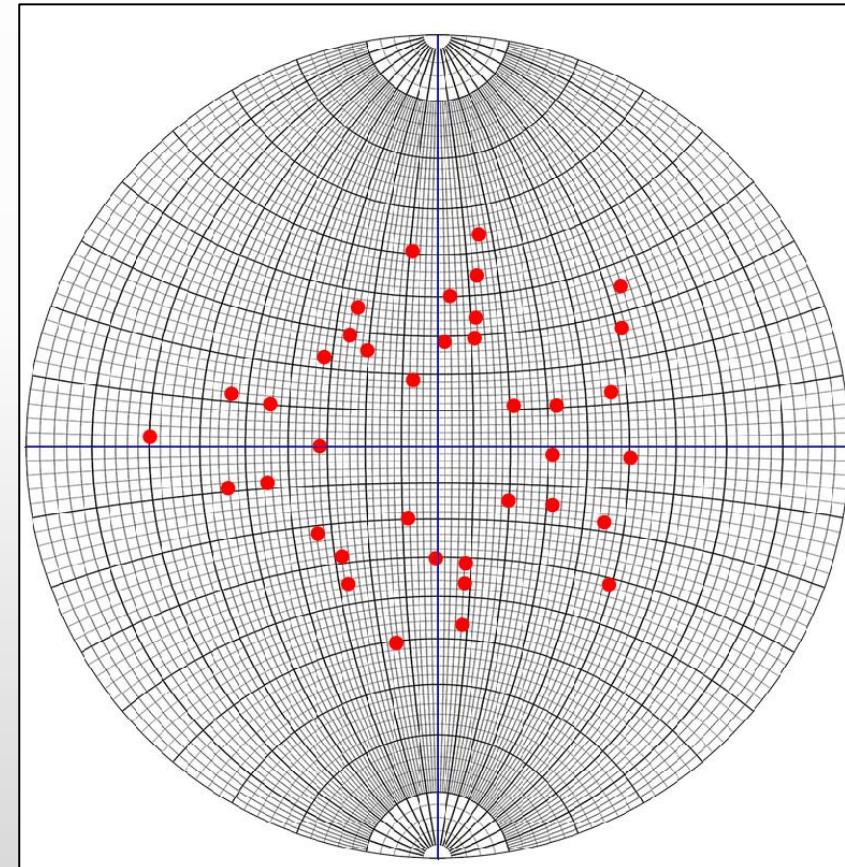
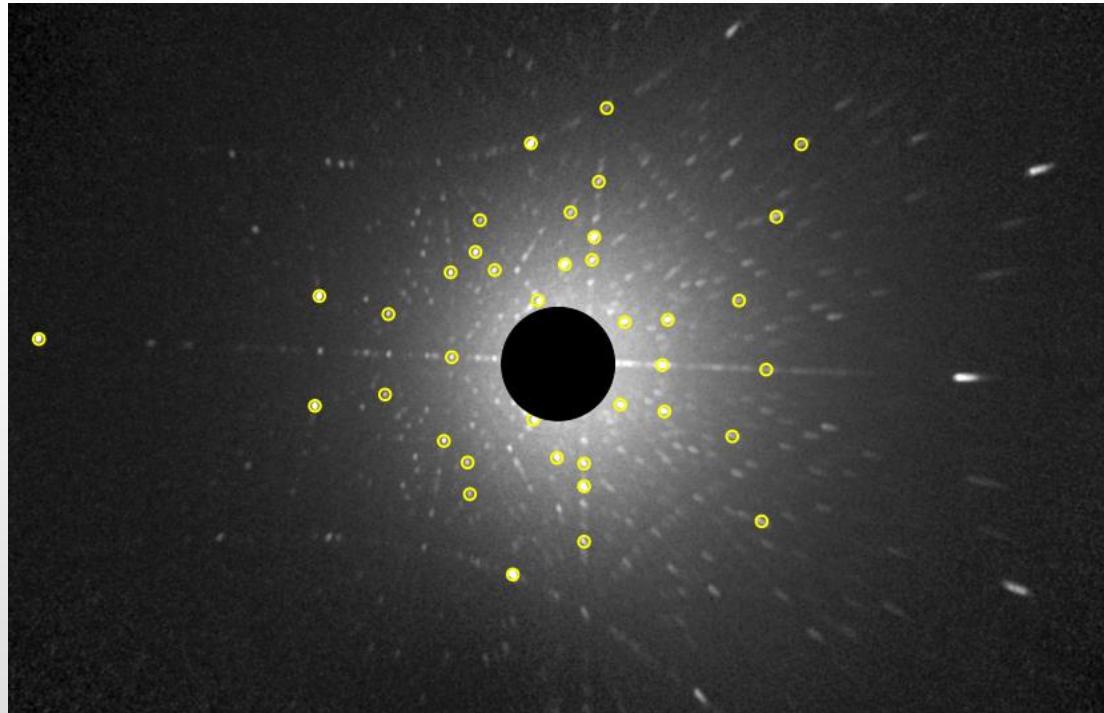
Topaz cross – zone axis for a particular set of five planes – visualization with VESTA



