# Course Details

## Course Overview:

**Course Code:**  MATH 550A

**Semester Year:**  Fall 2024

**Credit Hours:**  3

**Class Meeting Times: 12:30-1:45 p.m TTr**

**Class Location:**  Alderson Hall 152

**Instructor:** Brennan Sprinkle **Email:** bsprinkl@mines.edu

**Office Phone:** 303-525-5546 **Office Location:** Cheuvenet 230

**Office Hours:** Tuesday after class, Weds at 10:00a.m (also stop by my office whenever you’d like, I may be busy but we can surely find time to chat)

## Pre-requisites:

Multi-variable calculus, Some PDEs, ODEs, Some programming background (ideally MATLAB/Python),

## Course Description:

Partial differential equations can describe an enormous variety of physical systems and are typically hard to solve – but not numerically! I hope to convince you of both these points in this class: a) that PDEs are important to solve and understand b) and that often enough we can use computers solve and understand them (without too much trouble). There are lots of unique and interesting PDEs out there, but we’ll focus on the 3 linear PDEs that seem to show up the most: the Heat Equation, the Wave Equation, the Poisson equation. Using these three equations as a springboard, we’ll talk about a plurality of solution methods, why they work, and how we might extent these methods to other problems/PDEs.

## Learning Outcomes:

**As we study the Poisson equation we will learn how to**

* Set up a finite discretization and analyze error
* Impose different boundary conditions
* Impose boundary conditions on funny shaped domains (like a clown shaped domain)

**As we study the heat equation we will learn how to**

* Convert linear PDEs in higher spatial dimensions to linear algebra problems using finite difference methods
* Learn how to efficiently solve those linear algebra problems
* Solve non-linear ‘reaction diffusion’ equations related to the heat equation

**As we study the wave equation we will learn how to**

* Convert time dependent PDEs into ODEs and how to numerically solve ODEs
* Analyze the accuracy of these methods
* How to use a pseudo-spectral method to approximate functions and solve PDEs

**Returning to the Poisson equation we will learn how to**

* Find the weak form of a PDE and set up a finite element discretization
* Derive the ‘cotangent formula’ for the Laplacian on an arbitrary triangular mesh
* Solve PDEs in funny shaped domains (like a clown shaped domain)

## Assessments:

For each of the above units you will do a short worksheet with a few relevant problems, and a more involved ‘guided project’ type of assignment. Additionally there will be a self-guided final project on a topic of your choice.

* The guided projects will typically involve a few calculations (e.g deriving time step restrictions when solving a damped version of the wave equation) and some implementation of a simple numerical method (e.g a 2D finite difference scheme to solve the heat equation). The purpose is just to give you some guided practice on standard calculations or to fill in some gaps in the lectures notes that are best understood by grinding out a calculation on your own. **You will turn in a short (1-2) page writeup on the assignment that mostly consists of very through figures (with captions) and a description/explanation of those figures in the main text.** I’ll devote a few lectures to academic figure making, and gives lots of examples!
* For the final project, you’re free to work individually or in small teams. Midway through the semester you (and your team) will meet with me to decide on an interesting and relevant topic e.g when we’re talking about the heat equation, you may want to look at solutions to the Fitzhugh–Nagumo equation (which can have very cool spiraly solutions). This is also a great opportunity to kickstart a research project. **As a deliverable for those projects, you will give a five slide presentation to the class or a two page report with detailed figures.**

**The grade breakdown for the class is:**

-25% final project

-3x 10% short worksheets

-3x 15% guided projects

## **Rough timeline of worksheets and projects:**

I’ll post a worksheet close the beginning of every ‘unit’ of the class (e.g the Heat Equation) which will be due a week or two later. Following that I’ll post the ‘guided project’ for the given unit and it’ll be due roughly two weeks later. Around Midway through the semester I’ll schedule meetings with folks to discuss final projects, which will then be due before the end of the semester.

## Required Text:

**There are no required texts for this course. We will mostly work off of notes. But if you’re looking for nice texts on the topics, I like:**

* "[Spectral Methods in Matlab](https://epubs.siam.org/doi/book/10.1137/1.9780898719598)" by Nick Trefethen, which includes lots of [MATLAB codes](https://people.maths.ox.ac.uk/trefethen/spectral.html), and focuses on Fourier and Chebyshev representations for elliptic PDEs.
* "[Finite Difference Methods for Ordinary and Partial Differential Equations](https://epubs.siam.org/doi/book/10.1137/1.9780898717839)" by Randy LeVeque. Which is available freely in [PDF format](https://epubs.siam.org/doi/book/10.1137/1.9780898717839), and [Matlab and latex files](https://staff.washington.edu/rjl/fdmbook/) are available from the Randy's webpage.