CAPACITANCE AND DIELECTRICS

24.1. **IDENTIFY:** The capacitance depends on the geometry (area and plate separation) of the plates.

SET UP: For a parallel-plate capacitor, $V_{ab} = Ed$, $E = \dfrac{Q}{\varepsilon_0 A}$, and $C = \dfrac{Q}{V_{ab}}$.

EXECUTE: (a) $V_{ab} = Ed = (4.00 \times 10^6 \text{ V/m})(2.50 \times 10^{-3} \text{ m}) = 1.00 \times 10^4 \text{ V}$.

(b) Solving for the area gives

$$A = \frac{Q}{E\varepsilon_0} = \frac{80.0 \times 10^{-9} \text{ C}}{(4.00 \times 10^6 \text{ V/m})[8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)]} = 2.26 \times 10^{-3} \text{ m}^2 = 22.6 \text{ cm}^2.$$

(c) $C = \dfrac{Q}{V_{ab}} = \dfrac{80.0 \times 10^{-9} \text{ C}}{1.00 \times 10^4 \text{ V}} = 8.00 \times 10^{-12} \text{ F} = 8.00 \text{ pF}.$

EVALUATE: The capacitance is reasonable for laboratory capacitors, but the area is rather large.

24.2. **IDENTIFY and SET UP:** $C = \dfrac{\varepsilon_0 A}{d}$, $C = \dfrac{Q}{V}$ and $V = Ed$.

EXECUTE: (a) $C = \varepsilon_0 \dfrac{A}{d} = \varepsilon_0 \dfrac{0.000982 \text{ m}^2}{0.00328 \text{ m}} = 2.65 \text{ pF}.$

(b) $V = \dfrac{Q}{C} = \dfrac{4.35 \times 10^{-8} \text{ C}}{2.65 \times 10^{-12} \text{ F}} = 16.4 \text{ kV}.$

(c) $E = \dfrac{V}{d} = \dfrac{16.4 \times 10^3 \text{ V}}{0.00328 \text{ m}} = 5.00 \times 10^6 \text{ V/m}.$

EVALUATE: The electric field is uniform between the plates, at points that aren't close to the edges.

24.3. **IDENTIFY and SET UP:** It is a parallel-plate air capacitor, so we can apply the equations of Section 24.1.

EXECUTE: (a) $C = \dfrac{Q}{V_{ab}}$ so $V_{ab} = \dfrac{Q}{C} = \dfrac{0.148 \times 10^{-6} \text{ C}}{245 \times 10^{-12} \text{ F}} = 604 \text{ V}.$

(b) $C = \dfrac{\varepsilon_0 A}{d}$ so $A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(245 \times 10^{-12} \text{ F})(0.328 \times 10^{-3} \text{ m})}{8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 9.08 \times 10^{-3} \text{ m}^2 = 90.8 \text{ cm}^2.$

(c) $V_{ab} = Ed$ so $E = \dfrac{V_{ab}}{d} = \dfrac{604 \text{ V}}{0.328 \times 10^{-3} \text{ m}} = 1.84 \times 10^6 \text{ V/m}.$

(d) $E = \dfrac{\sigma}{\varepsilon_0}$ so $\sigma = E\varepsilon_0 = (1.84 \times 10^6 \text{ V/m})(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 1.63 \times 10^{-5} \text{ C/m}^2.$

EVALUATE: We could also calculate σ directly as Q/A. $\sigma = \dfrac{Q}{A} = \dfrac{0.148 \times 10^{-6} \text{ C}}{9.08 \times 10^{-3} \text{ m}^2} = 1.63 \times 10^{-5} \text{ C/m}^2$,

which checks.

24.4. **IDENTIFY:** $C = \varepsilon_0 \dfrac{A}{d}$ when there is air between the plates.

SET UP: $A = (3.0 \times 10^{-2} \text{ m})^2$ is the area of each plate.

EXECUTE: $C = \dfrac{(8.854 \times 10^{-12} \text{ F/m})(3.0 \times 10^{-2} \text{ m})^2}{5.0 \times 10^{-3} \text{ m}} = 1.59 \times 10^{-12} \text{ F} = 1.59 \text{ pF}.$

EVALUATE: C increases when A increases and C increases when d decreases.

24.5. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\varepsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0$ V.

EXECUTE: **(a)** $Q = CV_{ab} = (10.0 \times 10^{-6} \text{ F})(12.0 \text{ V}) = 1.20 \times 10^{-4} \text{ C} = 120 \ \mu\text{C}.$

(b) When d is doubled C is halved, so Q is halved. $Q = 60 \ \mu\text{C}.$

(c) If r is doubled, A increases by a factor of 4. C increases by a factor of 4 and Q increases by a factor of 4. $Q = 480 \ \mu\text{C}.$

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.6. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\varepsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, enough charge flows onto the plates to make $V_{ab} = 12.0$ V.

EXECUTE: **(a)** 12.0 V.

(b) (i) When d is doubled, C is halved. $V_{ab} = \dfrac{Q}{C}$ and Q is constant, so V doubles. $V = 24.0$ V.

(ii) When r is doubled, A increases by a factor of 4. V decreases by a factor of 4 and $V = 3.0$ V.

EVALUATE: The electric field between the plates is $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$. $V_{ab} = Ed$. When d is doubled E is unchanged and V doubles. When A is increased by a factor of 4, E decreases by a factor of 4 so V decreases by a factor of 4.

24.7. **IDENTIFY:** The energy stored in a capacitor depends on its capacitance, which in turn depends on its geometry.

SET UP: $C = Q/V$ for any capacitor, and $C = \dfrac{\varepsilon_0 A}{d}$ for a parallel-plate capacitor.

EXECUTE: **(a)** $C = \dfrac{Q}{V} = \dfrac{2.40 \times 10^{-10} \text{ C}}{42.0 \text{ V}} = 5.714 \times 10^{-12} \text{ F}.$ Using $C = \dfrac{\varepsilon_0 A}{d}$ gives

$d = \dfrac{\varepsilon_0 A}{C} = \dfrac{[8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)](6.80 \times 10^{-4} \text{ m}^2)}{5.714 \times 10^{-12} \text{ F}} = 1.05 \text{ mm}.$

(b) $d = 2.10 \times 10^{-3}$ m. $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{5.714 \times 10^{-12} \text{ F}}{2} = 2.857 \times 10^{-12} \text{ F}.$ $V = \dfrac{Q}{C}$, so $V = 2(42.0 \text{ V}) = 84.0$ V.

EVALUATE: Doubling the plate separation halves the capacitance, so twice the potential difference is required to keep the same charge on the plates.

24.8. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $V_{ab} = Ed$. $C = \dfrac{\varepsilon_0 A}{d}$.

SET UP: We want $E = 1.00 \times 10^4$ N/C when $V = 100$ V.

EXECUTE: (a) $d = \dfrac{V_{ab}}{E} = \dfrac{1.00 \times 10^2 \text{ V}}{1.00 \times 10^4 \text{ N/C}} = 1.00 \times 10^{-2} \text{ m} = 1.00 \text{ cm.}$

$A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(5.00 \times 10^{-12} \text{ F})(1.00 \times 10^{-2} \text{ m})}{8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 5.65 \times 10^{-3} \text{ m}^2.$ $A = \pi r^2$ so

$r = \sqrt{\dfrac{A}{\pi}} = 4.24 \times 10^{-2} \text{ m} = 4.24 \text{ cm.}$

(b) $Q = CV_{ab} = (5.00 \times 10^{-12} \text{ F})(1.00 \times 10^2 \text{ V}) = 5.00 \times 10^{-10} \text{ C} = 500 \text{ pC.}$

EVALUATE: $C = \dfrac{\varepsilon_0 A}{d}$. We could have a larger d, along with a larger A, and still achieve the required C

without exceeding the maximum allowed E.

24.9. IDENTIFY: Apply the results of Example 24.4. $C = Q/V$.

SET UP: $r_a = 0.50$ mm, $r_b = 5.00$ mm.

EXECUTE: (a) $C = \dfrac{L 2\pi\varepsilon_0}{\ln(r_b/r_a)} = \dfrac{(0.180 \text{ m})2\pi\varepsilon_0}{\ln(5.00/0.50)} = 4.35 \times 10^{-12} \text{ F.}$

(b) $V = Q/C = (10.0 \times 10^{-12} \text{ C})/(4.35 \times 10^{-12} \text{ F}) = 2.30 \text{ V.}$

EVALUATE: $\dfrac{C}{L} = 24.2$ pF. This value is similar to those in Example 24.4. The capacitance is determined

entirely by the dimensions of the cylinders.

24.10. IDENTIFY: Capacitance depends on the geometry of the object.

(a) **SET UP:** The capacitance of a cylindrical capacitor is $C = \dfrac{2\pi\varepsilon_0 L}{\ln(r_b/r_a)}$. Solving for r_b gives

$r_b = r_a e^{2\pi\varepsilon_0 L/C}.$

EXECUTE: Substituting in the numbers for the exponent gives

$$\dfrac{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.120 \text{ m})}{3.67 \times 10^{-11} \text{ F}} = 0.182.$$

Now use this value to calculate r_b: $r_b = r_a e^{0.182} = (0.250 \text{ cm})e^{0.182} = 0.300 \text{ cm.}$

(b) **SET UP:** For any capacitor, $C = Q/V$ and $\lambda = Q/L$. Combining these equations and substituting the
numbers gives $\lambda = Q/L = CV/L$.

EXECUTE: Numerically we get

$$\lambda = \dfrac{CV}{L} = \dfrac{(3.67 \times 10^{-11} \text{ F})(125 \text{ V})}{0.120 \text{ m}} = 3.82 \times 10^{-8} \text{ C/m} = 38.2 \text{ nC/m.}$$

EVALUATE: The distance between the surfaces of the two cylinders would be only 0.050 cm, which is just
0.50 mm. These cylinders would have to be carefully constructed.

24.11. IDENTIFY: We can use the definition of capacitance to find the capacitance of the capacitor, and then
relate the capacitance to geometry to find the inner radius.
(a) **SET UP:** By the definition of capacitance, $C = Q/V$.

EXECUTE: $C = \dfrac{Q}{V} = \dfrac{3.30 \times 10^{-9} \text{ C}}{2.20 \times 10^2 \text{ V}} = 1.50 \times 10^{-11} \text{ F} = 15.0 \text{ pF.}$

(b) **SET UP:** The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0 \dfrac{r_a r_b}{r_b - r_a}$.

EXECUTE: Solve for r_a and evaluate using $C = 15.0$ pF and $r_b = 4.00$ cm, giving $r_a = 3.09$ cm.

(c) SET UP: We can treat the inner sphere as a point charge located at its center and use Coulomb's law,

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$.

EXECUTE: $E = \dfrac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.30\times10^{-9} \text{ C})}{(0.0309 \text{ m})^2} = 3.12\times10^4 \text{ N/C}$.

EVALUATE: Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

24.12. IDENTIFY and SET UP: Use $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$ which was derived in Example 24.4. Then use $Q = CV$ to calculate Q.

EXECUTE: **(a)** Using $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$ gives

$\dfrac{C}{L} = \dfrac{2\pi(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)}{\ln[(3.5 \text{ mm})/(2.2 \text{ mm})]} = 1.2\times10^{-10} \text{ F/m} = 120 \text{ pF/m}$.

(b) $C = (1.20\times10^{-10} \text{ F/m})(2.8 \text{ m}) = 3.355\times10^{-10} \text{ F}$.

$Q = CV = (3.355\times10^{-10} \text{ F})(350\times10^{-3} \text{ V}) = 1.2\times10^{-10} \text{ C} = 120 \text{ pC}$.

The conductor at higher potential has the positive charge, so there is +120 pC on the inner conductor and −120 pC on the outer conductor.

EVALUATE: C depends only on the dimensions of the capacitor. Q and V are proportional.

24.13. IDENTIFY: Apply the results of Example 24.3. $C = Q/V$.

SET UP: $r_a = 15.0$ cm. Solve for r_b.

EXECUTE: **(a)** For two concentric spherical shells, the capacitance is $C = \dfrac{1}{k}\left(\dfrac{r_a r_b}{r_b - r_a}\right)$. $kCr_b - kCr_a = r_a r_b$

and $r_b = \dfrac{kCr_a}{kC - r_a} = \dfrac{k(116\times10^{-12} \text{ F})(0.150 \text{ m})}{k(116\times10^{-12} \text{ F}) - 0.150 \text{ m}} = 0.175 \text{ m} = 17.5 \text{ cm}$.

(b) $V = 220$ V and $Q = CV = (116\times10^{-12} \text{ F})(220 \text{ V}) = 2.55\times10^{-8} \text{ C} = 25.5 \text{ nC}$.

EVALUATE: A parallel-plate capacitor with $A = 4\pi r_a r_b = 0.33 \text{ m}^2$ and $d = r_b - r_a = 2.5\times10^{-2}$ m has

$C = \dfrac{\varepsilon_0 A}{d} = 117 \text{ pF}$, in excellent agreement with the value of C for the spherical capacitor.

24.14. IDENTIFY: Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots$ For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \dots$ $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the 5.0 μF and 8.0 μF capacitors in parallel is 13.0 μF. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.14. The equivalent capacitance of these three capacitors in series is 3.47 μF.

(b) $Q_{\text{tot}} = C_{\text{tot}}V = (3.47 \ \mu\text{F})(50.0 \text{ V}) = 174 \ \mu\text{C}$.

(c) Q_{tot} is the same as Q for each of the capacitors in the series combination shown in Figure 24.22, so Q for each of the capacitors is 174 μC.