9.2 Crystal of your choice

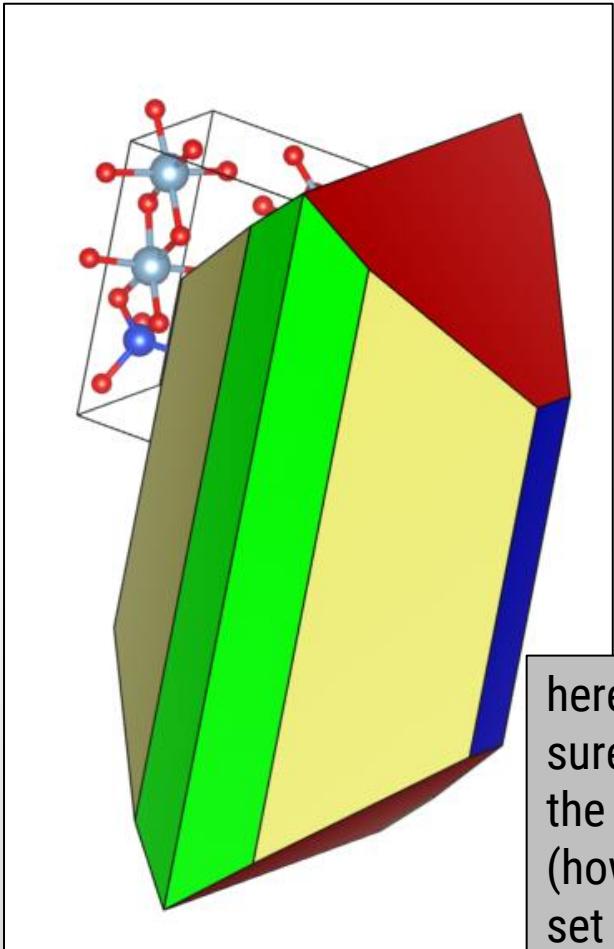
Topaz – zone axis for a particular set of five planes – fit spots

- We use a small python script to fit the spots (“less spots but no errors”)

