Problem set (ungraded)- Errors and Statistics

1. Let x be a continuous random variable governed by the following probability density function:
$$f(x) = \begin{cases} C(x^2 - 1) \ for \ 1 \leq x \leq 3 \\ \quad 0 \qquad elsewhere \end{cases}$$

   (a) Us the normalisation condition to find the value of C.

   (b) Find the mean of $f(x)$.

   (c) Find the second moment of $f(x)$.

   (d) Use $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$ to calculate the variance of $f(x)$.

2. A student monitors the activity of a radioactive source with a detector. The number of counts measured in a given time window obey the Poisson distribution $P_\mu(x) = e^{-\mu} \frac{\mu^x}{x!}$.

   (a) In the course of 10 minutes, the detector registers a total of 2540 counts. What is the corresponding rate R_{tot} (in counts per minute) and its uncertainty?

   (b) The student removes the source in order to record counts due to the background radiation. The detector registers 95 counts in the 3 minutes. What is the corresponding rate of the background, R_{bkg}, and its uncertainty?

   (c) What are the rate and its uncertainty due to the radioactive source alone?

3.

   (a) What is the name of the distribution that governs the number of non-overlapping events that have a low probability of occurring during a given time window?

   (b) A new medicine A is suspected of causing a rare disease. In a clinical trial with 1200 patients taking this medicine, 2 people contract this rare disease. What is the uncertainty in this number of incidents?

   (c) The clinical trial ran over a period of 2 years. Based on your answer in (b), state the number of incidents of this rare disease per 1000 people per year taking medicine A including its uncertainty.

   (d) In Ireland, 4870 people have contracted this disease in the last 3 years. The population in Ireland is 4.6 million. Calculate the incidence rate including its uncertainty per 1000 people per year.

   (e) Assuming that nobody in Ireland has taken medicine A, is there enough evidence to conclude that medicine A is causing this rare disease? Justify your answer.

4.

   (a) Given two independent variables p and q, with an associated uncertainty Δp and Δq, state the general error propagation formula that yields the uncertainty of a function $f(p, q)$ of these two variables.

   (b) The lens formula relates the focal length f to the image and object distance given by p and q, respectively:
$$f = \frac{pq}{p + q}$$
   Show that the error in f is given by
$$\Delta f = \frac{\sqrt{q^4 \Delta p^2 + p^4 \Delta q^2}}{(p + q)^2}$$

   (c) For $p = (3.1 \pm 0.1)cm$ and $q = (6.5 \pm 0.1)cm$, calculate the focal length including its uncertainty.