## 17

## TEMPERATURE AND HEAT

17.1. IDENTIFY and SET UP:  $T_F = \frac{9}{5}T_C + 32^\circ$ .

**EXECUTE:** (a)  $T_F = (9/5)(-62.8) + 32 = -81.0$ °F

**(b)** 
$$T_{\rm F} = (9/5)(56.7) + 32 = 134.1 \,^{\circ}\text{F}$$

(c) 
$$T_{\rm F} = (9/5)(31.1) + 32 = 88.0^{\circ} \text{F}$$

**EVALUATE:** Fahrenheit degrees are smaller than Celsius degrees, so it takes more F° than C° to express the difference of a temperature from the ice point.

17.2. **IDENTIFY** and **SET UP:** To convert a temperature between °C and K use  $T_C = T_K - 273.15$ . To convert from °F to °C, subtract 32° and multiply by 5/9. To convert from °C to °F, multiply by 9/5 and add 32°. To convert a temperature difference, use that Celsius and Kelvin degrees are the same size and that  $9 \text{ F}^\circ = 5 \text{ C}^\circ$ .

EXECUTE: (a) 
$$T_C = T_K - 273.15 = 310 - 273.15 = 36.9^{\circ}\text{C}; T_F = \frac{9}{5}T_C + 32^{\circ} = \frac{9}{5}(36.9^{\circ}) + 32^{\circ} = 98.4^{\circ}\text{F}.$$

**(b)** 
$$T_{\rm K} = T_{\rm C} + 273.15 = 40 + 273.15 = 313 \,\text{K}; \ T_{\rm F} = \frac{9}{5}T_{\rm C} + 32^{\circ} = \frac{9}{5}(40^{\circ}) + 32^{\circ} = 104^{\circ}\text{F}.$$

(c) 
$$7 \text{ C}^{\circ} = 7 \text{ K}$$
;  $7 \text{ C}^{\circ} = (7 \text{ C}^{\circ})(9 \text{ F}^{\circ}/5 \text{ C}^{\circ}) = 13 \text{ F}^{\circ}$ .

(d) 
$$4.0^{\circ}\text{C}$$
:  $T_{\text{F}} = \frac{9}{5}T_{\text{C}} + 32^{\circ} = \frac{9}{5}(4.0^{\circ}) + 32^{\circ} = 39.2^{\circ}\text{F}$ ;  $T_{\text{K}} = T_{\text{C}} + 273.15 = 4.0 + 273.15 = 277 \text{ K}$ .

$$-160^{\circ}\text{C}$$
:  $T_{\text{F}} = \frac{9}{5}T_{\text{C}} + 32^{\circ} = \frac{9}{5}(-160^{\circ}) + 32^{\circ} = -256^{\circ}\text{F}$ ;  $T_{\text{K}} = T_{\text{C}} + 273.15 = -160 + 273.15 = 113 \text{ K}$ .

(e) 
$$T_C = \frac{5}{9}(T_F - 32^\circ) = \frac{5}{9}(105^\circ - 32^\circ) = 41^\circ C$$
;  $T_K = T_C + 273.15 = 41 + 273.15 = 314 \text{ K}$ .

**EVALUATE:** Celsius-Fahrenheit conversions do not involve simple proportions due to the additive constant of 32°, but Celsius-Kelvin conversions require only simple addition/subtraction of 273.15.

17.3. **IDENTIFY:** Convert  $\Delta T$  between different scales.

SET UP:  $\Delta T$  is the same on the Celsius and Kelvin scales. 180 F° = 100 C°, so 1 C° =  $\frac{9}{5}$  F°.

EXECUTE: **(a)** 
$$\Delta T = 49.0 \text{ F}^{\circ}$$
.  $\Delta T = (49.0 \text{ F}^{\circ}) \left( \frac{1 \text{ C}^{\circ}}{\frac{9}{5} \text{ F}^{\circ}} \right) = 27.2 \text{ C}^{\circ}$ .

**(b)** 
$$\Delta T = -100 \text{ F}^{\circ}. \quad \Delta T = (-100.0 \text{ F}^{\circ}) \left( \frac{1 \text{ C}^{\circ}}{\frac{9}{5} \text{ F}^{\circ}} \right) = -55.6 \text{ C}^{\circ}$$

**EVALUATE:** The magnitude of the temperature change is larger in F° than in C°.

17.4. **IDENTIFY:** Set  $T_C = T_F$  and  $T_F = T_K$ .

**SET UP:** 
$$T_F = \frac{9}{5}T_C + 32^{\circ}C$$
 and  $T_K = T_C + 273.15 = \frac{5}{9}(T_F - 32^{\circ}) + 273.15$ 

**EXECUTE:** (a) 
$$T_F = T_C = T$$
 gives  $T = \frac{9}{5}T + 32^\circ$  and  $T = -40^\circ$ ;  $-40^\circ$ C =  $-40^\circ$ F.

**(b)** 
$$T_F = T_K = T$$
 gives  $T = \frac{5}{9}(T - 32^\circ) + 273.15$  and  $T = \frac{9}{4}(-(\frac{5}{9})(32^\circ) + 273.15) = 575^\circ$ ;  $575^\circ F = 575$  K.

**EVALUATE:** Since  $T_{\rm K} = T_{\rm C} + 273.15$  there is no temperature at which Celsius and Kelvin thermometers agree.

17.5. **IDENTIFY:** Convert  $\Delta T$  in kelvins to  $C^{\circ}$  and to  $F^{\circ}$ .

**SET UP:** 
$$1 \text{ K} = 1 \text{ C}^{\circ} = \frac{9}{5} \text{ F}^{\circ}$$

**EXECUTE:** (a) 
$$\Delta T_{\rm F} = \frac{9}{5} \Delta T_{\rm C} = \frac{9}{5} (-10.0 \text{ C}^{\circ}) = -18.0 \text{ F}^{\circ}$$

**(b)** 
$$\Delta T_{\rm C} = \Delta T_{\rm K} = -10.0 \, \, {\rm C}^{\circ}$$

**EVALUATE:** Kelvin and Celsius degrees are the same size. Fahrenheit degrees are smaller, so it takes more of them to express a given  $\Delta T$  value.

**17.6. IDENTIFY:** Convert  $T_{\rm K}$  to  $T_{\rm C}$  and then convert  $T_{\rm C}$  to  $T_{\rm F}$ .

**SET UP:** 
$$T_{\rm K} = T_{\rm C} + 273.15$$
 and  $T_{\rm F} = \frac{9}{5}T_{\rm C} + 32^{\circ}$ .

**EXECUTE:** (a) 
$$T_C = 400 - 273.15 = 127^{\circ}\text{C}$$
,  $T_F = (9/5)(126.85) + 32 = 260^{\circ}\text{F}$ 

**(b)** 
$$T_{\rm C} = 95 - 273.15 = -178^{\circ}\text{C}, \ T_{\rm F} = (9/5)(-178.15) + 32 = -289^{\circ}\text{F}$$

(c) 
$$T_C = 1.55 \times 10^7 - 273.15 = 1.55 \times 10^{7} \text{ °C}, \ T_F = (9/5)(1.55 \times 10^7) + 32 = 2.79 \times 10^{7} \text{ °F}$$

**EVALUATE:** All temperatures on the Kelvin scale are positive.  $T_{\rm C}$  is negative if the temperature is below the freezing point of water.

17.7. **IDENTIFY:** When the volume is constant,  $\frac{T_2}{T_1} = \frac{p_2}{p_1}$ , for T in kelvins.

SET UP:  $T_{\text{triple}} = 273.16 \text{ K}$ . Figure 17.7 in the textbook gives that the temperature at which  $CO_2$  solidifies is  $T_{CO_2} = 195 \text{ K}$ .

EXECUTE: 
$$p_2 = p_1 \left( \frac{T_2}{T_1} \right) = (1.35 \text{ atm}) \left( \frac{195 \text{ K}}{273.16 \text{ K}} \right) = 0.964 \text{ atm}$$

**EVALUATE:** The pressure decreases when T decreases.

17.8. **IDENTIFY:** Apply  $T = T_{\text{triple}} \frac{p}{p_{\text{triple}}} = (273.16 \text{ K}) \frac{p}{p_{\text{triple}}}$  and solve for p.

**SET UP:**  $p_{\text{triple}} = 325 \text{ mm of mercury}$ 

EXECUTE: 
$$p = (325.0 \text{ mm of mercury}) \left( \frac{373.15 \text{ K}}{273.16 \text{ K}} \right) = 444 \text{ mm of mercury}$$

**EVALUATE:** mm of mercury is a unit of pressure. Since  $T = T_{\text{triple}} \frac{p}{p_{\text{triple}}} = (273.16 \text{ K}) \frac{p}{p_{\text{triple}}}$  involves a

ratio of pressures, it is not necessary to convert the pressure to units of Pa.

17.9. **IDENTIFY** and **SET UP**: Fit the data to a straight line for p(T) and use this equation to find T when p = 0.

**EXECUTE:** (a) If the pressure varies linearly with temperature, then  $p_2 = p_1 + \gamma (T_2 - T_1)$ .

$$\gamma = \frac{p_2 - p_1}{T_2 - T_1} = \frac{6.50 \times 10^4 \text{ Pa} - 4.80 \times 10^4 \text{ Pa}}{100^{\circ}\text{C} - 0.01^{\circ}\text{C}} = 170.0 \text{ Pa/C}^{\circ}$$

Apply  $p = p_1 + \gamma (T - T_1)$  with  $T_1 = 0.01$ °C and p = 0 to solve for T.

$$0 = p_1 + \gamma (T - T_1)$$

$$T = T_1 - \frac{p_1}{\gamma} = 0.01$$
°C  $-\frac{4.80 \times 10^4 \text{ Pa}}{170 \text{ Pa/C}^\circ} = -282$ °C.

**(b)** Let  $T_1 = 100$ °C and  $T_2 = 0.01$ °C; use  $T_2/T_1 = p_2/p_1$  to calculate  $p_2$ , where T is in kelvins.

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right) = 6.50 \times 10^4 \text{ Pa} \left(\frac{0.01 + 273.15}{100 + 273.15}\right) = 4.76 \times 10^4 \text{ Pa}$$
; this differs from the  $4.80 \times 10^4 \text{ Pa}$  that was

measured so  $T_2/T_1 = p_2/p_1$  is not precisely obeyed.

**EVALUATE:** The answer to part (a) is in reasonable agreement with the accepted value of  $-273^{\circ}$ C.

**17.10. IDENTIFY:**  $1 \text{ K} = 1 \text{ C}^{\circ}$  and  $1 \text{ C}^{\circ} = \frac{9}{5} \text{ F}^{\circ}$ , so  $1 \text{ K} = \frac{9}{5} \text{ R}^{\circ}$ .

**SET UP:** On the Kelvin scale, the triple point is 273.16 K.

**EXECUTE:**  $T_{\text{triple}} = (9/5)273.16 \text{ K} = 491.69^{\circ}\text{R}.$ 

**EVALUATE:** One could also look at Figure 17.7 in the textbook and note that the Fahrenheit scale extends from  $-460^{\circ}$ F to  $+32^{\circ}$ F and conclude that the triple point is about  $492^{\circ}$ R.

17.11. **IDENTIFY:**  $\Delta L = L_0 \alpha \Delta T$ 

**SET UP:** For steel,  $\alpha = 1.2 \times 10^{-5} (\text{C}^{\circ})^{-1}$ 

EXECUTE:  $\Delta L = (1.2 \times 10^{-5} (\text{C}^{\circ})^{-1})(1410 \text{ m})(18.0 \,^{\circ}\text{C} - (-5.0 \,^{\circ}\text{C})) = +0.39 \text{ m}$ 

**EVALUATE:** The length increases when the temperature increases. The fractional increase is very small, since  $\alpha\Delta T$  is small.

17.12. IDENTIFY: Apply  $\Delta L = \alpha L_0 \Delta T$  and calculate  $\Delta T$ . Then  $T_2 = T_1 + \Delta T$ , with  $T_1 = 15.5$ °C.

**SET UP:** Table 17.1 gives  $\alpha = 1.2 \times 10^{-5} (\text{C}^{\circ})^{-1}$  for steel.

EXECUTE:  $\Delta T = \frac{\Delta L}{\alpha L_0} = \frac{0.471 \text{ ft}}{[1.2 \times 10^{-5} \text{ (C}^\circ)^{-1}][1671 \text{ ft}]} = 23.5 \text{ C}^\circ. \quad T_2 = 15.5^\circ\text{C} + 23.5 \text{ C}^\circ = 39.0^\circ\text{C}.$ 

**EVALUATE:** Since then the lengths enter in the ratio  $\Delta L/L_0$ , we can leave the lengths in ft.

17.13. **IDENTIFY:** Apply  $L = L_0(1 + \alpha \Delta T)$  to the diameter D of the penny.

**SET UP:**  $1 \text{ K} = 1 \text{ C}^{\circ}$ , so we can use temperatures in  $^{\circ}\text{C}$ .

**EXECUTE:** Death Valley:  $\alpha D_0 \Delta T = [2.6 \times 10^{-5} (\text{C}^\circ)^{-1}](1.90 \text{ cm})(28.0 \text{ C}^\circ) = 1.4 \times 10^{-3} \text{ cm}$ , so the

diameter is 1.9014 cm. Greenland:  $\alpha D_0 \Delta T = -3.6 \times 10^{-3}$  cm, so the diameter is 1.8964 cm.

**EVALUATE:** When T increases the diameter increases and when T decreases the diameter decreases.

**17.14. IDENTIFY:** Apply  $L = L_0(1 + \alpha \Delta T)$  to the diameter *d* of the rivet.

**SET UP:** For aluminum,  $\alpha = 2.4 \times 10^{-5} \text{ (C}^{\circ}\text{)}^{-1}$ . Let  $d_0$  be the diameter at  $-78.0^{\circ}\text{C}$  and d be the diameter at  $23.0^{\circ}\text{C}$ .

EXECUTE:  $d = d_0 + \Delta d = d_0 (1 + \alpha \Delta T) = (0.4500 \text{ cm})(1 + (2.4 \times 10^{-5} (\text{C}^\circ)^{-1} [23.0^\circ \text{C} - (-78.0^\circ \text{C})]).$ 

d = 0.4511 cm = 4.511 mm.

**EVALUATE:** We could have let  $d_0$  be the diameter at 23.0°C and d be the diameter at -78.0°C. Then  $\Delta T = -78.0$ °C -23.0°C.

**17.15. IDENTIFY:** Apply  $\Delta V = V_0 \beta \Delta T$ .

**SET UP:** For copper,  $\beta = 5.1 \times 10^{-5} (\text{C}^{\circ})^{-1}$ .  $\Delta V/V_0 = 0.150 \times 10^{-2}$ .

EXECUTE:  $\Delta T = \frac{\Delta V/V_0}{\beta} = \frac{0.150 \times 10^{-2}}{5.1 \times 10^{-5} (\text{C}^{\circ})^{-1}} = 29.4 \text{ C}^{\circ}. \ T_f = T_i + \Delta T = 49.4 ^{\circ}\text{C}.$ 

**EVALUATE:** The volume increases when the temperature increases.

17.16. **IDENTIFY:**  $\Delta V = \beta V_0 \Delta T$ . Use the diameter at  $-15^{\circ}$ C to calculate the value of  $V_0$  at that temperature.

**SET UP:** For a hemisphere of radius R, the volume is  $V = \frac{2}{3}\pi R^3$ . Table 17.2 gives  $\beta = 7.2 \times 10^{-5} \text{ (C}^{\circ})^{-1}$  for aluminum.

**EXECUTE:**  $V_0 = \frac{2}{3}\pi R^3 = \frac{2}{3}\pi (27.5 \text{ m})^3 = 4.356 \times 10^4 \text{ m}^3.$ 

$$\Delta V = (7.2 \times 10^{-5} (\text{C}^{\circ})^{-1})(4.356 \times 10^{4} \text{ m}^{3})[35^{\circ}\text{C} - (-15^{\circ}\text{C})] = 160 \text{ m}^{3}.$$

**EVALUATE:** We could also calculate  $R = R_0(1 + \alpha \Delta T)$  and calculate the new V from R. The increase in volume is  $V - V_0$ , but we would have to be careful to avoid round-off errors when two large volumes of nearly the same size are subtracted.

17.17. **IDENTIFY:** Apply  $\Delta V = V_0 \beta \Delta T$  to the volume of the flask and to the mercury. When heated, both the volume of the flask and the volume of the mercury increase.

**SET UP:** For mercury,  $\beta_{\text{Hg}} = 18 \times 10^{-5} (\text{C}^{\circ})^{-1}$ .

8.95 cm<sup>3</sup> of mercury overflows, so  $\Delta V_{\rm Hg} - \Delta V_{\rm glass} = 8.95 \text{ cm}^3$ .

EXECUTE:  $\Delta V_{\text{Hg}} = V_0 \beta_{\text{Hg}} \Delta T = (1000.00 \text{ cm}^3)(18 \times 10^{-5} \text{ (C}^\circ)^{-1})(55.0 \text{ C}^\circ) = 9.9 \text{ cm}^3.$ 

$$\Delta V_{\text{glass}} = \Delta V_{\text{Hg}} - 8.95 \text{ cm}^3 = 0.95 \text{ cm}^3.$$
  $\beta_{\text{glass}} = \frac{\Delta V_{\text{glass}}}{V_0 \Delta T} = \frac{0.95 \text{ cm}^3}{(1000.00 \text{ cm}^3)(55.0 \text{ C}^\circ)} = 1.7 \times 10^{-5} \text{ (C}^\circ)^{-1}.$ 

**EVALUATE:** The coefficient of volume expansion for the mercury is larger than for glass. When they are heated, both the volume of the mercury and the inside volume of the flask increase. But the increase for the mercury is greater and it no longer all fits inside the flask.

17.18. **IDENTIFY:** Apply  $\Delta V = V_0 \beta \Delta T$  to the tank and to the ethanol.

**SET UP:** For ethanol,  $\beta_e = 75 \times 10^{-5} \text{ (C}^{\circ})^{-1}$ . For steel,  $\beta_s = 3.6 \times 10^{-5} \text{ (C}^{\circ})^{-1}$ .

**EXECUTE:** The volume change for the tank is

$$\Delta V_{\rm s} = V_0 \beta_{\rm s} \Delta T = (1.90 \text{ m}^3)(3.6 \times 10^{-5} (\text{C}^\circ)^{-1})(-14.0 \text{ C}^\circ) = -9.576 \times 10^{-4} \text{ m}^3 = -0.9576 \text{ L}.$$

The volume change for the ethanol is

$$\Delta V_e = V_0 \beta_e \Delta T = (1.90 \text{ m}^3)(75 \times 10^{-5} \text{ (C}^\circ)^{-1})(-14.0 \text{ C}^\circ) = -1.995 \times 10^{-2} \text{ m}^3 = -19.95 \text{ L}.$$

The empty volume in the tank is  $\Delta V_{\rm e} - \Delta V_{\rm s} = -19.95~{\rm L} - (-0.9576~{\rm L}) = -19.0~{\rm L}$ . So 19.0 L of ethanol can be added to the tank.

**EVALUATE:** Both volumes decrease. But  $\beta_e > \beta_s$ , so the magnitude of the volume decrease for the ethanol is greater than it is for the tank.

17.19. IDENTIFY and SET UP: Use  $\Delta A = 2\alpha A_0 \Delta T$  to calculate  $\Delta A$  for the plate, and then  $A = A_0 + \Delta A$ .

EXECUTE: **(a)** 
$$A_0 = \pi r_0^2 = \pi \left(\frac{1.350 \text{ cm}}{2}\right)^2 = 1.431 \text{ cm}^2.$$

**(b)** Using 
$$\Delta A = 2\alpha A_0 \Delta T$$
, we have  $\Delta A = 2(1.2 \times 10^{-5} \text{ C}^{\circ -1})(1.431 \text{ cm}^2)(175^{\circ}\text{C} - 25^{\circ}\text{C}) = 5.15 \times 10^{-3} \text{ cm}^2$   
 $A = A_0 + \Delta A = 1.436 \text{ cm}^2$ .

**EVALUATE:** A hole in a flat metal plate expands when the metal is heated just as a piece of metal the same size as the hole would expand.

17.20. IDENTIFY: Apply  $\Delta L = L_0 \alpha \Delta T$  to the diameter  $D_{ST}$  of the steel cylinder and the diameter  $D_{BR}$  of the brass piston.

**SET UP:** For brass,  $\alpha_{BR} = 2.0 \times 10^{-5} (C^{\circ})^{-1}$ . For steel,  $\alpha_{ST} = 1.2 \times 10^{-5} (C^{\circ})^{-1}$ .