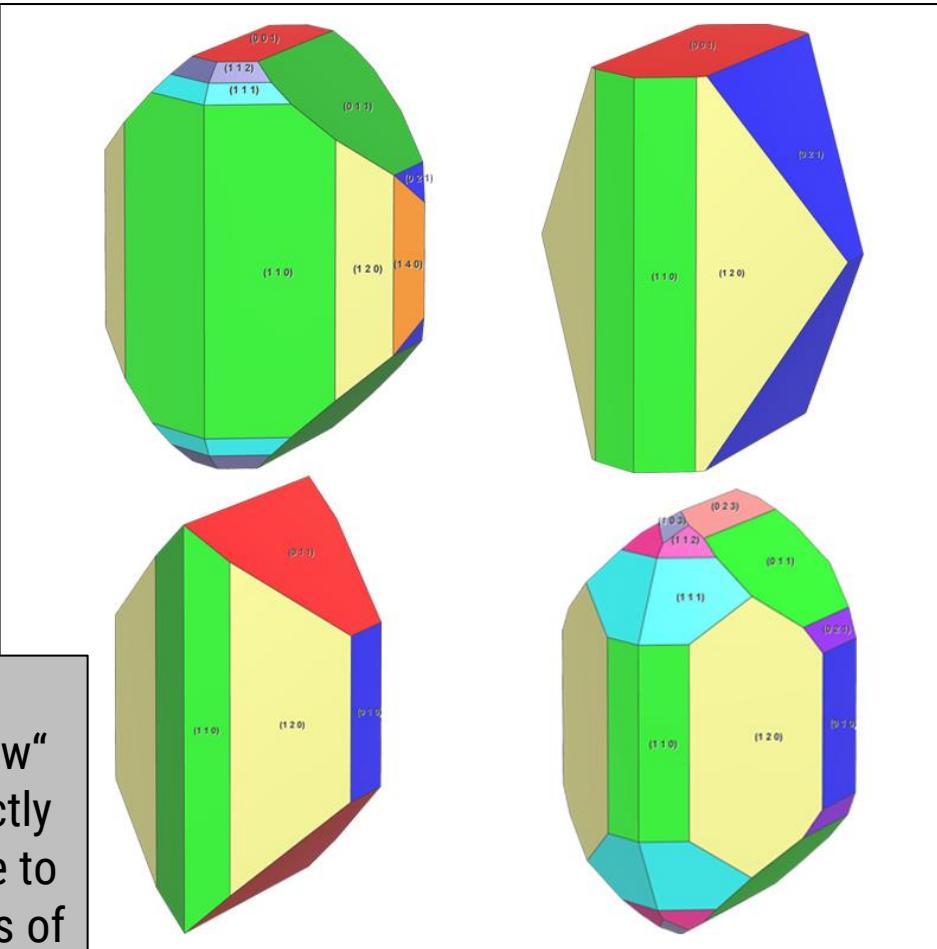


9.2 Crystal of your choice

Topaz – crystal shape



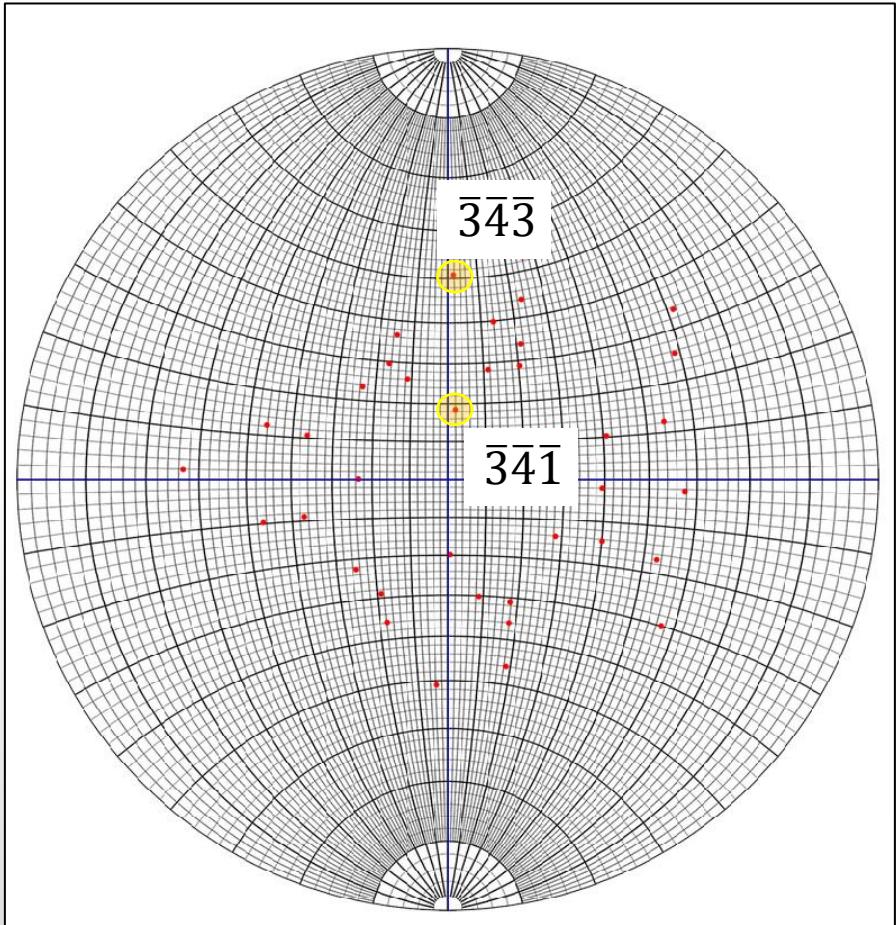
here we are not
sure how to „draw“
the shape correctly
(how do we have to
set the distances of
the planes?)



