MA1125 – Calculus
Tutorial problems #3

1. Show that there exists a real number $0 < x < \pi/2$ that satisfies the equation

$$x^3 \cos x + x^2 \sin x = 2.$$

2. For which values of a, b is the function f continuous at the point $x = 3$? Explain.

$$f(x) = \left\{ \begin{array}{cl} 2x^2 + ax + b & \text{if } x < 3 \\ 2a + b + 1 & \text{if } x = 3 \\ 5x^2 - bx + 2a & \text{if } x > 3 \end{array} \right\}.$$

3. Show that $f(x) = x^3 - 3x^2 + 1$ has three roots in the interval $(-1, 3)$. Hint: you need only consider the values that are attained by f at the integers $-1 \leq x \leq 3$.

4. Compute each of the following limits.

$$L = \lim_{x \to +\infty} \frac{2x^4 - 7x + 3}{3x^4 - 5x^2 + 1}, \qquad M = \lim_{x \to 2^-} \frac{2x^2 + 3x - 4}{3x^3 - 7x^2 + 4x - 4}.$$

5. Use the definition of the derivative to compute $f'(x_0)$ in each of the following cases.

$$f(x) = 3x^2, \qquad f(x) = 2/x, \qquad f(x) = (2x + 3)^2.$$

6. Show that there exists a real number $0 < x < \pi/2$ that satisfies the equation

$$x^2 + x - 1 = \sin x.$$

7. Show that $f(x) = 3x^3 - 5x + 1$ has three roots in the interval $(-2, 2)$. Hint: you need only consider the values that are attained by f at the integers $-2 \leq x \leq 2$.

8. Compute each of the following limits.

$$L = \lim_{x \to -\infty} \frac{6x^3 - 5x^2 + 7}{5x^4 - 3x + 1}, \qquad M = \lim_{x \to 2^+} \frac{x^3 + x^2 - 5x - 2}{x^3 - 5x^2 + 8x - 4}.$$

9. Use the Squeeze Theorem to show that $\lim_{x \to 0} x^2 \sin(1/x) = 0$.

10. Suppose that f is continuous with $f(0) < 1$. Show that there exists some $\delta > 0$ such that $f(x) < 1$ for all $-\delta < x < \delta$. Hint: use the ε-δ definition for some suitable ε.