

CS2100

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COMPUTER ORGANISATION

## Lecture #17

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# Combinational Circuits



**NUS**  
National University  
of Singapore

School of  
Computing

# Lecture #17: Combinational Circuits

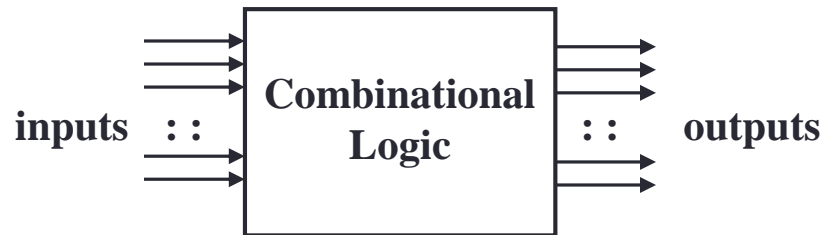
1. Introduction
2. Analysis Procedure
3. Design Methods
4. Gate-Level (SSI) Design
5. Block-Level Design
6. Summary of Arithmetic Circuits
7. Example: 6-Person Voting System
8. Magnitude Comparator
9. Circuit Delays

# 1. Introduction

- Two classes of logic circuits
  - Combinational
  - Sequential

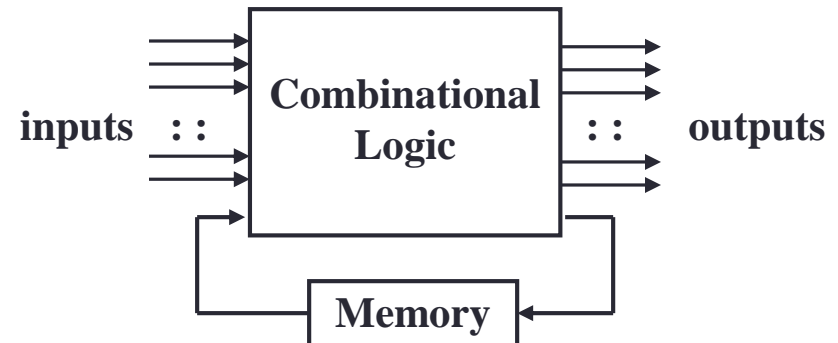
- **Combinational Circuit**

- Each output depends entirely on the immediate (present) inputs.



- **Sequential Circuit**

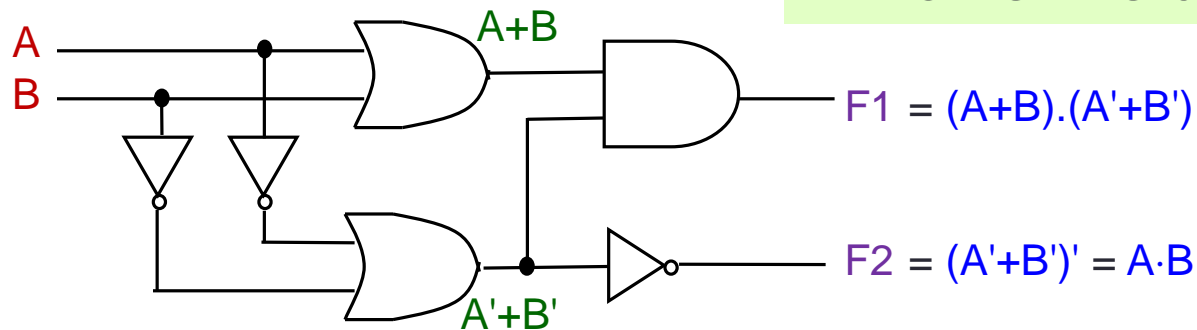
- Each output depends on both present inputs and state.



## 2. Analysis Procedure

- Given a combinational circuit, how do you analyze its function?

What is this circuit?



- Steps:
  1. Label the inputs and outputs.
  2. Obtain the functions of intermediate points and the outputs.
  3. Draw the truth table.
  4. Deduce the functionality of the circuit ➡ **Half adder.**

A	B	(A+B)	(A'+B')	F1	F2
0	0	0	1	0	0
0	1	1	1	1	0
1	0	1	1	1	0
1	1	1	0	0	1

# 3. Design Methods

- Different combinational circuit design methods:
  - Gate-level design method (with logic gates)
  - Block-level design method (with functional blocks)
- Design methods make use of logic gates and useful function blocks
  - These are available as Integrated Circuit (IC) chips.
  - Types of IC chips (based on packing density): SSI, MSI, LSI, VLSI, ULSI.
- Main objectives of circuit design:
  - Reduce cost (number of gates for small circuits; number of IC packages for complex circuits)
  - Increase speed
  - Design simplicity (re-use blocks where possible)

## 4. Gate-Level (SSI) Design: Half Adder (1/2)

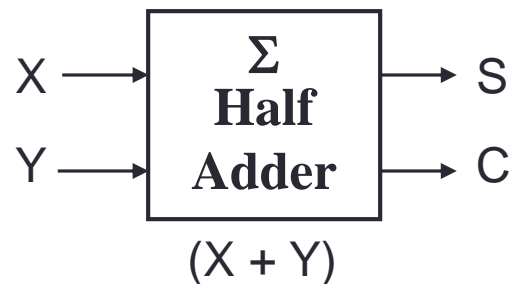
- Design procedure:

1. State problem

Example: Build a **Half Adder**.

2. Determine and label the inputs and outputs of circuit.

Example: Two inputs and two outputs labelled, as shown below.



X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

3. Draw the truth table.

## 4. Gate-Level (SSI) Design: Half Adder (2/2)

4. Obtain simplified Boolean functions.

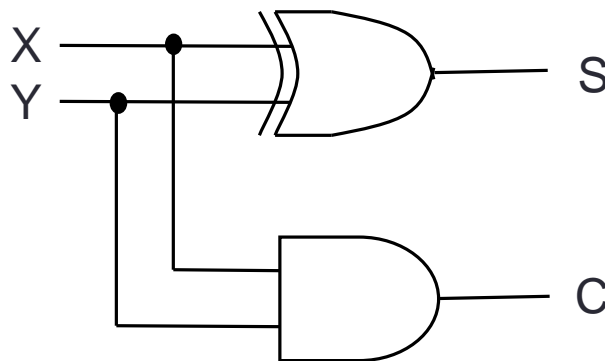
Example:  $C = X \cdot Y$

$$S = X' \cdot Y + X \cdot Y' = X \oplus Y$$

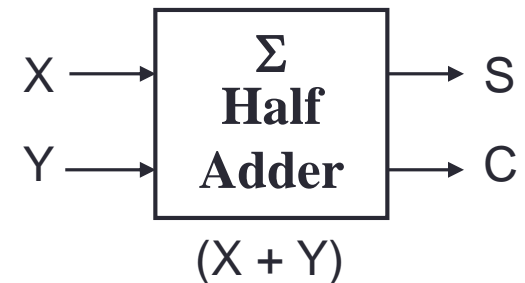
5. Draw the logic diagram.

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

### Half Adder



Block diagram  
of Half Adder



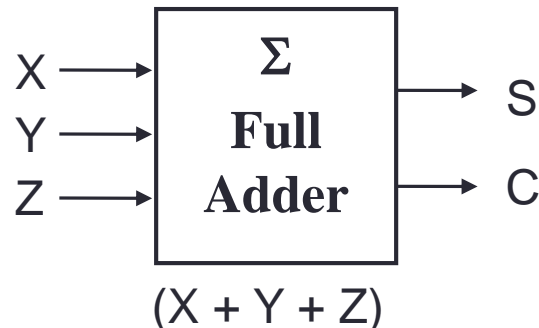
## 4. Gate-Level (SSI) Design: Full Adder (1/5)

- Half adder adds up only two bits.
- To add two binary numbers, we need to add 3 bits (including the *carry*).

- Example:

	1	1	1		carry
	0	0	1	1	X
+	0	1	1	1	Y
<hr/>					
	1	0	1	0	S
<hr/>					

- Need **Full Adder** (so called as it can be made from two half adders).





