UNIVERSITY OF DUBLIN

X-MA1241-1

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Theoretical Physics **Trinity Term 2013**
JF Mathematics
SF Two Subject Moderatorship

MA1241/MA1242 — CLASSICAL MECHANICS I/II

Thursday, May 2 Drawing Room 09.30 — 12.30

Dr. S. Kovacs

Credit will be given for the best 2 answers from each section.

All questions have equal weight.

'Formulae & tables' are available from the invigilators, if required.

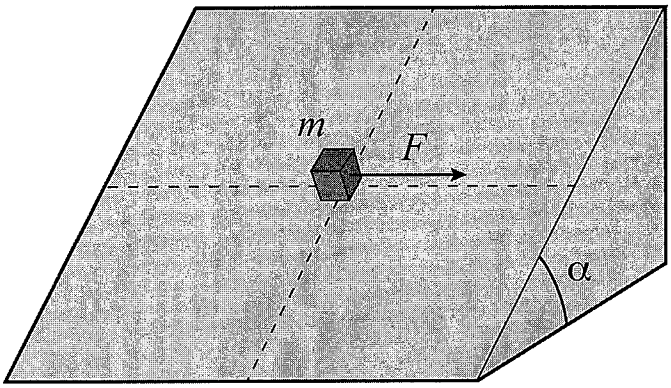
Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

The time allowed for this paper is 3 hours for students registered for both MA1241 and MA1242. For students registered for just one of the modules the time allowed is 2 hours.

Section MA1241 — Classical Mechanics I

Credit will be given for the best 2 questions answered in this section.

1. A small cube of mass m (to be treated as a point particle) is placed on a rough plane inclined at an angle α. Let μ be the friction coefficient between the cube and the plane.

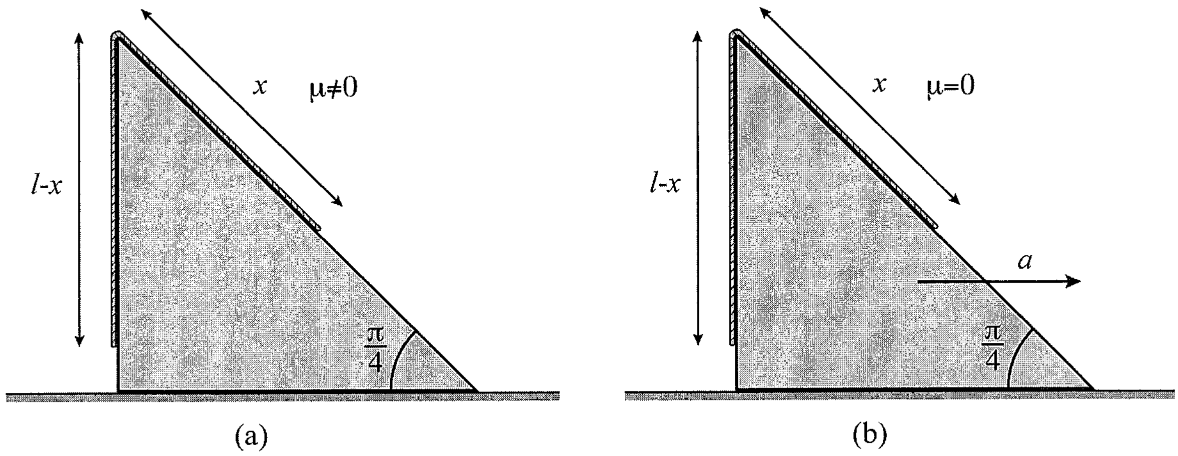


(a) Find the minimum value of the friction coefficient, $\mu_{\min}$, for which the cube remains in equilibrium without sliding under the effect of gravity.

(b) The cube is replaced by another cube of the same mass, but with friction coefficient with the plane equal to $\mu = 2\mu_{\min}$ (where $\mu_{\min}$ is the value computed in (a)). A constant horizontal force is applied to the cube as shown in the figure above.

Find the minimum value, $F_{\min}$, of the force that causes the cube to start sliding and determine in which direction the cube moves under such a force.

2. A rope of mass m and length l is placed on a wedge inclined at an angle $\theta = \frac{\pi}{4}$ as shown in the figure below. Let x be the length of the portion of rope lying on the inclined plane.

