JF PY1T10 Special Relativity

Lecture 6:

- Time Dilation

Summary of Lecture 5

Length Contraction

Proper / Rest Length (L_0): The length of an object in the object's rest frame (S).

Length of an object as measured from a moving frame of reference (L): the distance between the endpoints which are measured simultaneously.

$$\frac{L_0}{L} = \gamma$$

Does the Lorentz Contraction *really* take place?

All we can do is define what observations we need to take in order to measure the length of some object that is in motion relative to us. In this case we simply measure the positions of its ends at the same instant, as judged by us.

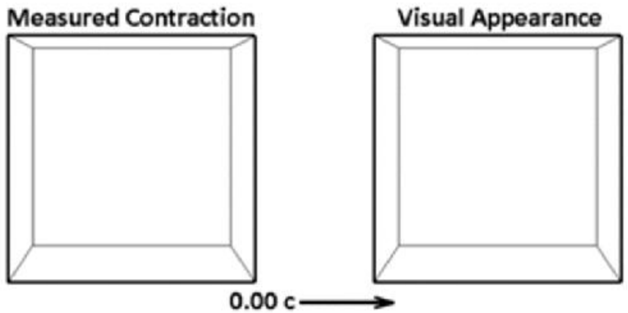
The contraction is not a property of matter, but something inherent in the measuring process. It is frame-dependent.

Summary of Lecture 5

Can we *see* the Lorentz Contraction? (Can you take a photo of it?)

Need to first consider the difference between observing (or measuring) and seeing:

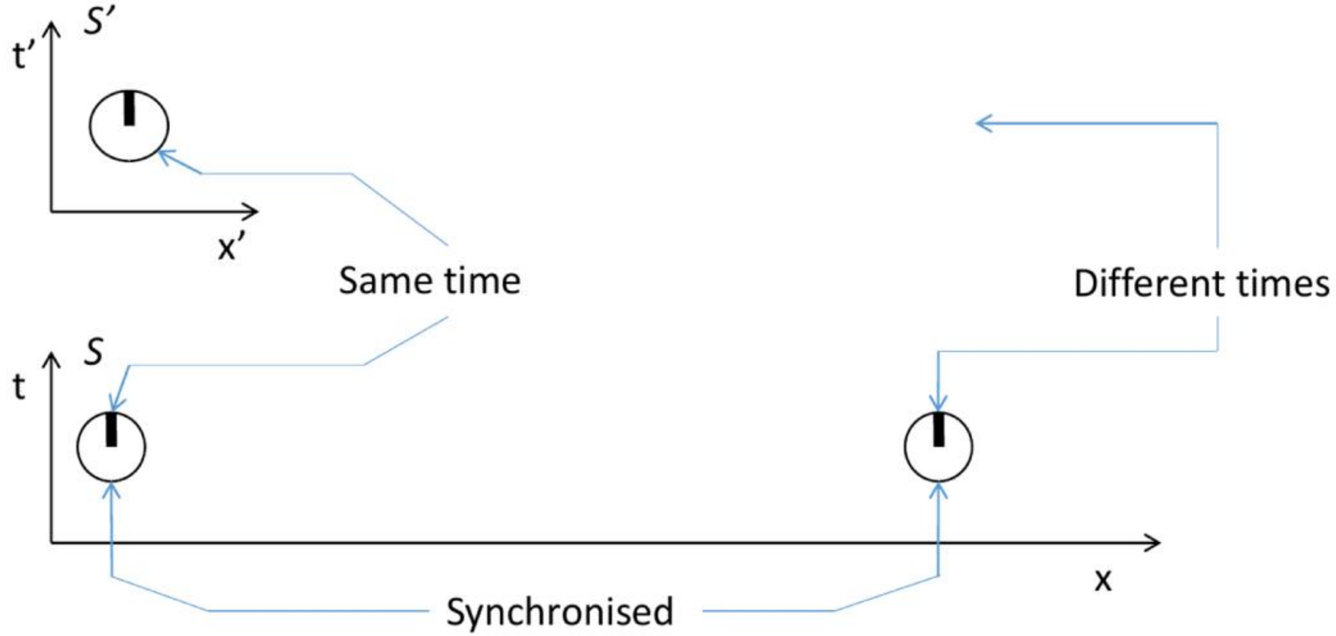
- Observing: Photons leave an object simultaneously
- Seeing: Photons arrive at the observer (camera sensor) simultaneously.



