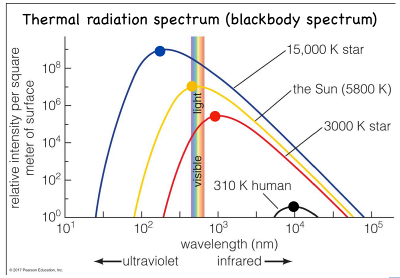
Lecture 7: Stellar masses and binary systems

Ch. 5 of “Astronomy: a Physical Perspective” (M. Kutner)

Prof Aline Vidotto

Quick recap of last lecture



Wien’s law:

$$\lambda_{\text{peak}} = \frac{2.9 \times 10^6}{T} \text{ nm}$$

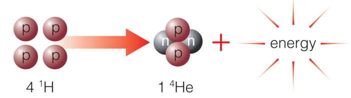
Stefan-Boltzmann law

$$F_\star = \sigma T^4$$

$$\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$$

Why is the sun (& stars) shining?

Nuclear reactions in the core release energy



- The power being generated is

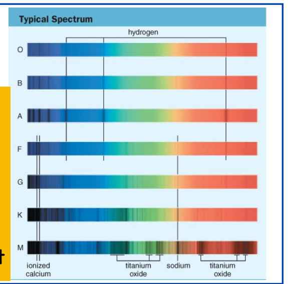
$$L_\star = \sigma T^4 \cdot (4\pi R_\star^2)$$

- The flux we detect in our telescopes is attenuated by distance:

$$F_{\text{detected}} = L_\star/(4\pi d^2)$$

Spectral classification of stars

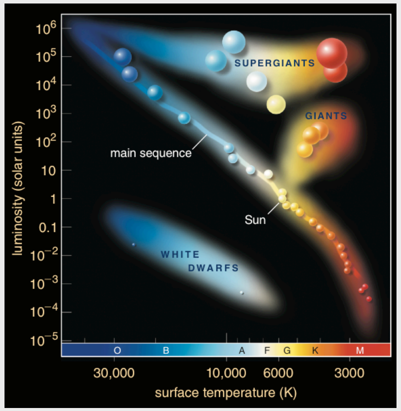
O B A F G K M — Hottest → Coolest



Hertzprung-Russell Diagram: “HR Diagram”

What is it?

- plot of luminosity versus temperature
- super important: temperature runs backwards
- super super important: this is a log-log plot



- Main characteristics:
  - Majority of stars are on the “main sequence”

$$L_\star = 4\pi R_\star^2 \sigma T^4$$

  - Stars with lower T and higher L than main-sequence stars must have larger radii. These stars are called **giants and supergiants.**
  - Stars with higher T and lower L than main-sequence stars must have smaller radii. These stars are called **white dwarfs**.

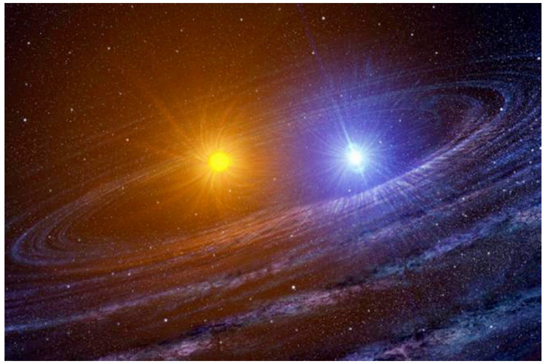
What we will cover today...

Goal: understand how the gravitational interactions in binary systems can be used to derive the masses of stars

Outline:

1. Binary star systems
2. Doppler effect in Astronomy
3. Deriving the stellar mass in binary star systems
4. Mass-luminosity-radius relationship in the main sequence phase

1. Binary star systems



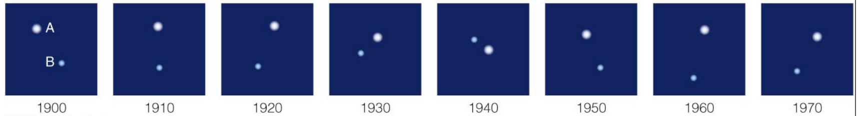
Binary stars are stars that are gravitationally bound to each other

