NOTE TO GRADER: I was not aware of the requirement to type out HW assignments until today's class (08/29). I typed out as much as I could, and will correct course on all future HW assignments.

Problem 1

SOLUTION

Part 1A

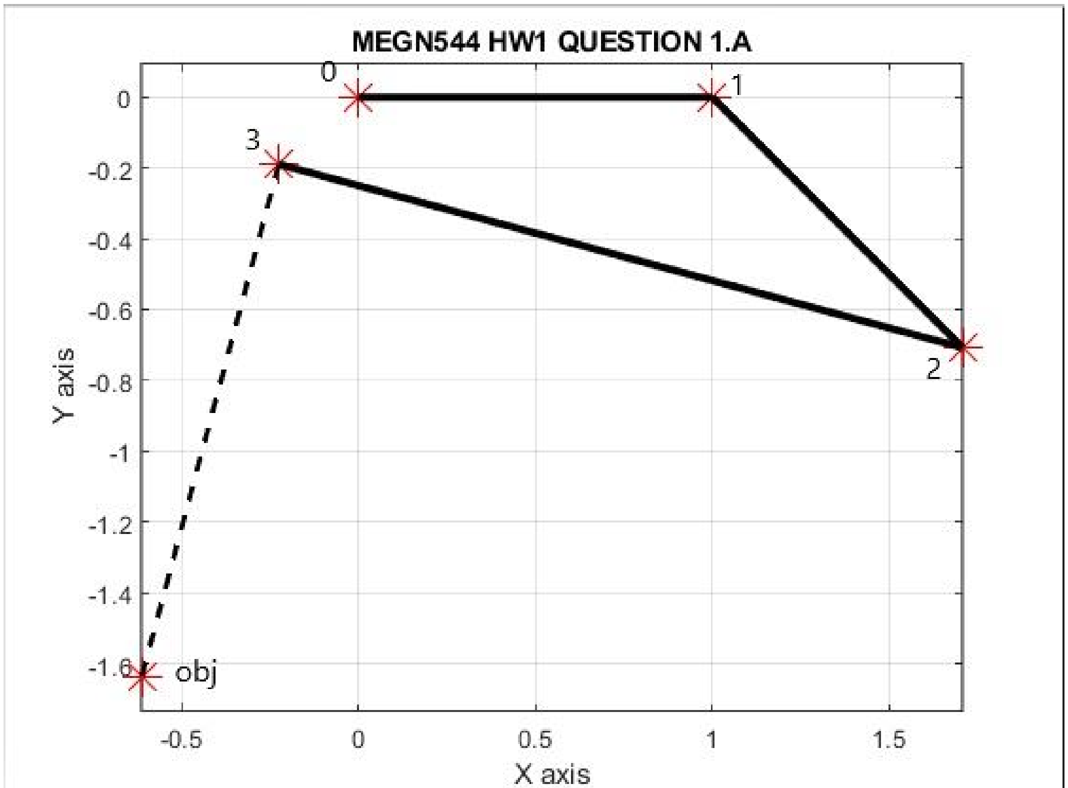


Figure 1: 1A

Part 1B

$$\vec{d_{01}}^{\,0} = [0, 1]^T$$

$$\vec{d_{12}}^{\,1} = [0.5, 0]^T$$