**EXECUTE:** (a) No, the brass expands more than the steel.

**(b)** Call  $D_0$  the inside diameter of the steel cylinder at 20°C. At 150°C,  $D_{ST} = D_{BR}$ .

 $D_0 + \Delta D_{\rm ST} = 25.000 \text{ cm} + \Delta D_{\rm BR}$ . This gives  $D_0 + \alpha_{\rm ST} D_0 \Delta T = 25.000 \text{ cm} + \alpha_{\rm BR} (25.000 \text{ cm}) \Delta T$ .

$$D_0 = \frac{25.000 \text{ cm}(1 + \alpha_{\text{BR}} \Delta T)}{1 + \alpha_{\text{ST}} \Delta T} = \frac{(25.000 \text{ cm})[1 + (2.0 \times 10^{-5} \text{ (C}^{\circ})^{-1})(130 \text{ C}^{\circ})]}{1 + (1.2 \times 10^{-5} \text{ (C}^{\circ})^{-1})(130 \text{ C}^{\circ})} = 25.026 \text{ cm}.$$

**EVALUATE:** The space inside the steel cylinder expands just like a solid piece of steel of the same size.

**17.21. IDENTIFY:** Apply  $\Delta L = L_0 \alpha \Delta T$  and stress  $= F/A = -Y \alpha \Delta T$ .

**SET UP:** For steel,  $\alpha = 1.2 \times 10^{-5} (\text{C}^{\circ})^{-1}$  and  $Y = 2.0 \times 10^{11} \text{ Pa.}$ 

EXECUTE: (a)  $\Delta L = L_0 \alpha \Delta T = (12.0 \text{ m})(1.2 \times 10^{-5} \text{ (C}^{\circ})^{-1})(42.0 \text{ C}^{\circ}) = 0.0060 \text{ m} = 6.0 \text{ mm}.$ 

**(b)** stress =  $-Y\alpha\Delta T = -(2.0 \times 10^{11} \text{ Pa})(1.2 \times 10^{-5} \text{ (C}^{\circ})^{-1})(42.0 \text{ C}^{\circ}) = -1.0 \times 10^{8} \text{ Pa}$ . The minus sign means the stress is compressive.

**EVALUATE:** Commonly occurring temperature changes result in very small fractional changes in length but very large stresses if the length change is prevented from occurring.

17.22. **IDENTIFY:** Apply stress =  $F/A = -Y\alpha\Delta T$  and solve for F.

**SET UP:** For brass,  $Y = 0.9 \times 10^{11}$  Pa and  $\alpha = 2.0 \times 10^{-5}$  (C°)<sup>-1</sup>.

EXECUTE: 
$$F = -Y\alpha\Delta T A = -(0.9 \times 10^{11} \text{ Pa})(2.0 \times 10^{-5} (\text{C}^{\circ})^{-1})(-110 \text{ C}^{\circ})(2.01 \times 10^{-4} \text{ m}^2) = 4.0 \times 10^4 \text{ N}$$

**EVALUATE:** A large force is required.  $\Delta T$  is negative and a positive tensile force is required.

17.23. **IDENTIFY** and **SET UP**: Apply  $Q = mc\Delta T$  to the kettle and water.

EXECUTE: kettle

$$Q = mc\Delta T$$
,  $c = 910 \text{ J/kg} \cdot \text{K}$  (from Table 17.3)

$$Q = (1.10 \text{ kg})(910 \text{ J/kg} \cdot \text{K})(85.0^{\circ}\text{C} - 20.0^{\circ}\text{C}) = 6.5065 \times 10^{4} \text{ J}$$

water

$$Q = mc\Delta T$$
,  $c = 4190 \text{ J/kg} \cdot \text{K}$  (from Table 17.3)

$$Q = (1.80 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(85.0^{\circ}\text{C} - 20.0^{\circ}\text{C}) = 4.902 \times 10^{5} \text{ J}$$

Total 
$$Q = 6.5065 \times 10^4 \text{ J} + 4.902 \times 10^5 \text{ J} = 5.55 \times 10^5 \text{ J}.$$

**EVALUATE:** Water has a much larger specific heat capacity than aluminum, so most of the heat goes into raising the temperature of the water.

17.24. IDENTIFY: The heat required is  $Q = mc\Delta T$ . P = 200 W = 200 J/s, which is energy divided by time.

**SET UP:** For water,  $c = 4.19 \times 10^3$  J/kg·K.

EXECUTE: (a) 
$$Q = mc\Delta T = (0.320 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(60.0 \text{ C}^\circ) = 8.04 \times 10^4 \text{ J}$$

**(b)** 
$$t = \frac{8.04 \times 10^4 \text{ J}}{200.0 \text{ J/s}} = 402 \text{ s} = 6.7 \text{ min}$$

**EVALUATE:** 0.320 kg of water has volume 0.320 L. The time we calculated in part (b) is consistent with our everyday experience.

17.25. IDENTIFY: Apply  $Q = mc\Delta T$  to find the heat that would raise the temperature of the student's body 7 C°.

**SET UP:** 1 W = 1 J/s

**EXECUTE:** Find Q to raise the body temperature from 37°C to 44°C.

$$Q = mc\Delta T = (70 \text{ kg})(3480 \text{ J/kg} \cdot \text{K})(7 \text{ C}^{\circ}) = 1.7 \times 10^6 \text{ J}.$$

$$t = \frac{1.7 \times 10^6 \text{ J}}{1200 \text{ J/s}} = 1400 \text{ s} = 23 \text{ min.}$$

**EVALUATE:** Heat removal mechanisms are essential to the well-being of a person.

**17.26. IDENTIFY** and **SET UP:** Use  $Q = mc \Delta T$ .

**EXECUTE:** (a)  $Q = mc\Delta T$ 

$$m = \frac{1}{2}(1.3 \times 10^{-3} \text{ kg}) = 0.65 \times 10^{-3} \text{ kg}$$

$$Q = (0.65 \times 10^{-3} \text{ kg})(1020 \text{ J/kg} \cdot \text{K})(37^{\circ}\text{C} - (-20^{\circ}\text{C})) = 38 \text{ J}.$$

**(b)** 20 breaths/min (60 min/1 h) = 1200 breaths/h

So 
$$Q = (1200)(38 \text{ J}) = 4.6 \times 10^4 \text{ J}.$$

**EVALUATE:** The heat loss rate is Q/t = 13 W.

17.27. **IDENTIFY:** Apply  $Q = mc\Delta T$ . m = w/g.

**SET UP:** The temperature change is  $\Delta T = 18.0 \text{ K}$ .

EXECUTE: 
$$c = \frac{Q}{m\Delta T} = \frac{gQ}{w\Delta T} = \frac{(9.80 \text{ m/s}^2)(1.25 \times 10^4 \text{ J})}{(28.4 \text{ N})(18.0 \text{ K})} = 240 \text{ J/kg} \cdot \text{K}.$$

**EVALUATE:** The value for c is similar to that for silver in Table 17.3, so it is a reasonable result.

**17.28. IDENTIFY:** The heat input increases the temperature of 2.5 gal/min of water from 10°C to 49°C. **SET UP:** 1.00 L of water has a mass of 1.00 kg, so

9.46 L/min = (9.46 L/min)(1.00 kg/L)(1 min/60 s) = 0.158 kg/s. For water,  $c = 4190 \text{ J/kg} \cdot \text{C}^{\circ}$ .

**EXECUTE:**  $Q = mc\Delta T$  so  $H = (Q/t) = (m/t)c\Delta T$ . Putting in the numbers gives

 $H = (0.158 \text{ kg/s})(4190 \text{ J/kg} \cdot \text{C}^{\circ})(49^{\circ}\text{C} - 10^{\circ}\text{C}) = 2.6 \times 10^{4} \text{ W} = 26 \text{ kW}.$ 

**EVALUATE:** The power requirement is large, the equivalent of 260 100-watt light bulbs, but this large power is needed only for short periods of time. The rest of the time, the unit uses no energy, unlike a conventional water heater, which continues to replace lost heat even when hot water is not needed.

**17.29. IDENTIFY** and **SET UP:** Set the change in gravitational potential energy equal to the quantity of heat added to the water.

EXECUTE: The change in mechanical energy equals the decrease in gravitational potential energy,

 $\Delta U = -mgh$ ;  $|\Delta U| = mgh$ .  $Q = |\Delta U| = mgh$  implies  $mc\Delta T = mgh$ 

 $\Delta T = gh/c = (9.80 \text{ m/s}^2)(225 \text{ m})/(4190 \text{ J/kg} \cdot \text{K}) = 0.526 \text{ K} = 0.526 \text{ C}^{\circ}$ 

**EVALUATE:** Note that the answer is independent of the mass of the object. Note also the small change in temperature that corresponds to this large change in height!

17.30. **IDENTIFY:** The work done by the brakes equals the initial kinetic energy of the train. Use the volume of the air to calculate its mass. Use  $Q = mc\Delta T$  applied to the air to calculate  $\Delta T$  for the air.

**SET UP:**  $K = \frac{1}{2}mv^2$ .  $m = \rho V$ .

**EXECUTE:** The initial kinetic energy of the train is  $K = \frac{1}{2}(25,000 \text{ kg})(15.5 \text{ m/s})^2 = 3.00 \times 10^6 \text{ J}.$ 

Therefore, Q for the air is  $3.00 \times 10^6$  J.  $m = \rho V = (1.20 \text{ kg/m}^3)(65.0 \text{ m})(20.0 \text{ m})(12.0 \text{ m}) = 1.87 \times 10^4 \text{ kg}.$ 

$$Q = mc\Delta T$$
 gives  $\Delta T = \frac{Q}{mc} = \frac{3.00 \times 10^6 \text{ J}}{(1.87 \times 10^4 \text{ kg})(1020 \text{ J/kg} \cdot \text{K})} = 0.157 \text{ C}^{\circ}.$ 

**EVALUATE:** The mass of air in the station is comparable to the mass of the train and the temperature rise is small.

17.31. IDENTIFY: Set the energy delivered to the nail equal to  $Q = mc\Delta T$  for the nail and solve for  $\Delta T$ .

**SET UP:** For aluminum,  $c = 0.91 \times 10^3$  J/kg·K.  $K = \frac{1}{2}mv^2$ .

**EXECUTE:** The kinetic energy of the hammer before it strikes the nail is

 $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.80 \text{ kg})(7.80 \text{ m/s})^2 = 54.8 \text{ J}$ . Each strike of the hammer transfers 0.60(54.8 J) = 32.9 J,

and with 10 strikes 
$$Q = 329 \text{ J}$$
.  $Q = mc\Delta T$  and  $\Delta T = \frac{Q}{mc} = \frac{329 \text{ J}}{(8.00 \times 10^{-3} \text{ kg})(0.91 \times 10^{3} \text{ J/kg} \cdot \text{K})} = 45.2 \text{ C}^{\circ}$ .

**EVALUATE:** This agrees with our experience that hammered nails get noticeably warmer.

17.32. IDENTIFY and SET UP: Use the power and time to calculate the heat input Q and then use  $Q = mc\Delta T$  to calculate c.

(a) EXECUTE: P = Q/t, so the total heat transferred to the liquid is Q = Pt = (65.0 W)(120 s) = 7800 J.

Then 
$$Q = mc\Delta T$$
 gives  $c = \frac{Q}{m\Delta T} = \frac{7800 \text{ K}}{0.780 \text{ kg}(22.54^{\circ}\text{C} - 18.55^{\circ}\text{C})} = 2.51 \times 10^{3} \text{ J/kg} \cdot \text{K}.$ 

**(b) EVALUATE:** Then the actual Q transferred to the liquid is less than 7800 J so the actual c is less than our calculated value; our result in part (a) is an overestimate.

**17.33. IDENTIFY:** Some of the kinetic energy of the bullet is transformed through friction into heat, which raises the temperature of the water in the tank.

**SET UP:** Set the loss of kinetic energy of the bullet equal to the heat energy Q transferred to the water.

 $Q = mc\Delta T$ . From Table 17.3, the specific heat of water is  $4.19 \times 10^3$  J/kg·C°.

**EXECUTE:** The kinetic energy lost by the bullet is

 $K_i - K_f = \frac{1}{2}m(v_i^2 - v_f^2) = \frac{1}{2}(15.0 \times 10^{-3} \text{ kg})[(865 \text{ m/s})^2 - (534 \text{ m/s})^2] = 3.47 \times 10^3 \text{ J}$ , so for the water

$$Q = 3.47 \times 10^3 \,\text{J}.$$
  $Q = mc\Delta T$  gives  $\Delta T = \frac{Q}{mc} = \frac{3.47 \times 10^3 \,\text{J}}{(13.5 \,\text{kg})(4.19 \times 10^3 \,\text{J/kg} \cdot \text{C}^\circ)} = 0.0613 \,\text{C}^\circ.$ 

**EVALUATE:** The heat energy required to change the temperature of ordinary-size objects is very large compared to the typical kinetic energies of moving objects.

**17.34. IDENTIFY:** The amount of heat lost by the boiling water is equal to the amount of heat gained by the water in the beaker.

SET UP: Calculate Q for each mass of water and set their algebraic sum equal to zero. Let the water you add have mass m. The 750 g of cold water has a temperature change of +65 C° and a heat flow  $Q_c$ . The mass m of water has a temperature change of -25 C° and a heat flow  $Q_h$ .  $Q = mc \Delta T$ .

EXECUTE: 
$$Q_c = mc \Delta T = (0.750 \text{ kg})(4190 \text{ J/kg} \cdot \text{C}^{\circ})(65 \text{ C}^{\circ}) = 2.043 \times 10^5 \text{ J}$$

$$Q_{\rm h} = mc \, \Delta T = m(4190 \, \text{J/kg} \cdot \text{C}^{\circ})(-25 \, \text{C}^{\circ}) = -(1.048 \times 10^5 \, \text{J/kg})m$$

$$Q_h + Q_c = 0$$
 so  $2.043 \times 10^5 \text{ J} + (-1.048 \times 10^5 \text{ J/kg}) m = 0$  and  $m = 1.95 \text{ kg} = 1950 \text{ g}$ .

**EVALUATE:** The amount of water we need to add (1950 g) is considerably greater than the water already in the beaker (750 g). This is reasonable because we want the final temperature to be closer to the 100°C temperature of the boiling water than to the original 10.0°C temperature of the original water.

- 17.35. IDENTIFY and SET UP: Heat comes out of the metal and into the water. The final temperature is in the range  $0 < T < 100^{\circ}\text{C}$ , so there are no phase changes.  $Q_{\text{system}} = 0$ .
  - (a) **EXECUTE:**  $Q_{\text{water}} + Q_{\text{metal}} = 0$

$$m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} + m_{\text{metal}} c_{\text{metal}} \Delta T_{\text{metal}} = 0$$

$$(1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(2.0 \text{ C}^{\circ}) + (0.500 \text{ kg})(c_{\text{metal}})(-78.0 \text{ C}^{\circ}) = 0$$

 $c_{\text{metal}} = 215 \text{ J/kg} \cdot \text{K}$ 

- **(b) EVALUATE:** Water has a larger specific heat capacity so stores more heat per degree of temperature change.
- (c) If some heat went into the styrofoam then  $Q_{\rm metal}$  should actually be larger than in part (a), so the true  $c_{\rm metal}$  is larger than we calculated; the value we calculated would be smaller than the true value.
- **17.36. IDENTIFY:** The heat that comes out of the person goes into the ice-water bath and causes some of the ice to melt.

SET UP: Normal body temperature is  $98.6^{\circ}\text{F} = 37.0^{\circ}\text{C}$ , so for the person  $\Delta T = -5 \text{ C}^{\circ}$ . The ice-water bath stays at  $0^{\circ}\text{C}$ . A mass m of ice melts and  $Q_{\text{ice}} = mL_{\text{f}}$ . From Table 17.4, for water  $L_{\text{f}} = 334 \times 10^3 \text{ J/kg}$ .

**EXECUTE:**  $Q_{\text{person}} = mc\Delta T = (70.0 \text{ kg})(3480 \text{ J/kg} \cdot \text{C}^{\circ})(-5.0 \text{ C}^{\circ}) = -1.22 \times 10^{6} \text{ J.}$  Therefore, the amount of

heat that goes into the ice is 
$$1.22 \times 10^6$$
 J.  $m_{\text{ice}} L_{\text{f}} = 1.22 \times 10^6$  J and  $m_{\text{ice}} = \frac{1.22 \times 10^6 \text{ J}}{334 \times 10^3 \text{ J/kg}} = 3.7 \text{ kg}.$ 

**EVALUATE:** If less ice than this is used, all the ice melts and the temperature of the water in the bath rises above 0°C.

**17.37. IDENTIFY:** The amount of heat lost by the iron is equal to the amount of heat gained by the water. The water must first be heated to 100°C and then vaporized.

**SET UP:** The relevant equations are  $Q = mc\Delta T$  and  $Q = L_v m$ . The specific heat of iron is

 $c_{\text{iron}} = 0.47 \times 10^3 \text{ J/(kg} \cdot \text{K)}$ , the specific heat of water is  $c_{\text{water}} = 4.19 \times 10^3 \text{ J/(kg} \cdot \text{K)}$ , and the heat of vaporization of water is  $L_v = 2256 \times 10^3 \text{ J/kg}$ .

**EXECUTE:** The iron cools:  $Q_{\text{iron}} = m_i c_i \Delta T_i$ .

The water warms and vaporizes: 
$$Q_{\text{water}} = c_{\text{w}} m_{\text{w}} \Delta T_{\text{w}} + m_{\text{w}} L_{\text{vw}} = m_{\text{w}} (c_{\text{w}} \Delta T_{\text{w}} + L_{\text{vw}})$$
.

Assume that all of the heat lost by the iron is gained by the water so that  $Q_{\text{water}} = -Q_{\text{iron}}$ . Equating the respective expressions for each Q and solving for  $m_{\text{w}}$  we obtain

$$m_{\rm w} = \frac{-m_{\rm i}c_{\rm i}\Delta T_{\rm i}}{c_{\rm w}\Delta T_{\rm w} + L_{\rm v_{\rm w}}} = \frac{-(1.20 \text{ kg})(0.47 \times 10^3 \text{ J/kg} \cdot \text{K})(120.0^{\circ}\text{C} - 650.0^{\circ}\text{C})}{(4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(100.0^{\circ}\text{C} - 15.0^{\circ}\text{C}) + 2256 \times 10^3 \text{ J/kg}} = 0.114 \text{ kg}.$$

**EVALUATE:** Note that only a relatively small amount of water is required to cause a very large temperature change in the iron. This is due to the high heat of vaporization and specific heat of water, and the relatively low specific heat capacity of iron.

17.38. **IDENTIFY:** The initial temperature of the ice and water mixture is  $0.0^{\circ}$ C. Assume all the ice melts. We will know that assumption is incorrect if the final temperature we calculate is less than  $0.0^{\circ}$ C. The net Q for the system (can, water, ice and lead) is zero.

SET UP: For copper,  $c_c = 390 \text{ J/kg} \cdot \text{K}$ . For lead,  $c_1 = 130 \text{ J/kg} \cdot \text{K}$ . For water,  $c_w = 4.19 \times 10^3 \text{ J/kg} \cdot \text{K}$  and  $L_f = 3.34 \times 10^5 \text{ J/kg}$ .

**EXECUTE:** For the copper can,  $Q_c = m_c c_c \Delta T_c = (0.100 \text{ kg})(390 \text{ J/kg} \cdot \text{K})(T - 0.0^{\circ}\text{C}) = (39.0 \text{ J/K})T$ .

For the water,  $Q_{\rm w} = m_{\rm w} c_{\rm w} \Delta T_{\rm w} = (0.160 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(T - 0.0^{\circ}\text{C}) = (670.4 \text{ J/K})T$ .

For the ice,  $Q_i = m_i L_f + m_i c_w \Delta T_w$ 

 $Q_i = (0.018 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) + (0.018 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(T - 0.0^{\circ}\text{C}) = 6012 \text{ J} + (75.4 \text{ J/K})T$ 

For the lead,  $Q_1 = m_1 c_1 \Delta T_1 = (0.750 \text{ kg})(130 \text{ J/kg} \cdot \text{K})(T - 255^{\circ}\text{C}) = (97.5 \text{ J/K})T - 2.486 \times 10^4 \text{ J}$ 

 $\Sigma Q = 0$  gives  $(39.0 \text{ J/K})T + (670.4 \text{ J/K})T + 6012 \text{ J} + (75.4 \text{ J/K})T + (97.5 \text{ J/K})T - 2.486 \times 10^4 \text{ J} = 0.$ 

$$T = \frac{1.885 \times 10^4 \text{ J}}{882.3 \text{ J/K}} = 21.4 \text{°C}.$$

EVALUATE: T > 0.0°C, which confirms that all the ice melts.

**17.39. IDENTIFY:** The heat lost by the cooling copper is absorbed by the water and the pot, which increases their temperatures.

