# Dual-Axis Tilting Quadrotor Aircraft

An investigation into the overactuatedness thereof

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# Abstract


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The aim of this project is to design, simulate and control a novel quadrotor platform which can articulate all 6 Degrees of Freedom by vectoring the propellers' directional thrust. To achieve this the strucuret of the air-frame must redirect those thrust vectors to any desired orientation. This means it has to transform its configuration during flight, redirecting lift actuators whilst still maintaining stable attitude & position control in spite of such relative motion. In view of this required articulation the proposal is to add 2 axes of extra actuation to each propeller. The effect of which is that each lift propeller can then be pitched or rolled relative to the body frame. This change, to what is an otherwise well understood and highly researched platform, produces an over-actuated control problem. Actuator allocation is the primary contribution of this paper with novel elements of non-linear (*state-space*) attitude control.

The structure of the dissertation first presents the design which the subsequent dynamics and control are derived with respect to. Following that the kinematics associated with rigid bodies are derived. Any unique effects that could apply to the design like gyroscopic, inertial and aerodynamic responses are investigated and then incorporated into the dynamics. Possible position and control algorithms are simulated and compared based on the plants' dynamics (*which include discretization effects on the system*). The relative performance of the controllers are evaluated but regular performance metrics for attitude and position control are ill-suited for such a system. Some time is spent discussing the consequence of this and how the controllers are actually evaluated. Finally the design is built and tested using readily available RC components and conclusions drawn on the success or failure of the design.

The purpose of the investigation is the practicality and feasibility of such a design, most importantly whether the added complexity of the mechanical system is a decent compromised for the added degrees of control actuation. The outcome of the build is ascertain if it's both economically (cost and control effort) feasible and practical to build such a prototype. The design and control treatment presented here are by no means optimal or the most exhaustive solutions, focus is placed on the system as a whole and not just one aspect of it.

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# Introduction

## 1.1 Foreword

### 1.1.1 A Brief Background to the Study

A popular topic for current control and automation research is that of quadrotor UAVs. Attitude control of a quadrotor poses a unique 6-DOF control problem, to be solved with an under-actuated 4-DOF system. As a result the $\phi$ pitch and $\theta$ roll plants aren't controllable. The attitude plant is often simplified around a stable operating point. The trimmed operation region is always at the inertial frames' origin; resulting in a zero-set point tracking problem. The highly coupled non-linear dynamics of a rigid body's translational and angular motions arise from gyroscopic torques [Section: 3.2.1] and Coriolis accelerations [Section: 3.2.2]. These effects are negligible around the origin[1], hence the origin trim point removes the systems' non-linear complexities. The control system can then reduce each state variable, $\vec{X}_b = \left[\phi\ \theta\ \psi\ x\ y\ z\right]^T$,to an isolated SISO plant.

As almost every recent quadrotor research paper will mention, the latest interest in the platform is because of recent emergences in availability of MEMS systems and low-cost microprocessors. Such advancements allow for real time state estimation and on-board control loop processing, all at a relatively low cost. Developmental progress in quadrotor and, to a lesser extent, the general UAV field has led to rapidly growing enthusiast community. Companies like HobbyKing [27] are now synonymous with providing custom made DIY hobbyist quadrotor kits, not just ready to fly commercial products like the DJI Phantom [17].

The avenues for potential application of both fixed wing and VTOL UAVs is expansive, supporting civil [50], agricultural [52] and security [35] industries. Quadrotor configurations provide a mechanically simple and low cost platform on which to test advanced aerospace control algorithms. Commercial drone use in industry is already a prolific emerging sector; especially in Southern Africa. Subsequently following the $8^{th}$ amendment of civil aviation laws [54], use of UAVs for commercial application has now been legalized. Any research into a non-trivial aspect of the field is going to extremely valuable.

Larger scale quadrotor, hexrotor and even octorotor UAVs are a popular intermediate choice for aerial cinematography due to their high payload capacity. Whilst still expensive, the cost of a commercial drone like the SteadiDrone Maverik [40] is far less than chartering a helicopter to achieve the same panoramic aerial scenes or on-site inspections. Another interesting application for UAVs is in the agricultural and cartography surveying sectors. One foreseeable issue which may hinder commercial drone sector progress is the inertial effects associated with large aerial bodies. When scaling up any vehicle, its performance is adversely affected if actuation rates aren't proportionately scaled.

[1] Expanded upon in Appendix:A

### 1.1.2 Research Questions & Hypotheses

The difficulty with control of a quadrotor is that fundamentally it's unstable and under-actuated, *which is empirically proven later with Layupanov Theorem in Chapter:4.* The quadrotor has only four controllable inputs, namely propeller rotational speeds $\Omega_{1,2,3,4}$ which are then abstracted[2] to virtual control inputs net torque, $\vec{\tau}_{net} = [\tau_\phi\ \tau_\theta\ \tau_\psi]^T$, and a perpendicular heave thrust $\vec{T}_{net} = \sum_{i=1}^{4} T(\Omega_i)$. Those four inputs have to affect both the linear X-Y-Z positions and angular pitch, roll and yaw angles, $\vec{\Upsilon} = [\phi\ \theta\ \psi]^T$. As a result of the under-actuation the attitude control problem becomes a zero set point problem as any other attempt to track attitude cannot be achieved.

The aim of this project is to implement quadrotor attitude and position set point tracking by solving the problem of its inherent under-actuation. Inspired by Boeing/Bell Helicopters' V22 Osprey and the tilting articulation of its propellers, the prototype design proposed here introduces two additional actuators for each of the quadrotors' lift propellers. Specifically, adding rotations about the X and Y axes for each motor/propeller pair. The result is a vectored 3 dimensional thrust force rather than a bound perpendicular lift force. The control problem is then posed as the design of net forces, $\vec{F}_{net} = [F_x\ F_y\ F_z]^T$, and torques, $\vec{\tau}_{net} = [\tau_\phi\ \tau_\theta\ \tau_\psi]^T$, such that for any given trajectory, $X_d = [x\ y\ z\ \psi\ \theta\ \phi]^T$, the error state $X_e = X_d - X_b$ asymptotically tends to $\vec{0}$.

$$lim_{t\to\infty} X_e = \vec{0} \quad \forall X \in \mathbb{R}^n \tag{1.1}$$

Where $n$ is the degrees of freedom. The over-actuation brings about the need for a control allocation scheme which distributes the 6 commanded system inputs (net torques and forces) among the actuator set (12 actuators) in order to optimize some objective function secondary to that of Eq:1.1.

Part of the control research question is the multivariable treatment of the system without making any simplifications to the non-linear dynamics involved in the quadrotors motion or making any assumptions about its operational conditions. Standard linearizations applied to the quadrotors' control plant don't hold true for the more aggressive manoeuvres; they're dependent on small angle approximation. Stable control law design will require expansion and simulation of existing kinematic models describing an aerial body and applying them to a quadrotor motion. Thereafter, design, development and control of this new actuator suite which is to be implemented on a quadrotor platform. The final key outcomes for the project are the simulation analysis and prototype construction of the proposed design.

Introducing relative motion within an unconstrained body will produce a lot of unwanted dynamics like inertial and gyroscopic responses, amongst others. A rotating propeller will respond to pitching much like a Control Moment Gyroscope [69] or a flywheel. A less trivial result is the aerodynamic torque produced from the propellers' aerofoil profile. Such induced responses occur in planes perpendicular to whatever the propellers' rotation exists in. These aspects are normally canceled out due to a basic quadrotors' co-planar propeller rotation. It's anticipated that a plant dependent control solution will have to compensate for these dynamics, which if left unaccounted for could potentially cause instability.

### 1.1.3 Significance of Study

Due to the huge popularity of quadrotor platforms as research tools, any work that improves the UAV & quadrotor general body of knowledge will prove to be valuable. With that being said, there is already a vast amount of existing research on linear and non-linear control techniques for regular quadrotor platforms. The attitude loop is the most common topic for control research, requiring an under-actuated solution and mostly linearized around the origin (See Appendix:A). Far less common is the application of optimal flight path and trajectory planning to quadrotor control. The uniqueness and difficulty of the quadrotor attitude control does not hold true for its position control, so standard

[2] The abstraction of which is explored in Appendix:A

techniques can be used for way point planning and the like once the attitude control problem has been answered.

The most significant aspect of this project is the attitude control, discussed later in Section:4.1. The over-actuation of the proposed design and, more critically, the manner in which the controllers' (virtual) output is distributed among those control effectors would appear to be the first of its kind. Otherwise known as control allocation, the requirements of the distribution algorithm(s) are outlined in Section:4.3. Dynamic set point attitude control for aerospace vehicles is not a subject heavily researched outside the satellite attitude control field. Even papers which propose similarly complex mechanical over-actuation (expanded upon in next in the literature review, Section:1.2) hardly broach the topic of attitude set points away from the origin.

Whilst the control plant (developed in Chapter:4) does indeed close both the position and attitude control loops, there is no consideration of trajectory generation nor flight path planning. Such topics are well discussed elsewhere in a far more concise and deliberate way than this project could ever hope to achieve. Once closed loop position and attitude control has been achieved, the control algorithms can be adjusted to account for higher order state derivative tracking needed for nodal way point planning. The heuristics involved with flight path planning are well documented elsewhere and their implementation is an academic task.

Where possible the system identification and control (design and allocation) for this project is kept both modular and generically applicable. The intention here is that its pertinence falls not only within the UAV field but to any aerospace or free body attitude control. Hopefully this investigation can be expanded upon with more in-depth research on one of the subsystems without compromising the functionality of the remainder of the whole plant.

Provisionally one possible outcome which the investigation could yield is insight into higher bandwidth actuation and thus a faster control response for larger aerospace bodies. Any standard quadrotor uses differential thrust to develop a torque about its body which suffers a second order inertial response when the propellers accelerate or decelerate, $\tau_{simplified} = \mathbb{I}_f \ \dot{\omega}_i$. Prioritizing pitching the propellers' principle axis of rotation rather than changes to the propellers speed could potentially improve the virtual control response. This is entirely dependent on how the allocator block is prioritized (presented in Section:4.3).

### 1.1.4 Scope and Limitations

#### Scope

Critical to this project is the conceptualized design and prototyping of a novel actuation suite to be used on a quadrotor platform. The express purpose of which is to apply set point attitude tracking control to the body. Stemming from this is an investigation into the kinematics that are potentially influenced by the design and the structures' relative motion. In order to apply correct control theory to achieve the attitude tracking on a physical prototype, the plant dynamics must first be identified so that plants responses can be approximated with confidence.