EVALUATE: The voltages across each capacitor in Figure 24.14 are $V_{10} = \dfrac{Q_{tot}}{C_{10}} = 17.4$ V,

$V_{13} = \dfrac{Q_{tot}}{C_{13}} = 13.4$ V, and $V_9 = \dfrac{Q_{tot}}{C_9} = 19.3$ V. $V_{10} + V_{13} + V_9 = 17.4$ V $+ 13.4$ V $+ 19.3$ V $= 50.1$ V. The sum

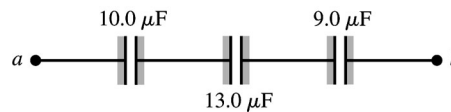of the voltages equals the applied voltage, apart from a small difference due to rounding.

Figure 24.14

24.15. **IDENTIFY:** For capacitors in series the voltage across the combination equals the sum of the voltages in the individual capacitors. For capacitors in parallel the voltage across the combination is the same as the voltage across each individual capacitor.
SET UP and **EXECUTE:** (a) Connect the capacitors in series so their voltages will add.
(b) $V = V_1 + V_2 + V_3 + \ldots = NV_1$, where N is the number of capacitors in the series combination, since the capacitors are identical. $N = \dfrac{V}{V_1} = \dfrac{500 \text{ V}}{0.10 \text{ V}} = 5000$.
EVALUATE: It requires many small cells to produce a large voltage surge.

24.16. **IDENTIFY:** The capacitors between b and c are in parallel. This combination is in series with the 15 pF capacitor.
SET UP: Let $C_1 = 15$ pF, $C_2 = 9.0$ pF and $C_3 = 11$ pF.
EXECUTE: (a) For capacitors in parallel, $C_{eq} = C_1 + C_2 + \ldots$ so $C_{23} = C_2 + C_3 = 20$ pF.

(b) $C_1 = 15$ pF is in series with $C_{23} = 20$ pF. For capacitors in series, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ so

$\dfrac{1}{C_{123}} = \dfrac{1}{C_1} + \dfrac{1}{C_{23}}$ and $C_{123} = \dfrac{C_1 C_{23}}{C_1 + C_{23}} = \dfrac{(15 \text{ pF})(20 \text{ pF})}{15 \text{ pF} + 20 \text{ pF}} = 8.6$ pF.

EVALUATE: For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

24.17. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
SET UP: Do parts (a) and (b) together. The capacitor network is drawn in Figure 24.17a.
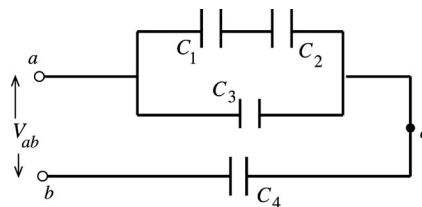
$C_1 = C_2 = C_3 = C_4 = 4.00 \ \mu\text{F}.$
$V_{ab} = 28.0$ V.

Figure 24.17a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_1 and C_2 are in series and are equivalent to C_{12} (Figure 24.17b).

$\dfrac{1}{C_{12}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}.$

Figure 24.17b

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.00 \times 10^{-6} \text{ F})(4.00 \times 10^{-6} \text{ F})}{4.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}} = 2.00 \times 10^{-6} \text{ F}.$$

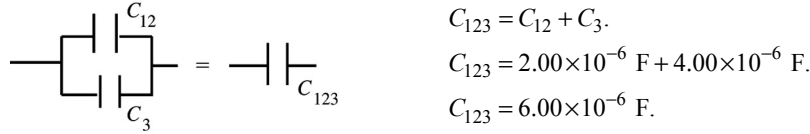C_{12} and C_3 are in parallel and are equivalent to C_{123} (Figure 24.17c).

$$C_{123} = C_{12} + C_3.$$
$$C_{123} = 2.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}.$$
$$C_{123} = 6.00 \times 10^{-6} \text{ F}.$$

Figure 24.17c

C_{123} and C_4 are in series and are equivalent to C_{1234} (Figure 24.17d).

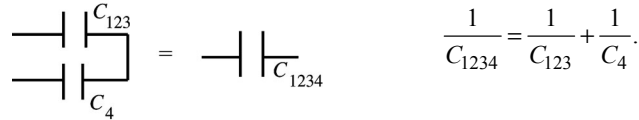$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4}.$$

Figure 24.17d

$$C_{1234} = \frac{C_{123} C_4}{C_{123} + C_4} = \frac{(6.00 \times 10^{-6} \text{ F})(4.00 \times 10^{-6} \text{ F})}{6.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}} = 2.40 \times 10^{-6} \text{ F}.$$

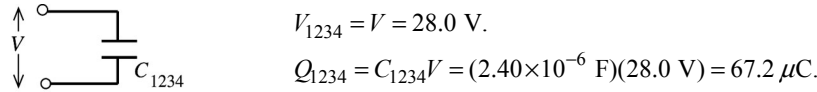The circuit is equivalent to the circuit shown in Figure 24.17e.

$$V_{1234} = V = 28.0 \text{ V}.$$
$$Q_{1234} = C_{1234} V = (2.40 \times 10^{-6} \text{ F})(28.0 \text{ V}) = 67.2 \ \mu\text{C}.$$

Figure 24.17e

Now build back up the original circuit, step by step. C_{1234} represents C_{123} and C_4 in series (Figure 24.17f).

$$Q_{123} = Q_4 = Q_{1234} = 67.2 \ \mu\text{C}$$
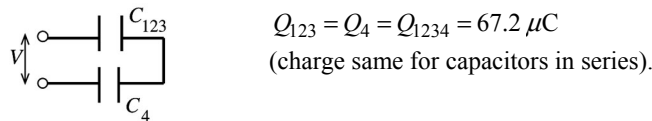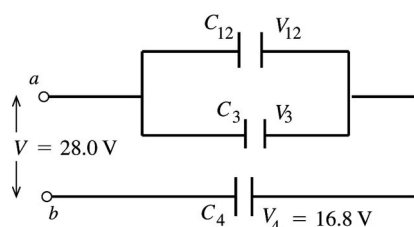(charge same for capacitors in series).

Figure 24.17f

Then $V_{123} = \dfrac{Q_{123}}{C_{123}} = \dfrac{67.2 \ \mu\text{C}}{6.00 \ \mu\text{F}} = 11.2 \text{ V}$.

$V_4 = \dfrac{Q_4}{C_4} = \dfrac{67.2 \ \mu\text{C}}{4.00 \ \mu\text{F}} = 16.8 \text{ V}.$

Note that $V_4 + V_{123} = 16.8 \text{ V} + 11.2 \text{ V} = 28.0 \text{ V}$, as it should.

Next consider the circuit as written in Figure 24.17g (next page).

$V_3 = V_{12} = 28.0 \text{ V} - V_4.$

$V_3 = 11.2 \text{ V}.$
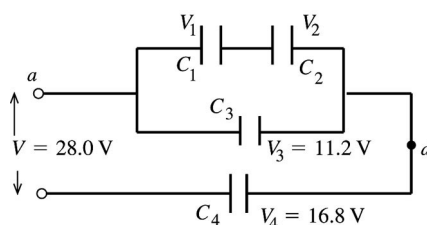
$Q_3 = C_3 V_3 = (4.00 \ \mu\text{F})(11.2 \text{ V}).$

$Q_3 = 44.8 \ \mu\text{C}.$

$Q_{12} = C_{12} V_{12} = (2.00 \ \mu\text{F})(11.2 \text{ V}).$

$Q_{12} = 22.4 \ \mu\text{C}.$

Figure 24.17g

Finally, consider the original circuit, as shown in Figure 24.17h.

$Q_1 = Q_2 = Q_{12} = 22.4 \ \mu\text{C}$

(charge same for capacitors in series).

$V_1 = \dfrac{Q_1}{C_1} = \dfrac{22.4 \ \mu\text{C}}{4.00 \ \mu\text{F}} = 5.6 \text{ V}.$

$V_2 = \dfrac{Q_2}{C_2} = \dfrac{22.4 \ \mu\text{C}}{4.00 \ \mu\text{F}} = 5.6 \text{ V}.$

Figure 24.17h

Note that $V_1 + V_2 = 11.2 \text{ V},$ which equals V_3 as it should.

Summary: $Q_1 = 22.4 \ \mu\text{C}, \ V_1 = 5.6 \text{ V}.$

$Q_2 = 22.4 \ \mu\text{C}, \ V_2 = 5.6 \text{ V}.$

$Q_3 = 44.8 \ \mu\text{C}, \ V_3 = 11.2 \text{ V}.$

$Q_4 = 67.2 \ \mu\text{C}, \ V_4 = 16.8 \text{ V}.$

(c) $V_{ad} = V_3 = 11.2 \text{ V}.$

EVALUATE: $V_1 + V_2 + V_4 = V,$ or $V_3 + V_4 = V.$ $Q_1 = Q_2, Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}.$

24.18. IDENTIFY: The two capacitors are in series. The equivalent capacitance is given by $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}.$

SET UP: For capacitors in series the charges are the same and the potentials add to give the potential across the network.

EXECUTE: (a) $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{(3.00 \times 10^{-6} \text{ F})} + \dfrac{1}{(5.00 \times 10^{-6} \text{ F})},$ so $C_{eq} = 1.875 \times 10^{-6}$ F. Then

$Q = VC_{eq} = (64.0 \text{ V})(1.875 \times 10^{-6} \text{ F}) = 1.20 \times 10^{-4}$ C $= 120 \ \mu\text{C}.$ Each capacitor has a charge of

1.20×10^{-4} C $= 120 \ \mu\text{C}.$

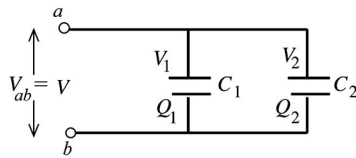**(b)** $V_1 = Q/C_1 = (1.20 \times 10^{-4}$ C$)/(3.0 \times 10^{-6}$ F$) = 40.0 \text{ V}.$

$V_2 = Q/C_2 = (1.20 \times 10^{-4}$ C$)/(5.0 \times 10^{-6}$ F$) = 24.0 \text{ V}.$

EVALUATE: $V_1 + V_2 = 64.0 \text{ V},$ which is equal to the applied potential $V_{ab}.$ The capacitor with the smaller C has the larger $V.$

24.19. IDENTIFY: The two capacitors are in parallel so the voltage is the same on each, and equal to the applied voltage $V_{ab}.$

SET UP: Do parts (a) and (b) together. The network is sketched in Figure 24.19 (next page).

EXECUTE: $V_1 = V_2 = V.$

$V_1 = 52.0$ V.

$V_2 = 52.0$ V.

Figure 24.19

$C = Q/V$ so $Q = CV.$

$Q_1 = C_1 V_1 = (3.00 \, \mu\text{F})(52.0 \text{ V}) = 156 \, \mu\text{C}.$ $Q_2 = C_2 V_2 = (5.00 \, \mu\text{F})(52.0 \text{ V}) = 260 \, \mu\text{C}.$

EVALUATE: To produce the same potential difference, the capacitor with the larger C has the larger $Q.$

24.20. **IDENTIFY:** For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V.$

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and $C_2.$

EXECUTE: **(a)** C_1 and C_2 are in parallel and so have the same potential across them:

$$V_1 = V_2 = \frac{Q_2}{C_2} = \frac{30.0 \times 10^{-6} \text{ C}}{3.00 \times 10^{-6} \text{ F}} = 10.0 \text{ V}.$$ Therefore, $Q_1 = V_1 C_1 = (10.0 \text{ V})(6.00 \times 10^{-6} \text{ F}) = 60.0 \times 10^{-6}$ C.

Since C_3 is in series with the parallel combination of C_1 and $C_2,$ its charge must be equal to their

combined charge: $Q_3 = 30.0 \times 10^{-6} \text{ C} + 60.0 \times 10^{-6} \text{ C} = 90.0 \times 10^{-6}$ C.

(b) The total capacitance is found from $\dfrac{1}{C_{eq}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00 \times 10^{-6} \text{ F}} + \dfrac{1}{5.00 \times 10^{-6} \text{ F}}$ and

$C_{eq} = 3.21 \mu\text{F}.$ $V_{ab} = \dfrac{Q_{tot}}{C_{eq}} = \dfrac{90.0 \times 10^{-6} \text{ C}}{3.21 \times 10^{-6} \text{ F}} = 28.0 \text{ V}.$

EVALUATE: $V_3 = \dfrac{Q_3}{C_3} = \dfrac{90.0 \times 10^{-6} \text{ C}}{5.00 \times 10^{-6} \text{ F}} = 18.0 \text{ V}.$ $V_{ab} = V_1 + V_3 = 10.0 \text{ V} + 18.0 \text{ V} = 28.0 \text{ V},$ as we just found.

24.21. **IDENTIFY:** Three of the capacitors are in series, and this combination is in parallel with the other two capacitors.

SET UP: For capacitors in series the voltages add and the charges are the same;

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots.$ For capacitors in parallel the voltages are the same and the charges add;

$C_{eq} = C_1 + C_2 + \dots.$ $C = \dfrac{Q}{V}.$

EXECUTE: **(a)** The equivalent capacitance of the 18.0 nF, 30.0 nF and 10.0 nF capacitors in series is 5.29 nF. When these capacitors are replaced by their equivalent we get the network sketched in Figure 24.21. The equivalent capacitance of these three capacitors in parallel is 19.3 nF, and this is the equivalent capacitance of the original network.
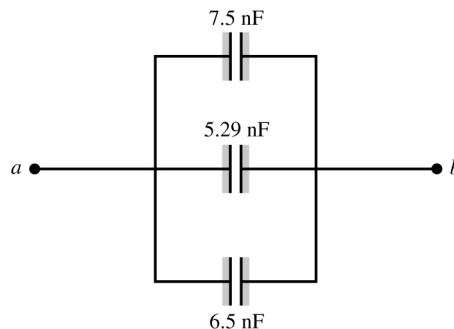

Figure 24.21

(b) $Q_{\text{tot}} = C_{\text{eq}}V = (19.3 \text{ nF})(25 \text{ V}) = 482 \text{ nC}.$

(c) The potential across each capacitor in the parallel network of Figure 24.21 is 25 V.

