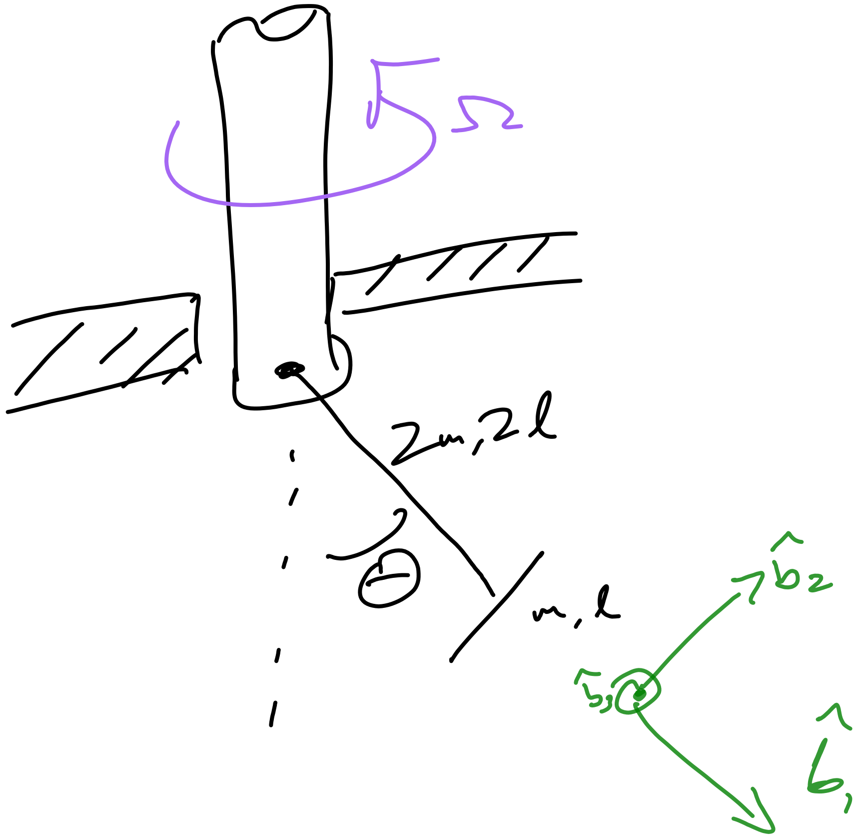


$$I = \begin{bmatrix} 1/12 & 0 & 0 \\ 0 & 20/3 & 0 \\ 0 & 0 & 27/4 \end{bmatrix} m l^2$$

(a)

$$\vec{\omega} = \Omega \hat{n}_2 + \dot{\theta} \hat{b}_3 =$$

$$\hat{n}_2 = -\cos\theta \hat{b}_1 + \sin\theta \hat{b}_2$$

$$\vec{\omega} = -\cos\theta \, \Omega \hat{b}_1 + \sin\theta \, \Omega \hat{b}_2 + \dot{\theta} \hat{b}_3$$

$$\vec{H} = -\tfrac{1}{12} m l^2 \Omega \cos\theta \, \hat{b}_1 + \tfrac{20}{3} \Omega m l^2 \sin\theta \, \hat{b}_2 + \tfrac{27}{4} m l^2 \dot{\theta} \hat{b}_3$$

(b) $$\frac{{}^N d\vec{H}}{dt} = \frac{{}^B d}{dt}(\vec{H}) + \vec{\omega}\times\vec{H}$$

$$= \begin{bmatrix} \frac{1}{12}ml^2\Omega\dot{\theta}\sin\theta \\ \frac{20}{3}ml^2\Omega\dot{\theta}\cos\theta \\ \frac{27}{4}ml^2\ddot{\theta} \end{bmatrix} + ml^2\begin{bmatrix} \frac{27}{4}\Omega\dot{\theta}\sin\theta - \frac{20}{3}\Omega\dot{\theta}\sin\theta \\ \frac{-1}{12}\Omega\dot{\theta}\cos\theta + \frac{27}{4}\Omega\dot{\theta}\cos\theta \\ -\frac{20}{3}\Omega^2\sin\theta\cos\theta + \frac{1}{12}\Omega^2\sin\theta\cos\theta \end{bmatrix}$$

$$\boxed{\frac{{}^N d}{dt}\vec{H} = ml^2\begin{bmatrix} \frac{1}{6}\Omega\dot{\theta}\sin\theta \\ \frac{40}{3}\Omega\dot{\theta}\cos\theta \\ \frac{27}{4}\ddot{\theta} - \frac{79}{12}\Omega^2\sin\theta\cos\theta \end{bmatrix}}$$

