

## NUCLEAR PHYSICS

**43.1. IDENTIFY and SET UP:** The pre-subscript is  $Z$ , the number of protons. The pre-superscript is the mass number  $A$ .  $A = Z + N$ , where  $N$  is the number of neutrons.

**EXECUTE:** (a)  ${}^{28}_{14}\text{Si}$  has 14 protons and 14 neutrons.

(b)  ${}^{85}_{37}\text{Rb}$  has 37 protons and 48 neutrons.

(c)  ${}^{205}_{81}\text{Tl}$  has 81 protons and 124 neutrons.

**EVALUATE:** The number of protons determines the chemical element.

**43.2. IDENTIFY:** The interaction energy of the nuclear spin angular momentum with the external field is  $U = -\mu_z B$ . The transition energy  $\Delta E$  for the neutron is related to the frequency and wavelength of the

photon by  $\Delta E = hf = \frac{hc}{\lambda}$ .

**SET UP:**  $|\mu_z| = 1.9130\mu_n$ , where  $\mu_n = 3.15245 \times 10^{-8} \text{ eV/T}$ .

**EXECUTE:** (a) As in Example 43.2,  $\Delta E = 2(1.9130)(3.15245 \times 10^{-8} \text{ eV/T})(2.30 \text{ T}) = 2.77 \times 10^{-7} \text{ eV}$ .

Since  $\vec{\mu}$  and  $\vec{S}$  are in opposite directions for a neutron, the antiparallel configuration is lower energy. This result is smaller than but comparable to that found in the example for protons.

(b)  $f = \frac{\Delta E}{h} = 66.9 \text{ MHz}$ ,  $\lambda = \frac{c}{f} = 4.48 \text{ m}$ .

**EVALUATE:**  $\Delta E$  and  $f$  for neutrons are smaller than the corresponding values for protons that were calculated in Example 43.2.

**43.3. IDENTIFY:** Calculate the spin magnetic energy shift for each spin state of the  $1s$  level. Calculate the energy splitting between these states and relate this to the frequency of the photons.

**SET UP:** When the spin component is parallel to the field the interaction energy is  $U = -\mu_z B$ . When the spin component is antiparallel to the field the interaction energy is  $U = +\mu_z B$ . The transition energy for a transition between these two states is  $\Delta E = 2\mu_z B$ , where  $\mu_z = 2.7928\mu_n$ . The transition energy is related to the photon frequency by  $\Delta E = hf$ , so  $2\mu_z B = hf$ .

**EXECUTE:**  $B = \frac{hf}{2\mu_z} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(22.7 \times 10^6 \text{ Hz})}{2(2.7928)(5.051 \times 10^{-27} \text{ J/T})} = 0.533 \text{ T}$

**EVALUATE:** This magnetic field is easily achievable. Photons of this frequency have wavelength  $\lambda = c/f = 13.2 \text{ m}$ . These are radio waves.

**43.4. IDENTIFY:** The binding energy  $E_B$  is the difference between the energy of the constituent nuclides and final nuclide, converted to energy units.

**SET UP:** Use the masses in Table 43.2 and  $m_n = 1.008665 \text{ u}$  for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons.  $1 \text{ u}$  is equivalent to  $931.5 \text{ MeV}$ .

For  ${}^{11}_5\text{B}$ :  $E_B(\text{B-11}) = 5m_H + 6m_n - m_{\text{B-11}}$ .

For  ${}^{11}_6\text{C}$ :  $E_B(\text{C-11}) = 6m_H + 5m_n - m_{\text{C-11}}$ .

**EXECUTE:** (a)  $^{11}_5\text{B}$ :  $E_B = [5(1.007825 \text{ u}) + 6(1.008665 \text{ u}) - 11.009305 \text{ u}](931.5 \text{ MeV/u}) = 76.21 \text{ MeV}$ .

$^{11}_6\text{C}$ :  $E_B = [6(1.007825 \text{ u}) + 5(1.008665 \text{ u}) - 11.011434 \text{ u}](931.5 \text{ MeV/u}) = 73.44 \text{ MeV}$ .

**EVALUATE:** (b) The  $^{11}_5\text{B}$  has a larger binding energy than the  $^{11}_6\text{C}$ . This is probably due to the fact that  $^{11}_5\text{B}$  has one fewer proton and one more neutron than  $^{11}_6\text{C}$ . The electrical repulsion between protons tends to decrease binding energy while the extra neutron tends to increase it without a corresponding electrical repulsion.

**43.5. (a) IDENTIFY:** Find the energy equivalent of the mass defect.

**SET UP:** A  $^{11}_5\text{B}$  atom has 5 protons,  $11 - 5 = 6$  neutrons, and 5 electrons. The mass defect therefore is

$$\Delta M = 5m_p + 6m_n + 5m_e - M(^{11}_5\text{B}).$$

**EXECUTE:**  $\Delta M = 5(1.0072765 \text{ u}) + 6(1.0086649 \text{ u}) + 5(0.0005485799 \text{ u}) - 11.009305 \text{ u} = 0.08181 \text{ u}$ . The energy equivalent is  $E_B = (0.08181 \text{ u})(931.5 \text{ MeV/u}) = 76.21 \text{ MeV}$ .

**(b) IDENTIFY and SET UP:** Eq. (43.11):  $E_B = C_1A - C_2A^{2/3} - C_3Z(Z-1)/A^{1/3} - C_4(A-2Z)^2/A$ .

The fifth term is zero since  $Z$  is odd but  $N$  is even.  $A = 11$  and  $Z = 5$ .

**EXECUTE:**  $E_B = (15.75 \text{ MeV})(11) - (17.80 \text{ MeV})(11)^{2/3} - (0.7100 \text{ MeV})5(4)/11^{1/3} - (23.69 \text{ MeV})(11-10)^2/11$ .  
 $E_B = +173.25 \text{ MeV} - 88.04 \text{ MeV} - 6.38 \text{ MeV} - 2.15 \text{ MeV} = 76.68 \text{ MeV}$ .

The percentage difference between the calculated and measured  $E_B$  is  $\frac{76.68 \text{ MeV} - 76.21 \text{ MeV}}{76.21 \text{ MeV}} = 0.6\%$ .

**EVALUATE:** Eq. (43.11) has a greater percentage accuracy for  $^{62}\text{Ni}$ . The semi-empirical mass formula is more accurate for heavier nuclei.

**43.6. IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

**SET UP:** 1 u is equivalent to 931.5 MeV.  $^{238}_{92}\text{U}$  has 92 protons, 146 neutrons and 238 nucleons.

**EXECUTE:** (a)  $146m_n + 92m_H - m_U = 1.93 \text{ u}$ .

(b)  $1.80 \times 10^3 \text{ MeV}$ .

(c) 7.57 MeV per nucleon (using 931.5 MeV/u and 238 nucleons).

**EVALUATE:** The binding energy per nucleon we calculated agrees with Figure 43.2 in the textbook.

**43.7. IDENTIFY:** The binding energy of the nucleus is the energy of its constituent particles minus the energy of the carbon-12 nucleus.

**SET UP:** In terms of the masses of the particles involved, the binding energy is

$$E_B = (6m_H + 6m_n - m_{\text{C-12}})c^2.$$

**EXECUTE:** (a) Using the values from Table 43.2, we get

$$E_B = [6(1.007825 \text{ u}) + 6(1.008665 \text{ u}) - 12.000000 \text{ u}](931.5 \text{ MeV/u}) = 92.16 \text{ MeV}.$$

(b) The binding energy per nucleon is  $(92.16 \text{ MeV})/(12 \text{ nucleons}) = 7.680 \text{ MeV/nucleon}$ .

(c) The energy of the C-12 nucleus is  $(12.0000 \text{ u})(931.5 \text{ MeV/u}) = 11178 \text{ MeV}$ . Therefore the percent of

the mass that is binding energy is  $\frac{92.16 \text{ MeV}}{11178 \text{ MeV}} = 0.8245\%$ .

**EVALUATE:** The binding energy of 92.16 MeV binds 12 nucleons. The binding energy per nucleon, rather than just the total binding energy, is a better indicator of the strength with which a nucleus is bound.

**43.8. IDENTIFY:** The binding energy  $E_B$  is the difference between the energy of the constituent nuclides and final nuclide, converted to energy units.

**SET UP:** Use the masses in Table 43.2 and  $m_n = 1.008665 \text{ u}$  for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

For an alpha particle:  $^4_2\text{He}$ :  $E_B(\alpha) = 2m_H + 2m_n - m_{\text{He-4}}$ .

For  $^{12}_6\text{C}$ :  $E_B(\text{C-12}) = 6m_H + 6m_n - m_{\text{C-12}}$ .

**EXECUTE:**  $^4_2\text{He}$ :  $E_B = [2(1.007825 \text{ u}) + 2(1.008665 \text{ u}) - 4.002603 \text{ u}](931.5 \text{ MeV/u}) = 28.296 \text{ MeV}$ , so 3 times the binding energy of the alpha particle is  $3(28.296 \text{ MeV}) = 84.889 \text{ MeV}$ .

$$^{12}_6\text{C}: E_B = [6(1.007825 \text{ u}) + 6(1.008665 \text{ u}) - 12.000000 \text{ u}](931.5 \text{ MeV/u}) = 92.163 \text{ MeV}.$$

**EVALUATE:** The binding energy of  $^{12}_6\text{C}$  is greater than 3 times the binding energy of an alpha particle.

This is reasonable since in addition to binding individual alpha particles, it takes additional energy to bind three of them in the nucleus to form carbon.

- 43.9. IDENTIFY:** Conservation of energy tells us that the initial energy (photon plus deuteron) is equal to the energy after the split (kinetic energy plus energy of the proton and neutron). Therefore the kinetic energy released is equal to the energy of the photon minus the binding energy of the deuteron.

**SET UP:** The binding energy of a deuteron is 2.224 MeV and the energy of the photon is  $E = hc/\lambda$ .

Kinetic energy is  $K = \frac{1}{2}mv^2$ .

**EXECUTE: (a)** The energy of the photon is

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{3.50 \times 10^{-13} \text{ m}} = 5.68 \times 10^{-13} \text{ J}.$$

The binding of the deuteron is  $E_B = 2.224 \text{ MeV} = 3.56 \times 10^{-13} \text{ J}$ . Therefore the kinetic energy is

$$K = (5.68 - 3.56) \times 10^{-13} \text{ J} = 2.12 \times 10^{-13} \text{ J} = 1.32 \text{ MeV}.$$

**(b)** The particles share the energy equally, so each gets half. Solving the kinetic energy for  $v$  gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06 \times 10^{-13} \text{ J})}{1.6605 \times 10^{-27} \text{ kg}}} = 1.13 \times 10^7 \text{ m/s}.$$

**EVALUATE:** Considerable energy has been released, because the particle speeds are in the vicinity of the speed of light.

- 43.10. IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

**SET UP:** 1 u is equivalent to 931.5 MeV.  $^{14}_7\text{N}$  has 7 protons and 7 neutrons.  $^4_2\text{He}$  has 2 protons and 2 neutrons.

**EXECUTE: (a)**  $7(m_p + m_n) - m_N = 0.112 \text{ u}$ , which is 105 MeV, or 7.48 MeV per nucleon.

**(b)** Similarly,  $2(m_p + m_n) - m_{\text{He}} = 0.03038 \text{ u} = 28.3 \text{ MeV}$ , or 7.07 MeV per nucleon.

**EVALUATE: (c)** The binding energy per nucleon is a little less for  $^4_2\text{He}$  than for  $^{14}_7\text{N}$ . This is in agreement with Figure 43.2 in the textbook.

- 43.11. IDENTIFY:** Use Eq. (43.11) to calculate the binding energy of two nuclei, and then calculate their binding energy per nucleon.

**SET UP and EXECUTE:**  $^{86}_{36}\text{Kr}$ :  $A = 86$  and  $Z = 36$ .  $N = A - Z = 50$ , which is even, so for the last term in Eq. (43.11) we use the plus sign. Putting the given number in the equation and using the values for the constants given in the textbook, we have

$$E_B = (15.75 \text{ MeV})(86) - (17.80 \text{ MeV})(86)^{2/3} - (0.71 \text{ MeV})\frac{(36)(35)}{86^{1/3}} \\ - (23.69 \text{ MeV})\frac{(86 - 72)^2}{86} + (39 \text{ MeV})(86)^{-4/3}.$$

$$E_B = 751.1 \text{ MeV} \text{ and } \frac{E_B}{A} = 8.73 \text{ MeV/nucleon}.$$

$^{180}_{73}\text{Ta}$ :  $A = 180$ ,  $Z = 73$ ,  $N = 180 - 73 = 107$ , which is odd.

$$E_B = (15.75 \text{ MeV})(180) - (17.80 \text{ MeV})(180)^{2/3} - (0.71 \text{ MeV})\frac{(73)(72)}{180^{1/3}} \\ - (23.69 \text{ MeV})\frac{(180 - 146)^2}{180} + (39 \text{ MeV})(180)^{-4/3}.$$

$$E_B = 1454.4 \text{ MeV} \text{ and } \frac{E_B}{A} = 8.08 \text{ MeV/nucleon}.$$

**EVALUATE:** The binding energy per nucleon is less for  $^{180}_{73}\text{Ta}$  than for  $^{86}_{36}\text{Kr}$ , in agreement with Figure 43.2.