## 4. Gate-Level (SSI) Design: Full Adder (2/5)

- Truth table:

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Note:

Z - carry in (to the current position)

C - carry out (to the next position)

C

	YZ	00	01	11	10
X					
0		0	0	1	0
1		0	1	1	1

- Using K-map, simplified SOP form:

$$C = X \cdot Y + X \cdot Z + Y \cdot Z$$

$$S = X' \cdot Y' \cdot Z + X' \cdot Y \cdot Z' + X \cdot Y' \cdot Z' + X \cdot Y \cdot Z$$

S

	YZ	00	01	11	10
X					
0		0	1	0	1
1		1	0	1	0

## 4. Gate-Level (SSI) Design: Full Adder (3/5)

- Alternative formulae using algebraic manipulation:

$$\begin{aligned}C &= X \cdot Y + X \cdot Z + Y \cdot Z \\&= X \cdot Y + (X + Y) \cdot Z \\&= X \cdot Y + ((X \oplus Y) + X \cdot Y) \cdot Z \\&= X \cdot Y + (X \oplus Y) \cdot Z + X \cdot Y \cdot Z \\&= \mathbf{X \cdot Y + (X \oplus Y) \cdot Z}\end{aligned}$$

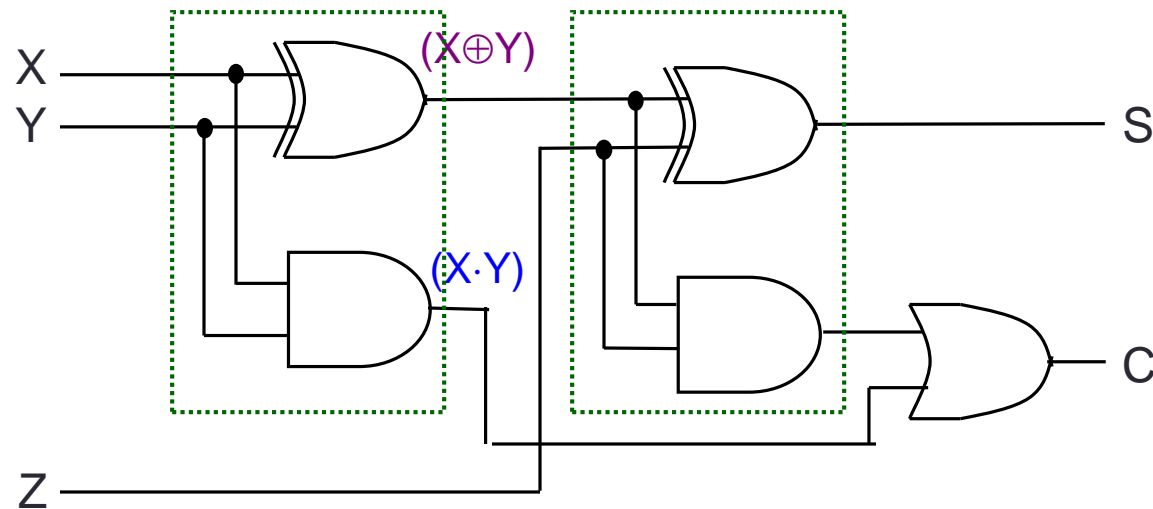
$$\begin{aligned}S &= X' \cdot Y' \cdot Z + X' \cdot Y \cdot Z' + X \cdot Y' \cdot Z' + X \cdot Y \cdot Z \\&= X' \cdot (Y' \cdot Z + Y \cdot Z') + X \cdot (Y' \cdot Z' + Y \cdot Z) \\&= X' \cdot (Y \oplus Z) + X \cdot (Y \oplus Z)' \\&= \mathbf{X \oplus (Y \oplus Z)}\end{aligned}$$

## 4. Gate-Level (SSI) Design: Full Adder (4/5)

- Circuit for above formulae:

$$C = X \cdot Y + (X \oplus Y) \cdot Z$$

$$S = X \oplus (Y \oplus Z) = (X \oplus Y) \oplus Z \text{ (XOR is associative)}$$



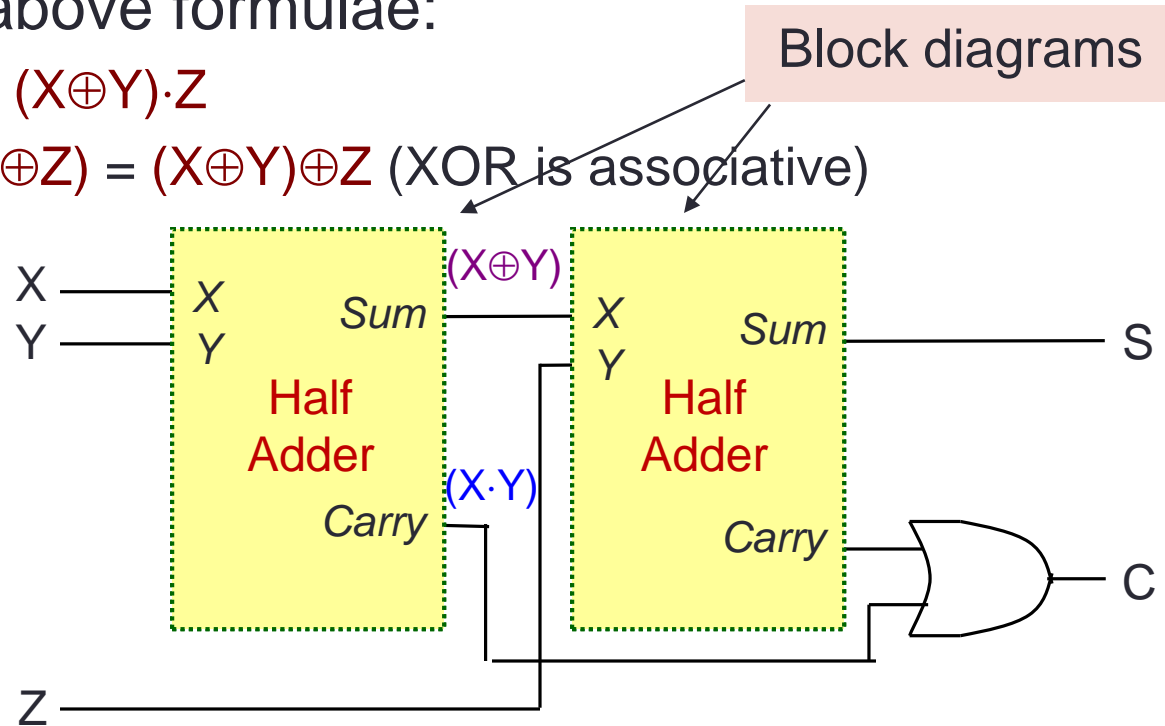
Full Adder made from two Half-Adders (+ an OR gate).

## 4. Gate-Level (SSI) Design: Full Adder (5/5)

- Circuit for above formulae:

$$C = X \cdot Y + (X \oplus Y) \cdot Z$$

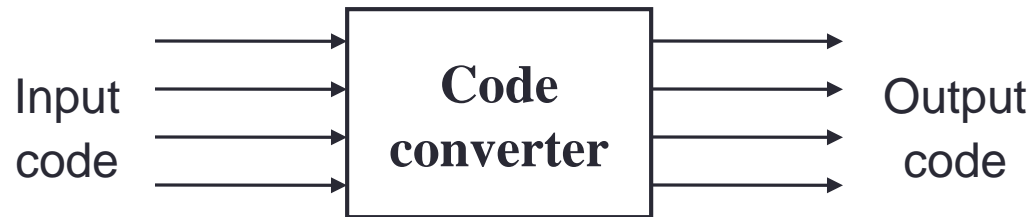
$$S = X \oplus (Y \oplus Z) = (X \oplus Y) \oplus Z \text{ (XOR is associative)}$$



Full Adder made from two Half-Adders (+ an OR gate).

## 4. Gate-Level (SSI) Design: Code Converters

- **Code converter** – takes an input code, translates to its equivalent output code.



- Example: **BCD to Excess-3 code converter**.
  - Input: BCD code
  - Output: Excess-3 code

