12

FLUID MECHANICS

12.1. **IDENTIFY:** Use $\rho = m/V$ to calculate the mass and then use $w = mg$ to calculate the weight.

SET UP: $\rho = m/V$ so $m = \rho V$ From Table 12.1, $\rho = 7.8\times10^3\ \text{kg/m}^3$.

EXECUTE: For a cylinder of length L and radius R,

$V = (\pi R^2)L = \pi(0.01425\ \text{m})^2(0.858\ \text{m}) = 5.474\times10^{-4}\ \text{m}^3$.

Then $m = \rho V = (7.8\times10^3\ \text{kg/m}^3)(5.474\times10^{-4}\ \text{m}^3) = 4.27\ \text{kg}$, and

$w = mg = (4.27\ \text{kg})(9.80\ \text{m/s}^2) = 41.8\ \text{N}$ (about 9.4 lbs). A cart is not needed.

EVALUATE: The rod is less than 1m long and less than 3 cm in diameter, so a weight of around 10 lbs seems reasonable.

12.2. **IDENTIFY:** The volume of the remaining object is the volume of a cube minus the volume of a cylinder, and it is this object for which we know the mass. The target variables are the density of the metal of the cube and the original weight of the cube.

SET UP: The volume of a cube with side length L is L^3, the volume of a cylinder of radius r and length L is $\pi r^2 L$, and density is $\rho = m/V$.

EXECUTE: **(a)** The volume of the metal left after the hole is drilled is the volume of the solid cube minus the volume of the cylindrical hole:

$V = L^3 - \pi r^2 L = (5.0\ \text{cm})^3 - \pi(1.0\ \text{cm})^2(5.0\ \text{cm}) = 109\ \text{cm}^3 = 1.09\times10^{-4}\ \text{m}^3$. The cube with the hole has mass $m = \dfrac{w}{g} = \dfrac{6.30\ \text{N}}{9.80\ \text{m/s}^2} = 0.6429\ \text{kg}$ and density $\rho = \dfrac{m}{V} = \dfrac{0.6429\ \text{kg}}{1.09\times10^{-4}\ \text{m}^3} = 5.9\times10^3\ \text{kg/m}^3$.

(b) The solid cube has volume $V = L^3 = 125\ \text{cm}^3 = 1.25\times10^{-4}\ \text{m}^3$ and mass

$m = \rho V = (5.9\times10^3\ \text{kg/m}^3)(1.25\times10^{-4}\ \text{m}^3) = 0.7372\ \text{kg}$. The original weight of the cube was $w = mg = 7.2\ \text{N}$.

EVALUATE: As Table 12.1 shows, the density of this metal is about twice that of aluminum and half that of lead, so it is reasonable.

12.3. **IDENTIFY:** $\rho = m/V$

SET UP: The density of gold is $19.3\times10^3\ \text{kg/m}^3$.

EXECUTE: $V = (5.0\times10^{-3}\ \text{m})(15.0\times10^{-3}\ \text{m})(30.0\times10^{-3}\ \text{m}) = 2.25\times10^{-6}\ \text{m}^3$.

$\rho = \dfrac{m}{V} = \dfrac{0.0158\ \text{kg}}{2.25\times10^{-6}\ \text{m}^3} = 7.02\times10^3\ \text{kg/m}^3$. The metal is not pure gold.

EVALUATE: The average density is only 36% that of gold, so at most 36% of the mass is gold.

12.4. **IDENTIFY:** Find the mass of gold that has a value of $\$1.00\times10^6$. Then use the density of gold to find the volume of this mass of gold.

SET UP: For gold, $\rho = 19.3\times10^3\ \text{kg/m}^3$. The volume V of a cube is related to the length L of one side by $V = L^3$.

EXECUTE: $m = (\$1.00\times10^6)\left(\dfrac{1\text{ troy ounce}}{\$1282}\right)\left(\dfrac{31.1035\times10^{-3}\text{ kg}}{1\text{ troy ounce}}\right) = 24.26\text{ kg}.$ $\rho = \dfrac{m}{V}$ so

$V = \dfrac{m}{\rho} = \dfrac{24.26\text{ kg}}{19.3\times10^3\text{ kg/m}^3} = 1.257\times10^{-3}\text{ m}^3.$ $L = V^{1/3} = 0.108\text{ m} = 10.8\text{ cm}.$

EVALUATE: The cube of gold would weigh about 50 lbs.

12.5. **IDENTIFY:** Apply $\rho = m/V$ to relate the densities and volumes for the two spheres.

SET UP: For a sphere, $V = \frac{4}{3}\pi r^3$. For lead, $\rho_1 = 11.3\times10^3\text{ kg/m}^3$ and for aluminum, $\rho_a = 2.7\times10^3\text{ kg/m}^3$.

EXECUTE: $m = \rho V = \frac{4}{3}\pi r^3\rho.$ Same mass means $r_a^3\rho_a = r_1^3\rho_1.$ $\dfrac{r_a}{r_1} = \left(\dfrac{\rho_1}{\rho_a}\right)^{1/3} = \left(\dfrac{11.3\times10^3}{2.7\times10^3}\right)^{1/3} = 1.6.$

EVALUATE: The aluminum sphere is larger, since its density is less.

12.6. **IDENTIFY:** Average density is $\rho = m/V$.

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. The sun has mass $M_{sun} = 1.99\times10^{30}\text{ kg}$ and radius $6.96\times10^8\text{ m}$.

EXECUTE: **(a)** $\rho = \dfrac{M_{sun}}{V_{sun}} = \dfrac{1.99\times10^{30}\text{ kg}}{\frac{4}{3}\pi(6.96\times10^8\text{ m})^3} = \dfrac{1.99\times10^{30}\text{ kg}}{1.412\times10^{27}\text{ m}^3} = 1.409\times10^3\text{ kg/m}^3$

(b) $\rho = \dfrac{1.99\times10^{30}\text{ kg}}{\frac{4}{3}\pi(2.00\times10^4\text{ m})^3} = \dfrac{1.99\times10^{30}\text{ kg}}{3.351\times10^{13}\text{ m}^3} = 5.94\times10^{16}\text{ kg/m}^3$

EVALUATE: For comparison, the average density of the earth is $5.5\times10^3\text{ kg/m}^3$. A neutron star is extremely dense.

12.7. **IDENTIFY:** $w = mg$ and $m = \rho V$. Find the volume V of the pipe.

SET UP: For a hollow cylinder with inner radius R_1, outer radius R_2, and length L the volume is $V = \pi(R_2^2 - R_1^2)L.$ $R_1 = 1.25\times10^{-2}\text{ m}$ and $R_2 = 1.75\times10^{-2}\text{ m}$.

EXECUTE: $V = \pi[(0.0175\text{ m})^2 - (0.0125\text{ m})^2](1.50\text{ m}) = 7.07\times10^{-4}\text{ m}^3.$

$m = \rho V = (8.9\times10^3\text{ kg/m}^3)(7.07\times10^{-4}\text{ m}^3) = 6.29\text{ kg}.$ $w = mg = 61.6\text{ N}.$

EVALUATE: The pipe weighs about 14 pounds.

12.8. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh.$

SET UP: Ocean water is seawater and has a density of $1.03\times10^3\text{ kg/m}^3$.

EXECUTE: $p - p_0 = (1.03\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(3200\text{ m}) = 3.23\times10^7\text{ Pa}.$

$p - p_0 = (3.23\times10^7\text{ Pa})\left(\dfrac{1\text{ atm}}{1.013\times10^5\text{ Pa}}\right) = 319\text{ atm}.$

EVALUATE: The gauge pressure is about 320 times the atmospheric pressure at the surface.

12.9. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh.$

SET UP: Freshwater has density $1.00\times10^3\text{ kg/m}^3$ and seawater has density $1.03\times10^3\text{ kg/m}^3$.

EXECUTE: **(a)** $p - p_0 = (1.00\times10^3\text{ kg/m}^3)(3.71\text{ m/s}^2)(500\text{ m}) = 1.86\times10^6\text{ Pa}.$

(b) $h = \dfrac{p - p_0}{\rho g} = \dfrac{1.86\times10^6\text{ Pa}}{(1.03\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 184\text{ m}$

EVALUATE: The pressure at a given depth is greater on earth because a cylinder of water of that height weighs more on earth than on Mars.

12.10. **IDENTIFY:** The difference in pressure at points with heights y_1 and y_2 is $p - p_0 = \rho g(y_1 - y_2)$. The outward force $F_\perp$ is related to the surface area A by $F_\perp = pA$.

SET UP: For blood, $\rho = 1.06\times10^{3}\ \text{kg/m}^3$. $y_1 - y_2 = 1.65$ m. The surface area of the segment is πDL, where $D = 1.50\times10^{-3}$ m and $L = 2.00\times10^{-2}$ m.

EXECUTE: **(a)** $p_1 - p_2 = (1.06\times10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.65\ \text{m}) = 1.71\times10^{4}$ Pa.

(b) The additional force due to this pressure difference is $\Delta F_\perp = (p_1 - p_2)A$.

$A = \pi DL = \pi(1.50\times10^{-3}\ \text{m})(2.00\times10^{-2}\ \text{m}) = 9.42\times10^{-5}\ \text{m}^2$.

$\Delta F_\perp = (1.71\times10^{4}\ \text{Pa})(9.42\times10^{-5}\ \text{m}^2) = 1.61$ N.

EVALUATE: The pressure difference is about $\frac{1}{6}$ atm.

12.11. **IDENTIFY:** Apply $p = p_0 + \rho gh$.

SET UP: Gauge pressure is $p - p_{\text{air}}$.

EXECUTE: The pressure difference between the top and bottom of the tube must be at least 5980 Pa in order to force fluid into the vein: $\rho gh = 5980$ Pa and

$$h = \frac{5980\ \text{Pa}}{\rho g} = \frac{5980\ \text{N/m}^2}{(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.581\ \text{m}.$$

EVALUATE: The bag of fluid is typically hung from a vertical pole to achieve this height above the patient's arm.

12.12. **IDENTIFY:** $p_0 = p_{\text{surface}} + \rho gh$ where p_{surface} is the pressure at the surface of a liquid and p_0 is the pressure at a depth *h* below the surface.

SET UP: The density of water is $1.00\times10^{3}\ \text{kg/m}^3$.

EXECUTE: **(a)** For the oil layer, $p_{\text{surface}} = p_{\text{atm}}$ and p_0 is the pressure at the oil-water interface.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = \rho gh = (600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.120\ \text{m}) = 706$ Pa

(b) For the water layer, $p_{\text{surface}} = 706\ \text{Pa} + p_{\text{atm}}$.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = 706\ \text{Pa} + \rho gh = 706\ \text{Pa} + (1.00\times10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.250\ \text{m}) = 3.16\times10^{3}$ Pa

EVALUATE: The gauge pressure at the bottom of the barrel is due to the combined effects of the oil layer and water layer. The pressure at the bottom of the oil layer is the pressure at the top of the water layer.

12.13. **IDENTIFY:** There will be a difference in blood pressure between your head and feet due to the depth of the blood.

SET UP: The added pressure is equal to ρgh.

EXECUTE: **(a)** $\rho gh = (1060\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.85\ \text{m}) = 1.92\times10^{4}$ Pa.

(b) This additional pressure causes additional outward force on the walls of the blood vessels in your brain.

EVALUATE: The pressure difference is about 1/5 atm, so it would be noticeable.

12.14. **IDENTIFY** and **SET UP:** Use $p_{\text{g}} = \rho gh$ to calculate the gauge pressure at this depth. Use $F = pA$ to calculate the force the inside and outside pressures exert on the window, and combine the forces as vectors to find the net force.

EXECUTE: **(a)** gauge pressure $= p - p_0 = \rho gh$ From Table 12.1 the density of seawater is $1.03\times10^{3}\ \text{kg/m}^3$, so

$$p - p_0 = \rho gh = (1.03\times10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(250\ \text{m}) = 2.52\times10^{6}\ \text{Pa}.$$

(b) The force on each side of the window is $F = pA$. Inside the pressure is p_0 and outside in the water the pressure is $p = p_0 + \rho gh$. The forces are shown in Figure 12.14 (next page).

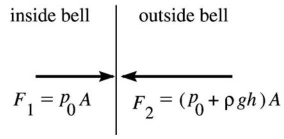


Figure 12.14

The net force is

$$F_2 - F_1 = (p_0 + \rho gh)A - p_0 A = (\rho gh)A$$

$$F_2 - F_1 = (2.52\times10^6 \text{ Pa})\pi(0.150 \text{ m})^2$$

$$F_2 - F_1 = 1.78\times10^5 \text{ N}$$

EVALUATE: The pressure at this depth is very large, over 20 times normal air pressure, and the net force on the window is huge. Diving bells used at such depths must be constructed to withstand these large forces.

12.15. IDENTIFY: The external pressure on the eardrum increases with depth in the ocean. This increased pressure could damage the eardrum.

SET UP: The density of seawater is $1.03\times10^3 \text{ kg/m}^3$. The area of the eardrum is $A = \pi r^2$, with $r = 4.1$ mm. The pressure increase with depth is $\Delta p = \rho gh$ and $F = pA$.

EXECUTE: $\Delta F = (\Delta p)A = \rho ghA$. Solving for h gives

$$h = \frac{\Delta F}{\rho g A} = \frac{1.5 \text{ N}}{(1.03\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)\pi(4.1\times10^{-3} \text{ m})^2} = 2.8 \text{ m}.$$

EVALUATE: 2.8 m is less than 10 ft, so it is probably a good idea to wear ear plugs if you scuba dive.

12.16. IDENTIFY and **SET UP:** Use $p = p_0 + \rho gh$ to calculate the pressure at the specified depths in the open tube. The pressure is the same at all points the same distance from the bottom of the tubes, so the pressure calculated in part (b) is the pressure in the tank. Gauge pressure is the difference between the absolute pressure and air pressure.

EXECUTE: $p_a = 980 \text{ millibar} = 9.80\times10^4 \text{ Pa}$

(a) Apply $p = p_0 + \rho gh$ to the right-hand tube. The top of this tube is open to the air so $p_0 = p_a$. The density of the liquid (mercury) is $13.6\times10^3 \text{ kg/m}^3$.

Thus $p = 9.80\times10^4 \text{ Pa} + (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0700 \text{ m}) = 1.07\times10^5 \text{ Pa}$.

(b) $p = p_0 + \rho gh = 9.80\times10^4 \text{ Pa} + (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0400 \text{ m}) = 1.03\times10^5 \text{ Pa}$.