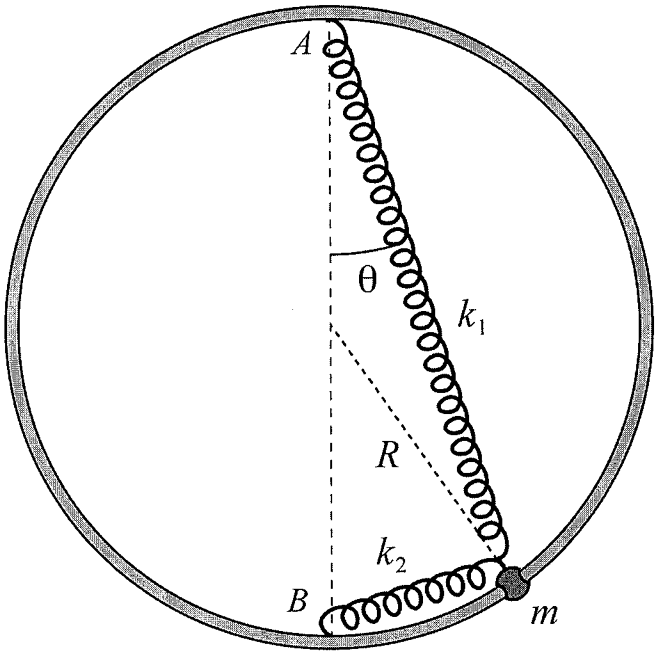


(a) Assuming the friction coefficient between the rope and the wedge to be μ, determine the minimum value of x for which the rope can remain in equilibrium on the wedge.

(b) Assume now the friction coefficient between rope and plane to be negligible. The wedge is given a constant horizontal acceleration, a. The hanging portion of rope, of length $l - x$, is constrained to remain in a vertical position along the wedge.

Determine the value of x for which the rope remains in equilibrium without sliding up or down as the wedge accelerates.

3. A bead of mass m is constrained to move (without friction) along a ring of radius R which is held fixed in a vertical plane. The bead is subject to gravity and it is connected to two massless springs whose other end points are attached to the ring at the antipodal points, A and B, along a vertical diameter. The springs have zero rest length and spring constants $k_1 = k$ and $k_2 = \alpha k$ respectively, with α a positive number.



(a) Find the values of the angle θ in the figure corresponding to stable and unstable equilibrium points for the bead for all possible values of the parameter α.

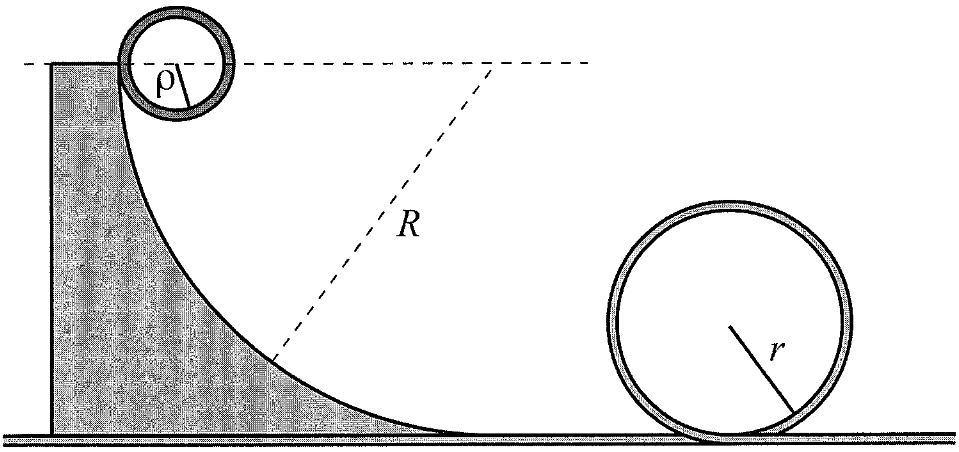
(b) The bead is placed in a stable equilibrium position and given a velocity v_0 along the ring.

Find the minimum value of v_0 that allows the bead to travel all the way to an unstable equilibrium point and compute the velocity of the bead as a function of the angle θ as it travels between the stable and unstable equilibrium points.

Section MA1242 — Classical Mechanics II

Credit will be given for the best 3 questions answered in this section.

4. A small hoop of mass m and radius ρ rolls down a block and onto a slide which forms a vertical circular loop of radius r as shown in the figure. The profile of the block is a quarter of a circle of radius R and the hoop rolls without slipping at all times.



Determine the minimum value of R for which the hoop can "loop the loop", i.e. travel along the circular path of radius r without ever losing contact, in the following two cases:

(a) The block is fixed to the plane.

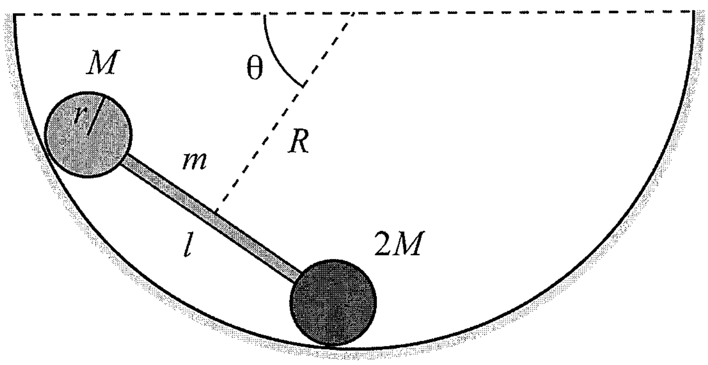
(b) The block has mass M and it can slide without friction on the horizontal plane, so that it is pushed to the left as the hoop rolls down.