Length contraction: Not a distortion caused by photon time delay. Actual measurement.

Effect of photon time delay evident to observer as rotation of the object (Terrell rotation).

Time Dilation "*Moving Clocks Run Slow*"



Three identical clocks:

Two clocks are at rest in *S* in different positions and are synchronised.

One clock is at rest in *S′*.

Origins of *S* and *S′* coincide at $t = t' = 0$.

Time Dilation

Frame *S'*: Two events at the same position, but different times (clock at rest in *S'*):

$$(t'_1, x') \rightarrow (t'_2, x')$$

Define the **proper time interval** $\boldsymbol{\tau_0} = t'_2 - t'_1$.

This is the time interval between two events which occur at the same point in *S'* (clock at rest)

Note that while there are an infinite number of inertial frames that these events occur in, there is only one inertial frame where these events occur at the same point – the frame that moves with the clock. This frame is unique, which is why we call the time interval between events in this frame the *proper time interval*.

Time Dilation

What is the time interval as measured in *S*?

$$\tau = t_2 - t_1$$
$$t_2 = \gamma(t_2' + {vx'}/{c^2})$$
$$t_1 = \gamma(t_1' + {vx'}/{c^2})$$

$$\therefore\ t_2 - t_1 = \gamma(t_2' - t_1')$$

$$\boxed{\tau = \gamma\tau_0}$$

But $\gamma \geq 1$, therefore $\tau > \tau_0$.

To an observer in *S*, the running clock in *S'* is running slow.

This is called ***Time Dilation***.

Age less rapidly.

Any periodic process can be considered a **clock:** pendulum, quartz crystal, half-life of radioactive particles etc.

Need clock travelling at $v \sim c$ to see the effect.

Rossi & Hall Experiment

Time dilation as predicted by Special Relativity is often verified by particle lifetime experiments.

- **1941** Rossi & Hall looked at decay of muons (elementary particle, similar to an electron but with much greater mass) produced by cosmic rays at top of atmosphere.
- They are unstable and decay into a electron, a neutrino and an antineutrino:

$$\mu\hat{}\pm \rightarrow\ e\hat{}\pm + v_1 + v_2$$

- They travel down toward Earth at $v \sim c$.
- We can detect both the muons and, later, the production of the electron as the muon decays.
- Can determine the time interval between arrival and decay.

Rossi & Hall Experiment

- Can predict what fraction of the muons should be lost through decay in a trip over a given distance l and duration $\approx l/c$
- Number of muons given by $N = N_0\, e^{-\frac{ln2\, t}{\tau_{1/2}}}$, where $\tau_{1/2}$ is the half-life of the decay ($\lambda = ln\quad /\tau_{1/2}$)
- Measure proper decay time in labs: $\tau_0 = 2.2 \times 10^{-6}$s
- Measure rate of arrival of mesons:
  - A) at top of mountain (2000m), R_T
  - B) at sea level, R_S.
- The muons survive the journey in far greater numbers than you would predict.

Rossi & Hall Experiment

- Rate of arrival of muons at mountain: $R_T = 563$ muons/hr.
- Time to travel down $= \frac{2000\text{ m}}{3\text{ x }10^8} = 6.5\text{ x }10^{-6}\text{s}$
- Knowing the half-life of the decay, we would expect an arrival rate of

$$R_S = 25 \text{ muons/hour.}$$

- But they measured 400 muons/hr!
- Why?

Rossi & Hall Experiment

Observed R_S $\gg$ predicted R_S because: $v \sim c$, therefore $\gamma \gg 1$

Mesons are moving clocks which are running slow:

- Observer on Earth measures $\tau \gg \tau_0$
- Less mesons have decayed and R_S is higher than expected.

- Can look at the experiment in another way – from a reference frame attached to the muon. In this reference frame, the muon is at rest. The Earth is moving towards it.

Rossi & Hall Experiment

The decay rate of the muons in this reference frame must be the same as measured in the lab (as all inertial frames are equivalent). But yet, more muons survive the journey than expected. Inconsistency??

From the viewpoint of the muons, the distance between the top of the mountain and sea-level has contracted!