- 43.12. IDENTIFY:** Compare the total mass on each side of the reaction equation. Neglect the masses of the neutrino and antineutrino.  
**SET UP:** 1 u is equivalent to 931.5 MeV.  
**EXECUTE:** (a) The energy released is the energy equivalent of  $m_n - m_p - m_e = 8.40 \times 10^{-4}$  u, or 783 keV.  
 (b)  $m_n > m_p$ , and the decay is not possible.  
**EVALUATE:**  $\beta^-$  and  $\beta^+$  particles have the same mass, equal to the mass of an electron.
- 43.13. IDENTIFY:** In each case determine how the decay changes  $A$  and  $Z$  of the nucleus. The  $\beta^+$  and  $\beta^-$  particles have charge but their nucleon number is  $A = 0$ .  
**(a) SET UP:**  $\alpha$ -decay:  $Z$  decreases by 2,  $A = N + Z$  decreases by 4 (an  $\alpha$  particle is a  ${}^4_2\text{He}$  nucleus).  
**EXECUTE:**  ${}^{239}_{94}\text{Pu} \rightarrow {}^4_2\text{He} + {}^{235}_{92}\text{U}$ .  
**(b) SET UP:**  $\beta^-$  decay:  $Z$  increases by 1,  $A = N + Z$  remains the same (a  $\beta^-$  particle is an electron,  ${}^0_{-1}\text{e}$ ).  
**EXECUTE:**  ${}^{24}_{11}\text{Na} \rightarrow {}^0_{-1}\text{e} + {}^{24}_{12}\text{Mg}$ .  
**(c) SET UP:**  $\beta^+$  decay:  $Z$  decreases by 1,  $A = N + Z$  remains the same (a  $\beta^+$  particle is a positron,  ${}^0_{+1}\text{e}$ ).  
**EXECUTE:**  ${}^{15}_8\text{O} \rightarrow {}^0_{+1}\text{e} + {}^{15}_7\text{N}$ .  
**EVALUATE:** In each case the total charge and total number of nucleons for the decay products equals the charge and number of nucleons for the parent nucleus; these two quantities are conserved in the decay.
- 43.14. IDENTIFY:** The energy released is equal to the mass defect of the initial and final nuclei.  
**SET UP:** The mass defect is equal to the difference between the initial and final masses of the constituent particles.  
**EXECUTE:** (a) The mass defect is  $238.050788 \text{ u} - 234.043601 \text{ u} - 4.002603 \text{ u} = 0.004584 \text{ u}$ . The energy released is  $(0.004584 \text{ u})(931.5 \text{ MeV/u}) = 4.270 \text{ MeV}$ .  
 (b) Take the ratio of the two kinetic energies, using the fact that  $K = p^2/2m$ :

$$\frac{K_{\text{Th}}}{K_{\alpha}} = \frac{\frac{p_{\text{Th}}^2}{2m_{\text{Th}}}}{\frac{p_{\alpha}^2}{2m_{\alpha}}} = \frac{m_{\alpha}}{m_{\text{Th}}} = \frac{4}{234}.$$

The kinetic energy of the Th is

$$K_{\text{Th}} = \frac{4}{234 + 4} K_{\text{Total}} = \frac{4}{238} (4.270 \text{ MeV}) = 0.07176 \text{ MeV} = 1.148 \times 10^{-14} \text{ J}.$$

Solving for  $v$  in the kinetic energy gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.148 \times 10^{-14} \text{ J})}{(234.043601)(1.6605 \times 10^{-27} \text{ kg})}} = 2.431 \times 10^5 \text{ m/s}.$$

**EVALUATE:** As we can see by the ratio of kinetic energies in part (b), the alpha particle will have a much higher kinetic energy than the thorium.

- 43.15. IDENTIFY:** Compare the mass of the original nucleus to the total mass of the decay products.  
**SET UP:** Subtract the electron masses from the neutral atom mass to obtain the mass of each nucleus.  
**EXECUTE:** If  $\beta^-$  decay of  ${}^{14}_6\text{C}$  is possible, then we are considering the decay  ${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + \beta^-$ .  
 $\Delta m = M({}^{14}_6\text{C}) - M({}^{14}_7\text{N}) - m_e$ .  
 $\Delta m = (14.003242 \text{ u} - 6(0.000549 \text{ u})) - (14.003074 \text{ u} - 7(0.000549 \text{ u})) - 0.0005491 \text{ u}$ .  
 $\Delta m = +1.68 \times 10^{-4} \text{ u}$ . So  $E = (1.68 \times 10^{-4} \text{ u})(931.5 \text{ MeV/u}) = 0.156 \text{ MeV} = 156 \text{ keV}$ .  
**EVALUATE:** In the decay the total charge and the nucleon number are conserved.
- 43.16. IDENTIFY:** In each reaction the nucleon number and the total charge are conserved.

**SET UP:** An  $\alpha$  particle has charge  $+2e$  and nucleon number 4. An electron has charge  $-e$  and nucleon number zero. A positron has charge  $+e$  and nucleon number zero.

**EXECUTE:** (a) A proton changes to a neutron, so the emitted particle is a positron ( $\beta^+$ ).

(b) The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle.

(c) A neutron changes to a proton, so the emitted particle is an electron ( $\beta^-$ ).

**EVALUATE:** We have considered the conservation laws. We have not determined if the decays are energetically allowed.

- 43.17. IDENTIFY:** The energy released is the energy equivalent of the difference in the masses of the original atom and the final atom produced in the capture. Apply conservation of energy to the decay products.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** (a) As in the example,  $(0.000897 \text{ u})(931.5 \text{ MeV/u}) = 0.836 \text{ MeV}$ .

(b)  $0.836 \text{ MeV} - 0.122 \text{ MeV} - 0.014 \text{ MeV} = 0.700 \text{ MeV}$ .

**EVALUATE:** We have neglected the rest mass of the neutrino that is emitted.

- 43.18. IDENTIFY:** Determine the energy released during tritium decay.

**SET UP:** In beta decay an electron,  $e^-$ , is emitted by the nucleus. The beta decay reaction is

${}^3_1\text{H} \rightarrow e^- + {}^3_2\text{He}$ . If neutral atom masses are used,  ${}^3_1\text{H}$  includes one electron and  ${}^3_2\text{He}$  includes two electrons.

One electron mass cancels and the other electron mass in  ${}^3_2\text{He}$  represents the emitted electron. Or, we can subtract the electron masses and use the nuclear masses. The atomic mass of  ${}^3_2\text{He}$  is 3.016029 u.

**EXECUTE:** (a) The mass of the  ${}^3_1\text{H}$  nucleus is  $3.016049 \text{ u} - 0.000549 \text{ u} = 3.015500 \text{ u}$ . The mass of the  ${}^3_2\text{He}$  nucleus is  $3.016029 \text{ u} - 2(0.000549 \text{ u}) = 3.014931 \text{ u}$ . The nuclear mass of  ${}^3_2\text{He}$  plus the mass of the emitted electron is  $3.014931 \text{ u} + 0.000549 \text{ u} = 3.015480 \text{ u}$ . This is slightly less than the nuclear mass for  ${}^3_1\text{H}$ , so the decay is energetically allowed.

(b) The mass decrease in the decay is  $3.015500 \text{ u} - 3.015480 \text{ u} = 2.0 \times 10^{-5} \text{ u}$ . Note that this can also be calculated as  $m({}^3_1\text{H}) - m({}^4_2\text{He})$ , where atomic masses are used. The energy released is  $(2.0 \times 10^{-5} \text{ u})(931.5 \text{ MeV/u}) = 0.019 \text{ MeV}$ . The total kinetic energy of the decay products is 0.019 MeV, or 19 keV.

**EVALUATE:** The energy is not shared equally by the decay products because they have unequal masses.

- 43.19. IDENTIFY and SET UP:**  $T_{1/2} = \frac{\ln 2}{\lambda}$  The mass of a single nucleus is  $124m_p = 2.07 \times 10^{-25} \text{ kg}$ .

$$|dN/dt| = 0.350 \text{ Ci} = 1.30 \times 10^{10} \text{ Bq}, \quad |dN/dt| = \lambda N.$$

$$\text{EXECUTE: } N = \frac{6.13 \times 10^{-3} \text{ kg}}{2.07 \times 10^{-25} \text{ kg}} = 2.96 \times 10^{22}; \quad \lambda = \frac{|dN/dt|}{N} = \frac{1.30 \times 10^{10} \text{ Bq}}{2.96 \times 10^{22}} = 4.39 \times 10^{-13} \text{ s}^{-1}.$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = 1.58 \times 10^{12} \text{ s} = 5.01 \times 10^4 \text{ y}.$$

**EVALUATE:** Since  $T_{1/2}$  is very large, the activity changes very slowly.

- 43.20. IDENTIFY:** The equation  $N = N_0 e^{-\lambda t}$  can be written as  $N = N_0 2^{-t/T_{1/2}}$ .

**SET UP:** The amount of elapsed time since the source was created is roughly 2.5 years.

**EXECUTE:** The current activity is  $N = (5000 \text{ Ci})2^{-(2.5 \text{ yr})/(5.271 \text{ yr})} = 3600 \text{ Ci}$ . The source is barely usable.

**EVALUATE:** Alternatively, we could calculate  $\lambda = \frac{\ln(2)}{T_{1/2}} = 0.132(\text{years})^{-1}$  and use  $N = N_0 e^{-\lambda t}$  directly to

obtain the same answer.

**43.21. IDENTIFY:** From the known half-life, we can find the decay constant, the rate of decay, and the activity.

**SET UP:**  $\lambda = \frac{\ln 2}{T_{1/2}}$ .  $T_{1/2} = 4.47 \times 10^9 \text{ yr} = 1.41 \times 10^{17} \text{ s}$ . The activity is  $\left| \frac{dN}{dt} \right| = \lambda N$ . The mass of one  $^{238}\text{U}$  is approximately  $238m_p$ .  $1 \text{ Ci} = 3.70 \times 10^{10} \text{ decays/s}$ .

**EXECUTE:** (a)  $\lambda = \frac{\ln 2}{1.41 \times 10^{17} \text{ s}} = 4.92 \times 10^{-18} \text{ s}^{-1}$ .

(b)  $N = \frac{|dN/dt|}{\lambda} = \frac{3.70 \times 10^{10} \text{ Bq}}{4.92 \times 10^{-18} \text{ s}^{-1}} = 7.52 \times 10^{27} \text{ nuclei}$ . The mass  $m$  of uranium is the number of nuclei times the mass of each one.  $m = (7.52 \times 10^{27})(238)(1.67 \times 10^{-27} \text{ kg}) = 2.99 \times 10^3 \text{ kg}$ .

(c)  $N = \frac{10.0 \times 10^{-3} \text{ kg}}{238m_p} = \frac{10.0 \times 10^{-3} \text{ kg}}{238(1.67 \times 10^{-27} \text{ kg})} = 2.52 \times 10^{22} \text{ nuclei}$ .

$\left| \frac{dN}{dt} \right| = \lambda N = (4.92 \times 10^{-18} \text{ s}^{-1})(2.52 \times 10^{22}) = 1.24 \times 10^5 \text{ decays/s}$ .

**EVALUATE:** Because  $^{238}\text{U}$  has a very long half-life, it requires a large amount (about 3000 kg) to have an activity of a 1.0 Ci.

**43.22. IDENTIFY:** From the half-life and mass of an isotope, we can find its initial activity rate. Then using the half-life, we can find its activity rate at a later time.

**SET UP:** The activity  $|dN/dt| = \lambda N$ .  $\lambda = \frac{\ln 2}{T_{1/2}}$ . The mass of one  $^{103}\text{Pd}$  nucleus is  $103m_p$ . In a time of one half-life the number of radioactive nuclei and the activity decrease by a factor of 2.

**EXECUTE:** (a)  $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(17 \text{ days})(24 \text{ h/day})(3600 \text{ s/h})} = 4.7 \times 10^{-7} \text{ s}^{-1}$ .

$N = \frac{0.250 \times 10^{-3} \text{ kg}}{103m_p} = 1.45 \times 10^{21}$ .  $|dN/dt| = (4.7 \times 10^{-7} \text{ s}^{-1})(1.45 \times 10^{21}) = 6.8 \times 10^{14} \text{ Bq}$ .

(b) 68 days is  $4T_{1/2}$  so the activity is  $(6.8 \times 10^{14} \text{ Bq})/2^4 = 4.2 \times 10^{13} \text{ Bq}$ .

**EVALUATE:** At the end of 4 half-lives, the activity rate is less than a tenth of its initial rate.

**43.23. IDENTIFY and SET UP:** As discussed in Section 43.4, the activity  $A = |dN/dt|$  obeys the same decay equation as  $N(t)$ :  $A = A_0 e^{-\lambda t}$ . For  $^{14}\text{C}$ ,  $T_{1/2} = 5730 \text{ y}$  and  $\lambda = \ln 2 / T_{1/2}$  so  $A = A_0 e^{-(\ln 2)t/T_{1/2}}$ ; calculate  $A$  at each  $t$ ;  $A_0 = 184 \text{ decays/min}$ .

**EXECUTE:** (a) For  $t = 1000 \text{ y}$ , we have  $A = (184 \text{ decays/min})e^{-(\ln 2)(1000 \text{ y})/(5730 \text{ y})} = 163 \text{ decays/min}$ .

(b) For  $t = 50,000 \text{ y}$ , the same equation gives  $A = 0.435 \text{ decays/min}$ .

**EVALUATE:** The time in part (b) is 8.73 half-lives, so the decay rate has decreased by a factor of  $(\frac{1}{2})^{8.73}$ .

**43.24. IDENTIFY and SET UP:** The decay rate decreases by a factor of 2 in a time of one half-life.

**EXECUTE:** (a) 24 days is 3 half-lives, so the activity at the end of that time is  $(325 \text{ Bq})/(2^3) = 40.6 \text{ Bq}$ , which rounds to 41 Bq.

(b) The activity is proportional to the number of radioactive nuclei, so the percent is

$\frac{17.0 \text{ Bq}}{40.6 \text{ Bq}} = 0.42 = 42\%$ .

