Financial Pricing Engine

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Abstract: The advancement of computing power has allowed mathematicians, statisticians, computer scientists and other professionals to solve complex problems using powerful machines. The progress in data science, more specifically in deep learning opened ways to solve high dimensional equations, bypassing the curse of dimensionality, a phenomena which illustrates the complexity of equations as they grow in degree. In this project we seek to use deep learning tools to price exotic financial products, combining stochastic partial differential equation and neural networks.

Keywords: deep learning, machine learning, financial mathematics, options, partial differential equations

1. Machine Learning

Machine learning is a collection of methods based on the idea of automating the decision making process of a computer. Programs are not created to perform specific tasks they can learn from data. Within machine learning, there are different areas of study ranging from data mining up to unsupervised learning and neural networks. In this project we'll focus on deep learning.

1.1. Deep Learning

Deep learning refers to the study of large, complex data sets through neural networks. The idea is that we feed the system a functions and neurons in deep layers will be able to interpret a result for such input after extensive "training". As researchers we do not know how the neurons reach such results. That is why neural networks are often referred as "black boxes". However, the practical result of machine learning has been astonishing and recognized by the scientific community. In this paper we use deep learning to solve high dimensional partial differential equations, a task that would be nearly impossible without the use of neural networks.

1.2. Target Functions

Target functions are functions that the machine learning algorithm seek to approximate in their predictions. These functions range from the simplest linear function up to high dimensional differential equations.

1.3. Linear Functions

Linear functions are considered the simplest form of functions. Their general representation is found below

$$f(x) = ax + b \tag{1}$$

where x is the independent variable and $f(x)$ is the dependent variable, a is the coefficient of the independent variable and b is the $f(x)$ intercept. In other words, b is the value of $f(x)$ when x equals 0.

1.4. Linear Regression

Linear regression is a linear functions that demonstrate the relationships between two variables. In this case, the independent variable is known as the predictor and response is the dependent variable. It is often shown in the form

$$Y = b_1 x + b_0 \tag{2}$$

where Y is the predictor, b_1 the coefficient of the response and b_0 the Y-intercept. In machine learning models, linear regression, like linear functions, take less time to compute and fairly simple neural networks can give precise results.

1.5. Quadratic Functions

Quadratic functions, unlike linear functions, possess relationships that go beyond the 1st degree, where the impact of the quadratic coefficient is of greater magnitude to the behavior of the function. They are shown in the form

$$f(x) = ax^2 + bx + c \tag{3}$$

where a is the quadratic coefficient, b the linear coefficient and c is the $f(x)$ intercept. As you can see in Appendix X, the increase in dimension complexity by 1 degree, makes the neural network fitting process much more complicated, highlighting the necessity for calibrations and neuron addition.

1.6. Black-Scholes Equation

The Black-Scholes is an equations designed to estimate the fair price for an option. An option is a financial product that gives the buyer the right but not the obligations to buy or sell a financial product at a certain price. This equation was designed by Black, Scholes and Merton and award a Nobel prize in economics.

1.7. Basket Options

1.8. Default Risk

2.

References