(c) Since $y_2 - y_1 = 4.00$ cm the pressure at the mercury surface in the left-hand end tube equals that calculated in part (b). Thus the absolute pressure of gas in the tank is 1.03×10^5 Pa.

(d) $p - p_0 = \rho gh = (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0400 \text{ m}) = 5.33\times10^3 \text{ Pa}$.

EVALUATE: If $p = p_0 + \rho gh$ is evaluated with the density of mercury and $p - p_a = 1 \text{ atm} = 1.01\times10^5$ Pa, then $h = 76$ cm. The mercury columns here are much shorter than 76 cm, so the gauge pressures are much less than 1.0×10^5 Pa.

12.17. IDENTIFY: Apply $p = p_0 + \rho gh$.

SET UP: For water, $\rho = 1.00\times10^3 \text{ kg/m}^3$.

EXECUTE: $p - p_{air} = \rho gh = (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(6.1 \text{ m}) = 6.0\times10^4 \text{ Pa}$.

EVALUATE: The pressure difference increases linearly with depth.

12.18. IDENTIFY: The gauge pressure of the person must be equal to the pressure due to the column of water in the straw.

SET UP: Apply $p = p_0 + \rho gh$.

EXECUTE: **(a)** The gauge pressure is $p_g = \rho gh = -(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.1 \text{ m}) = -1.1 \times 10^4$ Pa.

(b) In order for water to go up the straw, the pressure at the top of the straw must be lower than atmospheric pressure. Therefore the gauge pressure, $p - p_{atm}$, must be negative.

EVALUATE: The actual pressure is not negative, just the difference between the pressure at the top of the straw and atmospheric pressure.

12.19. **IDENTIFY:** $p = p_0 + \rho gh$. $F = pA$.

SET UP: For seawater, $\rho = 1.03 \times 10^3\ \text{kg/m}^3$.

EXECUTE: The force F that must be applied is the difference between the upward force of the water and the downward forces of the air and the weight of the hatch. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho gh)\,A - w = (1.03 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(30\ \text{m})(0.75\ \text{m}^2) - 300\ \text{N} = 2.27 \times 10^5\ \text{N}.$$

EVALUATE: The force due to the gauge pressure of the water is much larger than the weight of the hatch and would be impossible for the crew to apply it just by pushing.

12.20. **IDENTIFY** and **SET UP:** Apply $p = p_0 + \rho gh$ to the water and mercury columns. The pressure at the bottom of the water column is the pressure at the top of the mercury column.

EXECUTE: With just the mercury, the gauge pressure at the bottom of the cylinder is $p - p_0 = \rho_\text{m} g h_\text{m}$. With the water to a depth h_w, the gauge pressure at the bottom of the cylinder is $p - p_0 = \rho_\text{m} g h_\text{m} + \rho_\text{w} g h_\text{w}$. If this is to be double the first value, then $\rho_\text{w} g h_\text{w} = \rho_\text{m} g h_\text{m}$.

$$h_\text{w} = h_\text{m}(\rho_\text{m}/\rho_\text{w}) = (0.0800\ \text{m})(13.6 \times 10^3/1.00 \times 10^3) = 1.088\ \text{m}$$

The volume of water is $V = hA = (1.088\ \text{m})(12.0 \times 10^{-4}\ \text{m}^2) = 1.306 \times 10^{-3}\ \text{m}^3 = 1310\ \text{cm}^3 = 1.31\ \text{L}$.

EVALUATE: The density of mercury is 13.6 times the density of water and $(13.6)(8\ \text{cm}) = 109\ \text{cm}$, so the pressure increase from the top to the bottom of a 109-cm tall column of water is the same as the pressure increase from top to bottom for an 8-cm tall column of mercury.

12.21. **IDENTIFY:** The gauge pressure at the top of the oil column must produce a force on the disk that is equal to its weight.

SET UP: The area of the bottom of the disk is $A = \pi r^2 = \pi(0.150\ \text{m})^2 = 0.0707\ \text{m}^2$.

EXECUTE: **(a)** $p - p_0 = \dfrac{w}{A} = \dfrac{45.0\ \text{N}}{0.0707\ \text{m}^2} = 636\ \text{Pa}$.

(b) The increase in pressure produces a force on the disk equal to the increase in weight. By Pascal's law the increase in pressure is transmitted to all points in the oil.

(i) $\Delta p = \dfrac{83.0\ \text{N}}{0.0707\ \text{m}^2} = 1170\ \text{Pa}$. (ii) 1170 Pa

EVALUATE: The absolute pressure at the top of the oil produces an upward force on the disk but this force is partially balanced by the force due to the air pressure at the top of the disk.

12.22. **IDENTIFy:** Apply $p = p_0 + \rho gh$, where p_0 is the pressure at the surface of the fluid. Gauge pressure is $p - p_\text{air}$.

SET UP: For water, $\rho = 1.00 \times 10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** The pressure difference between the surface of the water and the bottom is due to the weight of the water and is still 2500 Pa after the pressure increase above the surface. But the surface pressure increase is also transmitted to the fluid, making the total difference from atmospheric pressure $2500\ \text{Pa} + 1500\ \text{Pa} = 4000\ \text{Pa}$.

(b) Initially, the pressure due to the water alone is $2500\ \text{Pa} = \rho gh$. Thus

$h = \dfrac{2500\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.255\ \text{m}$. To keep the bottom gauge pressure at 2500 Pa after the 1500 Pa increase at the surface, the pressure due to the water's weight must be reduced to 1000 Pa:

$h = \dfrac{1000\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.102\ \text{m}$. Thus the water must be lowered by $0.255\ \text{m} - 0.102\ \text{m} = 0.153\ \text{m}$.

EVALUATE: Note that ρgh, with $h = 0.153\ \text{m}$, is 1500 Pa.

12.23. **IDENTIFY:** $F_2 = \frac{A_2}{A_1}F_1$. F_2 must equal the weight $w = mg$ of the car.

SET UP: $A = \pi D^2/4$. D_1 is the diameter of the vessel at the piston where F_1 is applied and D_2 is the diameter at the car.

EXECUTE: $mg = \frac{\pi D_2^2/4}{\pi D_1^2/4}F_1$. $\frac{D_2}{D_1} = \sqrt{\frac{mg}{F_1}} = \sqrt{\frac{(1520\text{ kg})(9.80\text{ m/s}^2)}{125\text{ N}}} = 10.9$

EVALUATE: The diameter is smaller where the force is smaller, so the pressure will be the same at both pistons.

12.24. **IDENTIFY:** Apply $\Sigma F_y = ma_y$ to the piston, with $+y$ upward. $F = pA$.

SET UP: $1\text{ atm} = 1.013\times10^5$ Pa. The force diagram for the piston is given in Figure 12.24. p is the absolute pressure of the hydraulic fluid.

EXECUTE: $pA - w - p_{\text{atm}}A = 0$ and

$$p - p_{\text{atm}} = p_{\text{gauge}} = \frac{w}{A} = \frac{mg}{\pi r^2} = \frac{(1200\text{ kg})(9.80\text{ m/s}^2)}{\pi(0.15\text{ m})^2} = 1.7\times10^5\text{ Pa} = 1.7\text{ atm}$$

EVALUATE: The larger the diameter of the piston, the smaller the gauge pressure required to lift the car.

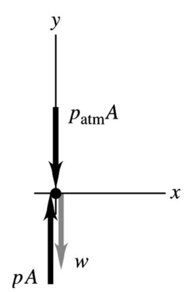


Figure 12.24

12.25. **IDENTIFY:** The force on an area A due to pressure p is $F_\perp = pA$. Use $p - p_0 = \rho gh$ to find the pressure inside the tank, at the bottom.

SET UP: $1\text{ atm} = 1.013\times10^5$ Pa. For benzene, $\rho = 0.90\times10^3\text{ kg/m}^3$. The area of the bottom of the tank is $\pi D^2/4$, where $D = 1.72$ m. The area of the vertical walls of the tank is πDL, where $L = 11.50$ m.

EXECUTE: **(a)** At the bottom of the tank,

$p = p_0 + \rho gh = 92(1.013\times10^5\text{ Pa}) + (0.90\times10^3\text{ kg/m}^3)(0.894)(9.80\text{ m/s}^2)(11.50\text{ m})$.

$p = 9.32\times10^6\text{ Pa} + 9.07\times10^4\text{ Pa} = 9.41\times10^6$ Pa. $F_\perp = pA = (9.41\times10^6\text{ Pa})\pi(1.72\text{ m})^2/4 = 2.19\times10^7$ N.

(b) At the outside surface of the bottom of the tank, the air pressure is

$p = (92)(1.013\times10^5\text{ Pa}) = 9.32\times10^6$ Pa. $F_\perp = pA = (9.32\times10^6\text{ Pa})\pi(1.72\text{ m})^2/4 = 2.17\times10^7$ N.

(c) $F_\perp = pA = 92(1.013\times10^5\text{ Pa})\pi(1.72\text{ m})(11.5\text{ m}) = 5.79\times10^8$ N

EVALUATE: Most of the force in part (a) is due to the 92 atm of air pressure above the surface of the benzene and the net force on the bottom of the tank is much less than the inward and outward forces.

12.26. **IDENTIFY:** The buoyant force B acts upward on the rock, opposing gravity. Archimedes's principle applies, and the forces must balance.

SET UP: $\rho = m/V$.

EXECUTE: With the rock of mass m in the water: $T + B = mg$. where T is the tension in the string. Call V the volume of the rock and ρ_w the density of water. By Archimedes's principle, $m = \rho_w V$, so we get $T + \rho_w Vg = mg$. Solving for V gives $V = (mg - T)/\rho_w g$. Now look at the rock in the liquid, where ρ is the density of the liquid. For the smallest density liquid, the rock is totally submerged, so the volume of liquid displaced is V. For floating we have $B = mg$, which gives $\rho gV = mg$. Solving for ρ and using the equation for V that we just found, we get

$$\rho = \frac{m}{\dfrac{mg-T}{\rho_{\text{w}}g}} = \frac{\rho_{\text{w}}mg}{mg-T} = \frac{(1000\ \text{kg/m}^3)(1.80\ \text{kg})(9.80\ \text{m/s}^2)}{(1.80\ \text{kg})(9.80\ \text{m/s}^2) - 12.8\ \text{N}} = 3640\ \text{kg/m}^3.$$

EVALUATE: In the water, the buoyant force was not enough to balance the weight of the rock, so there was a tension of 12.8 N in the string. In the new liquid, the buoyant force is equal to the weight. Therefore the liquid must be denser than water, which in fact it is.

12.27. IDENTIFY: By Archimedes's principle, the additional buoyant force will be equal to the additional weight (the man).

SET UP: $V = \dfrac{m}{\rho}$ where $dA = V$ and d is the additional distance the buoy will sink.

EXECUTE: With man on buoy must displace additional 80.0 kg of water.

$$V = \frac{m}{\rho} = \frac{80.0\ \text{kg}}{1030\ \text{kg/m}^3} = 0.07767\ \text{m}^3.\quad dA = V \text{ so } d = \frac{V}{A} = \frac{0.07767\ \text{m}^3}{\pi(0.450\ \text{m})^2} = 0.122\ \text{m}.$$

EVALUATE: We do not need to use the mass of the buoy because it is already floating and hence in balance.

12.28. IDENTIFY: Apply Newton's second law to the woman plus slab. The buoyancy force exerted by the water is upward and given by $B = \rho_{\text{water}}V_{\text{displ}}g$, where V_{displ} is the volume of water displaced.

SET UP: The floating object is the slab of ice plus the woman; the buoyant force must support both. The volume of water displaced equals the volume V_{ice} of the ice. The free-body diagram is given in Figure 12.28.

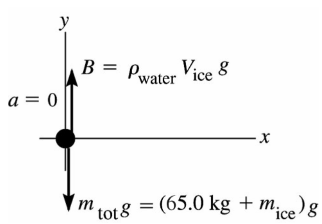


Figure 12.28

EXECUTE: $\Sigma F_y = ma_y$

$B - m_{\text{tot}}g = 0$

$\rho_{\text{water}}V_{\text{ice}}g = (65.0\ \text{kg} + m_{\text{ice}})g$

But $\rho = m/V$ so $m_{\text{ice}} = \rho_{\text{ice}}V_{\text{ice}}$

$$V_{\text{ice}} = \frac{65.0\ \text{kg}}{\rho_{\text{water}} - \rho_{\text{ice}}} = \frac{65.0\ \text{kg}}{1000\ \text{kg/m}^3 - 920\ \text{kg/m}^3} = 0.81\ \text{m}^3.$$

EVALUATE: The mass of ice is $m_{\text{ice}} = \rho_{\text{ice}}V_{\text{ice}} = 750$ kg.

12.29. IDENTIFY: Apply $\Sigma F_y = ma_y$ to the sample, with $+y$ upward. $B = \rho_{\text{water}}V_{\text{obj}}g$.

SET UP: $w = mg = 17.50$ N and $m = 1.79$ kg.

EXECUTE: $T + B - mg = 0$. $B = mg - T = 17.50\ \text{N} - 11.20\ \text{N} = 6.30\ \text{N}$.

$$V_{\text{obj}} = \frac{B}{\rho_{\text{water}}g} = \frac{6.30\ \text{N}}{(1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 6.43\times10^{-4}\ \text{m}^3.$$

$$\rho = \frac{m}{V} = \frac{1.79\ \text{kg}}{6.43\times10^{-4}\ \text{m}^3} = 2.78\times10^3\ \text{kg/m}^3.$$

EVALUATE: The density of the sample is greater than that of water and it doesn't float.

12.30. IDENTIFY: The upward buoyant force B exerted by the liquid equals the weight of the fluid displaced by the object. Since the object floats the buoyant force equals its weight.

SET UP: Glycerin has density $\rho_{gly} = 1.26\times10^3$ kg/m^3 and seawater has density $\rho_{sw} = 1.03\times10^3$ kg/m^3. Let V_{obj} be the volume of the apparatus. $g_E = 9.80$ m/s^2; $g_C = 5.40$ m/s^2. Let V_{sub} be the volume submerged on Caasi.

