National University of Singapore
School of Computing
CS3243 Introduction to AI

Tutorial 3: Informed Search

Issued: January 27, 2021 Discussion in: Week 5

Important Instructions:

- **Assignment 3** consists of **Question 5** from this tutorial.
- Your solutions for this tutorial must be TYPE-WRITTEN.
- You are to submit your solutions on LumiNUS by **Week 4, Saturday, 2359 hours**.
- Refer to LumiNUS for submission guidelines

Note: you may discuss the content of the questions with your classmates (outside your group). But each group should work out and write up ALL the solutions individually. If caught plagiarising, you may be awarded an F Grade for the module.

---

1. Prove that the tree-based variant of the A* search algorithm is optimal when an admissible heuristic is utilised.

2. Prove that the graph-based variant of the A* search algorithm is optimal when a consistent heuristic is utilised.

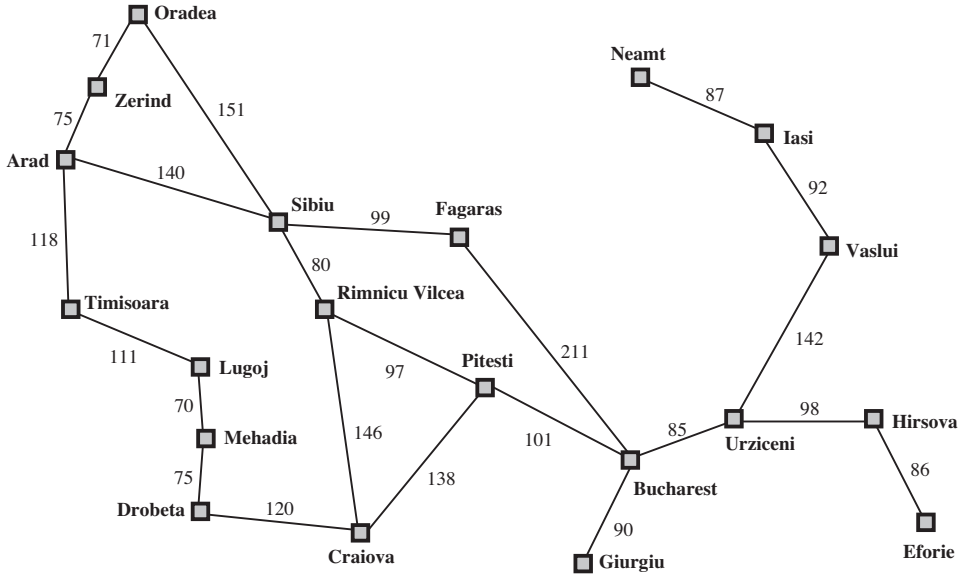
3. Refer to Figure 1 below. Apply the best-first search algorithm to find a path from Fagaras to Craiova, using the following evaluation function $f(n)$:

$$f(n) = g(n) + h(n)$$

where $h(n) = |h_{SLD}(\text{Craiova}) - h_{SLD}(n)|$ and $h_{SLD}(n)$ is the straight-line distance from any city n to Bucharest given in Figure 3.22 of AIMA 3rd edition (reproduced in Fig. 1).

(a) Trace the best-first search algorithm by showing the series of search trees as each node is expanded, based on the TREE−SEARCH algorithm below (Fig. 2).

(b) Prove that $h(n)$ is an admissible heuristic.



Values of h_{SLD} - straight-line distances to Bucharest

Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Figure 1: Graph of Romania.

function TREE-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

Figure 2: Tree search algorithm.

4. (a) Given that a heuristic h is such that $h(G) = 0$, where G is any goal state, prove that if h is consistent, then it must be admissible.

   (b) Give an example of an admissible heuristic function that is not consistent.

5. You have learned before that A* using graph search is optimal if $h(n)$ is consistent. Does this optimality still hold if $h(n)$ is admissible but inconsistent? Using the graph in Figure 3, let us now show that A* using graph search returns the non-optimal solution path (S, B, G) from start node S to goal node G with an admissible but inconsistent $h(n)$. We assume that $h(G) = 0$.

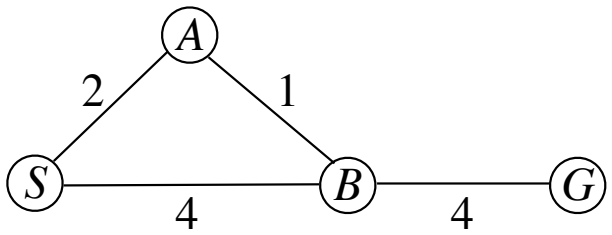


Figure 3: Graph.

Give nonnegative integer values for $h(A)$ and $h(B)$ such that A* using graph search returns the non-optimal solution path (S, B, G) from S to G with an admissible but inconsistent $h(n)$, and tie-breaking is not needed in A*.