Observer on Earth: Distance to travel = D_0, Lifetime = $\tau \gg \tau_0$

Observer on Meson: Distance to travel = $D << D_0$, Lifetime = τ_0

(L-F Contraction)

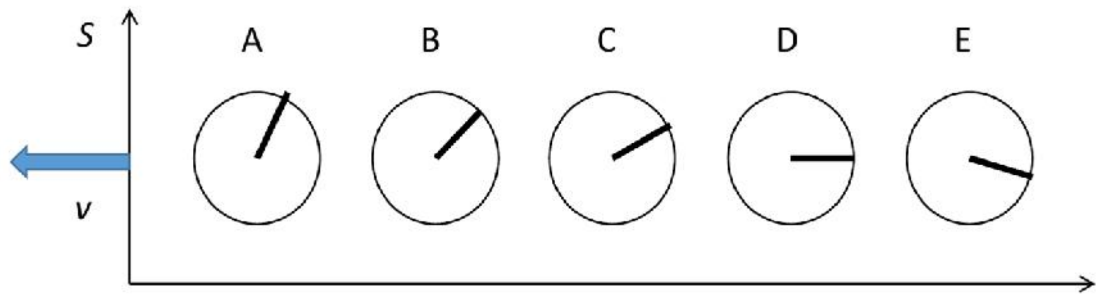
Time Dilation

For two frames in relative motion we can compare:

- **Proper time interval**: recorded by a single clock at rest in one frame, with
- **Improper time interval**: obtained by spatially separated clocks in the other frame.

Discussed more explicitly:

5 clocks equally spaced in *S* synchronised in *S*. Now suppose the whole system is moving to the left at speed *v* as viewed from *S'*.

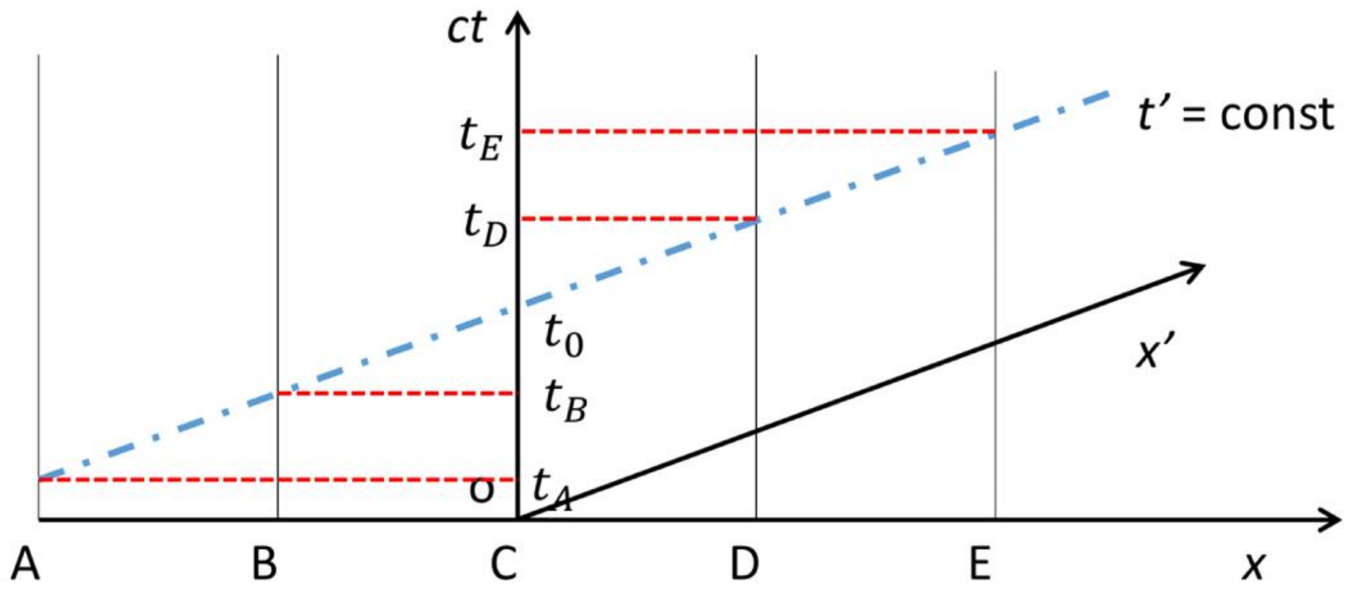


According to *S'* criterion for simultaneity, at any instance *t'* as measured in *S'*, the clocks show progressive difference in reading.

