Problem 1

SOLUTION

Figure 1 below shows multiple numerical solutions compared to the exact solution. If I were to redo this problem, I would pick better N values to show exactly when my approach converges, because with this current approach, my code converges much earlier than my highest N value. Only N = 10, 20 and 50 visibly deviate from the exact solution. Full transparency – there is no good reason to go up to N = 500. My approach converges much earlier than that, BUT my approach certainly converges correctly as desired.

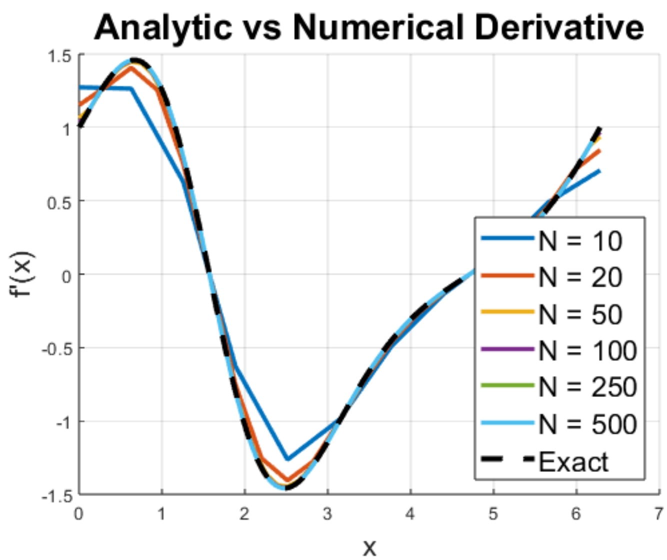

Figure 1: Numerical results compared to exact solution - Q1

Figure 2 shows a relative error convergence plot with both $p = \infty$ and $p = 2$. Reference slopes are plotted to show that $p = \infty$ converges parallel to $\frac{1}{N}$ on a log scale, and $p = 2$ converges parallel to $\frac{1}{N^{\frac{3}{2}}}$.

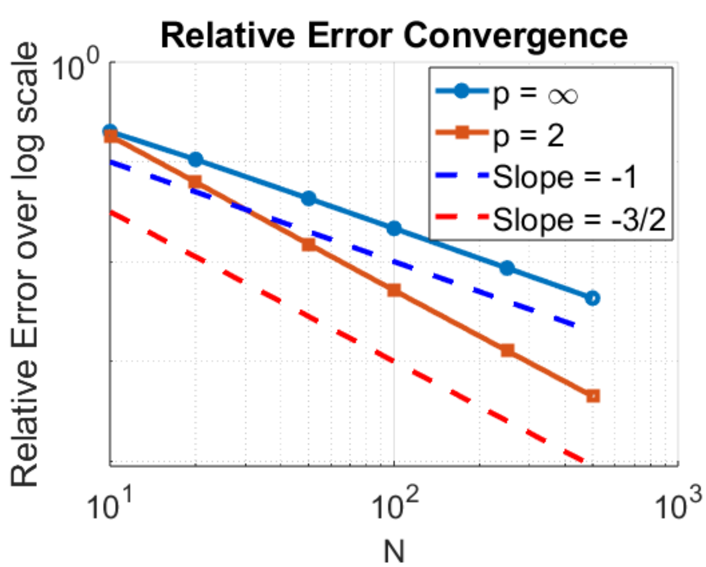

Figure 2: Relative Error Convergence

Problem 2

SOLUTION

Figure 3 below shows multiple numerical solutions compared to the exact solution for the second derivative of $\exp^{sin(x)}$. Again, arbitrary N values were chosen to populate the second figure for this problem with a sufficient amount of points. These points were not chosen to show exactly when my approach converges under a specific tolerance.

