TRINITY COLLEGE DUBLIN
THE UNIVERSITY OF DUBLIN

School of Mathematics

JF Mathematics
JF Theoretical Physics

Trinity Term 2015

MA1242 — Mechanics II

Tuesday, May 5th   Sports Centre   14.00 — 16.00

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.
All questions have equal weight.

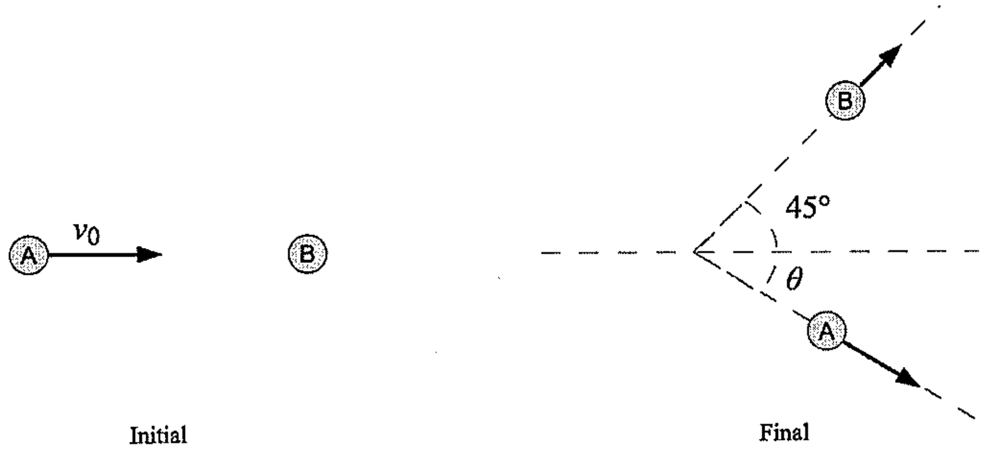
Materials Permitted for this Examination:

Formulae and Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Particle A of mass m has initial velocity v_0. After colliding with particle B of mass $2m$ initially at rest, the particles follow the paths shown in the sketch.

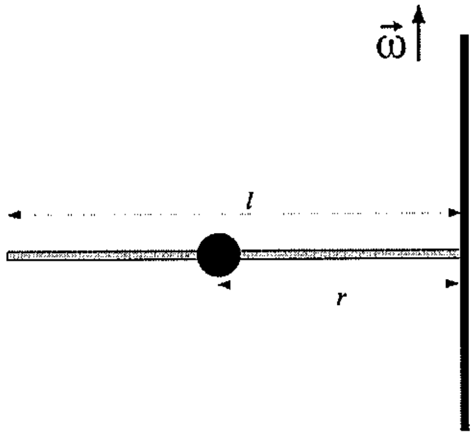


   (a) Determine the scattering angle θ of particle A assuming that the collision is elastic.

   (b) Determine the scattering angle Θ in the center of mass frame assuming that the collision is elastic.

   (c) Explain whether the angle θ increases or decreases if the collision is not elastic, but the scattering angle of particle B remains 45°. Same question for Θ.

2. A bead of mass m slides without friction on a horizontal wire of length l and mass M. The wire is uniform and rigid and rotates at constant angular speed ω. The distance of the bead to the vertical axis of rotation is denoted by r. See the figure below. Neglect gravity.



(a) Give the force diagram of the bead, and the corresponding equations of motion.

(b) Show that the general solution of $r(t)$ is:

$$r(t) = A\,e^{\omega t} + B\,e^{-\omega t},$$

where A and B are two constants determined by the initial conditions $r(0)$ and $\dot{r}(0)$. Let $r(0) = r_0$ with $0 < r_0 < l$. Determine $\dot{r}(0)$, such that the bead does not reach the pivot point nor the other end point of the wire for $t < \infty$.

(c) Determine the torque which needs to be exerted on the wire at the pivot point if the bead moves according to $r(t) = r_0\,\cosh(\omega t)$.

3. We approximate the trajectory of Halley's comet around the sun by an ellipse, with distance to the sun in the perihelion (the point of the ellipse closest to the sun) equal to $r_- =$0.586 AU and distance to the sun in the aphelion (the point of the ellipse farthest away from the sun) equal to $r_+ = 35.1$ AU. An Astronomical Unit (AU) is approximately 150×10^9 m, which is roughly equal to the distance between the earth and the sun. The general equation of the ellipse is given by:

$$r(\theta) = \frac{r_0}{1 - \varepsilon\cos\theta}.$$

   (a) Determine the eccentricity ε and the distance r_0 for the trajectory of Halley's comet.

   (b) Assume that:

      i. the orbit of the earth is a circle ($\varepsilon_{\text{earth}} = 0$),

      ii. the mass of the sun m_s is much larger than the mass of Halley's comet m_{H} and the mass of the earth m_e,

      iii. and that the answer is $r_0 = 1$ AU for part a).

   Express the speeds of Halley's comet in the perihelion and the aphelion, in terms of the speed v_e of the earth and ε.

   (c) With the same assumptions as above, express the ratio $\frac{T_{\mathrm{H}}}{T_e}$ of the periods of revolution around the sun of Halley's comet T_{H} and the earth T_e in terms of ε.