## 4. BCD to Excess-3 Code Converter (1/3)

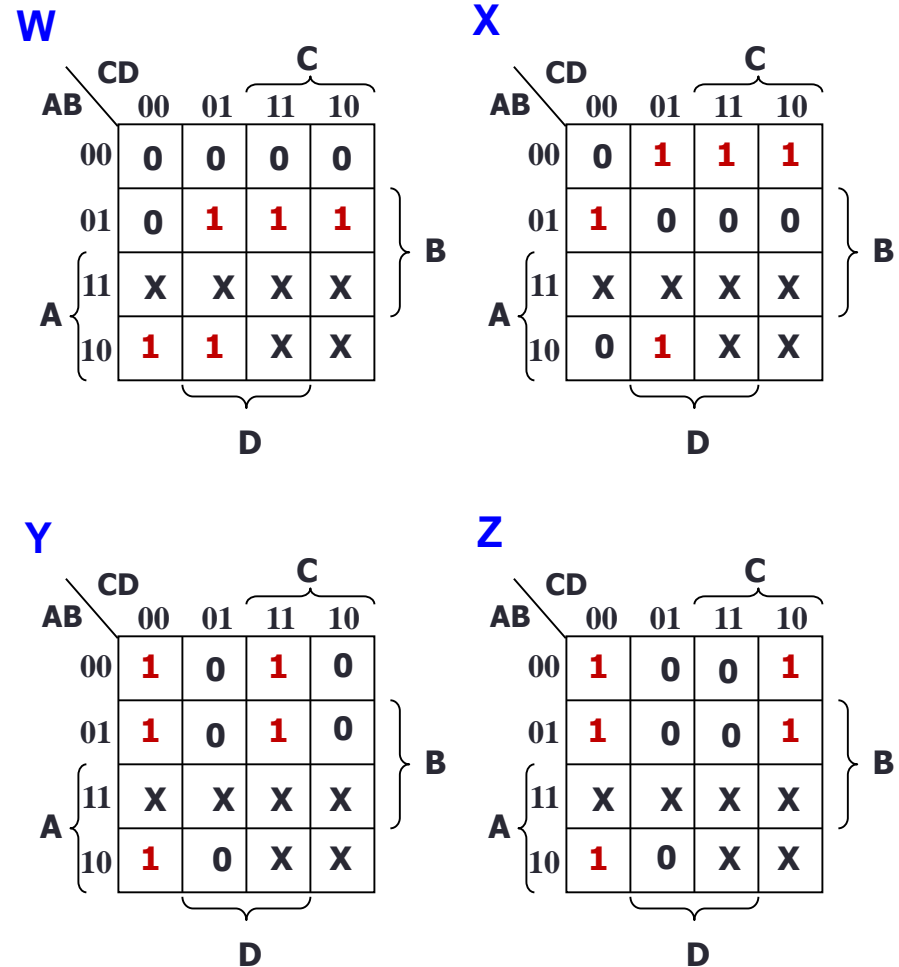
Digit	BCD code	Excess-3 code
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

# 4. BCD to Excess-3 Code Converter (2/3)

■ Truth table:

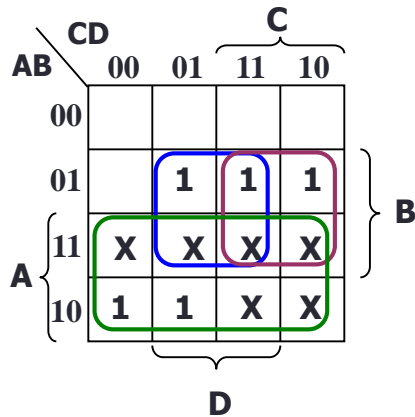
	BCD				Excess-3			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0
10	1	0	1	0	X	X	X	X
11	1	0	1	1	X	X	X	X
12	1	1	0	0	X	X	X	X
13	1	1	0	1	X	X	X	X
14	1	1	1	0	X	X	X	X
15	1	1	1	1	X	X	X	X

■ K-maps:

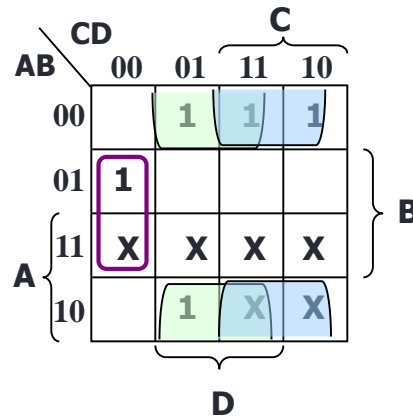


# 4. BCD to Excess-3 Code Converter (3/3)

W



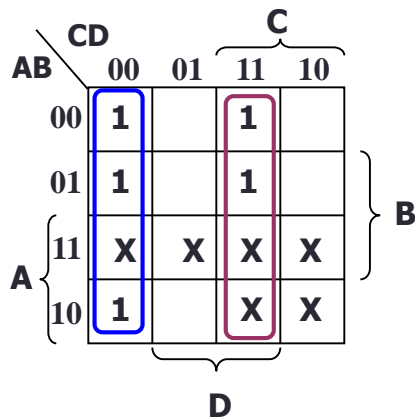
X



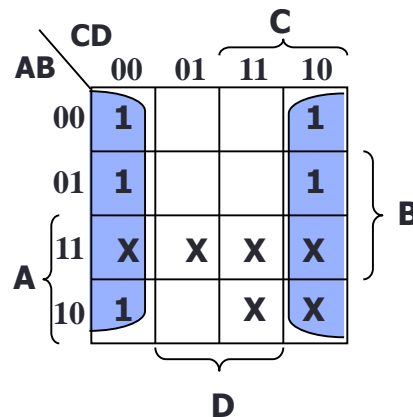
$$W = A + B \cdot C + B \cdot D$$

$$X = B' \cdot C + B' \cdot D + B \cdot C' \cdot D'$$

Y



Z



$$Y = C \cdot D + C' \cdot D'$$

$$Z = D'$$

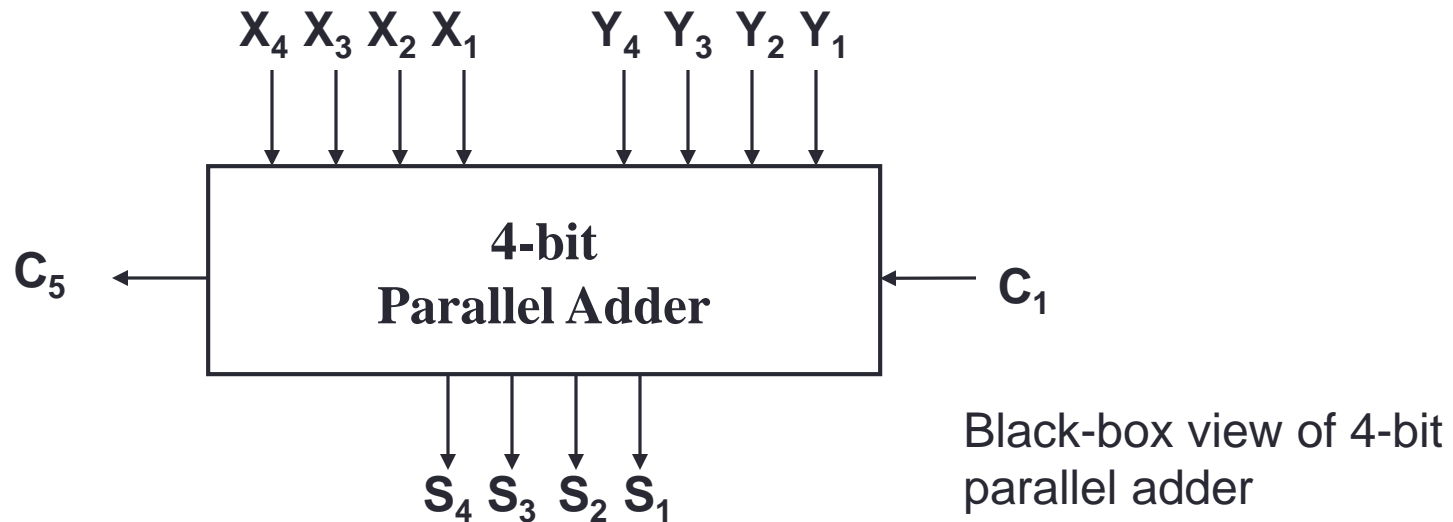


## 5. Block-Level Design

- More complex circuits can also be built using **block-level** method.
- In general, block-level design method (as opposed to gate-level design) relies on algorithms or formulae of the circuit, which are obtained by decomposing the main problem to sub-problems recursively (until small enough to be directly solved by blocks of circuits).
- First example shows how to create a 4-bit parallel adder using block-level design.
- Using **4-bit parallel adders** as building blocks, we can create the following:
  1. **BCD-to-Excess-3 Code Converter**
  2. **16-bit Parallel Adder**

## 5. 4-bit Parallel Adder (1/4)

- Consider a circuit to add two 4-bit numbers together and a carry-in, to produce a 5-bit result.



- 5-bit result is sufficient because the largest result is:  
 $1111_2 + 1111_2 + 1_2 = 11111_2$

## 5. 4-bit Parallel Adder (2/4)

- SSI design (gate-level design) technique should not be used here.
- Truth table for 9 inputs is too big:  $2^9 = 512$  rows!

$X_4X_3X_2X_1$	$Y_4Y_3Y_2Y_1$	$C_1$	$C_5$	$S_4S_3S_2S_1$
0 0 0 0	0 0 0 0	0	0	0 0 0 0
0 0 0 0	0 0 0 0	1	0	0 0 0 1
0 0 0 0	0 0 0 1	0	0	0 0 0 1
...	...	...	...	...
0 1 0 1	1 1 0 1	1	1	0 0 1 1
...	...	...	...	...
1 1 1 1	1 1 1 1	1	1	1 1 1 1

- Simplification becomes too complicated!