Aspects of the mechanical design are covered next in Section 2.2 but, beyond the cursory investigation, there is no scope for materials analysis or stress testing of the design. To the detriment of the project, the design will either produce an over-engineered or catastrophically under-engineered solution. The scope focuses mainly on the control application and embedded systems design, not the structural integrity of a proposed frame given the forces it may undergo. Physical measurements are only made for critical kinematics, such as inertial measurements for the second order gyroscopic and inertial dynamic responses.

As mentioned in the antecedent , Section: 1.1.3, trajectory & flight path planning are not ubiquitous with this dissertation. Derivations for the differential equations which dictate a 6-DOF body's movement are wholly applicable to any dynamic (rigid or otherwise) aerospace body, although some particular standards are used [sic Z-Y-X Euler Aerospace Sequence, 2.1]. Similarly the control plant is stabilized with non-linear state space control techniques, aided and justified by Lyupanov theorem. Alternative solutions through Model Predictive Control or Quantitative Feedback Theory could provide more refined or effective controllers, they aren't presented and remain open to further investigation. Quadrotor attitude control is commonly stabilized with feedback linearizations, decoupling plant around a trim point so that SISO techniques can be applied. A derivation of such a linearization is included in Appendix:A but beyond that there is no further discussion. Any comparison between non-zero and zero-set point attitude control of quadrotor is difficult as the fundamental objectives are in stark contrast with each other.

Arguably the most important and indeed novel aspect of this project is the control allocation. The system has 12 plant inputs and 6 output variables to be controlled and so there exists a family of actuator set $u \in \mathbb{U}$ solutions for each commanded input. Such a plant is classified as over-actuated. Ergo, there must be some logical process as to how those 12 inputs are articulated to achieve the desired 6 movements. Appropriate techniques are first investigated in Section:4.3 and compared before a final solution is implemented in Chapter:**??**. It is by no means a comprehensive investigation of every possible allocation scheme but rather an analysis of the sub-set of problems and design of what is regarded as a logical and pertinent approach.

With regards to the actual prototype design, in Section 2.2, it's assumed that certain aspects are a given certainty. Particularly the state estimation, updated through a 4-camera positioning system fused with a 6-axis IMU through Kalman Filtering, is assumed to precise and readily disposable at a consistent 50 Hz. Hence state estimation is included but is bereft of intricate detail, this is another topic which remains open to further investigation.

### Limitations

The biggest constraint faced by the design is the net weight of the assembled frame. Lift forces required to keep the body aloft are obviously dependent on the all up weight. Conventional wisdom has it that steady state actuator rates ought to be far less that saturation conditions. For stability to be guaranteed at all feasible operating conditions, the actuators must have sufficient headroom to still effect the desired control inputs. Conversely the structures' net weight is mostly dependent on the lift motors, being the heaviest part of the vehicle. A trade-off between these two factors makes designing the prototype a balancing act of compromise; added actuation is needed to produce the desired thrust vectoring. That added actuation is going to increase the weight which then requires more thrust force to ensure the vehicle remains airborne. Larger motors then need stronger actuators to effect the relative motion and overcome the bodies inertial response. It's a compromise between the weight of the body and the strength/quality of the actuation.

To forego the deliberation detailed above, a self imposed limitation applied to the design is to only make use of a predetermined motor, specifically a set of four Turnigy DST-700 BLDC motors. The Department of Electrical Engineering at University of Cape Town has a surplus of these from previous projects so it saves having to spec and purchase new motors. A direct consequence of this decision is that the net thrust disposable for actuation is limited to around 700g, $\approx 6.9$ N, per motor (see Section: 3.3.1). All other aspects of the prototype must adhere to this limitation. It's critical to ensure the control block doesn't induce over-saturation of the motor actuation, so the frame weight needs to be around 60-70% of the maximum available thrust. Saturation conditions are detailed later in Section: 4.3.

Another aspect of limitations produced by design decisions made, mostly to reduce prototype costs and complexity, is to use of 180° rotation servo motors. The servos are for individual motors' $\vec{X}_{M_i}$ axis pitch and $\vec{Y}_{M_i}$ axis roll actuations and act in lieu of BLDC or stepper motors. Any rotation beyond 360° requires closed loop position control and, unlike servos, will need slip rings to transmit power despite rotational movement. However the logistics of implementing such a design whilst maintaining an acceptable weight is almost impossible. Such an implementation is going to dramatically scale up the size of the prototype to accommodate for weight increases. Commercial camera stabilizing gimbals already make use of similar configurations but the I/O requirements from the flight controller $\mu$C already constricts the amount of expansion available.

Discrete elements for the whole system can potentially limit performance but are going to be mitigated if possible. For example analogue servos have an associated $1ms$ deadband from their $20Hz$ refresh rate. That can be addressed by using faster, albeit more expensive, digital servos which sample at 333 Hz. The prototypes' flight controller has to provide 12 PWM output compare channels for the 8 servos and 4 BLDC speed controllers. State updates from a ground control station and a fail safe 6Ch RC receiver module also need to be processed by the $\mu$C system. Particular attention is paid to the embedded system layout in Section:2.3.

## 1.2 Literature Review

### 1.2.1 Existing & Related Work

The field of transformable aerospace frames is not necessarily a new one, with many commercial examples having seen a lot of success over their operational life span. The most notable tilting-rotor vehicle is that of the Boeing/Bell V22 Osprey aircraft. First introduced in the field in 2007, the Osprey has the ability to pitch its two lift propellers forward to aid translational flight after vertically taking off or landing. In addition to this there have been a few papers published on similar tilting bi-rotor UAVs' (Fig: 1.1)[3] for research purposes.

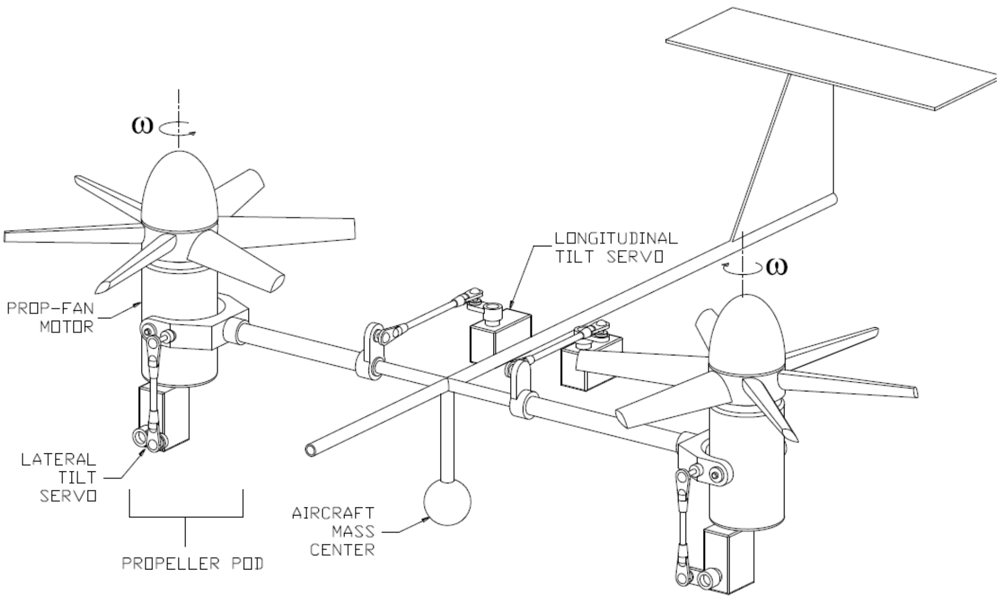



**Figure 1.1:** General Structure for Opposed Tilting Platform

[3]Image used from G. Gress: [23]

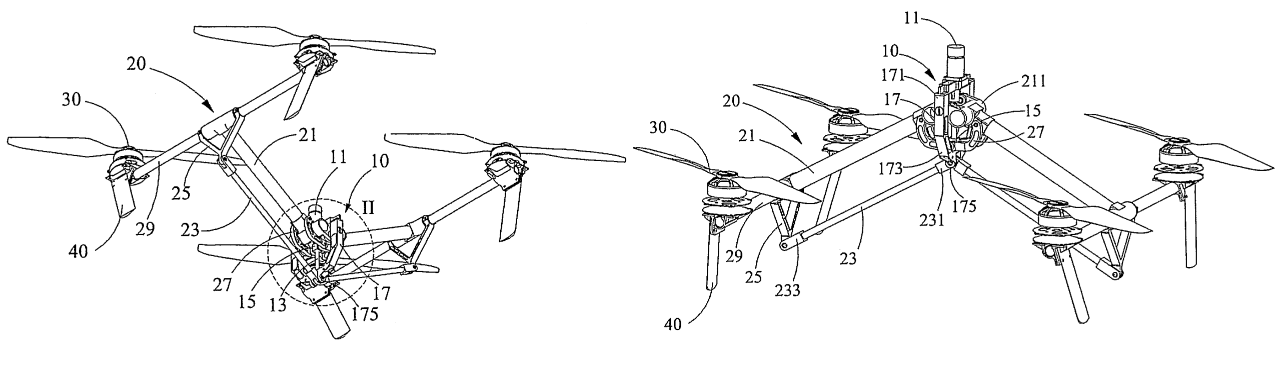
### Birotors

Research into birotor vehicles (Fig: 1.1) with ancilliary lift propeller actuation is often termed Opposed Active Tilting, *OAT*. Such a rotorcrafts' mechanical design applies either a single *oblique* 45° tilting axis relative to the body; [7,24,31], or a *lateral* tilting axis, adjacent to the body; [14,32,51,63]. Leading research is currently focussed on applying doubly actuated tilting axes to birotor UAVs. Dual axis Opposed Active Tilting or *dOAT* introduces vectored thrust with propeller pitch and roll motions to further expand the actuation suite, [2, 23]. A birotor is sometimes considered preferable to the multirotor platform due to its reduced controller effort. However the controller plant abstraction often detracts from the quality and effectiveness of its stability solution as a result of its' underactuation.