EXECUTE: On earth $B = \rho_{sw}(0.250V_{obj})g_E = mg_E$. $m = (0.250)\rho_{sw}V_{obj}$. On Caasi, $B = \rho_{gly}V_{sub}g_C = mg_C$. $m = \rho_{gyl}V_{sub}$. The two expressions for m must be equal, so

$$(0.250)V_{obj}\rho_{sw} = \rho_{gly}V_{sub} \text{ and } V_{sub} = \left(\frac{0.250\rho_{sw}}{\rho_{gly}}\right)V_{obj} = \left(\frac{[0.250][1.03\times10^3\ \text{kg/m}^3]}{1.26\times10^3\ \text{kg/m}^3}\right)V_{obj} = 0.204V_{obj}.$$

20.4% of the volume will be submerged on Caasi.

EVALUATE: Less volume is submerged in glycerin since the density of glycerin is greater than the density of seawater. The value of g on each planet cancels out and has no effect on the answer. The value of g changes the weight of the apparatus and the buoyant force by the same factor.

12.31. IDENTIFY: In air and in the liquid, the forces on the rock must balance. Archimedes's principle applies in the liquid.

SET UP: $B = \rho Vg$, $\rho = m/V$, call m the mass of the rock, V its volume, and ρ its density; T is the tension in the string and ρ_L is the density of the liquid.

EXECUTE: In air: $T = mg = \rho Vg$. $V = T/\rho g = (28.0\ \text{N})/[(1200\ \text{kg/m}^3)(9.80\ \text{m/s}^2)] = 0.00238\ \text{m}^3$.

In the liquid: $T + B = mg$, so

$T = mg - B = \rho Vg - \rho_L Vg = gV(\rho - \rho_L) = (9.80\ \text{m/s}^2)(0.00238\ \text{m}^3)(1200\ \text{kg/m}^3 - 750\ \text{kg/m}^3) = 10.5$ N.

EVALUATE: When the rock is in the liquid, the tension in the string is less than the tension when the rock is in air since the buoyant force helps balance some of the weight of the rock.

12.32. IDENTIFY: $B = \rho_{water}V_{obj}g$. The net force on the sphere is zero.

SET UP: The density of water is 1.00×10^3 kg/m^3.

EXECUTE: **(a)** $B = (1000\ \text{kg/m}^3)(0.650\ \text{m}^3)(9.80\ \text{m/s}^2) = 6.37\times10^3$ N

(b) $B = T + mg$ and $m = \dfrac{B-T}{g} = \dfrac{6.37\times10^3\ \text{N} - 1120\ \text{N}}{9.80\ \text{m/s}^2} = 536$ kg.

(c) Now $B = \rho_{water}V_{sub}g$, where V_{sub} is the volume of the sphere that is submerged. $B = mg$.

$$\rho_{water}V_{sub}g = mg \text{ and } V_{sub} = \frac{m}{\rho_{water}} = \frac{536\ \text{kg}}{1000\ \text{kg/m}^3} = 0.536\ \text{m}^3.\quad \frac{V_{sub}}{V_{obj}} = \frac{0.536\ \text{m}^3}{0.650\ \text{m}^3} = 0.824 = 82.4\%.$$

EVALUATE: The average density of the sphere is $\rho_{sph} = \dfrac{m}{V} = \dfrac{536\ \text{kg}}{0.650\ \text{m}^3} = 825\ \text{kg/m}^3$. $\rho_{sph} < \rho_{water}$, and that is why it floats with 82.4% of its volume submerged.

12.33. IDENTIFY and SET UP: Use $p = p_0 + \rho gh$ to calculate the gauge pressure at the two depths.

(a) The distances are shown in Figure 12.33a.

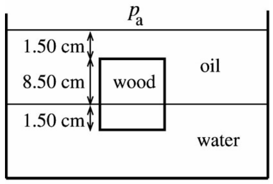


Figure 12.33a

EXECUTE: $p - p_0 = \rho gh$

The upper face is 1.50 cm below the top of the oil, so

$p - p_0 = (790\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0150\ \text{m})$

$p - p_0 = 116$ Pa

(b) The pressure at the interface is $p_{\text{interface}} = p_{\text{a}} + \rho_{\text{oil}} g(0.100 \text{ m})$. The lower face of the block is 1.50 cm below the interface, so the pressure there is $p = p_{\text{interface}} + \rho_{\text{water}} g(0.0150 \text{ m})$. Combining these two equations gives

$p - p_{\text{a}} = \rho_{\text{oil}} g(0.100 \text{ m}) + \rho_{\text{water}} g(0.0150 \text{ m})$

$p - p_{\text{a}} = [(790 \text{ kg/m}^3)(0.100 \text{ m}) + (1000 \text{ kg/m}^3)(0.0150 \text{ m})](9.80 \text{ m/s}^2)$

$p - p_{\text{a}} = 921 \text{ Pa}$

(c) IDENTIFY and **SET UP:** Consider the forces on the block. The area of each face of the block is $A = (0.100 \text{ m})^2 = 0.0100 \text{ m}^2$. Let the absolute pressure at the top face be p_{t} and the pressure at the bottom face be p_{b}. In $p = \dfrac{F_\perp}{A}$, use these pressures to calculate the force exerted by the fluids at the top and bottom of the block. The free-body diagram for the block is given in Figure 12.33b.

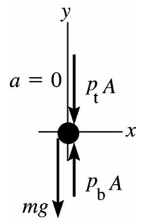


Figure 12.33b

EXECUTE: $\Sigma F_y = ma_y$

$p_{\text{b}} A - p_{\text{t}} A - mg = 0$

$(p_{\text{b}} - p_{\text{t}})A = mg$

Note that $(p_{\text{b}} - p_{\text{t}}) = (p_{\text{b}} - p_{\text{a}}) - (p_{\text{t}} - p_{\text{a}}) = 921 \text{ Pa} - 116 \text{ Pa} = 805 \text{ Pa}$; the difference in absolute pressures equals the difference in gauge pressures.

$$m = \frac{(p_{\text{b}} - p_{\text{t}})A}{g} = \frac{(805 \text{ Pa})(0.0100 \text{ m}^2)}{9.80 \text{ m/s}^2} = 0.821 \text{ kg}.$$

And then $\rho = m/V = 0.821 \text{ kg}/(0.100 \text{ m})^3 = 821 \text{ kg/m}^3$.

EVALUATE: We can calculate the buoyant force as $B = (\rho_{\text{oil}} V_{\text{oil}} + \rho_{\text{water}} V_{\text{water}})g$ where $V_{\text{oil}} = (0.0100 \text{ m}^2)(0.0850 \text{ m}) = 8.50 \times 10^{-4} \text{ m}^3$ is the volume of oil displaced by the block and $V_{\text{water}} = (0.0100 \text{ m}^2)(0.0150 \text{ m}) = 1.50 \times 10^{-4} \text{ m}^3$ is the volume of water displaced by the block. This gives $B = (0.821 \text{ kg})g$. The mass of water displaced equals the mass of the block.

12.34. IDENTIFY: The sum of the vertical forces on the ingot is zero. $\rho = m/V$. The buoyant force is $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: The density of aluminum is $2.7 \times 10^3 \text{ kg/m}^3$. The density of water is $1.00 \times 10^3 \text{ kg/m}^3$.

EXECUTE: **(a)** $T = mg = 89 \text{ N}$ so $m = 9.08 \text{ kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08 \text{ kg}}{2.7 \times 10^3 \text{ kg/m}^3} = 3.36 \times 10^{-3} \text{ m}^3 = 3.4 \text{ L}$.

(b) When the ingot is totally immersed in the water while suspended, $T + B - mg = 0$.

$B = \rho_{\text{water}} V_{\text{obj}} g = (1.00 \times 10^3 \text{ kg/m}^3)(3.36 \times 10^{-3} \text{ m}^3)(9.80 \text{ m/s}^2) = 32.9 \text{ N}$.

$T = mg - B = 89 \text{ N} - 32.9 \text{ N} = 56 \text{ N}$.

EVALUATE: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

12.35. IDENTIFY: The vertical forces on the rock sum to zero. The buoyant force equals the weight of liquid displaced by the rock. $V = \frac{4}{3}\pi R^3$.

SET UP: The density of water is $1.00 \times 10^3 \text{ kg/m}^3$.

EXECUTE: The rock displaces a volume of water whose weight is $39.2\text{ N} - 28.4\text{ N} = 10.8\text{ N}$. The mass of this much water is thus $10.8\text{ N}/(9.80\text{ m/s}^2) = 1.102\text{ kg}$ and its volume, equal to the rock's volume, is $\dfrac{1.102\text{ kg}}{1.00\times 10^3\text{ kg/m}^3} = 1.102\times 10^{-3}\text{ m}^3$. The weight of unknown liquid displaced is $39.2\text{ N} - 21.5\text{ N} = 17.7\text{ N}$, and its mass is $(17.7\text{ N})/(9.80\text{ m/s}^2) = 1.806\text{ kg}$. The liquid's density is thus $(1.806\text{ kg})/(1.102\times 10^{-3}\text{ m}^3) = 1.64\times 10^3\text{ kg/m}^3$.

EVALUATE: The density of the unknown liquid is a little more than 1.5 times the density of water.

12.36. IDENTIFY: The volume flow rate is Av.

SET UP: $Av = 0.750\text{ m}^3\text{/s}$. $A = \pi D^2/4$.

EXECUTE: **(a)** $v\pi D^2/4 = 0.750\text{ m}^3\text{/s}$. $v = \dfrac{4(0.750\text{ m}^3\text{/s})}{\pi(4.50\times 10^{-2}\text{ m})^2} = 472\text{ m/s}$.

(b) vD^2 must be constant, so $v_1D_1^2 = v_2D_2^2$. $v_2 = v_1\left(\dfrac{D_1}{D_2}\right)^2 = (472\text{ m/s})\left(\dfrac{D_1}{3D_1}\right)^2 = 52.4\text{ m/s}$.

EVALUATE: The larger the hole, the smaller the speed of the fluid as it exits.

12.37. IDENTIFY: Apply the equation of continuity.

SET UP: $A = \pi r^2$, $v_1A_1 = v_2A_2$.

EXECUTE: $v_2 = v_1(A_1/A_2)$. $A_1 = \pi(0.80\text{ cm})^2$, $A_2 = 20\pi(0.10\text{ cm})^2$. $v_2 = (3.0\text{ m/s})\dfrac{\pi(0.80)^2}{20\pi(0.10)^2} = 9.6\text{ m/s}$.

EVALUATE: The total area of the shower head openings is less than the cross-sectional area of the pipe, and the speed of the water in the shower head opening is greater than its speed in the pipe.

12.38. IDENTIFY: Apply the equation of continuity. The volume flow rate is vA.

SET UP: $1.00\text{ h} = 3600\text{ s}$. $v_1A_1 = v_2A_2$.

EXECUTE: **(a)** $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\text{ m/s})\left(\dfrac{0.070\text{ m}^2}{0.105\text{ m}^2}\right) = 2.3\text{ m/s}$

(b) $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\text{ m/s})\left(\dfrac{0.070\text{ m}^2}{0.047\text{ m}^2}\right) = 5.2\text{ m/s}$

(c) $V = v_1A_1t = (3.50\text{ m/s})(0.070\text{ m}^2)(3600\text{ s}) = 880\text{ m}^3$.

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

12.39. IDENTIFY and SET UP: Apply the continuity equation, $v_1A_1 = v_2A_2$. In part (a) the target variable is V. In part (b) solve for A and then from that get the radius of the pipe.

EXECUTE: **(a)** $vA = 1.20\text{ m}^3\text{/s}$

$$v = \frac{1.20\text{ m}^3\text{/s}}{A} = \frac{1.20\text{ m}^3\text{/s}}{\pi r^2} = \frac{1.20\text{ m}^3\text{/s}}{\pi(0.150\text{ m})^2} = 17.0\text{ m/s}$$

(b) $vA = 1.20\text{ m}^3\text{/s}$

$v\pi r^2 = 1.20\text{ m}^3\text{/s}$

$$r = \sqrt{\frac{1.20\text{ m}^3\text{/s}}{v\pi}} = \sqrt{\frac{1.20\text{ m}^3\text{/s}}{(3.80\text{ m/s})\pi}} = 0.317\text{ m}$$

EVALUATE: The speed is greater where the area and radius are smaller.

12.40. IDENTIFY: Narrowing the width of the pipe will increase the speed of flow of the fluid.

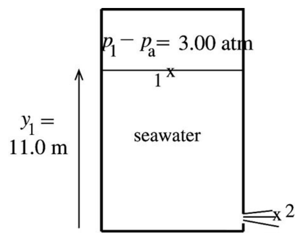
SET UP: The continuity equation is $A_1v_1 = A_2v_2$. $A = \frac{1}{4}\pi d^2$, where d is the pipe diameter.

EXECUTE: The continuity equation gives $\frac{1}{4}\pi d_1^2 v_1 = \frac{1}{4}\pi d_2^2 v_2$, so

$$v_2 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{2.50\text{ in.}}{1.00\text{ in.}}\right)^2 (6.00\text{ cm/s}) = 37.5\text{ cm/s}.$$

EVALUATE: To achieve the same volume flow rate the water flows faster in the smaller diameter pipe. Note that the pipe diameters entered in a ratio so there was no need to convert units.

12.41. IDENTIFY and **SET UP:**



Apply Bernoulli's equation with points 1 and 2 chosen as shown in Figure 12.41. Let $y = 0$ at the bottom of the tank so $y_1 = 11.0$ m and $y_2 = 0$. The target variable is v_2.

Figure 12.41

$$p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

$A_1 v_1 = A_2 v_2$, so $v_1 = (A_2/A_1)v_2$. But the cross-sectional area of the tank (A_1) is much larger than the cross-sectional area of the hole (A_2), so $v_1 << v_2$ and the $\frac{1}{2}\rho v_1^2$ term can be neglected.

EXECUTE: This gives $\frac{1}{2}\rho v_2^2 = (p_1 - p_2) + \rho g y_1$.

Use $p_2 = p_a$ and solve for v_2:

$$v_2 = \sqrt{2(p_1 - p_a)/\rho + 2gy_1} = \sqrt{\frac{2(3.039\times 10^5\text{ Pa})}{1030\text{ kg/m}^3} + 2(9.80\text{ m/s}^2)(11.0\text{ m})}$$

$v_2 = 28.4$ m/s

EVALUATE: If the pressure at the top surface of the water were air pressure, then Toricelli's theorem (Example: 12.8) gives $v_2 = \sqrt{2g(y_1 - y_2)} = 14.7$ m/s. The actual afflux speed is much larger than this due to the excess pressure at the top of the tank.

12.42. IDENTIFY: A change in the speed of the blood indicates that there is a difference in the cross-sectional area of the artery. Bernoulli's equation applies to the fluid.

SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. The two points are close together so we can neglect $\rho g(y_1 - y_2)$. $\rho = 1.06\times 10^3\text{ kg/m}^3$. The continuity equation is $A_1 v_1 = A_2 v_2$.

EXECUTE: Solve $p_1 - p_2 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$ for v_2:

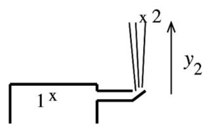
$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho} + v_1^2} = \sqrt{\frac{2(1.20\times 10^4\text{ Pa} - 1.15\times 10^4\text{ Pa})}{1.06\times 10^3\text{ kg/m}^3} + (0.300\text{ m/s})^2} = 1.0\text{ m/s} = 100\text{ cm/s}.$$

$v_2 = 1.0\text{ m/s} = 100\text{ cm/s}.$

The continuity equation gives $\dfrac{A_2}{A_1} = \dfrac{v_1}{v_2} = \dfrac{30\text{ cm/s}}{100\text{ cm/s}} = 0.30$. $A_2 = 0.30A_1$, so 70% of the artery is blocked.

EVALUATE: A 70% blockage reduces the blood speed from 100 cm/s to 30 cm/s, which should easily be detectable.

12.43. IDENTIFY and SET UP:



Apply Bernoulli's equation to points 1 and 2 as shown in Figure 12.43. Point 1 is in the mains and point 2 is at the maximum height reached by the stream, so $v_2 = 0$.

Figure 12.43

Solve for p_1 and then convert this absolute pressure to gauge pressure.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Let $y_1 = 0$, $y_2 = 15.0$ m. The mains have large diameter, so $v_1 \approx 0$.

Thus $p_1 = p_2 + \rho g y_2$.

But $p_2 = p_a$, so $p_1 - p_a = \rho g y_2 = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(15.0\ \text{m}) = 1.47\times10^5$ Pa.

EVALUATE: This is the gauge pressure at the bottom of a column of water 15.0 m high.

12.44. IDENTIFY: Toricelli's theorem says the speed of efflux is $v = \sqrt{2gh}$, where h is the distance of the small hole below the surface of the water in the tank. The volume flow rate is vA.

SET UP: $A = \pi D^2/4$, with $D = 6.00\times10^{-3}$ m.

EXECUTE: **(a)** $v = \sqrt{2(9.80\ \text{m/s}^2)(14.0\ \text{m})} = 16.6$ m/s

(b) $vA = (16.6\ \text{m/s})\pi(6.00\times10^{-3}\ \text{m})^2/4 = 4.69\times10^{-4}\ \text{m}^3/\text{s}$. A volume of $4.69\times10^{-4}\ \text{m}^3 = 0.469$ L is discharged each second.

EVALUATE: We have assumed that the diameter of the hole is much less than the diameter of the tank.

12.45. IDENTIFY: Apply Bernoulli's equation to the two points.

SET UP: $y_1 = y_2$. $v_1A_1 = v_2A_2$. $A_2 = 2A_1$.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. $v_2 = v_1\left(\frac{A_1}{A_2}\right) = (2.50\ \text{m/s})\left(\frac{A_1}{2A_1}\right) = 1.25$ m/s.

$$p_2 = p_1 + \tfrac{1}{2}\rho(v_1^2 - v_2^2) = 1.80\times10^4\ \text{Pa} + \tfrac{1}{2}(1000\ \text{kg/m}^3)[(2.50\ \text{m/s})^2 - (1.25\ \text{m/s})^2] = 2.03\times10^4\ \text{Pa}.$$

EVALUATE: The gauge pressure is higher at the second point because the water speed is less there.

12.46. IDENTIFY: Apply Bernoulli's equation to the two points.

SET UP: The continuity equation says $v_1A_1 = v_2A_2$. In Bernoulli's equation, either absolute or gauge pressures can be used at both points. $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

EXECUTE: Using $v_2 = \frac{1}{4}v_1$,

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = p_1 + \rho\left[\left(\frac{15}{32}\right)v_1^2 + g(y_1 - y_2)\right]$$

$$p_2 = 5.00\times10^4\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)\left(\frac{15}{32}(3.00\ \text{m/s})^2 + (9.80\ \text{m/s}^2)(11.0\ \text{m})\right) = 1.62\times10^5\ \text{Pa}.$$

EVALUATE: The decrease in speed and the decrease in height at point 2 both cause the pressure at point 2 to be greater than the pressure at point 1.

12.47. IDENTIFY and SET UP: Let point 1 be where $r_1 = 4.00$ cm and point 2 be where $r_2 = 2.00$ cm. The volume flow rate vA has the value $7200\ \text{cm}^3/\text{s}$ at all points in the pipe. Apply $v_1A_1 = v_2A_2$ to find the fluid speed at points 1 and 2 and then use Bernoulli's equation for these two points to find p_2.

EXECUTE: $v_1A_1 = v_1\pi r_1^2 = 7200\ \text{cm}^3$, so $v_1 = 1.43$ m/s

$v_2A_2 = v_2\pi r_2^2 = 7200\ \text{cm}^3/\text{s}$, so $v_2 = 5.73$ m/s

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$y_1 = y_2$ and $p_1 = 2.40\times10^5$ Pa, so $p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 2.25\times10^5$ Pa.

EVALUATE: Where the area decreases the speed increases and the pressure decreases.

12.48. IDENTIFY: $\rho = m/V$. Apply the equation of continuity and Bernoulli's equation to points 1 and 2.

SET UP: The density of water is 1 kg/L.

EXECUTE: **(a)** $\dfrac{(220)(0.355\text{ kg})}{60.0\text{ s}} = 1.30\text{ kg/s}.$

(b) The density of the liquid is $\dfrac{0.355\text{ kg}}{0.355\times10^{-3}\text{ m}^3} = 1000\text{ kg/m}^3$, and so the volume flow rate is

$\dfrac{1.30\text{ kg/s}}{1000\text{ kg/m}^3} = 1.30\times10^{-3}\text{ m}^3\text{/s} = 1.30\text{ L/s}.$ This result may also be obtained from

$\dfrac{(220)(0.355\text{ L})}{60.0\text{ s}} = 1.30\text{ L/s}.$

(c) $v_1 = \dfrac{1.30\times10^{-3}\text{ m}^3\text{/s}}{2.00\times10^{-4}\text{ m}^2} = 6.50\text{ m/s}.$ $v_2 = v_1/4 = 1.63\text{ m/s}.$

(d) $p_1 = p_2 + \dfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1).$

$p_1 = 152\text{ kPa} + (1000\text{ kg/m}^3)\left(\frac{1}{2}[(1.63\text{ m/s})^2 - (6.50\text{ m/s})^2] + (9.80\text{ m/s}^2)(-1.35\text{ m})\right).$ $p_1 = 119\text{ kPa}.$

EVALUATE: The increase in height and the increase in fluid speed at point 1 both cause the pressure at point 1 to be less than the pressure at point 2.

12.49. IDENTIFY: Increasing the cross-sectional area of the artery will increase the amount of blood that flows through it per second.

SET UP: The flow rate, $\dfrac{\Delta V}{\Delta t}$, is related to the radius R or diameter D of the artery by Poiseuille's law:

$\dfrac{\Delta V}{\Delta t} = \dfrac{\pi R^4}{8\eta}\left(\dfrac{p_1 - p_2}{L}\right) = \dfrac{\pi D^4}{128\eta}\left(\dfrac{p_1 - p_2}{L}\right).$ Assume the pressure gradient $(p_1 - p_2)/L$ in the artery remains the same.

EXECUTE: $(\Delta V/\Delta t)/D^4 = \dfrac{\pi}{128\eta}\left(\dfrac{p_1 - p_2}{L}\right) = \text{constant}$, so $(\Delta V/\Delta t)_{\text{old}}/D_{\text{old}}{}^4 = (\Delta V/\Delta t)_{\text{new}}/D_{\text{new}}{}^4$.

$(\Delta V/\Delta t)_{\text{new}} = 2(\Delta V/\Delta t)_{\text{old}}$ and $D_{\text{old}} = D$. This gives $D_{\text{new}} = D_{\text{old}}\left[\dfrac{(\Delta V/\Delta t)_{\text{new}}}{(\Delta V/\Delta t)_{\text{old}}}\right]^{1/4} = 2^{1/4}D = 1.19D.$

EVALUATE: Since the flow rate is proportional to D^4, a 19% increase in D doubles the flow rate.

12.50. IDENTIFY: Since a pressure difference is needed to keep the fluid flowing, there must be viscosity in the fluid.

SET UP: From Section 12.6, the pressure difference Δp over a length L of cylindrical pipe of radius R is proportional to L/R^4. In this problem, the length L is the same in both cases, so $R^4\,\Delta p$ must be constant. The target variable is the pressure difference.

EXECUTE: Since $R^4\,\Delta p$ is constant, we have $\Delta p_1 R_1^4 = \Delta p_2 R_2^4$.

$\Delta p_2 = \Delta p_1\left(\dfrac{R_1}{R_2}\right)^4 = (6.00\times10^4\text{ Pa})\left(\dfrac{0.21\text{ m}}{0.0700\text{ m}}\right)^4 = 4.86\times10^6\text{ Pa}.$

EVALUATE: The pipe is narrower, so the pressure difference must be greater.

12.51. IDENTIFY: $F = pA$, where A is the cross-sectional area presented by a hemisphere. The force F_{bb} that the body builder must apply must equal in magnitude the net force on each hemisphere due to the air inside and outside the sphere.

SET UP: $A = \pi \dfrac{D^2}{4}$.

EXECUTE: **(a)** $F_{\text{bb}} = (p_0 - p)\pi \dfrac{D^2}{4}$.

(b) The force on each hemisphere due to the atmosphere is

$\pi(5.00\times10^{-2}\text{ m})^2(1.013\times10^5\text{ Pa/atm})(0.975\text{ atm}) = 776\text{ N}$. The bodybuilder must exert this force on each hemisphere to pull them apart.

EVALUATE: The force is about 170 lbs, feasible only for a very strong person. The force required is proportional to the square of the diameter of the hemispheres.

12.52. **IDENTIFY:** Apply $p = p_0 + \rho gh$ and $\Delta V = -\dfrac{(\Delta p)V_0}{B}$, where B is the bulk modulus.

SET UP: Seawater has density $\rho = 1.03\times10^3\text{ kg/m}^3$. The bulk modulus of water is $B = 2.2\times10^9\text{ Pa}$. $p_{\text{air}} = 1.01\times10^5\text{ Pa}$.

EXECUTE:

(a) $p_0 = p_{\text{air}} + \rho gh = 1.01\times10^5\text{ Pa} + (1.03\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(10.92\times10^3\text{ m}) = 1.10\times10^8\text{ Pa}$

(b) At the surface 1.00 m^3 of seawater has mass $1.03\times10^3\text{ kg}$. At a depth of 10.92 km the change in volume is $\Delta V = -\dfrac{(\Delta p)V_0}{B} - \dfrac{(1.10\times10^8\text{ Pa})(1.00\text{ m}^3)}{2.2\times10^9\text{ Pa}} = -0.050\text{ m}^3$. The volume of this mass of water at this depth therefore is $V = V_0 + \Delta V = 0.950\text{ m}^3$. $\rho = \dfrac{m}{V} = \dfrac{1.03\times10^3\text{ kg}}{0.950\text{ m}^3} = 1.08\times10^3\text{ kg/m}^3$. The density is 5% larger than at the surface.

EVALUATE: For water B is small and a very large increase in pressure corresponds to a small fractional change in volume.

12.53. **IDENTIFY:** In part (a), the force is the weight of the water. In part (b), the pressure due to the water at a depth h is ρgh. $F = pA$ and $m = \rho V$.

SET UP: The density of water is $1.00\times10^3\text{ kg/m}^3$.

EXECUTE: **(a)** The weight of the water is

$\rho gV = (1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)((5.00\text{ m})(4.0\text{ m})(3.0\text{ m})) = 5.9\times10^5\text{ N}$.

(b) Integration gives the expected result that the force is what it would be if the pressure were uniform and equal to the pressure at the midpoint. If d is the depth of the pool and A is the area of one end of the pool, then $F = \rho gA\dfrac{d}{2} = (1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)((4.0\text{ m})(3.0\text{ m}))(1.50\text{ m}) = 1.76\times10^5\text{ N}$.

EVALUATE: The answer to part (a) can be obtained as $F = pA$, where $p = \rho gd$ is the gauge pressure at the bottom of the pool and $A = (5.0\text{ m})(4.0\text{ m})$ is the area of the bottom of the pool.

12.54. **IDENTIFY:** As the fish inflates its swim bladder, it changes its volume and hence the volume of water it displaces. This in turn changes the buoyant force on it, by Archimedes's principle.

SET UP: The buoyant force exerted by the water is $F_{\text{B}} = \rho_{\text{w}} gV_{\text{fish}}$. When the fish is fully submerged the buoyant force on it must equal its weight.

EXECUTE: **(a)** The average density of the fish is very close to the density of water.

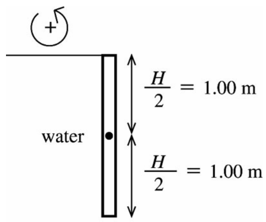
(b) Before inflation, $F_{\text{B}} = w = (2.75\text{ kg})(9.80\text{ m/s}^2) = 27.0\text{ N}$. When the volume increases by a factor of 1.10, the buoyant force also increases by a factor of 1.10 and becomes $(1.10)(27.0\text{ N}) = 29.7\text{ N}$.

(c) The water exerts an upward force 29.7 N and gravity exerts a downward force of 27.0 N so there is a net upward force of 2.7 N; the fish moves upward.

EVALUATE: Normally the buoyant force on the fish is equal to its weight, but if the fish inflates itself, the buoyant force increases and the fish rises.

12.55. IDENTIFY: Use $p = p_0 + \rho gh$ to find the gauge pressure versus depth, use $p = \dfrac{F_\perp}{A}$ to relate the pressure to the force on a strip of the gate, calculate the torque as force times moment arm, and follow the procedure outlined in the hint to calculate the total torque.

SET UP: The gate is sketched in Figure 12.55a.



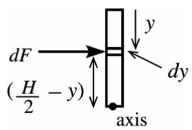
Let τ_u be the torque due to the net force of the water on the upper half of the gate, and τ_l be the torque due to the force on the lower half.

Figure 12.55a

With the indicated sign convention, τ_l is positive and τ_u is negative, so the net torque about the hinge is $\tau = \tau_l - \tau_u$. Let H be the height of the gate.

