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## Lecture #19

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# Sequential Logic



**NUS**  
National University  
of Singapore

School of  
Computing

# Lecture #19: Sequential Logic (1/2)

1. Introduction
2. Memory Elements
3. Latches
  - 3.1 *S-R* Latch
  - 3.2 *D* Latch
4. Flip-flops
  - 4.1 *S-R* Flip-flop
  - 4.2 *D* Flip-flop
  - 4.3 *J-K* Flip-flop
  - 4.4 *T* Flip-flop

# Lecture #19: Sequential Logic (2/2)

## 5. Asynchronous Inputs

## 6. Synchronous Sequential Circuit

### 6.1 Flip-flop Characteristic Tables

### 6.2 Analysis

### 6.3 Flip-flop Excitation Tables

### 6.4 Design

## 7. Memory

### 7.1 Memory Unit

### 7.2 Read/Write Operations

### 7.3 Memory Cell

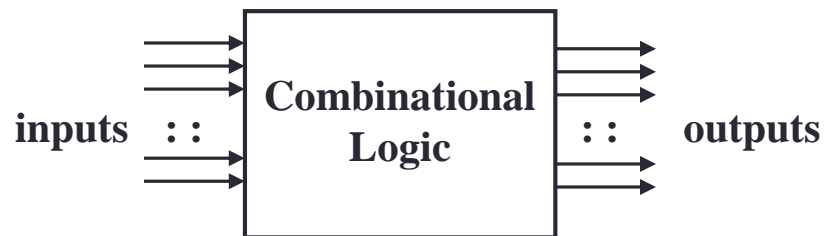
### 7.4 Memory Arrays

# 1. Introduction (1/2)

- Two classes of logic circuits
  - Combinational
  - Sequential

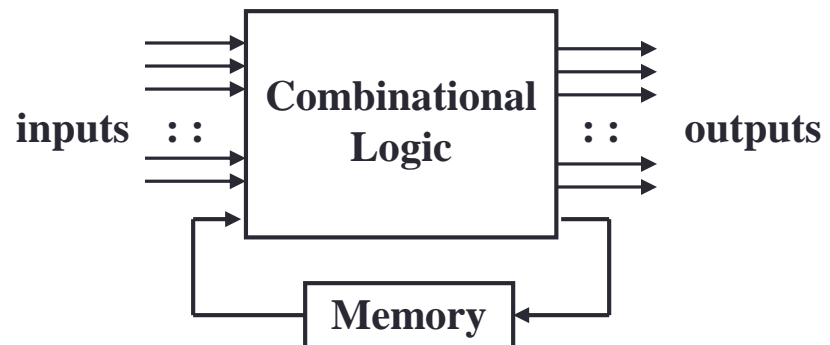
- **Combinational Circuit**

- Each output depends entirely on the immediate (present) inputs.



- **Sequential Circuit**

- Each output depends on both present inputs and state.

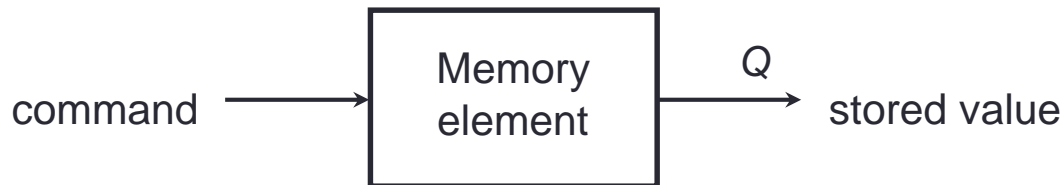


# 1. Introduction (2/2)

- Two types of sequential circuits:
  - **Synchronous**: outputs change only at specific time
  - **Asynchronous**: outputs change at any time
- Multivibrator: a class of sequential circuits
  - Bistable (2 stable states)
  - Monostable or one-shot (1 stable state)
  - Astable (no stable state)
- Bistable logic devices
  - **Latches** and **flip-flops**.
  - They differ in the methods used for changing their state.

## 2. Memory Elements (1/3)

- **Memory element:** a device which can remember value indefinitely, or change value on command from its inputs.



- **Characteristic table:**

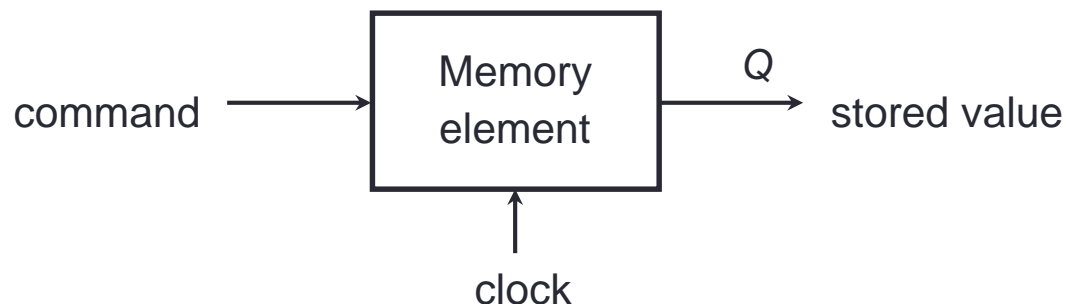
Command (at time $t$ )	$Q(t)$	$Q(t+1)$
Set	X	1
Reset	X	0
Memorise / No Change	0	0
	1	1

$Q(t)$  or  $Q$ : current state

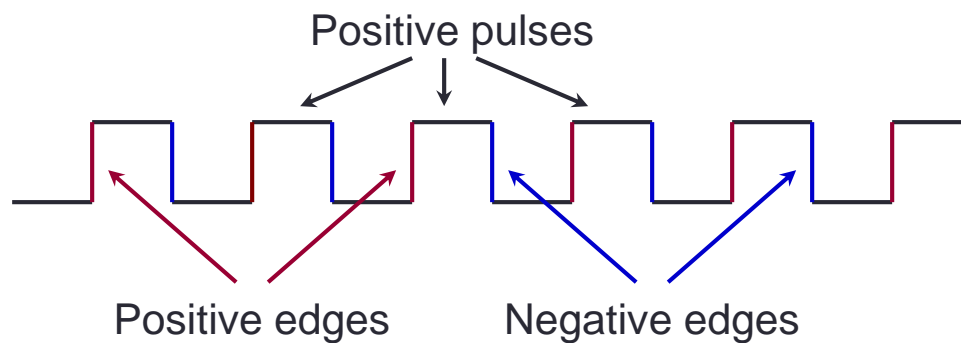
$Q(t+1)$  or  $Q^+$ : next state

## 2. Memory Elements (2/3)

- Memory element with clock.



- Clock** is usually a square wave.



## 2. Memory Elements (3/3)

- Two types of triggering/activation

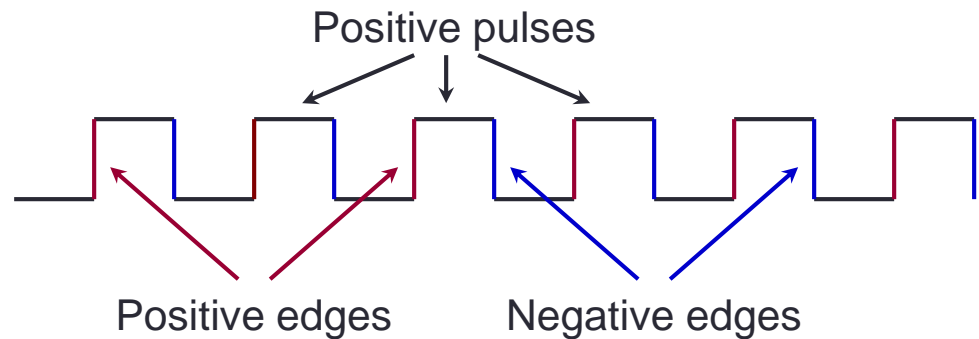
- Pulse-triggered
- Edge-triggered

- Pulse-triggered

- Latches
- ON = 1, OFF = 0

- Edge-triggered

- Flip-flops
- Positive edge-triggered (ON = from 0 to 1; OFF = other time)
- Negative edge-triggered (ON = from 1 to 0; OFF = other time)



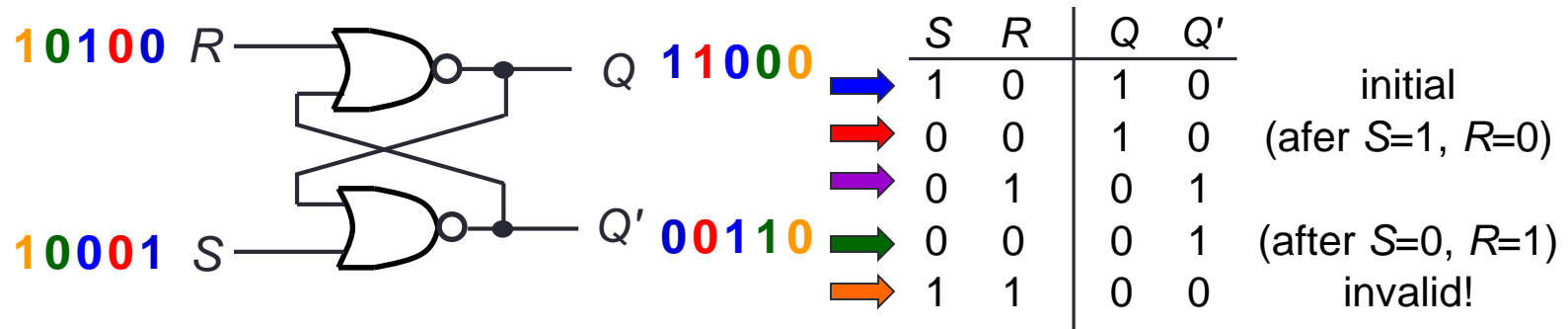


## 3.1 S-R Latch (1/3)

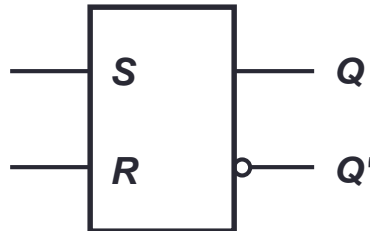
- Two inputs:  $S$  and  $R$ .
- Two complementary outputs:  $Q$  and  $Q'$ .
  - When  $Q = \text{HIGH}$ , we say latch is in **SET** state.
  - When  $Q = \text{LOW}$ , we say latch is in **RESET** state.
- For active-high input S-R latch (also known as NOR gate latch)
  - $R = \text{HIGH}$  and  $S = \text{LOW} \rightarrow Q$  becomes LOW (RESET state)
  - $S = \text{HIGH}$  and  $R = \text{LOW} \rightarrow Q$  becomes HIGH (SET state)
  - Both  $R$  and  $S$  are LOW  $\rightarrow$  No change in output  $Q$
  - Both  $R$  and  $S$  are HIGH  $\rightarrow$  Outputs  $Q$  and  $Q'$  are both LOW (invalid!)
- Drawback: invalid condition exists and must be avoided.

## 3.1 S-R Latch (2/3)

- Active-high input S-R latch:

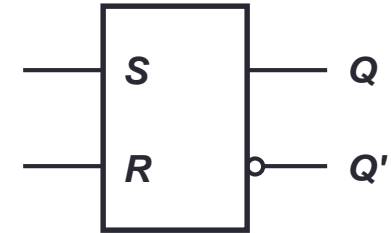


- Block diagram:



## 3.1 S-R Latch (3/3)

- **Characteristic table** for active-high input S-R latch:



S	R	Q	Q'	
0	0	NC	NC	No change. Latch remained in present state.
1	0	1	0	Latch SET.
0	1	0	1	Latch RESET.
1	1	0	0	Invalid condition.

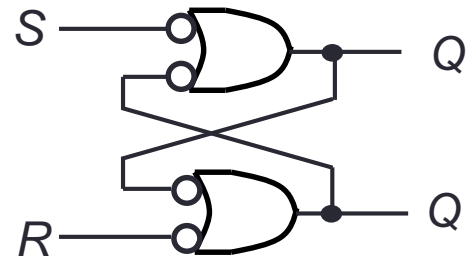
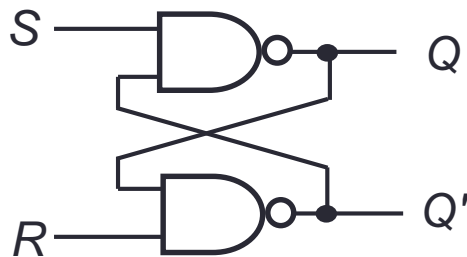
S	R	Q(t+1)	
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	indeterminate	

$$Q(t+1) = S + R' \cdot Q$$

$$S \cdot R = 0$$

## 3.1 Active-Low S-R Latch

- (You may skip this slide.)
- What we have seen is **active-high input** S-R latch.
- There are **active-low input** S-R latches, where NAND gates are used instead. See diagram on the left below.