$Q_{6.5} = C_{6.5}V_{6.5} = (6.5 \text{ nF})(25 \text{ V}) = 162 \text{ nC}.$

(d) 25 V.

EVALUATE: As with most circuits, we must go through a series of steps to simplify it as we solve for the unknowns.

24.22. **IDENTIFY:** Refer to Figure 24.10b in the textbook. For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \ldots$. For

capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$.

SET UP: The $11\,\mu\text{F}$, $4\,\mu\text{F}$ and replacement capacitor are in parallel and this combination is in series with the $9.0\,\mu\text{F}$ capacitor.

EXECUTE: $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{8.0\,\mu\text{F}} = \left(\dfrac{1}{(11 + 4.0 + x)\mu\text{F}} + \dfrac{1}{9.0\,\mu\text{F}} \right)$. $(15 + x)\mu\text{F} = 72\,\mu\text{F}$ and $x = 57\,\mu\text{F}$.

EVALUATE: Increasing the capacitance of the one capacitor by a large amount makes a small increase in the equivalent capacitance of the network.

24.23. **IDENTIFY and SET UP:** The energy density is given by $u = \frac{1}{2}\varepsilon_0 E^2$. Use $V = Ed$ to solve

for E.

EXECUTE: Calculate E: $E = \dfrac{V}{d} = \dfrac{400 \text{ V}}{5.00 \times 10^{-3} \text{ m}} = 8.00 \times 10^4 \text{ V/m}.$

Then $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(8.00 \times 10^4 \text{ V/m})^2 = 0.0283 \text{ J/m}^3.$

EVALUATE: E is smaller than the value in Example 24.8 by about a factor of 6 so u is smaller by about a factor of $6^2 = 36$.

24.24. **IDENTIFY:** Apply $C = Q/V$. $C = \dfrac{\varepsilon_0 A}{d}$. The work done to double the separation equals the change in the stored energy.

SET UP: $U = \dfrac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

EXECUTE: **(a)** $V = Q/C = (3.90\,\mu\text{C})/(920 \times 10^{-12} \text{ F}) = 4240 \text{ V} = 4.24 \text{ kV}.$

(b) $C = \dfrac{\varepsilon_0 A}{d}$ says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to $8480 \text{ V} = 8.48 \text{ kV}.$

(c) $U_i = \dfrac{Q^2}{2C} = \dfrac{(3.90 \times 10^{-6} \text{ C})^2}{2(920 \times 10^{-12} \text{ F})} = 8.27 \times 10^{-3} \text{ J} = 8.27 \text{ mJ}.$ If the separation is doubled while Q stays the

same, the capacitance halves, and the energy stored doubles to $2U_i$. The amount of work done to move the plates equals the difference in energy stored in the capacitor, so

$\Delta U = U_f - U_i = 2U_i - U_i = U_i = 8.27 \text{ mJ}.$

EVALUATE: The oppositely charged plates attract each other so positive work must be done by an external force to pull them farther apart.

24.25. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\varepsilon_0 A}{d}$. $V_{ab} = Ed$. The stored energy is $\frac{1}{2}QV$.

SET UP: $d = 1.50 \times 10^{-3} \text{ m}$. $1\,\mu\text{C} = 10^{-6} \text{ C}$

EXECUTE: **(a)** $C = \dfrac{0.0180 \times 10^{-6} \text{ C}}{200 \text{ V}} = 9.00 \times 10^{-11} \text{ F} = 90.0 \text{ pF}.$

(b) $C = \dfrac{\varepsilon_0 A}{d}$ so $A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(9.00 \times 10^{-11} \text{ F})(1.50 \times 10^{-3} \text{ m})}{8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 0.0152 \text{ m}^2$.

(c) $V = Ed = (3.0 \times 10^6 \text{ V/m})(1.50 \times 10^{-3} \text{ m}) = 4.5 \times 10^3 \text{ V} = 4.5 \text{ kV}$.

(d) Energy $= \frac{1}{2}QV = \frac{1}{2}(0.0180 \times 10^{-6} \text{ C})(200 \text{ V}) = 1.80 \times 10^{-6} \text{ J} = 1.80 \; \mu\text{J}$.

EVALUATE: We could also calculate the stored energy as $\dfrac{Q^2}{2C} = \dfrac{(0.0180 \times 10^{-6} \text{ C})^2}{2(9.00 \times 10^{-11} \text{ F})} = 1.80 \; \mu\text{J}$.

24.26. **IDENTIFY:** $C = \dfrac{\varepsilon_0 A}{d}$. The stored energy can be expressed either as $\dfrac{Q^2}{2C}$ or as $\dfrac{CV^2}{2}$, whichever is more convenient for the calculation.

SET UP: Since d is halved, C doubles.

EXECUTE: **(a)** If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

(b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.8 J, using $U = CV^2/2$, when V is held constant throughout.

EVALUATE: When the capacitor is disconnected, the stored energy decreases because of the positive work done by the attractive force between the plates. When the capacitor remains connected to the battery, $Q = CV$ tells us that the charge on the plates increases. The increased stored energy comes from the battery when it puts more charge onto the plates.

24.27. **IDENTIFY:** Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express U, Q, and E in terms of the capacitor voltage.

SET UP: Let the applied voltage be V. Let each capacitor have capacitance C. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage V.

EXECUTE: **(a)** <u>Series</u>: The voltage across each capacitor is $V/2$. The total energy stored is $U_s = 2(\frac{1}{2}C(V/2)^2) = \frac{1}{4}CV^2$.

<u>Parallel</u>: The voltage across each capacitor is V. The total energy stored is

$U_p = 2(\frac{1}{2}CV^2) = CV^2 \qquad \rightarrow \qquad U_p = 4U_s$.

(b) $Q = CV$ for a single capacitor with voltage V. $Q_s = 2[C(V/2)] = CV$; $Q_p = 2(CV) = 2CV$; $Q_p = 2Q_s$.

(c) $E = V/d$ for a capacitor with voltage V. $E_s = V/2d$; $E_p = V/d$; $E_p = 2E_s$.

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

24.28. **IDENTIFY:** The two capacitors are in series. $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$, $C = \dfrac{Q}{V}$, and $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same.

EXECUTE: **(a)** $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ so $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{(150 \text{ nF})(120 \text{ nF})}{150 \text{ nF} + 120 \text{ nF}} = 66.7 \text{ nF}$.

$Q = CV = (66.7 \text{ nF})(48 \text{ V}) = 3.2 \times 10^{-6} \text{ C} = 3.2 \; \mu\text{C}$.

(b) $Q = 3.2 \; \mu\text{C}$ for each capacitor.

(c) $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(66.7 \times 10^{-9} \text{ F})(48 \text{ V})^2 = 77 \; \mu\text{J}$.

(d) We know C and Q for each capacitor so rewrite U in terms of these quantities.

$U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$.

150 nF: $U = \dfrac{(3.2\times10^{-6}\ \text{C})^2}{2(150\times10^{-9}\ \text{F})} = 34\ \mu\text{J}.$

120 nF: $U = \dfrac{(3.2\times10^{-6}\ \text{C})^2}{2(120\times10^{-9}\ \text{F})} = 43\ \mu\text{J}.$

Note that $34\ \mu\text{J} + 43\ \mu\text{J} = 77\ \mu\text{J},$ the total stored energy calculated in part (c).

(e) 150 nF: $V = \dfrac{Q}{C} = \dfrac{3.2\times10^{-6}\ \text{C}}{150\times10^{-9}\ \text{F}} = 21\ \text{V}.$

120 nF: $V = \dfrac{Q}{C} = \dfrac{3.2\times10^{-6}\ \text{C}}{120\times10^{-9}\ \text{F}} = 27\ \text{V}.$

Note that these two voltages sum to 48 V, the voltage applied across the network.

EVALUATE: Since Q is the same, the capacitor with smaller C stores more energy $(U = Q^2/2C)$ and has a larger voltage $(V = Q/C)$.

24.29. **IDENTIFY:** The two capacitors are in parallel. $C_{eq} = C_1 + C_2.$ $C = \dfrac{Q}{V}.$ $U = \tfrac{1}{2}CV^2.$

SET UP: For capacitors in parallel, the voltages are the same and the charges add.

EXECUTE: (a) $C_{eq} = C_1 + C_2 = 35\ \text{nF} + 75\ \text{nF} = 110\ \text{nF}.$ $Q_{tot} = C_{eq}V = (110\times10^{-9}\ \text{F})(220\ \text{V}) = 24.2\ \mu\text{C}$

(b) $V = 220\ \text{V}$ for each capacitor.

35 nF: $Q_{35} = C_{35}V = (35\times10^{-9}\ \text{F})(220\ \text{V}) = 7.7\ \mu\text{C};$ 75 nF: $Q_{75} = C_{75}V = (75\times10^{-9}\ \text{F})(220\ \text{V}) = 16.5\ \mu\text{C}.$

Note that $Q_{35} + Q_{75} = Q_{tot}.$

(c) $U_{tot} = \tfrac{1}{2}C_{eq}V^2 = \tfrac{1}{2}(110\times10^{-9}\ \text{F})(220\ \text{V})^2 = 2.66\ \text{mJ}.$

(d) 35 nF: $U_{35} = \tfrac{1}{2}C_{35}V^2 = \tfrac{1}{2}(35\times10^{-9}\ \text{F})(220\ \text{V})^2 = 0.85\ \text{mJ};$

75 nF: $U_{75} = \tfrac{1}{2}C_{75}V^2 = \tfrac{1}{2}(75\times10^{-9}\ \text{F})(220\ \text{V})^2 = 1.81\ \text{mJ}.$ Since V is the same the capacitor with larger C stores more energy.

(e) 220 V for each capacitor.

EVALUATE: The capacitor with the larger C has the larger Q.

24.30. **IDENTIFY:** Capacitance depends on the geometry of the object.

(a) **SET UP:** The potential difference between the core and tube is $V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a).$ Solving for the

linear charge density gives $\lambda = \dfrac{2\pi\varepsilon_0 V}{\ln(r_b/r_a)} = \dfrac{4\pi\varepsilon_0 V}{2\ln(r_b/r_a)}.$

EXECUTE: Using the given values gives $\lambda = \dfrac{6.00\ \text{V}}{2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ln\left(\dfrac{2.00}{1.20}\right)} = 6.53\times10^{-10}\ \text{C/m}.$

(b) **SET UP:** $Q = \lambda L.$

EXECUTE: $Q = (6.53\times10^{-10}\ \text{C/m})(0.350\ \text{m}) = 2.29\times10^{-10}\ \text{C}.$

(c) **SET UP:** The definition of capacitance is $C = Q/V.$

EXECUTE: $C = \dfrac{2.29\times10^{-10}\ \text{C}}{6.00\ \text{V}} = 3.81\times10^{-11}\ \text{F}.$

(d) **SET UP:** The energy stored in a capacitor is $U = \tfrac{1}{2}CV^2.$

EXECUTE: $U = \tfrac{1}{2}(3.81\times10^{-11}\ \text{F})(6.00\ \text{V})^2 = 6.85\times10^{-10}\ \text{J}.$

EVALUATE: The stored energy could be converted to heat or other forms of energy.

24.31.   IDENTIFY: $U = \frac{1}{2}QV$. Solve for Q. $C = Q/V$.

SET UP: Example 24.4 shows that for a cylindrical capacitor, $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$.

EXECUTE: (a) $U = \frac{1}{2}QV$ gives $Q = \dfrac{2U}{V} = \dfrac{2(3.20 \times 10^{-9}\ \text{J})}{4.00\ \text{V}} = 1.60 \times 10^{-9}\ \text{C}$.

(b) $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$. Solving for r_b/r_a gives

$\dfrac{r_b}{r_a} = \exp(2\pi\varepsilon_0 L/C) = \exp(2\pi\varepsilon_0\ LV/Q) = \exp[2\pi\varepsilon_0 (15.0\ \text{m})(4.00\ \text{V})/(1.60 \times 10^{-9}\ \text{C})] = 8.05$.

The radius of the outer conductor is 8.05 times the radius of the inner conductor.

EVALUATE: When the ratio r_b/r_a increases, C/L decreases and less charge is stored for a given potential difference.

24.32.   IDENTIFY: Apply $u = \frac{1}{2}\varepsilon_0 E^2$.

SET UP: Example 24.3 shows that $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ between the conducting shells and that

$\dfrac{Q}{4\pi\varepsilon_0} = \left(\dfrac{r_a r_b}{r_b - r_a}\right) V_{ab}$.

EXECUTE: $E = \left(\dfrac{r_a r_b}{r_b - r_a}\right)\dfrac{V_{ab}}{r^2} = \left(\dfrac{(0.125\ \text{m})(0.148\ \text{m})}{0.148\ \text{m} - 0.125\ \text{m}}\right)\dfrac{120\ \text{V}}{r^2} = \dfrac{96.5\ \text{V} \cdot \text{m}}{r^2}$.

(a) For $r = 0.126\ \text{m}$, $E = 6.08 \times 10^3\ \text{V/m}$. $u = \frac{1}{2}\varepsilon_0 E^2 = 1.64 \times 10^{-4}\ \text{J/m}^3$.

(b) For $r = 0.147\ \text{m}$, $E = 4.47 \times 10^3\ \text{V/m}$. $u = \frac{1}{2}\varepsilon_0 E^2 = 8.85 \times 10^{-5}\ \text{J/m}^3$.

EVALUATE: (c) No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. E decreases as r increases, so u decreases also.