Birotor attitude control typically incorporates plant independent PD [7] and PID [51] controller schemes. Occasionally more computationally exhaustive and plant dependent Ideal and Adaptive backstepping controllers are exploited, presented in [31, 63] and [32] respectively. The coupling of a birotor vehicles' attitude system is more pronounced than that of a quadrotor, derived in Section: 3.2, and so feedback linearisation is almost always used. In an interesting progression from the norm, Lee et al, [37], proposed a PID co-efficient selection algorithm for a bi-rotor control block. Using a Particle Swarm Optimization techinque, similar to [71], the coefficients were globally optimized around a cost function. However their performance metric criterion is a basic ITAE[4] term and nothing more appropriate involving effects unique to flight systems. *PSO* algorithms iteratively search for a globally optimized solution and offer independent, derivative free optimization. Later on non-linear controller coefficient are also optimized using a PSO algorith, shown in Section:5.1.

### Quadrotors

Expanding on multirotor vehicles, the quadrotor UAV is a popular and well researched platform due to its mechanical simplicity. What would appear to be one of the first quadrotor research implementations, in 2002, is the X4-Flyer quadrotor, [26, 56]. Alternative iterations like the Microraptor [58] and STARMAC [28] quadcopters have been built and tested. A plethora of literature exists around quadrotor kinematics & control [4,10,15,41,57], however dedicated rigid body dynamic papers [43,53] provide better explanations of the kinematics. Often the plants' dynamics are simplified around an origin trim point and assumed to reduce into 6 SISO plants for each degree of freedom, Section: 3.1 and Appendix: A. Lately research projects incorporate aerodynamic effects like drag and propeller BEM theory into the plant model [12, 28, 60]. Although commonly neglected under standard opperating conditions, the higher fidelity models offer more precision without linearisations or assumptions, [5, 28].




**(a)** Inspire 1 articulated upwards

**(b)** Inspire 1 articulated downwards

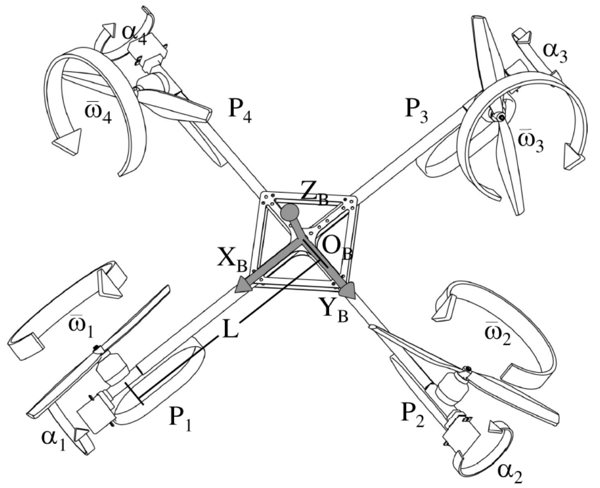
**Figure 1.2:** DJI Inspire 1

The only commercial example of a transforming quadrotor is the DJI Inspire1 [18], made by Shenzen

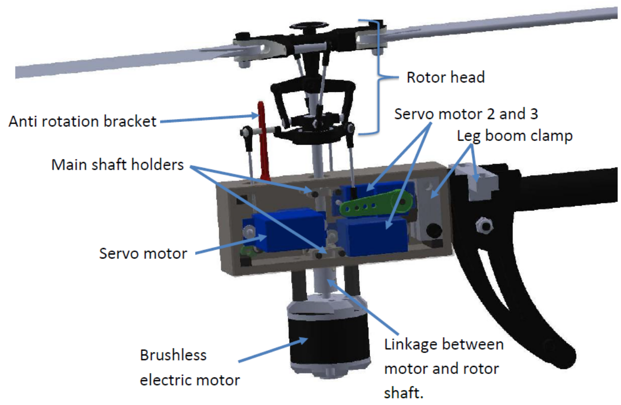
[4]Integral Time Additive Error

DJI Technologies (better known for the popular DJI Phantom drone). The Inspire can articulate its supporting arms up and down as shown in Fig:1.2 [5]. The purpose of such movements is to both alter the center of gravity and further expose a belly mounted camera gimbal for panoramic viewing angles. This transformation changes the moment of inertia about the body's center of gravity, in turn changing the inertial torque response induced by angular movements, an otherwise detrimental effect which makes researchers apprehensive of transformable aerospace frames. The range of "transformation" the frame can undergo is just limited to articulating the arms up and down.

In a similar fashion to the progression seen in birotor state-of-the-art, quadrotor research is engaging the topic of single and dual axis tilting articulation. First conceptualized and implemented on a prototype related to an ongoing project covered in two reports, [61, 62]. The authors M. Ryll et al.(2012, 2013) modified and tested a QuadroXL four rotor helicopter, propduced by MikroKopter [21], to actuate a single axis of tilt aligned with the frames arms (Fig:1.3a)[6]. Their proposed control solution, discussed in detail next in Sub-section:1.2.2, assumes no nominal linearised conditions around hover flight unlike a similar single axis tilting quadrotor prototype designed by Nemati, et al. (2012) [48]. The latter remains simulated but as yet untested.




**(a)** Single rotation axis aligned with the frames arm




**(b)** Cyclic-pitch & swashplate mechanism

**Figure 1.3**

An approach to improving quadrotor flight response is to alter the manner in which the thrust is actuated. Drawing from helicopter design, a project by Napsholm, (2013) [47], purported a quadrotor UAV that used swashplates for varying the propeller pitch and generating torque moments. The aim was a design which wasn't dependent power electronics to actuate variable thrust forces. Petrol motors were intended for use in place of BLDC motors. Furthermore, the design proposed a single axis of tilt actuation to each of the four motor modules. Whilst mechanically complex, Napsholm made use of existing RC helicopter components to design a rotor actuation bracket (Fig:1.3b). The cyclic-pitch swashplate used [49] could apply torques, $\tau_\phi$ and $\tau_\theta$, about the propellers' hub,*principle axis of rotation*, by altering the blades angle of attack throughout its rotational cycle. The actuation rate of such a configuration is far faster than that of a differential torque produced rolling/pitching motion.

Irrespective of the strong initial design in the early stages of his project, it would appear that Napsholms' research suffered due to time constraints. The introductory derivation on aerodynamic effects and deliberation over the design provide clear insight into the projects goals. However the control solution and system architecture, electronic and software, are significantly lacking. An introductory proposal of an MPC attitude control system detracted from the comprehensive dynamics discussed.

[5]Both images were sourced from the drones' patent, held by SZ DJI Tech Co [70]
[6]Image sourced from Modelling and Control of a Quadrotor UAV with tilting propellers, [61]

The project ended before testing, simulation and results could be obtained. Unfortunately, despite the novel over-actuated design, there was no discussion given on how the allocation would be performed.

Finally, the most crucial research to mention is that of a project completed by Pau Segui Gasco [20], which was a dual presented MSc project with Yazan Al-Rihani [1]. At the time of writing, this would appear to be the only project published pertaining to *over-actuation* in aerospace bodies implemented on a quadrotor platform. The research was split between the two authors who completed the control/electronic design and the mechanical platform design for their respective dissertations. Shown in Fig:1.4[7], the dual-axis articulation is achieved using an RC helicopter tail bracket and servo push-rod mechanism; reducing the mass of the articulated component but limiting the range of actuation. Considering the propellers as a spinning flywheel, the induced gyroscopic response can then be treated as an actuator plant. The commanded virtual control is then distributed by weighted inversion among the actuator set. The whole project justifies the extra actuation as redundancy but doesn't necessarily prove how such a redundancy could be beneficial.

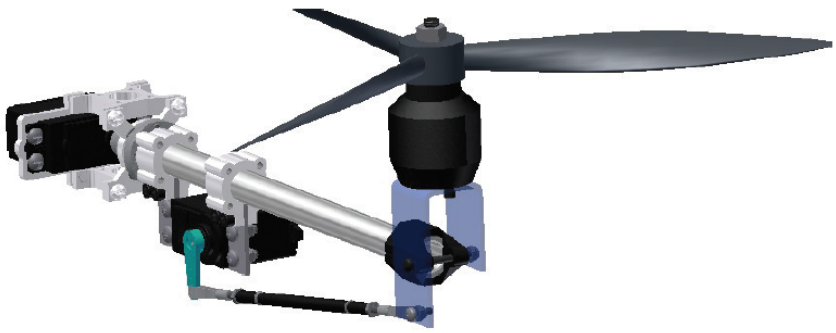


**Figure 1.4:** Dual-axis tilt-rotor mechanism

### 1.2.2 Notable Quadrotor Control Implementations

#### Quadcopter Attitude Control

Attitude control of a 6-DOF body is a topic best described by *The Attitude Control Problem* [66]. A rigid body that currently has an attitude state $\vec{E}_s$ and a desired state $\vec{E}_d$, the problem is to then find a torque control law:

$$\mu\tau = h(\vec{E}_s, \vec{E}_d, \vec{\omega}_s, \vec{\omega}_d) \tag{1.2}$$

Such that $lim\ \vec{E}_s \to \vec{E}_d$ and $lim\ \vec{\omega}_s \to \vec{\omega}_d$ asymptotically stabilize as $t \to \infty$. Depending on how the attitude is posed; with rotation matrices [34, 43, 53], quaternions [19, 22, 25, 34] or otherwise (Direct Cosine Matrix etc ...) the error sate $\Delta\vec{E}$[8]$= \vec{E}_d - \vec{E}_s$ could then differ to a (hamilton) multiplicative relationship. Note that here $\vec{E}$ is not necessarily an Euler set but any attitude representative state variable. Simulation and modelling papers often rely on Euler angle based rotation matrices for attitude representation, [9, 10, 42, 48, 59] without addressing the inherent singularity associated with such an attitude representation [sic Gimbal Lock, [64], Section:3.1.2]. The alternative quaternion attitude representation, first implemented on a quadrotor UAV in 2006 [65], is often used in lieu of rotation matrices but has its own caveat of *unwinding*, (Section:3.1.4), as a result of quaternions dual-coverage [45].

