# 29

# ELECTROMAGNETIC INDUCTION

**29.1.** **IDENTIFY:** The changing magnetic field causes a changing magnetic flux through the loop. This induces an emf in the loop which causes a current to flow in it.

**SET UP:** $|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right|$, $\Phi_B = BA\cos\phi$, $\phi = 0°$. $A$ is constant and $B$ is changing.

**EXECUTE:** **(a)** $|\varepsilon| = A\dfrac{dB}{dt} = (0.0900\ \text{m}^2)(0.190\ \text{T/s}) = 0.0171\ \text{V}$.

**(b)** $I = \dfrac{\varepsilon}{R} = \dfrac{0.0171\,\text{V}}{0.600\ \Omega} = 0.0285\ \text{A}$.

**EVALUATE:** These are small emfs and currents by everyday standards.

**29.2.** **IDENTIFY:** $|\varepsilon| = N\left|\dfrac{d\Phi_B}{dt}\right|$. $\Phi_B = BA\cos\phi$. $\Phi_B$ is the flux through each turn of the coil.

**SET UP:** $\phi_\text{i} = 0°$. $\phi_\text{f} = 90°$.

**EXECUTE:** **(a)** $\Phi_{B,\text{i}} = BA\cos 0° = (6.0\times 10^{-5}\ \text{T})(12\times 10^{-4}\ \text{m}^2)(1) = 7.2\times 10^{-8}\ \text{Wb}$. The total flux through the coil is $N\Phi_{B,\text{i}} = (200)(7.2\times 10^{-8}\ \text{Wb}) = 1.44\times 10^{-5}\ \text{Wb}$. $\Phi_{B,\text{f}} = BA\cos 90° = 0$.

**(b)** $|\varepsilon_\text{av}| = \left|\dfrac{N\Phi_\text{i} - N\Phi_\text{f}}{\Delta t}\right| = \dfrac{1.44\times 10^{-5}\ \text{Wb}}{0.040\ \text{s}} = 3.6\times 10^{-4}\ \text{V} = 0.36\ \text{mV}$.

**EVALUATE:** The average induced emf depends on how rapidly the flux changes.

**29.3.** **IDENTIFY** and **SET UP:** Use Faraday's law to calculate the average induced emf and apply Ohm's law to the coil to calculate the average induced current and charge that flows.

**(a) EXECUTE:** The magnitude of the average emf induced in the coil is $|\varepsilon_\text{av}| = N\left|\dfrac{\Delta\Phi_B}{\Delta t}\right|$. Initially, $\Phi_{B\text{i}} = BA\cos\phi = BA$. The final flux is zero, so $|\varepsilon_\text{av}| = N\dfrac{|\Phi_{B\text{f}} - \Phi_{B\text{i}}|}{\Delta t} = \dfrac{NBA}{\Delta t}$. The average induced current is $I = \dfrac{|\varepsilon_\text{av}|}{R} = \dfrac{NBA}{R\Delta t}$. The total charge that flows through the coil is $Q = I\Delta t = \left(\dfrac{NBA}{R\Delta t}\right)\Delta t = \dfrac{NBA}{R}$.

**EVALUATE:** The charge that flows is proportional to the magnetic field but does not depend on the time $\Delta t$.

**(b)** The magnetic stripe consists of a pattern of magnetic fields. The pattern of charges that flow in the reader coil tells the card reader the magnetic field pattern and hence the digital information coded onto the card.

**(c)** According to the result in part (a) the charge that flows depends only on the change in the magnetic flux and it does not depend on the rate at which this flux changes.

**29.4.** **IDENTIFY** and **SET UP:** Apply the result derived in Exercise 29.3: $Q = NBA/R$. In the present exercise the flux changes from its maximum value of $\Phi_B = BA$ to zero, so this equation applies. $R$ is the total resistance so here $R = 60.0\ \Omega + 45.0\ \Omega = 105.0\ \Omega$.

**EXECUTE:** $Q = \frac{NBA}{R}$ says $B = \frac{QR}{NA} = \frac{(3.56\times10^{-5}\ \text{C})(105.0\ \Omega)}{120(3.20\times10^{-4}\ \text{m}^2)} = 0.0973\ \text{T}.$

**EVALUATE:** A field of this magnitude is easily produced.

**29.5. IDENTIFY:** Apply Faraday's law.

**SET UP:** Let $+z$ be the positive direction for $\vec{A}$. Therefore, the initial flux is positive and the final flux is zero.

**EXECUTE:** **(a)** and **(b)** $\varepsilon = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{0-(1.5\ \text{T})\pi(0.120\ \text{m})^2}{2.0\times10^{-3}\ \text{s}} = +34\ \text{V}.$ Since $\varepsilon$ is positive and $\vec{A}$ is toward us, the induced current is counterclockwise.

**EVALUATE:** The shorter the removal time, the larger the average induced emf.

**29.6. IDENTIFY:** Apply $\varepsilon = -N\frac{d\Phi_B}{dt}$ and $I = \varepsilon/R$.

**SET UP:** $d\Phi_B/dt = AdB/dt$.

**EXECUTE:** **(a)** $|\varepsilon| = \frac{Nd\Phi_B}{dt} = NA\frac{d}{dt}(B) = NA\frac{d}{dt}((0.012\ \text{T/s})t + (3.00\times10^{-5}\ \text{T/s}^4)t^4).$

$|\varepsilon| = NA((0.012\ \text{T/s}) + (1.2\times10^{-4}\ \text{T/s}^4)t^3) = 0.0302\ \text{V} + (3.02\times10^{-4}\ \text{V/s}^3)t^3.$

**(b)** At $t = 5.00$ s, $|\varepsilon| = 0.0302\ \text{V} + (3.02\times10^{-4}\ \text{V/s}^3)(5.00\ \text{s})^3 = 0.0680\ \text{V}.$

$I = \frac{\varepsilon}{R} = \frac{0.0680\ \text{V}}{600\ \Omega} = 1.13\times10^{-4}\ \text{A}.$

**EVALUATE:** The rate of change of the flux is increasing in time, so the induced current is not constant but rather increases in time.

**29.7. IDENTIFY:** Calculate the flux through the loop and apply Faraday's law.

**SET UP:** To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance $r$ from the wire, is $B = \frac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

**EXECUTE:** **(a)** $B = \frac{\mu_0 i}{2\pi r}$, into the page.

**(b)** $d\Phi_B = BdA = \frac{\mu_0 i}{2\pi r}Ldr.$

**(c)** $\Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 iL}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 iL}{2\pi}\ln(b/a).$

**(d)** $|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{\mu_0 L}{2\pi}\ln(b/a)\frac{di}{dt}.$

**(e)** $|\varepsilon| = \frac{\mu_0(0.240\ \text{m})}{2\pi}\ln(0.360/0.120)(9.60\ \text{A/s}) = 5.06\times10^{-7}\ \text{V}.$

**EVALUATE:** The induced emf is proportional to the rate at which the current in the long straight wire is changing

**29.8. IDENTIFY:** Apply Faraday's law.

**SET UP:** Let $\vec{A}$ be upward in Figure E29.8 in the textbook.

**EXECUTE:** **(a)** $|\varepsilon_{\text{ind}}| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}(B_\perp A)\right|.$

$|\varepsilon_{\text{ind}}| = A\sin 60°\left|\frac{dB}{dt}\right| = A\sin 60°\left|\frac{d}{dt}\left((1.4\ \text{T})e^{-(0.057\text{s}^{-1})t}\right)\right| = (\pi r^2)(\sin 60°)(1.4\ \text{T})(0.057\ \text{s}^{-1})e^{-(0.057\ \text{s}^{-1})t}.$

$|\varepsilon_{\text{ind}}| = \pi(0.75\ \text{m})^2(\sin 60°)(1.4\ \text{T})(0.057\ \text{s}^{-1})e^{-(0.057\ \text{s}^{-1})t} = (0.12\ \text{V})\ e^{-(0.057\ \text{s}^{-1})t}.$

**(b)** $\varepsilon = \frac{1}{10}\varepsilon_0 = \frac{1}{10}(0.12\ \text{V}).$ $\frac{1}{10}(0.12\ \text{V}) = (0.12\ \text{V})\ e^{-(0.057\ \text{s}^{-1})t}.$ $\ln(1/10) = -(0.057\ \text{s}^{-1})t$ and $t = 40.4$ s.

**(c)** $\vec{B}$ is in the direction of $\vec{A}$ so $\Phi_B$ is positive. $B$ is getting weaker, so the magnitude of the flux is decreasing and $d\Phi_B/dt < 0$. Faraday's law therefore says $\varepsilon > 0$. Since $\varepsilon > 0$, the induced current must flow *counterclockwise* as viewed from above.

**EVALUATE:** The flux changes because the magnitude of the magnetic field is changing.

**29.9. IDENTIFY** and **SET UP:** Use Faraday's law to calculate the emf (magnitude and direction). The direction of the induced current is the same as the direction of the emf. The flux changes because the area of the loop is changing; relate $dA/dt$ to $dc/dt$, where $c$ is the circumference of the loop.

**(a) EXECUTE:** $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$.

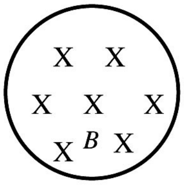
$\Phi_B = BA = (B/4\pi)c^2$.

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \left(\frac{B}{2\pi}\right)c\left|\frac{dc}{dt}\right|.$$

At $t = 9.0$ s, $c = 1.650\text{ m} - (9.0\text{ s})(0.120\text{ m/s}) = 0.570$ m.

$|\varepsilon| = (0.500\text{ T})(1/2\pi)(0.570\text{ m})(0.120\text{ m/s}) = 5.44$ mV.

**(b) SET UP:** The loop and magnetic field are sketched in Figure 29.9.




Take into the page to be the positive direction for $\vec{A}$.
Then the magnetic flux is positive.

**Figure 29.9**

**EXECUTE:** The positive flux is decreasing in magnitude; $d\Phi_B/dt$ is negative and $\varepsilon$ is positive. By the right-hand rule, for $\vec{A}$ into the page, positive $\varepsilon$ is clockwise.

**EVALUATE:** Even though the circumference is changing at a constant rate, $dA/dt$ is not constant and $|\varepsilon|$ is not constant. Flux $\otimes$ is decreasing so the flux of the induced current is $\otimes$ and this means that $I$ is clockwise, which checks.

