Defining Forcing Function (Bhargav)

General notes:

- This will be a forcing function that changes over time/crankshaft angle
    - Setting x-var for now as crankshaft angle
- Assuming that 4-stroke cycle is being used
- Force applied to the piston changes based on the stage in the cycle
- Transitions between stages probably can't be done continuously, so will likely need to model forcing function as a piecewise function, but hopefully equations in each stage are continuous
    - In code, will likely be a series of if-statements where each cycle is a crankshaft-angle-range
    - Each if-statement will likely have a forcing equation corresponding to that stage
- Thinking in terms of crankshaft angle (deg) starting at TDC for intake stroke, if it's being driven or being a driving force, and what forces are involved:
    - 0 - 180
        - Intake stroke (driven)
        - Forces: none
        - Note, 0deg is TDC and 180 is BDC, and so on…
    - 180 - 360
        - Compression stroke (driven)
        - Forces: increasing pressure
    - 360 - 540
        - Combustion (instantaneous)
        - Power stroke (driving)
        - Forces: Instant combustion pressure, then decreasing pressure
    - 540 - 720
        - Exhaust valve opens (instantaneous)
        - Exhaust stroke (driven)

            - Forces: none

- How will continuous motion of the single piston assembly be produced?
    - In the intake, compression, and exhaust stroke stages, the forcing function will apply the power stroke force to the piston head (to mimic the presence of 3 other piston assemblies), and then this piston assembly will do it itself during its power stroke stage
    - In every stage, the net force on the piston assembly will be power stroke - compression stroke, since these things are always happening simultaneously
    - So really we just need to simulate one stage (down motion will follow same profile as up motion, I think) and that's it…
        - Maybe we can add some animation effects to distinguish intake, compression, combustion, and exhaust strokes
    - Might have to do a timestep thing based on numerical integration, for how the force profile causes displacement (integration from accel to displacement)
        - Need to look at previous examples for how this was done
        - Otherwise, can probably just do a timestep simulation

4-stroke cycle stages:

- Purpose of this is to determine the forces acting on the piston at each stage; will figure out how to produce continuous motion of the piston later
    - Pretty sure continuous motion is driven by the constant power strokes…
    - Yup, that's what this video shows: How Car Engine Works | Autotechlabs
    - This means 4 piston assemblies are necessary to execute the 4-stroke cycle

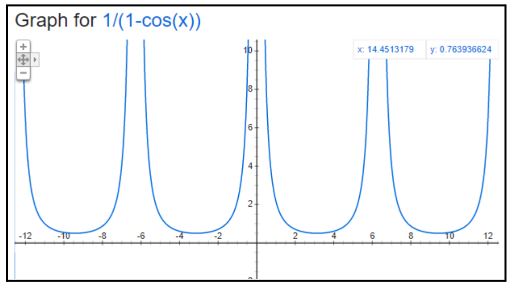
- ## Intake (down)
    - Constant pressure, increasing volume, constant temperature
    - Will assume 0 gage pressure (atmospheric pressure; no net forces here)

- ## Compression (up)
    - Increasing pressure, decreasing volume, increasing temperature
    - Need to model increases in pressure and temperature
        - Doing it like example 3.7.1 from here

- This example is for piston compression, so directly applicable
- Assumptions
  - Adiabatic compression
    - Means it happens without heat exchange w/ the surroundings
    - This is valid based on the example description: *"Adiabatic compressions actually occur in the cylinders of a car, where the compressions of the gas-air mixture take place so quickly that there is no time for the mixture to exchange heat with its environment"*
  - Quasi-static compression
    - Means that the process happens slowly enough to where there's internal thermodynamic equilibrium (no weird internal pressure gradients or anything)
    - Actual gas compression in engine cylinders is NOT quasi-static, but this is the assumption being made
  - Ideal gas
    - Means a *"a theoretical gas composed of many randomly moving point particles that are not subject to interparticle interactions"*
    - Ideal gases don't actually exist, but again, just an assumption we're making

- ## Finding pressure
  - $$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^{\gamma}$$
  - V = Ah, A cancels so V1/V2 = h1/h2
  - Need to relate h to crankshaft angle
    - From angle ranges specified, know that 0deg=TDC and 180deg=BDC
    - Based on crank length "r" and height "s" above TDC, h = s + r - rcos(theta)
      - This works for all theta

- h1=s+r-rcos(180) = s+2r (e.g. cylinder height; always true at the start of compression)
- h2=s+r-rcos(theta)
- Using F=A*P, F2 = AP1((s+2r)/(s+r-rcos(theta))^gamma
    - Thus, shape of the pressure vs crankshaft angle curve follows the form 1/(1-cos(x))

    - A: piston cross sectional area
    - P1: pressure at the start of the compression stage
        - = atmospheric (101325 Pa)
    - s: cylinder height above TDC
    - r: crank length
    - gamma: ratio of Cp/Cv for the gas mixture
        - = 1.4, from the example
        - From a quick google search, at 300K Cp=1.005 and Cv=0.718, so seems right
            - Temperature changes during compression, so these values probably do too, but will assume that the ratio stays constant
            - This is also the assumption the example probably makes
    - theta: crankshaft angle
        - This is the only independent variable in this equation

