UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS AND SCIENCE

SCHOOL OF MATHEMATICS

JF Maths/TP
SF TSM

Trinity Term 2014

MA1212 — LINEAR ALGEBRA II

Thursday, May 8 RDS-Main Hall 14:00 — 16:00

Dr. Paschalis Karageorgis

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.

1. Let $x_0 = 1$ and $y_0 = 5$. Suppose the sequences x_n, y_n are such that

$$x_n = 4x_{n-1} + y_{n-1}, \qquad y_n = 3x_{n-1} + 2y_{n-1}$$

for each integer $n \geq 1$. Determine both x_n and y_n explicitly in terms of n.

2. Find the Jordan form and a Jordan basis for the matrix

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 2 & 5 & 3 \\ -1 & -1 & 1 \end{bmatrix}.$$

3. Suppose that A is a 4×4 matrix whose column space is equal to its null space. Show that $A^2 = 0$ and then find the Jordan form of A.

4. Let Q be the quadratic form on $\mathbb{R}^3$ which is defined by the formula

$$Q(x, y, z) = 2x^2 + (a+4)y^2 + (a+1)z^2 + 2axy + 2(a-4)yz.$$

Find the values of the real parameter a for which the form is positive definite.

5. Suppose that A is a real symmetric matrix and let v_1, v_2 be eigenvectors of A with corresponding eigenvalues $\lambda_1 \neq \lambda_2$. Show that v_1 is perpendicular to v_2.

6. Let I_n denote the $n \times n$ identity matrix and let A be a real, invertible $n \times n$ matrix. Show that $A^t A$ is positive definite and that $I_n + A^t A$ is invertible.