## 5. 4-bit Parallel Adder (3/4)

- Alternative design possible.
- Addition formula for each pair of bits (with carry in),

$$C_{i+1}S_i = X_i + Y_i + C_i$$

has the same function as a full adder:

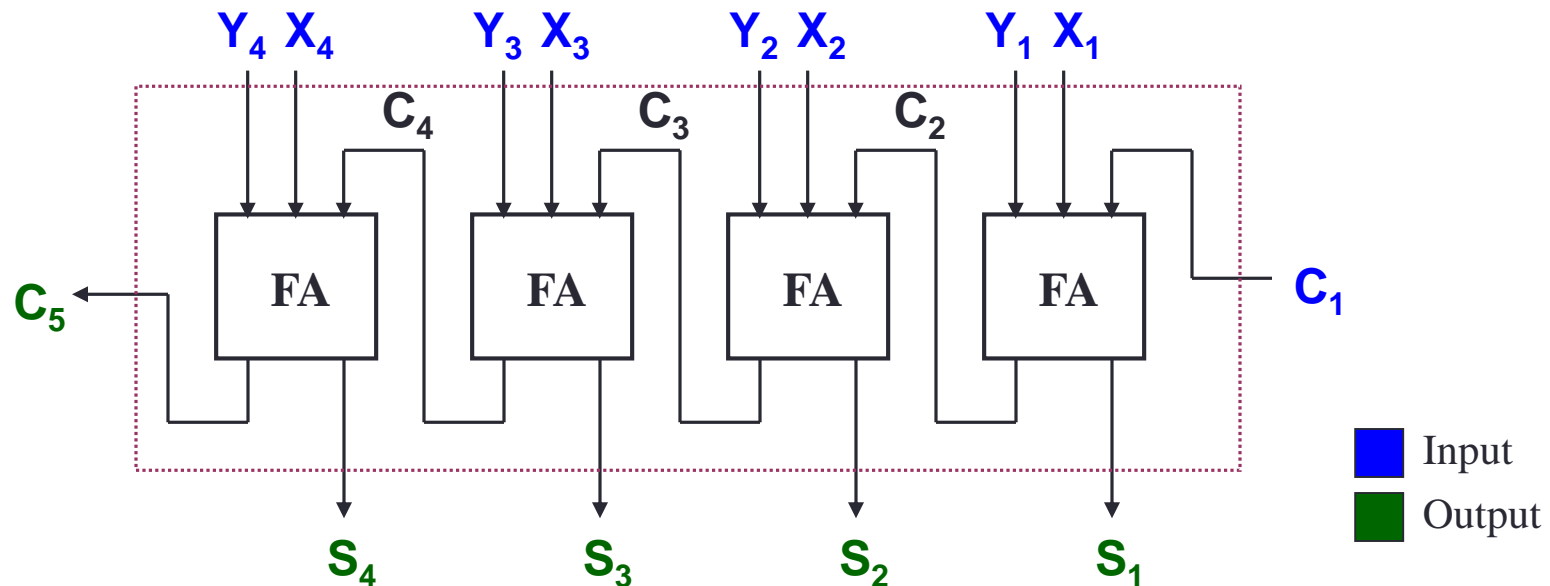
$$C_{i+1} = X_i \cdot Y_i + (X_i \oplus Y_i) \cdot C_i$$

$$S_i = X_i \oplus Y_i \oplus C_i$$

$$\begin{array}{rcl} C = & & 1\ 1\ 0\ 0 \\ X = & & 1\ 0\ 1\ 0 \\ Y = & & 1\ 1\ 1\ 1 \\ X + Y = & 1\ 1\ 0\ 0\ 1 \end{array}$$

## 5. 4-bit Parallel Adder (4/4)

- Cascading 4 full adders via their carries, we get:



- Note that carry is propagated by cascading the carry from one full adder to the next.
- Called **Parallel Adder** because inputs are presented simultaneously (in parallel). Also called **Ripple-Carry Adder**.

## 5. BCD to Excess-3 Converter: Revisit (1/2)

- Excess-3 code can be converted from BCD code using truth table:
- Gate-level design can be used since only 4 inputs.
- However, alternative design is possible.
- Use problem-specific formula:

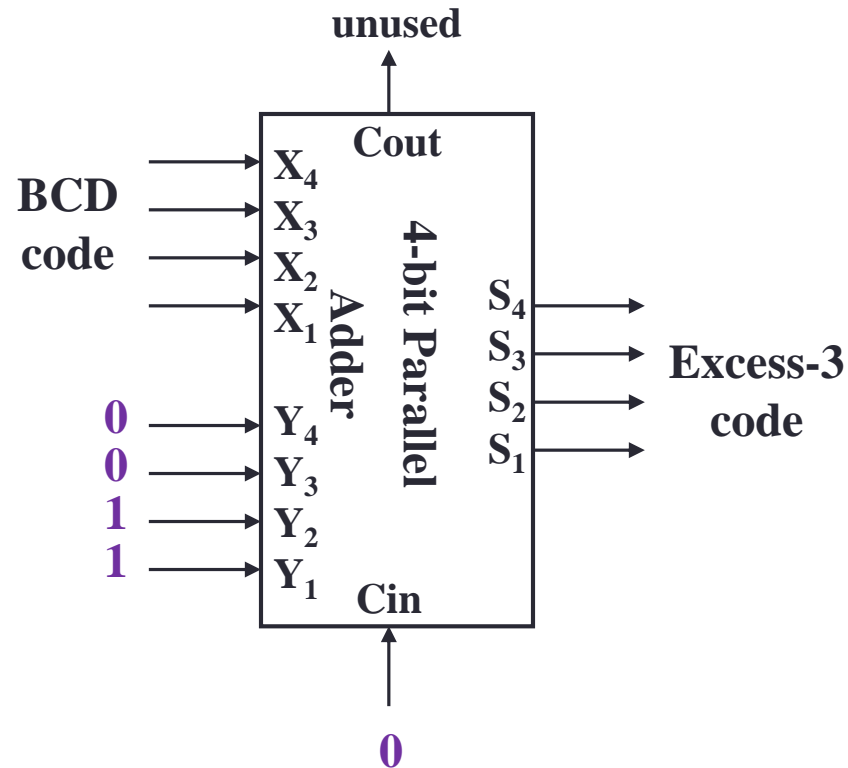
$$\begin{aligned} &\text{Excess-3 code} \\ &= \text{BCD Code} + 0011_2 \end{aligned}$$

	BCD				Excess-3			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0
10	1	0	1	0	X	X	X	X
11	1	0	1	1	X	X	X	X
12	1	1	0	0	X	X	X	X
13	1	1	0	1	X	X	X	X
14	1	1	1	0	X	X	X	X
15	1	1	1	1	X	X	X	X

## 5. BCD to Excess-3 Converter: Revisit (2/2)

- Block-level circuit:

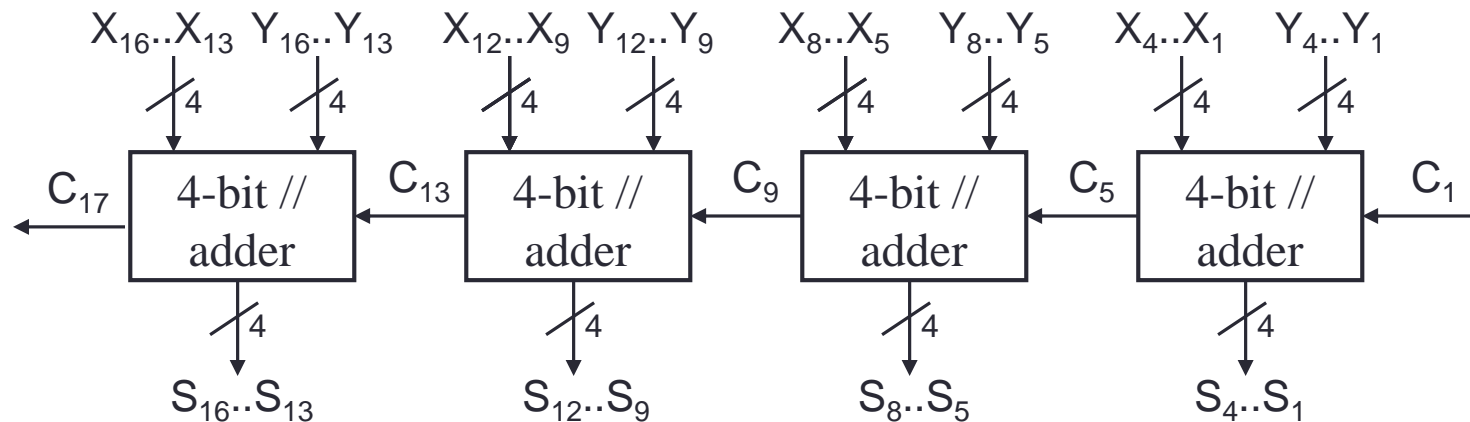
**A BCD to Excess-3  
Code Converter**



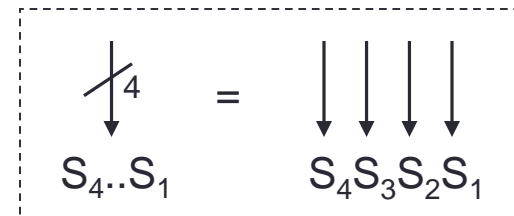
Note: In the lab, input 0 (low) is connected to GND, 1 (high) to Vcc.

