Monte Carlo Project

PRICING CALL OPTIONS WITH GBM SIMULATION AND COMPARISON WITH THE BLACK-SCHOLES

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OVERVIEW

- Background
- Code
- Results
- Next Steps

BACKGROUND: CALL OPTIONS

- Call options are contracts
- Gives buyers the right to purchase an asset at an agreed upon (strike) price after some time
- Market Price - Strike Price = Profit
- Allows people to leverage less capital for more gains
- Options themselves can be traded too before expiration

BACKGROUND: BLACK-SCHOLES

- The Black-Scholes is a PDE governing the price movements of derivative contracts
- Crucial for pricing call options
    - *V(S, t)* = option price as a function of asset price S and time t
    - *r* = risk-free interest rate
    - σ = historical volatility of stock

BLACK-SCHOLES EQUATION

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

BLACK-SCHOLES SOLUTION

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

BACKGROUND: BLACK-SCHOLES

- C is the analytical solutions to the Black-Scholes for call options
  - K = strike price
  - N = normal cdf
  - T = time to expiration
- First term: expected value of stock price
- Second term: expected value of strike price discounted by interest rate

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BACKGROUND: GBM

- GBM is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows brownian motion with drift
  - S = stock price at time t.
  - μ = drift rate of growth
  - σ = volatility of stock
  - W = Wiener process (or standard Brownian motion)
  - Z = standard normal rv

GEOMETRIC BROWNIAN MOTION MODEL (GBM)

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

GEOMETRIC BROWNIAN MOTION SOLUTION

$$S_t = S_0 \exp((\mu - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}Z)$$

CODE: MONTE-CARLO SIMULATION OF GBM

```python
weeks = 52
dt = T / weeks  # Time step
S = np.zeros((weeks + 1, N))  # Matrix to store price paths
S[0] = S0

# Generate the paths
for t in range(1, weeks + 1):
    # Apply Geometric Brownian Motion SDE
    Z = np.random.normal(size=N)
    S[t] = S[t - 1] * np.exp((r - 0.5 * sigma**2) * dt + sigma * np.sqrt(dt) * Z)
```

```python
# Initial Model Parameters
S0 = 100  # Initial spot price
r = 0.05  # Risk-free interest rate
sigma = 0.25  # Volatility of returns
K = 105  # Strike price
T = 1.0  # Time to maturity
```

$$S_t = S_0 \exp((\mu - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}Z)$$