50% or more of stellar systems are binary systems

There are different types of binary stars:

i. visual binaries
ii. astrometric binaries
iii. eclipsing binaries
iv. spectroscopic binaries

(i) Visual binary stars



- We see two stars orbiting around the common centre of mass
- The forces between the two stars do not affect the motion of the centre of mass. In the movies below, the centre of mass is at rest.

circular orbit

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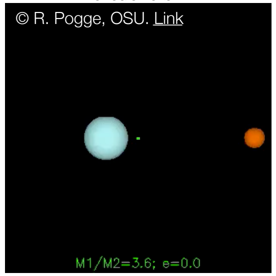

In both movies the primary star is of spectral type F0V and the secondary star of M0V.

mass ratio=3.6

eccentric orbit

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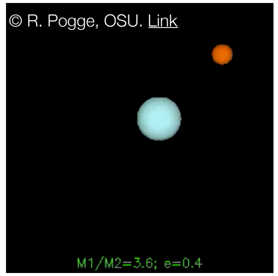
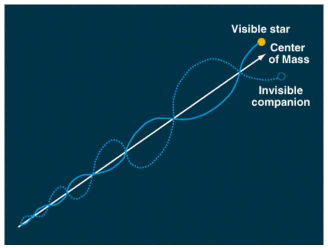
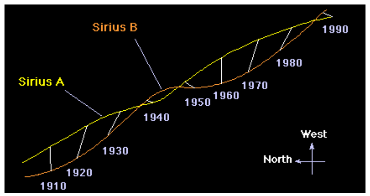


(ii) Astrometric binaries

- In astrometric binaries, we can only see the brighter star
- The stellar wobble can also be visible as changes in the star's apparent position in the sky.



When we follow the brighter star's path on the sky, we see that it wobbles back and forth across a straight path. This means that the star is moving in an orbit, so we can infer the presence of a companion



(iii) Eclipsing binary

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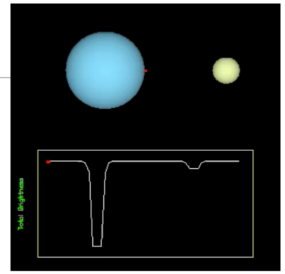
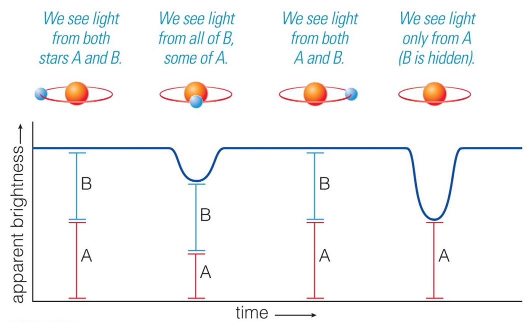
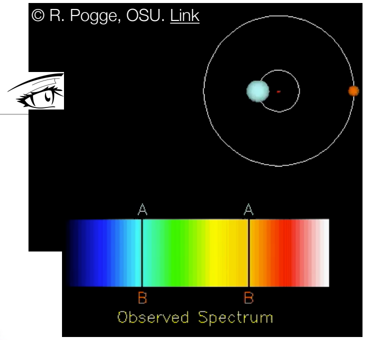


- Light from such a system periodically becomes brighter and then fainter.
- This occurs when the companion passes behind or in front of the main star → eclipses

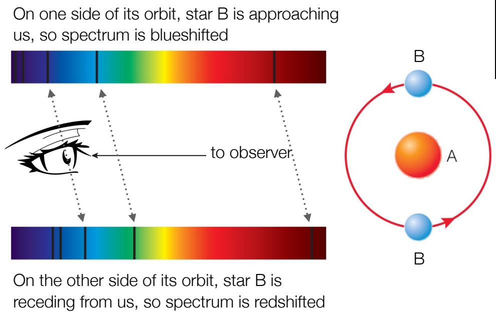


For eclipses to occur, orbit must be seen almost edge-on

(iv) Spectroscopic binary



- Two stars with Doppler shifts in opposite directions.
- Doppler shifts vary as stars orbit their common centre of mass



