Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Maths/TP/TSM **Trinity Term 2017**

MA1212 — Linear Algebra II

Saturday, May 6 RDS 14:00 – 16:00

Prof. Karageorgis

Instructions to Candidates:

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Find a matrix A that has v_1 as an eigenvector with eigenvalue $\lambda_1 = 1$ and v_2 as an eigenvector with eigenvalue $\lambda_2 = 3$ when

$$v_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \qquad v_2 = \begin{bmatrix} -4 \\ 5 \end{bmatrix}.$$

2. Find the Jordan form and a Jordan basis for the matrix

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 4 & 2 \\ 0 & -1 & 1 \end{bmatrix}.$$

3. The following matrix has eigenvalues $\lambda = 0, 1, 1$. Use this fact to find its Jordan form, its minimal polynomial and also its power A^{2017}.

$$A = \begin{bmatrix} -2 & 1 & 1 \\ -3 & 2 & 1 \\ -4 & 2 & 2 \end{bmatrix}.$$

4. Let P_1 be the space of all real polynomials of degree at most 1 and let

$$\langle f, g \rangle = \int_{-1}^{1} 3x \cdot f(x) g(x) \, dx \quad \text{for all } f, g \in P_1.$$

Find the matrix A of this bilinear form with respect to the standard basis and then find an orthogonal matrix B such that $B^t A B$ is diagonal.

5. Let A be a real symmetric matrix. Show that A is positive definite if and only if $A = P^t P$ for some invertible matrix P.