**SET UP:** For copper,  $c_c = 390 \text{ J/kg} \cdot \text{K}$ . For iron,  $c_i = 470 \text{ J/kg} \cdot \text{K}$ . For water,  $c_w = 4.19 \times 10^3 \text{ J/kg} \cdot \text{K}$ .

**EXECUTE:** For the copper pot,

 $Q_c = m_c c_c \Delta T_c = (0.500 \text{ kg})(390 \text{ J/kg} \cdot \text{K})(T - 20.0^{\circ}\text{C}) = (195 \text{ J/K})T - 3900 \text{ J}$ . For the block of iron,

 $Q_i = m_i c_i \Delta T_i = (0.250 \text{ kg})(470 \text{ J/kg} \cdot \text{K})(T - 85.0^{\circ}\text{C}) = (117.5 \text{ J/K})T - 9988 \text{ J. For the water,}$ 

 $Q_{\rm w} = m_{\rm w} c_{\rm w} \Delta T_{\rm w} = (0.170 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(T - 20.0^{\circ}\text{C}) = (712.3 \text{ J/K})T - 1.425 \times 10^{4} \text{ J}. \quad \Sigma Q = 0 \text{ gives}$ 

$$(195 \text{ J/K})T - 3900 \text{ J} + (117.5 \text{ J/K})T - 9988 \text{ J} + (712.3 \text{ J/K})T - 1.425 \times 10^4 \text{ J}. \quad T = \frac{2.814 \times 10^4 \text{ J}}{1025 \text{ J/K}} = 27.5^{\circ}\text{C}.$$

**EVALUATE:** The basic principle behind this problem is conservation of energy: no energy is lost; it is only transferred.

17.40. **IDENTIFY:** By energy conservation, the heat lost by the water is gained by the ice. This heat must first increase the temperature of the ice from  $-40.0^{\circ}$ C to the melting point of  $0.00^{\circ}$ C, then melt the ice, and finally increase its temperature to  $28.0^{\circ}$ C. The target variable is the mass of the water m.

**SET UP:**  $Q_{\text{ice}} = m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} + m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{w}} \Delta T_{\text{melted ice}}$  and  $Q_{\text{water}} = m c_{\text{w}} \Delta T_{\text{w}}$ .

**EXECUTE:** Using  $Q_{\text{ice}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{w}}\Delta T_{\text{melted ice}}$ , with the values given in the table in

the text, we have  $Q_{\text{ice}} = (0.200 \text{ kg})[2100 \text{ J/(kg} \cdot \text{C}^{\circ})](40.0 \text{C}^{\circ}) + (0.200 \text{ kg})(3.34 \times 10^5 \text{ J/kg})$ 

$$+(0.200 \text{ kg})[4190 \text{ J/(kg} \cdot \text{C}^{\circ})](28.0\text{C}^{\circ}) = 1.071 \times 10^{5} \text{ J}.$$

 $Q_{\text{water}} = mc_{\text{w}}\Delta T_{\text{w}} = m[4190 \text{ J/(kg} \cdot \text{C}^{\circ})](28.0^{\circ}\text{C} - 80.0^{\circ}\text{C}) = -(217,880 \text{ J/kg})m.$   $Q_{\text{ice}} + Q_{\text{water}} = 0 \text{ gives}$ 

 $1.071 \times 10^5 \text{ J} = (217,880 \text{ J/kg})m.$  m = 0.491 kg.

**EVALUATE:** There is about twice as much water as ice because the water must provide the heat not only to melt the ice but also to increase its temperature.

17.41. **IDENTIFY:** By energy conservation, the heat lost by the copper is gained by the ice. This heat must first increase the temperature of the ice from -20.0°C to the melting point of 0.00°C, then melt some of the ice. At the final thermal equilibrium state, there is ice and water, so the temperature must be 0.00°C. The target variable is the initial temperature of the copper.

**SET UP:** For temperature changes,  $Q = mc\Delta T$  and for a phase change from solid to liquid  $Q = mL_{\rm F}$ .

**EXECUTE:** For the ice.

 $Q_{\text{ice}} = (2.00 \text{ kg})[2100 \text{ J/(kg} \cdot \text{C}^\circ)](20.0 \text{C}^\circ) + (0.80 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) = 3.512 \times 10^5 \text{ J}$ . For the copper, using the specific heat from the table in the text gives

 $Q_{\text{copper}} = (6.00 \text{ kg})[390 \text{ J/(kg} \cdot \text{C}^\circ)](0^\circ\text{C} - T) = -(2.34 \times 10^3 \text{ J/C}^\circ)T$ . Setting the sum of the two heats equal to zero gives  $3.512 \times 10^5 \text{ J} = (2.34 \times 10^3 \text{ J/C}^\circ)T$ , which gives  $T = 150^\circ\text{C}$ .

**EVALUATE:** Since the copper has a smaller specific heat than that of ice, it must have been quite hot initially to provide the amount of heat needed.

17.42. IDENTIFY: Apply  $Q = mc\Delta T$  to each object. The net heat flow  $Q_{\text{system}}$  for the system (man, soft drink) is zero.

**SET UP:** The mass of 1.00 L of water is 1.00 kg. Let the man be designated by the subscript m and the "water" by w. *T* is the final equilibrium temperature.  $c_{\rm w} = 4190 \, \text{J/kg} \cdot \text{K}$ .  $\Delta T_{\rm K} = \Delta T_{\rm C}$ .

EXECUTE: (a)  $Q_{\text{system}} = 0$  gives  $m_{\text{m}}c_{\text{m}}\Delta T_{\text{m}} + m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}} = 0$ .  $m_{\text{m}}c_{\text{m}}(T - T_{\text{m}}) + m_{\text{w}}c_{\text{w}}(T - T_{\text{w}}) = 0$ .

$$m_{\mathrm{m}}c_{\mathrm{m}}(T_{\mathrm{m}}-T)=m_{\mathrm{w}}c_{\mathrm{w}}(T-T_{\mathrm{w}}). \text{ Solving for } T,\ T=\frac{m_{\mathrm{m}}c_{\mathrm{m}}T_{\mathrm{m}}+m_{\mathrm{w}}c_{\mathrm{w}}T_{\mathrm{w}}}{m_{\mathrm{m}}c_{\mathrm{m}}+m_{\mathrm{w}}c_{\mathrm{w}}}.$$

$$T = \frac{(70.0 \text{ kg})(3480 \text{ J/kg} \cdot \text{K})(37.0^{\circ}\text{C}) + (0.355 \text{ kg})(4190 \text{ J/kg} \cdot \text{C}^{\circ})(12.0^{\circ}\text{C})}{(70.0 \text{ kg})(3480 \text{ J/kg} \cdot \text{C}^{\circ}) + (0.355 \text{ kg})(4190 \text{ J/kg} \cdot \text{C}^{\circ})} = 36.85^{\circ}\text{C}$$

**(b)** The change in his body temperature is  $37.0^{\circ}\text{C} - 36.85^{\circ}\text{C} = 0.15 \text{ C}^{\circ}$ . It is possible a sensitive digital thermometer could measure this change since they can read to  $0.1^{\circ}\text{C}$ . It is best to refrain from drinking cold fluids prior to orally measuring a body temperature due to cooling of the mouth.

**EVALUATE:** Heat comes out of the body and its temperature falls. Heat goes into the soft drink and its temperature rises.

17.43. **IDENTIFY:** For the man's body,  $Q = mc\Delta T$ .

**SET UP:** From Exercise 17.42,  $\Delta T = 0.15 \, \text{C}^{\circ}$  when the body returns to 37.0°C.

**EXECUTE:** The rate of heat loss is  $7.00 \times 10^6$  J/day :  $7.00 \times 10^6$  J/day =  $\frac{mc\Delta T}{t}$  which gives

$$t = \frac{mc\Delta T}{7.00 \times 10^6 \text{ J/day}}. \quad t = \frac{(70.355 \text{ kg})(3480 \text{ J/kg} \cdot \text{C}^\circ)(0.15 \text{ C}^\circ)}{7.00 \times 10^6 \text{ J/day}} = 0.00525 \text{ d} = 7.6 \text{ min.}$$

**EVALUATE:** Even if all the BMR energy stays in the body, it takes the body several minutes to return to its normal temperature.

17.44. IDENTIFY: For a temperature change  $Q = mc\Delta T$  and for the liquid to solid phase change  $Q = -mL_{\rm f}$ .

SET UP: For water,  $c = 4.19 \times 10^3$  J/kg·K and  $L_f = 3.34 \times 10^5$  J/kg.

**EXECUTE:**  $Q = mc\Delta T - mL_{\rm f} = (0.290 \text{ kg})[(4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(-18.0 \text{ C}^{\circ}) - 3.34 \times 10^5 \text{ J/kg}] = -1.187 \times 10^5 \text{ J}$ , which rounds to  $-1.19 \times 10^5 \text{ J}$ .

The minus sign says  $1.19 \times 10^5$  J must be removed from the water.

$$(1.187 \times 10^5 \text{ J}) \left( \frac{1 \text{ cal}}{4.186 \text{ J}} \right) = 2.84 \times 10^4 \text{ cal} = 28.4 \text{ kcal.}$$

$$(1.187 \times 10^5 \text{ J}) \left( \frac{1 \text{ Btu}}{1055 \text{ J}} \right) = 113 \text{ Btu}.$$

**EVALUATE:** Q < 0 when heat comes out of an object. The equation  $Q = mc\Delta T$  puts in the correct sign automatically, from the sign of  $\Delta T = T_f - T_i$ . But in  $Q = \pm mL$  we must select the correct sign.

17.45. IDENTIFY and SET UP: Use  $Q = mc\Delta T$  for the temperature changes and Q = mL for the phase changes.

**EXECUTE:** Heat must be added to do the following:

ice at  $-10.0^{\circ}\text{C} \rightarrow \text{ice}$  at  $0^{\circ}\text{C}$ 

$$Q_{\text{ice}} = mc_{\text{ice}}\Delta T = (18.0 \times 10^{-3} \text{ kg})(2100 \text{ J/kg} \cdot \text{K})(0^{\circ}\text{C} - (-10.0^{\circ}\text{C})) = 378 \text{ J}$$

phase transition ice  $(0^{\circ}C) \rightarrow \text{liquid water } (0^{\circ}C) \text{(melting)}$ 

$$Q_{\text{melt}} = +mL_{\text{f}} = (18.0 \times 10^{-3} \text{ kg})(334 \times 10^{3} \text{ J/kg}) = 6.012 \times 10^{3} \text{ J}$$

water at  $0^{\circ}$ C (from melted ice)  $\rightarrow$  water at  $100^{\circ}$ C

$$Q_{\text{water}} = mc_{\text{water}} \Delta T = (18.0 \times 10^{-3} \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(100^{\circ}\text{C} - 0^{\circ}\text{C}) = 7.542 \times 10^{3} \text{ J}$$

phase transition water  $(100^{\circ}\text{C}) \rightarrow \text{steam} (100^{\circ}\text{C})(\text{boiling})$ 

$$Q_{\text{boil}} = +mL_{\text{v}} = (18.0 \times 10^{-3} \text{ kg})(2256 \times 10^{3} \text{ J/kg}) = 4.0608 \times 10^{4} \text{ J}$$

The total Q is 
$$Q = 378 \text{ J} + 6.012 \times 10^3 \text{ J} + 7.542 \times 10^3 \text{ J} + 4.068 \times 10^4 \text{ J} = 5.45 \times 10^4 \text{ J}$$

$$(5.45 \times 10^4 \text{ J})(1 \text{ cal}/4.186 \text{ J}) = 1.30 \times 10^4 \text{ cal}$$

$$(5.45 \times 10^4 \text{ J})(1 \text{ Btu}/1055 \text{ J}) = 51.7 \text{ Btu}.$$

**EVALUATE:** Q is positive and heat must be added to the material. Note that more heat is needed for the liquid to gas phase change than for the temperature changes.

17.46. **IDENTIFY:**  $Q = mc\Delta T$  for a temperature change and  $Q = +mL_f$  for the solid to liquid phase transition. The ice starts to melt when its temperature reaches 0.0°C. The system stays at 0.00°C until all the ice has melted.

SET UP: For ice, 
$$c = 2.10 \times 10^3$$
 J/kg·K. For water,  $L_f = 3.34 \times 10^5$  J/kg.

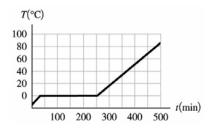
**EXECUTE:** (a) Q to raise the temperature of ice to  $0.00^{\circ}$ C:

$$Q = mc\Delta T = (0.550 \text{ kg})(2.10 \times 10^3 \text{ J/kg} \cdot \text{K})(15.0 \text{ C}^\circ) = 1.73 \times 10^4 \text{ J}. \quad t = \frac{1.73 \times 10^4 \text{ J}}{800.0 \text{ J/min}} = 21.7 \text{ min}.$$

**(b)** To melt all the ice requires 
$$Q = mL_f = (0.550 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) = 1.84 \times 10^5 \text{ J}.$$

$$t = \frac{1.84 \times 10^5 \text{ J}}{800.0 \text{ J/min}} = 230 \text{ min.}$$
 The total time after the start of the heating is 252 min.

EVALUATE: It takes much longer for the ice to melt than it takes the ice to reach the melting point.



**Figure 17.46** 

17.47. IDENTIFY and SET UP: The heat that must be added to a lead bullet of mass m to melt it is  $Q = mc\Delta T + mL_f$  ( $mc\Delta T$  is the heat required to raise the temperature from 25°C to the melting point of 327.3°C;  $mL_f$  is the heat required to make the solid  $\rightarrow$  liquid phase change.) The kinetic energy of the bullet if its speed is v is  $K = \frac{1}{2}mv^2$ .

**EXECUTE:** 
$$K = Q$$
 says  $\frac{1}{2}mv^2 = mc\Delta T + mL_f$ 

$$v = \sqrt{2(c\Delta T + L_{\rm f})}$$

$$v = \sqrt{2[(130 \text{ J/kg} \cdot \text{K})(327.3^{\circ}\text{C} - 25^{\circ}\text{C}) + 24.5 \times 10^{3} \text{ J/kg}]} = 357 \text{ m/s}$$

**EVALUATE:** This is a typical speed for a rifle bullet. A bullet fired into a block of wood does partially melt, but in practice not all of the initial kinetic energy is converted to heat that remains in the bullet.

17.48. IDENTIFY: For a temperature change,  $Q = mc\Delta T$ . For the vapor  $\rightarrow$  liquid phase transition,  $Q = -mL_v$ .

**SET UP:** For water, 
$$L_v = 2.256 \times 10^6$$
 J/kg and  $c = 4.19 \times 10^3$  J/kg·K.

**EXECUTE:** (a) 
$$Q = +m(-L_v + c\Delta T)$$

$$Q = +(25.0 \times 10^{-3} \text{ kg})(-2.256 \times 10^{6} \text{ J/kg} + [4.19 \times 10^{3} \text{ J/kg} \cdot \text{K}][-66.0 \text{ C}^{\circ}]) = -6.33 \times 10^{4} \text{ J/kg}$$

**(b)** 
$$Q = mc\Delta T = (25.0 \times 10^{-3} \text{ kg})(4.19 \times 10^{3} \text{ J/kg} \cdot \text{K})(-66.0 \text{ C}^{\circ}) = -6.91 \times 10^{3} \text{ J}.$$

(c) The total heat released by the water that starts as steam is nearly a factor of ten larger than the heat released by water that starts at 100°C. Steam burns are much more severe than hot-water burns.

**EVALUATE:** For a given amount of material, the heat for a phase change is typically much more than the heat for a temperature change.

17.49. **IDENTIFY:** Use  $Q = Mc\Delta T$  to find Q for a temperature rise from 34.0°C to 40.0°C. Set this equal to  $Q = mL_v$  and solve for m, where m is the mass of water the camel would have to drink.

**SET UP:**  $c = 3480 \text{ J/kg} \cdot \text{K}$  and  $L_v = 2.42 \times 10^6 \text{ J/kg}$ . For water, 1.00 kg has a volume 1.00 L. M = 400 kg is the mass of the camel.

**EXECUTE:** The mass of water that the camel saves is

$$m = \frac{Mc\Delta T}{L_{\rm v}} = \frac{(400 \text{ kg})(3480 \text{ J/kg} \cdot \text{K})(6.0 \text{ K})}{(2.42 \times 10^6 \text{ J/kg})} = 3.45 \text{ kg} \text{ which is a volume of } 3.45 \text{ L}.$$

**EVALUATE:** This is nearly a gallon of water, so it is an appreciable savings.

17.50. IDENTIFY: For a temperature change,  $Q = mc\Delta T$ . For the liquid  $\rightarrow$  vapor phase change,  $Q = +mL_v$ .

**SET UP:** The density of water is 1000 kg/m<sup>3</sup>.

**EXECUTE:** (a) The heat that goes into mass m of water to evaporate it is  $Q = +mL_v$ . The heat flow for the man is  $Q = m_{\text{man}} c \Delta T$ , where  $\Delta T = -1.00 \text{ C}^{\circ}$ .  $\Sigma Q = 0$  so  $mL_v + m_{\text{man}} c \Delta T = 0$  and

$$m = -\frac{m_{\text{man}}c\Delta T}{L_{\text{v}}} = -\frac{(70.0 \text{ kg})(3480 \text{ J/kg} \cdot \text{K})(-1.00 \text{ C}^{\circ})}{2.42 \times 10^{6} \text{ J/kg}} = 0.101 \text{ kg} = 101 \text{ g}.$$

**(b)**  $V = \frac{m}{\rho} = \frac{0.101 \text{ kg}}{1000 \text{ kg/m}^3} = 1.01 \times 10^{-4} \text{ m}^3 = 101 \text{ cm}^3$ . This is about 35% of the volume of a soft-drink can.

**EVALUATE:** Fluid loss by evaporation from the skin can be significant.

17.51. **IDENTIFY:** The asteroid's kinetic energy is  $K = \frac{1}{2}mv^2$ . To boil the water, its temperature must be raised to 100.0°C and the heat needed for the phase change must be added to the water.

**SET UP:** For water,  $c = 4190 \text{ J/kg} \cdot \text{K}$  and  $L_v = 2256 \times 10^3 \text{ J/kg}$ .

EXECUTE:  $K = \frac{1}{2}(2.60 \times 10^{15} \text{ kg})(32.0 \times 10^3 \text{ m/s})^2 = 1.33 \times 10^{24} \text{ J. } Q = mc\Delta T + mL_v.$ 

$$m = \frac{Q}{c\Delta T + L_{\rm v}} = \frac{1.33 \times 10^{22} \text{ J}}{(4190 \text{ J/kg} \cdot \text{K})(90.0 \text{ K}) + 2256 \times 10^3 \text{ J/kg}} = 5.05 \times 10^{15} \text{ kg}.$$

**EVALUATE:** The mass of water boiled is 2.5 times the mass of water in Lake Superior.

17.52. IDENTIFY:  $Q = mc\Delta T$  for a temperature change. The net Q for the system (sample, can and water) is zero.

SET UP: For water,  $c_w = 4.19 \times 10^3 \text{ J/kg} \cdot \text{K}$ . For copper,  $c_c = 390 \text{ J/kg} \cdot \text{K}$ .

EXECUTE: For the water,  $Q_{\text{w}} = m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}} = (0.200 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(7.1 \text{ C}^{\circ}) = 5.95 \times 10^3 \text{ J}.$ 

For the copper can,  $Q_c = m_c c_c \Delta T_c = (0.150 \text{ kg})(390 \text{ J/kg} \cdot \text{K})(7.1 \text{ C}^\circ) = 415 \text{ J}.$ 

For the sample,  $Q_s = m_s c_s \Delta T_s = (0.085 \text{ kg}) c_s (-73.9 \text{ C}^\circ)$ .

 $\Sigma Q = 0$  gives  $(0.085 \text{ kg})(-73.9 \text{ C}^{\circ})c_s + 415 \text{ J} + 5.95 \times 10^3 \text{ J} = 0$ .  $c_s = 1.01 \times 10^3 \text{ J/kg} \cdot \text{K}$ .

**EVALUATE:** Heat comes out of the sample and goes into the water and the can. The value of  $c_s$  we calculated is consistent with the values in Table 17.3.

17.53. **IDENTIFY** and **SET UP:** Heat flows out of the water and into the ice. The net heat flow for the system is zero. The ice warms to 0°C, melts, and then the water from the melted ice warms from 0°C to the final temperature. **EXECUTE:**  $Q_{\text{system}} = 0$ ; calculate Q for each component of the system: (Beaker has small mass says that  $Q = mc\Delta T$  for beaker can be neglected.)