(c)  $^{131}_{53}\text{I} \rightarrow ^0_{-1}\text{e} + ^{131}_{54}\text{Xe}$ . The nucleus  $^{131}_{54}\text{Xe}$  is produced.

**EVALUATE:** Both the activity and the number of radioactive nuclei present decrease by a factor of 2 in one half-life.

**43.25. IDENTIFY and SET UP:** Find  $\lambda$  from the half-life and the number  $N$  of nuclei from the mass of one nucleus and the mass of the sample. Then use Eq. (43.16) to calculate  $|dN/dt|$ , the number of decays per second.

**EXECUTE:** (a)  $|dN/dt| = \lambda N$ .

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(1.28 \times 10^9 \text{ y})(3.156 \times 10^7 \text{ s/1 y})} = 1.715 \times 10^{-17} \text{ s}^{-1}.$$

The mass of one  $^{40}\text{K}$  atom is approximately 40 u, so the number of  $^{40}\text{K}$  nuclei in the sample is

$$N = \frac{1.63 \times 10^{-9} \text{ kg}}{40 \text{ u}} = \frac{1.63 \times 10^{-9} \text{ kg}}{40(1.66054 \times 10^{-27} \text{ kg})} = 2.454 \times 10^{16}.$$

Then  $|dN/dt| = \lambda N = (1.715 \times 10^{-17} \text{ s}^{-1})(2.454 \times 10^{16}) = 0.421 \text{ decays/s}$ .

(b)  $|dN/dt| = (0.421 \text{ decays/s})(1 \text{ Ci}/(3.70 \times 10^{10} \text{ decays/s})) = 1.14 \times 10^{-11} \text{ Ci}$ .

**EVALUATE:** The very small sample still contains a very large number of nuclei. But the half-life is very large, so the decay rate is small.

**43.26. IDENTIFY:** Apply  $|dN/dt| = \lambda N$  to calculate  $N$ , the number of radioactive nuclei originally present in the spill. Since the activity  $A$  is proportional to the number of radioactive nuclei,  $N = N_0 e^{-\lambda t}$  leads to  $A = A_0 e^{-\lambda t}$ , where  $A$  is the activity.

**SET UP:** The mass of one  $^{131}\text{Ba}$  nucleus is about 131 u.

**EXECUTE:** (a)  $\left| \frac{dN}{dt} \right| = 400 \mu\text{Ci} = (400 \times 10^{-6})(3.70 \times 10^{10} \text{ s}^{-1}) = 1.48 \times 10^7 \text{ decays/s}$ .

$$T_{1/2} = \frac{\ln 2}{\lambda} \rightarrow \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12 \text{ d})(86,400 \text{ s/d})} = 6.69 \times 10^{-7} \text{ s}^{-1}.$$

$$\left| \frac{dN}{dt} \right| = \lambda N \Rightarrow N = \frac{|dN/dt|}{\lambda} = \frac{1.48 \times 10^7 \text{ decays/s}}{6.69 \times 10^{-7} \text{ s}^{-1}} = 2.21 \times 10^{13} \text{ nuclei. The mass of these } ^{131}\text{Ba} \text{ nuclei is}$$

$$m = (2.21 \times 10^{13} \text{ nuclei}) \times (131 \times 1.66 \times 10^{-27} \text{ kg/nucleus}) = 4.8 \times 10^{-12} \text{ kg} = 4.8 \times 10^{-9} \text{ g} = 4.8 \text{ ng}.$$

(b)  $A = A_0 e^{-\lambda t}$ .  $1 \mu\text{Ci} = (400 \mu\text{Ci}) e^{-\lambda t}$ .  $\ln(1/400) = -\lambda t$ .

$$t = -\frac{\ln(1/400)}{\lambda} = -\frac{\ln(1/400)}{6.69 \times 10^{-7} \text{ s}^{-1}} = 8.96 \times 10^6 \text{ s} \left( \frac{1 \text{ d}}{86,400 \text{ s}} \right) = 104 \text{ days}.$$

**EVALUATE:** The time is about 8.7 half-lives and the activity after that time is  $(400 \mu\text{Ci})\left(\frac{1}{2}\right)^{8.7} \approx 1 \mu\text{Ci}$ .

**43.27. IDENTIFY:** Apply  $A = A_0 e^{-\lambda t}$  and  $\lambda = \ln 2/T_{1/2}$ .

**SET UP:**  $\ln e^x = x$ .

**EXECUTE:**  $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$ .  $-\frac{(\ln 2)t}{T_{1/2}} = \ln(A/A_0)$ .

$$T_{1/2} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00 \text{ days})}{\ln(3091/8318)} = 2.80 \text{ days}.$$

**EVALUATE:** The activity has decreased by more than half and the elapsed time is more than one half-life.

**43.28. IDENTIFY:** The activity rate  $A$  decreases exponentially.

**SET UP:**  $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$  using  $\lambda = (\ln 2)/T_{1/2}$ .

**EXECUTE:**  $A = A_0 e^{-t(\ln 2)/T_{1/2}} = (0.376 \text{ Ci})e^{-(\ln 2)(120 \text{ min})/(43.0 \text{ min})} = 0.0543 \text{ Ci}$ .

**EXECUTE:** 2.00 h is 2.791 half-lives, so the activity is  $(0.376 \text{ Ci})(1/2)^{2.791} = 0.0543 \text{ Ci}$ , which agrees with our result.

**43.29. IDENTIFY and SET UP:** Apply  $|dN/dt| = \lambda N$  with  $\lambda = \ln 2/T_{1/2}$ . In one half-life, one half of the nuclei decay.

**EXECUTE:** (a)  $\left| \frac{dN}{dt} \right| = 7.56 \times 10^{11} \text{ Bq} = 7.56 \times 10^{11} \text{ decays/s.}$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(30.8 \text{ min})(60 \text{ s/min})} = 3.75 \times 10^{-4} \text{ s}^{-1}. N_0 = \frac{1}{\lambda} \left| \frac{dN}{dt} \right| = \frac{7.56 \times 10^{11} \text{ decay/s}}{3.75 \times 10^{-4} \text{ s}^{-1}} = 2.02 \times 10^{15} \text{ nuclei.}$$

(b) The number of nuclei left after one half-life is  $\frac{N_0}{2} = 1.01 \times 10^{15}$  nuclei, and the activity is half:

$$\left| \frac{dN}{dt} \right| = 3.78 \times 10^{11} \text{ decays/s.}$$

(c) After three half-lives (92.4 minutes) there is an eighth of the original amount, so  $N = 2.53 \times 10^{14}$  nuclei, and an eighth of the activity:  $\left| \frac{dN}{dt} \right| = 9.45 \times 10^{10} \text{ decays/s.}$

**EVALUATE:** Since the activity is proportional to the number of radioactive nuclei that are present, the activity is halved in one half-life.

**43.30. IDENTIFY:** Apply  $A = A_0 e^{-\lambda t}$ .

**SET UP:** From Example 43.9,  $\lambda = 3.83 \times 10^{-12} \text{ s}^{-1} = 1.21 \times 10^{-4} \text{ y}^{-1}$  for radiocarbon.

**EXECUTE:** The activity of the sample is  $\frac{2690 \text{ decays/min}}{(60 \text{ s/min})(0.500 \text{ kg})} = 89.7 \text{ Bq/kg}$ , while the activity of

atmospheric carbon is 255 Bq/kg (see Example 43.9). The age of the sample is then

$$t = -\frac{\ln(89.7/255)}{\lambda} = -\frac{\ln(89.7/255)}{1.21 \times 10^{-4} \text{ y}^{-1}} = 8640 \text{ y.}$$

**EVALUATE:** For  $^{14}\text{C}$ ,  $T_{1/2} = 5730 \text{ y}$ . The age is more than one half-life and the activity per kg of carbon is less than half the value when the tree died.

**43.31. IDENTIFY:** Knowing the equivalent dose in Sv, we want to find the absorbed energy.

**SET UP:** equivalent dose (Sv, rem) = RBE  $\times$  absorbed dose (Gy, rad); 100 rad = 1 Gy.

**EXECUTE:** (a) RBE = 1, so 0.25 mSv corresponds to 0.25 mGy.

$$\text{Energy} = (0.25 \times 10^{-3} \text{ J/kg})(5.0 \text{ kg}) = 1.2 \times 10^{-3} \text{ J.}$$

(b) RBE = 1 so 0.10 mGy = 10 mrad and 10 mrem.  $(0.10 \times 10^{-3} \text{ J/kg})(75 \text{ kg}) = 7.5 \times 10^{-3} \text{ J.}$

**EVALUATE:** (c)  $\frac{7.5 \times 10^{-3} \text{ J}}{1.2 \times 10^{-3} \text{ J}} = 6.2$ . Each chest x ray delivers only about 1/6 of the yearly background radiation energy.

**43.32. IDENTIFY and SET UP:** 1 Gy = 1 J/kg and is the SI unit of absorbed dose. 1 rad = 0.010 Gy. Sv is the SI unit for equivalent dose. Equivalent dose = RBE  $\times$  absorbed dose. Rem is the equivalent dose when the absorbed dose is in rad. For x rays, RBE = 1.0. For protons, RBE = 10.

**EXECUTE:** (a) 5.0 Gy, 500 rad. RBE = 1.0 so equivalent dose = absorbed dose. 5.0 Sv and 500 rem.

(b)  $(70.0 \text{ kg})(5.0 \text{ J/kg}) = 350 \text{ J.}$

(c) The absorbed dose and total absorbed energy are the same but the equivalent dose is 10 times larger. So the answers are: 5.0 Gy, 500 rad, 50 Sv, 5000 rem, 350 J.

**EVALUATE:** The same energy deposited by protons as x rays is ten times greater in its biological effect.

**43.33. IDENTIFY and SET UP:** The unit for absorbed dose is 1 rad = 0.01 J/kg = 0.01 Gy. Equivalent dose in rem is RBE times absorbed dose in rad.

**EXECUTE:** 1 rad =  $10^{-2}$  Gy, so 1 Gy = 100 rad and the dose was 500 rad.

$$\text{rem} = (\text{rad})(\text{RBE}) = (500 \text{ rad})(4.0) = 2000 \text{ rem. } 1 \text{ Gy} = 1 \text{ J/kg, so } 5.0 \text{ J/kg.}$$

**EVALUATE:** Gy, rad, and J/kg are all units of absorbed dose. Rem is a unit of equivalent dose, which depends on the RBE of the radiation.



- 43.34. IDENTIFY and SET UP:** The unit for absorbed dose is  $1 \text{ rad} = 0.01 \text{ J/kg} = 0.01 \text{ Gy}$ . Equivalent dose in rem is RBE times absorbed dose in rad.

**EXECUTE:** (a)  $\text{rem} = \text{rad} \times \text{RBE}$ .  $300 = x(10)$  and  $x = 30 \text{ rad}$ .

(b)  $1 \text{ rad}$  deposits  $0.010 \text{ J/kg}$ , so  $30 \text{ rad}$  deposit  $0.30 \text{ J/kg}$ . This radiation affects  $25 \text{ g}$  ( $0.025 \text{ kg}$ ) of tissue, so the total energy is  $(0.025 \text{ kg})(0.30 \text{ J/kg}) = 7.5 \times 10^{-3} \text{ J} = 7.5 \text{ mJ}$ .

(c)  $\text{RBE} = 1$  for  $\beta$ -rays, so  $\text{rem} = \text{rad}$ . Therefore  $30 \text{ rad} = 30 \text{ rem}$ .

**EVALUATE:** The same absorbed dose produces a larger equivalent dose when the radiation is neutrons than when it is electrons.

- 43.35. IDENTIFY and SET UP:** For x rays  $\text{RBE} = 1$  and the equivalent dose equals the absorbed dose.

**EXECUTE:** (a)  $175 \text{ krad} = 175 \text{ krem} = 1.75 \text{ kGy} = 1.75 \text{ kSv}$ .  $(1.75 \times 10^3 \text{ J/kg})(0.220 \text{ kg}) = 385 \text{ J}$ .

(b)  $175 \text{ krad} = 1.75 \text{ kGy}$ ;  $(1.50)(175 \text{ krad}) = 262.5 \text{ krem} = 2.625 \text{ kSv}$ . The energy deposited would be  $385 \text{ J}$ , the same as in (a).

**EVALUATE:** The energy required to raise the temperature of  $0.220 \text{ kg}$  of water  $1^\circ \text{C}$  is  $922 \text{ J}$ , and  $385 \text{ J}$  is less than this. The energy deposited corresponds to a very small amount of heating.

- 43.36. IDENTIFY and SET UP:** For x rays  $\text{RBE} = 1$  so the equivalent dose in Sv is the same as the absorbed dose in J/kg.

**EXECUTE:** One whole-body scan delivers  $(75 \text{ kg})(12 \times 10^{-3} \text{ J/kg}) = 0.90 \text{ J}$ . One chest x ray delivers

$(5.0 \text{ kg})(0.20 \times 10^{-3} \text{ J/kg}) = 1.0 \times 10^{-3} \text{ J}$ . It takes  $\frac{0.90 \text{ J}}{1.0 \times 10^{-3} \text{ J}} = 900$  chest x rays to deliver the same total

energy.

**EVALUATE:** For the CT scan the equivalent dose is much larger, and it is applied to the whole body.

- 43.37. IDENTIFY:** Apply  $|dN/dt| = \lambda N$ , with  $\lambda = \ln 2/T_{1/2}$ , to find the number of tritium atoms that were

ingested. Then use  $N = N_0 e^{-\lambda t}$  to find the number of decays in one week.

