# 31

# ALTERNATING CURRENT

**31.1.** **IDENTIFY:** The maximum current is the current amplitude, and it must not ever exceed 1.50 A.

**SET UP:** $I_{\text{rms}} = I/\sqrt{2}$. $I$ is the current amplitude, the maximum value of the current.

**EXECUTE:** $I = 1.50\text{A}$ gives $I_{\text{rms}} = \dfrac{1.50\text{ A}}{\sqrt{2}} = 1.06\text{ A}$.

**EVALUATE:** The current amplitude is larger than the root-mean-square current.

**31.2.** **IDENTIFY** and **SET UP:** Apply $I_{\text{rav}} = \dfrac{2}{\pi}I$ and $I_{\text{rms}} = \dfrac{I}{\sqrt{2}}$.

**EXECUTE:** **(a)** $I = \sqrt{2}I_{\text{rms}} = \sqrt{2}(2.10\text{ A}) = 2.97\text{ A}$.

**(b)** $I_{\text{rav}} = \dfrac{2}{\pi}I = \dfrac{2}{\pi}(2.97\text{ A}) = 1.89\text{ A}$.

**EVALUATE:** **(c)** The root-mean-square current is always greater than the rectified average, because squaring the current before averaging, and then taking the square root to get the root-mean-square value will always give a larger value than just averaging.

**31.3.** **IDENTIFY** and **SET UP:** Apply $V_{\text{rms}} = \dfrac{V}{\sqrt{2}}$.

**EXECUTE:** **(a)** $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45.0\text{ V}}{\sqrt{2}} = 31.8\text{ V}$.

**(b)** Since the voltage is sinusoidal, the average is zero.

**EVALUATE:** The voltage amplitude is larger than $V_{\text{rms}}$.

**31.4.** **IDENTIFY:** We want the phase angle for the source voltage relative to the current, and we want the capacitance if we know the current amplitude.

**SET UP:** $X_C = \dfrac{V}{I}$ and $X_C = \dfrac{1}{2\pi fC}$.

**EXECUTE:** **(a)** $\phi = -90°$. The source voltage lags the current by $90°$.

**(b)** $X_C = \dfrac{V}{I} = \dfrac{60.0\text{ V}}{5.30\text{ A}} = 11.3\ \Omega$. Solving $X_C = \dfrac{1}{2\pi fC}$ for $C$ gives

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(80.0\text{ Hz})(11.3\ \Omega)} = 1.76\times10^{-4}\text{ F}.$$

**EVALUATE:** This is a $176\text{-}\mu\text{F}$ capacitor, which is not unreasonable.

**31.5.** **IDENTIFY:** We want the phase angle for the source voltage relative to the current, and we want the inductance if we know the current amplitude.

**SET UP:** $X_L = \dfrac{V}{I}$ and $X_L = 2\pi fL$.

**EXECUTE:** **(a)** $\phi = +90°$. The source voltage leads the current by $90°$.

**(b)** $X_L = \dfrac{V}{I} = \dfrac{45.0\text{ V}}{3.90\text{ A}} = 11.54\ \Omega.$ Solving $X_L = 2\pi fL$ for *f* gives $f = \dfrac{X_L}{2\pi L} = \dfrac{11.54\ \Omega}{2\pi(9.50\times10^{-3}\text{ H})} = 193\text{ Hz}.$

**EVALUATE:** The angular frequency is about 1200 rad/s.

**31.6. IDENTIFY:** The reactance of capacitors and inductors depends on the angular frequency at which they are operated, as well as their capacitance or inductance.

**SET UP:** The reactances are $X_C = 1/\omega C$ and $X_L = \omega L.$

**EXECUTE:** **(a)** Equating the reactances gives $\omega L = \dfrac{1}{\omega C} \Rightarrow \omega = \dfrac{1}{\sqrt{LC}}.$

**(b)** Using the numerical values we get $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(5.00\text{ mH})(3.50\ \mu\text{F})}} = 7560\text{ rad/s}.$

$$X_C = X_L = \omega L = (7560\text{ rad/s})(5.00\text{ mH}) = 37.8\ \Omega.$$

**EVALUATE:** At other angular frequencies, the two reactances could be very different.

**31.7. IDENTIFY and SET UP:** Apply $X_C = \dfrac{1}{\omega C}$ and $V_C = IX_C.$

**EXECUTE:** $V = IX_C$ so $X_C = \dfrac{V}{I} = \dfrac{170\text{ V}}{0.850\text{ A}} = 200\ \Omega.$

$X_C = \dfrac{1}{\omega C}$ gives $C = \dfrac{1}{2\pi fX_C} = \dfrac{1}{2\pi(60.0\text{ Hz})(200\ \Omega)} = 1.33\times10^{-5}\text{ F} = 13.3\ \mu\text{F}.$

**EVALUATE:** The reactance relates the voltage amplitude to the current amplitude and is similar to Ohm's law.

**31.8. IDENTIFY:** The reactance of an inductor is $X_L = \omega L = 2\pi fL.$ The reactance of a capacitor is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}.$

**SET UP:** The frequency *f* is in Hz.

**EXECUTE:** **(a)** At 60.0 Hz, $X_L = 2\pi(60.0\text{ Hz})(0.450\text{ H}) = 170\ \Omega.$ $X_L$ is proportional to *f* so at 600 Hz, $X_L = 1700\ \Omega.$

**(b)** At 60.0 Hz, $X_C = \dfrac{1}{2\pi(60.0\text{ Hz})(2.50\times10^{-6}\text{ F})} = 1.06\times10^3\ \Omega.$ $X_C$ is proportional to $1/f$, so at 600 Hz, $X_C = 106\ \Omega.$

**(c)** $X_L = X_C$ says $2\pi fL = \dfrac{1}{2\pi fC}$ and $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.450\text{ H})(2.50\times10^{-6}\text{ F})}} = 150\text{ Hz}.$

**EVALUATE:** $X_L$ increases when *f* increases. $X_C$ decreases when *f* increases.

**31.9. IDENTIFY and SET UP:** Use $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}.$

**EXECUTE:** **(a)** $X_L = \omega L = 2\pi fL = 2\pi(80.0\text{ Hz})(3.00\text{ H}) = 1510\ \Omega.$

**(b)** $X_L = 2\pi fL$ gives $L = \dfrac{X_L}{2\pi f} = \dfrac{120\ \Omega}{2\pi(80.0\text{ Hz})} = 0.239\text{ H}.$

**(c)** $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(80.0\text{ Hz})(4.00\times10^{-6}\text{ F})} = 497\ \Omega.$

**(d)** $X_C = \dfrac{1}{2\pi fC}$ gives $C = \dfrac{1}{2\pi fX_C} = \dfrac{1}{2\pi(80.0\text{ Hz})(120\ \Omega)} = 1.66\times10^{-5}\text{ F}.$

**EVALUATE:** $X_L$ increases when $L$ increases; $X_C$ decreases when $C$ increases.

**31.10. IDENTIFY:** $V_L = I\omega L.$

**SET UP:** $\omega$ is the angular frequency, in rad/s. $f = \dfrac{\omega}{2\pi}$ is the frequency in Hz.

**EXECUTE:** $V_L = I\omega L = 2\pi fIL$, so $f = \dfrac{V_L}{2\pi IL} = \dfrac{(12.0\text{ V})}{2\pi(1.80\times10^{-3}\text{ A})(4.50\times10^{-4}\text{ H})} = 2.36\times10^6\text{ Hz} =$ 2.36 MHz.

**EVALUATE:** When $f$ is increased, $I$ decreases.

**31.11. IDENTIFY:** In an *L-R* ac circuit, we want to find out how the voltage across a resistor varies with time if we know how the voltage varies across the inductor.

**SET UP:** $v_L = -I\omega L\sin\omega t$ and $v_R = V_R\cos(\omega t)$.

**EXECUTE:** **(a)** $v_L = -I\omega L\sin\omega t$. $\omega = 480$ rad/s. $I\omega L = 12.0$ V.

$$I = \frac{12.0\text{ V}}{\omega L} = \frac{12.0\text{ V}}{(480\text{ rad/s})(0.180\text{ H})} = 0.1389\text{ A}.\quad V_R = IR = (0.1389\text{ A})(90.0\ \Omega) = 12.5\text{ V}.$$

$v_R = V_R\cos(\omega t) = (12.5\text{ V})\cos[(480\text{ rad/s})t]$.

**(b)** $v_R = (12.5\text{ V})\cos[(480\text{ rad/s})(2.00\times10^{-3}\text{ s})] = 7.17\text{ V}$.

**EVALUATE:** The instantaneous voltage (7.17 V) is less than the voltage amplitude (12.5 V).

**31.12. IDENTIFY:** Compare $v_C$ that is given in the problem to the general form $v_C = \dfrac{I}{\omega C}\sin\omega t$ and determine $\omega$.

**SET UP:** $X_C = \dfrac{1}{\omega C}$. $v_R = iR$ and $i = I\cos\omega t$.

**EXECUTE:** **(a)** $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(120\text{ rad/s})(4.80\times10^{-6}\text{ F})} = 1736\ \Omega$.

**(b)** $I = \dfrac{V_C}{X_C} = \dfrac{7.60\text{ V}}{1736\ \Omega} = 4.378\times10^{-3}\text{ A}$ and $i = I\cos\omega t = (4.378\times10^{-3}\text{ A})\cos[(120\text{ rad/s})t]$. Then

$v_R = iR = (4.38\times10^{-3}\text{ A})(250\ \Omega)\cos[(120\text{ rad/s})t] = (1.10\text{ V})\cos[(120\text{ rad/s})t]$.

**EVALUATE:** The voltage across the resistor has a different phase than the voltage across the capacitor.

**31.13. IDENTIFY and SET UP:** The voltage and current for a resistor are related by $v_R = iR$. Deduce the frequency of the voltage and use this in $X_L = \omega L$ to calculate the inductive reactance. The equation $v_L = I\omega L\cos(\omega t + 90°)$ gives the voltage across the inductor.

**EXECUTE:** **(a)** $v_R = (3.80\text{ V})\cos[(720\text{ rad/s})t]$.

$v_R = iR$, so $i = \dfrac{v_R}{R} = \left(\dfrac{3.80\text{ V}}{150\ \Omega}\right)\cos[(720\text{ rad/s})t] = (0.0253\text{ A})\cos[(720\text{ rad/s})t]$.

**(b)** $X_L = \omega L$.

$\omega = 720$ rad/s, $L = 0.250$ H, so $X_L = \omega L = (720\text{ rad/s})(0.250\text{ H}) = 180\ \Omega$.

**(c)** If $i = I\cos\omega t$ then $v_L = V_L\cos(\omega t + 90°)$ (from Eq. 31.10).

$V_L = I\omega L = IX_L = (0.02533\text{ A})(180\ \Omega) = 4.56\text{ V}$.

$v_L = (4.56\text{ V})\cos[(720\text{ rad/s})t + 90°]$.

But $\cos(a + 90°) = -\sin a$ (Appendix B), so $v_L = -(4.56\text{ V})\sin[(720\text{ rad/s})t]$.

**EVALUATE:** The current is the same in the resistor and inductor and the voltages are $90°$ out of phase, with the voltage across the inductor leading.

**31.14. IDENTIFY:** Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

**SET UP:** With no capacitor, $Z = \sqrt{R^2 + X_L^2}$ and $\tan\phi = \dfrac{X_L}{R}$. $X_L = \omega L$. $I = \dfrac{V}{Z}$. $V_L = IX_L$ and $V_R = IR$.

For an inductor, the voltage leads the current.

**EXECUTE:** **(a)** $X_L = \omega L = (250\text{ rad/s})(0.400\text{ H}) = 100\ \Omega$. $Z = \sqrt{(200\ \Omega)^2 + (100\ \Omega)^2} = 224\ \Omega$.

**(b)** $I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{224\ \Omega} = 0.134\text{ A}$.

**(c)** $V_R = IR = (0.134\text{ A})(200\ \Omega) = 26.8\text{ V}$. $V_L = IX_L = (0.134\text{ A})(100\ \Omega) = 13.4\text{ V}$.

**(d)** $\tan\phi = \dfrac{X_L}{R} = \dfrac{100\ \Omega}{200\ \Omega}$ and $\phi = +26.6°$. Since $\phi$ is positive, the source voltage leads the current.