Upper half of gate:

Calculate the torque due to the force on a narrow strip of height dy located a distance y below the top of the gate, as shown in Figure 12.55b. Then integrate to get the total torque.



The net force on the strip is $dF = p(y)\,dA$, where $p(y) = \rho gy$ is the pressure at this depth and $dA = W\,dy$ with $W = 4.00$ m.

$dF = \rho gyW\,dy$

Figure 12.55b

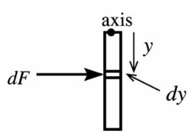
The moment arm is $(H/2 - y)$, so $d\tau = \rho gW(H/2 - y)y\,dy$.

$$\tau_u = \int_0^{H/2} d\tau = \rho gW\int_0^{H/2}(H/2 - y)y\,dy = \rho gW((H/4)y^2 - y^3/3)\Big|_0^{H/2}$$

$$\tau_u = \rho gW(H^3/16 - H^3/24) = \rho gW(H^3/48)$$

$$\tau_u = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(4.00\ \text{m})(2.00\ \text{m})^3/48 = 6.533\times10^3\ \text{N}\cdot\text{m}$$

Lower half of gate:



Consider the narrow strip shown in Figure 12.55c.

The depth of the strip is $(H/2 + y)$ so the force dF is

$dF = p(y)\,dA = \rho g(H/2 + y)W\,dy$.

Figure 12.55c

The moment arm is y, so $d\tau = \rho gW(H/2 + y)y\,dy$.

$$\tau_l = \int_0^{H/2} d\tau = \rho gW\int_0^{H/2}(H/2 + y)y\,dy = \rho gW((H/4)y^2 + y^3/3)\Big|_0^{H/2}$$

$\tau_1 = \rho g W(H^3/16 + H^3/24) = \rho g W(5H^3/48)$

$\tau_1 = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(4.00\ \text{m})5(2.00\ \text{m})^3/48 = 3.267\times10^4\ \text{N}\cdot\text{m}$

Then $\tau = \tau_1 - \tau_u = 3.267\times10^4\ \text{N}\cdot\text{m} - 6.533\times10^3\ \text{N}\cdot\text{m} = 2.61\times10^4\ \text{N}\cdot\text{m}.$

EVALUATE: The forces and torques on the upper and lower halves of the gate are in opposite directions so find the net value by subtracting the magnitudes. The torque on the lower half is larger than the torque on the upper half since pressure increases with depth.

12.56. IDENTIFY: The buoyant force B equals the weight of the air displaced by the balloon.

SET UP: $B = \rho_{\text{air}} V g$. Let g_M be the value of g for Mars. For a sphere $V = \frac{4}{3}\pi R^3$. The surface area of a sphere is given by $A = 4\pi R^2$. The mass of the balloon is $(5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

EXECUTE: **(a)** $B = mg_M$. $\rho_{\text{air}} V g_M = m g_M$. $\rho_{\text{air}} \frac{4}{3}\pi R^3 = (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

$$R = \frac{3(5.00\times10^{-3}\ \text{kg/m}^2)}{\rho_{\text{air}}} = 0.974\ \text{m}. \quad m = (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2) = 0.0596\ \text{kg}.$$

(b) $F_{\text{net}} = B - mg = ma$. $B = \rho_{\text{air}} V g = \rho_{\text{air}} \frac{4}{3}\pi R^3 g = (1.20\ \text{kg/m}^3)\left(\frac{4\pi}{3}\right)(0.974\ \text{m})^3(9.80\ \text{m/s}^2) = 45.5\ \text{N}.$

$$a = \frac{B - mg}{m} = \frac{45.5\ \text{N} - (0.0596\ \text{kg})(9.80\ \text{m/s}^2)}{0.0596\ \text{m}} = 754\ \text{m/s}^2,\ \text{upward.}$$

(c) $B = m_{\text{tot}} g$. $\rho_{\text{air}} V g = (m_{\text{balloon}} + m_{\text{load}})g$. $m_{\text{load}} = \rho_{\text{air}} \frac{4}{3}\pi R^3 - (5.00\times10^{-3}\ \text{kg/m}^2)4\pi R^2$.

$$m_{\text{load}} = (0.0154\ \text{kg/m}^3)\left(\frac{4\pi}{3}\right)(5[0.974\ \text{m}])^3 - (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi)(5[0.974\ \text{m}])^2$$

$$m_{\text{load}} = 7.45\ \text{kg} - 1.49\ \text{kg} = 5.96\ \text{kg}$$

EVALUATE: The buoyant force is proportional to R^3 and the mass of the balloon is proportional to R^2, so the load that can be carried increases when the radius of the balloon increases. We calculated the mass of the load. To find the weight of the load we would need to know the value of g for Mars.

12.57. IDENTIFY: The buoyant force on an object in a liquid is equal to the weight of the liquid it displaces.

SET UP: $V = \dfrac{m}{\rho}$.

EXECUTE: When it is floating, the ice displaces an amount of glycerin equal to its weight. From Table 12.1, the density of glycerin is $1260\ \text{kg/m}^3$. The volume of this amount of glycerin is $V = \dfrac{m}{\rho} = \dfrac{0.180\ \text{kg}}{1260\ \text{kg/m}^3} = 1.429\times10^{-4}\ \text{m}^3$. The ice cube produces 0.180 kg of water. The volume of this mass of water is $V = \dfrac{m}{\rho} = \dfrac{0.180\ \text{kg}}{1000\ \text{kg/m}^3} = 1.80\times10^{-4}\ \text{m}^3$. The volume of water from the melted ice is greater than the volume of glycerin displaced by the floating cube and the level of liquid in the cylinder rises. The distance the level rises is $\dfrac{1.80\times10^{-4}\ \text{m}^3 - 1.429\times10^{-4}\ \text{m}^3}{\pi(0.0350\ \text{m})^2} = 9.64\times10^{-3}\ \text{m} = 0.964\ \text{cm}.$

EVALUATE: The melted ice has the same mass as the solid ice, but a different density.

12.58. IDENTIFY: The pressure must be the same at the bottom of the tube. Therefore since the liquids have different densities, they must have difference heights.

SET UP: After the barrier is removed the top of the water moves downward a distance x and the top of the oil moves up a distance x, as shown in Figure 12.58. After the heights have changed, the gauge pressure at the bottom of each of the tubes is the same. The gauge pressure p at a depth h is $p - p_{\text{atm}} = \rho g h$.

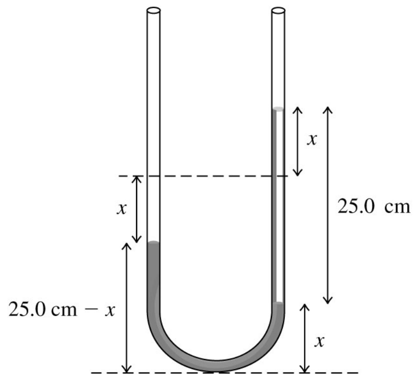


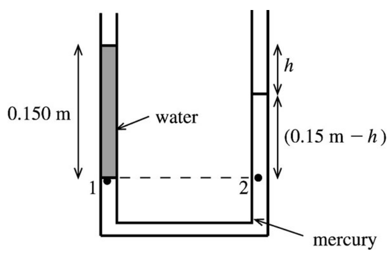
Figure 12.58

EXECUTE: The gauge pressure at the bottom of arm *A* of the tube is $p - p_{\text{atm}} = \rho_{\text{w}} g(25.0 \text{ cm} - x)$. The gauge pressure at the bottom of arm *B* of the tube is $p - p_{\text{atm}} = \rho_{\text{oil}} g(25.0 \text{ cm}) + \rho_{\text{w}} gx$. The gauge pressures must be equal, so $\rho_{\text{w}} g(25.0 \text{ cm} - x) = \rho_{\text{oil}} g(25.0 \text{ cm}) + \rho_{\text{w}} gx$. Dividing out *g* and using $\rho_{\text{oil}} = 0.80\rho_{\text{w}}$, we have $\rho_{\text{w}}(25.0 \text{ cm} - x) = 0.80\rho_{\text{w}}(25.0 \text{ cm}) + \rho_{\text{w}} x$. ρ_{w} divides out and leaves $25.0 \text{ cm} - x = 20.0 \text{ cm} + x$, so $x = 2.5$ cm. The height of fluid in arm *A* is $25.0 \text{ cm} - x = 22.5 \text{ cm}$ and the height in arm *B* is $25.0 \text{ cm} + x = 27.5 \text{ cm}$.

(b) (i) If the densities were the same there would be no reason for a difference in height and the height would be 25.0 cm on each side. (ii) The pressure exerted by the column of oil would be very small and the water would divide equally on both sides. The height in arm *A* would be 12.5 cm and the height in arm *B* would be $25.0 \text{ cm} + 12.5 \text{ cm} = 37.5 \text{ cm}$.

EVALUATE: The less dense fluid rises to a higher height, which is physically reasonable.

12.59. (a) IDENTIFY and **SET UP:**



Apply $p = p_0 + \rho gh$ to the water in the left-hand arm of the tube.
See Figure 12.59.

Figure 12.59

EXECUTE: $p_0 = p_{\text{a}}$, so the gauge pressure at the interface (point 1) is

$$p - p_{\text{a}} = \rho gh = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 1470 \text{ Pa}.$$

(b) IDENTIFY and **SET UP:** The pressure at point 1 equals the pressure at point 2. Apply Eq. (12.6) to the right-hand arm of the tube and solve for *h*.

EXECUTE: $p_1 = p_{\text{a}} + \rho_{\text{w}} g(0.150 \text{ m})$ and $p_2 = p_{\text{a}} + \rho_{\text{Hg}} g(0.150 \text{ m} - h)$

$p_1 = p_2$ implies $\rho_{\text{w}} g(0.150 \text{ m}) = \rho_{\text{Hg}} g(0.150 \text{ m} - h)$

$$0.150 \text{ m} - h = \frac{\rho_{\text{w}}(0.150 \text{ m})}{\rho_{\text{Hg}}} = \frac{(1000 \text{ kg/m}^3)(0.150 \text{ m})}{13.6 \times 10^3 \text{ kg/m}^3} = 0.011 \text{ m}$$

$$h = 0.150 \text{ m} - 0.011 \text{ m} = 0.139 \text{ m} = 13.9 \text{ cm}$$

EVALUATE: The height of mercury above the bottom level of the water is 1.1 cm. This height of mercury produces the same gauge pressure as a height of 15.0 cm of water.

12.60. **IDENTIFY:** Follow the procedure outlined in the hint. $F = pA$.

SET UP: The circular ring has area $dA = (2\pi R)dy$. The pressure due to the molasses at depth y is ρgy.

EXECUTE: $F = \int_0^h (\rho gy)(2\pi R)dy = \rho g\pi Rh^2$ where R and h are the radius and height of the tank. Using the given numerical values gives $F = 2.11\times10^8$ N.

EVALUATE: The net outward force is the area of the wall of the tank, $A = 2\pi Rh$, times the average pressure, the pressure $\rho gh/2$ at depth $h/2$.

12.61. **IDENTIFY:** Archimedes's principle applies.

SET UP: $\rho = m/V$, the buoyant force B is equal to the weight of the liquid displaced. Call m the mass of the block.

EXECUTE: **(a)** The volume of water displaced by the block is 80.0% of the volume of the block. Using $B = mg$: $\rho_w V_w g = mg$ gives $\rho_w (0.800 V_{block}) = m$. Therefore $V_{block} = m/(0.800\,\rho_w)$, so $V_{block} = (40.0\text{ kg})/[(0.800)(1000\text{ kg/m}^3)] = 0.0500\text{ m}^3$.

(b) With the maximum amount of bricks added on, the block is completely submerged but the bricks are not under water. Therefore $m_{bricks}g + mg = \rho_w V_{block} g$. Solving for m_{bricks} and putting in the numbers gives $m_{bricks} = \rho_w V_{block} - m = (1000\text{ kg/m}^3)(0.0500\text{ m}^3) - 40.0\text{ kg} = 10.0\text{ kg}$.

EVALUATE: If the bricks were to go under water, the buoyant force would increase because a greater volume of water would be displaced.

12.62. **IDENTIFY:** The buoyant force on the balloon must equal the total weight of the balloon fabric, the basket and its contents and the gas inside the balloon. $m_{gas} = \rho_{gas}V$. $B = \rho_{air}Vg$.

SET UP: The total weight, exclusive of the gas inside the balloon, is $900\text{ N} + 1700\text{ N} + 3200\text{ N} = 5800\text{ N}$.

EXECUTE: $5800\text{ N} + \rho_{gas}Vg = \rho_{air}Vg$ and $\rho_{gas} = 1.23\text{ kg/m}^3 - \dfrac{(5800\text{ N})}{(9.80\text{ m/s}^2)(2200\text{ m}^3)} = 0.96\text{ kg/m}^3$.

EVALUATE: The volume of a given mass of gas increases when the gas is heated, and the density of the gas therefore decreases.

12.63. **IDENTIFY:** Apply Newton's second law to the barge plus its contents. Apply Archimedes's principle to express the buoyancy force B in terms of the volume of the barge.

SET UP: The free-body diagram for the barge plus coal is given in Figure 12.63.

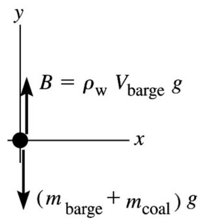


Figure 12.63

EXECUTE: $\sum F_y = ma_y$

$B - (m_{barge} + m_{coal})g = 0$

$\rho_w V_{barge} g = (m_{barge} + m_{coal})g$

$m_{coal} = \rho_w V_{barge} - m_{barge}$

$V_{barge} = (22\text{ m})(12\text{ m})(40\text{ m}) = 1.056\times10^4\text{ m}^3$

The mass of the barge is $m_{barge} = \rho_s V_s$, where s refers to steel.

From Table 12.1, $\rho_s = 7800\text{ kg/m}^3$. The volume V_s is 0.040 m times the total area of the five pieces of steel that make up the barge

$$V_s = (0.040\text{ m})[2(22\text{ m})(12\text{ m}) + 2(40\text{ m})(12\text{ m}) + (22\text{ m})(40\text{ m})] = 94.7\text{ m}^3.$$

Therefore, $m_{barge} = \rho_s V_s = (7800\text{ kg/m}^3)(94.7\text{ m}^3) = 7.39\times10^5\text{ kg}$.

Then $m_{coal} = \rho_w V_{barge} - m_{barge} = (1000\text{ kg/m}^3)(1.056\times10^4\text{ m}^3) - 7.39\times10^5\text{ kg} = 9.8\times10^6\text{ kg}$.

