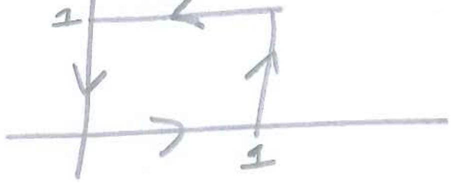
5.7 $m = 0.2$ kg

$$F_x(x,y) = 2p x^3 y^2$$

$$F_y(x,y) = x^q y$$

$$F_z(x,y) = 0$$

Choose a closed path C, e.g.



Since the force is conservative and the path is closed, we have

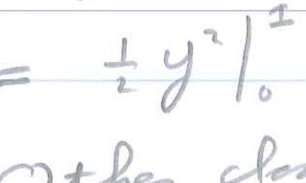
$$\oint \vec{F}\cdot d\vec{r} = 0$$

Explicit calculation gives $(q > 0$, otherwise divergence$)$

$$\int_0^1 F_x(x,0)\,dx + \int_0^1 F_y(1,y)\,dy + \int_1^0 F_x(x,1)\,dx + \int_1^0 F_y(0,y)\,dy$$

$$= \frac{1}{2} y^2 \Big|_0^1 + \frac{1}{2} p x^4 \Big|_1^0 = \frac{1}{2} - \frac{1}{2}p = 0 \Rightarrow p = 1$$

Other closed path



For the diagonal we have $(x,y) = (s,s)$

$$\Rightarrow \int_1^0 (2ps^5 + s^{q+1})\,ds = -\frac{p}{3} - \frac{1}{q+2}$$

$$\Rightarrow \oint \vec{F}\cdot d\vec{r} = \frac{1}{2} - \frac{p}{3} - \frac{1}{q+2}$$

with $p = 1$, we find $q = 4$.

$$\Rightarrow \boxed{p = 1 \ \& \ q = 4}$$

2. $\vec{V} = 6\hat{j}$

$$\left.\begin{aligned} F_x &= 2x^3y^2 \\ F_y &= x^4 y \\ F_z &= 0 \end{aligned}\right\} \quad \text{at } P = (1,2,0)$$

$$F_x = 8, \quad F_y = 2$$

$$F_{tang} = \frac{\vec{F}\cdot\vec{V}}{|\vec{V}|} = 2 = m\, a_{tang}$$

$$\Rightarrow \boxed{a_{tang} = 10 \text{ m/s}^2}$$

$$F_{perp} = 8N = \frac{mV^2}{R} \Rightarrow R = \frac{mV^2}{F_{perp}} = 0.9 \text{ m}$$

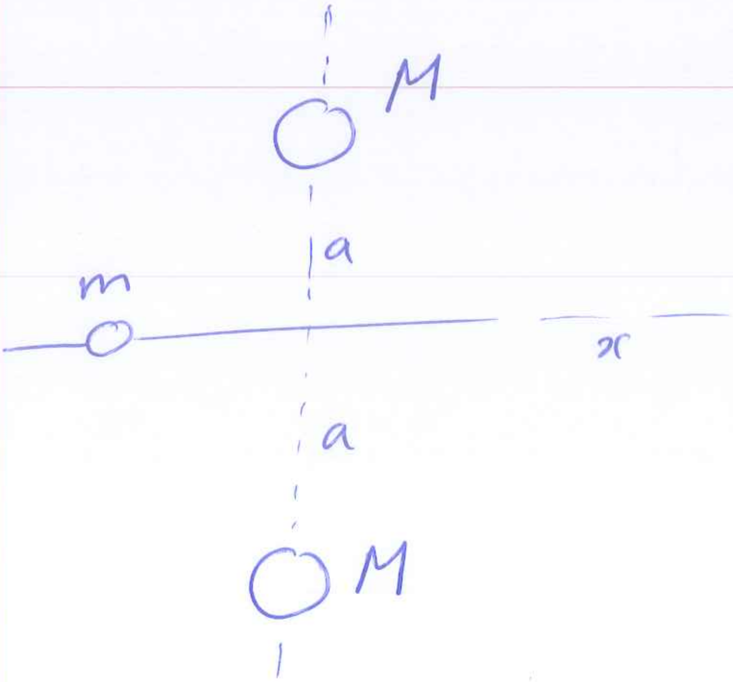
3. $$\int_O^P \vec{F}\cdot d\vec{r} = \int_0^1 F_x(x,0)\,dx + \int_0^2 F_y(1,y)\,dy$$

$$= 2\text{J} = -U(P) + U(O)$$

With $U(O) = 0$, we find

$$U(P) = 2\text{J}$$

5.13.



a) $$U(x) = -\frac{2GMm}{\sqrt{x^2+a^2}}$$

b) $$\frac{1}{2}mV_0^2 - \frac{1}{2}mV_i^2 = U(3a) - U(0)$$

$$= -\frac{2GMm}{\sqrt{10a^2}} + \frac{2GMm}{|a|}$$

$$= \frac{2(\sqrt{10}-1)GMm}{|a|}$$

$$V_0 = \sqrt{V_i^2 + \frac{4(\sqrt{10}-1)}{\sqrt{10}|a|}GM}$$