National University of Singapore
School of Computing
CS3243 Introduction to AI

Tutorial 5: Constraint Satisfaction Problems

Issued: February 14, 2021 Discussion in: Week 7

Important Instructions:

- **Assignment 5** consists of **Question 1** from this tutorial.
- Your solutions for this tutorial must be TYPE-WRITTEN.
- You are to submit your solutions on LumiNUS by **Recess Week, Saturday, 2359 hours**.
- Refer to LumiNUS for submission guidelines

Note: you may discuss the content of the questions with your classmates (outside your group). But each group should work out and write up ALL the solutions individually. If caught plagiarising, you may be awarded an F Grade for the module.

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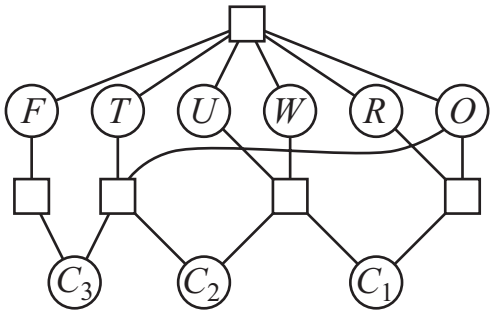
1. Consider the 4-queens problem on a 4×4 chess board. Suppose the leftmost column is column 1, and the topmost row is row 1. Let Q_i denote the row number of the queen in column i, $i = 1, 2, 3, 4$. Assume that variables are assigned in the order Q_1, Q_2, Q_3, Q_4, and the domain values of Q_i are tried in the order $1, 2, 3, 4$. Show a trace of the backtracking algorithm with forward checking to solve the 4-queens problem.

2. Show a trace of the backtracking algorithm with forward checking to solve the cryptarithmetic problem shown in Figure 1. Use the most constrained variable heuristic, and assume that the domain values (digits) are tried in ascending order (i.e., 0, 1, 2, $\cdots$).

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

(a)



(b)

Figure 1: Cryptarithmetic puzzle.

3. Consider the *item allocation problem*. We have a group of people $N = \{1, \ldots, n\}$, and a group of items $G = \{g_1, \ldots, g_m\}$. Each person $i \in N$ has a utility function $u_i : G \to \mathbb{R}_+$. The constraint is that every person is assigned *at most one item*, and each item is assigned to *at most one person*. An allocation simply says which person gets which item (if any).

   In what follows, you *must* use *only* the binary variables $x_{i,j} \in \{0, 1\}$, where $x_{i,j} = 1$ if person i receives the good g_j, and is 0 otherwise.

   (a) Write out the constraints: 'each person receives no more than item' and 'each item goes to at most one person', using only the $x_{i,j}$ variables[1].

   (b) Suppose that people are divided into *disjoint types* $N_1, \ldots, N_k$ (think of, say, genders or ethnicities), and items are divided into *disjoint blocks* $G_1, \ldots, G_\ell$. We further require that each N_p only be allowed to take no more than λ_{pq} items from block G_q. Write out this constraint using the $x_{i,j}$ variables. (Note that each N_i corresponds to the set of people who are of that person type.)

   (c) We say that player i *envies* player i' if the utility that player i has from their assigned item is strictly lower than the utility that player i has from the item assigned to player i'. Write out the constraints that ensure that in the allocation, no player envies any other player. You may assume that the validity constraints from (a) hold.

[1] You may use simple algebraic functions $-, +, \times, \div$, and numbers