## 5. 16-bit Parallel Adder

- Larger parallel adders can be built from smaller ones.
- Example: A **16-bit parallel adder** can be constructed from four 4-bit parallel adders:



**A 16-bit parallel adder**

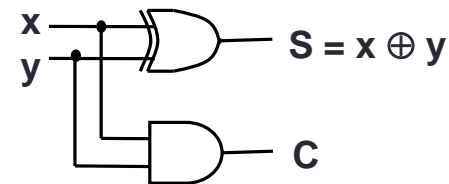
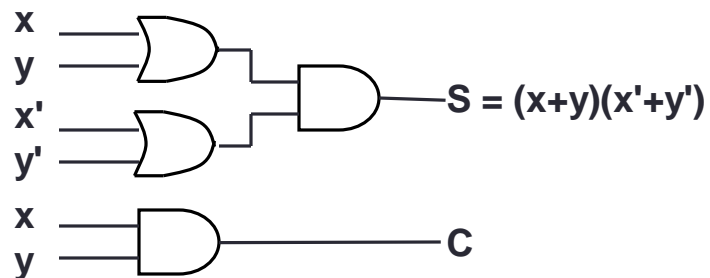
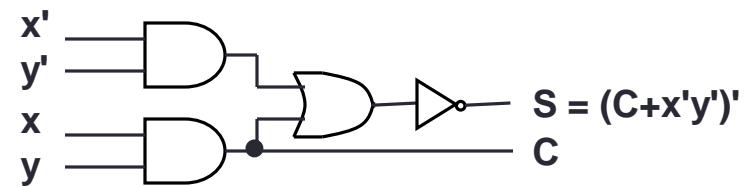
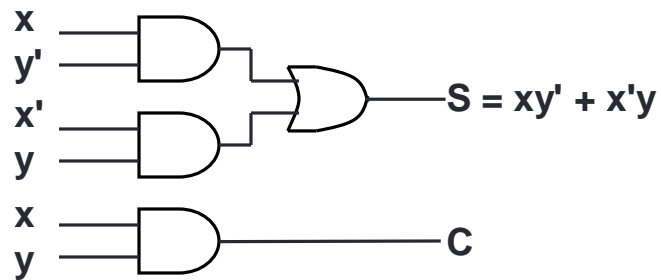
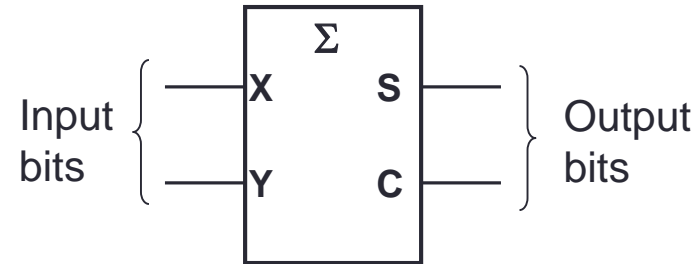




# 6. Summary of Arithmetic Circuits (1/4)

## ■ Half adder

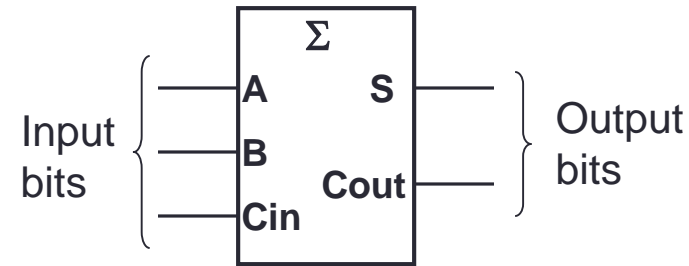
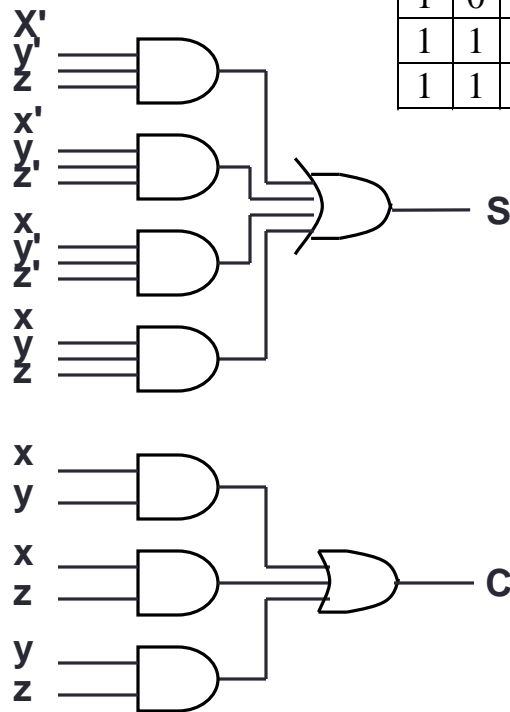
x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



# 6. Summary of Arithmetic Circuits (2/4)

## Full adder

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

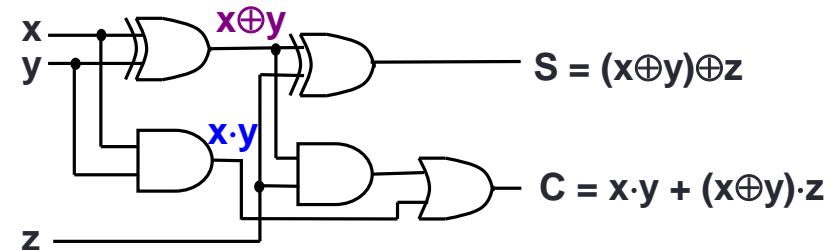


yz \ x	00	01	11	10
0			1	
1		1	1	1

$$C = xy + xz + yz$$

yz \ x	00	01	11	10
0		1		1
1	1		1	

$$S = x'y'z + x'yz' + xy'z' + xyz$$



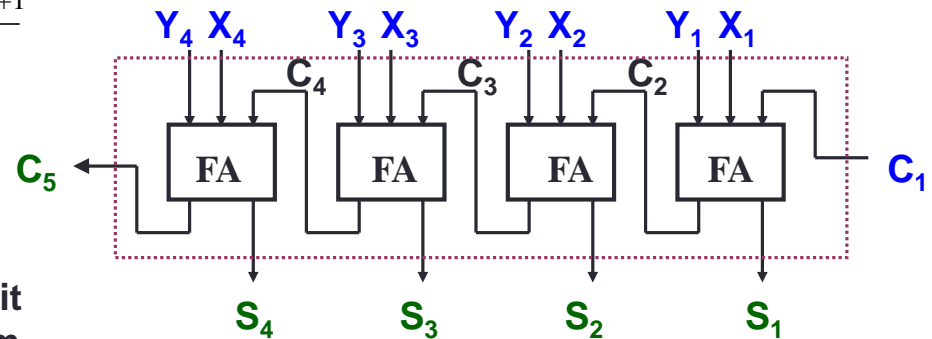
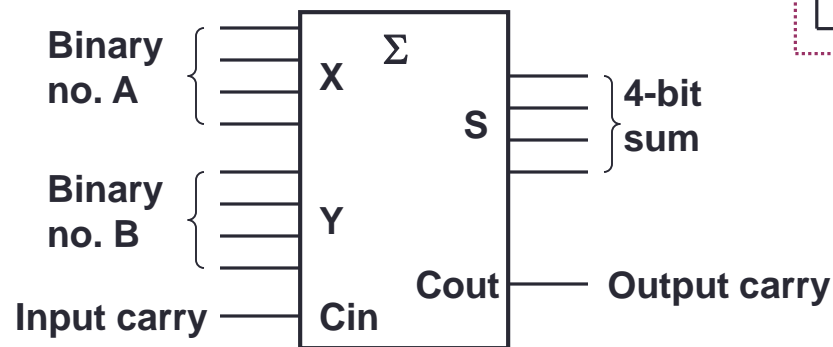
## 6. Summary of Arithmetic Circuits (3/4)