[7]Development of a Dual Axis Tilt Rotorcraft UAV: Modelling, Simulation and Control [20]

[8] *The Attitude Control* [66] describes these conventionally different error states

Quadrotor plant dynamics, as mentioned before, are often simplified; especially when represented with a 3-variable Euler angle set, $\vec{E} = [\phi\ \theta\ \psi]^T$. The coupled gyroscopic and Coriolis responses are both neglected when the angular velocity rate is small, $\vec{\omega}_b \approx 0$, and the inertial matrix is diagonal, $rk(\mathbb{I}_f) = x$ for $\in \mathbb{R}^x$. The consequence of which is the ineffectual deterioration of both the gyroscopic term, $\vec{\tau}_{gyro} = \vec{\omega}_b \times \mathbb{I}_b \vec{\omega}_b \approx 0$ and the Coriolis force term, $\vec{F}_{cor} = -\vec{\omega}_b \times \vec{a}_b \approx 0$ in the bodies dynamics [Chapter:3 for context]. Once the cross-product coupled terms are no longer of consequence, the 6 degrees of freedom, $[x\ y\ z\ \phi\ \theta\ \psi]^T$, can each be treated as an isolated SISO plant to be controlled with an appropriate technique. Quaternion represented attitude plants cannot easily be decomposed into individual single-input-single-output systems [quaternion dynamics in Section:3.1.3]. So a quaternion (combined four variable attitude state vector) is then used, $Q_b = [q_0\ \vec{q}\,]^T$ for the abstracted major loop plant.

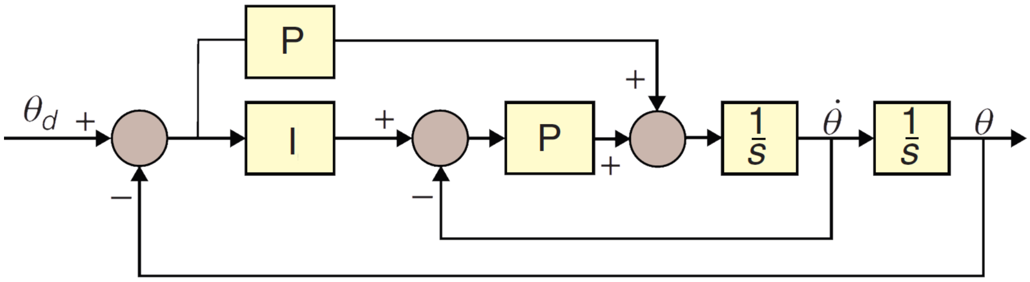



**Figure 1.5:** ArduCopter PI Euler Angle Attitude Control loop

Commercial flight controllers (Arducopter [3], Openpilot [38][9] etc ...) for custom fabricated UAV platforms all apply their own structured attitude controllers and state estimation algorithms, based on onboard hardware sensor fusion. The article *Build Your Own Quadrotor* [39] summarizes the control structures implemented on a range of common flight controllers. The most popular of which, the Arducopter, implements a feed-forward PI compensation controller (Fig:1.5)[10]. PI, PD and PID controllers are all easy and effective independent control solutions for general attitude plants once the systems have been linearised. Table:1.1 collectively lists the common attitude control blocks (not exclusively quadrotors UAVs but MAVs too) and which projects they've been implemented in, after which a critique on the more unique adaptations is given.

| Controller Type | Independent | Dependent | Total |
|---|---|---|---|
| PI | [66] | [66] | 2 |
| PD | [1,41] | [19,48] | 4 |
| PID | [9,11,57,61,66] | [28,59,66] | 8 |
| Lead | [15,56] | lead | 2 |
| IBC | [42,63][11] | [42] | 3 |
| ABC | [6,16,32,46] | | 4 |
| LQR | [11] | LQR | 1 |

**Table 1.1:** A Breakdown of common Attitude Controllers

In a collection of papers, written by Bouabdallah et al ... (2003,2004,2007) arguably the most prolific early quadrotor authors, a range of different control implementations are derived and reviewed. Their last paper (2007) [10] derived and pratically tested an Integral Backstepping attitude controller on an OS4 quadrotor. It builds on their research from an earlier paper (2003) [11] wherein an analysis of PID vs LQR attitude controllers in the context of quadrotors is posed. LQR controllers aim to optimize the controller effort (that being $||\mathbb{U}||$ or the $L_2$ norm of the plant input) and although, in theory, solving the assocaited Ricatti cost function may produce an optimial, stable and efficient control law it needs

[9] NOTE: OpenPilot's firmware branch is now maintained by LibrePilot
[10] Image sourced from *Build your own Quadrotor* [39]

exact plant matching. In practice, direct plant identification is difficult to achieve for a quadcopter or any aerospace body for that matter. The resultant controller in [11] achieved asymptotic stability but had poor steady state performance due to low confidence of the identified actuator dynamics and poor inertial measurements.

Adaptive Backstepping Control [68](any of the examples in Table:1.1) expands on nominal IBC fundamentals by introducing an added disturbance state term in the LCF used for the backstepping iteration. The drawback with this form of Backstepping approach is that, from the Lyupanov control theorem, a time derivative for the estimated disturbance (or an *update law*) is needed. Disturbance approximation has been investigated thoroughly but, for a signal without apriori information, some heuristic needs to be adopted with the approximation which usually involve compromise. In one example, [16], the authors implemented a statistical $proj(.)$ operator based technique. When used in adaptive control the projection operator [13], $proj(.)$, ensures a derivative based estimator is bounded for adaptive regression approxmation [55].

Although the control implementation isn't backstepping based, in [72], a sliding mode controller was used to compensate for the disturbances in an Unmanned Submersible Vehicle atttiude plant. The underwater current disturbances are approximated using a fuzzy logic system, specifically a *zero-order TSK* fuzzy controller. The TSK system has been proven to act in the same way as an Artificial Neural Network approximator [44]; where the TSK system is more comprehensible than the latter. Statistical analysis and investigation of approximators without a priori knowledge of a system are well beyond the scope of what this project hopes to achieve but are worth mentioning.

### Single/Dual Axis Control & Allocation

The extra actuation introduced with single and dual axis articulation provides room for extra control goals to be achieved as the order of actuation increases. Of the few papers published on tilting-axis quadrotors, PD controllers (Nemati et al.[2014] [48] and again in Gasco & Rihani [1, 20]) and PID controllers (Ryll et al.[2012,2013] [61,62]) are the norm for control blocks. For either of these systems there needs to be an allocation rule to distribute a commanded input amongst the actuator set. In [30], Johansen et al.[2012] describes the control allocation problem for a dynamic plant:
*Note in state space Equation:1.3a, it's assumed the plant input, $\tau$, has a multiplicative relationship with the response, $g(x,t,\tau) \iff g(x,t)\tau$.*

$$\dot{x} = f(x,t) + g(x,t)\tau \tag{1.3a}$$
$$y = l(x,t) \tag{1.3b}$$

With a state $x \in \mathbb{R}^n$ and $f(x,t)$ & $g(x,t)$ being the plants' dynamics and input responses respectively. In set point tracking, the output is then *tracking* the state $y = x$, and hence $y \in \mathbb{R}^n$. In an ideal well posed system the number of actuator inputs equals the number of controllable variable outputs; $dim(x) = dim(\tau) \in \mathbb{R}^n$. In the case where the input $\tau \in \mathbb{R}^m$, if $m > n$ the problem is then overactuated and a level of abstraction is needed; a virtual control input $\nu_d$ is designed by a control law $\nu_d = h(x_e,t)$ to affect dynamics. The goal is to then find a function that maps $\mathbb{R}^m \to \mathbb{R}^n$ for an actuator matrix $u \in \mathbb{U}^m$. An overactuated plant can be described as:

$$\dot{x} = f(x,t) + g(x,t)\nu_d,\ \nu_d \in \mathbb{R}^n \tag{1.4a}$$
$$\nu_c = B(x,t,u) \approx B(x,t)u,\ u \in \mathbb{U}^m,\ \nu_c \in \mathbb{R}^n \tag{1.4b}$$
$$y = x \tag{1.4c}$$

$B(x,t,u)$ is the effectiveness function which formulates how the actuator inputs $u$ relate to the virtual commanded input $\nu_c$. $B(x,t,u)$ can be abstracted to a multiplicative relationship $B(x,t)u$ if the plants' dynamics permit it, such that; $B(x,t) \in \mathbb{R}^{n\times m}$. For setpoint tracking the control law will design a desired virtual control input $\nu_d$, the allocation rule then has to solve $u$ for $\nu_c$ such that a slack variable $s = \nu_c - \nu_d$ is minimized:

$$\underset{u\in\mathbb{R}^m, s\in\mathbb{R}^n}{min} \|Q_s\| \text{ subject to } \nu_c - h(x_e,t) = \nu_c - \nu_d = s,\ u \in \mathbb{U} \tag{1.5}$$

Which ensures the commanded input $\nu_c$ tracks the desired control input $\nu_d$; $\nu_c \rightarrow \nu_d$ as per some cost function of the slack variable $Q_s$. Mostly the L2 norm, $\|Q_s\|$, is used. In an overactuated system it then follows that there is a set of inputs for each $\nu_c$. A unique actuator solution (rather than a family solution set) to Eq:1.5 needs a secondary objective function, $J(x, t, u)$. Eq:1.5 then becomes;

$$\underset{u \in \mathbb{R}^m, s \in \mathbb{R}^n}{min} (\|Q_s\| + J(x, t, u)) \text{ subject to } \nu_c - h(x_e, t) = s, u \in \mathbb{U} \tag{1.6}$$

Over-actuation is not something often applied to quadrotors and as a result rather than providing a comprehensive literature review of associated papers here (which are all mostly theoretical derivation), the contextual application and solutions to the above posed problems are expanded later in Section: 4.3.1. The only overactuated quadrotor (birotor dual-axis tilting makes the system critically actuated and so requires no allocation) literature which covers allocation of the given actuators is [1, 20], where the authors apply a weighted pseudo inverse (sic Moore Penrose Inverse [36]) allocation rule.

**Satellite Attitude Control**

Unconstrained attitude set-point tracking for 6-DOF bodies, quaternion based or otherwise, is a topic well covered in the field of satellite attitude control; [29, 33, 67]. The *status quo* for recent research is on non-linear adaptive attitude back-stepping control systems, wherein the adaptive update rule is the novel focus. Often plant uncertainty affects the inertia tensor of a satellite. In [29], the authors Wang Jia, et al. [2010], proposed applying adaptive back-stepping to compensate for steady state (asymmetric) inertial estimation errors. Alternatively, instead of deliberating on costly non-orbital prelaunch inertial measurements, [8] developed an algorithm for estimating the inertia tensor based on single axis controlled perturbations. However that does assume any initial estimates are sufficiently similar to the actual inertia of the body such that stability can be ensured.