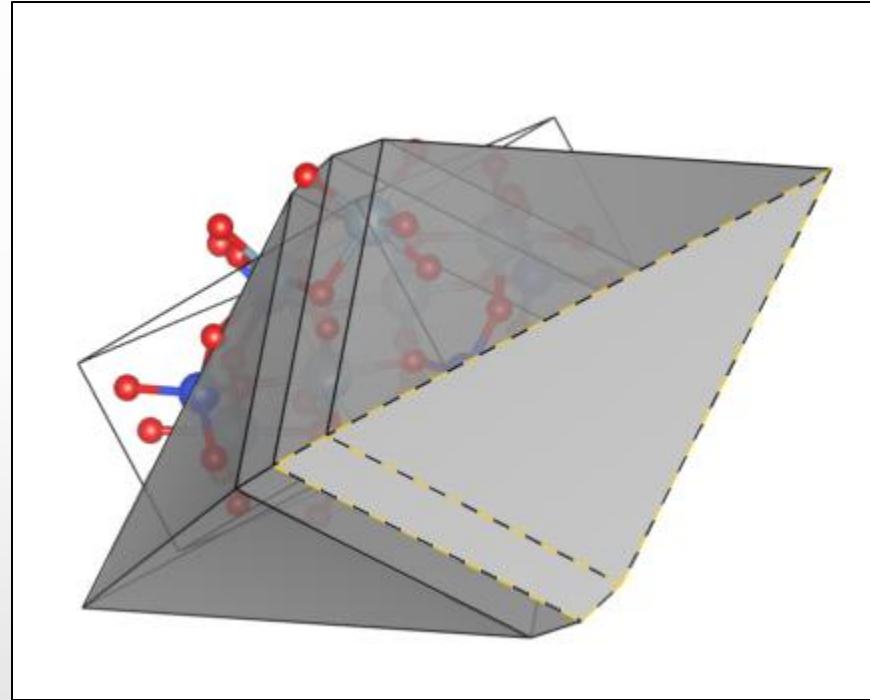
<http://www.meelis-bluetopaz.com/about-topaz.html>