**(e)** The phasor diagram is sketched in Figure 31.14.

**EVALUATE:** Note that $V_R + V_L$ is greater than $V$. The loop rule is satisfied at each instance of time but the voltages across $R$ and $L$ reach their maxima at different times.

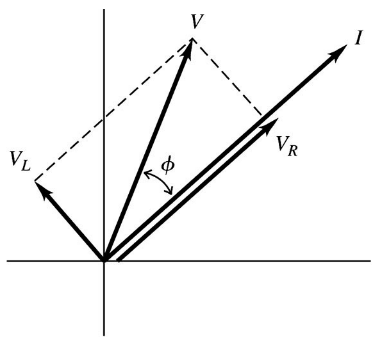



**Figure 31.14**

**31.15. IDENTIFY:** Apply the equations in Section 31.3.

**SET UP:** $\omega = 250\text{ rad/s}$, $R = 200\ \Omega$, $L = 0.400\text{ H}$, $C = 6.00\ \mu\text{F}$ and $V = 30.0\text{ V}$.

**EXECUTE:** **(a)** $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$$Z = \sqrt{(200\ \Omega)^2 + ((250\text{ rad/s})(0.400\text{ H}) - 1/((250\text{ rad/s})(6.00\times10^{-6}\text{ F})))^2} = 601\ \Omega.$$

**(b)** $I = \dfrac{V}{Z} = \dfrac{30\text{ V}}{601\ \Omega} = 0.0499\text{ A}$.

**(c)** $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{100\ \Omega - 667\ \Omega}{200\ \Omega}\right) = -70.6°$, and the voltage lags the current.

**(d)** $V_R = IR = (0.0499\text{ A})(200\ \Omega) = 9.98\text{ V}$; $V_L = I\omega L = (0.0499\text{ A})(250\text{ rad/s})(0.400\text{ H}) = 4.99\text{ V}$;

$$V_C = \frac{I}{\omega C} = \frac{(0.0499\text{ A})}{(250\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 33.3\text{ V}.$$

**EVALUATE:** **(e)** At any instant, $v = v_R + v_C + v_L$. But $v_C$ and $v_L$ are 180° out of phase, so $v_C$ can be larger than $v$ at a value of $t$, if $v_L + v_R$ is negative at that $t$.

**31.16. IDENTIFY:** For an *L-R-C* series ac circuit, we want to find the voltages and voltage amplitudes across all the circuit elements.

**SET UP:** $X_C = \dfrac{1}{\omega C}$, $X_L = \omega L$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $I = \dfrac{V}{Z}$ and $\tan\phi = \dfrac{X_L - X_C}{R}$. The instantaneous voltages are $v_R = V_R\cos(\omega t) = IR\cos(\omega t)$, $v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t)$, $v_C = V_C\sin(\omega t) = IX_C\sin(\omega t)$ and $v = V\cos(\omega t + \phi)$.

**EXECUTE:** $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(250\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 666.7\ \Omega$.

$X_L = \omega L = (250\text{ rad/s})(0.900\text{ H}) = 225\ \Omega$.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (225\ \Omega - 666.7\ \Omega)^2} = 484.9\ \Omega$.

$I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{484.9\ \Omega} = 0.06187\text{ A} = 61.87\text{ mA}$.

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{225\ \Omega - 666.7\ \Omega}{200\ \Omega} = -2.2085$ and $\phi = -1.146$ rad.

**(a)** $v_R = V_R\cos(\omega t) = IR\cos(\omega t) = (0.06187\ \text{A})(200\ \Omega)\cos[(250\ \text{rad/s})(20.0\times10^{-3}\ \text{s})] = 3.51\ \text{V}.$

$v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t) = -(0.06187\ \text{A})(225\ \Omega)\sin[(250\ \text{rad/s})(20.0\times10^{-3}\ \text{s})] = 13.35\ \text{V}.$

$v_C = V_C\sin(\omega t) = IX_C\sin(\omega t) = (0.06187\ \text{A})(666.7\ \Omega)\sin[(250\ \text{rad/s})(20.0\times10^{-3}\ \text{s})] = -39.55\ \text{V}.$

$v = V\cos(\omega t + \phi) = (30.0\ \text{V})\cos[(250\ \text{rad/s})(20.0\times10^{-3}\ \text{s}) - 1.146\ \text{rad}] = -22.70\ \text{V}.$

$v_R + v_L + v_C = 3.51\ \text{V} + 13.35\ \text{V} + (-39.55\ \text{V}) = -22.7\ \text{V}.$ $v_R + v_L + v_C$ is equal to $v$.

**(b)** $V_R = IR = (0.06187\ \text{A})(200\ \Omega) = 12.4$ V, $V_L = IX_L = (0.06187\ \text{A})(225\ \Omega) = 13.9$ V, and $V_C = IX_C = (0.06187\ \text{A})(666.7\ \Omega) = 41.2$ V.

$V_R + V_C + V_L = 12.4\ \text{V} + 41.2\ \text{V} + 13.9\ \text{V} = 67.5\ \text{V}.$ $V_R + V_C + V_L$ is not equal to $V$.

**EVALUATE:** The instantaneous voltages do add up to $v$ because they all occur at the same time, so they must add to $v$ by Kirchhoff's loop rule. The amplitudes do not add to $V$ because the maxima do not occur at the same time due to phase differences between the inductor, capacitor and resistor.

**31.17.** **IDENTIFY** and **SET UP:** Use the equation that preceeds Eq. (31.20): $V^2 = V_R^2 + (V_L - V_C)^2$.

**EXECUTE:** $V = \sqrt{(30.0\ \text{V})^2 + (50.0\ \text{V} - 90.0\ \text{V})^2} = 50.0\ \text{V}.$

**EVALUATE:** The equation follows directly from the phasor diagrams of Fig. 31.13 (b or c) in the textbook. Note that the voltage amplitudes do not simply add to give 170.0 V for the source voltage.

**31.18.** **IDENTIFY:** For an $L$-$R$ ac circuit, we want to use the resistance, voltage amplitude of the source and power in the resistor to find the impedance, the voltage amplitude across the inductor and the power factor.

**SET UP:** $P_{av} = \dfrac{1}{2}I^2R,\ \ Z = \dfrac{V}{I},\ \ V_R = IR,$ and $\tan\phi = \dfrac{X_L}{R}.$

**EXECUTE:** **(a)** $P_{av} = \dfrac{1}{2}I^2R.\ \ I = \sqrt{\dfrac{2P_{av}}{R}} = \sqrt{\dfrac{2(286\ \text{W})}{300\ \Omega}} = 1.381\ \text{A}.\ \ Z = \dfrac{V}{I} = \dfrac{500\ \text{V}}{1.381\ \text{A}} = 362\ \Omega.$

**(b)** $V_R = IR = (1.381\ \text{A})(300\ \Omega) = 414\ \text{V}.\ \ V_L = \sqrt{V^2 - V_R^2} = \sqrt{(500\ \text{V})^2 - (414\ \text{V})^2} = 280\ \text{V}.$

**(c)** $\tan\phi = \dfrac{X_L}{R} = \dfrac{V_L}{V_R} = \dfrac{280\ \text{V}}{414\ \text{V}}$ gives $\phi = 34.1°$. The power factor is $\cos\phi = 0.828$.

**EVALUATE:** The voltage amplitude across the resistor cannot exceed the voltage amplitude (500 V) of the ac source.

**31.19.** **IDENTIFY:** For a pure resistance, $P_{av} = V_{rms}I_{rms} = I_{rms}^2R$.

**SET UP:** 20.0 W is the average power $P_{av}$.

**EXECUTE:** **(a)** The average power is one-half the maximum power, so the maximum instantaneous power is 40.0 W.

**(b)** $I_{rms} = \dfrac{P_{av}}{V_{rms}} = \dfrac{20.0\ \text{W}}{120\ \text{V}} = 0.167\ \text{A}.$

**(c)** $R = \dfrac{P_{av}}{I_{rms}^2} = \dfrac{20.0\ \text{W}}{(0.167\ \text{A})^2} = 720\ \Omega.$

**EVALUATE:** We can also calculate the average power as $P_{av} = \dfrac{V_{R,rms}^2}{R} = \dfrac{V_{rms}^2}{R} = \dfrac{(120\ \text{V})^2}{720\ \Omega} = 20.0\ \text{W}.$

**31.20.** **IDENTIFY:** The average power supplied by the source is $P_{av} = V_{rms}I_{rms}\cos\phi$. The power consumed in the resistance is $P_{av} = I_{rms}^2R$.

**SET UP:** $\omega = 2\pi f = 2\pi(1.25\times10^3\ \text{Hz}) = 7.854\times10^3\ \text{rad/s}.\ \ X_L = \omega L = 157\ \Omega.\ \ X_C = \dfrac{1}{\omega C} = 909\ \Omega.$

**EXECUTE:** **(a)** First, let us find the phase angle between the voltage and the current: $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{157\ \Omega - 909\ \Omega}{350\ \Omega}$ and $\phi = -65.04°$. The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(350\ \Omega)^2 + (-752\ \Omega)^2} = 830\ \Omega$. The average power provided by the generator is then $P_{av} = V_{rms} I_{rms} \cos(\phi) = \dfrac{V_{rms}^2}{Z}\cos(\phi) = \dfrac{(120\ \text{V})^2}{830\ \Omega}\cos(-65.04°) = 7.32\ \text{W}$.

**(b)** The average power dissipated by the resistor is $P_R = I_{rms}^2 R = \left(\dfrac{120\ \text{V}}{830\ \Omega}\right)^2 (350\ \Omega) = 7.32\ \text{W}$.

**EVALUATE:** Conservation of energy requires that the answers to parts (a) and (b) are equal.

**31.21. IDENTIFY:** Relate the power factor to $R$ and $Z$ for an $L$-$R$-$C$ series ac circuit. Then use this result to find the voltage amplitude across a resistor.

**SET UP** and **EXECUTE:** **(a)** From Figure 31.13(a) or (b) in the textbook, $\cos\phi = \dfrac{IR}{IZ} = \dfrac{R}{Z}$.

**(b)** Using the result from (a) gives $Z = \dfrac{R}{\cos\phi}$. $I = \dfrac{V}{Z} = \dfrac{V\cos\phi}{R}$.

$V_R = IR = V\cos\phi = (90.0\ \text{V})\cos(-31.5°) = 76.7\ \text{V}$.

**EVALUATE:** The voltage amplitude for the resistor is less than the voltage amplitude of the ac source.

**31.22. IDENTIFY:** We want to relate the average power delivered by the source in an $L$-$R$-$C$ circuit to the rms current and resistance.

**SET UP:** From Exercise 31.21 we know that the power factor is $\cos\phi = \dfrac{R}{Z}$. We also know that $P_{av} = V_{rms} I_{rms}\cos\phi$.

**EXECUTE:** **(a)** $P_{av} = V_{rms} I_{rms}\cos\phi$. $\cos\phi = \dfrac{R}{Z}$ so $P_{av} = V_{rms} I_{rms}\dfrac{R}{Z}$. But $\dfrac{V_{rms}}{Z} = I_{rms}$ so $P_{av} = I_{rms}^2 R$.

**(b)** $I_{rms} = \dfrac{V_{rms}}{R}$ and $V_{rms} = V/\sqrt{2}$, so $P_{av} = \dfrac{V_{rms}^2}{R} = \dfrac{\left(\dfrac{36.0\ \text{V}}{\sqrt{2}}\right)^2}{96.0\ \Omega} = 6.75\ \text{W}$.

**EVALUATE:** The instantaneous power can be greater than 6.75 W at times, but it can also be less than that at other times, giving an average of 6.75 W.

**31.23. IDENTIFY** and **SET UP:** Use the equations of Section 31.3 to calculate $\phi$, $Z$, and $V_{rms}$. The average power delivered by the source is given by $P_{av} = I_{rms} V_{rms}\cos\phi$ and the average power dissipated in the resistor is $I_{rms}^2 R$.

**EXECUTE:** **(a)** $X_L = \omega L = 2\pi f L = 2\pi(400\ \text{Hz})(0.120\ \text{H}) = 301.6\ \Omega$.

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(400\ \text{Hz})(7.3\times10^{-6}\ \text{F})} = 54.51\ \Omega$.

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{301.6\ \Omega - 54.41\ \Omega}{240\ \Omega}$, so $\phi = +45.8°$. The power factor is $\cos\phi = +0.697$.

**(b)** $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(240\ \Omega)^2 + (301.6\ \Omega - 54.51\ \Omega)^2} = 344\ \Omega$.

