5

APPLYING NEWTON'S LAWS

5.1. **IDENTIFY:** $a = 0$ for each object. Apply $\Sigma F_y = ma_y$ to each weight and to the pulley.

SET UP: Take $+y$ upward. The pulley has negligible mass. Let T_r be the tension in the rope and let T_c be the tension in the chain.

EXECUTE: **(a)** The free-body diagram for each weight is the same and is given in Figure 5.1a. $\Sigma F_y = ma_y$ gives $T_r = w = 25.0$ N.

(b) The free-body diagram for the pulley is given in Figure 5.1b. $T_c = 2T_r = 50.0$ N.

EVALUATE: The tension is the same at all points along the rope.

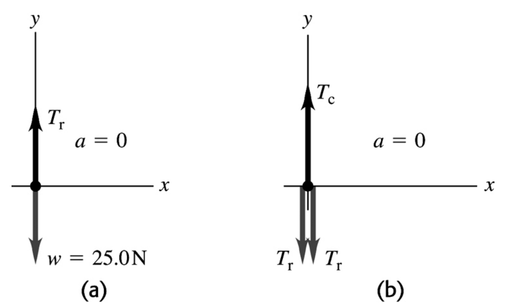


Figure 5.1

5.2. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to each weight.

SET UP: Two forces act on each mass: w down and $T(= w)$ up.

EXECUTE: In all cases, each string is supporting a weight w against gravity, and the tension in each string is w.

EVALUATE: The tension is the same in all three cases.

5.3. **IDENTIFY:** Both objects are at rest and $a = 0$. Apply Newton's first law to the appropriate object. The maximum tension T_{max} is at the top of the chain and the minimum tension is at the bottom of the chain.

SET UP: Let $+y$ be upward. For the maximum tension take the object to be the chain plus the ball. For the minimum tension take the object to be the ball. For the tension T three-fourths of the way up from the bottom of the chain, take the chain below this point plus the ball to be the object. The free-body diagrams in each of these three cases are sketched in Figure 5.3. $m_{b+c} = 75.0\text{ kg} + 26.0\text{ kg} = 101.0$ kg. $m_b = 75.0$ kg. m is the mass of three-fourths of the chain: $m = \frac{3}{4}(26.0\text{ kg}) = 19.5$ kg.

EXECUTE: **(a)** From Figure 5.3a, $\Sigma F_y = 0$ gives $T_{max} - m_{b+c}g = 0$ and $T_{max} = (101.0\text{ kg})(9.80\text{ m/s}^2) = 990$ N. From Figure 5.3b, $\Sigma F_y = 0$ gives $T_{min} - m_b g = 0$ and $T_{min} = (75.0\text{ kg})(9.80\text{ m/s}^2) = 735$ N.

(b) From Figure 5.3c, $\Sigma F_y = 0$ gives $T - (m + m_b)g = 0$ and $T = (19.5\text{ kg} + 75.0\text{ kg})(9.80\text{ m/s}^2) = 926$ N.

EVALUATE: The tension in the chain increases linearly from the bottom to the top of the chain.

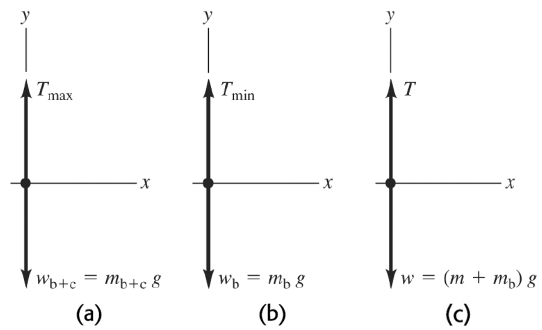


Figure 5.3

5.4. **IDENTIFY:** For the maximum tension, the patient is just ready to slide so static friction is at its maximum and the forces on him add to zero.

SET UP: **(a)** The free-body diagram for the person is given in Figure 5.4a. F is magnitude of the traction force along the spinal column and $w = mg$ is the person's weight. At maximum static friction, $f_s = \mu_s n$.

(b) The free-body diagram for the collar where the cables are attached is given in Figure 5.4b. The tension in each cable has been resolved into its x- and y-components.

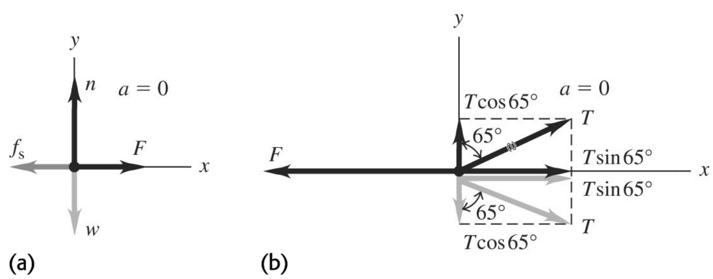


Figure 5.4

EXECUTE: **(a)** $n = w$ and $F = f_s = \mu_s n = 0.75w = 0.75(9.80\text{ m/s}^2)(78.5\text{ kg}) = 577\text{ N}$.

(b) $2T\sin 65° - F = 0$ so $T = \dfrac{F}{2\sin 65°} = \dfrac{0.75w}{2\sin 65°} = 0.41w = (0.41)(9.80\text{ m/s}^2)(78.5\text{ kg}) = 315\text{ N}$.

EVALUATE: The two tensions add up to 630 N, which is more than the traction force, because the cables do not pull directly along the spinal column.

5.5. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the frame.

SET UP: Let w be the weight of the frame. Since the two wires make the same angle with the vertical, the tension is the same in each wire. $T = 0.75w$.

EXECUTE: The vertical component of the force due to the tension in each wire must be half of the weight, and this in turn is the tension multiplied by the cosine of the angle each wire makes with the vertical.

$\dfrac{w}{2} = \dfrac{3w}{4}\cos\theta$ and $\theta = \arccos\frac{2}{3} = 48°$.

EVALUATE: If $\theta = 0°$, $T = w/2$ and $T \to \infty$ as $\theta \to 90°$. Therefore, there must be an angle where $T = 3w/4$.

5.6. **IDENTIFY:** Apply Newton's first law to the wrecking ball. Each cable exerts a force on the ball, directed along the cable.

SET UP: The force diagram for the wrecking ball is sketched in Figure 5.6.

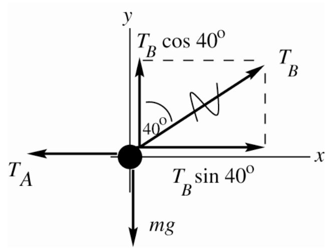


Figure 5.6

EXECUTE: **(a)** $\Sigma F_y = ma_y$

$T_B \cos 40° - mg = 0$

$$T_B = \frac{mg}{\cos 40°} = \frac{(3620 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 40°} = 4.63 \times 10^4 \text{ N} = 46.3 \text{ kN}$$

(b) $\Sigma F_x = ma_x$

$T_B \sin 40° - T_A = 0$

$T_A = T_B \sin 40° = 2.98 \times 10^4 \text{ N} = 29.8 \text{ kN}$

EVALUATE: If the angle 40° is replaced by 0° (cable *B* is vertical), then $T_B = mg$ and $T_A = 0$.

5.7. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the object and to the knot where the cords are joined.

SET UP: Let $+y$ be upward and $+x$ be to the right.

EXECUTE: **(a)** $T_C = w$, $T_A \sin 30° + T_B \sin 45° = T_C = w$, and $T_A \cos 30° - T_B \cos 45° = 0$. Since $\sin 45° = \cos 45°$, adding the last two equations gives $T_A(\cos 30° + \sin 30°) = w$, and so

$$T_A = \frac{w}{1.366} = 0.732w. \text{ Then, } T_B = T_A \frac{\cos 30°}{\cos 45°} = 0.897w.$$

(b) Similar to part (a), $T_C = w$, $-T_A \cos 60° + T_B \sin 45° = w$, and $T_A \sin 60° - T_B \cos 45° = 0$.

Adding these two equations, $T_A = \dfrac{w}{(\sin 60° - \cos 60°)} = 2.73w$, and $T_B = T_A \dfrac{\sin 60°}{\cos 45°} = 3.35w$.

EVALUATE: In part (a), $T_A + T_B > w$ since only the vertical components of T_A and T_B hold the object against gravity. In part (b), since T_A has a downward component T_B is greater than w.

5.8. **IDENTIFY:** Apply Newton's first law to the car.

SET UP: Use *x*- and *y*-coordinates that are parallel and perpendicular to the ramp.

EXECUTE: **(a)** The free-body diagram for the car is given in Figure 5.8 (next page). The vertical weight *w* and the tension *T* in the cable have each been replaced by their *x*- and *y*-components.

(b) $\Sigma F_x = 0$ gives $T \cos 31.0° - w \sin 25.0° = 0$ and

$$T = w\frac{\sin 25.0°}{\cos 31.0°} = (1130 \text{ kg})(9.80 \text{ m/s}^2)\frac{\sin 25.0°}{\cos 31.0°} = 5460 \text{ N}.$$

(c) $\Sigma F_y = 0$ gives $n + T \sin 31.0° - w \cos 25.0° = 0$ and

$$n = w \cos 25.0° - T \sin 31.0° = (1130 \text{ kg})(9.80 \text{ m/s}^2)\cos 25.0° - (5460 \text{ N})\sin 31.0° = 7220 \text{ N}$$

EVALUATE: We could also use coordinates that are horizontal and vertical and would obtain the same values of *n* and *T*.

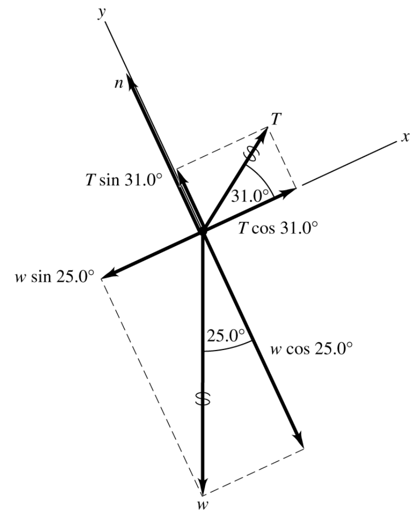


Figure 5.8

5.9. **IDENTIFY:** Since the velocity is constant, apply Newton's first law to the piano. The push applied by the man must oppose the component of gravity down the incline.

SET UP: The free-body diagrams for the two cases are shown in Figure 5.9. $\vec{F}$ is the force applied by the man. Use the coordinates shown in the figure.

EXECUTE: **(a)** $\Sigma F_x = 0$ gives $F - w\sin 19.0° = 0$ and $F = (180\text{ kg})(9.80\text{ m/s}^2)\sin 19.0° = 574\text{ N}$.

(b) $\Sigma F_y = 0$ gives $n\cos 19.0° - w = 0$ and $n = \dfrac{w}{\cos 19.0°}$. $\Sigma F_x = 0$ gives $F - n\sin 19.0° = 0$ and

$$F = \left(\frac{w}{\cos 19.0°}\right)\sin 19.0° = w\tan 19.0° = 607\text{ N}.$$

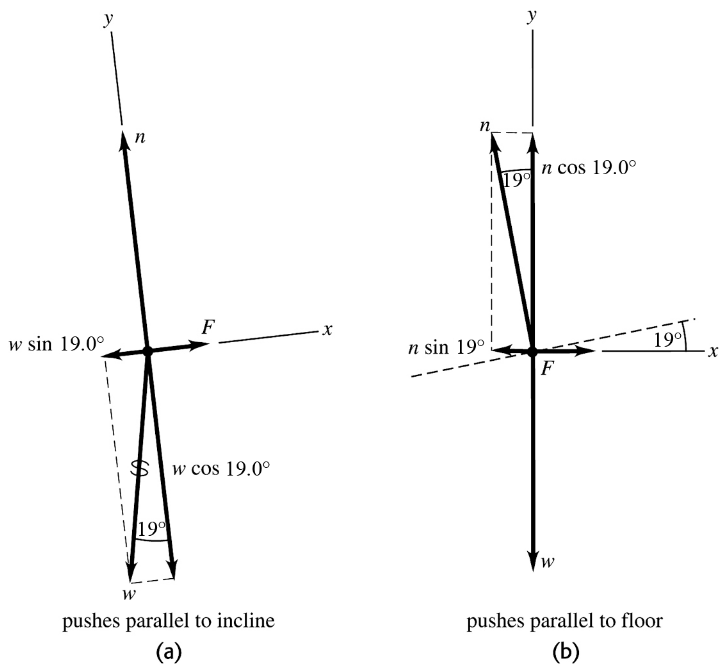


Figure 5.9

EVALUATE: When pushing parallel to the floor only part of the push is up the ramp to balance the weight of the piano, so you need a larger push in this case than if you push parallel to the ramp.

5.10. **IDENTIFY:** Apply Newton's first law to the hanging weight and to each knot. The tension force at each end of a string is the same.

(a) Let the tensions in the three strings be T, T', and T'', as shown in Figure 5.10a.

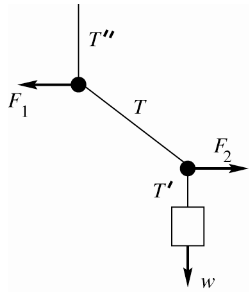


Figure 5.10a

SET UP: The free-body diagram for the block is given in Figure 5.10b.

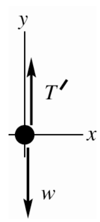


Figure 5.10b

EXECUTE:

$\Sigma F_y = 0$

$T' - w = 0$

$T' = w = 60.0\ \text{N}$

SET UP: The free-body diagram for the lower knot is given in Figure 5.10c.

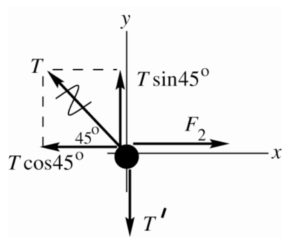


Figure 5.10c

EXECUTE:

$\Sigma F_y = 0$

$T\sin 45° - T' = 0$

$$T = \frac{T'}{\sin 45°} = \frac{60.0\ \text{N}}{\sin 45°} = 84.9\ \text{N}$$

(b) Apply $\Sigma F_x = 0$ to the force diagram for the lower knot:

$\Sigma F_x = 0$

$F_2 = T\cos 45° = (84.9\ \text{N})\cos 45° = 60.0\ \text{N}$

SET UP: The free-body diagram for the upper knot is given in Figure 5.10d.

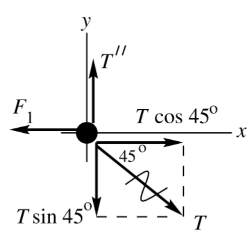


Figure 5.10d

EXECUTE:

$\Sigma F_x = 0$

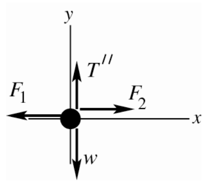
$T\cos 45° - F_1 = 0$

$F_1 = (84.9\ \text{N})\cos 45°$

$F_1 = 60.0\ \text{N}$

Note that $F_1 = F_2$.

EVALUATE: Applying $\Sigma F_y = 0$ to the upper knot gives $T'' = T\sin 45° = 60.0\ \text{N} = w$. If we treat the whole system as a single object, the force diagram is given in Figure 5.10e (next page).



$\Sigma F_x = 0$ gives $F_2 = F_1$, which checks

$\Sigma F_y = 0$ gives $T'' = w$, which checks

Figure 5.10e

5.11. **IDENTIFY:** We apply Newton's second law to the rocket and the astronaut in the rocket. A constant force means we have constant acceleration, so we can use the standard kinematics equations.

SET UP: The free-body diagrams for the rocket (weight w_r) and astronaut (weight w) are given in Figure 5.11. F_T is the thrust and n is the normal force the rocket exerts on the astronaut. The speed of sound is 331 m/s. We use $\Sigma F_y = ma_y$ and $v = v_0 + at$.

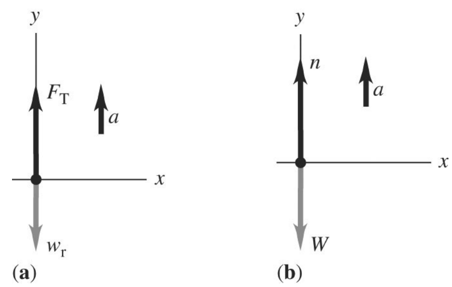


Figure 5.11

EXECUTE: **(a)** Apply $\Sigma F_y = ma_y$ to the rocket: $F_T - w_r = ma$. $a = 4g$ and $w_r = mg$, so $F = m\,(5g) = (2.25\times10^6\text{ kg})\,(5)\,(9.80\text{ m/s}^2) = 1.10\times10^8\text{ N}$.

(b) Apply $\Sigma F_y = ma_y$ to the astronaut: $n - w = ma$. $a = 4g$ and $m = \dfrac{w}{g}$, so $n = w + \left(\dfrac{w}{g}\right)(4g) = 5w$.

(c) $v_0 = 0$, $v = 331$ m/s and $a = 4g = 39.2\text{ m/s}^2$. $v = v_0 + at$ gives $t = \dfrac{v - v_0}{a} = \dfrac{331\text{ m/s}}{39.2\text{ m/s}^2} = 8.4\text{ s}$.

EVALUATE: The 8.4 s is probably an unrealistically short time to reach the speed of sound because you would not want your astronauts at the brink of blackout during a launch.

5.12. **IDENTIFY:** Apply Newton's second law to the rocket plus its contents and to the power supply. Both the rocket and the power supply have the same acceleration.

SET UP: The free-body diagrams for the rocket and for the power supply are given in Figure 5.12. Since the highest altitude of the rocket is 120 m, it is near to the surface of the earth and there is a downward gravity force on each object. Let $+y$ be upward, since that is the direction of the acceleration. The power supply has mass $m_{ps} = (15.5\text{ N})/(9.80\text{ m/s}^2) = 1.58\text{ kg}$.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the rocket gives $F - m_r g = m_r a$.

$$a = \frac{F - m_r g}{m_r} = \frac{1720\text{ N} - (125\text{ kg})(9.80\text{ m/s}^2)}{125\text{ kg}} = 3.96\text{ m/s}^2.$$

(b) $\Sigma F_y = ma_y$ applied to the power supply gives $n - m_{ps}g = m_{ps}a$.

$n = m_{ps}(g + a) = (1.58\text{ kg})(9.80\text{ m/s}^2 + 3.96\text{ m/s}^2) = 21.7\text{ N}$.

EVALUATE: The acceleration is constant while the thrust is constant, and the normal force is constant while the acceleration is constant. The altitude of 120 m is not used in the calculation.

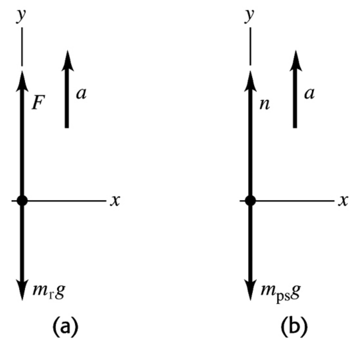


Figure 5.12

5.13. **IDENTIFY:** Use the kinematic information to find the acceleration of the capsule and the stopping time. Use Newton's second law to find the force F that the ground exerted on the capsule during the crash.

SET UP: Let $+y$ be upward. 311 km/h = 86.4 m/s. The free-body diagram for the capsule is given in Figure 5.13.

EXECUTE: $y - y_0 = -0.810\ \text{m}$, $v_{0y} = -86.4\ \text{m/s}$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{0 - (-86.4\ \text{m/s})^2}{2(-0.810)\ \text{m}} = 4610\ \text{m/s}^2 = 470g.$$

(b) $\Sigma F_y = ma_y$ applied to the capsule gives $F - mg = ma$ and

$$F = m(g + a) = (210\ \text{kg})(9.80\ \text{m/s}^2 + 4610\ \text{m/s}^2) = 9.70\times10^5\ \text{N} = 471w.$$

(c) $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y} + v_y} = \dfrac{2(-0.810\ \text{m})}{-86.4\ \text{m/s} + 0} = 0.0187\ \text{s}$

EVALUATE: The upward force exerted by the ground is much larger than the weight of the capsule and stops the capsule in a short amount of time. After the capsule has come to rest, the ground still exerts a force mg on the capsule, but the large 9.70×10^5 N force is exerted only for 0.0187 s.

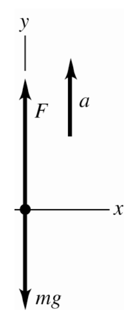


Figure 5.13

5.14. **IDENTIFY:** Apply Newton's second law to the three sleds taken together as a composite object and to each individual sled. All three sleds have the same horizontal acceleration a.

SET UP: The free-body diagram for the three sleds taken as a composite object is given in Figure 5.14a and for each individual sled in Figures 5.14b–d. Let $+x$ be to the right, in the direction of the acceleration. $m_{\text{tot}} = 60.0\ \text{kg}$.

EXECUTE: **(a)** $\Sigma F_x = ma_x$ for the three sleds as a composite object gives $P = m_{\text{tot}}a$ and

$$a = \frac{P}{m_{\text{tot}}} = \frac{190\ \text{N}}{60.0\ \text{kg}} = 3.17\ \text{m/s}^2.$$

(b) $\Sigma F_x = ma_x$ applied to the 10.0 kg sled gives $P - T_A = m_{10}a$ and

$T_A = P - m_{10}a = 190\text{ N} - (10.0\text{ kg})(3.17\text{ m/s}^2) = 158\text{ N}$. $\Sigma F_x = ma_x$ applied to the 30.0 kg sled gives

$T_B = m_{30}a = (30.0\text{ kg})(3.17\text{ m/s}^2) = 95.1\text{ N}$.

EVALUATE: If we apply $\Sigma F_x = ma_x$ to the 20.0 kg sled and calculate a from T_A and T_B found in part (b), we get $T_A - T_B = m_{20}a$. $a = \dfrac{T_A - T_B}{m_{20}} = \dfrac{158\text{ N} - 95.1\text{ N}}{20.0\text{ kg}} = 3.15\text{ m/s}^2$, which agrees closely with the value we calculated in part (a), the difference being due to rounding.

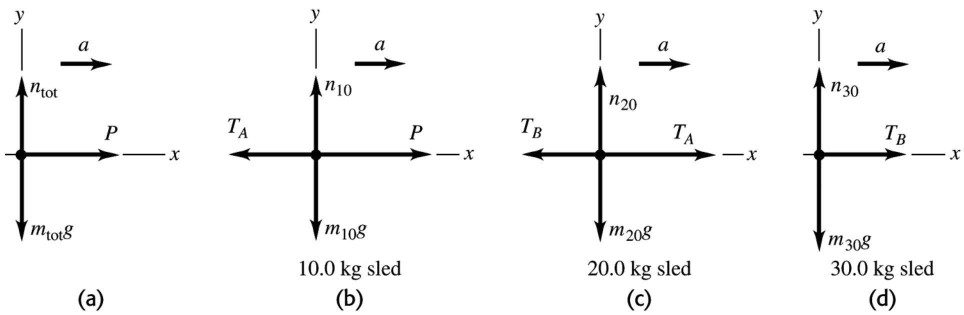


Figure 5.14

5.15. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the load of bricks and to the counterweight. The tension is the same at each end of the rope. The rope pulls up with the same force (T) on the bricks and on the counterweight. The counterweight accelerates downward and the bricks accelerate upward; these accelerations have the same magnitude.

(a) SET UP: The free-body diagrams for the bricks and counterweight are given in Figure 5.15.

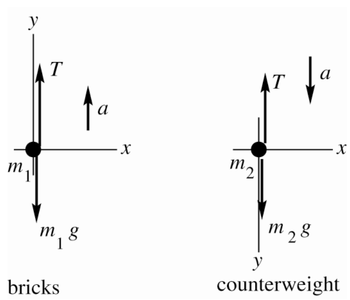


Figure 5.15

(b) EXECUTE: Apply $\Sigma F_y = ma_y$ to each object. The acceleration magnitude is the same for the two objects. For the bricks take $+y$ to be upward since $\vec{a}$ for the bricks is upward. For the counterweight take $+y$ to be downward since $\vec{a}$ is downward.