**SET UP:**  $1 \text{ rad} = 0.01 \text{ J/kg}$ .  $\text{rem} = \text{RBE} \times \text{rad}$ .

**EXECUTE:** (a) We need to know how many decays per second occur.

$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(12.3 \text{ y})(3.156 \times 10^7 \text{ s/y})} = 1.785 \times 10^{-9} \text{ s}^{-1}$ . The number of tritium atoms is

$N_0 = \frac{1}{\lambda} \left| \frac{dN}{dt} \right| = \frac{(0.35 \text{ Ci})(3.70 \times 10^{10} \text{ Bq/Ci})}{1.79 \times 10^{-9} \text{ s}^{-1}} = 7.2540 \times 10^{18}$  nuclei. The number of remaining nuclei after

one week is  $N = N_0 e^{-\lambda t} = (7.25 \times 10^{18}) e^{-(1.79 \times 10^{-9} \text{ s}^{-1})(7)(24)(3600 \text{ s})} = 7.2462 \times 10^{18}$  nuclei.

$\Delta N = N_0 - N = 7.8 \times 10^{15}$  decays. So the energy absorbed is

$E_{\text{total}} = \Delta N E_\gamma = (7.8 \times 10^{15})(5000 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = 6.25 \text{ J}$ . The absorbed dose is

$\frac{6.25 \text{ J}}{67 \text{ kg}} = 0.0932 \text{ J/kg} = 9.32 \text{ rad}$ . Since  $\text{RBE} = 1$ , then the equivalent dose is  $9.32 \text{ rem}$ .

**EVALUATE:** (b) In the decay, antineutrinos are also emitted. These are not absorbed by the body, and so some of the energy of the decay is lost.

- 43.38. IDENTIFY:**  $1 \text{ rem} = 0.01 \text{ Sv}$ . Equivalent dose in rem equals RBE times the absorbed dose in rad.

$1 \text{ rad} = 0.01 \text{ J/kg}$ . To change the temperature of water,  $Q = mc\Delta T$ .

**SET UP:** For water,  $c = 4190 \text{ J/kg} \cdot \text{K}$ .

**EXECUTE:** (a)  $5.4 \text{ Sv}(100 \text{ rem/Sv}) = 540 \text{ rem}$ .

(b) The RBE of 1 gives an absorbed dose of  $540 \text{ rad}$ .

(c) The absorbed dose is  $5.4 \text{ Gy}$ , so the total energy absorbed is  $(5.4 \text{ Gy})(65 \text{ kg}) = 351 \text{ J}$ . The energy required to raise the temperature of  $65 \text{ kg}$  by  $0.010^\circ \text{C}$  is  $(65 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(0.01^\circ \text{C}) = 3 \text{ kJ}$ .

**EVALUATE:** The amount of energy received corresponds to a very small heating of his body.

**43.39. IDENTIFY:** Each photon delivers energy. The energy of a single photon depends on its wavelength.

**SET UP:** equivalent dose (rem) = RBE  $\times$  absorbed dose (rad). 1 rad = 0.010 J/kg. For x rays, RBE = 1.

Each photon has energy  $E = \frac{hc}{\lambda}$ .

**EXECUTE:** (a)  $E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{0.0200 \times 10^{-9} \text{ m}} = 9.94 \times 10^{-15} \text{ J}$ . The absorbed energy is

$(5.00 \times 10^{10} \text{ photons})(9.94 \times 10^{-15} \text{ J/photon}) = 4.97 \times 10^{-4} \text{ J} = 0.497 \text{ mJ}$ .

(b) The absorbed dose is  $\frac{4.97 \times 10^{-4} \text{ J}}{0.600 \text{ kg}} = 8.28 \times 10^{-4} \text{ J/kg} = 0.0828 \text{ rad}$ . Since RBE = 1, the equivalent dose is 0.0828 rem.

**EVALUATE:** The amount of energy absorbed is rather small (only  $\frac{1}{2}$  mJ), but it is absorbed by only 600 g of tissue.

**43.40. IDENTIFY:** The reaction energy  $Q$  is  $Q = (M_A + M_B - M_C - M_D)c^2$ .

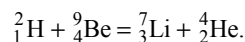
**SET UP:**  $Q = (m_{\text{H-1}} + m_{\text{H-3}} - m_{\text{H-2}} - m_{\text{H-2}})c^2$ . Use the masses in Table 43.2 and  $m_n = 1.008665 \text{ u}$  for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

**EXECUTE:**  $Q = [1.007825 \text{ u} + 3.016049 \text{ u} - 2(2.014102 \text{ u})](931.5 \text{ MeV/u}) = -4.033 \text{ MeV}$ .

**EVALUATE:** Since  $Q$  is negative, the initial energy is less than the final energy, so energy had to be put into the system to cause the reaction to occur. Therefore this is an endoergic reaction.

**43.41. (a) IDENTIFY and SET UP:** Determine X by balancing the charge and the nucleon number on the two sides of the reaction equation.

**EXECUTE:** X must have  $A = +2 + 9 - 4 = 7$  and  $Z = +1 + 4 - 2 = 3$ . Thus X is  ${}^7_3\text{Li}$  and the reaction is



(b) **IDENTIFY and SET UP:** Calculate the mass decrease and find its energy equivalent.

**EXECUTE:** If we use the neutral atom masses then there are the same number of electrons (five) in the reactants as in the products. Their masses cancel, so we get the same mass defect whether we use nuclear masses or neutral atom masses. The neutral atoms masses are given in Table 43.2. 1 u is equivalent to 931.5 MeV.

${}^2_1\text{H} + {}^9_4\text{Be}$  has mass  $2.014102 \text{ u} + 9.012182 \text{ u} = 11.26284 \text{ u}$ .

${}^7_3\text{Li} + {}^4_2\text{He}$  has mass  $7.016005 \text{ u} + 4.002603 \text{ u} = 11.018608 \text{ u}$ .

The mass decrease is  $11.26284 \text{ u} - 11.018608 \text{ u} = 0.007676 \text{ u}$ .

This corresponds to an energy release of  $(0.007676 \text{ u})(931.5 \text{ MeV/u}) = 7.150 \text{ MeV}$ .

(c) **IDENTIFY and SET UP:** Estimate the threshold energy by calculating the Coulomb potential energy when the  ${}^2_1\text{H}$  and  ${}^9_4\text{Be}$  nuclei just touch. Obtain the nuclear radii from  $R = R_0 A^{1/3}$ .

**EXECUTE:** The radius  $R_{\text{Be}}$  of the  ${}^9_4\text{Be}$  nucleus is  $R_{\text{Be}} = (1.2 \times 10^{-15} \text{ m})(9)^{1/3} = 2.5 \times 10^{-15} \text{ m}$ .

The radius  $R_{\text{H}}$  of the  ${}^2_1\text{H}$  nucleus is  $R_{\text{H}} = (1.2 \times 10^{-15} \text{ m})(2)^{1/3} = 1.5 \times 10^{-15} \text{ m}$ .

The nuclei touch when their center-to-center separation is

$$R = R_{\text{Be}} + R_{\text{H}} = 4.0 \times 10^{-15} \text{ m}.$$

The Coulomb potential energy of the two reactant nuclei at this separation is

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e(4e)}{r}.$$

$$U = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{4(1.602 \times 10^{-19} \text{ C})^2}{(4.0 \times 10^{-15} \text{ m})(1.602 \times 10^{-19} \text{ J/eV})} = 1.4 \text{ MeV}.$$

This is an estimate of the threshold energy for this reaction.

**EVALUATE:** The reaction releases energy but the total initial kinetic energy of the reactants must be 1.4 MeV in order for the reacting nuclei to get close enough to each other for the reaction to occur. The nuclear force is strong but is very short-range.

- 43.42. IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:**  $m_3^2\text{He} + m_2^1\text{H} - m_4^2\text{He} - m_1^1\text{H} = 1.97 \times 10^{-2} \text{ u}$ , so the energy released is 18.4 MeV.

**EVALUATE:** Using neutral atom masses includes three electron masses on each side of the reaction equation and the same result is obtained as if nuclear masses had been used.

- 43.43. IDENTIFY and SET UP:** The energy released is the energy equivalent of the mass decrease. 1 u is equivalent to 931.5 MeV. The mass of one  $^{235}\text{U}$  nucleus is  $235m_p$ .

**EXECUTE: (a)**  $^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow ^{144}_{56}\text{Ba} + ^{89}_{36}\text{Kr} + 3^1_0\text{n}$ . We can use atomic masses since the same number of electrons are included on each side of the reaction equation and the electron masses cancel. The mass decrease is  $\Delta M = m(^{235}_{92}\text{U}) + m({}^1_0\text{n}) - [m(^{144}_{56}\text{Ba}) + m(^{89}_{36}\text{Kr}) + 3m({}^1_0\text{n})]$ .

$\Delta M = 235.043930 \text{ u} + 1.0086649 \text{ u} - 143.922953 \text{ u} - 88.917631 \text{ u} - 3(1.0086649 \text{ u})$ ,  $\Delta M = 0.1860 \text{ u}$ . The energy released is  $(0.1860 \text{ u})(931.5 \text{ MeV/u}) = 173.3 \text{ MeV}$ .

**(b)** The number of  $^{235}\text{U}$  nuclei in 1.00 g is  $\frac{1.00 \times 10^{-3} \text{ kg}}{235m_p} = 2.55 \times 10^{21}$ . The energy released per gram is

$(173.3 \text{ MeV/nucleus})(2.55 \times 10^{21} \text{ nuclei/g}) = 4.42 \times 10^{23} \text{ MeV/g}$ .

**EVALUATE:** The energy released is  $7.1 \times 10^{10} \text{ J/kg}$ . This is much larger than typical heats of combustion, which are about  $5 \times 10^4 \text{ J/kg}$ .

- 43.44. IDENTIFY and SET UP:** 0.7% of naturally occurring uranium is the isotope  $^{235}\text{U}$ . The mass of one  $^{235}\text{U}$  nucleus is about  $235m_p$ .

**EXECUTE: (a)** The number of fissions needed is  $\frac{1.4 \times 10^{19} \text{ J}}{(200 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 4.38 \times 10^{29}$ . The

mass of  $^{235}\text{U}$  required is  $(4.38 \times 10^{29})(235m_p) = 1.7 \times 10^5 \text{ kg}$ .

**(b)**  $\frac{1.7 \times 10^5 \text{ kg}}{0.7 \times 10^{-2}} = 2.4 \times 10^7 \text{ kg}$ .

**EVALUATE:** The calculation assumes 100% conversion of fission energy to electrical energy.

- 43.45. IDENTIFY:** Charge and the number of nucleons are conserved in the reaction. The energy absorbed or released is determined by the mass change in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE: (a)**  $Z = 3 + 2 - 0 = 5$  and  $A = 4 + 7 - 1 = 10$ .

**(b)** The nuclide is a boron nucleus, and  $m_{\text{He}} + m_{\text{Li}} - m_{\text{n}} - m_{\text{B}} = -2.994 \times 10^{-3} \text{ u}$ , and so 2.79 MeV of energy is absorbed.

**EVALUATE:** The absorbed energy must come from the initial kinetic energy of the reactants.

- 43.46. IDENTIFY:** The charge and the nucleon number are conserved. The energy of the photon must be at least as large as the energy equivalent of the mass increase in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE: (a)**  $^{28}_{14}\text{Si} + \gamma \Rightarrow ^{24}_{12}\text{Mg} + ^4_Z\text{X}$ .  $A + 24 = 28$  so  $A = 4$ .  $Z + 12 = 14$  so  $Z = 2$ . X is an  $\alpha$  particle.

**(b)**  $-\Delta m = m(^{24}_{12}\text{Mg}) + m(^4_2\text{He}) - m(^{28}_{14}\text{Si}) = 23.985042 \text{ u} + 4.002603 \text{ u} - 27.976927 \text{ u} = 0.010718 \text{ u}$ .

$E_\gamma = (-\Delta m)c^2 = (0.010718 \text{ u})(931.5 \text{ MeV/u}) = 9.984 \text{ MeV}$ .

**EVALUATE:** The wavelength of the photon is

$$\lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{9.984 \times 10^6 \text{ eV}} = 1.24 \times 10^{-13} \text{ m} = 1.24 \times 10^{-4} \text{ nm. This is a gamma ray photon.}$$

- 43.47. IDENTIFY and SET UP:**  $m = \rho V$ . 1 gal = 3.788 L =  $3.788 \times 10^{-3} \text{ m}^3$ . The mass of a  $^{235}\text{U}$  nucleus is  $235m_p$ . 1 MeV =  $1.60 \times 10^{-13} \text{ J}$ .

**EXECUTE: (a)** For 1 gallon,  $m = \rho V = (737 \text{ kg/m}^3)(3.788 \times 10^{-3} \text{ m}^3) = 2.79 \text{ kg} = 2.79 \times 10^3 \text{ g}$ .

$$\frac{1.3 \times 10^8 \text{ J/gal}}{2.79 \times 10^3 \text{ g/gal}} = 4.7 \times 10^4 \text{ J/g.}$$

**(b)** 1 g contains  $\frac{1.00 \times 10^{-3} \text{ kg}}{235m_p} = 2.55 \times 10^{21}$  nuclei.

$$(200 \text{ MeV/nucleus})(1.60 \times 10^{-13} \text{ J/MeV})(2.55 \times 10^{21} \text{ nuclei}) = 8.2 \times 10^{10} \text{ J/g.}$$

**(c)** A mass of  $6m_p$  produces 26.7 MeV.

$$\frac{(26.7 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{6m_p} = 4.26 \times 10^{14} \text{ J/kg} = 4.26 \times 10^{11} \text{ J/g.}$$

**(d)** The total energy available would be  $(1.99 \times 10^{30} \text{ kg})(4.7 \times 10^7 \text{ J/kg}) = 9.4 \times 10^{37} \text{ J}$ .

$$\text{Power} = \frac{\text{energy}}{t} \text{ so } t = \frac{\text{energy}}{\text{power}} = \frac{9.4 \times 10^{37} \text{ J}}{3.86 \times 10^{26} \text{ W}} = 2.4 \times 10^{11} \text{ s} = 7600 \text{ y.}$$

**EVALUATE:** If the mass of the sun were all proton fuel, it would contain enough fuel to last

$$(7600 \text{ y}) \left( \frac{4.3 \times 10^{11} \text{ J/g}}{4.7 \times 10^4 \text{ J/g}} \right) = 7.0 \times 10^{10} \text{ y.}$$

- 43.48. IDENTIFY:** The minimum energy to remove a proton from the nucleus is equal to the energy difference between the two states of the nucleus (before and after proton removal).