**(c)** $V_{rms} = I_{rms} Z = (0.450\ \text{A})(344\ \Omega) = 155\ \text{V}$.

**(d)** $P_{av} = I_{rms} V_{rms}\cos\phi = (0.450\ \text{A})(155\ \text{V})(0.697) = 48.6\ \text{W}$.

**(e)** $P_{av} = I_{rms}^2 R = (0.450\ \text{A})^2(240\ \Omega) = 48.6\ \text{W}$.

**EVALUATE:** The average electrical power delivered by the source equals the average electrical power consumed in the resistor.

**(f)** All the energy stored in the capacitor during one cycle of the current is released back to the circuit in another part of the cycle. There is no net dissipation of energy in the capacitor.

**(g)** The answer is the same as for the capacitor. Energy is repeatedly being stored and released in the inductor, but no net energy is dissipated there.

**31.24.** **IDENTIFY** and **SET UP:** $P_{av} = V_{rms}I_{rms}\cos\phi$. $I_{rms} = \dfrac{V_{rms}}{Z}$. $\cos\phi = \dfrac{R}{Z}$.

**EXECUTE:** $I_{rms} = \dfrac{80.0\text{ V}}{105\ \Omega} = 0.762\text{ A}$. $\cos\phi = \dfrac{75.0\ \Omega}{105\ \Omega} = 0.714$. $P_{av} = (80.0\text{ V})(0.762\text{ A})(0.714) = 43.5\text{ W}$.

**EVALUATE:** Since the average power consumed by the inductor and by the capacitor is zero, we can also calculate the average power as $P_{av} = I_{rms}^2 R = (0.762\text{ A})^2(75.0\ \Omega) = 43.5\text{ W}$.

**31.25.** **IDENTIFY:** The angular frequency and the capacitance can be used to calculate the reactance $X_C$ of the capacitor. The angular frequency and the inductance can be used to calculate the reactance $X_L$ of the inductor. Calculate the phase angle $\phi$ and then the power factor is $\cos\phi$. Calculate the impedance of the circuit and then the rms current in the circuit. The average power is $P_{av} = V_{rms}I_{rms}\cos\phi$. On the average no power is consumed in the capacitor or the inductor, it is all consumed in the resistor.

**SET UP:** The source has rms voltage $V_{rms} = \dfrac{V}{\sqrt{2}} = \dfrac{45\text{ V}}{\sqrt{2}} = 31.8\text{ V}$.

**EXECUTE:** **(a)** $X_L = \omega L = (360\text{ rad/s})(15\times10^{-3}\text{ H}) = 5.4\ \Omega$.

$$X_C = \frac{1}{\omega C} = \frac{1}{(360\text{ rad/s})(3.5\times10^{-6}\text{ F})} = 794\ \Omega.\ \ \tan\phi = \frac{X_L - X_C}{R} = \frac{5.4\ \Omega - 794\ \Omega}{250\ \Omega}\ \text{ and }\ \phi = -72.4°.$$

The power factor is $\cos\phi = 0.302$.

**(b)** $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250\ \Omega)^2 + (5.4\ \Omega - 794\ \Omega)^2} = 827\ \Omega$. $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{31.8\text{ V}}{827\ \Omega} = 0.0385\text{ A}$.

$P_{av} = V_{rms}I_{rms}\cos\phi = (31.8\text{ V})(0.0385\text{ A})(0.302) = 0.370\text{ W}$.

**(c)** The average power delivered to the resistor is $P_{av} = I_{rms}^2 R = (0.0385\text{ A})^2(250\ \Omega) = 0.370\text{ W}$. The average power delivered to the capacitor and to the inductor is zero.

**EVALUATE:** On average the power delivered to the circuit equals the power consumed in the resistor. The capacitor and inductor store electrical energy during part of the current oscillation but each return the energy to the circuit during another part of the current cycle.

**31.26.** **IDENTIFY:** At resonance in an *L-R-C* ac circuit, we know the reactance of the capacitor and the voltage amplitude across it. From this information, we want to find the voltage amplitude of the source.

**SET UP:** At resonance, $Z = R$. $V_C = IX_C$.

**EXECUTE:** $I = \dfrac{V}{X_C} = \dfrac{600\text{ V}}{200\ \Omega} = 3.00\text{ A}$. $Z = R = 300\ \Omega$. $V = IZ = (3.00\text{ A})(300\ \Omega) = 900\text{ V}$.

**EVALUATE:** At resonance, $Z = R$, but $X_C$ is not zero.

**31.27.** **IDENTIFY** and **SET UP:** The current is largest at the resonance frequency. At resonance, $X_L = X_C$ and $Z = R$. For part (b), calculate *Z* and use $I = V/Z$.

**EXECUTE:** **(a)** $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 113\text{ Hz}$. $I = V/R = 15.0\text{ mA}$.

**(b)** $X_C = 1/\omega C = 500\ \Omega$. $X_L = \omega L = 160\ \Omega$.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (160\ \Omega - 500\ \Omega)^2} = 394.5\ \Omega$. $I = V/Z = 7.61\text{ mA}$. $X_C > X_L$ so the source voltage lags the current.

**EVALUATE:** $\omega_0 = 2\pi f_0 = 710\text{ rad/s}$. $\omega = 400\text{ rad/s}$ and is less than $\omega_0$. When $\omega < \omega_0$, $X_C > X_L$. Note that *I* in part (b) is less than *I* in part (a).

**31.28.** **IDENTIFY:** The impedance and individual reactances depend on the angular frequency at which the circuit is driven.

**SET UP:** The impedance is $Z=\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}$, the current amplitude is $I=V/Z$ and the instantaneous values of the potential and current are $v=V\cos(\omega t+\phi)$, where $\tan\phi=(X_L-X_C)/R$, and $i=I\cos\omega t$.

**EXECUTE:** **(a)** $Z$ is a minimum when $\omega L=\dfrac{1}{\omega C}$, which gives

$$\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(8.00\text{ mH})(12.5\ \mu\text{F})}}=3162\text{ rad/s, which rounds to 3160 rad/s. } Z=R=175\ \Omega.$$

**(b)** $I=V/Z=(25.0\text{ V})/(175\ \Omega)=0.143\text{ A}.$

**(c)** $i=I\cos\omega t=I/2$, so $\cos\omega t=\frac{1}{2}$, which gives $\omega t=60°=\pi/3$ rad. $v=V\cos(\omega t+\phi)$, where $\tan\phi=(X_L-X_C)/R=0/R=0$. So, $v=(25.0\text{ V})\cos\omega t=(25.0\text{ V})(1/2)=12.5\text{ V}.$

$v_R=Ri=(175\ \Omega)(1/2)(0.143\text{ A})=12.5\text{ V}.$

$$v_C=V_C\cos(\omega t-90°)=IX_C\cos(\omega t-90°)=\frac{0.143\text{ A}}{(3162\text{ rad/s})(12.5\ \mu\text{F})}\cos(60°-90°)=+3.13\text{ V}.$$

$v_L=V_L\cos(\omega t+90°)=IX_L\cos(\omega t+90°)=(0.143\text{ A})(3162\text{ rad/s})(8.00\text{ mH})\cos(60°+90°).$

$v_L=-3.13\text{ V}.$

**(d)** $v_R+v_L+v_C=12.5\text{ V}+(-3.13\text{ V})+3.13\text{ V}=12.5\text{ V}=v_{\text{source}}.$

**EVALUATE:** The instantaneous potential differences across all the circuit elements always add up to the value of the source voltage at that instant. In this case (resonance), the potentials across the inductor and capacitor have the same magnitude but are $180°$ out of phase, so they add to zero, leaving all the potential difference across the resistor.

**31.29.** **IDENTIFY** and **SET UP:** At the resonance frequency, $Z=R$. Use that $V=IZ$, $V_R=IR$, $V_L=IX_L$, and $V_C=IX_C$. $P_{\text{av}}$ is given by $P_{\text{av}}=\frac{1}{2}VI\cos\phi.$

**EXECUTE:** **(a)** $V=IZ=IR=(0.500\text{ A})(300\ \Omega)=150\text{ V}.$

**(b)** $V_R=IR=150\text{ V}.$

$X_L=\omega L=L(1/\sqrt{LC})=\sqrt{L/C}=2582\ \Omega;\ V_L=IX_L=1290\text{ V}.$

$X_C=1/(\omega C)=\sqrt{L/C}=2582\ \Omega;\ V_C=IX_C=1290\text{ V}.$

**(c)** $P_{\text{av}}=\frac{1}{2}VI\cos\phi=\frac{1}{2}I^2R$, since $V=IR$ and $\cos\phi=1$ at resonance.

$P_{\text{av}}=\frac{1}{2}(0.500\text{ A})^2(300\ \Omega)=37.5\text{ W}.$

**EVALUATE:** At resonance $V_L=V_C$. Note that $V_L+V_C>V$. However, at any instant $v_L+v_C=0$.

**31.30.** **IDENTIFY:** The current is maximum at the resonance frequency, so choose $C$ such that $\omega=50.0$ rad/s is the resonance frequency. At the resonance frequency $Z=R$.

**SET UP:** $V_L=I\omega L.$

**EXECUTE:** **(a)** The amplitude of the current is given by $I=\dfrac{V}{\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}}$. Thus, the current will have a maximum amplitude when $\omega L=\dfrac{1}{\omega C}$. Therefore,

$$C=\frac{1}{\omega^2L}=\frac{1}{(50.0\text{ rad/s})^2(3.00\text{ H})}=1.33\times10^{-4}\text{ F}=133\ \mu\text{F}.$$

**(b)** With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is $I = \frac{V}{R} = \frac{120\text{ V}}{400\ \Omega} = 0.300\text{ A}$. Thus, the amplitude of the voltage across the inductor is $V_L = I(\omega L) = (0.300\text{ A})(50.0\text{ rad/s})(3.00\text{ H}) = 45.0\text{ V}$.

**EVALUATE:** For the value of $C$ found in part (a), the resonance angular frequency is 50.0 rad/s.

**31.31. IDENTIFY and SET UP:** At resonance $X_L = X_C$, $\phi = 0$ and $Z = R$. $R = 150\ \Omega$, $L = 0.750$ H, $C = 0.0180\ \mu$F, $V = 150$ V

**EXECUTE:** **(a)** At the resonance frequency $X_L = X_C$ and from $\tan\phi = \frac{X_L - X_C}{R}$ we have that $\phi = 0°$ and the power factor is $\cos\phi = 1.00$.

**(b)** $P_{av} = \frac{1}{2}VI\cos\phi$ (Eq. 31.31).

At the resonance frequency $Z = R$, so $I = \frac{V}{Z} = \frac{V}{R}$.

$$P_{av} = \frac{1}{2}V\left(\frac{V}{R}\right)\cos\phi = \frac{1}{2}\frac{V^2}{R} = \frac{1}{2}\frac{(150\text{ V})^2}{150\ \Omega} = 75.0\text{ W}.$$

**EVALUATE:** **(c)** When $C$ and $f$ are changed but the circuit is kept on resonance, nothing changes in $P_{av} = V^2/(2R)$, so the average power is unchanged: $P_{av} = 75.0$ W. The resonance frequency changes but since $Z = R$ at resonance the current doesn't change.

**31.32. IDENTIFY:** The resonance angular frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$. $V_C = IX_C$. $V = IZ$.

**SET UP:** At resonance, $Z = R$.

**EXECUTE:** **(a)** $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.350\text{ H})(0.0120\times10^{-6}\text{ F})}} = 1.54\times10^4\text{ rad/s}.$

**(b)** $V = IZ = \left(\frac{V_C}{X_C}\right)Z = \left(\frac{V_C}{X_C}\right)R$. $X_C = \frac{1}{\omega C} = \frac{1}{(1.54\times10^4\text{ rad/s})(0.0120\times10^{-6}\text{ F})} = 5.41\times10^3\ \Omega.$

$$V = \left(\frac{670\text{ V}}{5.41\times10^3\ \Omega}\right)(400\ \Omega) = 49.5\text{ V}.$$

**EVALUATE:** The voltage amplitude for the capacitor is more than a factor of 10 times greater than the voltage amplitude of the source.