<u>bricks</u>: $\Sigma F_y = ma_y$

$T - m_1 g = m_1 a$

<u>counterweight</u>: $\Sigma F_y = ma_y$

$m_2 g - T = m_2 a$

Add these two equations to eliminate T:

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{28.0\text{ kg} - 15.0\text{ kg}}{15.0\text{ kg} + 28.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 2.96\text{ m/s}^2$$

(c) $T - m_1 g = m_1 a$ gives $T = m_1(a+g) = (15.0\text{ kg})(2.96\text{ m/s}^2 + 9.80\text{ m/s}^2) = 191\text{ N}$

As a check, calculate T using the other equation.

$m_2 g - T = m_2 a$ gives $T = m_2(g-a) = 28.0\text{ kg}(9.80\text{ m/s}^2 - 2.96\text{ m/s}^2) = 191\text{ N}$, which checks.

EVALUATE: The tension is 1.30 times the weight of the bricks; this causes the bricks to accelerate upward. The tension is 0.696 times the weight of the counterweight; this causes the counterweight to accelerate downward. If $m_1 = m_2$, $a = 0$ and $T = m_1 g = m_2 g$. In this special case the objects don't move. If $m_1 = 0$, $a = g$ and $T = 0$; in this special case the counterweight is in free fall. Our general result is correct in these two special cases.

5.16. **IDENTIFY:** In part (a) use the kinematic information and the constant acceleration equations to calculate the acceleration of the ice. Then apply $\Sigma\vec{F} = m\vec{a}$. In part (b) use $\Sigma\vec{F} = m\vec{a}$ to find the acceleration and use this in the constant acceleration equations to find the final speed.

SET UP: Figure 5.16 gives the free-body diagrams for the ice both with and without friction. Let $+x$ be directed down the ramp, so $+y$ is perpendicular to the ramp surface. Let ϕ be the angle between the ramp and the horizontal. The gravity force has been replaced by its x- and y-components.

EXECUTE: **(a)** $x - x_0 = 1.50$ m, $v_{0x} = 0$. $v_x = 2.50$ m/s. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x-x_0)} = \dfrac{(2.50\text{ m/s})^2 - 0}{2(1.50\text{ m})} = 2.08\text{ m/s}^2$. $\Sigma F_x = ma_x$ gives $mg\sin\phi = ma$ and $\sin\phi = \dfrac{a}{g} = \dfrac{2.08\text{ m/s}^2}{9.80\text{ m/s}^2}$.

$\phi = 12.3°$.

(b) $\Sigma F_x = ma_x$ gives $mg\sin\phi - f = ma$ and

$$a = \frac{mg\sin\phi - f}{m} = \frac{(8.00\text{ kg})(9.80\text{ m/s}^2)\sin 12.3° - 10.0\text{ N}}{8.00\text{ kg}} = 0.838\text{ m/s}^2.$$

Then $x - x_0 = 1.50$ m, $v_{0x} = 0$. $a_x = 0.838\text{ m/s}^2$ and $v_x^2 = v_{0x}^2 + 2a_x(x-x_0)$ gives

$$v_x = \sqrt{2a_x(x-x_0)} = \sqrt{2(0.838\text{ m/s}^2)(1.50\text{ m})} = 1.59\text{ m/s}$$

EVALUATE: With friction present the speed at the bottom of the ramp is less.

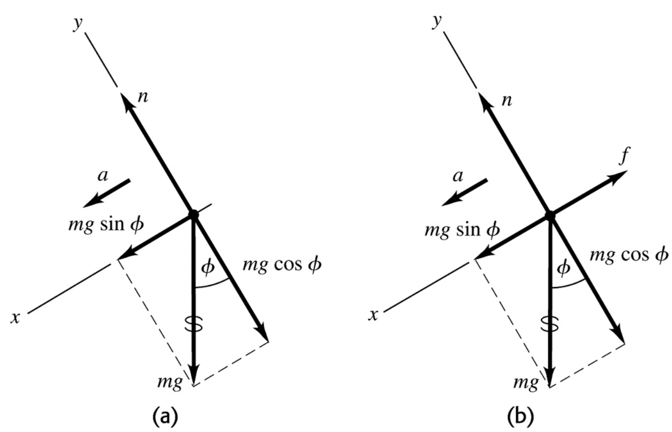


Figure 5.16

5.17. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Each block has the same magnitude of acceleration a.

SET UP: Assume the pulley is to the right of the 4.00 kg block. There is no friction force on the 4.00 kg block; the only force on it is the tension in the rope. The 4.00 kg block therefore accelerates to the right and the suspended block accelerates downward. Let $+x$ be to the right for the 4.00 kg block, so for it $a_x = a$, and let $+y$ be downward for the suspended block, so for it $a_y = a$.

EXECUTE: **(a)** The free-body diagrams for each block are given in Figures 5.17a and b.

(b) $\Sigma F_x = ma_x$ applied to the 4.00 kg block gives $T = (4.00\text{ kg})a$ and $a = \dfrac{T}{4.00\text{ kg}} = \dfrac{15.0\text{ N}}{4.00\text{ kg}} = 3.75\text{ m/s}^2$.

(c) $\Sigma F_y = ma_y$ applied to the suspended block gives $mg - T = ma$ and

$$m = \frac{T}{g-a} = \frac{15.0\text{ N}}{9.80\text{ m/s}^2 - 3.75\text{ m/s}^2} = 2.48\text{ kg}.$$

(d) The weight of the hanging block is $mg = (2.48\text{ kg})(9.80\text{ m/s}^2) = 24.3\text{ N}$. This is greater than the tension in the rope; $T = 0.617mg$.

EVALUATE: Since the hanging block accelerates downward, the net force on this block must be downward and the weight of the hanging block must be greater than the tension in the rope. Note that the blocks accelerate no matter how small m is. It is not necessary to have $m > 4.00$ kg, and in fact in this problem m is less than 4.00 kg.

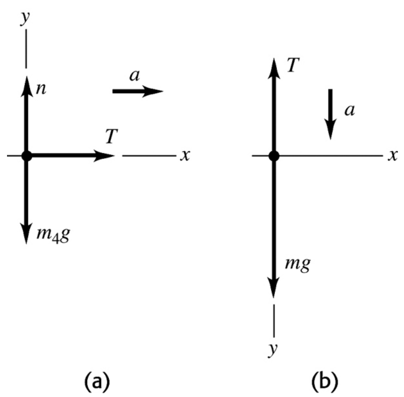


Figure 5.17

5.18. **IDENTIFY:** **(a)** Consider both gliders together as a single object, apply $\Sigma\vec{F} = m\vec{a}$, and solve for a. Use a in a constant acceleration equation to find the required runway length.

(b) Apply $\Sigma\vec{F} = m\vec{a}$ to the second glider and solve for the tension T_g in the towrope that connects the two gliders.

SET UP: In part (a), set the tension T_t in the towrope between the plane and the first glider equal to its maximum value, $T_t = 12{,}000$ N.

EXECUTE: **(a)** The free-body diagram for both gliders as a single object of mass $2m = 1400$ kg is given in Figure 5.18a. $\Sigma F_x = ma_x$ gives $T_t - 2f = (2m)a$ and $a = \dfrac{T_t - 2f}{2m} = \dfrac{12{,}000\text{ N} - 5000\text{ N}}{1400\text{ kg}} = 5.00\text{ m/s}^2$. Then

$a_x = 5.00\text{ m/s}^2$, $v_{0x} = 0$ and $v_x = 40$ m/s in $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = 160$ m.

(b) The free-body diagram for the second glider is given in Figure 5.18b.

$\Sigma F_x = ma_x$ gives $T_g - f = ma$ and $T_g = f + ma = 2500\text{ N} + (700\text{ kg})(5.00\text{ m/s}^2) = 6000$ N.

EVALUATE: We can verify that $\Sigma F_x = ma_x$ is also satisfied for the first glider.

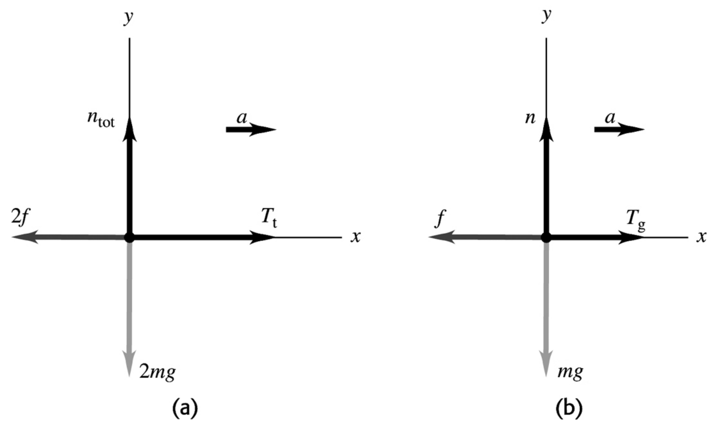


Figure 5.18

5.19. **IDENTIFY:** The maximum tension in the chain is at the top of the chain. Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of chain and boulder. Use the constant acceleration kinematic equations to relate the acceleration to the time.

SET UP: Let $+y$ be upward. The free-body diagram for the composite object is given in Figure 5.19. $T = 2.50w_{\text{chain}}$. $m_{\text{tot}} = m_{\text{chain}} + m_{\text{boulder}} = 1325\text{ kg}$.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $T - m_{\text{tot}}g = m_{\text{tot}}a$.

$$a = \frac{T - m_{\text{tot}}g}{m_{\text{tot}}} = \frac{2.50m_{\text{chain}}g - m_{\text{tot}}g}{m_{\text{tot}}} = \left(\frac{2.50m_{\text{chain}}}{m_{\text{tot}}} - 1\right)g$$

$$a = \left(\frac{2.50(575\text{ kg})}{1325\text{ kg}} - 1\right)(9.80\text{ m/s}^2) = 0.832\text{ m/s}^2$$

(b) Assume the acceleration has its maximum value: $a_y = 0.832\text{ m/s}^2$, $y - y_0 = 125\text{ m}$ and $v_{0y} = 0$.

$$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 \text{ gives } t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(125\text{ m})}{0.832\text{ m/s}^2}} = 17.3\text{ s}$$

EVALUATE: The tension in the chain is $T = 1.41\times10^4$ N and the total weight is 1.30×10^4 N. The upward force exceeds the downward force and the acceleration is upward.

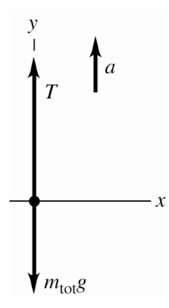


Figure 5.19

5.20. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of elevator plus student $(m_{\text{tot}} = 850\text{ kg})$ and also to the student $(w = 550\text{ N})$. The elevator and the student have the same acceleration.

SET UP: Let $+y$ be upward. The free-body diagrams for the composite object and for the student are given in Figure 5.20. T is the tension in the cable and n is the scale reading, the normal force the scale exerts on the student. The mass of the student is $m = w/g = 56.1\text{ kg}$.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the student gives $n - mg = ma_y$.

$a_y = \dfrac{n - mg}{m} = \dfrac{450\text{ N} - 550\text{ N}}{56.1\text{ kg}} = -1.78\text{ m/s}^2$. The elevator has a downward acceleration of 1.78 m/s^2.

(b) $a_y = \dfrac{670\text{ N} - 550\text{ N}}{56.1\text{ kg}} = 2.14\text{ m/s}^2$.

(c) $n = 0$ means $a_y = -g$. The student should worry; the elevator is in free fall.

(d) $\Sigma F_y = ma_y$ applied to the composite object gives $T - m_{\text{tot}}g = m_{\text{tot}}a_y$. $T = m_{\text{tot}}(a_y + g)$. In part (a), $T = (850\text{ kg})(-1.78\text{ m/s}^2 + 9.80\text{ m/s}^2) = 6820\text{ N}$. In part (c), $a_y = -g$ and $T = 0$.

EVALUATE: In part (b), $T = (850\text{ kg})(2.14\text{ m/s}^2 + 9.80\text{ m/s}^2) = 10{,}150\text{ N}$. The weight of the composite object is 8330 N. When the acceleration is upward the tension is greater than the weight and when the acceleration is downward the tension is less than the weight.

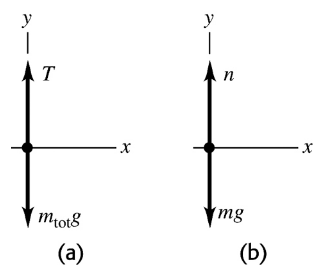


Figure 5.20

5.21. **IDENTIFY:** While the person is in contact with the ground, he is accelerating upward and experiences two forces: gravity downward and the upward force of the ground. Once he is in the air, only gravity acts on him so he accelerates downward. Newton's second law applies during the jump (and at all other times).
SET UP: Take $+y$ to be upward. After he leaves the ground the person travels upward 60 cm and his acceleration is $g = 9.80\text{ m/s}^2$, downward. His weight is w so his mass is w/g. $\Sigma F_y = ma_y$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply to the jumper.

EXECUTE: **(a)** $v_y = 0$ (at the maximum height), $y - y_0 = 0.60\text{ m}$, $a_y = -9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.60\text{ m})} = 3.4\text{ m/s}$.

(b) The free-body diagram for the person while he is pushing up against the ground is given in Figure 5.21 (next page).

(c) For the jump, $v_{0y} = 0$, $v_y = 3.4\text{ m/s}$ (from part (a)), and $y - y_0 = 0.50\text{ m}$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{(3.4\text{ m/s})^2 - 0}{2(0.50\text{ m})} = 11.6\text{ m/s}^2$. $\Sigma F_y = ma_y$ gives $n - w = ma$.

$n = w + ma = w\left(1 + \dfrac{a}{g}\right) = 2.2w$.

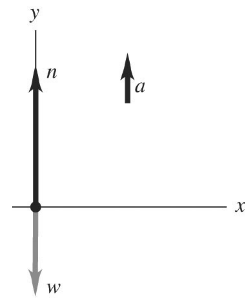


Figure 5.21

EVALUATE: To accelerate the person upward during the jump, the upward force from the ground must exceed the downward pull of gravity. The ground pushes up on him because he pushes down on the ground.

5.22. **IDENTIFY:** Acceleration and velocity are related by $a_y = \frac{dv_y}{dt}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the rocket.

SET UP: Let $+y$ be upward. The free-body diagram for the rocket is sketched in Figure 5.22. $\vec{F}$ is the thrust force.

EXECUTE: **(a)** $v_y = At + Bt^2$. $a_y = A + 2Bt$. At $t = 0$, $a_y = 1.50\ \text{m/s}^2$ so $A = 1.50\ \text{m/s}^2$. Then $v_y = 2.00\ \text{m/s}$ at $t = 1.00\ \text{s}$ gives $2.00\ \text{m/s} = (1.50\ \text{m/s}^2)(1.00\ \text{s}) + B(1.00\ \text{s})^2$ and $B = 0.50\ \text{m/s}^3$.

(b) At $t = 4.00\ \text{s}$, $a_y = 1.50\ \text{m/s}^2 + 2(0.50\ \text{m/s}^3)(4.00\ \text{s}) = 5.50\ \text{m/s}^2$.

(c) $\Sigma F_y = ma_y$ applied to the rocket gives $T - mg = ma$ and $T = m(a+g) = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 5.50\ \text{m/s}^2) = 3.89\times10^4\ \text{N}$. $T = 1.56w$.

(d) When $a = 1.50\ \text{m/s}^2$, $T = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 1.50\ \text{m/s}^2) = 2.87\times10^4\ \text{N}$.

EVALUATE: During the time interval when $v(t) = At + Bt^2$ applies the magnitude of the acceleration is increasing, and the thrust is increasing.

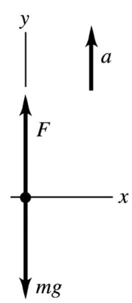


Figure 5.22

5.23. **IDENTIFY:** We know the external forces on the box and want to find the distance it moves and its speed. The force is not constant, so the acceleration will not be constant, so we cannot use the standard constant-acceleration kinematics formulas. But Newton's second law will apply.

SET UP: First use Newton's second law to find the acceleration as a function of time: $a_x(t) = \frac{F_x}{m}$. Then integrate the acceleration to find the velocity as a function of time, and next integrate the velocity to find the position as a function of time.

EXECUTE: Let $+x$ be to the right. $a_x(t)=\dfrac{F_x}{m}=\dfrac{(-6.00\ \text{N/s}^2)t^2}{2.00\ \text{kg}}=-(3.00\ \text{m/s}^4)t^2$. Integrate the acceleration to find the velocity as a function of time: $v_x(t)=-(1.00\ \text{m/s}^4)t^3+9.00\ \text{m/s}$. Next integrate the velocity to find the position as a function of time: $x(t)=-(0.250\ \text{m/s}^4)t^4+(9.00\ \text{m/s})t$. Now use the given values of time.

(a) $v_x=0$ when $(1.00\ \text{m/s}^4)t^3=9.00\ \text{m/s}$. This gives $t=2.08$ s. At $t=2.08$ s,

$x=(9.00\ \text{m/s})(2.08\ \text{s})-(0.250\ \text{m/s}^4)(2.08\ \text{s})^4=18.72\ \text{m}-4.68\ \text{m}=14.0\ \text{m}.$

(b) At $t=3.00$ s, $v_x(t)=-(1.00\ \text{m/s}^4)(3.00\ \text{s})^3+9.00\ \text{m/s}=-18.0\ \text{m/s}$, so the speed is 18.0 m/s.

EVALUATE: The box starts out moving to the right. But because the acceleration is to the left, it reverses direction and v_x is negative in part (b).

5.24. **IDENTIFY:** We know the position of the crate as a function of time, so we can differentiate to find its acceleration. Then we can apply Newton's second law to find the upward force.

SET UP: $v_y(t)=dy/dt$, $a_y(t)=dv_y/dt$, and $\Sigma F_y=ma_y$.

EXECUTE: Let $+y$ be upward. $dy/dt=v_y(t)=2.80\ \text{m/s}+(1.83\ \text{m/s}^3)t^2$ and $dv_y/dt=a_y(t)=(3.66\ \text{m/s}^3)t$. At $t=4.00$ s, $a_y=14.64\ \text{m/s}^2$. Newton's second law in the y direction gives $F-mg=ma$. Solving for F gives $F=49\ \text{N}+(5.00\ \text{kg})(14.64\ \text{m/s}^2)=122\ \text{N}$.

EVALUATE: The force is greater than the weight since it is accelerating the crate upwards.

5.25. **IDENTIFY:** At the maximum tilt angle, the patient is just ready to slide down, so static friction is at its maximum and the forces on the patient balance.

SET UP: Take $+x$ to be down the incline. At the maximum angle $f_s=\mu_s n$ and $\Sigma F_x=ma_x=0$.

EXECUTE: The free-body diagram for the patient is given in Figure 5.25. $\Sigma F_y=ma_y$ gives $n=mg\cos\theta$. $\Sigma F_x=0$ gives $mg\sin\theta-\mu_s n=0$. $mg\sin\theta-\mu_s mg\cos\theta=0$. $\tan\theta=\mu_s$ so $\theta=50°$.

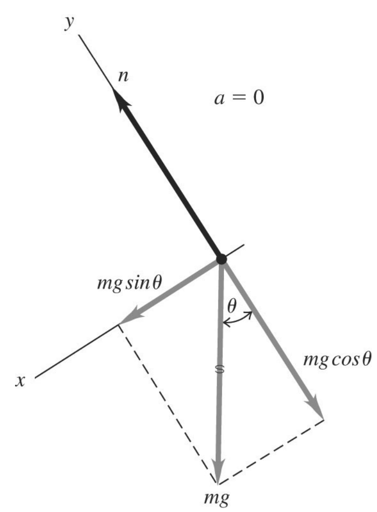


Figure 5.25

EVALUATE: A larger angle of tilt would cause more blood to flow to the brain, but it would also cause the patient to slide down the bed.

5.26. **IDENTIFY:** $f_s\le\mu_s n$ and $f_k=\mu_k n$. The normal force n is determined by applying $\Sigma\vec{F}=m\vec{a}$ to the block. Normally, $\mu_k\le\mu_s$. f_s is only as large as it needs to be to prevent relative motion between the two surfaces.

SET UP: Since the table is horizontal, with only the block present $n=135$ N. With the brick on the block, $n=270$ N.

EXECUTE: **(a)** The friction is static for $P=0$ to $P=75.0$ N. The friction is kinetic for $P>75.0$ N.

(b) The maximum value of f_s is $\mu_s n$. From the graph the maximum f_s is $f_s = 75.0\text{ N}$, so $\mu_s = \frac{\max f_s}{n} = \frac{75.0\text{ N}}{135\text{ N}} = 0.556$. $f_k = \mu_k n$. From the graph, $f_k = 50.0\text{ N}$ and $\mu_k = \frac{f_k}{n} = \frac{50.0\text{ N}}{135\text{ N}} = 0.370$.

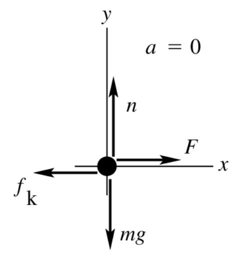
(c) When the block is moving the friction is kinetic and has the constant value $f_k = \mu_k n$, independent of P. This is why the graph is horizontal for $P > 75.0\text{ N}$. When the block is at rest, $f_s = P$ since this prevents relative motion. This is why the graph for $P < 75.0\text{ N}$ has slope $+1$.

(d) max f_s and f_k would double. The values of *f* on the vertical axis would double but the shape of the graph would be unchanged.

EVALUATE: The coefficients of friction are independent of the normal force.

5.27. **(a) IDENTIFY:** Constant speed implies $a = 0$. Apply Newton's first law to the box. The friction force is directed opposite to the motion of the box.

SET UP: Consider the free-body diagram for the box, given in Figure 5.27a. Let $\vec{F}$ be the horizontal force applied by the worker. The friction is kinetic friction since the box is sliding along the surface.



EXECUTE:

$\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

so $f_k = \mu_k n = \mu_k mg$

Figure 5.27a

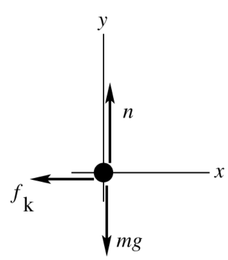
$\Sigma F_x = ma_x$

$F - f_k = 0$

$F = f_k = \mu_k mg = (0.20)(16.8\text{ kg})(9.80\text{ m/s}^2) = 33\text{ N}$

(b) IDENTIFY: Now the only horizontal force on the box is the kinetic friction force. Apply Newton's second law to the box to calculate its acceleration. Once we have the acceleration, we can find the distance using a constant acceleration equation. The friction force is $f_k = \mu_k mg$, just as in part (a).

SET UP: The free-body diagram is sketched in Figure 5.27b.



EXECUTE:

$\Sigma F_x = ma_x$

$-f_k = ma_x$

$-\mu_k mg = ma_x$

$a_x = -\mu_k g = -(0.20)(9.80\text{ m/s}^2) = -1.96\text{ m/s}^2$

Figure 5.27b

Use the constant acceleration equations to find the distance the box travels:

$v_x = 0$, $v_{0x} = 3.50\text{ m/s}$, $a_x = -1.96\text{ m/s}^2$, $x - x_0 = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (3.50\text{ m/s})^2}{2(-1.96\text{ m/s}^2)} = 3.1\text{ m}$$

EVALUATE: The normal force is the component of force exerted by a surface perpendicular to the surface. Its magnitude is determined by $\Sigma\vec{F} = m\vec{a}$. In this case n and mg are the only vertical forces and $a_y = 0$, so $n = mg$. Also note that f_k and n are proportional in magnitude but perpendicular in direction.

5.28. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box.

