Problem 1

SOLUTION
The answers provided were computed in MATLAB using the following fomula.

$$\frac{d}{dt}(R) = \dot{R} = [\vec{\omega}]_x \times R$$

PART A

$$\dot{R} = \begin{bmatrix} -0.866 & -0.5 & 0 \\ 0.5 & -0.866 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

PART B

$$\dot{R} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -0.5 & 0.866 & 0 \end{bmatrix}$$

PART C

$$\dot{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0.866 & 0.5 & 0 \end{bmatrix}$$

PART D

$$\dot{R} = \begin{bmatrix} -0.4740 & 0.2177 & 0.3706 \\ 0.6435 & -0.1783 & -0.6308 \\ 0.2667 & -0.8829 & 0.6193 \end{bmatrix}$$

Problem 2

SOLUTION
Given: $\vec{d}^0_{01}, R^0_1, \vec{v}^0_{01}, \vec{a}^0_{01}, \vec{\omega}^0_{01}$

Find: $\vec{\alpha}^1_{02}, \vec{a}^1_{02}$

Assuming we are ignoring air resistance, because we are given no information about drag coefficients...

Note $\vec{r}^1_{12} = 0.1\hat{x}_1$

$$\vec{r}^0_{12} = R^0_1 \vec{r}^1_{12} = [0.05, 0.0866, 0]^T$$

$$\vec{r}^0_{02} = \vec{d}^0_{01} + \vec{r}^0_{12} = [20.05, 15.0866, 10]^T$$

$$\vec{a}^0_1 = \vec{a}_0 + \vec{a}^0_{01} + \vec{\alpha}^0_{01} \times \vec{r}^0_{01} + \vec{\omega}^0_{01} \times (\vec{\omega}^0_{01} \times \vec{r}^0_{01}) + 2(\vec{\omega}^0_{01} \times \vec{v}^0_{01})$$

Note that the inertial observer is not accelerating, so $\vec{a}_0 = 0$. Simplifying, we have

$$\vec{a}_1^0 = \vec{a}_{01}^0 + \vec{\alpha}_{01}^0 \times \vec{d}_{01}^0 + \vec{\omega}_{01}^0 \times (\vec{\omega}_{01}^0 \times \vec{d}_{01}^0) + 2(\vec{\omega}_{01}^0 \times \vec{v}_{01}^0)$$

Given the R_1^0 matrix is just a rotation about the $\hat{z}^0$ direction, the body $\hat{z}^1$ axis of the vehicle must be parallel to the world $\hat{z}^0$ axis, (IE we know the quad-rotor is not pitching). Considering the nature of quad-rotors, quad-copters can only accelerate laterally in an inertial $x-y$ plane when the angular velocities of each rotor are NOT parallel (or anti-parallel) to the inertial $\hat{z}^0$ direction (because we are assuming no drag due to air resistance). Therefore, it must be true that the instantaneous angular acceleration of the quad-copter is $\vec{0}$. Therefore,

$$\vec{a}_1^0 = \vec{a}_{01}^0 + \vec{\omega}_{01}^0 \times (\vec{\omega}_{01}^0 \times \vec{d}_{01}^0) + 2(\vec{\omega}_{01}^0 \times \vec{v}_{01}^0) = \begin{bmatrix} -129.8 \\ -33.35 \\ 0.1 \end{bmatrix} [\frac{m}{s^2}]$$

Because the quad-rotor is not prysmatic, my final answer is

$$\vec{a}_{02}^1 = \vec{a}_1^0 = \begin{bmatrix} -129.8 \\ -33.35 \\ 0.1 \end{bmatrix} [\frac{m}{s^2}]$$

Problem 3

SOLUTION

Joint	$\vec{\omega}_i^0$	$\vec{v}_i^0$	$\vec{a}_i^0$
0	$[0, 0, 0.5]^T$	$\vec{0}$	$\vec{0}$
1	$[0, 1, 0.5]^T$	$\vec{0}$	$\vec{0}$
2	$[0, -0.5, 0.5]^T$	$[-0.8125, 0.1083, 0.1083]^T$	$[-0.1083, -0.4063, -0.4063]^T$
3	$[0, -0.5, 0.5]^T$	$[-1.8125, 0.2815, 0.2815]^T$	$[-0.1732, -0.5. -0.5]^T$

I got $\vec{\omega}_i$ from inspection. From $\vec{\omega}_i$, I used the following expressions to fill in the rest of the chart.

$$\vec{v}_i^0 = \vec{v}_{i-1}^0 + \vec{\omega}_i^0 \times \vec{r}_i^0$$

$$\vec{a}_i^0 = \vec{\omega} \times (\vec{\omega}_i^0 \times \vec{r}_i^0)$$

This acceleration expression is true because: (1) joint 0 is not accelerating, so the reference frame is not accelerating. (2) the arms are not prysmatic, so there is no relative or Coriolis acceleration. (3) there is no angular acceleration anywhere, IE $\vec{\alpha}_i = 0$. Therefore, the acceleration of each point relative to joint 0 is solely centripetal acceleration.