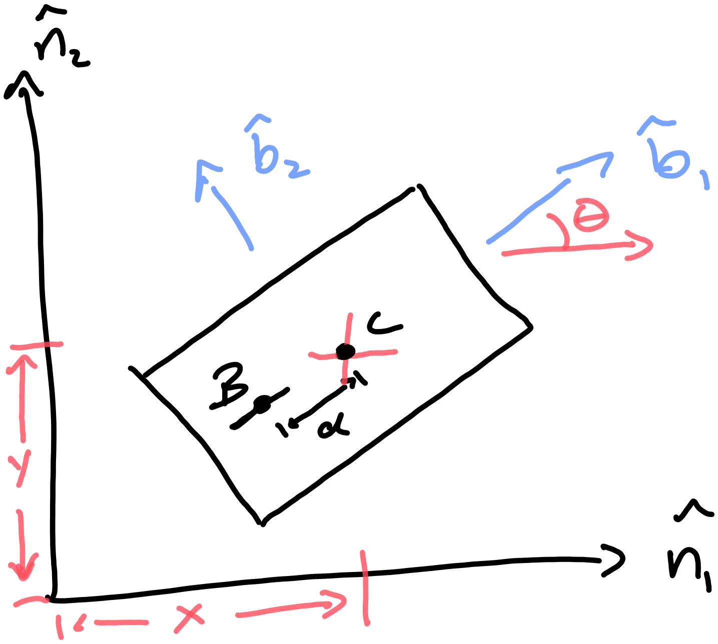
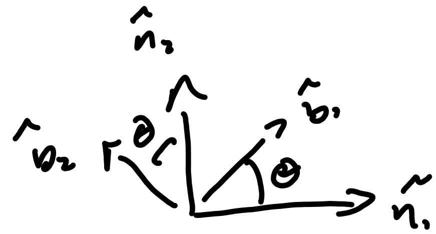



$\hat b_1 = \hat n_1 \cos\theta + \hat n_2 \sin\theta$

(a)

$$\phi = \vec v_B \cdot \hat b_2 = 0 \qquad \vec r_B = x\hat n_1 + y\hat n_2 - d\,\hat b_1$$

$$\dot{\vec r}_B = \dot x\,\hat n_1 + \dot y\,\hat n_2 - d\dot\theta\,\hat b_2$$

$\hat n_1 \cdot \hat b_2 = -\sin\theta$

$\hat n_2 \cdot \hat b_2 = \cos\theta$

$$\phi = -\dot x \sin\theta + \dot y \cos\theta - d\dot\theta = 0$$

$$\dot\phi = -\ddot x \sin\theta + \ddot y \cos\theta - d\ddot\theta - \dot x\dot\theta\cos\theta - \dot y\dot\theta\sin\theta = 0$$

$a_{1x} = -\sin\theta$

$a_{1y} = \cos\theta$

$a_{1\theta} = -d$

$$T = \frac{1}{2} m \dot{\vec r}_c \cdot \dot{\vec r}_c + \frac{1}{2} I_c \omega^2 = \frac{1}{2} m\left(\dot x^2 + \dot y^2\right) + \frac{1}{2} I_c \dot\theta^2$$

$$V = 0$$

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) - \frac{\partial L}{\partial x} = -\lambda \sin\theta$$

$$\boxed{\begin{aligned} m\ddot x &= -\lambda\sin\theta \\ m\ddot y &= \lambda\cos\theta \\ I_c\ddot\theta &= -d\lambda \end{aligned}}$$

$$\frac{\lambda}{m}(\sin^2\theta + \cos^2\theta) + \frac{d^2}{I_c}\lambda - \dot{x}\dot{\theta}\cos\theta - \dot{y}\dot{\theta}\sin\theta$$

Problem setup
(constraint equation, velocity, choice of coordinates, etc.)

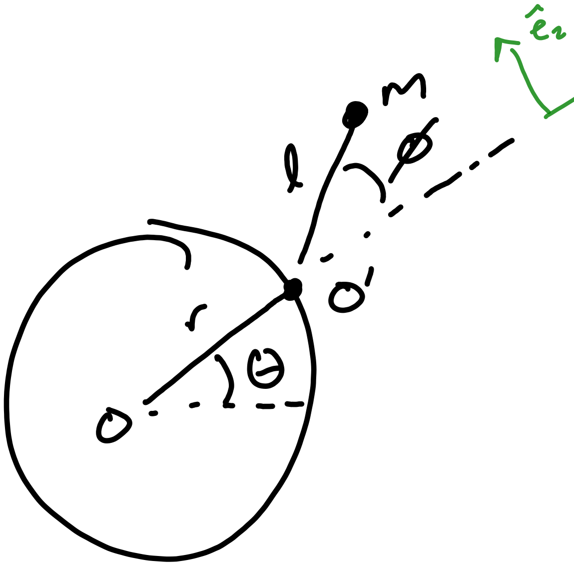
$$\lambda\left(1 + \frac{md^2}{I_c}\right) = m\dot{x}\dot{\theta}\cos\theta + \dot{y}\dot{\theta}\sin\theta$$

$$\boxed{\lambda = \frac{m\dot{\theta}(\dot{x}\cos\theta + \dot{y}\sin\theta)}{1 + md^2/I_c}}$$