SET UP: Since the only vertical forces are n and w, the normal force on the box equals its weight. Static friction is as large as it needs to be to prevent relative motion between the box and the surface, up to its maximum possible value of $f_s^{max} = \mu_s n$. If the box is sliding then the friction force is $f_k = \mu_k n$.

EXECUTE: **(a)** If there is no applied force, no friction force is needed to keep the box at rest.

(b) $f_s^{max} = \mu_s n = (0.40)(40.0\text{ N}) = 16.0\text{ N}$. If a horizontal force of 6.0 N is applied to the box, then $f_s = 6.0\text{ N}$ in the opposite direction.

(c) The monkey must apply a force equal to f_s^{max}, 16.0 N.

(d) Once the box has started moving, a force equal to $f_k = \mu_k n = 8.0\text{ N}$ is required to keep it moving at constant velocity.

(e) $f_k = 8.0\text{ N}$. $a = (18.0\text{ N} - 8.0\text{ N})/(40.0\text{ N}/9.80\text{ m/s}^2) = 2.45\text{ m/s}^2$

EVALUATE: $\mu_k < \mu_s$ and less force must be applied to the box to maintain its motion than to start it moving.

5.29. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the crate. $f_s \le \mu_s n$ and $f_k = \mu_k n$.

SET UP: Let $+y$ be upward and let $+x$ be in the direction of the push. Since the floor is horizontal and the push is horizontal, the normal force equals the weight of the crate: $n = mg = 441\text{ N}$. The force it takes to start the crate moving equals max f_s and the force required to keep it moving equals f_k.

EXECUTE: **(a)** max $f_s = 313\text{ N}$, so $\mu_s = \dfrac{313\text{ N}}{441\text{ N}} = 0.710$. $f_k = 208\text{ N}$, so $\mu_k = \dfrac{208\text{ N}}{441\text{ N}} = 0.472$.

(b) The friction is kinetic. $\Sigma F_x = ma_x$ gives $F - f_k = ma$ and

$F = f_k + ma = 208\text{ N} + (45.0\text{ kg})(1.10\text{ m/s}^2) = 258\text{ N}$.

(c) (i) The normal force now is $mg = 72.9\text{ N}$. To cause it to move,

$F = \max f_s = \mu_s n = (0.710)(72.9\text{ N}) = 51.8\text{ N}$.

(ii) $F = f_k + ma$ and $a = \dfrac{F - f_k}{m} = \dfrac{258\text{ N} - (0.472)(72.9\text{ N})}{45.0\text{ kg}} = 4.97\text{ m/s}^2$

EVALUATE: The kinetic friction force is independent of the speed of the object. On the moon, the mass of the crate is the same as on earth, but the weight and normal force are less.

5.30. **IDENTIFY:** Newton's second law applies to the rocks on the hill. When they are moving, kinetic friction acts on them, but when they are at rest, static friction acts.

SET UP: Use coordinates with axes parallel and perpendicular to the incline, with $+x$ in the direction of the acceleration. $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y = 0$.

EXECUTE: With the rock sliding up the hill, the friction force is down the hill. The free-body diagram is given in Figure 5.30a.

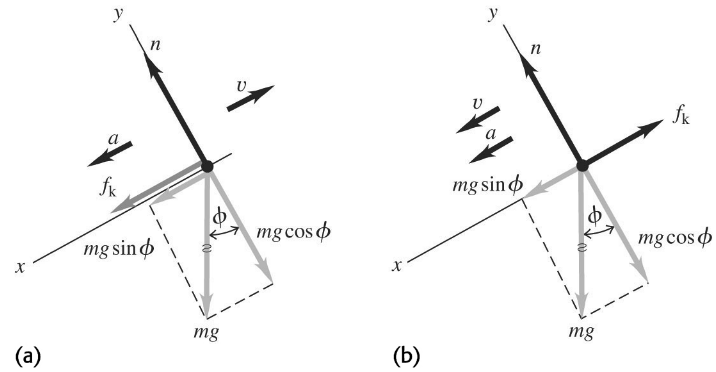


Figure 5.30

$\Sigma F_y = ma_y = 0$ gives $n = mg\cos\phi$ and $f_k = \mu_k n = \mu_k mg\cos\phi$. $\Sigma F_x = ma_x$ gives $mg\sin\phi + \mu_k mg\cos\phi = ma$.

$a = g(\sin\phi + \mu_k\cos\phi) = (9.80\text{ m/s}^2)[\sin 36° + (0.45)\cos 36°]$. $a = 9.33\text{ m/s}^2$, down the incline.

(b) The component of gravity down the incline is $mg\sin\phi = 0.588mg$. The maximum possible static friction force is $f_s = \mu_s n = \mu_s mg\cos\phi = 0.526mg$. f_s can't be as large as $mg\sin\phi$ and the rock slides back down. As the rock slides down, f_k is up the incline. The free-body diagram is given in Figure 5.30b. $\Sigma F_y = ma_y = 0$ gives $n = mg\cos\phi$ and $f_k = \mu_k n = \mu_k mg\cos\phi$. $\Sigma F_x = ma_x$ gives $mg\sin\phi - \mu_k mg\cos\phi = ma$, so $a = g(\sin\phi - \mu_k\cos\phi) = 2.19\text{ m/s}^2$, down the incline.

EVALUATE: The acceleration down the incline in (a) is greater than that in (b) because in (a) the static friction and gravity are both acting down the incline, whereas in (b) friction is up the incline, opposing gravity which still acts down the incline.

5.31. **IDENTIFY:** A 10.0-kg box is pushed on a ramp, causing it to accelerate. Newton's second law applies.

SET UP: Choose the *x*-axis along the surface of the ramp and the *y*-axis perpendicular to the surface. The only acceleration of the box is in the *x*-direction, so $\Sigma F_x = ma_x$ and $\Sigma F_y = 0$. The external forces acting on the box are the push *P* along the surface of the ramp, friction f_k, gravity *mg*, and the normal force *n*. The ramp rises at 55.0° above the horizontal, and $f_k = \mu_k n$. The friction force opposes the sliding, so it is directed up the ramp in part (a) and down the ramp in part (b).

EXECUTE: **(a)** Applying $\Sigma F_y = 0$ gives $n = mg\cos(55.0°)$, so the force of kinetic friction is $f_k = \mu_k n = (0.300)(10.0\text{ kg})(9.80\text{ m/s}^2)(\cos 55.0°) = 16.86\text{ N}$. Call the +*x*-direction down the ramp since that is the direction of the acceleration of the box. Applying $\Sigma F_x = ma_x$ gives $P + mg\sin(55.0°) - f_k = ma$. Putting in the numbers gives $(10.0\text{ kg})a = 120\text{ N} + (98.0\text{ N})(\sin 55.0°) - 16.86\text{ N}$; $a = 18.3\text{ m/s}^2$.

(b) Now *P* is up the up the ramp and f_k is down the ramp, but the other force components are unchanged, so $f_k = 16.86\text{ N}$ as before. We now choose +*x* to be up the ramp, so $\Sigma F_x = ma_x$ gives $P - mg\sin(55.0°) - f_k = ma$. Putting in the same numbers as before gives $a = 2.29\text{ m/s}^2$.

EVALUATE: Pushing up the ramp produces a much smaller acceleration than pushing down the ramp because gravity helps the downward push but opposes the upward push.

5.32. **IDENTIFY:** For the shortest time, the acceleration is a maximum, so the toolbox is just ready to slide relative to the bed of the truck. The box is at rest relative to the truck, but it is accelerating relative to the ground because the truck is accelerating. Therefore Newton's second law will be useful.
SET UP: If the truck accelerates to the right the static friction force on the box is to the right, to try to prevent the box from sliding relative to the truck. The free-body diagram for the box is given in

Figure 5.32. The maximum acceleration of the box occurs when f_s has its maximum value, so $f_s = \mu_s n$. If the box doesn't slide, its acceleration equals the acceleration of the truck. The constant-acceleration equation $v_x = v_{0x} + a_x t$ applies.

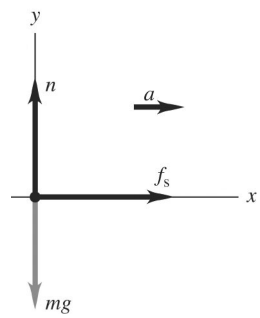


Figure 5.32

EXECUTE: $n = mg$. $\Sigma F_x = ma_x$ gives $f_s = ma$ so $\mu_s mg = ma$ and $a = \mu_s g = 6.37\ \text{m/s}^2$. $v_{0x} = 0$, $v_x = 30.0\ \text{m/s}$. $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{30.0\ \text{m/s} - 0}{6.37\ \text{m/s}^2} = 4.71\ \text{s}$.

EVALUATE: If the truck has a smaller acceleration it is still true that $f_s = ma$, but now $f_s < \mu_s n$.

5.33. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the composite object consisting of the two boxes and to the top box. The friction the ramp exerts on the lower box is kinetic friction. The upper box doesn't slip relative to the lower box, so the friction between the two boxes is static. Since the speed is constant the acceleration is zero.

SET UP: Let $+x$ be up the incline. The free-body diagrams for the composite object and for the upper box are given in Figure 5.33. The slope angle ϕ of the ramp is given by $\tan\phi = \dfrac{2.50\ \text{m}}{4.75\ \text{m}}$, so $\phi = 27.76°$. Since the boxes move down the ramp, the kinetic friction force exerted on the lower box by the ramp is directed up the incline. To prevent slipping relative to the lower box the static friction force on the upper box is directed up the incline. $m_{\text{tot}} = 32.0\ \text{kg} + 48.0\ \text{kg} = 80.0\ \text{kg}$.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the composite object gives $n_{\text{tot}} = m_{\text{tot}} g\cos\phi$ and $f_k = \mu_k m_{\text{tot}} g\cos\phi$. $\Sigma F_x = ma_x$ gives $f_k + T - m_{\text{tot}} g\sin\phi = 0$ and

$T = (\sin\phi - \mu_k\cos\phi)m_{\text{tot}}g = (\sin 27.76° - [0.444]\cos 27.76°)(80.0\ \text{kg})(9.80\ \text{m/s}^2) = 57.1\ \text{N}$.

The person must apply a force of 57.1 N, directed up the ramp.

(b) $\Sigma F_x = ma_x$ applied to the upper box gives $f_s = mg\sin\phi = (32.0\ \text{kg})(9.80\ \text{m/s}^2)\sin 27.76° = 146\ \text{N}$, directed up the ramp.

EVALUATE: For each object the net force is zero.

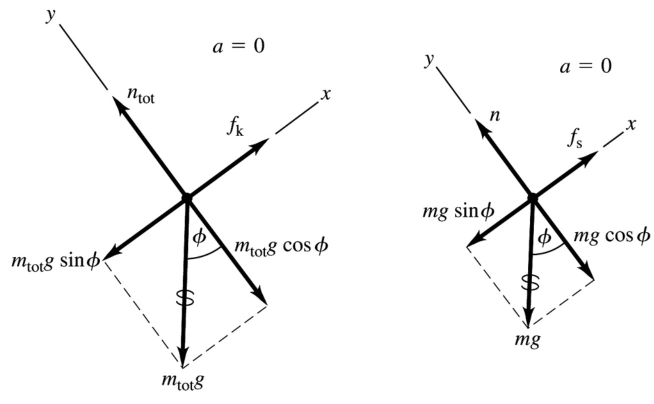


Figure 5.33

5.34. **IDENTIFY:** Constant speed means zero acceleration for each block. If the block is moving, the friction force the tabletop exerts on it is kinetic friction. Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: The free-body diagrams and choice of coordinates for each block are given by Figure 5.34. $m_A = 4.59$ kg and $m_B = 2.55$ kg.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ with $a_y = 0$ applied to block B gives $m_B g - T = 0$ and $T = 25.0$ N. $\Sigma F_x = ma_x$ with $a_x = 0$ applied to block A gives $T - f_k = 0$ and $f_k = 25.0$ N. $n_A = m_A g = 45.0$ N and $\mu_k = \dfrac{f_k}{n_A} = \dfrac{25.0\text{ N}}{45.0\text{ N}} = 0.556.$

(b) Now let A be block A plus the cat, so $m_A = 9.18$ kg. $n_A = 90.0$ N and $f_k = \mu_k n = (0.556)(90.0\text{ N}) = 50.0$ N. $\sum F_x = ma_x$ for A gives $T - f_k = m_A a_x$. $\sum F_y = ma_y$ for block B gives $m_B g - T = m_B a_y$. a_x for A equals a_y for B, so adding the two equations gives $m_B g - f_k = (m_A + m_B)a_y$ and $a_y = \dfrac{m_B g - f_k}{m_A + m_B} = \dfrac{25.0\text{ N} - 50.0\text{ N}}{9.18\text{ kg} + 2.55\text{ kg}} = -2.13\text{ m/s}^2$. The acceleration is upward and block B slows down.

EVALUATE: The equation $m_B g - f_k = (m_A + m_B)a_y$ has a simple interpretation. If both blocks are considered together then there are two external forces: $m_B g$ that acts to move the system one way and f_k that acts oppositely. The net force of $m_B g - f_k$ must accelerate a total mass of $m_A + m_B$.

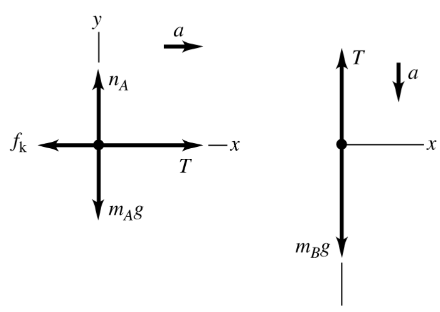


Figure 5.34

5.35. **IDENTIFY:** Use $\Sigma\vec{F} = m\vec{a}$ to find the acceleration that can be given to the car by the kinetic friction force. Then use a constant acceleration equation.

SET UP: Take $+x$ in the direction the car is moving.

EXECUTE: **(a)** The free-body diagram for the car is shown in Figure 5.35. $\Sigma F_y = ma_y$ gives $n = mg$. $\Sigma F_x = ma_x$ gives $-\mu_k n = ma_x$. $-\mu_k mg = ma_x$ and $a_x = -\mu_k g$. Then $v_x = 0$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = -\dfrac{v_{0x}^2}{2a_x} = +\dfrac{v_{0x}^2}{2\mu_k g} = \dfrac{(28.7\text{ m/s})^2}{2(0.80)(9.80\text{ m/s}^2)} = 52.5\text{ m}$.

(b) $v_{0x} = \sqrt{2\mu_k g(x - x_0)} = \sqrt{2(0.25)(9.80\text{ m/s}^2)52.5\text{ m}} = 16.0\text{ m/s}$

EVALUATE: For constant stopping distance $\dfrac{v_{0x}^2}{\mu_k}$ is constant and v_{0x} is proportional to $\sqrt{\mu_k}$. The answer to part (b) can be calculated as $(28.7\text{ m/s})\sqrt{0.25/0.80} = 16.0\text{ m/s}$.

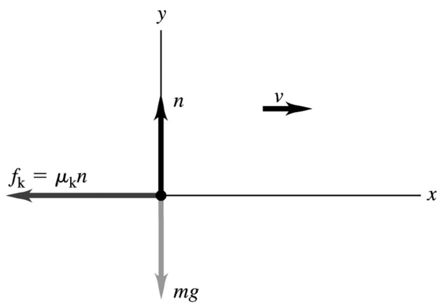


Figure 5.35

5.36. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box. When the box is ready to slip the static friction force has its maximum possible value, $f_s = \mu_s n$.

SET UP: Use coordinates parallel and perpendicular to the ramp.

EXECUTE: **(a)** The normal force will be $w\cos\alpha$ and the component of the gravitational force along the ramp is $w\sin\alpha$. The box begins to slip when $w\sin\alpha > \mu_s w\cos\alpha$, or $\tan\alpha > \mu_s = 0.35$, so slipping occurs at $\alpha = \arctan(0.35) = 19.3°$.

(b) When moving, the friction force along the ramp is $\mu_k w\cos\alpha$, the component of the gravitational force along the ramp is $w\sin\alpha$, so the acceleration is

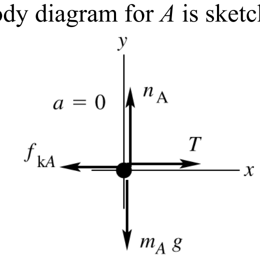
$$(w\sin\alpha - w\mu_k\cos\alpha)/m = g(\sin\alpha - \mu_k\cos\alpha) = 0.92\text{ m/s}^2.$$

(c) Since $v_{0x} = 0$, $2ax = v^2$, so $v = (2ax)^{1/2}$, or $v = [(2)(0.92\text{ m/s}^2)(5\text{ m})]^{1/2} = 3\text{ m/s}$.

EVALUATE: When the box starts to move, friction changes from static to kinetic and the friction force becomes smaller.

5.37. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each crate. The rope exerts force T to the right on crate A and force T to the left on crate B. The target variables are the forces T and F. Constant v implies $a = 0$.

SET UP: The free-body diagram for A is sketched in Figure 5.37a.



EXECUTE:

$\Sigma F_y = ma_y$

$n_A - m_A g = 0$

$n_A = m_A g$

$f_{kA} = \mu_k n_A = \mu_k m_A g$

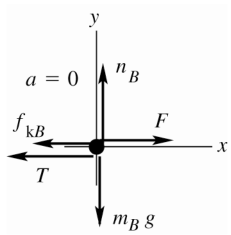
Figure 5.37a

$\Sigma F_x = ma_x$

$T - f_{kA} = 0$

$T = \mu_k m_A g$

SET UP: The free-body diagram for B is sketched in Figure 5.37b.



EXECUTE:
$\Sigma F_y = ma_y$
$n_B - m_B g = 0$
$n_B = m_B g$
$f_{kB} = \mu_k n_B = \mu_k m_B g$

Figure 5.37b

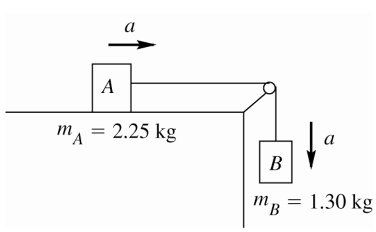
$\Sigma F_x = ma_x$
$F - T - f_{kB} = 0$
$F = T + \mu_k m_B g$
Use the first equation to replace T in the second:
$F = \mu_k m_A g + \mu_k m_B g.$
(a) $F = \mu_k (m_A + m_B)g$
(b) $T = \mu_k m_A g$
EVALUATE: We can also consider both crates together as a single object of mass $(m_A + m_B)$. $\Sigma F_x = ma_x$ for this combined object gives $F = f_k = \mu_k (m_A + m_B)g$, in agreement with our answer in part (a).

5.38. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the box.
SET UP: Let $+y$ be upward and $+x$ be horizontal, in the direction of the acceleration. Constant speed means $a = 0$.
EXECUTE: **(a)** There is no net force in the vertical direction, so $n + F\sin\theta - w = 0$, or $n = w - F\sin\theta = mg - F\sin\theta$. The friction force is $f_k = \mu_k n = \mu_k (mg - F\sin\theta)$. The net horizontal force is $F\cos\theta - f_k = F\cos\theta - \mu_k (mg - F\sin\theta)$, and so at constant speed,

$$F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

(b) Using the given values, $F = \dfrac{(0.35)(90\text{ kg})(9.80\text{ m/s}^2)}{(\cos 25° + (0.35)\sin 25°)} = 290\text{ N}.$
EVALUATE: If $\theta = 0°$, $F = \mu_k mg$.

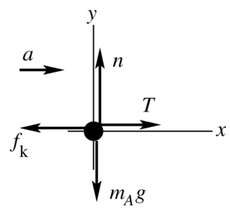
5.39. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to each block. The target variables are the tension T in the cord and the acceleration a of the blocks. Then a can be used in a constant acceleration equation to find the speed of each block. The magnitude of the acceleration is the same for both blocks.
SET UP: The system is sketched in Figure 5.39a.



For each block take a positive coordinate direction to be the direction of the block's acceleration.

Figure 5.39a

block on the table: The free-body is sketched in Figure 5.39b (next page).

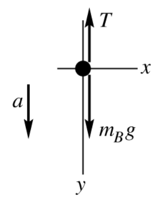


EXECUTE:
$\Sigma F_y = ma_y$
$n - m_A g = 0$
$n = m_A g$
$f_k = \mu_k n = \mu_k m_A g$

Figure 5.39b

$\Sigma F_x = ma_x$
$T - f_k = m_A a$
$T - \mu_k m_A g = m_A a$

SET UP: <u>hanging block</u>: The free-body is sketched in Figure 5.39c.



EXECUTE:
$\Sigma F_y = ma_y$
$m_B g - T = m_B a$
$T = m_B g - m_B a$

Figure 5.39c

(a) Use the second equation in the first

$$m_B g - m_B a - \mu_k m_A g = m_A a$$

$$(m_A + m_B)a = (m_B - \mu_k m_A)g$$

$$a = \frac{(m_B - \mu_k m_A)g}{m_A + m_B} = \frac{(1.30\text{ kg} - (0.45)(2.25\text{ kg}))(9.80\text{ m/s}^2)}{2.25\text{ kg} + 1.30\text{ kg}} = 0.7937\text{ m/s}^2$$

SET UP: Now use the constant acceleration equations to find the final speed. Note that the blocks have the same speeds. $x - x_0 = 0.0300$ m, $a_x = 0.7937\text{ m/s}^2$, $v_{0x} = 0$, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

EXECUTE: $v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.7937\text{ m/s}^2)(0.0300\text{ m})} = 0.218\text{ m/s} = 21.8\text{ cm/s}.$

(b) $T = m_B g - m_B a = m_B(g - a) = 1.30\text{ kg}(9.80\text{ m/s}^2 - 0.7937\text{ m/s}^2) = 11.7\text{ N}$

Or, to check, $T - \mu_k m_A g = m_A a.$

$T = m_A(a + \mu_k g) = 2.25\text{ kg}(0.7937\text{ m/s}^2 + (0.45)(9.80\text{ m/s}^2)) = 11.7\text{ N},$ which checks.

EVALUATE: The force T exerted by the cord has the same value for each block. $T < m_B g$ since the hanging block accelerates downward. Also, $f_k = \mu_k m_A g = 9.92$ N. $T > f_k$ and the block on the table accelerates in the direction of T.

5.40. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the ball. At the terminal speed, $f = mg$.

SET UP: The fluid resistance is directed opposite to the velocity of the object. At half the terminal speed, the magnitude of the frictional force is one-fourth the weight.

EXECUTE: **(a)** If the ball is moving up, the frictional force is down, so the magnitude of the net force is $(5/4)w$ and the acceleration is $(5/4)g$, down.

(b) While moving down, the frictional force is up, and the magnitude of the net force is $(3/4)w$ and the acceleration is $(3/4)g$, down.

EVALUATE: The frictional force is less than mg in each case and in each case the net force is downward and the acceleration is downward.

5.41. **(a) IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the crate. Constant v implies $a = 0$. Crate moving says that the friction is kinetic friction. The target variable is the magnitude of the force applied by the woman.

SET UP: The free-body diagram for the crate is sketched in Figure 5.41.

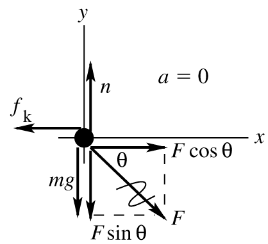


Figure 5.41

EXECUTE:

$\Sigma F_y = ma_y$

$n - mg - F\sin\theta = 0$

$n = mg + F\sin\theta$

$f_k = \mu_k n = \mu_k mg + \mu_k F\sin\theta$

$\Sigma F_x = ma_x$

$F\cos\theta - f_k = 0$

$F\cos\theta - \mu_k mg - \mu_k F\sin\theta = 0$

$F(\cos\theta - \mu_k\sin\theta) = \mu_k mg$

$$F = \frac{\mu_k mg}{\cos\theta - \mu_k\sin\theta}$$

(b) IDENTIFY and SET UP: "Start the crate moving" means the same force diagram as in part (a), except that μ_k is replaced by μ_s. Thus $F = \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$.