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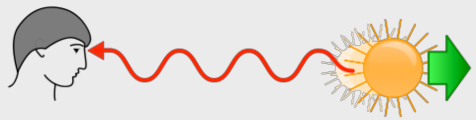
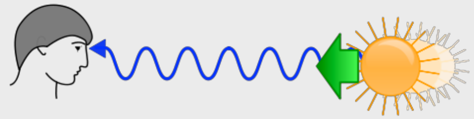
Note: Often only one Doppler shift (of one component) can be observed, because the other component is too faint.

2. Doppler effect in Astronomy

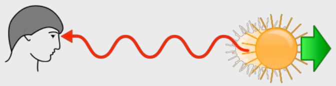
Receding source causes wavelength to increase (waves are '*stretched*'): redshift



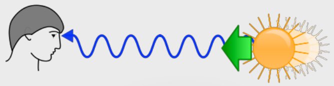


Approaching source causes wavelength to decrease (waves are '*squashed*'): blueshift

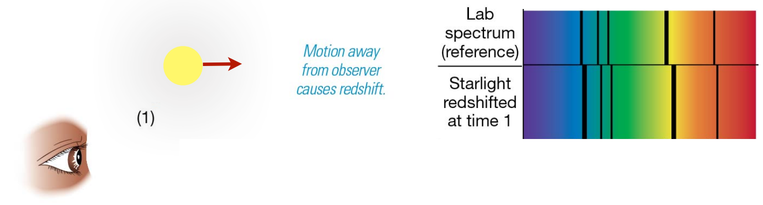
Doppler effect in Astronomy



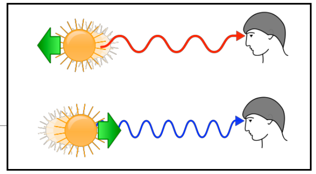
Receding star causes wavelength to increase (waves are '*stretched*'): redshift



Approaching star causes wavelength to decrease (waves are '*squashed*'): blueshift



Doppler effect in Astronomy



- λ: wavelength at which a signal is received
- λ_0: wavelength at which a signal is emitted (*rest* wavelength)
- $\Delta\lambda = \lambda - \lambda_0$: wavelength shift

$$\boxed{\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c}}$$

derive this equation for them??

$v_r > 0$ implies that $\Delta\lambda > 0$: wavelength is shifted to the red (redshift)

$v_r < 0$ implies that $\Delta\lambda < 0$: wavelength is shifted to the blue (blueshift)

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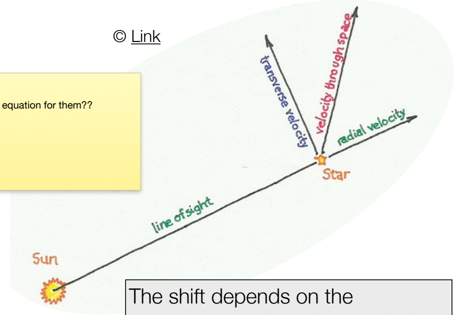
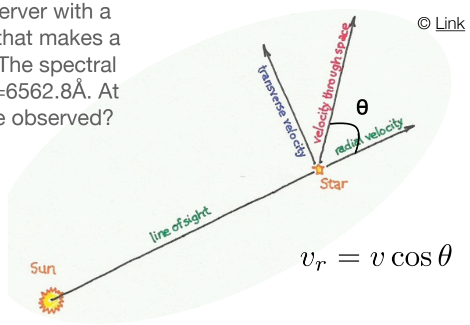


The shift depends on the component of the relative velocity along the line joining the source and the observer: we call this component the **radial velocity** v_r.