25.33.   IDENTIFY: $C = KC_0$. $U = \frac{1}{2}CV^2$.

SET UP: $C_0 = 12.5\ \mu\text{F}$ is the value of the capacitance without the dielectric present.

EXECUTE: (a) With the dielectric, $C = (3.75)(12.5\ \mu\text{F}) = 46.9\ \mu\text{F}$.

<u>Before</u>: $U = \frac{1}{2}C_0 V^2 = \frac{1}{2}(12.5 \times 10^{-6}\ \text{F})(24.0\ \text{V})^2 = 3.60\ \text{mJ}$.

<u>After</u>: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9 \times 10^{-6}\ \text{F})(24.0\ \text{V})^2 = 13.5\ \text{mJ}$.

(b) $\Delta U = 13.5\ \text{mJ} - 3.6\ \text{mJ} = 9.9\ \text{mJ}$. The energy increased.

EVALUATE: The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.

24.34.   IDENTIFY: $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

SET UP: $V = Ed$ holds both with and without the dielectric.

EXECUTE: (a) $V = Ed = (3.00 \times 10^4\ \text{V/m})(1.50 \times 10^{-3}\ \text{m}) = 45.0\ \text{V}$.

$Q = C_0 V = (8.00 \times 10^{-12}\ \text{F})(45.0\ \text{V}) = 3.60 \times 10^{-10}\ \text{C} = 360\ \text{pC}$.

(b) With the dielectric, $C = KC_0 = (2.70)(8.00\ \text{pF}) = 21.6\ \text{pF}$. V is still 45.0 V, so

$Q = CV = (21.6 \times 10^{-12}\ \text{F})(45.0\ \text{V}) = 9.72 \times 10^{-10}\ \text{C} = 972\ \text{pC}$.

EVALUATE: The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. Q increases by a factor of K.

24.35. **IDENTIFY** and **SET UP:** Q is constant so we can apply Eq. (24.14). The charge density on each surface of the dielectric is given by $\sigma_i = \sigma(1-1/K)$.

EXECUTE: $E = \dfrac{E_0}{K}$ so $K = \dfrac{E_0}{E} = \dfrac{3.20\times10^5 \text{ V/m}}{2.50\times10^5 \text{ V/m}} = 1.28$.

(a) $\sigma_i = \sigma(1-1/K)$.

$\sigma = \varepsilon_0 E_0 = (8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.20\times10^5 \text{ N/C}) = 2.833\times10^{-6} \text{ C/m}^2$.

$\sigma_i = (2.833\times10^{-6} \text{ C/m}^2)(1-1/1.28) = 6.20\times10^{-7} \text{ C/m}^2$.

(b) As calculated above, $K = 1.28$.

EVALUATE: The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

24.36. **IDENTIFY:** Capacitance depends on geometry, and the introduction of a dielectric increases the capacitance.

SET UP: For a parallel-plate capacitor with dielectric, $C = K\varepsilon_0 A/d$.

EXECUTE: **(a)** Solving for d gives

$$d = \frac{K\varepsilon_0 A}{C} = \frac{(3.0)(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.22 \text{ m})(0.28 \text{ m})}{1.0\times10^{-9} \text{ F}} = 1.64\times10^{-3} \text{ m} = 1.64 \text{ mm}.$$

Dividing this result by the thickness of a sheet of paper gives $\dfrac{1.64 \text{ mm}}{0.20 \text{ mm/sheet}} \approx 8$ sheets.

(b) Solving for the area of the plates gives $A = \dfrac{Cd}{K\varepsilon_0} = \dfrac{(1.0\times10^{-9} \text{ F})(0.012 \text{ m})}{(3.0)(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 0.45 \text{ m}^2$.

(c) Teflon has a smaller dielectric constant (2.1) than the posterboard, so she will need more area to achieve the same capacitance.

EVALUATE: The use of dielectric makes it possible to construct reasonable-sized capacitors since the dielectric increases the capacitance by a factor of K.

24.37. **IDENTIFY** and **SET UP:** For a parallel-plate capacitor with a dielectric we can use the equation $C = K\varepsilon_0 A/d$. Minimum A means smallest possible d. d is limited by the requirement that E be less than 1.60×10^7 V/m when V is as large as 5500 V.

EXECUTE: $V = Ed$ so $d = \dfrac{V}{E} = \dfrac{5500 \text{ V}}{1.60\times10^7 \text{ V/m}} = 3.44\times10^{-4} \text{ m}$.

Then $A = \dfrac{Cd}{K\varepsilon_0} = \dfrac{(1.25\times10^{-9} \text{ F})(3.44\times10^{-4} \text{ m})}{(3.60)(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 0.0135 \text{ m}^2$.

EVALUATE: The relation $V = Ed$ applies with or without a dielectric present. A would have to be larger if there were no dielectric.

24.38. **IDENTIFY:** We can model the cell wall as a large sheet carrying equal but opposite charges, which makes it equivalent to a parallel-plate capacitor.

SET UP: With air between the layers, $E_0 = \dfrac{Q}{\varepsilon_0 A} = \dfrac{\sigma}{\varepsilon_0}$ and $V_0 = E_0 d$. The energy density in the electric field is $u = \frac{1}{2}\varepsilon_0 E^2$. The volume of a shell of thickness t and average radius R is $4\pi R^2 t$. The volume of a solid sphere of radius R is $\frac{4}{3}\pi R^3$. With the dielectric present, $E = \dfrac{E_0}{K}$ and $V = \dfrac{V_0}{K}$.

EXECUTE: **(a)** $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{0.50\times10^{-3} \text{ C/m}^2}{8.854\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)} = 5.6\times10^7 \text{ V/m}$.

(b) $V_0 = E_0 d = (5.6\times10^7 \text{ V/m})(5.0\times10^{-9} \text{ m}) = 0.28 \text{ V}$. The outer wall of the cell is at higher potential, since it has positive charge.

(c) For the cell, $V_{\text{cell}} = \frac{4}{3}\pi R^3$, which gives $R = \left(\frac{3V_{\text{cell}}}{4\pi}\right)^{1/3} = \left(\frac{3(10^{-16}\ \text{m}^3)}{4\pi}\right)^{1/3} = 2.9\times10^{-6}$ m. The volume

of the cell wall is $V_{\text{wall}} = 4\pi R^2 t = 4\pi(2.9\times10^{-6}\ \text{m})^2(5.0\times10^{-9}\ \text{m}) = 5.3\times10^{-19}\ \text{m}^3$. The energy density in

the cell wall is $u_0 = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}[8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)](5.6\times10^7\ \text{V/m})^2 = 1.39\times10^4\ \text{J/m}^3$. The total

electric-field energy in the cell wall is $(1.39\times10^4\ \text{J/m}^3)(5.3\times10^{-19}\ \text{m}^3) = 7\times10^{-15}$ J.

(d) $E = \dfrac{E_0}{K} = \dfrac{5.6\times10^7\ \text{V/m}}{5.4} = 1.0\times10^7$ V/m and $V = \dfrac{V_0}{K} = \dfrac{0.28\ \text{V}}{5.4} = 0.052$ V.

EVALUATE: To a first approximation, many biological structures can be modeled as basic circuit elements.

24.39. IDENTIFY: $C = Q/V$. $C = KC_0$. $V = Ed$.

SET UP: Table 24.1 gives $K = 3.1$ for mylar.

EXECUTE: **(a)** $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times10^{-7}\ \text{F})(12\ \text{V}) = 6.3\times10^{-6}$ C.

(b) $\sigma_i = \sigma(1-1/K)$ so $Q_i = Q(1-1/K) = (9.3\times10^{-6}\ \text{C})(1-1/3.1) = 6.3\times10^{-6}$ C.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: $E = V/d$ and V is constant so E doesn't change when the dielectric is inserted.

24.40. IDENTIFY and SET UP: The energy density is due to the electric field in the dielectric. $u = \frac{1}{2}\varepsilon E^2$, where

$\varepsilon = K\varepsilon_0$. $V = Ed$. In this case, $E = 0.800E_{\text{m}}$.

EXECUTE: **(a)** Using $u = \frac{1}{2}\varepsilon E^2$ with $\varepsilon = K\varepsilon_0$, we have

$u = (1/2)(2.6)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)[(0.800)(2.0\times10^7\ \text{V/m})]^2 = 2945\ \text{J/m}^3$, which rounds to 2900 J/m^3.

(b) First get the plate separation d: $V = Ed$ gives

$d = V/E = (500\ V)/[(0.800)(2.0\times10^7\ \text{V/m})] = 3.125\times10^{-5}$ m.

The stored energy is $U = u\times\text{volume} = uAd$, so

$A = U/ud = (0.200\times10^{-3}\ \text{J})/[(2945\ \text{J/m}^3)(3.125\times10^{-5}\text{m})] = 2.2\times10^{-3}\ \text{m}^2 = 22\ \text{cm}^2$.

EVALUATE: If this capacitor has square plates, their dimensions would be $x = (22\ \text{cm}^2)^{1/2} = 4.7$ cm on each side. This is considerably larger than ordinary laboratory capacitors used in circuits.

24.41. (a) IDENTIFY and SET UP: Since the capacitor remains connected to the power supply the potential

difference doesn't change when the dielectric is inserted. Use $U = \frac{1}{2}CV^2$ to calculate V and combine it

with $K = C/C_0$ to obtain a relation between the stored energies and the dielectric constant and use this to calculate K.

EXECUTE: Before the dielectric is inserted $U_0 = \frac{1}{2}C_0V^2$ so $V = \sqrt{\dfrac{2U_0}{C_0}} = \sqrt{\dfrac{2(1.85\times10^{-5}\ \text{J})}{360\times10^{-9}\ \text{F}}} = 10.1$ V.

(b) $K = C/C_0$.

$U_0 = \frac{1}{2}C_0V^2$, $U = \frac{1}{2}CV^2$ so $C/C_0 = U/U_0$.

$K = \dfrac{U}{U_0} = \dfrac{1.85\times10^{-5}\ \text{J} + 2.32\times10^{-5}\ \text{J}}{1.85\times10^{-5}\ \text{J}} = 2.25$.

EVALUATE: K increases the capacitance and then from $U = \frac{1}{2}CV^2$, with V constant an increase in C

gives an increase in U.

24.42. IDENTIFY: $C = KC_0$. $C = Q/V$. $V = Ed$.

SET UP: Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn't change.

EXECUTE: **(a)** The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (the battery is still connected), $\dfrac{C_{\text{after}}}{C_{\text{before}}} = \dfrac{Q_{\text{after}}}{Q_{\text{before}}} = \dfrac{45.0 \text{ pC}}{25.0 \text{ pC}} = K = 1.80.$

(b) The area of the plates is $\pi r^2 = \pi(0.0300 \text{ m})^2 = 2.827 \times 10^{-3} \text{ m}^2$ and the separation between them is thus $d = \dfrac{\varepsilon_0 A}{C} = \dfrac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)}{12.5 \times 10^{-12} \text{ F}} = 2.00 \times 10^{-3} \text{ m}.$ Before the dielectric is inserted,

$C = \dfrac{\varepsilon_0 A}{d} = \dfrac{Q}{V}$ and $V = \dfrac{Qd}{\varepsilon_0 A} = \dfrac{(25.0 \times 10^{-12} \text{ C})(2.00 \times 10^{-3} \text{ m})}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)} = 2.00 \text{ V}.$ The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

(c) Before the dielectric is inserted, $E = \dfrac{Q}{\varepsilon_0 A} = \dfrac{25.0 \times 10^{-12} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)} = 1000 \text{ N/C}.$

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

EVALUATE: $E = \dfrac{V}{d} = \dfrac{2.00 \text{ V}}{2.00 \times 10^{-3} \text{ m}} = 1000 \text{ N/C},$ whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need d to calculate E because V is the potential difference between points separated by distance d.

24.43.   **IDENTIFY:**   Apply $\oint K\vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\varepsilon_0}$ to calculate E. $V = Ed$ and $C = Q/V$ apply whether there is a dielectric between the plates or not.

(a) SET UP: Apply $\oint K\vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\varepsilon_0}$ to the dashed surface in Figure 24.43.
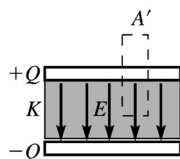
EXECUTE: $\oint K\vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\varepsilon_0}.$

$\oint K\vec{E} \cdot d\vec{A} = KEA'.$

since $E = 0$ outside the plates

$Q_{\text{encl-free}} = \sigma A' = (Q/A)A'.$

Figure 24.43

Thus $KEA' = \dfrac{(Q/A)A'}{\varepsilon_0}$ and $E = \dfrac{Q}{\varepsilon_0 AK}.$

SET UP and EXECUTE: **(b)** $V = Ed = \dfrac{Qd}{\varepsilon_0 AK}.$

(c) $C = \dfrac{Q}{V} = \dfrac{Q}{Qd/\varepsilon_0 AK} = K\dfrac{\varepsilon_0 A}{d} = KC_0.$

EVALUATE: Our result shows that $K = C/C_0,$ which is Eq. (24.12).

24.44.   **IDENTIFY:**   Gauss's law in dielectrics has the same form as in vacuum except that the electric field is multiplied by a factor of K and the charge enclosed by the Gaussian surface is the free charge. The capacitance of an object depends on its geometry.

(a) SET UP: The capacitance of a parallel-plate capacitor is $C = K\varepsilon_0 A/d$ and the charge on its plates is $Q = CV.$

EXECUTE: First find the capacitance:

$$C = \dfrac{K\varepsilon_0 A}{d} = \dfrac{(2.1)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0225 \text{ m}^2)}{1.00 \times 10^{-3} \text{ m}} = 4.18 \times 10^{-10} \text{ F}.$$

Now find the charge on the plates: $Q = CV = (4.18 \times 10^{-10} \text{ F})(12.0 \text{ V}) = 5.02 \times 10^{-9} \text{ C}.$

(b) SET UP: Gauss's law within the dielectric gives $KEA = Q_{\text{free}}/\varepsilon_0$.