CODE: BLACK-SCHOLES CALCULATION

```
def black_scholes(S0, K, T, r, sigma, option_type="call"):
    d1 = (np.log(S0 / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    if option_type == "call":
        price = (S0 * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2))
    else:
        price = (K * np.exp(-r * T) * norm.cdf(-d2) - S0 * norm.cdf(-d1))
    return price
```

```
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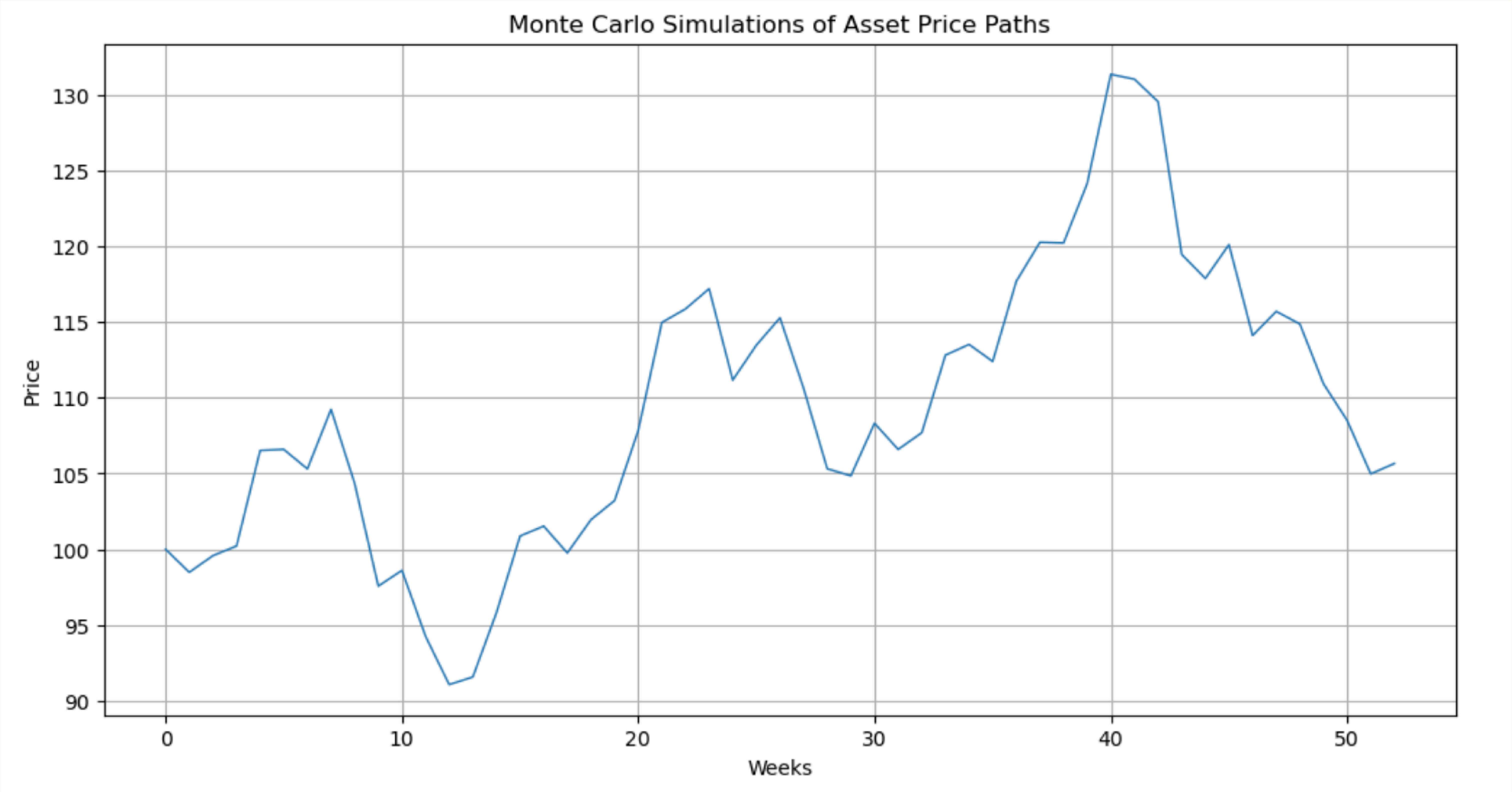
RESULTS : MONTE-CARLO



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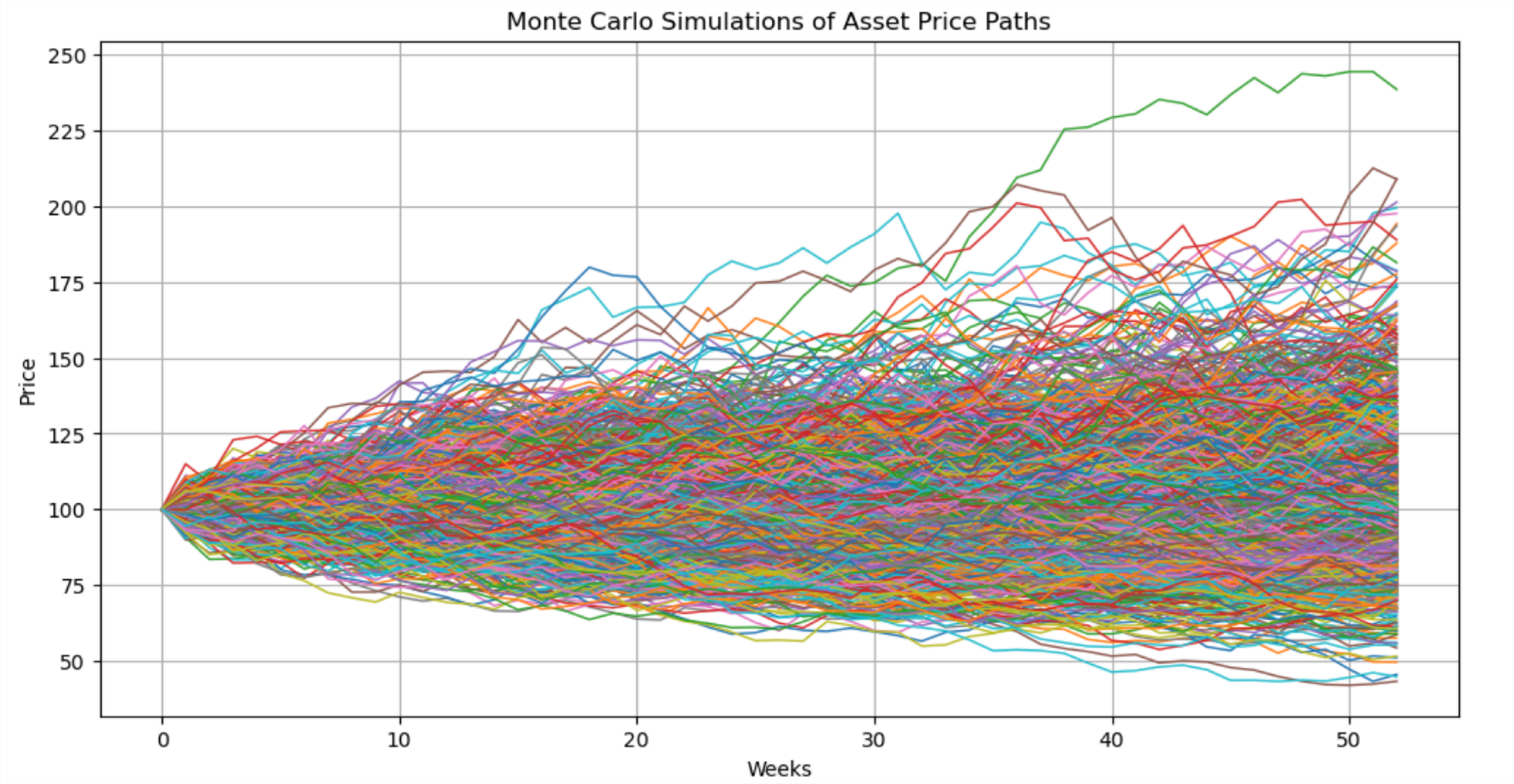

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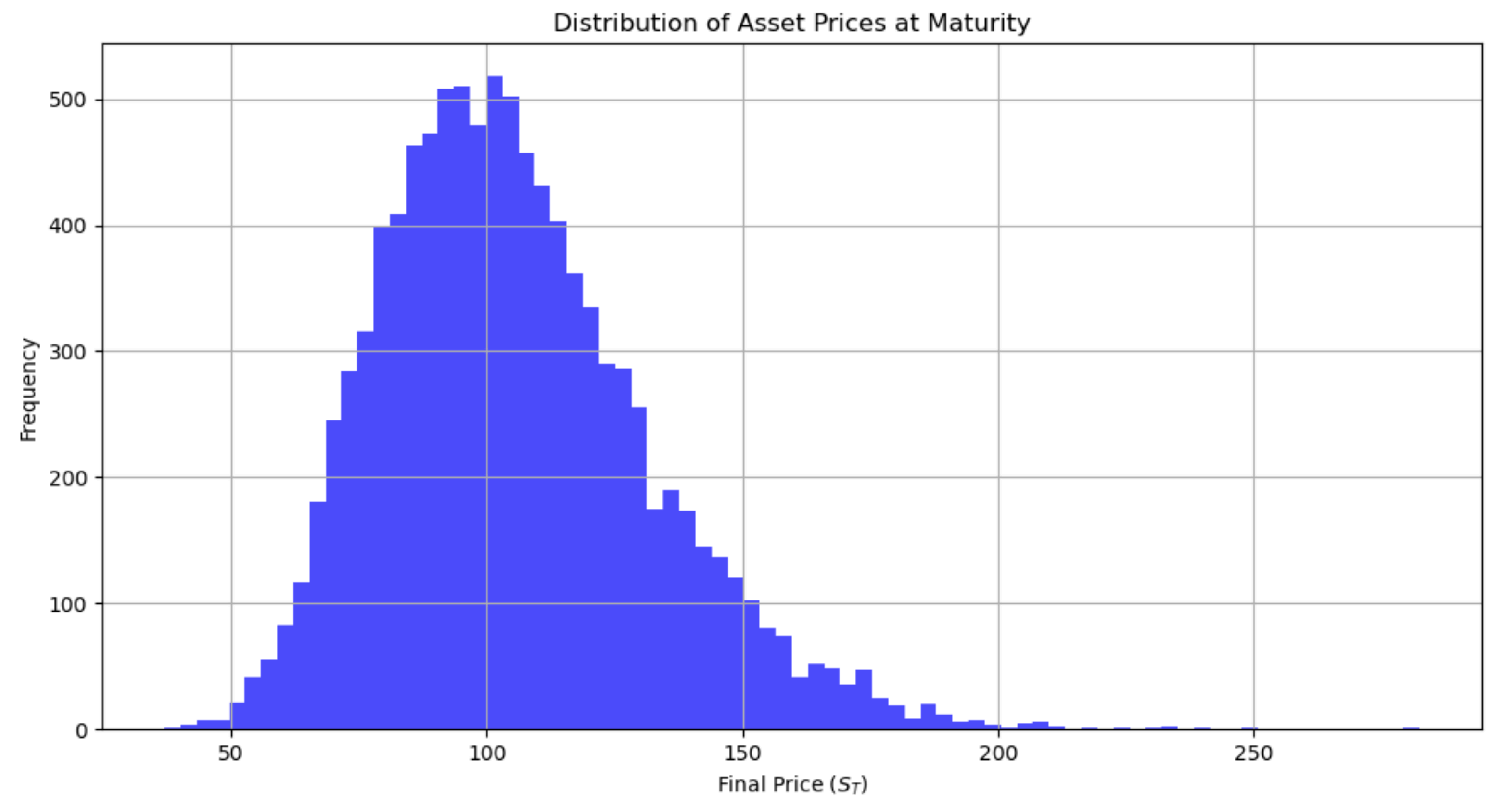