Example: moving source

Å: Angstrom
1Å = 0.1 nm
1Å = 10^{-10}m

A star is moving away from an observer with a speed of v=10 km/s, along a path that makes a θ=30º angle with the line-of-sight. The spectral line "Hα" has a rest wavelength λ_0=6562.8Å. At what wavelength will the Hα line be observed?



$$v_r = v\cos\theta$$

Solution: The radial velocity is

$$v_r = +10\cos(30^o) = 8.66 \text{ km/s}$$

Given that c = 3x10^5 km/s, thus

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c} = \frac{8.66}{3\times10^5} = 2.9\times10^{-5}$$

$$\Delta\lambda = 2.9\times10^{-5}\lambda_0 = 0.19\text{Å}$$

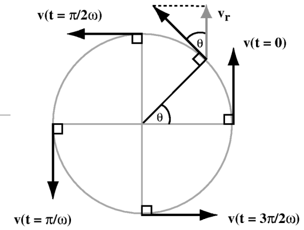
$$\boxed{\lambda = \lambda_0 + \Delta\lambda = 6563.0\text{Å}}$$

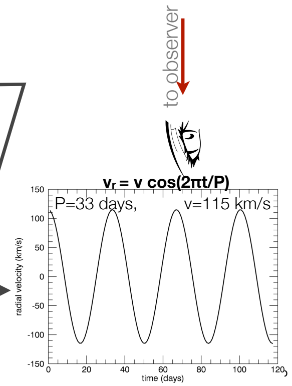
Homework: If the observer is also moving directly away from the star at a speed of 20 km/s, at what wavelength will the Hα line be observed?

Doppler shift caused by a star in circular orbit

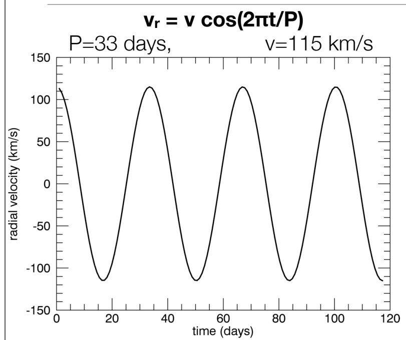
- At t=0, star is moving directly away the observer with a velocity v. Hence: v_r=v
- The speed *v* of the star remains constant, but the direction changes, so the radial velocity v_r changes.
  - As the star moves the component of the velocity along the line of sight is v_r=vcosθ
- The angle θ keeps track of how far around the circle the star has gone: **θ=ωt**. ω is the angular velocity of the star, so its period is P=2π/ω. Thus: v_r=v cos(ωt) = v cos(2πt/P)
- The spectral lines shift back and forth with a shift given by

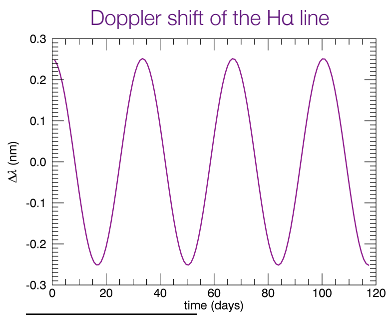
$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c} = \frac{v\cos(2\pi t/P)}{c}$$



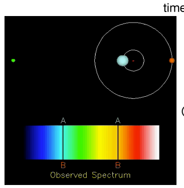


Doppler shift caused by a star in circular orbit (cont.)





$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c} = \frac{v\cos(2\pi t/P)}{c}$$



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Doppler shift: effect of the inclination "i" of the orbital plane

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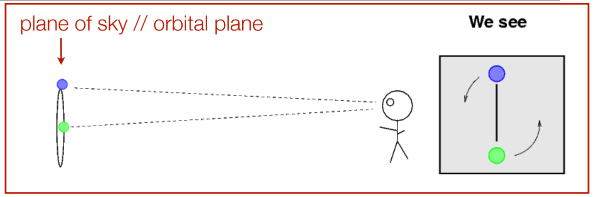
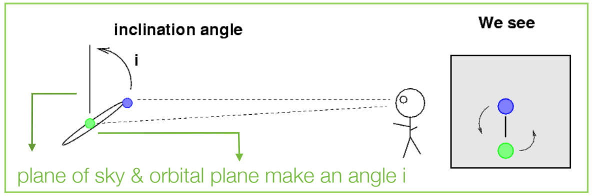
If the orbital plane is contained in the plane of sky, the velocity of the star projected along the line-of-sight is zero. Hence v_r=0 → **no Doppler shift!**





If the orbital plane makes an angle i with the plane of sky, the velocity of the star projected along the line-of-sight is vsin(i). So:
v_r= v cos(2πt/P) sin(i)

In the limit where the plane of the orbit is perpendicular to the plane of the sky, i=90º and sin(i) =1. We recover the result from our previous example: v_r= v cos(2πt/P).

