

TRINITY COLLEGE DUBLIN
THE UNIVERSITY OF DUBLIN

School of Mathematics

JF Mathematics
JF Theoretical Physics
JF TSM - Mathematics

Trinity Term 2015

MA1111 — Linear Algebra I

Wednesday, May 13 Sports Centre 09.30 — 11.30

Prof. V. Dotsenko

Instructions to Candidates:

ATTEMPT ALL QUESTIONS

The number of marks you can get for a complete solution to any individual question is written next to the question. A complete solution to a question includes coherent explanations of answers you give.

Unless otherwise specified, you may use all statements proved in class without proof; when using some statement, you should formulate it clearly, e.g. "in class, we proved that a square matrix A is invertible if and only if $\det(A) \neq 0$ ".

Materials Permitted for this Examination:

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. (a) (15 points) Using elementary row operations, compute the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 7 & 49 \end{pmatrix}.$$

- (b) (10 points) Find a polynomial $f(t)$ of degree at most 2 for which $f(1) = 1$, $f(2) = 0$, $f(7) = 11$.

2. (25 points) Describe all possible values of i , j , k and l for which the term

$$a_{56}a_{64}a_{l1}a_{27}a_{k2}a_{35}a_{ij}$$

has the coefficient 1 in the expansion of the determinant of a 7×7 matrix with entries a_{ij} , $1 \leq i, j \leq 7$.

3. (a) (10 points) Under which condition is a system of vectors f_1, \dots, f_m of a vector space V called a spanning set? Prove that if a system of vectors f_1, \dots, f_m is a spanning set, then it remains a spanning set after being extended by an arbitrary vector v from V .
- (b) (15 points) Assume that the system of vectors u , v , and w (all belonging to the same vector space V) forms a spanning set. Prove that then the system of vectors $u + v$, $u - w$, $2v + w$ also forms a spanning set. What are possible values of $\dim V$ in this situation? Explain your answer.
4. (25 points) A 3×3 -grid whose cells are filled in with 9 real numbers is called a magic square if all its row sums are pairwise equal, and equal to all of its column sums. Explain how to view the set of all magic squares as a subspace of \mathbb{R}^9 , compute the dimension of this subspace, and find a basis of this subspace.