Useful formulae:

The moment of inertia of a hoop of mass m and radius ρ with respect to an axis through the centre and perpendicular to the plane of the hoop is

$$I = m\rho^2$$

5. A dumbbell consists of two spheres of radius r and mass M and $2M$ respectively connected by a rigid rod of mass m and length l. The dumbbell can move on the internal surface of a fixed cylinder of radius R, which lies with its axis in a horizontal position. Let the length of the rod be $l = R - 3r$ and assume that the dumbbell is constrained to move in a vertical plane perpendicular to the axis of the cylinder.



(a) Determine the minimum value of the friction coefficient between the sphere of mass $2M$ and the surface of the cylinder which allows the dumbbell to remain in equilibrium when placed at an angle $\theta = \frac{\pi}{6}$

(b) Find the equilibrium position for the system if the friction coefficient between the spheres and the cylinder is negligible and compute the period of the small oscillations of the system about its stable equilibrium point.

Useful formulae:

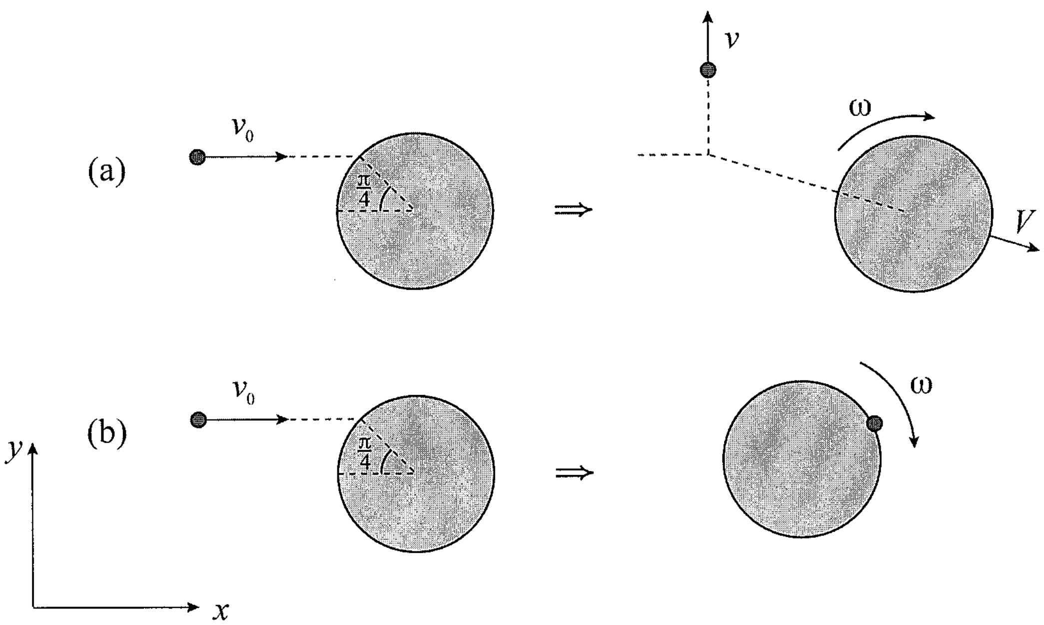
The moment of inertia of a sphere of mass M and radius r with respect to an axis through its centre is

$$I_s = \frac{2}{5}Mr^2$$

The moment of inertia of a rod of mass m and length l with respect to a perpendicular axis through its mid point is

$$I_r = \frac{1}{12}ml^2$$

6. A point particle of mass m and a disk of radius R and mass $2m$ move on a frictionless horizontal plane. The particle is given an initial velocity v_0 in the positive x direction and collides with the disk, which is initially at rest, as shown in the figure below.



(a) In an elastic collision the particle is observed to be deflected so that its final non-zero velocity, v, is in the positive y direction.

Compute, in this case, v, the velocity of the centre of mass of the disk, $\vec{V}$, and its angular velocity about the centre of mass after the collision.

(b) If the collision is completely inelastic, i.e. the particle sticks to the edge of the disk, compute the velocity of the centre of mass of the system of disk and particle and its angular velocity about the centre of mass after the collision.

Useful formulae:

The moment of inertia of a disk of mass M and radius R about an axis through the centre and perpendicular to the plane of the disk is

$$I = \frac{1}{2}MR^2$$