Conceptual question

If a distant galaxy has a substantial redshift (as viewed from our galaxy), then anyone living in that galaxy would see a substantial redshift in a spectrum of the Milky Way Galaxy.

(a) Yes, and the redshifts would be the same.

(b) Yes, but we would measure a higher redshift than they would.

(c) Yes, but we would measure a lower redshift than they would.

(d) No, they would not measure a redshift toward us.

(e) No, they would measure a blueshift.

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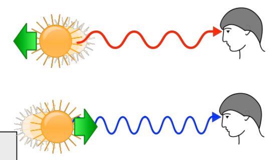
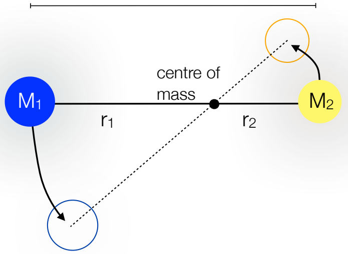


Explanation: The shift depends on the component of the relative velocity along the line joining the source and the observer. The relative velocity is the same for someone living in the Milk Way or for someone living in the distant galaxy.

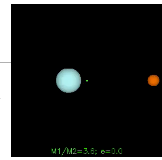
3. Deriving the stellar mass in binary star systems

$$R = r_1 + r_2$$

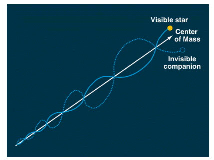


For any binary system, we determine the period of the orbit directly

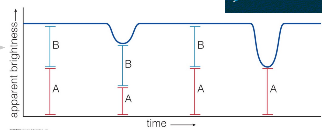
- visual binaries: we can measure the period (and also the angular distance between both stars)



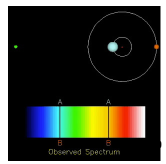
- astrometric binary: we can see how long it takes for the wobble to go through a full cycle



- eclipsing binary: we can see how long it takes for the lightcurve to go through a full cycle (as one star eclipses the other)



- spectroscopic binaries: we can see how long it takes for the Doppler shifts to go through a full cycle



If we know the orbital period P, then we can derive the stellar masses using Kepler's 3rd law

- Recall: In the case of planets orbiting around the star, Kepler's 3rd law is:

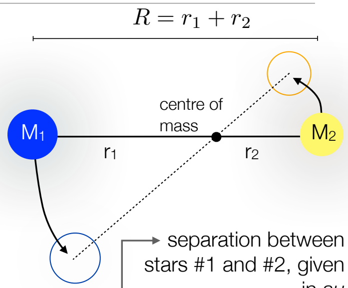
$$M_\star = \frac{4\pi^2}{G}\frac{R^3}{P^2}$$

- Or, rewriting equation in more useful units:

$$\frac{M_\star}{M_\odot} = \frac{\left(\frac{R}{1\ \mathrm{au}}\right)^3}{\left(\frac{P}{1\ \mathrm{yr}}\right)^2}$$

- For binary systems, the masses of each component of the system are comparable and Kepler's law becomes

$$\boxed{M_1 + M_2 = \frac{4\pi^2}{G}\frac{R^3}{P^2}}$$



separation between stars #1 and #2, given in *au*

$$\boxed{\frac{M_1 + M_2}{M_\odot} = \frac{\left(\frac{R}{1\ \mathrm{au}}\right)^3}{\left(\frac{P}{1\ \mathrm{yr}}\right)^2}}$$

period of the orbit given in *years*

The centre of mass

Note: The separation R is only directly derived for visual binaries (provided distance is known). When we measure Doppler shifts, we measure the velocities of the star. So it is useful to derive a relation between $\{r_1, r_2\}$ and $\{v_1, v_2\}$.

