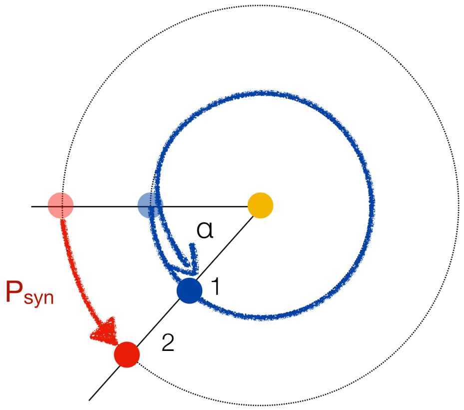
Lecture 6: Physical properties of stars

Read: Ch. 5.1,5.2, 5.4, 14.1, 14.2, 15.1 of “The Cosmic Perspective” (Bennett et al.)

Prof Aline Vidotto

Quick recap of last lecture

Measuring periods is SS planets:



Their relative (or synoptic) angular speed is

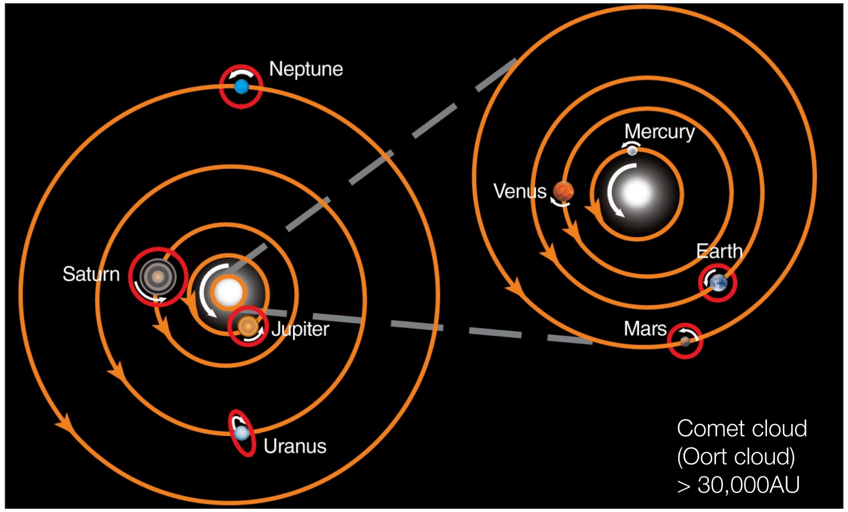
$$\Omega_{\rm rel} := \Omega_{\rm syn} = \Omega_1 - \Omega_2$$

$$\frac{1}{P_{\rm syn}} = \frac{1}{P_1} - \frac{1}{P_2}$$

Measuring distances is SS planets:

Jovian planets:
Jupiter (5AU), Saturn (10AU),
Uranus (20AU), Neptune (30AU)

Terrestrial planets:
Mercury, Venus, Earth,
Mars < 1.5AU



Angular momentum in the SS:

- orbital $\vec{L} = \vec{r} \times \vec{p}$

$$L_{\rm orb} = M_p r_{\rm orb}^2 2\pi / P_{\rm orb}$$

- rotational $\vec{L} = I\vec{\Omega}$

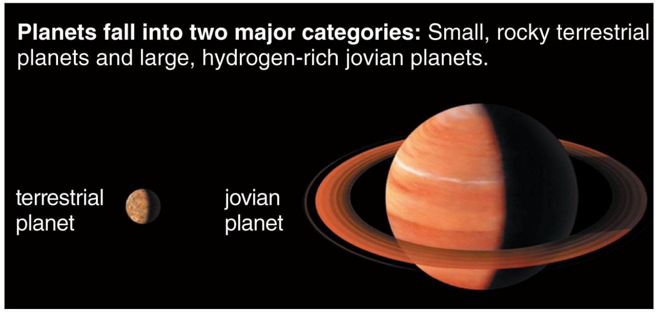
$$L_{\rm rot} = 2/5 M_p R_p^2 2\pi / P_{\rm rot}$$

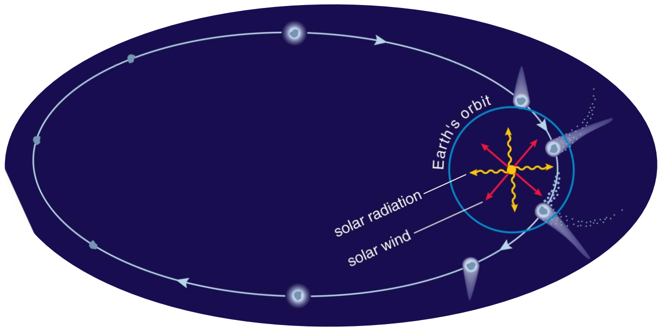
$$L_{\rm tot} = L_{\rm sun} + L_{\rm planets}$$

- for the Sun: mostly rotational
- for the planets: mostly orbital

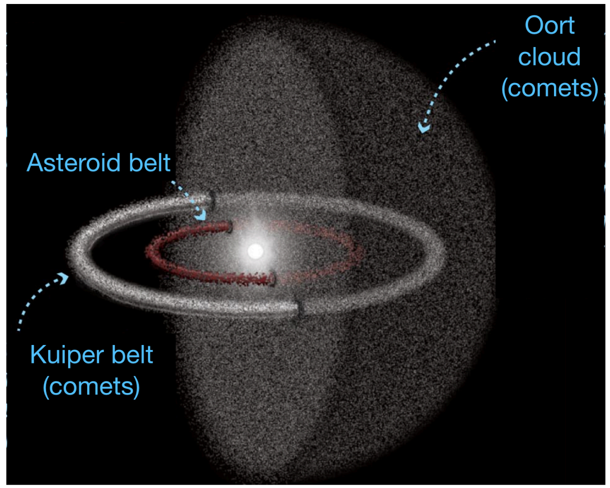
Solar system inventory:

- Two major planet types





- Rocky asteroids & icy comets



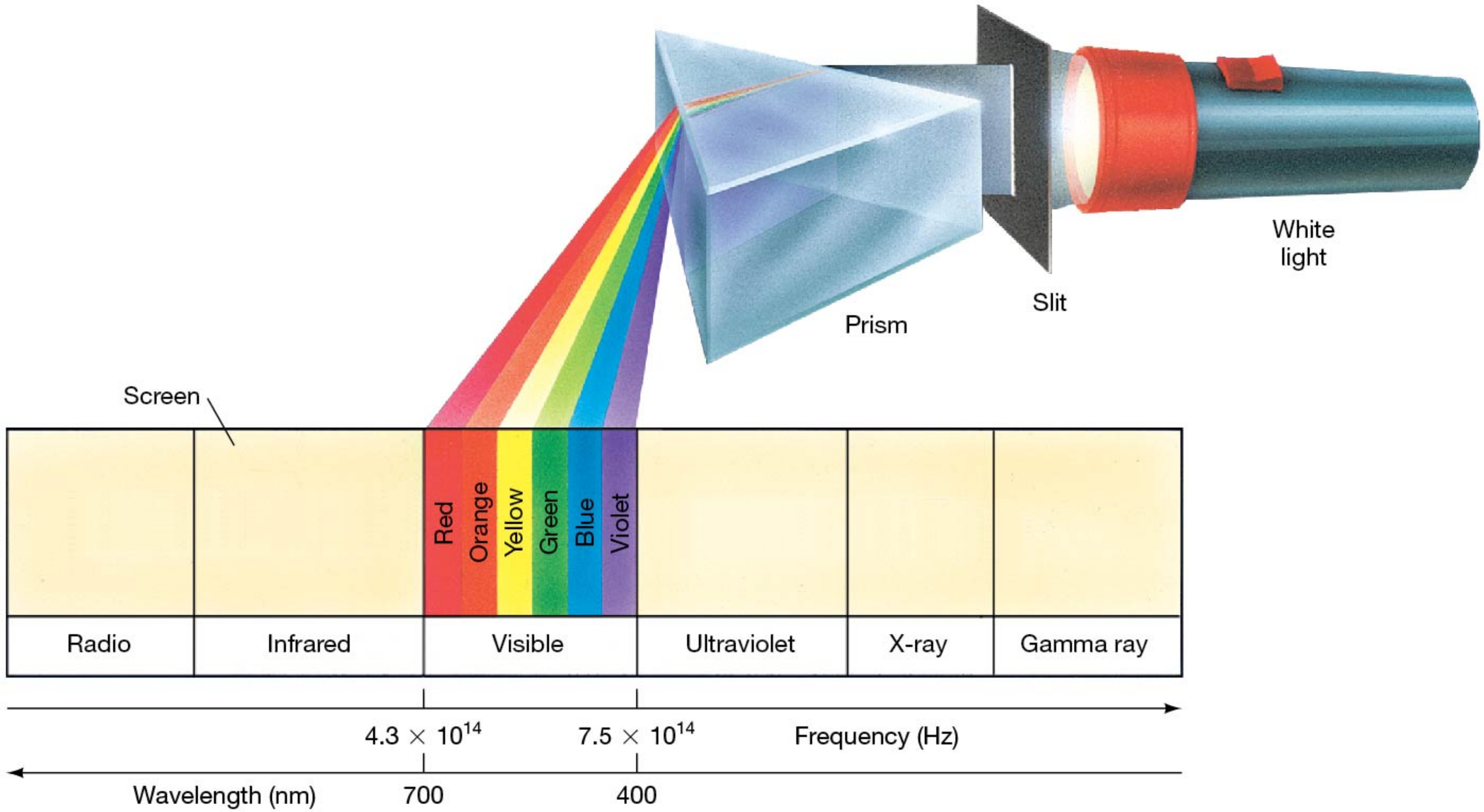
- Comets grow 2 tails when close to the sun: ion tail (solar wind), dust tail (solar radiation)

What we will cover today…

Goal: learn how to derive the main physical properties of the Sun and stars

1. The electromagnetic spectrum
2. Physical properties of the Sun
3. Physical properties of stars

1. The electromagnetic spectrum



Wavelength, frequency and energy of light

- Wave-particle duality: Light can act either like a wave or like a particle.
- Particles of light are called **photons**.
  - Each photon has a wavelength λ and a frequency *f*. All light travels with speed c=3x10^8 m/s. Hence:

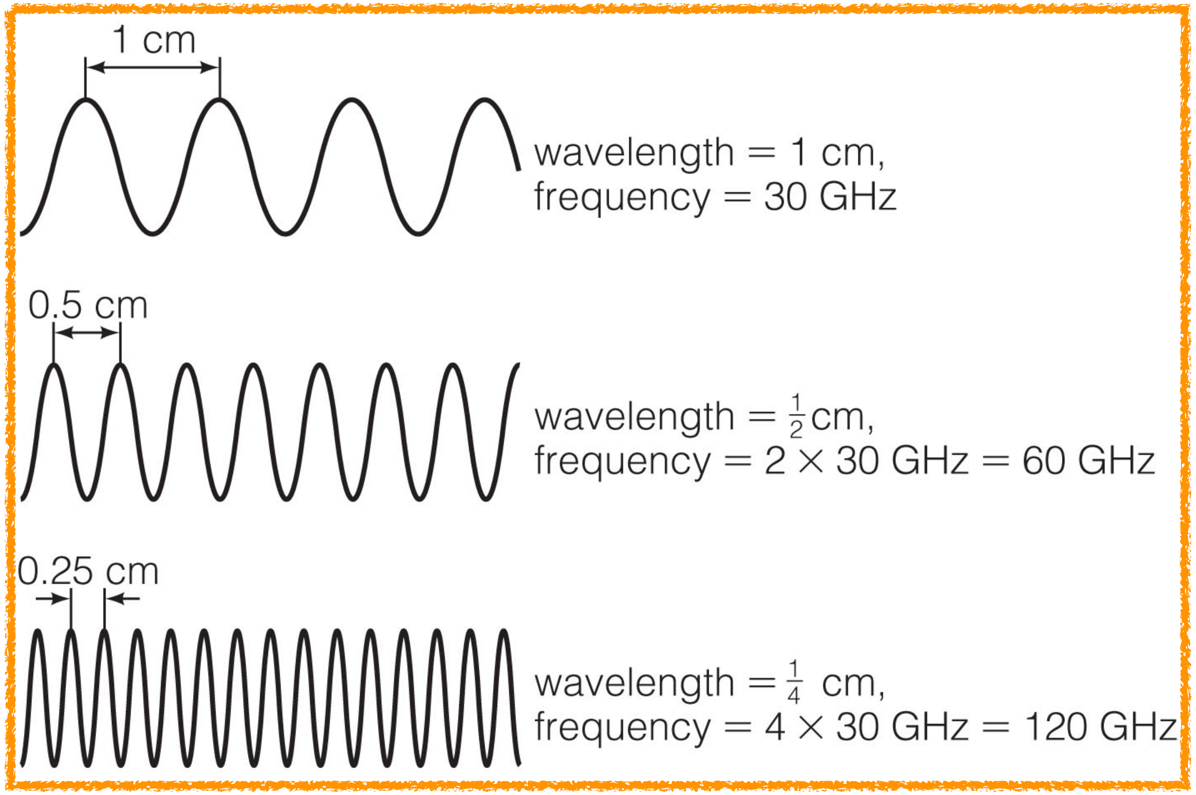
$$\lambda f = c$$

  - The energy of a photon depends on its frequency

$$E = hf$$

  - h: Planck's constant: h=6.626 × 10^{-34} J·s

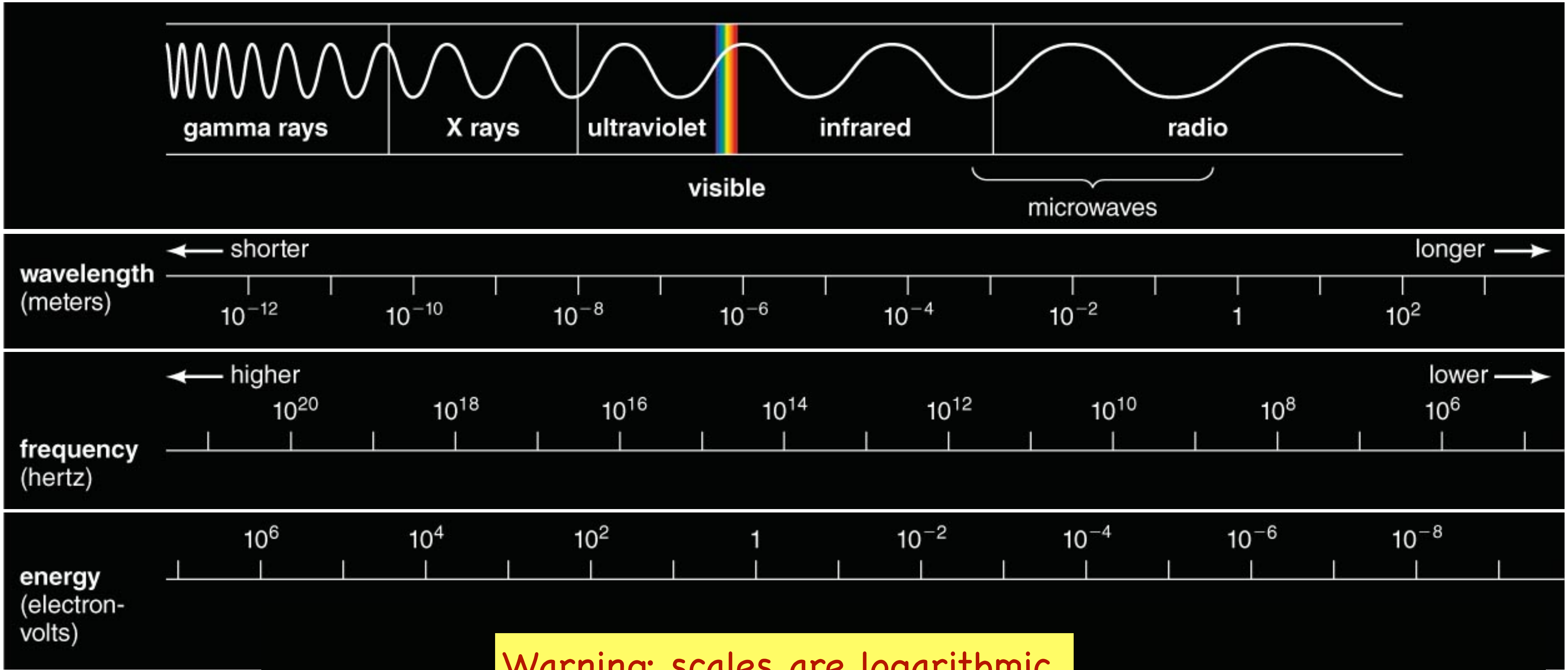
$$E = \frac{hc}{\lambda}$$



The Electromagnetic Spectrum

Human eyes cannot see most forms of light!

- The visible spectrum is only a small part of the electromagnetic spectrum
- Different parts of the full electromagnetic spectrum have different names, but there is no limit on possible wavelengths.
- Different colours of light are distinguished by their frequency and wavelength.

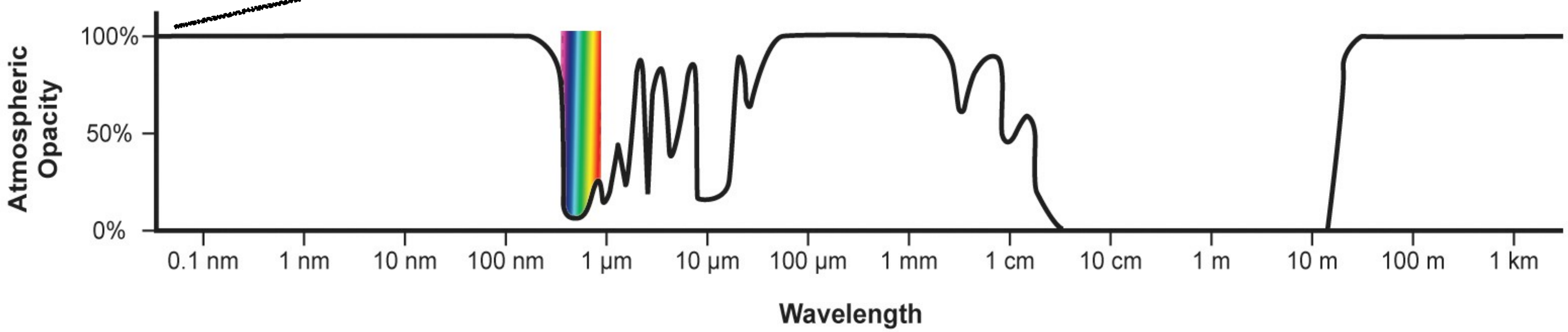


Warning: scales are logarithmic

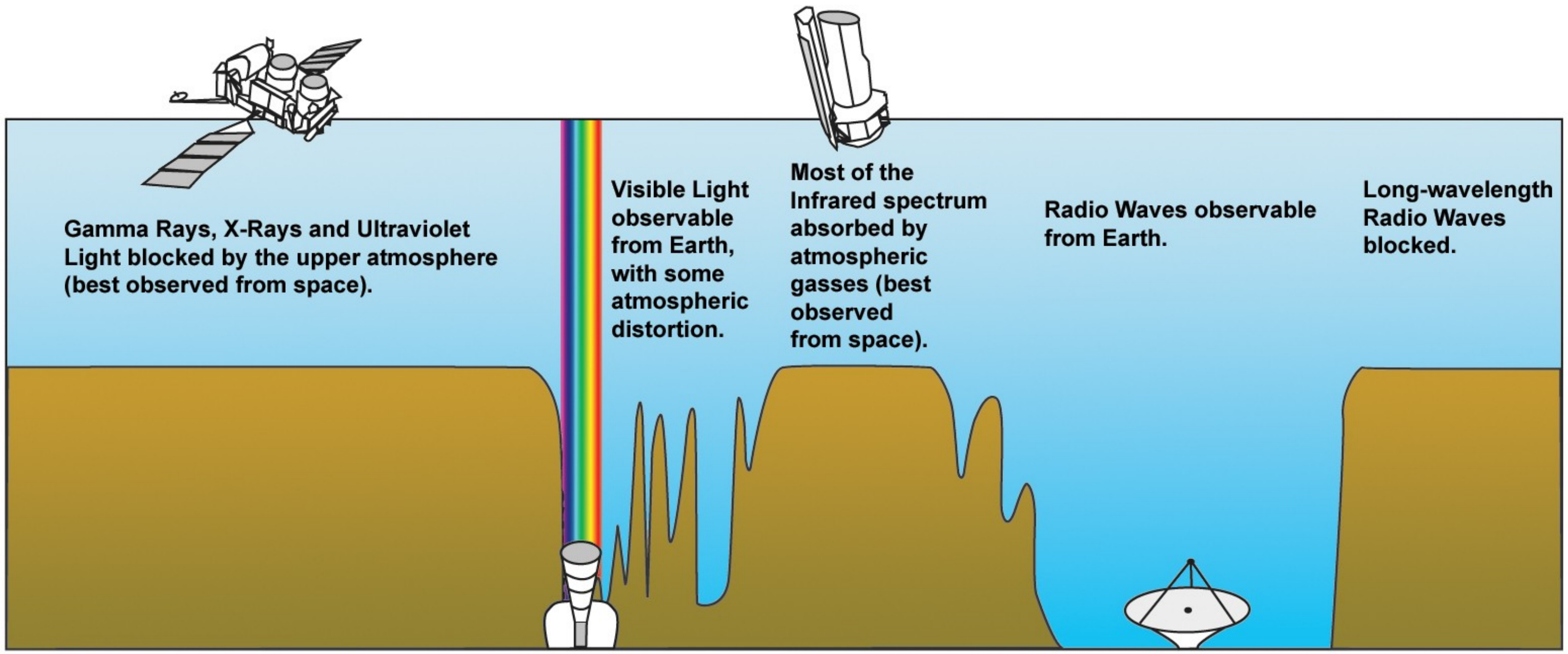
Atmospheric opacity

- Earth's atmosphere is only transparent (i.e., not opaque) at a few wavelengths: the visible, the near infrared, and the part of the radio spectrum with frequencies higher than the AM band.