9.2 Crystal of your choice

Topaz – angle between planes



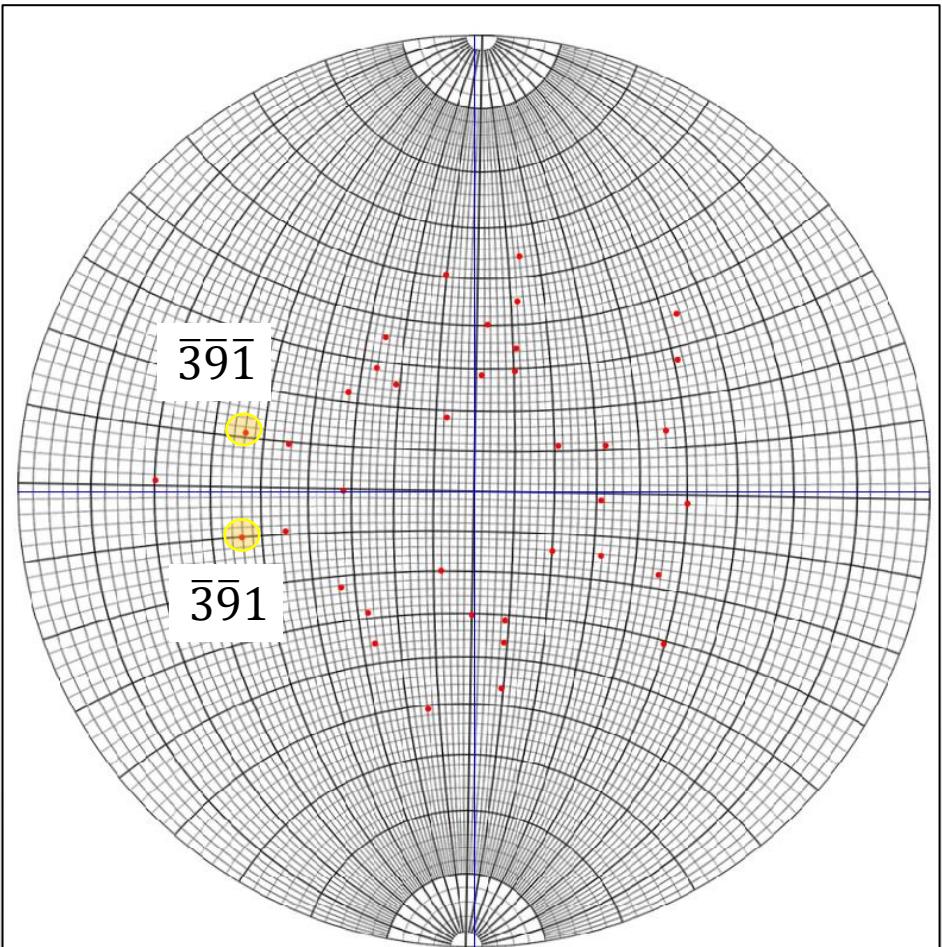
→ $(16.0 \pm 1.0)^\circ$



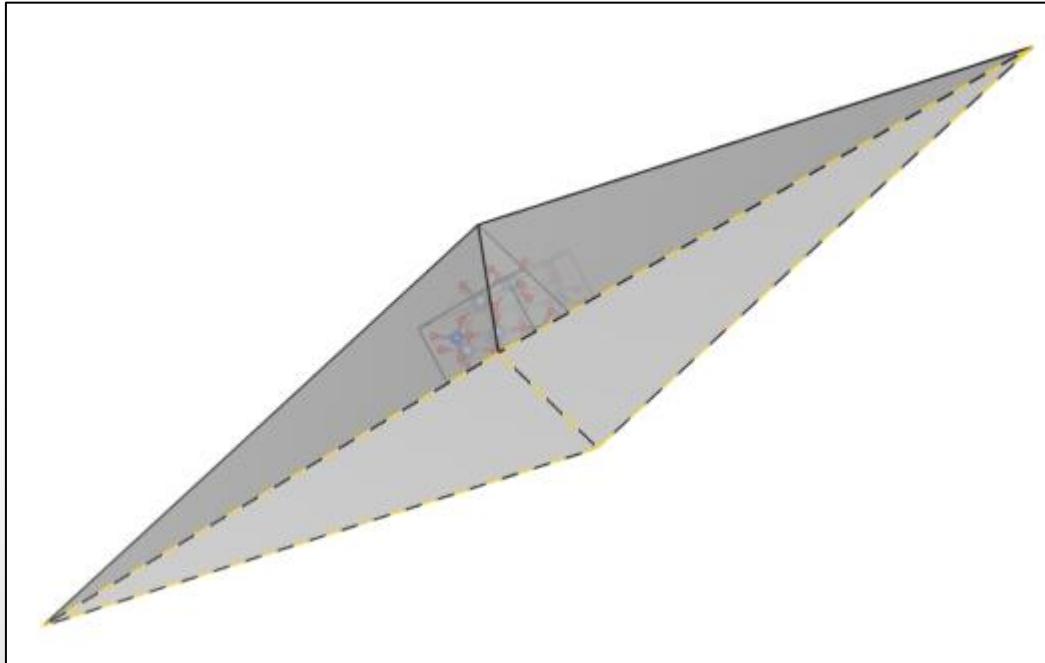
Angle between $(-3 -4 -1)$ and $(-3 -4 -3)$ = 15.9963°

9.2 Crystal of your choice

Topaz – angle between planes



→ $(10.5 \pm 1.0)^\circ$



Angle between (-3 -9 1) and (-3 -9 -1) = 11.4431°

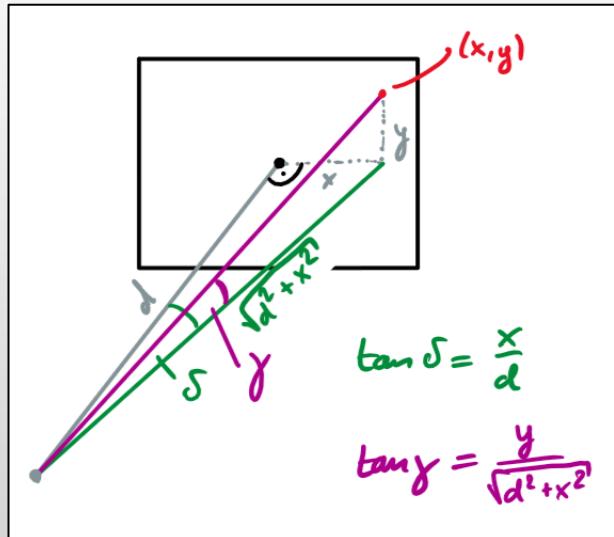
9.3 General tasks

Stereographic projection

- Projection: $(x, y) \rightarrow (\gamma, \delta)$
- Assume the coordinate centre to be $(x_0, y_0) = (0,0)$

$$\delta = \begin{cases} 0, & x = 0 \\ \arctan \frac{x}{d}, & x \geq 0 \end{cases}$$

$$\gamma = \arctan \frac{y}{\sqrt{d^2 + x^2}}$$



- Mathematica:

```
delta[X1_, Y1_] := If[X1 - X0 == 0, 0, ArcTan[X2[X1, Y1] / d] * 180 / Pi]
```

```
gamma[X1_, Y1_] := ArcTan[Y2[X1, Y1] / Sqrt[d^2 + (X2[X1, Y1])^2]] * 180 / Pi
```

- $X0, Y0$: coordinate center
- $X2, Y2$: functions that calculate the X and Y value with respect to the coordinate center and the respective quadrants