**29.10. IDENTIFY:** Rotating the coil changes the angle between it and the magnetic field, which changes the magnetic flux through it. This change induces an emf in the coil.

**SET UP:** $\varepsilon_{av} = N\left|\dfrac{\Delta\Phi_B}{\Delta t}\right|$, $\Phi_B = BA\cos\phi$. $\phi$ is the angle between the normal to the loop and $\vec{B}$, so $\phi_i = 90.0° - 37.0° = 53.0°$ and $\phi_f = 0°$.

**EXECUTE:** $$\varepsilon_{av} = \frac{NBA|\cos\phi_f - \cos\phi_i|}{\Delta t} = \frac{(80)(1.70\text{ T})(0.250\text{ m})(0.400\text{ m})}{0.0600\text{ s}}|\cos 0° - \cos 53.0°| = 90.3\text{ V}.$$

**EVALUATE:** The flux changes because the orientation of the coil relative to the magnetic field changes, even though the field remains constant.

**29.11. IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

**SET UP:** The flux through a coil is $\Phi_B = NBA\cos\phi$ and the induced emf is $\varepsilon = -d\Phi_B/dt$.

**EXECUTE:** **(a)** $|\varepsilon| = d\Phi_B/dt = d[A(B_0 + bx)]/dt = bA\,dx/dt = bAv$.

**(b)** Clockwise

**(c)** Same answers except the current is counterclockwise.

**EVALUATE:** Even though the coil remains within the magnetic field, the flux through it changes because the strength of the field is changing.

**29.12. IDENTIFY** and **SET UP:** Use Faraday's law, $\varepsilon_{av} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right|$. The flux change is due to the changing magnetic field.

**EXECUTE:** $\mathcal{E}_{\text{av}} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = A\left|\dfrac{\Delta B}{\Delta t}\right| = \pi r^2\left|\dfrac{\Delta B}{\Delta t}\right| = \pi(0.011\text{ m})^2\left(\dfrac{8.0\text{ T}-0}{20\text{ s}}\right) = 1.5\times10^{-4}\text{ V}.$

**EVALUATE:** This is 0.15 mV, which is rather small. Since $P \propto V^2$, this emf may not produce much heating.

**29.13.** **IDENTIFY:** Apply the results of Example 29.3.

**SET UP:** $\mathcal{E}_{\text{max}} = NBA\omega$.

**EXECUTE:** $\omega = \dfrac{\mathcal{E}_{\text{max}}}{NBA} = \dfrac{2.40\times10^{-2}\text{ V}}{(120)(0.0750\text{ T})(0.016\text{ m})^2} = 10.4\text{ rad/s}.$

**EVALUATE:** We may also express $\omega$ as $99.3\text{ rev/min}$ or $1.66\text{ rev/s}$.

**29.14.** **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

**SET UP:** The flux through a coil is $\Phi_B = NBA\cos\phi$ and the induced emf is $\mathcal{E} = -d\Phi_B/dt$.

**EXECUTE:** The flux is constant in each case, so the induced emf is zero in all cases.

**EVALUATE:** Even though the coil is moving within the magnetic field and has flux through it, this flux is not *changing*, so no emf is induced in the coil.

**29.15.** **IDENTIFY** and **SET UP:** The field of the induced current is directed to oppose the change in flux.

**EXECUTE:** **(a)** The field is into the page and is increasing so the flux is increasing. The field of the induced current is out of the page. To produce field out of the page the induced current is counterclockwise.

**(b)** The field is into the page and is decreasing so the flux is decreasing. The field of the induced current is into the page. To produce field into the page the induced current is clockwise.

**(c)** The field is constant so the flux is constant and there is no induced emf and no induced current.

**EVALUATE:** The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

**29.16.** **IDENTIFY** and **SET UP:** Use Lenz's law. The induced current flows so as to oppose the flux change that is inducing it. The magnetic field due to $I$ is out of the page for loops $A$ and $C$ and into the page for loops $B$ and $D$. The field is constant since $I$ is constant, so any flux change is due to the motion of the loops.

**EXECUTE:** **(a)** *A:* The loop is moving away from the wire, so the magnetic field through the loop is getting weaker. This results in decreasing flux through the loop. Since the field is out of the page, the induced current flows in a direction so that its magnetic field inside the loop will be out of the page, which is a counterclockwise direction.

*B:* The flux through the loop is decreasing with the magnetic field into the page, so the induced current is clockwise.

*C:* The flux through the loop is constant, so there is no induced current.

*D:* The flux through the loop is increasing with the field into the page, so the induced current is counterclockwise.

**(b)** *A:* The flux is decreasing, so the loop is pulled toward the wire to increase the flux through the loop.

*B:* The flux is decreasing, so the loop is pulled toward the wire to increase the flux through the loop.

*C:* No current is induced, so there is no force.

*D:* The flux is increasing, so the loop is repelled by the wire to decrease the flux through the loop.

**EVALUATE:** In part (b), look at the direction of the force on the segment of each loop closest to the wire. For $A$ and $B$, the induced current is in the same direction as $I$, so the wire attracts these loops. For $D$ the induced current is opposite to $I$, so the wire repels the loop. For $C$ there is no induced current, so there is no force.

**29.17.** **IDENTIFY** and **SET UP:** Use the right-hand rule to find the direction of the magnetic field due to the long wire at the location of each loop. Lenz's law says that the magnetic field of the induced current is directed to oppose the change in flux through the circuit. Since the current $I$ is decreasing, the flux through each coil is decreasing, so the induced current flows to oppose this flux decrease.

**EXECUTE:** **(a)** The magnetic field of the long wire is directed out of the page at $C$ and into the page at $A$. When the current decreases, the magnetic field decreases. Therefore, the magnetic field of the induced current in loop $C$ is directed out of the page inside the loop, to oppose the decrease in flux out of the page due to the current in the long wire. To produce magnetic field in this direction, the induced current in $C$ is counterclockwise. The magnetic field of the induced current in loop $A$ is directed into the page inside the loop, to oppose the decrease in flux into the page due to the current in the long wire. To produce a magnetic field in this direction, the induced current in $A$ is clockwise.

**(b)** The through both coils A and C is decreasing, so they will be pulled toward the long wire to oppose this decrease.

**EVALUATE:** As a check on the answer in (b), look at the current in the section of each loop that is nearest to the wire. For both loops, this induced current is in the same direction as the current $I$ in the wire. When two parallel wires carry current in the same direction, they attract each other, which agrees with our answer in (b).

**29.18.** **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.

**SET UP** and **EXECUTE:** The magnetic field is outward through the round coil and is decreasing, so the magnetic field due to the induced current must also point outward to oppose this decrease. Therefore the induced current is counterclockwise.

**EVALUATE:** Careful! Lenz's law does not say that the induced current flows to oppose the magnetic flux. Instead it says that the current flows to oppose the *change* in flux.

**29.19.** **IDENTIFY** and **SET UP:** Apply Lenz's law, in the form that states that the flux of the induced current tends to oppose the change in flux.

**EXECUTE:** **(a)** With the switch closed the magnetic field of coil $A$ is to the right at the location of coil $B$. When the switch is opened the magnetic field of coil $A$ goes away. Hence by Lenz's law the field of the current induced in coil $B$ is to the right, to oppose the decrease in the flux in this direction. To produce magnetic field that is to the right the current in the circuit with coil $B$ must flow through the resistor in the direction $a$ to $b$.

**(b)** With the switch closed the magnetic field of coil $A$ is to the right at the location of coil $B$. This field is stronger at points closer to coil $A$ so when coil $B$ is brought closer the flux through coil $B$ increases. By Lenz's law the field of the induced current in coil $B$ is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil $B$ must flow through the resistor in the direction $b$ to $a$.

**(c)** With the switch closed the magnetic field of coil $A$ is to the right at the location of coil $B$. The current in the circuit that includes coil $A$ increases when $R$ is decreased and the magnetic field of coil $A$ increases when the current through the coil increases. By Lenz's law the field of the induced current in coil $B$ is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil $B$ must flow through the resistor in the direction $b$ to $a$.

**EVALUATE:** In parts (b) and (c) the change in the circuit causes the flux through circuit $B$ to increase and in part (a) it causes the flux to decrease. Therefore, the direction of the induced current is the same in parts (b) and (c) and opposite in part (a).

**29.20.** **IDENTIFY:** Apply Lenz's law.

**SET UP:** The field of the induced current is directed to oppose the change in flux in the secondary circuit.

**EXECUTE:** **(a)** The magnetic field in $A$ is to the left and is increasing. The flux is increasing so the field due to the induced current in $B$ is to the right. To produce magnetic field to the right, the induced current flows through $R$ from right to left.

**(b)** The magnetic field in $A$ is to the right and is decreasing. The flux is decreasing so the field due to the induced current in $B$ is to the right. To produce magnetic field to the right the induced current flows through $R$ from right to left.

**(c)** The magnetic field in $A$ is to the right and is increasing. The flux is increasing so the field due to the induced current in $B$ is to the left. To produce magnetic field to the left the induced current flows through $R$ from left to right.

**EVALUATE:** The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

**29.21.** **IDENTIFY** and **SET UP:** Lenz's law requires that the flux of the induced current opposes the change in flux.

**EXECUTE:** **(a)** The magnetic field is out of the page and increasing, so the induced current should flow so that its field is into the page, so the induced current is clockwise.

**(b)** The current reaches a constant value so $\Phi_B$ is constant. $d\Phi_B/dt = 0$ and there is no induced current.

**(c)** The magnetic field is out of the page and is decreasing, so the induced current should flow that its magnetic field is out of the page. Thus the induced current is counterclockwise.

**EVALUATE:** Only a change in flux produces an induced current. The induced current is in one direction when the current in the outer ring is increasing and is in the opposite direction when that current is decreasing.

**29.22.** **IDENTIFY:** The changing flux through the loop due to the changing magnetic field induces a current in the wire. Energy is dissipated by the resistance of the wire due to the induced current in it.

**SET UP:** The magnitude of the induced emf is $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$, $P = I^2 R$, $I = \varepsilon/R$.

**EXECUTE:** **(a)** $\vec{B}$ is out of page and $\Phi_B$ is decreasing, so the field of the induced current is directed out of the page inside the loop and the induced current is counterclockwise.

