JF PY1T10 Special Relativity

Lecture 5

Length Contraction

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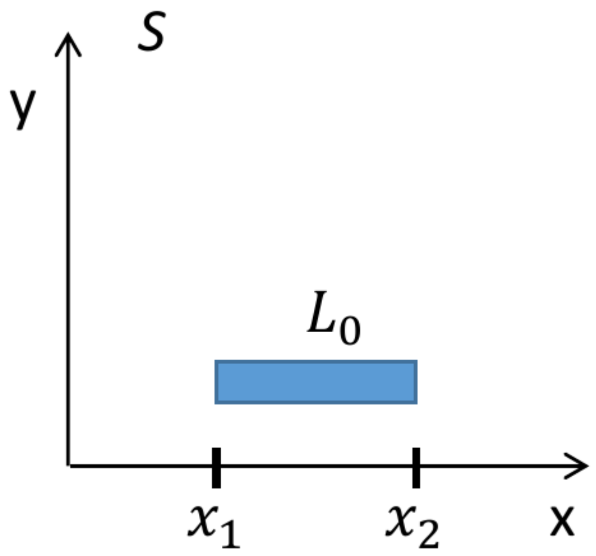
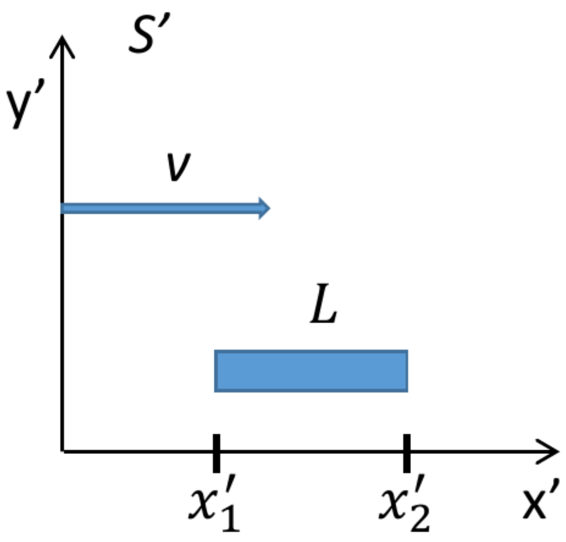




Rod parallel to *v* and *x*-axis is at rest in *S*.

Ends at x_1, x_2 independent of *t*.

Rest length = $L_0 = x_2 - x_1$.

This is the length of the rod as measured in an inertial frame where it is not moving (i.e. in *S*).

Viewed from *S′* the rod moves with speed -*v* parallel to *x′*-axis.

Define the length in this frame, L, to be distance between x_1' and x_2', which are measured *simultaneously* in *S′*, i.e. they have the same value of *t′*.

Length Contraction

In S', length is:

$$L = x'_{2,t'} - x'_{1,t'}$$

Note: t' is the same for both.

Use LT:

$$x_2 = \gamma(x'_2 + vt')$$
$$x_1 = \gamma(x'_1 + vt')$$

Then:

$$L_0 = x_2 - x_1 = \gamma(x'_2 - x'_1) = \gamma L$$

Or:

$$\frac{L_0}{L} = \gamma; \quad L < L_0 \; if \; v > 0$$

This is a **Lorentz-Fitzgerald Contraction**

Exercise: Show that if the rod is at rest in S', the length in S is less than the length measured in S'.

Length Contraction

Note that the events $(x_1', t'), (x_2', t')$ are not simultaneous as measured in S:

LT: $t_2 - t_1 = \gamma\left(t' + \frac{vx_2'}{c^2}\right) - \gamma\left(t' + \frac{vx_1'}{c^2}\right) = \frac{\gamma v(x_2' - x_1')}{c^2} \neq 0 \; if \; v > 0$

$\therefore\ t_2 \neq t_1$ when $v > 0$

If the rod perpendicular lies perpendicular to *v*, there is no change of length since $y = y', z = z'$.

"Has the rod really contracted?"

What is real is what is measured.

What if someone looks at the rod?

At a given instant he sees a rod but photons from the far end will have left earlier than photons from the near end. But this is not the conditions of the measurement.

Difference between Observing & Seeing?

Observation: The simultaneous measurement of the positions of a number of points on an object. If using light, the photons should leave the object at a certain time in the observer's frame. However, they may arrive at the observer at a different time, due to the finite speed of light. Data must be corrected to calculate true measurements of the object.

Seeing (or photographing): Photons arrive simultaneously at the observer (or camera), having left from different positions on the object at different times. In general, the object appears distorted. Visual appearance may be different to observed and measured shape.

Observing: photons *leave* an object simultaneously.

Seeing: photons *arrive* at the observer simultaneously.

Difference between Observing & Seeing?

In **seeing** an object, photons from the far end of it left earlier than photons from the near end. They all arrive at the viewer at the same time, making the object appear as it is at rest *but rotated*.