The volume of this mass of coal is $V_{\text{coal}} = m_{\text{coal}}/\rho_{\text{coal}} = 9.8\times10^6\ \text{kg}/1500\ \text{kg/m}^3 = 6500\ \text{m}^3$; this is less than V_{barge} so it will fit into the barge.

EVALUATE: The buoyancy force B must support both the weight of the coal and also the weight of the barge. The weight of the coal is about 13 times the weight of the barge. The buoyancy force increases when more of the barge is submerged, so when it holds the maximum mass of coal the barge is fully submerged.

12.64. **IDENTIFY:** For a floating object, the buoyant force equals the weight of the object. $B = \rho_{\text{fluid}}V_{\text{submerged}}g$.

SET UP: Water has density $\rho = 1.00\ \text{g/cm}^3$.

EXECUTE: **(a)** The volume displaced must be that which has the same weight and mass as the ice,

$$\frac{16.4\ \text{g}}{1.00\ \text{g/cm}^3} = 16.4\ \text{cm}^3.$$

(b) No; when melted, the cube produces the same volume of water as was displaced by the floating cube, and the water level does not change.

(c) $\dfrac{16.4\ \text{g}}{1.05\ \text{g/cm}^3} = 15.6\ \text{cm}^3$.

(d) The melted water takes up more volume than the salt water displaced, and so $16.4\ \text{cm}^3 - 15.6\ \text{cm}^3 = 0.8\ \text{cm}^3$ flows over.

EVALUATE: The volume of water from the melted cube is less than the volume of the ice cube, but the cube floats with only part of its volume submerged.

12.65. **IDENTIFY:** Apply Newton's second law to the car. The buoyancy force is given by Archimedes's principle.

(a) SET UP: The free-body diagram for the floating car is given in Figure 12.65. (V_{sub} is the volume that is submerged.)

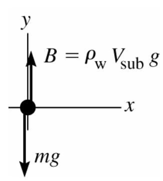


Figure 12.65

EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_{\text{w}}V_{\text{sub}}g - mg = 0$

$V_{\text{sub}} = m/\rho_{\text{w}} = (900\ \text{kg})/(1000\ \text{kg/m}^3) = 0.900\ \text{m}^3$

$V_{\text{sub}}/V_{\text{obj}} = (0.900\ \text{m}^3)/(3.0\ \text{m}^3) = 0.30 = 30\%$

EVALUATE: The average density of the car is $(900\ \text{kg})/(3.0\ \text{m}^3) = 300\ \text{kg/m}^3$. $\rho_{\text{car}}/\rho_{\text{water}} = 0.30$; this equals $V_{\text{sub}}/V_{\text{obj}}$.

(b) SET UP: When the car starts to sink it is fully submerged and the buoyant force is equal to the weight of the car plus the water that is inside it.

EXECUTE: When the car is fully submerged $V_{\text{sub}} = V$, the volume of the car, and

$B = \rho_{\text{water}}Vg = (1000\ \text{kg/m}^3)(3.0\ \text{m}^3)(9.80\ \text{m/s}^2) = 2.94\times10^4\ \text{N}$.

The weight of the car is $mg = (900\ \text{kg})(9.80\ \text{m/s}^2) = 8820\ \text{N}$.

Thus the weight of the water in the car when it sinks is the buoyant force minus the weight of the car itself:

$m_{\text{water}} = (2.94\times10^4\ \text{N} - 8820\ \text{N})/(9.80\ \text{m/s}^2) = 2.10\times10^3\ \text{kg}$

And $V_{\text{water}} = m_{\text{water}}/\rho_{\text{water}} = (2.10\times10^3\ \text{kg})/(1000\ \text{kg/m}^3) = 2.10\ \text{m}^3$

The fraction this is of the total interior volume is $(2.10\ \text{m}^3)/(3.00\ \text{m}^3) = 0.70 = 70\%$.

EVALUATE: The average density of the car plus the water inside it is

$(900\ \text{kg} + 2100\ \text{kg})/(3.0\ \text{m}^3) = 1000\ \text{kg/m}^3$, so $\rho_{\text{car}} = \rho_{\text{water}}$ when the car starts to sink.

12.66. **IDENTIFY:** For a floating object the buoyant force equals the weight of the object. The buoyant force when the wood sinks is $B = \rho_{\text{water}} V_{\text{tot}} g$, where V_{tot} is the volume of the wood plus the volume of the lead. $\rho = m/V$.

SET UP: The density of lead is $11.3 \times 10^3\ \text{kg/m}^3$.

EXECUTE: $V_{\text{wood}} = (0.600\ \text{m})(0.250\ \text{m})(0.080\ \text{m}) = 0.0120\ \text{m}^3$.

$m_{\text{wood}} = \rho_{\text{wood}} V_{\text{wood}} = (700\ \text{kg/m}^3)(0.0120\ \text{m}^3) = 8.40\ \text{kg}$.

$B = (m_{\text{wood}} + m_{\text{lead}})g$. Using $B = \rho_{\text{water}} V_{\text{tot}} g$ and $V_{\text{tot}} = V_{\text{wood}} + V_{\text{lead}}$ gives

$\rho_{\text{water}}(V_{\text{wood}} + V_{\text{lead}})g = (m_{\text{wood}} + m_{\text{lead}})g$. $m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}}$ then gives

$\rho_{\text{water}} V_{\text{wood}} + \rho_{\text{water}} V_{\text{lead}} = m_{\text{wood}} + \rho_{\text{lead}} V_{\text{lead}}$.

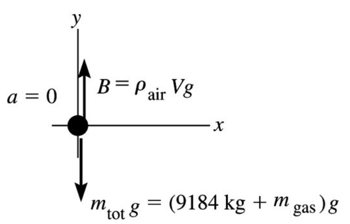
$$V_{\text{lead}} = \frac{\rho_{\text{water}} V_{\text{wood}} - m_{\text{wood}}}{\rho_{\text{lead}} - \rho_{\text{water}}} = \frac{(1000\ \text{kg/m}^3)(0.0120\ \text{m}^3) - 8.40\ \text{kg}}{11.3 \times 10^3\ \text{kg/m}^3 - 1000\ \text{kg/m}^3} = 3.50 \times 10^{-4}\ \text{m}^3.$$

$m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}} = 3.95\ \text{kg}$.

EVALUATE: The volume of the lead is only 2.9% of the volume of the wood. If the contribution of the volume of the lead to F_{B} is neglected, the calculation is simplified: $\rho_{\text{water}} V_{\text{wood}} g = (m_{\text{wood}} + m_{\text{lead}})g$ and $m_{\text{lead}} = 3.6\ \text{kg}$. The result of this calculation is in error by about 9%.

12.67. **(a) IDENTIFY:** Apply Newton's second law to the airship. The buoyancy force is given by Archimedes's principle; the fluid that exerts this force is the air.

SET UP: The free-body diagram for the dirigible is given in Figure 12.67. The lift corresponds to a mass $m_{\text{lift}} = (90 \times 10^3\ \text{N})/(9.80\ \text{m/s}^2) = 9.184 \times 10^3\ \text{kg}$. The mass m_{tot} is $9.184 \times 10^3\ \text{kg}$ plus the mass m_{gas} of the gas that fills the dirigible. B is the buoyant force exerted by the air.



EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{tot}} g = 0$

$\rho_{\text{air}} V g = (9.184 \times 10^3\ \text{kg} + m_{\text{gas}})g$

Figure 12.67

Write m_{gas} in terms of V: $m_{\text{gas}} = \rho_{\text{gas}} V$ and let g divide out; the equation becomes

$\rho_{\text{air}} V = 9.184 \times 10^3\ \text{kg} + \rho_{\text{gas}} V$.

$$V = \frac{9.184 \times 10^3\ \text{kg}}{1.20\ \text{kg/m}^3 - 0.0899\ \text{kg/m}^3} = 8.27 \times 10^3\ \text{m}^3$$

EVALUATE: The density of the airship is less than the density of air and the airship is totally submerged in the air, so the buoyancy force exceeds the weight of the airship.

(b) SET UP: Let m_{lift} be the mass that could be lifted.

EXECUTE: From part (a), $m_{\text{lift}} = (\rho_{\text{air}} - \rho_{\text{gas}})V = (1.20\ \text{kg/m}^3 - 0.166\ \text{kg/m}^3)(8.27 \times 10^3\ \text{m}^3) = 8550\ \text{kg}$.

The lift force is $m_{\text{lift}} g = (8550\ \text{kg})(9.80\ \text{m/s}^2) = 83.8\ \text{kN}$.

EVALUATE: The density of helium is less than that of air but greater than that of hydrogen. Helium provides lift, but less lift than hydrogen. Hydrogen is not used because it is highly explosive in air.

12.68. **IDENTIFY:** The buoyant force on the boat is equal to the weight of the water it displaces, by Archimedes's principle.

SET UP: $F_{\text{B}} = \rho_{\text{fluid}} g V_{\text{sub}}$, where V_{sub} is the volume of the object that is below the fluid's surface.

EXECUTE: **(a)** The boat floats, so the buoyant force on it equals the weight of the object: $F_B = mg$. Using Archimedes's principle gives $\rho_w gV = mg$ and $V = \dfrac{m}{\rho_w} = \dfrac{5750\text{ kg}}{1.00\times 10^3\text{ kg/m}^3} = 5.75\text{ m}^3$.

(b) $F_B = mg$ and $\rho_w gV_{sub} = mg$. $V_{sub} = 0.80V = 4.60\text{ m}^3$, so the mass of the floating object is $m = \rho_w V_{sub} = (1.00\times 10^3\text{ kg/m}^3)(4.60\text{ m}^3) = 4600\text{ kg}$. He must throw out $5750\text{ kg} - 4600\text{ kg} = 1150\text{ kg}$.

EVALUATE: He must throw out 20% of the boat's mass.

12.69. **IDENTIFY:** After the water leaves the hose the only force on it is gravity. Use conservation of energy to relate the initial speed to the height the water reaches. The volume flow rate is Av.

SET UP: $A = \pi D^2/4$

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$ gives $v = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(28.0\text{ m})} = 23.4\text{ m/s}$.

$(\pi D^2/4)v = 0.500\text{ m/s}^3$. $D = \sqrt{\dfrac{4(0.500\text{ m/s}^3)}{\pi v}} = \sqrt{\dfrac{4(0.500\text{ m/s}^3)}{\pi(23.4\text{ m/s})}} = 0.165\text{ m} = 16.5\text{ cm}$.

(b) D^2v is constant so if D is twice as great, then v is decreased by a factor of 4. h is proportional to v^2, so h is decreased by a factor of 16. $h = \dfrac{28.0\text{ m}}{16} = 1.75\text{ m}$.

EVALUATE: The larger the diameter of the nozzle the smaller the speed with which the water leaves the hose and the smaller the maximum height.

12.70. **IDENTIFY:** Archimedes's principle applies. The buoyant force is due to the water displaced by the completely submerged life preserver and the partially submerged person.

SET UP: The buoyant force is equal to the weight of the sea water displaced by the life preserver and 80% of the person's volume. $\rho = m/V$. The buoyant force balances the weight of the life preserver and the person. The density of sea water is 1030 kg/m^3.

EXECUTE: Balancing the forces on the person-life preserver system gives $B = W_p + W_{LP}$. Using $\rho = m/V$ gives

$\rho_w(V_{LP} + 0.800V_p) = \rho_p V_p + \rho_{LP} V_{LP}$.

For the person, we have $V_p = m_p/\rho_p$ so the last equation becomes

$\rho_w(V_{LP} + 0.800m_p/\rho_p) = m_p + \rho_{LP} V_{LP}$. Solving for ρ_{LP} gives

$\rho_{LP} = [\rho_w(V_{LP} + 0.800m_p/\rho_p) - m_p]/V_{LP}$. Putting in the numbers gives

$$\rho_{LP} = \frac{(1030\text{ kg/m}^3)\left[0.0400\text{ m}^3 + \dfrac{(0.800)(75.0\text{ kg})}{980\text{ kg/m}^3}\right] - 75.0\text{ kg}}{0.0400\text{ m}^3} = 732\text{ kg/m}^3.$$

EVALUATE: The density of the life preserver is less than that of sea water, which it must be if it is to provide buoyancy to the person in the water.

12.71. **IDENTIFY:** As water flows from the tank, the water level changes. This affects the speed with which the water flows out of the tank and the pressure at the bottom of the tank.

SET UP: Bernoulli's equation, $p_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$, and the continuity equation, $A_1v_1 = A_2v_2$, both apply.

EXECUTE: **(a)** Let point 1 be at the surface of the water in the tank and let point 2 be in the stream of water that is emerging from the tank. $p_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$. $v_1 = \dfrac{\pi d_2^2}{\pi d_1^2}v_2$, with $d_2 = 0.0200\text{ m}$ and $d_1 = 2.00\text{ m}$. $v_1 << v_2$ so the $\frac{1}{2}\rho v_1^2$ term can be neglected. $v_2 = \sqrt{\dfrac{2p_0}{\rho} + 2gh}$, where $h = y_1 - y_2$ and $p_0 = p_1 - p_2 = 5.00\times 10^3\text{ Pa}$. Initially $h = h_0 = 0.800\text{ m}$ and when the tank has drained

$h = 0$. At $t = 0$, $v_2 = \sqrt{\dfrac{2(5.00\times10^3\text{ Pa})}{1000\text{ kg/m}^3} + 2(9.8\text{ m/s}^2)(0.800\text{ m})} = \sqrt{10 + 15.68}\text{ m/s} = 5.07\text{ m/s}$. If the tank is open to the air, $p_0 = 0$ and $v_2 = 3.96$ m/s. The ratio is 1.28.