**(a) SET UP:**  $^{12}_6\text{C} = ^1_1\text{H} + ^{11}_5\text{B}$ .  $\Delta m = m(^1_1\text{H}) + m(^{11}_5\text{B}) - m(^{12}_6\text{C})$ . The electron masses cancel when neutral atom masses are used.

**EXECUTE:**  $\Delta m = 1.007825 \text{ u} + 11.009305 \text{ u} - 12.000000 \text{ u} = 0.01713 \text{ u}$ . The energy equivalent of this mass increase is  $(0.01713 \text{ u})(931.5 \text{ MeV/u}) = 16.0 \text{ MeV}$ .

**(b) SET UP and EXECUTE:** We follow the same procedure as in part (a).

$$\Delta M = 6M_{\text{H}} + 6M_{\text{n}} - ^{12}_6\text{M} = 6(1.007825 \text{ u}) + 6(1.008665 \text{ u}) - 12.000000 \text{ u} = 0.09894 \text{ u.}$$

$$E_{\text{B}} = (0.09894 \text{ u})(931.5 \text{ MeV/u}) = 92.16 \text{ MeV. } \frac{E_{\text{B}}}{A} = 7.68 \text{ MeV/u.}$$

**EVALUATE:** The proton removal energy is about twice the binding energy per nucleon.

- 43.49. IDENTIFY:** The minimum energy to remove a proton or a neutron from the nucleus is equal to the energy difference between the two states of the nucleus, before and after removal.

**(a) SET UP:**  $^{17}_8\text{O} = ^1_0\text{n} + ^{16}_8\text{O}$ .  $\Delta m = m(^1_0\text{n}) + m(^{16}_8\text{O}) - m(^{17}_8\text{O})$ . The electron masses cancel when neutral atom masses are used.

**EXECUTE:**  $\Delta m = 1.008665 \text{ u} + 15.994915 \text{ u} - 16.999132 \text{ u} = 0.004448 \text{ u}$ . The energy equivalent of this mass increase is  $(0.004448 \text{ u})(931.5 \text{ MeV/u}) = 4.14 \text{ MeV}$ .

**(b) SET UP and EXECUTE:** Following the same procedure as in part (a) gives

$$\Delta M = 8M_{\text{H}} + 9M_{\text{n}} - ^{17}_8\text{M} = 8(1.007825 \text{ u}) + 9(1.008665 \text{ u}) - 16.999132 \text{ u} = 0.1415 \text{ u.}$$

$$E_{\text{B}} = (0.1415 \text{ u})(931.5 \text{ MeV/u}) = 131.8 \text{ MeV. } \frac{E_{\text{B}}}{A} = 7.75 \text{ MeV/nucleon.}$$

**EVALUATE:** The neutron removal energy is about half the binding energy per nucleon.

**43.50. IDENTIFY:** At steady-state, the rate of production of  $^{110}_{47}\text{Ag}$  is equal to the rate at which it decays. The decay follows an exponential law but the rate of production is a constant rate.  $\lambda = (\ln 2)/T_{1/2}$ .

**SET UP:** The rate of decay is  $R_{\text{decay}} = -dN/dt = \lambda N$ . The rate of production is a constant 8400 Bq. Equating the two rates gives

$$R_{\text{decay}} = R_{\text{prod}} \rightarrow \lambda N = R_{\text{prod}} \rightarrow N = R_{\text{prod}}/\lambda = R_{\text{prod}}T_{1/2}/(\ln 2).$$

**EXECUTE: (a)** Using the result we just found for  $N$  gives

$$R_{\text{prod}}T_{1/2}/(\ln 2) = (8400 \text{ s}^{-1})(24.6 \text{ s})/(\ln 2) = 2.98 \times 10^5 \text{ nuclei.}$$

**(b)** The change in  $N$  during time  $dt$  is

$dN = (R_{\text{prod}} - R_{\text{decay}})dt$ , where  $R_{\text{prod}} = 8400 \text{ Bq} = \text{constant}$  and  $R_{\text{decay}} = dN/dt = \lambda N$ . Separating variables and

integrating gives  $\int_0^{N_f} \frac{dN}{R - \lambda N} = \int_0^T dt$ . Carrying out the integral and combining terms gives

$$-\frac{1}{\lambda} [\ln(R - \lambda N_f) - \ln R] = T \rightarrow -\frac{1}{\lambda} \ln\left(1 - \frac{\lambda N_f}{R}\right) = T.$$

Solving for  $N_f$  and using  $\lambda = (\ln 2)/T_{1/2}$  gives

$$N_f = \frac{R}{\lambda}(1 - e^{-\lambda T}) = \frac{RT_{1/2}}{\ln 2}(1 - e^{-\lambda T}) = \frac{(8400 \text{ Bq})(24.6 \text{ s})}{\ln 2}(1 - e^{-(\ln 2)(24.6 \text{ s})/(24.6 \text{ s})}) = 1.49 \times 10^5 \text{ nuclei.}$$

**EVALUATE:** In one half-life, the number of  $^{110}_{47}\text{Ag}$  nuclei has reached  $\frac{1}{2}$  of its steady-state value.

**43.51. IDENTIFY:** Use the decay scheme and half-life of  $^{90}\text{Sr}$  to find out the product of its decay and the amount left after a given time.

**SET UP:** The particle emitted in  $\beta^-$  decay is an electron,  ${}^0_{-1}\text{e}$ . In a time of one half-life, the number of radioactive nuclei decreases by a factor of 2.  $6.25\% = \frac{1}{16} = 2^{-4}$ .

**EXECUTE: (a)**  $^{90}_{38}\text{Sr} \rightarrow {}^0_{-1}\text{e} + ^{90}_{39}\text{Y}$ . The daughter nucleus is  $^{90}_{39}\text{Y}$ .

**(b)** 56 y is  $2T_{1/2}$  so  $N = N_0/2^2 = N_0/4$ ; 25% is left.

**(c)**  $\frac{N}{N_0} = 2^{-n}$ ;  $\frac{N}{N_0} = 6.25\% = \frac{1}{16} = 2^{-4}$  so  $t = 4T_{1/2} = 112 \text{ y}$ .

**EVALUATE:** After half a century,  $\frac{1}{4}$  of the  $^{90}\text{Sr}$  would still be left!

**43.52. IDENTIFY:** Calculate the mass defect for the decay. Example 43.5 uses conservation of linear momentum to determine how the released energy is divided between the decay partners.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** The  $\alpha$ -particle will have  $\frac{226}{230}$  of the released energy (see Example 43.5).

$$\frac{226}{230}(m_{\text{Th}} - m_{\text{Ra}} - m_{\text{He-4}}) = 5.032 \times 10^{-3} \text{ u or } 4.69 \text{ MeV.}$$

**EVALUATE:** Most of the released energy goes to the  $\alpha$  particle, since its mass is much less than that of the daughter nucleus.

**43.53. (a) IDENTIFY and SET UP:** The heavier nucleus will decay into the lighter one.

**EXECUTE:**  $^{25}_{13}\text{Al}$  will decay into  $^{25}_{12}\text{Mg}$ .

**(b) IDENTIFY and SET UP:** Determine the emitted particle by balancing  $A$  and  $Z$  in the decay reaction.

**EXECUTE:** This gives  $^{25}_{13}\text{Al} \rightarrow ^{25}_{12}\text{Mg} + {}^0_{+1}\text{e}$ . The emitted particle must have charge  $+e$  and its nucleon number must be zero. Therefore, it is a  $\beta^+$  particle, a positron.

**(c) IDENTIFY and SET UP:** Calculate the energy defect  $\Delta M$  for the reaction and find the energy equivalent of  $\Delta M$ . Use the nuclear masses for  $^{25}_{13}\text{Al}$  and  $^{25}_{12}\text{Mg}$ , to avoid confusion in including the correct number of electrons if neutral atom masses are used.

**EXECUTE:** The nuclear mass for  $^{25}_{13}\text{Al}$  is

$$M_{\text{nuc}}(^{25}_{13}\text{Al}) = 24.990428 \text{ u} - 13(0.000548580 \text{ u}) = 24.983296 \text{ u}.$$

The nuclear mass for  $^{25}_{12}\text{Mg}$  is  $M_{\text{nuc}}(^{25}_{12}\text{Mg}) = 24.985837 \text{ u} - 12(0.000548580 \text{ u}) = 24.979254 \text{ u}.$

The mass defect for the reaction is

$$\Delta M = M_{\text{nuc}}(^{25}_{13}\text{Al}) - M_{\text{nuc}}(^{25}_{12}\text{Mg}) - M(^0_+e) = 24.983296 \text{ u} - 24.979254 \text{ u} - 0.00054858 \text{ u} = 0.003493 \text{ u}.$$

$$Q = (\Delta M)c^2 = 0.003493 \text{ u}(931.5 \text{ MeV}/1 \text{ u}) = 3.254 \text{ MeV}.$$

**EVALUATE:** The mass decreases in the decay and energy is released. Note:  $^{25}_{13}\text{Al}$  can also decay into

$^{25}_{12}\text{Mg}$  by the electron capture:  $^{25}_{13}\text{Al} + ^0_{-1}e \rightarrow ^{25}_{12}\text{Mg}$ . The  $^0_{-1}e$  electron in the reaction is an orbital electron in the neutral  $^{25}_{13}\text{Al}$  atom. The mass defect can be calculated using the nuclear masses:

$$\Delta M = M_{\text{nuc}}(^{25}_{13}\text{Al}) + M(^0_{-1}e) - M_{\text{nuc}}(^{25}_{12}\text{Mg}) = 24.983296 \text{ u} + 0.00054858 \text{ u} - 24.979254 \text{ u} = 0.004591 \text{ u}.$$

$Q = (\Delta M)c^2 = (0.004591 \text{ u})(931.5 \text{ MeV}/1 \text{ u}) = 4.277 \text{ MeV}$ . The mass decreases in the decay and energy is released.

**43.54. IDENTIFY:** Calculate the mass change in the decay. If the mass decreases the decay is energetically allowed.

**SET UP:** Example 43.5 shows how the released energy is distributed among the decay products for  $\alpha$  decay.

**EXECUTE:** (a)  $m_{^{210}_{84}\text{Po}} - m_{^{206}_{82}\text{Pb}} - m_{^4_2\text{He}} = 5.806 \times 10^{-3} \text{ u}$ , or  $Q = 5.41 \text{ MeV}$ . The energy of the alpha particle is  $(206/210)$  times this, or  $5.30 \text{ MeV}$  (see Example 43.5).

(b)  $m_{^{210}_{84}\text{Po}} - m_{^{209}_{83}\text{Bi}} - m_{^1_1\text{H}} = -5.350 \times 10^{-3} \text{ u} < 0$ , so the decay is not possible.

(c)  $m_{^{210}_{84}\text{Po}} - m_{^{209}_{84}\text{Po}} - m_{\text{n}} = -8.221 \times 10^{-3} \text{ u} < 0$ , so the decay is not possible.

(d)  $m_{^{210}_{85}\text{At}} > m_{^{210}_{84}\text{Po}}$ , so the decay is not possible.

(e)  $m_{^{210}_{83}\text{Bi}} + 2m_{\text{e}} > m_{^{210}_{84}\text{Po}}$ , so the decay is not possible.

**EVALUATE:** Of the decay processes considered in the problem, only  $\alpha$  decay is energetically allowed for  $^{210}_{84}\text{Po}$ .

**43.55. IDENTIFY and SET UP:** The amount of kinetic energy released is the energy equivalent of the mass change

in the decay.  $m_{\text{e}} = 0.0005486 \text{ u}$  and the atomic mass of  $^{14}_7\text{N}$  is  $14.003074 \text{ u}$ . The energy equivalent of  $1 \text{ u}$  is  $931.5 \text{ MeV}$ .  $^{14}_6\text{C}$  has a half-life of  $T_{1/2} = 5730 \text{ y} = 1.81 \times 10^{11} \text{ s}$ . The RBE for an electron is  $1.0$ .

**EXECUTE:** (a)  $^{14}_6\text{C} \rightarrow \text{e}^- + ^{14}_7\text{N} + \bar{\nu}_{\text{e}}$ .

(b) The mass decrease is  $\Delta M = m(^{14}_6\text{C}) - [m_{\text{e}} + m(^{14}_7\text{N})]$ . Use nuclear masses, to avoid difficulty in accounting for atomic electrons. The nuclear mass of  $^{14}_6\text{C}$  is  $14.003242 \text{ u} - 6m_{\text{e}} = 13.999950 \text{ u}$ . The nuclear mass of  $^{14}_7\text{N}$  is  $14.003074 \text{ u} - 7m_{\text{e}} = 13.999234 \text{ u}$ .

$\Delta M = 13.999950 \text{ u} - 13.999234 \text{ u} - 0.000549 \text{ u} = 1.67 \times 10^{-4} \text{ u}$ . The energy equivalent of  $\Delta M$  is  $0.156 \text{ MeV}$ .