**(b)** $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$. The current due to the emf is

$I = \frac{|\varepsilon|}{R} = \frac{\pi r^2}{R}\left|\frac{dB}{dt}\right| = \frac{\pi(0.0480\text{ m})^2}{0.160\ \Omega}(0.680\text{ T/s}) = 0.03076\text{ A}$. The rate of energy dissipation is

$P = I^2 R = (0.03076\text{ A})^2(0.160\ \Omega) = 1.51\times10^{-4}\text{ W}$.

**EVALUATE:** Both the current and resistance are small, so the power is also small.

**29.23.** **IDENTIFY:** The changing flux through the loop due to the changing magnetic field induces a current in the wire.

**SET UP:** The magnitude of the induced emf is $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$, $I = \varepsilon/R$.

**EXECUTE:** $\vec{B}$ is into the page and $\Phi_B$ is increasing, so the field of the induced current is directed out of the page inside the loop and the induced current is counterclockwise.

$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right| = \pi(0.0250\text{ m})^2(0.380\text{ T/s}^3)(3t^2) = (2.238\times10^{-3}\text{ V/s}^2)t^2$.

$I = \frac{|\varepsilon|}{R} = (5.739\times10^{-3}\text{ A/s}^2)t^2$. When $B = 1.33\text{ T}$, we have $1.33\text{ T} = (0.380\text{ T/s}^3)t^3$, which gives

$t = 1.518\text{ s}$. At this $t$, $I = (5.739\times10^{-3}\text{ A/s}^2)(1.518\text{ s})^2 = 0.0132\text{ A}$.

**EVALUATE:** As the field changes, the current will also change.

**29.24.** **IDENTIFY:** The magnetic flux through the loop is decreasing, so an emf will be induced in the loop, which will induce a current in the loop. The magnetic field will exert a force on the loop due to this current.

**SET UP:** The motional $\varepsilon$ is $\varepsilon = vBL$, $I = \varepsilon/R$, and $F_B = ILB$.

**EXECUTE:** Use $I = \frac{\varepsilon}{R} = \frac{BLv}{R}$ and $F_B = ILB$.

$F_B = ILB = v\frac{B^2L^2}{R} = \frac{3.00\text{ m/s}}{0.600\ \Omega}(2.40\text{ T})^2(0.0150\text{ m})^2 = 6.48\times10^{-3}\text{ N} = 6.48\text{ mN}$.

$\vec{B}$ is into the page and $\Phi_B$ is decreasing, so the field of the induced current is into the page inside the loop and the induced current is clockwise. Using $\vec{F} = I\vec{l}\times\vec{B}$, we see that the force on the left-hand end of the loop to be to the left.

**EVALUATE:** The force is very small by everyday standards.

**29.25.** **IDENTIFY:** A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.

**SET UP:** The induced emf is $\varepsilon = vBL\sin\phi$, where $\phi$ is the angle between the velocity and the magnetic field.

**EXECUTE:** **(a)** $\varepsilon = vBL\sin\phi = (5.00\text{ m/s})(0.450\text{ T})(0.300\text{ m})(\sin 90°) = 0.675\text{ V}$

**(b)** The positive charges are moved to end $b$, so $b$ is at the higher potential.

**(c)** $E = V/L = (0.675\text{ V})/(0.300\text{ m}) = 2.25\text{ V/m}$. The direction of $\vec{E}$ is from $b$ to $a$.

**(d)** The positive charges are pushed to $b$, so $b$ has an excess of positive charge.

**(e)** (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.

**EVALUATE:** The motional emf is large enough to have noticeable effects in some cases.

**29.26.** **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

**SET UP:** The flux through a coil is $\Phi_B = NBA\cos\phi$ and the induced emf is $\varepsilon = -d\Phi_B/dt$.

**EXECUTE:** **(a)** and **(c)** The magnetic flux is constant, so the induced emf is zero.

**(b)** The area inside the field is changing. If we let $x$ be the length (along the 30.0-cm side) in the field, then $A = (0.400\text{ m})x$. $\Phi_B = BA = B(0.400\text{ m})x$.

$$|\varepsilon| = |d\Phi_B/dt| = B\,d[(0.400\text{ m})x]/dt = B(0.400\text{ m})dx/dt = B(0.400\text{ m})v.$$

$$\varepsilon = (1.25\text{ T})(0.400\text{ m})(0.0200\text{ m/s}) = 0.0100\text{ V}.$$

**EVALUATE:** It is not *flux* that induces an emf, but rather a *rate of change* of the flux. The induced emf in part (b) is small enough to be ignored in many instances.

**29.27. IDENTIFY:** $\varepsilon = vBL$.

**SET UP:** $L = 5.00\times10^{-2}$ m. 1 mph = 0.4470 m/s.

**EXECUTE:** $v = \dfrac{\varepsilon}{BL} = \dfrac{1.50\text{ V}}{(0.650\text{ T})(5.00\times10^{-2}\text{ m})} = 46.2\text{ m/s} = 103\text{ mph}.$

**EVALUATE:** This is a large speed and not practical. It is also difficult to produce a 5.00-cm wide region of 0.650 T magnetic field.

**29.28. IDENTIFY:** $\varepsilon = vBL$.

**SET UP:** 1 mph = 0.4470 m/s. $1\text{ G} = 10^{-4}$ T.

**EXECUTE:** **(a)** $\varepsilon = (180\text{ mph})\left(\dfrac{0.4470\text{ m/s}}{1\text{ mph}}\right)(0.50\times10^{-4}\text{ T})(1.5\text{ m}) = 6.0\text{ mV}.$ This is much too small to be noticeable.

**(b)** $\varepsilon = (565\text{ mph})\left(\dfrac{0.4470\text{ m/s}}{1\text{ mph}}\right)(0.50\times10^{-4}\text{ T})(64.4\text{ m}) = 0.813\text{ V}.$ This is too small to be noticeable.

**EVALUATE:** Even though the speeds and values of $L$ are large, the earth's field is small and motional emfs due to the earth's field are not important in these situations.

**29.29. IDENTIFY** and **SET UP:** $\varepsilon = vBL$. Use Lenz's law to determine the direction of the induced current. The force $F_{\text{ext}}$ required to maintain constant speed is equal and opposite to the force $F_I$ that the magnetic field exerts on the rod because of the current in the rod.

**EXECUTE:** **(a)** $\varepsilon = vBL = (7.50\text{ m/s})(0.800\text{ T})(0.500\text{ m}) = 3.00\text{ V}.$

**(b)** $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from $b$ to $a$ in the rod.

**(c)** $I = \dfrac{\varepsilon}{R} = \dfrac{3.00\text{ V}}{1.50\ \Omega} = 2.00\text{ A}.$ $F_I = ILB\sin\phi = (2.00\text{ A})(0.500\text{ m})(0.800\text{ T})\sin 90° = 0.800\text{ N}.$ $\vec{F}_I$ is to the left. To keep the bar moving to the right at constant speed an external force with magnitude $F_{\text{ext}} = 0.800$ N and directed to the right must be applied to the bar.

**(d)** The rate at which work is done by the force $F_{\text{ext}}$ is $F_{\text{ext}}v = (0.800\text{ N})(7.50\text{ m/s}) = 6.00\text{ W}.$ The rate at which thermal energy is developed in the circuit is $I^2R = (2.00\text{ A})^2(1.50\ \Omega) = 6.00\text{ W}.$ These two rates are equal, as is required by conservation of energy.

**EVALUATE:** The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

**29.30. IDENTIFY:** Use the three approaches specified in the problem for determining the direction of the induced current. $I = \varepsilon/R$. The induced potential across a moving bar is $\varepsilon = vBL$.

**SET UP:** Let $\vec{A}$ be directed into the figure, so a clockwise emf is positive.

**EXECUTE:** **(a)** $\varepsilon = vBL = (5.0\text{ m/s})(0.750\text{ T})(0.650\text{ m}) = 2.438\text{ V},$ which rounds to 2.4 V.

**(b)** (i) Let $q$ be a positive charge in the moving bar, as shown in Figure 29.30a. The magnetic force on this charge is $\vec{F} = q\vec{v}\times\vec{B}$, which points *upward*. This force pushes the current in a *counterclockwise* direction through the circuit.

(ii) $\Phi_B$ is positive and is increasing in magnitude, so $d\Phi_B/dt > 0$. Then by Faraday's law $\varepsilon < 0$ and the emf and induced current are counterclockwise.

(iii) The flux through the circuit is increasing, so the induced current must cause a magnetic field out of the paper to oppose this increase. Hence this current must flow in a *counterclockwise sense*, as shown in Figure 29.30b.

**(c)** $\mathcal{E} = RI$. $I = \dfrac{\mathcal{E}}{R} = \dfrac{2.438 \text{ V}}{25.0\ \Omega} = 0.09752 \text{ A}$, which rounds to 98 mA.

**EVALUATE:** All three methods agree on the direction of the induced current.

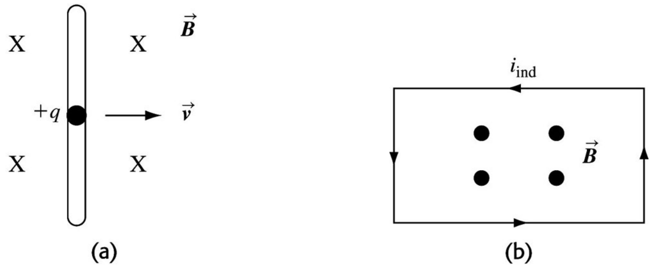



**Figure 29.30**

**29.31. IDENTIFY:** The motion of the bar due to the applied force causes a motional emf to be induced across the ends of the bar, which induces a current through the bar. The magnetic field exerts a force on the bar due to this current.

**SET UP:** The applied force is to the left and equal to $F_{\text{applied}} = F_B = ILB$. $\mathcal{E} = BvL$ and $I = \dfrac{\mathcal{E}}{R} = \dfrac{BvL}{R}$.

**EXECUTE:** **(a)** $\vec{B}$ out of page and $\Phi_B$ decreasing, so the field of the induced current is out of the page inside the loop and the induced current is counterclockwise.

**(b)** Combining $F_{\text{applied}} = F_B = ILB$ and $\mathcal{E} = BvL$, we have $I = \dfrac{\mathcal{E}}{R} = \dfrac{BvL}{R}$. $F_{\text{applied}} = \dfrac{vB^2L^2}{R}$. The rate at which this force does work is $P_{\text{applied}} = F_{\text{applied}}v = \dfrac{(vBL)^2}{R} = \dfrac{[(5.90 \text{ m/s})(0.650 \text{ T})(0.360 \text{ m})]^2}{45.0\ \Omega} = 0.0424 \text{ W}$.