(b) $v_1 = -\dfrac{dh}{dt} = \dfrac{A_2}{A_1}v_2 = \left(\dfrac{d_2}{d_1}\right)^2\sqrt{\dfrac{2p_0}{\rho} + 2gh} = \left(\dfrac{d_2}{d_1}\right)^2\sqrt{2g}\sqrt{\dfrac{p_0}{g\rho} + h}$. Separating variables gives $\dfrac{dh}{\sqrt{\dfrac{p_0}{g\rho} + h}} = -\left(\dfrac{d_2}{d_1}\right)^2\sqrt{2g}\,dt$. We now must integrate $\displaystyle\int_{h_0}^{0}\frac{dh'}{\sqrt{\dfrac{p_0}{g\rho} + h'}} = -\left(\frac{d_2}{d_1}\right)^2\sqrt{2g}\int_0^t dt'$. To do the left-hand side integral, make the substitution $u = \dfrac{p_0}{g\rho} + h'$, which makes $du = dh'$. The integral is then of the form $\displaystyle\int\frac{du}{u^{1/2}}$, which can be readily integrated using $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1}$. The result is $2\left(\sqrt{\dfrac{p_0}{g\rho}} - \sqrt{\dfrac{p_0}{g\rho} + h_0}\right) = -\left(\dfrac{d_2}{d_1}\right)^2\sqrt{2g}\,t$. Solving for *t* gives $t = \left(\dfrac{d_1}{d_2}\right)^2\sqrt{\dfrac{2}{g}}\left(\sqrt{\dfrac{p_0}{g\rho} + h_0} - \sqrt{\dfrac{p_0}{g\rho}}\right)$. Since $\dfrac{p_0}{g\rho} = \dfrac{5.00\times10^3\text{ Pa}}{(9.8\text{ m/s}^2)(1000\text{ kg/m}^3)} = 0.5102\text{ m}$, we get

$t = \left(\dfrac{2.00}{0.0200}\right)^2\sqrt{\dfrac{2}{9.8\text{ m/s}^2}}\left(\sqrt{0.5102\text{ m} + 0.800\text{ m}} - \sqrt{0.5102}\text{ m}\right) = 1.944\times10^3\text{ s} = 32.4\text{ min}$. When $p_0 = 0$,

$t = \left(\dfrac{2.00}{0.0200}\right)^2\sqrt{\dfrac{2}{9.8\text{ m/s}^2}}\left(\sqrt{0.800\text{ m}}\right) = 4.04\times10^3\text{ s} = 67.3\text{ min}$. The ratio is 2.08.

EVALUATE: Both ratios are greater than one because a surface pressure greater than atmospheric pressure causes the water to drain with a greater speed and in a shorter time than if the surface were open to the atmosphere with a pressure of one atmosphere.

12.72. IDENTIFY: $B = \rho V_A g$. Apply Newton's second law to the beaker, liquid and block as a combined object and also to the block as a single object.

SET UP: Take $+y$ upward. Let F_D and F_E be the forces corresponding to the scale reading.

EXECUTE: Forces on the combined object: $F_D + F_E - (w_A + w_B + w_C) = 0$. $w_A = F_D + F_E - w_B - w_C$. *D* and *E* read mass rather than weight, so write the equation as $m_A = m_D + m_E - m_B - m_C$. $m_D = F_D/g$ is the reading in kg of scale *D*; a similar statement applies to m_E.

$m_A = 3.50\text{ kg} + 7.50\text{ kg} - 1.00\text{ kg} - 1.80\text{ kg} = 8.20\text{ kg}$.

Forces on *A*: $B + F_D - w_A = 0$. $\rho V_A g + F_D - m_A g = 0$. $\rho V_A + m_D = m_A$.

$$\rho = \frac{m_A - m_D}{V_A} = \frac{8.20\text{ kg} - 3.50\text{ kg}}{3.80\times10^{-3}\text{ m}^3} = 1.24\times10^3\text{ kg/m}^3$$

(b) *D* reads the mass of *A*: 8.20 kg. *E* reads the total mass of *B* and *C*: 2.80 kg.

EVALUATE: The sum of the readings of the two scales remains the same.

12.73. IDENTIFY: Apply $\sum F_y = ma_y$ to the ball, with $+y$ upward. The buoyant force is given by Archimedes's principle.

SET UP: The ball's volume is $V = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(12.0\text{ cm})^3 = 7238\text{ cm}^3$. As it floats, it displaces a weight of water equal to its weight.

EXECUTE: **(a)** By pushing the ball under water, you displace an additional amount of water equal to 76.0% of the ball's volume or $(0.760)(7238\text{ cm}^3) = 5501\text{ cm}^3$. This much water has a mass of

$5501\ g = 5.501\ \text{kg}$ and weighs $(5.501\ \text{kg})(9.80\ \text{m/s}^2) = 53.9\ \text{N}$, which is how hard you'll have to push to submerge the ball.

(b) The upward force on the ball in excess of its own weight was found in part (a): 53.9 N. The ball's mass is equal to the mass of water displaced when the ball is floating:

$$(0.240)(7238\ \text{cm}^3)(1.00\ \text{g/cm}^3) = 1737\ \text{g} = 1.737\ \text{kg},$$

and its acceleration upon release is thus $a = \dfrac{F_{\text{net}}}{m} = \dfrac{53.9\ \text{N}}{1.737\ \text{kg}} = 31.0\ \text{m/s}^2.$

EVALUATE: When the ball is totally immersed the upward buoyant force on it is much larger than its weight.

12.74. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the barrel, with $+y$ upward. The buoyant force on the barrel is given by Archimedes's principle.

SET UP: $\rho_{\text{av}} = m_{\text{tot}}/V$. An object floats in a fluid if its average density is less than the density of the fluid. The density of seawater is $1030\ \text{kg/m}^3$.

EXECUTE: **(a)** The average density of a filled barrel is

$\dfrac{m_{\text{oil}} + m_{\text{steel}}}{V} = \rho_{\text{oil}} + \dfrac{m_{\text{steel}}}{V} = 750\ \text{kg/m}^3 + \dfrac{15.0\ \text{kg}}{0.120\ \text{m}^3} = 875\ \text{kg/m}^3$, which is less than the density of seawater, so the barrel floats.

(b) The fraction above the surface is

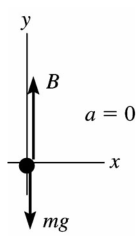
$$1 - \frac{\rho_{\text{av}}}{\rho_{\text{water}}} = 1 - \frac{875\ \text{kg/m}^3}{1030\ \text{kg/m}^3} = 0.150 = 15.0\%.$$

(c) The average density is $910\ \text{kg/m}^3 + \dfrac{32.0\ \text{kg}}{0.120\ \text{m}^3} = 1172\ \text{kg/m}^3$, which means the barrel sinks. In order to lift it, a tension $T = w_{\text{tot}} - B = (1177\ \text{kg/m}^3)(0.120\ \text{m}^3)(9.80\ \text{m/s}^2) - (1030\ \text{kg/m}^3)(0.120\ \text{m}^3)(9.80\ \text{m/s}^2)$ $= 173\ \text{N}$ is required.

EVALUATE: When the barrel floats, the buoyant force B equals its weight, w. In part (c) the buoyant force is less than the weight and $T = w - B$.

12.75. **IDENTIFY:** Apply Newton's second law to the block. In part (a), use Archimedes's principle for the buoyancy force. In part (b), use $p = p_0 + \rho gh$ to find the pressure at the lower face of the block and then use $p = \dfrac{F_\perp}{A}$ to calculate the force the fluid exerts.

(a) SET UP: The free-body diagram for the block is given in Figure 12.75a.



EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_{\text{L}} V_{\text{sub}} g = \rho_{\text{B}} V_{\text{obj}} g$

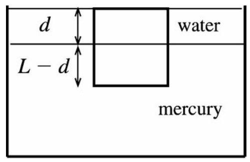
Figure 12.75a

The fraction of the volume that is submerged is $V_{\text{sub}}/V_{\text{obj}} = \rho_{\text{B}}/\rho_{\text{L}}$.

Thus the fraction that is *above* the surface is $V_{\text{above}}/V_{\text{obj}} = 1 - \rho_{\text{B}}/\rho_{\text{L}}$.

EVALUATE: If $\rho_{\text{B}} = \rho_{\text{L}}$ the block is totally submerged as it floats.

(b) SET UP: Let the water layer have depth d, as shown in Figure 12.75b.



EXECUTE: $p = p_0 + \rho_w g d + \rho_L g(L-d)$

Applying $\sum F_y = ma_y$ to the block gives $(p - p_0)A - mg = 0.$

Figure 12.75b

$[\rho_w g d + \rho_L g(L-d)]A = \rho_B L A g$

A and g divide out and $\rho_w d + \rho_L(L-d) = \rho_B L$

$d(\rho_w - \rho_L) = (\rho_B - \rho_L)L$

$$d = \left(\frac{\rho_L - \rho_B}{\rho_L - \rho_w}\right)L$$

(c) $d = \left(\dfrac{13.6\times10^3\ \text{kg/m}^3 - 7.8\times10^3\ \text{kg/m}^3}{13.6\times10^3\ \text{kg/m}^3 - 1000\ \text{kg/m}^3}\right)(0.100\ \text{m}) = 0.0460\ \text{m} = 4.60\ \text{cm}$

EVALUATE: In the expression derived in part (b), if $\rho_B = \rho_L$ the block floats in the liquid totally submerged and no water needs to be added. If $\rho_L \to \rho_w$ the block continues to float with a fraction $1 - \rho_B/\rho_w$ above the water as water is added, and the water never reaches the top of the block $(d \to \infty)$.

12.76. IDENTIFY: For the floating tanker, the buoyant force equals its total weight. The buoyant force is given by Archimedes's principle.

SET UP: When the metal is in the tanker, it displaces its weight of water and after it has been pushed overboard it displaces its volume of water.

EXECUTE: **(a)** The change in height Δy is related to the displaced volume ΔV by $\Delta y = \dfrac{\Delta V}{A}$, where A is the surface area of the water in the lock. ΔV is the volume of water that has the same weight as the metal, so $\Delta y = \dfrac{\Delta V}{A} = \dfrac{w/(\rho_{\text{water}} g)}{A} = \dfrac{w}{\rho_{\text{water}} g A} = \dfrac{(2.50\times10^6\ \text{N})}{(1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)[(60.0\ \text{m})(20.0\ \text{m})]} = 0.213\ \text{m}.$

(b) In this case, ΔV is the volume of the metal; in the above expression, ρ_{water} is replaced by $\rho_{\text{metal}} = 7.20\rho_{\text{water}}$, which gives , $\Delta y' = \dfrac{\Delta y}{7.2}$, so

$\Delta y - \Delta y' = \Delta y - \dfrac{\Delta y}{7.20} = \Delta y\left(1 - \dfrac{1}{7.20}\right) = (0.213\ \text{m})\left(1 - \dfrac{1}{7.20}\right) = 0.183$ m; the water level falls this amount.

EVALUATE: The density of the metal is greater than the density of water, so the volume of water that has the same weight as the steel is greater than the volume of water that has the same volume as the steel.

12.77. IDENTIFY: After leaving the tank, the water is in free fall, with $a_x = 0$ and $a_y = +g$.

SET UP: The speed of efflux is $\sqrt{2gh}$.

EXECUTE: **(a)** The time it takes any portion of the water to reach the ground is $t = \sqrt{\dfrac{2(H-h)}{g}}$, in which time the water travels a horizontal distance $R = vt = 2\sqrt{h(H-h)}$.

(b) Note that if $h' = H - h$, $h'(H - h') = (H - h)h$, and so $h' = H - h$ gives the same range. A hole $H - h$ below the water surface is a distance h above the bottom of the tank.

EVALUATE: For the special case of $h = H/2$, $h = h'$ and the two points coincide. For the upper hole the speed of efflux is less but the time in the air during the free fall is greater.

12.78. **IDENTIFY:** Bernoulli's equation applies to the water.

SET UP: First use Bernoulli's equation, $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$, to find the speed of the water as it enters the hold. Then use $dV/dt = Av$ to find the rate at which water flows into the hole, and then solve for the time for 10.0 L to flow in. The pressure at the top of the water is the same as the pressure of the cabin into which the water flows (atmospheric pressure), and the speed of the water at the surface is zero. 1 m^3 = 1000 L.

EXECUTE: **(a)** Using the above conditions, Bernoulli's equation gives

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p_0 + 0 + 0 = p_0 + \tfrac{1}{2}\rho v^2 + \rho g y$$

Solving for v gives $v = \sqrt{-2gy}$. The rate at which water enters ship is $V/t = Av = A\sqrt{-2gy}$. Thus

$$t = V/Av = \frac{V}{A\sqrt{-2gy}} = \frac{10\times10^{-3}\ \text{m}^3}{(1.20\times10^{-4}\ \text{m}^2)\sqrt{-2(9.80\ \text{m/s}^2)(-0.900\ \text{m})}} = 19.8\ \text{s}.$$

(b) The mass of 10.0 L of water is 10 kg, which will have no appreciable effect on the weight of a ship.

EVALUATE: The speed at which the water enters the hole is the same as if it had just fallen a distance of 0.900 m.

12.79. **IDENTIFY:** As you constrict the hose, you decrease its area, but the equation of continuity applies to the water.

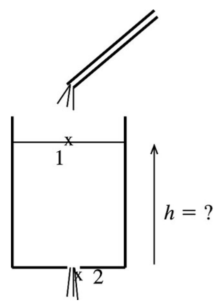
SET UP: $A_1 v_1 = A_2 v_2$. The distance traveled by a projectile that is fired from a height h with an initial horizontal velocity v is $x = vt$ where $t = \sqrt{\dfrac{2h}{g}}$.

EXECUTE: Since h is fixed, t does not change as we constrict the nozzle. Looking at the ratio of distances we obtain $\dfrac{x_1}{x_2} = \dfrac{v_1 t}{v_2 t} = \dfrac{A_2}{A_1} = \dfrac{\pi r_2^2}{\pi r_1^2}$, which gives $x_1 = x_2\left(\dfrac{r_2}{r_1}\right)^2 = (0.950\ \text{m})\left(\dfrac{1.80\ \text{cm}}{0.750\ \text{cm}}\right)^2 = 5.47\ \text{m}.$

EVALUATE: A smaller constriction results in a higher exit velocity, which results in a greater range, so our result is plausible.

12.80. **IDENTIFY:** Use Bernoulli's equation to find the velocity with which the water flows out the hole.

SET UP: The water level in the vessel will rise until the volume flow rate into the vessel, $2.40\times10^{-4}\ \text{m}^3/\text{s}$, equals the volume flow rate out the hole in the bottom.



Let points 1 and 2 be chosen as in Figure 12.80.

Figure 12.80

EXECUTE: Bernoulli's equation: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Volume flow rate out of hole equals volume flow rate from tube gives that $v_2 A_2 = 2.40\times10^{-4}\ \text{m}^3/\text{s}$ and

$$v_2 = \frac{2.40\times10^{-4}\ \text{m}^3/\text{s}}{1.50\times10^{-4}\ \text{m}^2} = 1.60\ \text{m/s}$$

$A_1 \gg A_2$ and $v_1 A_1 = v_2 A_2$ says that $\frac{1}{2}\rho v_1^2 \ll \frac{1}{2}\rho v_2^2$; neglect the $\frac{1}{2}\rho v_1^2$ term.

