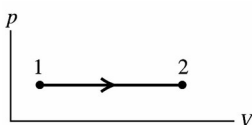


## THE FIRST LAW OF THERMODYNAMICS

- 19.1. (a) IDENTIFY and SET UP:** The pressure is constant and the volume increases.



The  $pV$ -diagram is sketched in Figure 19.1.

**Figure 19.1**

**(b)**  $W = \int_{V_1}^{V_2} p dV$ .

Since  $p$  is constant,  $W = p \int_{V_1}^{V_2} dV = p(V_2 - V_1)$ .

The problem gives  $T$  rather than  $p$  and  $V$ , so use the ideal gas law to rewrite the expression for  $W$ .

**EXECUTE:**  $pV = nRT$  so  $p_1V_1 = nRT_1$ ,  $p_2V_2 = nRT_2$ ; subtracting the two equations gives

$$p(V_2 - V_1) = nR(T_2 - T_1).$$

Thus  $W = nR(T_2 - T_1)$  is an alternative expression for the work in a constant pressure process for an ideal gas.

$$\text{Then } W = nR(T_2 - T_1) = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(107^\circ\text{C} - 27^\circ\text{C}) = +1330 \text{ J}.$$

**EVALUATE:** The gas expands when heated and does positive work.

- 19.2. IDENTIFY:** At constant pressure,  $W = p\Delta V = nR\Delta T$ . Since the gas is doing work, it must be expanding, so  $\Delta V$  is positive, which means that  $\Delta T$  must also be positive.

**SET UP:**  $R = 8.3145 \text{ J/mol} \cdot \text{K}$ .  $\Delta T$  has the same numerical value in kelvins and in  $^\circ\text{C}$ .

$$\text{EXECUTE: } \Delta T = \frac{W}{nR} = \frac{2.40 \times 10^3 \text{ J}}{(6 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})} = 48.1 \text{ K. } \Delta T_K = \Delta T_C \text{ and}$$

$$T_2 = 27.0^\circ\text{C} + 48.1^\circ\text{C} = 75.1^\circ\text{C}.$$

**EVALUATE:** When  $W > 0$  the gas expands. When  $p$  is constant and  $V$  increases,  $T$  increases.

- 19.3. IDENTIFY:** Example 19.1 shows that for an isothermal process  $W = nRT \ln(p_1/p_2)$ .  $pV = nRT$  says  $V$  decreases when  $p$  increases and  $T$  is constant.

**SET UP:**  $T = 65.0 + 273.15 = 338.15 \text{ K}$ .  $p_2 = 3p_1$ .

**EXECUTE: (a)** The  $pV$ -diagram is sketched in Figure 19.3 (next page).

$$\text{(b) } W = (2.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(338.15 \text{ K}) \ln\left(\frac{p_1}{3p_1}\right) = -6180 \text{ J}.$$

**EVALUATE:** Since  $V$  decreases,  $W$  is negative.

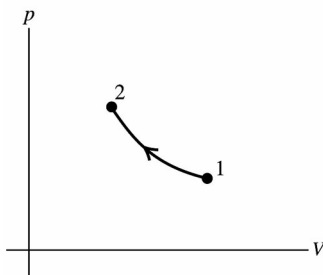


Figure 19.3

- 19.4. IDENTIFY:** The work done in a cycle is the area enclosed by the cycle in a  $pV$  diagram.

**SET UP:** (a)  $1 \text{ mm of Hg} = 133.3 \text{ Pa}$ .  $p_{\text{gauge}} = p - p_{\text{air}}$ . In calculating the enclosed area only changes in pressure enter and you can use gauge pressure.  $1 \text{ L} = 10^{-3} \text{ m}^3$ .

(b) Since  $pV = nRT$  and  $T$  is constant, the maximum number of moles of air in the lungs is when  $pV$  is a maximum. In the ideal gas law the absolute pressure  $p = p_{\text{gauge}} + p_{\text{air}}$  must be used.

$p_{\text{air}} = 760 \text{ mm of Hg}$ .  $1 \text{ mm of Hg} = 1 \text{ torr}$ .

**EXECUTE:** (a) By counting squares and noting that the area of 1 square is  $(1 \text{ mm of Hg})(0.1 \text{ L})$ , we estimate that the area enclosed by the cycle is about  $7.5 (\text{mm of Hg}) \cdot \text{L} = 1.00 \text{ N} \cdot \text{m}$ . The net work done is positive.

(b) The maximum  $pV$  is when  $p = 11 \text{ torr} + 760 \text{ torr} = 771 \text{ torr} = 1.028 \times 10^5 \text{ Pa}$  and

$V = 1.4 \text{ L} = 1.4 \times 10^{-3} \text{ m}^3$ . The maximum  $pV$  is  $(pV)_{\text{max}} = 144 \text{ N} \cdot \text{m}$ .  $pV = nRT$  so

$$n_{\text{max}} = \frac{(pV)_{\text{max}}}{RT} = \frac{144 \text{ N} \cdot \text{m}}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.059 \text{ mol}.$$

**EVALUATE:** While inhaling the gas does positive work on the lungs, but while exhaling the lungs do work on the gas, so the net work is positive.

- 19.5. IDENTIFY:** Example 19.1 shows that for an isothermal process  $W = nRT \ln(p_1/p_2)$ . Solve for  $p_1$ .

**SET UP:** For a compression ( $V$  decreases)  $W$  is negative, so  $W = -392 \text{ J}$ .  $T = 295.15 \text{ K}$ .

**EXECUTE:** (a)  $\frac{W}{nRT} = \ln\left(\frac{p_1}{p_2}\right)$ .  $\frac{p_1}{p_2} = e^{W/nRT}$ .

$$\frac{W}{nRT} = \frac{-392 \text{ J}}{(0.305 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(295.15 \text{ K})} = -0.5238.$$

$$p_1 = p_2 e^{W/nRT} = (1.76 \text{ atm})e^{-0.5238} = 1.04 \text{ atm}.$$

(b) In the process the pressure increases and the volume decreases. The  $pV$ -diagram is sketched in Figure 19.5.

**EVALUATE:**  $W$  is the work done by the gas, so when the surroundings do work on the gas,  $W$  is negative. The gas was compressed at constant temperature, so its pressure must have increased, which means that  $p_1 < p_2$ , which is what we found.

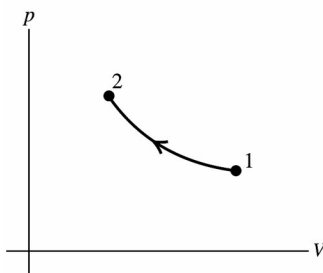


Figure 19.5

- 19.6. (a) **IDENTIFY and SET UP:** The  $pV$ -diagram is sketched in Figure 19.6.

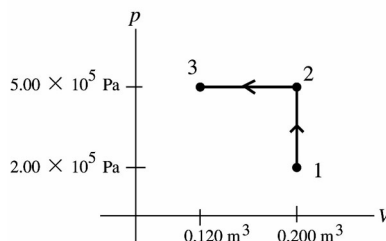


Figure 19.6

- (b) Calculate  $W$  for each process, using the expression for  $W$  that applies to the specific type of process.

**EXECUTE:**  $1 \rightarrow 2$ ,  $\Delta V = 0$ , so  $W = 0$

$2 \rightarrow 3$

$p$  is constant; so  $W = p \Delta V = (5.00 \times 10^5 \text{ Pa})(0.120 \text{ m}^3 - 0.200 \text{ m}^3) = -4.00 \times 10^4 \text{ J}$  ( $W$  is negative since the volume decreases in the process.)

$$W_{\text{tot}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} = -4.00 \times 10^4 \text{ J}$$

**EVALUATE:** The volume decreases so the total work done is negative.

- 19.7. **IDENTIFY:** Calculate  $W$  for each step using the appropriate expression for each type of process.

**SET UP:** When  $p$  is constant,  $W = p \Delta V$ . When  $\Delta V = 0$ ,  $W = 0$ .

**EXECUTE: (a)**  $W_{13} = p_1(V_2 - V_1)$ ,  $W_{32} = 0$ ,  $W_{24} = p_2(V_1 - V_2)$  and  $W_{41} = 0$ . The total work done by the system is  $W_{13} + W_{32} + W_{24} + W_{41} = (p_1 - p_2)(V_2 - V_1)$ , which is the area in the  $pV$  plane enclosed by the loop.

(b) For the process in reverse, the pressures are the same, but the volume changes are all the negatives of those found in part (a), so the total work is negative of the work found in part (a).

**EVALUATE:** When  $\Delta V > 0$ ,  $W > 0$  and when  $\Delta V < 0$ ,  $W < 0$ .

- 19.8. **IDENTIFY:** The gas is undergoing an isobaric compression, so its temperature and internal energy must be decreasing.

**SET UP:** The  $pV$  diagram shows that in the process the volume decreases while the pressure is constant.

$$1 \text{ L} = 10^{-3} \text{ m}^3 \text{ and } 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa.}$$

**EXECUTE: (a)**  $pV = nRT$ .  $n$ ,  $R$  and  $p$  are constant so  $\frac{V}{T} = \frac{nR}{p} = \text{constant}$ .  $\frac{V_a}{T_a} = \frac{V_b}{T_b}$ .

$$V_b = V_a \left( \frac{T_b}{T_a} \right) = (0.500 \text{ L}) \left( \frac{T_a/4}{T_a} \right) = 0.125 \text{ L.}$$

(b) For a constant pressure process,  $W = p \Delta V = (1.50 \text{ atm})(0.125 \text{ L} - 0.500 \text{ L})$  and

$$W = (-0.5625 \text{ L} \cdot \text{atm}) \left( \frac{10^{-3} \text{ m}^3}{1 \text{ L}} \right) \left( \frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}} \right) = -57.0 \text{ J. } W \text{ is negative since the volume decreases.}$$

Since  $W$  is negative, work is done on the gas.

(c) For an ideal gas,  $\Delta U = nC_V \Delta T$  so  $U$  decreases when  $T$  decreases. The internal energy of the gas decreases because the temperature decreases.

(d) For a constant pressure process,  $Q = nC_p \Delta T$ .  $T$  decreases so  $\Delta T$  is negative and  $Q$  is therefore negative. Negative  $Q$  means heat leaves the gas.

**EVALUATE:**  $W = nR \Delta T$  and  $Q = nC_p \Delta T$ .  $C_p > R$ , so more energy leaves as heat than is added by work done on the gas, and the internal energy of the gas decreases.

- 19.9. **IDENTIFY:**  $\Delta U = Q - W$ . For a constant pressure process,  $W = p \Delta V$ .

**SET UP:**  $Q = +1.15 \times 10^5 \text{ J}$ , since heat enters the gas.

**EXECUTE: (a)**  $W = p \Delta V = (1.65 \times 10^5 \text{ Pa})(0.320 \text{ m}^3 - 0.110 \text{ m}^3) = 3.47 \times 10^4 \text{ J.}$

(b)  $\Delta U = Q - W = 1.15 \times 10^5 \text{ J} - 3.47 \times 10^4 \text{ J} = 8.04 \times 10^4 \text{ J}$ .

**EVALUATE:** (c)  $W = p\Delta V$  for a constant pressure process and  $\Delta U = Q - W$  both apply to any material.

The ideal gas law wasn't used and it doesn't matter if the gas is ideal or not.

- 19.10. IDENTIFY:** The type of process is not specified. We can use  $\Delta U = Q - W$  because this applies to all processes. Calculate  $\Delta U$  and then from it calculate  $\Delta T$ .

**SET UP:**  $Q$  is positive since heat goes into the gas;  $Q = +1500 \text{ J}$ .

$W$  is positive since gas expands;  $W = +2100 \text{ J}$ .

**EXECUTE:**  $\Delta U = 1500 \text{ J} - 2100 \text{ J} = -600 \text{ J}$ .

We can also use  $\Delta U = n(\frac{3}{2}R)\Delta T$  since this is true for any process for an ideal gas.

$$\Delta T = \frac{2\Delta U}{3nR} = \frac{2(-600 \text{ J})}{3(5.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})} = -9.62 \text{ C}^\circ$$

$$T_2 = T_1 + \Delta T = 127^\circ\text{C} + (-9.62 \text{ C}^\circ) = 117^\circ\text{C}.$$

**EVALUATE:** More energy leaves the gas in the expansion work than enters as heat. The internal energy therefore decreases, and for an ideal gas this means the temperature decreases. We didn't have to convert  $\Delta T$  to kelvins since  $\Delta T$  is the same on the Kelvin and Celsius scales.

- 19.11. IDENTIFY:** Part  $ab$  is isochoric, but  $bc$  is not any of the familiar processes.

**SET UP:**  $pV = nRT$  determines the Kelvin temperature of the gas. The work done in the process is the area under the curve in the  $pV$  diagram.  $Q$  is positive since heat goes into the gas.  $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ .

$$1 \text{ L} = 1 \times 10^{-3} \text{ m}^3. \quad \Delta U = Q - W.$$

**EXECUTE:** (a) The lowest  $T$  occurs when  $pV$  has its smallest value. This is at point  $a$ , and

$$T_a = \frac{p_a V_a}{nR} = \frac{(0.20 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(2.0 \text{ L})(1.0 \times 10^{-3} \text{ m}^3/\text{L})}{(0.0175 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 278 \text{ K}.$$

(b)  $a$  to  $b$ :  $\Delta V = 0$  so  $W = 0$ .

$b$  to  $c$ : The work done by the gas is positive since the volume increases. The magnitude of the work is the area under the curve so  $W = \frac{1}{2}(0.50 \text{ atm} + 0.30 \text{ atm})(6.0 \text{ L} - 2.0 \text{ L})$  and

$$W = (1.6 \text{ L} \cdot \text{atm})(1 \times 10^{-3} \text{ m}^3/\text{L})(1.013 \times 10^5 \text{ Pa/atm}) = 162 \text{ J}.$$

(c) For  $abc$ ,  $W = 162 \text{ J}$ .  $\Delta U = Q - W = 215 \text{ J} - 162 \text{ J} = 53 \text{ J}$ .