**31.33. IDENTIFY:** At resonance $Z = R$ and $X_L = X_C$.

**SET UP:** $\omega_0 = \frac{1}{\sqrt{LC}}$. $V = IZ$. $V_R = IR$, $V_L = IX_L$ and $V_C = V_L$.

**EXECUTE:** **(a)** $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.280\text{ H})(4.00\times10^{-6}\text{ F})}} = 945\text{ rad/s}.$

**(b)** $I = 1.70$ A at resonance, so $R = Z = \frac{V}{I} = \frac{120\text{ V}}{1.70\text{ A}} = 70.6\ \Omega.$

**(c)** At resonance, $V_R = 120$ V, $V_L = V_C = I\omega L = (1.70\text{ A})(945\text{ rad/s})(0.280\text{ H}) = 450\text{ V}.$

**EVALUATE:** At resonance, $V_R = V$ and $V_L - V_C = 0$.

**31.34. IDENTIFY:** Let $I_1$, $V_1$ and $I_2$, $V_2$ be rms values for the primary and secondary. A transformer transforms voltages according to $\frac{V_2}{V_1} = \frac{N_2}{N_1}$. The effective resistance of a secondary circuit of resistance $R$ is $R_{eff} = \frac{R}{(N_2/N_1)^2}$. Resistance $R$ is related to $P_{av}$ and $V_{rms}$ by $P_{av} = \frac{V_{rms}^2}{R}$. Conservation of energy requires $P_{av,1} = P_{av,2}$ so $V_1 I_1 = V_2 I_2$.

**SET UP:** Let $V_1 = 240\text{ V}$ and $V_2 = 120\text{ V}$, so $P_{2,\text{av}} = 1600\text{ W}$. These voltages are rms.

**EXECUTE:** **(a)** $V_1 = 240\text{ V}$ and we want $V_2 = 120\text{ V}$, so use a step-down transformer with $N_2/N_1 = \frac{1}{2}$.

**(b)** $P_{\text{av}} = V_1 I_1$, so $I_1 = \dfrac{P_{\text{av}}}{V_1} = \dfrac{1600\text{ W}}{240\text{ V}} = 6.67\text{ A}$.

**(c)** The resistance $R$ of the blower is $R = \dfrac{V_2^2}{P_{\text{av}}} = \dfrac{(120\text{ V})^2}{1600\text{ W}} = 9.00\ \Omega$. The effective resistance of the blower is

$$R_{\text{eff}} = \frac{9.00\ \Omega}{(1/2)^2} = 36.0\ \Omega.$$

**EVALUATE:** $I_2 = V_2/R_2 = (120\text{ V})/(9.00\ \Omega) = 13.3\text{ A}$, so $I_2 V_2 = (13.3\text{ A})(120\text{ V}) = 1600\text{ W}$. Energy is provided to the primary at the same rate that it is consumed in the secondary. Step-down transformers step up resistance and the current in the primary is less than the current in the secondary.

**31.35. IDENTIFY** and **SET UP:** The equation $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ relates the primary and secondary voltages to the number of turns in each. $I = V/R$ and the power consumed in the resistive load is $I_{\text{rms}}^2 = V_{\text{rms}}^2/R$. Let $I_1, V_1$ and $I_2, V_2$ be rms values for the primary and secondary.

**EXECUTE:** **(a)** $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ so $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{120\text{ V}}{12.0\text{ V}} = 10.$

**(b)** $I_2 = \dfrac{V_2}{R} = \dfrac{12.0\text{ V}}{5.00\ \Omega} = 2.40\text{ A}.$

**(c)** $P_{\text{av}} = I_2^2 R = (2.40\text{ A})^2(5.00\ \Omega) = 28.8\text{ W}.$

**(d)** The power drawn from the line by the transformer is the 28.8 W that is delivered by the load.

$$P_{\text{av}} = \frac{V_1^2}{R} \text{ so } R = \frac{V_1^2}{P_{\text{av}}} = \frac{(120\text{ V})^2}{28.8\text{ W}} = 500\ \Omega.$$

And $\left(\dfrac{N_1}{N_2}\right)^2 (5.00\ \Omega) = (10)^2(5.00\ \Omega) = 500\ \Omega$, as was to be shown.

**EVALUATE:** The resistance is "transformed." A load of resistance $R$ connected to the secondary draws the same power as a resistance $(N_1/N_2)^2 R$ connected directly to the supply line, without using the transformer.

**31.36. IDENTIFY:** $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}}$ and $P_{\text{av},1} = P_{\text{av},2}$. $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2}$. Let $I_1, V_1$ and $I_2, V_2$ be rms values for the primary and secondary.

**SET UP:** $V_1 = 120\text{ V}$. $V_2 = 13{,}000\text{ V}$.

**EXECUTE:** **(a)** $\dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{13{,}000\text{ V}}{120\text{ V}} = 108.$

**(b)** $P_{\text{av}} = V_2 I_2 = (13{,}000\text{ V})(8.50\times10^{-3}\text{ A}) = 110\text{ W}.$

**(c)** $I_1 = \dfrac{P_{\text{av}}}{V_1} = \dfrac{110\text{ W}}{120\text{ V}} = 0.917\text{ A}.$

**EVALUATE:** Since the power supplied to the primary must equal the power delivered by the secondary, in a step-up transformer the current in the primary is greater than the current in the secondary.

**31.37. IDENTIFY** and **SET UP:** Use $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ to relate $L$ and $R$ to $\phi$. The voltage across the coil leads the current in it by $52.3°$, so $\phi = +52.3°$.

**EXECUTE:** $\tan\phi = \dfrac{X_L - X_C}{R}$. But there is no capacitance in the circuit so $X_C = 0$. Thus

$\tan\phi = \dfrac{X_L}{R}$ and $X_L = R\tan\phi = (48.0\ \Omega)\tan 52.3° = 62.1\ \Omega$.

$X_L = \omega L = 2\pi fL$ so $L = \dfrac{X_L}{2\pi f} = \dfrac{62.1\ \Omega}{2\pi(80.0\ \text{Hz})} = 0.124\ \text{H}$.

**EVALUATE:** $\phi > 45°$ when $(X_L - X_C) > R$, which is the case here.

**31.38. IDENTIFY and SET UP:** The impedance is $Z = \sqrt{R^2 + X_L^2}$. where $X_L = \omega L$.

**EXECUTE:** The resistance of the solenoid is $R = V/I = (48.0\ \text{V})/(5.50\ \text{A}) = 8.727\ \Omega$. Under the given conditions, the impedance of the solenoid is $Z = V/I = (48.0\ \text{V})/(3.60\ \text{A}) = 13.33\ \Omega$. We also know that $Z = \sqrt{R^2 + X_L^2} = \sqrt{(8.727\ \Omega)^2 + X_L^2} = 13.33\ \Omega$. Solving for $X_L$ gives $X_L = 10.08\ \Omega$.

Therefore $X_L = \omega L = (20.0\ \text{rad/s})L = 10.08\ \Omega$. So $L = X_L/\omega = (10.08\ \Omega)/(20\ \text{rad/s}) = 0.504\ \text{H}$.

**EVALUATE:** This is larger than typical laboratory inductors, but reasonable for a solenoid.

**31.39. IDENTIFY:** An *L-R-C* ac circuit operates at resonance. We know *L*, *C*, and *V* and want to find *R*.

**SET UP:** At resonance, $Z = R$ and $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$. $X_C = \dfrac{1}{\omega C}$, $I = V/Z$.

**EXECUTE:** $\omega = \dfrac{1}{\sqrt{LC}} = 633.0\ \text{rad/s}$ $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(633\ \text{rad/s})(4.80\times 10^{-6}\ \text{F})} = 329.1\ \Omega$.

$I = \dfrac{V_C}{X_C} = \dfrac{80.0\ \text{V}}{329.1\ \Omega} = 0.2431\ \text{A}$. At resonance $Z = R$, so $I = \dfrac{V}{R}$. $R = \dfrac{V}{I} = \dfrac{56.0\ \text{V}}{0.2431\ \text{A}} = 230\ \Omega$.

**EVALUATE:** At resonance, the impedance is a minimum.

**31.40. IDENTIFY:** $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$. $V_{\text{rms}} = I_{\text{rms}}R$. $V_{C,\text{rms}} = I_{\text{rms}}X_C$. $V_{L,\text{rms}} = I_{\text{rms}}X_L$.

**SET UP:** $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{30.0\ \text{V}}{\sqrt{2}} = 21.2\ \text{V}$.

**EXECUTE: (a)** $\omega = 200\ \text{rad/s}$, so $X_L = \omega L = (200\ \text{rad/s})(0.400\ \text{H}) = 80.0\ \Omega$ and

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(200\ \text{rad/s})(6.00\times 10^{-6}\ \text{F})} = 833\ \Omega$. $Z = \sqrt{(200\ \Omega)^2 + (80.0\ \Omega - 833\ \Omega)^2} = 779\ \Omega$.

$I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{21.2\ \text{V}}{779\ \Omega} = 0.0272\ \text{A}$. $V_1$ reads $V_{R,\text{rms}} = I_{\text{rms}}R = (0.0272\ \text{A})(200\ \Omega) = 5.44\ \text{V}$. $V_2$ reads $V_{L,\text{rms}} = I_{\text{rms}}X_L = (0.0272\ \text{A})(80.0\ \Omega) = 2.18\ \text{V}$. $V_3$ reads $V_{C,\text{rms}} = I_{\text{rms}}X_C = (0.0272\ \text{A})(833\ \Omega) = 22.7\ \text{V}$.

$V_4$ reads $\left|\dfrac{V_L - V_C}{\sqrt{2}}\right| = |V_{L,\text{rms}} - V_{C,\text{rms}}| = |2.18\ \text{V} - 22.7\ \text{V}| = 20.5\ \text{V}$. $V_5$ reads $V_{\text{rms}} = 21.2\ \text{V}$.

**(b)** $\omega = 1000\ \text{rad/s}$ so $X_L = \omega L = (5)(80.0\ \Omega) = 400\ \Omega$ and $X_C = \dfrac{1}{\omega C} = \dfrac{833\ \Omega}{5} = 167\ \Omega$.

$Z = \sqrt{(200\ \Omega)^2 + (400\ \Omega - 167\ \Omega)^2} = 307\ \Omega$. $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{21.2\ \text{V}}{307\ \Omega} = 0.0691\ \text{A}$. $V_1$ reads $V_{R,\text{rms}} = 13.8\ \text{V}$.

$V_2$ reads $V_{L,\text{rms}} = 27.6\ \text{V}$. $V_3$ reads $V_{C,\text{rms}} = 11.5\ \text{V}$.

$V_4$ reads $|V_{L,\text{rms}} - V_{C,\text{rms}}| = |27.6\ \text{V} - 11.5\ \text{V}| = 16.1\ \text{V}$. $V_5$ reads $V_{\text{rms}} = 21.2\ \text{V}$.

**EVALUATE:** The resonance frequency for this circuit is $\omega_0 = \dfrac{1}{\sqrt{LC}} = 645\ \text{rad/s}$. $200\ \text{rad/s}$ is less than the resonance frequency and $X_C > X_L$. $1000\ \text{rad/s}$ is greater than the resonance frequency and $X_L > X_C$.

**31.41. IDENTIFY:** We can use geometry to calculate the capacitance and inductance, and then use these results to calculate the resonance angular frequency.

**Set Up:** The capacitance of an air-filled parallel plate capacitor is $C = \dfrac{\varepsilon_0 A}{d}$. The inductance of a long solenoid is $L = \dfrac{\mu_0 A N^2}{l}$. The inductor has $N = (125 \text{ coils/cm})(9.00 \text{ cm}) = 1125$ coils. The resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$. $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$. $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$.

**Execute:** $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.50\times10^{-2}\ \text{m})^2}{8.00\times10^{-3}\ \text{m}} = 2.24\times10^{-12}\ \text{F}.$

$L = \dfrac{\mu_0 A N^2}{l} = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.250\times10^{-2}\ \text{m})^2(1125)^2}{9.00\times10^{-2}\ \text{m}} = 3.47\times10^{-4}\ \text{H}.$

$\omega_0 = \dfrac{1}{\sqrt{(3.47\times10^{-4}\ \text{H})(2.24\times10^{-12}\ \text{F})}} = 3.59\times10^{7}\ \text{rad/s}.$

**Evaluate:** The result is a rather high angular frequency.

**31.42.** **Identify:** Use geometry to calculate the self-inductance of the toroidal solenoid. Then find its reactance and use this to find the impedance, and finally the current amplitude, of the circuit.

**Set Up:** $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, $X_L = 2\pi f L$, $Z = \sqrt{R^2 + X_L^2}$, and $I = V/Z$.