Satellite actuator sets tend to include additional redundant effectors, to ensure fault tollerance and reliability, and hence require control allocation. Seen in the paper [33]; the authors, Kristiansen et al. [2005], address the over-actuation with direct and well-matched inversion before applying quaternion based back-stepping for attitude control. A quadratic pseudo inverse solution to Eq:1.6 is:

$$u = B^{\dagger}({\tau_a}^b - D{\omega_{ib}}^b) \tag{1.7a}$$

$$B^{\dagger} = B^T(BB^T)^{-1} \tag{1.7b}$$

Where $B$ is the control effectiveness matrix and $B^{\dagger}$ is such that $BB^{\dagger} = \mathbb{I}$. Specifically $B^{\dagger}$ is the general *pseudo* inverse of $B$ (more on inversions in Sec:4.3). It's assumed there's a multiplicative relationship between the input, $u \in \mathbb{U}$, and the input effectiveness matrix in Eq:1.4b. The controller designed actuator torque ${\tau_a}^b$ then dictates the input $u$ as in Eq:1.7a. Much like the over-actuation previously discussed W.R.T quadcopters; the pseudo inversion method of control distribution applies quadratic optimization to the allocation slack cost function, Eq:1.5.

# Chapter 2

# Prototype Design

## 2.1 Conventions Used

The attitude conventions used for the systems' dynamic derivations in the following Chapter:3 are first briefly discussed here. Often these aspects are omitted or assumed to be known already. It's important to clearly and unambiguously define a standard set of framing conventions to avoid uncertainty later. Rotation matrices are included but focus remains on the *contrast* between a rotation and transformation operation. Both [25] and [53] provide an in depth and thorough explanation of rotation matrices and DCM attitude representation if such concepts are unfamiliar to the reader.

### 2.1.1 Reference Frames Convention

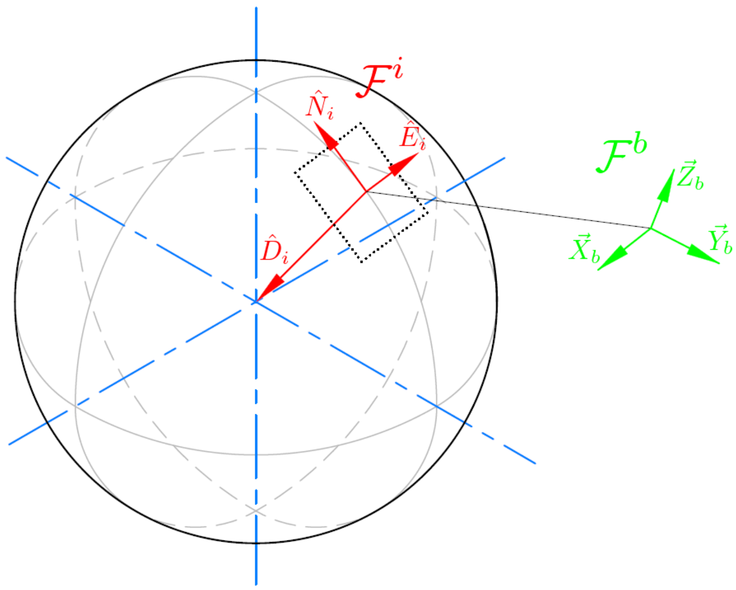



**Figure 2.1:** Inertial and Body Reference Frames

Euler (aerospace) frames are used for principle inertial and body coordinates (Fig:2.1). The inertial frame, $\mathcal{F}^i$, is aligned such that the $\vec{X}_i$ axis is in the $\hat{N}$orth direction, $\vec{Y}_i$ is in the $\hat{E}$ast direction and $\vec{Z}_i$ is in the $\hat{D}$ownward direction[1]. The body frame, $\mathcal{F}^b$, then has both $\vec{X}_b$ and $\vec{Y}_b$ aligned with two perpendicular arms of the quadrotors' body and the $\vec{Z}_b$ axis in the body's normal direction (Fig:2.4). The body frames' axes and their relation to the prototype design are highlighted next in Section:2.1.2.

[1]In orbital sequences this would be toward the Earths' center. Sometimes referred to as the NED convention

Frame superscripts $i$ and $b$ represent inertial and body frames respectively whilst vector subscripts imply the reference frame in which the vectors' coordinates exists in.

Relative angular displacement between two frames is commonly measured by the three angle Euler set. The Euler angles $[\phi\ \theta\ \psi]^T$ represents rotations about the $\vec{X}$,$\vec{Y}$ and $\vec{Z}$ axes respectively. Depending on how the rotation sequence is formulated, those angles can be used to construct rotation matrices which give relation to vectors or can transform coordinates. The generic equation to rotate a vector $\vec{v}$ about a (normalized) axis $\hat{n}$ by some angle $\mu$ is given by[2]:

$$\vec{v}\,' = \big(1 - cos(\mu)\big)\big(\vec{v} \cdot \hat{n}\big)\hat{n} + cos(\mu)\vec{v} + sin(\mu)\big(\hat{n} \times \vec{v}\big) \tag{2.1}$$

Which, when $\hat{n}$ is either $\vec{X}$,$\vec{Y}$ or $\vec{Z}$ axes, can be simplified to produce the common rotation matrices $\mathbb{R}_x(\phi)$,$\mathbb{R}_y(\theta)$ and $\mathbb{R}_z(\psi)$. Multiplication by a rotation matrix $\mathbb{R}(\cdot)$ applies a left-handed*rotation* operator, the resultant vector still exists in the same reference frame;

$$\vec{v}\,' = \mathbb{R}_x(\phi)\vec{v} \tag{2.2a}$$
$$\vec{v}\,', \vec{v} \in \mathcal{F}^1 \tag{2.2b}$$

*No subscripts are used in Eq: 2.2 to indicate reference frame ownership because all vectors are in the same frame*

A *transformation* changes the vectors reference frame. The transformation is a rotation by an angle of the difference between the resulting and principle reference frames. A transformation from frame $\mathcal{F}^1$ to $\mathcal{F}^2$, differing by an angle of $\phi$ about the $\vec{X}$ axis is then:

$$\vec{v}_2 = \mathbb{R}_x(-\phi)\vec{v}_1 \tag{2.3a}$$
$$\vec{v}_2 \in \mathcal{F}^2 \text{ and } \vec{v}_1 \in \mathcal{F}^1 \tag{2.3b}$$

The distinction between Eq:2.2 and Eq:2.3 is the sense of the angular operand, and hence the effect it has on the argument vector. The transformation of a vector from $\mathcal{F}^i$ to $\mathcal{F}^b$ is the product of three sequential operations about each of the axes. Because each subsequent rotation is applied relative to a new intermediate frame, the sequence of axial rotations will effect the Euler set. Any consequences of that chosen order is something well documented in *Quaternions and Rotation Sequence*, [34]. In this dissertation the ZYX sequence is used. Hence a transformation of a vector $\vec{v}$ from the inertial to the body frame is applied by:

$$\vec{v}_b = \mathbb{R}_i^b(\psi, \theta, \phi)\vec{v}_i \tag{2.4a}$$
$$\mathbb{R}_i^b \triangleq \mathbb{R}_z(-\psi)\mathbb{R}_y(-\theta)\mathbb{R}_x(-\phi) \tag{2.4b}$$
$$\mathbb{R}_z(-\psi)\mathbb{R}_y(-\theta)\mathbb{R}_x(-\phi) \iff \mathbb{R}_x(\phi)\mathbb{R}_y(\theta)\mathbb{R}_z(\psi) \tag{2.4c}$$

The relation in Eq:2.4c is as an inversion (*transpose*) of the rotation matrix. A rotation matrix's inverse can be used interchangeably to maintain a positive sense of the rotational angle. To ensure clarity it's adopted that a negative angular sense implies a *transformation* to a different reference frame. Where applicable, the order of rotation will indicate the sequence direction and an angular sign differentiates a rotation or transformation operation.

An inherent singularity does exists with such attitude representations. Indeed Quaternions are used later in Sec: 3.1.3 in lieu of Euler angles. Euler angular attitude representation is, however, easily understood and well suited to the conventional distinctions made here. Quaternion operations are similarly sequenced in the ZYX order:

$$\mathbb{R}_i^b \iff Q_b^* \otimes (.) \otimes Q_b \tag{2.5a}$$
$$Q_b^* \triangleq Q_z^* Q_y^* Q_x^* \text{ and } Q_b \triangleq Q_x Q_y Q_z \tag{2.5b}$$

With $\otimes$ being the Hamilton product (or quaternion operator). Each quaternion, $Q_i$, is a unit quaternion about that $\hat{i}^{th}$ axis. The operator and subsequent quaternion kinematics are defined later in Sec: 3.1.3.

[2]Derived and proven in *Quadrotor Dynamics and Control* [57]

### 2.1.2 Motor Axis Layout

Fundamentally the whole structure, although treated as rigid in the kinematics, consists of multiple relative bodies. Each propeller and motor pair is actuated by two servos. If the propeller rotates about the motors' $\vec{Z}$ axis, then two servos are aligned with $\vec{Y}$ and $\vec{X}$ axes to pitch and roll the propeller away from its principle axis of rotation. Each of the four motors have their own reference frame, $\mathcal{F}^{M_i}$, aligned as in Fig:2.2.