$$\vec{d_{23}}^{\,2} = [-2, 0]^T$$

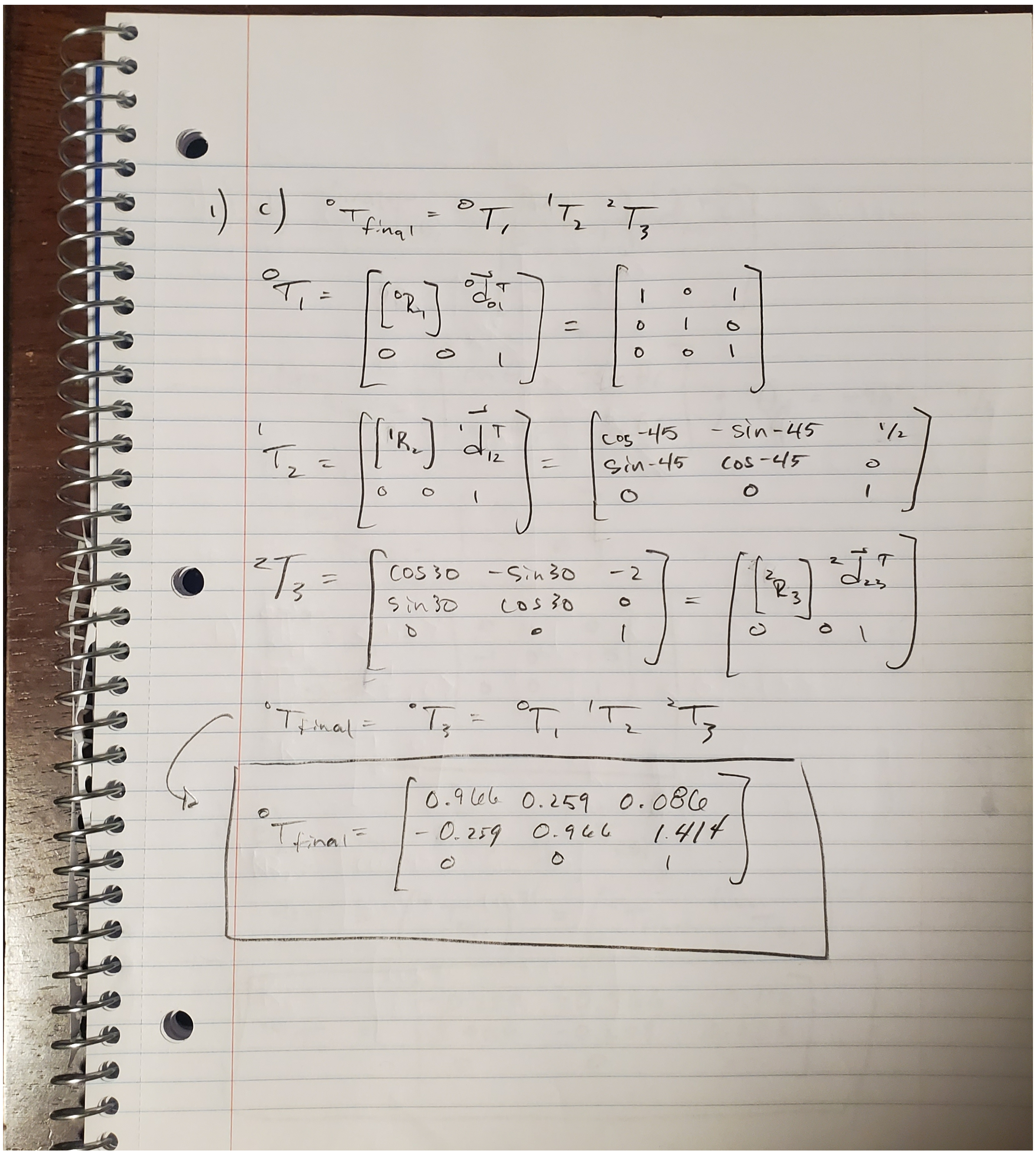
$$\vec{d_{obj}}^{\,final} = [0, -1.5]^T$$

$$R_1^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} cos(-45^o) & -sin(-45^o) \\ sin(-45^o) & cos(-45^o) \end{bmatrix}$$

$$R_3^2 = \begin{bmatrix} cos(30^o) & -sin(30^o) \\ sin(30^o) & cos(30^o) \end{bmatrix}$$