100%: light does not pass through the atmosphere



- Our atmosphere absorbs a lot of the electromagnetic radiation impinging on it.



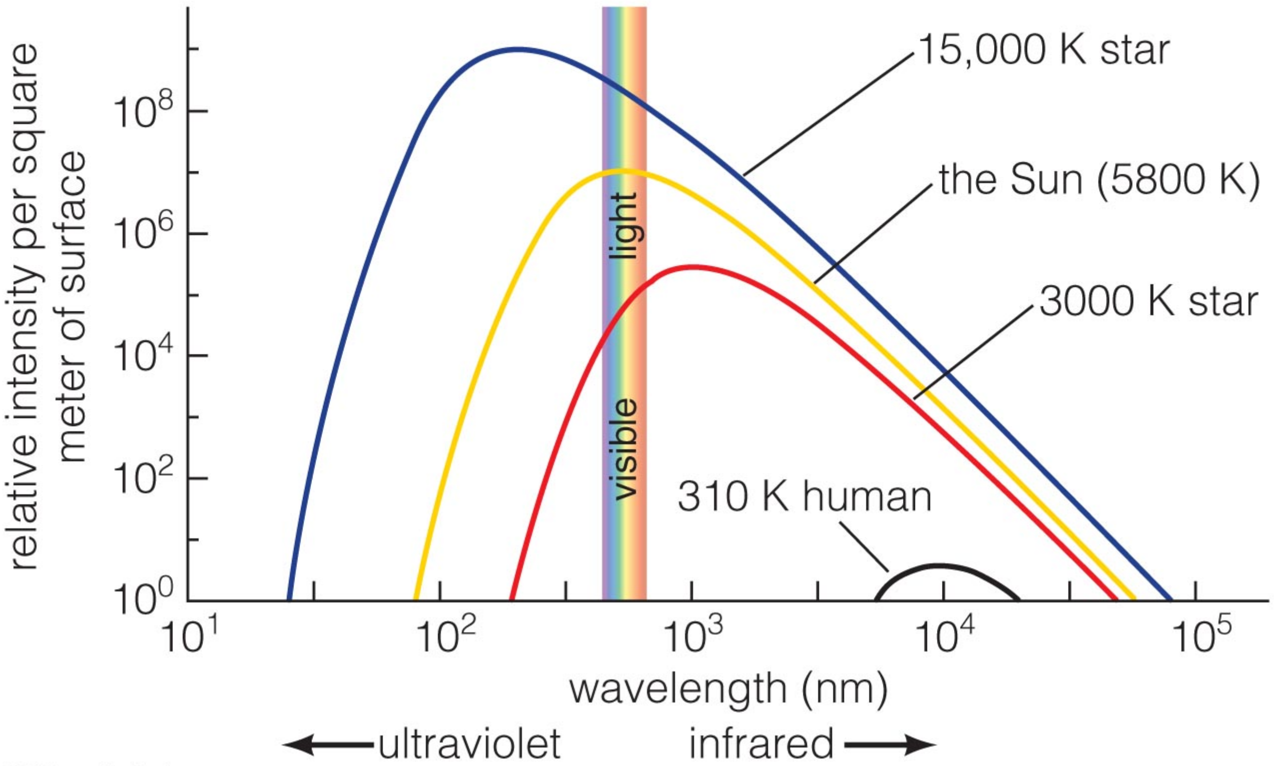
- Astronomy at other wavelengths must be done above the atmosphere (use telescopes in orbit of the Earth).
- Ex: Hubble, XMM-Newton, Chandra, etc

Thermal radiation spectrum (blackbody)

Spectrum: intensity or energy flux as a function of λ or f.

- Nearly all large or dense objects emit thermal radiation, including stars, planets, you.
- An object's thermal radiation spectrum depends on only one property: **temperature**. Although the curve shifts with temperature, the 'bell' shape remains the same!

Thermal radiation spectrum (blackbody spectrum)



Thermal radiation is called **blackbody** radiation.

Blackbody: idealised body that absorbs all the radiation falling upon it. In thermal equilibrium, a blackbody must reemit the same amount of energy as it absorbs.

Note: The Planck equation for a blackbody will be derived in your Modern Physics lectures

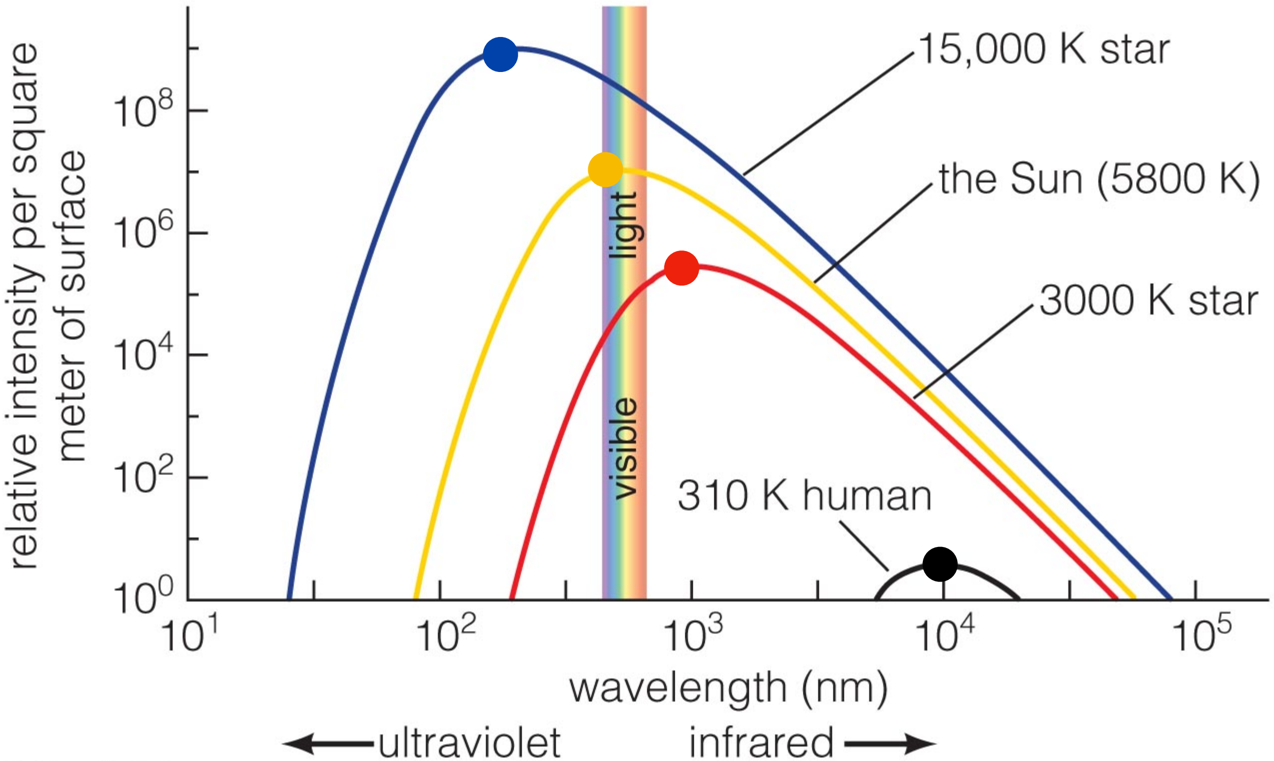
Law's of thermal radiation: Wien's law

1. Peak wavelength is inversely proportional to temperature

$$\lambda_{\text{peak}} = \frac{2.9 \times 10^6}{T}\ \text{nm}$$

→ wavelength of maximum intensity in nanometers

Thermal radiation spectrum (blackbody spectrum)



Note: Wien's law is derived by finding the maximum of the blackbody curves:

$$d(\text{Intensity})/d\lambda = 0$$

(Modern Physics lectures)

●● denote the peaks of
●● the radiation

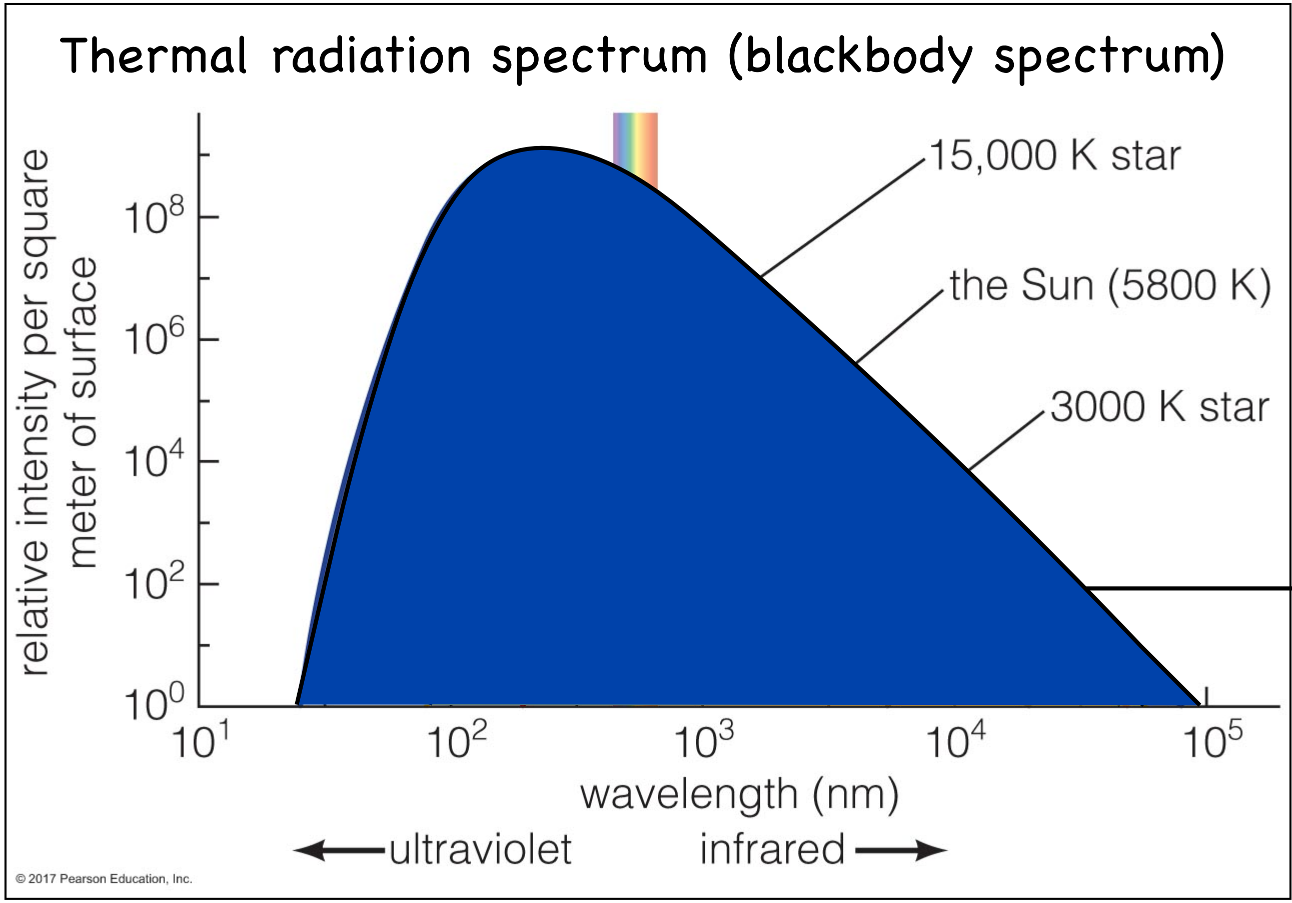
Law's of thermal radiation: Stefan-Boltzmann law