**EVALUATE:** The power is small because the magnetic force is usually small compared to everyday forces.

**29.32. IDENTIFY:** The motion of the bar due to the applied force causes a motional emf to be induced across the ends of the bar, which induces a current through the bar and through the resistor. This current dissipates energy in the resistor.

**SET UP:** $P_R = I^2R$, $\mathcal{E} = BvL = IR$.

**EXECUTE:** **(a)** $\vec{B}$ is out of the page and $\Phi_B$ is increasing, so the field of the induced current is into the page inside the loop and the induced current is clockwise.

**(b)** $P_R = I^2R$ so $I = \sqrt{\dfrac{P_R}{R}} = \sqrt{\dfrac{0.840 \text{ W}}{45.0\ \Omega}} = 0.1366 \text{ A}$. $I = \dfrac{\text{emf}}{R} = \dfrac{BvL}{R}$.

$$v = \frac{IR}{BL} = \frac{(0.1366 \text{ A})(45.0\ \Omega)}{(0.650 \text{ T})(0.360 \text{ m})} = 26.3 \text{ m/s}.$$

**EVALUATE:** This speed is around 60 mph, so it would not be very practical to generate energy this way.

**29.33. IDENTIFY:** The motion of the bar causes an emf to be induced across its ends, which induces a current in the circuit.

**SET UP:** $\mathcal{E} = BvL$, $I = \mathcal{E}/R$.

**EXECUTE:** $\vec{F}_B$ on the bar is to the left so $\vec{v}$ is to the right. Using $\mathcal{E} = BvL$ and $I = \mathcal{E}/R$, we have

$$I = \frac{BvL}{R}. \quad v = \frac{IR}{BL} = \frac{(1.75 \text{ A})(6.00\ \Omega)}{(1.20 \text{ T})(0.250 \text{ m})} = 35.0 \text{ m/s}.$$

**EVALUATE:** This speed is greater than 60 mph!

**29.34.** **IDENTIFY:** A motional emf is induced across the blood vessel.

**SET UP** and **EXECUTE:** **(a)** Each slab of flowing blood has maximum width $d$ and is moving perpendicular to the field with speed $v$. $\mathcal{E} = vBL$ becomes $\mathcal{E} = vBd$.

**(b)** $B = \dfrac{\mathcal{E}}{vd} = \dfrac{1.0\times10^{-3}\ \text{V}}{(0.15\ \text{m/s})(5.0\times10^{-3}\ \text{m})} = 1.3\ \text{T}.$

**(c)** The blood vessel has cross-sectional area $A = \pi d^2/4$. The volume of blood that flows past a cross section of the vessel in time $t$ is $\pi(d^2/4)vt$. The volume flow rate is volume/time $= R = \pi d^2 v/4$. $v = \dfrac{\mathcal{E}}{Bd}$ so $R = \dfrac{\pi d^2}{4}\left(\dfrac{\mathcal{E}}{Bd}\right) = \dfrac{\pi \mathcal{E} d}{4B}$.

**EVALUATE:** A very strong magnetic field (1.3 T) is required to produce a small potential difference of only 1 mV.

**29.35.** **IDENTIFY:** While the circuit is entering and leaving the region of the magnetic field, the flux through it will be changing. This change will induce an emf in the circuit.

**SET UP:** When the loop is entering or leaving the region of magnetic field the flux through it is changing and there is an induced emf. The magnitude of this induced emf is $\mathcal{E} = BLv$. The length $L$ is 0.750 m. When the loop is totally within the field the flux through the loop is not changing so there is no induced emf. The induced current has magnitude $I = \dfrac{\mathcal{E}}{R}$ and direction given by Lenz's law.

**EXECUTE:** **(a)** $I = \dfrac{\mathcal{E}}{R} = \dfrac{BLv}{R} = \dfrac{(1.25\ \text{T})(0.750\ \text{m})(3.0\ \text{m/s})}{12.5\ \Omega} = 0.225\ \text{A}$. The magnetic field through the loop is directed out of the page and is increasing, so the magnetic field of the induced current is into the page inside the loop and the induced current is clockwise.

**(b)** The flux is not changing so $\mathcal{E}$ and $I$ are zero.

**(c)** $I = \dfrac{\mathcal{E}}{R} = 0.225\ \text{A}$. The magnetic field through the loop is directed out of the page and is decreasing, so the magnetic field of the induced current is out of the page inside the loop and the induced current is counterclockwise.

**(d)** Let clockwise currents be positive. At $t = 0$ the loop is entering the field. It is totally in the field at time $t_a$ and beginning to move out of the field at time $t_b$. The graph of the induced current as a function of time is sketched in Figure 29.35.

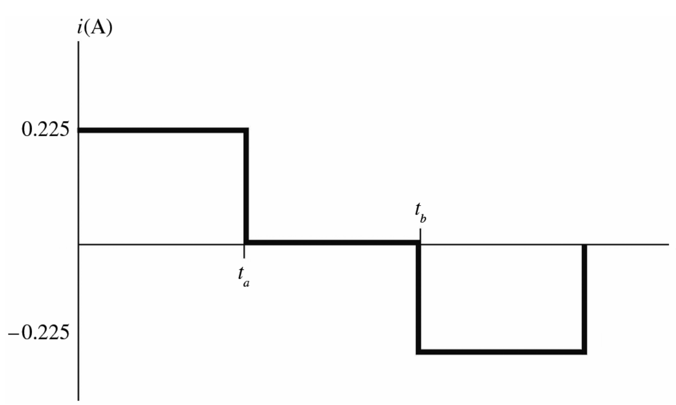



**Figure 29.35**

**EVALUATE:** Even though the circuit is moving throughout all parts of this problem, an emf is induced in it only when the flux through it is changing. While the coil is entirely within the field, the flux is constant, so no emf is induced.

**29.36. IDENTIFY:** A changing magnetic flux through a coil induces an emf in that coil, which means that an electric field is induced in the material of the coil.

**SET UP:** According to Faraday's law, the induced electric field obeys the equation $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$.

**EXECUTE:** **(a)** For the magnitude of the induced electric field, Faraday's law gives

$$E 2\pi r = d(B\pi r^2)/dt = \pi r^2\, dB/dt.$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{0.0225\text{ m}}{2}(0.250\text{ T/s}) = 2.81\times 10^{-3}\text{ V/m}.$$

**(b)** The field points toward the south pole of the magnet and is decreasing, so the induced current is counterclockwise.

**EVALUATE:** This is a very small electric field compared to most others found in laboratory equipment.

**29.37. IDENTIFY:** Apply $\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$.

**SET UP:** Evaluate the integral for a path which is a circle of radius $r$ and concentric with the solenoid. The magnetic field of the solenoid is confined to the region inside the solenoid, so $B(r) = 0$ for $r > R$.

**EXECUTE:** **(a)** $\frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r_1^2 \frac{dB}{dt}$.

**(b)** $E = \frac{1}{2\pi r_1}\frac{d\Phi_B}{dt} = \frac{\pi r_1^2}{2\pi r_1}\frac{dB}{dt} = \frac{r_1}{2}\frac{dB}{dt}$. The direction of $\vec{E}$ is shown in Figure 29.37a.

**(c)** All the flux is within $r < R$, so outside the solenoid $E = \frac{1}{2\pi r_2}\frac{d\Phi_B}{dt} = \frac{\pi R^2}{2\pi r_2}\frac{dB}{dt} = \frac{R^2}{2r_2}\frac{dB}{dt}$.

**(d)** The graph is sketched in Figure 29.37b.

**(e)** At $r = R/2$, $|\varepsilon| = \frac{d\Phi_B}{dt} = \pi(R/2)^2\frac{dB}{dt} = \frac{\pi R^2}{4}\frac{dB}{dt}$.

**(f)** At $r = R$, $|\varepsilon| = \frac{d\Phi_B}{dt} = \pi R^2\frac{dB}{dt}$.

**(g)** At $r = 2R$, $|\varepsilon| = \frac{d\Phi_B}{dt} = \pi R^2\frac{dB}{dt}$.

**EVALUATE:** The emf is independent of the distance from the center of the cylinder at all points outside it. Even though the magnetic field is zero for $r > R$, the induced electric field is nonzero outside the solenoid and a nonzero emf is induced in a circular turn that has $r > R$.

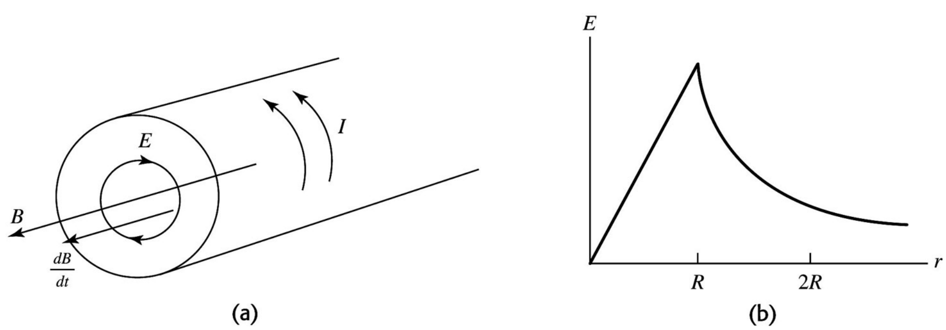
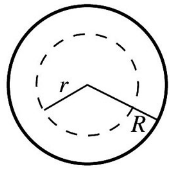



**Figure 29.37**

**29.38. IDENTIFY:** Use $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to calculate the induced electric field $E$ at a distance $r$ from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.

**SET UP:** The end view of the solenoid is sketched in Figure 29.38.




Let $R$ be the radius of the solenoid.

**Figure 29.38**

Apply $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to an integration path that is a circle of radius $r$, where $r < R$. We need to calculate just the magnitude of $E$ so we can take absolute values.

**EXECUTE:** **(a)** $\left|\oint \vec{E} \cdot d\vec{l}\right| = E(2\pi r)$.

$\Phi_B = B\pi r^2, \left|-\dfrac{d\Phi_B}{dt}\right| = \pi r^2 \left|\dfrac{dB}{dt}\right|$.

$\left|\oint \vec{E} \cdot d\vec{l}\right| = \left|-\dfrac{d\Phi_B}{dt}\right|$ implies $E(2\pi r) = \pi r^2 \left|\dfrac{dB}{dt}\right|$.