**Execute:** $L = \dfrac{\mu_0 N^2 A}{2\pi r} = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(2900)^2(0.450\times10^{-4}\ \text{m}^2)}{9.00\times10^{-2}\ \text{m}} = 8.41\times10^{-4}\ \text{H}.$

$X_L = 2\pi f L = (2\pi)(495\ \text{Hz})(8.41\times10^{-4}\ \text{H}) = 2.616\ \Omega.$ $Z = \sqrt{R^2 + X_L^2} = 3.832\ \Omega.$

$I = \dfrac{V}{Z} = \dfrac{24.0\ \text{V}}{3.832\ \Omega} = 6.26\ \text{A}.$

**Evaluate:** The inductance is physically reasonable.

**31.43.** **Identify** and **Set Up:** Source voltage lags current so it must be that $X_C > X_L$.

**Execute:** **(a)** We must add an inductor in series with the circuit. When $X_C = X_L$ the power factor has its maximum value of unity, so calculate the additional $L$ needed to raise $X_L$ to equal $X_C$.

**(b)** Power factor $\cos\phi$ equals 1 so $\phi = 0$ and $X_C = X_L$. Calculate the present value of $X_C - X_L$ to see how much more $X_L$ is needed: $R = Z\cos\phi = (60.0\ \Omega)(0.720) = 43.2\ \Omega$

$\tan\phi = \dfrac{X_L - X_C}{R}$ so $X_L - X_C = R\tan\phi.$

$\cos\phi = 0.720$ gives $\phi = -43.95°$ ($\phi$ is negative since the voltage lags the current).

Then $X_L - X_C = R\tan\phi = (43.2\ \Omega)\tan(-43.95°) = -41.64\ \Omega.$

Therefore need to add $41.64\ \Omega$ of $X_L$.

$X_L = \omega L = 2\pi f L$ and $L = \dfrac{X_L}{2\pi f} = \dfrac{41.64\ \Omega}{2\pi(50.0\ \text{Hz})} = 0.133\ \text{H}$, amount of inductance to add.

**Evaluate:** From the information given we can't calculate the original value of $L$ in the circuit, just how much to add. When this $L$ is added the current in the circuit will increase.

**31.44.** **Identify:** $X_L = \omega L$. $P_{av} = V_{rms} I_{rms} \cos\phi$.

**Set Up:** $f = 120$ Hz; $\omega = 2\pi f$.

**Execute:** **(a)** $X_L = \omega L \Rightarrow L = \dfrac{X_L}{\omega} = \dfrac{250\ \Omega}{2\pi(120\ \text{Hz})} = 0.332\ \text{H}.$

**(b)** $Z = \sqrt{R^2 + X_L^2} = \sqrt{(400\ \Omega)^2 + (250\ \Omega)^2} = 472\ \Omega.$ $\cos\phi = \dfrac{R}{Z}$ and $I_{rms} = \dfrac{V_{rms}}{Z}$. $P_{av} = \dfrac{V_{rms}^2}{Z}\dfrac{R}{Z}$, so

$V_{rms} = Z\sqrt{\dfrac{P_{av}}{R}} = (472\ \Omega)\sqrt{\dfrac{450\ \text{W}}{400\ \Omega}} = 501\ \text{V}.$

**EVALUATE:** $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{501\text{ V}}{472\ \Omega} = 1.06\text{ A}$. We can calculate $P_{\text{av}}$ as $I_{\text{rms}}^2 R = (1.06\text{ A})^2(400\ \Omega) = 450\text{ W}$, which checks.

**31.45.** **IDENTIFY:** We know the impedances and the average power consumed. From these we want to find the power factor and the rms voltage of the source.

**SET UP:** $P = I_{\text{rms}}^2 R$. $\cos\phi = \dfrac{R}{Z}$. $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $V_{\text{rms}} = I_{\text{rms}} Z$.

**EXECUTE:** **(a)** $I_{\text{rms}} = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{60.0\text{ W}}{300\ \Omega}} = 0.447\text{ A}$. $Z = \sqrt{(300\ \Omega)^2 + (500\ \Omega - 300\ \Omega)^2} = 361\ \Omega$.

$\cos\phi = \dfrac{R}{Z} = \dfrac{300\ \Omega}{361\ \Omega} = 0.831$.

**(b)** $V_{\text{rms}} = I_{\text{rms}} Z = (0.447\text{ A})(361\ \Omega) = 161\text{ V}$.

**EVALUATE:** The voltage amplitude of the source is $V_{\text{rms}}\sqrt{2} = 228\text{ V}$.

**31.46.** **IDENTIFY** and **SET UP:** $X_C = \dfrac{1}{\omega C}$. $X_L = \omega L$.

**EXECUTE:** **(a)** $\dfrac{1}{\omega_1 C} = \omega_1 L$ and $LC = \dfrac{1}{\omega_1^2}$. At angular frequency $\omega_2$,

$\dfrac{X_L}{X_C} = \dfrac{\omega_2 L}{1/\omega_2 C} = \omega_2^2 LC = (2\omega_1)^2 \dfrac{1}{\omega_1^2} = 4$. $X_L > X_C$.

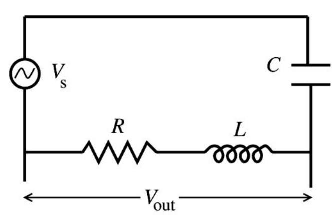
**(b)** At angular frequency $\omega_3$, $\dfrac{X_L}{X_C} = \omega_3^2 LC = \left(\dfrac{\omega_1}{3}\right)^2 \left(\dfrac{1}{\omega_1^2}\right) = \dfrac{1}{9}$. $X_C > X_L$.

**(c)** The resonance angular frequency $\omega_0$ is the value of $\omega$ for which $X_C = X_L$, so $\omega_0 = \omega_1$.

**EVALUATE:** When $\omega$ increases, $X_L$ increases and $X_C$ decreases. When $\omega$ decreases, $X_L$ decreases and $X_C$ increases.

**31.47.** **IDENTIFY** and **SET UP:** Express $Z$ and $I$ in terms of $\omega$, $L$, $C$, and $R$. The voltages across the resistor and the inductor are $90°$ out of phase, so $V_{\text{out}} = \sqrt{V_R^2 + V_L^2}$.

**EXECUTE:** The circuit is sketched in Figure 31.47.




$$X_L = \omega L,\ X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

**Figure 31.47**

$$V_{\text{out}} = I\sqrt{R^2 + X_L^2} = I\sqrt{R^2 + \omega^2 L^2} = V_s \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V_{\text{out}}}{V_s} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$\underline{\omega \text{ small:}}$

As $\omega$ gets small, $R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2 \to \dfrac{1}{\omega^2 C^2}$, $R^2 + \omega^2 L^2 \to R^2$.

Therefore $\dfrac{V_{out}}{V_s} \to \sqrt{\dfrac{R^2}{(1/\omega^2 C^2)}} = \omega RC$ as $\omega$ becomes small.

$\underline{\omega \text{ large:}}$

As $\omega$ gets large, $R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2 \to R^2 + \omega^2 L^2 \to \omega^2 L^2$, $R^2 + \omega^2 L^2 \to \omega^2 L^2$.

Therefore, $\dfrac{V_{out}}{V_s} \to \sqrt{\dfrac{\omega^2 L^2}{\omega^2 L^2}} = 1$ as $\omega$ becomes large.

**EVALUATE:** $V_{out}/V_s \to 0$ as $\omega$ becomes small, so there is $V_{out}$ only when the frequency $\omega$ of $V_s$ is large. If the source voltage contains a number of frequency components, only the high frequency ones are passed by this filter.

**31.48. IDENTIFY:** $V = V_C = IX_C$. $I = V/Z$.

**SET UP:** $X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$.

**EXECUTE:** $V_{out} = V_C = \dfrac{I}{\omega C} \Rightarrow \dfrac{V_{out}}{V_s} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

If $\omega$ is large: $\dfrac{V_{out}}{V_s} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \dfrac{1}{\omega C\sqrt{(\omega L)^2}} = \dfrac{1}{(LC)\omega^2}$.

If $\omega$ is small: $\dfrac{V_{out}}{V_s} \approx \dfrac{1}{\omega C\sqrt{(1/\omega C)^2}} = \dfrac{\omega C}{\omega C} = 1$.

**EVALUATE:** When $\omega$ is large, $X_C$ is small and $X_L$ is large so $Z$ is large and the current is small. Both factors in $V_C = IX_C$ are small. When $\omega$ is small, $X_C$ is large and the voltage amplitude across the capacitor is much larger than the voltage amplitudes across the resistor and the inductor.

**31.49. IDENTIFY:** $I = V/Z$ and $P_{av} = \frac{1}{2}I^2 R$.

**SET UP:** $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

**EXECUTE:** **(a)** $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

**(b)** $P_{av} = \dfrac{1}{2}I^2 R = \dfrac{1}{2}\left(\dfrac{V}{Z}\right)^2 R = \dfrac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2}$.

**(c)** The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \dfrac{1}{\omega_0 C}$, so $\omega_0 = \dfrac{1}{\sqrt{LC}}$.

**(d)** The average power is

$$P_{av} = \frac{(100\text{ V})^2 (200\ \Omega)/2}{(200\ \Omega)^2 + \left[\omega(2.00\text{ H}) - 1/[\omega(0.500\times 10^{-6}\text{ F})]\right]^2} = \frac{1{,}000{,}000\,\omega^2}{40{,}000\,\omega^2 + (2\omega^2 - 2{,}000{,}000\text{ s}^{-2})^2}\text{ W}.$$

The graph of $P_{av}$ versus $\omega$ is sketched in Figure 31.49.

**EVALUATE:** Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light purple curve in Figure 31.19 in the textbook.

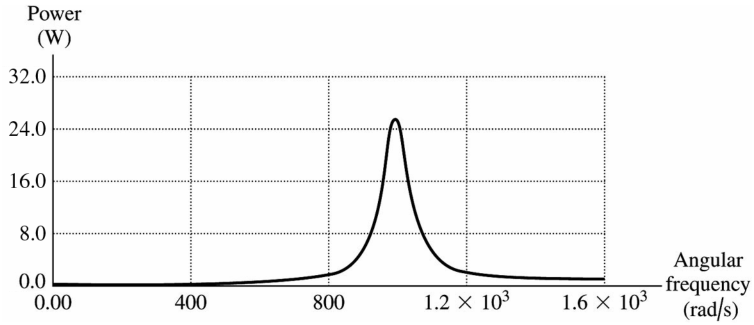



**Figure 31.49**

**31.50. IDENTIFY:** $V_L = I\omega L$ and $V_C = \dfrac{I}{\omega C}$.

**SET UP:** Problem 31.49 shows that $I = \dfrac{V}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

**EXECUTE:** **(a)** $V_L = I\omega L = \dfrac{V\omega L}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

**(b)** $V_C = \dfrac{I}{\omega C} = \dfrac{V}{\omega C\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

**(c)** The graphs are given in Figure 31.50.

**EVALUATE:** **(d)** When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \dfrac{1}{\sqrt{LC}} = 1000\ \text{rad/s}$, the two voltages are equal, and are a maximum, 1000 V.

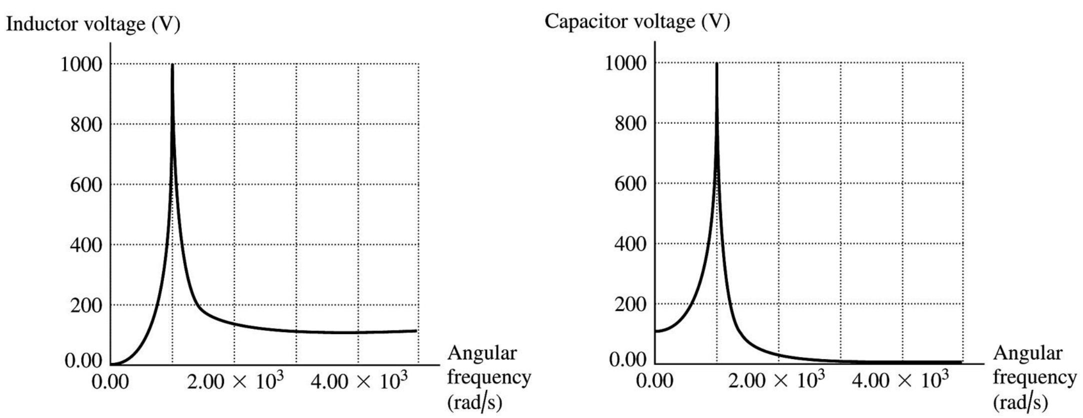



**Figure 31.50**

**31.51. IDENTIFY:** We know $R$, $X_C$, and $\phi$ so $\tan\phi = \dfrac{X_L - X_C}{R}$ tells us $X_L$. Use $P_{\text{av}} = I_{\text{rms}}^2 R$ to calculate $I_{\text{rms}}$. Then calculate $Z$ and use $V_{\text{rms}} = I_{\text{rms}} Z$ to calculate $V_{\text{rms}}$ for the source.