**EVALUATE:** 215 J of heat energy went into the gas. 53 J of energy stayed in the gas as increased internal energy and 162 J left the gas as work done by the gas on its surroundings.

- 19.12. IDENTIFY and SET UP:** Calculate  $W$  using the equation for a constant pressure process. Then use  $\Delta U = Q - W$  to calculate  $Q$ .

**EXECUTE:** (a)  $W = \int_{V_1}^{V_2} p \, dV = p(V_2 - V_1)$  for this constant pressure process.

$$W = (1.80 \times 10^5 \text{ Pa})(1.20 \text{ m}^3 - 1.70 \text{ m}^3) = -9.00 \times 10^4 \text{ J}. \quad (\text{The volume decreases in the process, so } W \text{ is negative.})$$

(b)  $\Delta U = Q - W$ .  $Q = \Delta U + W = -1.40 \times 10^5 \text{ J} + (-9.00 \times 10^4 \text{ J}) = -2.30 \times 10^5 \text{ J}$ . Negative  $Q$  means heat flows out of the gas.

(c) **EVALUATE:**  $W = \int_{V_1}^{V_2} p \, dV = p(V_2 - V_1)$  (constant pressure) and  $\Delta U = Q - W$  apply to *any* system,

not just to an ideal gas. We did not use the ideal gas equation, either directly or indirectly, in any of the calculations, so the results are the same whether the gas is ideal or not.

- 19.13. IDENTIFY:** We read values from the  $pV$ -diagram and use the ideal gas law, as well as the first law of thermodynamics.

**SET UP:** Use  $pV = nRT$  to calculate  $T$  at each point. The work done in a process is the area under the curve in the  $pV$  diagram.  $\Delta U = Q - W$  for all processes.

**EXECUTE:** (a)  $pV = nRT$  so  $T = \frac{pV}{nR}$ .

$$\text{point } a: T_a = \frac{(2.0 \times 10^5 \text{ Pa})(0.010 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 535 \text{ K}$$

$$\text{point } b: T_b = \frac{(5.0 \times 10^5 \text{ Pa})(0.070 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 9350 \text{ K}$$

$$\text{point } c: T_c = \frac{(8.0 \times 10^5 \text{ Pa})(0.070 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 15,000 \text{ K}$$

(b) The work done by the gas is positive since the volume increases. The magnitude of the work is the area under the curve:

$$W = \frac{1}{2}(2.0 \times 10^5 \text{ Pa} + 5.0 \times 10^5 \text{ Pa})(0.070 \text{ m}^3 - 0.010 \text{ m}^3) = 2.1 \times 10^4 \text{ J}$$

(c)  $\Delta U = Q - W$  so  $Q = \Delta U + W = 15,000 \text{ J} + 2.1 \times 10^4 \text{ J} = 3.6 \times 10^4 \text{ J}$ .

EVALUATE:  $Q$  is positive so heat energy goes into the gas.

19.14. IDENTIFY:  $\Delta U = Q - W$ . For a constant pressure process,  $W = p\Delta V$ .

SET UP:  $Q = +2.20 \times 10^6 \text{ J}$ ;  $Q > 0$  since this amount of heat goes into the water.

$$p = 2.00 \text{ atm} = 2.03 \times 10^5 \text{ Pa}.$$

$$\text{EXECUTE: (a)} \quad W = p\Delta V = (2.03 \times 10^5 \text{ Pa})(0.824 \text{ m}^3 - 1.00 \times 10^{-3} \text{ m}^3) = 1.67 \times 10^5 \text{ J}$$

$$\text{(b)} \quad \Delta U = Q - W = 2.20 \times 10^6 \text{ J} - 1.67 \times 10^5 \text{ J} = 2.03 \times 10^6 \text{ J}.$$

EVALUATE:  $2.20 \times 10^6 \text{ J}$  of energy enters the water.  $1.67 \times 10^5 \text{ J}$  of energy leaves the materials through expansion work and the remainder stays in the material as an increase in internal energy.

19.15. IDENTIFY: Apply  $\Delta U = Q - W$  to the gas.

SET UP: For the process,  $\Delta V = 0$ .  $Q = +700 \text{ J}$  since heat goes into the gas.

EXECUTE: (a) Since  $\Delta V = 0$ ,  $W = 0$ .

$$\text{(b)} \quad pV = nRT \text{ says } \frac{p}{T} = \frac{nR}{V} = \text{constant. Since } p \text{ doubles, } T \text{ doubles. } T_b = 2T_a.$$

$$\text{(c)} \quad \text{Since } W = 0, \Delta U = Q = +700 \text{ J. } U_b = U_a + 700 \text{ J}.$$

EVALUATE: For an ideal gas, when  $T$  increases,  $U$  increases.

19.16. IDENTIFY:  $\Delta U = Q - W$ .

SET UP:  $Q < 0$  when heat leaves the gas.

EXECUTE: For an isothermal process,  $\Delta U = 0$ , so  $W = Q = -410 \text{ J}$ .

EVALUATE: In a compression the volume decreases and  $W < 0$ .

19.17. IDENTIFY: For a constant pressure process,  $W = p\Delta V$ ,  $Q = nC_p\Delta T$ , and  $\Delta U = nC_V\Delta T$ .  $\Delta U = Q - W$  and  $C_p = C_V + R$ . For an ideal gas,  $p\Delta V = nR\Delta T$ .

SET UP: From Table 19.1,  $C_V = 28.46 \text{ J/mol} \cdot \text{K}$ .

EXECUTE: (a) The  $pV$  diagram is shown in Figure 19.17 (next page).

$$\text{(b)} \quad W = pV_2 - pV_1 = nR(T_2 - T_1) = (0.250 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(100.0 \text{ K}) = 208 \text{ J}.$$

(c) The work is done on the piston.

(d) Since  $\Delta U = nC_V\Delta T$  holds for any process, we have

$$\Delta U = nC_V\Delta T = (0.250 \text{ mol})(28.46 \text{ J/mol} \cdot \text{K})(100.0 \text{ K}) = 712 \text{ J}.$$

(e) Either  $Q = nC_p\Delta T$  or  $Q = \Delta U + W$  gives  $Q = 920 \text{ J}$  to three significant figures.

(f) The lower pressure would mean a correspondingly larger volume, and the net result would be that the work done would be the same as that found in part (b).

EVALUATE:  $W = nR\Delta T$ , so  $W$ ,  $Q$  and  $\Delta U$  all depend only on  $\Delta T$ . When  $T$  increases at constant pressure,  $V$  increases and  $W > 0$ .  $\Delta U$  and  $Q$  are also positive when  $T$  increases.

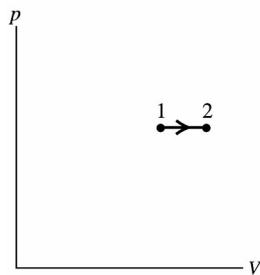


Figure 19.17

**19.18. IDENTIFY:** For constant volume  $Q = nC_V\Delta T$ . For constant pressure,  $Q = nC_p\Delta T$ . For any process of an ideal gas,  $\Delta U = nC_V\Delta T$ .

**SET UP:**  $R = 8.315 \text{ J/mol} \cdot \text{K}$ . For helium,  $C_V = 12.47 \text{ J/mol} \cdot \text{K}$  and  $C_p = 20.78 \text{ J/mol} \cdot \text{K}$ .

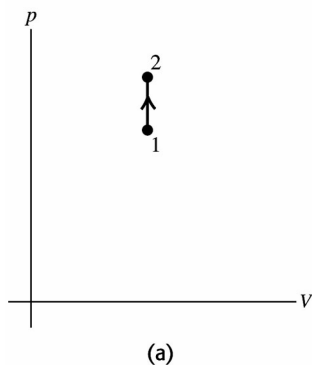
**EXECUTE:** (a)  $Q = nC_V\Delta T = (0.0100 \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(40.0 \text{ C}^\circ) = 4.99 \text{ J}$ . The  $pV$ -diagram is sketched in Figure 19.18a.

(b)  $Q = nC_p\Delta T = (0.0100 \text{ mol})(20.78 \text{ J/mol} \cdot \text{K})(40.0 \text{ C}^\circ) = 8.31 \text{ J}$ . The  $pV$ -diagram is sketched in Figure 19.18b.

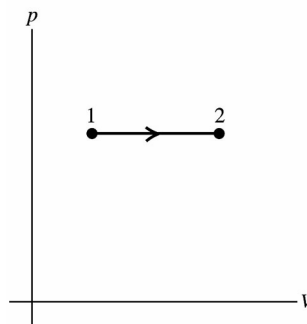
(c) More heat is required for the constant pressure process.  $\Delta U$  is the same in both cases. For constant volume  $W = 0$  and for constant pressure  $W > 0$ . The additional heat energy required for constant pressure goes into expansion work.

(d)  $\Delta U = nC_V\Delta T = 4.99 \text{ J}$  for both processes.  $\Delta U$  is path independent and for an ideal gas depends only on  $\Delta T$ .

**EVALUATE:**  $C_p = C_V + R$ , so  $C_p > C_V$ .



(a)



(b)

Figure 19.18

**19.19. IDENTIFY:** For constant volume,  $Q = nC_V\Delta T$ . For constant pressure,  $Q = nC_p\Delta T$ .

**SET UP:** From Table 19.1 in the text,  $C_V = 20.76 \text{ J/mol} \cdot \text{K}$  and  $C_p = 29.07 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE:** (a) Using  $Q = nC_V\Delta T$ ,  $\Delta T = \frac{Q}{nC_V} = \frac{645 \text{ J}}{(0.185 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})} = 167.9 \text{ K}$  and  $T = 948 \text{ K}$ .

The  $pV$ -diagram is sketched in Figure 19.19a.

(b) Using  $Q = nC_p\Delta T$ ,  $\Delta T = \frac{Q}{nC_p} = \frac{645 \text{ J}}{(0.185 \text{ mol})(29.07 \text{ J/mol} \cdot \text{K})} = 119.9 \text{ K}$  and  $T = 900 \text{ K}$ .

The  $pV$ -diagram is sketched in Figure 19.19b.

**EVALUATE:** At constant pressure some of the heat energy added to the gas leaves the gas as expansion work and the internal energy change is less than if the same amount of heat energy is added at constant volume.  $\Delta T$  is proportional to  $\Delta U$ .

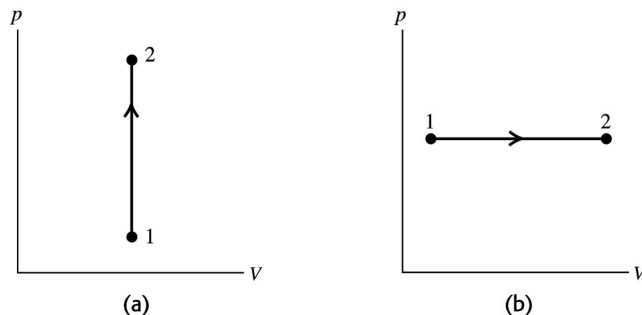


Figure 19.19

**19.20. IDENTIFY:** For an ideal gas,  $\Delta U = C_V \Delta T$ , and at constant pressure,  $p\Delta V = nR\Delta T$ .

**SET UP:**  $C_V = \frac{3}{2}R$  for a monatomic gas.

**EXECUTE:**  $\Delta U = n(\frac{3}{2}R)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}(4.00 \times 10^4 \text{ Pa})(8.00 \times 10^{-3} \text{ m}^3 - 2.00 \times 10^{-3} \text{ m}^3) = 360 \text{ J}$ .

**EVALUATE:**  $W = nR\Delta T = \frac{2}{3}\Delta U = 240 \text{ J}$ .  $Q = nC_p\Delta T = n(\frac{5}{2}R)\Delta T = \frac{5}{3}\Delta U = 600 \text{ J}$ . 600 J of heat energy flows into the gas. 240 J leaves as expansion work and 360 J remains in the gas as an increase in internal energy.

**19.21. IDENTIFY:**  $\Delta U = Q - W$ . For an ideal gas,  $\Delta U = C_V \Delta T$ , and at constant pressure,  $W = p\Delta V = nR\Delta T$ .

**SET UP:**  $C_V = \frac{3}{2}R$  for a monatomic gas.

**EXECUTE:**  $\Delta U = n(\frac{3}{2}R)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}W$ . Then  $Q = \Delta U + W = \frac{5}{2}W$ , so  $W/Q = \frac{2}{5}$ .

**EVALUATE:** For diatomic or polyatomic gases,  $C_V$  is a different multiple of  $R$  and the fraction of  $Q$  that is used for expansion work is different.

**19.22. IDENTIFY:** Apply  $pV = nRT$  to calculate  $T$ . For this constant pressure process,  $W = p\Delta V$ .  $Q = nC_p\Delta T$ . Use  $\Delta U = Q - W$  to relate  $Q$ ,  $W$ , and  $\Delta U$ .

**SET UP:**  $2.50 \text{ atm} = 2.53 \times 10^5 \text{ Pa}$ . For a monatomic ideal gas,  $C_V = 12.47 \text{ J/mol} \cdot \text{K}$  and  $C_p = 20.78 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE:** (a)  $T_1 = \frac{pV_1}{nR} = \frac{(2.53 \times 10^5 \text{ Pa})(3.20 \times 10^{-2} \text{ m}^3)}{(3.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 325 \text{ K}$ .

$T_2 = \frac{pV_2}{nR} = \frac{(2.53 \times 10^5 \text{ Pa})(4.50 \times 10^{-2} \text{ m}^3)}{(3.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 456 \text{ K}$ .

(b)  $W = p\Delta V = (2.53 \times 10^5 \text{ Pa})(4.50 \times 10^{-2} \text{ m}^3 - 3.20 \times 10^{-2} \text{ m}^3) = 3.29 \times 10^3 \text{ J}$ .

(c)  $Q = nC_p\Delta T = (3.00 \text{ mol})(20.78 \text{ J/mol} \cdot \text{K})(456 \text{ K} - 325 \text{ K}) = 8.17 \times 10^3 \text{ J}$ .