0.250 kg of water: cools from 75.0°C to 40.0°C

$$Q_{\text{water}} = mc\Delta T = (0.250 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(40.0^{\circ}\text{C} - 75.0^{\circ}\text{C}) = -3.666 \times 10^{4} \text{ J}.$$

ice: warms to 0°C; melts; water from melted ice warms to 40.0°C

$$\begin{split} &Q_{\text{ice}} = mc_{\text{ice}}\Delta T + mL_{\text{f}} + mc_{\text{water}}\Delta T. \\ &Q_{\text{ice}} = m[(2100 \text{ J/kg} \cdot \text{K})(0^{\circ}\text{C} - (-20.0^{\circ}\text{C})) + 334 \times 10^{3} \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(40.0^{\circ}\text{C} - 0^{\circ}\text{C})]. \\ &Q_{\text{ice}} = (5.436 \times 10^{5} \text{ J/kg})m. \quad Q_{\text{system}} = 0 \text{ says } Q_{\text{water}} + Q_{\text{ice}} = 0. \quad -3.666 \times 10^{4} \text{ J} + (5.436 \times 10^{5} \text{ J/kg})m = 0. \end{split}$$

$$m = \frac{3.666 \times 10^4 \text{ J}}{5.436 \times 10^5 \text{ J/kg}} = 0.0674 \text{ kg}.$$

**EVALUATE:** Since the final temperature is 40.0°C we know that all the ice melts and the final system is all liquid water. The mass of ice added is much less than the mass of the 75°C water; the ice requires a large heat input for the phase change.

17.54. IDENTIFY and SET UP: Large block of ice implies that ice is left, so  $T_2 = 0$ °C (final temperature). Heat comes out of the ingot and into the ice. The net heat flow is zero. The ingot has a temperature change and the ice has a phase change.

**EXECUTE:**  $Q_{\text{system}} = 0$ ; calculate Q for each component of the system:

ingot

$$Q_{\text{ingot}} = mc\Delta T = (4.00 \text{ kg})(234 \text{ J/kg} \cdot \text{K})(0^{\circ}\text{C} - 750^{\circ}\text{C}) = -7.02 \times 10^{5} \text{ J}$$

ice

 $Q_{\text{ice}} = +mL_{\text{f}}$ , where m is the mass of the ice that changes phase (melts)

$$Q_{\text{system}} = 0 \text{ says } Q_{\text{ingot}} + Q_{\text{ice}} = 0$$

$$-7.02 \times 10^5 \text{ J} + m(334 \times 10^3 \text{ J/kg}) = 0$$

$$m = \frac{7.02 \times 10^5 \text{ J}}{334 \times 10^3 \text{ J/kg}} = 2.10 \text{ kg}$$

**EVALUATE:** The liquid produced by the phase change remains at 0°C since it is in contact with ice.

17.55. **IDENTIFY:** Set  $Q_{\text{system}} = 0$ , for the system of water, ice, and steam.  $Q = mc\Delta T$  for a temperature change and  $Q = \pm mL$  for a phase transition.

**SET UP:** For water, 
$$c = 4190 \text{ J/kg} \cdot \text{K}$$
,  $L_{\rm f} = 334 \times 10^3 \text{ J/kg}$  and  $L_{\rm v} = 2256 \times 10^3 \text{ J/kg}$ .

**EXECUTE:** The steam both condenses and cools, and the ice melts and heats up along with the original water.  $m_i L_f + m_i c(28.0 \text{ C}^\circ) + m_w c(28.0 \text{ C}^\circ) - m_{\text{steam}} L_v + m_{\text{steam}} c(-72.0 \text{ C}^\circ) = 0$ . The mass of steam needed is

$$m_{\text{steam}} = \frac{(0.450 \text{ kg})(334 \times 10^3 \text{ J/kg}) + (2.85 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(28.0 \text{ C}^\circ)}{2256 \times 10^3 \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(72.0 \text{ C}^\circ)} = 0.190 \text{ kg}.$$

**EVALUATE:** Since the final temperature is greater than 0.0°C, we know that all the ice melts.

**17.56. IDENTIFY:** At steady state, the rate of heat flow is the same throughout both rods, as well as out of the boiling water and into the ice-water mixture. The heat that flows into the ice-water mixture goes only into melting ice since the temperature remains at 0.00°C.

**SET UP:** For steady state heat flow,  $\frac{Q}{t} = \frac{kA\Delta T}{L}$ . The heat to melt ice is  $Q = mL_f$ .

EXECUTE: (a)  $\frac{Q}{t} = \frac{kA\Delta T}{L}$  is the same for both of the rods. Using the physical properties of brass and

copper from the tables in the text, we have

$$\frac{[109.0 \text{ W/(m \cdot K)}](100.0^{\circ}\text{C} - T)}{0.300 \text{ m}} = \frac{[385.0 \text{ W/(m \cdot K)}](T - 0.0^{\circ}\text{C})}{0.800 \text{ m}}.$$

Solving for T gives T = 43.0°C.

**(b)** The heat entering the ice-water mixture is

$$Q = \frac{kAt\Delta T}{L} = \frac{[109.0 \text{ W/(m \cdot K)}](0.00500 \text{ m}^2)(300.0 \text{ s})(100.0^{\circ}\text{C} - 43.0^{\circ}\text{C})}{0.300 \text{ m}}. \quad Q = 3.1065 \times 10^4 \text{ J. Then}$$

$$Q = mL_f \text{ so } m = \frac{3.1065 \times 10^4 \text{ J}}{3.34 \times 10^5 \text{ J/kg}} = 0.0930 \text{ kg} = 93.0 \text{ g.}$$

**EVALUATE:** The temperature of the interface between the two rods is between the two extremes (0°C and 100°C), but not midway between them.

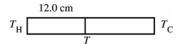
**17.57. IDENTIFY** and **SET UP:** The temperature gradient is  $(T_{\rm H} - T_{\rm C})/L$  and can be calculated directly. Use  $H = kA(T_{\rm H} - T_{\rm C})/L$  to calculate the heat current H. In part (c) use H from part (b) and apply  $H = kA(T_{\rm H} - T_{\rm C})/L$  to the 12.0-cm section of the left end of the rod.  $T_2 = T_{\rm H}$  and  $T_1 = T$ , the target variable.

**EXECUTE**: **(a)** Temperature gradient =  $(T_H - T_C)/L = (100.0^{\circ}\text{C} - 0.0^{\circ}\text{C})/0.450 \text{ m} = 222 \text{ C}^{\circ}/\text{m} = 222 \text{ K/m}$ 

**(b)** 
$$H = kA(T_H - T_C)/L$$
. From Table 17.5,  $k = 385 \text{ W/m} \cdot \text{K}$ , so

$$H = (385 \text{ W/m} \cdot \text{K})(1.25 \times 10^{-4} \text{ m}^2)(222 \text{ K/m}) = 10.7 \text{ W}.$$

(c) H = 10.7 W for all sections of the rod.



## **Figure 17.57**

Apply  $H = kA\Delta T/L$  to the 12.0 cm section (Figure 17.57):  $T_{\rm H} - T = LH/kA$  and  $T = T_{\rm H} - LH/Ak = 100.0$ °C  $-\frac{(0.120 \text{ m})(10.7 \text{ W})}{(1.25 \times 10^{-4} \text{ m}^2)(385 \text{ W/m} \cdot \text{K})} = 73.3$ °C

**EVALUATE:** H is the same at all points along the rod, so  $\Delta T/\Delta x$  is the same for any section of the rod with length  $\Delta x$ . Thus  $(T_{\rm H}-T)/(12.0~{\rm cm})=(T_{\rm H}-T_{\rm C})/(45.0~{\rm cm})$  gives that  $T_{\rm H}-T=26.7~{\rm C}^{\circ}$  and  $T=73.3^{\circ}{\rm C}$ , as we already calculated.

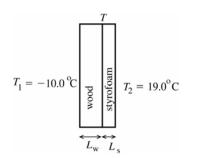
17.58. IDENTIFY: For a melting phase transition,  $Q = mL_f$ . The rate of heat conduction is  $\frac{Q}{t} = \frac{kA(T_H - T_C)}{L}$ .

**SET UP:** For water,  $L_f = 3.34 \times 10^5$  J/kg.

EXECUTE: The heat conducted by the rod in 10.0 min is  $Q = mL_f = (8.50 \times 10^{-3} \text{ kg})(3.34 \times 10^5 \text{ J/kg}) =$ 

2.84×10<sup>3</sup> J. 
$$\frac{Q}{t} = \frac{2.84 \times 10^3 \text{ J}}{600 \text{ s}} = 4.73 \text{ W}.$$
  $k = \frac{(Q/t)L}{A(T_{\text{H}} - T_{\text{C}})} = \frac{(4.73 \text{ W})(0.600 \text{ m})}{(1.25 \times 10^{-4} \text{ m}^2)(100 \text{ C}^\circ)} = 227 \text{ W/m} \cdot \text{K}.$ 

**EVALUATE:** The heat conducted by the rod is the heat that enters the ice and produces the phase change. **17.59. IDENTIFY** and **SET UP:** Call the temperature at the interface between the wood and the styrofoam T. The heat current in each material is given by  $H = kA(T_H - T_C)/L$ .



See Figure 17.59.

Heat current through the wood:  $H_{\rm w} = k_{\rm w} A (T-T_1) L_{\rm w}$ Heat current through the styrofoam:  $H_{\rm s} = k_{\rm s} A (T_2-T)/L_{\rm s}$  In steady-state heat does not accumulate in either material. The same heat has to pass through both materials in succession, so  $H_{\rm w} = H_{\rm s}$ .

**EXECUTE:** (a) This implies  $k_w A(T-T_1)/L_w = k_s A(T_2-T)/L_s$ 

$$k_{\rm w}L_{\rm s}(T-T_1) = k_{\rm s}L_{\rm w}(T_2-T)$$

$$T = \frac{k_{\rm w} L_{\rm s} T_1 + k_{\rm s} L_{\rm w} T_2}{k_{\rm w} L_{\rm s} + k_{\rm s} L_{\rm w}} = \frac{-0.0176 \text{ W} \cdot {}^{\circ}\text{C/K} + 0.01539 \text{ W} \cdot {}^{\circ}\text{C/K}}{0.00257 \text{ W/K}} = -0.86 {}^{\circ}\text{C}.$$

**EVALUATE:** The temperature at the junction is much closer in value to  $T_1$  than to  $T_2$ . The styrofoam has a very small k, so a larger temperature gradient is required for than for wood to establish the same heat current.

**(b) IDENTIFY** and **SET UP:** Heat flow per square meter is  $\frac{H}{A} = k \left( \frac{T_{\text{H}} - T_{\text{C}}}{L} \right)$ . We can calculate this either

for the wood or for the styrofoam; the results must be the same.

**EXECUTE:** wood

$$\frac{H_{\rm w}}{A} = k_{\rm w} \frac{T - T_1}{L_{\rm w}} = (0.080 \text{ W/m} \cdot \text{K}) \frac{-0.86 \text{°C} - (-10.0 \text{°C})}{0.030 \text{ m}} = 24 \text{ W/m}^2.$$

stvrofoam

$$\frac{H_s}{A} = k_s \frac{T_2 - T}{L_s} = (0.027 \text{ W/m} \cdot \text{K}) \frac{19.0^{\circ}\text{C} - (-0.86^{\circ}\text{C})}{0.022 \text{ m}} = 24 \text{ W/m}^2.$$

**EVALUATE:** *H* must be the same for both materials and our numerical results show this. Both materials are good insulators and the heat flow is very small.

17.60. IDENTIFY:  $\frac{Q}{t} = \frac{kA(T_{\text{H}} - T_{\text{C}})}{L}$ 

SET UP:  $T_{\rm H} - T_{\rm C} = 175^{\circ}{\rm C} - 35^{\circ}{\rm C}$ . 1 K = 1 C°, so there is no need to convert the temperatures to kelvins.

EXECUTE: **(a)**  $\frac{Q}{t} = \frac{(0.040 \text{ W/m} \cdot \text{K})(1.40 \text{ m}^2)(175^{\circ}\text{C} - 35^{\circ}\text{C})}{4.0 \times 10^{-2} \text{ m}} = 196 \text{ W}.$ 

(b) The power input must be 196 W, to replace the heat conducted through the walls.

**EVALUATE:** The heat current is small because k is small for fiberglass.

17.61. **IDENTIFY:** There is a temperature difference across the skin, so we have heat conduction through the skin.

**SET UP:** Apply  $H = kA \frac{T_{\text{H}} - T_{\text{C}}}{L}$  and solve for k.

EXECUTE:  $k = \frac{HL}{A(T_{\text{H}} - T_{\text{C}})} = \frac{(75 \text{ W})(0.75 \times 10^{-3} \text{ m})}{(2.0 \text{ m}^2)(37^{\circ}\text{C} - 30.0^{\circ}\text{C})} = 4.0 \times 10^{-3} \text{ W/m} \cdot \text{C}^{\circ}.$ 

**EVALUATE:** This is a small value; skin is a poor conductor of heat. But the thickness of the skin is small, so the rate of heat conduction through the skin is not small.

17.62. IDENTIFY:  $\frac{Q}{t} = \frac{k A \Delta T}{L}$ . Q/t is the same for both sections of the rod.

**SET UP:** For copper,  $k_c = 385 \text{ W/m} \cdot \text{K}$ . For steel,  $k_s = 50.2 \text{ W/m} \cdot \text{K}$ .

EXECUTE: (a) For the copper section,  $\frac{Q}{t} = \frac{(385 \text{ W/m} \cdot \text{K})(4.00 \times 10^{-4} \text{ m}^2)(100 \text{°C} - 65.0 \text{°C})}{1.00 \text{ m}} = 5.39 \text{ J/s}.$ 

**(b)** For the steel section,  $L = \frac{k A\Delta T}{(Q/t)} = \frac{(50.2 \text{ W/m} \cdot \text{K})(4.00 \times 10^{-4} \text{ m}^2)(65.0 \text{°C} - 0 \text{°C})}{5.39 \text{ J/s}} = 0.242 \text{ m}.$ 

**EVALUATE:** The thermal conductivity for steel is much less than that for copper, so for the same  $\Delta T$  and A, a smaller L for steel would be needed for the same heat current as in copper.

17.63. IDENTIFY and SET UP: The heat conducted through the bottom of the pot goes into the water at 100°C to convert it to steam at 100°C. We can calculate the amount of heat flow from the mass of material that changes phase. Then use  $H = kA(T_H - T_C)/L$  to calculate  $T_H$ , the temperature of the lower surface of the pan.

**EXECUTE:** 
$$Q = mL_v = (0.390 \text{ kg})(2256 \times 10^3 \text{ J/kg}) = 8.798 \times 10^5 \text{ J}$$

$$H = Q/t = 8.798 \times 10^5 \text{ J/180 s} = 4.888 \times 10^3 \text{ J/s}$$

Then 
$$H = kA(T_{\rm H} - T_{\rm C})/L$$
 says that  $T_{\rm H} - T_{\rm C} = \frac{HL}{kA} = \frac{(4.888 \times 10^3 \text{ J/s})(8.50 \times 10^{-3} \text{ m})}{(50.2 \text{ W/m} \cdot \text{K})(0.150 \text{ m}^2)} = 5.52 \text{ C}^{\circ}$ 

$$T_{\rm H} = T_{\rm C} + 5.52 \, \, \text{C}^{\circ} = 100 \, \, ^{\circ}\text{C} + 5.52 \, \, \text{C}^{\circ} = 105.5 \, \, ^{\circ}\text{C}.$$

**EVALUATE:** The larger  $T_{\rm H} - T_{\rm C}$  is the larger H is and the faster the water boils.

**17.64. IDENTIFY:** Apply  $H = kA(T_H - T_C)/L$  and solve for A.

**SET UP:** The area of each circular end of a cylinder is related to the diameter *D* by  $A = \pi R^2 = \pi (D/2)^2$ . For steel, k = 50.2 W/m·K. The boiling water has T = 100°C, so  $\Delta T = 300$  K.

EXECUTE: 
$$\frac{Q}{t} = kA\frac{\Delta T}{L}$$
 and 190 J/s =  $(50.2 \text{ W/m} \cdot \text{K})A\left(\frac{300 \text{ K}}{0.500 \text{ m}}\right)$ . This gives  $A = 6.308 \times 10^{-3} \text{ m}^2$ ,

and 
$$D = \sqrt{4A/\pi} = \sqrt{4(6.308 \times 10^{-3} \text{ m}^2)/\pi} = 8.96 \times 10^{-2} \text{ m} = 8.96 \text{ cm}.$$

**EVALUATE:** *H* increases when *A* increases.

**17.65. IDENTIFY:** Assume the temperatures of the surfaces of the window are the outside and inside temperatures. Use the concept of thermal resistance. For part (b) use the fact that when insulating materials are in layers, the *R* values are additive.

**SET UP:** From Table 17.5,  $k = 0.8 \text{ W/m} \cdot \text{K}$  for glass. R = L/k.

EXECUTE: (a) For the glass, 
$$R_{\text{glass}} = \frac{5.20 \times 10^{-3} \text{ m}}{0.8 \text{ W/m} \cdot \text{K}} = 6.50 \times 10^{-3} \text{ m}^2 \cdot \text{K/W}.$$

$$H = \frac{A(T_{\rm H} - T_{\rm C})}{R} = \frac{(1.40 \text{ m})(2.50 \text{ m})(39.5 \text{ K})}{6.50 \times 10^{-3} \text{ m}^2 \cdot \text{K/W}} = 2.1 \times 10^4 \text{ W}$$

**(b)** For the paper, 
$$R_{\text{paper}} = \frac{0.750 \times 10^{-3} \text{ m}}{0.05 \text{ W/m} \cdot \text{K}} = 0.015 \text{ m}^2 \cdot \text{K/W}$$
. The total *R* is

$$R = R_{\text{glass}} + R_{\text{paper}} = 0.0215 \text{ m}^2 \cdot \text{K/W}. \quad H = \frac{A(T_{\text{H}} - T_{\text{C}})}{R} = \frac{(1.40 \text{ m})(2.50 \text{ m})(39.5 \text{ K})}{0.0215 \text{ m}^2 \cdot \text{K/W}} = 6.4 \times 10^3 \text{ W}.$$

**EVALUATE:** The layer of paper decreases the rate of heat loss by a factor of about 3.

**17.66. IDENTIFY:** The rate of energy radiated per unit area is  $\frac{H}{A} = e\sigma T^4$ .

**SET UP:** A perfect blackbody has e = 1.

EXECUTE: (a) 
$$\frac{H}{4} = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(273 \text{ K})^4 = 315 \text{ W/m}^2$$

**(b)** 
$$\frac{H}{4} = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(2730 \text{ K})^4 = 3.15 \times 10^6 \text{ W/m}^2$$

**EVALUATE:** When the Kelvin temperature increases by a factor of 10 the rate of energy radiation increases by a factor of  $10^4$ .

**17.67. IDENTIFY:** The pot loses energy by blackbody radiation, but it is not an ideal blackbody. The surrounding atmosphere also acts like a blackbody at 20.0°C radiating back into the pot.

**SET UP:** Assume that the walls of the coffee pot are of negligible thickness so that the surface temperature of the pot will be nearly the same as the water it contains. We can find the surface area of the pot from its known

volume of 
$$0.75 \text{ L} = 7.5 \times 10^{-4} \text{ m}^3$$
. We know that  $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$  where  $T = 95^\circ = 368 \text{ K}$  and

$$T_{\rm s} = 20.0^{\circ} = 293$$
 K. Finally, we know that for a sphere  $A = 4\pi R^2$  and  $V = \frac{4}{3}\pi R^3$ .

**EXECUTE:** First find the radius of the pot from its volume:

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3(7.5 \times 10^{-4} \text{ m}^3)}{4\pi}\right)^{1/3} = 0.05636 \text{ m}.$$

Next find the surface area of the pot:

$$A = 4\pi R^2 = 4\pi (0.05636 \text{ m})^2 = 0.0399 \text{ m}^2$$

Finally, we determine the rate of thermal emission:

$$H_{\text{net}} = e\sigma A(T^4 - T_s^4) = (0.60)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.0399 \text{ m}^2)[(368 \text{ K})^4 - (293 \text{ K})^4] = 15 \text{ W}.$$

**EVALUATE:** There must be a temperature difference between the outside and inside surface of the pot for heat conduction to occur. But if the thickness of the pot is small, this temperature difference is also small and we can assume that the temperature of the two surfaces is the same. If the pot were in outer space, the "air" temperature would be essentially 0 K, so it would lose heat at a much faster rate than we just found.

17.68. **IDENTIFY:** The net heat current is  $H = Ae\sigma(T^4 - T_s^4)$ . A power input equal to H is required to maintain constant temperature of the sphere.