**SET UP:** Source voltage lags current so $\phi = -54.0°$. $X_C = 350\ \Omega$, $R = 180\ \Omega$, $P_{\text{av}} = 140\ \text{W}$.

**EXECUTE:** **(a)** $\tan\phi = \dfrac{X_L - X_C}{R}$.

$X_L = R\tan\phi + X_C = (180\ \Omega)\tan(-54.0°) + 350\ \Omega = -248\ \Omega + 350\ \Omega = 102\ \Omega.$

**(b)** $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = I_{\text{rms}}^2 R$ (Exercise 31.22). $I_{\text{rms}} = \sqrt{\dfrac{P_{\text{av}}}{R}} = \sqrt{\dfrac{140\ \text{W}}{180\ \Omega}} = 0.882\ \text{A}.$

**(c)** $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 306\ \Omega.$

$V_{\text{rms}} = I_{\text{rms}} Z = (0.882\ \text{A})(306\ \Omega) = 270\ \text{V}.$

**EVALUATE:** We could also use $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$.

$V_{\text{rms}} = \dfrac{P_{\text{av}}}{I_{\text{rms}}\cos\phi} = \dfrac{140\ \text{W}}{(0.882\ \text{A})\cos(-54.0°)} = 270\ \text{V}$, which agrees. The source voltage lags the current when $X_C > X_L$, and this agrees with what we found.

**31.52. IDENTIFY** and **SET UP:** For an *L-R-C* circuit, $\tan\phi = \dfrac{X_L - X_C}{R}$. $P = I_{\text{rms}}^2 R$. $V_{\text{rms}} = I_{\text{rms}} Z$. $\cos\phi = \dfrac{R}{Z}$. For this circuit, we know that $\phi = +40.0°$.

**EXECUTE:** **(a)** $X_L = R\tan\phi + X_C = (200\ \Omega)\tan 40.0° + 400\ \Omega = 568\ \Omega.$

**(b)** $I_{\text{rms}} = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{150\ \text{W}}{200\ \Omega}} = 0.866\ \text{A}.$

**(c)** $Z = \dfrac{R}{\cos\phi} = \dfrac{200\ \Omega}{\cos 40.0°} = 261\ \Omega.$ $V_{\text{rms}} = I_{\text{rms}} Z = (0.866\ \text{A})(261\ \Omega) = 226\ \text{V}.$

**EVALUATE:** The voltage amplitude is $V_{\text{rms}}\sqrt{2} = (226\ \text{V})\sqrt{2} = 320\ \text{V}.$

**31.53. IDENTIFY** and **SET UP:** Calculate $Z$ and $I = V/Z$.

**EXECUTE:** **(a)** For $\omega = 800\ \text{rad/s}$:

$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{(500\ \Omega)^2 + \{(800\ \text{rad/s})(2.0\ \text{H}) - 1/[(800\ \text{rad/s})(5.0\times10^{-7}\ \text{F})]\}^2}$. $Z = 1030\ \Omega.$

$I = \dfrac{V}{Z} = \dfrac{100\ \text{V}}{1030\ \Omega} = 0.0971\ \text{A}.$ $V_R = IR = (0.0971\ \text{A})(500\ \Omega) = 48.6\ \text{V},$

$V_C = IX_C = \dfrac{I}{\omega C} = \dfrac{0.0971\ \text{A}}{(800\ \text{rad/s})(5.0\times10^{-7}\ \text{F})} = 243\ \text{V}$ and

$V_L = I\omega L = (0.0971\ \text{A})(800\ \text{rad/s})(2.00\ \text{H}) = 155\ \text{V}.$ $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right) = -60.9°.$ The graph of each voltage versus time is given in Figure 31.53a.

**(b)** Repeating exactly the same calculations as above for $\omega = 1000\ \text{rad/s}$:

$Z = R = 500\ \Omega$; $\phi = 0$; $I = 0.200\ \text{A}$; $V_R = V = 100\ \text{V}$; $V_C = V_L = 400\ \text{V}$. The graph of each voltage versus time is given in Figure 31.53b.

**(c)** Repeating exactly the same calculations as part (a) for $\omega = 1250\ \text{rad/s}$:

$Z = 1030\ \Omega$; $\phi = +60.9°$; $I = 0.0971\ \text{A}$; $V_R = 48.6\ \text{V}$; $V_C = 155\ \text{V}$; $V_L = 243\ \text{V}$. The graph of each voltage versus time is given in Figure 31.53c.

**EVALUATE:** The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.00\ \text{H})(0.500\ \mu\text{F})}} = 1000\ \text{rad/s}.$ For $\omega < \omega_0$ the phase angle is negative and for $\omega > \omega_0$ the phase angle is positive.

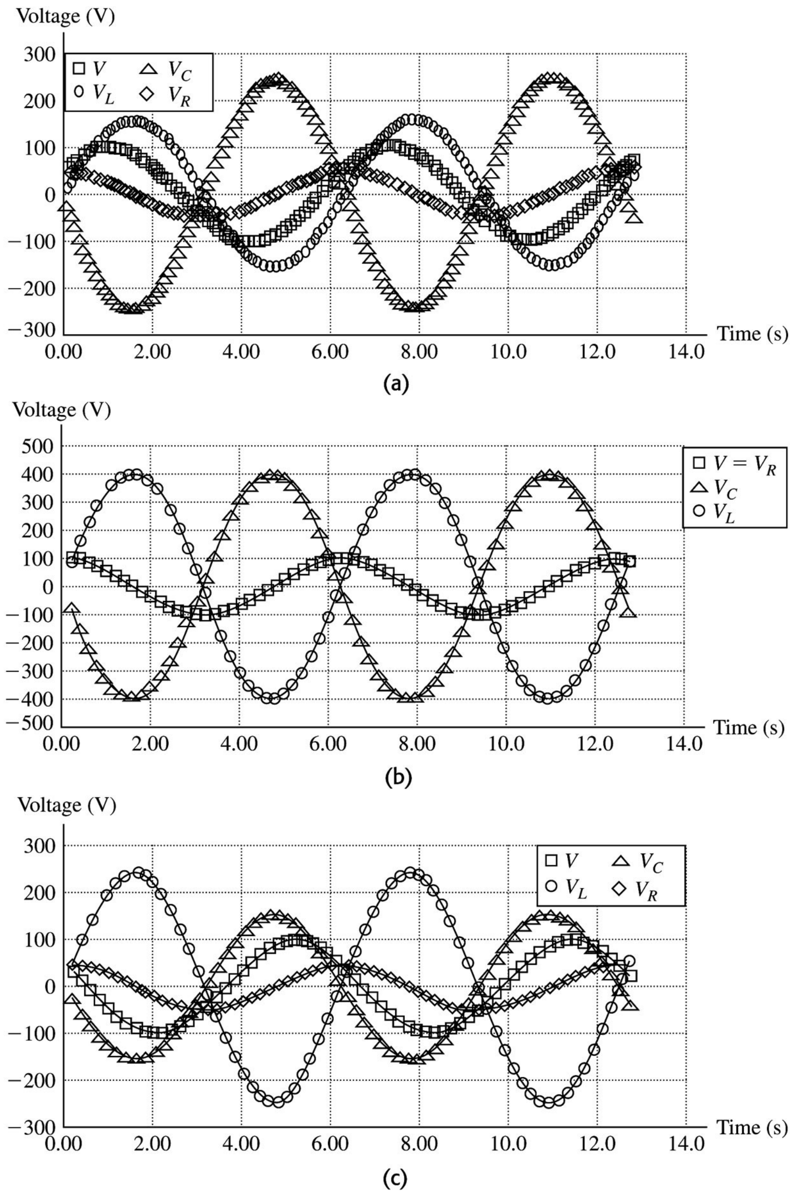



**Figure 31.53**

**31.54.** **IDENTIFY:** At any instant of time the same rules apply to the parallel ac circuit as to the parallel dc circuit: the voltages are the same and the currents add.

**SET UP:** For a resistor the current and voltage in phase. For an inductor the voltage leads the current by $90°$ and for a capacitor the voltage lags the current by $90°$.

**EXECUTE:** **(a)** The parallel *L-R-C* circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.

**(b)** $i_R = \dfrac{v}{R}$ is always in phase with the voltage. $i_L = \dfrac{v}{\omega L}$ lags the voltage by $90°$, and $i_C = v\omega C$ leads the voltage by $90°$. The phasor diagram is sketched in Figure 31.54 (next page).

**(c)** From the diagram, $I^2 = I_R^2 + (I_C - I_L)^2 = \left(\frac{V}{R}\right)^2 + \left(V\omega C - \frac{V}{\omega L}\right)^2.$

**(d)** From part (c): $I = V\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$. But $I = \frac{V}{Z}$, so $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$.

**EVALUATE:** For large $\omega$, $Z \to \frac{1}{\omega C}$. The current in the capacitor branch is much larger than the current in the other branches. For small $\omega$, $Z \to \omega L$. The current in the inductive branch is much larger than the current in the other branches.

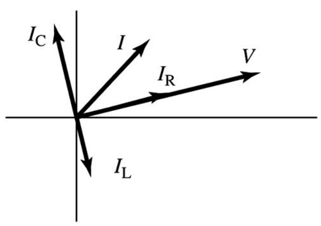



**Figure 31.54**

**31.55. IDENTIFY:** Apply the expression for $1/Z$ from Problem 31.54.

**SET UP:** From Problem 31.54, $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$.

**EXECUTE:** **(a)** Using $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$, we see that the impedance $Z$ is a maximum when the square root is a minimum, and that occurs when $\omega C - \frac{1}{\omega L} = 0$. But that occurs when $\omega = \frac{1}{\sqrt{LC}}$, which is the resonance angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$. Since $I = V/Z$, the current is then a minimum when $Z$ is a maximum.

**(b)** Using the result from part (a) gives $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.300\text{ H})(0.100\times10^{-6}\text{ F})}} = 5770\text{ rad/s}$.

**(c)** At resonance, $Z = R = 100\ \Omega$, so $I = V/R = (240\text{ V})/(100\ \Omega) = 2.40\text{ A}$.

**(d)** At resonance, the amplitude of the current in the resistor is $I = V/R = (240\text{ V})/(100\ \Omega) = 2.40\text{ A}$.

**(e)** At resonance, $X_L = \omega L = (5770\text{ rad/s})(0.300\text{ H}) = 1730\ \Omega$, which is also $X_C$. The amplitude of the maximum current through the inductor is $I = V/X_L = (240\text{ V})/(1730\ \Omega) = 0.139\text{ A}$.

**(f)** Since we are at resonance, $X_L = X_C = 1730$ A. Therefore $I = V/X_C = (240\text{ V})/(1730\ \Omega) = 0.139\text{ A}$.

**EVALUATE:** The parallel circuit is sketched in Figure 31.55. At resonance, $|i_C| = |i_L|$ and at any instant of time these two currents are in opposite directions. Therefore, the net current between $a$ and $b$ is always zero. If the inductor and capacitor each have some resistance, and these resistances aren't the same, then it is no longer true that $i_C + i_L = 0$. The result in part (a) for a parallel *L-R-C* circuit at resonance that the impedance is a maximum and the current is a minimum is the opposite of the behavior of a series *L-R-C* circuit at resonance.

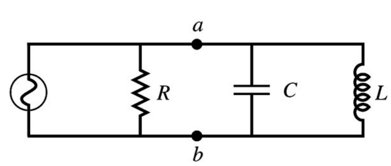



**Figure 31.55**

**31.56.** **IDENTIFY:** Refer to the results and the phasor diagram in Problem 31.54. The source voltage is applied across each parallel branch.