EXECUTE: $F \to \infty$ if $\cos\theta - \mu_s\sin\theta = 0$. This gives $\mu_s = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$.

EVALUATE: $\vec{F}$ has a downward component so $n > mg$. If $\theta = 0$ (woman pushes horizontally), $n = mg$ and $F = f_k = \mu_k mg$.

5.42. **IDENTIFY and SET UP:** Apply $v_t = \sqrt{\dfrac{mg}{D}}$.

EXECUTE: **(a)** Solving for D in terms of v_t, $D = \dfrac{mg}{v_t^2} = \dfrac{(80\text{ kg})(9.80\text{ m/s}^2)}{(42\text{ m/s})^2} = 0.44\text{ kg/m}$.

(b) $v_t = \sqrt{\dfrac{mg}{D}} = \sqrt{\dfrac{(45\text{ kg})(9.80\text{ m/s}^2)}{(0.25\text{ kg/m})}} = 42\text{ m/s}$.

EVALUATE: "Terminal speed depends on the mass of the falling object."

5.43. **IDENTIFY:** Since the stone travels in a circular path, its acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle. The only horizontal force on the stone is the tension of the string. Set the tension in the string equal to its maximum value.

SET UP: $\sum F_x = ma_x$ gives $T = m\dfrac{v^2}{R}$.

EXECUTE: **(a)** The free-body diagram for the stone is given in Figure 5.43 (next page). In the diagram the stone is at a point to the right of the center of the path.

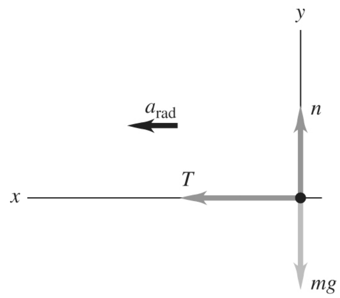


Figure 5.43

(b) Solving for v gives $v = \sqrt{\dfrac{TR}{m}} = \sqrt{\dfrac{(60.0\text{ N})(0.90\text{ m})}{0.80\text{ kg}}} = 8.2\text{ m/s}.$

EVALUATE: The tension is directed toward the center of the circular path of the stone. Gravity plays no role in this case because it is a vertical force and the acceleration is horizontal.

5.44. **IDENTIFY:** The wrist exerts a force on the hand causing the hand to move in a horizontal circle. Newton's second law applies to the hand.

SET UP: Each hand travels in a circle of radius 0.750 m and has mass (0.0125)(52 kg) = 0.65 kg and weight 6.4 N. The period for each hand is $T = (1.0\text{ s})/(2.0) = 0.50\text{ s}.$ Let $+x$ be toward the center of the circular path. The speed of the hand is $v = 2\pi R/T$, the radial acceleration is $a_{rad} = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$, and $\sum F_x = ma_x = ma_{rad}$.

EXECUTE: **(a)** The free-body diagram for one hand is given in Figure 5.44. $\vec{F}$ is the force exerted on the hand by the wrist. This force has both horizontal and vertical components.

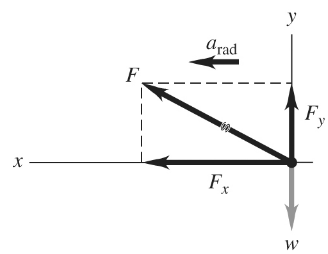


Figure 5.44

(b) $a_{rad} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(0.750\text{ m})}{(0.50\text{ s})^2} = 118\text{ m/s}^2$, so $F_x = ma_{rad} = (0.65\text{ kg})(118\text{ m/s}^2) = 77\text{ N}.$

(c) $\dfrac{F}{w} = \dfrac{77\text{ N}}{6.4\text{ N}} = 12,$ so the horizontal force from the wrist is 12 times the weight of the hand.

EVALUATE: The wrist must also exert a vertical force on the hand equal to the weight of the hand.

5.45. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car. It has acceleration $\vec{a}_{rad}$, directed toward the center of the circular path.

SET UP: The analysis is the same as in Example 5.23.

EXECUTE: **(a)** $F_A = m\left(g + \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 + \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = 61.8\text{ N}.$

(b) $F_B = m\left(g - \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 - \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = -30.4\text{ N},$ where the minus sign indicates that the track pushes down on the car. The magnitude of this force is 30.4 N.

EVALUATE: $|F_A| > |F_B|$. $|F_A| - 2mg = |F_B|$.

5.46. **IDENTIFY:** The acceleration of the car at the top and bottom is toward the center of the circle, and Newton's second law applies to it.

SET UP: Two forces are acting on the car, gravity and the normal force. At point B (the top), both forces are toward the center of the circle, so Newton's second law gives $mg + n_B = ma$. At point A (the bottom), gravity is downward but the normal force is upward, so $n_A - mg = ma$.

EXECUTE: Solving the equation at B for the acceleration gives

$a = \dfrac{mg + n_B}{m} = \dfrac{(0.800\text{ kg})(9.8\text{ m/s}^2) + 6.00\text{ N}}{0.800\text{ kg}} = 17.3\text{ m/s}^2$. Solving the equation at A for the normal force gives $n_A = m(g + a) = (0.800\text{ kg})(9.8\text{ m/s}^2 + 17.3\text{ m/s}^2) = 21.7\text{ N}$.

EVALUATE: The normal force at the bottom is greater than at the top because it must balance the weight in addition to accelerate the car toward the center of its track.

5.47. **IDENTIFY:** A model car travels in a circle so it has radial acceleration, and Newton's second law applies to it.

SET UP: We use $\Sigma\vec{F} = m\vec{a}$, where the acceleration is $a_{\text{rad}} = \dfrac{v^2}{R}$ and the time T for one revolution is $T = 2\pi R/v$.

EXECUTE: At the bottom of the track, taking $+y$ upward, $\Sigma\vec{F} = m\vec{a}$ gives $n - mg = ma$, where n is the normal force. This gives $2.50mg - mg = ma$, so $a = 1.50g$. The acceleration is $a_{\text{rad}} = \dfrac{v^2}{R}$, so

$v = \sqrt{aR} = \sqrt{(1.50)(9.80\text{ m/s}^2)(5.00\text{ m})} = 8.573$ m/s, so $T = 2\pi R/v = 2\pi(5.00\text{ m})/(8.573\text{ m}) = 3.66$ s.

EVALUATE: We never need the mass of the car because we know the acceleration as a fraction of g and the force as a fraction of mg.

5.48. **IDENTIFY:** Since the car travels in an arc of a circle, it has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the arc. The only horizontal force on the car is the static friction force exerted by the roadway. To calculate the minimum coefficient of friction that is required, set the static friction force equal to its maximum value, $f_s = \mu_s n$. Friction is static friction because the car is not sliding in the radial direction.

SET UP: The free-body diagram for the car is given in Figure 5.48 (next page). The diagram assumes the center of the curve is to the left of the car.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $n = mg$. $\Sigma F_x = ma_x$ gives $\mu_s n = m\dfrac{v^2}{R}$. $\mu_s mg = m\dfrac{v^2}{R}$ and

$$\mu_s = \frac{v^2}{gR} = \frac{(25.0\text{ m/s})^2}{(9.80\text{ m/s}^2)(170\text{ m})} = 0.375$$

(b) $\dfrac{v^2}{\mu_s} = Rg = \text{constant}$, so $\dfrac{v_1^2}{\mu_{s1}} = \dfrac{v_2^2}{\mu_{s2}}$. $v_2 = v_1\sqrt{\dfrac{\mu_{s2}}{\mu_{s1}}} = (25.0\text{ m/s})\sqrt{\dfrac{\mu_{s1}/3}{\mu_{s1}}} = 14.4\text{ m/s}$.

EVALUATE: A smaller coefficient of friction means a smaller maximum friction force, a smaller possible acceleration and therefore a smaller speed.

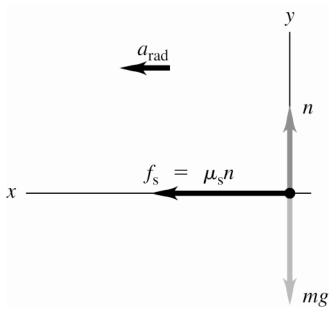


Figure 5.48

5.49. **IDENTIFY:** Apply Newton's second law to the car in circular motion, assume friction is negligible.

SET UP: The acceleration of the car is $a_{rad} = v^2/R$. As shown in the text, the banking angle β is given by $\tan\beta = \frac{v^2}{gR}$. Also, $n = mg/\cos\beta$. 65.0 mi/h = 29.1 m/s.

EXECUTE: **(a)** $\tan\beta = \frac{(29.1\text{ m/s})^2}{(9.80\text{ m/s}^2)(225\text{ m})}$ and $\beta = 21.0°$. The expression for $\tan\beta$ does not involve the mass of the vehicle, so the truck and car should travel at the same speed.

(b) For the car, $n_{car} = \frac{(1125\text{ kg})(9.80\text{ m/s}^2)}{\cos 21.0°} = 1.18\times10^4\text{ N}$ and $n_{truck} = 2n_{car} = 2.36\times10^4\text{ N}$, since $m_{truck} = 2m_{car}$.

EVALUATE: The vertical component of the normal force must equal the weight of the vehicle, so the normal force is proportional to *m*.

5.50. **IDENTIFY:** The acceleration of the person is $a_{rad} = v^2/R$, directed horizontally to the left in the figure in the problem. The time for one revolution is the period $T = \frac{2\pi R}{v}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the person.

SET UP: The person moves in a circle of radius $R = 3.00\text{ m} + (5.00\text{ m})\sin 30.0° = 5.50\text{ m}$. The free-body diagram is given in Figure 5.50. $\vec{F}$ is the force applied to the seat by the rod.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $F\cos 30.0° = mg$ and $F = \frac{mg}{\cos 30.0°}$. $\Sigma F_x = ma_x$ gives $F\sin 30.0° = m\frac{v^2}{R}$. Combining these two equations gives

$v = \sqrt{Rg\tan\theta} = \sqrt{(5.50\text{ m})(9.80\text{ m/s}^2)\tan 30.0°} = 5.58\text{ m/s}$. Then the period is

$T = \frac{2\pi R}{v} = \frac{2\pi(5.50\text{ m})}{5.58\text{ m/s}} = 6.19\text{ s}$.

(b) The net force is proportional to *m* so in $\Sigma\vec{F} = m\vec{a}$ the mass divides out and the angle for a given rate of rotation is independent of the mass of the passengers.

EVALUATE: The person moves in a horizontal circle so the acceleration is horizontal. The net inward force required for circular motion is produced by a component of the force exerted on the seat by the rod.

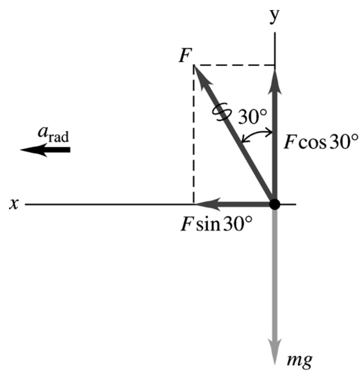


Figure 5.50

5.51. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of the person plus seat. This object moves in a horizontal circle and has acceleration a_{rad}, directed toward the center of the circle.

SET UP: The free-body diagram for the composite object is given in Figure 5.51. Let $+x$ be to the right, in the direction of $\vec{a}_{rad}$. Let $+y$ be upward. The radius of the circular path is $R = 7.50$ m. The total mass is $(255\text{ N} + 825\text{ N})/(9.80\text{ m/s}^2) = 110.2\text{ kg}$. Since the rotation rate is $28.0\text{ rev/min} = 0.4667\text{ rev/s}$, the period T is $\dfrac{1}{0.4667\text{ rev/s}} = 2.143\text{ s}$.

EXECUTE: $\Sigma F_y = ma_y$ gives $T_A\cos 40.0° - mg = 0$ and $T_A = \dfrac{mg}{\cos 40.0°} = \dfrac{255\text{ N} + 825\text{ N}}{\cos 40.0°} = 1410\text{ N}$.

$\Sigma F_x = ma_x$ gives $T_A\sin 40.0° + T_B = ma_{rad}$ and

$$T_B = m\frac{4\pi^2 R}{T^2} - T_A\sin 40.0° = (110.2\text{ kg})\frac{4\pi^2(7.50\text{ m})}{(2.143\text{ s})^2} - (1410\text{ N})\sin 40.0° = 6200\text{ N}$$

The tension in the horizontal cable is 6200 N and the tension in the other cable is 1410 N.

EVALUATE: The weight of the composite object is 1080 N. The tension in cable A is larger than this since its vertical component must equal the weight. The tension in cable B is less than ma_{rad} because part of the required inward force comes from a component of the tension in cable A.

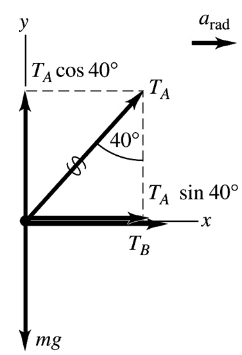


Figure 5.51

5.52. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the button. The button moves in a circle, so it has acceleration a_{rad}.

SET UP: We apply Newton's second law to the horizontal and vertical motion. Vertically we get $n = w$, and horizontally we get $\mu_s mg = mv^2/R$. Combining these equations gives $\mu_s = \dfrac{v^2}{Rg}$. Also, $v = 2\pi R/T$.

EXECUTE: **(a)** $\mu_s = \frac{v^2}{Rg}$. Expressing v in terms of the period T, $v = \frac{2\pi R}{T}$ so $\mu_s = \frac{4\pi^2 R}{T^2 g}$. A platform speed of 40.0 rev/min corresponds to a period of 1.50 s, so $\mu_s = \frac{4\pi^2 (0.220\text{ m})}{(1.50\text{ s})^2 (9.80\text{ m/s}^2)} = 0.394.$

(b) For the same coefficient of static friction, the maximum radius is proportional to the square of the period (longer periods mean slower speeds, so the button may be moved farther out) and so is inversely proportional to the square of the speed. Thus, at the higher speed, the maximum radius is

$(0.220\text{ m})\left(\frac{40.0}{60.0}\right)^2 = 0.0978\text{ m.}$

EVALUATE: $a_{rad} = \frac{4\pi^2 R}{T^2}$. The maximum radial acceleration that friction can give is $\mu_s mg$. At the faster rotation rate T is smaller so R must be smaller to keep a_{rad} the same.

5.53. **IDENTIFY:** The acceleration due to circular motion is $a_{rad} = \frac{4\pi^2 R}{T^2}$.

SET UP: $R = 400$ m. $1/T$ is the number of revolutions per second.

EXECUTE: **(a)** Setting $a_{rad} = g$ and solving for the period T gives

$$T = 2' \sqrt{\frac{R}{g}} = 2' \sqrt{\frac{400\text{ m}}{9.80\text{ m/s}^2}} = 40.1\text{ s,}$$

so the number of revolutions per minute is $(60\text{ s/min})/(40.1\text{ s}) = 1.5\text{ rev/min}$.

(b) The lower acceleration corresponds to a longer period, and hence a lower rotation rate, by a factor of the square root of the ratio of the accelerations, $T' = (1.5\text{ rev/min}) \times \sqrt{3.70/9.8} = 0.92\text{ rev/min}$.

EVALUATE: In part (a) the tangential speed of a point at the rim is given by $a_{rad} = \frac{v^2}{R}$, so $v = \sqrt{Ra_{rad}} = \sqrt{Rg} = 62.6\text{ m/s}$; the space station is rotating rapidly.

5.54. **IDENTIFY:** $T = \frac{2\pi R}{v}$. The apparent weight of a person is the normal force exerted on him by the seat he is sitting on. His acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle.

SET UP: The period is $T = 60.0$ s. The passenger has mass $m = w/g = 90.0$ kg.

EXECUTE: **(a)** $v = \frac{2\pi R}{T} = \frac{2\pi(50.0\text{ m})}{60.0\text{ s}} = 5.24\text{ m/s}$. Note that $a_{rad} = \frac{v^2}{R} = \frac{(5.24\text{ m/s})^2}{50.0\text{ m}} = 0.549\text{ m/s}^2$.

(b) The free-body diagram for the person at the top of his path is given in Figure 5.54a. The acceleration is downward, so take $+y$ downward. $\Sigma F_y = ma_y$ gives $mg - n = ma_{rad}$.

$n = m(g - a_{rad}) = (90.0\text{ kg})(9.80\text{ m/s}^2 - 0.549\text{ m/s}^2) = 833\text{ N}$.

The free-body diagram for the person at the bottom of his path is given in Figure 5.54b. The acceleration is upward, so take $+y$ upward. $\Sigma F_y = ma_y$ gives $n - mg = ma_{rad}$ and $n = m(g + a_{rad}) = 931\text{ N}$.

(c) Apparent weight $= 0$ means $n = 0$ and $mg = ma_{rad}$. $g = \frac{v^2}{R}$ and $v = \sqrt{gR} = 22.1\text{ m/s}$. The time for one revolution would be $T = \frac{2\pi R}{v} = \frac{2\pi(50.0\text{ m})}{22.1\text{ m/s}} = 14.2\text{ s}$. Note that $a_{rad} = g$.

(d) $n = m(g + a_{rad}) = 2mg = 2(882\text{ N}) = 1760\text{ N}$, twice his true weight.

EVALUATE: At the top of his path his apparent weight is less than his true weight and at the bottom of his path his apparent weight is greater than his true weight.

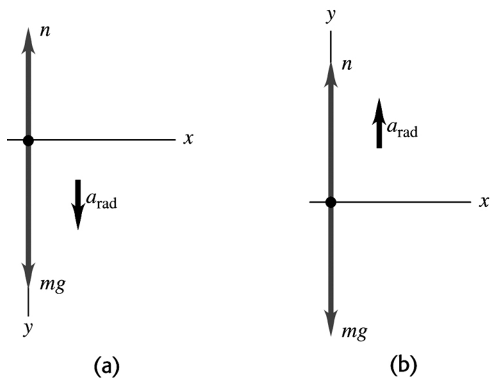
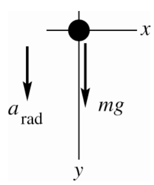


Figure 5.54

5.55. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the motion of the pilot. The pilot moves in a vertical circle. The apparent weight is the normal force exerted on him. At each point $\vec{a}_{rad}$ is directed toward the center of the circular path.

(a) SET UP: "the pilot feels weightless" means that the vertical normal force n exerted on the pilot by the chair on which the pilot sits is zero. The force diagram for the pilot at the top of the path is given in Figure 5.55a.



EXECUTE:

$\Sigma F_y = ma_y$

$mg = ma_{rad}$

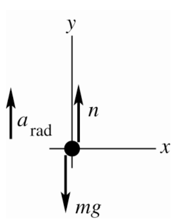
$g = \dfrac{v^2}{R}$

Figure 5.55a

Thus $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(150\text{ m})} = 38.34\text{ m/s}$

$$v = (38.34\text{ m/s})\left(\frac{1\text{ km}}{10^3\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 138\text{ km/h}$$

(b) SET UP: The force diagram for the pilot at the bottom of the path is given in Figure 5.55b. Note that the vertical normal force exerted on the pilot by the chair on which the pilot sits is now upward.



EXECUTE:

$\Sigma F_y = ma_y$

$n - mg = m\dfrac{v^2}{R}$

$n = mg + m\dfrac{v^2}{R}$

This normal force is the pilot's apparent weight.

Figure 5.55b

$w = 700\text{ N},$ so $m = \dfrac{w}{g} = 71.43\text{ kg}$

$$v = (280\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right) = 77.78\text{ m/s}$$

Thus $n = 700\text{ N} + 71.43\text{ kg}\dfrac{(77.78\text{ m/s})^2}{150\text{ m}} = 3580\text{ N}.$

EVALUATE: In part (b), $n > mg$ since the acceleration is upward. The pilot feels he is much heavier than when at rest. The speed is not constant, but it is still true that $a_{\text{rad}} = v^2/R$ at each point of the motion.

5.56. **IDENTIFY:** $a_{\text{rad}} = v^2/R,$ directed toward the center of the circular path. At the bottom of the dive, $\vec{a}_{\text{rad}}$ is upward. The apparent weight of the pilot is the normal force exerted on her by the seat on which she is sitting.

SET UP: The free-body diagram for the pilot is given in Figure 5.56.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{v^2}{R}$ gives $R = \dfrac{v^2}{a_{\text{rad}}} = \dfrac{(95.0\text{ m/s})^2}{4.00(9.80\text{ m/s}^2)} = 230\text{ m}.$

(b) $\Sigma F_y = ma_y$ gives $n - mg = ma_{\text{rad}}.$

$n = m(g + a_{\text{rad}}) = m(g + 4.00g) = 5.00mg = (5.00)(50.0\text{ kg})(9.80\text{ m/s}^2) = 2450\text{ N}$

EVALUATE: Her apparent weight is five times her true weight, the force of gravity the earth exerts on her.

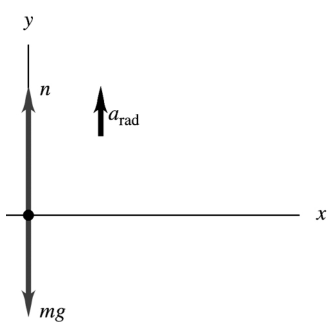
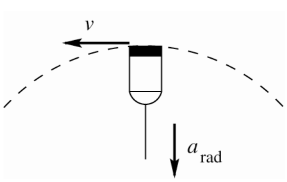


Figure 5.56

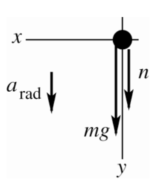
5.57. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the water. The water moves in a vertical circle. The target variable is the speed v; we will calculate a_{rad} and then get v from $a_{\text{rad}} = v^2/R.$

SET UP: Consider the free-body diagram for the water when the pail is at the top of its circular path, as shown in Figures 5.57a and b.



The radial acceleration is in toward the center of the circle so at this point is downward. n is the downward normal force exerted on the water by the bottom of the pail.

Figure 5.57a



EXECUTE:
$\Sigma F_y = ma_y$

$n + mg = m\dfrac{v^2}{R}$

Figure 5.57b

At the minimum speed the water is just ready to lose contact with the bottom of the pail, so at this speed, $n \to 0.$ (Note that the force n cannot be upward.)

With $n \to 0$ the equation becomes $mg = m\dfrac{v^2}{R}$. $v = \sqrt{gR} = \sqrt{(9.80 \text{ m/s}^2)(0.600 \text{ m})} = 2.42 \text{ m/s}$.

EVALUATE: At the minimum speed $a_{\text{rad}} = g$. If v is less than this minimum speed, gravity pulls the water (and bucket) out of the circular path.

5.58. **IDENTIFY:** The ball has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the circular path. When the ball is at the bottom of the swing, its acceleration is upward.

SET UP: Take $+y$ upward, in the direction of the acceleration. The bowling ball has mass $m = w/g = 7.27 \text{ kg}$.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(4.20 \text{ m/s})^2}{3.80 \text{ m}} = 4.64 \text{ m/s}$, upward.

(b) The free-body diagram is given in Figure 5.58. $\Sigma F_y = ma_y$ gives $T - mg = ma_{\text{rad}}$.

$T = m(g + a_{\text{rad}}) = (7.27 \text{ kg})(9.80 \text{ m/s}^2 + 4.64 \text{ m/s}^2) = 105 \text{ N}$

EVALUATE: The acceleration is upward, so the net force is upward and the tension is greater than the weight.

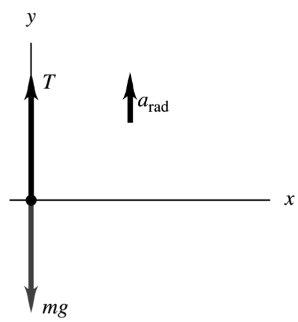


Figure 5.58

5.59. **IDENTIFY:** Since the arm is swinging in a circle, objects in it are accelerated toward the center of the circle, and Newton's second law applies to them.

