

Advanced Calculus

MA1132

Tutorial Exercises 8

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To be completed before and during tutorials of Friday, 29. March

1. Consider the region R defined by

$$R = \{(x, y), | x^2 + y^2 \leq 9\}.$$

Let $f(x, y) = 3 - \sqrt{x^2 + y^2}$. Calculate

$$\iint_R f(x, y) dA.$$

2. Consider the region R that is inside the curve $r = 4 \cos \theta$ and outside the curve (lemniscate) $r^2 = -8 \cos 2\theta$, where r and θ are the polar coordinates: $x = r \cos \theta$, $y = r \sin \theta$.

- (a) What is the curve $r = 4 \cos \theta$?
- (b) Sketch the region R .
- (c) Find the area of R .

3. Find the volume V of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, the cylinders $az = x^2$, $a > 0$, $x^2 + y^2 = b^2$, and located in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$.

4. Find the surface area of the portion of the surface $z = 2x + y^2$ that is above the triangular region with vertices $(0, 0)$, $(0, 1)$ and $(1, 1)$.

5. Find the surface area of the portion of the paraboloid $2z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 8$.

6. Find a parametric representation of

- (a) the elliptic cone

$$z = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}.$$

- (b) the hyperboloid of one sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$$