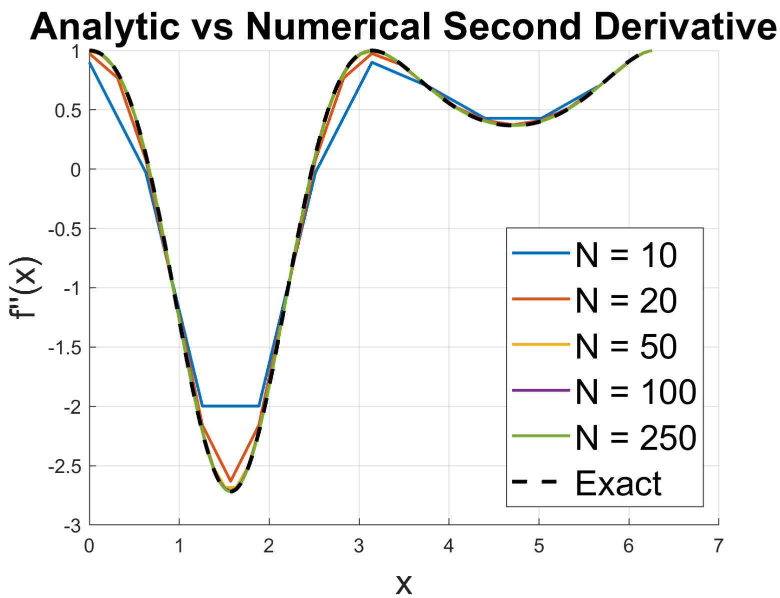

Figure 3: Numerical results compared to exact solution - Q2

Figure 4 shows the error converges with second order. Both figures 3 and 4 successfully capture the trend of convergence where the error shrinks as N increases.

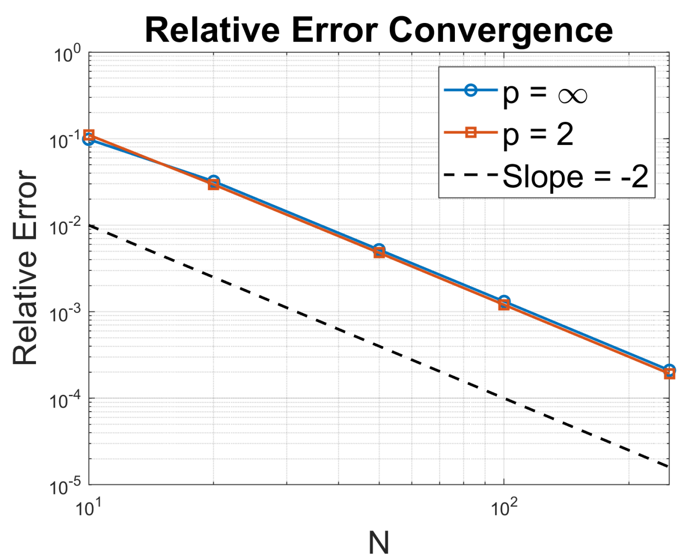

Figure 4: Relative Error Convergence

Problem 3

SOLUTION

Figure 5 below shows multiple numerical solutions compared to the exact solution of the given differential equation for problem 3 of this assignment. These N values were chosen more appropriately than my choices for the first two problems, but they still do not relate to convergence under an arbitrarily tolerance. I will make this adjustment for future assignments this semester.

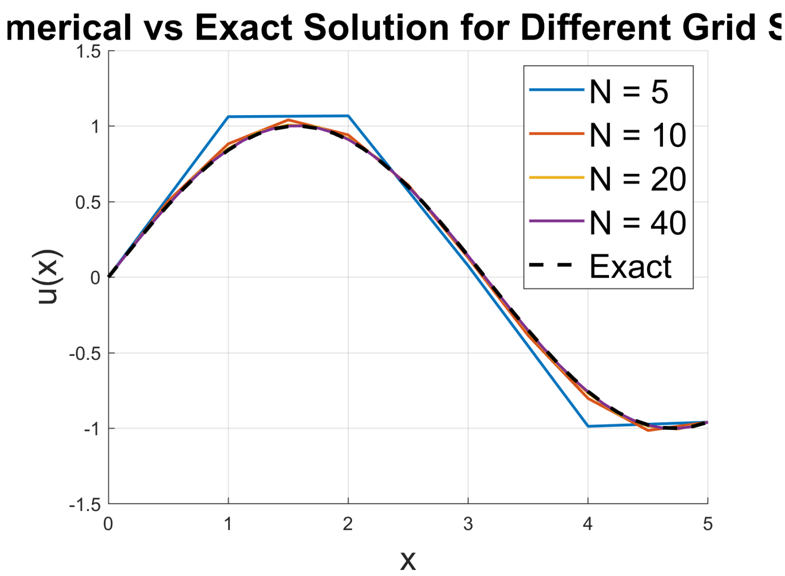

Figure 5: Numerical results compared to exact solution - Q3

Figure 6 shows the error converges with second order. Both figures 5 and 6 successfully capture the trend of convergence where the error shrinks as N increases. A reference slope of -2 is plotted in figure 6 below to show that this method successfully converges with second order.