### ■ 4-bit parallel adder

Subscript $i$	4	3	2	1	
Input carry	0	1	1	0	$C_i$
Augend	1	0	1	1	$A_i$
Addend	0	0	1	1	$B_i$
Sum	1	1	1	0	$S_i$
Output carry	0	0	1	1	$C_{i+1}$

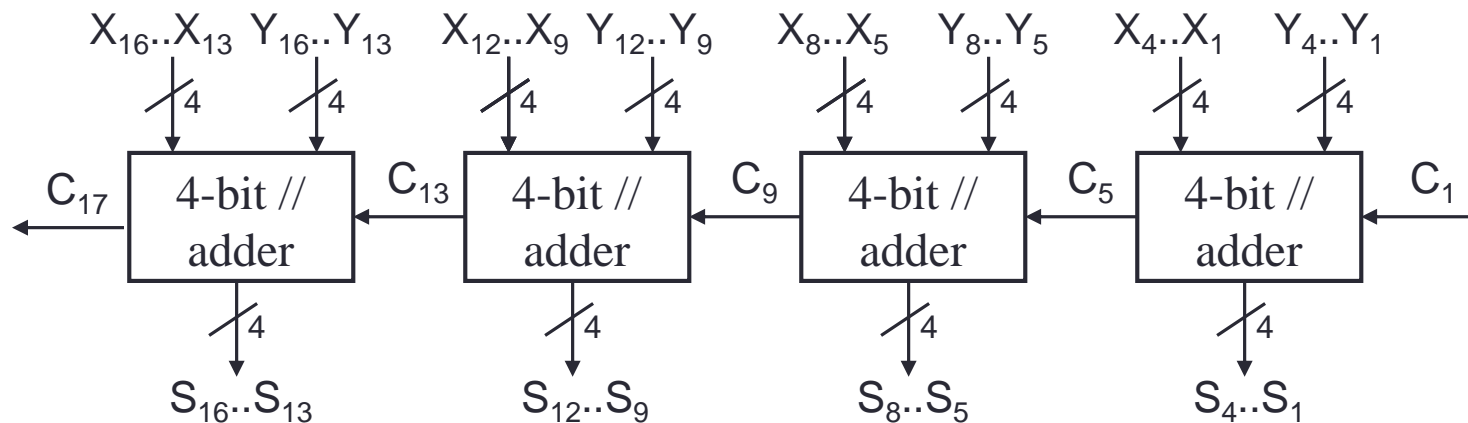
2 ways:

- ◆ Serial (one FA)
- ◆ Parallel ( $n$  FAs for  $n$  bits)



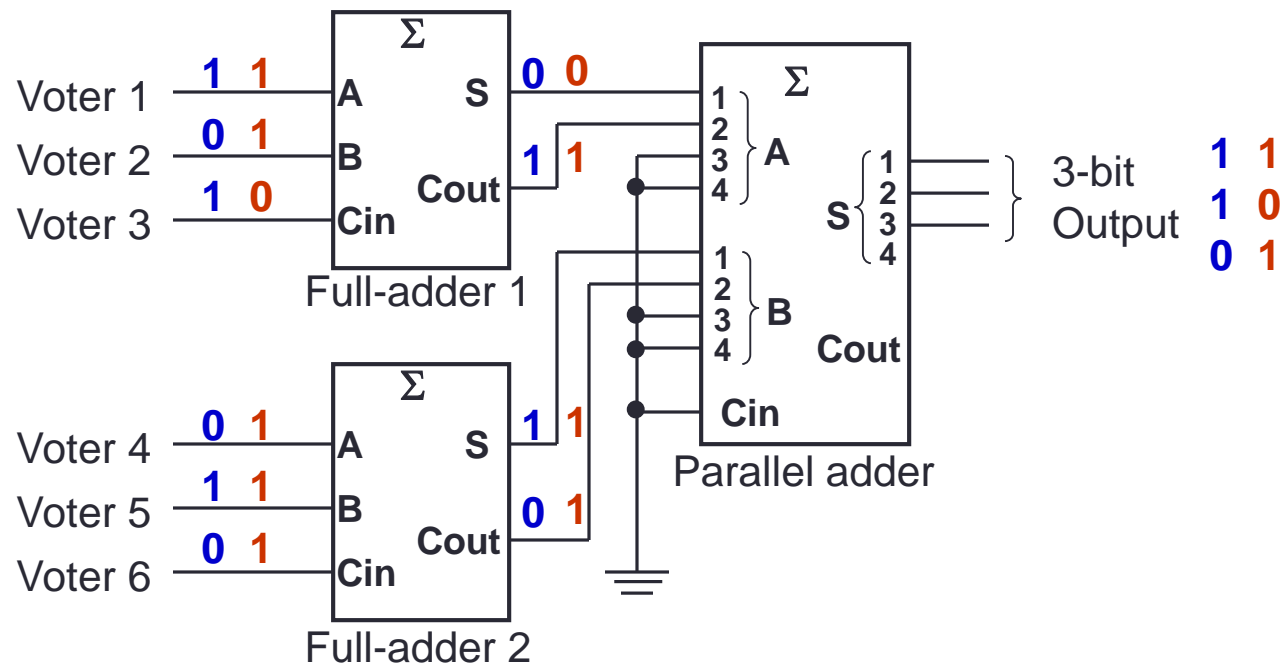
## 6. Summary of Arithmetic Circuits (4/4)

- Cascading 4 full adders (FAs) gives a 4-bit parallel adder.
  - Classical method: 9 input variables  $\rightarrow 2^9 = 512$  rows in truth table!
- Cascading method can be extended to larger adders.
  - Example: **16-bit parallel adder**.



# 7. Example: 6-Person Voting System

- Application: **6-person voting system**.
  - Use FAs and a 4-bit parallel adder.
  - Each FA can sum up to 3 votes.



## 8. Magnitude Comparator (1/4)

- **Magnitude comparator**: compares 2 unsigned values  $A$  and  $B$ , to check if  $A > B$ ,  $A = B$ , or  $A < B$ .
- To design an  $n$ -bit magnitude comparator using classical method, it would require  $2^{2n}$  rows in truth table!
- We shall exploit regularity in our design.
- Question: How do we compare two 4-bit unsigned values  $A (a_3a_2a_1a_0)$  and  $B (b_3b_2b_1b_0)$ ?