2. Emitted power per square meter of surface of the object is proportional to fourth power of temperature

$$F = \sigma T^4$$

units: W/m²

Power per area is called Surface energy Flux (or just flux)



Stefan-Boltzmann constant:

$$\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$$

Note: flux is related to the integral of the blackbody curves over all wavelengths

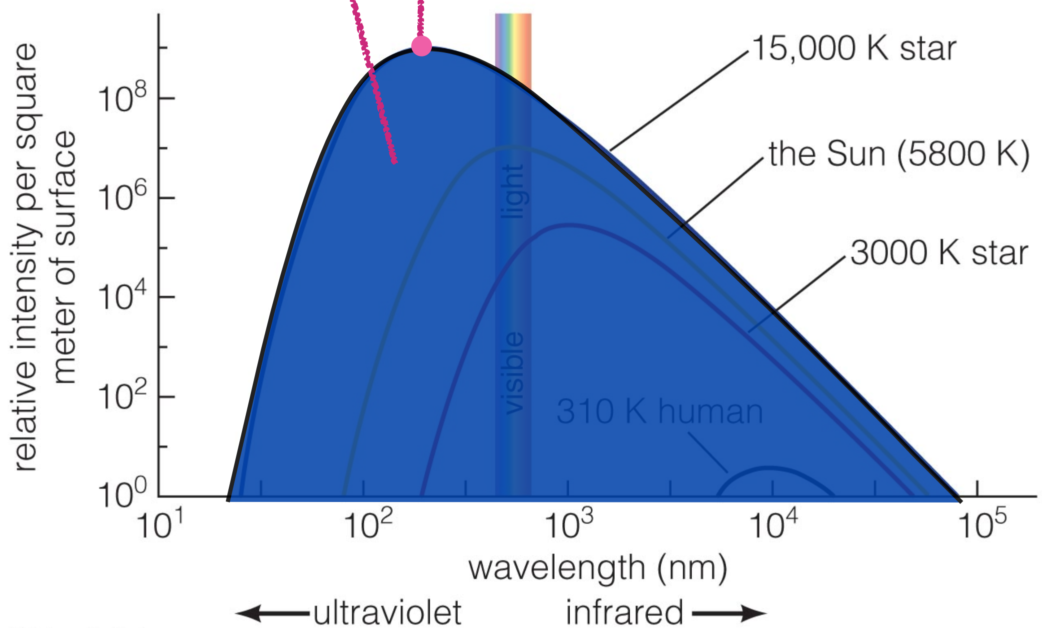
flux ∝ ∫(Intensity) dλ

(Modern Physics lectures)

Example: thermal spectrum of a star

- Find the surface energy flux (emitted power per square meter of surface) and the wavelength of peak intensity for a 15,000K star.

$$F_\star = \sigma T^4 = (5.67 \times 10^{-8}) \cdot (15000)^4$$
$$= 2.87 \times 10^9 \ \mathrm{W/m^2}$$

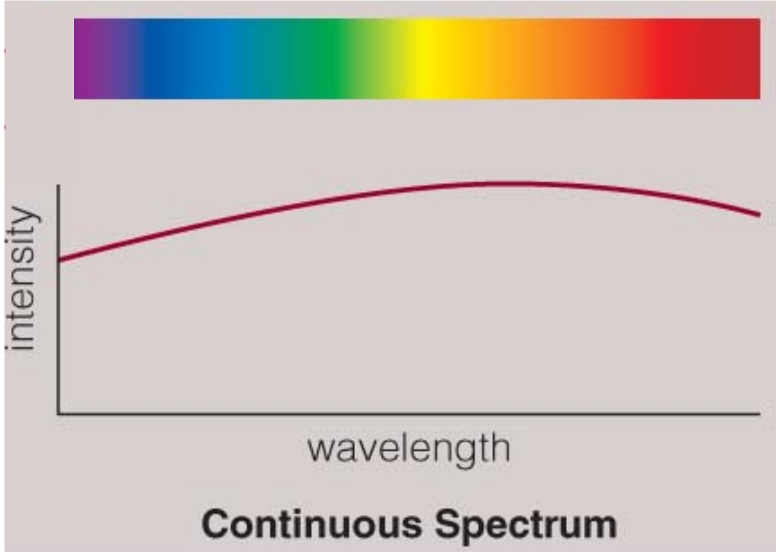


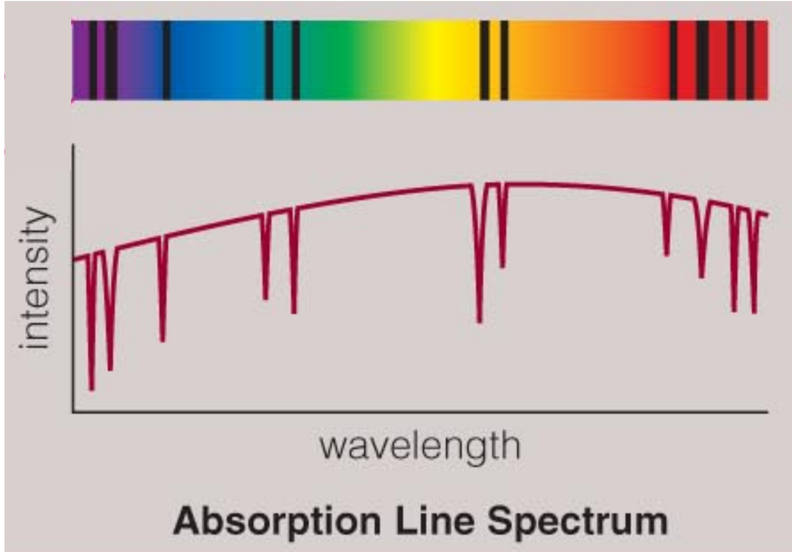
$$\lambda_{\text{peak}} = \frac{2.9 \times 10^6}{T} \ \text{nm}$$
$$= \frac{2.9 \times 10^6}{15000}$$
$$= 190 \ \text{nm}$$

In which region of the electromagnetic spectrum is this wavelength?
<u>Homework:</u> Calculate the frequency and energy of this radiation?

Spectral lines

- First order approximation: stellar spectrum is that of a blackbody
- However, at closer inspection, stellar spectrum shows dark “absorption” lines





- Spectral lines are fingerprints of chemical elements.
  - Each line is formed by an atomic line transition (2nd year Physics!)
  - Each transition depends on the photon energy, frequency and wavelength

Each type of atom has a unique spectral fingerprint → Observing the fingerprints in a spectrum tells us which kinds of atoms are present.

Why don’t we glow in the dark?

(a) People do not emit any kind of light.
(b) People only emit light that is invisible to our eyes.
(c) People are too small to emit enough light for us to see.
(d) People do not contain enough radioactive material.

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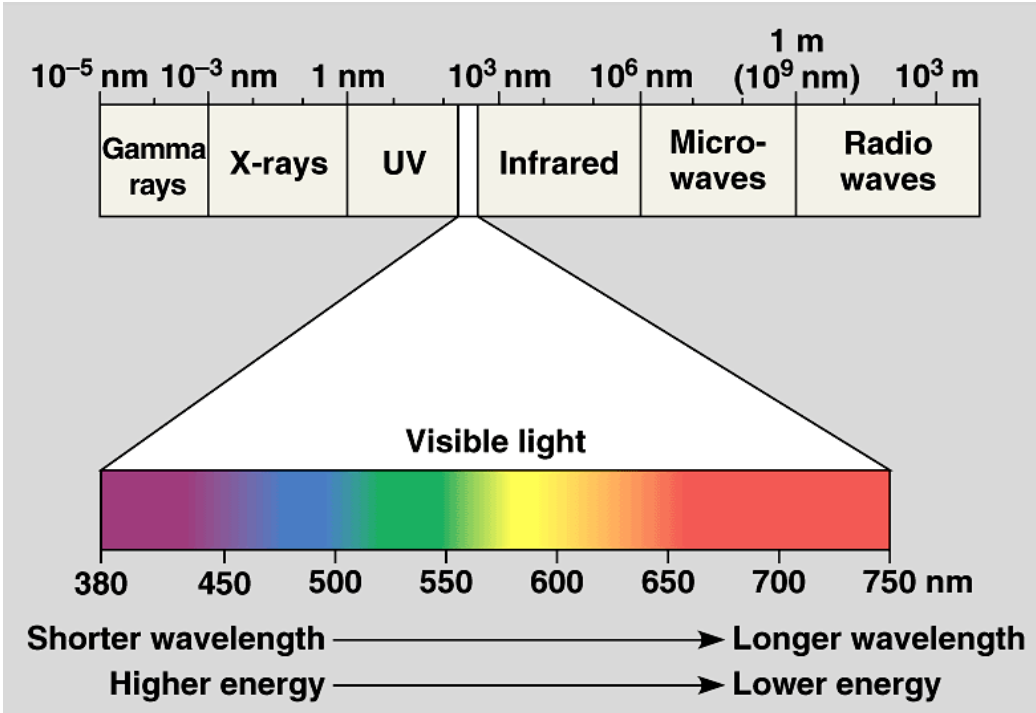
Our peak wavelength is

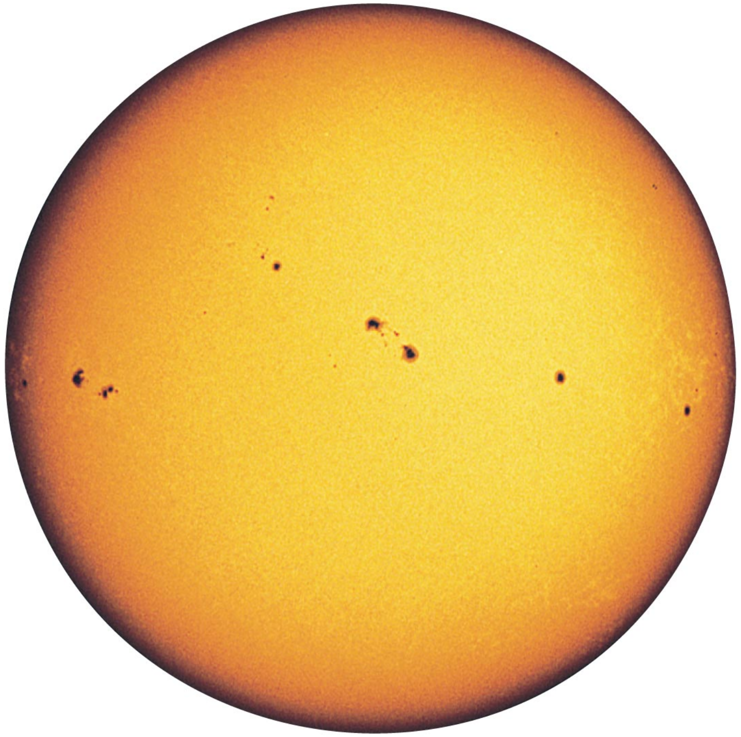
$$T = 37^{o}\mathrm{C} = 310.15\ \mathrm{K}$$

$$\lambda_{\text{peak}} = \frac{2.9 \times 10^6}{T}\ \mathrm{nm}$$

$$= \frac{2.9 \times 10^6}{310.15} = 9350\ \mathrm{nm}$$



2. Physical properties of the Sun



Mass:
2×10^{30} kg
(300,000 Earth's mass)