Part 1C



1) c) ${}^0T_{final} = {}^0T_1 \; {}^1T_2 \; {}^2T_3$

$${}^0T_1 = \begin{bmatrix} \left[{}^0R_1\right] & {}^0\vec{d}_{01}^{\,T} \\ 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \left[{}^1R_2\right] & {}^1\vec{d}_{12}^{\,T} \\ 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos -45 & -\sin -45 & 1/2 \\ \sin -45 & \cos -45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} \cos 30 & -\sin 30 & -2 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \left[{}^2R_3\right] & {}^2\vec{d}_{23}^{\,T} \\ 0 \quad 0 & 1 \end{bmatrix}$$

$${}^0T_{final} = {}^0T_3 = {}^0T_1 \; {}^1T_2 \; {}^2T_3$$

$$\boxed{{}^0T_{final} = \begin{bmatrix} 0.966 & 0.259 & 0.086 \\ -0.259 & 0.966 & 1.414 \\ 0 & 0 & 1 \end{bmatrix}}$$

Figure 2: 1C

Part's 1D and 1E

1) d) $${}^{final}T_{obj} = \begin{bmatrix} \left[{}^{final}R_{obj}\right] & {}^{f}\vec{d}_{obj}^{\,T} \\ 0 \quad 0 & 1 \end{bmatrix}$$

$${}^{final}R_{obj} = \begin{bmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{bmatrix} ; \quad \theta_f = -90^\circ = -\pi/2$$

$${}^{final}R_{obj} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\boxed{{}^{final}T_{obj} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1.5 \\ 0 & 0 & 1 \end{bmatrix}}$$

1) e) $${}^{0}T_{obj} = {}^{0}T_{final}\;{}^{final}T_{obj}$$

$${}^{obj}T_{0} = \left({}^{0}T_{obj}\right)^{-1} = \left({}^{0}T_{final}\;{}^{final}T_{obj}\right)^{-1}$$

$$\boxed{{}^{obj}T_{0} = \begin{bmatrix} -0.259 & -0.966 & -0.112 \\ 0.966 & -0.259 & 0.283 \\ 0 & 0 & 1 \end{bmatrix}}$$

