A binomial tree model is given as below:

- Let X be a random walk with transition probability

$$p(X_n = (j+1)\Delta x | X_{n-1} = j\Delta x) = p, \quad p(X_n = (j-1)\Delta x | X_{n-1} = j\Delta x) = 1 - p.$$

A $\mathrm{CRR}(N; s, r, \sigma, T)$ model assumes Stock price S satisfies with the following additional conditions:

- S starts from s;
- $X_t = \ln S_t$ follows the random walk with transition probability to upper leg probability

$$\hat{p} = \frac{e^{r\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}$$

  and down leg probability $1 - \hat{p}$.

We consider an option with

$$\text{type} = \text{call, maturity } T = 1 \text{ and strike } K = 100.$$

Its underlying stock has

$$\text{spot price } S_0 = 100 \text{ and volatility } \sigma = 20\%.$$

Moreover, the interest rate is 5%.

You are expected to try the following tasks:

1. Compute BSM call value.

2. Compute the value of $\mathrm{CRR}(N = 2000)$ and observe if it is sufficiently close to BSM value?

3. Next, we want to demonstrate the convergence

$$\mathrm{CRR(N)} \to \mathrm{BSM}, \text{ as } N \to \infty.$$

   To do that, we take the number of steps $N = 10 + 20k$, where k ranges over all integers in $\{0, 1, \ldots, 49\}$.

   pseudocode

   - plot a graph of the mapping, for $k \in range(50)$

   $$k \mapsto CRR(10 + 20k).$$

   - on the same figure, plot a horizontal line with the level of BSM value.
   - observe if the CRR curve actually converging to the BSM line?

4. Repeat the above convergence demonstration by replacing the number of steps by

   $$N = 10 + 25k.$$

   Observe the CRR curve converging to BSM line in the same manner?