EXECUTE: Solving for E gives

$$E = \frac{Q_{\text{free}}}{KA\varepsilon_0} = \frac{5.02 \times 10^{-9} \text{ C}}{(2.1)(0.0225 \text{ m}^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 1.20 \times 10^4 \text{ N/C}.$$

(c) SET UP: Without the Teflon and the voltage source, the charge is unchanged but the potential increases, so $C = \varepsilon_0 A/d$ and Gauss's law now gives $EA = Q/\varepsilon_0$.

EXECUTE: First find the capacitance:

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0225 \text{ m}^2)}{1.00 \times 10^{-3} \text{ m}} = 1.99 \times 10^{-10} \text{ F}.$$

The potential difference is $V = \dfrac{Q}{C} = \dfrac{5.02 \times 10^{-9} \text{ C}}{1.99 \times 10^{-10} \text{ F}} = 25.2$ V. From Gauss's law, the electric field is

$$E = \frac{Q}{\varepsilon_0 A} = \frac{5.02 \times 10^{-9} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0225 \text{ m}^2)} = 2.52 \times 10^4 \text{ N/C}.$$

EVALUATE: The dielectric reduces the electric field inside the capacitor because the electric field due to the dipoles of the dielectric is opposite to the external field due to the free charge on the plates.

24.45.   IDENTIFY: $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

SET UP: $E = 0.95U$.

EXECUTE: (a) The power output is 2.70×10^5 W, and 95% of the original energy is converted, so

$$E = Pt = (2.70 \times 10^5 \text{ W})(1.48 \times 10^{-3} \text{ s}) = 400 \text{ J}. \quad U = \frac{400 \text{ J}}{0.95} = 421 \text{ J}.$$

(b) $U = \frac{1}{2}CV^2$ so $C = \dfrac{2U}{V^2} = \dfrac{2(421 \text{ J})}{(125 \text{ V})^2} = 0.054$ F.

EVALUATE: For a given V, the stored energy increases linearly with C.

24.46.   IDENTIFY and SET UP: $C = \dfrac{\varepsilon_0 A}{d}$. $C = Q/V$. $V = Ed$. $U = \frac{1}{2}CV^2$. With the battery disconnected, Q is constant. When the separation d is doubled, C is halved.

EXECUTE: (a) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{\varepsilon_0 (0.12 \text{ m})^2}{3.7 \times 10^{-3} \text{ m}} = 3.446 \times 10^{-11}$ F, which rounds to 34 pF.

(b) $Q = CV = (3.446 \times 10^{-11} \text{ F})(12 \text{ V}) = 4.135 \times 10^{-10}$ C, which rounds to 410 pC.

(c) $E = V/d = (12 \text{ V})/(3.7 \times 10^{-3} \text{ m}) = 3200$ V/m.

(d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(3.446 \times 10^{-11} \text{ F})(12 \text{ V})^2 = 2.48 \times 10^{-9}$ J, which rounds to 2.5 nJ.

(e) If the battery is disconnected, so the charge remains constant, and the plates are pulled farther apart to 0.0074 m, then the calculations above can be carried out just as before, and we find:

(a) $C = 1.7 \times 10^{-11}$ F $= 17$ pF.

(b) $Q = 4.1 \times 10^{-10}$ C $= 410$ pC.

(c) $E = 3200$ V/m.

(d) $U = \dfrac{Q^2}{2C} = \dfrac{(4.1 \times 10^{-10} \text{ C})^2}{2(1.7 \times 10^{-11} \text{ F})} = 5.0 \times 10^{-9}$ J $= 5.0$ nJ.

EVALUATE: Q is unchanged. $E = \dfrac{Q}{\varepsilon_0 A}$ so E is therefore unchanged. U doubles because C is halved with Q unchanged. The additional stored energy comes from the work done by the force that pulled the plates apart.

24.47. **IDENTIFY:** $C = \dfrac{\varepsilon_0 A}{d}$.

SET UP: $A = 4.2 \times 10^{-5}$ m^2. The original separation between the plates is $d = 0.700 \times 10^{-3}$ m. d' is the separation between the plates at the new value of C.

EXECUTE: $C_0 = \dfrac{A\varepsilon_0}{d} = \dfrac{(4.20 \times 10^{-5} \text{ m}^2)\varepsilon_0}{7.00 \times 10^{-4} \text{ m}} = 5.31 \times 10^{-13}$ F. The new value of C is

$C = C_0 + 0.25$ pF $= 7.81 \times 10^{-13}$ F. But $C = \dfrac{A\varepsilon_0}{d'}$, so $d' = \dfrac{A\varepsilon_0}{C} = \dfrac{(4.20 \times 10^{-5} \text{ m}^2)\varepsilon_0}{7.81 \times 10^{-13} \text{ F}} = 4.76 \times 10^{-4}$ m.

Therefore the key must be depressed by a distance of 7.00×10^{-4} m $- 4.76 \times 10^{-4}$ m $= 0.224$ mm.

EVALUATE: When the key is depressed, d decreases and C increases.

24.48. **IDENTIFY:** $C = KC_0 = K\varepsilon_0 \dfrac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0$ cm$^2 = 1.0 \times 10^{-4}$ m^2.

EXECUTE: (a) $C = (10) \dfrac{(8.85 \times 10^{-12} \text{ F/m})(1.0 \times 10^{-4} \text{ m}^2)}{7.5 \times 10^{-9} \text{ m}} = 1.18 \ \mu$F per cm^2.

(b) $E = \dfrac{V}{d} = \dfrac{85 \text{ mV}}{7.5 \times 10^{-9} \text{ m}} = 1.13 \times 10^7$ V/m.

EVALUATE: The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

24.49. **IDENTIFY:** Some of the charge from the original capacitor flows onto the uncharged capacitor until the potential differences across the two capacitors are the same.

SET UP: $C = \dfrac{Q}{V_{ab}}$. Let $C_1 = 20.0 \ \mu$F and $C_2 = 10.0 \ \mu$F. The energy stored in a capacitor is

$\frac{1}{2}QV_{ab} = \frac{1}{2}CV_{ab}^2 = \dfrac{Q^2}{2C}$.

EXECUTE: (a) The initial charge on the $20.0 \ \mu$F capacitor is

$Q = C_1(800 \text{ V}) = (20.0 \times 10^{-6} \text{ F})(800 \text{ V}) = 0.0160$ C.

(b) In the final circuit, charge Q is distributed between the two capacitors and $Q_1 + Q_2 = Q$. The final

circuit contains only the two capacitors, so the voltage across each is the same, $V_1 = V_2$. $V = \dfrac{Q}{C}$ so $V_1 = V_2$

gives $\dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$. $Q_1 = \dfrac{C_1}{C_2}Q_2 = 2Q_2$. Using this in $Q_1 + Q_2 = 0.0160$ C gives $3Q_2 = 0.0160$ C and

$Q_2 = 5.33 \times 10^{-3}$ C. $Q = 2Q_2 = 1.066 \times 10^{-2}$ C. $V_1 = \dfrac{Q_1}{C_1} = \dfrac{1.066 \times 10^{-2} \text{ C}}{20.0 \times 10^{-6} \text{ F}} = 533$ V.

$V_2 = \dfrac{Q_2}{C_2} = \dfrac{5.33 \times 10^{23} \text{ C}}{10.0 \times 10^{26} \text{ F}} = 533$ V. The potential differences across the capacitors are the same, as they

should be.

(c) Energy $= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$ gives

Energy $= \frac{1}{2}(20.0 \times 10^{-6} \text{ F} + 10.0 \times 10^{-6} \text{ F})(533 \text{ V})^2 = 4.26$ J.

(d) The $20.0 \ \mu$F capacitor initially has energy $= \frac{1}{2}C_1V^2 = \frac{1}{2}(20.0 \times 10^{-6} \text{ F})(800 \text{ V})^2 = 6.40$ J. The decrease in stored energy that occurs when the capacitors are connected is 6.40 J $- 4.26$ J $= 2.14$ J.

EVALUATE: The decrease in stored energy is because of conversion of electrical energy to other forms during the motion of the charge when it becomes distributed between the two capacitors. Thermal energy is generated by the current in the wires and energy is emitted in electromagnetic waves.

24.50.   IDENTIFY: Initially the capacitors are connected in parallel to the source and we can calculate the charges Q_1 and Q_2 on each. After they are reconnected to each other the total charge is $Q = Q_2 - Q_1$. $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.

SET UP: After they are reconnected, the charges add and the voltages are the same, so $C_{eq} = C_1 + C_2$, as for capacitors in parallel.

EXECUTE: Originally $Q_1 = C_1 V_1 = (9.0 \ \mu F) \ (64 \ V) = 5.8 \times 10^{-4} \ C = 580 \ \mu C$, and

$Q_2 = C_2 V_2 = (4.0 \ \mu F)(64 \ V) = 2.6 \times 10^{-4} \ C = 260 \ \mu C$. $C_{eq} = C_1 + C_2 = 13.0 \ \mu F$. The original energy stored

is $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(13.0 \times 10^{-6} \ F)(64 \ V)^2 = 2.662 \times 10^{-2} \ J$. Disconnect and flip the capacitors, so now the

total charge is $Q = Q_2 - Q_1 = 3.20 \times 10^{-4} \ C$ and the equivalent capacitance is still the same, $C_{eq} = 13.0 \ \mu F$.

The new energy stored is $U = \frac{Q^2}{2C_{eq}} = \frac{(3.20 \times 10^{-4} \ C)^2}{2(13.0 \times 10^{-6} \ F)} = 3.983 \times 10^{-3} \ J$. The change in stored energy is

$\Delta U = 3.983 \times 10^{-3} \ J - 2.662 \times 10^{-2} \ J = -2.3 \times 10^{-2} \ J = -0.023 \ J$.

EVALUATE: When they are reconnected, charge flows and thermal energy is generated and energy is radiated as electromagnetic waves.

24.51.   IDENTIFY: Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same; $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$ For

capacitors in parallel the voltages are the same and the charges add; $C_{eq} = C_1 + C_2 + \ldots$ $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

EXECUTE: (a) Find C_{eq} for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.51.
$U_{tot} = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(2.19 \times 10^{-6} \ F)(12.0 \ V)^2 = 1.58 \times 10^{-4} \ J = 158 \ \mu J$.

(b) From Figure 24.51c, $Q_{tot} = C_{eq}V = (2.19 \times 10^{-6} \ F)(12.0 \ V) = 2.63 \times 10^{-5} \ C$. From Figure 24.51b,

$Q_{4.8} = 2.63 \times 10^{-5} \ C$. $V_{4.8} = \frac{Q_{4.8}}{C_{4.8}} = \frac{2.63 \times 10^{-5} \ C}{4.80 \times 10^{-6} \ F} = 5.48 \ V$.

$U_{4.8} = \frac{1}{2}CV^2 = \frac{1}{2}(4.80 \times 10^{-6} \ F)(5.48 \ V)^2 = 7.21 \times 10^{-5} \ J = 72.1 \ \mu J$.

This one capacitor stores nearly half the total stored energy.

EVALUATE: $U = \frac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest C stores the greatest amount of energy.
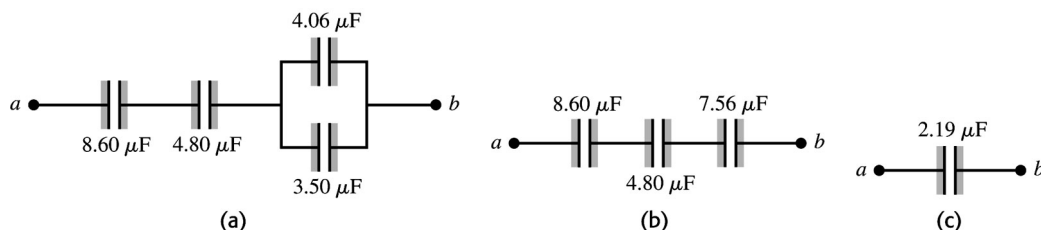

Figure 24.51

24.52. **IDENTIFY** and **SET UP:** The charge Q is the same on capacitors in series, and the potential V is the same for capacitors in parallel. C_1 is in series with C_2, and that combination is in parallel with C_3. The C_1-C_2-C_3 combination is in series with C_4. $V = Q/C$.
EXECUTE: **(a)** Since C_1 and C_2 are in series, and that combination is in parallel with C_3, the potential difference across the C_1-C_2 combination is the same as the potential difference across C_3, which is 40.0 V. Also, $Q_1 = Q_2 = Q$.
$V_1 + V_2 = 40.0$ V.
$Q/C_1 + Q/C_2 = 40.0$ V.
$Q/(6.00\ \mu\text{F}) + Q/(3.00\ \mu\text{F}) = 40.0$ V.
$Q = 80.0\ \mu\text{C}$.
Therefore
$V_1 = Q/C_1 = (80.0\ \mu\text{C})/(6.00\ \mu\text{F}) = 13.3$ V.
$V_2 = Q/C_2 = (80.0\ \mu\text{C})/(3.00\ \mu\text{F}) = 26.7$ V.
(b) First get the charge Q_4 on C_4. We know that $Q_1 = Q_2 = Q = 80.0\ \mu\text{C}$. We also have
$Q_3 = C_3 V_3 = (4.00\ \mu\text{F})(40.0\ \text{V}) = 160\ \mu\text{C}$.
$Q_4 = Q + Q_3 = 80.0\ \mu\text{C} + 160\ \mu\text{C} = 240\ \mu\text{C}$.
$V_4 = Q_4/C_4 = (240\ \mu\text{C})/(8.00\ \mu\text{F}) = 30.0$ V.
(c) $V_{ab} = V_3 + V_4 = 40.0\ \text{V} + 30.0\ \text{V} = 70.0$ V.
EVALUATE: C_3 and C_4 are *not* in parallel, so $V_3 \neq V_4$.