**SET UP:** The surface area of a sphere is  $4\pi r^2$ .

EXECUTE: 
$$H = 4\pi (0.0150 \text{ m})^2 (0.35)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(3000 \text{ K})^4 - (290 \text{ K})^4] = 4.54 \times 10^3 \text{ W}.$$

**EVALUATE:** Since 3000 K > 290 K and H is proportional to  $T^4$ , the rate of emission of heat energy is much greater than the rate of absorption of heat energy from the surroundings.

**17.69. IDENTIFY:** Use  $H = Ae\sigma T^4$  to calculate A.

**SET UP:**  $H = Ae\sigma T^4$  so  $A = H/e\sigma T^4$ 

150-W and all electrical energy consumed is radiated says H = 150 W.

EXECUTE: 
$$A = \frac{150 \text{ W}}{(0.35)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(2450 \text{ K})^4} = 2.1 \times 10^{-4} \text{ m}^2 (1 \times 10^4 \text{ cm}^2/1 \text{ m}^2) = 2.1 \text{ cm}^2$$

**EVALUATE:** Light-bulb filaments are often in the shape of a tightly wound coil to increase the surface area; larger A means a larger radiated power H.

**17.70. IDENTIFY:** Apply  $H = Ae\sigma T^4$  and calculate A.

**SET UP:** For a sphere of radius R,  $A = 4\pi R^2$ .  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ . The radius of the earth is  $R_{\rm E} = 6.37 \times 10^6 \text{ m}$ , the radius of the sun is  $R_{\rm sun} = 6.96 \times 10^8 \text{ m}$ , and the distance between the earth and the sun is  $r = 1.50 \times 10^{11} \text{ m}$ .

**EXECUTE:** The radius is found from 
$$R = \sqrt{\frac{A}{4\pi}} = \sqrt{\frac{H/(\sigma T^4)}{4\pi}} = \sqrt{\frac{H}{4\pi\sigma}} \frac{1}{T^2}$$
.

(a) 
$$R_{\rm a} = \sqrt{\frac{(2.7 \times 10^{32} \text{ W})}{4\pi (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)}} \frac{1}{(11,000 \text{ K})^2} = 1.61 \times 10^{11} \text{ m}$$

**(b)** 
$$R_{\rm b} = \sqrt{\frac{(2.10 \times 10^{23} \text{ W})}{4\pi (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)}} \frac{1}{(10,000 \text{ K})^2} = 5.43 \times 10^6 \text{ m}$$

**EVALUATE: (c)** The radius of Procyon B is comparable to that of the earth, and the radius of Rigel is comparable to the earth-sun distance.

17.71. **IDENTIFY:** Use  $\Delta L = L_0 \alpha \Delta T$  to find the change in diameter of the sphere and the change in length of the cable. Set the sum of these two increases in length equal to 2.00 mm.

**SET UP:**  $\alpha_{\text{brass}} = 2.0 \times 10^{-5} \text{ K}^{-1} \text{ and } \alpha_{\text{steel}} = 1.2 \times 10^{-5} \text{ K}^{-1}.$ 

**EXECUTE:**  $\Delta L = (\alpha_{\text{brass}} L_{0,\text{brass}} + \alpha_{\text{steel}} L_{0,\text{steel}}) \Delta T$ .

$$\Delta T = \frac{2.00 \times 10^{-3} \text{ m}}{(2.0 \times 10^{-5} \text{ K}^{-1})(0.350 \text{ m}) + (1.2 \times 10^{-5} \text{ K}^{-1})(10.5 \text{ m})} = 15.0 \text{ C}^{\circ}. \quad T_2 = T_1 + \Delta T = 35.0 ^{\circ}\text{C}.$$

**EVALUATE:** The change in diameter of the brass sphere is 0.10 mm. This is small, but should not be neglected.

17.72. **IDENTIFY:** Apply  $\Delta L = L_0 \alpha \Delta T$  to the radius of the hoop. The thickness of the space equals the increase in radius of the hoop.

SET UP: The earth has radius  $R_{\rm E} = 6.37 \times 10^6$  m and this is the initial radius  $R_0$  of the hoop. For steel,  $\alpha = 1.2 \times 10^{-5}$  K<sup>-1</sup>. 1 K = 1 C°.

**EXECUTE:** The increase in the radius of the hoop would be

$$\Delta R = R\alpha\Delta T = (6.37 \times 10^6 \text{ m})(1.2 \times 10^{-5} \text{ K}^{-1})(0.5 \text{ K}) = 38 \text{ m}.$$

**EVALUATE:** Even though  $\Delta R$  is large, the fractional change in radius,  $\Delta R/R_0$ , is very small.

- 17.73. **IDENTIFY** and **SET UP:** Use the temperature difference in  $M^{\circ}$  and in  $C^{\circ}$  between the melting and boiling points of mercury to relate  $M^{\circ}$  to  $C^{\circ}$ . Also adjust for the different zero points on the two scales to get an equation for  $T_{M}$  in terms of  $T_{C}$ .
  - (a) EXECUTE: normal melting point of mercury:  $-39^{\circ}\text{C} = 0.0^{\circ}\text{M}$

normal boiling point of mercury: 357°C = 100.0°M

 $100.0 \text{ M}^{\circ} = 396 \text{ C}^{\circ} \text{ so } 1 \text{ M}^{\circ} = 3.96 \text{ C}^{\circ}$ 

Zero on the M scale is -39 on the C scale, so to obtain  $T_{\rm C}$  multiply  $T_{\rm M}$  by 3.96 and then subtract 39°:  $T_{\rm C} = 3.96T_{\rm M} - 39^{\circ}$ 

Solving for  $T_{\rm M}$  gives  $T_{\rm M} = \frac{1}{3.96} (T_{\rm C} + 39^{\circ})$ 

The normal boiling point of water is  $100^{\circ}$ C;  $T_{\rm M} = \frac{1}{3.96} (100^{\circ} + 39^{\circ}) = 35.1^{\circ}$ M.

**(b)**  $10.0 \text{ M}^{\circ} = 39.6 \text{ C}^{\circ}$ 

**EVALUATE:** A M° is larger than a C° since it takes fewer of them to express the difference between the boiling and melting points for mercury.

17.74. **IDENTIFY:**  $v = \sqrt{F/\mu} = \sqrt{FL/m}$ . For the fundamental,  $\lambda = 2L$  and  $f = \frac{v}{\lambda} = \frac{1}{2}\sqrt{\frac{F}{mL}}$ . F, v and  $\lambda$  change

when T changes because L changes.  $\Delta L = L\alpha \Delta T$ , where L is the original length.

**SET UP:** For copper,  $\alpha = 1.7 \times 10^{-5} (\text{C}^{\circ})^{-1}$ .

**EXECUTE:** (a) We can use differentials to find the frequency change because all length changes are small percents.  $\Delta f \approx \frac{\partial f}{\partial L} \Delta L$  (only L changes due to heating).

$$\Delta f = \frac{1}{2} \frac{1}{2} (F/mL)^{-1/2} (F/m) (-1/L^2) \Delta L = -\frac{1}{2} \left( \frac{1}{2} \sqrt{\frac{F}{mL}} \right) \frac{\Delta L}{L} = -\frac{1}{2} f \frac{\Delta L}{L}.$$

 $\Delta f = -\frac{1}{2}(\alpha \Delta T)f = -\frac{1}{2}(1.7 \times 10^{-5} (\text{C}^{\circ})^{-1})(40 \text{ C}^{\circ})(440 \text{ Hz}) = -0.15 \text{ Hz}$ . The frequency decreases since the length increases.

**(b)** 
$$\Delta v = \frac{\partial v}{\partial L} \Delta L$$
.

$$\frac{\Delta v}{v} = \frac{\frac{1}{2} (FL/m)^{-1/2} (F/m) \Delta L}{\sqrt{FL/m}} = \frac{\Delta L}{2L} = \frac{\alpha \Delta T}{2} = \frac{1}{2} (1.7 \times 10^{-5} (\text{C}^{\circ})^{-1}) (40 \text{ C}^{\circ}) = 3.4 \times 10^{-4} = 0.034\%.$$

(c) 
$$\lambda = 2L$$
 so  $\Delta \lambda = 2\Delta L \rightarrow \frac{\Delta \lambda}{\lambda} = \frac{2\Delta L}{2L} = \frac{\Delta L}{L} = \alpha \Delta T$ .

$$\frac{\Delta \lambda}{\lambda} = (1.7 \times 10^{-5} \text{ (C}^{\circ})^{-1})(40 \text{ C}^{\circ}) = 6.8 \times 10^{-4} = 0.068\%. \ \lambda \text{ increases.}$$

**EVALUATE:** The wave speed and wavelength increase when the length increases and the frequency decreases. The percentage change in the frequency is -0.034%. The fractional change in all these quantities is very small.

17.75. IDENTIFY and SET UP: Use  $\Delta V = V_0 \beta \Delta T$  for the volume expansion of the oil and of the cup. Both the volume of the cup and the volume of the olive oil increase when the temperature increases, but  $\beta$  is larger for the oil so it expands more. When the oil starts to overflow,  $\Delta V_{\text{oil}} = \Delta V_{\text{glass}} + (3.00 \times 10^{-3} \text{ m})A$ , where A is the cross-sectional area of the cup.

EXECUTE:  $\Delta V_{\text{oil}} = V_{0,\text{oil}} \beta_{\text{oil}} \Delta T = (9.7 \text{ cm}) A \beta_{\text{oil}} \Delta T$ .  $\Delta V_{\text{glass}} = V_{0,\text{glass}} \beta_{\text{glass}} \Delta T = (10.0 \text{ cm}) A \beta_{\text{glass}} \Delta T$ .  $(9.7 \text{ cm}) A \beta_{\text{oil}} \Delta T = (10.0 \text{ cm}) A \beta_{\text{glass}} \Delta T + (0.300 \text{ cm}) A$ . The A divides out. Solving for  $\Delta T$  gives  $\Delta T = 47.4 \text{ C}^{\circ}$ .  $T_2 = T_1 + \Delta T = 69.4^{\circ}\text{C}$ .

**EVALUATE:** If the expansion of the cup is neglected, the olive oil will have expanded to fill the cup when  $(0.300 \text{ cm})A = (9.7 \text{ cm})A\beta_{\text{oil}}\Delta T$ , so  $\Delta T = 45.5 \text{ C}^{\circ}$  and  $T_2 = 77.5^{\circ}\text{C}$ . Our result is somewhat higher than this. The cup also expands but not as much since  $\beta_{\text{glass}} << \beta_{\text{oil}}$ .

**17.76. IDENTIFY:** As the tape changes temperature, the distances between the markings will increase, thus making the readings inaccurate.

**SET UP:** For steel,  $\alpha = 1.2 \times 10^{-5} (\text{C}^{\circ})^{-1}$ . The two points that match the length of the object are 25.970 m apart at 20.0°C. Find the distance between them at 5.00°C. For linear expansion,  $L = L_0 (1 + \alpha \Delta T)$ .

**EXECUTE:**  $L = L_0(1 + \alpha \Delta T) = (25.970 \text{ m})[1 + (1.2 \times 10^{-5} (\text{C}^{\circ})^{-1})(5.00^{\circ}\text{C} - 20.0^{\circ}\text{C})] = 25.965 \text{ m}$ . The true distance between the points is 25.965 m.

**EVALUATE:** The error in measurement is 25.970 m - 25.965 m = 0.005 m = 5 mm. This is not likely to be a very serious error in a measurement of nearly 30 m. If greater precision is needed, some sort of laser measuring device would probably be used.

17.77. IDENTIFY and SET UP: Call the metals A and B. Use the data given to calculate  $\alpha$  for each metal.

**EXECUTE:**  $\Delta L = L_0 \alpha \Delta T$  so  $\alpha = \Delta L/(L_0 \Delta T)$ 

metal A: 
$$\alpha_A = \frac{\Delta L}{L_0 \Delta T} = \frac{0.0650 \text{ cm}}{(30.0 \text{ cm})(100 \text{ C}^\circ)} = 2.167 \times 10^{-5} \text{ (C}^\circ)^{-1}$$

metal B: 
$$\alpha_B = \frac{\Delta L}{L_0 \Delta T} = \frac{0.0350 \text{ cm}}{(30.0 \text{ cm})(100 \text{ C}^\circ)} = 1.167 \times 10^{-5} (\text{C}^\circ)^{-1}$$

**EVALUATE:**  $L_0$  and  $\Delta T$  are the same, so the rod that expands the most has the larger  $\alpha$ .

**IDENTIFY** and **SET UP:** Now consider the composite rod (Figure 17.77). Apply  $\Delta L = L_0 \alpha \Delta T$ . The target variables are  $L_A$  and  $L_B$ , the lengths of the metals A and B in the composite rod.

$$\begin{array}{c|c}
L_A & 30.0 \text{ cm} - L_A \\
\hline
A & B & \Delta T = 100 \text{ C}^{\circ} \\
\Delta L = 0.058 \text{ cm}
\end{array}$$

**Figure 17.77** 

**EXECUTE:** 
$$\Delta L = \Delta L_A + \Delta L_B = (\alpha_A L_A + \alpha_B L_B) \Delta T$$

$$\Delta L/\Delta T = \alpha_A L_A + \alpha_B (0.300 \text{ m} - L_A)$$

$$L_A = \frac{\Delta L/\Delta T - (0.300 \text{ m})\alpha_B}{\alpha_A - \alpha_B} = \frac{(0.058 \times 10^{-2} \text{ m})/(100 \text{ C}^\circ) - (0.300 \text{ m})(1.167 \times 10^{-5} (\text{C}^\circ)^{-1})}{1.00 \times 10^{-5} (\text{C}^\circ)^{-1}} = 23.0 \text{ cm}$$

$$L_B = 30.0 \text{ cm} - L_A = 30.0 \text{ cm} - 23.0 \text{ cm} = 7.0 \text{ cm}$$

**EVALUATE:** The expansion of the composite rod is similar to that of rod A, so the composite rod is mostly metal A.

17.78. **IDENTIFY:** Apply  $\Delta V = V_0 \beta \Delta T$  to the gasoline and to the volume of the tank.

**SET UP:** For aluminum,  $\beta = 7.2 \times 10^{-5} \text{ K}^{-1}$ .  $1 \text{ L} = 10^{-3} \text{ m}^3$ .

**EXECUTE:** (a) The lost volume, 2.6 L, is the difference between the expanded volume of the fuel and the tanks, and the maximum temperature difference is

$$\Delta T = \frac{\Delta V}{(\beta_{\text{fuel}} - \beta_{\text{A1}})V_0} = \frac{(2.6 \times 10^{-3} \text{ m}^3)}{(9.5 \times 10^{-4} \text{ (C}^\circ)^{-1} - 7.2 \times 10^{-5} \text{ (C}^\circ)^{-1})(106.0 \times 10^{-3} \text{ m}^3)} = 28 \text{ C}^\circ.$$

The maximum temperature was 32°C.

**(b)** No fuel can spill if the tanks are filled just before takeoff.

**EVALUATE:** Both the volume of the gasoline and the capacity of the tanks increased when T increased. But  $\beta$  is larger for gasoline than for aluminum so the volume of the gasoline increased more. When the tanks have returned to  $4.0^{\circ}$ C on Sunday morning there is 2.6 L of air space in the tanks.

17.79. **IDENTIFY:** The change in length due to heating is  $\Delta L_T = L_0 \alpha \Delta T$  and this need not equal  $\Delta L$ . The change

in length due to the tension is  $\Delta L_F = \frac{FL_0}{AY}$ . Set  $\Delta L = \Delta L_F + \Delta L_T$ .

SET UP: 
$$\alpha_{\text{brass}} = 2.0 \times 10^{-5} \ (\text{C}^{\circ})^{-1}$$
.  $\alpha_{\text{steel}} = 1.5 \times 10^{-5} \ (\text{C}^{\circ})^{-1}$ .  $Y_{\text{steel}} = 20 \times 10^{10} \ \text{Pa}$ .

**EXECUTE:** (a) The change in length is due to the tension and heating.  $\frac{\Delta L}{L_0} = \frac{F}{AY} + \alpha \Delta T$ . Solving for F/A,

$$\frac{F}{A} = Y \left( \frac{\Delta L}{L_0} - \alpha \Delta T \right).$$

**(b)** The brass bar is given as "heavy" and the wires are given as "fine," so it may be assumed that the stress in the bar due to the fine wires does not affect the amount by which the bar expands due to the temperature increase. This means that  $\Delta L$  is not zero, but is the amount  $\alpha_{\text{brass}} L_0 \Delta T$  that the brass expands, and so

$$\frac{F}{4} = Y_{\text{steel}} (\alpha_{\text{brass}} - \alpha_{\text{steel}}) \Delta T = (20 \times 10^{10} \text{ Pa})(2.0 \times 10^{-5} (\text{C}^{\circ})^{-1} - 1.2 \times 10^{-5} (\text{C}^{\circ})^{-1})(120 \text{ C}^{\circ}) = 1.92 \times 10^{8} \text{ Pa}.$$

**EVALUATE:** The length of the brass bar increases more than the length of the steel wires. The wires remain taut and are under tension when the temperature of the system is raised above 20°C.

17.80. IDENTIFY and SET UP:  $v = \sqrt{F/\mu}$ . The coefficient of linear expansion  $\alpha$  is defined by  $\Delta L = L_0 \alpha \Delta T$ .

This can be combined with  $Y = \frac{F/A}{\Delta L/L_0}$  to give  $\Delta F = -Y\alpha A\Delta T$  for the change in tension when the

temperature changes by  $\Delta T$ . Combine the two equations and solve for  $\alpha$ .

**EXECUTE:** 
$$v_1 = \sqrt{F/\mu}$$
,  $v_1^2 = F/\mu$  and  $F = \mu v_1^2$ 

The length and hence  $\mu$  stay the same but the tension decreases by  $\Delta F = -Y\alpha A\Delta T$ .

$$v_2 = \sqrt{(F + \Delta F)/\mu} = \sqrt{(F - Y\alpha A\Delta T)/\mu}$$

$$v_2^2 = F/\mu - Y\alpha A \Delta T/\mu = v_1^2 - Y\alpha A \Delta T/\mu$$

And  $\mu = m/L$  so  $A/\mu = AL/m = V/m = 1/\rho$ . (A is the cross-sectional area of the wire, V is the volume of a

length L.) Thus 
$$v_1^2 - v_2^2 = \alpha(Y\Delta T/\rho)$$
 and  $\alpha = \frac{v_1^2 - v_2^2}{(Y/\rho)\Delta T}$ .

**EVALUATE:** When *T* increases the tension decreases and *v* decreases.

**17.81.** (a) **IDENTIFY** and **SET UP:** The diameter of the ring undergoes linear expansion (increases with *T*) just like a solid steel disk of the same diameter as the hole in the ring. Heat the ring to make its diameter equal to 2.5020 in.

EXECUTE: 
$$\Delta L = \alpha L_0 \Delta T$$
 so  $\Delta T = \frac{\Delta L}{L_0 \alpha} = \frac{0.0020 \text{ in.}}{(2.5000 \text{ in.})(1.2 \times 10^{-5} (\text{C}^\circ)^{-1})} = 66.7 \text{ C}^\circ$ 

$$T = T_0 + \Delta T = 20.0^{\circ}\text{C} + 66.7 \text{ C}^{\circ} = 87^{\circ}\text{C}$$

**(b) IDENTIFY** and **SET UP:** Apply the linear expansion equation to the diameter of the brass shaft and to the diameter of the hole in the steel ring.

**EXECUTE:** 
$$L = L_0(1 + \alpha \Delta T)$$

Want  $L_{\rm s}$  (steel) =  $L_{\rm b}$  (brass) for the same  $\Delta T$  for both materials:  $L_{0\rm s}(1+\alpha_{\rm s}\Delta T)=L_{0\rm b}(1+\alpha_{\rm b}\Delta T)$  so

$$L_{0s} + L_{0s}\alpha_{s}\Delta T = L_{0b} + L_{0b}\alpha_{b}\Delta T$$

$$\Delta T = \frac{L_{0b} - L_{0s}}{L_{0s}\alpha_{s} - L_{0b}\alpha_{b}} = \frac{2.5020 \text{ in.} - 2.5000 \text{ in.}}{(2.5000 \text{ in.})(1.2 \times 10^{-5} (\text{C}^{\circ})^{-1}) - (2.5020 \text{ in.})(2.0 \times 10^{-5} (\text{C}^{\circ})^{-1})}$$

$$\Delta T = \frac{0.0020}{3.00 \times 10^{-5} - 5.00 \times 10^{-5}} \text{ C}^{\circ} = -100 \text{ C}^{\circ}$$

$$T = T_0 + \Delta T = 20.0^{\circ}\text{C} - 100 \text{ C}^{\circ} = -80^{\circ}\text{C}$$

**EVALUATE:** Both diameters decrease when the temperature is lowered but the diameter of the brass shaft decreases more since  $\alpha_b > \alpha_s$ ;  $|\Delta L_b| - |\Delta L_s| = 0.0020$  in.