(d)  $\Delta U = Q - W = 4.88 \times 10^3 \text{ J}$ .

**EVALUATE:** We could also calculate  $\Delta U$  as

$\Delta U = nC_V\Delta T = (3.00 \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(456 \text{ K} - 325 \text{ K}) = 4.90 \times 10^3 \text{ J}$ , which agrees with the value we calculated in part (d).

**19.23. IDENTIFY:**  $\Delta U = Q - W$ . Apply  $Q = nC_p\Delta T$  to calculate  $C_p$ . Apply  $\Delta U = nC_V\Delta T$  to calculate  $C_V$ .

$\gamma = C_p/C_V$ .

**SET UP:**  $\Delta T = 15.0 \text{ C}^\circ = 15.0 \text{ K}$ . Since heat is added,  $Q = +970 \text{ J}$ .

**EXECUTE:** (a)  $\Delta U = Q - W = +970 \text{ J} - 223 \text{ J} = 747 \text{ J}$ .

$$(b) C_p = \frac{Q}{n\Delta T} = \frac{970 \text{ J}}{(1.75 \text{ mol})(15.0 \text{ K})} = 37.0 \text{ J/mol} \cdot \text{K}. \quad C_V = \frac{\Delta U}{n\Delta T} = \frac{747 \text{ J}}{(1.75 \text{ mol})(15.0 \text{ K})} = 28.5 \text{ J/mol} \cdot \text{K}.$$

$$\gamma = \frac{C_p}{C_V} = \frac{37.0 \text{ J/mol} \cdot \text{K}}{28.5 \text{ J/mol} \cdot \text{K}} = 1.30.$$

**EVALUATE:** The value of  $\gamma$  we calculated is similar to the values given in Tables 19.1 for polyatomic gases.

**19.24. IDENTIFY:**  $C_p = C_V + R$  and  $\gamma = \frac{C_p}{C_V}$ .

**SET UP:**  $R = 8.315 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE:**  $C_p = C_V + R$ .  $\gamma = \frac{C_p}{C_V} = 1 + \frac{R}{C_V}$ .  $C_V = \frac{R}{\gamma - 1} = \frac{8.315 \text{ J/mol} \cdot \text{K}}{0.127} = 65.5 \text{ J/mol} \cdot \text{K}$ . Then

$$C_p = C_V + R = 73.8 \text{ J/mol} \cdot \text{K}.$$

**EVALUATE:** The value of  $C_V$  is about twice the values for the polyatomic gases in Table 19.1. A propane molecule has more atoms and hence more internal degrees of freedom than the polyatomic gases in the table.

**19.25. IDENTIFY:** Calculate  $W$  and  $\Delta U$  and then use the first law to calculate  $Q$ .

(a) **SET UP:**  $W = \int_{V_1}^{V_2} p dV$

$$pV = nRT \text{ so } p = nRT/V$$

$$W = \int_{V_1}^{V_2} (nRT/V) dV = nRT \int_{V_1}^{V_2} dV/V = nRT \ln(V_2/V_1) \text{ (work done during an isothermal process).}$$

**EXECUTE:**  $W = (0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(350 \text{ K}) \ln(0.25V_1/V_1) = (436.5 \text{ J}) \ln(0.25) = -605 \text{ J}$ .

**EVALUATE:**  $W$  for the gas is negative, since the volume decreases.

(b) **SET UP:**  $\Delta U = nC_V \Delta T$  for any ideal gas process.

**EXECUTE:**  $\Delta T = 0$  (isothermal) so  $\Delta U = 0$ .

**EVALUATE:**  $\Delta U = 0$  for any ideal gas process in which  $T$  doesn't change.

(c) **SET UP:**  $\Delta U = Q - W$

**EXECUTE:**  $\Delta U = 0$  so  $Q = W = -605 \text{ J}$ . ( $Q$  is negative; the gas liberates 605 J of heat to the surroundings.)

**EVALUATE:**  $Q = nC_V \Delta T$  is only for a constant volume process, so it doesn't apply here.

$Q = nC_p \Delta T$  is only for a constant pressure process, so it doesn't apply here.

**19.26. IDENTIFY:** For an adiabatic process of an ideal gas,  $p_1 V_1^\gamma = p_2 V_2^\gamma$ , no heat enters or leaves the gas. The ideal gas law still applies.

**SET UP:**  $p_1 V_1^\gamma = p_2 V_2^\gamma$ ,  $pV = nRT$ ,  $W = \int_{V_1}^{V_2} p dV$ . For an ideal monatomic gas,  $\gamma = 5/3$ .

**EXECUTE:** Solving  $p_1 V_1^\gamma = p_2 V_2^\gamma$  for  $p_2$  and rearranging gives  $p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma$ , so we need to find  $V_2$ .

Applying  $W = \int_{V_1}^{V_2} p dV$  to an adiabatic process, we use the fact that  $pV^\gamma = \text{constant}$ . In this case, the

constant is  $p_1 V_1^\gamma$  (since we know  $p_1$  and  $V_1$ ), and we'll call it  $K$  for the time being. This tells us that

$$p = K/V^\gamma. \text{ Using this in the integral, we get } W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} K/V^\gamma dV = \frac{K}{1-\gamma} (V_2^{1-\gamma} - V_1^{1-\gamma}).$$

$$K = p_1 V_1^\gamma = (2500 \text{ Pa})(2.10 \text{ m}^3)^{5/3} = 8609 \text{ N} \cdot \text{m}^3, \quad W = 1480 \text{ J}, \text{ and } \gamma = 5/3. \text{ Putting in these numbers and}$$

$$\text{solving for } V_2 \text{ gives } V_2 = 2.8697 \text{ m}^3. \text{ Putting this value into } p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma \text{ gives}$$



$$p_2 = (2500 \text{ Pa}) \left( \frac{2.10 \text{ m}^3}{2.8697 \text{ m}^3} \right)^{5/3} = 1490 \text{ Pa} = 1.49 \text{ kPa}.$$

**EVALUATE:** The pressure dropped because the gas expanded adiabatically and did work, so our result is reasonable. An alternative approach is the following: We know that  $Q = 0$  and  $\Delta U = n \left( \frac{3}{2} R \right) \Delta T$ . We have

$W = -\Delta U$ , so  $\Delta U = -1480 \text{ J}$ . Therefore  $-1480 \text{ J} = n \left( \frac{3}{2} R \right) \Delta T$ , which gives  $\Delta T = -23.73 \text{ K}$ . The ideal gas law gives  $T_1 = p_1 V_1 / nR = (2500 \text{ Pa})(2.10 \text{ m}^3) / [(5.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})] = 126.3 \text{ K}$ . Therefore  $T_2 = T_1 + \Delta T = +102.6 \text{ K}$ . Using  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$  gives

$$V_2^{2/3} = V_1^{2/3} \left( \frac{T_1}{T_2} \right) = (2.10 \text{ m}^3) \left( \frac{126.3 \text{ K}}{102.6 \text{ K}} \right) = 2.019 \text{ m}^2, \text{ so } V_2 = (2.019 \text{ m}^2)^{3/2} = 2.869 \text{ m}^3. \text{ Therefore}$$

$$p_2 = nRT_2/V_2 = (5.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(102.6 \text{ K})/(2.869 \text{ m}^3) = 1490 \text{ Pa} = 1.49 \text{ kPa}.$$

**19.27. IDENTIFY:** For an adiabatic process of an ideal gas,  $p_1 V_1^\gamma = p_2 V_2^\gamma$ ,  $W = \frac{1}{\gamma-1} (p_1 V_1 - p_2 V_2)$ , and

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}.$$

**SET UP:** For a monatomic ideal gas  $\gamma = 5/3$ .

**EXECUTE: (a)**  $p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma = (1.50 \times 10^5 \text{ Pa}) \left( \frac{0.0800 \text{ m}^3}{0.0400 \text{ m}^3} \right)^{5/3} = 4.76 \times 10^5 \text{ Pa}.$

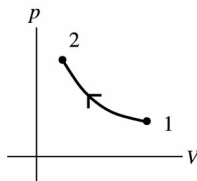
**(b)** This result may be substituted into  $W = \frac{1}{\gamma-1} p_1 V_1 (1 - (V_1/V_2)^{\gamma-1})$ , or, substituting the above form for

$$p_2, \quad W = \frac{1}{\gamma-1} p_1 V_1 (1 - (V_1/V_2)^{\gamma-1}) = \frac{3}{2} (1.50 \times 10^5 \text{ Pa})(0.0800 \text{ m}^3) \left( 1 - \left( \frac{0.0800}{0.0400} \right)^{2/3} \right) = -1.06 \times 10^4 \text{ J}.$$

**(c)** From  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ ,  $(T_2/T_1) = (V_2/V_1)^{\gamma-1} = (0.0800/0.0400)^{2/3} = 1.59$ , and since the final temperature is higher than the initial temperature, the gas is heated.

**EVALUATE:** In an adiabatic compression  $W < 0$  since  $\Delta V < 0$ .  $Q = 0$  so  $\Delta U = -W$ .  $\Delta U > 0$  and the temperature increases.

**19.28. IDENTIFY and SET UP: (a)** In the process the pressure increases and the volume decreases. The  $pV$ -diagram is sketched in Figure 19.28.



**Figure 19.28**

**(b)** For an adiabatic process for an ideal gas  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ ,  $p_1 V_1^\gamma = p_2 V_2^\gamma$ , and  $pV = nRT$ .

**EXECUTE:** From the first equation,  $T_2 = T_1 (V_1/V_2)^{\gamma-1} = (293 \text{ K})(V_1/0.0900V_1)^{1.4-1}$

$$T_2 = (293 \text{ K})(11.11)^{0.4} = 768 \text{ K} = 495^\circ\text{C}$$

(Note: In the equation  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$  the temperature *must* be in kelvins.)

$$p_1 V_1^\gamma = p_2 V_2^\gamma \text{ implies } p_2 = p_1 (V_1/V_2)^\gamma = (1.00 \text{ atm})(V_1/0.0900V_1)^{1.4}$$

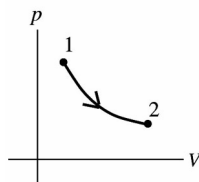
$$p_2 = (1.00 \text{ atm})(11.11)^{1.4} = 29.1 \text{ atm}$$

**EVALUATE:** Alternatively, we can use  $pV = nRT$  to calculate  $p_2$ :  $n, R$  constant implies

$$pV/T = nR = \text{constant} \quad \text{so} \quad p_1V_1/T_1 = p_2V_2/T_2.$$

$$p_2 = p_1(V_1/V_2)(T_2/T_1) = (1.00 \text{ atm})(V_1/0.0900V_1)(768 \text{ K}/293 \text{ K}) = 29.1 \text{ atm, which checks.}$$

- 19.29. (a) IDENTIFY and SET UP:** In the expansion the pressure decreases and the volume increases. The  $pV$ -diagram is sketched in Figure 19.29.



**Figure 19.29**

**(b) Adiabatic means  $Q = 0$ .**

Then  $\Delta U = Q - W$  gives  $W = -\Delta U = -nC_V \Delta T = nC_V(T_1 - T_2)$ .

$$C_V = 12.47 \text{ J/mol} \cdot \text{K} \quad (\text{Table 19.1}).$$

$$\text{EXECUTE: } W = (0.450 \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(66.0^\circ\text{C} - 10.0^\circ\text{C}) = +314 \text{ J.}$$

$W$  positive for  $\Delta V > 0$  (expansion)

$$\text{(c) } \Delta U = -W = -314 \text{ J.}$$

**EVALUATE:** There is no heat energy input. The energy for doing the expansion work comes from the internal energy of the gas, which therefore decreases. For an ideal gas, when  $T$  decreases,  $U$  decreases.

- 19.30. IDENTIFY:** Assume the expansion is adiabatic.  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$  relates  $V$  and  $T$ . Assume the air behaves as an ideal gas, so  $\Delta U = nC_V\Delta T$ . Use  $pV = nRT$  to calculate  $n$ .

**SET UP:** For air,  $C_V = 29.76 \text{ J/mol} \cdot \text{K}$  and  $\gamma = 1.40$ .  $V_2 = 0.800V_1$ .  $T_1 = 293.15 \text{ K}$ .  $p_1 = 2.026 \times 10^5 \text{ Pa}$ .

For a sphere,  $V = \frac{4}{3}\pi r^3$ .

$$\text{EXECUTE: (a) } T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = (293.15 \text{ K}) \left( \frac{V_1}{0.800V_1} \right)^{0.40} = 320.5 \text{ K} = 47.4^\circ\text{C}.$$

$$\text{(b) } V_1 = \frac{4}{3}\pi r^3 = \frac{4\pi}{3}(0.1195 \text{ m})^3 = 7.15 \times 10^{-3} \text{ m}^3.$$

$$n = \frac{p_1V_1}{RT_1} = \frac{(2.026 \times 10^5 \text{ Pa})(7.15 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})} = 0.594 \text{ mol.}$$

$$\Delta U = nC_V\Delta T = (0.594 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(321 \text{ K} - 293 \text{ K}) = 345 \text{ J.}$$

**EVALUATE:** We could also use  $\Delta U = -W = -\frac{1}{\gamma-1}(p_1V_1 - p_2V_2)$  to calculate  $\Delta U$ , if we first found  $p_2$  from  $pV = nRT$ .

- 19.31. IDENTIFY:** Combine  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$  with  $pV = nRT$  to obtain an expression relating  $T$  and  $p$  for an adiabatic process of an ideal gas.