9.3 General tasks

Maximum hkl

- The energy of the electrons is

$$E_{e^-} = eU_a$$

- Assumption: The emission of the X-ray photons happens without energy loss
→ maximal energy of the photons

$$\frac{2\pi\hbar c}{\lambda_{min}} = E_X = E_{e^-} = eU_a$$

- With the well-known formula $k = \frac{2\pi}{\lambda}$ this results in

$$k_{max} = \frac{eU_a}{\hbar c}$$

- The formula for the lattice distance is

$$d_{hkl} = \frac{2\pi}{G}$$

with $G = |\vec{k} - \vec{k}_0|$ the

- Looking at the Ewald sphere one sees that

$$|\vec{k} - \vec{k}_0|_{max} \leq 2k_0 = 2k_{max}$$

- With the previous formulas this results in

$$d_{hkl} = \frac{2\pi}{G} \geq \frac{2\pi}{2k_{max}} = \frac{\pi\hbar c}{eU_a}$$

- For a simple cubic crystal

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

- This gives us the limiting condition

$$\boxed{\sqrt{h^2 + k^2 + l^2} \leq \frac{eU_a a}{\pi\hbar c}}$$

9.3 General tasks

Maximum hkl - Pyrite

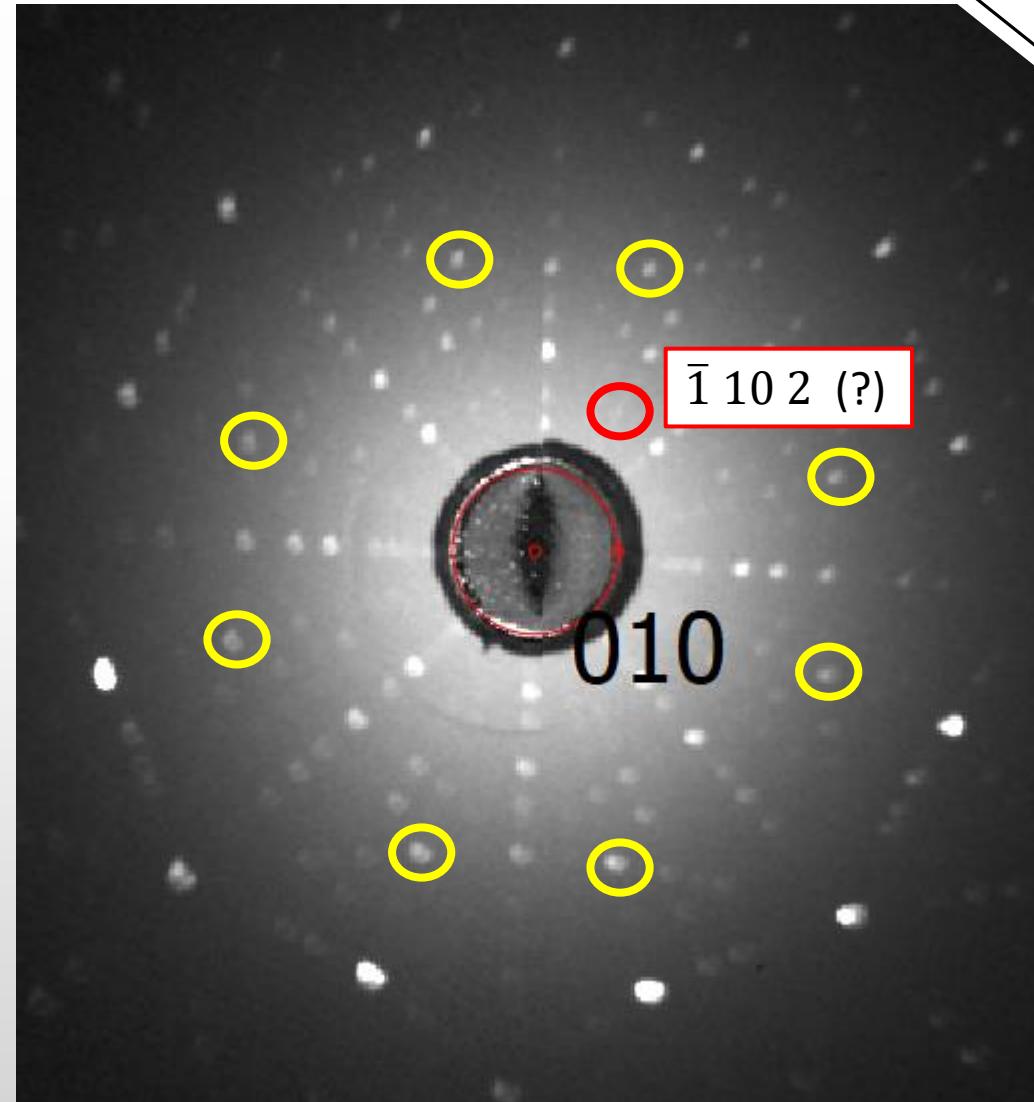
- Pyrite (sc) has the lattice constant $a = 5.4179\text{\AA}$
- Assuming that the images are taken at the optimal acceleration voltage $U_a = 30\text{kV}$ we get the limit