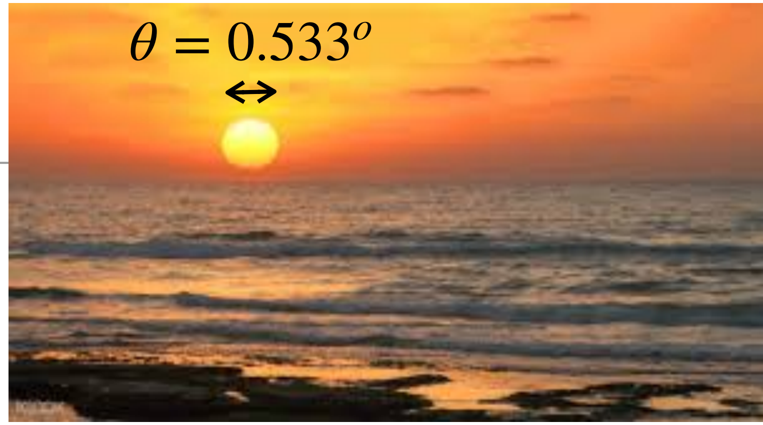
Derived using Kepler's 3rd law

Radius:
6.96×10^{8} m
(109 times Earth's radius)

Luminosity (power):
3.84×10^{26} watts

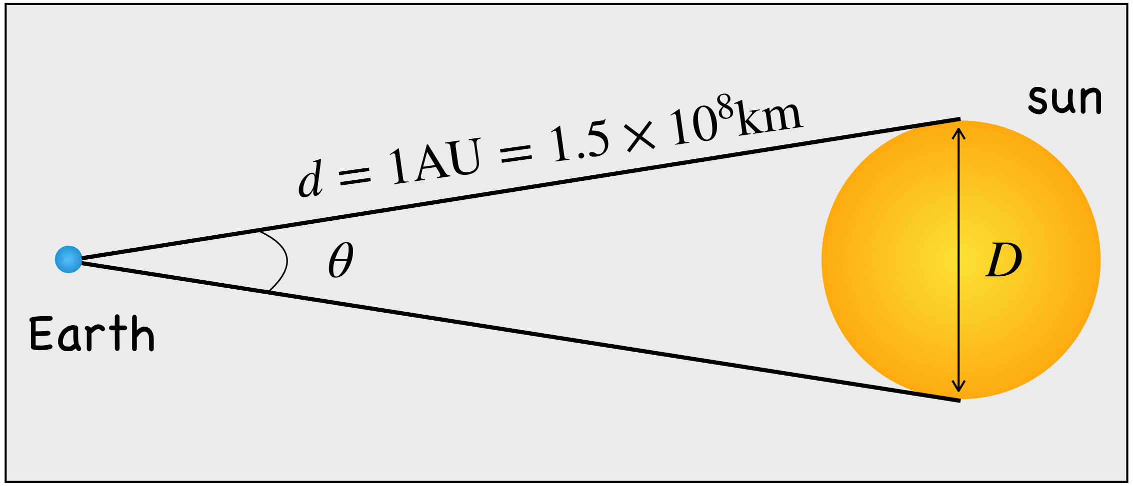
We will derive next

The size of the Sun



- The Sun subtends an angular size of

$$\theta = 0.533^o$$
$$= 0.533\frac{\pi}{180^o} = 9.3 \times 10^{-3} \text{ rad}$$



- For small angles (in radians), we have that

$$\theta = \frac{D}{d}$$

$$D = \theta d$$
$$= (9.33 \times 10^{-3}) \cdot (1.5 \times 10^8\text{km})$$
$$= 1.39 \times 10^6\text{km}$$

- The solar radius is $R_\odot = \dfrac{D}{2}$

$$R_\odot = 6.96 \times 10^5 \text{ km}$$

$$\boxed{R_\odot = 6.96 \times 10^8 \text{ m}}$$

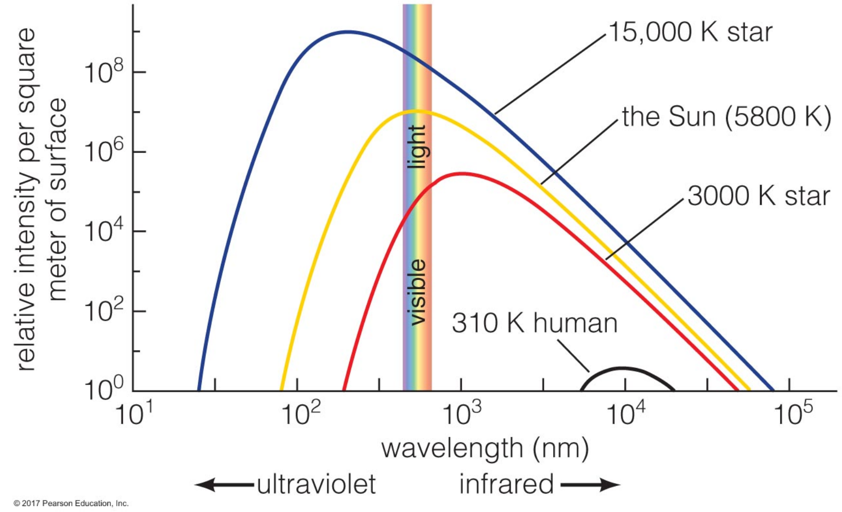
Solar temperature, flux and luminosity

- The Sun emits a 'blackbody' spectrum with λ_{peak} ~ 500 nm. What is its temperature?

$$\lambda_{\rm peak} = \frac{2.9\times10^6}{T}\ \rm nm$$

Wien's law

$$T = \frac{2.9\times10^6}{\lambda_{\rm peak}} = \frac{2.9\times10^6}{500} = 5800\ \rm K$$



- What is the surface energy flux of the Sun?

Stefan-Boltzmann law

$$F_\odot = \sigma T^4 = (5.67\times10^{-8})\cdot(5800^4) = 6.4\times10^7 \rm W/m^2$$

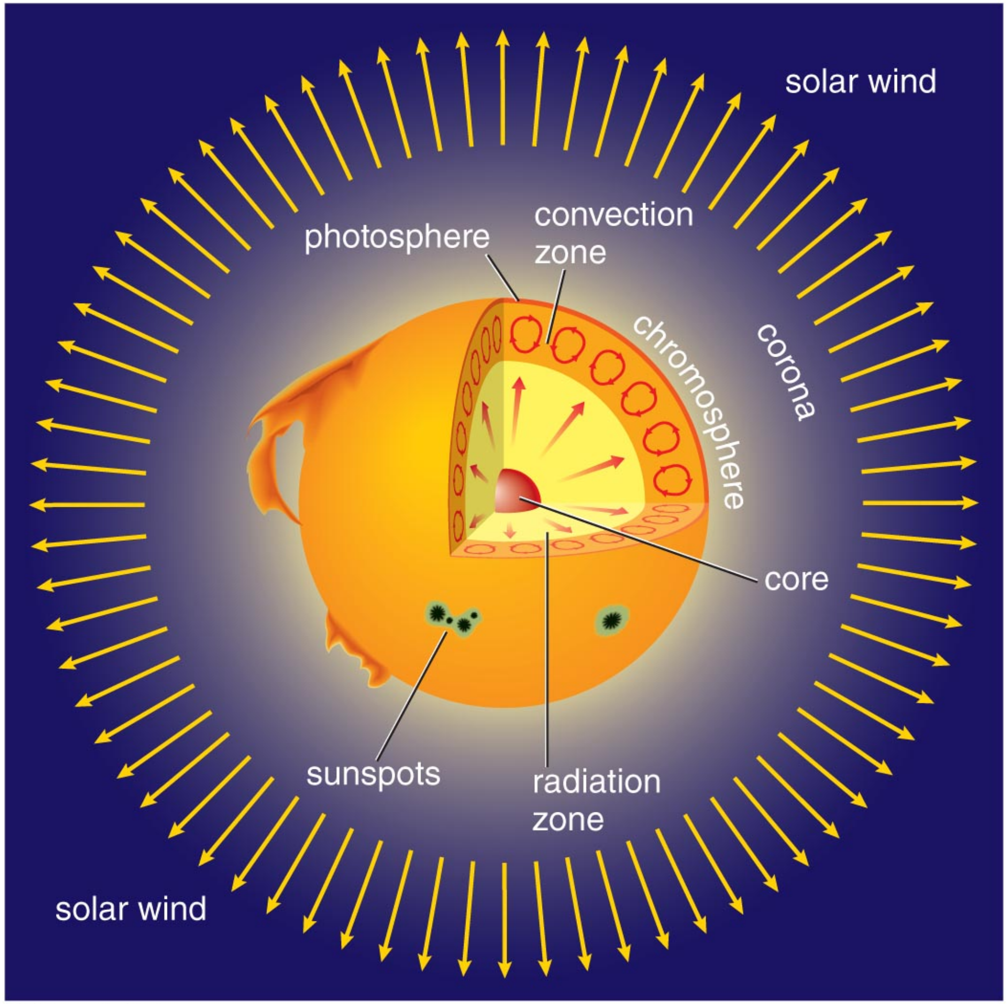
- What is its total emitted power? power = flux · area

$$R_\odot = 6.96\times10^8\ \rm m$$

$$L_\odot = F_\odot\cdot(4\pi R_\odot^2) = (6.4\times10^7)\cdot(4\pi(6.96\times10^8)^2) = 3.9\times10^{26}\ \rm W$$

Very important: In Astronomy, power is called luminosity

The structure of the Sun (exterior)