**SET UP:**  $T_1 = 299.15 \text{ K}$ .

$$\text{EXECUTE: } V = \frac{nRT}{p} \quad \text{so} \quad T_1 \left( \frac{nRT_1}{p_1} \right)^{\gamma-1} = T_2 \left( \frac{nRT_2}{p_2} \right)^{\gamma-1} \quad \text{and} \quad \frac{T_1^\gamma}{p_1^{\gamma-1}} = \frac{T_2^\gamma}{p_2^{\gamma-1}}.$$

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} = (299.15 \text{ K}) \left( \frac{0.850 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right)^{0.4/1.4} = 284.8 \text{ K} = 11.6^\circ\text{C}.$$

**EVALUATE:** For an adiabatic process of an ideal gas, when the pressure decreases the temperature decreases.

**19.32. IDENTIFY:**  $pV = nRT$  For an adiabatic process,  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ .

**SET UP:** For an ideal monatomic gas,  $\gamma = 5/3$ .

**EXECUTE:** (a)  $T = \frac{pV}{nR} = \frac{(1.00 \times 10^5 \text{ Pa})(2.50 \times 10^{-3} \text{ m}^3)}{(0.1 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})} = 301 \text{ K}.$

(b) (i) Isothermal: If the expansion is *isothermal*, the process occurs at constant temperature and the final temperature is the same as the initial temperature, namely 301 K.  $p_2 = p_1(V_1/V_2) = \frac{1}{2}p_1 = 5.00 \times 10^4 \text{ Pa}.$

(ii) Isobaric:  $\Delta p = 0$  so  $p_2 = 1.00 \times 10^5 \text{ Pa}.$   $T_2 = T_1(V_2/V_1) = 2T_1 = 602 \text{ K}.$

(iii) Adiabatic: Using  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ ,  $T_2 = \frac{T_1V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{(301 \text{ K})(V_1)^{0.67}}{(2V_1)^{0.67}} = (301 \text{ K})(\frac{1}{2})^{0.67} = 189 \text{ K}.$  Then

$$pV = nRT \text{ gives } p_2 = 3.14 \times 10^4 \text{ Pa}.$$

**EVALUATE:** In an isobaric expansion,  $T$  increases. In an adiabatic expansion,  $T$  decreases.

**19.33. IDENTIFY and SET UP:** For an ideal gas,  $pV = nRT$ . The work done is the area under the path in the  $pV$ -diagram.

**EXECUTE:** (a) The product  $pV$  increases and this indicates a temperature increase.

(b) The work is the area in the  $pV$  plane bounded by the blue line representing the process and the verticals at  $V_a$  and  $V_b$ . The area of this trapezoid is

$$\frac{1}{2}(p_b + p_a)(V_b - V_a) = \frac{1}{2}(2.40 \times 10^5 \text{ Pa})(0.0400 \text{ m}^3) = 4800 \text{ J}.$$

**EVALUATE:** The work done is the average pressure,  $\frac{1}{2}(p_1 + p_2)$ , times the volume increase.

**19.34. IDENTIFY:** Use  $pV = nRT$  to calculate  $T$ .  $W$  is the area under the process in the  $pV$ -diagram. Use  $\Delta U = nC_V\Delta T$  and  $\Delta U = Q - W$  to calculate  $Q$ .

**SET UP:** In state  $c$ ,  $p_c = 2.0 \times 10^5 \text{ Pa}$  and  $V_c = 0.0040 \text{ m}^3$ . In state  $a$ ,  $p_a = 4.0 \times 10^5 \text{ Pa}$  and  $V_a = 0.0020 \text{ m}^3$ .

**EXECUTE:** (a)  $T_c = \frac{p_cV_c}{nR} = \frac{(2.0 \times 10^5 \text{ Pa})(0.0040 \text{ m}^3)}{(0.500 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 192 \text{ K}.$

(b)  $W = \frac{1}{2}(4.0 \times 10^5 \text{ Pa} + 2.0 \times 10^5 \text{ Pa})(0.0030 \text{ m}^3 - 0.0020 \text{ m}^3) + (2.0 \times 10^5 \text{ Pa})(0.0040 \text{ m}^3 - 0.0030 \text{ m}^3)$   
 $W = +500 \text{ J}.$  500 J of work is done by the gas.

(c)  $T_a = \frac{p_aV_a}{nR} = \frac{(4.0 \times 10^5 \text{ Pa})(0.0020 \text{ m}^3)}{(0.500 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 192 \text{ K}.$  For the process,  $\Delta T = 0$ , so  $\Delta U = 0$  and

$$Q = W = +500 \text{ J}.$$
 500 J of heat enters the system.

**EVALUATE:** The work done by the gas is positive since the volume increases.

**19.35. IDENTIFY:** We can read the values from the  $pV$ -diagram and apply the ideal gas law and the first law of thermodynamics.

**SET UP:** At each point  $pV = nRT$ , with  $T = 85 + 273 = 358 \text{ K}.$  For an isothermal process of an ideal gas,  $W = nRT \ln(V_2/V_1)$ .  $\Delta U = nC_V\Delta T$  for any ideal gas process.

**EXECUTE:** (a) At point  $b$ ,  $p = 0.200 \text{ atm} = 2.026 \times 10^4 \text{ Pa}$  and  $V = 0.100 \text{ m}^3$

$$n = \frac{pV}{RT} = \frac{(2.026 \times 10^4 \text{ Pa})(0.100 \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(358 \text{ K})} = 0.681 \text{ moles}.$$

(b)  $n$ ,  $R$ , and  $T$  are constant so  $p_aV_a = p_bV_b$ .

$$V_a = V_b \left( \frac{p_b}{p_a} \right) = (0.100 \text{ m}^3) \left( \frac{0.200 \text{ atm}}{0.600 \text{ atm}} \right) = 0.0333 \text{ m}^3.$$

$$(c) W = nRT \ln (V_b/V_a) = (0.681 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(358 \text{ K}) \ln \left( \frac{0.100 \text{ m}^3}{0.0333 \text{ m}^3} \right) = 2230 \text{ J} = 2.23 \text{ kJ}$$

$W$  is positive and corresponds to work done by the gas.

(d)  $\Delta U = nC_V \Delta T$  so for an isothermal process ( $\Delta T = 0$ ),  $\Delta U = 0$ .

EVALUATE:  $W$  is positive when the volume increases, so the area under the curve is positive. For any isothermal process,  $\Delta U = 0$ .

**19.36. IDENTIFY:** Segment  $ab$  is isobaric,  $bc$  is isochoric, and  $ca$  is isothermal.

SET UP: He is a monatomic gas so  $C_V = \frac{3}{2}R$  and  $C_p = \frac{5}{2}R$ . For any process of an ideal gas,

$\Delta U = nC_V \Delta T$ . For an isothermal process of an ideal gas,  $\Delta U = 0$  so  $Q = W = nRT \ln(V_2/V_1)$ .

EXECUTE: (a) Apply  $pV = nRT$  to states  $a$  and  $c$ .  $T_a = T_c$  so  $nRT$  is constant and  $p_a V_a = p_c V_c$ .

$$p_a = p_c \left( \frac{V_c}{V_a} \right) = (2.0 \times 10^5 \text{ Pa}) \left( \frac{0.040 \text{ m}^3}{0.010 \text{ m}^3} \right) = 8.0 \times 10^5 \text{ Pa}.$$

$$(b) T_a = \frac{p_a V_a}{nR} = \frac{(8.0 \times 10^5 \text{ Pa})(0.010 \text{ m}^3)}{(3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 296 \text{ K};$$

$$T_b = \frac{p_b V_b}{nR} = \frac{(8.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3)}{(3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 1184 \text{ K};$$

$$T_c = \frac{p_c V_c}{nR} = \frac{(2.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3)}{(3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 296 \text{ K} = T_a.$$

(c)  $ab$ :  $Q = nC_p \Delta T = (3.25 \text{ mol})\left(\frac{5}{2}\right)(8.315 \text{ J/mol} \cdot \text{K})(1184 \text{ K} - 296 \text{ K}) = 6.00 \times 10^4 \text{ J}$ ; heat enters the gas.

$bc$ :  $Q = nC_V \Delta T = (3.25 \text{ mol})\left(\frac{3}{2}\right)(8.315 \text{ J/mol} \cdot \text{K})(296 \text{ K} - 1184 \text{ K}) = -3.60 \times 10^4 \text{ J}$ ; heat leaves the gas.

$ca$ :  $Q = nRT \ln \left( \frac{V_a}{V_c} \right) = (3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(296 \text{ K}) \ln \left( \frac{0.010 \text{ m}^3}{0.040 \text{ m}^3} \right) = -1.11 \times 10^4 \text{ J}$ ; heat leaves the gas.

(d)  $ab$ :  $\Delta U = nC_V \Delta T = (3.25 \text{ mol})\left(\frac{3}{2}\right)(8.315 \text{ J/mol} \cdot \text{K})(1184 \text{ K} - 296 \text{ K}) = 3.60 \times 10^4 \text{ J}$ ; the internal energy increased.

$bc$ :  $\Delta U = nC_V \Delta T = (3.25 \text{ mol})\left(\frac{3}{2}\right)(8.315 \text{ J/mol} \cdot \text{K})(296 \text{ K} - 1184 \text{ K}) = -3.60 \times 10^4 \text{ J}$ ; the internal energy decreased.

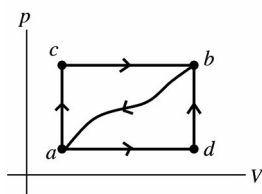
$ca$ :  $\Delta T = 0$  so  $\Delta U = 0$ .

EVALUATE: As we saw in (d), for any closed path on a  $pV$  diagram,  $\Delta U = 0$  because we are back at the same values of  $P$ ,  $V$ , and  $T$ .

**19.37. IDENTIFY:** Use  $\Delta U = Q - W$  and the fact that  $\Delta U$  is path independent.

$W > 0$  when the volume increases,  $W < 0$  when the volume decreases, and  $W = 0$  when the volume is constant.  $Q > 0$  if heat flows into the system.

SET UP: The paths are sketched in Figure 19.37.



$$Q_{acb} = +90.0 \text{ J} \text{ (positive since heat flows in)}$$

$$W_{acb} = +60.0 \text{ J} \text{ (positive since } \Delta V > 0 \text{)}$$

Figure 19.37

EXECUTE: (a)  $\Delta U = Q - W$

$\Delta U$  is path independent;  $Q$  and  $W$  depend on the path.

$$\Delta U = U_b - U_a$$

This can be calculated for any path from  $a$  to  $b$ , in particular for path  $acb$ :

$$\Delta U_{a \rightarrow b} = Q_{acb} - W_{acb} = 90.0 \text{ J} - 60.0 \text{ J} = 30.0 \text{ J}.$$

Now apply  $\Delta U = Q - W$  to path  $adb$ ;  $\Delta U = 30.0 \text{ J}$  for this path also.

$$W_{adb} = +15.0 \text{ J} \text{ (positive since } \Delta V > 0)$$

$$\Delta U_{a \rightarrow b} = Q_{adb} - W_{adb} \text{ so } Q_{adb} = \Delta U_{a \rightarrow b} + W_{adb} = 30.0 \text{ J} + 15.0 \text{ J} = +45.0 \text{ J}.$$

(b) Apply  $\Delta U = Q - W$  to path  $ba$ :  $\Delta U_{b \rightarrow a} = Q_{ba} - W_{ba}$

$$W_{ba} = -35.0 \text{ J} \text{ (negative since } \Delta V < 0)$$

$$\Delta U_{b \rightarrow a} = U_a - U_b = -(U_b - U_a) = -\Delta U_{a \rightarrow b} = -30.0 \text{ J}$$

$$\text{Then } Q_{ba} = \Delta U_{b \rightarrow a} + W_{ba} = -30.0 \text{ J} - 35.0 \text{ J} = -65.0 \text{ J}.$$

( $Q_{ba} < 0$ ; the system liberates heat.)

(c)  $U_a = 0$ ,  $U_d = 8.0 \text{ J}$

$$\Delta U_{a \rightarrow b} = U_b - U_a = +30.0 \text{ J, so } U_b = +30.0 \text{ J}.$$

process  $a \rightarrow d$

$$\Delta U_{a \rightarrow d} = Q_{ad} - W_{ad}$$

$$\Delta U_{a \rightarrow d} = U_d - U_a = +8.0 \text{ J}$$

$W_{adb} = +15.0 \text{ J}$  and  $W_{adb} = W_{ad} + W_{db}$ . But the work  $W_{db}$  for the process  $d \rightarrow b$  is zero since  $\Delta V = 0$  for that process. Therefore  $W_{ad} = W_{adb} = +15.0 \text{ J}$ .

Then  $Q_{ad} = \Delta U_{a \rightarrow d} + W_{ad} = +8.0 \text{ J} + 15.0 \text{ J} = +23.0 \text{ J}$  (positive implies heat absorbed).

process  $d \rightarrow b$

$$\Delta U_{d \rightarrow b} = Q_{db} - W_{db}$$

$$W_{db} = 0, \text{ as already noted.}$$

$$\Delta U_{d \rightarrow b} = U_b - U_d = 30.0 \text{ J} - 8.0 \text{ J} = +22.0 \text{ J}.$$

Then  $Q_{db} = \Delta U_{d \rightarrow b} + W_{db} = +22.0 \text{ J}$  (positive; heat absorbed).

**EVALUATE:** The signs of our calculated  $Q_{ad}$  and  $Q_{db}$  agree with the problem statement that heat is absorbed in these processes.

**19.38. IDENTIFY:**  $\Delta U = Q - W$ .

**SET UP:**  $W = 0$  when  $\Delta V = 0$ .

**EXECUTE:** For each process,  $Q = \Delta U + W$ . No work is done in the processes  $ab$  and  $dc$ , and so

$W_{bc} = W_{abc} = 450 \text{ J}$  and  $W_{ad} = W_{adc} = 120 \text{ J}$ . The heat flow for each process is: for  $ab$ ,  $Q = 90 \text{ J}$ . For  $bc$ ,  $Q = 440 \text{ J} + 450 \text{ J} = 890 \text{ J}$ . For  $ad$ ,  $Q = 180 \text{ J} + 120 \text{ J} = 300 \text{ J}$ . For  $dc$ ,  $Q = 350 \text{ J}$ . Heat is absorbed in each process. Note that the arrows representing the processes all point in the direction of increasing temperature (increasing  $U$ ).