$$\sqrt{h^2 + k^2 + l^2} \leq 26.22$$

- The highest planes we clearly observed on Pyrite are the planes of the combinations 3,9,1

$$\sqrt{3^2 + 9^2 + 1^2} \approx 9.5 < 26.22$$

- Explanation:
higher planes
 \leftrightarrow smaller plane distances $d_{hkl} \sim \frac{1}{\sqrt{h^2+k^2+l^2}}$
 \rightarrow one needs smaller $\lambda \leftrightarrow$ higher energy $E \sim \frac{1}{\lambda}$
 But: These are not the planes that are chosen for the fits (this is not a problem)

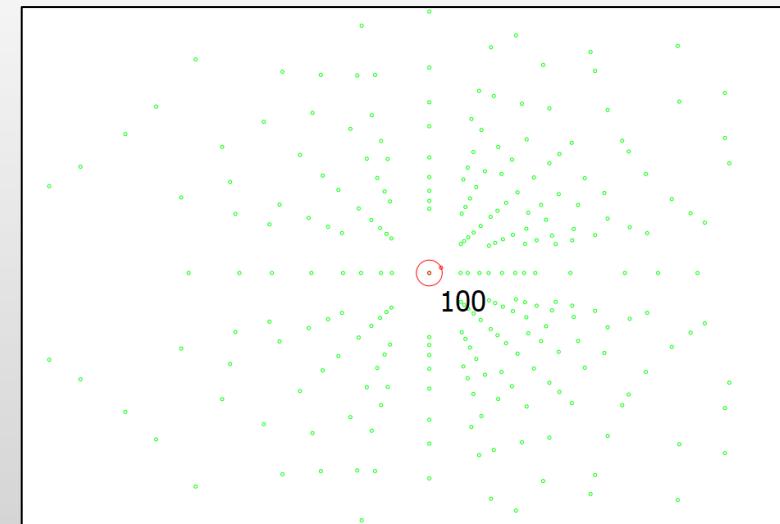
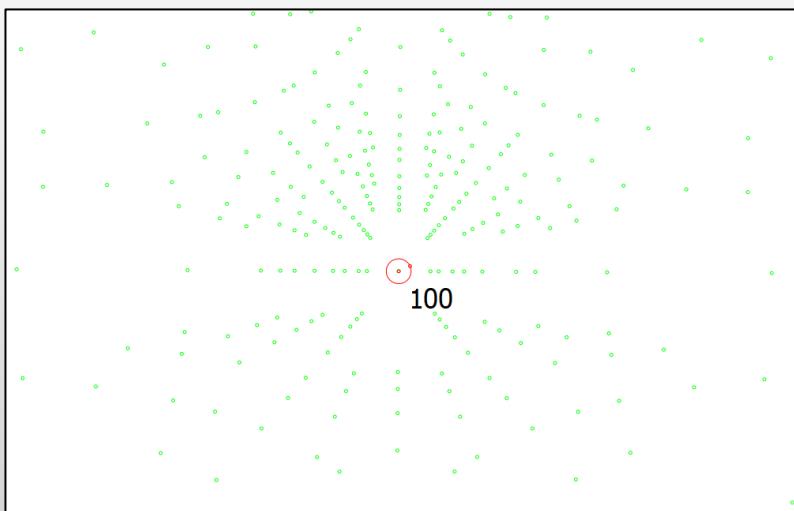
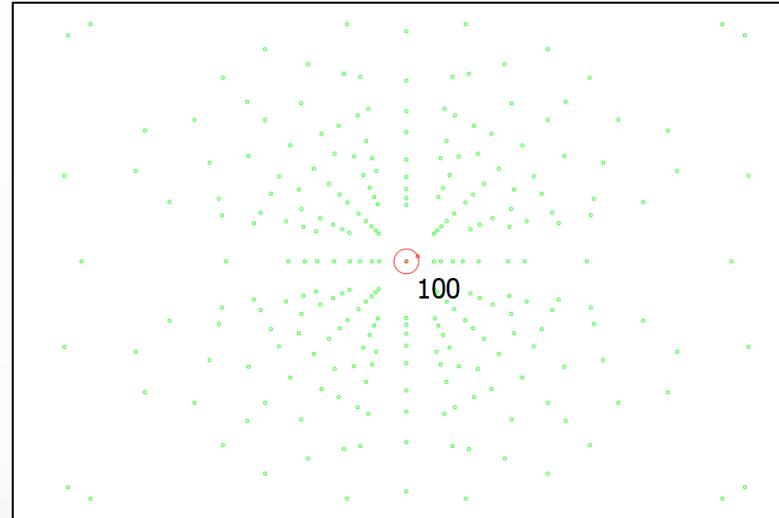