17.82. IDENTIFY: Calculate the total food energy value for one doughnut.  $K = \frac{1}{2}mv^2$ .

**SET UP:** 1 cal = 4.186 J

**EXECUTE:** (a) The energy is (2.0 g)(4.0 kcal/g) + (17.0 g)(4.0 kcal/g) + (7.0 g)(9.0 kcal/g) = 139 kcal. The time required is (139 kcal)/(510 kcal/h) = 0.273 h = 16.4 min.

**(b)**  $v = \sqrt{2K/m} = \sqrt{2(139 \times 10^3 \text{ cal})(4.186 \text{ J/cal})/(60 \text{ kg})} = 139 \text{ m/s} = 501 \text{ km/h}.$ 

**EVALUATE:** When we set K = Q, we must express Q in J, so we can solve for v in m/s.

**17.83. IDENTIFY:** The heat generated by shivering goes into the woman's body and therefore raises her temperature.

SET UP: Find the heat Q to raise the body temperature 1.0 °C and find the time it takes to produce this much heat energy at a rate of  $(290 \text{ W/m}^2)(1.8 \text{ m}^2) = 522 \text{ J/s}$ .  $P = \frac{Q}{t}$  and  $Q = mc \Delta T$ .

EXECUTE: (a) 
$$Q = mc \Delta T = (68 \text{ kg})(3500 \text{ J/kg})(1.0 \text{ C}^{\circ}) = 2.38 \times 10^{5} \text{ J}.$$
  $P = \frac{Q}{t} \text{ so } t = \frac{Q}{P} = \frac{Q}{t} = \frac{Q}{t}$ 

$$\frac{2.38 \times 10^5 \,\mathrm{J}}{522 \,\mathrm{J/s}} = 456 \,\mathrm{s}.$$

**EVALUATE:** The time found is 4.6 min. During this time, the body would also be losing heat through radiation, so the temperature rise would actually be less than  $1.0 \, \text{C}^{\circ}$ .

17.84. **IDENTIFY:**  $Q_{\text{system}} = 0$ . Assume that the normal melting point of iron is above 745°C so the iron initially is solid.

SET UP: For water,  $c = 4190 \text{ J/kg} \cdot \text{K}$  and  $L_v = 2256 \times 10^3 \text{ J/kg}$ . For solid iron,  $c = 470 \text{ J/kg} \cdot \text{K}$ .

**EXECUTE:** The heat released when the iron slug cools to  $100^{\circ}$ C is  $Q = mc\Delta T =$ 

 $(0.1000 \text{ kg})(470 \text{ J/kg} \cdot \text{K})(645 \text{ K}) = 3.03 \times 10^4 \text{ J}$ . The heat absorbed when the temperature of the water is raised to  $100^{\circ}\text{C}$  is  $Q = mc\Delta T = (0.0850 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(80.0 \text{ K}) = 2.85 \times 10^4 \text{ J}$ . This is less than the heat released from the iron and  $3.03 \times 10^4 \text{ J} - 2.85 \times 10^4 \text{ J} = 1.81 \times 10^3 \text{ J}$  of heat is available for converting some of the liquid water at  $100^{\circ}\text{C}$  to vapor. The mass m of water that boils is

$$m = \frac{1.81 \times 10^3 \text{ J}}{2256 \times 10^3 \text{ J/kg}} = 8.01 \times 10^{-4} \text{ kg} = 0.801 \text{ g}.$$

- (a) The final temperature is 100°C.
- **(b)** There is 85.0 g 0.801 g = 84.2 g of liquid water remaining, so the final mass of the iron and remaining water is 184.2 g.

**EVALUATE:** If we ignore the phase change of the water and write

 $m_{\text{iron}}c_{\text{iron}}(T-745^{\circ}\text{C}) + m_{\text{water}}c_{\text{water}}(T-20.0^{\circ}\text{C}) = 0$ , when we solve for T we will get a value slightly larger than 100°C. That result is unphysical and tells us that some of the water changes phase.

17.85. IDENTIFY and SET UP: To calculate Q, use  $Q = mc\Delta T$  in the form dQ = nCdT and integrate, using

C(T) given in the problem.  $C_{av}$  is obtained from  $C = \frac{1}{n} \frac{dQ}{dT}$  using the finite temperature range instead of

an infinitesimal dT.

**EXECUTE:** (a) dQ = nCdT

$$Q = n \int_{T_1}^{T_2} C \, dT = n \int_{T_1}^{T_2} k(T^3/\theta^3) dT = (nk/\theta^3) \int_{T_1}^{T_2} T^3 \, dt = (nk/\theta^3) \left(\frac{1}{4}T^4 \Big|_{T_1}^{T_2}\right)$$

$$Q = \frac{nk}{4\theta^3} (T_2^4 - T_1^4) = \frac{(1.50 \text{ mol})(1940 \text{ J/mol} \cdot \text{K})}{4(281 \text{ K})^3} ((40.0 \text{ K})^4 - (10.0 \text{ K})^4) = 83.6 \text{ J}.$$

17-21

**(b)** 
$$C_{\text{av}} = \frac{1}{n} \frac{\Delta Q}{\Delta T} = \frac{1}{1.50 \text{ mol}} \left( \frac{83.6 \text{ J}}{40.0 \text{ K} - 10.0 \text{ K}} \right) = 1.86 \text{ J/mol} \cdot \text{K}$$

(c) 
$$C = k(T/\theta)^3 = (1940 \text{ J/mol} \cdot \text{K})(40.0 \text{ K/281 K})^3 = 5.60 \text{ J/mol} \cdot \text{K}$$

**EVALUATE:** C is increasing with T, so C at the upper end of the temperature integral is larger than its average value over the interval.

17.86. IDENTIFY: For a temperature change,  $Q = mc\Delta T$ , and for the liquid  $\rightarrow$  solid phase change,  $Q = -mL_f$ .

SET UP: The volume  $V_{\rm w}$  of the water determines its mass.  $m_{\rm w} = \rho_{\rm w} V_{\rm w}$ . For water,  $\rho_{\rm w} = 1000 \text{ kg/m}^3$ ,  $c = 4190 \text{ J/kg} \cdot \text{K}$ , and  $L_{\rm f} = 334 \times 10^3 \text{ J/kg}$ .

**EXECUTE:** Set the heat energy that flows into the water equal to the final gravitational potential energy.  $L_f \rho_w V_w + c_w \rho_w V_w \Delta T = mgh$ . Solving for h gives

$$h = \frac{(1000 \text{ kg/m}^3)(1.90 \times 0.800 \times 0.240 \text{ m}^3)[334 \times 10^3 \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(37.0 \text{ C}^\circ)]}{(70.0 \text{ kg})(9.80 \text{ m/s}^2)}$$

$$h = 2.60 \times 10^5 \text{ m} = 260 \text{ km}.$$

**EVALUATE:** The heat associated with temperature and phase changes corresponds to a very large amount of mechanical energy.

17.87. **IDENTIFY:** Apply  $Q = mc\Delta T$  to the air in the room.

**SET UP:** The mass of air in the room is  $m = \rho V = (1.20 \text{ kg/m}^3)(3200 \text{ m}^3) = 3840 \text{ kg}$ . 1 W = 1 J/s.

**EXECUTE:** (a)  $Q = (3000 \text{ s})(140 \text{ students})(100 \text{ J/s} \cdot \text{student}) = 4.20 \times 10^7 \text{ J}.$ 

**(b)** 
$$Q = mc\Delta T$$
.  $\Delta T = \frac{Q}{mc} = \frac{4.20 \times 10^7 \text{ J}}{(3840 \text{ kg})(1020 \text{ J/kg} \cdot \text{K})} = 10.7 \text{ C}^{\circ}$ .

(c) 
$$\Delta T = (10.7 \text{ C}^\circ) \left( \frac{280 \text{ W}}{100 \text{ W}} \right) = 30.0 \text{ C}^\circ.$$

**EVALUATE:** In the absence of a cooling mechanism for the air, the air temperature would rise significantly.

**17.88. IDENTIFY:** dQ = nCdT so for the temperature change  $T_1 \to T_2$ ,  $Q = n \int_{T_1}^{T_2} C(T) dT$ .

SET UP:  $\int dT = T$  and  $\int TdT = \frac{1}{2}T^2$ . Express  $T_1$  and  $T_2$  in kelvins:  $T_1 = 300$  K,  $T_2 = 500$  K.

**EXECUTE:** Denoting C by C = a + bT, a and b independent of temperature, integration gives

$$Q = n \left[ a(T_2 - T_1) + \frac{b}{2} (T_2^2 - T_1^2) \right].$$

 $Q = (3.00 \text{ mol})[(29.5 \text{ J/mol} \cdot \text{K})(500 \text{ K} - 300 \text{ K}) + (4.10 \times 10^{-3} \text{ J/mol} \cdot \text{K}^2)((500 \text{ K})^2 - (300 \text{ K})^2)].$   $Q = 1.97 \times 10^4 \text{ J}.$ 

**EVALUATE:** If C is assumed to have the constant value 29.5 J/mol·K, then  $Q = 1.77 \times 10^4$  J for this temperature change. At  $T_1 = 300$  K, C = 32.0 J/mol·K and at  $T_2 = 500$  K, C = 33.6 J/mol·K. The average value of C is 32.8 J/mol·K. If C is assumed to be constant and to have this average value, then  $Q = 1.97 \times 10^4$  J, which is equal to the correct value.

**17.89. IDENTIFY:** The energy generated in the body is used to evaporate water, which prevents the body from overheating.

**SET UP:** Energy is (power)(time); calculate the heat energy Q produced in one hour. The mass m of water that vaporizes is related to Q by  $Q = mL_v$ . 1.0 kg of water has a volume of 1.0 L.

EXECUTE: (a)  $Q = (0.80)(500 \text{ W})(3600 \text{ s}) = 1.44 \times 10^6 \text{ J}$ . The mass of water that evaporates each hour is

$$m = \frac{Q}{L_{\rm v}} = \frac{1.44 \times 10^6 \text{ J}}{2.42 \times 10^6 \text{ J/kg}} = 0.60 \text{ kg}.$$

**(b)** (0.60 kg/h)(1.0 L/kg) = 0.60 L/h. The number of bottles of water is  $\frac{0.60 \text{ L/h}}{0.750 \text{ L/bottle}} = 0.80 \text{ bottles/h}$ .

**EVALUATE:** It is not unreasonable to drink 8/10 of a bottle of water per hour during vigorous exercise.

**17.90. IDENTIFY:** If it cannot be gotten rid of in some way, the metabolic energy transformed to heat will increase the temperature of the body.

SET UP: From Problem 17.89,  $Q = 1.44 \times 10^6 \,\text{J}$  and  $m = 70 \,\text{kg}$ .  $Q = mc\Delta T$ . Convert the temperature change in C° to F° using that  $9 \,\text{F}^\circ = 5 \,\text{C}^\circ$ .

EXECUTE: (a)  $Q = mc\Delta T$  so  $\Delta T = \frac{Q}{mc} = \frac{1.44 \times 10^6 \text{ J}}{(70 \text{ kg})(3480 \text{ J/kg} \cdot \text{C}^\circ)} = 5.9 \text{ C}^\circ.$ 

**(b)** 
$$\Delta T = (5.9^{\circ}\text{C}) \left( \frac{9 \text{ F}^{\circ}}{5 \text{ C}^{\circ}} \right) = 10.6^{\circ}\text{F}. \ T = 98.6^{\circ}\text{F} + 10.6 \text{ F}^{\circ} = 109^{\circ}\text{F}.$$

**EVALUATE:** A temperature this high can cause heat stroke and be lethal.

17.91. IDENTIFY and SET UP: The heat produced from the reaction is  $Q_{\text{reaction}} = mL_{\text{reaction}}$ , where  $L_{\text{reaction}}$  is the heat of reaction of the chemicals.

 $Q_{\text{reaction}} = W + \Delta U_{\text{spray}}$ 

**EXECUTE:** For a mass m of spray,  $W = \frac{1}{2}mv^2 = \frac{1}{2}m(19 \text{ m/s})^2 = (180.5 \text{ J/kg})m$  and

 $\Delta U_{\text{spray}} = Q_{\text{spray}} = mc\Delta T = m(4190 \text{ J/kg} \cdot \text{K})(100^{\circ}\text{C} - 20^{\circ}\text{C}) = (335,200 \text{ J/kg})m.$ 

Then  $Q_{\text{reaction}} = (180 \text{ J/kg} + 335,200 \text{ J/kg})m = (335,380 \text{ J/kg})m$  and  $Q_{\text{reaction}} = mL_{\text{reaction}}$  implies  $mL_{\text{reaction}} = (335,380 \text{ J/kg})m$ .

The mass m divides out and  $L_{\text{reaction}} = 3.4 \times 10^5 \text{ J/kg}.$ 

**EVALUATE:** The amount of energy converted to work is negligible for the two significant figures to which the answer should be expressed. Almost all of the energy produced in the reaction goes into heating the compound.

17.92. IDENTIFY: For a temperature change  $Q = mc\Delta T$ . For the vapor  $\rightarrow$  liquid phase transition,  $Q = -mL_v$ .

**SET UP:** For water,  $c = 4190 \text{ J/kg} \cdot \text{K}$  and  $L_v = 2256 \times 10^3 \text{ J/kg}$ .

**EXECUTE:** The requirement that the heat supplied in each case is the same gives

 $m_{\rm w}c_{\rm w}\Delta T_{\rm w} = m_{\rm s}(c_{\rm w}\Delta T_{\rm s} + L_{\rm v})$ , where  $\Delta T_{\rm w} = 42.0$  K and  $\Delta T_{\rm s} = 65.0$  K. The ratio of the masses is

$$\frac{m_{\rm s}}{m_{\rm w}} = \frac{c_{\rm w}\Delta T_{\rm w}}{c_{\rm w}\Delta T_{\rm s} + L_{\rm v}} = \frac{(4190 \text{ J/kg} \cdot \text{K})(42.0 \text{ K})}{(4190 \text{ J/kg} \cdot \text{K})(65.0 \text{ K}) + 2256 \times 10^3 \text{ J/kg}} = 0.0696,$$

so  $0.0696 \ \text{kg}$  of steam supplies the same heat as  $1.00 \ \text{kg}$  of water.

**EVALUATE:** Note the heat capacity of water is used to find the heat lost by the condensed steam, since the phase transition produces liquid water at an initial temperature of 100°C.

17.93. **IDENTIFY:** The heat lost by the water is equal to the amount of heat gained by the ice. First calculate the amount of heat the water could give up if it is cooled to 0.0°C. Then see how much heat it would take to melt all of the ice. If the heat to melt the ice is less than the heat the water would give up, the ice all melts and then the resulting water is heated to some final temperature.

**SET UP:**  $Q = mc\Delta T$  and  $Q = mL_f$ .

**EXECUTE:** (a) Heat from water if cooled to  $0.0^{\circ}$ C:  $Q = mc \Delta T$ 

$$Q = mc\Delta T = (1.50 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(28.0 \text{ K}) = 1.760 \times 10^5 \text{ J}$$

Heat to melt all of the ice:  $Q = mc\Delta T + mL_f = m(c\Delta T + L_f)$ 

$$Q = (0.600 \text{ kg})[(2100 \text{ J/kg} \cdot \text{K})(22.0 \text{ K}) + 3.34 \times 10^5 \text{ J/kg}] = 2.276 \times 10^5 \text{ J}$$

Since the heat required to melt all the ice is greater than the heat available by cooling the water to  $0.0^{\circ}$ C, not all the ice will melt.

**(b)** Since not all the ice melts, the final temperature of the water (and ice) will be  $0.0^{\circ}$ C. So the heat from the water will melt only part of the ice. Call m the mass of the melted ice. Therefore

 $Q_{\text{from water}} = 1.760 \times 10^5 \text{ J} = (0.600 \text{ kg})(2100 \text{ J/kg} \cdot \text{K})(22.0 \text{ K}) + m(3.34 \times 10^5 \text{ J/kg})$ , which gives m = 0.444 kg, which is the amount of ice that melts. The mass of ice remaining is 0.600 kg - 0.444 kg = 0.156 kg. The final temperature will be  $0.0^{\circ}\text{C}$  since some ice remains in the water.

**EVALUATE:** An alternative approach would be to assume that all the ice melts and find the final temperature of the water in the container. This actually comes out to be negative, which is not possible if all the ice melts. Therefore not all the ice could have melted. Once you know this, proceed as in part (b).

**17.94. IDENTIFY:** The amount of heat lost by the soft drink and mug is equal to the heat gained by the ice. The ice must first be heated to 0.0°C, then melted, and finally the resulting water heated to the final temperature of the system.

SET UP: Assume that all the ice melts. If we calculate  $T_f < 0$ , we will know this assumption is incorrect.

For aluminum,  $c_a = 910 \text{ J/kg} \cdot \text{K}$ . For water,  $L_f = 3.35 \times 10^5 \text{ J/kg}$  and  $c_w = 4.19 \times 10^3 \text{ J/kg} \cdot \text{K}$ . For ice,

 $c_i = 2100 \text{ J/kg} \cdot \text{K}$ . The density of water is  $1.00 \times 10^3 \text{ kg/m}^3$ , so 1.00 L of water has mass 1.00 kg.

**EXECUTE:** For the soft drink:

$$Q_{\rm w} = m_{\rm w} c_{\rm w} \Delta T_{\rm w} = (2.00 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(T - 20.0^{\circ}\text{C}) = (8380 \text{ J/K})T - 1.676 \times 10^5 \text{ J}.$$

For the mug: 
$$Q_a = m_a c_a \Delta T_a = (0.257 \text{ kg})(910 \text{ J/kg} \cdot \text{K})(T - 20.0^{\circ}\text{C}) = (234 \text{ J/K})T - 4.68 \times 10^3 \text{ J}.$$

For the ice:  $Q_i = m_i c_i \Delta T_i + m_i L_f + m_i c_w \Delta T_w$ .

$$Q_i = (0.120 \text{ kg})[(2100 \text{ J/kg} \cdot \text{K})(15.0 \text{ C}^\circ) + 3.34 \times 10^5 \text{ J/kg} + (4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(T - 0 \text{ C}^\circ)].$$

$$Q_1 = 4.386 \times 10^4 \text{ J} + (503 \text{ J/K})T.$$

$$\Sigma Q = 0$$
 gives  $(8380 \text{ J/K})T - 1.676 \times 10^5 \text{ J} + (234 \text{ J/K})T - 4.68 \times 10^3 \text{ J} + 4.386 \times 10^4 \text{ J} + (503 \text{ J/K})T = 0.$ 

$$T = \frac{1.284 \times 10^5 \,\mathrm{J}}{9117 \,\mathrm{J/K}} = 14.1 \,\mathrm{^{\circ}C}.$$

**EVALUATE:** T > 0°C, so our assumption that all the ice melts is correct. Note that the ice and the water from the melted ice have different specific heat capacities.

**17.95. IDENTIFY** and **SET UP:** Assume that all the ice melts and that all the steam condenses. If we calculate a final temperature T that is outside the range 0°C to 100°C then we know that this assumption is incorrect. Calculate Q for each piece of the system and then set the total  $Q_{\text{system}} = 0$ .

**EXECUTE:** (a) copper can (changes temperature from  $0.0^{\circ}$  to T; no phase change):

$$Q_{\text{can}} = mc\Delta T = (0.446 \text{ kg})(390 \text{ J/kg} \cdot \text{K})(T - 0.0^{\circ}\text{C}) = (173.9 \text{ J/K})T$$

<u>ice</u> (melting phase change and then the water produced warms to *T*):

$$Q_{\text{ice}} = +mL_f + mc\Delta T = (0.0950 \text{ kg})(334 \times 10^3 \text{ J/kg}) + (0.0950 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(T - 0.0^{\circ}\text{C})$$

$$Q_{\text{ice}} = 3.173 \times 10^4 \text{ J} + (398.0 \text{ J/K})T.$$

steam (condenses to liquid and then water produced cools to *T*):

$$Q_{\text{steam}} = -mL_{\text{v}} + mc\Delta T = -(0.0350 \text{ kg})(2256 \times 10^3 \text{ J/kg}) + (0.0350 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(T - 100.0^{\circ}\text{C})$$

$$Q_{\text{steam}} = -7.896 \times 10^4 \text{ J} + (146.6 \text{ J/K})T - 1.466 \times 10^4 \text{ J} = -9.362 \times 10^4 \text{ J} + (146.6 \text{ J/K})T$$

$$Q_{\text{system}} = 0$$
 implies  $Q_{\text{can}} + Q_{\text{ice}} + Q_{\text{steam}} = 0$ .

$$(173.9 \text{ J/K})T + 3.173 \times 10^4 \text{ J} + (398.0 \text{ J/K})T - 9.362 \times 10^4 \text{ J} + (146.6 \text{ J/K})T = 0$$

$$(718.5 \text{ J/K})T = 6.189 \times 10^4 \text{ J}$$

$$T = \frac{6.189 \times 10^4 \text{ J}}{718.5 \text{ J/K}} = 86.1 \text{°C}.$$

**(b)** No ice, no steam, and 0.0950 kg + 0.0350 kg = 0.130 kg of liquid water.

**EVALUATE:** This temperature is between 0°C and 100°C so our assumptions about the phase changes being complete were correct.