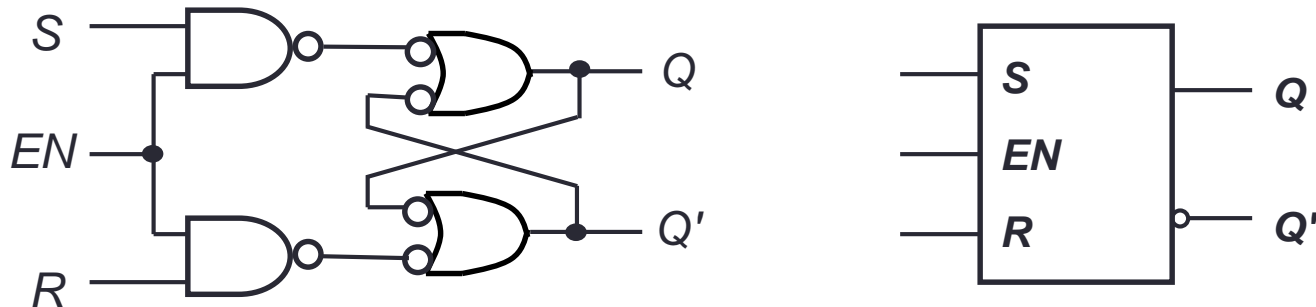


- In this case,
  - when  $R=0$  and  $S=1$ , the latch is reset (i.e.  $Q$  becomes 0)
  - when  $R=1$  and  $S=0$ , the latch is set (i.e.  $Q$  becomes 1)
  - when  $S=R=1$ , it is a no-change command.
  - when  $S=R=0$ , it is an invalid command.
- Sometimes, we use the alternative gate diagram for the NAND gate. See diagram on the right above. (This appears in more complex latches/flip-flops in the later slides.)

(Sometimes, the inputs are labelled as  $S'$  and  $R'$ .)

## 3.1 Gated S-R Latch

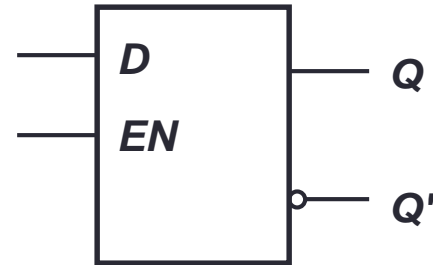
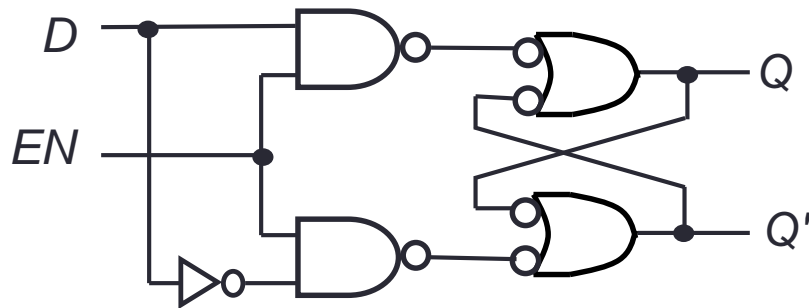
- S-R latch + *enable input* ( $EN$ ) and 2 NAND gates → a **gated S-R latch**.



- Outputs change (if necessary) only when  $EN$  is high.

## 3.2 Gated $D$ Latch (1/2)

- Make input  $R$  equal to  $S'$   $\rightarrow$  **gated  $D$  latch**.
- $D$  latch eliminates the undesirable condition of invalid state in the  $S$ - $R$  latch.



## 3.2 Gated $D$ Latch (2/2)

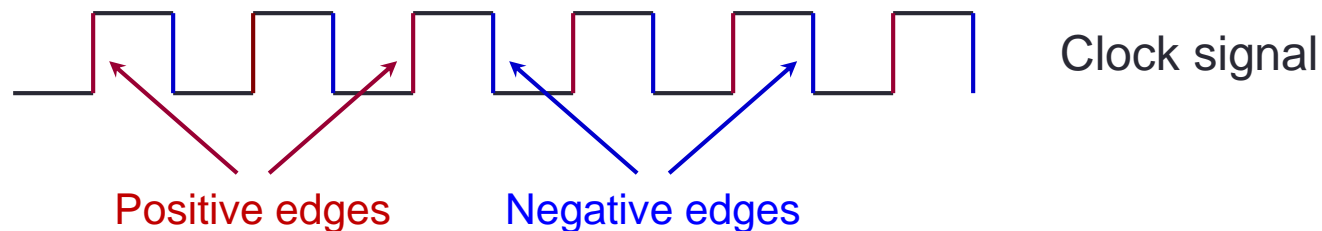
- When  $EN$  is high,
  - $D = \text{HIGH} \rightarrow$  latch is SET
  - $D = \text{LOW} \rightarrow$  latch is RESET
- Hence when  $EN$  is high,  $Q$  “follows” the  $D$  (data) input.
- **Characteristic table:**

$EN$	$D$	$Q(t+1)$	
1	0	0	Reset
1	1	1	Set
0	X	$Q(t)$	No change

When  $EN=1$ ,  $Q(t+1) = D$

## 4. Flip-flops (1/2)

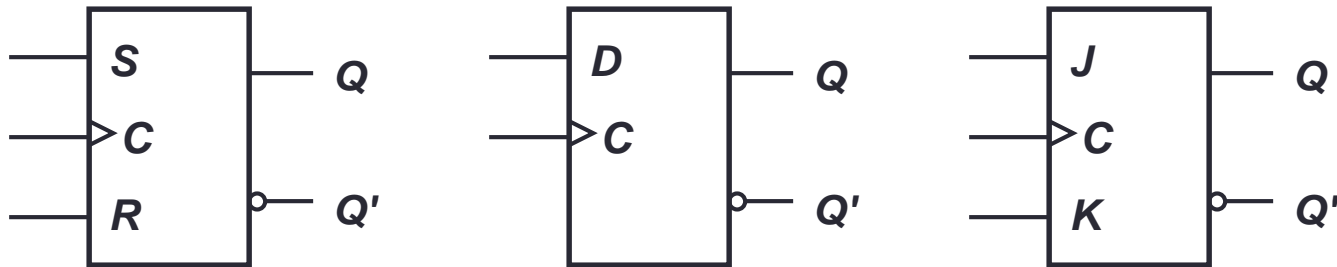
- Flip-flops are synchronous bistable devices.
- Output changes state at a specified point on a triggering input called the **clock**.
- Change state either at the positive (rising) edge, or at the negative (falling) edge of the clock signal.



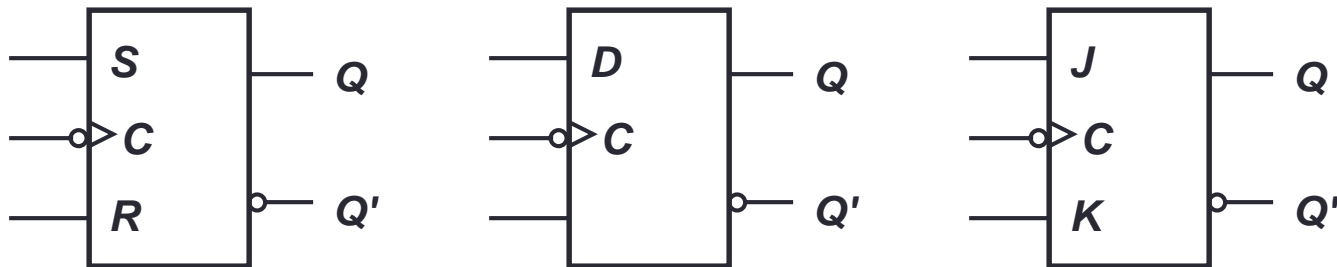


## 4. Flip-flops (2/2)

- *S-R* flip-flop, *D* flip-flop, and *J-K* flip-flop.
- Note the “>” symbol at the clock input.



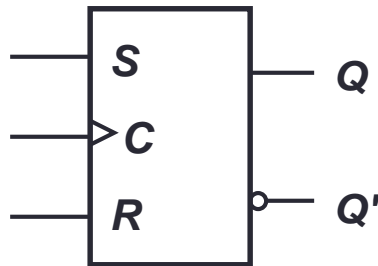
Positive edge-triggered flip-flops



Negative edge-triggered flip-flops

## 4.1 S-R Flip-flop

- **S-R flip-flop**: On the triggering edge of the clock pulse,
  - $R = \text{HIGH}$  and  $S = \text{LOW} \rightarrow Q$  becomes LOW (RESET state)
  - $S = \text{HIGH}$  and  $R = \text{LOW} \rightarrow Q$  becomes HIGH (SET state)
  - Both  $R$  and  $S$  are LOW  $\rightarrow$  No change in output  $Q$
  - Both  $R$  and  $S$  are HIGH  $\rightarrow$  Invalid!
- **Characteristic table** of positive edge-triggered S-R flip-flop:



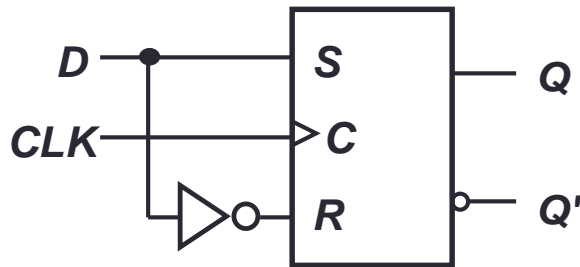
S	R	CLK	$Q(t+1)$	Comments
0	0	X	$Q(t)$	No change
0	1	$\uparrow$	0	Reset
1	0	$\uparrow$	1	Set
1	1	$\uparrow$	?	Invalid

X = irrelevant ("don't care")

$\uparrow$  = clock transition LOW to HIGH

## 4.2 D Flip-flop (1/2)

- **D flip-flop**: Single input  $D$  (data). On the triggering edge of the clock pulse,
  - $D = \text{HIGH} \rightarrow Q$  becomes HIGH (SET state)
  - $D = \text{LOW} \rightarrow Q$  becomes LOW (RESET state)
- Hence,  $Q$  “follows”  $D$  at the clock edge.
- Convert S-R flip-flop into a  $D$  flip-flop: add an inverter.



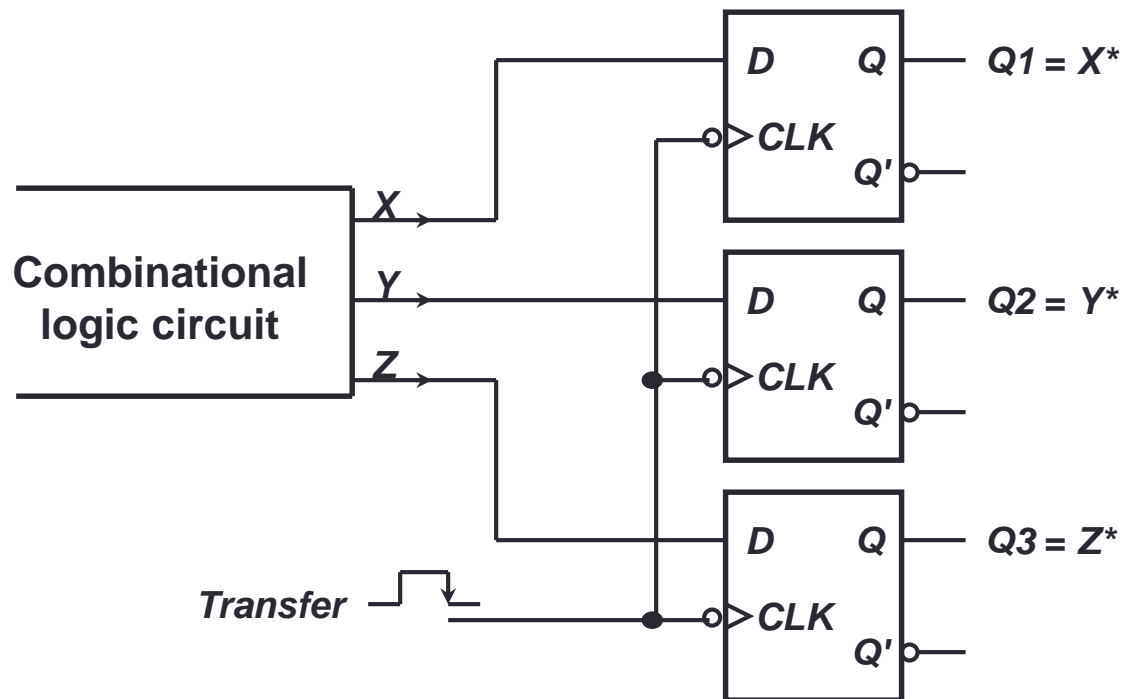
A positive edge-triggered D flip-flop formed with an S-R flip-flop.

$D$	$CLK$	$Q(t+1)$	Comments
1	↑	1	Set
0	↑	0	Reset

↑ = clock transition LOW to HIGH

## 4.2 D Flip-flop (2/2)

- Application: Parallel data transfer.
  - To transfer logic-circuit outputs  $X$ ,  $Y$ ,  $Z$  to flip-flops  $Q1$ ,  $Q2$  and  $Q3$  for storage.