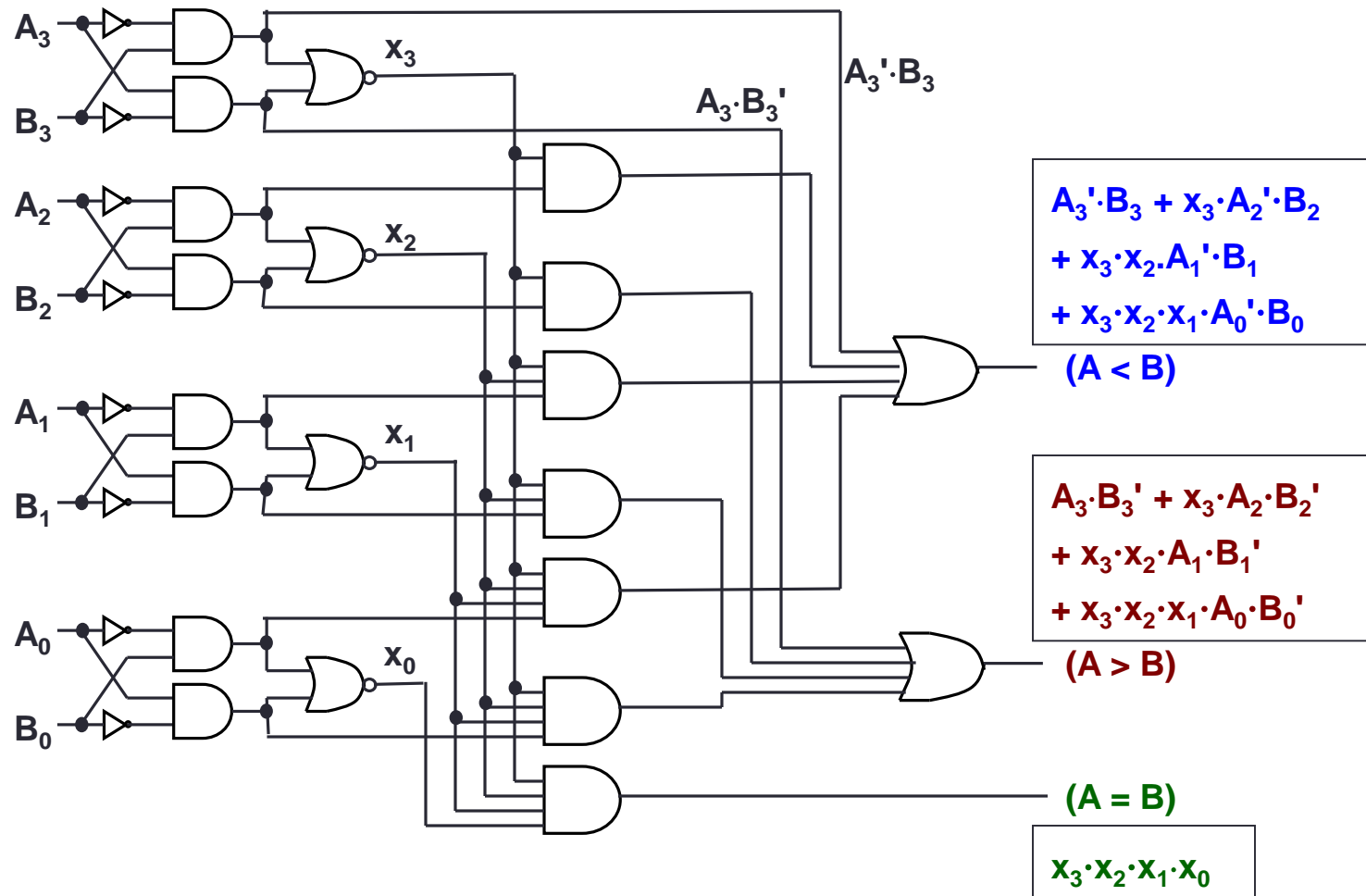
If  $(a_3 > b_3)$  then  $A > B$

If  $(a_3 < b_3)$  then  $A < B$

If  $(a_3 = b_3)$  then if  $(a_2 > b_2)$  ...

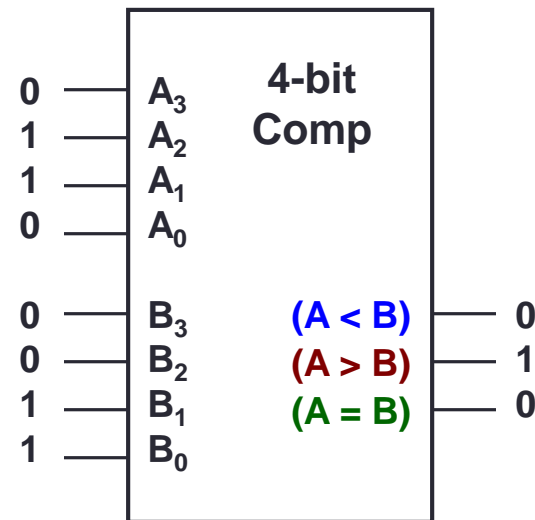
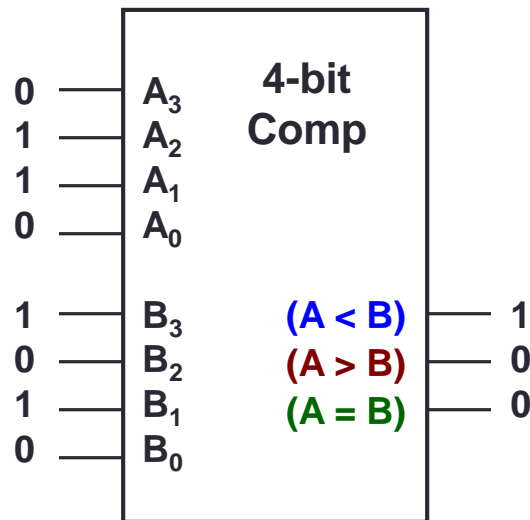
# 8. Magnitude Comparator (2/4)

Let  $A = A_3A_2A_1A_0$ ,  $B = B_3B_2B_1B_0$ ;  $x_i = A_i \cdot B_i + A_i' \cdot B_i'$



## 8. Magnitude Comparator (3/4)

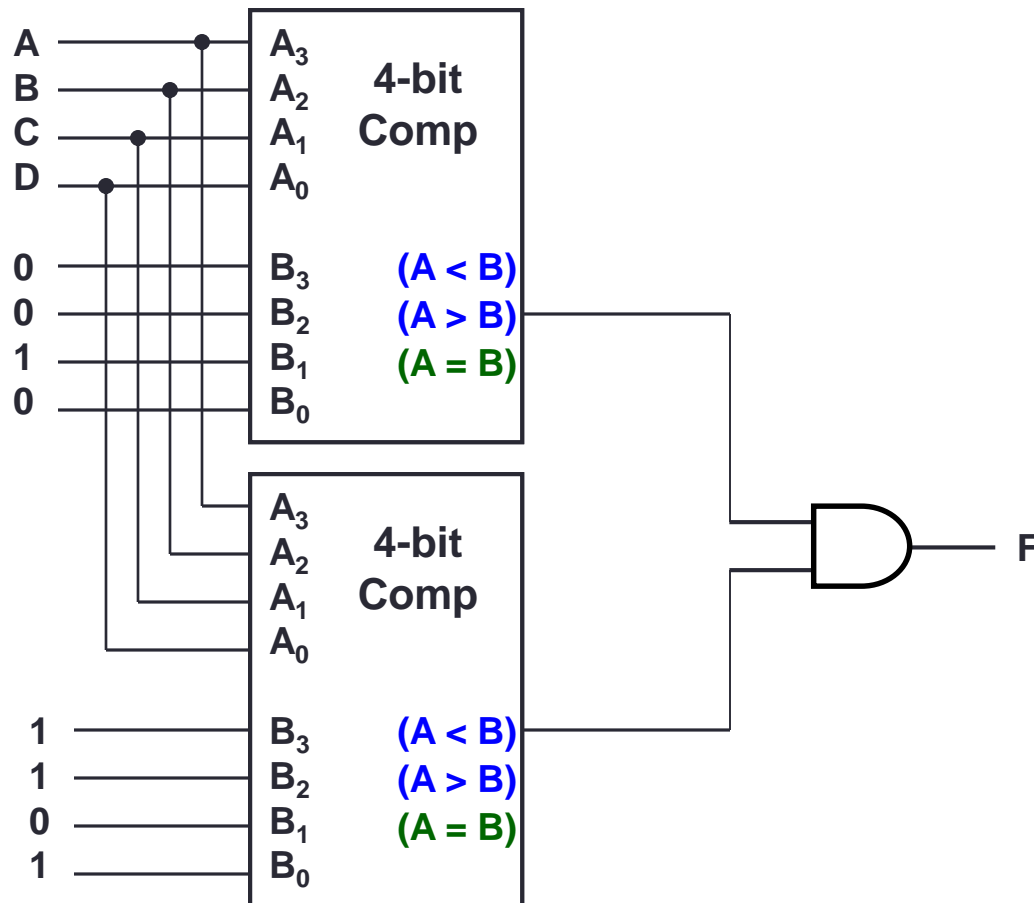
- Block diagram of a 4-bit magnitude comparator





## 8. Magnitude Comparator (4/4)

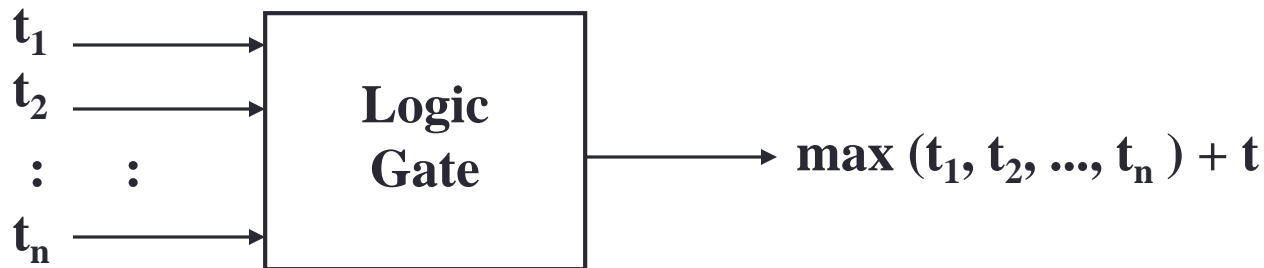
- A function  $F$  accepts a 4-bit binary value  $ABCD$ , and returns 1 if  $3 \leq ABCD \leq 12$ , or 0 otherwise. How would you implement  $F$  using magnitude comparators and a suitable logic gate?