**EVALUATE:**  $\Delta U$  is path independent so is the same for paths  $adc$  and  $abc$ .  $Q_{adc} = 300 \text{ J} + 350 \text{ J} = 650 \text{ J}$ .  $Q_{abc} = 90 \text{ J} + 890 \text{ J} = 980 \text{ J}$ .  $Q$  and  $W$  are path dependent and are different for these two paths.

**19.39. IDENTIFY:** Use  $pV = nRT$  to calculate  $T_c/T_a$ . Calculate  $\Delta U$  and  $W$  and use  $\Delta U = Q - W$  to obtain  $Q$ .

**SET UP:** For path  $ac$ , the work done is the area under the line representing the process in the  $pV$ -diagram.

$$\text{EXECUTE: (a) } \frac{T_c}{T_a} = \frac{p_c V_c}{p_a V_a} = \frac{(1.0 \times 10^5 \text{ J})(0.060 \text{ m}^3)}{(3.0 \times 10^5 \text{ J})(0.020 \text{ m}^3)} = 1.00. \quad T_c = T_a.$$

(b) Since  $T_c = T_a$ ,  $\Delta U = 0$  for process  $abc$ . For  $ab$ ,  $\Delta V = 0$  and  $W_{ab} = 0$ . For  $bc$ ,  $p$  is constant and  $W_{bc} = p\Delta V = (1.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3) = 4.0 \times 10^3 \text{ J}$ . Therefore,  $W_{abc} = +4.0 \times 10^3 \text{ J}$ . Since  $\Delta U = 0$ ,  $Q = W = +4.0 \times 10^3 \text{ J}$ .  $4.0 \times 10^3 \text{ J}$  of heat flows into the gas during process  $abc$ .

(c)  $W = \frac{1}{2}(3.0 \times 10^5 \text{ Pa} + 1.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3) = +8.0 \times 10^3 \text{ J}$ .  $Q_{ac} = W_{ac} = +8.0 \times 10^3 \text{ J}$ .

**EVALUATE:** The work done is path dependent and is greater for process  $ac$  than for process  $abc$ , even though the initial and final states are the same.

**19.40. IDENTIFY:** For a cycle,  $\Delta U = 0$  and  $Q = W$ . Calculate  $W$ .

**SET UP:** The magnitude of the work done by the gas during the cycle equals the area enclosed by the cycle in the  $pV$ -diagram.

**EXECUTE:** (a) The cycle is sketched in Figure 19.40.

(b)  $|W| = (3.50 \times 10^4 \text{ Pa} - 1.50 \times 10^4 \text{ Pa})(0.0435 \text{ m}^3 - 0.0280 \text{ m}^3) = +310 \text{ J}$ . More negative work is done for  $cd$  than positive work for  $ab$  and the net work is negative.  $W = -310 \text{ J}$ .

(c)  $Q = W = -310 \text{ J}$ . Since  $Q < 0$ , the net heat flow is out of the gas.

**EVALUATE:** During each constant pressure process  $W = p\Delta V$  and during the constant volume process  $W = 0$ .

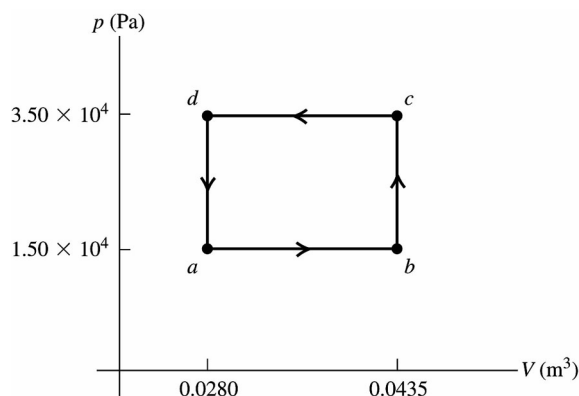


Figure 19.40

**19.41. IDENTIFY:** Use the first law of thermodynamics to relate  $Q_{\text{tot}}$  to  $W_{\text{tot}}$  for the cycle.

Calculate  $W_{ab}$  and  $W_{bc}$  and use what we know about  $W_{\text{tot}}$  to deduce  $W_{ca}$ .

(a) **SET UP and EXECUTE:** We aren't told whether the pressure increases or decreases in process  $bc$ . The two possibilities for the cycle are sketched in Figure 19.41.

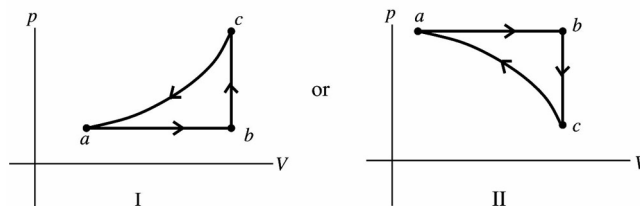


Figure 19.41

In cycle I, the total work is negative and in cycle II the total work is positive. For a cycle,  $\Delta U = 0$ , so

$$Q_{\text{tot}} = W_{\text{tot}}.$$

The net heat flow for the cycle is out of the gas, so heat  $Q_{\text{tot}} < 0$  and  $W_{\text{tot}} < 0$ . Sketch I is correct.

(b) **SET UP and EXECUTE:**  $W_{\text{tot}} = Q_{\text{tot}} = -800 \text{ J}$

$$W_{\text{tot}} = W_{ab} + W_{bc} + W_{ca}$$

$$W_{bc} = 0 \text{ since } \Delta V = 0.$$

$$W_{ab} = p\Delta V \text{ since } p \text{ is constant. But since it is an ideal gas, } p\Delta V = nR\Delta T.$$

$$W_{ab} = nR(T_b - T_a) = 1660 \text{ J}$$

$$W_{ca} = W_{\text{tot}} - W_{ab} = -800 \text{ J} - 1660 \text{ J} = -2460 \text{ J}.$$

**EVALUATE:** In process *ca* the volume decreases and the work  $W$  is negative.

- 19.42. IDENTIFY:** Apply the appropriate expression for  $W$  for each type of process.  $pV = nRT$  and

$$C_p = C_V + R.$$

**SET UP:**  $R = 8.315 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE:** Path *ac* has constant pressure, so  $W_{ac} = p\Delta V = nR\Delta T$ , and

$$W_{ac} = nR(T_c - T_a) = (3 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(492 \text{ K} - 300 \text{ K}) = 4.789 \times 10^3 \text{ J}.$$

Path *cb* is adiabatic ( $Q = 0$ ), so  $W_{cb} = Q - \Delta U = -\Delta U = -nC_V\Delta T$ , and using  $C_V = C_p - R$ ,

$$W_{cb} = -n(C_p - R)(T_b - T_c) = -(3 \text{ mol})(29.1 \text{ J/mol} \cdot \text{K} - 8.3145 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 492 \text{ K}) = -6.735 \times 10^3 \text{ J}.$$

Path *ba* has constant volume, so  $W_{ba} = 0$ . So the total work done is

$$W = W_{ac} + W_{cb} + W_{ba} = 4.789 \times 10^3 \text{ J} - 6.735 \times 10^3 \text{ J} + 0 = -1.95 \times 10^3 \text{ J}.$$

**EVALUATE:**  $W > 0$  when  $\Delta V > 0$ ,  $W < 0$  when  $\Delta V < 0$  and  $W = 0$  when  $\Delta V = 0$ .

- 19.43. IDENTIFY:** Segment *ab* is isochoric, *bc* is isothermal, and *ca* is isobaric.

**SET UP:** For *bc*,  $\Delta T = 0$ ,  $\Delta U = 0$ , and  $Q = W = nRT \ln(V_c/V_b)$ . For ideal  $\text{H}_2$  (diatomic),  $C_V = \frac{5}{2}R$  and

$$C_p = \frac{7}{2}R. \Delta U = nC_V \Delta T \text{ for any process of an ideal gas.}$$

**EXECUTE: (a)**  $T_b = T_c$ . For states *b* and *c*,  $pV = nRT = \text{constant}$  so  $p_b V_b = p_c V_c$  and

$$V_c = V_b \left( \frac{p_b}{p_c} \right) = (0.20 \text{ L}) \left( \frac{2.0 \text{ atm}}{0.50 \text{ atm}} \right) = 0.80 \text{ L}.$$

$$\text{(b) } T_a = \frac{p_a V_a}{nR} = \frac{(0.50 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(0.20 \times 10^{-3} \text{ m}^3)}{(0.0040 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 305 \text{ K}. V_a = V_b \text{ so for states } a \text{ and } b,$$

$$\frac{T}{p} = \frac{V}{nR} = \text{constant so } \frac{T_a}{p_a} = \frac{T_b}{p_b}. T_b = T_c = T_a \left( \frac{p_b}{p_a} \right) = (305 \text{ K}) \left( \frac{2.0 \text{ atm}}{0.50 \text{ atm}} \right) = 1220 \text{ K}; T_c = 1220 \text{ K}.$$

**(c) ab:**  $Q = nC_V \Delta T = n(\frac{5}{2}R) \Delta T$ , which gives

$$Q = (0.0040 \text{ mol})(\frac{5}{2})(8.315 \text{ J/mol} \cdot \text{K})(1220 \text{ K} - 305 \text{ K}) = +76 \text{ J}. Q \text{ is positive and heat goes into the gas.}$$

*ca:*  $Q = nC_p \Delta T = n(\frac{7}{2}R) \Delta T$ , which gives

$$Q = (0.0040 \text{ mol})(\frac{7}{2})(8.315 \text{ J/mol} \cdot \text{K})(305 \text{ K} - 1220 \text{ K}) = -107 \text{ J}. Q \text{ is negative and heat comes out of the gas.}$$

*bc:*  $Q = W = nRT \ln(V_c/V_b)$ , which gives

$$Q = (0.0040 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(1220 \text{ K}) \ln(0.80 \text{ L}/0.20 \text{ L}) = 56 \text{ J}. Q \text{ is positive and heat goes into the gas.}$$

**(d) ab:**  $\Delta U = nC_V \Delta T = n(\frac{5}{2}R) \Delta T$ , which gives

$$\Delta U = (0.0040 \text{ mol})(\frac{5}{2})(8.315 \text{ J/mol} \cdot \text{K})(1220 \text{ K} - 305 \text{ K}) = +76 \text{ J}. \text{ The internal energy increased.}$$

*bc:*  $\Delta T = 0$  so  $\Delta U = 0$ . The internal energy does not change.

*ca:*  $\Delta U = nC_V \Delta T = n(\frac{5}{2}R) \Delta T$ , which gives

$$\Delta U = (0.0040 \text{ mol})(\frac{5}{2})(8.315 \text{ J/mol} \cdot \text{K})(305 \text{ K} - 1220 \text{ K}) = -76 \text{ J}. \text{ The internal energy decreased.}$$

**EVALUATE:** The net internal energy change for the complete cycle  $a \rightarrow b \rightarrow c \rightarrow a$  is

$$\Delta U_{\text{tot}} = +76 \text{ J} + 0 + (-76 \text{ J}) = 0. \text{ For any complete cycle the final state is the same as the initial state and}$$

the net internal energy change is zero. For the cycle the net heat flow is

$Q_{\text{tot}} = +76 \text{ J} + (-107 \text{ J}) + 56 \text{ J} = +25 \text{ J}. \Delta U_{\text{tot}} = 0$  so  $Q_{\text{tot}} = W_{\text{tot}}$ . The net work done in the cycle is positive and this agrees with our result that the net heat flow is positive.

- 19.44 IDENTIFY:** The segments  $ab$  and  $bc$  are not any of the familiar ones, such as isothermal, isobaric, or isochoric, but  $ac$  is isobaric.

**SET UP:** For helium,  $C_V = 12.47 \text{ J/mol} \cdot \text{K}$  and  $C_p = 20.78 \text{ J/mol} \cdot \text{K}$ .  $\Delta U = Q - W$ .  $W$  is the area under the  $p$  versus  $V$  curve.  $\Delta U = nC_V\Delta T$  for any process of an ideal gas.

**EXECUTE:** (a)  $W = \frac{1}{2}(1.0 \times 10^5 \text{ Pa} + 3.5 \times 10^5 \text{ Pa})(0.0060 \text{ m}^3 - 0.0020 \text{ m}^3)$   
 $+ \frac{1}{2}(1.0 \times 10^5 \text{ Pa} + 3.5 \times 10^5 \text{ Pa})(0.0100 \text{ m}^3 - 0.0060 \text{ m}^3) = 1800 \text{ J}.$

Find  $\Delta T = T_c - T_a$ .  $p$  is constant so  $\Delta T = \frac{p\Delta V}{nR} = \frac{(1.0 \times 10^5 \text{ Pa})(0.0100 \text{ m}^3 - 0.0020 \text{ m}^3)}{(\frac{1}{3} \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 289 \text{ K}$ . Then

$$\Delta U = nC_V\Delta T = (\frac{1}{3} \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(289 \text{ K}) = 1.20 \times 10^3 \text{ J}.$$

$$Q = \Delta U + W = 1.20 \times 10^3 \text{ J} + 1800 \text{ J} = 3.00 \times 10^3 \text{ J}. \quad Q > 0, \text{ so this heat is transferred into the gas.}$$

(b) This process is isobaric, so  $Q = nC_p\Delta T = (\frac{1}{3} \text{ mol})(20.78 \text{ J/mol} \cdot \text{K})(289 \text{ K}) = 2.00 \times 10^3 \text{ J}$ .  $Q > 0$ , so this heat is transferred into the gas.

(c)  $Q$  is larger in part (a).

**EVALUATE:**  $\Delta U$  is the same in parts (a) and (b) because the initial and final states are the same, but in (a) more work is done.

- 19.45. IDENTIFY:** Use  $Q = nC_V\Delta T$  to calculate the temperature change in the constant volume process and use  $pV = nRT$  to calculate the temperature change in the constant pressure process. The work done in the constant volume process is zero and the work done in the constant pressure process is  $W = p\Delta V$ . Use  $Q = nC_p\Delta T$  to calculate the heat flow in the constant pressure process.  $\Delta U = nC_V\Delta T$ , or  $\Delta U = Q - W$ .