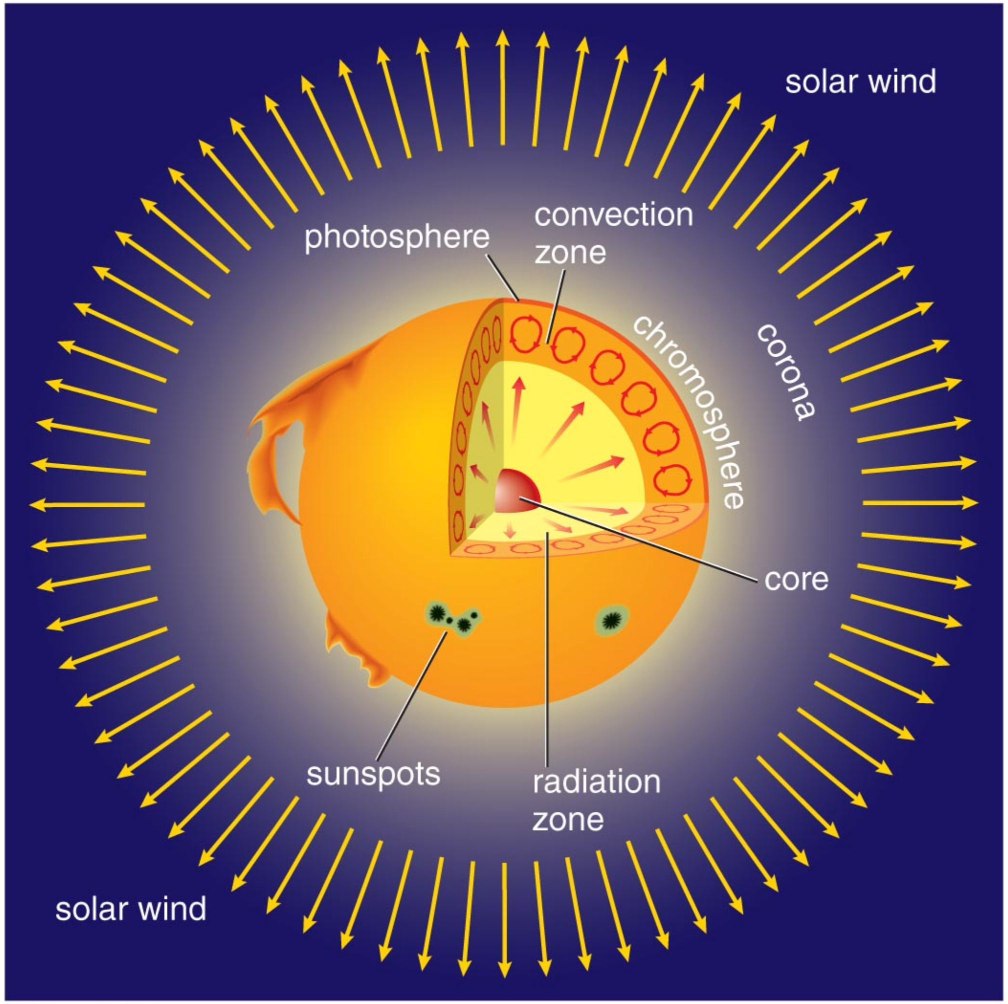


- **Solar wind:** A flow of charged particles from the surface of the Sun
- **Corona:** Outermost layer of solar atmosphere; T~1 million K
- **Chromosphere:** Middle layer of solar atmosphere; T ~ 10^4 - 10^5 K
- **Photosphere:**
  - Visible surface of Sun: T=5800 K (check "blackbody radiation")
  - sunspots: dark spots created by intense magnetic fields. Why do they appear darker?
  - sunspots are cooler (4000K) than photosphere

$$\frac{F_{\text{spot}}}{F_{\text{photo}}} = \frac{\sigma T^4_{\text{spot}}}{\sigma T^4_{\text{photo}}} = \frac{4000^4}{5800^4} = 0.2$$

Sunspots emit 0.2 times the photospheric flux (making them appear darker)

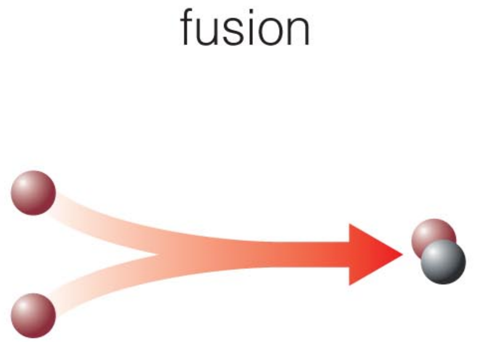
The structure of the Sun (interior)

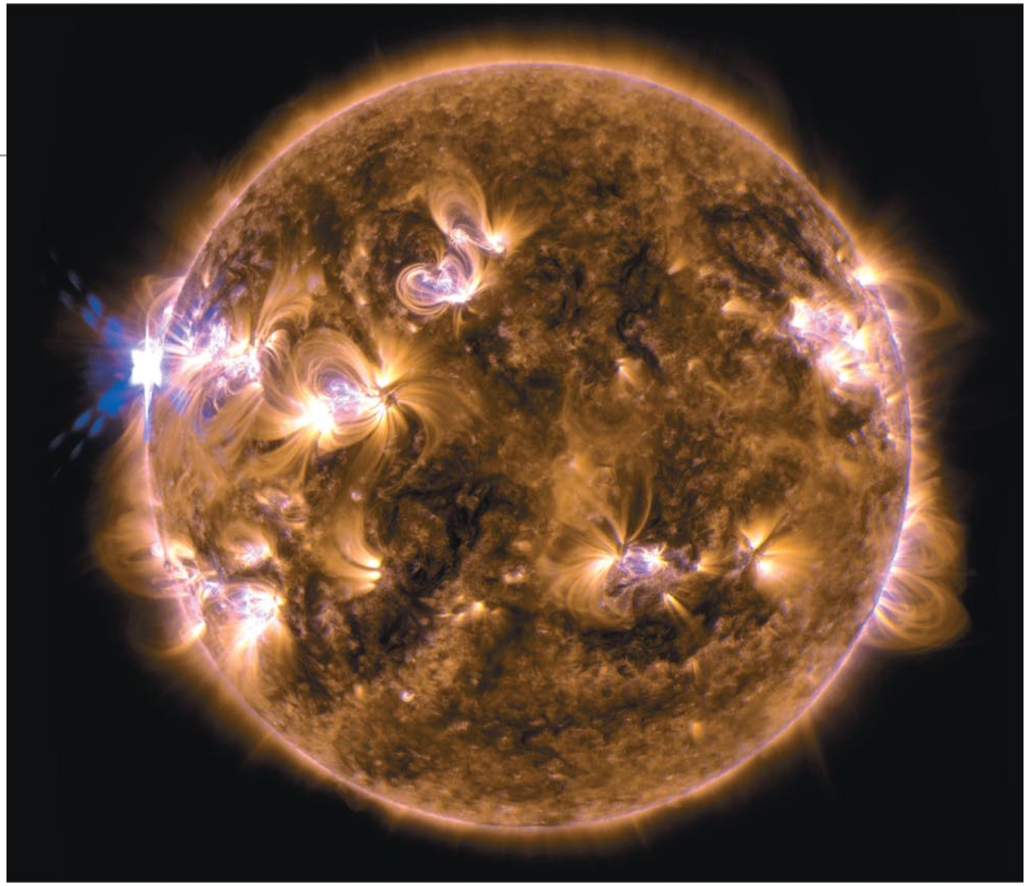


- **Convection Zone:**
  - Energy transported upward by rising hot gas
- **Radiation Zone:**
  - Energy transported upward by photons
- **Core:**
  - Energy generated by nuclear fusion; T ~ 15 million K

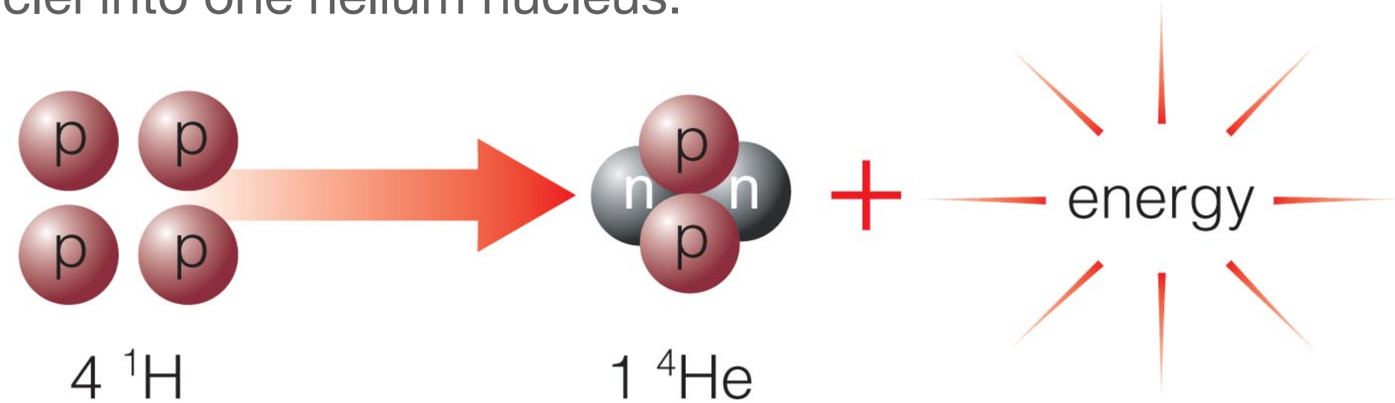
Why is the Sun shining?

- Sun is powered by nuclear fusion
  - Small nuclei stick together to make a bigger one.





- The Sun releases energy by fusing four hydrogen nuclei into one helium nucleus.



$$\text{energy} = \text{mass} \times c^2$$

Mass-energy conservation in Hydrogen fusion

- Four hydrogen nuclei (protons) that go into the overall reaction have a slightly greater mass than the helium nucleus that comes out

$$m_p = 1.6726 \times 10^{-27} \text{ kg} \longrightarrow 4 \times m_p = 6.690 \times 10^{-27} \text{ kg}$$

$$m_{He} = 6.643 \times 10^{-27} \text{ kg}$$

$$\Delta m = 4 \times m_p - m_{He} = (6.690 - 6.643) \times 10^{-27} \text{ kg} = 0.047 \times 10^{-27} \text{ kg}$$

- The Sun becomes 0.047×10^{-27} kg lighter after each nuclear reaction!
- This mass is converted in energy, which is released in each nuclear reaction:

$$E = \Delta m \cdot c^2 = (0.047 \times 10^{-27}) \cdot (3 \times 10^8)^2 = 4.2 \times 10^{-12} \text{ J (per nuclear reaction)}$$

- The sun's luminosity is 3.84×10^{26} W $= 3.84 \times 10^{26}$ J/s. Thus, the number of nuclear reactions happening at each second is

$$\frac{\text{Sun's energy per second}}{\text{energy per reaction}} = \frac{3.84 \times 10^{26} \text{J/s}}{4.2 \times 10^{-12} \text{ J}} = 9 \times 10^{37} \text{ reactions/s}$$

9×10^{37} atoms of Helium are created per second inside the sun!

<u>Homework</u>: calculate how much mass "disappears" from the Sun per second

The energy emitted by the Sun is produced

(a) uniformly throughout the whole Sun.

(b) throughout the whole Sun, but more in the centre than at the surface, as $1/r^2$.

(c) in a very small region at the very centre of the Sun.

(d) from radioactive elements created in the Big Bang (early universe).

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3. Physical properties of stars

Stars are also powered by nuclear reactions in their central parts.