SET UP: $R = 0.700 \text{ m}$. A 45° angle is $\frac{1}{8}$ of a full rotation, so in $\frac{1}{2}$ s a hand travels through a distance of $\frac{1}{8}(2\pi R)$. In (c) use coordinates where $+y$ is upward, in the direction of $\vec{a}_{\text{rad}}$ at the bottom of the swing.

The acceleration is $a_{\text{rad}} = \dfrac{v^2}{R}$.

EXECUTE: **(a)** $v = \dfrac{1}{8}\left(\dfrac{2\pi R}{0.50 \text{ s}}\right) = 1.10 \text{ m/s}$ and $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(1.10 \text{ m/s})^2}{0.700 \text{ m}} = 1.73 \text{ m/s}^2$.

(b) The free-body diagram is shown in Figure 5.59. F is the force exerted by the blood vessel.

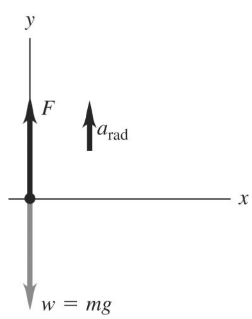


Figure 5.59

(c) $\Sigma F_y = ma_y$ gives $F - w = ma_{\text{rad}}$ and

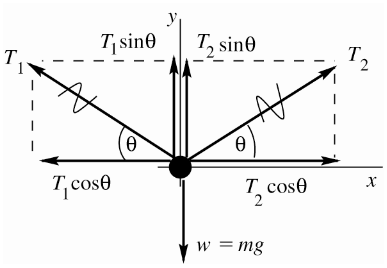
$F = m(g + a_{\text{rad}}) = (1.00\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2 + 1.73\ \text{m/s}^2) = 1.15\times10^{-2}\ \text{N}$, upward.

(d) When the arm hangs vertically and is at rest, $a_{\text{rad}} = 0$ so $F = w = mg = 9.8\times10^{-3}\ \text{N}$.

EVALUATE: The acceleration of the hand is only about 20% of g, so the increase in the force on the blood drop when the arm swings is about 20%.

5.60. **IDENTIFY:** Apply Newton's first law to the person. Each half of the rope exerts a force on him, directed along the rope and equal to the tension T in the rope.

SET UP: **(a)** The force diagram for the person is given in Figure 5.60.



T_1 and T_2 are the tensions in each half of the rope.

Figure 5.60

EXECUTE: $\Sigma F_x = 0$

$T_2\cos\theta - T_1\cos\theta = 0$

This says that $T_1 = T_2 = T$ (The tension is the same on both sides of the person.)

$\Sigma F_y = 0$

$T_1\sin\theta + T_2\sin\theta - mg = 0$

But $T_1 = T_2 = T$, so $2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta} = \frac{(90.0\ \text{kg})(9.80\ \text{m/s}^2)}{2\sin10.0^\circ} = 2540\ \text{N}$$

(b) The relation $2T\sin\theta = mg$ still applies but now we are given that $T = 2.50\times10^4\ \text{N}$ (the breaking strength) and are asked to find θ.

$$\sin\theta = \frac{mg}{2T} = \frac{(90.0\ \text{kg})(9.80\ \text{m/s}^2)}{2(2.50\times10^4\ \text{N})} = 0.01764,\quad \theta = 1.01^\circ.$$

EVALUATE: $T = mg/(2\sin\theta)$ says that $T = mg/2$ when $\theta = 90^\circ$ (rope is vertical).

$T \to \infty$ when $\theta \to 0$ since the upward component of the tension becomes a smaller fraction of the tension.

5.61. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the knot.

SET UP: $a = 0$. Use coordinates with axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the knot is sketched in Figure 5.61.

T_1 is more vertical so supports more of the weight and is larger. You can also see this from $\Sigma F_x = ma_x$:

$T_2\cos40^\circ - T_1\cos60^\circ = 0$. $T_2\cos40^\circ - T_1\cos60^\circ = 0$.

(b) T_1 is larger so set $T_1 = 5000\ \text{N}$. Then $T_2 = T_1/1.532 = 3263.5\ \text{N}$. $\Sigma F_y = ma_y$ gives

$T_1\sin60^\circ + T_2\sin40^\circ = w$ and $w = 6400\ \text{N}$.

EVALUATE: The sum of the vertical components of the two tensions equals the weight of the suspended object. The sum of the tensions is greater than the weight.

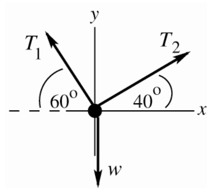


Figure 5.61

5.62. **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to each object. Constant speed means $a = 0$.

SET UP: The free-body diagrams are sketched in Figure 5.62. T_1 is the tension in the lower chain, T_2 is the tension in the upper chain and $T = F$ is the tension in the rope.

EXECUTE: The tension in the lower chain balances the weight and so is equal to *w*. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to *w* and the tension in the rope, which equals *F*, is $w/2$. Then, the downward force on the upper pulley due to the rope is also *w*, and so the upper chain exerts a force *w* on the upper pulley, and the tension in the upper chain is also *w*.

EVALUATE: The pulley combination allows the worker to lift a weight *w* by applying a force of only $w/2$.

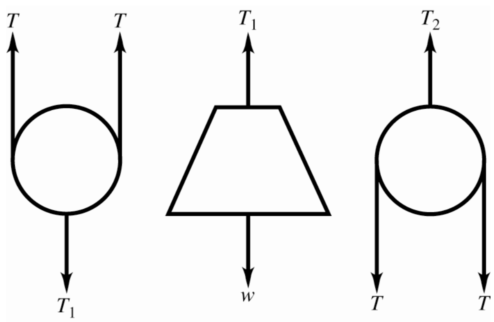


Figure 5.62

5.63. **IDENTIFY:** The engine is hanging at rest, so its acceleration is zero which means that the forces on it must balance. We balance horizontal components and vertical components.

SET UP: In addition to the tensions in the four cables shown in the text, gravity also acts on the engine. Call $+x$ horizontally to the right and $+y$ vertically upward, and call θ the angle that cable *C* makes with cable *D*. The mass of the engine is 409 kg and the tension T_A in cable *A* is 722 N.

EXECUTE: The tension in cable *D* is the only force balancing gravity on the engine, so $T_D = mg$. In the *x*-direction, we have $T_A = T_C \sin\theta$, which gives $T_C = T_A/\sin\theta = (722\text{ N})/(\sin 37.1°) = 1197\text{ N}$. In the *y*-direction, we have $T_B - T_D - T_C\cos\theta = 0$, which gives $T_B = (409\text{ kg})(9.80\text{ m/s}^2) + (1197\text{ N})\cos(37.1°) = 4963\text{ N}$. Rounding to 3 significant figures gives $T_B = 4960$ N and $T_C = 1200$ N.

EVALUATE: The tension in cable *B* is greater than the weight of the engine because cable *C* has a downward component that *B* must also balance.

5.64. **IDENTIFY:** Apply Newton's first law to the ball. Treat the ball as a particle.

SET UP: The forces on the ball are gravity, the tension in the wire and the normal force exerted by the surface. The normal force is perpendicular to the surface of the ramp. Use *x*- and *y*-axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the ball is given in Figure 5.64 (next page). The normal force has been replaced by its *x* and *y* components.

(b) $\Sigma F_y = 0$ gives $n\cos 35.0° - w = 0$ and $n = \dfrac{mg}{\cos 35.0°} = 1.22mg$.

(c) $\Sigma F_x = 0$ gives $T - n\sin 35.0° = 0$ and $T = (1.22mg)\sin 35.0° = 0.700mg$.

EVALUATE: Note that the normal force is greater than the weight, and increases without limit as the angle of the ramp increases toward 90°. The tension in the wire is $w\tan\phi$, where ϕ is the angle of the ramp and T also increases without limit as $\phi \to 90°$.

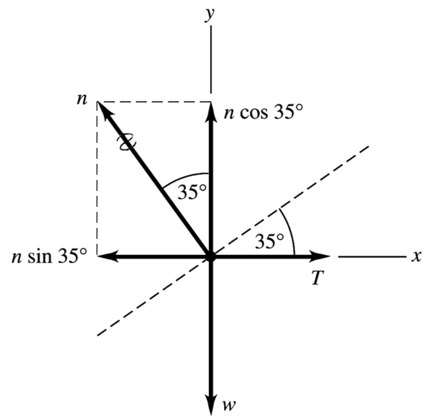


Figure 5.64

5.65. **IDENTIFY:** Apply Newton's first law to the ball. The force of the wall on the ball and the force of the ball on the wall are related by Newton's third law.

SET UP: The forces on the ball are its weight, the tension in the wire, and the normal force applied by the wall. To calculate the angle ϕ that the wire makes with the wall, use Figure 5.65a: $\sin\phi = \dfrac{16.0\text{ cm}}{46.0\text{ cm}}$ and $\phi = 20.35°$

EXECUTE: **(a)** The free-body diagram is shown in Figure 5.65b. Use the x and y coordinates shown in the figure. $\Sigma F_y = 0$ gives $T\cos\phi - w = 0$ and $T = \dfrac{w}{\cos\phi} = \dfrac{(45.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 20.35°} = 470\text{ N}$

(b) $\Sigma F_x = 0$ gives $T\sin\phi - n = 0$. $n = (470\text{ N})\sin 20.35° = 163\text{ N}$. By Newton's third law, the force the ball exerts on the wall is 163 N, directed to the right.

EVALUATE: $n = \left(\dfrac{w}{\cos\phi}\right)\sin\phi = w\tan\phi$. As the angle ϕ decreases (by increasing the length of the wire), T decreases and n decreases.

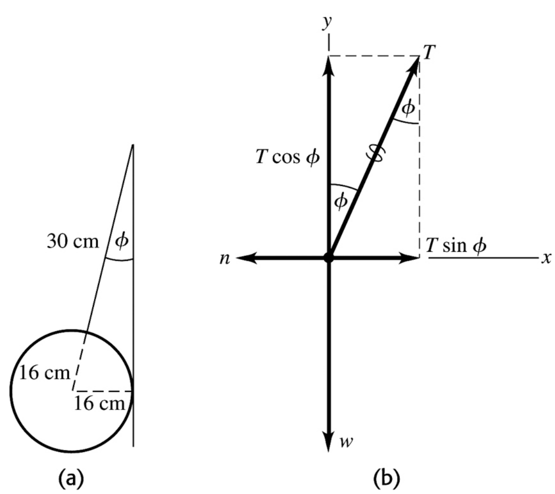


Figure 5.65

5.66. **IDENTIFY:** In each rough patch, the kinetic friction (and hence the acceleration) is constant, but the constants are different in the two patches. Newton's second law applies, as well as the constant-acceleration kinematics formulas in each patch.

SET UP: Choose the $+y$-axis upward and the $+x$-axis in the direction of the velocity.

EXECUTE: **(a)** Find the velocity and time when the box is at $x = 2.00$ m. Newton's second law tells us that $n = mg$ and $-f_k = ma_x$ which gives $-\mu_k mg = ma_x$; $a_x = -\mu_k g = -(0.200)(9.80\text{ m/s}^2) = -1.96\text{ m/s}^2$. Now use the kinematics equations involving v_x. Using $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ we get

$v_x = \sqrt{(4.00\text{ m/s})^2 + 2(-1.96\text{ m/s}^2)(2.00\text{ m})} = 2.857\text{ m/s}$. Now solve the equation $v_x = v_{0x} + a_x t$ for t to get $t = (2.857\text{ m/s} - 4.00\text{ m/s})/(-1.96\text{ m/s}^2) = 0.5834\text{ s}$.

Now look at the motion in the section for which $\mu_k = 0.400$: $a_x = -(0.400)(9.80\text{ m/s}^2) = -3.92\text{ m/s}^2$, $v_x = 0$, $v_{0x} = 2.857$ m/s. Solving $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ for $x - x_0$ gives $x - x_0 = -(2.857\text{ m/s})^2/[2(-3.92\text{ m/s}^2)] = 1.041$ m. The box is at the point $x = 2.00\text{ m} + 1.041\text{ m} = 3.04$ m.

Solving $v_x = v_{0x} + a_x t$ for t gives $t = (-2.857\text{ m/s})/(-3.92\text{ m/s}^2) = 0.7288$ s. The total time is $0.5834\text{ s} + 0.7288\text{ s} = 1.31$ s.

EVALUATE: We cannot do this problem in a single process because the acceleration, although constant in each patch, is different in the two patches.

5.67. **IDENTIFY:** Kinematics will give us the acceleration of the person, and Newton's second law will give us the force (the target variable) that his arms exert on the rest of his body.

SET UP: Let the person's weight be W, so $W = 680$ N. Assume constant acceleration during the speeding up motion and assume that the body moves upward 15 cm in 0.50 s while speeding up. The constant-acceleration kinematics formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ and $\Sigma F_y = ma_y$ apply. The free-body diagram for the person is given in Figure 5.67. F is the force exerted on him by his arms.

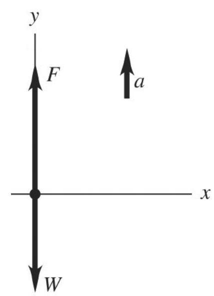


Figure 5.67

EXECUTE: $v_{0y} = 0$, $y - y_0 = 0.15$ m, $t = 0.50$ s. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.15\text{ m})}{(0.50\text{ s})^2} = 1.2\text{ m/s}^2.\ \ \Sigma F_y = ma_y \text{ gives } F - W = ma.\ \ m = \frac{W}{g}, \text{ so}$$

$$F = W\left(1 + \frac{a}{g}\right) = 1.12W = 762\text{ N}.$$

EVALUATE: The force is greater than his weight, which it must be if he is to accelerate upward.

5.68. **IDENTIFY:** The force is time-dependent, so the acceleration is not constant. Therefore we must use calculus instead of the standard kinematics formulas. Newton's second law applies.

SET UP: The acceleration is the time derivative of the velocity and $\Sigma F_y = ma_y$.

EXECUTE: Differentiating the velocity gives $a_y = dv_y/dt = 2.00\text{ m/s}^2 + (1.20\text{ m/s}^3)t$. Find the time when $v_y = 9.00$ m/s: $9.00\text{ m/s} = (2.00\text{ m/s}^2)t + (0.600\text{ m/s}^3)t^2$. Solving this quadratic for t and taking the positive value gives $t = 2.549$ s. At this time the acceleration is $a = 2.00\text{ m/s}^2 + (1.20\text{ m/s}^3)(2.549\text{ s}) = 5.059\text{ m/s}^2$. Now apply Newton's second law to the box, calling T the tension in the rope: $T - mg = ma$, which gives $T = m(g + a) = (2.00\text{ kg})(9.80\text{ m/s}^2 + 5.059\text{ m/s}^2) = 29.7$ N.

EVALUATE: The tension is greater than the weight of the box, which it must be to accelerate the box upward. As time goes on, the acceleration, and hence the tension, would increase.

5.69. IDENTIFY: We know the forces on the box and want to find information about its position and velocity. Newton's second law will give us the box's acceleration.

SET UP: $a_y(t) = \dfrac{\Sigma F_y}{m}$. We can integrate the acceleration to find the velocity and the velocity to find the position. At an altitude of several hundred meters, the acceleration due to gravity is essentially the same as it is at the earth's surface.

EXECUTE: Let $+y$ be upward. Newton's second law gives $T - mg = ma_y$, so $a_y(t) = (12.0\ \text{m/s}^3)t - 9.8\ \text{m/s}^2$. Integrating the acceleration gives $v_y(t) = (6.00\ \text{m/s}^3)t^2 - (9.8\ \text{m/s}^2)t$.

(a) (i) At $t = 1.00$ s, $v_y = -3.80$ m/s. (ii) At $t = 3.00$ s, $v_y = 24.6$ m/s.

(b) Integrating the velocity gives $y - y_0 = (2.00\ \text{m/s}^3)t^3 - (4.9\ \text{m/s}^2)t^2$. $v_y = 0$ at $t = 1.63$ s. At $t = 1.63$ s, $y - y_0 = 8.71\ \text{m} - 13.07\ \text{m} = -4.36$ m.

(c) Setting $y - y_0 = 0$ and solving for t gives $t = 2.45$ s.

EVALUATE: The box accelerates and initially moves downward until the tension exceeds the weight of the box. Once the tension exceeds the weight, the box will begin to accelerate upward and will eventually move upward, as we saw in part (b).

5.70. IDENTIFY: We can use the standard kinematics formulas because the force (and hence the acceleration) is constant, and we can use Newton's second law to find the force needed to cause that acceleration. Kinetic friction, not static friction, is acting.

SET UP: From kinematics, we have $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and $\Sigma F_x = ma_x$ applies. Forces perpendicular to the ramp balance. The force of kinetic friction is $f_\text{k} = \mu_\text{k} mg\cos\theta$.

EXECUTE: Call $+x$ upward along the surface of the ramp. Kinematics gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(8.00\ \text{m})}{(6.00\ \text{s})^2} = 0.4444\ \text{m/s}^2$. $\Sigma F_x = ma_x$ gives $F - mg\sin\theta - \mu_\text{k} mg\cos\theta = ma_x$. Solving for F and putting in the numbers for this problem gives

$F = m(a_x + g\sin\theta + \mu_k mg\cos\theta) = (5.00\ \text{kg})(0.4444\ \text{m/s}^2 + 4.9\ \text{m/s}^2 + 3.395\ \text{m/s}^2) = 43.7$ N.

EVALUTE: As long as the box is moving, only kinetic friction, not static friction, acts on it. The force is less than the weight of the box because only part of the box's weight acts down the ramp. We should also investigate if the force is great enough to start the box moving in the first place. In that case, static friction would have it maximum value, so $f_\text{s} = \mu_\text{s} n$. The force F in this would be $F = \mu_\text{s} mg\cos(30°) + mg\sin(30°) = mg(\mu_\text{s}\cos 30° + \sin 30°) = (5.00\ \text{kg})(9.80\ \text{m/s}^2)[(0.43)(\cos 30°) + \sin 30°] = 42.7$ N. Since the force we found is 43.7 N, it is great enough to overcome static friction and cause the box to move.

5.71. IDENTIFY: The system of boxes is accelerating, so we apply Newton's second law to each box. The friction is kinetic friction. We can use the known acceleration to find the tension and the mass of the second box.

SET UP: The force of friction is $f_\text{k} = \mu_\text{k} n$, $\Sigma F_x = ma_x$ applies to each box, and the forces perpendicular to the surface balance.

EXECUTE: **(a)** Call the $+x$-axis along the surface. For the 5 kg block, the vertical forces balance, so $n + F\sin 53.1° - mg = 0$, which gives $n = 49.0\ \text{N} - 31.99\ \text{N} = 17.01$ N. The force of kinetic friction is $f_\text{k} = \mu_\text{k} n = 5.104$ N. Applying Newton's second law along the surface gives $F\cos 53.1° - T - f_\text{k} = ma$. Solving for T gives $T = F\cos 53.1° - f_\text{k} - ma = 24.02\ \text{N} - 5.10\ \text{N} - 7.50\ \text{N} = 11.4$ N.

(b) For the second box, $T - f_k = ma$. $T - \mu_\text{k} mg = ma$. Solving for m gives

$$m = \frac{T}{\mu_\text{k} g + a} = \frac{11.42\ \text{N}}{(0.3)(9.8\ \text{m/s}^2) + 1.5\ \text{m/s}^2} = 2.57\ \text{kg}.$$

EVALUATE: The normal force for box B is less than its weight due to the upward pull, but the normal force for box A is equal to its weight because the rope pulls horizontally on A.

5.72. **IDENTIFY:** The horizontal force has a component up the ramp and a component perpendicular to the surface of the ramp. The upward component causes the upward acceleration and the perpendicular component affects the normal force on the box. Newton's second law applies. The forces perpendicular to the surface balance.

SET UP: Balance forces perpendicular to the ramp: $n - mg\cos\theta - F\sin\theta = 0.$ Applying Newton's second law parallel to the ramp surface gives $F\cos\theta - f_\text{k} - mg\sin\theta = ma.$

EXECUTE: Using the above equations gives $n = mg\cos\theta + F\sin\theta.$ The force of friction is $f_\text{k} = \mu_\text{k} n,$ so $f_\text{k} = \mu_\text{k}(mg\cos\theta + F\sin\theta).$ $F\cos\theta - \mu_\text{k} mg\cos\theta - \mu_\text{k} F\sin\theta - mg\sin\theta = ma.$ Solving for F gives

$$F = \frac{m(a + \mu_\text{k} g\cos\theta + g\sin\theta)}{\cos\theta - \mu_\text{k}\sin\theta}.$$ Putting in the numbers, we get

$$F = \frac{(6.00\text{ kg})[3.60\text{ m/s}^2 + (0.30)(9.80\text{ m/s}^2)\cos 37.0° + (9.80\text{ m/s}^2)\sin 37.0°]}{\cos 37.0° - (0.30)\sin 37.0°} = 115\text{ N}$$

EVALUATE: Even though the push is horizontal, it can cause a vertical acceleration because it causes the normal force to have a vertical component greater than the vertical component of the box's weight.

5.73. **IDENTIFY:** Newton's second law applies to the box.

SET UP: $f_\text{k} = \mu_\text{k} n,$ $\Sigma F_x = ma_x$, and $\Sigma F_y = ma_y$ apply to the box. Take the +x-axis down the surface of the ramp and the +y-axis perpendicular to the surface upward.

EXECUTE: $\Sigma F_y = ma_y$ gives $n + F\sin(33.0°) - mg\cos(33.0°) = 0$, which gives $n = 51.59$ N. The friction force is $f_\text{k} = \mu_\text{k} n = (0.300)(51.59\text{ N}) = 15.48$ N. Parallel to the surface we have $\Sigma F_x = ma_x$ which gives $F\cos(33.0°) + mg\sin(33.0°) - f_\text{k} = ma$, which gives $a = 6.129\text{ m/s}^2$. Finally the velocity formula gives us $v_x = v_{0x} + a_x t = 0 + (6.129\text{ m/s}^2)(2.00\text{ s}) = 12.3$ m/s.

EVALUATE: Even though F is horizontal and mg is vertical, it is best to choose the axes as we have done, rather than horizontal-vertical, because the acceleration is then in the x-direction. Taking x and y to be horizontal-vertical would give the acceleration x- and y-components, which would complicate the solution.

5.74. **IDENTIFY:** This is a system having constant acceleration, so we can use the standard kinematics formulas as well as Newton's second law to find the unknown mass m_2.

SET UP: Newton's second law applies to each block. The standard kinematics formulas can be used to find the acceleration because the acceleration is constant. The normal force on m_1 is $m_1 g\cos\alpha,$ so the force of friction on it is $f_\text{k} = \mu_\text{k} m_1 g\cos\alpha.$

EXECUTE: Standard kinematics gives the acceleration of the system to be $a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(12.0\text{ m})}{(3.00\text{ s})^2} = 2.667\text{ m/s}^2.$ For m_1, $n = m_1 g\cos\alpha = 117.7\text{ N},$ so the friction force on m_1 is $f_\text{k} = (0.40)(117.7\text{ N}) = 47.08\text{ N}.$ Applying Newton's second law to m_1 gives $T - f_\text{k} - m_1 g\sin\alpha = m_1 a,$ where T is the tension in the cord. Solving for T gives $T = f_\text{k} + m_1 g\sin\alpha + m_1 a = 47.08\text{ N} + 156.7\text{ N} + 53.34\text{ N} = 257.1\text{ N}.$ Newton's second law for m_2 gives $m_2 g - T = m_2 a,$ so $m_2 = \dfrac{T}{g - a} = \dfrac{257.1\text{ N}}{9.8\text{ m/s}^2 - 2.667\text{ m/s}^2} = 36.0\text{ kg}.$

EVALUATE: We could treat these blocks as a two-block system. Newton's second law would then give $m_2 g - m_1 g\sin\alpha - \mu_\text{k} m_1 g\cos\alpha = (m_1 + m_2)a,$ which gives the same result as above.

