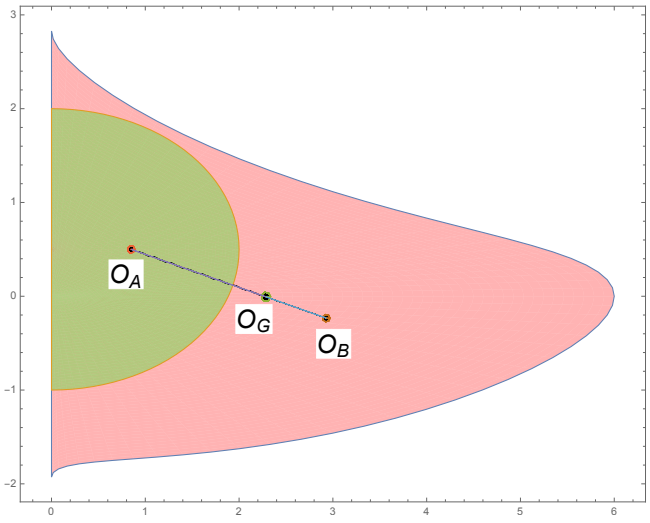
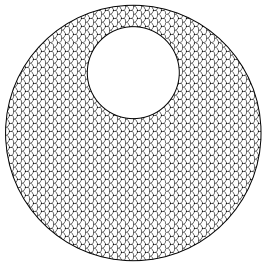
Module MA2341 (Frolov), Advanced Mechanics I
Homework Sheet 9
Each set of homework questions is worth 100 marks

Problem 1. Let the rigid body G be a composition of two rigid bodies A and B, see the picture below. Express the inertia tensor $I_{ik}^{(A)}$ of the rigid body A defined with respect to its centre of mass O_A through the inertia tensors $I_{ik}^{(G)}$ and $I_{ik}^{(B)}$ of the rigid bodies G and B. Assume that the location of the centres of mass O_G, O_A, O_B, and the masses of A and B are known.



Problem 2. Let the rigid body G be a homogeneous solid cylinder of radius R and of height H. Let the rigid body A be obtained by cutting out from G a cylinder B of radius r whose axes of symmetry is parallel to the axes of G. The distance between the axis of G and the cylinder B is $a \leq R - r$.



(a) Find the principal moments of inertia of the rigid body A.

(b) Find the frequency of small oscillations of the rigid body A about a horizontal axis perpendicular to the line connecting the centres of mass of G and B and passing through the centre of

(i) the cylinder G, (ii) the cylinder B.

Problem 3. Consider the system in problem 4 of Par. 32 (Landau and Lifshitz page 103).

(a) Find the Lagrangian and equations of motion of the system.

(b) Determine the angular velocity of the rod AB the moment before it hits the ground if its initial angular velocity is $\sqrt{3g/l}$.

Bonus question (each bonus question is worth extra 25 marks)

A homogeneous cone of mass m, height h, and base radius r can roll without slipping on an inclined plane with its tip fixed at one point. The plane forms angle ϕ with the horizontal plane. Find the Lagrangian and equations of motion of this system, and the frequency of small oscillations.