24.53. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.
SET UP: The network is sketched in Figure 24.53a.
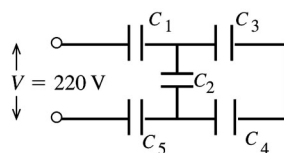
$C_1 = C_5 = 8.4\ \mu\text{F}$.
$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$.

Figure 24.53a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.53b):
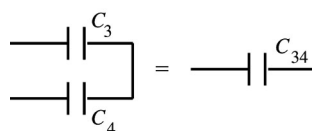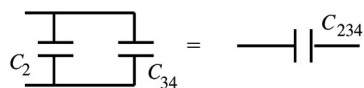

$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}.$$

$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3 C_4}.$$

Figure 24.53b

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}.$$

C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.53c):

$C_{234} = C_2 + C_{34}$.
$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}$.
$C_{234} = 6.3\ \mu\text{F}$.

Figure 24.53c

$C_1, C_5,$ and C_{234} are in series and can be replaced by C_{eq} (Figure 24.53d):

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}.$$

$$\frac{1}{C_{eq}} = \frac{2}{8.4 \ \mu F} + \frac{1}{6.3 \ \mu F}.$$

$$C_{eq} = 2.5 \ \mu F.$$

Figure 24.53d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.
(b) IDENTIFY and **SET UP:** In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
EXECUTE: The equivalent circuit is drawn in Figure 24.53e.

$$Q_{eq} = C_{eq}V.$$

$$Q_{eq} = (2.5 \ \mu F)(220 \ V) = 550 \ \mu C.$$

Figure 24.53e

$Q_1 = Q_5 = Q_{234} = 550 \ \mu C$ (capacitors in series have same charge).

$$V_1 = \frac{Q_1}{C_1} = \frac{550 \ \mu C}{8.4 \ \mu F} = 65 \ V.$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550 \ \mu C}{8.4 \ \mu F} = 65 \ V.$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550 \ \mu C}{6.3 \ \mu F} = 87 \ V.$$

Now draw the network as in Figure 24.53f.
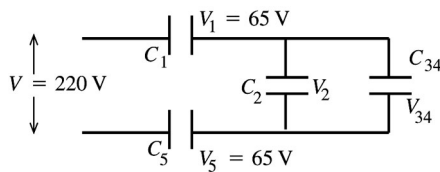
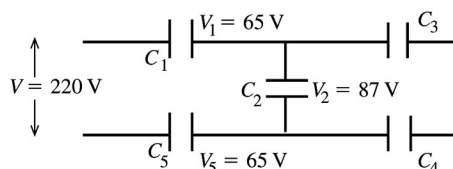

$V_2 = V_{34} = V_{234} = 87 \ V$
capacitors in parallel have the same potential.

Figure 24.53f

$$Q_2 = C_2V_2 = (4.2 \ \mu F)(87 \ V) = 370 \ \mu C.$$

$$Q_{34} = C_{34}V_{34} = (2.1 \ \mu F)(87 \ V) = 180 \ \mu C.$$

Finally, consider the original circuit (Figure 24.53g).

$Q_3 = Q_4 = Q_{34} = 180 \ \mu C$
capacitors in series have the same charge.

Figure 24.53g

$$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu C}{4.2\ \mu F} = 43\text{ V.}$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu C}{4.2\ \mu F} = 43\text{ V.}$$

Summary: $Q_1 = 550\ \mu C$, $V_1 = 65$ V.

$Q_2 = 370\ \mu C$, $V_2 = 87$ V.

$Q_3 = 180\ \mu C$, $V_3 = 43$ V.

$Q_4 = 180\ \mu C$, $V_4 = 43$ V.

$Q_5 = 550\ \mu C$, $V_5 = 65$ V.

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220$ V (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$.

24.54. **IDENTIFY** and **SET UP:** The total stored energy is $U = \frac{1}{2}CV^2$, and the energy density is $u = U/(\text{volume})$.

The volume of a cylinder is $\pi r^2 l$. $u = \frac{1}{2}\varepsilon E^2$, where $\varepsilon = K\varepsilon_0$.

EXECUTE: **(a)** $U = \frac{1}{2}CV^2 = (1/2)(3000\text{ F})(2.7\text{ V})^2 = 1.09 \times 10^4$ J, which rounds to 1.1×10^4 J.

(b) 3.0 Wh $= (3.0\text{ J/s})(3600\text{ s}) = 1.1 \times 10^4$ J, which agrees with our result in (a) within the accuracy of the given numbers.

(c) $u = U/(\text{volume}) = U/(\pi r^2 l) = (1.09 \times 10^4\text{ J})/[\pi(0.030\text{ m})^2(0.135\text{ m})] = 2.9 \times 10^7$ J/m^3.

(d) For polyester, $K = 3.3$ and $E_m = 6 \times 10^7$ V/m, so

$$u = \frac{1}{2}K\varepsilon_0 E^2 = (1/2)(3.3)(8.854 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(6 \times 10^7\text{ V/m})^2 = 5.3 \times 10^4\text{ J/m}^3.$$

$u/u_{\text{polyester}} = (2.9 \times 10^7\text{ J/m}^3)/(5.3 \times 10^4\text{ J/m}^3) = 540$, so this capacitor can have over 500 times the energy density of a polyester capacitor.

EVALUATE: It requires only 2.7 V to give this capacitor a stored energy of 1.1×10^4 J. For a typical

1.0-μF capacitor, the voltage would be $V = (2U/C)^{1/2} = [2(1.11 \times 0^4\text{ J})/(1.0 \times 10^{-6}\text{ F})] = 1.5 \times 10^5$ V $=$

150 kV. That's quite a difference from 2.7 V!

24.55. **IDENTIFY:** Capacitors in series carry the same charge, while capacitors in parallel have the same potential difference across them.

SET UP: $V_{ab} = 150$ V, $Q_1 = 150\ \mu C$, $Q_3 = 450\ \mu C$, and $V = Q/C$.

EXECUTE: $C_1 = 3.00\ \mu F$ so $V_1 = \frac{Q_1}{C_1} = \frac{150\ \mu C}{3.00\ \mu F} = 50.0$ V and $V_1 = V_2 = 50.0$ V. $V_1 + V_3 = V_{ab}$ so

$V_3 = 100$ V. $C_3 = \frac{Q_3}{V_3} = \frac{450\ \mu C}{100\text{ V}} = 4.50\ \mu F$. $Q_1 + Q_2 = Q_3$ so $Q_2 = Q_3 - Q_1 = 450\ \mu C - 150\ \mu C = 300\ \mu C$

and $C_2 = \frac{Q_2}{V_2} = \frac{300\ \mu C}{50.0\text{ V}} = 6.00\ \mu F$.

EVALUATE: Capacitors in parallel only carry the same charge if they have the same capacitance.

24.56. **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open, each pair of 3.00 μF and 6.00 μF capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed, each pair of 3.00 μF and 6.00 μF capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: **(a)** With the switch open $C_{eq} = \left(\left(\frac{1}{3\ \mu F} + \frac{1}{6\ \mu F} \right)^{-1} + \left(\frac{1}{3\ \mu F} + \frac{1}{6\ \mu F} \right)^{-1} \right) = 4.00\ \mu F.$

$Q_{\text{total}} = C_{eq}V = (4.00\ \mu F)(210\text{ V}) = 8.40 \times 10^{-4}$ C. By symmetry, each capacitor carries 4.20×10^{-4} C. The

voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140$ V and $V_{ac} = Q/C_6 = 70$ V.

$V_{cd} = V_{ad} - V_{ac} = 70$ V.

(b) When the switch is closed, the points c and d must be at the same potential, so the equivalent

capacitance is $C_{eq} = \left(\dfrac{1}{(3.00 + 6.00)\ \mu\text{F}} + \dfrac{1}{(3.00 + 6.00)\ \mu\text{F}} \right)^{-1} = 4.5\ \mu\text{F}.$

$Q_{total} = C_{eq}V = (4.50\ \mu\text{F})(210\ \text{V}) = 9.5\times10^{-4}$ C, and each capacitor has the same potential difference of

105 V (again, by symmetry).

(c) Consider the $C_3 = 3.00\ \mu\text{F}$ and $C_6 = 6.00\ \mu\text{F}$ capacitors in the upper branch of the network. The

only way for the net charge Q_{net} on the negative plate of C_3 and the positive plate of C_6 to change is

by charge to flow through the switch. With the switch open all four capacitors have the same charge

and $Q_{net} = 0$. With the switch closed the charge on C_3 is $Q_3 = (3.00\ \mu\text{F})(105\ \text{V}) = 315\ \mu\text{C}$ and the

charge on C_6 is $Q_6 = (6.00\ \mu\text{F})(105\ \text{V}) = 630\ \mu\text{C}$ and $Q_{net} = Q_2 - Q_1 = 315\ \mu\text{C}$. Therefore, the change in

Q_{net} is $315\ \mu\text{C}$ and this is the amount of charge that flowed through the switch when it was closed.

EVALUATE: When the switch is closed the charge must redistribute to make points c and d be at the same potential.

24.57. **(a) IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.57a.
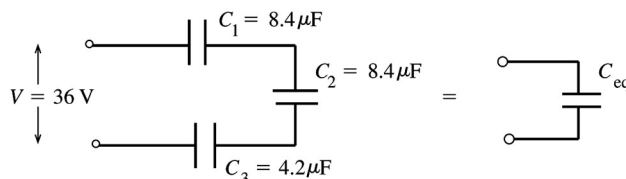

Figure 24.57a

EXECUTE: $C_3 = C_1/2$ so $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{4}{8.4\ \mu\text{F}}$ and $C_{eq} = 8.4\ \mu\text{F}/4 = 2.1\ \mu\text{F}.$

$Q = C_{eq}V = (2.1\ \mu\text{F})(36\ \text{V}) = 76\ \mu\text{C}.$

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76\ \mu\text{C}.$

EVALUATE: The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

(b) IDENTIFY and SET UP: Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36$ V.

EXECUTE: $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3.$

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3).$

But also $V_1 + V_2 + V_3 = V = 36$ V, so $U = \frac{1}{2}QV = \frac{1}{2}(76\ \mu\text{C})(36\ \text{V}) = 1.4\times10^{-3}$ J.

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) IDENTIFY: The charges on the plates redistribute to make the potentials across each capacitor the same.

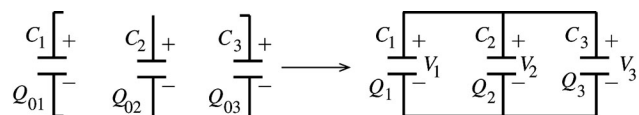**SET UP:** The capacitors before and after they are connected are sketched in Figure 24.57b.

Figure 24.57b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76 \ \mu C) = 228 \ \mu C$. Thus $Q_1 + Q_2 + Q_3 = 228 \ \mu C$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4 \ \mu F/4.2 \ \mu F) = 2Q_3$.

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228 \ \mu C$.

$5Q_3 = 228 \ \mu C$ and $Q_3 = 45.6 \ \mu C$, $Q_1 = Q_2 = 91.2 \ \mu C$

Then $V_1 = \dfrac{Q_1}{C_1} = \dfrac{91.2 \ \mu C}{8.4 \ \mu F} = 11$ V, $V_2 = \dfrac{Q_2}{C_2} = \dfrac{91.2 \ \mu C}{8.4 \ \mu F} = 11$ V, and $V_3 = \dfrac{Q_3}{C_3} = \dfrac{45.6 \ \mu C}{4.2 \ \mu F} = 11$ V.

The voltage across each capacitor in the parallel combination is 11 V.

(d) $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $V_1 = V_2 = V_3$ so $U = \frac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \frac{1}{2}(11 \text{ V})(228 \ \mu C) = 1.3 \times 10^{-3}$ J.

EVALUATE: This is less than the original energy of 1.4×10^{-3} J. The stored energy has decreased, as in Example 24.7.

24.58.   **IDENTIFY:** $C = \dfrac{\varepsilon_0 A}{d}$. $C = \dfrac{Q}{V}$. $V = Ed$. $U = \frac{1}{2}QV$.

SET UP: $d = 3.0 \times 10^3$ m. $A = \pi r^2$, with $r = 1.0 \times 10^3$ m.

EXECUTE: **(a)** $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)\pi(1.0 \times 10^3 \text{ m})^2}{3.0 \times 10^3 \text{ m}} = 9.3 \times 10^{-9}$ F.

(b) $V = \dfrac{Q}{C} = \dfrac{20 \text{ C}}{9.3 \times 10^{-9} \text{ F}} = 2.2 \times 10^9$ V.

(c) $E = \dfrac{V}{d} = \dfrac{2.2 \times 10^9 \text{ V}}{3.0 \times 10^3 \text{ m}} = 7.3 \times 10^5$ V/m.

(d) $U = \frac{1}{2}QV = \frac{1}{2}(20 \text{ C})(2.2 \times 10^9 \text{ V}) = 2.2 \times 10^{10}$ J.

EVALUATE: Thunderclouds involve very large potential differences and large amounts of stored energy.

24.59.   **IDENTIFY:**   Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

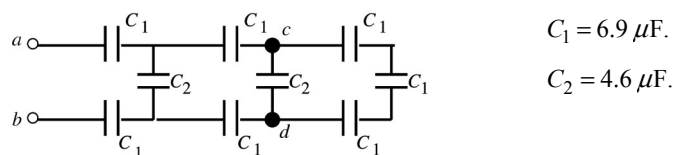**(a) SET UP:**   The network is sketched in Figure 24.59a.