They have a **surface flux**

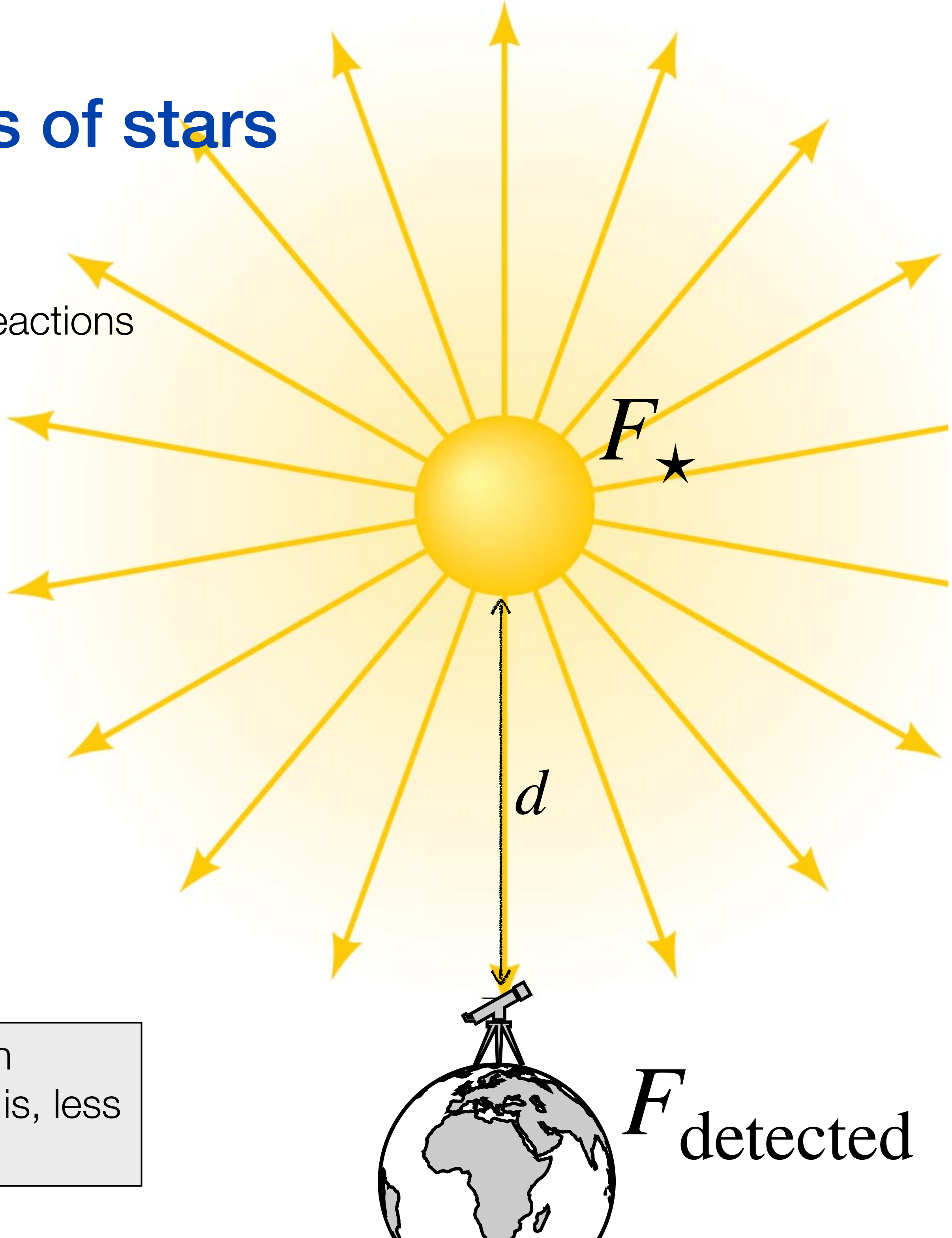
$$F_\star = \sigma T^4$$

and a **luminosity**

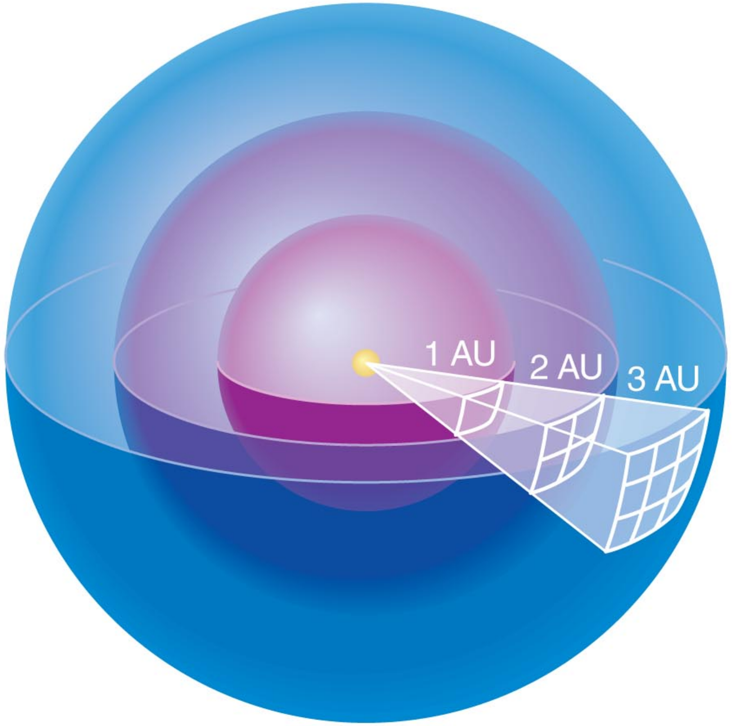
$$L_\star = F_\star \cdot (4\pi R_\star^2)$$

However, the **flux we detect** in our telescopes is <u>not</u> F_{star}.

The detected flux is **attenuated** with distance: the farther away an object is, less radiation (flux) we detect.



How do we measure stellar luminosities?



- The amount of luminosity passing through each sphere is the same.
- Area of sphere: 4π (distance)^2
- Divide luminosity by area to get **detected flux** (aka brightness).

$$\text{Detected flux} = \frac{\text{Luminosity}}{4\pi(\text{distance})^2}$$

$$F_{\text{detected}} = \frac{L_\star}{4\pi d^2}$$

- We can determine a star's luminosity if we can measure its distance and flux (apparent brightness):

$$\boxed{L_\star = F_{\text{detected}} 4\pi d^2}$$

<u>Physical units:</u>

• Luminosity: Amount of power a star radiates (energy per second = W)

• Detected flux or apparent brightness: amount of energy that reaches Earth (energy per second per square meter = W/m^2)

Example: detected flux of a star

- Calculate the flux we detect of a 15,000K star that is at a distance of 20 pc. Assume this star has a radius of 5 $R_\odot$.

$$1 \text{ pc} = 2 \times 10^5 \text{ AU} = 3.08 \times 10^{16} \text{ m}$$

- We already calculated the surface flux of a 15,000K star:

$$F_\star = \sigma T^4 \quad = 2.87 \times 10^9 \text{ W/m}^2$$

Important: same physical units, but different quantities!

- Its luminosity is

$$L_\star = F_\star \cdot (4\pi R_\star^2) = 2.87 \times 10^9 \cdot 4\pi(5 \times 6.96 \times 10^8)^2$$

$$= 4.37 \times 10^{29} \text{ W}$$

- The flux we detect with our telescopes is

$$F_{\text{detected}} = \frac{L_\star}{4\pi d^2} = \frac{4.37 \times 10^{29}}{4\pi(20\text{pc})^2} = \frac{4.37 \times 10^{29}}{4\pi(20 \cdot 3.08 \times 10^{16})^2} = 9 \times 10^{-8} \text{ W/m}^2$$

Solar irradiance at Earth

- The total solar irradiance (or the solar constant) is the amount of of solar energy received per square meter of area, every second, at Earth.
  - this is a measure of flux received, similar to what we calculated for the flux detected from other stars:

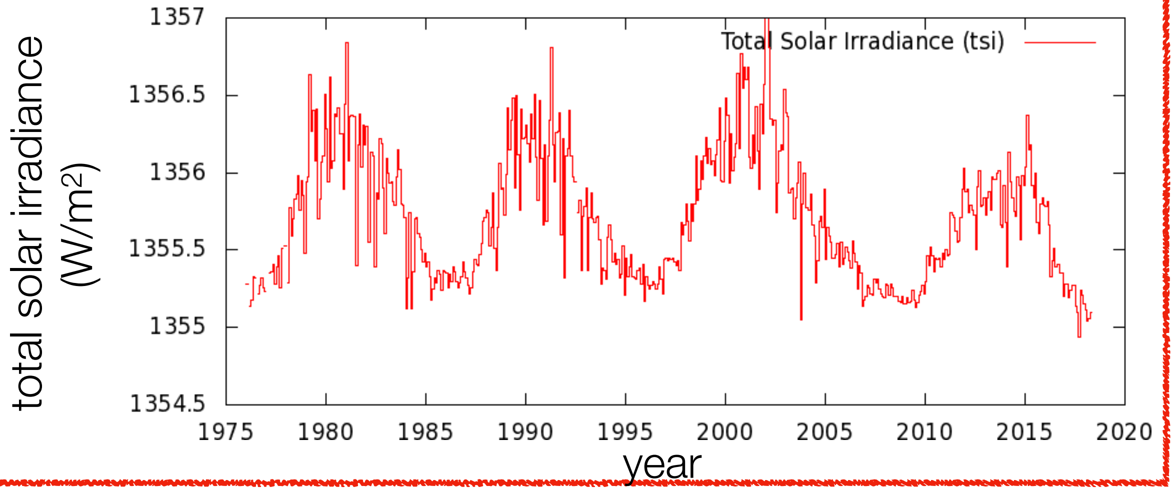
$$F_{\text{detected}} = \frac{L_\star}{4\pi d^2}$$

- Using the solar luminosity and Sun-Earth distance (1AU), the solar constant is:

$$F = \frac{L_\odot}{4\pi d^2} = \frac{3.84 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} \simeq 1350 \mathrm{W/m^2}$$

→ Relevant for climate studies

• The solar constant is in fact not exactly constant…
• There are small variations in the solar luminosity with a period of 11 years (solar cycle)



Calculating stellar radii

- We can rarely measure stellar radii directly, so we use the laws of thermal radiation to derive the radius of stars.

$$L_\star = F_\star \cdot (4\pi R_\star^2) \longrightarrow L_\star = \sigma T^4 \cdot (4\pi R_\star^2)$$

$$\boxed{R_\star = \sqrt{\frac{L_\star}{4\pi\sigma T^4}}}$$

- Example: The red supergiant star Betelgeuse has a luminosity of 120,000 L$_\odot$ and a surface temperature of about 3650K. Calculate its radius.