5.75. **IDENTIFY:** Newton's second law applies, as do the constant-acceleration kinematics equations.

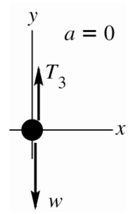
SET UP: Call the +x-axis horizontal and to the right and the +y-axis vertically upward. $\Sigma F_y = ma_y$ and $\Sigma F_x = ma_x$ both apply to the book.

EXECUTE: The book has no horizontal motion, so $\Sigma F_x = ma_x = 0$, which gives us the normal force n: $n = F\cos(60.0°)$. The kinetic friction force is $f_k = \mu_k n = (0.300)(96.0\text{ N})(\cos 60.0°) = 14.4\text{ N}$. In the vertical direction, we have $\Sigma F_y = ma_y$, which gives $F\sin(60.0°) - mg - f_k = ma$. Solving for a gives us $a = [(96.0\text{ N})(\sin 60.0°) - 49.0\text{ N} - 14.4\text{ N}]/(5.00\text{ kg}) = 3.948\text{ m/s}^2$ upward. Now the velocity formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(3.948\text{ m/s}^2)(0.400\text{ m})} = 1.78\text{ m/s}$.

EVALUATE: Only the upward component of the force F makes the book accelerate upward, while the horizontal component of T is the magnitude of the normal force.

5.76. **IDENTIFY:** The system is in equilibrium. Apply Newton's first law to block A, to the hanging weight and to the knot where the cords meet. Target variables are the two forces.

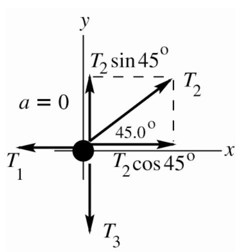
(a) SET UP: The free-body diagram for the hanging block is given in Figure 5.76a.



EXECUTE:
$\Sigma F_y = ma_y$
$T_3 - w = 0$
$T_3 = 12.0\text{ N}$

Figure 5.76a

SET UP: The free-body diagram for the knot is given in Figure 5.76b.

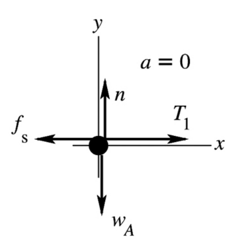


EXECUTE:
$\Sigma F_y = ma_y$
$T_2 \sin 45.0° - T_3 = 0$
$$T_2 = \frac{T_3}{\sin 45.0°} = \frac{12.0\text{ N}}{\sin 45.0°}$$
$T_2 = 17.0\text{ N}$

Figure 5.76b

$\Sigma F_x = ma_x$
$T_2 \cos 45.0° - T_1 = 0$
$T_1 = T_2 \cos 45.0° = 12.0\text{ N}$

SET UP: The free-body diagram for block A is given in Figure 5.76c.



EXECUTE:
$\Sigma F_x = ma_x$
$T_1 - f_s = 0$
$f_s = T_1 = 12.0\text{ N}$

Figure 5.76c

EVALUATE: Also can apply $\Sigma F_y = ma_y$ to this block:

$$n - w_A = 0$$
$$n = w_A = 60.0\text{ N}$$

Then $\mu_s n = (0.25)(60.0\text{ N}) = 15.0\text{ N}$; this is the maximum possible value for the static friction force. We see that $f_s < \mu_s n$; for this value of w the static friction force can hold the blocks in place.

(b) SET UP: We have all the same free-body diagrams and force equations as in part (a) but now the static friction force has its largest possible value, $f_s = \mu_s n = 15.0\ \text{N}$. Then $T_1 = f_s = 15.0\ \text{N}$.

EXECUTE: From the equations for the forces on the knot

$T_2 \cos 45.0° - T_1 = 0$ implies $T_2 = T_1/\cos 45.0° = \dfrac{15.0\ \text{N}}{\cos 45.0°} = 21.2\ \text{N}$

$T_2 \sin 45.0° - T_3 = 0$ implies $T_3 = T_2 \sin 45.0° = (21.2\ \text{N})\sin 45.0° = 15.0\ \text{N}$

And finally $T_3 - w = 0$ implies $w = T_3 = 15.0\ \text{N}$.

EVALUATE: Compared to part (a), the friction is larger in part (b) by a factor of $(15.0/12.0)$ and w is larger by this same ratio.

5.77. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: Constant speed means $a = 0$. When the blocks are moving, the friction force is f_k and when they are at rest, the friction force is f_s.

EXECUTE: **(a)** The tension in the cord must be $m_2 g$ in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2 g = (m_1 g \sin\alpha + \mu_k m_1 g \cos\alpha)$ and $m_2 = m_1(\sin\alpha + \mu_k \cos\alpha)$.

(b) In this case, the friction force acts in the same direction as the tension on the block of mass m_1, so $m_2 g = (m_1 g \sin\alpha - \mu_k m_1 g \cos\alpha)$, or $m_2 = m_1(\sin\alpha - \mu_k \cos\alpha)$.

(c) Similar to the analysis of parts (a) and (b), the largest m_2 could be is $m_1(\sin\alpha + \mu_s \cos\alpha)$ and the smallest m_2 could be is $m_1(\sin\alpha - \mu_s \cos\alpha)$.

EVALUATE: In parts (a) and (b) the friction force changes direction when the direction of the motion of m_1 changes. In part (c), for the largest m_2 the static friction force on m_1 is directed down the incline and for the smallest m_2 the static friction force on m_1 is directed up the incline.

5.78. **IDENTIFY:** The net force at any time is $F_{\text{net}} = ma$.

SET UP: At $t = 0$, $a = 62g$. The maximum acceleration is $140g$ at $t = 1.2\ \text{ms}$.

EXECUTE: **(a)** $F_{\text{net}} = ma = 62mg = 62(210\times10^{-9}\ \text{kg})(9.80\ \text{m/s}^2) = 1.3\times10^{-4}\ \text{N}$. This force is 62 times the flea's weight.

(b) $F_{\text{net}} = 140mg = 2.9\times10^{-4}\ \text{N}$, at $t = 1.2\ \text{ms}$.

(c) Since the initial speed is zero, the maximum speed is the area under the $a_x - t$ graph. This gives 1.2 m/s.

EVALUATE: a is much larger than g and the net external force is much larger than the flea's weight.

5.79. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Use Newton's third law to relate forces on A and on B.

SET UP: Constant speed means $a = 0$.

EXECUTE: **(a)** Treat A and B as a single object of weight $w = w_A + w_B = 1.20\ \text{N} + 3.60\ \text{N} = 4.80\ \text{N}$. The free-body diagram for this combined object is given in Figure 5.79a. $\Sigma F_y = ma_y$ gives $n = w = 4.80\ \text{N}$. $f_k = \mu_k n = (0.300)(4.80\ \text{N}) = 1.44\ \text{N}$. $\Sigma F_x = ma_x$ gives $F = f_k = 1.44\ \text{N}$.

(b) The free-body force diagrams for blocks A and B are given in Figure 5.79b. n and f_k are the normal and friction forces applied to block B by the tabletop and are the same as in part (a). f_{kB} is the friction force that A applies to B. It is to the right because the force from A opposes the motion of B. n_B is the downward force that A exerts on B. f_{kA} is the friction force that B applies to A. It is to the left because block B wants A to move with it. n_A is the normal force that block B exerts on A. By Newton's third law, $f_{kB} = f_{kA}$ and these forces are in opposite directions. Also, $n_A = n_B$ and these forces are in opposite directions.

$\Sigma F_y = ma_y$ for block A gives $n_A = w_A = 1.20\ \text{N}$, so $n_B = 1.20\ \text{N}$.

$f_{kA} = \mu_k n_A = (0.300)(1.20\ \text{N}) = 0.360\ \text{N}$, and $f_{kB} = 0.360\ \text{N}$.

$\Sigma F_x = ma_x$ for block A gives $T = f_{kA} = 0.360$ N.

$\Sigma F_x = ma_x$ for block B gives $F = f_{kB} + f_k = 0.360\text{ N} + 1.44\text{ N} = 1.80$ N.

EVALUATE: In part (a) block A is at rest with respect to B and it has zero acceleration. There is no horizontal force on A besides friction, and the friction force on A is zero. A larger force F is needed in part (b), because of the friction force between the two blocks.

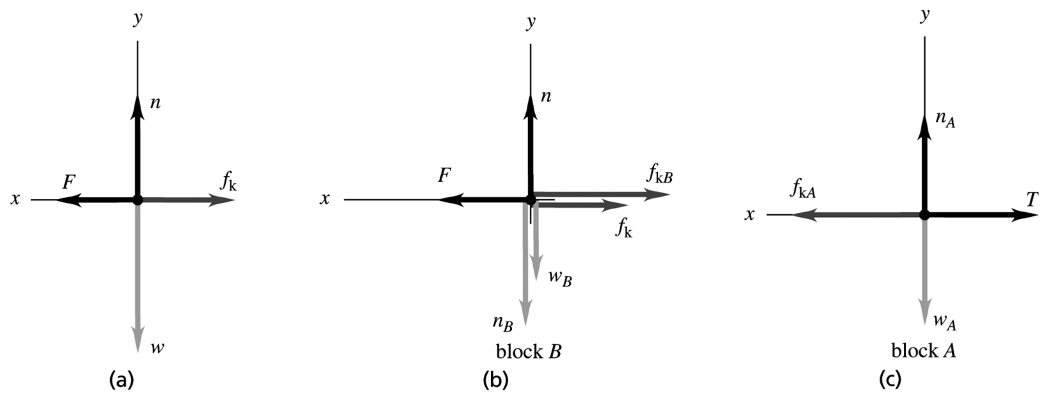


Figure 5.79

5.80. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the passenger to find the maximum allowed acceleration. Then use a constant acceleration equation to find the maximum speed.
SET UP: The free-body diagram for the passenger is given in Figure 5.80.
EXECUTE: $\Sigma F_y = ma_y$ gives $n - mg = ma$. $n = 1.6mg$, so $a = 0.60g = 5.88\text{ m/s}^2$.

$y - y_0 = 3.0$ m, $a_y = 5.88\text{ m/s}^2$, $v_{0y} = 0$ so $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = 5.9$ m/s.

EVALUATE: A larger final speed would require a larger value of a_y, which would mean a larger normal force on the person.

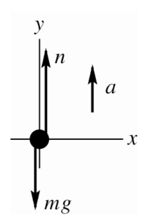


Figure 5.80

5.81. **IDENTIFY:** $a = dv/dt$. Apply $\Sigma\vec{F} = m\vec{a}$ to yourself.
SET UP: The reading of the scale is equal to the normal force the scale applies to you.
EXECUTE: The elevator's acceleration is $a = \dfrac{dv(t)}{dt} = 3.0\text{ m/s}^2 + 2(0.20\text{ m/s}^3)t = 3.0\text{ m/s}^2 + (0.40\text{ m/s}^3)t$.

At $t = 4.0$ s, $a = 3.0\text{ m/s}^2 + (0.40\text{ m/s}^3)(4.0\text{ s}) = 4.6\text{ m/s}^2$. From Newton's second law, the net force on you is $F_{net} = F_{scale} - w = ma$ and $F_{scale} = w + ma = (64\text{ kg})(9.8\text{ m/s}^2) + (64\text{ kg})(4.6\text{ m/s}^2) = 920$ N.
EVALUATE: a increases with time, so the scale reading is increasing.

5.82. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the hammer. Since the hammer is at rest relative to the bus, its acceleration equals that of the bus.
SET UP: The free-body diagram for the hammer is given in Figure 5.82.

EXECUTE: $\Sigma F_y = ma_y$ gives $T\sin 56.0° - mg = 0$ so $T\sin 56.0° = mg$. $\Sigma F_x = ma_x$ gives $T\cos 56.0° = ma$.

Divide the second equation by the first: $\dfrac{a}{g} = \dfrac{1}{\tan 56.0°}$ and $a = 6.61\ \text{m/s}^2$.

EVALUATE: When the acceleration increases, the angle between the rope and the ceiling of the bus decreases, and the angle the rope makes with the vertical increases.

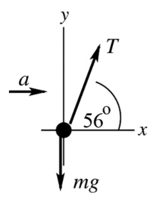
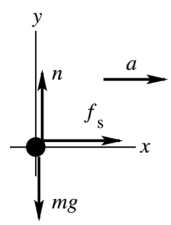


Figure 5.82

5.83. **IDENTIFY:** First calculate the maximum acceleration that the static friction force can give to the case. Apply $\Sigma \vec{F} = m\vec{a}$ to the case.

(a) SET UP: The static friction force is to the right in Figure 5.83a (northward) since it tries to make the case move with the truck. The maximum value it can have is $f_s = \mu_s N$.



EXECUTE:

$\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

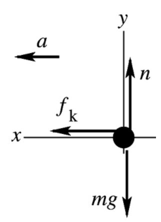
$f_s = \mu_s n = \mu_s mg$

Figure 5.83a

$\Sigma F_x = ma_x$. $f_s = ma$. $\mu_s mg = ma$. $a = \mu_s g = (0.30)(9.80\ \text{m/s}^2) = 2.94\ \text{m/s}^2$. The truck's acceleration is less than this so the case doesn't slip relative to the truck; the case's acceleration is $a = 2.20\ \text{m/s}^2$ (northward). Then $f_s = ma = (40.0\ \text{kg})(2.20\ \text{m/s}^2) = 88.0\ \text{N}$, northward.

(b) IDENTIFY: Now the acceleration of the truck is greater than the acceleration that static friction can give the case. Therefore, the case slips relative to the truck and the friction is kinetic friction. The friction force still tries to keep the case moving with the truck, so the acceleration of the case and the friction force are both southward. The free-body diagram is sketched in Figure 5.83b.

SET UP:



EXECUTE:

$\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_k = \mu_k mg = (0.20)(40.0\ \text{kg})(9.80\ \text{m/s}^2)$

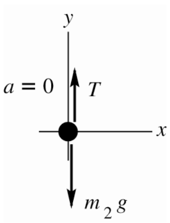
$f_k = 78\ \text{N}$, southward

Figure 5.83b

EVALUATE: $f_k = ma$ implies $a = \dfrac{f_k}{m} = \dfrac{78\text{ N}}{40.0\text{ kg}} = 2.0\text{ m/s}^2$. The magnitude of the acceleration of the case is less than that of the truck and the case slides toward the front of the truck. In both parts (a) and (b) the friction is in the direction of the motion and accelerates the case. Friction opposes *relative* motion between two surfaces in contact.

5.84. **IDENTIFY:** Apply Newton's first law to the rope. Let m_1 be the mass of that part of the rope that is on the table, and let m_2 be the mass of that part of the rope that is hanging over the edge. $(m_1 + m_2 = m,$ the total mass of the rope). Since the mass of the rope is not being neglected, the tension in the rope varies along the length of the rope. Let T be the tension in the rope at that point that is at the edge of the table.

SET UP: The free-body diagram for the hanging section of the rope is given in Figure 5.84a.



EXECUTE:

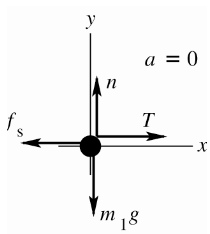
$\Sigma F_y = ma_y$

$T - m_2 g = 0$

$T = m_2 g$

Figure 5.84a

SET UP: The free-body diagram for that part of the rope that is on the table is given in Figure 5.84b.



EXECUTE:

$\Sigma F_y = ma_y$

$n - m_1 g = 0$

$n = m_1 g$

Figure 5.84b

When the maximum amount of rope hangs over the edge the static friction has its maximum value:

$f_s = \mu_s n = \mu_s m_1 g$

$\Sigma F_x = ma_x$

$T - f_s = 0$

$T = \mu_s m_1 g$

Use the first equation to replace T:

$m_2 g = \mu_s m_1 g$

$m_2 = \mu_s m_1$

The fraction that hangs over is $\dfrac{m_2}{m} = \dfrac{\mu_s m_1}{m_1 + \mu_s m_1} = \dfrac{\mu_s}{1+\mu_s}$.

EVALUATE: As $\mu_s \to 0$, the fraction goes to zero and as $\mu_s \to \infty$, the fraction goes to unity.

5.85. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the point where the three wires join and also to one of the balls. By symmetry the tension in each of the 35.0 cm wires is the same.

SET UP: The geometry of the situation is sketched in Figure 5.85a. The angle ϕ that each wire makes with the vertical is given by $\sin\phi = \dfrac{12.5\text{ cm}}{47.5\text{ cm}}$ and $\phi = 15.26°$. Let T_A be the tension in the vertical wire and let T_B be the tension in each of the other two wires. Neglect the weight of the wires. The free-body diagram for the left-hand ball is given in Figure 5.85b and for the point where the wires join in Figure 5.85c. n is the force one ball exerts on the other.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the ball gives $T_B \cos\phi - mg = 0$.

$T_B = \dfrac{mg}{\cos\phi} = \dfrac{(15.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 15.26°} = 152\text{ N}$. Then $\Sigma F_y = ma_y$ applied in Figure 5.85c gives $T_A - 2T_B \cos\phi = 0$ and $T_A = 2(152\text{ N})\cos\phi = 294\text{ N}$.

(b) $\Sigma F_x = ma_x$ applied to the ball gives $n - T_B \sin\phi = 0$ and $n = (152\text{ N})\sin 15.26° = 40.0\text{ N}$.

EVALUATE: T_A equals the total weight of the two balls.

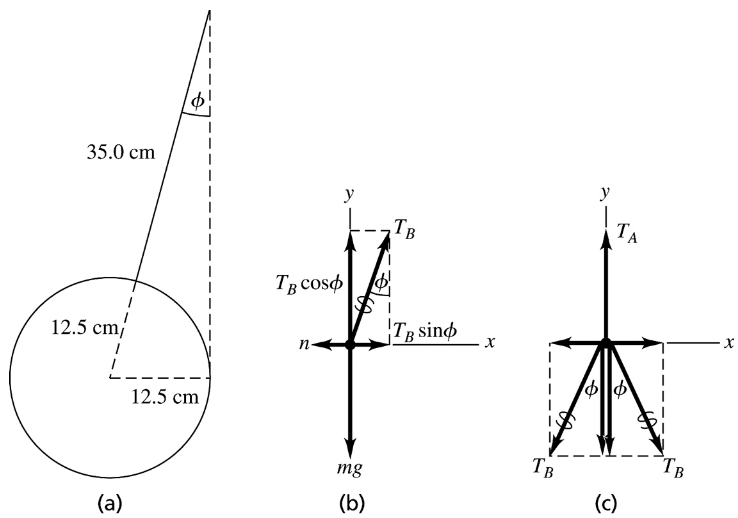


Figure 5.85

5.86. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car to calculate its acceleration. Then use a constant acceleration equation to find the initial speed.

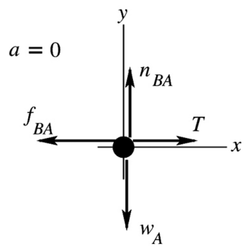
SET UP: Let $+x$ be in the direction of the car's initial velocity. The friction force f_k is then in the $-x$-direction. $192\text{ ft} = 58.52\text{ m}$.

EXECUTE: $n = mg$ and $f_k = \mu_k mg$. $\Sigma F_x = ma_x$ gives $-\mu_k mg = ma_x$ and $a_x = -\mu_k g = -(0.750)(9.80\text{ m/s}^2) = -7.35\text{ m/s}^2$. $v_x = 0$ (stops), $x - x_0 = 58.52\text{ m}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = \sqrt{-2a_x(x - x_0)} = \sqrt{-2(-7.35\text{ m/s}^2)(58.52\text{ m})} = 29.3\text{ m/s} = 65.5\text{ mi/h}$. He was guilty.

EVALUATE: $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = -\dfrac{v_{0x}^2}{2a_x}$. If his initial speed had been 45 mi/h he would have stopped in $\left(\dfrac{45\text{ mi/h}}{65.5\text{ mi/h}}\right)^2 (192\text{ ft}) = 91\text{ ft}$.

5.87. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Forces between the blocks are related by Newton's third law. The target variable is the force F. Block B is pulled to the left at constant speed, so block A moves to the right at constant speed and $a = 0$ for each block.

SET UP: The free-body diagram for block A is given in Figure 5.87a. n_{BA} is the normal force that B exerts on A. $f_{BA} = \mu_k n_{BA}$ is the kinetic friction force that B exerts on A. Block A moves to the right relative to B, and f_{BA} opposes this motion, so f_{BA} is to the left. Note also that F acts just on B, not on A.



EXECUTE:

$\Sigma F_y = ma_y$

$n_{BA} - w_A = 0$

$n_{BA} = 1.90 \text{ N}$

$f_{BA} = \mu_k n_{BA} = (0.30)(1.90 \text{ N}) = 0.57 \text{ N}$

Figure 5.87a

$\Sigma F_x = ma_x$. $T - f_{BA} = 0$. $T = f_{BA} = 0.57$ N.

SET UP: The free-body diagram for block B is given in Figure 5.87b.

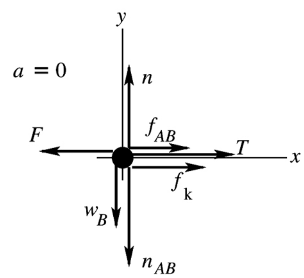


Figure 5.87b

EXECUTE: n_{AB} is the normal force that block A exerts on block B. By Newton's third law n_{AB} and n_{BA} are equal in magnitude and opposite in direction, so $n_{AB} = 1.90$ N. f_{AB} is the kinetic friction force that A exerts on B. Block B moves to the left relative to A and f_{AB} opposes this motion, so f_{AB} is to the right. $f_{AB} = \mu_k n_{AB} = (0.30)(1.90 \text{ N}) = 0.57$ N. n and f_k are the normal and friction force exerted by the floor on block B; $f_k = \mu_k n$. Note that block B moves to the left relative to the floor and f_k opposes this motion, so f_k is to the right.

$\Sigma F_y = ma_y$: $n - w_B - n_{AB} = 0$. $n = w_B + n_{AB} = 4.20 \text{ N} + 1.90 \text{ N} = 6.10$ N. Then

$f_k = \mu_k n = (0.30)(6.10 \text{ N}) = 1.83$ N. $\Sigma F_x = ma_x$: $f_{AB} + T + f_k - F = 0$.

$F = T + f_{AB} + f_k = 0.57 \text{ N} + 0.57 \text{ N} + 1.83 \text{ N} = 3.0$ N.

EVALUATE: Note that f_{AB} and f_{BA} are a third law action-reaction pair, so they must be equal in magnitude and opposite in direction and this is indeed what our calculation gives.

5.88. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box. Compare the acceleration of the box to the acceleration of the truck and use constant acceleration equations to describe the motion.

SET UP: Both objects have acceleration in the same direction; take this to be the $+x$-direction.

EXECUTE: If the box were to remain at rest relative to the truck, the friction force would need to cause an acceleration of 2.20 m/s^2; however, the maximum acceleration possible due to static friction is $(0.19)(9.80 \text{ m/s}^2) = 1.86 \text{ m/s}^2$, and so the box will move relative to the truck; the acceleration of the box would be $\mu_k g = (0.15)(9.80 \text{ m/s}^2) = 1.47 \text{ m/s}^2$. The difference between the distance the truck moves and the distance the box moves (i.e., the distance the box moves relative to the truck) will be 1.80 m after a time

$$t = \sqrt{\frac{2\Delta x}{a_{\text{truck}} - a_{\text{box}}}} = \sqrt{\frac{2(1.80 \text{ m})}{(2.20 \text{ m/s}^2 - 1.47 \text{ m/s}^2)}} = 2.221 \text{ s}.$$

In this time, the truck moves $\frac{1}{2}a_{\text{truck}}t^2 = \frac{1}{2}(2.20 \text{ m/s}^2)(2.221 \text{ s})^2 = 5.43$ m.

