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CS3243 Introduction to Artificial Intelligence
Assignment 2:

Prove that *any deterministic search algorithm* will, in the worst case, search the entire state space. More formally, prove the following theorem

Theorem 1. *Let $\mathcal{A}$ be some complete, deterministic search algorithm. Then for any search problem defined by a finite connected graph $G = \langle V, E \rangle$ (where V is the set of possible states and E are the transition edges between them), there exists a choice of start node s_0 and goal node g so that $\mathcal{A}$ searches through the entire graph G.*

Since the number of vertices are finite, and the search algorithm is deterministic, then if t

Inductive approach: set the start node to some arbitrary node s and the goal node to some arbitrary node g_1. Then run A on G with s and g_1. Let U_1 be the set of unvisited nodes. A will have searched through the entire graph of G if and only if U_1 is empty. If so, we are done. Otherwise, choose an arbitrary node g_2 in U_1 and repeat this process. In general, when we run A with starting node s and goal g_n, we construct U_n. Then, if $|U_n| = 0$ we are done, otherwise we choose $g_{(n+1)}$ from U_n. Since A is complete, all nodes are reachable. Since A is deterministic, $U_n \supseteq U_{(n+1)}$. Since $g_{(n+1)} \in U_{(n+1)}$ and $g_{(n+1)} \notin U_n$, then $U_n \supset U_{(n+1)}$ and $|U_n| > |U_{(n+1)}|$. Let the number of nodes in G be c, which must be finite. Then, inductively, there must be some $t \leq c$ such that $|U_1| > |U_2| > \ldots > |U_t| = 0$, since $U_1 \supset U_2 \supset \ldots \supset U_t$. In this case, there are no unvisited nodes and all nodes have been visited (A has searched through the entire graph G). The choices of start node and goal node that produces this result are s and G_t respectively.