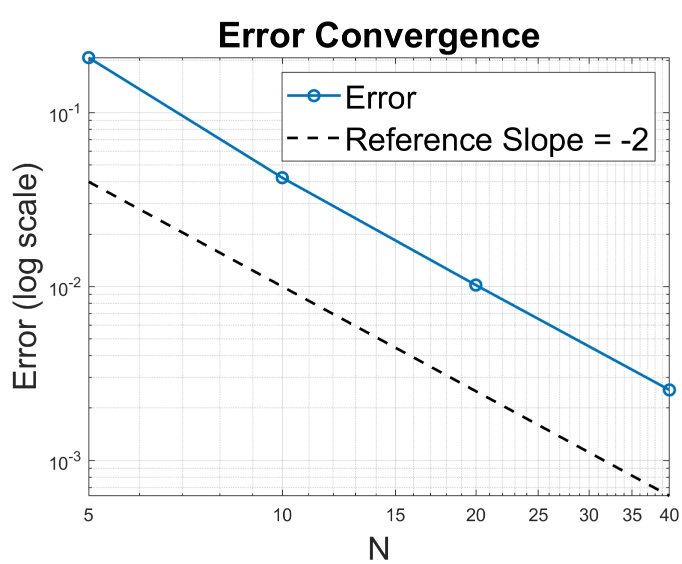

Figure 6: Relative Error Convergence

Problem 4

SOLUTION

Dirichlet Conditions

To show results for this problem, 5 N values were chosen to later build an error convergence plot (N = 16, 32, 64, 128, 256). All 5 numerical solutions are plotted in a 3D subplot to compare to the exact solution.