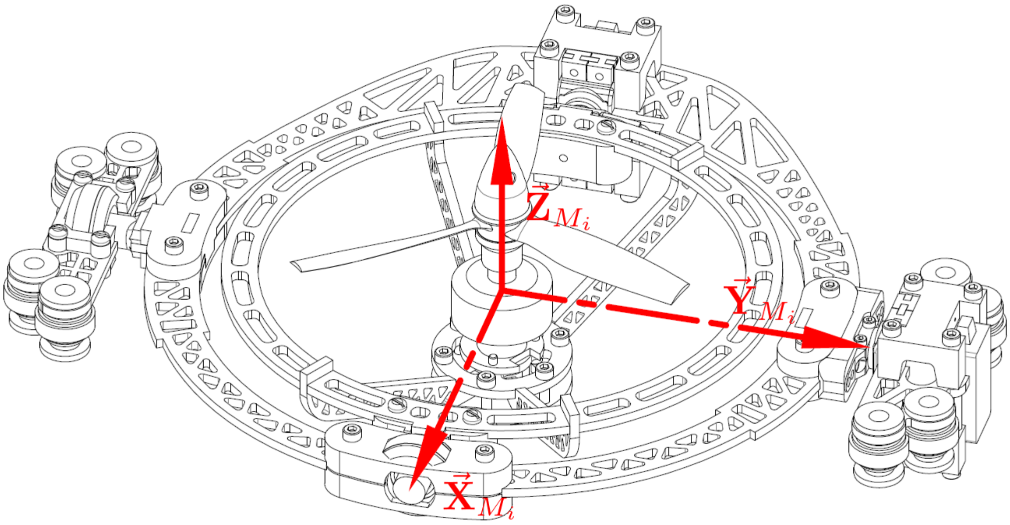



**Figure 2.2:** Aligned Motor Frame Axes

The motor frames, numbered $1-4$, transform to the body frame first by an angle of $\lambda_i$ about the $\vec{X}_{M_i}$ axis. Then by $\eta_i$ about the $\vec{Y}_{M_i'}$ axis in an intermediate $M_i'$ frame. The second servo actuates $\eta_i$ to produce a second intermediate frame $M_i''$, the servo is fixed in the $M_i'$ frame. Finally there is a relative $\vec{Z}_{M_i''}$ rotation between $\mathcal{F}^b$ and $\mathcal{F}^{M_i''}$. The layout of all four motor modules are such that the $\vec{Z}$ axis transformation between intermediate frame $\mathcal{F}^{M_i''}$ and $\mathcal{F}^b$ are all constants; $0,\ 90°,\ 180°$ or $270°$.

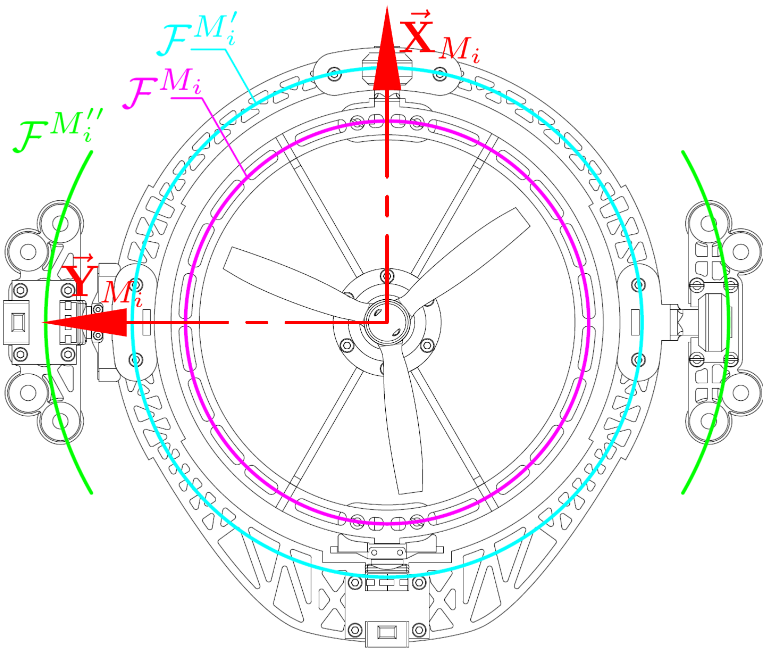



**Figure 2.3:** Intermediate Motor Frames

The four motor modules are aligned relative to the body's XYZ axes as show in Fig:2.4. Modules 1 and 3 have their X-axes in the positive and negative X direction of the body frame. Similarly Modules 2 and 4 have their X-axes in the positive and negative Y directions of the body frame.

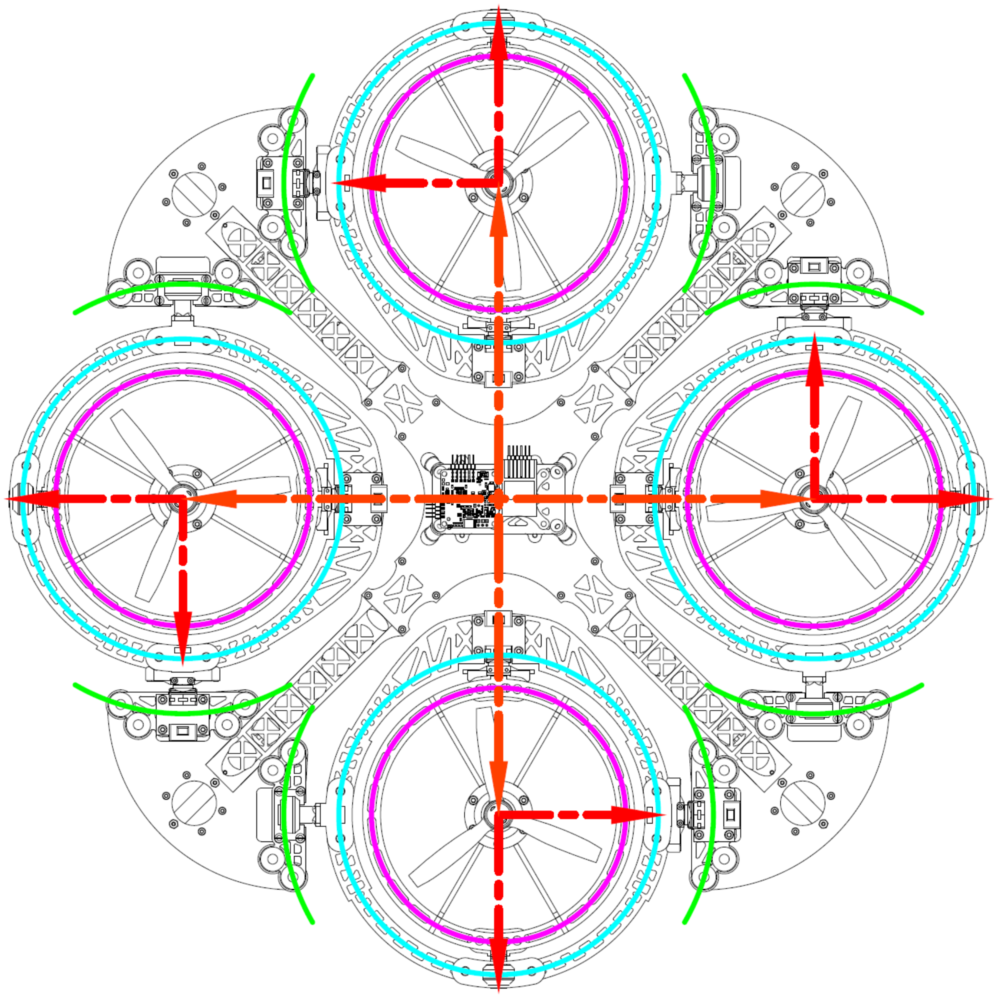


**Figure 2.4:** Body Frame Axes Layout

Transformation relationships from each of the motor frames to the body can be characterized as:

$$\vec{v}_b = \mathbb{R}_z(-\sigma_i)\mathbb{R}_y(-\eta_i)\mathbb{R}_z(-\lambda_i)\vec{v}_{M_i}, \ \sigma_i \in [0, 90^\circ, 180^\circ, 270^\circ] \tag{2.6a}$$

$$\mathbb{R}_z = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}, \begin{bmatrix}0 & -1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}, \begin{bmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 1\end{bmatrix} \text{ for } i \in [1,2,3,4] \text{ respectively} \tag{2.6b}$$

The actuator space,including propeller speed $\omega_i$ $[rps]$, is then $(\in \mathbb{R}^{12})^3$, or rather $\mathbb{U} \in \mathbb{R}^{12}$. The actuator input set $u \in \mathbb{U}$ is then structured as:

$$u_{\in\mathbb{U}} = \left[\omega_1 \ \lambda_1 \ \eta_1 \ \ldots \ \omega_4 \ \lambda_4 \ \eta_4\right]^T \tag{2.7}$$

## 2.2 Design

The actual prototype went through multiple different design iterations, all aimed at using mostly off the shelf RC components and optimizing construction costs. Important design aspects are discussed

[3]Disambiguation: An omission of axial subscript on the $\mathbb{R}$ symbol implies a real space of the superscript dimension.

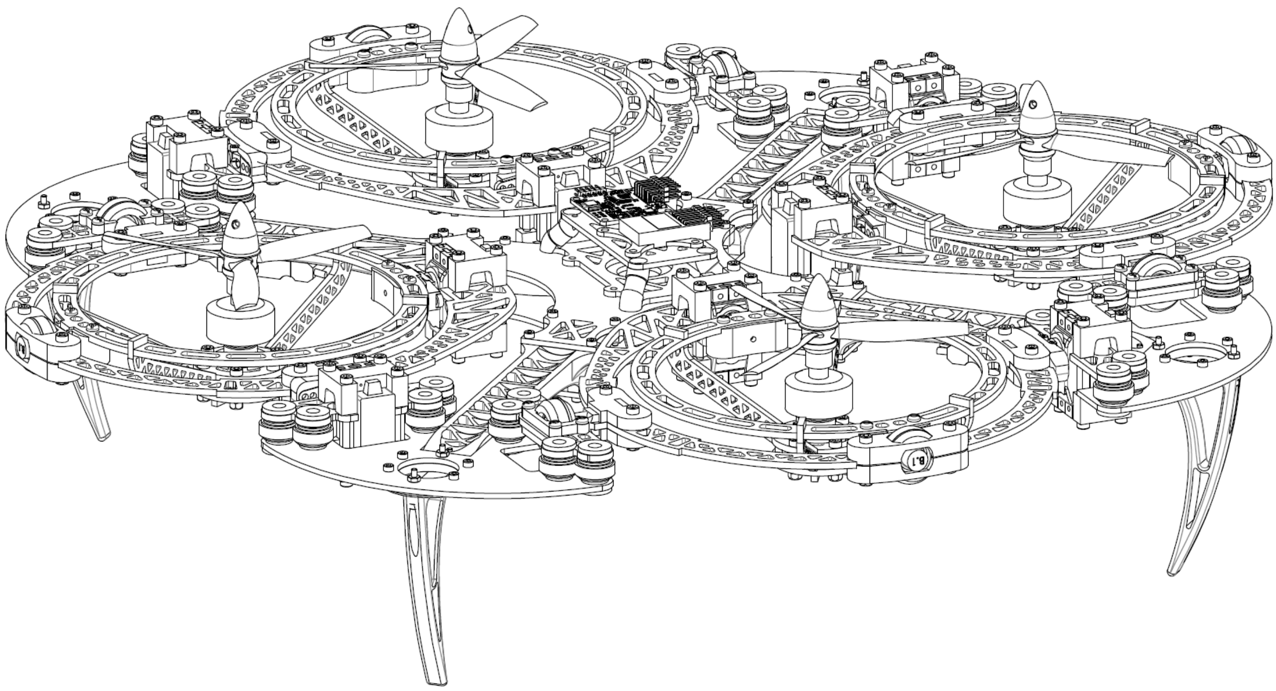


**Figure 2.5:** Isometric layout of the designed prototype

here to give context to some of the dynamics derived in the next chapter. Sub-system identification and associated electrical characteristics are presented here. A review of the physical prototype built and control loop implementation is given in Chapter:6 along with actual flight test results.