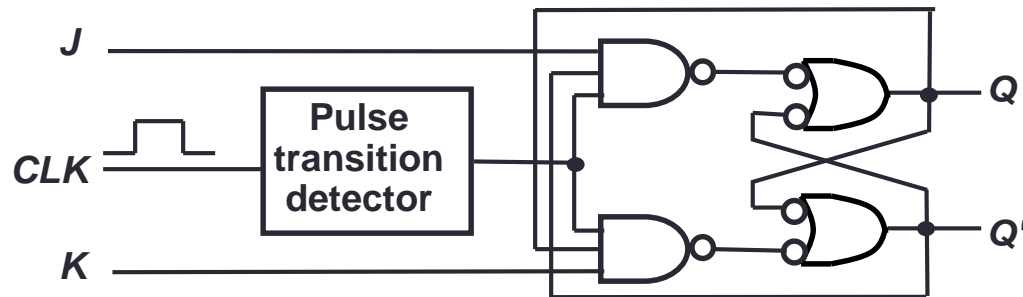
\* After occurrence of negative-going transition

## 4.3 *J-K* Flip-flop (1/2)

- *J-K* flip-flop:  $Q$  and  $Q'$  are fed back to the pulse-steering NAND gates.
- No invalid state.
- Include a toggle state
  - $J = \text{HIGH}$  and  $K = \text{LOW} \rightarrow Q$  becomes HIGH (SET state)
  - $K = \text{HIGH}$  and  $J = \text{LOW} \rightarrow Q$  becomes LOW (RESET state)
  - Both  $J$  and  $K$  are LOW  $\rightarrow$  No change in output  $Q$
  - Both  $J$  and  $K$  are HIGH  $\rightarrow$  Toggle

## 4.3 J-K Flip-flop (2/2)

- J-K flip-flop circuit:



- Characteristic table:

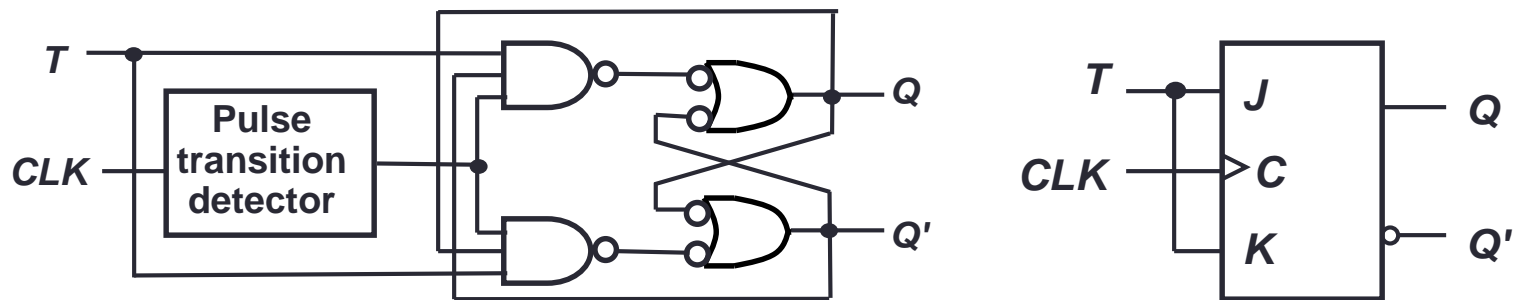
J	K	CLK	$Q(t+1)$	Comments
0	0	↑	$Q(t)$	No change
0	1	↑	0	Reset
1	0	↑	1	Set
1	1	↑	$Q(t)'$	Toggle

$$Q(t+1) = J \cdot Q' + K' \cdot Q$$

Q	J	K	$Q(t+1)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

## 4.4 $T$ Flip-flop

- $T$  flip-flop: Single input version of the  $J$ - $K$  flip-flop, formed by tying both inputs together.



- Characteristic table:

$T$	$CLK$	$Q(t+1)$	Comments
0	$\uparrow$	$Q(t)$	No change
1	$\uparrow$	$Q(t)'$	Toggle

$$Q(t+1) = T \cdot Q' + T' \cdot Q$$

$Q$	$T$	$Q(t+1)$
0	0	0
0	1	1
1	0	1
1	1	0

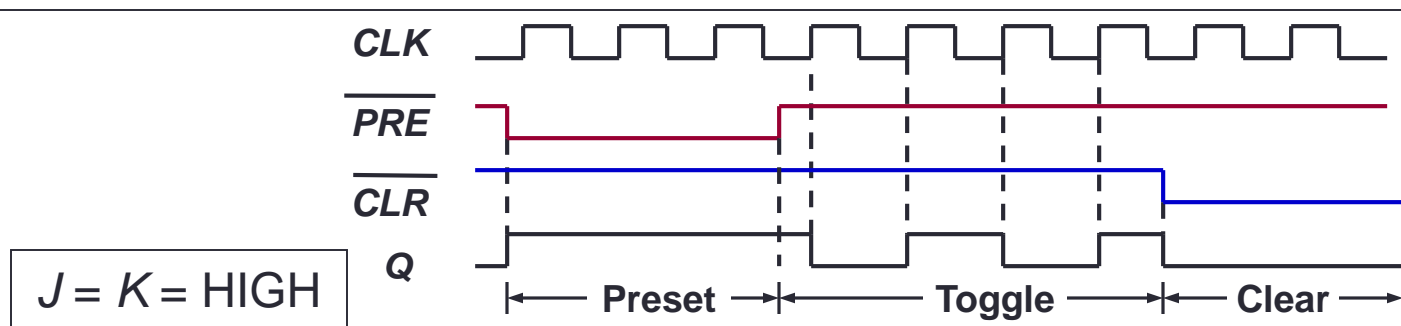
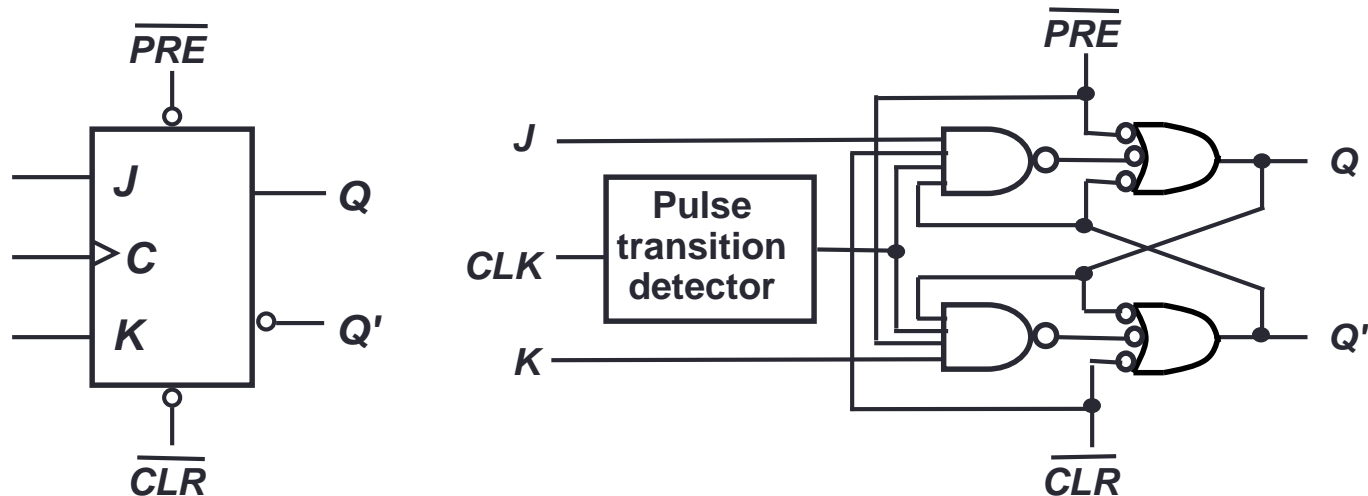
## 5. Asynchronous Inputs (1/2)

- $S$ - $R$ ,  $D$  and  $J$ - $K$  inputs are **synchronous inputs**, as data on these inputs are transferred to the flip-flop's output only on the triggered edge of the clock pulse.
- **Asynchronous** inputs affect the state of the flip-flop independent of the clock; example: *preset* ( $PRE$ ) and *clear* ( $CLR$ ) [or *direct set* ( $SD$ ) and *direct reset* ( $RD$ )].
- When  $PRE$ =HIGH,  $Q$  is immediately set to HIGH.
- When  $CLR$ =HIGH,  $Q$  is immediately cleared to LOW.
- Flip-flop in normal operation mode when both  $PRE$  and  $CLR$  are LOW.



## 5. Asynchronous Inputs (2/2)

- A  $J$ - $K$  flip-flop with active-low PRESET and CLEAR asynchronous inputs.



## 6. Synchronous Sequential Circuits

- Building blocks: **logic gates** and **flip-flops**.
- Flip-flops make up the **memory** while the gates form one or more combinational sub-circuits.
- We have discussed *S-R* flip-flop, *J-K* flip-flop, *D* flip-flop and *T* flip-flop.

# 6.1 Flip-flop Characteristic Tables

- Each type of flip-flop has its own behaviour, shown by its **characteristic table**.

$J$	$K$	$Q(t+1)$	Comments
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q(t)'$	Toggle

$S$	$R$	$Q(t+1)$	Comments
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	?	Unpredictable

$D$	$Q(t+1)$
0	0      Reset
1	1      Set

$T$	$Q(t+1)$
0	$Q(t)$ No change
1	$Q(t)'$ Toggle

## 6.2 Sequential Circuits: Analysis (1/7)

- Given a sequential circuit diagram, we can analyze its behaviour by deriving its *state table* and hence its *state diagram*.
- Requires *state equations* to be derived for the flip-flop inputs, as well as *output functions* for the circuit outputs other than the flip-flops (if any).
- We use  $\mathbf{A}(t)$  and  $\mathbf{A}(t+1)$  (or simply  $\mathbf{A}$  and  $\mathbf{A}^+$ ) to represent the present state and next state, respectively, of a flip-flop represented by  $A$ .

## 6.2 Sequential Circuits: Analysis (2/7)

- Example using  $D$  flip-flops

State equations:

$$A^+ = A \cdot x + B \cdot x$$

$$B^+ = A' \cdot x$$

Output function:

$$y = (A + B) \cdot x'$$

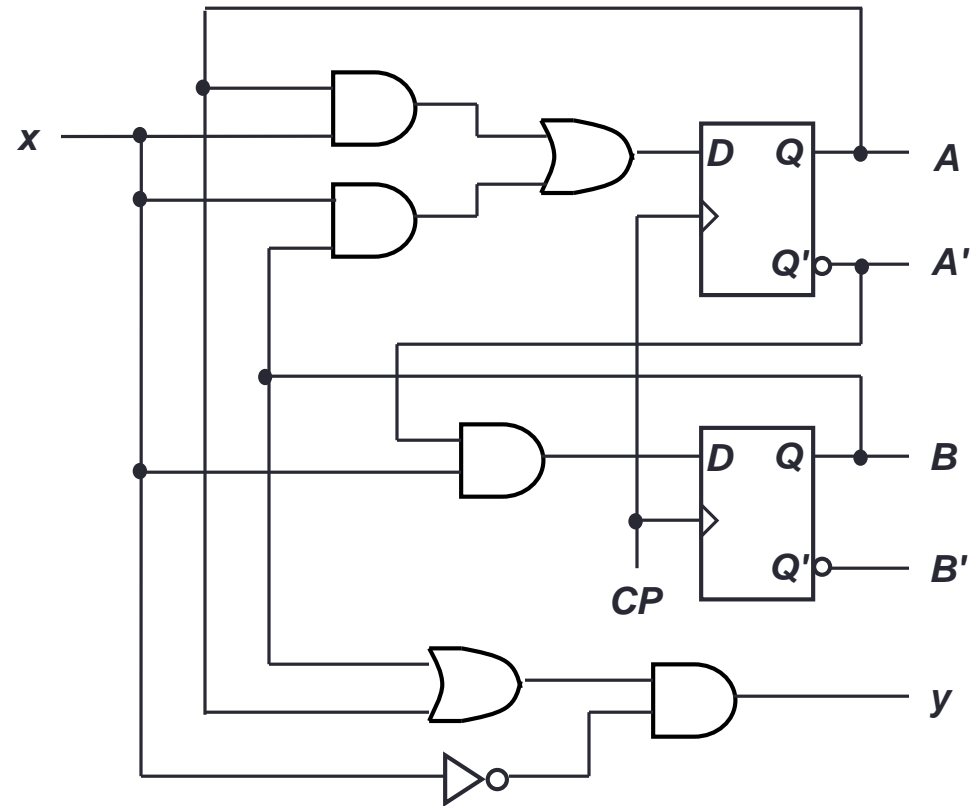


Figure 1

## 6.2 Sequential Circuits: Analysis (3/7)

- From the *state equations* and *output function*, we derive the **state table**, consisting of all possible binary combinations of present states and inputs.
- State table
  - Similar to truth table.
  - Inputs and present state on the left side.
  - Outputs and next state on the right side.
- $m$  flip-flops and  $n$  inputs  $\rightarrow 2^{m+n}$  rows.