- Finding temperature

- $$T_2 = \left(\frac{p_2 V_2}{p_1 V_1}\right) T_1$$

            - P1: known (see above)
            - T1: temperature at the start of compression stage
                - = ambient (25C = 298.15K)
            - V2/V1: reciprocal of above; known
            - P2: this is at peak compression, so can simplify compression expression to get this (without needing to know theta)
        - Here, both pressure and temperature can be found instantaneously and subsequently for any crankshaft angle

- ## Combustion
    - Increasing pressure, constant volume, increasing temperature
    - Called "heat addition" in the Otto cycle
    - This will be assumed to be an instantaneous step increase in pressure on the piston head
    - Will assume there's conservation of mass within the piston assembly and that ideal gas law still applies, so from P=nRT/V, n,R,V are all constant, so T is the only thing that can increase pressure here
    - Need to model peak temperature and pressure
        - For the heat addition (combustion) stage, have the formula q2,3 = mCv(T3-T2)
            - If m, Cv, T2 and q2,3 are known, then can calculate T3; q2,3 is the only unknown
        - Standard heats of combustion for different compounds can be looked up
            - For gasoline, heat of combustion is about 47 kJ/g
            - From google, ideal stoichiometric ratio of a gasoline engine is 14.7:1, meaning 14.7g of air required to burn 1g of gasoline
            - How to find heat of combustion for air-fuel mixture? Is it simply finding the mass of gasoline in the cylinder based on the total number of moles and stoichiometric ratio, and multiplying that by the gasoline heat of combustion? Does air contribute nothing?
                - This is a bit hard to find on google, so going to assume yes this is how it works
        - Can use ideal gas law to figure out number of moles that we start with for the air-fuel mixture, based on intake stage assumptions
            - n = PV/RT

- P = ambient pressure (see compression section)
- V = A(s+2r) (calculation from compression section)
- R = 8.3145 1/(mol*K)
- T = ambient temperature (see compression section)
- So, n=PA(s+2r)/RT
    - This can be calculated instantaneously; not dependent on compression calcs

■ Finding q2,3

- Calculate moles of gasoline in cylinder: n/15.7
- Assuming an average molecular weight of 0.1 kg/mol for gasoline, based on a quick google search
- Calculate grams of gasoline in cylinder (m): (n/15.7)(0.1)
- Heat of combustion (kJ) of gasoline in cylinder: q2,3=(n/15.7)(0.1)(47)
    - This can be calculated instantaneously; not dependent on compression calcs

■ Finding temperature

- T3=q2,3/(m*Cv)+T2
- q2,3: known now
- m: known now
- T2: known from compression section
- Cv = 0.718, found in compression calcs
- This can be calculated instantaneously, based on T2 from compression section

■ Finding pressure

- This is using ideal gas law, from state 2 (start of combustion) and state 3 (end of combustion)
- From gay lussac's law (still assuming an ideal gas):

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

            - P3=P2T3/T2
            - P2: known from compression section
            - T2: known from compression section
            - T3: known from above
            - This can be calculated instantaneously, based on compression section calcs and T3 above

- ## Power stroke (down)
    - Decreasing pressure, increasing volume, decreasing temperature
    - Note, there is pressure acting on the piston head all throughout this step
    - Need to model decreases in pressure and temperature
        - Note: compression and power strokes occur simultaneously in a 4-stroke cycle engine
            - Compression stroke theta is 180-360deg
            - Power stroke theta is 360-540deg
        - The higher T and thus P after the combustion reaction are the differences from the compression stroke conditions
        - This is about how pressure on the piston head and temperature in the cylinder change with crankshaft angle, so these should be the same formulas as in the compression section but just with different T and P
        - ### Finding pressure
            - $$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^{\gamma}$$
            - Using state 3 (start of power stroke) and state 4 (end of power stroke)
            - V = Ah, A cancels so V3/V4 = h3/h4
            - h3=s+r-rcos(360) = s (always true at the start of power stroke)
            - h4=s+r-rcos(theta)
            - Using F=A*P, F4 = AP3(s/(s+r-rcos(theta)))^gamma
                - P3: known from end of combustion stage (see above)

- (All other vars the same as from compression stage)
- This can be calculated instantaneously, based on P3 and theta
- theta from compression stroke will be different than theta here, for power stroke, since the former goes from 180-360deg, while the latter goes from 360-540deg
- Will need to do something like “theta_p = theta_c + 180” or something

- Finding temperature
  - $$T_2 = \left(\frac{p_2 V_2}{p_1 V_1}\right) T_1$$
  - Using same states mentioned in pressure section above
  - P3: known (see combustion section)
  - T3: known (see combustion section)
  - V4/V3: reciprocal of above; known
  - P4: known at this point
  - This can be calculated instantaneously, based on P3 and P4

- Exhaust valve opens
  - Decreasing pressure (to atmospheric pressure), constant volume, decreasing temperature (to atmospheric temperature)
  - Called "heat rejection" in the Otto cycle
  - This will be assumed to be an instantaneous drop in pressure and temperature (no forces here)
- Exhaust stroke (up)
  - Constant pressure, decreasing volume, decreasing temperature?
  - Will assume 0 gage pressure (atmospheric pressure; no net forces here)

Input parameters

Specific to forcing function

- r: crank length
- s: height above TDC
- d: piston diameter
- P1: atmospheric pressure
- T1: atmospheric temperature
- Cp: specific heat of constant pressure, for air-fuel mixture
- Cv: specific heat of constant volume, for air-fuel mixture
- Stoichiometric ratio: ratio of air to gasoline of air-fuel mixture