$E = \frac{1}{2} r \left|\dfrac{dB}{dt}\right|$.

$B = \mu_0 n I$, so $\dfrac{dB}{dt} = \mu_0 n \dfrac{dI}{dt}$.

Thus $E = \frac{1}{2} r \mu_0 n \dfrac{dI}{dt} = \frac{1}{2}(0.00500 \text{ m})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(900 \text{ m}^{-1})(36.0 \text{ A/s}) = 1.02 \times 10^{-4} \text{ V/m}$.

**(b)** $r = 0.0100$ cm is still inside the solenoid so the expression in part (a) applies.

$E = \frac{1}{2} r \mu_0 n \dfrac{dI}{dt} = \frac{1}{2}(0.0100 \text{ m})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(900 \text{ m}^{-1})(36.0 \text{ A/s}) = 2.04 \times 10^{-4} \text{ V/m}$.

**EVALUATE:** Inside the solenoid $E$ is proportional to $r$, so $E$ doubles when $r$ doubles.

**29.39.** **IDENTIFY:** Apply $E = \dfrac{1}{2\pi r}\left|\dfrac{d\Phi_B}{dt}\right|$ with $\Phi_B = \mu_0 n i A$.

**SET UP:** $A = \pi r^2$, where $r = 0.0110$ m. In $E = \dfrac{1}{2\pi r}\left|\dfrac{d\Phi_B}{dt}\right|$, $r = 0.0350$ m.

**EXECUTE:** $|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right| = \left|\dfrac{d}{dt}(BA)\right| = \left|\dfrac{d}{dt}(\mu_0 n i A)\right| = \mu_0 n A \left|\dfrac{di}{dt}\right|$ and $|\varepsilon| = E(2\pi r)$. Therefore, $\left|\dfrac{di}{dt}\right| = \dfrac{E 2\pi r}{\mu_0 n A}$.

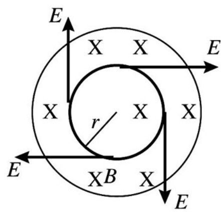
$$\left|\frac{di}{dt}\right| = \frac{(8.00 \times 10^{-6} \text{ V/m}) 2\pi (0.0350 \text{ m})}{\mu_0 (400 \text{ m}^{-1}) \pi (0.0110 \text{ m})^2} = 9.21 \text{ A/s}.$$

**EVALUATE:** Outside the solenoid the induced electric field decreases with increasing distance from the axis of the solenoid.

**29.40.** **IDENTIFY:** Use $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to calculate the induced electric field $E$ and use this $E$ in $\varepsilon = \oint \vec{E} \cdot d\vec{l}$ to calculate $\varepsilon$ between two points.

**SET UP** and **EXECUTE:** **(a)** Because of the axial symmetry and the absence of any electric charge, the field lines are concentric circles.

**(b)** See Figure 29.40.




$\vec{E}$ is tangent to the ring. The direction of $\vec{E}$ (clockwise or counterclockwise) is the direction in which current will be induced in the ring.

**Figure 29.40**

Use the sign convention for Faraday's law to deduce this direction. Let $\vec{A}$ be into the paper. Then $\Phi_B$ is positive. $B$ decreasing then means $\dfrac{d\Phi_B}{dt}$ is negative, so by $\varepsilon = -\dfrac{d\Phi_B}{dt}$, $\varepsilon$ is positive and therefore clockwise. Thus $\vec{E}$ is clockwise around the ring. To calculate $E$ apply $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to a circular path that coincides with the ring.

$\oint \vec{E} \cdot d\vec{l} = E(2\pi r)$.

$\Phi_B = B\pi r^2$; $\left|\dfrac{d\Phi_B}{dt}\right| = \pi r^2 \left|\dfrac{dB}{dt}\right|$.

$E(2\pi r) = \pi r^2 \left|\dfrac{dB}{dt}\right|$ and $E = \frac{1}{2} r \left|\dfrac{dB}{dt}\right| = \frac{1}{2}(0.100\text{ m})(0.0350\text{ T/s}) = 1.75\times10^{-3}\text{ V/m}$.

**(c)** The induced emf has magnitude

$\varepsilon = \oint \vec{E} \cdot d\vec{l} = E(2\pi r) = (1.75\times10^{-3}\text{ V/m})(2\pi)(0.100\text{ m}) = 1.100\times10^{-3}\text{ V}$. Then

$I = \dfrac{\varepsilon}{R} = \dfrac{1.100\times10^{-3}\text{ V}}{4.00\,\Omega} = 2.75\times10^{-4}\text{ A}$.

**(d)** Points $a$ and $b$ are separated by a distance around the ring of $\pi r$ so

$\varepsilon = E(\pi r) = (1.75\times10^{-3}\text{ V/m})(\pi)(0.100\text{ m}) = 5.50\times10^{-4}\text{ V}$.

**(e)** The ends are separated by a distance around the ring of $2\pi r$ so $\varepsilon = 1.10\times10^{-3}\text{ V}$ as calculated in part (c).

**EVALUATE:** The induced emf, calculated from Faraday's law and used to calculate the induced current, is associated with the induced electric field integrated around the total circumference of the ring.

**29.41. IDENTIFY:** Apply Faraday's law in the form $|\varepsilon_{\text{av}}| = N\left|\dfrac{\Delta\Phi_B}{\Delta t}\right|$.

**SET UP:** The magnetic field of a large straight solenoid is $B = \mu_0 nI$ inside the solenoid and zero outside. $\Phi_B = BA$, where $A$ is $8.00\text{ cm}^2$, the cross-sectional area of the long straight solenoid.

**EXECUTE:** $|\varepsilon_{\text{av}}| = N\left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = \left|\dfrac{NA(B_{\text{f}} - B_{\text{i}})}{\Delta t}\right| = \dfrac{NA\mu_0 nI}{\Delta t}$.

$$\varepsilon_{\text{av}} = \frac{\mu_0(12)(8.00\times10^{-4}\text{ m}^2)(9000\text{ m}^{-1})(0.350\text{ A})}{0.0400\text{ s}} = 9.50\times10^{-4}\text{ V}.$$

**EVALUATE:** An emf is induced in the second winding even though the magnetic field of the solenoid is zero at the location of the second winding. The changing magnetic field induces an electric field outside the solenoid and that induced electric field produces the emf.

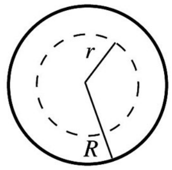
**29.42. IDENTIFY and SET UP:** The equations $i_{\text{C}} = \dfrac{dq}{dt} = \varepsilon\dfrac{d\Phi_E}{dt}$ and $i_{\text{D}} = \varepsilon\dfrac{d\Phi_E}{dt}$ show that $i_{\text{C}} = i_{\text{D}}$ and also relate $i_{\text{D}}$ to the rate of change of the electric field flux between the plates. Use this to calculate $dE/dt$ and apply the generalized form of Ampere's law to calculate $B$.

**EXECUTE:** **(a)** $i_{\text{C}} = i_{\text{D}}$, so $j_{\text{D}} = \dfrac{i_{\text{D}}}{A} = \dfrac{i_{\text{C}}}{A} = \dfrac{0.520\text{ A}}{\pi r^2} = \dfrac{0.520\text{ A}}{\pi(0.0400\text{ m})^2} = 103\text{ A/m}^2$.

**(b)** $j_D = \epsilon_0 \dfrac{dE}{dt}$ so $\dfrac{dE}{dt} = \dfrac{j_D}{\epsilon_0} = \dfrac{103\text{ A/m}^2}{8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 1.16\times10^{13}\text{ V/m}\cdot\text{s}.$

**SET UP** and **EXECUTE:** **(c)** Apply Ampere's law $\oint \vec{B}\cdot d\vec{l} = \mu_0(i_C + i_D)_{\text{encl}}$ to a circular path with radius $r = 0.0200$ m.

An end view of the solenoid is given in Figure 29.42.




By symmetry the magnetic field is tangent to the path and constant around it.

**Figure 29.42**

Thus $\oint \vec{B}\,.\,d\vec{l} = \oint B dl = B\int dl = B(2\pi r).$

$i_C = 0$ (no conduction current flows through the air space between the plates)

The displacement current enclosed by the path is $j_D \pi r^2$.

Thus $B(2\pi r) = \mu_0(j_D \pi r^2)$ and

$B = \frac{1}{2}\mu_0 j_D r = \frac{1}{2}(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(103\text{ A/m}^2)(0.0200\text{ m}) = 1.30\times10^{-6}\text{ T} = 1.30\ \mu\text{T}.$

**(d)** $B = \frac{1}{2}\mu_0 j_D r$. Now $r$ is $\frac{1}{2}$ the value in (c), so $B$ is also $\frac{1}{2}$ its value in (c):

$B = \frac{1}{2}(1.30\times10^{-6}\text{ T}) = 0.650\times10^{-7}\text{ T} = 0.650\ \mu\text{T}.$

**EVALUATE:** The definition of displacement current allows the current to be continuous at the capacitor. The magnetic field between the plates is zero on the axis $(r = 0)$ and increases as $r$ increases.

**29.43.** **IDENTIFY:** $q = CV$. For a parallel-plate capacitor, $C = \dfrac{\epsilon A}{d}$, where $\epsilon = K\epsilon_0$. $i_C = dq/dt$. $j_D = \epsilon\dfrac{dE}{dt}$.

**SET UP:** $E = q/\epsilon A$ so $dE/dt = i_C/\epsilon A$.

**EXECUTE:** **(a)** $q = CV = \left(\dfrac{\epsilon A}{d}\right)V = \dfrac{(4.70)\epsilon_0(3.00\times10^{-4}\text{ m}^2)(120\text{ V})}{2.50\times10^{-3}\text{ m}} = 5.99\times10^{-10}\text{ C}.$

**(b)** $\dfrac{dq}{dt} = i_C = 6.00\times10^{-3}\text{ A}.$

**(c)** $j_D = \epsilon\dfrac{dE}{dt} = K\epsilon_0\dfrac{i_C}{K\epsilon_0 A} = \dfrac{i_C}{A} = j_C$, so $i_D = i_C = 6.00\times10^{-3}\text{ A}.$

**EVALUATE:** $i_D = i_C$, so Kirchhoff's junction rule is satisfied where the wire connects to each capacitor plate.

**29.44.** **IDENTIFY** and **SET UP:** Use $i_C = q/t$ to calculate the charge $q$ that the current has carried to the plates in time $t$. The equations $V = Ed$ and $E = \dfrac{\sigma}{\epsilon_0}$ relate $q$ to the electric field $E$ and the potential difference between the plates. The displacement current density is $j_D = \epsilon\dfrac{dE}{dt}$.