$$L_\star = 120000 L_\odot = 120000 \times 3.8 \times 10^{26} = 4.6 \times 10^{31} \text{ W}$$

$$R_\star = \sqrt{\frac{L_\star}{4\pi\sigma T^4}} = \sqrt{\frac{4.6 \times 10^{31}}{4\pi(5.67 \times 10^{-8})(3650^4)}} = 5.9 \times 10^{11} \text{ m}$$

$$= 3.9\text{AU}$$

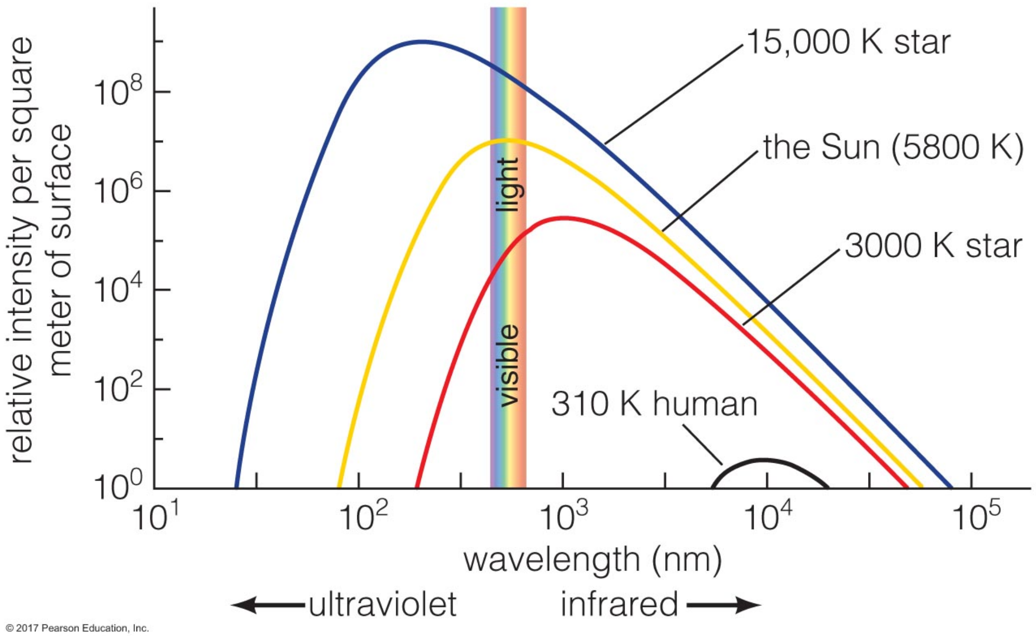
This is almost four times the Sun-Earth distance, which means that the orbits of all the inner planets of our solar system could fit easily inside Betelgeuse!

Colours of stars tells us about their temperatures

Remember Wien's law?

Blue stars:	~ 20,000 K
White stars:	~ 10,000 K
Yellow stars:	~ 6,000 K
Orange stars:	~ 4,000 K
Red stars:	~ 3,000 K

Thermal radiation spectrum (blackbody spectrum)



Stars have different colours

Stellar classification

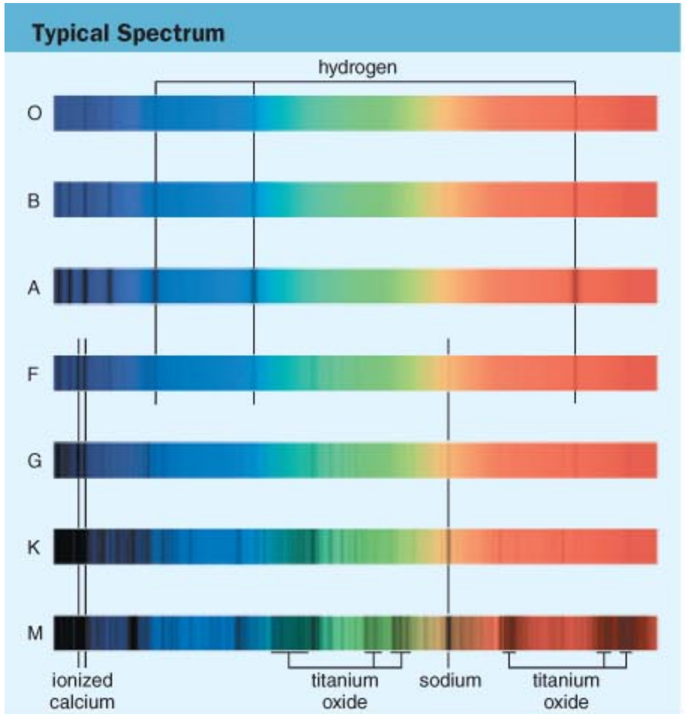
- Because of the different temperatures of the stars, they have different absorption lines in their spectrum
- The strength of the line gives us a way to classify stars into **Spectral types**

TABLE 10.2 Spectral Classes

Spectral Class	Temperature (K)
O	30,000
B	20,000
A	10,000
F	7000
G	6000
K	4000
M	3000

Hottest

Coolest



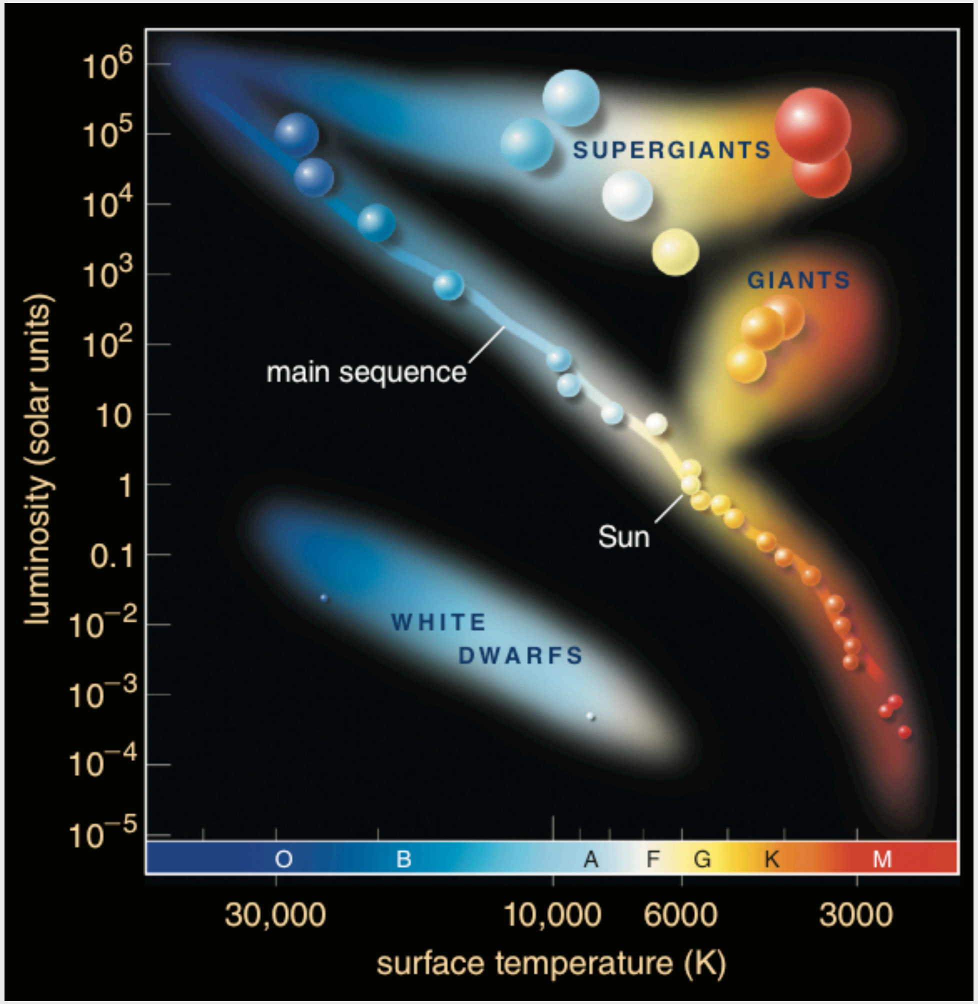
- Example: Sun is spectral type G2

you need to memorise the order of spectral types!
O B A F G K M

Hertzprung-Russell Diagram: “HR Diagram”

What is it?

- plot of luminosity versus temperature
- super important: temperature runs backwards
- super super important: this is a log-log plot



- Main characteristics:
  - Majority of stars are on the “main sequence”

$$L_\star = 4\pi R_\star^2 \sigma T^4$$

  - Stars with lower T and higher L than main-sequence stars must have larger radii. These stars are called **giants and supergiants.**
  - Stars with higher T and lower L than main-sequence stars must have smaller radii. These stars are called **white dwarfs**.

What are the two most important intrinsic properties for classifying stars?

(a) Distance and surface temperature

(b) Luminosity and surface temperature

(c) Distance and luminosity

(d) Mass and age

(e) Distance and colour

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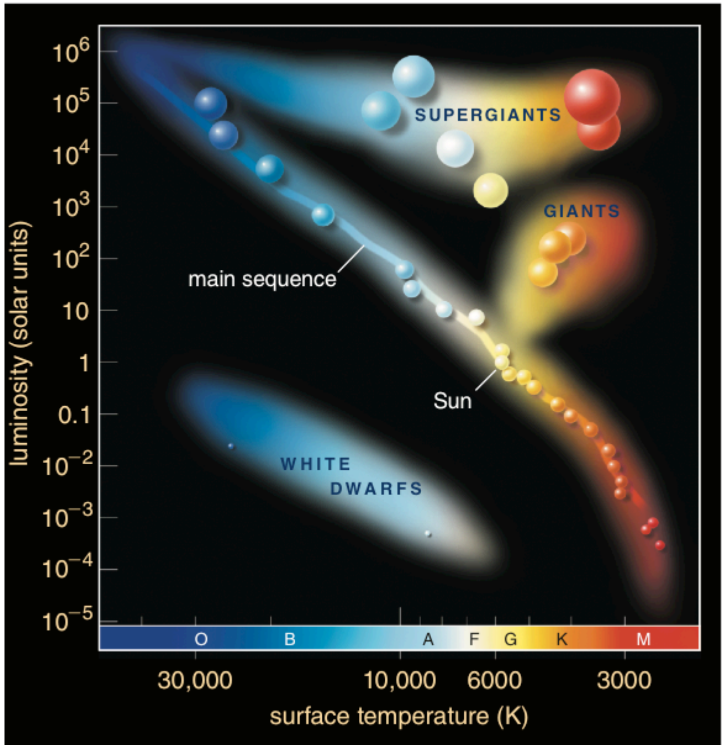

Explanation: The H–R diagram plots stars based on their luminosities and surface temperatures.

What is the difference between the thermal radiation (a continuous spectrum) of a hotter and a cooler star (assuming they have the same size)?

(a) The hotter star produces more energy at all wavelengths.

(b) The peak of the spectrum of the hotter stars shifts redward.

(c) The peak of the spectrum of the hotter stars shifts blueward.

(d) A and B

(e) A and C

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Which is hottest?

(a) a blue star

(b) a red star

(c) a planet that emits only infrared light

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Homework

1. Mastering Physics assignment:
   - is available NOW
   - Deadline: **15 March (Friday)**
   - late assignment will not receive any credits
   - this is a **long** assignment. do not leave it until the last day to work through it
   - **No deadline extension**

TABLE 14.1 Basic Properties of the Sun

Radius (R_{Sun})	696,000 km (about 109 times the radius of Earth)
Mass (M_{Sun})	2×10^{30} kg (about 300,000 times the mass of Earth)
Luminosity (L_{Sun})	3.8×10^{26} watts
Composition (by percentage of mass)	70% hydrogen, 28% helium, 2% heavier elements
Rotation rate	25 days (equator) to 30 days (poles)
Surface temperature	5800 K (average); 4000 K (sunspots)
Core temperature	15 million K