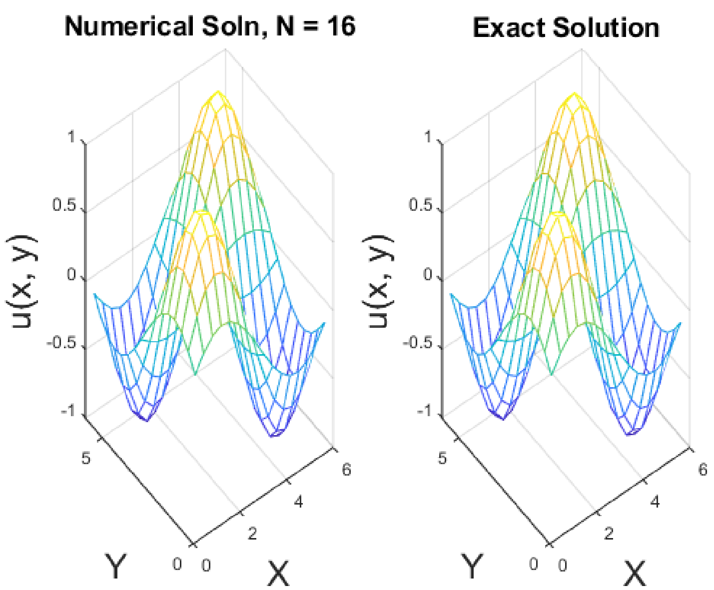

Figure 7: Relative Error Convergence

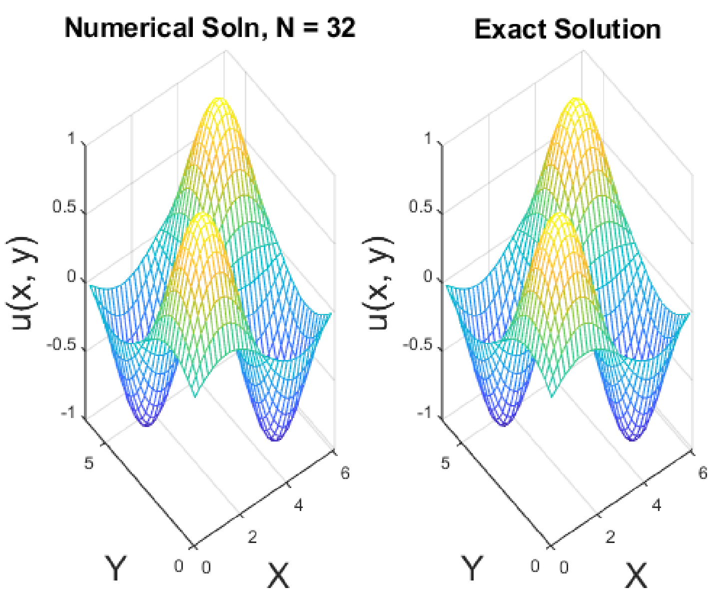

Figure 8: Relative Error Convergence

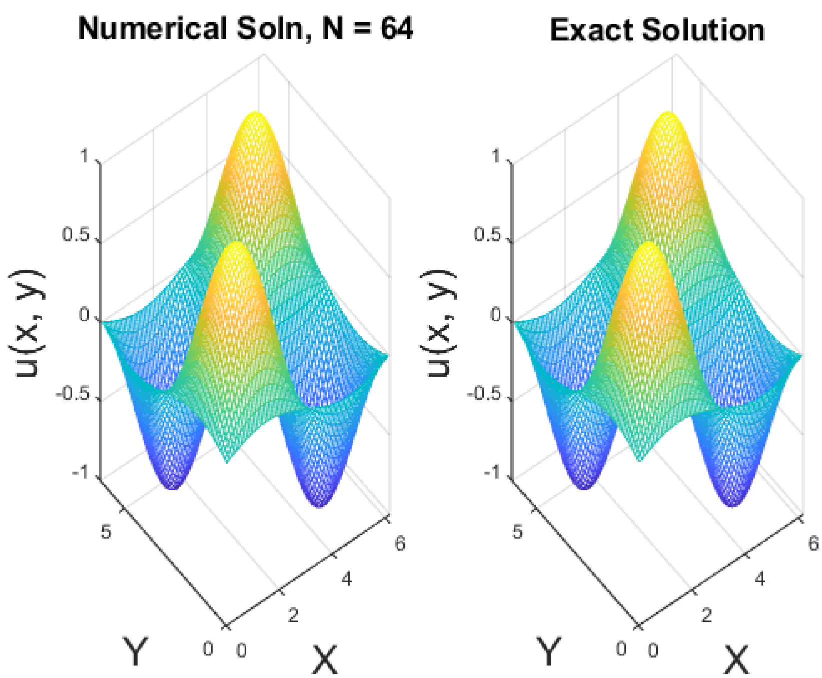

Figure 9: Relative Error Convergence

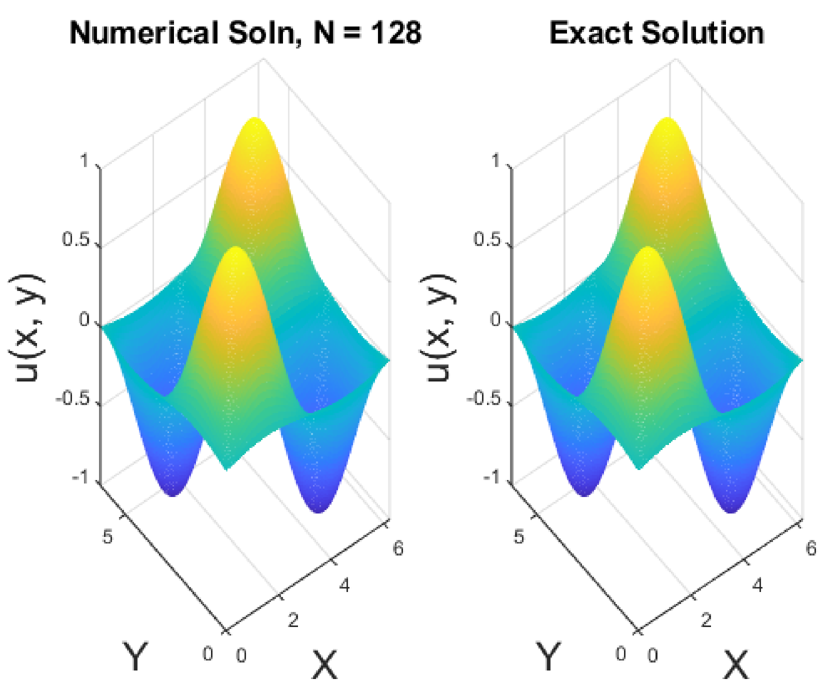

Figure 10: Relative Error Convergence

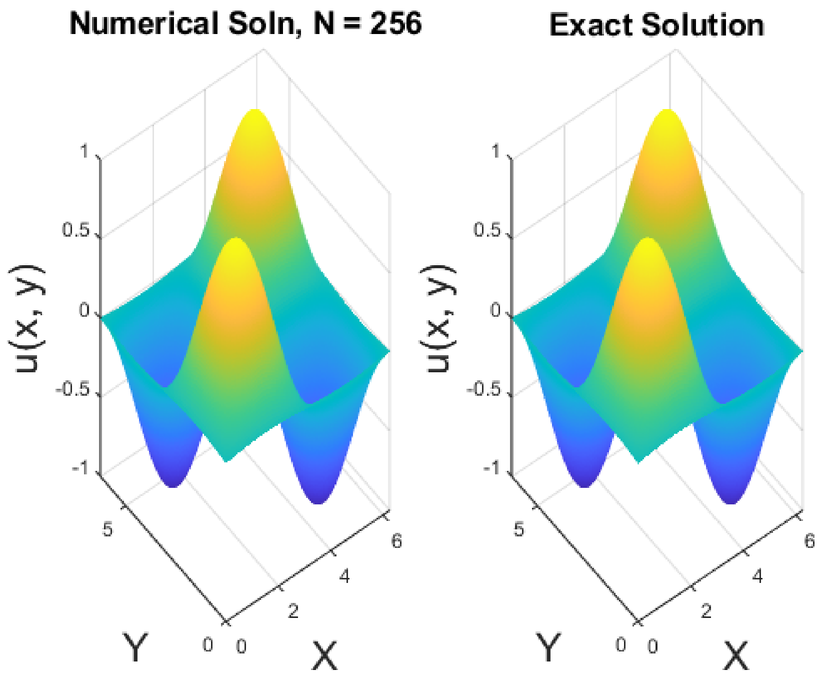

Figure 11: Relative Error Convergence

Figure 12 below shows the error converges with second order. A reference slope of -2 is plotted in figure 6 below to show that this method successfully converges with second order.