**EXECUTE:** **(a)** $i_C = 1.80\times10^{-3}$ A.

$q = 0$ at $t = 0$.

The amount of charge brought to the plates by the charging current in time $t$ is

$q = i_C t = (1.80\times10^{-3}\text{ A})(0.500\times10^{-6}\text{ s}) = 9.00\times10^{-10}\text{ C}.$

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{9.00\times10^{-10}\ \text{C}}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00\times10^{-4}\ \text{m}^2)} = 2.03\times10^5\ \text{V/m}.$$

$V = Ed = (2.03\times10^5\ \text{V/m})(2.00\times10^{-3}\text{m}) = 406\ \text{V}.$

**(b)** $E = q/\epsilon_0 A.$

$$\frac{dE}{dt} = \frac{dq/dt}{\epsilon_0 A} = \frac{i_C}{\epsilon_0 A} = \frac{1.80\times10^{-3}\ \text{A}}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00\times10^{-4}\ \text{m}^2)} = 4.07\times10^{11}\ \text{V/m}\cdot\text{s}.$$

Since $i_C$ is constant $dE/dt$ does not vary in time.

**(c)** $j_D = \epsilon_0 \dfrac{dE}{dt}$ (with $\epsilon$ replaced by $\epsilon_0$ since there is vacuum between the plates).

$j_D = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.07\times10^{11}\ \text{V/m}\cdot\text{s}) = 3.60\ \text{A/m}^2.$

$i_D = j_D A = (3.60\ \text{A/m}^2)(5.00\times10^{-4}\ \text{m}^2) = 1.80\times10^{-3}\ \text{A};\ i_D = i_C.$

**EVALUATE:** $i_C = i_D$. The constant conduction current means the charge $q$ on the plates and the electric field between them both increase linearly with time and $i_D$ is constant.

**29.45. IDENTIFY:** Apply $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}.$

**SET UP:** For magnetic fields less than the critical field, there is no internal magnetic field. For fields greater than the critical field, $\vec{B}$ is very nearly equal to $\vec{B}_0$.

**EXECUTE:** **(a)** The external field is less than the critical field, so inside the superconductor $\vec{B} = 0$ and

$\vec{M} = -\dfrac{\vec{B}_0}{\mu_0} = -\dfrac{(0.130\ \text{T})\hat{i}}{\mu_0} = -(1.03\times10^5\ \text{A/m})\hat{i}$. Outside the superconductor, $\vec{B} = \vec{B}_0 = (0.130\ \text{T})\hat{i}$ and

$\vec{M} = 0.$

**(b)** The field is greater than the critical field and $\vec{B} = \vec{B}_0 = (0.260\ \text{T})\hat{i}$, both inside and outside the superconductor.

**EVALUATE:** Below the critical field the external field is expelled from the superconducting material.

**29.46. IDENTIFY:** The 4.00-cm long left side of the loop is a bar moving in a magnetic field, so an emf is induced across its ends. This emf causes current to flow through the loop, and the external magnetic field exerts a force on the bar due to the current in it. Ohm's law applies to the circuit and Newton's second law applies to the loop.

**SET UP:** The induced potential across the left-end side is $\varepsilon = vBL$, the magnetic force on the bar is $F_{mag}$ = $ILB$, and Ohm's law is $\varepsilon = IR$. Newton's second law is $\Sigma\vec{F} = m\vec{a}$. The flux through the loop is decreasing, so the induced current is clockwise. Alternatively, the magnetic force on positive charge in the moving left-end bar is upward, by the right-hand rule, which also gives a clockwise current. Therefore the magnetic force on the 4.00-cm segment is to the left, opposite to $\vec{F}_{ext}$.

**EXECUTE:** **(a)** Combining the equations discussed in the set up, the magnetic force on the 4.00-cm bar (and on the loop) is

$F_{mag} = ILB = (\varepsilon/R)LB = (vBL/R)LB = v(BL)^2/R.$

Newton's second law gives

$F_{ext} - F_{mag} = ma.$

$ma = F_{ext} - v(BL)^2/R.$

$(0.0240\ \text{kg})a = 0.180\ \text{N} - (0.0300\ \text{m/s})[(2.90\ \text{T})(0.0400\ \text{m})]^2/(0.00500\ \Omega).$

$a = 4.14\ \text{m/s}^2.$

**(b)** At terminal speed $v_T$, $F_{mag} = F_{ext}$.

$v_T(BL)^2/R = F_{ext}.$

$v_T = RF_{ext}/(BL)^2 = (0.00500\ \Omega)(0.180\ \text{N})/[(2.90\ \text{T})(0.0400\ \text{m})]^2 = 0.0669\ \text{m/s} = 6.69\ \text{cm/s}$. The speed is constant thereafter, so the acceleration is zero.

**(c)** $a = F_{ext}/m = (0.180\ \text{N})/(0.0240\ \text{kg}) = 7.50\ \text{m/s}^2.$

**EVALUATE:** The acceleration is acceleration is constant once the loop is out of the magnetic field. But while it is partly in the field, the acceleration is not constant because the current changes as the speed changes and this causes the magnetic force to vary.

**29.47. IDENTIFY:** Apply Faraday's law and Lenz's law.

**SET UP:** For a discharging $RC$ circuit, $i(t)=\frac{V_0}{R}e^{-t/RC}$, where $V_0$ is the initial voltage across the capacitor. The resistance of the small loop is $(25)(0.600\text{ m})(1.0\ \Omega/\text{m})=15.0\ \Omega$.

**EXECUTE:** **(a)** The large circuit is an $RC$ circuit with a time constant of $\tau=RC=(10\ \Omega)(20\times10^{-6}\text{ F})=200\ \mu\text{s}$. Thus, the current as a function of time is $i=((100\text{ V})/(10\ \Omega))\,e^{-t/200\,\mu\text{s}}$. At $t=200\ \mu s$, we obtain $i=(10\text{ A})(e^{-1})=3.7\text{ A}$.

**(b)** Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop and referring to the solution of Exercise 29.7 we obtain $\Phi_B=\int_c^{c+a}\frac{\mu_0 ib}{2\pi r}dr=\frac{\mu_0 ib}{2\pi}\ln\left(1+\frac{a}{c}\right)$.

Therefore, the emf induced in the small loop at $t=200\ \mu\text{s}$ is $\varepsilon=-N\frac{d\Phi_B}{dt}=-\frac{N\mu_0 b}{2\pi}\ln\left(1+\frac{a}{c}\right)\frac{di}{dt}$.

$\varepsilon=-\frac{(25)(4\pi\times10^{-7}\text{ Wb/A}\cdot\text{m}^2)(0.200\text{ m})}{2\pi}\ln(3.0)\left(-\frac{3.7\text{ A}}{200\times10^{-6}\text{ s}}\right)=+20.0\text{ mV}$. Thus, the induced current in the small loop is $i'=\frac{\varepsilon}{R}=\frac{20.0\text{ mV}}{15.0\ \Omega}=1.33\text{ mA}$.

**(c)** The magnetic field from the large loop is directed out of the page within the small loop.The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

**EVALUATE:** **(d)** Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop—as can be seen by comparing the distance $c$ to the dimensions of the large loop.

**29.48. IDENTIFY:** The changing current in the large $RC$ circuit produces a changing magnetic flux through the small circuit, which induces an emf in the small circuit. This emf causes a current in the small circuit.

**SET UP:** For a charging $RC$ circuit, $i(t)=\frac{\varepsilon}{R}e^{-t/RC}$, where $\varepsilon$ is the emf (90.0 V) added to the large circuit.

Exercise 29.7 shows that $\Phi_B=\frac{\mu_0 ib}{2\pi}\ln(1+a/c)$ for each turn of the small circuit, and $\varepsilon_{\text{induced}}=-\frac{d\Phi_B}{dt}$.

**EXECUTE:** $\frac{d\Phi_B}{dt}=\frac{\mu_0 b}{2\pi}\ln(1+a/c)\frac{di}{dt}$. $\frac{di}{dt}=-\frac{\varepsilon}{R^2C}e^{-t/RC}$ and

$|\varepsilon_{\text{induced}}|=N\left|\frac{d\Phi_B}{dt}\right|=\frac{N\mu_0 b}{2\pi}\ln(1+a/c)\frac{\varepsilon}{R^2C}e^{-t/RC}=\frac{N\mu_0 b}{2\pi}\ln(1+a/c)\frac{1}{RC}i$. The resistance of the small loop is $(25)(0.600\text{ m})(1.0\ \Omega/\text{m})=15\ \Omega$.

$$|\varepsilon_{\text{induced}}|=(25)(2.00\times10^{-7}\text{ T}\cdot\text{m/A})(0.200\text{ m})\ln(1+10.0/5.0)\frac{1}{(10\ \Omega)(20\times10^{-6}\text{ F})}(5.00\text{ A}).$$

$|\varepsilon_{\text{induced}}|=0.02747\text{ V}$. The induced current is $\frac{|\varepsilon_{\text{induced}}|}{R}=\frac{0.02747\text{ V}}{15\ \Omega}=1.83\times10^{-3}\text{ A}=1.83\text{ mA}$, which rounds to 1.8 mA. The current in the large loop is counterclockwise. The magnetic field through the small loop is into the page and the flux is decreasing, so the magnetic field due to the induced current in the small loop is into the page and the induced current in the small loop is clockwise.

**EVALUATE:** The answer is actually independent of $N$ because the emf induced in the small coil is proportional to $N$ and the resistance of that coil is also proportional to $N$. Since $I=\varepsilon/R$, the $N$ will cancel out.

**29.49. IDENTIFY:** The changing current in the solenoid will cause a changing magnetic field (and hence changing flux) through the secondary winding, which will induce an emf in the secondary coil.

**SET UP:** The magnetic field of the solenoid is $B = \mu_0 ni$, and the induced emf is $|\varepsilon| = N\left|\frac{d\Phi_B}{dt}\right|$.

**EXECUTE:** $B = \mu_0 ni = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(90.0\times 10^2\ \text{m}^{-1})(0.160\ \text{A/s}^2)t^2 = (1.810\times 10^{-3}\ \text{T/s}^2)t^2$. The total flux through secondary winding is $(5.0)B(2.00\times 10^{-4}\ \text{m}^2) = (1.810\times 10^{-6}\ \text{Wb/s}^2)t^2$.