$C_1 = 6.9 \ \mu F.$

$C_2 = 4.6 \ \mu F.$

Figure 24.59a

EXECUTE: Simplify the network by replacing the capacitor combinations by their equivalents. Make the replacement shown in Figure 24.59b.
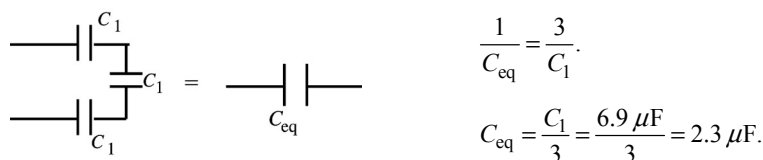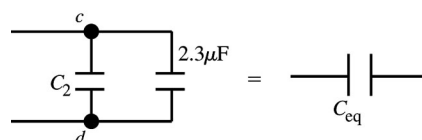

$\dfrac{1}{C_{eq}} = \dfrac{3}{C_1}.$

$C_{eq} = \dfrac{C_1}{3} = \dfrac{6.9 \ \mu F}{3} = 2.3 \ \mu F.$

Figure 24.59b

Next make the replacement shown in Figure 24.59c.

$C_{eq} = 2.3\ \mu F + C_2.$

$C_{eq} = 2.3\ \mu F + 4.6\ \mu F = 6.9\ \mu F.$

Figure 24.59c

Make the replacement shown in Figure 24.59d.
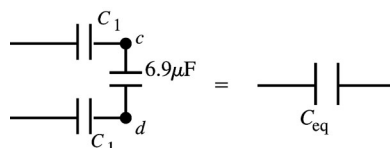
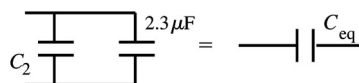$\dfrac{1}{C_{eq}} = \dfrac{2}{C_1} + \dfrac{1}{6.9\ \mu F} = \dfrac{3}{6.9\ \mu F}.$

$C_{eq} = 2.3\ \mu F.$

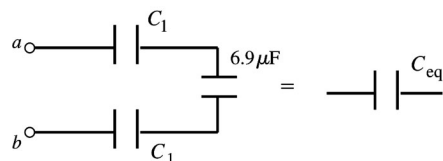**Figure 24.59d**

Make the replacement shown in Figure 24.59e.

$C_{eq} = C_2 + 2.3\ \mu F = 4.6\ \mu F + 2.3\ \mu F.$

$C_{eq} = 6.9\ \mu F.$

Figure 24.59e

Make the replacement shown in Figure 24.59f.
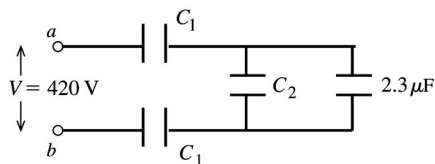

$\dfrac{1}{C_{eq}} = \dfrac{2}{C_1} + \dfrac{1}{6.9\ \mu F} = \dfrac{3}{6.9\ \mu F}.$

$C_{eq} = 2.3\ \mu F.$
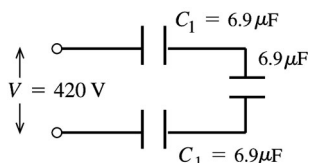
Figure 24.59f

(b) SET UP and **EXECUTE**:   Consider the network as drawn in Figure 24.59g.

From part (a) $2.3\ \mu F$ is the equivalent
capacitance of the rest of the network.

Figure 24.59g

The equivalent network is shown in Figure 24.59h.
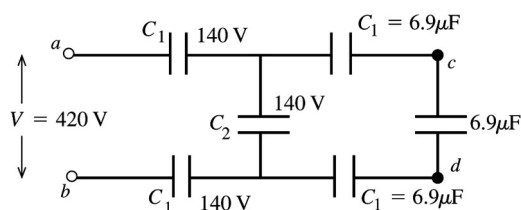

The capacitors are in series, so all
three capacitors have the same Q.

Figure 24.59h

But here all three have the same C, so by $V = Q/C$ all three must have the same V. The three voltages must add to 420 V, so each capacitor has $V = 140$ V. The 6.9 μF to the right is the equivalent of C_2 and the 2.3 μF capacitor in parallel, so $V_2 = 140$ V. (Capacitors in parallel have the same potential difference.)

Hence $Q_1 = C_1V_1 = (6.9\ \mu\text{F})(140\ \text{V}) = 9.7 \times 10^{-4}$ C and $Q_2 = C_2V_2 = (4.6\ \mu\text{F})(140\ \text{V}) = 6.4 \times 10^{-4}$ C.

(c) From the potentials deduced in part (b) we have the situation shown in Figure 24.59i.

From part (a) 6.9 μF is the equivalent capacitance of the rest of the network.

Figure 24.59i

The three right-most capacitors are in series and therefore have the same charge. But their capacitances are also equal, so by $V = Q/C$ they each have the same potential difference. Their potentials must sum to 140 V, so the potential across each is 47 V and $V_{cd} = 47$ V.

EVALUATE: In each capacitor network the rules for combining V for capacitors in series and parallel are obeyed. Note that $V_{cd} < V$, in fact $V - 2(140\ \text{V}) - 2(47\ \text{V}) = V_{cd}$.

24.60. **IDENTIFY:** Find the total charge on the capacitor network when it is connected to the battery. This is the amount of charge that flows through the signal device when the switch is closed.
Circuit (a):
SET UP: For capacitors in parallel, $C_{eq} = C_1 + C_2 + C_3 + \ldots$

EXECUTE: $C_{equiv} = C_1 + C_2 + C_3 = 60.0\ \mu\text{F}$. $Q = CV = (60.0\ \mu\text{F})(120\ \text{V}) = 7200\ \mu\text{C}$.

EVALUATE: More charge is stored by the three capacitors in parallel than would be stored in each capacitor used alone.
Circuit (b):

SET UP: $C_{equiv} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} \right)^{-1}$.

EXECUTE: $C_{equiv} = 5.45\ \mu\text{F}$. $Q = (5.45\ \mu\text{F})(120\text{V}) = 654\ \mu\text{C}$.

EVALUATE: Less charge is stored by the three capacitors in series than would be stored in each capacitor used alone.

24.61. **(a) IDENTIFY and SET UP:** Q is constant. $C = KC_0$; use $C = Q/V_{ab}$ to relate the dielectric constant K to the ratio of the voltages without and with the dielectric.
EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$.

without the dielectric: $V_0 = Q/C_0$.

$V_0/V = K$, so $K = (45.0\ \text{V})/(11.5\ \text{V}) = 3.91$.

EVALUATE: Our analysis agrees with Eq. (24.13).
(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area $2A/3$ and air between the plates and one with area $A/3$ and dielectric between the plates.
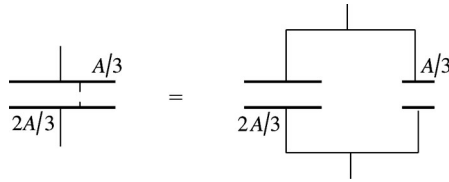**SET UP:** The equivalent network is shown in Figure 24.61 (next page).

Figure 24.61

EXECUTE: Let $C_0 = \varepsilon_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3, C_2 = 2C_0/3$;
$C_{eq} = C_1 + C_2 = (C_0/3)(K + 2)$.

$$V = \frac{Q}{C_{eq}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0 \text{ V})\left(\frac{3}{5.91}\right) = 22.8 \text{ V}.$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

24.62. **IDENTIFY:** This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$.

SET UP: For capacitors in series, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

EXECUTE: **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\dfrac{\varepsilon_0 A}{(d-a)/2} = \dfrac{\varepsilon_0 A}{d-a}$.

(b) $C = \dfrac{\varepsilon_0 A}{d-a} = \dfrac{\varepsilon_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}$.

EVALUATE: **(c)** As $a \to 0$, $C \to C_0$. The metal slab has no effect if it is very thin. And as $a \to d$, $C \to \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance y parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large C.

24.63. **IDENTIFY:** Capacitors in series carry the same charge, but capacitors in parallel have the same potential difference across them.

SET UP: $V_{ab} = 48.0 \text{ V}$. $C = Q/V$ and $U = \frac{1}{2}CV^2$. For capacitors in parallel, $C = C_1 + C_2$, and for capacitors in series, $1/C = 1/C_1 + 1/C_2$.

EXECUTE: Using $U = \frac{1}{2}CV^2$ gives $C = \dfrac{2U}{V^2} = \dfrac{2(2.90\times10^{-3} \text{ J})}{(48.0 \text{ V})^2} = 2.517\times10^{-6} \text{ F}$, which is the equivalent capacitance of the network. The equivalent capacitance for C_1 and C_2 in series is

$C_{12} = \frac{1}{2}(4.00 \text{ }\mu\text{F}) = 2.00 \text{ }\mu\text{F}$. If C_{123} is the equivalent capacitance for C_{12} and C_3 in parallel, then

$\dfrac{1}{C_{123}} + \dfrac{1}{C_4} = \dfrac{1}{C}$. Solving for C_{123} gives

$\dfrac{1}{C_{123}} = \dfrac{1}{C} - \dfrac{1}{C_4} = \dfrac{1}{2.517\times10^{-6} \text{ F}} - \dfrac{1}{8.00\times10^{-6} \text{ F}} = 2.722\times10^5 \text{ F}^{-1}$, so $C_{123} = 3.673\times10^{-6} \text{ F}$.

$C_{12} + C_3 = C_{123}$. $C_3 = C_{123} - C_{12} = 3.673 \text{ }\mu\text{F} - 2.00 \text{ }\mu\text{F} = 1.67 \text{ }\mu\text{F}$.

EVALUATE: As with most circuits, it is necessary to solve them in a series of steps rather than using a single step.

24.64. **IDENTIFY:** The electric field energy density is $u = \frac{1}{2}\varepsilon_0 E^2$. $U = \frac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)^2 = \frac{\lambda^2}{8\pi^2\varepsilon_0 r^2}$.

(b) $U = \int u \, dV = 2\pi L \int u r \, dr = \frac{L\lambda^2}{4\pi\varepsilon_0} \int_{r_a}^{r_b} \frac{dr}{r}$ and $\frac{U}{L} = \frac{\lambda^2}{4\pi\varepsilon_0} \ln(r_b/r_a)$.

(c) $U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi\varepsilon_0 L} \ln(r_b/r_a) = \frac{\lambda^2 L}{4\pi\varepsilon_0} \ln(r_b/r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U = \frac{Q^2}{2C}$ to calculate C/L and would obtain the same result as in Example 24.4.

24.65. **IDENTIFY:** The two slabs of dielectric are in series with each other.

SET UP: The capacitor is equivalent to C_1 and C_2 in series, so $\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C}$, which gives $C = \frac{C_1 C_2}{C_1 + C_2}$.

EXECUTE: With $d = 1.90$ mm, $C_1 = \frac{K_1\varepsilon_0 A}{d}$ and $C_2 = \frac{K_2\varepsilon_0 A}{d}$.

$C = \left(\frac{K_1 K_2}{K_1 + K_2}\right)\frac{\varepsilon_0 A}{d} = \left(\frac{(4.7)(2.6)}{4.7 + 2.6}\right)\frac{(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.0800 \text{ m})^2}{1.90\times10^{-3} \text{ m}} = 4.992\times10^{-11} \text{ F}$.

$U = \frac{1}{2}CV^2 = \frac{1}{2}(4.992\times10^{-11} \text{ F})(86.0 \text{ V})^2 = 1.85\times10^{-7} \text{ J}$.

EVALUATE: The dielectrics increase the capacitance, allowing the capacitor to store more energy than if it were air-filled.

24.66. **IDENTIFY:** The capacitor is equivalent to two capacitors in parallel, as shown in Figure 24.66.
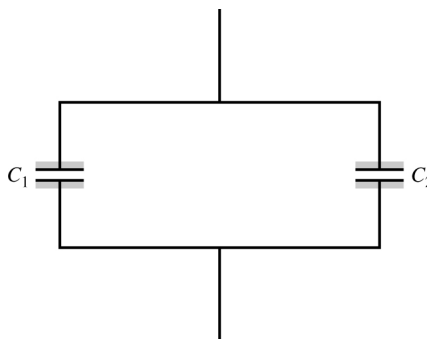

Figure 24.66

SET UP: Each of these two capacitors have plates that are 12.0 cm by 6.0 cm. For a parallel-plate capacitor with dielectric filling the volume between the plates, $C = K\varepsilon_0 \frac{A}{d}$. For two capacitors in parallel, $C = C_1 + C_2$. The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: (a) $C = C_1 + C_2$.

$C_2 = \varepsilon_0 \dfrac{A}{d} = \dfrac{(8.854 \times 10^{-12} \text{ F/m})(0.120 \text{ m})(0.060 \text{ m})}{4.50 \times 10^{-3} \text{ m}} = 1.42 \times 10^{-11} \text{ F}$.

$C_1 = KC_2 = (3.40)(1.42 \times 10^{-11} \text{ F}) = 4.83 \times 10^{-11} \text{ F}$. $C = C_1 + C_2 = 6.25 \times 10^{-11} \text{ F} = 62.5 \text{ pF}$.

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.25 \times 10^{-11} \text{ F})(18.0 \text{ V})^2 = 1.01 \times 10^{-8} \text{ J}$.

(c) Now $C_1 = C_2$ and $C = 2(1.42 \times 10^{-11} \text{ F}) = 2.84 \times 10^{-11} \text{ F}$.

$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.84 \times 10^{-11} \text{ F})(18.0 \text{ V})^2 = 4.60 \times 10^{-9} \text{ J}$.

EVALUATE: The plexiglass increases the capacitance and that increases the energy stored for the same voltage across the capacitor.