**17.96. IDENTIFY:** The final amount of ice is less than the initial mass of water, so water remains and the final temperature is 0°C. The ice added warms to 0°C and heat comes out of water to convert that water to ice.

Conservation of energy says  $Q_i + Q_w = 0$ , where  $Q_i$  and  $Q_w$  are the heat flows for the ice that is added and for the water that freezes.

SET UP: Let  $m_i$  be the mass of ice that is added and  $m_w$  is the mass of water that freezes. The mass of ice increases by 0.434 kg, so  $m_i + m_w = 0.434$  kg. For water,  $L_f = 334 \times 10^3$  J/kg and for ice

 $c_i = 2100 \text{ J/kg} \cdot \text{K}$ . Heat comes out of the water when it freezes, so  $Q_w = -mL_f$ .

**EXECUTE:**  $Q_{\rm i} + Q_{\rm w} = 0$  gives  $m_{\rm i} c_{\rm i} (15.0 \text{ C}^{\circ}) + (-m_{\rm w} L_{\rm f}) = 0$ ,  $m_{\rm w} = 0.434 \text{ kg} - m_{\rm i}$ , so

 $m_i c_i (15.0 \text{ C}^\circ) + (-0.434 \text{ kg} + m_i) L_f = 0.$ 

 $m_{\rm i} = \frac{(0.434 \text{ kg})L_{\rm f}}{c_{\rm i}(15.0 \text{ C}^{\circ}) + L_{\rm f}} = \frac{(0.434 \text{ kg})(334 \times 10^3 \text{ J/kg})}{(2100 \text{ J/kg} \cdot \text{K})(15.0 \text{ K}) + 334 \times 10^3 \text{ J/kg}} = 0.397 \text{ kg. } 0.397 \text{ kg of ice was added.}$ 

**EVALUATE:** The mass of water that froze when the ice at -15.0C° was added was 0.884 kg - 0.450 kg - 0.397 kg = 0.037 kg.

17.97. **IDENTIFY** and **SET UP:** Heat comes out of the steam when it changes phase and heat goes into the water and causes its temperature to rise.  $Q_{\text{system}} = 0$ . First determine what phases are present after the system has come to a uniform final temperature.

**EXECUTE:** (a) Heat that must be removed from steam if all of it condenses is

$$Q = -mL_v = -(0.0400 \text{ kg})(2256 \times 10^3 \text{ J/kg}) = -9.02 \times 10^4 \text{ J}$$

Heat absorbed by the water if it heats all the way to the boiling point of 100°C:

 $Q = mc\Delta T = (0.200 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(50.0 \text{ C}^{\circ}) = 4.19 \times 10^4 \text{ J}$ 

**(b)** Mass of steam that condenses is  $m = Q/L_v = 4.19 \times 10^4 \text{ J}/2256 \times 10^3 \text{ J/kg} = 0.0186 \text{ kg}$ . Thus there is 0.0400 kg – 0.0186 kg = 0.0214 kg of steam left. The amount of liquid water is 0.0186 kg + 0.200 kg = 0.219 kg.

**EVALUATE:** The water can't absorb enough heat for all the steam to condense. Steam is left and the final temperature then must be 100°C.

**17.98. IDENTIFY:** Heat is conducted out of the body. At steady state, the rate of heat flow is the same in both layers (fat and fur).

**SET UP:** Since the model is only a crude approximation to a bear, we will make the simplifying assumption that the surface area of each layer is constant and given by the surface area of a sphere of radius 1.5 m. Let the temperature of the fat-air boundary be T. A section of the two layers is sketched in Figure 17.98. A Kelvin degree is the same size as a Celsius degree, so  $W/m \cdot K$  and  $W/m \cdot C^{\circ}$  are equivalent units. At steady state the heat current through each layer is equal to 50 W. The area of each

layer is  $A = 4\pi r^2$ , with r = 0.75 m. Use  $H = kA \frac{T_H - T_C}{L}$ .

**Figure 17.98** 

**EXECUTE:** (a) Apply  $H = kA \frac{T_{\text{H}} - T_{\text{C}}}{L}$  to the fat layer and solve for  $T_{\text{C}} = T$ . For the fat layer  $T_{\text{H}} = 31^{\circ}\text{C}$ .

$$T = T_{\rm H} - \frac{HL}{kA} = 31^{\circ}\text{C} - \frac{(50 \text{ W})(4.0 \times 10^{-2} \text{ m})}{(0.20 \text{ W/m} \cdot \text{K})(4\pi)(0.75 \text{ m})^2} = 31^{\circ}\text{C} - 1.4^{\circ}\text{C} = 29.6^{\circ}\text{C}.$$

**(b)** Apply 
$$H = kA \frac{T_{\rm H} - T_{\rm C}}{L}$$
 to the air layer and solve for  $L = L_{\rm air}$ . For the air layer  $T_{\rm H} = T = 29.6$ °C and

$$T_{\rm C} = 2.7^{\circ} \text{C}.$$
  $L = \frac{kA(T_{\rm H} - T_{\rm C})}{H} = \frac{(0.024 \text{ W/m} \cdot \text{K})(4\pi)(0.75 \text{ m})^2(29.6^{\circ} \text{C} - 2.7^{\circ} \text{C})}{50 \text{ W}} = 9.1 \text{ cm}.$ 

**EVALUATE:** The thermal conductivity of air is much lass than the thermal conductivity of fat, so the temperature gradient for the air must be much larger to achieve the same heat current. So, most of the temperature difference is across the air layer.

17.99. IDENTIFY: Apply 
$$H = kA \frac{T_H - T_C}{L}$$
.

**SET UP:** For the glass use L = 12.45 cm, to account for the thermal resistance of the air films on either side of the glass.

EXECUTE: **(a)** 
$$H = (0.120 \text{ W/m} \cdot \text{K}) (2.00 \times 0.95 \text{ m}^2) \left( \frac{28.0 \text{ C}^{\circ}}{5.0 \times 10^{-2} \text{ m} + 1.8 \times 10^{-2} \text{ m}} \right) = 93.9 \text{ W}.$$

**(b)** The heat flow through the wood part of the door is reduced by a factor of 
$$1 - \frac{(0.50)^2}{(2.00 \times 0.95)} = 0.868$$
, so

it becomes 81.5 W. The heat flow through the glass is

$$H_{\text{glass}} = (0.80 \text{ W/m} \cdot \text{K})(0.50 \text{ m})^2 \left(\frac{28.0 \text{ C}^{\circ}}{12.45 \times 10^{-2} \text{ m}}\right) = 45.0 \text{ W}, \text{ and so the ratio is } \frac{81.5 + 45.0}{93.9} = 1.35.$$

**EVALUATE:** The single-pane window produces a significant increase in heat loss through the door. (See Problem 17.101).

**17.100. IDENTIFY:** Apply 
$$H = kA \frac{\Delta T}{L}$$
 and solve for  $k$ .

**SET UP:** *H* equals the power input required to maintain a constant interior temperature.

EXECUTE: 
$$k = H \frac{L}{A\Delta T} = (180 \text{ W}) \frac{(3.9 \times 10^{-2} \text{ m})}{(2.18 \text{ m}^2)(65.0 \text{ K})} = 5.0 \times 10^{-2} \text{ W/m} \cdot \text{K}.$$

**EVALUATE:** Our result is consistent with the values for insulating solids in Table 17.5.

**17.101. IDENTIFY** and **SET UP:** Use *H* written in terms of the thermal resistance *R*:  $H = A\Delta T/R$ , where R = L/k and  $R = R_1 + R_2 + ...$  (additive).

**EXECUTE:** single pane:  $R_s = R_{glass} + R_{film}$ , where  $R_{film} = 0.15 \text{ m}^2 \cdot \text{K/W}$  is the combined thermal resistance of the air films on the room and outdoor surfaces of the window.

$$R_{\text{glass}} = L/k = (4.2 \times 10^{-3} \text{ m})/(0.80 \text{ W/m} \cdot \text{K}) = 0.00525 \text{ m}^2 \cdot \text{K/W}$$

Thus 
$$R_s = 0.00525 \text{ m}^2 \cdot \text{K/W} + 0.15 \text{ m}^2 \cdot \text{K/W} = 0.1553 \text{ m}^2 \cdot \text{K/W}.$$

double pane:  $R_d = 2R_{glass} + R_{air} + R_{film}$ , where  $R_{air}$  is the thermal resistance of the air space between the

panes. 
$$R_{\text{air}} = L/k = (7.0 \times 10^{-3} \text{ m})/(0.024 \text{ W/m} \cdot \text{K}) = 0.2917 \text{ m}^2 \cdot \text{K/W}$$

Thus 
$$R_{\rm d} = 2(0.00525~{\rm m}^2\cdot{\rm K/W}) + 0.2917~{\rm m}^2\cdot{\rm K/W} + 0.15~{\rm m}^2\cdot{\rm K/W} = 0.4522~{\rm m}^2\cdot{\rm K/W}$$

$$H_s = A\Delta T/R_s$$
,  $H_d = A\Delta T/R_d$ , so  $H_s/H_d = R_d/R_s$  (since A and  $\Delta T$  are same for both)

$$H_{\rm s}/H_{\rm d} = (0.4522 \text{ m}^2 \cdot \text{K/W})/(0.1553 \text{ m}^2 \cdot \text{K/W}) = 2.9$$

**EVALUATE:** The heat loss is about a factor of 3 less for the double-pane window. The increase in *R* for a double pane is due mostly to the thermal resistance of the air space between the panes.

17.102. IDENTIFY: Apply  $H = \frac{kA\Delta T}{L}$  to each rod. Conservation of energy requires that the heat current through

the copper equals the sum of the heat currents through the brass and the steel.

**SET UP:** Denote the quantities for copper, brass and steel by 1, 2, and 3, respectively, and denote the temperature at the junction by  $T_0$ .

**EXECUTE:** (a)  $H_1 = H_2 + H_3$ . Using  $H = kA(T_H - T_C)/L$  and dividing by the common area gives

$$\frac{k_1}{L_1}(100^{\circ}\text{C} - T_0) = \frac{k_2}{L_2}T_0 + \frac{k_3}{L_3}T_0. \text{ Solving for } T_0 \text{ gives } T_0 = \frac{(k_1/L_1)}{(k_1/L_1) + (k_2/L_2) + (k_3/L_3)}(100^{\circ}\text{C}). \text{ Substitution}$$

of numerical values gives  $T_0 = 78.4$ °C.

**(b)** Using  $H = \frac{kA}{L}\Delta T$  for each rod, with  $\Delta T_1 = 21.6$  C°,  $\Delta T_2 = \Delta T_3 = 78.4$  C° gives  $H_1 = 12.8 \text{ W}, H_2 = 9.50 \text{ W} \text{ and } H_3 = 3.30 \text{ W}.$ 

**EVALUATE:** In part (b),  $H_1$  is seen to be the sum of  $H_2$  and  $H_3$ .

17.103. IDENTIFY: At steady state, the heat current is the same in all parts of the composite rod.

**SET UP:** Apply  $H = kA(T_H - T_C)/L$  to each segment of the rod. Let A be the aluminum rod, B the brass, and C the copper.  $T_1$  is the temperature at the brass-copper interface, and  $T_2$  is the temperature at the copper-aluminum interface. The other end of the brass rod is at 100.0°C and the other end of the aluminum rod is at 0.0°C.

EXECUTE: **(a)** and **(b)**  $H_{\rm B} = H_{\rm C}$ :  $\frac{k_{\rm B}A(100^{\circ}{\rm C} - T_1)}{L_{\rm R}} = \frac{k_{\rm C}A(T_1 - T_2)}{L_{\rm C}}$ 

$$H_{\rm B} = H_{\rm A}$$
:  $\frac{k_{\rm B}A(100^{\circ}{\rm C} - T_1)}{L_{\rm B}} = \frac{k_{\rm A}A(T_2 - 0^{\circ}{\rm C})}{L_{\Delta}} = \frac{k_{\rm A}AT_2}{L_{\Delta}}$ 

 $H_{\rm B}=H_{\rm A}; \quad \frac{k_{\rm B}A(100^{\circ}{\rm C}-T_{\rm I})}{L_{\rm B}}=\frac{k_{\rm A}A(T_2-0^{\circ}{\rm C})}{L_{\rm A}}=\frac{k_{\rm A}AT_2}{L_{\rm A}}$  Cancel the areas A and put in the following numbers:  $k_{\rm A}=205~{\rm W/m\cdot K},~k_{\rm B}=109~{\rm W/m\cdot K},~k_{\rm C}=385~{\rm W/m\cdot K},$  $L_{\rm A} = 24.0 \text{ cm} = 0.240 \text{ m}, L_{\rm B} = 12.0 \text{ cm} = 0.120 \text{ m}, L_{\rm C} = 18.0 \text{ cm} = 0.180 \text{ m}$ . Solving the two heat current equations simultaneously gives  $T_1 = 59.809$ °C, which rounds to 59.8°C, and  $T_2 = 42.740$ °C, which rounds to 42.7°C.

(c) For the aluminum section,  $H = \frac{k_A A T_2}{L_A}$ . Putting in the numbers and temperature from (b) gives

 $H_A = (2.30 \text{ cm}^2)(1 \text{ m}/100 \text{ cm})^2(205 \text{ W/m} \cdot \text{K}) (42.740^{\circ}\text{C})/(0.240 \text{ m}) = 8.40 \text{ J/s} = 8.40 \text{ W}.$ 

EVALUATE: As a check, we can calculate the heat current in the copper and brass to see if they agree with our answer in (c). For copper we have

$$H_{\rm C} = \frac{k_{\rm C}A(T_1 - T_2)}{L_{\rm C}} = (385 \text{ W/m} \cdot \text{K})(2.30 \times 10^{-4} \text{ m}^2)(59.809^{\circ}\text{C} - 42.740^{\circ}\text{C})/(0.180 \text{ m}) = 8.40 \text{ W, which}$$

agrees with our answer in (c). As a double check, we could also do the brass.

17.104. IDENTIFY: The nonmechanical part of the basal metabolic rate (i.e., the heat) leaves the body by radiation from the surface.

**SET UP:** In the radiation equation,  $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$ , the temperatures must be in kelvins; e = 1.0, T = 30°C = 303 K, and  $T_s = 18$ °C = 291 K. Call the basal metabolic rate BMR.

**EXECUTE:** (a)  $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$ .

 $H_{\text{net}} = (2.0 \text{ m}^2)(1.0)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)([303 \text{ K}]^4 - [291 \text{ K}]^4) = 140 \text{ W}.$ 

**(b)** (0.80)BMR = 140 W, so BMR = 180 W.

**EVALUATE:** If the emissivity of the skin were less than 1.0, the body would radiate less so the BMR would have to be lower than we found in (b).

- 17.105. (a) IDENTIFY and EXECUTE: Heat must be conducted from the water to cool it to 0°C and to cause the phase transition. The entire volume of water is not at the phase transition temperature, just the upper surface that is in contact with the ice sheet.
  - **(b) IDENTIFY:** The heat that must leave the water in order for it to freeze must be conducted through the layer of ice that has already been formed.

**SET UP:** Consider a section of ice that has area A. At time t let the thickness be h. Consider a short time interval t to t + dt. Let the thickness that freezes in this time be dh. The mass of the section that freezes in the time interval dt is  $dm = \rho dV = \rho A dh$ . The heat that must be conducted away from this mass of water to freeze it is  $dQ = dmL_f = (\rho AL_f)dh$ .  $H = dQ/dt = kA(\Delta T/h)$ , so the heat dQ conducted in time dt

throughout the thickness h that is already there is  $dQ = kA \left( \frac{T_{\rm H} - T_{\rm C}}{h} \right) dt$ . Solve for dh in terms of dt and

integrate to get an expression relating h and t.

**EXECUTE:** Equate these expressions for dQ.

$$\rho A L_{\rm f} dh = kA \left(\frac{T_{\rm H} - T_{\rm C}}{h}\right) dt$$

$$h dh = \left(\frac{k(T_{\rm H} - T_{\rm C})}{\rho L_{\rm f}}\right) dt$$

Integrate from t = 0 to time t. At t = 0 the thickness h is zero.

$$\int_{0}^{h} h \, dh = \left[ k (T_{\rm H} - T_{\rm C}) / \rho L_{\rm f} \right] \int_{0}^{t} dt$$

$$\frac{1}{2}h^2 = \frac{k(T_{\rm H} - T_{\rm C})}{\rho L_{\rm f}}t$$
 and  $h = \sqrt{\frac{2k(T_{\rm H} - T_{\rm C})}{\rho L_{\rm f}}}\sqrt{t}$ 

The thickness after time t is proportional to  $\sqrt{t}$ .

- (c) The expression in part (b) gives  $t = \frac{h^2 \rho L_{\rm f}}{2k(T_{\rm H} T_{\rm C})} = \frac{(0.25 \text{ m})^2 (920 \text{ kg/m}^3)(334 \times 10^3 \text{ J/kg})}{2(1.6 \text{ W/m} \cdot \text{K})(0^{\circ}\text{C} (-10^{\circ}\text{C}))} = 6.0 \times 10^5 \text{ s}$ t = 170 h.
- (d) Find t for h = 40 m. t is proportional to  $h^2$ , so  $t = (40 \text{ m}/0.25 \text{ m})^2 (6.00 \times 10^5 \text{ s}) = 1.5 \times 10^{10} \text{ s}$ . This is about 500 years. With our current climate this will not happen.

**EVALUATE:** As the ice sheet gets thicker, the rate of heat conduction through it decreases. Part (d) shows that it takes a very long time for a moderately deep lake to totally freeze.

**17.106. IDENTIFY:**  $I_1 r_1^2 = I_2 r_2^2$ . Apply  $H = Ae\sigma T^4$  to the sun.

**SET UP:**  $I_1 = 1.50 \times 10^3 \text{ W/m}^2 \text{ when } r = 1.50 \times 10^{11} \text{ m}.$ 

**EXECUTE:** (a) The energy flux at the surface of the sun is

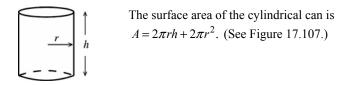
$$I_2 = (1.50 \times 10^3 \text{ W/m}^2) \left( \frac{1.50 \times 10^{11} \text{ m}}{6.96 \times 10^8 \text{ m}} \right)^2 = 6.97 \times 10^7 \text{ W/m}^2.$$

**(b)** Solving Eq. (17.25) with 
$$e = 1$$
,  $T = \left[ \frac{H}{A} \frac{1}{\sigma} \right]^{\frac{1}{4}} = \left[ \frac{6.97 \times 10^7 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \right]^{\frac{1}{4}} = 5920 \text{ K}.$ 

**EVALUATE:** The total power output of the sun is  $P = 4\pi r_2^2 I_2 = 4 \times 10^{26}$  W.

**17.107. IDENTIFY** and **SET UP:** Use  $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$  to find the net heat current into the can due to radiation. Use Q = Ht to find the heat that goes into the liquid helium, set this equal to mL and solve for the mass m of helium that changes phase.

**EXECUTE:** Calculate the net rate of radiation of heat from the can.  $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$ .



**Figure 17.107** 

$$A = 2\pi r (h+r) = 2\pi (0.045 \text{ m})(0.250 \text{ m} + 0.045 \text{ m}) = 0.08341 \text{ m}^2.$$
  
$$H_{\text{net}} = (0.08341 \text{ m}^2)(0.200)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)((4.22 \text{ K})^4 - (77.3 \text{ K})^4)$$

 $H_{\text{net}} = -0.0338 \text{ W}$  (the minus sign says that the net heat current is into the can). The heat that is put into the can by radiation in one hour is  $Q = -(H_{\text{net}})t = (0.0338 \text{ W})(3600 \text{ s}) = 121.7 \text{ J}$ . This heat boils a mass m

of helium according to the equation 
$$Q = mL_f$$
, so  $m = \frac{Q}{L_f} = \frac{121.7 \text{ J}}{2.09 \times 10^4 \text{ J/kg}} = 5.82 \times 10^{-3} \text{ kg} = 5.82 \text{ g}.$ 

**EVALUATE:** In the expression for the net heat current into the can the temperature of the surroundings is raised to the fourth power. The rate at which the helium boils away increases by about a factor of  $(293/77)^4 = 210$  if the walls surrounding the can are at room temperature rather than at the temperature of the liquid nitrogen.