## 6.2 Sequential Circuits: Analysis (4/7)

- **State table** for circuit of Figure 1:

State equations:

$$A^+ = A \cdot x + B \cdot x$$

$$B^+ = A' \cdot x$$

Output function:

$$y = (A + B) \cdot x'$$

Present State		Input	Next State		Output
A	B		A <sup>+</sup>	B <sup>+</sup>	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

## 6.2 Sequential Circuits: Analysis (5/7)

- Alternative form of state table:

Full table

Present State		Input	Next State		Output
<i>A</i>	<i>B</i>		<i>A</i> <sup>+</sup>	<i>B</i> <sup>+</sup>	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Compact table

Present State	Next State		Output	
	<i>x</i> =0	<i>x</i> =1	<i>x</i> =0	<i>x</i> =1
<i>AB</i>	<i>A</i> <sup>+</sup> <i>B</i> <sup>+</sup>	<i>A</i> <sup>+</sup> <i>B</i> <sup>+</sup>	<i>y</i>	<i>y</i>
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0



## 6.2 Sequential Circuits: Analysis (6/7)

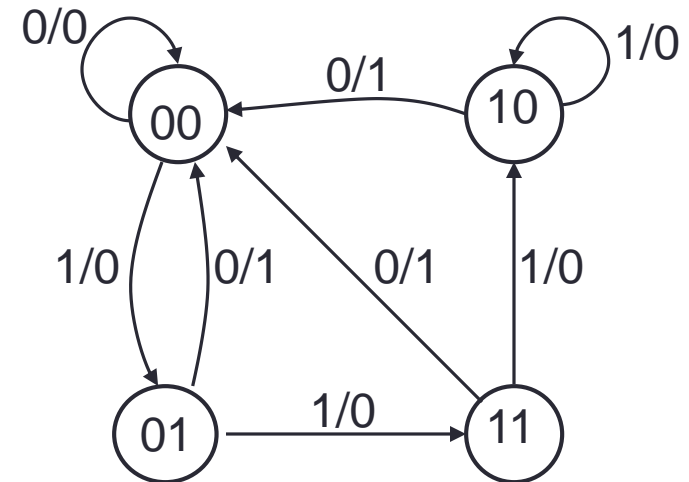
- From the *state table*, we can draw the *state diagram*.
- State diagram
  - Each state is denoted by a circle.
  - Each arrow (between two circles) denotes a transition of the sequential circuit (a row in state table).
  - A label of the form  $a/b$  is attached to each arrow where  $a$  (if there is one) denotes the inputs while  $b$  (if there is one) denotes the outputs of the circuit in that transition.
- Each combination of the flip-flop values represents a state. Hence,  $m$  flip-flops  $\rightarrow$  up to  $2^m$  states.

## 6.2 Sequential Circuits: Analysis (7/7)

- **State diagram** of the circuit of Figure 1:

Present State	Next State		Output	
	$x=0$	$x=1$	$x=0$	$x=1$
$AB$	$A^+B^+$	$A^+B^+$	$y$	$y$
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

**DONE!**



## 6.2 Flip-flop Input Functions (1/3)

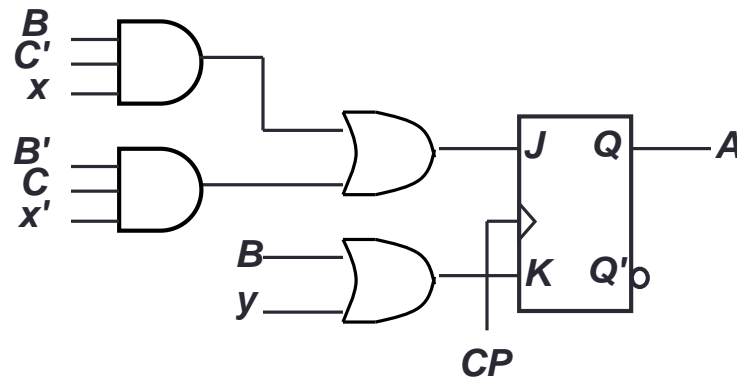
- The outputs of a sequential circuit are functions of the present states of the flip-flops and the inputs. These are described algebraically by the *circuit output functions*.
  - In Figure 1:  $y = (A + B) \cdot x'$
- The part of the circuit that generates inputs to the flip-flops are described algebraically by the *flip-flop input functions* (or *flip-flop input equations*).
- The flip-flop input functions determine the next state generation.
- From the flip-flop input functions and the characteristic tables of the flip-flops, we obtain the next states of the flip-flops.

## 6.2 Flip-flop Input Functions (2/3)

- Example: circuit with a *JK* flip-flop.
- We use 2 letters to denote each flip-flop input: the first letter denotes the input of the flip-flop (*J* or *K* for *J-K* flip-flop, *S* or *R* for *S-R* flip-flop, *D* for *D* flip-flop, *T* for *T* flip-flop) and the second letter denotes the name of the flip-flop.

$$JA = B \cdot C' \cdot x + B' \cdot C \cdot x'$$

$$KA = B + y$$



## 6.2 Flip-flop Input Functions (3/3)

- In Figure 1, we obtain the following **state equations** by observing that  $Q^+ = DQ$  for a  $D$  flip-flop:

$$A^+ = A \cdot x + B \cdot x \quad (\text{since } DA = A \cdot x + B \cdot x)$$

$$B^+ = A' \cdot x \quad (\text{since } DB = A' \cdot x)$$

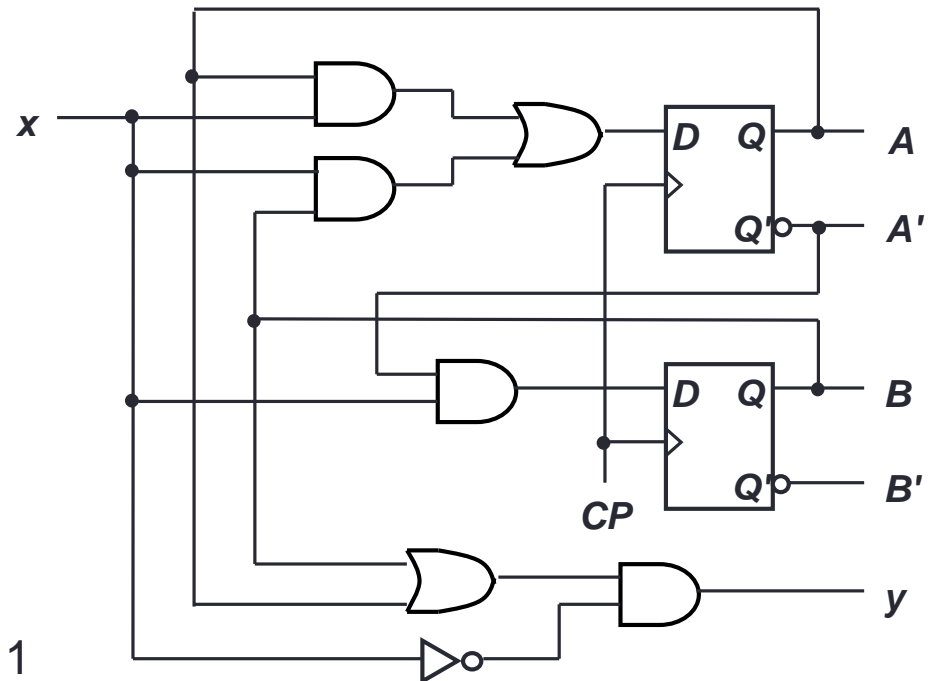
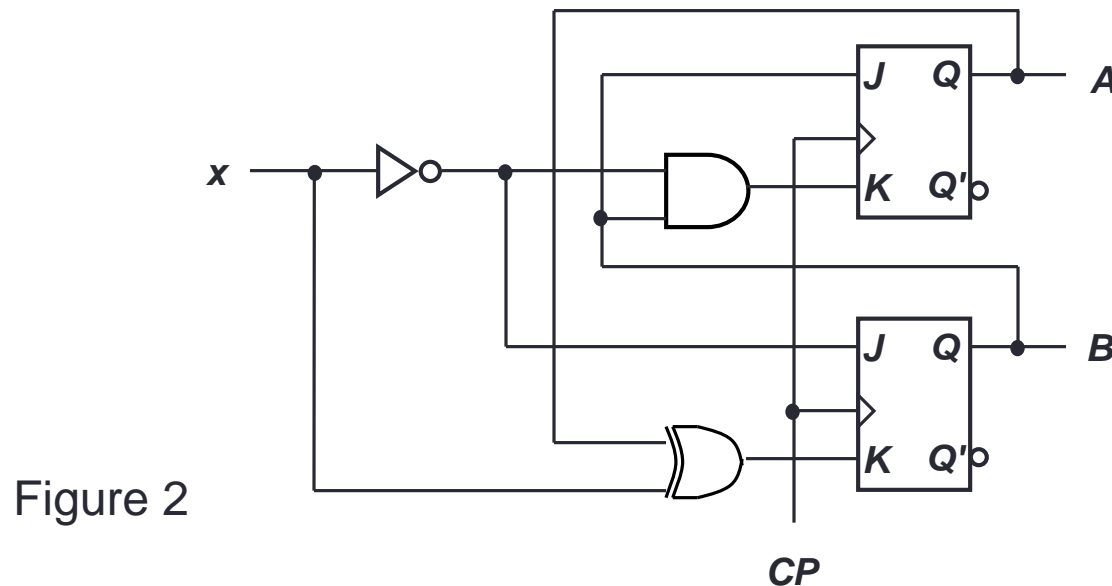


Figure 1

## 6.2 Analysis: Example #2 (1/3)

- Given Figure 2, a sequential circuit with two  $J$ - $K$  flip-flops  $A$  and  $B$ , and one input  $x$ .



- Obtain the **flip-flop input functions** from the circuit:

$$JA = B$$

$$JB = x'$$

$$KA = B \cdot x'$$

$$KB = A' \cdot x + A \cdot x' = A \oplus x$$

## 6.2 Analysis: Example #2 (2/3)

$$JA = B$$

$$KA = B \cdot x'$$

$$JB = x'$$

$$KB = A' \cdot x + A \cdot x' = A \oplus x$$

- Fill the **state table** using the above functions, knowing the characteristics of the flip-flops used.

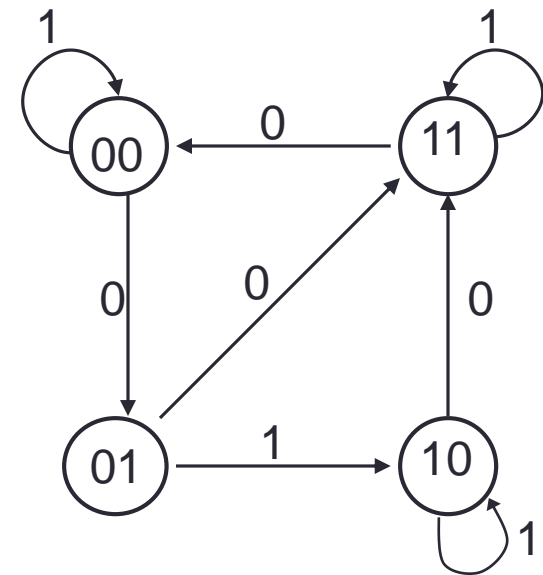
<i>J</i>	<i>K</i>	<i>Q(t+1)</i>	Comments
0	0	<i>Q(t)</i>	No change
0	1	0	Reset
1	0	1	Set
1	1	<i>Q(t)'</i>	Toggle

Present state		Input <i>x</i>	Next state		Flip-flop inputs			
<i>A</i>	<i>B</i>		<i>A</i> <sup>+</sup>	<i>B</i> <sup>+</sup>	<i>JA</i>	<i>KA</i>	<i>JB</i>	<i>KB</i>
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

## 6.2 Analysis: Example #2 (3/3)

- Draw the **state diagram** from the state table.

Present state		Input $x$	Next state		Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$	$JA$	$KA$	$JB$	$KB$
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0





## 6.2 Analysis: Example #3 (1/3)

- Derive the state table and state diagram of this circuit.

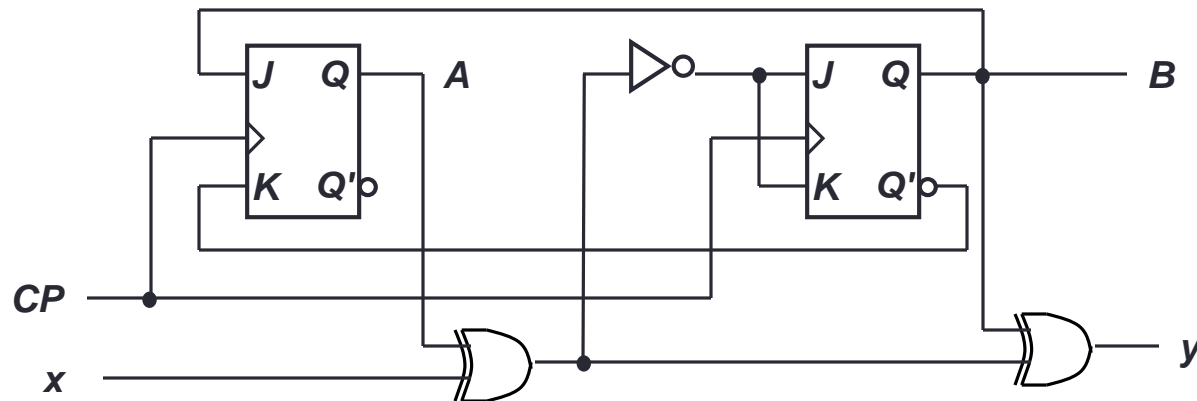


Figure 3

- Flip-flop input functions:

$$JA = B$$

$$KA = B'$$

$$JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$$

## 6.2 Analysis: Example #3 (2/3)

- Flip-flop input functions:

$$JA = B$$

$$JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$$

$$KA = B'$$

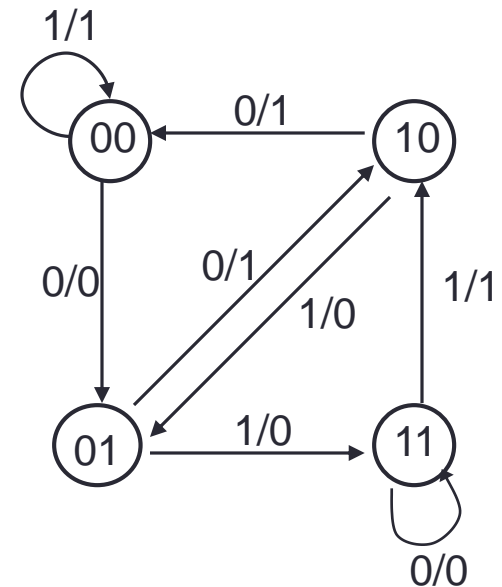
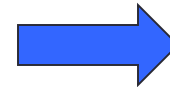
- State table:

Present state		Input $x$	Next state		Output $y$	Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$		$JA$	$KA$	$JB$	$KB$
0	0	0	0	1	0	0	1	1	1
0	0	1	0	0	1	0	1	0	0
0	1	0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	0	0	0
1	0	0	0	0	1	0	1	0	0
1	0	1	0	1	0	0	1	1	1
1	1	0	1	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1	1

## 6.2 Analysis: Example #3 (3/3)

### ■ State diagram:

Present state		Input $x$	Next state		Output $y$	Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$		$JA$	$KA$	$JB$	$KB$
0	0	0	0	1	0	0	1	1	1
0	0	1	0	0	1	0	1	0	0
0	1	0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	0	0	0
1	0	0	0	0	1	0	1	0	0
1	0	1	0	1	0	0	1	1	1
1	1	0	1	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1	1



## 6.3 Flip-flop Excitation Tables (1/2)

- *Analysis*: Starting from a circuit diagram, derive the state table or state diagram.
- *Design*: Starting from a set of specifications (in the form of state equations, state table, or state diagram), derive the logic circuit.
- *Characteristic tables* are used in analysis.
- *Excitation tables* are used in design.