24.67.   **IDENTIFY:**   The object is equivalent to two identical capacitors in parallel, where each has the same area A, plate separation d and dielectric with dielectric constant K.

SET UP: For each capacitor in the parallel combination, $C = \dfrac{\varepsilon_0 A}{d}$.

EXECUTE: (a) The charge distribution on the plates is shown in Figure 24.67.

(b) $C = 2\left(\dfrac{\varepsilon_0 A}{d}\right) = \dfrac{2(4.2)\varepsilon_0 (0.120 \text{ m})^2}{4.5 \times 10^{-4} \text{ m}} = 2.38 \times 10^{-9} \text{ F}$.

EVALUATE: If two of the plates are separated by both sheets of paper to form a capacitor,

$C = \dfrac{\varepsilon_0 A}{2d} = \dfrac{2.38 \times 10^{-9} \text{ F}}{4}$, smaller by a factor of 4 compared to the capacitor in the problem.
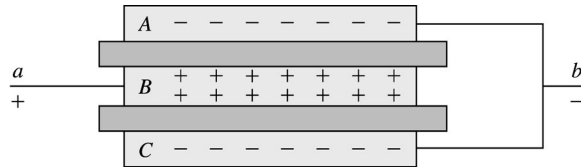

Figure 24.67

24.68.   **IDENTIFY:**   The system is equivalent to two capacitors in parallel. One of the capacitors has plate separation d, plate area $w(L - h)$ and air between the plates. The other has the same plate separation d, plate area wh and dielectric constant K.

SET UP: Define K_{eff} by $C_{\text{eq}} = \dfrac{K_{\text{eff}}\varepsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: (a) The capacitors are in parallel, so $C = \dfrac{\varepsilon_0 w(L - h)}{d} + \dfrac{K\varepsilon_0 wh}{d} = \dfrac{\varepsilon_0 wL}{d}\left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$. This

gives $K_{\text{eff}} = \left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$.

(b) For gasoline, with $K = 1.95$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{4}\right) = 1.24$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{2}\right) = 1.48$;

$\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{3L}{4}\right) = 1.71$.

(c) For methanol, with $K = 33$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{4}\right) = 9$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{2}\right) = 17$;

$\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{3L}{4}\right) = 25$.

(d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

EVALUATE: When $h = 0$, $K_{eff} = 1$. When $h = L$, $K_{eff} = K$.

24.69. **IDENTIFY** and **SET UP:** For two capacitors in series, $\dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{C_{eq}}$, which gives $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$. For two

capacitors in parallel, $C_{eq} = C_1 + C_2$. $C = Q/V$. The stored energy can be written as $U = \frac{1}{2}CV^2$ or

$U = \dfrac{Q^2}{2C}$.

EXECUTE: **(a)** When connected in series, the stored energy is 0.0400 J, so

$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\dfrac{C_1 C_2}{C_1 + C_2}\right)V^2 = \frac{1}{2}\left(\dfrac{C_1 C_2}{C_1 + C_2}\right)(200.0 \text{ V})^2 = 0.0400 \text{ J}$, which gives

$\dfrac{C_1 C_2}{C_1 + C_2} = 2.00 \ \mu F$.

When connected in parallel, the stored energy is 0.180 J, so

$U = \frac{1}{2}CV^2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2)(200.0 \text{ V})^2 = 0.180 \text{ J}$.

$C_1 + C_2 = 9.00 \ \mu F$.

Solving the two equations for C_1 and C_2 gives $C_1 = 6.00 \ \mu F$ and $C_2 = 3.00 \ \mu F$.

(b) When the capacitors are in series, both have the same charge. The stored energy is $U = \dfrac{Q^2}{2C}$, so the

capacitor with the *smaller* capacitance stores more energy, which is C_2.

(c) When the capacitors are in parallel, the potential across them is the same. The stored energy is

$U = \frac{1}{2}CV^2$, so the capacitor with the *larger* capacitance stores the most energy, which is C_1.

EVALUATE: When the two capacitors are connected in parallel, they can store considerably more energy than when in series.

24.70. **IDENTIFY** and **SET UP:** The presence of the dielectric affects the charge and energy in the capacitor for a given potential difference. $V = Ed$, $Q = CV$, $K = C/C_0$, $U = \frac{1}{2}CV^2$. We use the values for K and E_m from Table 24.2. In this case, $E = 0.500E_m$ and $d = 2.50$ mm $= 0.00250$ m.

EXECUTE: **(a)** Using $U = \frac{1}{2}CV^2$, $C = KC_0$, $V = Ed$, and $E = 0.500E_m$, the stored energy is

$U = \frac{1}{2}CV^2 = \frac{1}{2}KC_0(Ed)^2 = \frac{1}{2}KC_0(0.500E_m d)^2$.

For polycarbonate, $K = 2.8$ and $E_m = 3 \times 10^7$ V/m. Therefore the stored energy is

$U = (1/2)[(2.8)(6.00 \times 10^{-12} \text{ F})][(0.500)(3 \times 10^7 \text{ V/m})(0.00250 \text{ m})]^2 = 1.18 \times 10^{-2} \text{ J}$, which rounds to 12 mJ.

Using similar calculations for the other materials, the results for U are:
12 mJ (polycarbonate)
56 mJ (polyester)
51 mJ (polypropylene)
4.9 mJ (polystyrene)
2.2 mJ (pyrex)

(b) $Q = CV = KC_0(Ed) = KC_0(0.500E_m)d$.

For polycarbonate we have

$Q = (2.8)(6.00 \times 10^{-12} \text{ F})(0.500)(3 \times 10^7 \text{ V/m})(0.00250 \text{ m}) = 6.3 \times 10^{-7} \text{ C} = 0.63 \ \mu C$.

Similar calculations for the other materials yield:
0.63 μC (polycarbonate)
1.5 μC (polyester)
1.2 μC (polypropylene)
0.39 μC (polystyrene)
0.35 μC (pyrex)

(c) $V = Ed = 0.500E_{\mathrm{m}}d$. For polycarbonate this gives

$V = (0.500)(3 \times 10^7 \text{ V/m})(0.00250 \text{ m}) = 3.8 \times 10^4 \text{ V} = 38 \text{ kV}$.

Similar calculations for the other materials yield:

38 kV (polycarbonate)

75 kV (polyester)

88 kV (polypropylene)

25 kV (polystyrene)

13 kV (pyrex)

EVALUATE: **(d)** Polyester is best for maximum energy storage and maximum charge, but polypropylene is best for maximum voltage. No single material is best for all three categories. As so often occurs, the choice of materials is a trade-off.

24.71. **IDENTIFY and SET UP:** For a parallel-plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$. The stored energy can be expressed as

$$U = \tfrac{1}{2}CV^2 \text{ or } U = \frac{Q^2}{2C}.$$

EXECUTE: **(a)** If the battery remains connected, V remains constant, so it is useful to write the energy in terms of V and C:

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2 = \frac{\varepsilon_0 A V^2}{2} \cdot \frac{1}{d}.$$

If the battery is disconnected, Q remains constant, so it is useful to write the energy in terms of Q and C:

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2\left(\dfrac{\varepsilon_0 A}{d}\right)} = \left(\frac{Q^2}{2\varepsilon_0 A}\right)d.$$

The graph shows a linear relationship between U and $1/d$, so it must represent the case where the battery remains connected to the capacitor.

(b) In a graph of U versus $1/d$ for the equation $U = \dfrac{\varepsilon_0 A V^2}{2} \cdot \dfrac{1}{d}$, the slope should be equal to $\dfrac{\varepsilon_0 A V^2}{2}$.

Choosing points on the graph in the problem, the slope is $\dfrac{(73 - 18) \times 10^{-9} \text{ J}}{20.0 \text{ cm}^{-1} - 5.0 \text{ cm}^{-1}} = 3.67 \times 10^{-11} \text{ J} \cdot \text{m}$.

Solving for A gives

$A = 2(\text{slope})/\varepsilon_0 V^2 = 2(3.67 \times 10^{-11} \text{ J} \cdot \text{m})/(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(24.0 \text{ V})^2] = 0.014 \text{ m}^2 = 144 \text{ cm}^2$.

(c) <u>With the battery connected</u>: $U = \dfrac{\varepsilon_0 A V^2}{2} \cdot \dfrac{1}{d}$, so as we increase d from 0.0500 cm to 0.400 cm, the energy *decreases* since V remains constant.

<u>With the battery disconnected</u>: $U = \left(\dfrac{Q^2}{2\varepsilon_0 A}\right)d$, so as we increase d, the energy *increases* since Q does not change. Therefore there is more energy stored with the battery *disconnected* as d is increased.

EVALUATE: If this capacitor were square, its plates would be 12 cm×12 cm. This is a reasonable size for a piece of apparatus for use in a laboratory and could easily be manufactured.

24.72. **IDENTIFY:** The system can be considered to be two capacitors in parallel, one with plate area $L(L - x)$ and air between the plates and one with area Lx and dielectric filling the space between the plates.

SET UP: $C = \dfrac{K\varepsilon_0 A}{d}$ for a parallel-plate capacitor with plate area A.

EXECUTE: **(a)** $C = \dfrac{\varepsilon_0}{D}\big[(L - x)L + xKL\big] = \dfrac{\varepsilon_0 L}{D}\big[L + (K - 1)x\big]$.

(b) $dU = \frac{1}{2}(dC)V^2$, where $C = C_0 + \frac{\varepsilon_0 L}{D}(-dx + dxK)$, with $C_0 = \frac{\varepsilon_0 L}{D}\left[L + (K-1)x\right]$. This gives

$$dU = \frac{1}{2}\left(\frac{\varepsilon_0 L dx}{D}(K-1)\right)V^2 = \frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

(c) If the charge is kept constant on the plates, then $Q = \frac{\varepsilon_0 LV}{D}\left[L + (K-1)x\right]$ and

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C_0 V^2\left(\frac{C}{C_0}\right). \quad U \approx \frac{C_0 V^2}{2}\left(1 - \frac{\varepsilon_0 L}{DC_0}(K-1)dx\right) \text{ and } \Delta U = U - U_0 = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

(d) Since $dU = -Fdx = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion dx,

meaning that the slab feels a force pushing it out.

EVALUATE: **(e)** When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for dU in part (c) to $dU = -Fdx$ gives

$$F = \frac{(K-1)\varepsilon_0 V^2 L}{2D}.$$

24.73. **IDENTIFY** and **SET UP:** The potential difference is $V = 30$ mV – (–70 mV) = 100 mV, and $Q = CV$.
    EXECUTE: $Q = CV$ gives $Q/\text{cm}^2 = (C/\text{cm}^2)V = (1 \ \mu\text{F/cm}^2)(100 \text{ mV})(1 \text{ mol}/10^5 \text{ C}) = 10^{-12}$ mol/cm^2, which is choice (c).
    EVALUATE: This charge produces a potential difference of 100 mV = 0.1 V, which is certainly measurable using ordinary laboratory meters.

24.74. **IDENTIFY** and **SET UP:** The change in concentration of Na$^+$ ions is equal to the added charge divided by the volume of the spherical egg. The original concentration of ions is given as 30 mmol/L. We use the answer from Problem 24.73 to get the added charge.
    EXECUTE: The added charge is $(10^{-12}$ mol/cm$^2)$(surface area of egg) = $(10^{-12}$ mol/cm$^2)(4\pi R^2)$, and the original volume of the egg is $(4\pi/3)R^3$. Therefore the change in concentration is
    $(10^{-12}$ mol/cm$^2)(4\pi R^2)/\left[(4\pi/3)R^3\right] = 3(10^{-12}$ mol/cm$^2)/R = 3(10^{-12}$ mol/cm$^2)/(100\times10^{-4}\text{cm}) =$

    3×10^{10} mol/cm$^3 = 3\times10^{-5}$ mmol/L.

    The fractional change in the concentrations is $(3\times10^{-5}$ mmol/L)/(30 mmol/L) $= 10^{-5}$, which is 1 part in 10^5. Therefore choice (b) is correct.
    EVALUATE: As a percent, this change is $10^{-3}\% = 0.001\%$, which is quite small yet certainly important for the organism.

24.75. **IDENTIFY** and **SET UP:** The calcium Ca^{2+} ions carry twice the charge of the Na$^+$ ions.
    EXECUTE: The charge to produce the given voltage change would be the same as with Na$^+$, so we would need only half as many Ca^{2+} ions to accomplish this. Thus choice (a) is correct.
    EVALUATE: Ca^{+2} ions are nearly twice as heavy as Na$^+$ ions, so they may not move as readily as the sodium ions.

24.76. **IDENTIFY** and **SET UP:** The energy is needed to change the potential from 30 mV to –70 mV.
    $U = \frac{1}{2}CV^2$. The capacitance is $(1 \ \mu\text{F/cm}^2)$(surface area of egg).

    EXECUTE: For a spherical egg, the surface area is $4\pi R^2$, so the capacitance is
    $C = (1 \ \mu\text{F/cm}^2)(4\pi R^2) = (1 \ \mu\text{F/cm}^2)(4\pi)(100\times10^{-4} \text{ cm})^2 = 1.26\times10^{-9}$ F.

    The change in stored energy is
    $\Delta U = \frac{1}{2}CV_2^2 - \frac{1}{2}CV_1^2 = \frac{1}{2}C(V_2^2 - V_1^2)$.

    $\Delta U = (1/2)(1.26\times10^{-9} \text{ F})[(-70\times10^{-3} \text{ V})^2 - (30\times10^{-3} \text{ V})^2] = 2.5\times10^{-12} \text{ J} = 2.5 \text{ pJ} \approx 3 \text{ pJ}$, which makes choice (d) the correct one.
    EVALUATE: The actual energy required would probably be greater than 2.5 pJ, depending on the process by which the charging is accomplished, but our value is the minimum energy needed.