(c) The mass of carbon is  $(0.18)(75 \text{ kg}) = 13.5 \text{ kg}$ . From Example 43.9, the activity due to  $1 \text{ g}$  of carbon in a living organism is  $0.255 \text{ Bq}$ . The number of decay/s due to  $13.5 \text{ kg}$  of carbon is  $(13.5 \times 10^3 \text{ g})(0.255 \text{ Bq/g}) = 3.4 \times 10^3 \text{ decays/s}$ .

(d) Each decay releases  $0.156 \text{ MeV}$  so  $3.4 \times 10^3 \text{ decays/s}$  releases  $530 \text{ MeV/s} = 8.5 \times 10^{-11} \text{ J/s}$ .

(e) The total energy absorbed in 1 year is  $(8.5 \times 10^{-11} \text{ J/s})(3.156 \times 10^7 \text{ s}) = 2.7 \times 10^{-3} \text{ J}$ . The absorbed dose is  $\frac{2.7 \times 10^{-3} \text{ J}}{75 \text{ kg}} = 3.6 \times 10^{-5} \text{ J/kg} = 36 \mu\text{Gy} = 3.6 \text{ mrad}$ . With  $\text{RBE} = 1.0$ , the equivalent dose is  $36 \mu\text{Sv} = 3.6 \text{ mrem}$ .

**EVALUATE:** Section 43.5 says that background radiation exposure is about 1.0 mSv per year. The radiation dose calculated in this problem is much less than this.

- 43.56. IDENTIFY and SET UP:**  $m_\pi = 264m_e = 2.40 \times 10^{-28} \text{ kg}$ . The total energy of the two photons equals the rest mass energy  $m_\pi c^2$  of the pion.

**EXECUTE: (a)**  $E_{\text{ph}} = \frac{1}{2} m_\pi c^2 = \frac{1}{2} (2.40 \times 10^{-28} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 1.08 \times 10^{-11} \text{ J} = 67.5 \text{ MeV}$ .

$$E_{\text{ph}} = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{E_{\text{ph}}} = \frac{1.24 \times 10^{-6} \text{ eV} \cdot \text{m}}{67.5 \times 10^6 \text{ eV}} = 1.84 \times 10^{-14} \text{ m} = 18.4 \text{ fm}.$$

These are gamma ray photons, so they have  $\text{RBE} = 1.0$ .

**(b)** Each pion delivers  $2(1.08 \times 10^{-11} \text{ J}) = 2.16 \times 10^{-11} \text{ J}$ .

The absorbed dose is  $200 \text{ rad} = 2.00 \text{ Gy} = 2.00 \text{ J/kg}$ .

The energy deposited is  $(25 \times 10^{-3} \text{ kg})(2.00 \text{ J/kg}) = 0.050 \text{ J}$ .

The number of  $\pi^0$  mesons needed is  $\frac{0.050 \text{ J}}{2.16 \times 10^{-11} \text{ J/meson}} = 2.3 \times 10^9$  mesons.

**EVALUATE:** Note that charge is conserved in the decay since the pion is neutral. If the pion is initially at rest the photons must have equal momenta in opposite directions so the two photons have the same  $\lambda$  and are emitted in opposite directions. The photons also have equal energies since they have the same momentum and  $E = pc$ .

- 43.57. IDENTIFY and SET UP:** The mass defect is  $E_B/c^2$ .

**EXECUTE:**  $m_{11\text{C}} - m_{11\text{B}} - 2m_e = 1.03 \times 10^{-3} \text{ u}$ . Decay is energetically possible.

**EVALUATE:** The energy released in the decay is  $(1.03 \times 10^{-3} \text{ u})(931.5 \text{ MeV/u}) = 0.959 \text{ MeV}$ .

- 43.58. IDENTIFY:** Assume the activity is constant during the year and use the given value of the activity to find the number of decays that occur in one year. Absorbed dose is the energy absorbed per mass of tissue. Equivalent dose is RBE times absorbed dose.

**SET UP:** For  $\alpha$  particles,  $\text{RBE} = 20$  (from Table 43.3).

**EXECUTE:**  $(0.52 \times 10^{-6} \text{ Ci})(3.7 \times 10^{10} \text{ Bq/Ci})(3.156 \times 10^7 \text{ s}) = 6.07 \times 10^{11} \alpha$  particles. The absorbed dose is  $\frac{(6.07 \times 10^{11})(4.0 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(0.50 \text{ kg})} = 0.78 \text{ Gy} = 78 \text{ rad}$ . The equivalent dose is

$$(20)(78 \text{ rad}) = 1600 \text{ rem}.$$

**EVALUATE:** The equivalent dose is 16 Sv. This is large enough for significant damage to the person.

- 43.59. IDENTIFY and SET UP:** One-half of the sample decays in a time of  $T_{1/2}$ .

**EXECUTE: (a)**  $\frac{10 \times 10^9 \text{ y}}{200,000 \text{ y}} = 5.0 \times 10^4$ .

**(b)**  $\left(\frac{1}{2}\right)^{5.0 \times 10^4}$ . This exponent is too large for most hand-held calculators. But  $\left(\frac{1}{2}\right) = 10^{-0.301}$ , so

$$\left(\frac{1}{2}\right)^{5.0 \times 10^4} = (10^{-0.301})^{5.0 \times 10^4} = 10^{-15,000}.$$

**EVALUATE:** For  $N = 1$  after 16 billion years,  $N_0 = 10^{15,000}$ . The mass of this many  $^{99}\text{Tc}$  nuclei would be  $(99)(1.66 \times 10^{-27} \text{ kg})(10^{15,000}) = 10^{14,750} \text{ kg}$ , which is immense, far greater than the mass of any star.

- 43.60. IDENTIFY:** One rad of absorbed dose is 0.01 J/kg. The equivalent dose in rem is the absorbed dose in rad times the RBE. For part (c) apply  $|dN/dt| = \lambda N$  with  $\lambda = \frac{\ln 2}{T_{1/2}}$ .

**SET UP:** For  $\alpha$  particles, RBE = 20 (Table 43.3).

**EXECUTE:** (a)  $(7.75 \times 10^{12})(4.77 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})/(70.0 \text{ kg}) = 0.0846 \text{ Gy} = 8.46 \text{ rad}$ .

(b)  $(20)(8.46 \text{ rad}) = 169 \text{ rem}$ .

(c)  $\left| \frac{dN}{dt} \right| = \frac{m}{Am_p} \frac{\ln(2)}{T_{1/2}} = 1.17 \times 10^9 \text{ Bq} = 31.6 \text{ mCi}$ .

(d)  $t = \frac{7.75 \times 10^{12}}{1.17 \times 10^9 \text{ Bq}} = 6.62 \times 10^3 \text{ s}$ , which is about 1.8 hours.

**EVALUATE:** The time in part (d) is so small in comparison with the half-life that the decrease in activity of the source may be neglected.

- 43.61. IDENTIFY:** Use  $N = N_0 e^{-\lambda t}$  to relate the initial number of radioactive nuclei,  $N_0$ , to the number,  $N$ , left after time  $t$ .

**SET UP:** We have to be careful; after  $^{87}\text{Rb}$  has undergone radioactive decay it is no longer a rubidium atom. Let  $N_{85}$  be the number of  $^{85}\text{Rb}$  atoms; this number doesn't change. Let  $N_0$  be the number of  $^{87}\text{Rb}$  atoms on earth when the solar system was formed. Let  $N$  be the present number of  $^{87}\text{Rb}$  atoms.

**EXECUTE:** The present measurements say that  $0.2783 = N/(N + N_{85})$ .

$(N + N_{85})(0.2783) = N$ , so  $N = 0.3856 N_{85}$ . The percentage we are asked to calculate is  $N_0/(N_0 + N_{85})$ .

$N$  and  $N_0$  are related by  $N = N_0 e^{-\lambda t}$  so  $N_0 = e^{+\lambda t} N$ .

$$\text{Thus } \frac{N_0}{N_0 + N_{85}} = \frac{N e^{\lambda t}}{N e^{\lambda t} + N_{85}} = \frac{(0.3856 e^{\lambda t}) N_{85}}{(0.3856 e^{\lambda t}) N_{85} + N_{85}} = \frac{0.3856 e^{\lambda t}}{0.3856 e^{\lambda t} + 1}.$$

$$t = 4.6 \times 10^9 \text{ y}; \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4.75 \times 10^{10} \text{ y}} = 1.459 \times 10^{-11} \text{ y}^{-1}.$$

$$e^{\lambda t} = e^{(1.459 \times 10^{-11} \text{ y}^{-1})(4.6 \times 10^9 \text{ y})} = e^{0.06711} = 1.0694.$$

$$\text{Thus } \frac{N_0}{N_0 + N_{85}} = \frac{(0.3856)(1.0694)}{(0.3856)(1.0694) + 1} = 29.2\%.$$

**EVALUATE:** The half-life for  $^{87}\text{Rb}$  is a factor of 10 larger than the age of the solar system, so only a small fraction of the  $^{87}\text{Rb}$  nuclei initially present have decayed; the percentage of rubidium atoms that are radioactive is only a bit less now than it was when the solar system was formed.

- 43.62. IDENTIFY:** Apply  $N = N_0 e^{-\lambda t}$ , with  $\lambda = \frac{\ln 2}{T_{1/2}}$ .

**SET UP:** Let 1 refer to  $^{15}_8\text{O}$  and 2 refer to  $^{19}_8\text{O}$ .  $\frac{N_1}{N_2} = \frac{e^{-\lambda_1 t}}{e^{-\lambda_2 t}}$ , since  $N_0$  is the same for the two isotopes.

$$e^{-\lambda t} = e^{-(\ln 2/T_{1/2})t} = (e^{-\ln 2})^{t/T_{1/2}} = \left(\frac{1}{2}\right)^{t/T_{1/2}}. \quad \frac{N_1}{N_2} = \left(\frac{1}{2}\right)^{(t/(T_{1/2})_1)/(t/(T_{1/2})_2)} = 2^{t\left(\frac{1}{(T_{1/2})_2} - \frac{1}{(T_{1/2})_1}\right)}.$$

**EXECUTE:** (a) After 3.0 min = 180 s, the ratio of the number of nuclei is

$$N_1/N_2 = \frac{2^{-180/122.2}}{2^{-180/26.9}} = 2^{(180)\left(\frac{1}{26.9} - \frac{1}{122.2}\right)} = 2^{(180)(0.02899)} = 37.2.$$

(b) After 12.0 min = 720 s, the ratio is  $N_1/N_2 = 2^{(720)(0.02899)} = 1.92 \times 10^6$ .

**EVALUATE:** The  $^{19}_8\text{O}$  nuclei decay at a greater rate, so the ratio  $N(^{15}_8\text{O})/N(^{19}_8\text{O})$  increases with time.



- 43.63. IDENTIFY and SET UP:** Find the energy emitted and the energy absorbed each second. Convert the absorbed energy to absorbed dose and to equivalent dose.

**EXECUTE: (a)** First find the number of decays each second:

$$2.6 \times 10^{-4} \text{ Ci} \left( \frac{3.70 \times 10^{10} \text{ decays/s}}{1 \text{ Ci}} \right) = 9.6 \times 10^6 \text{ decays/s. The average energy per decay is } 1.25 \text{ MeV, and}$$

one-half of this energy is deposited in the tumor. The energy delivered to the tumor per second then is

$$\frac{1}{2} (9.6 \times 10^6 \text{ decays/s})(1.25 \times 10^6 \text{ eV/decay})(1.602 \times 10^{-19} \text{ J/eV}) = 9.6 \times 10^{-7} \text{ J/s.}$$

**(b)** The absorbed dose is the energy absorbed divided by the mass of the tissue:

$$\frac{9.6 \times 10^{-7} \text{ J/s}}{0.200 \text{ kg}} = (4.8 \times 10^{-6} \text{ J/kg} \cdot \text{s})(1 \text{ rad}/(0.01 \text{ J/kg})) = 4.8 \times 10^{-4} \text{ rad/s.}$$

**(c)** equivalent dose (REM) = RBE  $\times$  absorbed dose (rad). In one second the equivalent dose is

$$(0.70)(4.8 \times 10^{-4} \text{ rad}) = 3.4 \times 10^{-4} \text{ rem.}$$

**(d)**  $(200 \text{ rem})/(3.4 \times 10^{-4} \text{ rem/s}) = (5.9 \times 10^5 \text{ s})(1 \text{ h}/3600 \text{ s}) = 164 \text{ h} = 6.9 \text{ days.}$

**EVALUATE:** The activity of the source is small so that absorbed energy per second is small and it takes several days for an equivalent dose of 200 rem to be absorbed by the tumor. A 200-rem dose equals 2.00 Sv and this is large enough to damage the tissue of the tumor.

- 43.64. IDENTIFY:** The tritium (H-3) decays to He-3. The ratio of the number of He-3 atoms to H-3 atoms allows us to calculate the time since the decay began, which is when the H-3 was formed by the nuclear explosion. The H-3 decay is exponential.

**SET UP:** The number of tritium (H-3) nuclei decreases exponentially as  $N_{\text{H}} = N_{0,\text{H}} e^{-\lambda t}$ , with a half-life of 12.3 years. The amount of He-3 present after a time  $t$  is equal to the original amount of tritium minus the number of tritium nuclei that are still undecayed after time  $t$ .

**EXECUTE:** The number of He-3 nuclei after time  $t$  is

$$N_{\text{He}} = N_{0,\text{H}} - N_{\text{H}} = N_{0,\text{H}} - N_{0,\text{H}} e^{-\lambda t} = N_{0,\text{H}} (1 - e^{-\lambda t}).$$

Taking the ratio of the number of He-3 atoms to the number of tritium (H-3) atoms gives

$$\frac{N_{\text{He}}}{N_{\text{H}}} = \frac{N_{0,\text{H}} (1 - e^{-\lambda t})}{N_{0,\text{H}} e^{-\lambda t}} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1.$$

Solving for  $t$  gives  $t = \frac{\ln(1 + N_{\text{He}}/N_{\text{H}})}{\lambda}$ . Using the given numbers and  $T_{1/2} = \frac{\ln 2}{\lambda}$ , we have

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12.3 \text{ y}} = 0.0563/\text{y} \text{ and } t = \frac{\ln(1 + 4.3)}{0.0563/\text{y}} = 30 \text{ years.}$$

**EVALUATE:** One limitation on this method would be that after many years the ratio of H to He would be too small to measure accurately.

