Advanced Calculus
MA1132
Tutorial Exercises 3

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To be completed before and during tutorials of Friday, 15. February

1. For each of the following, either find the limit or show it doesn't exist.

   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4}{x^4+y^2}$.

   (b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-2xy+y^2}{x-y}$.

   (c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2-x^3-xy^2}{x^2+y^2}$.

   (d) $\lim\limits_{(x,y)\to(1,0)} \dfrac{x\sin(y)}{y^2-1}$.

   (e) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$.

   (f) $\lim\limits_{(x,y)\to(2,1)} \dfrac{x^3-2x^2-xy^2+2y^2}{x^3-2x^2+xy^2-2y^2+x-2}$.

   (g) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+y}{|x|+|y|}$.

   (h) $\lim\limits_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1}$.

   (i) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$.

2. For each of the limits in Question 1 that exist at the given point, using the expression you evaluated as a rule, define a function that is continuous at that point.

3. For the following functions

   i. sketch the domain of f (you may use/check with Mathematica). Use solid lines for portions of the boundary included in the domain and dashed lines for portions not included. Determine whether the domain is an open set, closed set, or neither;

   ii. give the sets where each of the following functions are continuous.

   (a) $f(x,y)=\sin(x+y)$.

   (b) $f(x,y)=\dfrac{x^2+y^2}{x^2-3x+2}$.

(c) $f(x,y) = \dfrac{xy}{xy}$.

(d) $f(x,y,z) = \dfrac{1}{x^2+z^2-10}$.

4. For each of the following functions, calculate all the first and second order partial derivatives, and verify that the corresponding mixed second order partial derivatives are the same.

   (a) $f(x,y) = \ln(1+y^2e^{2x})$.

   (b) $f(x,y) = \dfrac{x^2+y^2}{x+y}$.

   (c) $f(x,y) = \ln(x^2y\sin x)$.

   (d) $f(x,y,z) = x^2y\cos z$.

5. For each of the following functions, by differentiating implicitly, find each of the first order partial derivatives. You should regard z (or w if it appears) as the dependent variable, and you should leave your answer in terms of the independent and the dependent variables.

   (a) $\ln(2x^2+y-z^3) = x$.

   (b) $y^2 + z\cos(xyz) = 0$.

   (c) $e^{xy}\sinh z - z^2x + y = 0$.

   (d) $\ln(2x^2+yz^3+3w) = z$.

6. Sketch the level curve $z = k$ for the specified values of k

$$z = 8x^2 - 8x + 2y^2 + 4y, \qquad k = -2, -1, 2$$

7. A cone is the union of a set of half-lines that start at a common apex point and go through a base which can be any parametric curve. Show that the graph of $z = \sqrt{x^2-4x+4y^2+8y+8}$ is a cone. Where is its apex? What can be chosen as its base? Sketch the base.

8. Show that the hyperboloid of one sheet $x^2+y^2-z^2 = 1$ is a connected surface, that is any two points of the hyperboloid can be connected by a curve which lies on the hyperboloid. Sketch the hyperboloid.

9. Sketch (by hand) the graph of the function and identify it (and then you can check with Mathematica)

   (a) $z = -\sqrt{2x-x^2-y^2}$

   (b) $z = \sqrt{2y-y^2}$