Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JS & SS Mathematics
JS Theoretical Physics
Moderatorship

Trinity Term 2017

MA1242 — Mechanics II

Thursday, May 18 **Exam Hall** **09:30 — 11:30**

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.

All questions have equal weight.

'Formulae & Tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination - please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. A hoop of radius R and mass M rolls on a horizontal surface with constant speed V. The plane of the hoop makes an angle α with the vertical, and the hoop traces out a circle of radius b on the surface. We ignore the difference between the radius of the circle traced out by the contact point between surface and hoop, and the circle traced out by its center of mass.

   (a) Make a sketch of the rolling hoop with the friction, normal and gravitational forces in the sketch. Include also the direction of the spin angular momentum $\vec{L}_s$ due to rotation around the axis through its center of mass and orthogonal to the plane of the hoop.

   (b) Give the equations of motions of the hoop and express the friction force f in terms of M, V and b. Determine also the magnitude of the torque τ of the forces with respect to the center of mass of the hoop.

   (c) Argue that the magnitude of τ should equal

   $$\tau = \cos(\alpha)\, L_s\, \Omega,$$

   where $L_s = |\vec{L}_s|$ and Ω is the angular speed of the motion around the circle with radius b.

   (d) Express $\tan(\alpha)$ in terms of V, b and g.

2. At the equator, one throws a particle vertically upwards with initial speed v. The speed v is small such that the maximal height h of the particle above the surface of the earth is very small compared to the radius R_e of the earth. Recall the formula for the Coriolis force

$$\vec{F}_{\text{Cor}} = -2m\vec{\Omega} \times \vec{v}, \tag{1}$$

where $\vec{\Omega}$ is the angular velocity of the rotating coordinate system, and $\vec{v}$ is the velocity relative to the rotating coordinate system.

   (a) Define suitable polar coordinates (r, θ). Show that the $\hat{\theta}$ component of the equation of motion of the particle is equivalent to

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -2\Omega\dot{r}$$

   (b) Give $\dot{r}$ as a function of time t. Take $t = 0$ at the time of the throw. Assuming that r is well approximated by R_e, and that $\dot{\theta} \ll \Omega$, express $\ddot{\theta}$ as a function of t.

   (c) Solve for θ as a function of t, using the expression found under (b).

   (d) Determine the distance d between the point where the particle departs from the surface of the earth and the point where it returns to the surface of the earth. In which direction is the displacement?

Hint: The acceleration in polar coordinates is given by:

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}.$$

3. The Great Comet of 1997 (or Comet Hale-Bopp) passed its perihelion on April 1st 1997 at a distance $r_- = 0.194$ Astronomical Units (AU) from the center of the Sun. To describe its motion in a simple model, we assume that the comet only experiences the gravitational attraction from the Sun, and that its mass is very small compared to that of the Sun. The speed of the comet in the perihelion is given as:

$$v_0 = \alpha\sqrt{\frac{GM}{r_-}},$$

where α is a positive constant, G is the gravitational constant and M is the mass of the Sun.

(a) Determine in terms of M, m, G and r_-:

i. the magnitude of the angular momentum of the comet (with respect to the origin $r = 0$),

ii. and the total mechanical energy (take the potential energy $U(r)$ such that $\lim_{r\to\infty} U(r) = 0$).

(b) Determine the constants ε and r_0 in the equation of the trajectory,

$$r = \frac{r_0}{1 - \varepsilon\cos(\theta)},$$

in terms of α and r_-.

(c) Assume $\alpha = 1.4125$. Determine the orbital period of the Comet.

Some useful constants:

Astronomical Unit: 149.6 10^9 m

Mass Sun: 1.989 10^{30} kg

Gravitational constant: 6.674 10^{-11} m^3 kg^{-1} s^{-2}