EVALUATE: To prevent the box from sliding off the truck the coefficient of static friction would have to be $\mu_s = (2.20\text{ m/s}^2)/g = 0.224$.

5.89. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Parts (a) and (b) will be done together.

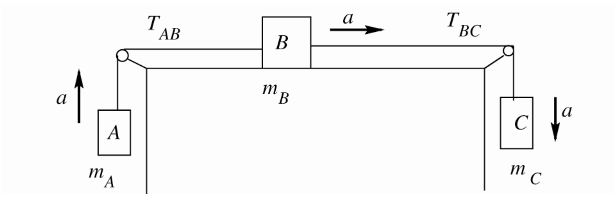


Figure 5.89a

Note that each block has the same magnitude of acceleration, but in different directions. For each block let the direction of $\vec{a}$ be a positive coordinate direction.

SET UP: The free-body diagram for block *A* is given in Figure 5.89b.

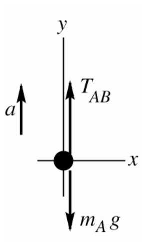


Figure 5.89b

EXECUTE:

$\Sigma F_y = ma_y$

$T_{AB} - m_A g = m_A a$

$T_{AB} = m_A(a+g)$

$T_{AB} = 4.00\text{ kg}(2.00\text{ m/s}^2 + 9.80\text{ m/s}^2) = 47.2\text{ N}$

SET UP: The free-body diagram for block *B* is given in Figure 5.89c.

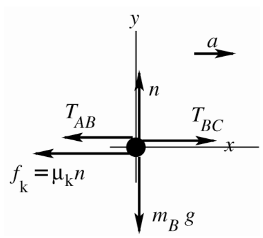


Figure 5.89c

EXECUTE:

$\Sigma F_y = ma_y$

$n - m_B g = 0$

$n = m_B g$

$f_k = \mu_k n = \mu_k m_B g = (0.25)(12.0\text{ kg})(9.80\text{ m/s}^2) = 29.4\text{ N}$

$\Sigma F_x = ma_x$

$T_{BC} - T_{AB} - f_k = m_B a$

$T_{BC} = T_{AB} + f_k + m_B a = 47.2\text{ N} + 29.4\text{ N} + (12.0\text{ kg})(2.00\text{ m/s}^2)$

$T_{BC} = 100.6\text{ N}$

SET UP: The free-body diagram for block *C* is sketched in Figure 5.89d (next page).

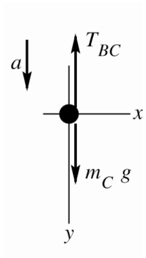


Figure 5.89d

EXECUTE:

$\Sigma F_y = ma_y$

$m_C g - T_{BC} = m_C a$

$m_C(g-a) = T_{BC}$

$$m_C = \frac{T_{BC}}{g-a} = \frac{100.6\text{ N}}{9.80\text{ m/s}^2 - 2.00\text{ m/s}^2} = 12.9\text{ kg}$$

EVALUATE: If all three blocks are considered together as a single object and $\Sigma\vec{F} = m\vec{a}$ is applied to this combined object, $m_C g - m_A g - \mu_k m_B g = (m_A + m_B + m_C)a$. Using the values for μ_k, m_A and m_B given in the problem and the mass m_C we calculated, this equation gives $a = 2.00\text{ m/s}^2$, which checks.

5.90. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. They have the same magnitude of acceleration, a.

SET UP: Consider positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block).

EXECUTE: **(a)** The forces along the inclines and the accelerations are related by $T - (100\text{ kg})g\sin 30.0° = (100\text{ kg})a$ and $(50\text{ kg})g\sin 53.1° - T = (50\text{ kg})a$, where T is the tension in the cord and a the mutual magnitude of acceleration. Adding these relations, $(50\text{ kg}\sin 53.1° - 100\text{ kg}\sin 30.0°)g = (50\text{ kg} + 100\text{ kg})a$, or $a = -0.067g$. Since a comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, a would be $+0.067g$.

(b) $a = 0.067(9.80\text{ m/s}^2) = 0.658\text{ m/s}^2$.

(c) Substituting the value of a (including the proper sign, depending on choice of coordinates) into either of the above relations involving T yields 424 N.

EVALUATE: For part (a) we could have compared $mg\sin\theta$ for each block to determine which direction the system would move.

5.91. **IDENTIFY:** Let the tensions in the ropes be T_1 and T_2.

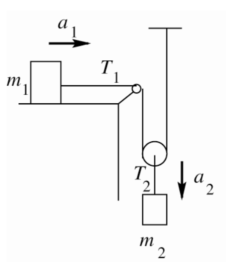
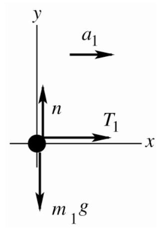


Figure 5.91a

Consider the forces on each block. In each case take a positive coordinate direction in the direction of the acceleration of that block.

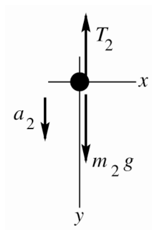
SET UP: The free-body diagram for m_1 is given in Figure 5.91b.



EXECUTE:
$\Sigma F_x = ma_x$
$T_1 = m_1a_1$

Figure 5.91b

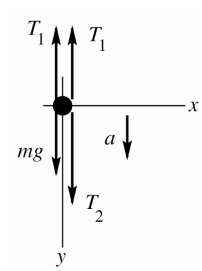
SET UP: The free-body diagram for m_2 is given in Figure 5.91c.



EXECUTE:
$\Sigma F_y = ma_y$
$m_2g - T_2 = m_2a_2$

Figure 5.91c

This gives us two equations, but there are four unknowns (T_1, T_2, a_1 and a_2) so two more equations are required.
SET UP: The free-body diagram for the moveable pulley (mass m) is given in Figure 5.91d.



EXECUTE:
$\Sigma F_y = ma_y$
$mg + T_2 - 2T_1 = ma$

Figure 5.91d

But our pulleys have negligible mass, so $mg = ma = 0$ and $T_2 = 2T_1$. Combine these three equations to eliminate T_1 and T_2: $m_2g - T_2 = m_2a_2$ gives $m_2g - 2T_1 = m_2a_2$. And then with $T_1 = m_1a_1$ we have $m_2g - 2m_1a_1 = m_2a_2$.
SET UP: There are still two unknowns, a_1 and a_2. But the accelerations a_1 and a_2 are related. In any time interval, if m_1 moves to the right a distance d, then in the same time m_2 moves downward a distance $d/2$. One of the constant acceleration kinematic equations says $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$, so if m_2 moves half the distance it must have half the acceleration of m_1: $a_2 = a_1/2$, or $a_1 = 2a_2$.
EXECUTE: This is the additional equation we need. Use it in the previous equation and get
$m_2g - 2m_1(2a_2) = m_2a_2$.
$a_2(4m_1 + m_2) = m_2g$
$a_2 = \dfrac{m_2g}{4m_1 + m_2}$ and $a_1 = 2a_2 = \dfrac{2m_2g}{4m_1 + m_2}$.
EVALUATE: If $m_2 \to 0$ or $m_1 \to \infty$, $a_1 = a_2 = 0$. If $m_2 \gg m_1$, $a_2 = g$ and $a_1 = 2g$.

5.92. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to block B, to block A and B as a composite object, and to block C. If A and B slide together all three blocks have the same magnitude of acceleration.
SET UP: If A and B don't slip, the friction between them is static. The free-body diagrams for block B, for blocks A and B, and for C are given in Figure 5.92. Block C accelerates downward and A and B accelerate

to the right. In each case take a positive coordinate direction to be in the direction of the acceleration. Since block *A* moves to the right, the friction force f_s on block *B* is to the right, to prevent relative motion between the two blocks. When *C* has its largest mass, f_s has its largest value: $f_s = \mu_s n$.

EXECUTE: $\Sigma F_x = ma_x$ applied to the block *B* gives $f_s = m_B a$. $n = m_B g$ and $f_s = \mu_s m_B g$. $\mu_s m_B g = m_B a$ and $a = \mu_s g$. $\Sigma F_x = ma_x$ applied to blocks $A+B$ gives $T = m_{AB}a = m_{AB}\mu_s g$. $\Sigma F_y = ma_y$ applied to block *C* gives $m_C g - T = m_C a$. $m_C g - m_{AB}\mu_s g = m_C \mu_s g$. $m_C = \dfrac{m_{AB}\mu_s}{1-\mu_s} = (5.00\text{ kg} + 8.00\text{ kg})\left(\dfrac{0.750}{1-0.750}\right) = 39.0\text{ kg}$.

EVALUATE: With no friction from the tabletop, the system accelerates no matter how small the mass of *C* is. If m_C is less than 39.0 kg, the friction force that *A* exerts on *B* is less than $\mu_s n$. If m_C is greater than 39.0 kg, blocks *C* and *A* have a larger acceleration than friction can give to block *B*, and *A* accelerates out from under *B*.

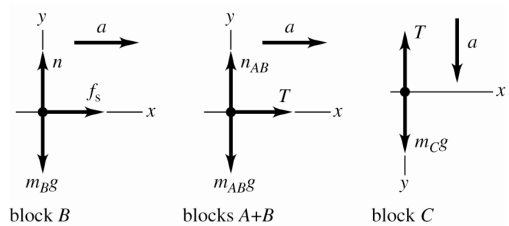


Figure 5.92

5.93. **IDENTIFY:** Apply the method of Exercise 5.15 to calculate the acceleration of each object. Then apply constant acceleration equations to the motion of the 2.00 kg object.

SET UP: After the 5.00 kg object reaches the floor, the 2.00 kg object is in free fall, with downward acceleration *g*.

EXECUTE: The 2.00-kg object will accelerate upward at $g\dfrac{5.00\text{ kg} - 2.00\text{ kg}}{5.00\text{ kg} + 2.00\text{ kg}} = 3g/7$, and the 5.00-kg object will accelerate downward at $3g/7$. Let the initial height above the ground be h_0. When the large object hits the ground, the small object will be at a height $2h_0$, and moving upward with a speed given by $v_0^2 = 2ah_0 = 6gh_0/7$. The small object will continue to rise a distance $v_0^2/2g = 3h_0/7$, and so the maximum height reached will be $2h_0 + 3h_0/7 = 17h_0/7 = 1.46$ m above the floor , which is 0.860 m above its initial height.

EVALUATE: The small object is 1.20 m above the floor when the large object strikes the floor, and it rises an additional 0.26 m after that.

5.94. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box.

SET UP: The box has an upward acceleration of $a = 1.90\text{ m/s}^2$.

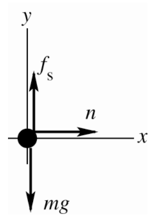
EXECUTE: The floor exerts an upward force *n* on the box, obtained from $n - mg = ma$, or $n = m(a+g)$. The friction force that needs to be balanced is

$$\mu_k n = \mu_k m(a+g) = (0.32)(36.0\text{ kg})(1.90\text{ m/s}^2 + 9.80\text{ m/s}^2) = 135\text{ N}.$$

EVALUATE: If the elevator were not accelerating the normal force would be $n = mg$ and the friction force that would have to be overcome would be 113 N. The upward acceleration increases the normal force and that increases the friction force.

5.95. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block. The cart and the block have the same acceleration. The normal force exerted by the cart on the block is perpendicular to the front of the cart, so is horizontal and to the right. The friction force on the block is directed so as to hold the block up against the downward pull of gravity. We want to calculate the minimum *a* required, so take static friction to have its maximum value, $f_s = \mu_s n$.

SET UP: The free-body diagram for the block is given in Figure 5.95.



EXECUTE:
$\Sigma F_x = ma_x$
$n = ma$
$f_s = \mu_s n = \mu_s ma$

Figure 5.95

$\Sigma F_y = ma_y$: $f_s - mg = 0$

$\mu_s ma = mg$, so $a = g/\mu_s$.

EVALUATE: An observer on the cart sees the block pinned there, with no reason for a horizontal force on it because the block is at rest relative to the cart. Therefore, such an observer concludes that $n = 0$ and thus $f_s = 0$, and he doesn't understand what holds the block up against the downward force of gravity. The reason for this difficulty is that $\Sigma\vec{F} = m\vec{a}$ does not apply in a coordinate frame attached to the cart. This reference frame is accelerated, and hence not inertial. The smaller μ_s is, the larger a must be to keep the block pinned against the front of the cart.

5.96. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: Use coordinates where $+x$ is directed down the incline.

EXECUTE: **(a)** Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; i.e., the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\text{ kg})g(\sin 30° - (0.25)\cos 30°) - T = (4.00\text{ kg})a$, or $11.11\text{ N} - T = (4.00\text{ kg})a$, and similarly for the larger, $15.44\text{ N} + T = (8.00\text{ kg})a$. Adding these two relations, $26.55\text{ N} = (12.00\text{ kg})a$, $a = 2.21\text{ m/s}^2$.

(b) Substitution into either of the above relations gives $T = 2.27\text{ N}$.

(c) The string will be slack. The 4.00-kg block will have $a = 2.78\text{ m/s}^2$ and the 8.00-kg block will have $a = 1.93\text{ m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

EVALUATE: If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The 8.00-kg block would have a smaller acceleration even though it has a larger mass, since it has a larger μ_k.

5.97. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the plank.

SET UP: Both objects have $a = 0$.

EXECUTE: Let n_B be the normal force between the plank and the block and n_A be the normal force between the block and the incline. Then, $n_B = w\cos\theta$ and $n_A = n_B + 3w\cos\theta = 4w\cos\theta$. The net frictional force on the block is $\mu_k(n_A + n_B) = \mu_k 5w\cos\theta$. To move at constant speed, this must balance the component of the block's weight along the incline, so $3w\sin\theta = \mu_k 5w\cos\theta$, and

$\mu_k = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 37° = 0.452$.

EVALUATE: In the absence of the plank the block slides down at constant speed when the slope angle and coefficient of friction are related by $\tan\theta = \mu_k$. For $\theta = 36.9°$, $\mu_k = 0.75$. A smaller μ_k is needed when the plank is present because the plank provides an additional friction force.

5.98. **IDENTIFY:** Apply Newton's second law to Jack in the Ferris wheel.

SET UP: $\Sigma\vec{F} = m\vec{a}$ and Jack's acceleration is $a_{rad} = v^2/R$, and $v = 2\pi R/T$. At the highest point, the normal force that the chair exerts on Jack is ¼ of his weight, or $0.25mg$. Take $+y$ downward.

EXECUTE: $\Sigma F_y = ma_y$ gives $mg - n = mv^2/R$. $mg - 0.25mg = mv^2/R$, so $v^2/R = 0.75g$. Using $T = 2\pi R/T$, we get $v^2/R = 4\pi^2 R/T^2$. Therefore $4\pi^2 R/T^2 = 0.750g$. $T = 1/(0.100\text{ rev/s}) = 10.0\text{ s/rev}$, so $R = (0.750g)T^2/(4\pi^2) = (0.750)(9.80\text{ m/s}^2)[(10.0\text{ s})/(2\pi)]^2 = 18.6\text{ m}$.

EVALUATE: This Ferris wheel would be about 120 ft in diameter, which is certainly large but not impossible.

5.99. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the automobile.

SET UP: The "correct" banking angle is for zero friction and is given by $\tan\beta = \frac{v_0^2}{gR}$, as derived in the text. Use coordinates that are vertical and horizontal, since the acceleration is horizontal.

EXECUTE: For speeds larger than v_0, a frictional force is needed to keep the car from skidding. In this case, the inward force will consist of a part due to the normal force *n* and the friction force f; $n\sin\beta + f\cos\beta = ma_{\text{rad}}$. The normal and friction forces both have vertical components; since there is no vertical acceleration, $n\cos\beta - f\sin\beta = mg$. Using $f = \mu_s n$ and $a_{\text{rad}} = \frac{v^2}{R} = \frac{(1.5v_0)^2}{R} = 2.25\, g\tan\beta$, these two relations become $n\sin\beta + \mu_s n\cos\beta = 2.25\, mg\tan\beta$ and $n\cos\beta - \mu_s n\sin\beta = mg$. Dividing to cancel *n* gives $\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = 2.25\tan\beta$. Solving for μ_s and simplifying yields $\mu_s = \frac{1.25\sin\beta\cos\beta}{1 + 1.25\sin^2\beta}$. Using $\beta = \arctan\left(\frac{(20\text{ m/s})^2}{(9.80\text{ m/s}^2)(120\text{ m})}\right) = 18.79°$ gives $\mu_s = 0.34$.

EVALUATE: If μ_s is insufficient, the car skids away from the center of curvature of the roadway, so the friction is inward.

5.100. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car. The car moves in the arc of a horizontal circle, so $\vec{a} = \vec{a}_{\text{rad}}$, directed toward the center of curvature of the roadway. The target variable is the speed of the car. a_{rad} will be calculated from the forces and then *v* will be calculated from $a_{\text{rad}} = v^2/R$.

(a) To keep the car from sliding up the banking the static friction force is directed down the incline. At maximum speed the static friction force has its maximum value $f_s = \mu_s n$.

SET UP: The free-body diagram for the car is sketched in Figure 5.100a.

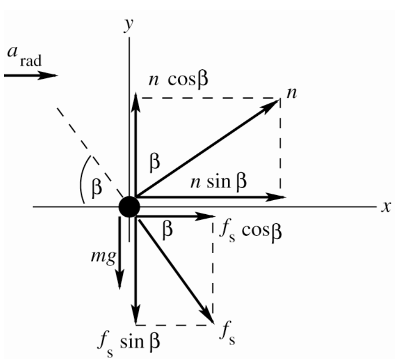


Figure 5.100a

EXECUTE:

$\Sigma F_y = ma_y$

$n\cos\beta - f_s\sin\beta - mg = 0$

But $f_s = \mu_s n$, so

$n\cos\beta - \mu_s n\sin\beta - mg = 0$

$$n = \frac{mg}{\cos\beta - \mu_s\sin\beta}$$

$\Sigma F_x = ma_x$

$n\sin\beta + \mu_s n\cos\beta = ma_{\text{rad}}$

$n(\sin\beta + \mu_s\cos\beta) = ma_{\text{rad}}$

Use the ΣF_y equation to replace n:

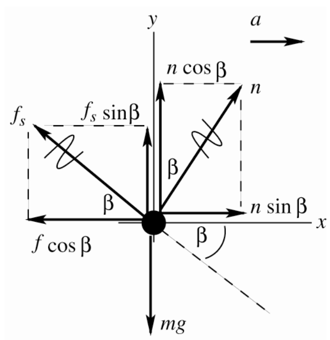
$$\left(\frac{mg}{\cos\beta-\mu_s\sin\beta}\right)(\sin\beta+\mu_s\cos\beta)=ma_{rad}$$

$$a_{rad}=\left(\frac{\sin\beta+\mu_s\cos\beta}{\cos\beta-\mu_s\sin\beta}\right)g=\left(\frac{\sin 25°+(0.30)\cos 25°}{\cos 25°-(0.30)\sin 25°}\right)(9.80\text{ m/s}^2)=8.73\text{ m/s}^2$$

$a_{rad}=v^2/R$ implies $v=\sqrt{a_{rad}R}=\sqrt{(8.73\text{ m/s}^2)(50\text{ m})}=21$ m/s.

(b) IDENTIFY: To keep the car from sliding *down* the banking the static friction force is directed up the incline. At the minimum speed the static friction force has its maximum value $f_s=\mu_s n$.

SET UP: The free-body diagram for the car is sketched in Figure 5.100b.



The free-body diagram is identical to that in part (a) except that now the components of f_s have opposite directions. The force equations are all the same except for the opposite sign for terms containing μ_s.

Figure 5.100b

EXECUTE: $a_{rad}=\left(\frac{\sin\beta-\mu_s\cos\beta}{\cos\beta+\mu_s\sin\beta}\right)g=\left(\frac{\sin 25°-(0.30)\cos 25°}{\cos 25°+(0.30)\sin 25°}\right)(9.80\text{ m/s}^2)=1.43\text{ m/s}^2$

$v=\sqrt{a_{rad}R}=\sqrt{(1.43\text{ m/s}^2)(50\text{ m})}=8.5$ m/s.

EVALUATE: For v between these maximum and minimum values, the car is held on the road at a constant height by a static friction force that is less than $\mu_s n$. When $\mu_s\to 0$, $a_{rad}=g\tan\beta$. Our analysis agrees with the result of the banking derived in the text for this special case.

5.101. **IDENTIFY:** Apply $\Sigma\vec{F}=m\vec{a}$ to each block.

SET UP: For block B use coordinates parallel and perpendicular to the incline. Since they are connected by ropes, blocks A and B also move with constant speed.

EXECUTE: **(a)** The free-body diagrams are sketched in Figure 5.101 (next page).

(b) The blocks move with constant speed, so there is no net force on block A; the tension in the rope connecting A and B must be equal to the frictional force on block A, $T_1=(0.35)(25.0\text{ N})=8.8$ N.

(c) The weight of block C will be the tension in the rope connecting B and C; this is found by considering the forces on block B. The components of force along the ramp are the tension in the first rope (8.8 N, from part (b)), the component of the weight along the ramp, the friction on block B and the tension in the second rope. Thus, the weight of block C is

$$w_C=8.8\text{ N}+w_B(\sin 36.9°+\mu_k\cos 36.9°)=8.8\text{ N}+(25.0\text{ N})(\sin 36.9°+(0.35)\cos 36.9°)=30.8\text{ N}$$

The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight w of blocks A and B, $w_C=w(\mu_k+(\sin\theta+\mu_k\cos\theta))$, giving the same result.

(d) Applying Newton's second law to the remaining masses (B and C) gives:

$$a=g(w_C-\mu_k w_B\cos\theta-w_B\sin\theta)/(w_B+w_C)=1.54\text{ m/s}^2.$$

EVALUATE: Before the rope between A and B is cut the net external force on the system is zero. When the rope is cut the friction force on A is removed from the system and there is a net force on the system of blocks B and C.

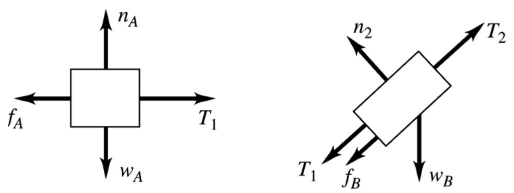


Figure 5.101

5.102. IDENTIFY: The analysis of this problem is similar to that of the conical pendulum in the text.

SET UP: As shown in the text for a conical pendulum, $\tan\beta = \dfrac{a_{\text{rad}}}{g} = \dfrac{v^2}{Rg}$.

EXECUTE: Solving for v in terms of β and R,

$v = \sqrt{gR\tan\beta} = \sqrt{(9.80\text{ m/s}^2)(50.0\text{ m})\tan 30.0°} = 16.8\text{ m/s},$ about 60.6 km/h.

EVALUATE: The greater the speed of the bus the larger will be the angle β, so T will have a larger horizontal, inward component.

5.103. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$, with $f = kv$.

SET UP: Follow the analysis that leads to the equation $v_y = v_t[1 - e^{-(k/m)t}]$, except now the initial speed is $v_{0y} = 3mg/k = 3v_t$ rather than zero.

EXECUTE: The separated equation of motion has a lower limit of $3v_t$ instead of zero; specifically,

$$\int_{3v_t}^{v} \frac{dv}{v - v_t} = \ln\frac{v_t - v}{-2v_t} = \ln\left(\frac{v}{2v_t} - \frac{1}{2}\right) = -\frac{k}{m}t, \text{ or } v = 2v_t\left[\frac{1}{2} + e^{-(k/m)t}\right]$$

where $v_t = mg/k$.

EVALUATE: As $t \to \infty$ the speed approaches v_t. The speed is always greater than v_t and this limit is approached from above.