- From the definition of the centre-of-mass, we have:

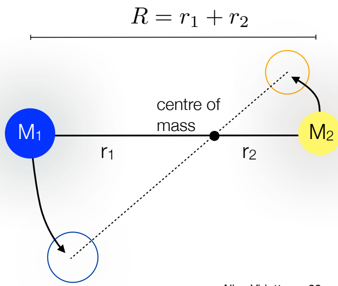
$$M_1 r_1 = M_2 r_2 \quad \rightarrow \quad \frac{M_1}{M_2} = \frac{r_2}{r_1}$$

- Differentiating with respect to time:

$$M_1 v_1 = M_2 v_2 \quad \rightarrow \quad \frac{M_1}{M_2} = \frac{v_2}{v_1}$$

- Therefore

$$\frac{v_2}{v_1} = \frac{r_2}{r_1}$$



An alternative form of Kepler's 3rd law using stellar velocities instead of distances

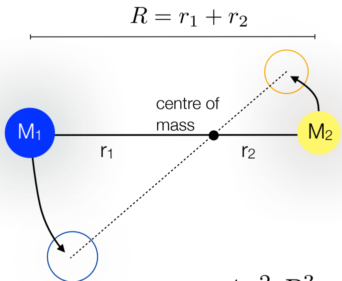
- The period of the orbit is related to the radius r and the speed v:

$$\mathrm{v} = \frac{\text{circumference}}{\text{period}} = \frac{2\pi \mathrm{r}}{\mathrm{P}}$$

- Since the periods of both stars must be the same

$$v_1 = \frac{2\pi r_1}{P} \qquad v_2 = \frac{2\pi r_2}{P}$$

$$\underbrace{r_1 + r_2}_{=R} = \frac{P}{2\pi}(v_1 + v_2)$$

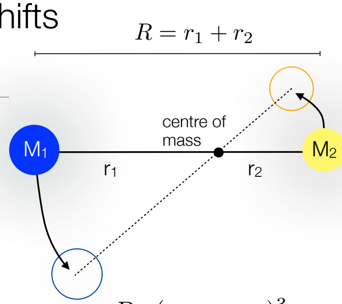


Kepler's law: $M_1 + M_2 = \frac{4\pi^2}{G}\frac{R^3}{P^2}$

$$\boxed{M_1 + M_2 = \frac{P}{2\pi G}(v_1 + v_2)^3}$$

Example: Binary star Doppler shifts

A spectroscopic binary system is observed to have a period of 10 yr. The radial velocity of these two stars are v_{1r}=10km/s and v_{2r}=20 km/s. Find the masses of these two stars if the inclination of the orbit is 90º.



Solution: As v_r=vsin(i), we have that the (projected) radial velocity of each star is

$$v_{r1} = v_1 \sin(i)$$
$$v_{r2} = v_2 \sin(i)$$

Substituting this into Kepler's law:

$$M_1 + M_2 = \frac{P}{2\pi G}(v_1 + v_2)^3$$

$$M_1 + M_2 = \frac{P}{2\pi G}\frac{(v_{1r} + v_{2r})^3}{\sin^3 i}$$

$$M_1 + M_2 = 10.2 M_\odot \quad \text{(i=90º)}$$

From the centre of mass, we have $\frac{M_1}{M_2} = \frac{v_2}{v_1} = 2$

We solve these two equations to find: M_1=6.8M_{sun} and M_2=3.4M_{sun}.

Homework: If the inclination of the orbit were i=45º, how would this change the mass ratio and the total mass?

Conceptual question

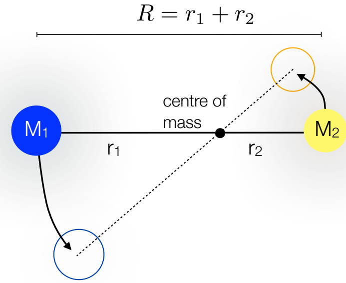
What properties of a binary star system are needed to determine the masses of the stars?

(a) stellar size and orbit size
(b) orbit size and spectral type
(c) stellar size and spectral type
(d) orbit size and orbit period
(e) orbit period and stellar size

Conceptual question

What properties of a binary star system are needed to determine the masses of the stars?