Let's see how this happens.

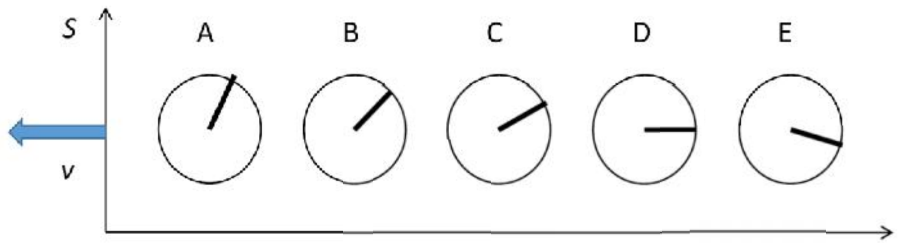
Time Dilation

First draw a space-time graph, where $x = 0$ is the clock at *C*.



Draw x'-axis and specify instant t' in S', by drawing a line parallel to the x'-axis.

Time Dilation



By LT: $$t' = \gamma(t - \frac{vx}{c^2})$$

If clock C at x = 0 in S reads t_0 at t' then:

$$t' = \gamma\, t_0$$

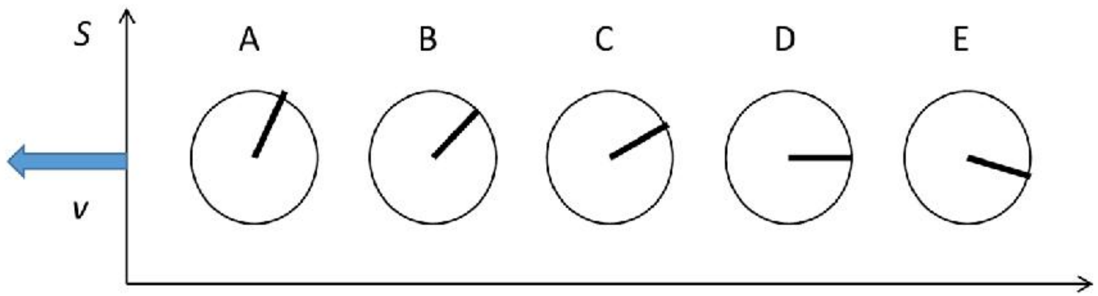
$$\gamma\, t' = \gamma\left(t - \frac{t - vx}{c^2}\right)$$

Or $$t(x) = t_0 + \frac{vx}{c^2}$$

As judged from S' the clocks show progressive error.

Time Dilation

This is related to the radio signal procedure for synchronising clocks (defining simultaneity).



If signal is sent out from C, then for observers in *S'*, A and B are moving away from it, but D and E are running to meet it.

But observers in *S* are will not be aware of this, so they set clocks D and E ($x > 0$) too far ahead and A and B ($x < 0$) too far behind.

Concept Question

Ulysses wants to verify time dilation. He has two very accurate synchronised atomic clocks; one is placed on a rocket heading into deep space at a speed of 0.6c while the other is kept on Earth. The time on the rocket is regularly signalled back to Earth.

Once the delay in the signal reaching Earth is taken into account, which of the following is observed on Earth according to Special Relativity?

- A. Seconds on the rocket clock are longer than seconds on Earth.
- B. Seconds on Earth are longer than seconds on the rocket clock.
- C. Seconds on Earth are the same as seconds on the rocket clock.
- D. You cannot predict if there is change in clock times or the magnitude or direction of that change.

Concept Question

A meter stick with a speed of 0.8c moves past an observer. In the observer's reference frame, how long does it take the stick to pass the observer ?

A. 1.6 ns

B. 2.5 ns

C. 4.2 ns

D. 6.9 ns

E. 8.3 ns

Concept Question

Kim sits beside a clock X and looks at a clock B which is two light seconds away. Kim is at rest with respect to both clocks. Which statement is correct about the times Kim sees on the two clocks?

A. Kim sees the same time on both clocks.

B. Kim sees different times the two clocks, but we cannot tell by how much.

C. Kim sees that clock X is 2 seconds behind clock Y

D. Kim sees that clock X is 2 seconds ahead of clock Y