## 6.3 Flip-flop Excitation Tables (1/2)

- *Excitation tables*: given the required transition from present state to next state, determine the flip-flop input(s).

$Q$	$Q^+$	$J$	$K$
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

$JK$  Flip-flop

$Q$	$Q^+$	$S$	$R$
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

$SR$  Flip-flop

$Q$	$Q^+$	$D$
0	0	0
0	1	1
1	0	0
1	1	1

$D$  Flip-flop

$Q$	$Q^+$	$T$
0	0	0
0	1	1
1	0	1
1	1	0

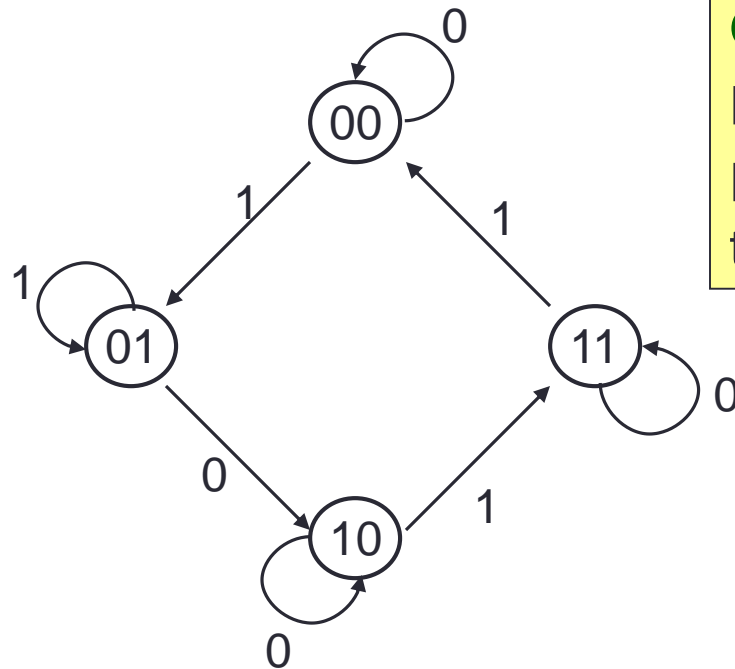
$T$  Flip-flop

## 6.4 Sequential Circuits: Design

- Design procedure:
  - Start with circuit specifications – description of circuit behaviour, usually a state diagram or state table.
  - Derive the state table.
  - Perform state reduction if necessary.
  - Perform state assignment.
  - Determine number of flip-flops and label them.
  - Choose the type of flip-flop to be used.
  - Derive circuit excitation and output tables from the state table.
  - Derive circuit output functions and flip-flop input functions.
  - Draw the logic diagram.

## 6.4 Design: Example #1 (1/5)

- Given the following state diagram, design the sequential circuit using *JK* flip-flops.



### Questions:

How many flip-flops are needed?

How many input variable are there?

### Answers:

Two flip-flops.

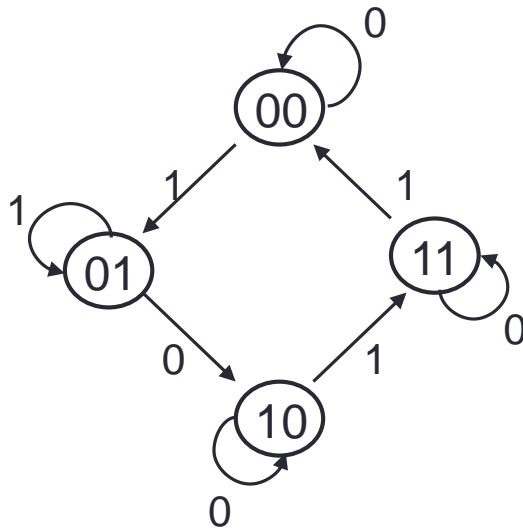
Let's call them *A* and *B*.

One input variable.

Let's call it *x*.

## 6.4 Design: Example #1 (2/5)

- Circuit state/excitation table, using  $JK$  flip-flops.



$Q$	$Q^+$	$J$	$K$
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

$JK$  Flip-flop's  
excitation table.

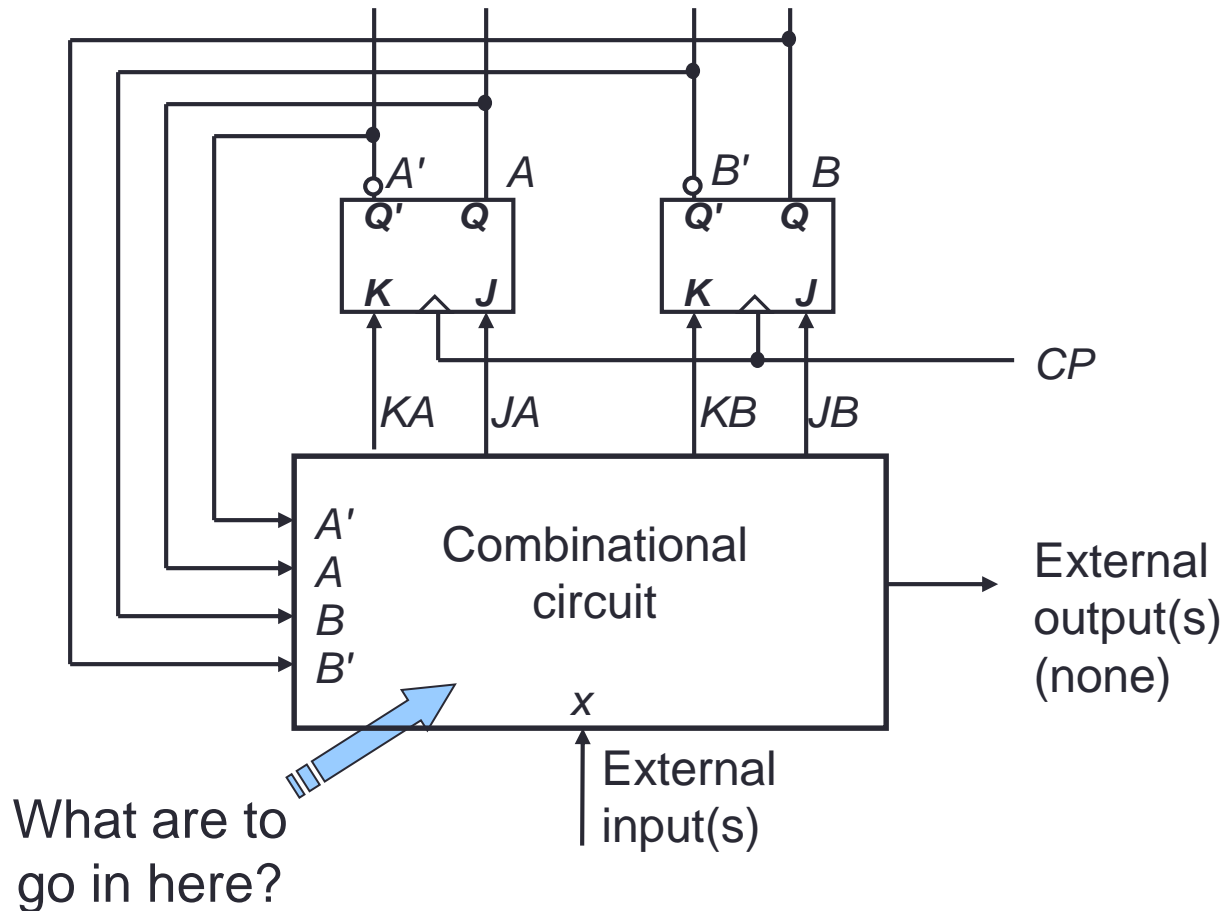
Present State	Next State	
	$x=0$	$x=1$
$AB$	$A^+B^+$	$A^+B^+$
00	00	01
01	10	01
10	10	11
11	11	00

Present state		Input	Next state		Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$	$JA$	$KA$	$JB$	$KB$
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1



## 6.4 Design: Example #1 (3/5)

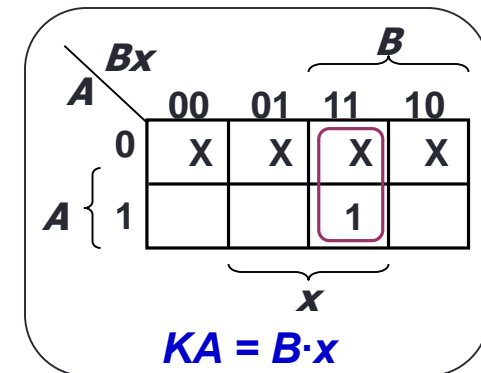
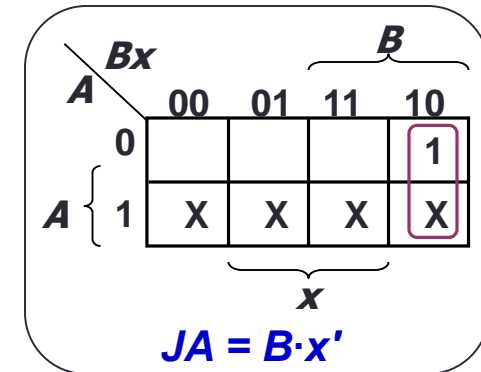
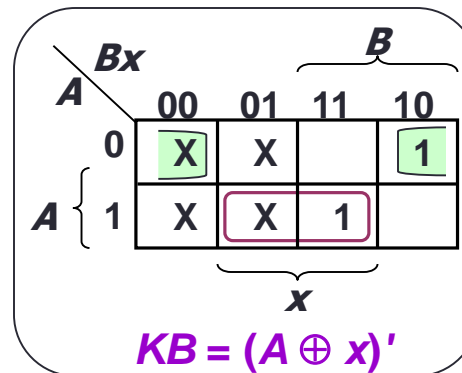
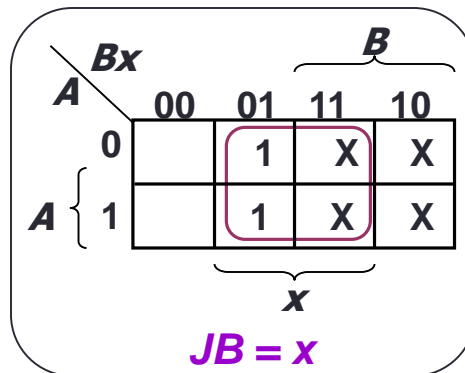
- Block diagram.



## 6.4 Design: Example #1 (4/5)

- From **state table**, get **flip-flop input functions**.

Present state		Input $x$	Next state		Flip-flop inputs			
$A$	$B$		$A^+$	$B^+$	$JA$	$KA$	$JB$	$KB$
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1



## 6.4 Design: Example #1 (5/5)

- Flip-flop input functions:

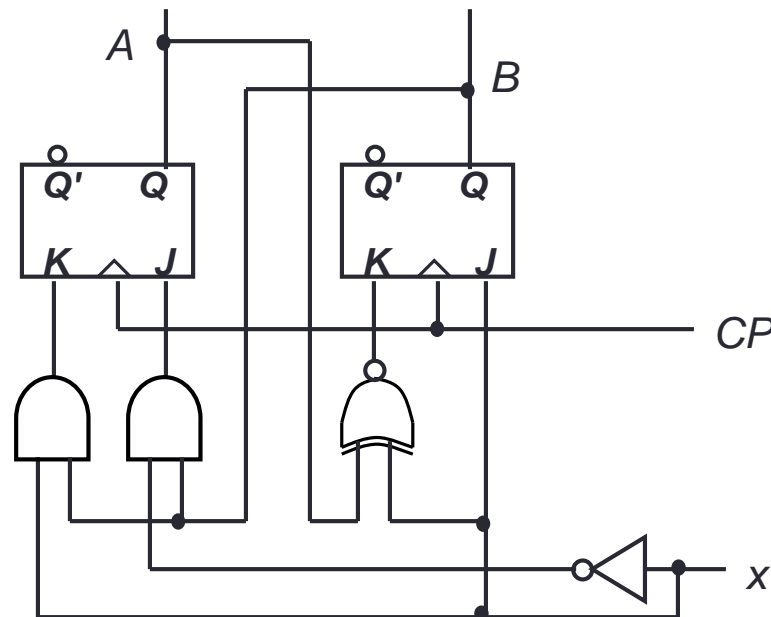
$$JA = B \cdot x'$$

$$JB = x$$

$$KA = B \cdot x$$

$$KB = (A \oplus x)'$$

- Logic diagram:



## 6.4 Design: Example #2 (1/3)

- Using  $D$  flip-flops, design the circuit based on the state table below. (Exercise: Design it using  $JK$  flip-flops.)