Figure 3: 1D and 1E

Problem 2

SOLUTION

Part's 2A and 2B

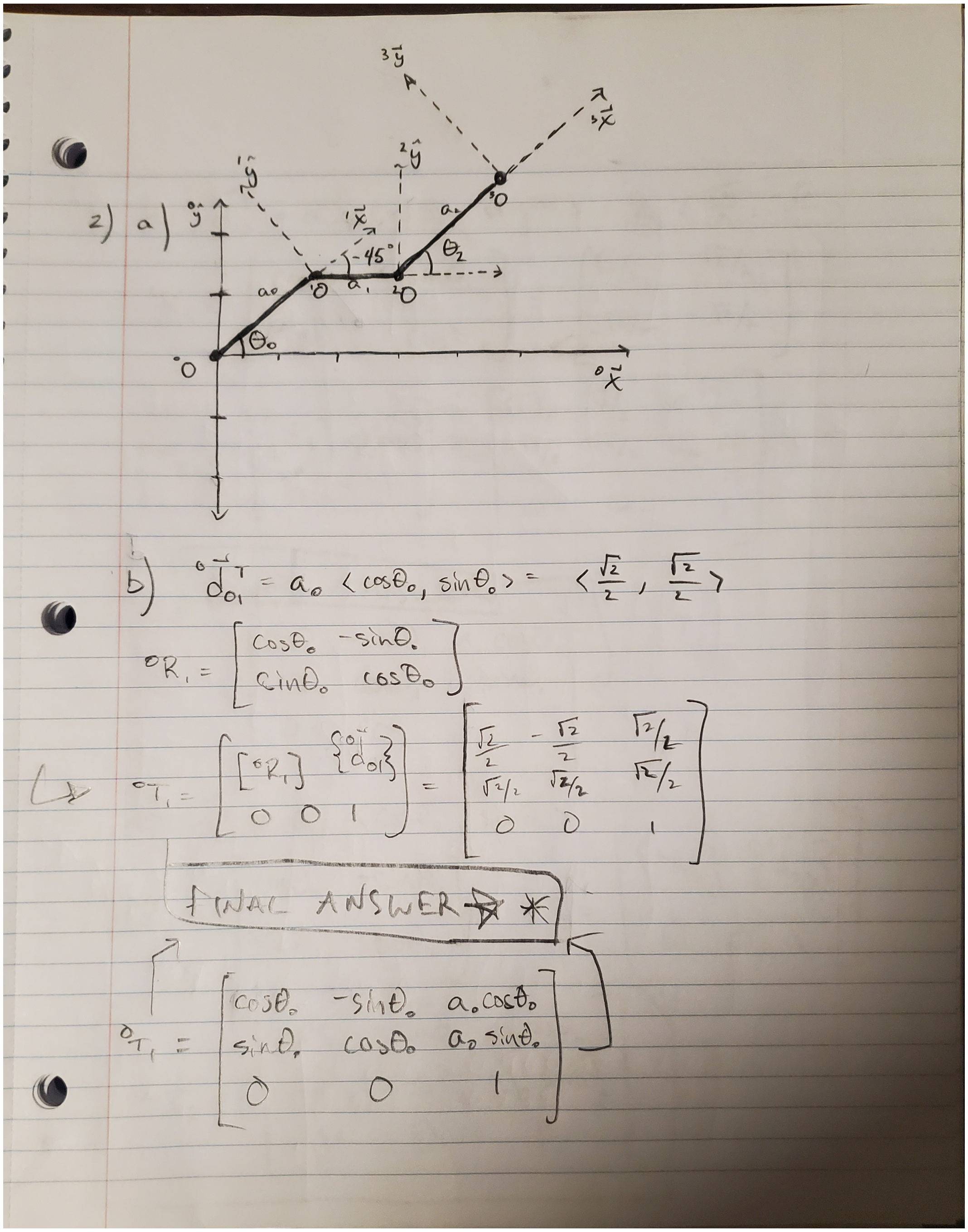
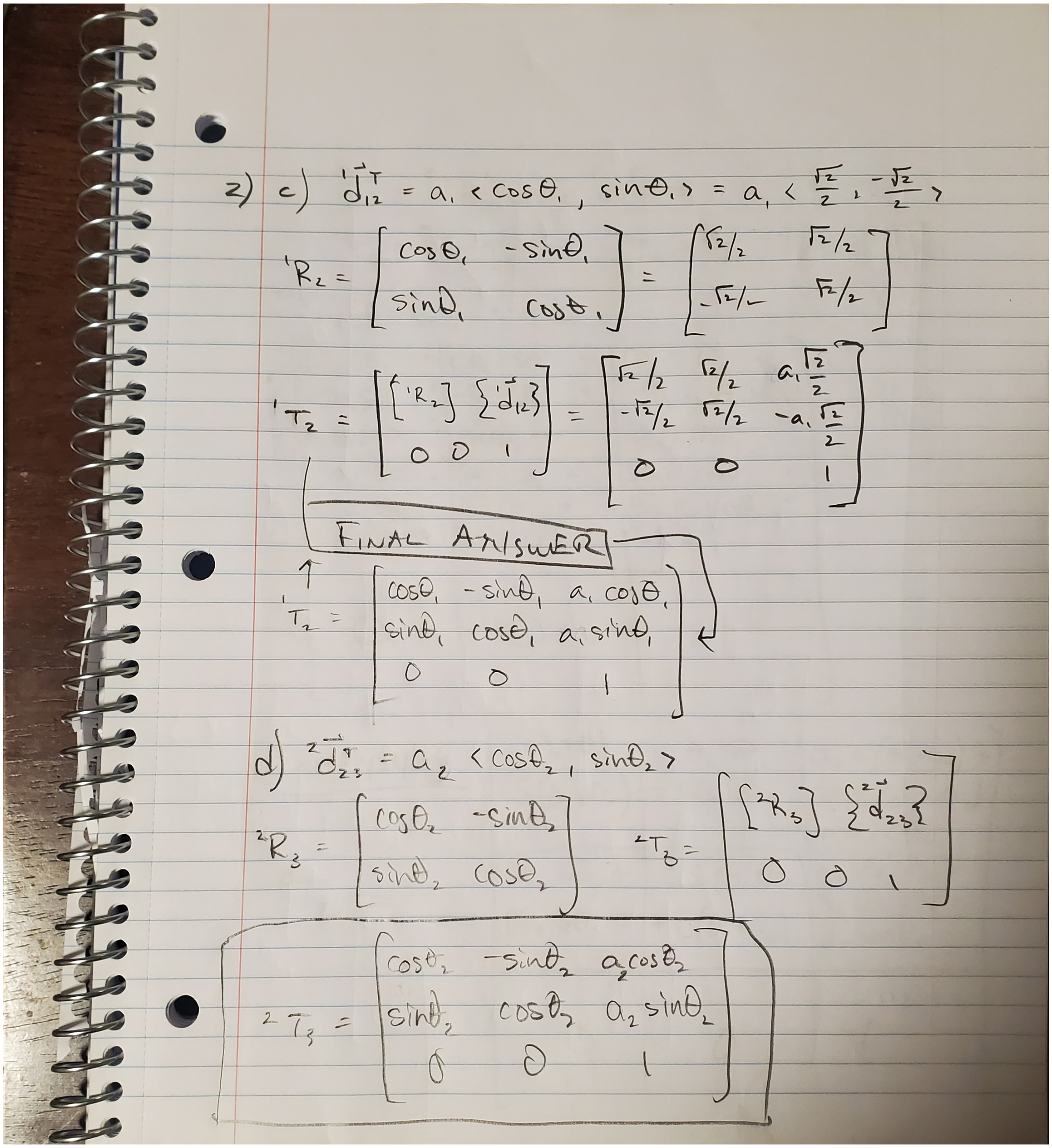


