

Faculty of Engineering, Mathematics and Science School of Mathematics

JF Maths/TP/TSM

Trinity Term 2018

MA1111 — Linear Algebra I

Wednesday, May 16

Exam Hall

09:30 - 11:30

Paschalis Karageorgis

Instructions to Candidates:

Attempt all questions. All questions are weighted equally. Non-programmable calculators are permitted for this examination. Formulae and tables are available from the invigilators, if needed.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Find a quadratic polynomial, say $f(x) = ax^2 + bx + c$, such that

$$f(1) = 2,$$
 $f(2) = 6,$ $f(3) = 8.$

2. Show that the matrix A is invertible and determine its inverse.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{bmatrix}.$$

- 3. Let A_n denote the $n \times n$ matrix whose diagonal entries are equal to 2 and all other entries are equal to 5. Compute the determinant of A_n .
- 4. Find a linear transformation $T \colon \mathbb{R}^3 \to \mathbb{R}^2$ such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\2\end{bmatrix}, \qquad T\left(\begin{bmatrix}1\\2\\3\end{bmatrix}\right) = \begin{bmatrix}7\\5\end{bmatrix}, \qquad T\left(\begin{bmatrix}1\\2\\2\end{bmatrix}\right) = \begin{bmatrix}1\\3\end{bmatrix}.$$

5. Suppose that the columns of an $m \times n$ matrix A form a complete set for \mathbb{R}^m . Show that there exists some $n \times m$ matrix B such that AB is the identity matrix.