5.104. IDENTIFY: The block has acceleration $a_{\text{rad}} = v^2/r$, directed to the left in the figure in the problem. Apply $\Sigma\vec{F} = m\vec{a}$ to the block.

SET UP: The block moves in a horizontal circle of radius $r = \sqrt{(1.25\text{ m})^2 - (1.00\text{ m})^2} = 0.75\text{ m}$. Each string makes an angle θ with the vertical. $\cos\theta = \dfrac{1.00\text{ m}}{1.25\text{ m}}$, so $\theta = 36.9°$. The free-body diagram for the block is given in Figure 5.104. Let $+x$ be to the left and let $+y$ be upward.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $T_u\cos\theta - T_l\cos\theta - mg = 0$.

$$T_l = T_u - \frac{mg}{\cos\theta} = 80.0\text{ N} - \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 31.0\text{ N}.$$

(b) $\Sigma F_x = ma_x$ gives $(T_u + T_l)\sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{r(T_u + T_l)\sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(80.0\text{ N} + 31.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 3.53\text{ m/s}.$ The number of revolutions per second is $\dfrac{v}{2\pi r} = \dfrac{3.53\text{ m/s}}{2\pi(0.75\text{ m})} = 0.749\text{ rev/s} = 44.9\text{ rev/min}.$

(c) If $T_l \to 0$, $T_u \cos\theta = mg$ and $T_u = \dfrac{mg}{\cos\theta} = \dfrac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 49.0\text{ N}$. $T_u \sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{rT_u \sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(49.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 2.35\text{ m/s}$. The number of revolutions per minute is

$(44.9\text{ rev/min})\left(\dfrac{2.35\text{ m/s}}{3.53\text{ m/s}}\right) = 29.9\text{ rev/min}$.

EVALUATE: The tension in the upper string must be greater than the tension in the lower string so that together they produce an upward component of force that balances the weight of the block.

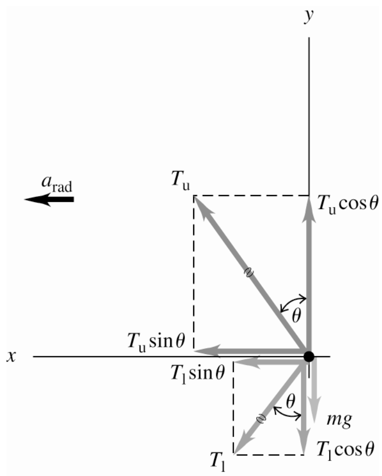


Figure 5.104

5.105. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the person. The person moves in a horizontal circle so his acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle. The target variable is the coefficient of static friction between the person and the surface of the cylinder.

$v = (0.60\text{ rev/s})\left(\dfrac{2\pi R}{1\text{ rev}}\right) = (0.60\text{ rev/s})\left(\dfrac{2\pi(2.5\text{ m})}{1\text{ rev}}\right) = 9.425\text{ m/s}$

(a) SET UP: The problem situation is sketched in Figure 5.105a.

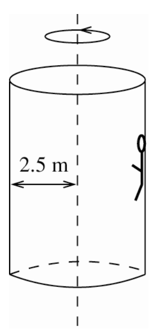
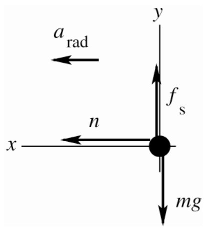


Figure 5.105a



The free-body diagram for the person is sketched in Figure 5.105b.
The person is held up against gravity by the static friction force exerted on him by the wall.
The acceleration of the person is a_{rad}, directed in toward the axis of rotation.

Figure 5.105b

(b) EXECUTE: To calculate the minimum μ_s required, take f_s to have its maximum value, $f_s = \mu_s n$.

$\Sigma F_y = ma_y$: $f_s - mg = 0$

$\mu_s n = mg$

$\Sigma F_x = ma_x$: $n = mv^2/R$

Combine these two equations to eliminate n: $\mu_s mv^2/R = mg$

$$\mu_s = \frac{Rg}{v^2} = \frac{(2.5\text{ m})(9.80\text{ m/s}^2)}{(9.425\text{ m/s})^2} = 0.28$$

(c) EVALUATE: No, the mass of the person divided out of the equation for μ_s. Also, the smaller μ_s is, the larger v must be to keep the person from sliding down. For smaller μ_s the cylinder must rotate faster to make n large enough.

5.106. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the person and to the cart.

SET UP: The apparent weight, w_{app} is the same as the upward force on the person exerted by the car seat.

EXECUTE: (a) The apparent weight is the actual weight of the person minus the centripetal force needed to keep him moving in his circular path:

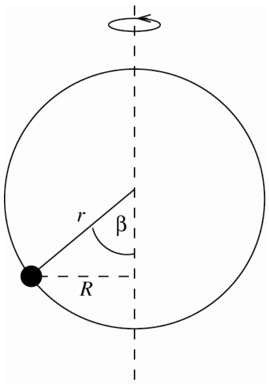
$$w_{app} = mg - \frac{mv^2}{R} = (70\text{ kg})\left[(9.8\text{ m/s}^2) - \frac{(12\text{ m/s})^2}{40\text{ m}}\right] = 434\text{ N}.$$

(b) The cart will lose contact with the surface when its apparent weight is zero; i.e., when the road no longer has to exert any upward force on it: $mg - \dfrac{mv^2}{R} = 0$. $v = \sqrt{Rg} = \sqrt{(40\text{ m})(9.8\text{ m/s}^2)} = 19.8\text{ m/s}$. The answer doesn't depend on the cart's mass, because the centripetal force needed to hold it on the road is proportional to its mass and so to its weight, which provides the centripetal force in this situation.

EVALUATE: At the speed calculated in part (b), the downward force needed for circular motion is provided by gravity. For speeds greater than this, more downward force is needed and there is no source for it and the cart leaves the circular path. For speeds less than this, less downward force than gravity is needed, so the roadway must exert an upward vertical force.

5.107. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the circular motion of the bead. Also use $a_{rad} = 4\pi^2 R/T^2$ to relate a_{rad} to the period of rotation T.

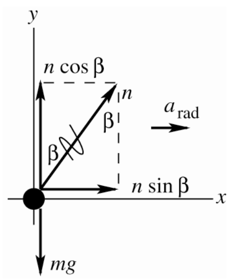
SET UP: The bead and hoop are sketched in Figure 5.107a.



The bead moves in a circle of radius $R = r\sin\beta$. The normal force exerted on the bead by the hoop is radially inward.

Figure 5.107a

The free-body diagram for the bead is sketched in Figure 5.107b.



EXECUTE:

$\Sigma F_y = ma_y$

$n\cos\beta - mg = 0$

$n = mg/\cos\beta$

$\Sigma F_x = ma_x$

$n\sin\beta = ma_{rad}$

Figure 5.107b

Combine these two equations to eliminate n:

$$\left(\frac{mg}{\cos\beta}\right)\sin\beta = ma_{rad}$$

$$\frac{\sin\beta}{\cos\beta} = \frac{a_{rad}}{g}$$

$a_{rad} = v^2/R$ and $v = 2\pi R/T$, so $a_{rad} = 4\pi^2 R/T^2$, where T is the time for one revolution.

$R = r\sin\beta$, so $a_{rad} = \dfrac{4\pi^2 r\sin\beta}{T^2}$

Use this in the above equation: $\dfrac{\sin\beta}{\cos\beta} = \dfrac{4\pi^2 r\sin\beta}{T^2 g}$

This equation is satisfied by $\sin\beta = 0$, so $\beta = 0$, or by $\dfrac{1}{\cos\beta} = \dfrac{4\pi^2 r}{T^2 g}$, which gives $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$.

(a) 4.00 rev/s implies $T = (1/4.00)\ \text{s} = 0.250\ \text{s}$

Then $\cos\beta = \dfrac{(0.250\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2(0.100\ \text{m})}$ and $\beta = 81.1°$.

(b) This would mean $\beta = 90°$. But $\cos 90° = 0$, so this requires $T \to 0$. So β approaches $90°$ as the hoop rotates very fast, but $\beta = 90°$ is not possible.

(c) 1.00 rev/s implies $T = 1.00\ \text{s}$

The $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$ equation then says $\cos\beta = \dfrac{(1.00\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})} = 2.48,$ which is not possible. The only way to have the $\Sigma\vec{F} = m\vec{a}$ equations satisfied is for $\sin\beta = 0.$ This means $\beta = 0;$ the bead sits at the bottom of the hoop.

EVALUATE: $\beta \to 90°$ as $T \to 0$ (hoop moves faster). The largest value T can have is given by $T^2 g/(4\pi^2 r) = 1$ so $T = 2\pi\sqrt{r/g} = 0.635$ s. This corresponds to a rotation rate of $(1/0.635)$ rev/s $= 1.58$ rev/s. For a rotation rate less than 1.58 rev/s, $\beta = 0$ is the only solution and the bead sits at the bottom of the hoop. Part (c) is an example of this.

5.108. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the combined object of motorcycle plus rider.

SET UP: The object has acceleration $a_{\text{rad}} = v^2/r,$ directed toward the center of the circular path.

EXECUTE: **(a)** For the tires not to lose contact, there must be a downward force on the tires. Thus, the (downward) acceleration at the top of the sphere must exceed *mg*, so $m\dfrac{v^2}{R} > mg,$ and $v > \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(13.0\text{ m})} = 11.3\text{ m/s}.$

(b) The (upward) acceleration will then be 4*g*, so the upward normal force must be $5mg = 5(110\text{ kg})(9.80\text{ m/s}^2) = 5390$ N.

EVALUATE: At any nonzero speed the normal force at the bottom of the path exceeds the weight of the object.

5.109. **IDENTIFY:** The block begins to move when static friction has reached its maximum value. After that, kinetic friction acts and the block accelerates, obeying Newton's second law.

SET UP: $\Sigma F_x = ma_x$ and $f_{s,max} = \mu_s n$, where n is the normal force (the weight of the block in this case).

EXECUTE: **(a) & (b)** $\Sigma F_x = ma_x$ gives $T - \mu_k mg = ma$. The graph with the problem shows the acceleration a of the block versus the tension T in the cord. So we solve the equation from Newton's second law for a versus T, giving $a = (1/m)T - \mu_k g$. Therefore the slope of the graph will be $1/m$ and the intercept with the vertical axis will be $-\mu_k g$. Using the information given in the problem for the best-fit equation, we have $1/m = 0.182\text{ kg}^{-1}$, so $m = 5.4945$ kg and $-\mu_k g = -2.842\text{ m/s}^2,$ so $\mu_k = 0.290$.

When the block is just ready to slip, we have $f_{s,max} = \mu_s n$, which gives $\mu_s = (20.0\text{ N})/[(5.4945\text{ kg})(9.80\text{ m/s}^2)] = 0.371$.

(c) On the Moon, g is less than on earth, but the mass m of the block would be the same as would μ_k. Therefore the slope $(1/m)$ would be the same, but the intercept $(-\mu_k g)$ would be less negative.

EVALUATE: Both coefficients of friction are reasonable or ordinary materials, so our results are believable.

5.110. **IDENTIFY:** Near the top of the hill the car is traveling in a circular arc, so it has radial acceleration and Newton's second law applies. We have measurements for the force the car exerts on the road at various speeds.

SET UP: The acceleration of the car is $a_{\text{rad}} = v^2/R$ and $\Sigma F_y = ma_y$ applies to the car. Let the $+y$-axis be downward, since that is the direction of the acceleration of the car.

EXECUTE: **(a)** Apply $\Sigma F_y = ma_y$ to the car at the top of the hill: $mg - n = mv^2/R$, where n is the force the road exerts on the car (which is the same as the force the car exerts on the road). Solving for n gives $n = mg - (m/R)v^2$. So if we plot n versus v^2, we should get a straight line having slope equal to $-m/R$ and intercept with the vertical axis at mg. We could make a table of v^2 and n using the given numbers given with the problem, or we could use graphing software. The resulting graph is shown in Figure 5.110.

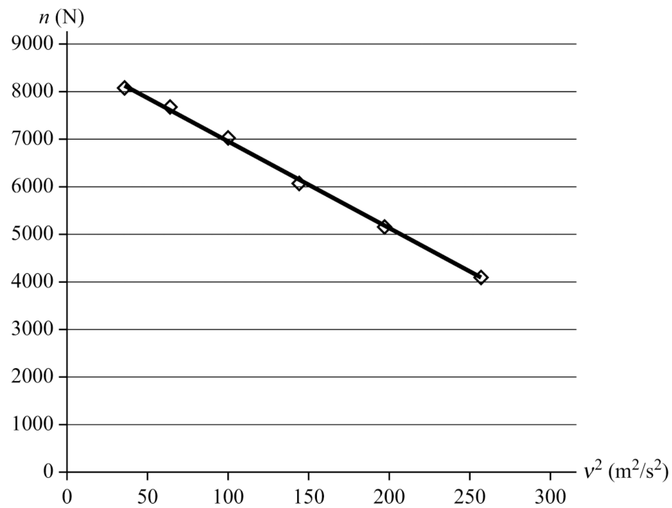


Figure 5.110

(b) The best-fit equation for the graph in Figure 5.110 is $n = [-18.12 \text{ N/(m/s)}^2]v^2 + 8794$ N. Therefore $mg = 8794$ N, which gives $m = (8794 \text{ N})/(9.80 \text{ m/s}^2) = 897$ kg.
The slope is equal to $-m/R$, so $R = -m/\text{slope} = -(897 \text{ kg})/[-18.12 \text{ N/(m/s)}^2] = 49.5$ m.

(c) At the maximum speed, $n = 0$. Using $mg - n = mv^2/R$, this gives $v = \sqrt{gR} = \sqrt{(9.80 \text{ m/s}^2)(49.5 \text{ m})} =$ 22.0 m/s.

EVALUATE: We can double check (c) using our graph. Putting $n = 0$ into the best-fit equation, we get $v = \sqrt{(8794 \text{ N})(18.14 \text{ N}\cdot\text{s}^2/\text{m}^2)} = 22.0$ m/s, which checks. Also 22 m/s is about 49 mph, which is not an unreasonabled speed on a hill.

5.111. **IDENTIFY:** A cable pulling parallel to the surface of a ramp accelerates 2170-kg metal blocks up a ramp that rises at 40.0° above the horizontal. Newton's second law applies to the blocks, and the constant-acceleration kinematics formulas can be used.

SET UP: Call the $+x$-axis parallel to the ramp surface pointing upward because that is the direction of the acceleration of the blocks, and let the y-axis be perpendicular to the surface. There is no acceleration in the y-direction. $\Sigma F_x = ma_x$, $f_k = \mu_k n$, and $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

EXECUTE: **(a)** First use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to find the acceleration of a block. Since $v_{0x} = 0$, we have $a_x = 2(x - x_0)/t^2 = 2(8.00 \text{ m})/(4.20 \text{ s})^2 = 0.9070 \text{ m/s}^2$. The forces in the y-direction balance, so $n = mg\cos(40.0°)$, so $f_k = (0.350)(2170 \text{ kg})(9.80 \text{ m/s}^2)\cos(40.0°) = 5207$ N. Using $\Sigma F_x = ma_x$, we have $T - mg\sin(40.0°) - f_k = ma$. Solving for T gives
$T = (2170 \text{ kg})(9.80 \text{ m/s}^2)\sin(40.0°) + 5207 \text{ N} + (2170 \text{ kg})(0.9070 \text{ m/s}^2) = 2.13\times10^4 \text{ N} = 21.3$ kN.
From the table shown with the problem, this tension is greater than the safe load of a ½ inch diameter cable (which is 19.0 kN), so we need to use a 5/8-inch cable.

(b) We assume that the safe load (SL) is proportional to the cross-sectional area of the cable, which means that SL $\propto \pi(D/2)^2 \propto (\pi/4)D^2$, where D is the diameter of the cable. Therefore a graph of SL versus D^2 should give a straight line. We could use the data given in the table with the problem to make the graph by hand, or we could use graphing software. The resulting graph is shown in Figure 5.111 (next page). The best-fit line has a slope of 74.09 kN/in.^{2} and a y-intercept of 0.499 kN. For a cable of diameter $D = 9/16$ in., this equation gives SL $= (74.09 \text{ kN/in.}^2)(9/16 \text{ in.})^2 + 0.499 \text{ kN} = 23.9$ kN.

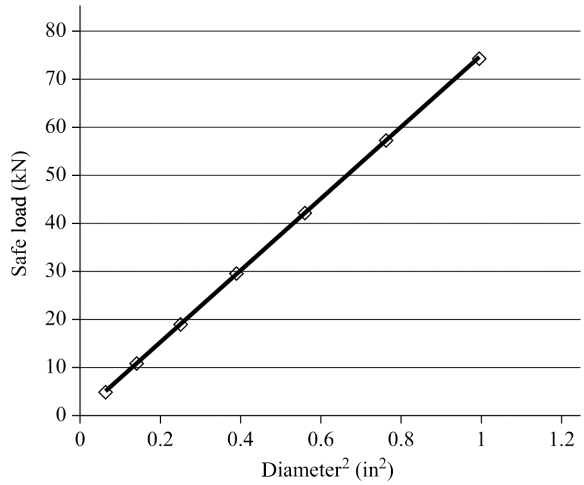


Figure 5.111

(c) The acceleration is now zero, so the forces along the surface balance, giving $T + f_s = mg\sin(40.0°)$. Using the numbers we get $T = 3.57$ kN.

(d) The tension at the top of the cable must accelerate the block and the cable below it, so the tension at the top would be larger. For a 5/8-inch cable, the mass per meter is 0.98 kg/m, so the 9.00-m long cable would have a mass of (0.98 kg/m)(9.00 m) = 8.8 kg. This is only 0.4% of the mass of the block, so neglecting the cable weight has little effect on accuracy.

EVALUATE: It is reasonable that the safe load of a cable is proportional to its cross-sectional area. If we think of the cable as consisting of many tiny strings each pulling, doubling the area would double the number of strings.

5.112. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: For both parts, take the x-direction to be horizontal and positive to the right, and the y-direction to be vertical and positive upward. The normal force between the block and the wedge is n; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is A, and the components of acceleration of the block are a_x and a_y.

EXECUTE: **(a)** The equations of motion are then $MA = -n\sin\alpha$, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$. Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, A, a_x, a_y and n. Solution is possible with the imposition of the relation between A, a_x and a_y. An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is a_y, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle α, so the relation needed is $\dfrac{a_y}{a_x - A} = -\tan\alpha$. At this point, algebra is unavoidable. A possible approach is to eliminate a_x by noting that $a_x = -\dfrac{M}{m}A$, using this in the kinematic constraint to eliminate a_y and then eliminating n. The results are:

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+m)\ \tan\alpha}{(M+m)\ \tan\alpha + (M/\tan\alpha)}$$

(b) When $M \gg m, A \to 0,$ as expected (the large block won't move). Also,

$a_x \to \dfrac{g}{\tan\alpha + (1/\tan\alpha)} = g\dfrac{\tan\alpha}{\tan^2\alpha + 1} = g\sin\alpha\cos\alpha$ which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha.$

(c) The trajectory is a straight line with slope $-\left(\dfrac{M+m}{M}\right)\tan\alpha.$

EVALUATE: If $m \gg M,$ our general results give $a_x = 0$ and $a_y = -g.$ The massive block accelerates straight downward, as if it were in free fall.

5.113. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: From Problem 5.112, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$ for the block. $a_y = 0$ gives $a_x = g\tan\alpha.$

EXECUTE: If the block is not to move vertically, both the block and the wedge have this horizontal acceleration and the applied force must be $F = (M+m)a = (M+m)g\tan\alpha.$

EVALUATE: $F \to 0$ as $\alpha \to 0$ and $F \to \infty$ as $\alpha \to 90°.$

5.114. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each of the three masses and to the pulley B.

SET UP: Take all accelerations to be positive downward. The equations of motion are straightforward, but the kinematic relations between the accelerations, and the resultant algebra, are not immediately obvious. If the acceleration of pulley B is $a_B,$ then $a_B = -a_3,$ and a_B is the average of the accelerations of masses 1 and 2, or $a_1 + a_2 = 2a_B = -2a_3.$

EXECUTE: **(a)** There can be no net force on the massless pulley B, so $T_C = 2T_A.$ The five equations to be solved are then $m_1 g - T_A = m_1 a_1,\ \ m_2 g - T_A = m_2 a_2,\ \ m_3 g - T_C = m_3 a_3,\ \ a_1 + a_2 + 2a_3 = 0$ and $2T_A - T_C = 0$. These are five equations in five unknowns, and may be solved by standard means. The accelerations a_1 and a_2 may be eliminated using $2a_3 = -(a_1 + a_2) = -[2g - T_A((1/m_1) + (1/m_2))].$ The tension T_A may be eliminated by using $T_A = (1/2)T_C = (1/2)m_3(g - a_3).$

Combining and solving for a_3 gives $a_3 = g\dfrac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

(b) The acceleration of the pulley B has the same magnitude as a_3 and is in the opposite direction.

(c) $a_1 = g - \dfrac{T_A}{m_1} = g - \dfrac{T_C}{2m_1} = g - \dfrac{m_3}{2m_1}(g - a_3).$ Substituting the above expression for a_3 gives

$a_1 = g\dfrac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

(d) A similar analysis (or, interchanging the labels 1 and 2) gives $a_2 = g\dfrac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

(e), **(f)** Once the accelerations are known, the tensions may be found by substitution into the appropriate equation of motion, giving $T_A = g\dfrac{4m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}, T_C = g\dfrac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

(g) If $m_1 = m_2 = m$ and $m_3 = 2m,$ all of the accelerations are zero, $T_C = 2mg$ and $T_A = mg.$ All masses and pulleys are in equilibrium, and the tensions are equal to the weights they support, which is what is expected.

EVALUATE: It is useful to consider special cases. For example, when $m_1 = m_2 \gg m_3$ our general result gives $a_1 = a_2 = +g$ and $a_3 = g.$

5.115. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the ball at each position.

SET UP: When the ball is at rest, $a = 0$. When the ball is swinging in an arc it has acceleration component $a_{\text{rad}} = \dfrac{v^2}{R}$, directed inward.

EXECUTE: Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos\beta = w$ or $T_A = w/\cos\beta$. At point B, the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w\cos\beta$ and the ratio $(T_B/T_A) = \cos^2\beta$.

EVALUATE: At point B the net force on the ball is not zero; the ball has a tangential acceleration.

5.116. **IDENTIFY:** The forces must balance for the person not to slip.

SET UP and EXECUTE: As was done in earlier problems, balancing forces parallel to and perpendicular to the surface of the rock leads to the equation $\mu_s = \tan\theta = 1.2$, so $\theta = 50°$, which is choice (b).

EVALUATE: The condition $\mu_s = \tan\theta$ applies only when the person is just ready to slip, which would be the case at the maximum angle.

5.117. **IDENTIFY:** Friction changes from static friction to kinetic friction.

SET UP and EXECUTE: When she slipped, static friction must have been at its maximum value, and that was enough to support her weight just before she slipped. But the kinetic friction will be less than the maximum static friction, so the kinetic friction force will not be enough to balance her weight down the incline. Therefore she will slide down the surface and continue to accelerate downward, making (b) the correct choice.

EVALUATE: Shoes with a greater coefficient of static friction would enable her to walk more safely.

5.118. **IDENTIFY:** The person pushes off horizontally and acclerates herself, so Newton's second law applies.

SET UP and EXECUTE: She runs horizontally, so her vertical acceleration is zero, which makes the normal force n due to the ground equal to her weight mg. In the horizontal direction, static friction accelerates her forward, and it must be its maximum value to achieve her maximum acceleration. Therefore $f_s = ma = \mu_s n = \mu_s mg$, which gives $a = \mu_s g = 1.2g$, making (d) the correct choice.

EVALUATE: Shoes with more friction would allow her to accelerate even faster.