**SET UP:** For  $\text{N}_2$ ,  $C_V = 20.76 \text{ J/mol} \cdot \text{K}$  and  $C_p = 29.07 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE:** (a) For process  $ab$ ,  $\Delta T = \frac{Q}{nC_V} = \frac{1.36 \times 10^4 \text{ J}}{(2.50 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})} = 262.0 \text{ K}$ .  $T_a = 293.1 \text{ K}$ , so

$$T_b = 555 \text{ K}. \quad pV = nRT \text{ says } T \text{ doubles when } V \text{ doubles and } p \text{ is constant, so}$$

$$T_c = 2(555 \text{ K}) = 1110 \text{ K} = 837^\circ\text{C}.$$

(b) For process  $ab$ ,  $W_{ab} = 0$ . For process  $bc$ ,

$$W_{bc} = p\Delta V = nR\Delta T = (2.50 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1110 \text{ K} - 555 \text{ K}) = 1.153 \times 10^4 \text{ J} = 11.5 \text{ kJ}.$$

$$W = W_{ab} + W_{bc} = 1.15 \times 10^4 \text{ J} = 11.5 \text{ kJ}.$$

(c) For process  $bc$ ,  $Q = nC_p\Delta T = (2.50 \text{ mol})(29.07 \text{ J/mol} \cdot \text{K})(1110 \text{ K} - 555 \text{ K}) = 4.03 \times 10^4 \text{ J} = 40.3 \text{ kJ}$ .

(d)  $\Delta U = nC_V\Delta T = (2.50 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(1110 \text{ K} - 293 \text{ K}) = 4.24 \times 10^4 \text{ J} = 42.4 \text{ kJ}$ .

**EVALUATE:** The total  $Q$  is  $1.36 \times 10^4 \text{ J} + 4.03 \times 10^4 \text{ J} = 5.39 \times 10^4 \text{ J} = 53.9 \text{ kJ}$ .

$\Delta U = Q - W = 5.39 \times 10^4 \text{ J} - 1.15 \times 10^4 \text{ J} = 4.24 \times 10^4 \text{ J} = 42.4 \text{ kJ}$ , which agrees with our results in part (d).

- 19.46. IDENTIFY:** For a constant pressure process,  $Q = nC_p\Delta T$ .  $\Delta U = Q - W$ .  $\Delta U = nC_V\Delta T$  for any ideal gas process.

**SET UP:** For  $\text{N}_2$ ,  $C_V = 20.76 \text{ J/mol} \cdot \text{K}$  and  $C_p = 29.07 \text{ J/mol} \cdot \text{K}$ .  $Q < 0$  if heat comes out of the gas.

**EXECUTE:** (a)  $n = \frac{Q}{C_p\Delta T} = \frac{-2.5 \times 10^4 \text{ J}}{(29.07 \text{ J/mol} \cdot \text{K})(-40.0 \text{ K})} = 21.5 \text{ mol}.$

(b)  $\Delta U = nC_V\Delta T = Q(C_V/C_p) = (-2.5 \times 10^4 \text{ J})(20.76/29.07) = -1.79 \times 10^4 \text{ J}.$

(c)  $W = Q - \Delta U = -7.15 \times 10^3 \text{ J}.$



(d)  $\Delta U$  is the same for both processes, and if  $\Delta V = 0$ ,  $W = 0$  and  $Q = \Delta U = -1.79 \times 10^4$  J.

EVALUATE: For a given  $\Delta T$ ,  $Q$  is larger in magnitude when the pressure is constant than when the volume is constant.

- 19.47. **IDENTIFY:**  $pV = nRT$ . For an isothermal process  $W = nRT \ln(V_2/V_1)$ . For a constant pressure process,  $W = p\Delta V$ .

SET UP:  $1 \text{ L} = 10^{-3} \text{ m}^3$ .

EXECUTE: (a) The  $pV$ -diagram is sketched in Figure 19.47.

(b) At constant temperature, the product  $pV$  is constant, so

$$V_2 = V_1(p_1/p_2) = (1.5 \text{ L}) \left( \frac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}} \right) = 6.00 \text{ L.}$$

The final pressure is given as being the same as

$$p_3 = p_2 = 2.5 \times 10^4 \text{ Pa.}$$

The final volume is the same as the initial volume, so  $T_3 = T_1(p_3/p_1) = 75.0 \text{ K}$ .

(c) Treating the gas as ideal, the work done in the first process is  $W = nRT \ln(V_2/V_1) = p_1 V_1 \ln(p_1/p_2)$ .

$$W = (1.00 \times 10^5 \text{ Pa})(1.5 \times 10^{-3} \text{ m}^3) \ln \left( \frac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}} \right) = 208 \text{ J.}$$

For the second process,  $W = p_2(V_3 - V_2) = p_2(V_1 - V_2) = p_2 V_1(1 - (p_1/p_2))$ .

$$W = (2.50 \times 10^4 \text{ Pa})(1.5 \times 10^{-3} \text{ m}^3) \left( 1 - \frac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}} \right) = -113 \text{ J.}$$

The total work done is  $208 \text{ J} - 113 \text{ J} = 95 \text{ J}$ .

(d) Heat at constant volume. No work would be done by the gas or on the gas during this process.

EVALUATE: When the volume increases,  $W > 0$ . When the volume decreases,  $W < 0$ .

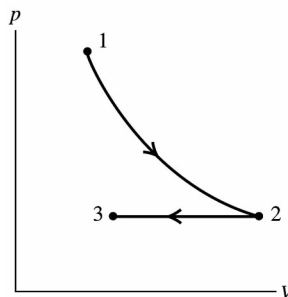


Figure 19.47

- 19.48. **IDENTIFY:**  $\Delta V = \beta V_0 \Delta T$ .  $W = p\Delta V$  since the applied pressure (air pressure) is constant.  $Q = mc_p \Delta T$ .  $\Delta U = Q - W$ .

SET UP: For copper,  $\beta = 5.1 \times 10^{-5} (\text{C}^\circ)^{-1}$ ,  $c_p = 390 \text{ J/kg} \cdot \text{K}$ , and  $\rho = 8.90 \times 10^3 \text{ kg/m}^3$ .

EXECUTE: (a)  $\Delta V = \beta \Delta T V_0 = (5.1 \times 10^{-5} (\text{C}^\circ)^{-1})(70.0 \text{ C}^\circ)(2.00 \times 10^{-2} \text{ m}^3) = 2.86 \times 10^{-8} \text{ m}^3$ .

(b)  $W = p\Delta V = 2.88 \times 10^{-3} \text{ J}$ .

(c)  $Q = mc_p \Delta T = \rho V_0 c_p \Delta T = (8.9 \times 10^3 \text{ kg/m}^3)(8.00 \times 10^{-6} \text{ m}^3)(390 \text{ J/kg} \cdot \text{K})(70.0 \text{ C}^\circ) = 1944 \text{ J}$ .

(d) To three figures,  $\Delta U = Q = 1940 \text{ J}$ .

(e) Under these conditions, the difference is not substantial, since  $W$  is much less than  $Q$ .

EVALUATE:  $\Delta U = Q - W$  applies to any material. For solids the expansion work is much less than  $Q$ .

- 19.49. **IDENTIFY:** For an adiabatic process of an ideal gas,  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ .  $pV = nRT$ .

SET UP: For air,  $\gamma = 1.40 = \frac{7}{5}$ .

**EXECUTE:** (a) As the air moves to lower altitude its density increases; under an adiabatic compression, the temperature rises. If the wind is fast-moving,  $Q$  is not as likely to be significant, and modeling the process as adiabatic (no heat loss to the surroundings) is more accurate.

(b)  $V = \frac{nRT}{p}$ , so  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$  gives  $T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$ . The temperature at the higher pressure is

$$T_2 = T_1 (p_1/p_2)^{(\gamma-1)/\gamma} = (258.15 \text{ K}) \left[ (8.12 \times 10^4 \text{ Pa}) / (5.60 \times 10^4 \text{ Pa}) \right]^{2/7} = 287.1 \text{ K} = 13.9^\circ\text{C} \text{ so the temperature would rise by } 11.9^\circ\text{C}.$$

**EVALUATE:** In an adiabatic compression,  $Q = 0$  but the temperature rises because of the work done on the gas.

**19.50. IDENTIFY:** The process is adiabatic. Apply  $p_1 V_1^\gamma = p_2 V_2^\gamma$  and  $pV = nRT$ .  $Q = 0$  so

$$\Delta U = -W = -\frac{1}{\gamma-1} (p_1 V_1 - p_2 V_2).$$

**SET UP:** For ideal monatomic helium,  $\gamma = 5/3 = 1.667$ .  $p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ .

$$V_1 = 2.00 \times 10^3 \text{ m}^3. \quad p_2 = 0.900 \text{ atm} = 9.117 \times 10^4 \text{ Pa}. \quad T_1 = 288.15 \text{ K}.$$

**EXECUTE:** (a)  $V_2^\gamma = V_1^\gamma \left( \frac{p_1}{p_2} \right)$ .  $V_2 = V_1 \left( \frac{p_1}{p_2} \right)^{1/\gamma} = (2.00 \times 10^3 \text{ m}^3) \left( \frac{1.00 \text{ atm}}{0.900 \text{ atm}} \right)^{1/1.67} = 2.13 \times 10^3 \text{ m}^3$ .

(b)  $pV = nRT$  gives  $\frac{T_1}{p_1 V_1} = \frac{T_2}{p_2 V_2}$ .

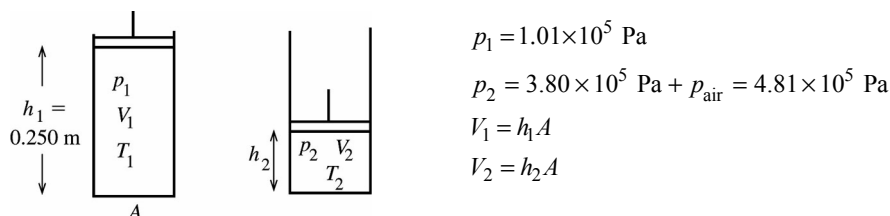
$$T_2 = T_1 \left( \frac{p_2}{p_1} \right) \left( \frac{V_2}{V_1} \right) = (288.15 \text{ K}) \left( \frac{0.900 \text{ atm}}{1.00 \text{ atm}} \right) \left( \frac{2.13 \times 10^3 \text{ m}^3}{2.00 \times 10^3 \text{ m}^3} \right) = 276.2 \text{ K} = 3.0^\circ\text{C}.$$

$$(c) \Delta U = -\frac{(1.013 \times 10^5 \text{ Pa})(2.00 \times 10^3 \text{ m}^3) - (9.117 \times 10^4 \text{ Pa})(2.13 \times 10^3 \text{ m}^3)}{1.667 - 1} = -1.25 \times 10^7 \text{ J}.$$

**EVALUATE:** The internal energy decreases when the temperature decreases.

**19.51. IDENTIFY:** Assume that the gas is ideal and that the process is adiabatic. Apply  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$  and  $p_1 V_1^\gamma = p_2 V_2^\gamma$  to relate pressure and volume and temperature and volume. The distance the piston moves is related to the volume of the gas. Use  $W = nC_V(T_1 - T_2)$  to calculate  $W$ .

(a) **SET UP:**  $\gamma = C_p/C_V = (C_V + R)/C_V = 1 + R/C_V = 1.40$ . The two positions of the piston are shown in Figure 19.51.



**Figure 19.51**

**EXECUTE:** For an adiabatic process of an ideal gas,  $p_1 V_1^\gamma = p_2 V_2^\gamma$ .

$$p_1 h_1^\gamma A^\gamma = p_2 h_2^\gamma A^\gamma$$

$$h_2 = h_1 \left( \frac{p_1}{p_2} \right)^{1/\gamma} = (0.250 \text{ m}) \left( \frac{1.01 \times 10^5 \text{ Pa}}{4.81 \times 10^5 \text{ Pa}} \right)^{1/1.40} = 0.08199 \text{ m}$$

The piston has moved a distance  $h_1 - h_2 = 0.250 \text{ m} - 0.08199 \text{ m} = 0.168 \text{ m}$ .

**(b) SET UP:**  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_1 h_1^{\gamma-1} A^{\gamma-1} = T_2 h_2^{\gamma-1} A^{\gamma-1}$$

**EXECUTE:**  $T_2 = T_1 \left( \frac{h_1}{h_2} \right)^{\gamma-1} = 300.1 \text{ K} \left( \frac{0.250 \text{ m}}{0.08199 \text{ m}} \right)^{0.40} = 468.7 \text{ K} = 196^\circ\text{C}.$

**(c) SET UP and EXECUTE:**  $W = nC_V(T_1 - T_2)$  gives

$W = (20.0 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(300.1 \text{ K} - 468.7 \text{ K}) = -7.01 \times 10^4 \text{ J} = -70.1 \text{ kJ}.$  This is the work done *by* the gas. The work done *on* the gas by the pump is  $+70.1 \text{ kJ}.$

**EVALUATE:** In an adiabatic compression of an ideal gas the temperature increases. In any compression the work  $W$  done by the gas is negative.