Figure 4: 2A and 2B

Part's 2C and 2D



2) c) $${}^1\vec{d}_{12}^{\,T} = a_1 \langle \cos\theta_1 , \sin\theta_1 \rangle = a_1 \langle \frac{\sqrt{2}}{2} , -\frac{\sqrt{2}}{2} \rangle$$

$${}^1R_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \left[{}^1R_2\right] & \left\{{}^1\vec{d}_{12}\right\} \\ 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & a_1\frac{\sqrt{2}}{2} \\ -\sqrt{2}/2 & \sqrt{2}/2 & -a_1\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

FINAL ANSWER

$${}^1T_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & a_1\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix}$$

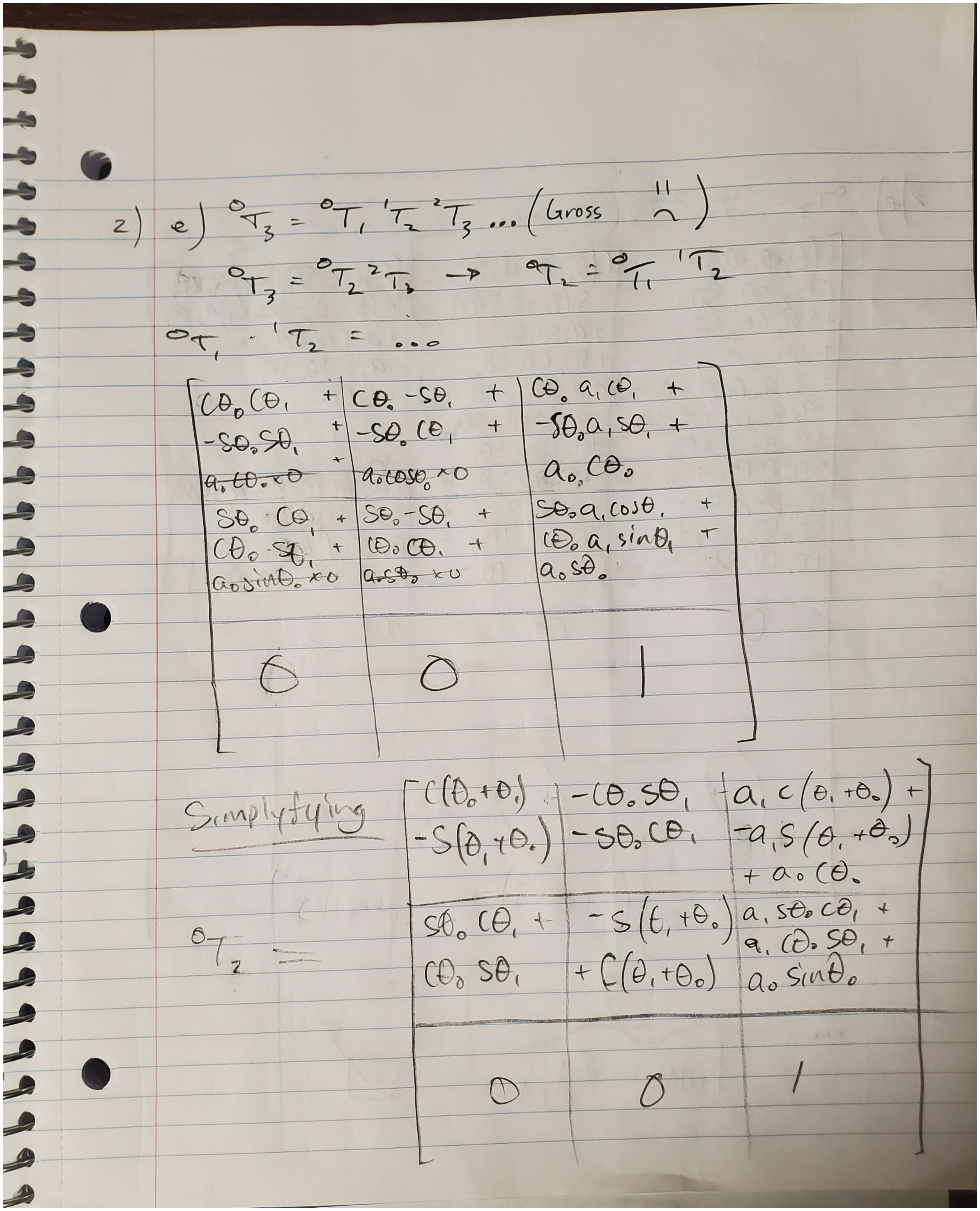
d) $${}^2\vec{d}_{23}^{\,T} = a_2 \langle \cos\theta_2 , \sin\theta_2 \rangle$$

$${}^2R_3 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \qquad {}^2T_3 = \begin{bmatrix} \left[{}^2R_3\right] & \left\{{}^2\vec{d}_{23}\right\} \\ 0 \quad 0 & 1 \end{bmatrix}$$

$$\boxed{{}^2T_3 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & a_2\sin\theta_2 \\ 0 & 0 & 1 \end{bmatrix}}$$

Figure 5: 2C and 2D

Part 2E


2) e) ${}^0T_3 = {}^0T_1\, {}^1T_2\, {}^2T_3 \dots$ (Gross)