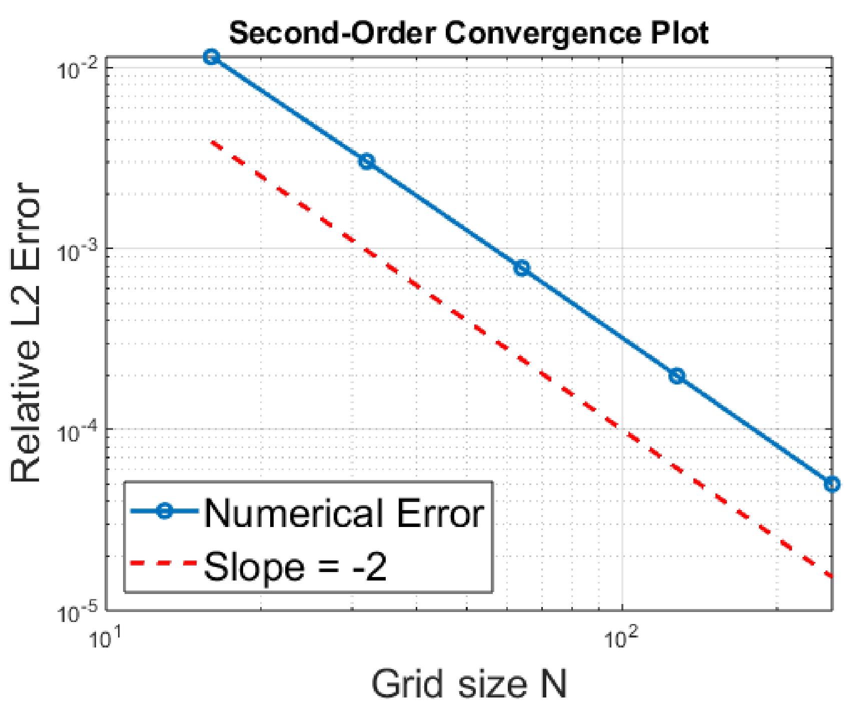

Figure 12: Relative Error Convergence

Neumann Conditions

Neumann conditions introduce new ghost points at the domain boundaries, which add more points into the domain than we actually care about solving. In a linear algebra sense where

$$A\vec{u} = \vec{x}$$

This linear algebra is no longer computable. A is no longer invertible because it's no longer square, it's width is no longer the same length as $\vec{u}$ (which is required to be computable) and MATLAB no longer lets me use backslash to do this matrix math.