$$\sqrt{h^2 + k^2 + l^2} \leq \frac{eU_a a}{\pi \hbar c}$$

9.3 General tasks

Tilted detector – poorly collimated beam – variable sample-detector distance

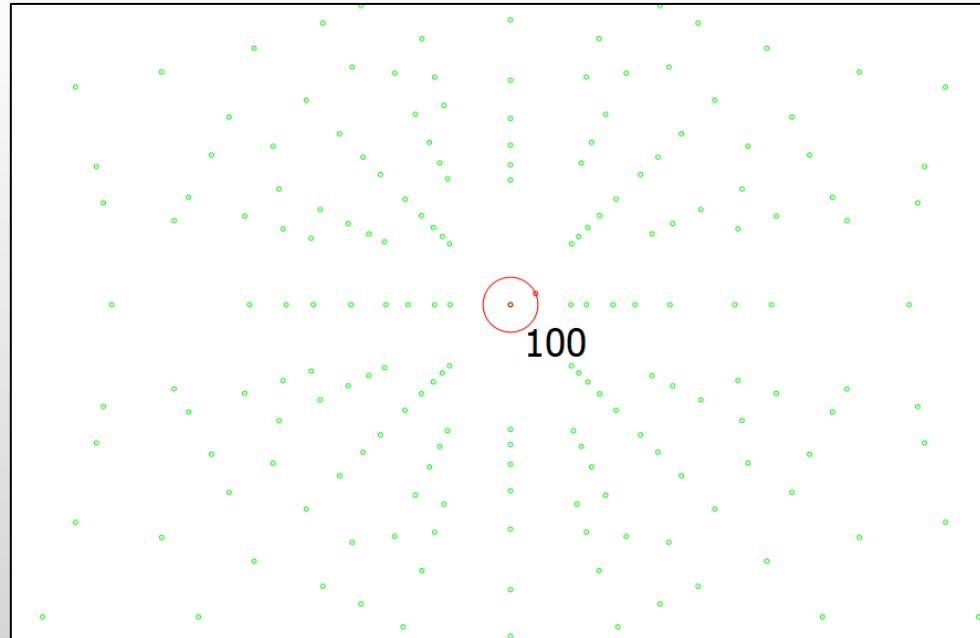
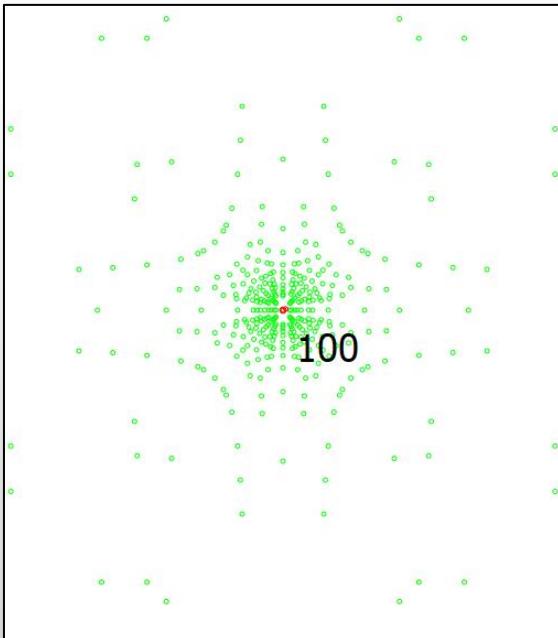
- Tilted detector:
 - “Distorted” images
 - Symmetry gets lost



9.3 General tasks

Tilted detector – poorly collimated beam – variable sample-detector distance

- Poorly collimated beam:
 - the angle of incidence is not fixed
 - the spots get washed out
- Variable sample-detector distance:
 - Larger distance → larger spot distance



9.3 General tasks

Experimental errors

- Previously discussed: poorly collimated beam
- Wrong settings in CLIP (can be seen if angle of planes is totally wrong)
- Pixel-accuracy
- Accuracy of the Wulff-net
- Staining on the crystal surface