- 43.65. IDENTIFY and SET UP:** The number of radioactive nuclei left after time  $t$  is given by  $N = N_0 e^{-\lambda t}$ . The problem says  $N/N_0 = 0.29$ ; solve for  $t$ .

**EXECUTE:**  $0.29 = e^{-\lambda t}$  so  $\ln(0.29) = -\lambda t$  and  $t = -\ln(0.29)/\lambda$ . Example 43.9 gives

$$\lambda = 1.209 \times 10^{-4} \text{ y}^{-1} \text{ for } ^{14}\text{C. Thus } t = \frac{-\ln(0.29)}{1.209 \times 10^{-4} \text{ y}} = 1.0 \times 10^4 \text{ y.}$$

**EVALUATE:** The half-life of  $^{14}\text{C}$  is 5730 y, so our calculated  $t$  is about 1.75 half-lives, so the fraction remaining is around  $\left(\frac{1}{2}\right)^{1.75} = 0.30$ .

- 43.66. IDENTIFY:** In terms of the number  $\Delta N$  of cesium atoms that decay in one week and the mass  $m = 1.0 \text{ kg}$ , the equivalent dose is  $3.5 \text{ Sv} = \frac{\Delta N}{m} ((\text{RBE})_{\gamma} E_{\gamma} + (\text{RBE})_{\text{e}} E_{\text{e}})$ .

**SET UP:** 1 day =  $8.64 \times 10^4$  s. 1 year =  $3.156 \times 10^7$  s.

**EXECUTE:**  $3.5 \text{ Sv} = \frac{\Delta N}{m} ((1)(0.66 \text{ MeV}) + (1.5)(0.51 \text{ MeV})) = \frac{\Delta N}{m} (2.283 \times 10^{-13} \text{ J})$ , so

$$\Delta N = \frac{(1.0 \text{ kg})(3.5 \text{ Sv})}{(2.283 \times 10^{-13} \text{ J})} = 1.535 \times 10^{13}. \quad \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{(30.07 \text{ y})(3.156 \times 10^7 \text{ sec/y})} = 7.30 \times 10^{-10} \text{ sec}^{-1}.$$

$$\Delta N = |dN/dt|t = \lambda Nt, \text{ so } N = \frac{\Delta N}{\lambda t} = \frac{1.535 \times 10^{13}}{(7.30 \times 10^{-10} \text{ s}^{-1})(7 \text{ days})(8.64 \times 10^4 \text{ s/day})} = 3.48 \times 10^{16}.$$

**EVALUATE:** We have assumed that  $|dN/dt|$  is constant during a time of one week. That is a very good approximation, since the half-life is much greater than one week.

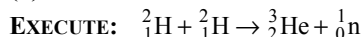
**43.67. (a) IDENTIFY and SET UP:** Use  $R = R_0 A^{1/3}$  to calculate the radius  $R$  of a  ${}^2_1\text{H}$  nucleus. Calculate the

Coulomb potential energy  $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$  of the two nuclei when they just touch.

**EXECUTE:** The radius of  ${}^2_1\text{H}$  is  $R = (1.2 \times 10^{-15} \text{ m})(2)^{1/3} = 1.51 \times 10^{-15} \text{ m}$ . The barrier energy is the Coulomb potential energy of two  ${}^2_1\text{H}$  nuclei with their centers separated by twice this distance:

$$U = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.602 \times 10^{-19} \text{ C})^2}{2(1.51 \times 10^{-15} \text{ m})} = 7.64 \times 10^{-14} \text{ J} = 0.48 \text{ MeV}.$$

**(b) IDENTIFY and SET UP:** Find the energy equivalent of the mass decrease.



If we use neutral atom masses there are two electrons on each side of the reaction equation, so their masses cancel. The neutral atom masses are given in Table 43.2.

$${}^2_1\text{H} + {}^2_1\text{H} \text{ has mass } 2(2.014102 \text{ u}) = 4.028204 \text{ u}$$

$${}^3_2\text{He} + {}^1_0\text{n} \text{ has mass } 3.016029 \text{ u} + 1.008665 \text{ u} = 4.024694 \text{ u}$$

The mass decrease is  $4.028204 \text{ u} - 4.024694 \text{ u} = 3.510 \times 10^{-3} \text{ u}$ . This corresponds to a liberated energy of  $(3.510 \times 10^{-3} \text{ u})(931.5 \text{ MeV/u}) = 3.270 \text{ MeV}$ , or  $(3.270 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV}) = 5.239 \times 10^{-13} \text{ J}$ .

**(c) IDENTIFY and SET UP:** We know the energy released when two  ${}^2_1\text{H}$  nuclei fuse. Find the number of reactions obtained with one mole of  ${}^2_1\text{H}$ .

**EXECUTE:** Each reaction takes two  ${}^2_1\text{H}$  nuclei. Each mole of  $\text{D}_2$  has  $6.022 \times 10^{23}$  molecules, so  $6.022 \times 10^{23}$  pairs of atoms. The energy liberated when one mole of deuterium undergoes fusion is  $(6.022 \times 10^{23})(5.239 \times 10^{-13} \text{ J}) = 3.155 \times 10^{11} \text{ J/mol}$ .

**EVALUATE:** The energy liberated per mole is more than a million times larger than from chemical combustion of one mole of hydrogen gas.

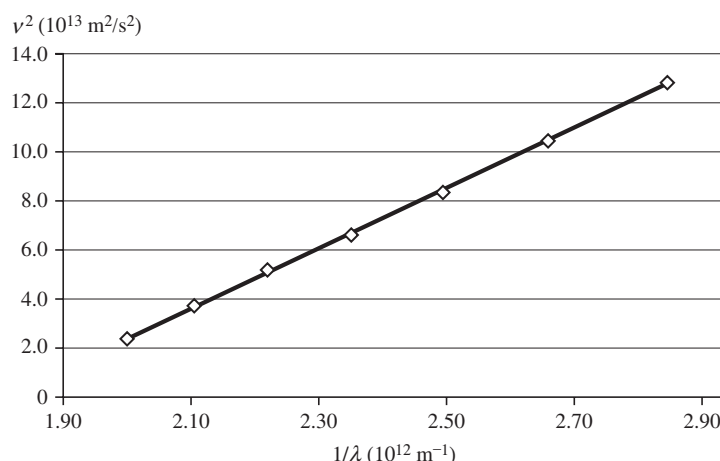
**43.68. IDENTIFY:** The energy of the photon is equal to the sum of the kinetic energies of the proton and the neutron plus the binding energy of the deuteron.

**SET UP:**  $\frac{hc}{\lambda} = K_p + K_n + E_B$ . Since the proton and neutron have equal kinetic energy, this equation

$$\text{becomes } hc/\lambda = 2K_p + E_B = 2\left(\frac{1}{2} m_p v^2\right) + E_B. \text{ Solving for } v^2 \text{ gives } v^2 = \frac{hc}{m_p} \cdot \frac{1}{\lambda} - \frac{E_B}{m_p}.$$

**EXECUTE: (a)** Figure 43.68 shows the graph of  $v^2$  versus  $1/\lambda$  for the data given in the problem. As shown above, the equation for  $v^2$  versus  $1/\lambda$  is  $v^2 = \frac{hc}{m_p} \cdot \frac{1}{\lambda} - \frac{E_B}{m_p}$ . The graph of  $v^2$  versus  $1/\lambda$  should be a

straight line with slope equal to  $hc/m_p$  and y-intercept equal to  $-E_B/m_p$ . The slope of the best-fit straight line for our graph is  $119.9 \text{ m}^3/\text{s}^2$  and the y-intercept is  $-2.156 \times 10^{14} \text{ m}^2/\text{s}^2$ .

**Figure 43.68**

(b) Using the values for the slope and y-intercept for our graph, we have

$$hc/m_p = \text{slope} \rightarrow m_p = hc/(\text{slope}).$$

$$m_p = (6.626 \times 10^{-34} \text{ J}\cdot\text{s})(2.998 \times 10^8 \text{ m/s}) / (119.9 \text{ m}^3/\text{s}^2) = 1.66 \times 10^{-27} \text{ kg}.$$

The y-intercept gives us the binding energy:  $-E_B/m_p = y\text{-intercept}$ , so  $E_B = -m_p(y\text{-intercept})$ .

$$E_B = -(1.66 \times 10^{-27} \text{ kg})(-2.156 \times 10^{14} \text{ m}^2/\text{s}^2) = 3.58 \times 10^{-13} \text{ J} = 2.23 \times 10^6 \text{ eV} = 2.23 \text{ MeV}.$$

**EVALUATE:** The binding energy of a deuteron is  $E_B = (m_{\text{H-1}} + m_n - m_{\text{D}})c^2$ , so

$$E_B = (1.007825 \text{ u} + 1.008665 \text{ u} - 2.014102 \text{ u})(931.5 \text{ MeV/u}) = 2.22 \text{ MeV}, \text{ so we are very close. Our results for } m_p \text{ are very close to the accepted value for the proton mass of } 1.67 \times 10^{-27} \text{ kg}.$$

**43.69. IDENTIFY:** Apply  $\left| \frac{dN}{dt} \right| = \lambda N_0 e^{-\lambda t}$ , with  $\lambda = \frac{\ln 2}{T_{1/2}}$ .

**SET UP:**  $\ln |dN/dt| = \ln \lambda N_0 - \lambda t$ .

**EXECUTE: (a)** A least-squares fit to log of the activity vs. time gives a slope of magnitude

$$\lambda = 0.5995 \text{ h}^{-1}, \text{ for a half-life of } \frac{\ln 2}{\lambda} = 1.16 \text{ h}.$$

(b) The initial activity is  $N_0 \lambda$ , and this gives  $N_0 = \frac{(2.00 \times 10^4 \text{ Bq})}{(0.5995 \text{ hr}^{-1})(1 \text{ hr}/3600 \text{ s})} = 1.20 \times 10^8$ .

(c)  $N = N_0 e^{-\lambda t} = 1.81 \times 10^6$ .

**EVALUATE:** The activity decreases by about  $\frac{1}{2}$  in the first hour, so the half-life is about 1 hour.

**43.70. IDENTIFY:** We cannot tell much from the raw data, but we know for radioactive decay of a single nuclide the activity rate  $A$  decreases as  $A = A_0 e^{-\lambda t} = |dN/dt|$ .

**SET UP:**  $\left| \frac{dN}{dt} \right| = \lambda N = \lambda N_0 e^{-\lambda t}$  for a single nuclide. Since  $A = A_0 e^{-\lambda t}$ , it follows that  $\ln A = \ln A_0 - \lambda t$ .

Therefore if a single nuclide is present, a graph of  $\ln A$  versus  $t$  should be a straight line with slope equal to  $-\lambda$ . We start with a graph of  $\ln A$  versus  $t$  for the data given in the problem. Figure 43.70a shows this graph.

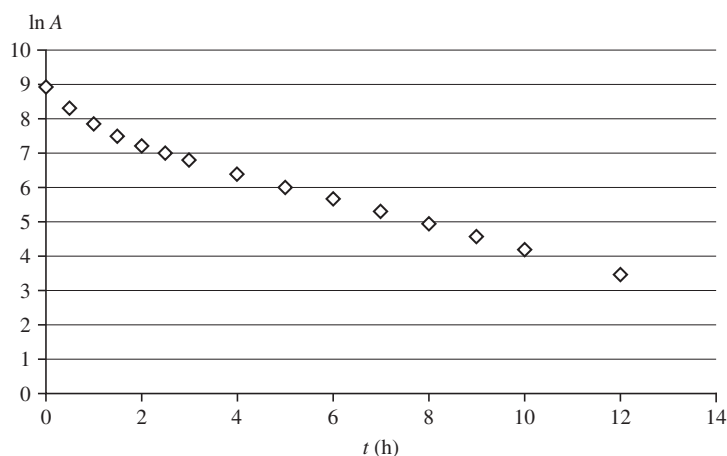


Figure 43.70a

**EXECUTE: (a)** The graph in Figure 43.70a is not a straight line, which suggests that the sample contains more than one nuclide. However after about 4.0 h, the graph does become a straight line. This suggests that a short-lived nuclide has decreased in activity, leaving only a single long-lived nuclide. To investigate the long-lived nuclide, we assume that nearly all the decays from 5.0 h on are due to this nuclide. So we make a graph of  $\log A$  versus  $t$  for the decays from 5.0 h to 12.0 h. The result is shown in Figure 43.70b. This graph is a straight line, suggesting that our hypothesis is correct.