Measure y from the bottom of the bucket, so $y_2 = 0$ and $y_1 = h$.

$p_1 = p_2 = p_a$ (air pressure)

Then $p_a + \rho gh = p_a + \frac{1}{2}\rho v_2^2$ and $h = v_2^2/2g = (1.60 \text{ m/s})^2/2(9.80 \text{ m/s}^2) = 0.131 \text{ m} = 13.1 \text{ cm}$

EVALUATE: The greater the flow rate into the bucket, the larger v_2 will be at equilibrium and the higher the water will rise in the bucket.

12.81. IDENTIFY: Apply Bernoulli's equation and the equation of continuity.

SET UP: The speed of efflux is $\sqrt{2gh}$, where h is the distance of the hole below the surface of the fluid.

EXECUTE: **(a)** $v_3A_3 = \sqrt{2g(y_1 - y_3)}A_3 = \sqrt{2(9.80 \text{ m/s}^2)(8.00 \text{ m})}(0.0160 \text{ m}^2) = 0.200 \text{ m}^3/\text{s}$.

(b) Since p_3 is atmospheric pressure, the gauge pressure at point 2 is

$p_2 = \frac{1}{2}\rho(v_3^2 - v_2^2) = \frac{1}{2}\rho v_3^2\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = \frac{8}{9}\rho g(y_1 - y_3)$, using the expression for v_3 found above.

Substitution of numerical values gives $p_2 = 6.97 \times 10^4$ Pa.

EVALUATE: We could also calculate p_2 by applying Bernoulli's equation to points 1 and 2.

12.82 IDENTIFY: Apply Bernoulli's equation to the air in the hurricane.

SET UP: For a particle a distance r from the axis, the angular momentum is $L = mvr$.

EXECUTE: **(a)** Using the constancy of angular momentum, the product of the radius and speed is constant, so the speed at the rim is about $(200 \text{ km/h})\left(\frac{30}{350}\right) = 17 \text{ km/h}$.

(b) The pressure is lower at the eye, by an amount

$$\Delta p = \frac{1}{2}(1.2 \text{ kg/m}^3)((200 \text{ km/h})^2 - (17 \text{ km/h})^2)\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)^2 = 1.8 \times 10^3 \text{ Pa}.$$

(c) $\frac{v^2}{2g} = 160$ m.

(d) The pressure difference at higher altitudes is even greater.

EVALUATE: According to Bernoulli's equation, the pressure decreases when the fluid velocity increases.

12.83. IDENTIFY: Apply Bernoulli's equation and the equation of continuity.

SET UP: The speed of efflux at point D is $\sqrt{2gh_1}$.

EXECUTE: Applying the equation of continuity to points at C and D gives that the fluid speed is $\sqrt{8gh_1}$ at C. Applying Bernoulli's equation to points A and C gives that the gauge pressure at C is $\rho gh_1 - 4\rho gh_1 = -3\rho gh_1$, and this is the gauge pressure at the surface of the fluid at E. The height of the fluid in the column is $h_2 = 3h_1$.

EVALUATE: The gauge pressure at C is less than the gauge pressure ρgh_1 at the bottom of tank A because of the speed of the fluid at C.

12.84. **(a) IDENTIFY:** Apply constant acceleration equations to the falling liquid to find its speed as a function of the distance below the outlet. Then apply $v_1A_1 = v_2A_2$ to relate the speed to the radius of the stream.

SET UP:

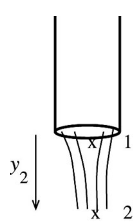


Figure 12.84

Let point 1 be at the end of the pipe and let point 2 be in the stream of liquid at a distance y_2 below the end of the tube, as shown in Figure 12.84.

Consider the free fall of the liquid. Take $+y$ to be downward.

Free fall implies $a_y = g$. v_y is positive, so replace it by the speed v.

EXECUTE: $v_2^2 = v_1^2 + 2a(y - y_0)$ gives $v_2^2 = v_1^2 + 2gy_2$ and $v_2 = \sqrt{v_1^2 + 2gy_2}$.

Equation of continuity says $v_1 A_1 = v_2 A_2$

And since $A = \pi r^2$ this becomes $v_1 \pi r_1^2 = v_2 \pi r_2^2$ and $v_2 = v_1 (r_1/r_2)^2$.

Use this in the above to eliminate v_2: $v_1 (r_1^2/r_2^2) = \sqrt{v_1^2 + 2gy_2}$

$r_2 = r_1 \sqrt{v_1}/(v_1^2 + 2gy_2)^{1/4}$

To correspond to the notation in the problem, let $v_1 = v_0$ and $r_1 = r_0$, since point 1 is where the liquid first leaves the pipe, and let r_2 be r and y_2 be y. The equation we have derived then becomes

$r = r_0 \sqrt{v_0}/(v_0^2 + 2gy)^{1/4}$.

(b) $v_0 = 1.20$ m/s

We want the value of y that gives $r = \frac{1}{2} r_0$, or $r_0 = 2r$.

The result obtained in part (a) says $r^4 (v_0^2 + 2gy) = r_0^4 v_0^2$.

Solving for y gives $y = \dfrac{[(r_0/r)^4 - 1] v_0^2}{2g} = \dfrac{(16-1)(1.20\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 1.10$ m.

EVALUATE: The equation derived in part (a) says that r decreases with distance below the end of the pipe.

12.85. IDENTIFY and **SET UP:** We are given the densities of elements in the table and look up their atomic masses in Appendix D.

EXECUTE: For example, for aluminum, the density is 2.7 g/cm^3 and the atomic mass is 26.98 g/mol.

(a) Figure 12.85 shows the graph of density versus atomic mass.

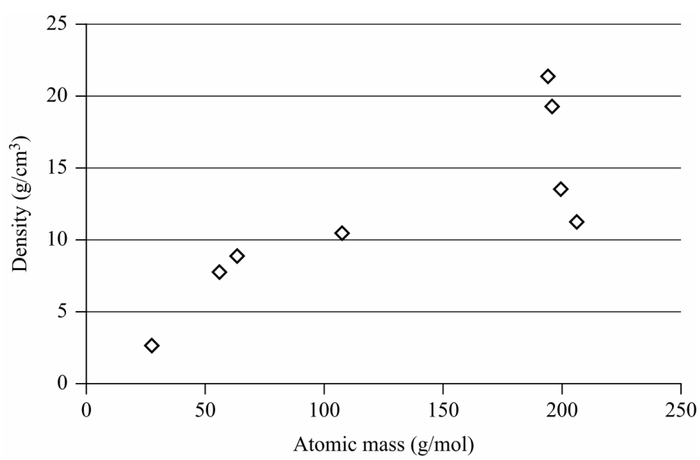


Figure 12.85

(b) From the graph, we see that there is no obvious mathematical relation between the two variables. No straight line or simple curve can be fitted to the data points.

(c) Density depends not only on atomic mass, but also on how tightly atoms are packed together. This packing is determined by the electrical interactions between atoms.

EVALUATE: Not all data can be reduced to straight-line graphs!

12.86. IDENTIFY: Archimedes's principle applies. For a floating object, the buoyant force balances the weight.

SET UP: Call m_b the mass of the block and m_n the mass of n quarters, so $m_n = nM$, where $M = 5.670$ g is the mass of a single quarter. $m_b + m_n$. The submerged volume is $(L - h)L^2$, where $L = 8.0$ cm.

EXECUTE: The forces balance, so $(m_b + m_n)g = \rho_{liq} L^2(L - h)g$. Solving for h gives $h = \dfrac{\rho_{liq}L^3 - m_b}{\rho_{liq}L^2} - \left(\dfrac{M}{\rho_{liq}L^2}\right)n$. If we plot h versus n, the slope is $-\dfrac{M}{\rho_{liq}L^2}$ and the y-intercept is $\dfrac{\rho_{liq}L^3 - m_b}{\rho_{liq}L^2}$. For the graph in the problem, we can calculate the slope by reading points from the graph. The top and bottom points seem easiest to read, so they give a slope of (1.2 cm – 3.0 cm)/25 = 0.072 cm. (Due to uncertainties in reading the graph, your answers may differ slightly from the ones here.) Using this slope, we get ρ_{liq} = –(5.670 g)/[(–0.072 cm)(8.0 cm)^2] = 1.24 g/cm^3, which rounds to 1.2 g/cm^3 = 1200 kg/m^3.

(b) Simplifying the y-intercept gives y-intercept = $L - m_b/[(\rho_{liq})(L^2)]$. From the graph, the y-intercept is 3.0 cm, so we have 3.0 cm = 8.0 cm – m_b/[(1.24 g/cm^3)(8.0 cm)^2], which gives m_b = 400 g = 0.40 kg.

EVALUATE: The unknown liquid is 20% denser than water.

12.87. IDENTIFY: Bernoulli's equation applies. We have free-fall projectile motion after the liquid leaves the tank. The pressure at the hole where the liquid exits is atmospheric pressure p_0. The absolute pressure at the top of the liquid is $p_g + p_0$.

SET UP: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

EXECUTE: **(a)** The graph of R^2 versus p_g is shown in Figure 12.87.

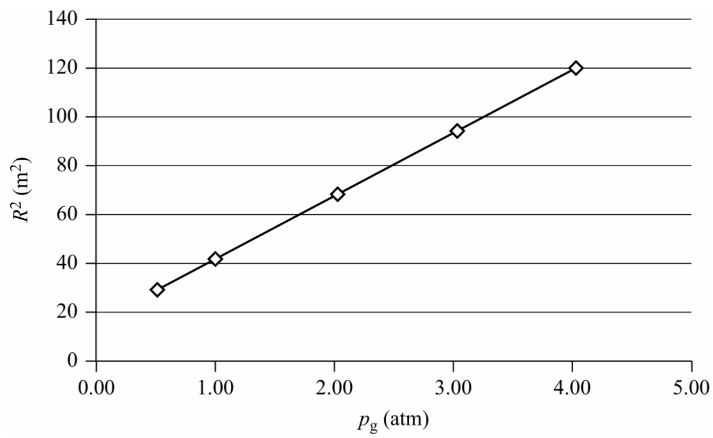


Figure 12.87

Applying Bernoulli's equation between the top and bottom of the liquid in the tank gives $p_0 + p_g + \rho gh = \frac{1}{2}\rho v^2 + p_0$, which simplifies to $p_g + \rho gh = \frac{1}{2}\rho v^2$.

The free-fall motion after leaving the tank gives $vt = R$ and $y = \frac{1}{2}gt^2$, where y = 50.0 cm. Eliminating t between these two equations gives $v^2 = (\rho g/4y)R^2$. Putting this into the result from Bernoulli's equation gives $p_g + \rho gh = (\rho g/4y)R^2$. Solving for R^2 in terms of h gives $R^2 = (4y/\rho g)\ p_g + 4yh$.

This is the equation of a straight line of slope $4y/\rho g$, which gives $\rho = 4y/[g(\text{slope})]$ and y-intercept $4yh$. The best-fit equation is R^2 = (25.679 m^2/atm) p_g + 16.385 m^2. The y-intercept gives us h: $4yh$ = y-intercept, so h = (y-intercept)/(4y) = (16.385 m^2)/[4(0.500 m)] = 8.2 m. And the density is ρ = 4y/[g(slope)] = 4(0.500 m)/[(9.80 m/s^2)(25.679 m^2/atm)(1 atm/1.01 × 10^5 Pa)] = 803 kg/m^3.

EVALUATE: The liquid is about 80% as dense as water, and h = 8.2 m which is about 25 ft, so this is a rather large tank.

12.88. IDENTIFY: Apply Bernoulli's equation to the fluid in the siphon.

SET UP: The efflux speed from a small hole a distance h below the surface of fluid in a large open tank is $\sqrt{2gh}$.

EXECUTE: **(a)** The fact that the water first moves upward before leaving the siphon does not change the efflux speed, $\sqrt{2gh}$.

(b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_a - \rho g(H+h)$, where the assumption that the cross-sectional area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p=0$ and solving for H gives $H=(p_a/\rho g)-h$.

EVALUATE: The analysis shows that $H+h<\dfrac{p_a}{\rho g}$, so there is also a limitation on $H+h$. For water and normal atmospheric pressure, $\dfrac{p_a}{\rho g}=10.3\text{ m}$.

12.89. IDENTIFY and **SET UP:** One atmosphere of pressure is 760 mm Hg. The gauge pressure is $p_g=\rho gh$.

EXECUTE: Since 1 atm is 760 mm Hg, the pressure is (150 mm/750 mm)P_{atm}. Solving for the depth h gives $h=\dfrac{P}{\rho g}=\dfrac{\left(\dfrac{150\text{ mm}}{760\text{ mm}}\right)(1.01\times10^5\text{ Pa})}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)}=2.0\text{ m}$, which is choice (b).

EVALUATE: This result is reasonable since an elephant can have its chest several meters under water.

12.90. IDENTIFY and **SET UP:** Use the definition of pressure. $\Delta p = F/A$.

EXECUTE: $\Delta p = F/A$ = (24,000 N)/[π(0.60 m)^2] = 21,200 Pa. Converting to mm Hg gives $21{,}200\text{ Pa }[(760\text{ mm Hg})/(1.01\times10^5\text{ Pa})]=160\text{ mm Hg}$, which is choice (a).

EVALUATE: The force (24,000 N) is large, but the pressure cannot be too large since the area of the diaphragm is quite large.

12.91. IDENTIFY and **SET UP:** The gauge pressure increases with depth, and since the force is proportional to the pressure, so does the force. $p_g=\rho gh$ and $p = F/A$.

EXECUTE: The force is $F = pA = (\rho gh)A$, which tells us that the force increases linearly with distance, as in choice (a).

EVALUATE: The diaphragm experiences greater force as the elephant goes into deeper water.

12.92. IDENTIFY and **SET UP:** The gauge pressure depends on the density of the liquid, $p_g=\rho gh$.

EXECUTE: Since $p_g=\rho gh$, a denser liquid will exert a greater pressure. Since salt water is denser than fresh water, the gauge pressure at a given depth will be greater in salt water than in freshwater. Therefore the maximum depth in salt water would be less than in freshwater, which is choice (b).

EVALUATE: Although the pressure in salt water would be greater than in freshwater, it would not be much greater since the density of seawater (1030 kg/m^3) is only slightly greater than that of freshwater (1000 kg/m^3).