Present state		Input	Next state		Output
$A$	$B$		$A^+$	$B^+$	
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	0	0	0

## 6.4 Design: Example #2 (2/3)

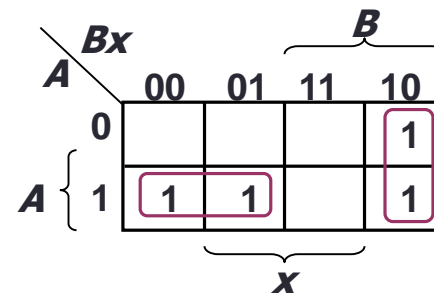
- Determine expressions for flip-flop inputs and the circuit output  $y$ .

Present state		Input $x$	Next state		Output $y$
$A$	$B$		$A^+$	$B^+$	
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	0	0	0

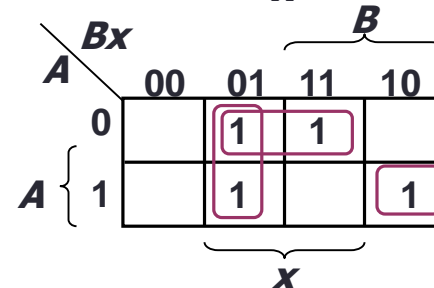
$$DA(A,B,x) = \Sigma m(2,4,5,6)$$

$$DB(A,B,x) = \Sigma m(1,3,5,6)$$

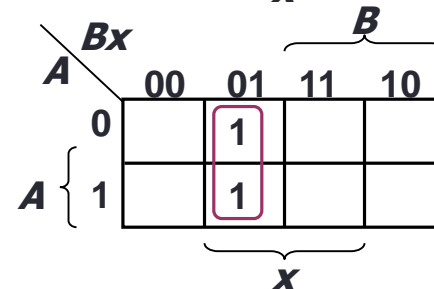
$$y(A,B,x) = \Sigma m(1,5)$$



$$DA = A \cdot B' + B \cdot x'$$



$$DB = A' \cdot x + B' \cdot x + A \cdot B \cdot x'$$



$$y = B' \cdot x$$

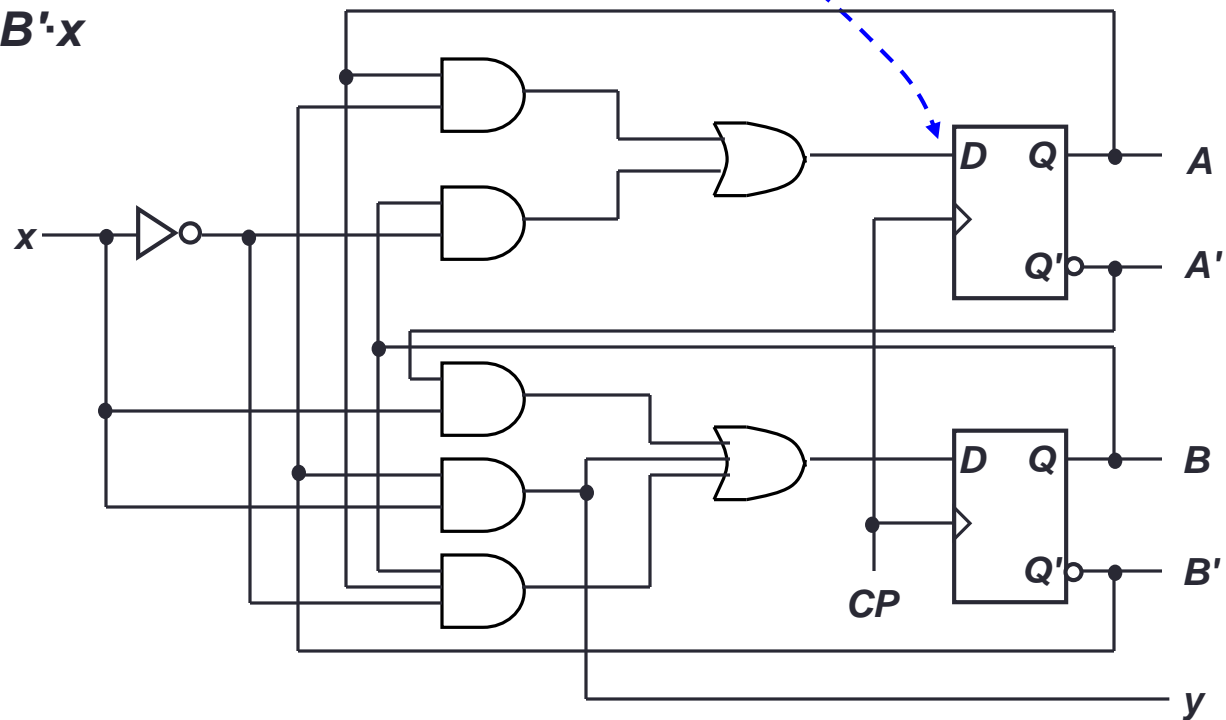
## 6.4 Design: Example #2 (3/3)

- From derived expressions, draw logic diagram:

$$DA = A \cdot B' + B \cdot x'$$

$$DB = A' \cdot x + B' \cdot x + A \cdot B \cdot x'$$

$$y = B' \cdot x$$



## 6.4 Design: Example #3 (1/4)

- Design involving unused states.

Present state				Next state				Flip-flop inputs							Output
state			Input		state										
A	B	C	x		A <sup>+</sup>	B <sup>+</sup>	C <sup>+</sup>	SA	RA	SB	RB	SC	RC		y
0	0	1	0		0	0	1	0	X	0	X	X	0		0
0	0	1	1		0	1	0	0	X	1	0	0	1		0
0	1	0	0		0	1	1	0	X	X	0	1	0		0
0	1	0	1		1	0	0	1	0	0	1	0	X		0
0	1	1	0		0	0	1	0	X	0	1	X	0		0
0	1	1	1		1	0	0	1	0	0	1	0	1		0
1	0	0	0		1	0	1	X	0	0	X	1	0		0
1	0	0	1		1	0	0	X	0	0	X	0	X		1
1	0	1	0		0	0	1	0	1	0	X	X	0		0
1	0	1	1		1	0	0	X	0	0	X	0	1		1

Given these

## Derive these

Are there other unused states?

Unused state 000:

[illegible]

## 6.4 Design: Example #3 (2/4)

- From state table, obtain expressions for flip-flop inputs.

**SA =  $B \cdot x$**

$AB \backslash Cx$		$C$			
		00	01	11	10
$A$	00	X	X		
	01		1	1	
	11	X	X	X	X
	10	X	X	X	

$x$

$B$

**RA =  $C \cdot x'$**

$AB \backslash Cx$		$C$			
		00	01	11	10
$A$	00	X	X	X	X
	01	X			X
	11	X	X	X	X
	10				1

$x$

$B$

**SB =  $A' \cdot B' \cdot x$**

$AB \backslash Cx$		$C$			
		00	01	11	10
$A$	00	X	X	1	
	01	X			
	11	X	X	X	X
	10				

$x$

$B$

**RB =  $B \cdot C + B \cdot x$**

$AB \backslash Cx$		$C$			
		00	01	11	10
$A$	00	X	X		X
	01		1	1	1
	11	X	X	X	X
	10	X	X	X	X

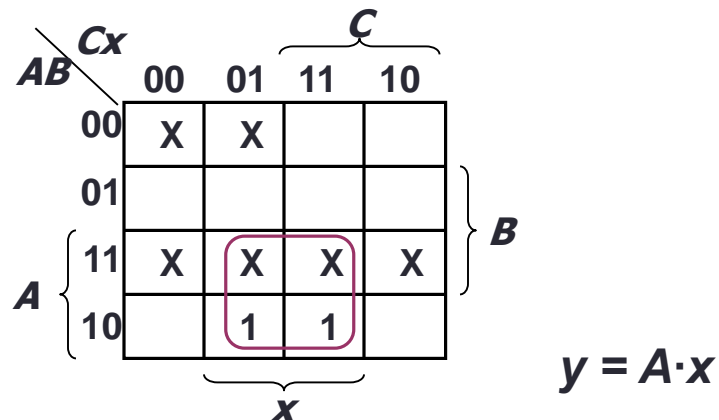
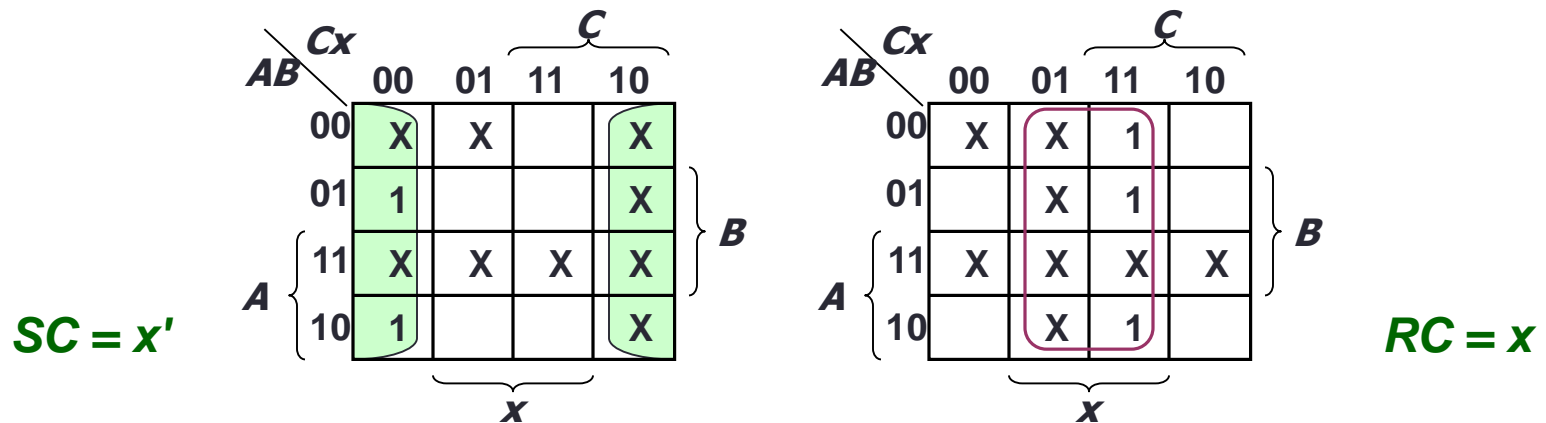
$x$

$B$



## 6.4 Design: Example #3 (3/4)

- From state table, obtain expressions for flip-flop inputs (cont'd).



## 6.4 Design: Example #3 (4/4)

- From derived expressions, draw the logic diagram:

$$SA = B \cdot x$$

$$RA = C \cdot x'$$

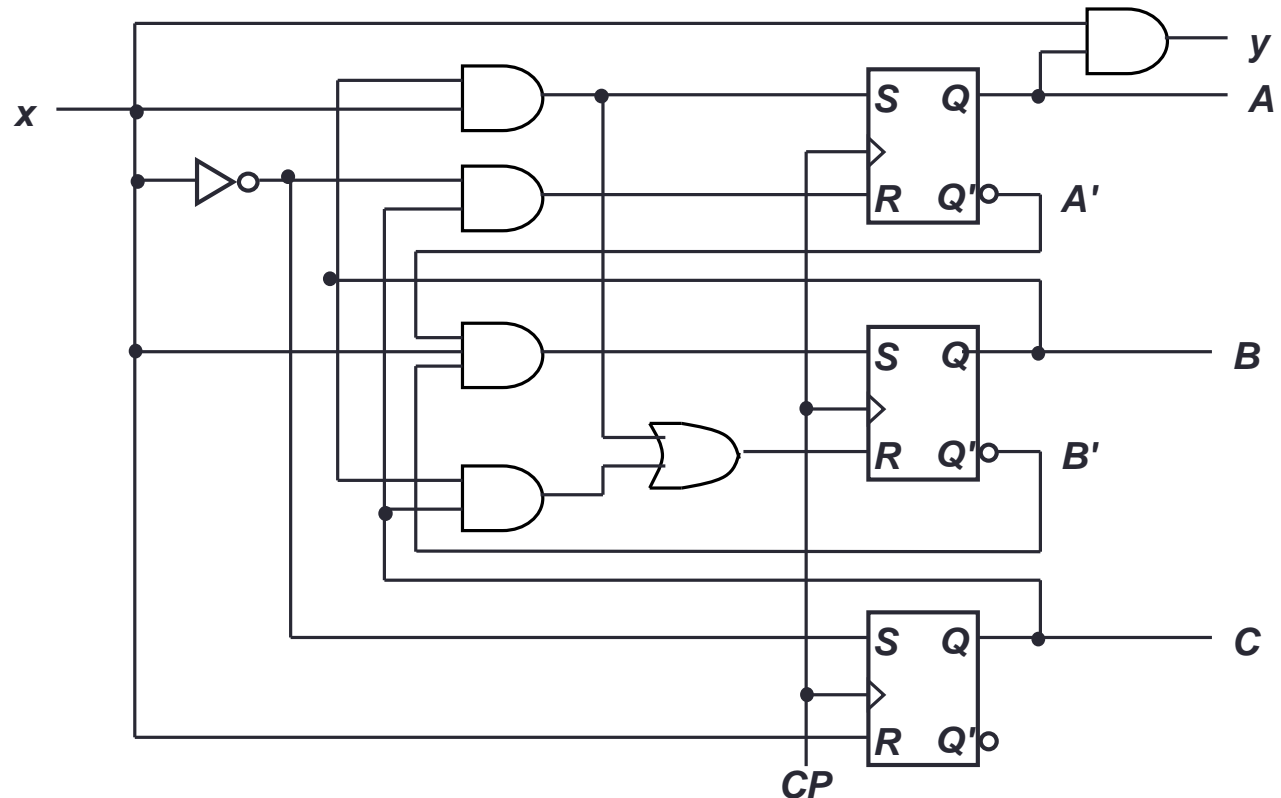
$$SB = A' \cdot B' \cdot x$$

$$RB = B \cdot C + B \cdot x$$

$$SC = x'$$

$$RC = x$$

$$y = A \cdot x$$

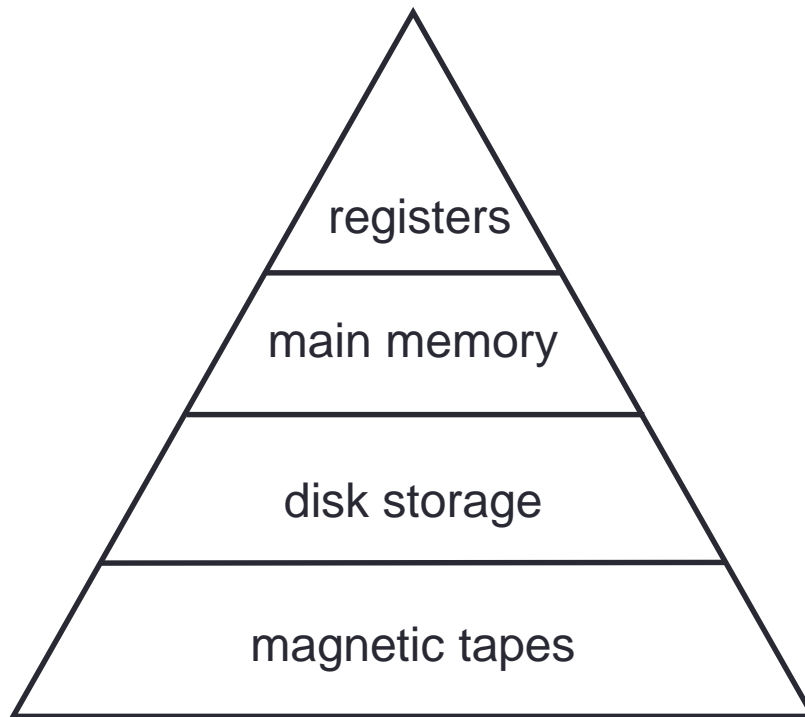


## 7. Memory (1/4)

- Memory stores programs and data.
- Definitions:
  - 1 byte = 8 bits
  - 1 word: in multiple of bytes, a unit of transfer between main memory and registers, usually size of register.
  - 1 KB (kilo-bytes) =  $2^{10}$  bytes; 1 MB (mega-bytes) =  $2^{20}$  bytes; 1 GB (giga-bytes) =  $2^{30}$  bytes; 1 TB (tera-bytes) =  $2^{40}$  bytes.
- **Desirable properties:** fast access, large capacity, economical cost, non-volatile.
- However, most memory devices do not possess all these properties.

## 7. Memory (2/4)

### Memory hierarchy



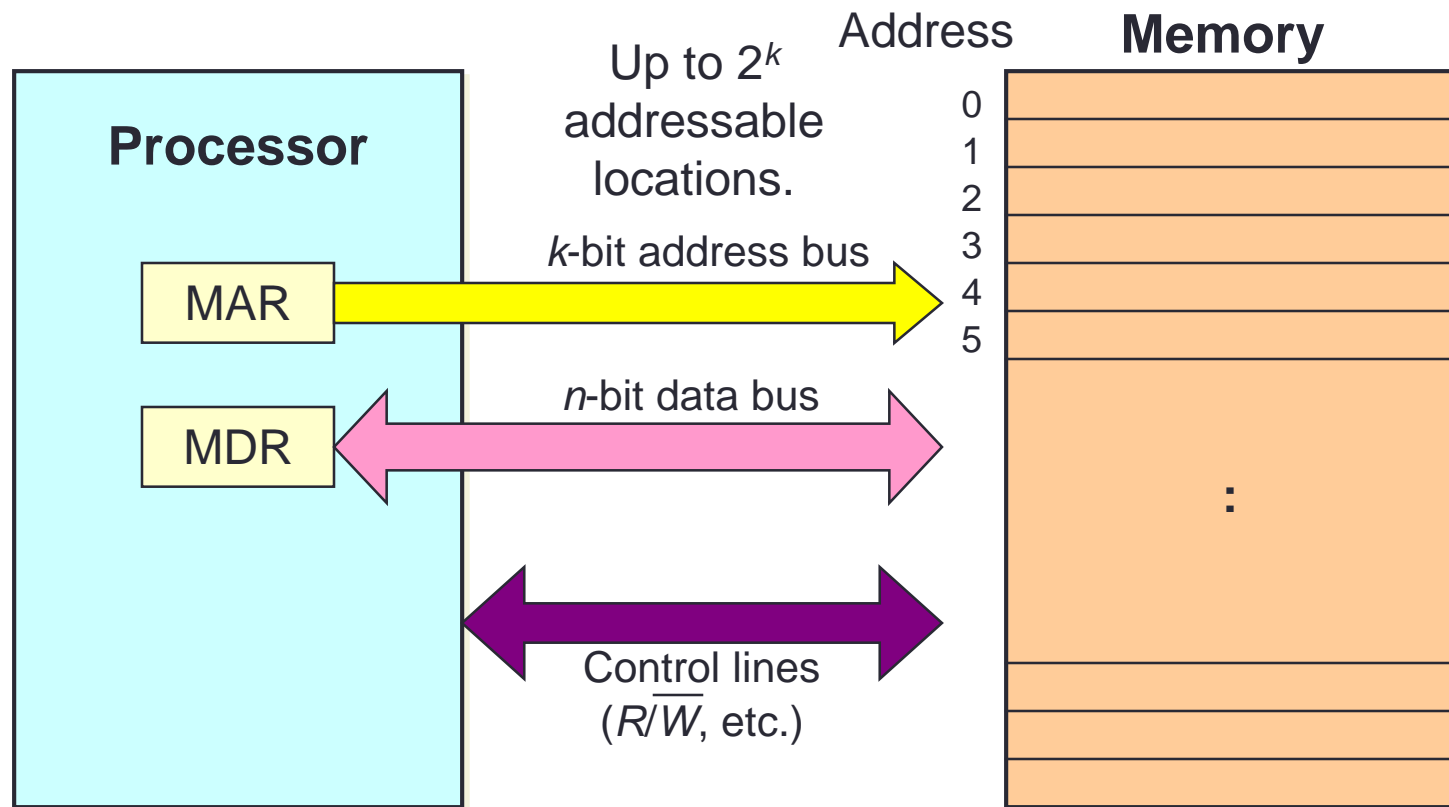
Fast, expensive  
(small numbers),  
volatile



Slow, cheap  
(large numbers),  
non-volatile

## 7. Memory (3/4)

### Data transfer

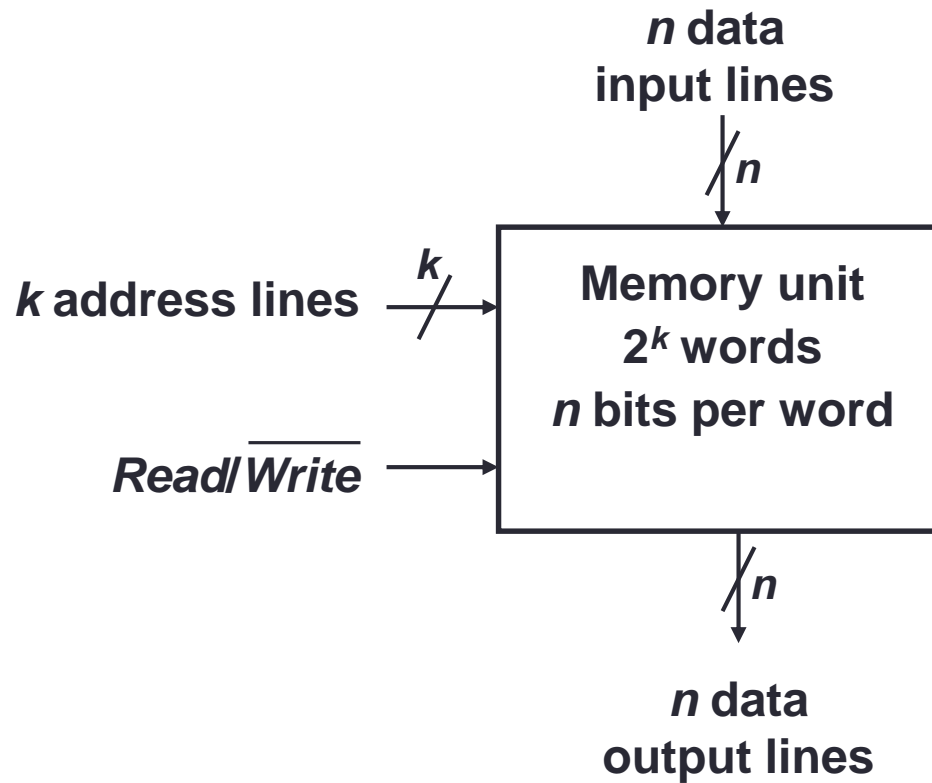


## 7. Memory (4/4)

- A memory unit stores binary information in groups of bits called *words*.
- The data consists of  $n$  lines (for  $n$ -bit words). **Data input lines** provide the information to be stored (*written*) into the memory, while **data output lines** carry the information out (*read*) from the memory.
- The **address** consists of  $k$  lines which specify which word (among the  $2^k$  words available) to be selected for reading or writing.
- The control lines *Read* and *Write* (usually combined into a single control line *Read/Write*) specifies the direction of transfer of the data.

## 7.1 Memory Unit

- Block diagram of a memory unit:



## 7.2 Read/Write Operations

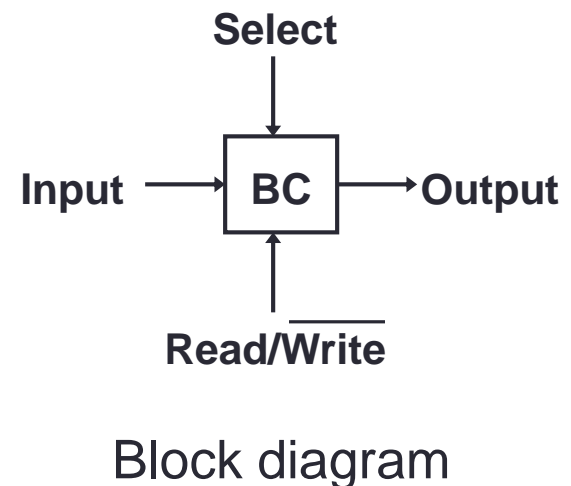
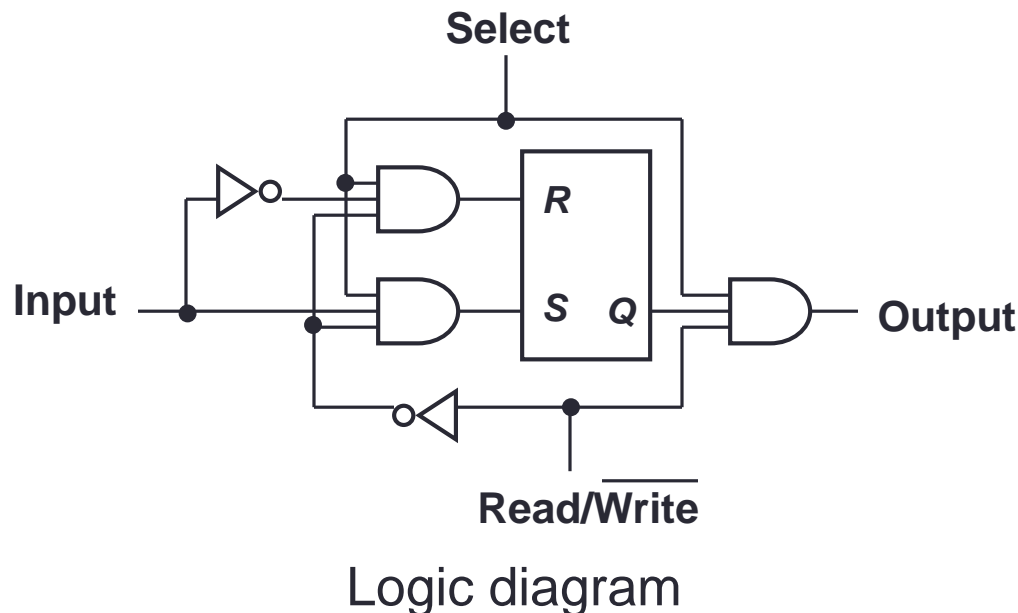
- **Write** operation:
  - Transfers the address of the desired word to the address lines.
  - Transfers the data bits (the word) to be stored in memory to the data input lines.
  - Activates the *Write* control line (set *Read/Write* to 0).
- **Read** operation:
  - Transfers the address of the desired word to the address lines.
  - Activates the *Read* control line (set *Read/Write* to 1).