Note: when an object appears shorter due to Lorentz-Fitzgerald contraction, this is not a distortion caused by photon time delay. This is an actual measurement from **observing** the object, and its length, while shortened, is still a correct physical characteristic caused by relativistic speeds.

Distortions due to finite travel times of photons are generally rotations of the object.

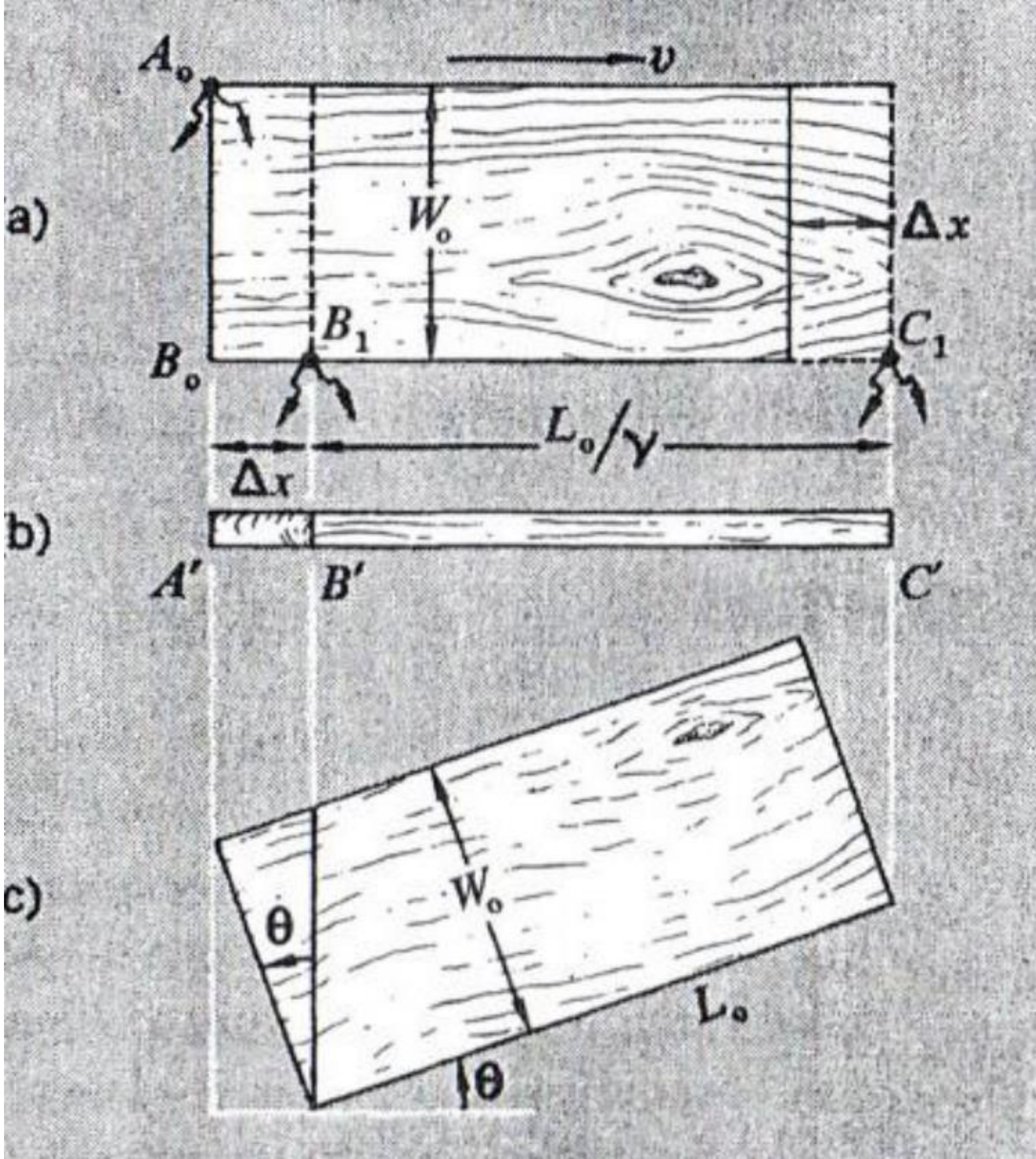
Example:

Wooden board moving with speed v parallel to its length L_0 :

View in its own plane in direction perpendicular to v.

a) Rectangular object moving with speed *v* parallel to *x*.
b) The apparent positions of the corners *A*, *B*, *C* as recorded at a given instant by a distant observer looking in the *y* direction in the plane of the object.
c) It is inferred that the object is rotated in its own plane, but not Lorentz-contracted.

-- “Special Relativity” by A. P. French.



Apparent Object Rotation

Light from rear corner A_0 has to travel distance W_0 more than light from B_0 .