(a) stellar size and orbit size
(b) orbit size and spectral type
(c) stellar size and spectral type
(d) orbit size R and orbit period P
(e) orbit period and stellar size



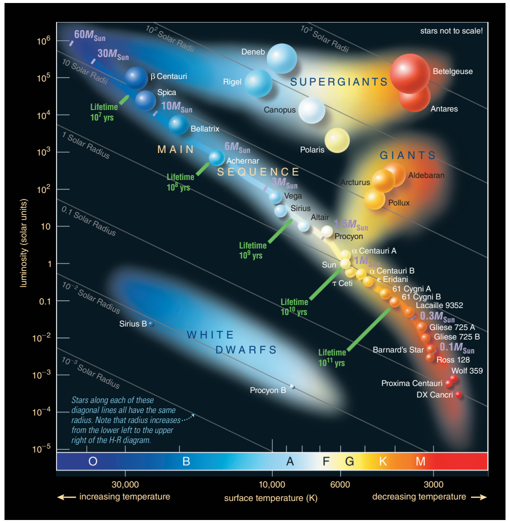
Explanation: Recall the use of Kepler's 3rd law:

$$M_1 + M_2 = \frac{4\pi^2}{G}\frac{R^3}{P^2}$$

Note: Alternatively, one could also work out the masses of the stars if orbital period P and stellar (radial) velocities were known

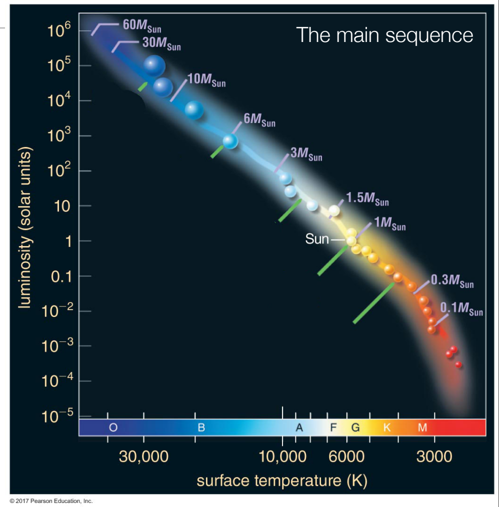
$$M_1 + M_2 = \frac{P}{2\pi G}(v_1 + v_2)^3$$

4. Mass-luminosity-radius relationship in the main sequence phase

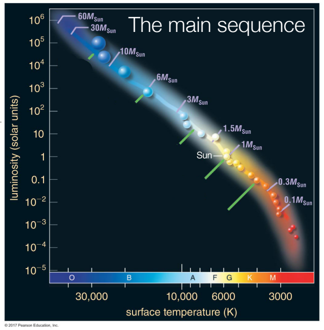


The main sequence is a mass sequence

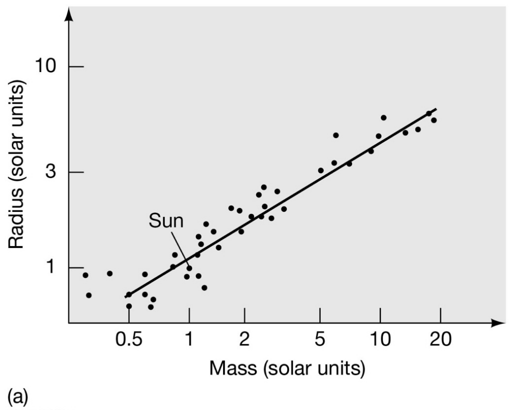
- From many binary studies, we have a good idea of the mass of main-sequence stars
  - cool stars → low masses → low luminosities
  - hot stars → high masses → high luminosities
- Once mass is specified, we know its point on the main sequence.
- From this, we derive a mass-luminosity relation
- From its temperature and luminosity, we derive radius: mass-radius relation