$|\varepsilon| = N\left|\frac{d\Phi_B}{dt}\right| = (3.619\times 10^{-6}\ \text{V/s})t$. $i = 3.20\ \text{A}$ says $3.20\ \text{A} = (0.160\ \text{A/s}^2)t^2$ and $t = 4.472\ \text{s}$. This gives

$|\varepsilon| = (3.619\times 10^{-6}\ \text{V/s})(4.472\ \text{s}) = 1.62\times 10^{-5}\ \text{V}$.

**EVALUATE:** This a very small voltage, about $16\ \mu\text{V}$.

**29.50.** **IDENTIFY:** Apply Faraday's law.
**SET UP:** For rotation about the $y$-axis the situation is the same as in Examples 29.3 and 29.4 and we can apply the results from those examples.
**EXECUTE:** **(a)** Rotating about the $y$-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$\varepsilon_{\text{max}} = \omega BA = (35.0\ \text{rad/s})(0.320\ \text{T})(6.00\times 10^{-2}\ \text{m}^2) = 0.672\ \text{V}$.

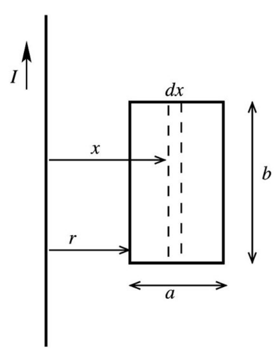
**(b)** Rotating about the $x$-axis: $\frac{d\Phi_B}{dt} = 0$ and $\varepsilon = 0$.

**(c)** Rotating about the $z$-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$\varepsilon_{\text{max}} = \omega BA = (35.0\ \text{rad/s})(0.320\ \text{T})(6.00\times 10^{-2}\ \text{m}^2) = 0.672\ \text{V}$.

**EVALUATE:** The maximum emf is the same if the loop is rotated about an edge parallel to the $z$-axis as it is when it is rotated about the $z$-axis.

**29.51.** **(a) IDENTIFY:** (i) $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right|$. The flux is changing because the magnitude of the magnetic field of the wire decreases with distance from the wire. Find the flux through a narrow strip of area and integrate over the loop to find the total flux.
**SET UP:**




Consider a narrow strip of width $dx$ and a distance $x$ from the long wire, as shown in Figure 29.51a. The magnetic field of the wire at the strip is $B = \mu_0 I/2\pi x$.
The flux through the strip is $d\Phi_B = Bb\,dx = (\mu_0 Ib/2\pi)(dx/x)$.

**Figure 29.51a**

**EXECUTE:** The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\frac{\mu_0 Ib}{2\pi}\right)\int_r^{r+a}\frac{dx}{x}$.

$$\Phi_B = \left(\frac{\mu_0 Ib}{2\pi}\right)\ln\left(\frac{r+a}{r}\right).$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dr}\frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi}\left(-\frac{a}{r(r+a)}\right)v.$$

$$|\varepsilon| = \frac{\mu_0 Iabv}{2\pi r(r+a)}.$$

(ii) **IDENTIFY:** $\varepsilon = Bvl$ for a bar of length $l$ moving at speed $v$ perpendicular to a magnetic field $B$. Calculate the induced emf in each side of the loop, and combine the emfs according to their polarity.
**SET UP:** The four segments of the loop are shown in Figure 29.51b.

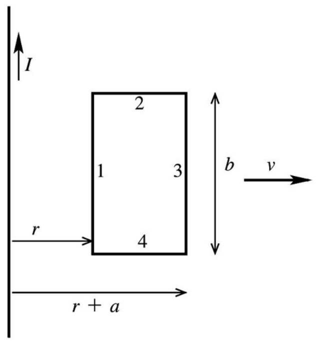



**Figure 29.51b**

**EXECUTE:** The emf in each side of the loop is $\varepsilon_1 = \left(\dfrac{\mu_0 I}{2\pi r}\right)vb$,

$\varepsilon_3 = \left(\dfrac{\mu_0 I}{2\pi(r+a)}\right)vb, \quad \varepsilon_2 = \varepsilon_4 = 0.$

Both emfs $\varepsilon_1$ and $\varepsilon_3$ are directed toward the top of the loop so oppose each other. The net emf is

$$\varepsilon = \varepsilon_1 - \varepsilon_3 = \frac{\mu_0 Ivb}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right) = \frac{\mu_0 Iabv}{2\pi r(r+a)}.$$

This expression agrees with what was obtained in (i) using Faraday's law.
**(b)** (i) **IDENTIFY** and **SET UP:** The flux of the induced current opposes the change in flux.
**EXECUTE:** $\vec{B}$ is $\otimes$. $\Phi_B$ is decreasing, so the flux $\Phi_{\text{ind}}$ of the induced current is $\otimes$ and the current is clockwise.
(ii) **IDENTIFY** and **SET UP:** Use the right-hand rule to find the force on the positive charges in each side of the loop. The forces on positive charges in segments 1 and 3 of the loop are shown in Figure 29.51c.

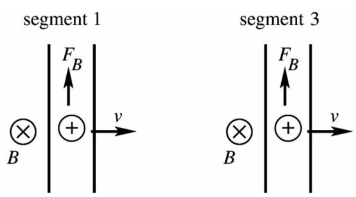



**Figure 29.51c**

**EXECUTE:** $B$ is larger at segment 1 since it is closer to the long wire, so $F_B$ is larger in segment 1 and the induced current in the loop is clockwise. This agrees with the direction deduced in (i) using Lenz's law.
**(c) EVALUATE:** When $v = 0$ the induced emf should be zero; the expression in part (a) gives this. When $a \to 0$ the flux goes to zero and the emf should approach zero; the expression in part (a) gives this. When $r \to \infty$ the magnetic field through the loop goes to zero and the emf should go to zero; the expression in part (a) gives this.

**29.52. IDENTIFY:** Apply the results of Example 29.3, generalized to $N$ loops: $\varepsilon_{\max} = N\omega BA$. $v = r\omega$.
**SET UP:** In the expression for $\varepsilon_{\max}$, $\omega$ must be in rad/s. 30 rpm = 3.14 rad/s.
**EXECUTE:** **(a)** Solving for $A$ we obtain $A = \dfrac{\varepsilon_{\max}}{\omega NB} = \dfrac{9.0\ \text{V}}{(3.14\ \text{rad/s})(2000\ \text{turns})(8.0\times10^{-5}\ \text{T})} = 18\ \text{m}^2.$

**(b)** Assuming a point on the coil at maximum distance from the axis of rotation we have

$$v = r\omega = \sqrt{\frac{A}{\pi}}\omega = \sqrt{\frac{18\ \text{m}^2}{\pi}}(3.14\ \text{rad/s}) = 7.5\ \text{m/s}.$$

**EVALUATE:** The device is not very feasible. The coil would need a rigid frame and the effects of air resistance would be appreciable.

**29.53.** **IDENTIFY:** Apply Faraday's law in the form $\mathcal{E}_{\text{av}} = -N\dfrac{\Delta\Phi_B}{\Delta t}$ to calculate the average emf. Apply Lenz's law to calculate the direction of the induced current.

**SET UP:** $\Phi_B = BA.$ The flux changes because the area of the loop changes.

**EXECUTE:** **(a)** $\mathcal{E}_{\text{av}} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = B\left|\dfrac{\Delta A}{\Delta t}\right| = B\dfrac{\pi r^2}{\Delta t} = (1.35\text{ T})\dfrac{\pi(0.0650/2\text{ m})^2}{0.250\text{ s}} = 0.0179\text{ V} = 17.9\text{ mV}.$

**(b)** Since the magnetic field is directed into the page and the magnitude of the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point *a* through the resistor to point *b*.

**EVALUATE:** Faraday's law can be used to find the direction of the induced current. Let $\vec{A}$ be into the page. Then $\Phi_B$ is positive and decreasing in magnitude, so $d\Phi_B/dt < 0.$ Therefore $\mathcal{E} > 0$ and the induced current is clockwise around the loop.

**29.54.** **IDENTIFY:** The movement of the rod causes an emf to be induced across its ends, which causes a current to flow through the circuit. The magnetic field exerts a force on this current.

**SET UP:** The magnetic force is $F_{\text{mag}} = ILB,$ the induced emf is $\mathcal{E} = vBL.$ $\sum F = ma$ applies to the rod, and $a = dv/dt.$

**EXECUTE:** The net force on the rod is $F - iLB = ma.$ $i = \dfrac{vBL}{R}.$ $F - \dfrac{vB^2L^2}{R} = ma.$ $F - \dfrac{vB^2L^2}{R} = m\dfrac{dv}{dt}.$

Integrating to find the time gives $\dfrac{F}{m}\displaystyle\int_0^t dt' = \int_0^v \dfrac{dv'}{1 - \dfrac{v'B^2L^2}{FR}},$ which gives $\dfrac{Ft}{m} = -\dfrac{FR}{B^2L^2}\ln\left(1 - \dfrac{vB^2L^2}{FR}\right).$

Solving for $t$ and putting in the numbers gives

$$t = -\frac{Rm}{B^2L^2}\ln\left(1 - \frac{vB^2L^2}{FR}\right) = -(0.120\text{ kg})(888.9\text{ s/kg})\ln\left(1 - \frac{25.0\text{ m/s}}{(1.90\text{ N})(888.9\text{ s/kg})}\right) = 1.59\text{ s}.$$

**EVALUATE:** We cannot use the constant-acceleration kinematics formulas because as the speed *v* of the rod changes, the magnetic force on it also changes. Therefore the acceleration of the rod is not constant.

**29.55.** **IDENTIFY:** Find the magnetic field at a distance *r* from the center of the wire. Divide the rectangle into narrow strips of width *dr*, find the flux through each strip and integrate to find the total flux.