${}^0T_3 = {}^0T_2\, {}^2T_3 \rightarrow {}^0T_2 = {}^0T_1\, {}^1T_2$

${}^0T_1 \cdot {}^1T_2 = \dots$

$$\begin{bmatrix} C\theta_0 C\theta_1 + -S\theta_0 S\theta_1 + \cancel{a_0 C\theta_0 \times 0} & C\theta_0 -S\theta_1 + -S\theta_0 C\theta_1 + \cancel{a_0 \cos\theta_0 \times 0} & C\theta_0 a_1 C\theta_1 + -S\theta_0 a_1 S\theta_1 + a_0 C\theta_0 \\ S\theta_0 C\theta_1 + C\theta_0 S\theta_1 + \cancel{a_0 \sin\theta_0 \times 0} & S\theta_0 -S\theta_1 + C\theta_0 C\theta_1 + \cancel{a_0 S\theta_0 \times 0} & S\theta_0 a_1 \cos\theta_1 + C\theta_0 a_1 \sin\theta_1 + a_0 S\theta_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simplifying

$${}^0T_2 = \begin{bmatrix} C(\theta_0+\theta_1) - S(\theta_1+\theta_0) & -C\theta_0 S\theta_1 - S\theta_0 C\theta_1 & a_1 C(\theta_1+\theta_0) + -a_1 S(\theta_1+\theta_0) + a_0 C\theta_0 \\ S\theta_0 C\theta_1 + C\theta_0 S\theta_1 & -S(\theta_1+\theta_0) + C(\theta_1+\theta_0) & a_1 S\theta_0 C\theta_1 + a_1 C\theta_0 S\theta_1 + a_0 \sin\theta_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 6: 2E

Still on Part 2E,

$$T^0_3 = T^0_2 * T^2_3$$

From the T_2^0 already shown, let T_2^0 be equal to:

$$T_2^0 = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} C(\theta_2)T_{11} - S(\theta_2)T_{12} & -S(\theta_2)T_{11} + C(\theta_2)T_{12} & a_2C(\theta_2)T_{11} + a_2S(\theta_2)T_{12} + T_{13} \\ T_{21}C(\theta_2) + T_{22}S(\theta_2) & -S(\theta_2)T_{21} + T_{22}C(\theta_2) & a_2C(\theta_2)T_{21} + a_2S(\theta_2)T_{22} + T_{23} \\ T_{31}C(\theta_2) + T_{32}S(\theta_2) & -S(\theta_2)T_{31} + T_{32}C(\theta_2) & a_2C(\theta_2)T_{31} + a_2S(\theta_2)T_{32} + T_{33} \end{bmatrix}$$

Problem 3

SOLUTION
For this problem, I will use this general equation:

$$T_j^i = \begin{bmatrix} [R_j^i] & \vec{d_{ij}}^{\,i} \\ \vec{0}^T & 1 \end{bmatrix}$$

Part 3A

$$\theta_0 = 0$$

$$R_1^0 = I_{2x2}$$

$$\vec{d_{01}}^{\,0} = [2, 1]^T$$

$$T_1^0 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 3B

$$\theta_1 = \frac{\pi}{4}$$

$$R_2^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 \\ S\theta_1 & C\theta_1 \end{bmatrix}$$

$$\vec{d_{12}}^{\,1} = [0, 0]^T$$

$$T_2^1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 3C

$$\theta_2 = 0$$

$$R_2^3 = I_{2x2}$$

$$\vec{d_{23}}^{\,2} = [2, 0]^T$$

$$T_3^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 3D

$$\theta_3 = \frac{3\pi}{4}$$

$$R_3^4 = \begin{bmatrix} C\theta_3 & -S\theta_3 \\ S\theta_3 & C\theta_3 \end{bmatrix}$$

$$\vec{d_{34}}^{\,3} = [0, 0]^T$$

$$T_4^3 = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 3E

$$\theta_4 = 0$$

$$R_4^5 = I_{2x2}$$

$$\vec{d_{45}}^{\,4} = [0, 3]^T$$

$$T_5^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 3F

$$T_5^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_4^5$$

Using Matlab,

$$T_5^0 = \begin{bmatrix} 0 & 1 & 3.4142 \\ -1 & 0 & -0.5858 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 3G