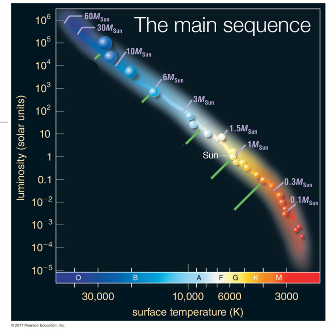
Mass-radius relation for stars in the main sequence



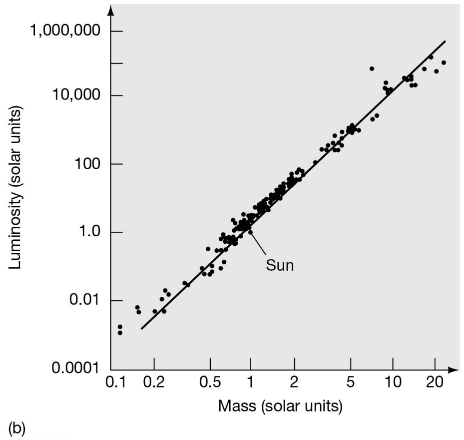
More massive stars have larger **radius**



Mass-luminosity relation for stars in the main sequence



More massive stars have higher **luminosities**



- A linear relation in log-log plot is the equivalent of a power-law relation in linear-linear plot
- For the mass-luminosity relation:

$$\frac{L_\star}{L_\odot} = \left[\frac{M_\star}{M_\odot}\right]^{3.5}$$

Main-sequence lifetime: "t"

- The main sequence is the phase when stars are burning H into He in their cores

$$L_\star = \frac{E}{t} = \frac{(fM_\star)c^2}{t}$$

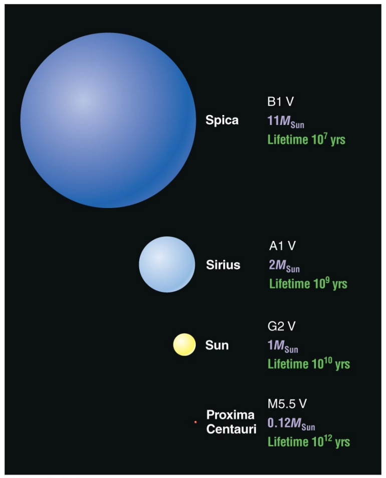
fraction of stellar mass converted to energy

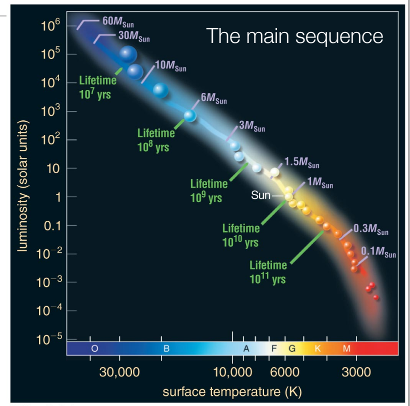
- Thus
- $$t = \frac{(fM_\star)c^2}{L_\star}$$ In solar units: $$\frac{t}{t_\odot} = \frac{M_\star}{M_\odot}\frac{L_\odot}{L_\star}$$
- $t_\odot$ is the time the Sun spends in the main sequence. $t_\odot \approx 10^{10}$yr.
- Using the mass-luminosity relation,

$$\frac{L_\star}{L_\odot} = \left[\frac{M_\star}{M_\odot}\right]^{3.5} \longrightarrow \boxed{t = 10^{10}\left(\frac{M_\star}{M_\odot}\right)^{-2.5} \text{yr}}$$

The **higher** the mass, the **faster** is the evolution during the main sequence phase

Comparing different stars in the main sequence





$$t = 10^{10}\left(\frac{M_\star}{M_\odot}\right)^{-2.5} \text{yr}$$

Conceptual question

True or False? Stars that begin their lives with the most mass live longer than less massive stars because it takes them a lot longer to use up their hydrogen fuel.

(a) True, with more hydrogen to burn, massive stars can live for billions of years.

(b) True, low mass stars run out of hydrogen very quickly and have very short lifetimes.

(c) False, stars have similar lifetimes despite their different masses.

(d) False, more massive stars are much more luminous than low mass stars and use up their hydrogen faster, even though they have more of it.

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