### 2.2.1 Articulation

**(a)** Motor Module Supporting Assembly

**(b)** Motor Frame Damping Support Assemblies

**Figure 2.6:** Motor Assembly

The novel aspect of the design is that each of the motor modules can be redirected independently. Two servos affixed onto sequential rings to pitch and roll the substructures' axes. A gyroscope-like frame surrounds each motor/propeller pair to allow for the extra movements. Each servo has a coaxial support bearing and all mounting brackets are 3D printed from CAD models. Each module is designed such that the centre of mass occurs at the intersection of both rotating axes. As a result when servos induce relative rotation of the bodies, there is no torque arm from unbalanced mass

(Fig:2.6a). The primary frame has silicon damping balls between brackets which attach to the motor gyroscope assembly (Fig:2.6b). There is a complete bill of materials for all the 3D printed parts used in Appendix:**??**

The propellers rotational plane is not exactly the aligned with the plane made by the $\vec{X}_{M_i}$ and $\vec{Y}_{M_i}$ rotational servo axes (Fig:2.7). The offset is approximately (mm) and has to be considered when evaluating pitch/roll gyroscopic torque responses.Power for the BLDC and servo motors is supplied conventionally and the transmission lines are affixed such that they don't impede rotation. The servos themselves have a range of 180°, positioned such that a zero$^{\text{th}}$ offset aligns the motor modules with the body frame and has a ±90° range.

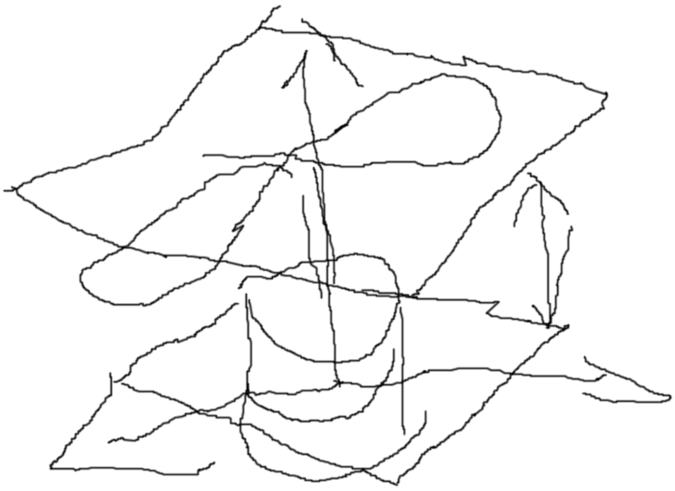


**Figure 2.7:** Difference between propeller and motor planes

The servos used are metal gear Corona DS-339MG digital servos, as a resul

### 2.2.2 Inertial Matrix Function

### 2.2.3 Overall Aspects

#### Vibration Damping

#### Landing Skids

#### Motors & ESCs

## 2.3 System Layout

### 2.3.1 Electrical Design

### 2.3.2 Component Characteristics

# Chapter 3

# Kinematics & Dynamics

## 3.1 Rigid Body Dynamics

### 3.1.1 Lagrange Derivation

### 3.1.2 Rotation Matrix Peculiarities

### 3.1.3 Quaternion Dynamics

### 3.1.4 The Unwinding Problem

## 3.2 Non-linearities

### 3.2.1 Gyroscopic Torques

### 3.2.2 Coriolis Acceleration

### 3.2.3 Inertial Matrix

## 3.3 Aerodynamics

### 3.3.1 Thrust Forces & Propeller Torques

### 3.3.2 Drag

### 3.3.3 Conning & Flapping

### 3.3.4 Vortex Ring State

## 3.4 Consolidated Model

# Chapter 4

# Control Treatment

## Control Plant & Discussion

#### Control Plant Inputs

#### Model Dependent & Independent Controllers

## 4.1 Attitude Control

### 4.1.1 The Attitude Control Problem

### 4.1.2 Quaternion Based Controllers

#### PD Controller

#### Auxilliary Plant Controller

#### PID Controller

### 4.1.3 Non-linear Controllers

#### Ideal Back-stepping Controller

#### Adaptive Back-stepping Controller

#### Lyupanov Derived Ideal Controller

## 4.2 Position Control

### 4.2.1 Backstepping Position Controller

## 4.3 Controller Allocation

### 4.3.1 Non-linear Plant Control Allocation

### 4.3.2 Pseudo Inverse Allocator

# Chapter 5

# Simulations & Results

## 5.1 Controller Tuning

### 5.1.1 Partical Swarm Based Optimization

### 5.1.2 Performance Metric

### 5.1.3 Global & Local Minima

### 5.1.4 Fmincon Differences

## 5.2 Simulation Block

## 5.3 Optimized Controller Comparisons

### 5.3.1 Allocator Performance

### 5.3.2 Attitude Control Results

### 5.3.3 Autopilot Outcome

# Chapter 6

# Prototype Flight Results

# Appendix A

# Standard Quadrotor Dynamics

# Bibliography

[1] Yazan Al-Rihani. *Development of a dual axis tilt rotorcraft uav: Design, prototyping and control.*, volume 1. Cranfield University: School of Engineering, 2012.

[2] N. Amiri, A. Ramirez-Serrano, and Davies R. Modelling of opposed lateral and longitudinal tilting dual-fan unmanned aerial vehicle. *International Federation of Automatic Control*, pages 2054–2059, September 2011.

[3] APMCopter. Arducopter main page. Website: http://www.arducopter.co.uk/, 6 2016. Arducopter (APM) Official Website.

[4] E. Balasubramanian and R. Vasantharaj. Dynamic modelling and control of quadrotor. *International Journal of Engineering and Technology*, pages 63–39, February 2013.

[5] M. Bangura and R. Mahony. Non-linear dynamic modelling for high performance control of a quadrotor. In *Australasian Conference on Robotics and Automation, Victoria University of Wellington.* Victoria Unversity of Wellington, 12 2012. Published in Conference Proceedings.

[6] Mohd Ariffanan Basri, Abdul R. Husain, and Kumeresan A. Danapalasingam. Intelligent adaptive backstepping control for mimo uncertain non-linear quadrotor helicopter systems. *Institute of Measurement Control Transactions*, pages 1–17, 2014.

[7] Charles Blouin and Eric Lanteigne. Pitch control on an oblique active tilting bi-rotor. *International Conference on Unmanned Aircraft Systems*, pages 791–799, May 2014.

[8] R. Bodrany, W. Steyn, and M. Crawford. In-orbit estimation of the inertia matrix and thruster parameters of uosat-12. In *Conference on Small Satellites*, volume 14, pages 1–11. American Institute of Aeronautics and Astronautics, 2000.

[9] Hossein Bolandi, Mohammed Rezaei, Rezo Mohsenipour, Hossein Nemati, and Seed Majid Smailzadeh. Attitude control of a quadrotor with optimized pid. *Intelligent Control and Automation*, pages 335–342, August 2013.

[10] S. Bouabdallah and R. Siegwart. Full control of a quadrotor. *IEEE International Conference on Intelligent Robots and Systems*, pages 153–158, 11 2007. Written for Autonomous Systems Lab at Swiss Federal Institute of Technology.

[11] Samir Bouabdallah, Andre Noth, and Roland Siegward. Pid vs lq control technqiues applies to an indoor micro quadrotor. *IEEE International Conference on Intelligent Robots and Systems*, pages 2451–2456, 9 2004.

[12] J. Brandt and M. Selig. Propeller performance data at low reynolds numbers. *American Institute of Aeronautics and Astronautics Sciences Meeting, 49th*, pages 1–18, January 2011.

[13] Jian Chen, Aman Behal, and Darren M. Dawson. Adaptive output feedback control for a class of mimo nonlinear systems. In *Proceedings of the American Control Conference*, pages 5300–5306, Minneapolis, Minnesota, US, 6 2006. American Control Conference.

[14] Arindam B. Chowdhury, Anil Kulhare, and Guarav Raina. A generalized control method for tilt-rotor uav stabilization. *IEEE International COnference on Cyber Technology in Automation, Control and Intelligent Systems*, pages 309–314, May 2012.

[15] R.F. de Olivera, F.T. de Salvi, and E.M. Belo. Dynamic modelling, simulation and control of an autonomous quadcopter aircraft. *International Congress of Mechanical Engineering*, pages 1–9, November 2009.

[16] Chen Diao, Bin Xian, Qiang Yin, Wei Zeng, Haotao Li, and Yungao Yang. A nonlinear adaptive control approach for quadrotor uavs. In *Asian Control Conference Proceedings*, volume 8, pages 223–228, Kaohsiung, Taiwan, 5 2011. Asian Control Conference.

[17] DJI Drones. Dji inspire one, 2016.

[18] DJI Drones. Dji phantom, 2016.

[19] Emil Fresk and George Nikolakopoulos. Full quaternion based attitude control for a quadrotor. *European Control Conference*, pages 3864–3869, 6 2013.

[20] Pau. S Gasco. *Development of a Dual Axis Tilt Rotorcraft UAV: Modelling, Simulation and Control*, volume 1. Cranfield University: School of Engineering, 2012.

[21] HiSystems GmbH. Mikrokopter quadroxl, 2016.

[22] Basile Graf. Quaternions and dynamics. Publication for Mathematics - Dynamical Systems, 2 2007.

[23] Gary R. Gress. Lift fans as gyroscopes for controlling compact vtol air vehicles: Overview and development status of oblique active tilting. In *American Helicopter Society Annual Forum*, volume 63, Virginia Beach, 5 2007. American Helicopter Society, American Helicopter Society Inc. Forum Proceedings.