**19.52. IDENTIFY:** For constant pressure,  $W = p\Delta V$ . For an adiabatic process of an ideal gas,

$$W = \frac{C_V}{R}(p_1 V_1 - p_2 V_2) \text{ and } p_1 V_1^\gamma = p_2 V_2^\gamma.$$

**SET UP:**  $\gamma = \frac{C_p}{C_V} = \frac{C_p + C_V}{C_V} = 1 + \frac{R}{C_V}.$

**EXECUTE:** **(a)** The  $pV$ -diagram is sketched in Figure 19.52.

**(b)** The work done is  $W = p_0(2V_0 - V_0) + \frac{C_V}{R}(p_0(2V_0) - p_3(4V_0)).$   $p_3 = p_0(2V_0/4V_0)^\gamma$  and so

$$W = p_0 V_0 \left[ 1 + \frac{C_V}{R}(2 - 2^{2-\gamma}) \right]. \text{ Note that } p_0 \text{ is the absolute pressure.}$$

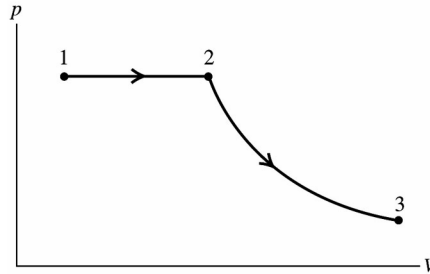
**(c)** The most direct way to find the temperature is to find the ratio of the final pressure and volume to the

original and treat the air as an ideal gas.  $p_3 = p_2 \left( \frac{V_2}{V_3} \right)^\gamma = p_1 \left( \frac{V_2}{V_3} \right)^\gamma$ , since  $p_1 = p_2$ . Then

$$T_3 = T_0 \frac{p_3 V_3}{p_1 V_1} = T_0 \left( \frac{V_2}{V_3} \right)^\gamma \left( \frac{V_3}{V_1} \right) = T_0 \left( \frac{1}{2} \right)^\gamma 4 = T_0 (2)^{2-\gamma}.$$

**(d)** Since  $n = \frac{p_0 V_0}{RT_0}$ ,  $Q = \frac{p_0 V_0}{RT_0} (C_V + R)(2T_0 - T_0) = p_0 V_0 \left( \frac{C_V}{R} + 1 \right).$  This amount of heat flows into the gas, since  $Q > 0$ .

**EVALUATE:** In the isobaric expansion the temperature doubles and in the adiabatic expansion the temperature decreases. If the gas is diatomic, with  $\gamma = \frac{7}{5}$ ,  $2 - \gamma = \frac{3}{5}$  and  $T_3 = 1.52T_0$ ,  $W = 2.21p_0 V_0$ , and  $Q = 3.50p_0 V_0$ .  $\Delta U = 1.29p_0 V_0$ .  $\Delta U > 0$  and this is consistent with an increase in temperature.



**Figure 19.52**

**19.53. IDENTIFY:** In each case calculate either  $\Delta U$  or  $Q$  for the specific type of process and then apply the first law.

**(a) SET UP:** isothermal ( $\Delta T = 0$ )  $\Delta U = Q - W$ ;  $W = +450 \text{ J}$ . For any process of an ideal gas,

$$\Delta U = nC_V \Delta T.$$

**EXECUTE:** Therefore, for an ideal gas, if  $\Delta T = 0$  then  $\Delta U = 0$  and  $Q = W = +450$  J.

**(b) SET UP:** adiabatic ( $Q = 0$ )

$$\Delta U = Q - W; \quad W = +450 \text{ J.}$$

**EXECUTE:**  $Q = 0$  says  $\Delta U = -W = -450$  J.

**(c) SET UP:** isobaric  $\Delta p = 0$

Use  $W$  to calculate  $\Delta T$  and then calculate  $Q$ .

**EXECUTE:**  $W = p\Delta V = nR\Delta T$ ;  $\Delta T = W/nR$

$$Q = nC_p\Delta T \text{ and for a monatomic ideal gas } C_p = \frac{5}{2}R.$$

$$\text{Thus } Q = n\frac{5}{2}R\Delta T = (5Rn/2)(W/nR) = 5W/2 = +1125 \text{ J.}$$

$$\Delta U = nC_V\Delta T \text{ for any ideal gas process and } C_V = C_p - R = \frac{3}{2}R.$$

$$\text{Thus } \Delta U = 3W/2 = +675 \text{ J.}$$

**EVALUATE:** 450 J of energy leaves the gas when it performs expansion work. In the isothermal process this energy is replaced by heat flow into the gas and the internal energy remains the same. In the adiabatic process the energy used in doing the work decreases the internal energy. In the isobaric process 1125 J of heat energy enters the gas, 450 J leaves as the work done and 675 J remains in the gas as increased internal energy.

**19.54. IDENTIFY:**  $pV = nRT$ . For the isobaric process,  $W = p\Delta V = nR\Delta T$ . For the isothermal process,

$$W = nRT \ln\left(\frac{V_2}{V_1}\right).$$

**SET UP:**  $R = 8.315 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE:** **(a)** The  $pV$  diagram for these processes is sketched in Figure 19.54.

**(b)** Find  $T_2$ . For process  $1 \rightarrow 2$ ,  $n$ ,  $R$  and  $p$  are constant so  $\frac{T}{V} = \frac{p}{nR} = \text{constant}$ .  $\frac{T_1}{V_1} = \frac{T_2}{V_2}$  and

$$T_2 = T_1 \left(\frac{V_2}{V_1}\right) = (355 \text{ K})(2) = 710 \text{ K.}$$

**(c)** The maximum pressure is for state 3. For process  $2 \rightarrow 3$ ,  $n$ ,  $R$  and  $T$  are constant.  $p_2V_2 = p_3V_3$  and

$$p_3 = p_2 \left(\frac{V_2}{V_3}\right) = (2.40 \times 10^5 \text{ Pa})(2) = 4.80 \times 10^5 \text{ Pa.}$$

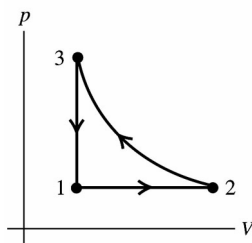
**(d)** process  $1 \rightarrow 2$ :  $W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = 738 \text{ J}$ .

$$\text{process } 2 \rightarrow 3: W = nRT \ln\left(\frac{V_3}{V_2}\right) = (0.250 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(710 \text{ K}) \ln\left(\frac{1}{2}\right) = -1023 \text{ J.}$$

process  $3 \rightarrow 1$ :  $\Delta V = 0$  and  $W = 0$ .

The total work done is  $738 \text{ J} + (-1023 \text{ J}) = -285 \text{ J}$ . This is the work done by the gas. The work done on the gas is 285 J.

**EVALUATE:** The final pressure and volume are the same as the initial pressure and volume, so the final state is the same as the initial state. For the cycle,  $\Delta U = 0$  and  $Q = W = -285 \text{ J}$ . During the cycle, 285 J of heat energy must leave the gas.



**Figure 19.54**

**19.55. IDENTIFY and SET UP:** Use the ideal gas law, the first law of thermodynamics, and expressions for  $Q$  and  $W$  for specific types of processes.

**EXECUTE: (a)** initial expansion (state 1  $\rightarrow$  state 2)

$$p_1 = 2.40 \times 10^5 \text{ Pa}, \quad T_1 = 355 \text{ K}, \quad p_2 = 2.40 \times 10^5 \text{ Pa}, \quad V_2 = 2V_1$$

$$pV = nRT; \quad T/V = p/nR = \text{constant, so } T_1/V_1 = T_2/V_2 \text{ and } T_2 = T_1(V_2/V_1) = 355 \text{ K}(2V_1/V_1) = 710 \text{ K}$$

$$\Delta p = 0 \text{ so } W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = +738 \text{ J}$$

$$Q = nC_p\Delta T = (0.250 \text{ mol})(29.17 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = +2590 \text{ J}$$

$$\Delta U = Q - W = 2590 \text{ J} - 738 \text{ J} = 1850 \text{ J}$$

**(b)** At the beginning of the final cooling process (cooling at constant volume),  $T = 710 \text{ K}$ . The gas returns to its original volume and pressure, so also to its original temperature of 355 K.

$$\Delta V = 0 \text{ so } W = 0$$

$$Q = nC_V\Delta T = (0.250 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(355 \text{ K} - 710 \text{ K}) = -1850 \text{ J}$$

$$\Delta U = Q - W = -1850 \text{ J}.$$

**(c)** For any ideal gas process  $\Delta U = nC_V\Delta T$ . For an isothermal process  $\Delta T = 0$ , so  $\Delta U = 0$ .

**EVALUATE:** The three processes return the gas to its initial state, so  $\Delta U_{\text{total}} = 0$ ; our results agree with this.

**19.56. IDENTIFY:**  $pV = nRT$ . For an adiabatic process of an ideal gas,  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ .

**SET UP:** For  $\text{N}_2$ ,  $\gamma = 1.40$ .

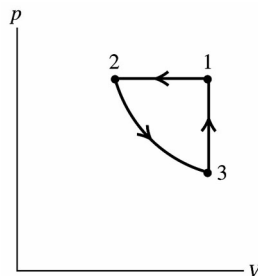
**EXECUTE: (a)** The  $pV$ -diagram is sketched in Figure 19.56.

**(b)** At constant pressure, halving the volume halves the Kelvin temperature, and the temperature at the beginning of the adiabatic expansion is 150 K. The volume doubles during the adiabatic expansion, and from Eq. (19.22), the temperature at the end of the expansion is  $(150 \text{ K})(1/2)^{0.40} = 114 \text{ K}$ .

**(c)** The minimum pressure occurs at the end of the adiabatic expansion (state 3). During the final heating the volume is held constant, so the minimum pressure is proportional to the Kelvin temperature,

$$p_{\text{min}} = (1.80 \times 10^5 \text{ Pa})(114 \text{ K}/300 \text{ K}) = 6.82 \times 10^4 \text{ Pa}.$$

**EVALUATE:** In the adiabatic expansion the temperature decreases.



**Figure 19.56**

**19.57. IDENTIFY:** Use the appropriate expressions for  $Q$ ,  $W$ , and  $\Delta U$  for each type of process.  $\Delta U = Q - W$  can also be used.

**SET UP:** For  $\text{N}_2$ ,  $C_V = 20.76 \text{ J/mol} \cdot \text{K}$  and  $C_p = 29.07 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE: (a)**  $W = p\Delta V = nR\Delta T = (0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(-150 \text{ K}) = -187 \text{ J}$ ,

$$Q = nC_p\Delta T = (0.150 \text{ mol})(29.07 \text{ J/mol} \cdot \text{K})(-150 \text{ K}) = -654 \text{ J}, \quad \Delta U = Q - W = -467 \text{ J}.$$

**(b)** From Eq. (19.26), using the expression for the temperature found in Problem 19.56,

$$W = \frac{1}{0.40}(0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(150 \text{ K})[1 - (1/2)^{0.40}] = 113 \text{ J}. \quad Q = 0 \text{ for an adiabatic process, and}$$

$$\Delta U = Q - W = -W = -113 \text{ J}.$$

**(c)**  $\Delta V = 0$ , so  $W = 0$ . Using the temperature change as found in Problem 19.66 part (b),

$$Q = nC_V\Delta T = (0.150 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 113.7 \text{ K}) = 580 \text{ J} \text{ and } \Delta U = Q - W = Q = 580 \text{ J}.$$

**EVALUATE:** For each process we could also use  $\Delta U = nC_V\Delta T$  to calculate  $\Delta U$ .

**19.58. IDENTIFY:** Use the appropriate expression for  $W$  for each type of process.

**SET UP:** For a monatomic ideal gas,  $\gamma = 5/3$  and  $C_V = 3R/2$ .

**EXECUTE:** (a)  $W = nRT \ln(V_2/V_1) = nRT \ln(3) = 3.29 \times 10^3 \text{ J}$ .

(b)  $Q = 0$  so  $W = -\Delta U = -nC_V\Delta T$ .  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$  gives  $T_2 = T_1(1/3)^{2/3}$ . Then  $W = nC_VT_1(1 - (1/3)^{2/3}) = 2.33 \times 10^3 \text{ J}$ .

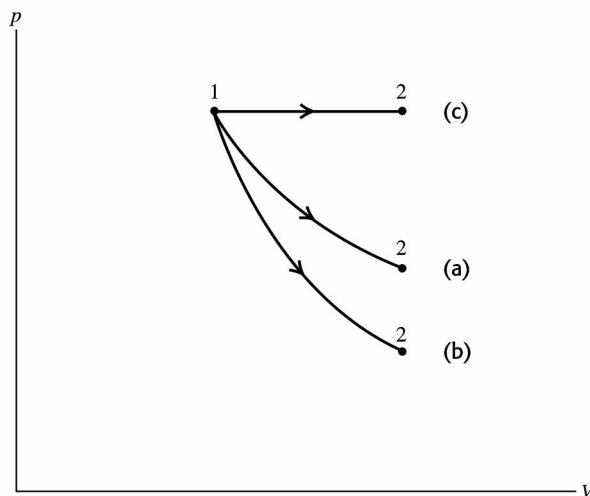
(c)  $V_2 = 3V_1$ , so  $W = p\Delta V = 2pV_1 = 2nRT_1 = 6.00 \times 10^3 \text{ J}$ .

(d) Each process is shown in Figure 19.58. The most work done is in the isobaric process, as the pressure is maintained at its original value. The least work is done in the adiabatic process.

(e) The isobaric process involves the most work and the largest temperature increase, and so requires the most heat. Adiabatic processes involve no heat transfer, and so the magnitude is zero.

(f) The isobaric process doubles the Kelvin temperature, and so has the largest change in internal energy. The isothermal process necessarily involves no change in internal energy.

**EVALUATE:** The work done is the area under the path for the process in the  $pV$ -diagram. Figure 19.58 shows that the work done is greatest in the isobaric process and least in the adiabatic process.



**Figure 19.58**

**19.59. IDENTIFY:** For an adiabatic process, no heat enters or leaves the gas. An isochoric process takes place at constant volume, and an isobaric process takes place at constant pressure. The first law of thermodynamics applies.

**SET UP:** For any process, including an isochoric process,  $Q = nC_V\Delta T$ , and for an isobaric process,  $Q = nC_p\Delta T$ .  $Q = \Delta U + W$ .

**EXECUTE:** (a) Process  $a$  is adiabatic since no heat goes into or out of the system. In processes  $b$  and  $c$ , the temperature change is the same, but more heat goes into the gas for process  $c$ . Since the change in internal energy is the same for both  $b$  and  $c$ , some of the heat in  $c$  must be doing work, but not in  $b$ . Therefore  $b$  is isochoric and  $c$  is isobaric. To summarize:  $a$  is adiabatic,  $b$  is isochoric,  $c$  is isobaric.