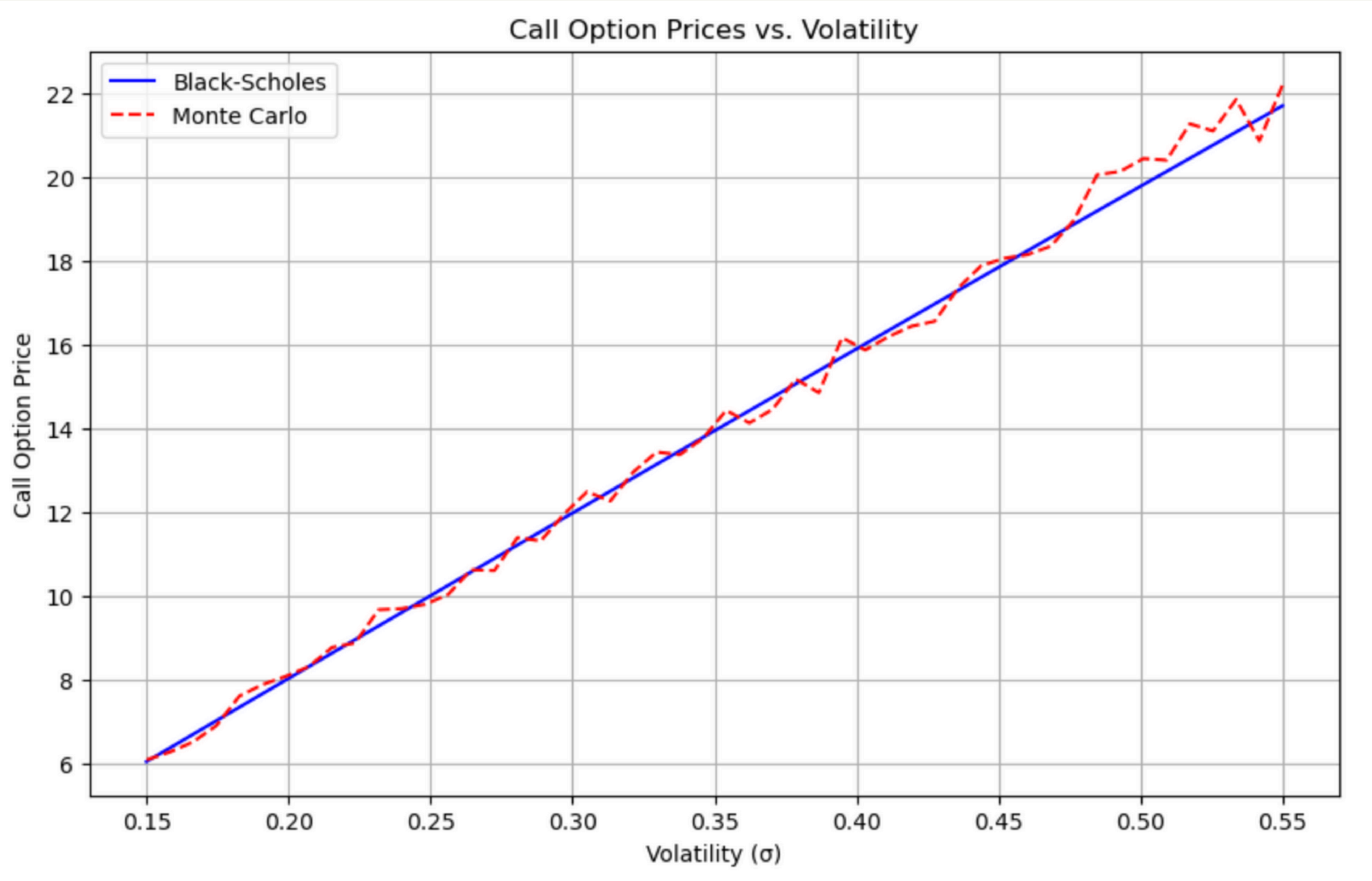


RESULTS : COMPARISON WITH BLACK-SCHOLES



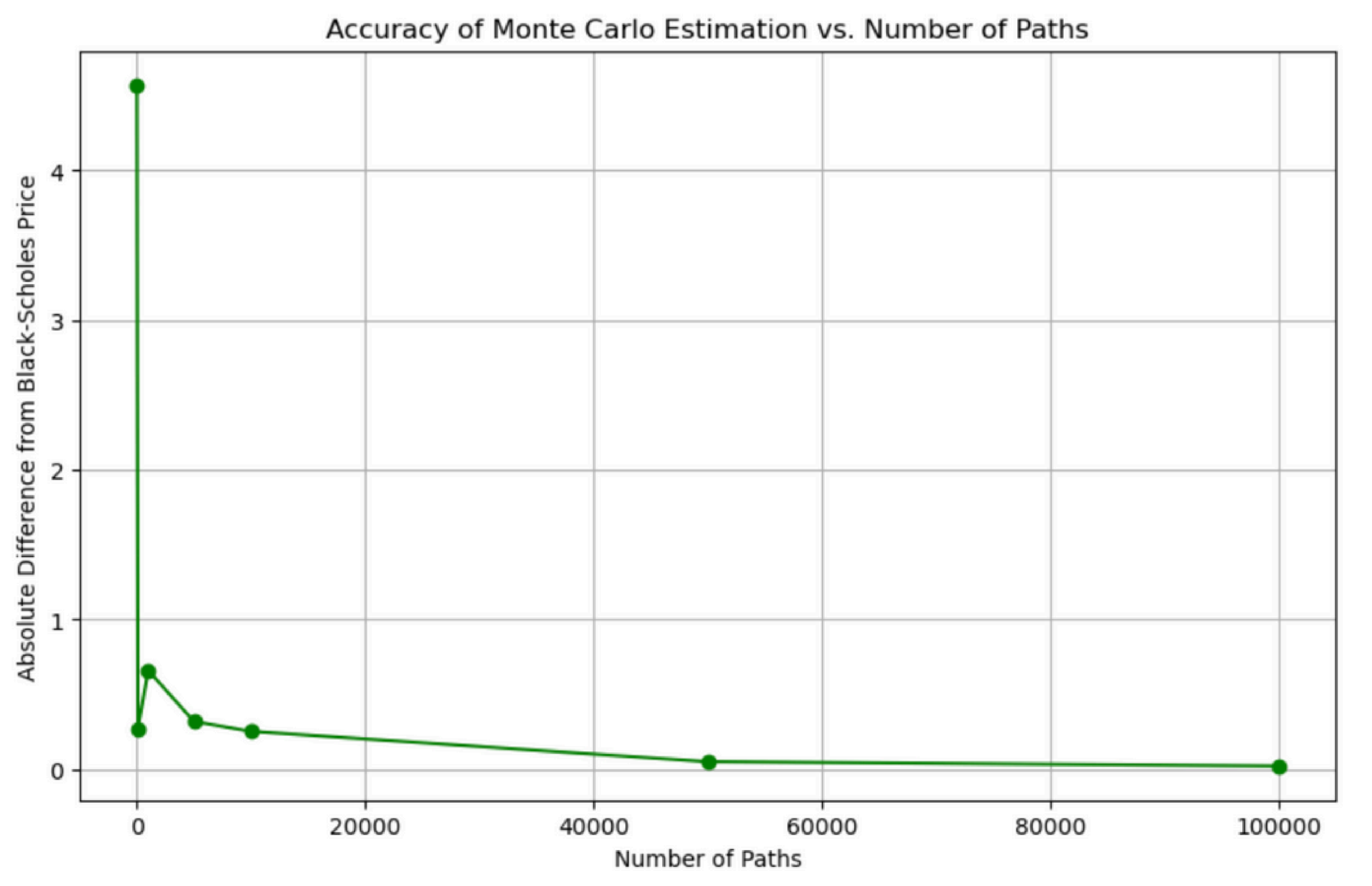
Volatility Level: 0.15
Black-Scholes Call Price: 6.04
Monte Carlo Call Price: 6.01

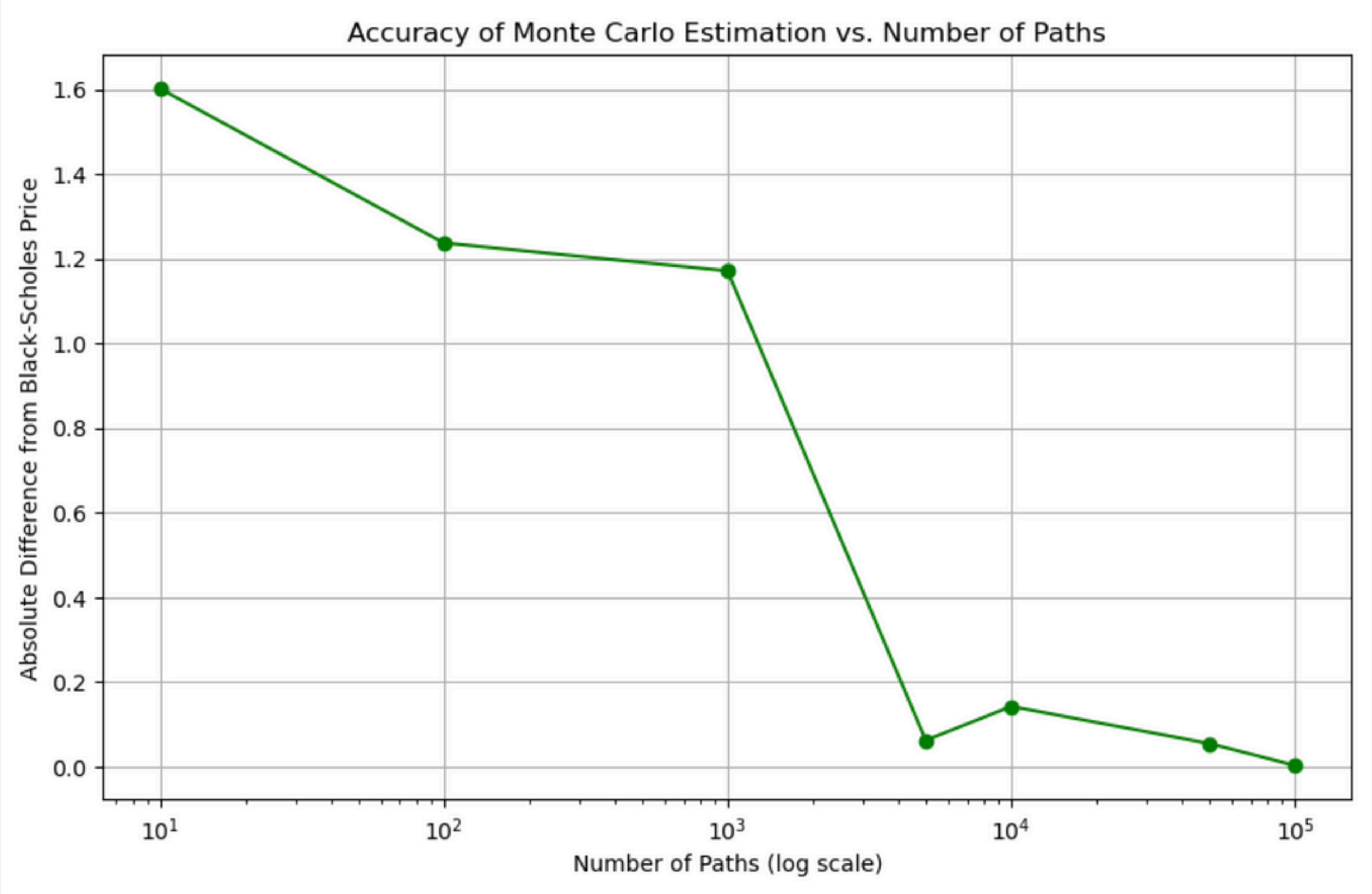
Volatility Level: 0.35
Black-Scholes Call Price: 13.94
Monte Carlo Call Price: 14.06

Volatility Level: 0.25
Black-Scholes Call Price: 10.00
Monte Carlo Call Price: 10.26

Volatility Level: 0.45
Black-Scholes Call Price: 17.85
Monte Carlo Call Price: 17.87

RESULTS : VARYING NUMBER OF PRICE PATHS





NEXT STEPS

- Account dividends paid by the underlying asset
- Explore volatility that changes as a function of time as oppose to static, such as the Heston model

Thank you!

REFERENCES

- *Chen, Kinder*. "Black-Scholes Model and Monte Carlo Simulation." Medium.
- *Jiang, Qiwu*. 2019. "Comparison of Black–Scholes Model and Monte-Carlo Simulation on Stock Price Modeling." In Advances in Economics, Business and Management Research, volume 109, 135-137. International Conference on Economic Management and Cultural Industry (ICEMCI 2019). Atlantis Press.
- *Meding, Isak, and Viking Zandhoff Westerlund*. 2021. "Pricing European Options with the Black-Scholes and Monte Carlo Methods: A Comparative Study." Bachelor’s thesis, University of Gothenburg, School of Business, Economics and Law, Department of Business Administration.