**SET UP:** $V = \sqrt{2}V_{\text{rms}} = 254.6\text{ V}$.

**EXECUTE:** **(a)** $I_R = \dfrac{V}{R} = \dfrac{254.6\text{ V}}{400\ \Omega} = 0.636\text{ A}$.

**(b)** $I_C = V\omega C = (254.6\text{ V})(360\text{ rad/s})(6.00\times10^{-6}\text{ F}) = 0.550\text{ A}$.

**(c)** $\phi = \arctan\left(\dfrac{I_C}{I_R}\right) = \arctan\left(\dfrac{0.550\text{ A}}{0.636\text{ A}}\right) = 40.8°$.

**(d)** $I = \sqrt{I_R^2 + I_C^2} = \sqrt{(0.636\text{ A})^2 + (0.550\text{ A})^2} = 0.841\text{ A}$.

**(e)** Leads since $\phi > 0$.

**EVALUATE:** The phasor diagram shows that the current in the capacitor always leads the source voltage.

**31.57.** **IDENTIFY:** The average power depends on the phase angle $\phi$.

**SET UP:** The average power is $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$, and the impedance is $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

**EXECUTE:** **(a)** $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi = \frac{1}{2}(V_{\text{rms}}I_{\text{rms}})$, which gives $\cos\phi = \frac{1}{2}$, so $\phi = \pi/3 = 60°$.

$\tan\phi = (X_L - X_C)/R$, which gives $\tan 60° = (\omega L - 1/\omega C)/R$. Using $R = 75.0\ \Omega$, $L = 5.00\text{ mH}$ and $C = 2.50\ \mu\text{F}$ and solving for $\omega$ we get $\omega = 28760\text{ rad/s} = 28{,}800\text{ rad/s}$.

**(b)** $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L = (28{,}760\text{ rad/s})(5.00\text{ mH}) = 144\ \Omega$ and

$X_C = 1/\omega C = 1/[(28{,}760\text{ rad/s})(2.50\ \mu\text{F})] = 13.9\ \Omega$, giving $Z = \sqrt{(75\ \Omega)^2 + (144\ \Omega - 13.9\ \Omega)^2} = 150\ \Omega$;

$I = V/Z = (15.0\text{ V})/(150\ \Omega) = 0.100\text{ A}$ and $P_{\text{av}} = \frac{1}{2}VI\cos\phi = \frac{1}{2}(15.0\text{ V})(0.100\text{ A})(1/2) = 0.375\text{ W}$.

**EVALUATE:** All this power is dissipated in the resistor because the average power delivered to the inductor and capacitor is zero.

**31.58.** **IDENTIFY** and **SET UP:** The maximum energy in the inductor depends on the current amplitude in the inductor. $U_L = \frac{1}{2}LI^2$ and $U_C = \frac{1}{2}CV^2$. The impedance of a series *L-R-C* circuit is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2},\quad X_C = \frac{1}{\omega C}.$$

**EXECUTE:** **(a)** Use $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ to find the impedance of the circuit.

$$Z = \sqrt{(60.0\ \Omega)^2 + \left[(120\text{ rad/s})(0.800\text{ H}) - \frac{1}{(120\text{ rad/s})(3.00\times10^{-4}\text{ F})}\right]^2} = 90.85\ \Omega.$$

The amplitude of the current is therefore $I = V/Z = (90.0\text{ V})/(90.85\ \Omega) = 0.9906\text{ A}$, so the maximum energy stored in the inductor is $U_L = \frac{1}{2}LI^2 = (1/2)(0.800\text{ H})(0.9906\text{ A})^2 = 0.393\text{ J}$.

**(b)** The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2$, but the capacitor voltage is 90° out of phase with the current. Thus when the current is a maximum, the voltage across the capacitor is zero, so the energy stored in the capacitor is also zero.

**(c)** The capacitor stores its maximum energy when it is at maximum voltage, which is

$V_C = IX_C = I\dfrac{1}{\omega C} = (0.9906\text{ A})\left[\dfrac{1}{(120\text{ rad/s})(3.00\times10^{-4}\text{ F})}\right] = 27.52\text{ V}$. The maximum energy in the capacitor at this time is $U_C = \frac{1}{2}CV^2 = (1/2)(3.00\times10^{-4}\text{ F})(27.52\text{ V})^2 = 0.114\text{ J}$.

**EVALUATE:** The maximum energy stored in the inductor is not the same as in the capacitor due to the presence of resistance.

**31.59. IDENTIFY and SET UP:** The equation $V_C = IX_C$ allows us to calculate $I$ and then $V = IZ$ gives $Z$. Solve $Z=\sqrt{R^2+(X_L-X_C)^2}$ for $X_L$.

**EXECUTE: (a)** $V_C = IX_C$ so $I=\dfrac{V_C}{X_C}=\dfrac{360\text{ V}}{480\ \Omega}=0.750\text{ A}.$

**(b)** $V = IZ$ so $Z=\dfrac{V}{I}=\dfrac{120\text{ V}}{0.750\text{ A}}=160\ \Omega.$

**(c)** $Z^2=R^2+(X_L-X_C)^2.$

$X_L-X_C=\pm\sqrt{Z^2-R^2}$, so

$X_L=X_C\pm\sqrt{Z^2-R^2}=480\ \Omega\pm\sqrt{(160\ \Omega)^2-(80.0\ \Omega)^2}=480\ \Omega\pm139\ \Omega.$

$X_L=619\ \Omega$ or $341\ \Omega.$

**EVALUATE: (d)** $X_C=\dfrac{1}{\omega C}$ and $X_L=\omega L$. At resonance, $X_C=X_L$. As the frequency is lowered below the resonance frequency $X_C$ increases and $X_L$ decreases. Therefore, for $\omega<\omega_0$, $X_L<X_C$. So for $X_L=341\ \Omega$ the angular frequency is less than the resonance angular frequency. $\omega$ is greater than $\omega_0$ when $X_L=619\ \Omega$. But at these two values of $X_L$, the magnitude of $X_L-X_C$ is the same so $Z$ and $I$ are the same. In one case $(X_L=691\ \Omega)$ the source voltage leads the current and in the other $(X_L=341\ \Omega)$ the source voltage lags the current.

**31.60. IDENTIFY and SET UP:** The capacitive reactance is $X_C=\dfrac{1}{\omega C}$, the inductive reactance is $X_L=\omega L$, and the impedance of an $L$-$R$-$C$ series circuit is $Z=\sqrt{R^2+(X_L-X_C)^2}$.

**EXECUTE: (a)** The current amplitude is $I = V/R = (135\text{ V})/(90.0\ \Omega) = 1.50\text{ A}$.

**(b)** The voltage amplitude across the inductor is $V_L = IX_L = (1.50\ \Omega)(320\ \Omega) = 480\text{ V}$.

**(c)** The impedance is $Z = V/I = (240\text{ V})/(1.50\text{ A}) = 160\ \Omega$. We also know that the impedance is $Z=\sqrt{R^2+(X_L-X_C)^2}$. We know that $X_L = 320\ \Omega$, so we can find $X_C$.

$160\ \Omega=\sqrt{(90.0\ \Omega)^2+(320\ \Omega-X_C)^2}$. Squaring and solving for $X_C$ gives two values, $X_C = 188\ \Omega$ and $X_C = 452\ \Omega$.

**(d)** At resonance, $\omega L=\dfrac{1}{\omega C}$. $X_C < X_L$ for $\omega > \omega_{\text{res}}$ and $X_C > X_L$ for $\omega < \omega_{\text{res}}$. In this circuit, $X_L = 320\ \Omega$, so $\omega < \omega_{\text{res}}$ for $X_C = 452\ \Omega$.

**EVALUATE:** Due to the square of $(X_L - X_C)$ in the impedance, we get two possibilities in (c).

**31.61. IDENTIFY:** At resonance, $Z=R$. $I=V/R$. $V_R=IR$, $V_C=IX_C$ and $V_L=IX_L$. $U_C=\tfrac{1}{2}CV_C^2$ and $U_L=\tfrac{1}{2}LI^2$.

**SET UP:** The amplitudes of each time-dependent quantity correspond to the maximum values of those quantities.

**EXECUTE: (a)** $I=\dfrac{V}{Z}=\dfrac{V}{\sqrt{R^2+(\omega L-1/\omega C)^2}}$. At resonance $\omega L=\dfrac{1}{\omega C}$ and $I_{\max}=\dfrac{V}{R}$.

**(b)** $V_C=IX_C=\dfrac{V}{R\omega_0 C}=\dfrac{V}{R}\sqrt{\dfrac{L}{C}}.$

**(c)** $V_L=IX_L=\dfrac{V}{R}\omega_0 L=\dfrac{V}{R}\sqrt{\dfrac{L}{C}}.$

**(d)** $U_C = \frac{1}{2}CV_C^2 = \frac{1}{2}C\frac{V^2}{R^2}\frac{L}{C} = \frac{1}{2}L\frac{V^2}{R^2}.$

**(e)** $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\frac{V^2}{R^2}.$

**EVALUATE:** At resonance $V_C = V_L$ and the maximum energy stored in the inductor equals the maximum energy stored in the capacitor.

**31.62. IDENTIFY:** Apply $V_{\text{rms}} = I_{\text{rms}}Z$.

**SET UP:** $\omega_0 = \frac{1}{\sqrt{LC}}$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

**EXECUTE:** **(a)** $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.80\text{ H})(9.00\times10^{-7}\text{ F})}} = 786\text{ rad/s}.$

**(b)** $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$Z = \sqrt{(300\ \Omega)^2 + ((786\text{ rad/s})(1.80\text{ H}) - 1/[(786\text{ rad/s})(9.00\times10^{-7}\text{ F})])^2} = 300\ \Omega.$

$I_{\text{rms-0}} = \frac{V_{\text{rms}}}{Z} = \frac{60\text{ V}}{300\ \Omega} = 0.200\text{ A}.$

**(c)** We want $I = \frac{1}{2}I_{\text{rms-0}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$. $R^2 + (\omega L - 1/\omega C)^2 = \frac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2}$.

$\omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} + R^2 - \frac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2} = 0$ and $(\omega^2)^2 L^2 + \omega^2\left(R^2 - \frac{2L}{C} - \frac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2}\right) + \frac{1}{C^2} = 0.$

Substituting in the values for this problem, the equation becomes $(\omega^2)^2(3.24) + \omega^2(-4.27\times10^6) + 1.23\times10^{12} = 0$. Solving this quadratic equation in $\omega^2$ we find $\omega^2 = 8.90\times10^5\text{ rad}^2/\text{s}^2$ or $4.28\times10^5\text{ rad}^2/\text{s}^2$ and $\omega = 943\text{ rad/s}$ or $654\text{ rad/s}$.

**(d)** (i) $R = 300\ \Omega$, $I_{\text{rms-0}} = 0.200\text{ A}$, $|\omega_1 - \omega_2| = 289\text{ rad/s}$. (ii) $R = 30\ \Omega$, $I_{\text{rms-0}} = 2\text{A}$, $|\omega_1 - \omega_2| = 28\text{ rad/s}$. (iii) $R = 3\ \Omega$, $I_{\text{rms-0}} = 20\text{ A}$, $|\omega_1 - \omega_2| = 2.88\text{ rad/s}$.

**EVALUATE:** The width gets smaller as $R$ gets smaller; $I_{\text{rms}-0}$ gets larger as $R$ gets smaller.

**31.63. IDENTIFY:** $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$ and $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$. Calculate $Z$. $R = Z\cos\phi$.

**SET UP:** $f = 50.0\text{ Hz}$ and $\omega = 2\pi f$. The power factor is $\cos\phi$.

**EXECUTE:** **(a)** $P_{\text{av}} = \frac{V_{\text{rms}}^2}{Z}\cos\phi$. $Z = \frac{V_{\text{rms}}^2\cos\phi}{P_{\text{av}}} = \frac{(120\text{ V})^2(0.560)}{(220\text{ W})} = 36.7\ \Omega.$

$R = Z\cos\phi = (36.7\ \Omega)(0.560) = 20.6\ \Omega.$

**(b)** $Z = \sqrt{R^2 + X_L^2} \cdot X_L = \sqrt{Z^2 - R^2} = \sqrt{(36.7\ \Omega)^2 - (20.6\ \Omega)^2} = 30.4\ \Omega$. But $\phi = 0$ is at resonance, so the inductive and capacitive reactances equal each other. Therefore we need to add $X_C = 30.4\ \Omega$. $X_C = \frac{1}{\omega C}$ therefore gives $C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50.0\text{ Hz})(30.4\ \Omega)} = 1.05\times10^{-4}\text{ F}.$

**(c)** At resonance, $P_{\text{av}} = \frac{V^2}{R} = \frac{(120\text{ V})^2}{20.6\ \Omega} = 699\text{ W}.$

**EVALUATE:** $P_{\text{av}} = I_{\text{rms}}^2 R$ and $I_{\text{rms}}$ is maximum at resonance, so the power drawn from the line is maximum at resonance.