(b)  $Q_b = nC_V\Delta T$  and  $Q_c = nC_p\Delta T$ . Subtracting gives  $Q_c - Q_b = nC_p\Delta T - nC_V\Delta T = n(C_p - C_V)\Delta T = nR\Delta T = 20 \text{ J}$ . Solving for  $\Delta T$  gives  $\Delta T = (20 \text{ J})/nR = (20 \text{ J})/[(0.300 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})] = 8.0^\circ\text{C}$ , so  $T_2 = 20.0^\circ\text{C} + 8.0^\circ\text{C} = 28.0^\circ\text{C}$ .

(c)  $\frac{Q_c}{Q_b} = \frac{nC_p\Delta T}{nC_V\Delta T} = \frac{C_p}{C_V} = \gamma = \frac{50 \text{ J}}{30 \text{ J}} = \frac{5}{3}$ . Since  $\gamma = 5/3$ , the gas must be monatomic, in which case we have

$C_V = 3/2 R$  and  $C_p = 5/2 R$ . Therefore

Process  $a$ :  $Q = \Delta U + W$  gives  $0 = nC_V\Delta T + W$ .

$W = -n(3/2 R)\Delta T = -(0.300 \text{ mol})(3/2)(8.314 \text{ J/mol} \cdot \text{K})(8.0 \text{ K}) = -30 \text{ J}$ .

Process *b*: The volume is constant, so  $W = 0$ .

Process *c*:  $Q = \Delta U + W$ .  $\Delta U$  is the same as for process *a* because  $\Delta T$  is the same, so we have  $50 \text{ J} = 30 \text{ J} + W$ , which gives  $W = 20 \text{ J}$ .

(d) The greatest work has the greatest volume change. Using the results of part (c), process *a* has the greatest amount of work and hence the greatest volume change.

(e) The volume is increasing if  $W$  is positive. Therefore

Process *a*:  $W$  is negative, so the volume decreases.

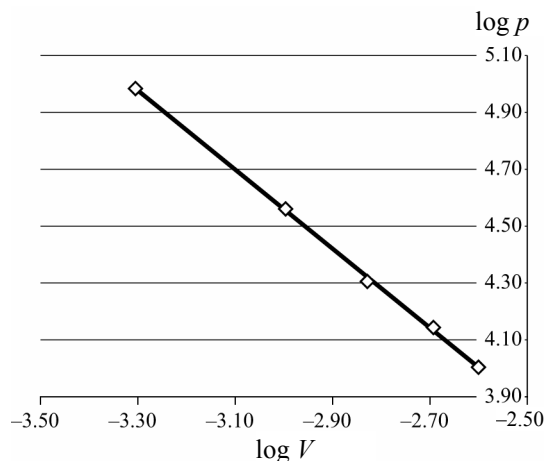
Process *b*:  $W = 0$  so the volume stays the same.

Process *c*:  $W$  is positive, so the volume increases.

**EVALUATE:** In Process *a*, no heat enters the gas, yet its temperature increases. This means that work must have been done on the gas, as we found.

- 19.60. IDENTIFY and SET UP:** The cylinder is insulated, so no heat can go into or out of the gas, which makes this an adiabatic process. For an adiabatic process,  $pV^\gamma = \text{constant}$ , and the ideal gas law,  $pV = nRT$ , also applies.

**EXECUTE:** (a) The graph of  $\log p$  versus  $\log V$  is shown in Figure 19.60.



**Figure 19.60**

For an adiabatic process,  $pV^\gamma = \text{constant}$ . Taking logs of both sides of this equation gives

$\log(pV^\gamma) = \log p + \log V^\gamma = \log p + \gamma \log V = \log(\text{constant})$ . Solving for  $\log p$  gives

$\log p = -\gamma \log V + \log(\text{constant})$ . Therefore a graph of  $\log p$  versus  $\log V$  should be a straight line having a slope equal to  $-\gamma$ .

(b) The equation of the best-fit line in the graph is  $\log p = -1.3946 \log V + 0.381$ , so  $-\gamma = -1.3946$ , so  $\gamma = 1.4$ . This is the adiabatic constant for a diatomic gas, so this gas must be diatomic.

(c) The ideal gas law gives  $pV = nRT$ , so  $nR = pV/T$ . Using the first set of data points in the table gives  $nR = (0.101 \text{ atm})(2.50 \text{ L})/(293.15 \text{ K}) = 8.61 \times 10^{-4} \text{ L} \cdot \text{atm}/\text{K}$ , so  $1/nR = 1160 \text{ K}/\text{L} \cdot \text{atm}$ . Using this number, we can calculate  $T$  for the rest of the pairs of points in the table. For example, for the next set of points, we have  $T = pV/nR = (1160 \text{ K}/\text{L} \cdot \text{atm})(2.02 \text{ L})(0.139 \text{ atm}) = 326 \text{ K}$ . We do likewise for the other pairs of points. The results are:

For 1.48 L, 0.202 atm:  $T = 347 \text{ K}$ .

For 1.01 L, 0.361 atm:  $T = 423 \text{ K}$ .

For 0.50 L, 0.952 atm:  $T = 553 \text{ K}$ .

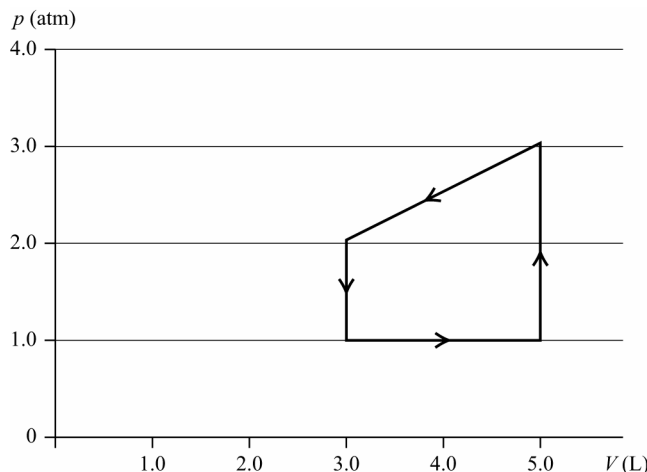
As the volume decreases, the temperature is increasing, so the temperature is increasing during compression.

**EVALUATE:** This result confirms that during an adiabatic compression, the gas temperature increases because work is being done on the gas.

- 19.61. IDENTIFY and SET UP:** We have information on the pressure and volume of the gas during the process, but we know almost nothing else about the gas. We do know that the first law of thermodynamics must

apply to the gas during this process, so  $Q = \Delta U + W$ , and the work done by the gas is  $W = \int_{V_1}^{V_2} p dV$ . If  $W$  is positive, the gas does work, but if  $W$  is negative, work is done on the gas.

**EXECUTE:** (a) Figure 19.61 shows the  $pV$ -diagram for this process. On the  $pV$ -diagram, we see that the graph is a closed figure; the gas begins and ends in the same state.



**Figure 19.61**

(b) Applying  $Q = \Delta U + W$ , we see that  $\Delta U = 0$  because the gas ends up at the same state from which it

began. Therefore  $Q = W$ .  $W = \int_{V_1}^{V_2} p dV$ , so the work is the area under the curve on a  $pV$ -diagram. For a closed cycle such as this one, the work is the area enclosed within the diagram. We calculate this work geometrically:  $|W| = \text{area (rectangle)} + \text{area (triangle)} = (2.0 \text{ L})(1.0 \text{ atm}) + \frac{1}{2} (2.0 \text{ L})(1.0 \text{ atm}) = 3.0 \text{ L} \cdot \text{atm} = 300 \text{ J}$ . But the net work is negative, so  $Q = -3.0 \text{ L} \cdot \text{atm} = -300 \text{ J}$ . Since  $Q$  is negative, heat flows out of the gas.

**EVALUATE:** We know that the work is negative because in the upper part of the diagram, the volume is decreasing, which means that the gas is being compressed.

- 19.62. IDENTIFY:**  $m = \rho V$ . The density of air is given by  $\rho = \frac{pM}{RT}$ . For an adiabatic process,  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ .  $pV = nRT$ .

**SET UP:** Using  $V = \frac{nRT}{p}$  in  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$  gives  $T_1 p_1^{1-\gamma} = T_2 p_2^{1-\gamma}$ .

**EXECUTE:** (a) The  $pV$ -diagram is sketched in Figure 19.62.

(b) The final temperature is the same as the initial temperature, and the density is proportional to the absolute pressure. The mass needed to fill the cylinder is then

$$m = \rho_0 V \frac{p}{p_{\text{air}}} = (1.23 \text{ kg/m}^3)(575 \times 10^{-6} \text{ m}^3) \frac{1.45 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} = 1.02 \times 10^{-3} \text{ kg}.$$

Without the turbocharger or intercooler the mass of air at  $T = 15.0^\circ\text{C}$  and  $p = 1.01 \times 10^5 \text{ Pa}$  in a cylinder is  $m = \rho_0 V = 7.07 \times 10^{-4} \text{ kg}$ . The increase in power is proportional to the increase in mass of air in the

cylinder; the percentage increase is  $\frac{1.02 \times 10^{-3} \text{ kg}}{7.07 \times 10^{-4} \text{ kg}} - 1 = 0.44 = 44\%$ .



(c) The temperature after the adiabatic process is  $T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma}$ . The density becomes

$\rho = \rho_0 \left( \frac{T_1}{T_2} \right) \left( \frac{p_2}{p_1} \right) = \rho_0 \left( \frac{p_2}{p_1} \right)^{(1-\gamma)/\gamma} \left( \frac{p_2}{p_1} \right) = \rho_0 \left( \frac{p_2}{p_1} \right)^{1/\gamma}$ . The mass of air in the cylinder is

$$m = (1.23 \text{ kg/m}^3)(575 \times 10^{-6} \text{ m}^3) \left( \frac{1.45 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right)^{1/1.40} = 9.16 \times 10^{-4} \text{ kg},$$

The percentage increase in power is  $\frac{9.16 \times 10^{-4} \text{ kg}}{7.07 \times 10^{-4} \text{ kg}} - 1 = 0.30 = 30\%$ .

**EVALUATE:** The turbocharger and intercooler each have an appreciable effect on the engine power.

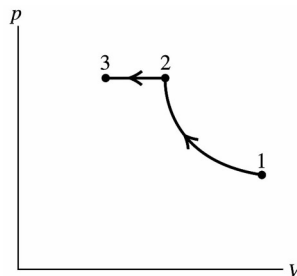


Figure 19.62

**19.63. IDENTIFY and SET UP:** The gas is cooled at constant volume. The ideal gas law applies, so  $pV = nRT$ . At

constant volume, this becomes  $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ .

**EXECUTE:** Solving  $\frac{p_1}{T_1} = \frac{p_2}{T_2}$  for  $p_2$  gives  $p_2 = p_1 \frac{T_2}{T_1} = (2000 \text{ psi}) \frac{268 \text{ K}}{293 \text{ K}} = 1830 \text{ psi}$ , which is choice (c).

**EVALUATE:** As the temperature decreases, the pressure decreases, as expected.

**19.64. IDENTIFY and SET UP:** The rapid expansion of the gas is an adiabatic process.

**EXECUTE:** The work the gas does comes from its internal energy, so its temperature decreases, causing some of it to condense. Therefore choice (d) is correct.

**EVALUATE:** This is the same principle used by snow-making machines.

**19.65. IDENTIFY and SET UP:** The gas is initially a gauge pressure of 2000 psi (absolute pressure of 2014.7 psi). It will continue to flow out until it is at the same absolute pressure as the outside air, which is 1.0 atm, or 14.7 psi. So we need to find the volume the gas would occupy at 1.0 atm of absolute pressure. The ideal gas law,  $pV = nRT$ , applies to the gas, and the temperature is constant during this process.

**EXECUTE:** For an isothermal process,  $T$  is constant, so  $pV = nRT$  can be put into the form

$$V_2 = V_1 \frac{p_1}{p_2} = (500 \text{ L}) \frac{2014.7 \text{ psi}}{14.7 \text{ psi}} = 6.85 \times 10^4 \text{ L}. \text{ The volume of gas lost is therefore}$$

$$6.85 \times 10^4 \text{ L} - 500 \text{ L} = 6.80 \times 10^4 \text{ L}. \text{ The gas flows at a constant rate of } 8.2 \text{ L/min, so}$$

$$(8.2 \text{ L/min})t = 6.80 \times 10^4 \text{ L}, \text{ which gives } t = 830 \text{ s} = 140 \text{ h, which is choice (d).}$$

**EVALUATE:** The rate of flow might not be uniform as the gas approaches 1.0 atm, but for most of the time under high pressure, it should be reasonable to assume that the flow rate can be held constant.

**19.66. IDENTIFY and SET UP:** The oxygen and the  $\text{N}_2\text{O}$  are at the same temperature in the same container.

Therefore to have a 50%/50% mixture by volume, they should have equal numbers of moles.  $pV = nRT$  applies.

**EXECUTE:** The molecular mass of  $\text{N}_2\text{O}$  is  $(28 + 16) \text{ g/mol} = 44 \text{ g/mol}$ . The amount present is  $1.7 \text{ kg} = 1700 \text{ g}$ , which is  $1700/44 = 38.64 \text{ mol}$ . Therefore the  $\text{O}_2$  must also contain 38.64 mol. The temperature is

$20^{\circ}\text{C} = 293\text{ K}$ . The pressure is  $50\text{ psi} + 14.7\text{ psi} = 64.7\text{ psi}$ . Since  $1.0\text{ atm} = 14.7\text{ psi} = 1.01 \times 10^5\text{ Pa}$ , converting gives  $64.7\text{ psi} = 4.445 \times 10^5\text{ Pa}$ . Using  $pV = nRT$ , we have

$V = nRT/p = (38.64\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})/(4.445 \times 10^5\text{ Pa}) = 0.21\text{ m}^3$ , which is choice (a).

**EVALUATE:** The mixture is 50%/50% by volume, but not by weight, since  $\text{N}_2\text{O}$  is heavier than  $\text{O}_2$ .