[24] Gary R. Gress. *Passive Stabilization of VTOL Aircraft Having Obliquely Tilting Propellers.* University of Calgary, Department of Mechanical Engineering, Calgary, Alberta, 2014.

[25] Karsten Groekatthfer and Zizung Yoon. Introudction into quaternions for spacecraft attitude representation. *Technical University of Berlin: Department of Astronautics and Aeronatuics*, pages 1–16, May 2012.

[26] N. Guenard, T. Hamel, and V. Moreau. Dynamic modelling and control strategy for an x4-flyer. *International Conference on Control and Automation*, pages 141–146, June 2005.

[27] HobbyKing.com. Hobby king: The ultimate hobby experience, 2016.

[28] G. Hoffmann, H. Huang, S. Waslander, and C. Tomlin. Quadrotor helicopter flight dynamics and control: Theory and experiment. In *Guidance, Navigation and Control Conference and Exhibit*, pages 1–19, Hilton Head, South Carolina, 8 2010. American Institute of Aeronautics and Astronautics, American Institute of Aeronautics and Astronautics. Derivation of advanced aerodynamic affects on STARMAC Quadrotor Prototype.

[29] W. Jia, Z. Ming, Y. Zhiwei, and L. Bin. Adaptive back-stepping lpv control of satellite attitude maneuvers with sum of squares. In *World Congress on Intelligent Control and Automation*, volume 8, pages 1747–1752. IEEE, 7 2010.

[30] Tor A. Johansen and Thor I. Fossen. Control allocation - a survery. *Automatica*, 45:10871103, 11 2012. Prepared for: Department of Engineering Cybernetics - Norwegian University of Science and Technology.

[31] Farid Kendoul, Isabelle Fantoni, and Rogelio Lozano. Modeling and control of a small autonomous aircraft having two tilting rotors. *IEEE Conference on Decision and COntrol*, pages 8144–8149, December 2005.

[32] P. Krishnamurthy and F. Khorrami. Adaptive backstepping and theta-d based controllers for a tilt-rotor aircraft. *Mediterranean Conference on Control and Automation*, pages 540–545, June 2011.

[33] Raymond Kristiansen and Per J. Nicklasson. Satellite attitude control by quaternion-based backstepping. *American Control Coference*, N/A:907–912, 6 2005. Published by Department of Computer Science, Electrical Engineering and Space Technology; Narvik University College.

[34] Jack B. Kuipers. *Quaternions and Rotation Sequences: A Prier with Aplication to Orbital Aerospace and Virtual Reality*, pages 127–143. Princeton University Press, September 2002. Used for Quaternion and Rotation Matrix reference.

[35] Peter Lambert. Nakazawa, banton and jin, bai x. Technical Report N/A, Computer and Electrical Engineering: University of Victoria, Victoria, Canada, 12 2013.

[36] Prof Allan J. Laub. The moore-penrose pseudo inverse. UCLA Math33A Course Content, UCLA, Los Angeles, 3 2008. Course Notes cited from `http://www.math.ucla.edu/~laub/33a.2.12s/mppseudoinverse.pdf`.

[37] Jang-Ho Lee, Byoung-Mun Min, and Eung-Tai Kim. Autopilot design of tilt-rotor uav using particle swarm optimization method. *International COnference on Control, Automation and Systems*, pages 1629–1633, October 2007.

[38] LibrePilot. Openpilot/librepilot wiki. Website: http://opwiki.readthedocs.io/en/latest/index.html, 5 2016. Information wiki page for LibrePilot/OpenPilot firmware.

[39] Hyon Lim, Jaemann Park, Daewon Lee, and H.J. Kim. Build your own quadrotor. *IEEE ROBOTICS & AUTOMATION MAGAZINE*, pages 33–45, 9 2012. Publication on Opensource Autopilot systems.

[40] SteadiDrone PTY LTD. Steadidrone home, 2016.

[41] Teppo Luukkonen. Modelling and control of a quadcopter. Master's thesis, Aalto University: School of Science, Eepso, Finland, 8 2011. Independent research project in applied mathematics.

[42] Tarek Madani and Abdelaziz Benallegue. Backstepping control for a quadrotor helicopter. *International COnference on Intelligent Robots and Systems*, pages 3255–3260, October 2006.

[43] I. Mandre. Rigid body dynamics using euler's equations, rungekutta and quaternions, 2 2006.

[44] Carlos J. Mantas and Jose M. Puche. Artificial neural networks are zero-order tsk fuzzy systems. In *IEE Transactions on Fuzzy Systems*, volume 16, pages 630–644, 6 2008.

[45] Christopher G. Mayhew, Ricardo G. Sanfelice, and Andrew R. Teel. On quaternion based attitude control and the unwinding phenomenon. *American Control Conference*, pages 299–304, June 2011.

[46] Ashfaq A. Mian and Wang Daoboo. Modelling and backstepping-based nonlinear control of a 6dof quadrotor helicopter. *Chinese Journal of Aeronautics*, 21:261–268, 3 2008. Simulated Backstepping Control.

[47] Svein Rivli Napsholm. Prototype of a tiltrotor helicopter. Master's thesis, Norwegian University of Science and Technology: Department of Engineering Cybernetics, Norway, 1 2013.

[48] A. Nemati and M. Kumar. Modeling and control of a single axis tilting quadcopter. *American Control Conference*, pages 3077–3082, June 2014.

[49] Kenzo Nonami, Farid Kendoul, Satoshi Suzuki, Wei Wang, and Daisuke Nakazawa. *Autonomous Flying Robots: Unmanned Aerial Vehicles and Micro Aerial Vehicles*, chapter 2, pages 44–48. Springer Japan, 1 edition, 2010. References to Cyclic-Pitch Control relevant subsections.

[50] Gustavo P. Oliveira. Quadcopter civil applications. Master's thesis, Informatics and Computer Engineering: University of Portugal, Portugal, 2 2014.

[51] Christos Papachristos, Kostas Alexis, and Anthony Tzes. Design and experimental attitude control of an unmanned tilt-rotor aerial vehicle. *International Conference on Advanced Robotics*, pages 465–470, June 2011.

[52] Parth N. Patel, Malav A. Patel, Rahul M. Faldu, and Yash R. Dave. Quadcopter for agricultural surveilance. In *Advance in Electronic and Electrical Engineering*, volume 3, India, 2013.

[53] J. Peraire and S. Widnall. 3d rigid body dynamics: Euler angles. Lecture notes for Dynamics Course, 2009. Dynamics course notes, fall 2007.

[54] D. Peters. Eigth amendment of the civil aviation regulations. Goverment Gazette Notice, 5 2015. In Amendment to the Civil Aviation Act, 2009 (Act No.13 of 2009).

[55] Jean-Baptiste Pomet and Laurent Praly. Adaptive nonlinear regulation: Estimation from the lyupanov equation. In *IEEE Transactions on Automatic Control*, volume 37, pages 729–740. IEEE, 6 1992.

[56] P. Pounds, R. Mahony, P. Hynes, and J. Roberts. Design of a four-rotor aerial robot. *Australasian Conference on Robotics and Automation*, pages 145–150, November 2002.

[57] Beard Randal. Quadrotor dynamics and control. Report, Brigham Young University, 2 2008. Part of the Electrical and Computer Engineering Commons.

[58] O. Rawashdeh, H.C. Yang, R. AbouSleiman, and B. Sababha. Microraptor: A low cost autonomous quadrotor system. *International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, pages 1–8, August 2009.

[59] Anastasia Razinkove, Igor Gaponov, and Hyun-Chan Cho. Adaptive control over quadcopter uav under disturbances. *International Conference on Control, Automation and Systems*, pages 386–390, October 2014.

[60] M.K. Rwigema. Propeller blade element momentum theory with vortex wake deflection. *International Congress of the Aeronautical Sciences, 27th*, pages 1–9, January 2010.

[61] M. Ryll, H. Bulthoff, and P. Robuffo Giordano. Modelling and control of a quadrotor uav with tilting propellers. *IEEE International Conference on Robotics and Automation*, pages 4606–4613, May 2012.

[62] M. Ryll, H. Bulthoff, and P. Robuffo Giordano. First flight tests of a quadrotor uav with tilting propellers. *IEEE International Conference on Robotics and Automation*, pages 295–302, May 2013.

[63] A. Sanchez, J. Escareo, O. Garcia, and R. Lozano. Autonomous hovering of a noncyclic tiltrotor uav: Modeling, control and implementation. *The International Federation of Automatic Control*, pages 803–808, July 2008.

[64] Puneet Singla, Daniele Mortari, and John L. Junkins. How to avoid singularity when using euler angles? *Advances in the Astronautical Sciences*, pages 1409–1426, January 2005.

[65] Abdelhamid Tayebi and Stephen McGilvray. Attitude stabilization of a vtol quadrotor aircraft. *IEEE Transactions on Control Systems Technology*, pages 562–571, May 2006.

[66] John Ting-Yung Wen and Kenneth Kreutz-Delgado. The attitude control problem. *IEEE Transactions on Automatic Control*, pages 1148–1162, October 1991.

[67] P. Tsiotras, M. Corless, and J.m Longuski. A novel approach to the attitude control of axisymmetric spacecraft. *Automatica*, 31:1099–1112, 3 1995. Control Automatica, Printed in Great Britan.

[68] E. van Kampen and M. M. van Paassen. Ae4301: Automatic flight control system design. Delft Centre for Systems and Control; MSc Notes, 1 2008. Course Notes cited from: `http://aerostudents.com/master/advancedFlightControl.php`.

[69] Ronny Votel and Doug Sinclair. Comparison of control moment gyros and reaction wheels for small earth-observing satellites. In *Conference on Small Satellites*, volume 26, pages 1–7. Utah State University, 8 2012. Open access on AIAA conference website.

[70] Tao Wang, Tao Zhao, Du Hao, and Mingxi Wang. Transformable aerial vehicle, 09 2014.

[71] X. Xiaozhu, L. Zaozhen, and C. Weining. Intelligent adaptive backstepping controller design based on the adaptive particle swarm optimization. *Chinese Control and Decision Conference*, pages 13–17, September 2009.

[72] Song Xin and Zou Zaojian. A fuzzy sliding mode controller with adaptive disturance approximation for an underwater robot. In *International Asia Conference on Informatics in Control, Automation and Robotics*, volume 2, pages 50–53, 10 2010.