**31.64. IDENTIFY and SET UP:** We use $I = V/Z$, $X_L = \omega L$, and $Z = \sqrt{R^2 + X_L^2}$.

**EXECUTE:** $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + X_L^2}}$. Squaring and rearranging gives $\dfrac{1}{I^2} = \left(\dfrac{L}{V}\right)^2 \omega^2 + \left(\dfrac{R}{V}\right)^2$. Therefore a graph of $1/I^2$ versus $\omega^2$ should be a straight line having slope equal to $(L/V)^2$ and $y$-intercept equal to $(R/V)^2$. We can find the slope using two convenient points on the graph, giving

slope = $\dfrac{(21.0 - 9.0)\ \text{A}^{-2}}{(3500 - 500)\ \text{rad}^2/\text{s}^2} = 4.00\times10^{-3}\ \text{s}^2/\text{rad}^2\cdot\text{A}^2.$

Solving for $L$ gives $L = V\sqrt{\text{slope}} = (12.0\ \text{V})\sqrt{4.00\times10^{-3}\ \text{s}^2/\text{rad}^2\cdot\text{A}^2} = 0.759\ \text{H}.$

Extending the line, we find the $y$-intercept is 7.0 $\text{A}^{-2}$. Using this value to solve for $R$ gives

$R = V\sqrt{y\text{-intercept}} = (12.0\ \text{V})\sqrt{7.00\,\text{A}^{-2}} = 32\ \Omega.$

**EVALUATE:** These are reasonable values for $L$ and $R$ for a large solenoid, so we're confident in the results.

**31.65. IDENTIFY and SET UP:** For an $L$-$R$-$C$ series circuit, the maximum current occurs at resonance, and the resonance angular frequency is $\omega_{\text{res}} = \dfrac{1}{\sqrt{LC}}$.

**EXECUTE:** At resonance, the angular frequency is $\omega_{\text{res}} = \dfrac{1}{\sqrt{LC}}$. Squaring gives $\omega_{\text{res}}^2 = \dfrac{1}{L}\cdot\dfrac{1}{C}$, so a graph of $\omega_{\text{res}}^2$ versus $1/C$ should be a straight line with a slope equal to $1/L$. Using two convenient points on the graph, we find the slope to be $\dfrac{(25.0 - 1.00)\times10^4\ \text{rad}^2/\text{s}^2}{(4.50 - 1.75)\times10^3\ \text{F}^{-1}} = 5.455\ \text{F/s}^2.$ Solving for $L$ gives $L = (\text{slope})^{-1} = (5.455\ \text{F/s}^2)^{-1} = 0.183$ H, which rounds to 0.18 H, since we cannot determine the slope of the graph in the text with anything better than 2 significant figures. To find $R$, we realize that at resonance $Z = R$, so $R = V/I = (90.0\ \text{V})/(4.50\ \text{A}) = 20.0\ \Omega$.

**EVALUATE:** These are reasonable values for $L$ and $R$ for a laboratory solenoid.

**31.66. IDENTIFY and SET UP:** For an $L$-$R$-$C$ series circuit, $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ and the power factor is $\cos\phi = R/Z$.

**EXECUTE:** **(a)** $\cos\phi = R/Z$, so $R = Z\cos\phi$.

At 80 Hz: $R = (15\ \Omega)\cos(-71°) = 4.88\ \Omega$

At 160 Hz: $R = (13\ \Omega)\cos(67°) = 5.08\ \Omega$

The average resistance is $(4.88\ \Omega + 5.08\ \Omega)/2 = 5.0\ \Omega$.

**(b)** We use $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ with $R = 5.0\ \Omega$ from part (a).

<u>At 80 Hz</u>: $\tan(-71°) = \dfrac{2\pi(80\ \text{Hz})L - \dfrac{1}{2\pi(80\ \text{Hz})C}}{5.0\ \Omega}.$

$-14.52 = 160\pi\ \text{Hz}\ L - 1/[(160\pi\ \text{Hz})C]$. Eq. (1)

<u>At 160 Hz</u>: $\tan(67°) = \dfrac{2\pi(160\ \text{Hz})L - \dfrac{1}{2\pi(160\ \text{Hz})C}}{5.0\ \Omega}.$

$11.78 = 320\pi\ \text{Hz}\ L - 1/[(320\pi\ \text{Hz})C]$. Eq. (2)

Multiply Eq. (1) by –2 and add it to Eq. (2), giving
$2(14.52) + 11.78 = (1/C)(1/80\pi - 1/320\pi)$.
$C = 7.31\times10^{-5}$ F, which rounds to $C = 73\ \mu$F.
Substituting this result into either Eq. (1) or Eq. (2) gives $L = 25.3$ mH, which rounds to $L = 25$ mH.

**(c)** The resonance angular frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$, so the resonance frequency is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.53\times10^{-2}\ \text{H})(73.1\times10^{-6}\ \text{F})}} = 117\ \text{Hz}.$$

At resonance, $Z = R = 5.0\ \Omega$ and $\phi = 0$.

**EVALUATE:** It is only at resonance that $Z = R$, not at the other frequencies.

**31.67. IDENTIFY:** $p_R = i^2R$. $p_L = iL\dfrac{di}{dt}$. $p_C = \dfrac{q}{C}i$.

**SET UP:** $i = I\cos\omega t$.

**EXECUTE:** **(a)** $p_R = i^2R = I^2\cos^2(\omega t)R = V_RI\cos^2(\omega t) = \dfrac{1}{2}V_RI(1+\cos(2\omega t))$.

$$P_{\text{av}}(R) = \frac{1}{T}\int_0^T p_R dt = \frac{V_RI}{2T}\int_0^T [1+\cos(2\omega t)]dt = \frac{V_RI}{2T}[t]_0^T = \tfrac{1}{2}V_RI.$$

**(b)** $p_L = Li\dfrac{di}{dt} = -\omega LI^2\cos(\omega t)\sin(\omega t) = -\tfrac{1}{2}V_LI\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{\text{av}}(L) = 0$.

**(c)** $p_C = \dfrac{q}{C}i = v_Ci = V_CI\sin(\omega t)\cos(\omega t) = \tfrac{1}{2}V_CI\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{\text{av}}(C) = 0$.

**(d)** $p = p_R + p_L + p_c = V_RI\cos^2(\omega t) - \tfrac{1}{2}V_LI\sin(2\omega t) + \tfrac{1}{2}V_CI\sin(2\omega t)$ and

$p = I\cos\omega t(V_R\cos\omega t - V_L\sin\omega t + V_C\sin\omega t)$. But $\cos\phi = \dfrac{V_R}{V}$ and $\sin\phi = \dfrac{V_L - V_C}{V}$, so

$p = VI\cos\omega t(\cos\phi\cos\omega t - \sin\phi\sin\omega t)$, at any instant of time.

**EVALUATE:** At an instant of time the energy stored in the capacitor and inductor can be changing, but there is no net consumption of electrical energy in these components.

**31.68. IDENTIFY:** $V_L = IX_L$. $\dfrac{dV_L}{d\omega} = 0$ at the $\omega$ where $V_L$ is a maximum. $V_C = IX_C$. $\dfrac{dV_C}{d\omega} = 0$ at the $\omega$ where $V_C$ is a maximum.

**SET UP:** Problem 31.49 shows that $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

**EXECUTE:** **(a)** $V_R = \text{maximum}$ when $V_C = V_L \Rightarrow \omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$.

**(b)** $V_L = \text{maximum}$ when $\dfrac{dV_L}{d\omega} = 0$. Therefore: $\dfrac{dV_L}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V\omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)$.

$$0 = \frac{VL}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V\omega^2 L(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{\left[R^2 + (\omega L - 1/\omega C)^2\right]^{3/2}}.\quad R^2 + (\omega L - 1/\omega C)^2 = \omega^2(L^2 - 1/\omega^4 C^2).$$

$$R^2 + \frac{1}{\omega^2C^2} - \frac{2L}{C} = -\frac{1}{\omega^2C^2}.\quad \frac{1}{\omega^2} = LC - \frac{R^2C^2}{2} \text{ and } \omega = \frac{1}{\sqrt{LC - R^2C^2/2}}.$$

**(c)** $V_C = \text{maximum}$ when $\dfrac{dV_C}{d\omega} = 0$. Therefore: $\dfrac{dV_C}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)$.

$0 = -\dfrac{V}{\omega^2 C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \dfrac{V(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{C(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}$. $R^2 + (\omega L - 1/\omega C)^2 = -\omega^2(L^2 - 1/\omega^4 C^2)$.

$R^2 + \omega^2 L^2 - \dfrac{2L}{C} = -\omega^2 L^2$ and $\omega = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{2L^2}}$.

**EVALUATE:** $V_L$ is maximum at a frequency greater than the resonance frequency and $V_C$ is a maximum at a frequency less than the resonance frequency. These frequencies depend on $R$, as well as on $L$ and on $C$.

**31.69.** **IDENTIFY** and **SET UP:** We are told that the platinum electrode behaves like an ideal capacitor in series with the resistance of the fluid. The impedance of an *R-C* circuit is $Z = \sqrt{R^2 + X_C^2}$, where $X_C = \dfrac{1}{\omega C}$.

**EXECUTE:** For a dc signal we have $\omega = 2\pi f = 0$. Using $X_C = \dfrac{1}{\omega C}$ we see that as $\omega \to 0$ we have $X_C \to \infty$, and so $Z \to \infty$. The correct choice is (b).

**EVALUATE:** The oscillation period of such a circuit is $T = 1/f$, so $T \to \infty$ as $\omega \to 0$.

**31.70.** **IDENTIFY** and **SET UP:** We are told that the platinum electrode behaves like an ideal capacitor in series with the resistance of the fluid, which is given by $R_A = \rho/(10a)$, where $\rho = 100\ \Omega \cdot \text{cm} = 1\ \Omega \cdot \text{m}$ and $d = 2a = 20\ \mu\text{m}$. We know that $X_C = \dfrac{1}{\omega C}$, where we are given $C = 10\ \text{nF} = 10^{-8}\ \text{F}$ and $\omega = 2\pi f = 2\pi[(5000/\pi)\text{Hz}] = 10^4\ \text{rad/s}$. For an *R-C* circuit we know that the impedance is given by $Z = \sqrt{R^2 + X_C^2}$.

**EXECUTE:** $R_A = \rho/(10a) = (1\ \Omega \cdot \text{m})/[10(10^{-5}\ \text{m})] = 10^4\ \Omega$. The capacitive reactance is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(10^4\ \text{rad/s})(10^{-8}\ \text{F})} = 10^4\ \Omega$. Thus the impedance is $Z = \sqrt{R^2 + X_C^2} = \sqrt{(10^4\ \Omega)^2 + (10^4\ \Omega)^2} = \sqrt{2} \times (10^4\ \Omega)$, so the correct choice is (c).

**EVALUATE:** In this case, the capacitance contributes as much to the impedance as the resistance does.

**31.71.** **IDENTIFY** and **SET UP:** We know that $V_{\text{rms}} = \dfrac{V}{\sqrt{2}}$, where $V$ is the amplitude (peak value) of the voltage. According to the problem, the peak-to-peak voltage $V_{\text{pp}}$ is the difference between the two extreme values of voltage.

**EXECUTE:** Since the voltage oscillates between $+V$ and $-V$ the peak-to-peak voltage is $V_{\text{pp}} = V - (-V) = 2V = 2\sqrt{2}V_{\text{rms}}$. Thus, the correct answer is (d).

**EVALUATE:** The voltage amplitude is half the peak-to-peak voltage.

**31.72.** **IDENTIFY** and **SET UP:** The impedance of an *R-C* circuit is $Z = \sqrt{R^2 + X_C^2}$, where $X_C = \dfrac{1}{\omega C}$.

**EXECUTE:** As the frequency of oscillation gets very large, $X_C$ gets very small, so the impedance approaches the access resistance $R$. So the impedance approaches a constant but nonzero value, which is choice (c).

**EVALUATE:** For high oscillation frequency, the access resistance has more effect on the circuit than the capacitance does.