(c) $$\Sigma\vec{M} = l\hat{b}_1\times(-2mg\hat{n}_2) + 2l\hat{b}_1\times(-mg\hat{n}_2) + \tau\,\hat{n}_2$$

$$= -4lmg\sin\theta\,\hat{b}_3 + \tau(-\cos\theta\,\hat{b}_1 + \sin\theta\,\hat{b}_2)$$

(d) $\hat{b}_1$: $\frac{1}{6}ml^2\Omega\dot{\theta}\sin\theta = -\tau\cos\theta$

$\hat{b}_2$: $\frac{40}{3}ml^2\Omega\dot{\theta}\cos\theta = \tau\sin\theta$

can solve for $\tau(\theta,\dot{\theta})$

$\hat{b}_3$: $ml^2\left(\frac{27}{4}\ddot{\theta} - \frac{79}{12}\Omega^2\sin\theta\cos\theta\right) = -4lmg\sin\theta$

$$\ddot{\theta} = \frac{79}{81}\Omega^2\sin\theta\cos\theta - \frac{16}{27}\frac{g}{l}\sin\theta$$

(d) equilibria

$$\ddot{\Theta} = \underbrace{\sin\Theta}_{\Theta = 0} \underbrace{\left(\frac{79}{81} \Omega^2 \cos\Theta - \frac{16}{27} \frac{g}{l} \right)}_{\text{goes to zero when}}$$

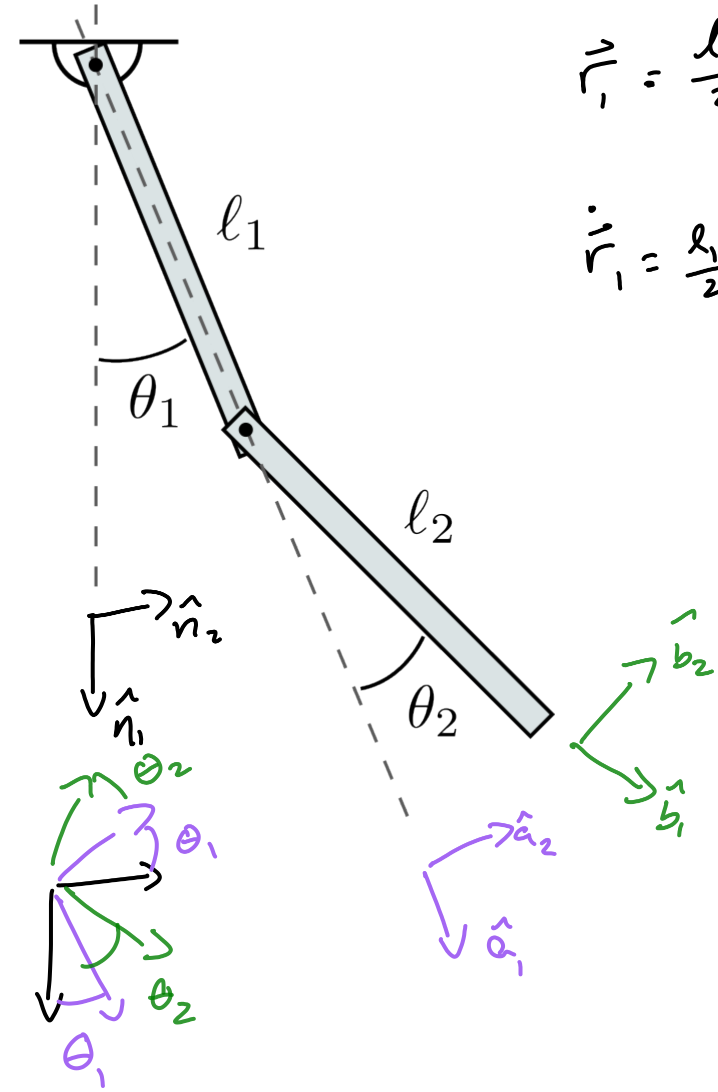
$$\frac{79}{81} \Omega^2 \cos\Theta - \frac{16}{27} \frac{g}{l} = 0$$

Only possible when $\cos\Theta \leq |1|$

$$\cos\Theta = \frac{48}{79} \frac{g}{l} \frac{1}{\Omega^2}$$

bifurcation: $\frac{48}{79} \frac{g}{l} \frac{1}{\Omega^2} \leq 1 \xrightarrow{\text{occurs when}} \Omega^2 \geq \frac{48}{79} \frac{g}{l}$

$$\boxed{\Omega_{CR} = \sqrt{\frac{48}{79} \frac{g}{l}}}$$



$$\vec{r}_1 = \frac{l_1}{2}\hat{a}_1 \qquad \vec{r}_2 = l_1\hat{a}_1 + \frac{l_2}{2}\hat{b}_1$$