Memory Enable	<i>Read/Write</i>	Memory Operation
0	X	None
1	0	Write to selected word
1	1	Read from selected word



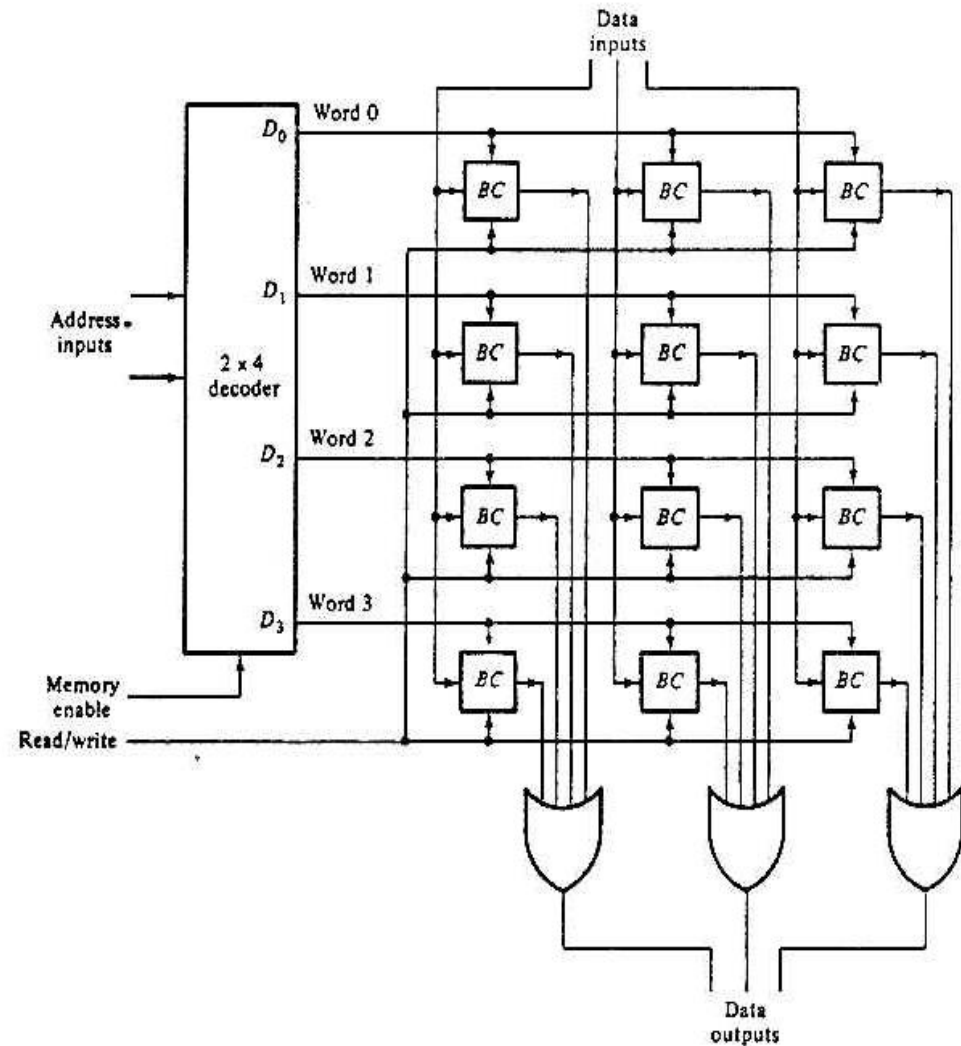
## 7.3 Memory Cell

- Two types of RAM
  - Static RAMs use flip-flops as the memory cells.
  - Dynamic RAMs use capacitor charges to represent data. Though simpler in circuitry, they have to be constantly refreshed.
- A single memory cell of the static RAM has the following logic and block diagrams:



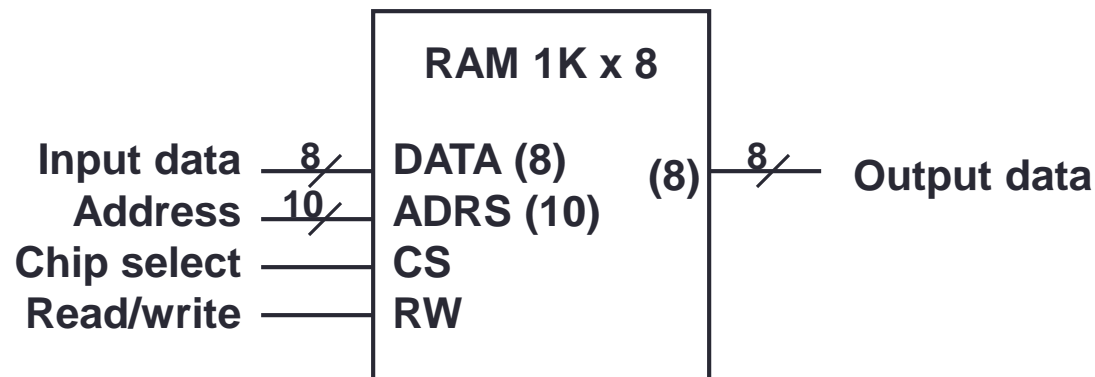
## 7.4 Memory Arrays (1/4)

- Logic construction of a **4×3 RAM** (with decoder and OR gates):



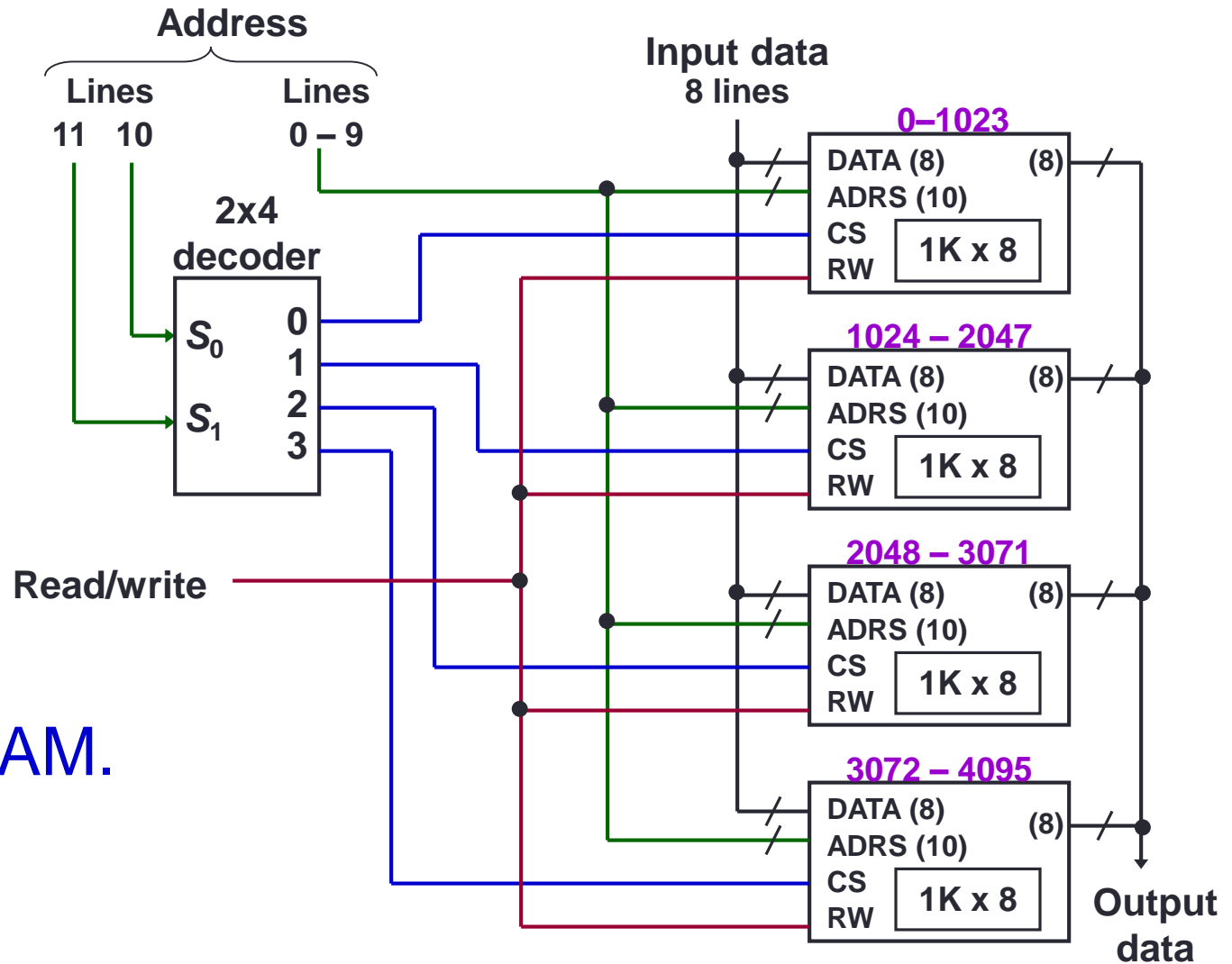
## 7.4 Memory Arrays (2/4)

- An array of RAM chips: memory chips are combined to form larger memory.
- A **1K × 8-bit RAM chip**:



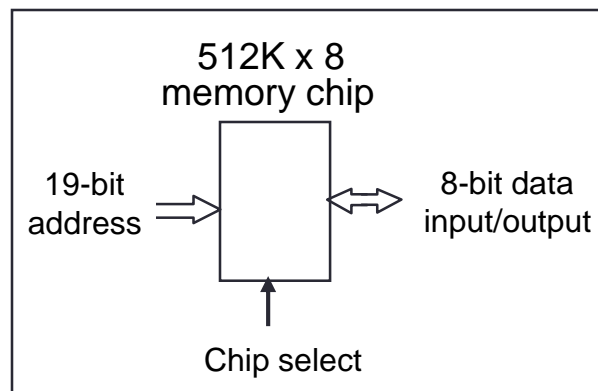
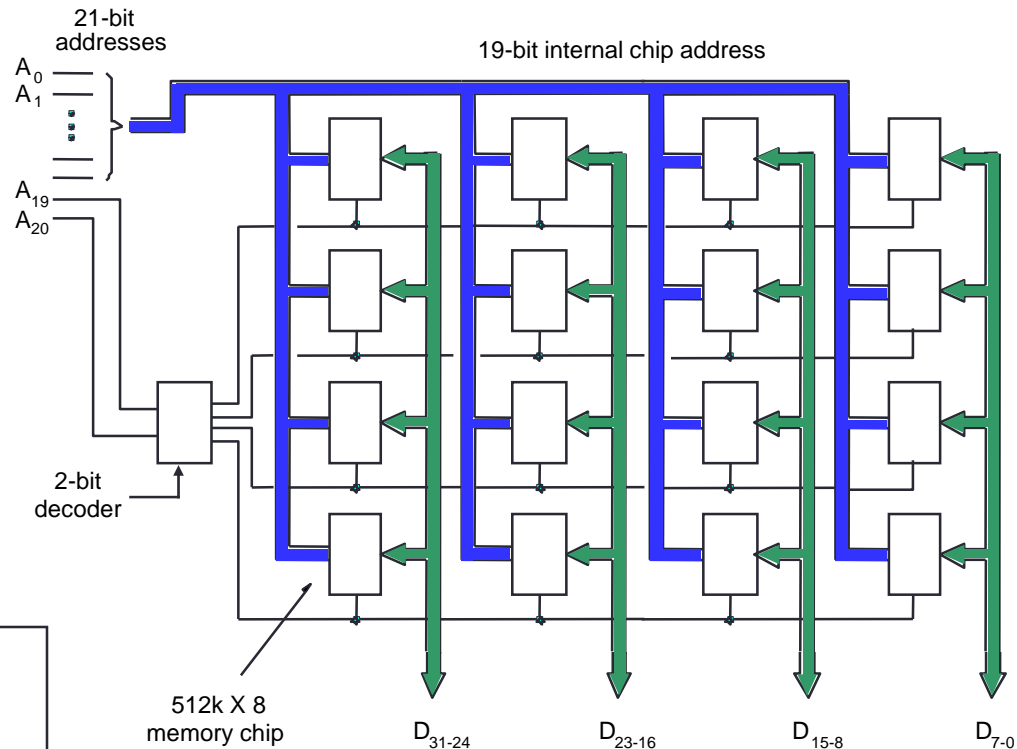
Block diagram of a 1K x 8 RAM chip

## 7.4 Memory Arrays (3/4)



- 4K × 8 RAM.

## 7.4 Memory Arrays (4/4)



- **2M × 32** memory module
  - Using 512K × 8 memory chips.

# End of File