To reach eye at the same time, it must start earlier by time $\Delta t = \frac{W_0}{c}$

But in Δt board has moved distance $\Delta x = v\Delta t = \frac{vW_0}{c}$

Near edge has contracted length $\frac{L_0}{\gamma}$. (B_1 and C_1 are equidistant from observer).

Thus viewer sees foreshortened view of edge AB and contracted view of BC.

$$A'B' = \frac{vW_0}{c}$$

$$B'C' = L_0\left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

Suppose the board is at rest but turned about axis perpendicular to plane by angle θ. Then projected lengths of edges are $W_0\sin\theta$ and $L_0\cos\theta$.

These correspond to the values above if $\sin\theta = v/c$.

We see the board rotated.

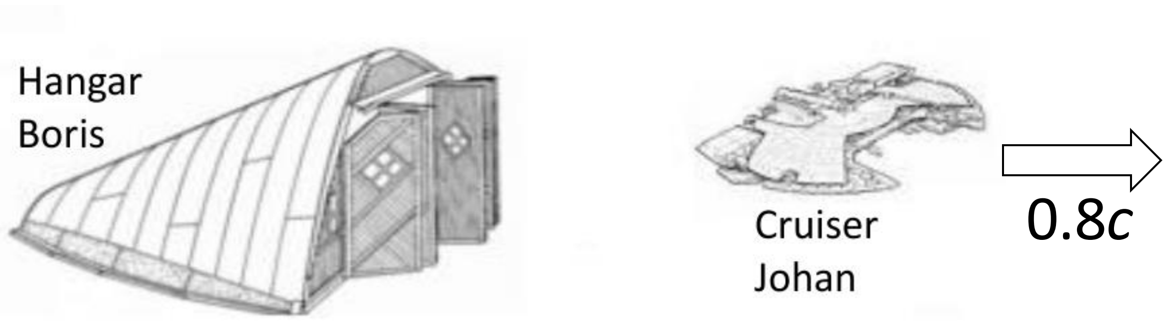
In general, we cannot see the Lorentz contraction.

Concept Question 1

Johan is in a **600m** long space cruiser.

Boris is standing in the **2km** long space hangar.

Johan flies at **0.8c** away from Boris, as shown in the diagram.



Which statement is correct about Boris and Johan's observations of each other?

A. Johan's cruiser is only 360m long in Boris's reference frame, while the hangar is 2km long in Johan's reference frame.

B. Johan's cruiser is only 360m long in Boris's reference frame, while the hangar is 1.2km long in Johan's reference frame.

C. Johan's cruiser is 600m long in Boris's reference frame, while the hangar is 2km long in Johan's reference frame.

D. Johan's cruiser is 600m long in Boris's reference frame, while the hangar is 1.2km long in Johan's reference frame.

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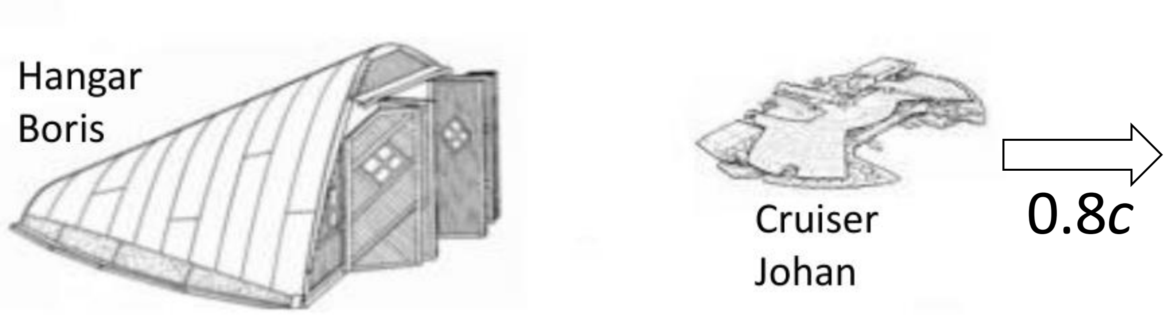

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Concept Question 2

Terri is flying at a speed of $0.6c$ ($\gamma = 1.25$) towards a chasm.

Earlier, when at rest w.r.t. the chasm, she measured it to be 4.5km long.

Terri’s craft is 4km long.

Which of the following is true from Terri’s frame of reference?

A. Terry’s craft is 5km long and the front of Terri’s craft leaves the chasm before the back enters.

B. The chasm is now 3.6km long and the front of Terri’s craft leaves the chasm before the back enters.

C. Terry’s craft is now 3.2km long and Terri’s craft fits entirely in the chasm

D. The chasm is now 5.625km and Terri’s craft fits entirely in the chasm

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Which of the following is true from Terri's frame of reference?

A. Terry's craft is 5km long and the front of Terri's craft leaves the chasm before the back enters.

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C. Terry's craft is now 3.2km long and Terri's craft fits entirely in the chasm

D. The chasm is now 5.625km and Terri's craft fits entirely in the chasm