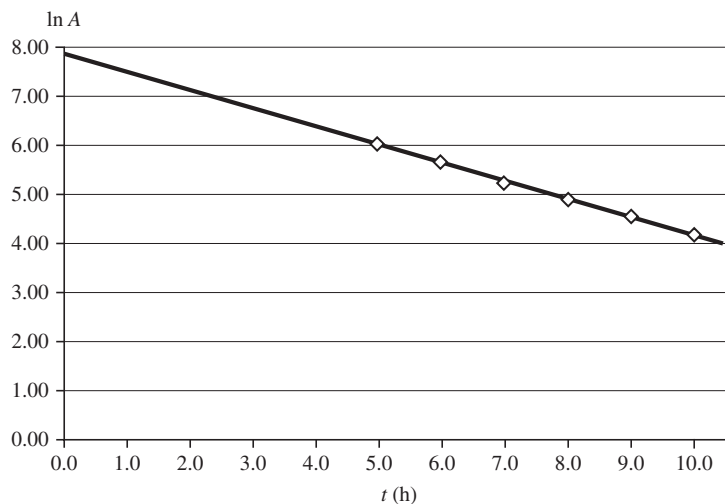
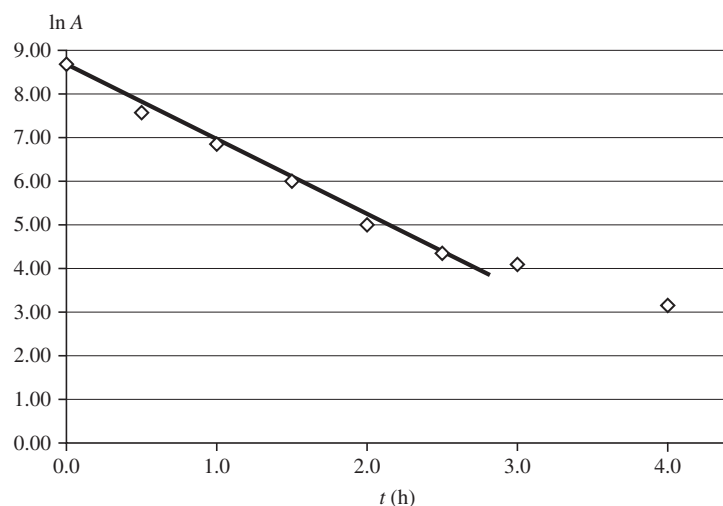


Figure 43.70b

To investigate the short-lived nuclide (or nuclides), we use our graph in Figure 43.70b to determine the activity of the long-lived nuclide during the first 4.0 h. We then subtract these numbers from the total decay rate to determine the decay rate due to the short-lived nuclide. The table shows these results.

Time (h)	Total rate (dec/s)	$A$ (dec/s) for long-lived nuclide	$\ln A$ for long-lived nuclide	$A$ (dec/s) for short-lived nuclide	$\ln A$ for short-lived nuclide
0	7500	2440	7.8	5060	8.53
0.5	4120	2320	7.75	1800	7.50
1.0	2570	1720	7.45	850	6.75
1.5	1790	1410	7.25	380	5.94
2.0	1350	1210	7.1	140	4.94
2.5	1070	992	6.9	78	4.36
3.0	872	6.7	812	60	4.09
4.0	596	572	6.35	24	3.18

Now we use the data in our table to graph  $\ln A$  versus  $t$  for the short-lived nuclide. This graph is shown in Figure 43.70c. The last two points on the graph are unreliable because the decay rate is very small. For the rest of the points, the graph is a straight line. Therefore, since our two graphs of  $\ln A$  versus  $t$  yield straight lines, it appears that our sample contains a minimum of two different nuclides, one with a short half-life and one with a long half-life.



**Figure 43.70c**

**(b) Long-lived nuclide:** The slope of the graph in Figure 43.70b is  $-0.3636 \text{ h}^{-1}$ , so  $\lambda_{\text{long}} = -\text{slope} = 0.3636 \text{ h}^{-1}$ .

Using  $\lambda = (\ln 2)/T_{1/2}$ , the half-life is  $T_{1/2} = (\ln 2)/\lambda_{\text{long}} = (\ln 2)/(0.3636 \text{ h}^{-1}) = 1.9 \text{ h}$ .

**Short-lived nuclide:** The slope of the graph in Figure 43.70c is  $-1.78 \text{ h}^{-1}$ , so  $\lambda_{\text{short}} = 1.78 \text{ h}^{-1}$ . The half-life is  $T_{1/2} = (\ln 2)/(1.78 \text{ h}^{-1}) = 0.39 \text{ h}$ .

**(c)**  $\left| \frac{dN}{dt} \right| = \lambda N$ , so  $N = \frac{|dN/dt|}{\lambda}$ .

**Long-lived nuclide:**  $N_0 = (2440 \text{ Bq})/[(0.3636 \text{ h}^{-1})(1 \text{ h}/3600 \text{ s})] = 2.4 \times 10^7$  nuclei.

**Short-lived nuclide:**  $N_0 = (5060 \text{ Bq})/[(1.78 \text{ h}^{-1})(1 \text{ h}/3600 \text{ s})] = 1.0 \times 10^7$  nuclei.

**(d)** Use  $N = N_0 e^{-\lambda t}$  for each nuclide.

**Long-lived nuclide:**  $N = N_0 e^{-\lambda_{\text{long}} t} = (2.4 \times 10^7) e^{-(0.3636 \text{ h}^{-1})(5.0 \text{ h})} = 3.9 \times 10^6$  nuclei.

**Short-lived nuclide:**  $N = N_0 e^{-\lambda_{\text{short}} t} = (1.0 \times 10^7) e^{-(1.78 \text{ h}^{-1})(5.0 \text{ h})} = 1.4 \times 10^3$  nuclei.

**EVALUATE:** After 5.0 h, the number of shorter-lived nuclei is much less than the number of longer-lived nuclei. The ratio of the number of short-lived to the number of long-lived nuclei is

$$\frac{N_{\text{short}}}{N_{\text{long}}} = \frac{1.0 \times 10^7 e^{-\lambda_{\text{short}} t}}{2.4 \times 10^7 e^{-\lambda_{\text{long}} t}} = 0.42 e^{-(\lambda_{\text{short}} - \lambda_{\text{long}})t}. \text{ Since } \lambda_{\text{short}} > \lambda_{\text{long}}, \text{ this ratio keeps decreasing with time.}$$

- 43.71. IDENTIFY:** Apply  $A = A_0 e^{-\lambda t}$ , where  $A$  is the activity and  $\lambda = (\ln 2)/T_{1/2}$ . This equation can be written as  $A = A_0 2^{-(t/T_{1/2})}$ . The activity of the engine oil is proportional to the mass worn from the piston rings.

**SET UP:**  $1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$ .

**EXECUTE:** The activity of the original iron, after 1000 hours of operation, would be  $(9.4 \times 10^{-6} \text{ Ci})(3.7 \times 10^{10} \text{ Bq/Ci})2^{-(1000 \text{ h})/[(45 \text{ d})(24 \text{ h/d})]} = 1.8306 \times 10^5 \text{ Bq}$ . The activity of the oil is 84 Bq, or  $4.5886 \times 10^{-4}$  of the total iron activity, and this must be the fraction of the mass worn, or mass of  $4.59 \times 10^{-2} \text{ g}$ . The rate at which the piston rings lost their mass is then  $4.59 \times 10^{-5} \text{ g/h}$ .

**EVALUATE:** This method is very sensitive and can measure very small amounts of wear.

- 43.72. IDENTIFY and SET UP:** Follow the procedure outlined in the problem. Solve the differential equation  $dN_2/dt = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 N_2$ , where  $N_{10}$  is the initial number of  ${}^{234}_{92}\text{U}$  nuclei and  $N_2$  is the number of  ${}^{230}_{88}\text{Th}$  nuclei. Assume a solution of the form  $N_2(t) = N_{10}(h_1 e^{-\lambda_1 t} + h_2 e^{-\lambda_2 t})$  and follow the suggestions in the problem.

**EXECUTE: (a)**  $N_2(0) = 0 = N_{10}(h_1 + h_2) \rightarrow h_2 = -h_1$ .

**(b)** Take the derivative of  $N_2(t)$ :  $dN_2/dt = N_{10}(-\lambda_1 h_1 e^{-\lambda_1 t} - \lambda_2 h_2 e^{-\lambda_2 t})$ . Now substitute  $dN/dt$  and  $N(t)$  into the original differential equation.

$$N_{10}(-\lambda_1 h_1 e^{-\lambda_1 t} - \lambda_2 h_2 e^{-\lambda_2 t}) = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 (h_1 e^{-\lambda_1 t} + h_2 e^{-\lambda_2 t}).$$

Collect coefficients of  $e^{-\lambda_1 t}$  and  $e^{-\lambda_2 t}$ .

$$N_{10}(\lambda_1 h_1 + \lambda_1 - \lambda_2 h_2) e^{-\lambda_1 t} + N_{10}(\lambda_2 h_2 - \lambda_2 h_2) e^{-\lambda_2 t} = 0.$$

Setting the coefficients equal to zero gives  $h_1 = \frac{\lambda_1}{\lambda_2 - \lambda_1}$ . Putting this result into the equation for  $N_2(t)$  and

$$\text{using the fact that } h_2 = -h_1 \text{ gives } N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$

**(c)** The initial number of  ${}^{234}_{92}\text{U}$  atoms is

$$N_{10} = (30.0 \text{ g})[(6.022 \times 10^{23} \text{ atoms})/(234 \text{ g})] = 7.7205 \times 10^{22} \text{ atoms}.$$

Using  $\lambda = (\ln 2)/T_{1/2}$ , we get  $\lambda_1 = (\ln 2)/(2.46 \times 10^5 \text{ y})$  and  $\lambda_2 = (\ln 2)/(7.54 \times 10^4 \text{ y})$ , which gives

$$\frac{\lambda_1}{\lambda_2 - \lambda_1} = 0.44197. \text{ At time } t = 2.46 \times 10^5 \text{ y, the number } N_2 \text{ of } {}^{230}_{88}\text{Th} \text{ atoms is}$$

$$N_2 = (7.7205 \times 10^{22} \text{ atoms})(0.44197)[e^{-(\ln 2)} - e^{-(\ln 2)(2.46)/(0.754)}] = 1.3506 \times 10^{22} \text{ atoms}.$$

The mass  $M$  of all these  ${}^{230}_{88}\text{Th}$  atoms is

$$M = (1.3506 \times 10^{22} \text{ atoms})[(1 \text{ mol})/(6.022 \times 10^{23} \text{ atoms})][(230 \text{ g})/\text{mol}] = 5.16 \text{ g}.$$

**EVALUATE:** The  ${}^{234}_{92}\text{U}$  decays with a half-life of  $2.46 \times 10^5 \text{ y}$ , so at this time half the original uranium, or 15.0 g, is still left but only 5.16 g of  ${}^{230}_{88}\text{Th}$  is present because it has continued to decay after being formed.

- 43.73. IDENTIFY and SET UP:** The reaction is  ${}^{130}_{52}\text{Te} + \text{X} \rightarrow {}^{131}_{52}\text{Te}$ .

**EXECUTE:** X must have no charge since Z remains 52, and it must increase the atomic weight from 130 to 131, so it must be a neutron, which is choice (a).

**EVALUATE:** The other reactions in the choices all start out with the wrong isotope, so they cannot be correct.

**43.74. IDENTIFY and SET UP:** The reaction is  ${}^{131}_{52}\text{Te} \rightarrow {}^{131}_{53}\text{I} + \text{X}$ .

**EXECUTE:** The mass number 131 does not change, but the atomic number goes from 52 to 53, so the nucleus gained a charge of +1 (or lost a charge of -1). Therefore X must be a  $\beta^-$  particle, which is choice (b).

**EVALUATE:** Alpha decay would change the mass number  $A$  by 4 units,  $\beta^+$  decay would decrease the atomic number  $Z$  by 1 unit, and gamma decay would not affect  $Z$ .

**43.75. IDENTIFY and SET UP:** A thyroid treatment administers 3.7 GBq of  ${}^{131}_{53}\text{I}$ , which has a half-life of

$$8.04 \text{ days. } \lambda = (\ln 2)/T_{1/2} \text{ and } |dN/dt| = \lambda N = \frac{\ln 2}{T_{1/2}} N.$$

**EXECUTE:** Solve for  $N$  and put in the numbers.

$$N = \frac{T_{1/2} |dN/dt|}{\ln 2} = (8.04 \text{ d})(24 \times 3600 \text{ s/d})(3.7 \times 10^9 \text{ decays/s})/(\ln 2) = 3.7 \times 10^{15} \text{ atoms, which is choice (d).}$$

**EVALUATE:** This amount is  $(3.7 \times 10^{15} \text{ atoms})/(6.02 \times 10^{23} \text{ atoms/mol}) = 6.1 \times 10^{-9} \text{ mol} \approx 6 \text{ nanomoles}$ .

**43.76. IDENTIFY and SET UP:** The reaction is  ${}^{123}_{52}\text{Te} + \text{p} \rightarrow {}^{123}_{53}\text{I} + \text{n}$ . Calculate the reaction energy  $Q$  to find out if the reaction is exoergic or endoergic.

**EXECUTE:**  $Q = (M_{\text{Te}} + M_{\text{p}} - M_{\text{I}} - M_{\text{n}})c^2$ . Using the values from the problem gives

$Q = (122.904270 \text{ u} + 1.007825 \text{ u} - 122.905589 \text{ u} - 1.008665 \text{ u})c^2 = -0.002159 \text{ u}c^2$ , so the reaction is endoergic. We must put in energy to cause the reaction, which means that the proton must have a minimum kinetic energy, so choice (d) is correct.

**EVALUATE:**  $|Q| = (0.002159 \text{ u})(931.5 \text{ MeV/u}) = 2.011 \text{ MeV}$ , so the proton must have at least 2.011 MeV of kinetic energy to cause the reaction.

**43.77. IDENTIFY and SET UP:**  ${}^{131}_{53}\text{I}$  undergoes  $\beta^-$  decay, but  ${}^{123}_{53}\text{I}$  undergoes gamma decay.

**EXECUTE:** From Table 43.3 we see that the RBE for gamma rays is 1, but the RBE for electrons is 1.0-1.5, so the electrons could cause more tissue damage than the gamma rays. This makes choice (b) the best one.

**EVALUATE:** The higher the RBE, the more likely it is that tissue damage could occur.