Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Mathematics
JF Theoretical Physics

Trinity Term 2018

MA1241 — Mechanics I

Friday, May 4 **Sports Centre** **9:30 — 11:30**

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.

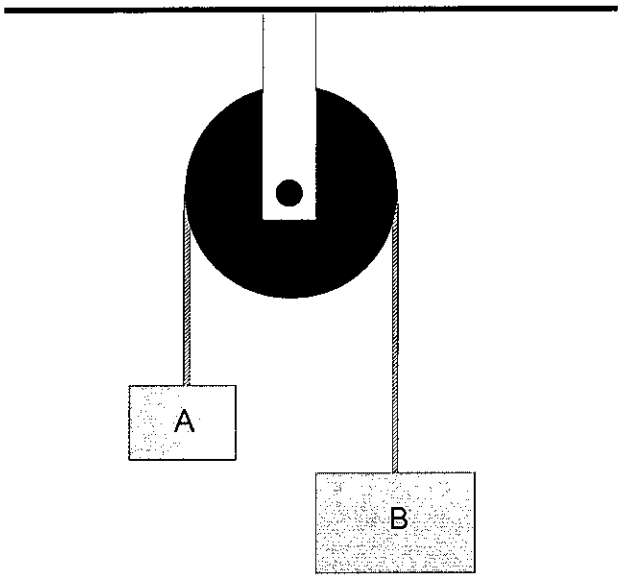
All questions have equal weight.

'Formulae & Tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination - please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Consider the Atwood machine in the figure below. The masses of the two blocks A and B are respectively M_A and M_B. The radius of the pulley is R and moves without friction.



(a) Assume that the mass of the pulley is negligible. Determine the tension in the rope in terms of the masses and the gravitational acceleration g.

(b) Determine also the acceleration of block B, assuming that the mass of the pulley is negligible.

(c) Consider now a massive pulley of mass M_{p}, and assume that the pulley can be well approximated by a disk. Give the equation for the angular acceleration of the pulley in terms of R, M_p and the tensions in the left and right side of the rope.

(d) Give the acceleration of block B in terms of the masses and g.

Hint: The moment of inertia of a disk with radius R and mass M with respect to its central, orthogonal axis is $I = \frac{1}{2}MR^2$.

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2. A solid cylinder is thrown on a surface with speed v_0 orthogonal to its longitudinal axis. Initially it slides without rolling, but due to friction it begins to roll.

   (a) List which of the following quantities is conserved for the cylinder:

      i. momentum,

      ii. angular momentum w.r.t. a point O on the surface,

      iii. energy.

      Motivate your answer.

   (b) Determine its speed when the cylinder rolls without sliding.

   (c) How much of the initial kinetic energy is dissipated by the friction force?

   Hint: Recall that the moment of inertia I_0 of a solid cylinder (mass M, radius R) with respect to its longitudinal axis, equals $\frac{1}{2}MR^2$.

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3. Let a tautochrone curve (inverted cycloid) be parametrized by

$$x(s) = \frac{\ell}{4}(s + \sin(s)), \qquad z(s) = \frac{\ell}{4}(1 - \cos(s)),$$

with $s \in [-\pi, \pi]$. A particle of mass m moves along the tautochrone curve under the influence of the gravitational force $-mg\,\hat{\mathrm{k}}$. The maximum value of s for the trajectory of the particle is s_0.

(a) Give the total mechanical energy of the particle.

(b) Determine the kinetic and potential energy of the particle in terms of $\dot{w}(t)$ and/or $w(t)$, with $w(t) = \ell \sin(s(t)/2)$

(c) Show that the motion of the particle is periodic and that the period T is independent of s_0.

(d) Determine T.

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