Problem 1

SOLUTION

Assuming the density of Aluminum, ρ_{Al}, is $2710\frac{kg}{m^3}$. Also assuming the density of the plate is uniformly distributed.

$$V = \pi R^2 t$$

$$m = \rho_{Al} V = \rho_{Al} \pi R^2 t = 0.4257[kg]$$

PART A

In the disk's body frame,

$$I_{disk} = \begin{bmatrix} \frac{1}{12}mt^2 + \frac{1}{4}mR^2 & 0 & 0 \\ 0 & \frac{1}{12}mt^2 + \frac{1}{4}mR^2 & 0 \\ 0 & 0 & \frac{1}{2}mR^2 \end{bmatrix} = \begin{bmatrix} 0.0011 & 0 & 0 \\ 0 & 0.0011 & 0 \\ 0 & 0 & 0.0021 \end{bmatrix}$$

PART B

In the inertial frame,

$$I^0_{disk} = R^0_{disk} I_{disk} (R^0_{disk})^T \approx \begin{bmatrix} 0.0011 & 0 & 0 \\ 0 & 0.0011 & 0 \\ 0 & 0 & 0.0021 \end{bmatrix}$$

PART C

$$\Sigma \vec{T} = [\vec{\omega}]_x (I_0 \vec{\omega} + I_0 \vec{\alpha}) = \vec{0} \rightarrow \vec{\alpha} = -\vec{\omega} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} [\frac{rad}{s^2}]$$

Where $\vec{\alpha}$ and $\vec{\omega}$ are both in the world/zero frame.

$$\Sigma \vec{F} = m\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r}$$

$$\rightarrow |\vec{a}| = \frac{1}{m}|\omega^2| r + \frac{1}{m}|\vec{\alpha}| r = 0.1349[\frac{m}{s^2}]$$

Note $\rightarrow$ this is true for any $\vec{r}$ on the disk.

PART D

$$\Sigma \vec{F} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix} = m\vec{a}$$

$$\vec{a} = \frac{1}{m} \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.2350 \\ 0.4698 \\ 0.7047 \end{bmatrix} [\frac{m}{s^2}]$$

$$\Sigma \vec{T} = \begin{bmatrix} 0.4 \\ 0.5 \\ 0.6 \end{bmatrix} = [\vec{\omega}]_x (I_0 \vec{\omega} + I_0 \vec{\alpha})$$

$$\vec{\alpha} = I_0^{-1}[\vec{\omega}]_x^{-1}(\begin{bmatrix}0.4\\0.5\\0.6\end{bmatrix} - [\vec{\omega}]_x I_0 \vec{\omega})$$

$$\vec{\alpha} = \begin{bmatrix}454.5455\\528.1579\\354.8940\end{bmatrix} [\frac{rad}{s^2}]$$