MA1125 – Calculus
Tutorial problems #6

1. Let $a_1, a_2, \ldots, a_n$ be some given constants and let f be the function defined by

$$f(x) = (x - a_1)^2 + (x - a_2)^2 + \ldots + (x - a_n)^2.$$

Show that $f(x)$ becomes minimum when x is equal to $\overline{x} = (a_1 + a_2 + \ldots + a_n)/n$.

2. Find the global minimum and the global maximum values that are attained by

$$f(x) = 3x^4 - 16x^3 + 18x^2 - 1, \qquad 0 \leq x \leq 2.$$

3. Find the linear approximation to the function f at the point x_0 in the case that

$$f(x) = \frac{(x^2+1)^4 \cdot e^{x^2-1}}{\sqrt{3x+1}}, \qquad x_0 = 1.$$

4. The top of a 5m ladder is sliding down a wall at the rate of 0.25 m/sec. How fast is the base sliding away from the wall when the top lies 3 metres above the ground?

5. Let $n > 0$ be a given constant. Show that $x^n \ln x \geq -\frac{1}{ne}$ for all $x > 0$.

6. Find the global minimum and the global maximum values that are attained by

$$f(x) = x^2 \cdot e^{4-2x}, \qquad -1 \leq x \leq 2.$$

7. Find the point on the graph of $y = 2\sqrt{x}$ which lies closest to the point $(2, 1)$.

8. Find the largest possible area for a rectangle that is inscribed inside a semicircle of radius $r > 0$, if one side of the rectangle lies along the diameter of the semicircle.

9. Two cars are driving in opposite directions along two parallel roads which are 300m apart. If one is driving at 50 m/sec and the other is driving at 30 m/sec, how fast is the distance between them changing 5 seconds after they pass one another?

10. Show that $f(x) = x^4 + 5x - 1$ has a unique root in $(0, 1)$ and use Newton's method with initial guess $x_1 = 0$ to approximate this root within two decimal places.