MA1125 – Calculus
Tutorial problems #5

1. Show that the polynomial $f(x) = x^3 - 5x^2 - 8x + 1$ has exactly one root in $(0, 1)$.

2. Let $b > 1$ be a given constant. Use the mean value theorem to show that

$$1 - \frac{1}{b} < \ln b < b - 1.$$

3. Compute each of the following limits.

$$L_1 = \lim_{x\to 2} \frac{2x^3 - 5x^2 + 5x - 6}{3x^3 - 5x^2 - 4}, \qquad L_2 = \lim_{x\to\infty} \frac{\ln x}{x^2}, \qquad L_3 = \lim_{x\to 0} (x + \cos x)^{1/x}.$$

4. For which values of x is $f(x) = (\ln x)^2$ increasing? For which values is it concave up?

5. Find the intervals on which f is increasing/decreasing and the intervals on which f is concave up/down. Use this information to sketch the graph of f.

$$f(x) = \frac{x^2}{x^2 + 3}.$$

6. Show that the polynomial $f(x) = x^3 + x^2 - 5x + 1$ has exactly two roots in $(0, 2)$.

7. Use the mean value theorem for the case $f(x) = \sqrt{x + 4}$ to show that

$$2 + \frac{1}{2} < \sqrt{7} < 2 + \frac{3}{4}.$$

8. Compute each of the following limits.

$$L_1 = \lim_{x\to 2} \frac{x^3 - 5x^2 + 8x - 4}{x^3 - 3x^2 + 4}, \qquad L_2 = \lim_{x\to 1} \frac{\ln x}{x^4 - 1}, \qquad L_3 = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}.$$

9. For which values of x is $f(x) = e^{-2x^2}$ increasing? For which values is it concave up?

10. Show that there exists a unique number $1 < x < \pi$ such that $x^3 = 3 \sin x + 1$.