From Inspection,

$$\vec{p_5}^0 = \vec{d_{01}}^0 + \vec{d_{12}}^0 + \vec{d_{23}}^0 + \vec{d_{34}}^0 + \vec{d_{45}}^0$$

$$\vec{p_5}^0 = \vec{d_{01}}^0 + \vec{d_{23}}^0 + \vec{d_{45}}^0$$

$$\vec{d_{01}}^0 = Given$$

$$\vec{d_{23}}^0 = T_1^0 * T_2^1 * \vec{d_{23}}^2$$

$$\vec{d_{45}}^0 = T_1^0 * T_2^1 * T_2^3 * T_3^4 * \vec{d_{45}}^4$$

$$\vec{d_{05}}^0 = \vec{d_{01}}^0 + \vec{d_{23}}^0 + \vec{d_{45}}^0$$

$$\vec{d_{05}}^0 = [3.414, -0.586]^T$$

Part 3H

Let $\theta_1 = \frac{\pi}{4}$ and $\theta_3 = \frac{3\pi}{4}$.

$$R_5^0 = R_1^0 * R_2^1 * R_3^2 * R_3^4 * R_4^5$$

Note: for R_1^0, R_2^3 and R_4^5, $\theta = 0$. So,

$$R_5^0 = R_2^1 * R_3^4$$

$$R_5^0 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 3I

$$\vec{d_{05}}^0 = \vec{d_{01}}^0 + \vec{d_15}^0$$

$$\vec{d_{15}}^0 = \vec{d_{05}}^0 - \vec{d_{01}}^0$$

$$\begin{bmatrix} \vec{d_{15}}^0 \\ 1 \end{bmatrix} = [T_5^0] \begin{bmatrix} \vec{d_{15}}^5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \vec{d_{15}}^5 \\ 1 \end{bmatrix} = [T_5^0]^{-1} \begin{bmatrix} \vec{d_{15}}^0 \\ 1 \end{bmatrix} = [T_0^5] \begin{bmatrix} \vec{d_{15}}^5 \\ 1 \end{bmatrix}$$

$$\vec{d_{15}}^5 = \begin{bmatrix} -1.414 \\ 1.586 \end{bmatrix}$$

The total displacement from O_1 to O_5 is $||\vec{d_{15}}^5|| = \sqrt{(1.414)^2 + (1.586)^2} = 2.125$. WNote that T_5^0 is equal to the product of all the former transformation matrices already calculated earlier in this problem, and $T_0^5 = [T_5^0]^{-1}$

Problem 4

SOLUTION

Part 4A

NO. NOT A ROTATION MATRIX. The determinant is negative one instead of positive one.

Part 4B

YES. This matrix has orthogonal rows/cols, units rows/cols, and a determinant of positive 1.

Part 4C

YES. This matrix has orthogonal rows/cols, units rows/cols, and a determinant of positive 1.

Part 4D

NO. NOT A ROTATION MATRIX. The determinant is negative one instead of positive one.

Problem 5

SOLUTION

Part 5A

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

This is the only possibility that makes all rows and columns unit and orthogonal.

Part 5B

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

This is again the only possibility that makes all rows and columns unit and orthogonal, but the entry has to be negative so that the determinant can come out to be positive.

Part 5C

$$\begin{bmatrix} 0.3698 & 0.3353 & 0.8665 \\ -0.9276 & 0.1860 & 0.3240 \\ -0.0524 & 0.9236 & 0.3797 \end{bmatrix}$$

I got this answer bu finding the entry in each row that would make it unit. For example:

$$B_i = \sqrt{1 - A_i^2 - C_i^2}$$

Part 5D

$$\begin{bmatrix} 0.4094 & 0.1166 & 0.9049 \\ -0.2039 & 0.9550 & 0.2153 \\ 0.8893 & 0.2727 & -0.3672 \end{bmatrix}$$

Again, for every row or column, it must be true that

$$A_i^2 + B_i^2 + C_i^2 = 1$$