$$\vec{\omega}_{A/N} = \dot{\theta}_1\hat{a}_3 \qquad \vec{\omega}_{B/N} = (\dot{\theta}_1 + \dot{\theta}_2)\hat{b}_3$$

$$\dot{\vec{r}}_1 = \frac{l_1}{2}\dot{\theta}_1\hat{a}_2 \qquad \dot{\vec{r}}_2 = l_1\dot{\theta}_1\hat{a}_2 + \frac{l_2}{2}(\dot{\theta}_1 + \dot{\theta}_2)\hat{b}_2$$

$$T = \frac{1}{2} m_1\left(\frac{l_1}{2}\right)^2\dot{\theta}_1^2 + \frac{1}{2}\left(\frac{1}{12}m_1 l_1^2\right)\dot{\theta}_1^2 + \frac{1}{2}m_2\left(l_1^2\dot{\theta}_1^2 + \left(\frac{l_2}{2}\right)^2(\dot{\theta}_1+\dot{\theta}_2)^2 + \cos\theta_2\, l_1 l_2\dot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2)\right) + \frac{1}{2}\left(\frac{1}{12}m_2 l_2^2 (\dot{\theta}_1+\dot{\theta}_2)^2\right)$$

$$T = \frac{1}{6}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{6}m_2 l_2^2(\dot{\theta}_1+\dot{\theta}_2)^2 + \frac{1}{2}m_2 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_1 l_2\dot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2)\cos\theta_2$$

$$V = -m_1 g\frac{l_1}{2}\cos(\theta_1) - m_2 g\left(l_1\cos\theta_1 + \frac{l_2}{2}\cos(\theta_1+\theta_2)\right)$$

$$L = T - V$$

1)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right) - \frac{\partial L}{\partial\theta_1} = 0$$

$$\frac{1}{3}m_1 l_1^2\ddot{\theta}_1 + \frac{1}{3}m_2 l_2^2(\ddot{\theta}_1+\ddot{\theta}_2) + m_2 l_1^2\ddot{\theta}_1 + \frac{1}{2}m_2 l_1 l_2\Big((2\ddot{\theta}_1+\ddot{\theta}_2)\cos\theta_2 - (2\dot{\theta}_1+\dot{\theta}_2)\dot{\theta}_2\sin\theta_2\Big) - g l_1\left(\frac{m_1}{2} + m_2\right)\sin\theta_1 - \frac{m_2 g}{2}\sin(\theta_1+\theta_2) = 0$$

2) $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

$$\frac{1}{3} m_2 l_2^2 \left(\ddot\theta_1 + \ddot\theta_2\right) + \frac{1}{2} m_2 l_1 l_2 \left(\ddot\theta_1 \cos\theta_2 - \cancel{\dot\theta_1 \dot\theta_2 \sin\theta_2}\right)$$

$$+ \frac{1}{2} m_2 l_1 l_2 \left(\dot\theta_1^2 + \cancel{\dot\theta_2 \dot\theta_1}\right)\sin\theta_2 - \frac{m_2 g}{2} \sin\left(\theta_1 + \theta_2\right) = 0$$

$$M(\vec\theta) = \begin{bmatrix} \frac{1}{3}\left(m_1 l_1^2 + m_2 l_2^2\right) + m_2 l_1^2 + m_2 l_1 l_2 \cos\theta_2 & \frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \\ \frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 & \frac{1}{3} m_2 l_2^2 \end{bmatrix}$$

$$h(\vec\theta, \dot{\vec\theta}) = \begin{bmatrix} -\frac{m_2}{2} l_1 l_2 \left(2\dot\theta_1 + \dot\theta_2\right)\dot\theta_2 \sin\theta_2 - g l_1 \left(\frac{m_1}{2} + m_2\right)\sin\theta_1 - \frac{m_2 g}{2}\sin\left(\theta_1 + \theta_2\right) \\ \frac{1}{2} m_2 l_1 l_2 \dot\theta_1^2 \sin\theta_2 - \frac{m_2 g}{2}\sin\left(\theta_1 + \theta_2\right) \end{bmatrix}$$

$$M(\vec\theta)\begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix} + h(\vec\theta, \dot{\vec\theta}) = 0$$

Equilibria: $h(\vec{\Theta}, \dot{\vec{\Theta}}) = 0$

when $\dot{\Theta}_1 = \dot{\Theta}_2 = 0$ <u>and</u>

$\sin\Theta_2 = 0$, $\sin(\Theta_1 + \Theta_2) = 0$,

$\sin(\Theta_1) = 0$

so, eq when $\dot{\vec{\Theta}} = \vec{0}$

$$\vec{\Theta} = \{(0,0), (0,\pi), (\pi,0), (\pi,\pi)\}$$

state vector:

$$\vec{x} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{-M^{-1}\vec{h}} \\ 0 & 0 & & \end{bmatrix}$$