Problem approach
(Used lagrangian, Lagrange's equations, Lagrange multipliers)

Problem solution
(Correct lamda from setup, correct figures from setup)

Reflection
(Thoughts shared about approach/ usefulness are thoughtful, clear, etc.




$$\vec{r} = r + l\cos\phi\, \hat{e}_1 + l\sin\phi\, \hat{e}_2$$

$$\dot{\vec{r}} = (-l\sin\phi\,\dot{\phi})\hat{e}_1 + l\cos\phi\,\dot{\phi}\,\hat{e}_2 + \omega(r + l\cos\phi)\hat{e}_2 - l\omega\sin\phi\,\hat{e}_1$$

$$= -l\sin\phi\,(\dot{\phi}+\omega)\hat{e}_1 + \left(r\omega + l\cos\phi(\dot{\phi}+\omega)\right)\hat{e}_2$$

$$L = T = \frac{1}{2}m\left|\dot{\vec{r}}\cdot\dot{\vec{r}}\right| = l^2(\dot{\phi}+\omega)^2\sin^2\phi + l^2(\dot{\phi}+\omega)^2\cos^2\phi + r^2\omega^2 + 2rl\omega(\dot{\phi}+\omega)\cos\phi$$

$$L = \frac{ml^2}{2}(\dot{\phi}+\omega)^2 + \frac{mr^2\omega^2}{2} + mrl\omega(\dot{\phi}+\omega)\cos\phi$$

$$p = \frac{\partial L}{\partial\dot{\phi}} = ml^2(\dot{\phi}+\omega) + mrl\omega\cos\phi$$

$$\dot{\phi} = \frac{p}{ml^2} - \omega\left(1 + \frac{r}{l}\cos\phi\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\phi}}\right) - \frac{\partial L}{\partial\phi} = 0$$

$$ml^2\ddot{\phi} - mrl\omega\dot{\phi}\sin\phi + mrl\omega(\dot{\phi}+\omega)\sin\phi = 0$$

$$ml^2\ddot{\phi} + mrl\omega^2\sin\phi = 0$$

$$\boxed{\ddot{\phi} + \frac{r}{l}\omega^2\sin\phi = 0}$$

$$H = p\dot{\phi} - L(\phi,\dot{\phi})$$

$$= ml^2(\dot{\phi}+\omega)\dot{\phi} + \cancel{mrl\omega\dot{\phi}}\cos\phi - \frac{1}{2}ml^2\underbrace{(\dot{\phi}+\omega)^2}_{\dot{\phi}^2+2\dot{\phi}\omega+\omega^2} - \frac{mr^2\omega^2}{2} - mrl\omega(\cancel{\dot{\phi}}+\omega)\cos\phi$$

$$= \frac{1}{2}ml^2\dot{\phi}^2 - \frac{1}{2}m(r^2+l^2)\omega^2 - mrl\omega^2\cos\phi$$

$$= \frac{p^2}{2ml^2} - p\omega\left(1+\frac{r}{l}\cos\phi\right) + \frac{ml^2}{2}\omega^2\overbrace{\left(1+\frac{r}{l}\cos\phi\right)^2}^{\cancel{1}+\cancel{2\frac{r}{l}\cos\phi}+\frac{r^2}{l^2}\cos^2\phi} - \frac{ml^2}{2}\omega^2\left(\cancel{1}+\frac{r^2}{l^2}\right) \cancel{- mrl\omega^2\cos\phi}$$

$$= \frac{p^2}{2ml^2} - p\omega\left(1+\frac{r}{l}\cos\phi\right) + \frac{mr^2\omega^2}{2}\underbrace{(\cos^2\phi - 1)}_{-\sin^2\phi}$$

$$= \frac{p^2}{2ml^2} - p\omega\left(1+\frac{r}{l}\cos\phi\right) - \frac{mr^2\omega^2}{2}\sin^2\phi$$

$$\dot{\phi} = \frac{\partial H}{\partial p} = \frac{p}{ml^2} - \omega\left(1 + \frac{r}{l}\cos\phi\right)$$

$$\dot{p} = -\frac{\partial H}{\partial \phi} = -p\omega\frac{r}{l}\sin\phi + mr^2\omega^2\sin\phi\cos\phi$$

$$= r\omega\sin\phi\left(\frac{-p}{l} + mr\omega\cos\phi\right)$$