**17.108. IDENTIFY:** We have blackbody radiation. The sphere at 41.0°C (314 K) radiates into the box, but the box at 30.0°C (303 K) radiates back into the sphere. All temperatures must be in kelvins.

**SET UP:** The rate at which heat is radiated by a blackbody in surroundings is  $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$ . The target variable is the emissivity e for part (a). We know that  $H_{\text{net}} = 0.660$  J/s for the sphere, and  $A = 4\pi r^2$  for a sphere.

**EXECUTE:** (a) Using  $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$ , we put in the numbers and solve for the emissivity *e*. Using  $H_{\text{net}} = 0.660 \text{ J/s}$ ,  $A = 4\pi r^2 = 4\pi (0.0320 \text{ m})^2 = 0.012868 \text{ m}^2$ , T = 314 K,  $T_s = 303 \text{ K}$ , and  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ , we get e = 0.700. Note that this number has no units.

**(b)** Using  $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$ , we get

$$H = (0.700)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.012868 \text{ m}^2)[(355 \text{ K})^4 - (303 \text{ K})^4] = 3.81 \text{ J/s} = 3.81 \text{ W}.$$

**EVALUATE:** The ratio of powers is (3.81 W)/(0.660 W) = 5.77. This is much less than  $2^4$ , which is 16. The temperature must be in kelvins, so a temperature of 82°C (355 K) is not double 41°C (314 K). The temperature ratio is only 355/314 = 1.13.

**17.109. IDENTIFY:** The jogger radiates heat but the air radiates heat back into the jogger.

**SET UP:** The emissivity of a human body is taken to be 1.0. In the equation for the radiation heat current,  $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$ , the temperatures must be in kelvins.

**EXECUTE:** (a)  $P_{\text{jog}} = (0.80)(1300 \text{ W}) = 1.04 \times 10^3 \text{ J/s}.$ 

**(b)**  $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$ , which gives

 $H_{\text{net}} = (1.85 \text{ m}^2)(1.00)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)([306 \text{ K}]^4 - [313 \text{ K}]^4) = -87.1 \text{ W}$ . The person gains 87.1 J of heat each second by radiation.

(c) The total excess heat per second is 1040 J/s + 87 J/s = 1130 J/s.

(d) In 1 min = 60 s, the runner must dispose of  $(60 \text{ s})(1130 \text{ J/s}) = 6.78 \times 10^4 \text{ J}$ . If this much heat goes to evaporate water, the mass m of water that evaporates in one minute is given by  $Q = mL_v$ , so

$$m = \frac{Q}{L_v} = \frac{6.78 \times 10^4 \text{ J}}{2.42 \times 10^6 \text{ J/kg}} = 0.028 \text{ kg} = 28 \text{ g}.$$

(e) In a half-hour, or 30 minutes, the runner loses (30 min)(0.028 kg/min) = 0.84 kg. The runner must

drink 0.84 L, which is 
$$\frac{0.84 \text{ L}}{0.750 \text{ L/bottle}} = 1.1 \text{ bottles}.$$

**EVALUATE:** The person *gains* heat by radiation since the air temperature is greater than his skin temperature.

**17.110. IDENTIFY:** Heat  $Q_i$  goes into the ice when it warms to 0°C, melts, and the resulting water warms to the final temperature  $T_f$ . Heat  $Q_{ow}$  comes out of the ocean water when it cools to  $T_f$ . Conservation of energy gives  $Q_i + Q_{ow} = 0$ .

SET UP: For ice,  $c_i = 2100 \text{ J/kg} \cdot \text{K}$ . For water,  $L_f = 334 \times 10^3 \text{ J/kg}$  and  $c_w = 4190 \text{ J/kg} \cdot \text{K}$ . Let m be the total mass of the water on the earth's surface. So  $m_i = 0.0175m$  and  $m_{ow} = 0.975m$ .

**EXECUTE:** 
$$Q_i + Q_{ow} = 0$$
 gives  $m_i c_i (30 \text{ C}^\circ) + m_i L_f + m_i c_w T_f + m_{ow} c_w (T_f - 5.00^\circ \text{C}) = 0$ .

$$T_{\rm f} = \frac{-m_{\rm i} c_{\rm i}(30~{\rm C}^\circ) - m_{\rm i} L_{\rm f} + m_{\rm ow} c_{\rm w}(5.00~{\rm C}^\circ)}{(m_{\rm i} + m_{\rm ow}) c_{\rm w}}.$$

$$T_{\rm f} = \frac{-(0.0175m)(2100 \text{ J/kg} \cdot \text{K})(30 \text{ K}) - (0.0175m)(334 \times 10^3 \text{ J/kg}) + (0.975m)(4190 \text{ J/kg} \cdot \text{K})(5.00 \text{ K})}{(0.0175m + 0.975m)(4190 \text{ J/kg} \cdot \text{K})}$$

$$T_{\rm f} = \frac{1.348 \times 10^4 \text{ J/kg}}{4.159 \times 10^3 \text{ J/kg} \cdot \text{K}} = 3.24^{\circ}\text{C}. \text{ The temperature decrease is } 1.76 \text{ C}^{\circ}.$$

$$T_{\rm f} = \frac{1.348 \times 10^4 \text{ J/kg}}{4.159 \times 10^3 \text{ J/kg} \cdot \text{K}} = 3.24 \text{°C}$$
. The temperature decrease is 1.76 °C.

**EVALUATE:** The mass of ice in the icecaps is much less than the mass of the water in the oceans, but much more heat is required to change the phase of 1 kg of ice than to change the temperature of 1 kg of water 1 C°, so the lowering of the temperature of the oceans would be appreciable.

17.111. IDENTIFY: The latent heat of fusion  $L_f$  is defined by  $Q = mL_f$  for the solid  $\rightarrow$  liquid phase transition. For a temperature change,  $Q = mc\Delta T$ .

SET UP: At t = 1 min the sample is at its melting point and at t = 2.5 min all the sample has melted. **EXECUTE:** (a) It takes 1.5 min for all the sample to melt once its melting point is reached and the heat input during this time interval is  $(1.5 \text{ min})(10.0 \times 10^3 \text{ J/min}) = 1.50 \times 10^4 \text{ J}$ .  $Q = mL_f$ .

$$L_{\rm f} = \frac{Q}{m} = \frac{1.50 \times 10^4 \text{ J}}{0.500 \text{ kg}} = 3.00 \times 10^4 \text{ J/kg}.$$

**(b)** The liquid's temperature rises 30 °C in 1.5 min.  $Q = mc\Delta T$ .

$$c_{\text{liquid}} = \frac{Q}{m\Delta T} = \frac{1.50 \times 10^4 \text{ J}}{(0.500 \text{ kg})(30 \text{ C}^\circ)} = 1.00 \times 10^3 \text{ J/kg} \cdot \text{K}.$$

The solid's temperature rises 15 °C in 1.0 min. 
$$c_{\text{solid}} = \frac{Q}{m\Delta T} = \frac{1.00 \times 10^4 \text{ J}}{(0.500 \text{ kg})(15 \text{ C}^\circ)} = 1.33 \times 10^3 \text{ J/kg} \cdot \text{K}.$$

**EVALUATE:** The specific heat capacities for the liquid and solid states are different. The values of c and  $L_{\rm f}$  that we calculated are within the range of values in Tables 17.3 and 17.4.

17.112. IDENTIFY: The heat lost by the water is equal to the heat gained by the liquid and the cup. The specific heat capacities do not change over the temperature ranges involved. No phase changes are involved. SET UP:  $Q = mc \Delta T$ , and  $Q_L + Q_m + Q_w = 0$ , where L is for the liquid, m is for the metal, and w is for

**EXECUTE:** For the first experiment, 0.500 kg of the liquid are used.  $Q_L + Q_m + Q_w = 0$  gives  $0 = (0.500 \text{ kg})c_1(58.1^{\circ}\text{C} - 20.0^{\circ}\text{C}) + (0.200 \text{ kg})c_m(58.1^{\circ}\text{C} - 20.0^{\circ}\text{C})$  $+ (0.500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(58.1^{\circ}\text{C} - 80.0^{\circ}\text{C})$ 

For the second experiment, 1.00 kg of the liquid is used.  $Q_L + Q_m + Q_w = 0$  gives  $0 = (1.00 \text{ kg})c_1(49.3^{\circ}\text{C} - 20.0^{\circ}\text{C}) + (0.200 \text{ kg})c_m(49.3^{\circ}\text{C} - 20.0^{\circ}\text{C})$  $+ (0.500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(49.3^{\circ}\text{C} - 80.0^{\circ}\text{C})$ 

The two equations from the two experiments simplify to

 $19.050c_{\rm L} + 7.620c_{\rm m} = 45,880.5$ 

$$29.3c_L + 5.86c_m = 64{,}316.5$$

Solving these two equations simultaneously gives  $c_m = 1067 \text{ J/kg} \cdot \text{K}$ , which rounds to 1070 J/kg · K, and  $c_L = 1981.6 \text{ J/kg} \cdot \text{K}$ , which rounds to 1980 J/kg · K.

**EVALUATE:** The liquid has about half the specific heat capacity of water. The metal has a specific heat capacity of 1070 J/kg·K, which is a bit more than that of aluminum. So both answers are physically reasonable.

17.113. IDENTIFY: At steady state, the heat current in both bars is the same when they are connected end-to-end. The heat to melt the ice is the heat conducted through the bars.

**SET UP:** 
$$Q = mL$$
 and  $H = kA \frac{T_{\rm H} - T_{\rm C}}{L}$ .

**EXECUTE:** With bar *A* alone: 0.109 kg of ice melts in 45.0 min = (45.0)(60) s. Therefore the heat current is  $H = mL_{g}/t = (0.109 \text{ kg})(334 \times 10^{5} \text{ J/kg})/[(45.0)(60) \text{ s}] = 13.48 \text{ J/s} = 13.48 \text{ W}$ . Applying this result to the

heat flow in bar A gives  $H = kA \frac{T_H - T_C}{L}$ . Solving for  $k_A$  gives  $k_A = HL/A(T_H - T_C)$ . Numerically we get

 $k_A = (13.48 \text{ W})(0.400 \text{ m})/[(2.50 \times 10^{-4} \text{ m}^2)(100 \text{ C}^\circ)] = 215.7 \text{ W/m} \cdot \text{K}$ , which rounds to 216 W/m·K. With the two bars end-to-end: The heat current is the same in both bars, so  $H_A = H_B$ . Using

$$H = kA \frac{T_{\rm H} - T_{\rm C}}{L} \text{ for each bar, we get } \frac{k_{\rm A}A(100^{\circ}{\rm C} - 62.4^{\circ}{\rm C})}{L} = \frac{k_{\rm B}A(62.4^{\circ}{\rm C} - 0^{\circ}{\rm C})}{L}. \text{ Using our result for } k_{\rm A} = \frac{k_{\rm B}A(62.4^{\circ}{\rm C} - 0^{\circ}{\rm C})}{L}$$

and canceling A and L, we get  $k_B = 130 \text{ W/m} \cdot \text{K}$ 

**EVALUATE:**  $k_A = 216$  W/m·K, which is slightly larger than that of aluminum, and  $k_B = 130$  W/m·K, which is between that of aluminum and brass. Therefore these results are physically reasonable.

**17.114. IDENTIFY:** The rate in (iv) is given by  $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$ , with T = 309 K and  $T_s = 320 \text{ K}$ . The heat absorbed in the evaporation of water is Q = mL.

**SET UP:** 
$$m = \rho V$$
, so  $\frac{m}{V} = \rho$ .

EXECUTE: (a) The rates are: (i) 280 W,

- (ii)  $(54 \text{ J/h} \cdot \text{C}^{\circ} \cdot \text{m}^2)(1.5 \text{ m}^2)(11 \text{ C}^{\circ})/(3600 \text{ s/h}) = 0.248 \text{ W},$
- (iii)  $(1400 \text{ W/m}^2)(1.5 \text{ m}^2) = 2.10 \times 10^3 \text{ W},$
- (iv)  $(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.5 \text{ m}^2)((320 \text{ K})^4 (309 \text{ K})^4) = 116 \text{ W}.$

The total is 2.50 kW, with the largest portion due to radiation from the sun.

**(b)** 
$$\frac{P}{\rho L_{\rm v}} = \frac{2.50 \times 10^3 \text{ W}}{(1000 \text{ kg/m}^3)(2.42 \times 10^6 \text{ J/kg} \cdot \text{K})} = 1.03 \times 10^{-6} \text{ m}^3/\text{s}$$
. This is equal to 3.72 L/h.

(c) Redoing the above calculations with e = 0 and the decreased area gives a power of 945 W and a corresponding evaporation rate of 1.4 L/h. Wearing reflective clothing helps a good deal. Large areas of loose-weave clothing also facilitate evaporation.

**EVALUATE:** The radiant energy from the sun absorbed by the area covered by clothing is assumed to be zero, since  $e \approx 0$  for the clothing and the clothing reflects almost all the radiant energy incident on it. For the same reason, the exposed skin area is the area used in applying  $H_{\text{net}} = e\sigma A(T^4 - T_{\text{s}}^4)$ .

**17.115. IDENTIFY:** Apply the equation  $H = kA(T_H - T_C)/L$ . For a cylindrical surface, the area A in this equation is a function of the distance r from the central axis, and the material must be considered as a series of shells with thickness dr and a temperature difference dT between the inside and outside of the shell. The heat current will be a constant, and must be found by integrating a differential equation.

**SET UP:** The surface area of the curved side of a cylinder is  $2\pi rL$ . When x = 1,  $\ln(1+x) \approx x$ .

**EXECUTE:** (a) For a cylindrical shell,  $H = kA(T_{\rm H} - T_{\rm C})/L$  becomes  $H = k(2\pi rL)\frac{dT}{dr}$  or  $\frac{Hdr}{2\pi r} = kLdT$ .

Between the limits r = a and r = b, this integrates to  $\frac{H}{2\pi} \ln(b/a) = kL(T_2 - T_1)$ , or  $H = \frac{2\pi k L(T_2 - T_1)}{\ln(b/a)}$ .

**(b)** Using 
$$H = k(2\pi rL)\frac{dT}{dr}$$
 from part (a), we integrate:  $\int_{a}^{r} \frac{Hdr}{k2\pi rL} = -\int_{T_2}^{T} dT$ , which gives

 $\frac{H}{2\pi kL}\ln(r/a) = -(T - T_2) = T_2 - T.$  Solving for T and using H from part (a) gives

$$T = T_2 - \frac{H}{2\pi kL} \ln(r/a) = \frac{(T_2 - T_1)2\pi kL \ln(r/a)}{2\pi kL \ln(b/a)} = T_2 - \frac{(T_2 - T_1)\ln(r/a)}{\ln(b/a)}, \text{ which can also be written as}$$

$$T(r) = T_2 + (T_1 - T_2) \frac{\ln(r/a)}{\ln(b/a)}.$$

(c) For a thin-walled cylinder, 
$$a \approx b$$
, so  $\frac{b-a}{a} \ll 1$ . We can write  $\frac{b}{a} = 1 + \frac{b-a}{a}$ , and the log term becomes

$$\ln\left(\frac{b}{a}\right) = \ln\left(1 + \frac{b-a}{a}\right) \approx \frac{b-a}{a}$$
 using  $\ln(1+x) \approx x$  for  $x << 1$ . Therefore the rate of heat flow becomes

$$H = \frac{(T_2 - T_1)2\pi kL}{\frac{b - a}{a}} = k(2\pi aL)\frac{T_2 - T_1}{b - a}.$$
 This is equivalent to Eq. (17.21) in which  $A = 2\pi aL$ , which is the

surface area of the thin cylindrical shell of radius a and length L.

(d) For steady-state heat flow, the rate of flow through the cork is the same as through the Styrofoam.

Using our result from part (a), we have  $H_C = H_S$ , which gives

$$(140^{\circ}\text{C} - T)2\pi k_{\text{C}}L/(\ln 4/2) = (T - 15^{\circ}\text{C})2\pi k_{\text{S}}L/(\ln 6/4)$$

Cancelling  $2\pi L$  and using the given values for  $k_{\rm C}$  and  $k_{\rm S}$ , we get  $T=73\,^{\circ}{\rm C}$ .

(e) Use the result of part (a) for H.

$$H = \frac{(140^{\circ}\text{C} - 73^{\circ}\text{C})2\pi (0.0400 \text{ W/m} \cdot \text{K})(2.00 \text{ m})}{\ln(4/2)} = 49 \text{ W}.$$

**EVALUATE:** As a check, calculate *H* in the Styrofoam.  $\Delta T = 73^{\circ}\text{C} - 15^{\circ}\text{C} = 58 \text{ C}^{\circ}$ .

$$H_{\rm S} = \frac{2\pi (2.00 \text{ m})(0.027 \text{ W/m} \cdot \text{K})}{\ln(6.00/4.00)} (58 \text{ C}^{\circ}) = 49 \text{ W}$$
. This is the same as we just found, which it should be

for steady-state flow.

**17.116. IDENTIFY:** The cryoprotectant must cool from 22°C to -20°C, then it must freeze, and finally it must cool from -20°C to 77 K (-196°C).

**SET UP:** We use 
$$Q = mc\Delta T$$
 and  $Q = mL_f$ .  $Q_{tot} = Q_1 + Q_2 + Q_3$ .

EXECUTE: 
$$Q_1 = mc\Delta T = (35 \times 10^{-3} \text{ kg})(4500 \text{ J/kg} \cdot \text{K})(42 \text{ C}^\circ) = 6.6 \text{ kJ}$$

$$Q_2 = mL_f = (35 \times 10^{-3} \text{ kg})(2.80 \times 10^5 \text{ J/kg}) = 9.8 \text{ kJ}$$

$$Q_3 = mc\Delta T = (35 \times 10^{-3} \text{ kg})(2000 \text{ J/kg} \cdot \text{K})[-196^{\circ}\text{C} - (-20^{\circ}\text{C})] = 12.3 \text{ kJ}$$

$$Q_{\text{tot}} = 6.6 \text{ kJ} + 9.8 \text{ kJ} + 12.3 \text{ kJ} = 28.7 \text{ kJ} \approx 29 \text{ kJ} = 2.9 \times 10^4 \text{ J}$$
, which is choice (b).

**EVALUATE:** This is a large amount of heat for such a small sample, but it must change its temperature from 22°C to –196°C and also freeze.

17.117. IDENTIFY and SET UP: The graph shows that the specific heat of the solid decreases with temperature, so its average value is less than the  $2.0 \times 10^3$  J/kg·K shown in the table.

**EXECUTE:** Since the average specific heat is less than the value in the table, less heat will need to come out of it to bring it into equilibrium with the cold plate. Therefore a shorter time will be needed for it to come to equilibrium with the cold plate, which is choice (a).

**EVALUATE:** The average value of the specific heat is about 1500 J/kg·K between  $-20^{\circ}$ C and  $-200^{\circ}$ C, so the difference in time could be important enough to be concerned about. As we saw in the previous problem, the heat to cool the cryoprotectant from  $-20^{\circ}$ C down to  $-196^{\circ}$ C was the largest contribution to the total heat.

**17.118. IDENTIFY:** The rate of heat conduction should be the same in both cases, but the area in the second case is 4 times the area in the first case since the linear dimensions are both doubled.

SET UP: 
$$H = kA \frac{T_H - T_C}{I}$$
,  $H_1 = H_2$ ,  $A_2 = 4A_1$ ,  $k$  and  $T_H - T_C$  are the same in both cases.

**EXECUTE:** Calculate 
$$H_1$$
 and  $H_2$  and equate them:  $kA_1 \frac{\Delta T}{L_1} = kA_2 \frac{\Delta T}{L_2} = k(4A_1) \frac{\Delta T}{L_2}$ . Cancelling common

factors gives  $L_2 = 4L_1$ , so the layer should be 4 times as thick as it was for the smaller plates, which is choice (d).

**EVALUATE:** If we doubled only one dimension, then  $L_2$  would have to be only twice as thick as for the smaller plates.

17.119. IDENTIFY and SET UP: Heat from the cryoprotectant *and* the environment enters the cold plate. You measure the amount of heat that enters the cold plate, and assume that it all came from the cryoprotectant. EXECUTE: You think that more heat entered the plate from the cryoprotectant than actually did so. This will make you think that the specific heat of the cryoprotectant is greater than it actually is, which is choice (a).

## 17-32 Chapter 17

**EVALUATE:** Heat from the environment could also be entering the cryoprotectant, but since the cold plate is on average colder than the cryoprotectant, more heat will enter the cold plate than will enter the cryoprotectant, so the two effects will not cancel each other out. You will still measure a specific heat that is greater than the actual value.