## 9. Circuit Delays (1/5)

- Given a logic gate with delay  $t$ . If inputs are stable at times  $t_1, t_2, \dots, t_n$ , then the earliest time in which the output will be stable is:

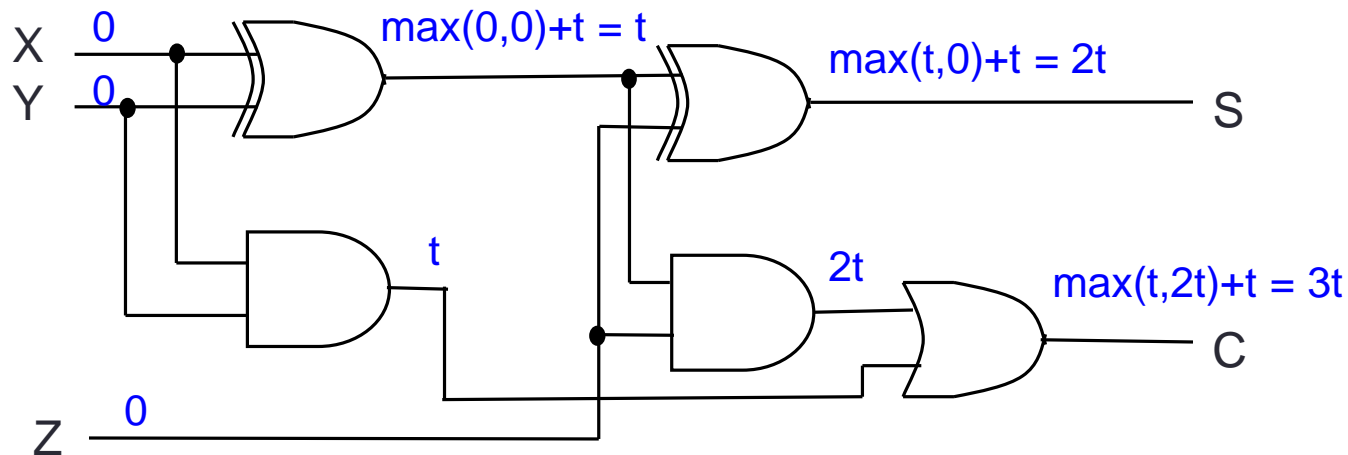
$$\max(t_1, t_2, \dots, t_n) + t$$



- To calculate the delays of all outputs of a combinational circuit, repeat above rule for all gates.

## 9. Circuit Delays (2/5)

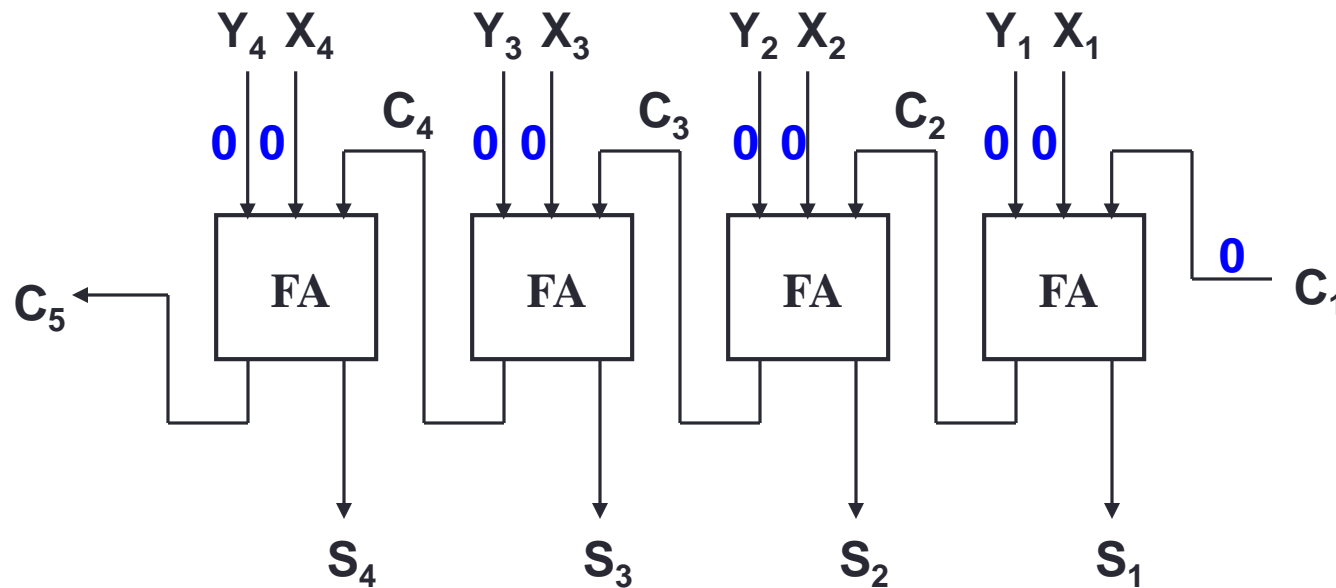
- As a simple example, consider the full adder circuit where all inputs are available at time 0. Assume each gate has delay  $t$ .



- Outputs **S** and **C** experience delays of  **$2t$**  and  **$3t$**  respectively.

## 9. Circuit Delays (3/5)

- More complex example: 4-bit parallel adder.



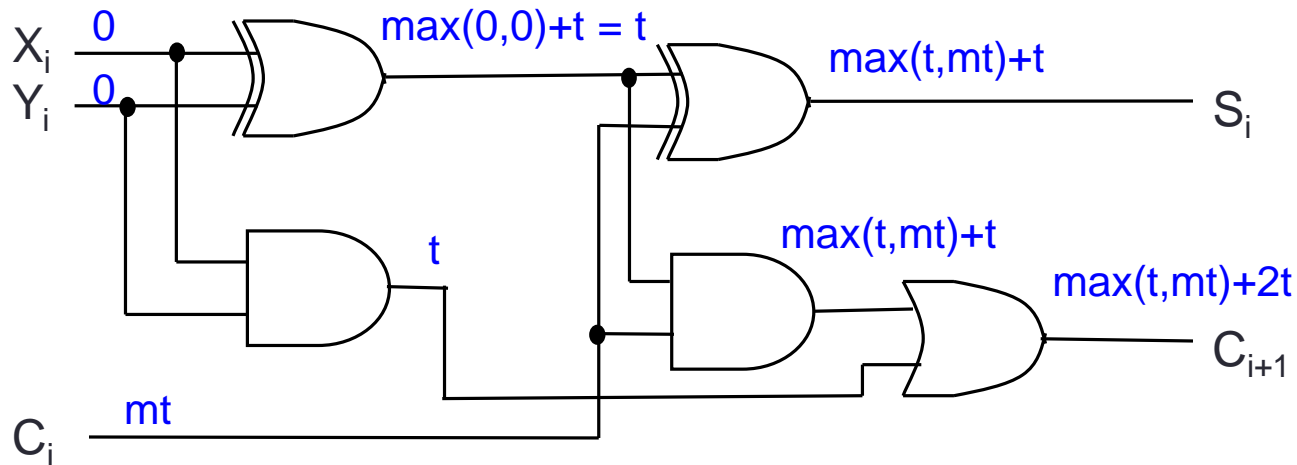
## 9. Circuit Delays (4/5)

- Analyse the delay for the repeated block.

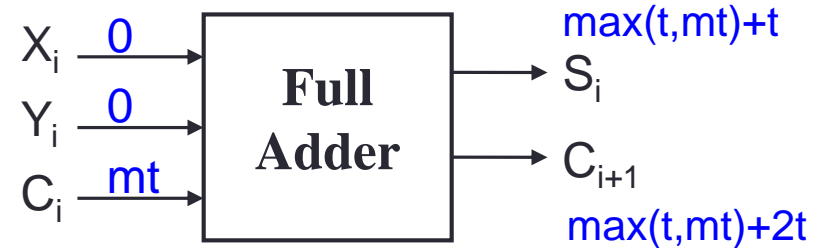


where  $X_i, Y_i$  are stable at  $0t$ , while  $C_i$  is assumed to be stable at  $mt$ .

- Performing the delay calculation:



## 9. Circuit Delays (5/5)



- Calculating:

When  $i=1$ ,  $m=0$ ;  $S_1 = 2t$  and  $C_2 = 3t$

When  $i=2$ ,  $m=3$ ;  $S_2 = 4t$  and  $C_3 = 5t$

When  $i=3$ ,  $m=5$ ;  $S_3 = 6t$  and  $C_4 = 7t$

When  $i=4$ ,  $m=7$ ;  $S_4 = 8t$  and  $C_5 = 9t$

- In general, an  $n$ -bit ripple-carry parallel adder will experience the following delay times:

$$S_n = ( (n-1)2 + 2 ) t$$

$$C_{n+1} = ( (n-1)2 + 3 ) t$$

- Propagation delay of ripple-carry parallel adders is proportional to the number of bits it handles.

- Maximum delay:  $( (n-1)2 + 3 ) t$

# Quick Review Questions

- DLD pages 128 – 129  
Questions 6-1 to 6-4.



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