**SET UP:** Example 28.8 uses Ampere's law to show that the magnetic field inside the wire, a distance $r$ from the axis, is $B(r) = \mu_0 Ir/2\pi R^2.$

**EXECUTE:** Consider a small strip of length *W* and width *dr* that is a distance *r* from the axis of the wire, as shown in Figure 29.55. The flux through the strip is $d\Phi_B = B(r)W\,dr = \dfrac{\mu_0 IW}{2\pi R^2}r\,dr.$ The total flux through the rectangle is $\Phi_B = \displaystyle\int d\Phi_B = \left(\frac{\mu_0 IW}{2\pi R^2}\right)\int_0^R r\,dr = \frac{\mu_0 IW}{4\pi}.$

**EVALUATE:** Note that the result is independent of the radius *R* of the wire.

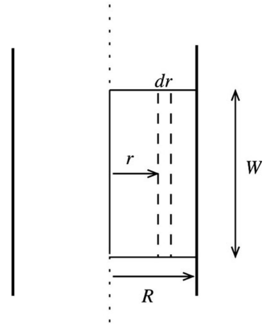



**Figure 29.55**

**29.56. IDENTIFY:** Apply Newton's second law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for $v$. At the terminal speed, $a = 0$.

**SET UP:** The induced emf in the loop has a magnitude $BLv$. The induced emf is counterclockwise, so it opposes the voltage of the battery, $\varepsilon$.

**EXECUTE:** **(a)** The net current in the loop is $I = \dfrac{\varepsilon - BLv}{R}$. The acceleration of the bar is $a = \dfrac{F}{m} = \dfrac{ILB\sin(90°)}{m} = \dfrac{(\varepsilon - BLv)LB}{mR}$. To find $v(t)$, set $\dfrac{dv}{dt} = a = \dfrac{(\varepsilon - BLv)LB}{mR}$ and solve for $v$ using the method of separation of variables:

$\int_0^v \dfrac{dv}{(\varepsilon - BLv)} = \int_0^t \dfrac{LB}{mR}dt \rightarrow v = \dfrac{\varepsilon}{BL}(1 - e^{-B^2L^2t/mR}) = (14\text{ m/s})(1 - e^{-t/6.0\text{ s}})$. The graph of $v$ versus $t$ is sketched in Figure 29.56. Note that the graph of this function is similar in appearance to that of a charging capacitor.

**(b)** Just after the switch is closed, $v = 0$ and $I = \varepsilon/R = 2.4\text{ A}$, $F = ILB = 2.074\text{ N}$, and $a = F/m = 2.3\text{ m/s}^2$.

**(c)** When $v = 2.0\text{ m/s}$, $a = \dfrac{[12\text{ V} - (2.4\text{ T})(0.36\text{ m})(2.0\text{ m/s})](0.36\text{ m})(2.4\text{ T})}{(0.90\text{ kg})(5.0\ \Omega)} = 2.0\text{ m/s}^2$.

**(d)** Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\dfrac{\varepsilon}{BL} = \dfrac{12\text{ V}}{(2.4\text{ T})(0.36\text{ m})} = 14\text{ m/s}$, which makes the acceleration zero.

**EVALUATE:** The current in the circuit is clockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

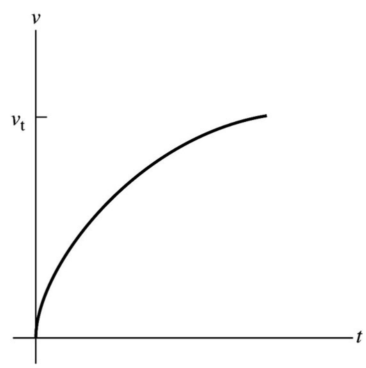
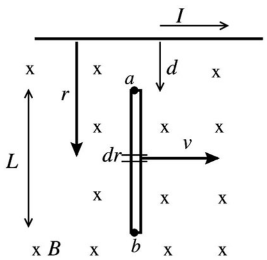



**Figure 29.56**

**29.57. (a)** and **(b) IDENTIFY** and **SET UP:**




The magnetic field of the wire is given by $B = \dfrac{\mu_0 I}{2\pi r}$ and varies along the length of the bar. At every point along the bar $\vec{B}$ has direction into the page. Divide the bar up into thin slices, as shown in Figure 29.57a.

**Figure 29.57a**

**EXECUTE:** The emf $d\varepsilon$ induced in each slice is given by $d\varepsilon = \vec{v}\times\vec{B}\cdot d\vec{l}$. $\vec{v}\times\vec{B}$ is directed toward the wire, so $d\varepsilon = -vB\,dr = -v\left(\dfrac{\mu_0 I}{2\pi r}\right)dr$. The total emf induced in the bar is

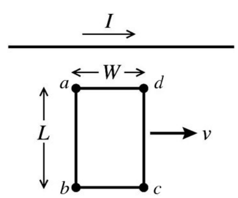
$$V_{ba} = \int_a^b d\varepsilon = -\int_d^{d+L}\left(\frac{\mu_0 I v}{2\pi r}\right)dr = -\frac{\mu_0 I v}{2\pi}\int_d^{d+L}\frac{dr}{r} = -\frac{\mu_0 I v}{2\pi}[\ln(r)]_d^{d+L}.$$

$$V_{ba} = -\frac{\mu_0 I v}{2\pi}(\ln(d+L)-\ln(d)) = -\frac{\mu_0 I v}{2\pi}\ \ln(1+L/d).$$

**EVALUATE:** The minus sign means that $V_{ba}$ is negative, point *a* is at higher potential than point *b*. (The force $\vec{F} = q\vec{v}\times\vec{B}$ on positive charge carriers in the bar is towards *a*, so *a* is at higher potential.) The potential difference increases when *I* or *v* increase, or *d* decreases.

**(c) IDENTIFY:** Use Faraday's law to calculate the induced emf.

**SET UP:** The wire and loop are sketched in Figure 29.57b.




**EXECUTE:** As the loop moves to the right the magnetic flux through it doesn't change. Thus $\varepsilon = -\dfrac{d\Phi_B}{dt} = 0$ and $I = 0$.

**Figure 29.57b**

**EVALUATE:** This result can also be understood as follows. The induced emf in section *ab* puts point *a* at higher potential; the induced emf in section *dc* puts point *d* at higher potential. If you travel around the loop then these two induced emf's sum to zero. There is no emf in the loop and hence no current.

**29.58. IDENTIFY:** Apply Faraday's law to calculate the magnitude and direction of the induced emf.

**SET UP:** Let $\vec{A}$ be directed out of the page in the figure with the problem in the textbook. This means that counterclockwise emf is positive.

**EXECUTE: (a)** $\Phi_B = BA = B_0\pi r_0^2\left[1-3(t/t_0)^2+2(t/t_0)^3\right]$.

**(b)** $\varepsilon = -\dfrac{d\Phi_B}{dt} = -B_0\pi r_0^2\dfrac{d}{dt}\left[1-3(t/t_0)^2+2(t/t_0)^3\right] = -\dfrac{B_0\pi r_0^2}{t_0}\left[-6(t/t_0)+6(t/t_0)^2\right]$.

$\varepsilon = -\dfrac{6\,B_0\pi r_0^2}{t_0}\left(\left(\dfrac{t}{t_0}\right)^2-\left(\dfrac{t}{t_0}\right)\right)$. At $t = 5.0\times10^{-3}\text{s}$,

$\varepsilon = -\dfrac{6B_0\pi(0.0420\text{ m})^2}{0.010\text{ s}}\left(\left(\dfrac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)^2-\left(\dfrac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)\right) = 0.0665\text{ V}$. $\varepsilon$ is positive so it is counterclockwise.

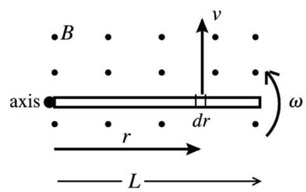
**(c)** $I = \dfrac{\varepsilon}{R_{\text{total}}} \Rightarrow R_{\text{total}} = r+R = \dfrac{\varepsilon}{I} \Rightarrow r = \dfrac{0.0665\text{ V}}{3.0\times10^{-3}\text{ A}} - 12\ \Omega = 10.2\ \Omega$.

**(d)** Evaluating the emf at $t = 1.21\times10^{-2}$ s and using the equations of part (b), $\varepsilon = -0.0676$ V, and the current flows clockwise, from *b* to *a* through the resistor.

**(e)** $\varepsilon = 0$ when $0 = \left(\left(\dfrac{t}{t_0}\right)^2-\left(\dfrac{t}{t_0}\right)\right)$. $1 = \dfrac{t}{t_0}$ and $t = t_0 = 0.010$ s.

**EVALUATE:** At $t = t_0$, $B = 0$. At $t = 5.00\times10^{-3}$ s, $\vec{B}$ is in the $+\hat{k}$-direction and is decreasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is counterclockwise. At $t = 0.0121$ s, $\vec{B}$ is in the $+\hat{k}$-direction and is increasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is clockwise. These results for the direction of $\mathcal{E}$ agree with the results we obtained from Faraday's law.

**29.59.** **IDENTIFY:** Use the expression for motional emf to calculate the emf induced in the rod.
**SET UP:** **(a)** The rotating rod is shown in Figure 29.59a.




The emf induced in a thin slice is $d\mathcal{E} = \vec{v}\times\vec{B}\cdot d\vec{l}$.

**Figure 29.59a**

**EXECUTE:** Assume that $\vec{B}$ is directed out of the page. Then $\vec{v}\times\vec{B}$ is directed radially outward and $dl = dr$, so $\vec{v}\times\vec{B}\cdot d\vec{l} = vB\,dr$.

$v = r\omega$ so $d\mathcal{E} = \omega Br\,dr$.

The $d\mathcal{E}$ for all the thin slices that make up the rod are in series so they add:

$$\mathcal{E} = \int d\mathcal{E} = \int_0^L \omega Br\,dr = \tfrac{1}{2}\omega BL^2 = \tfrac{1}{2}(8.80\ \text{rad/s})(0.650\ \text{T})(0.240\ \text{m})^2 = 0.165\ \text{V}.$$

**EVALUATE:** $\mathcal{E}$ increases with $\omega$, $B$, or $L^2$.

**(b) SET UP and EXECUTE:** No current flows so there is no $IR$ drop in potential. Thus the potential difference between the ends equals the emf of 0.165 V calculated in part (a).

**(c) SET UP:** The rotating rod is shown in Figure 29.59b.

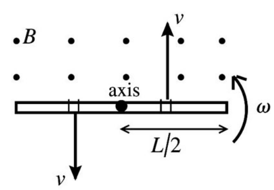



**Figure 29.59b**

**EXECUTE:** The emf between the center of the rod and each end is $\mathcal{E} = \tfrac{1}{2}\omega B(L/2)^2 = \tfrac{1}{4}(0.165\ \text{V}) = 0.0412\ \text{V}$, with the direction of the emf from the center of the rod toward each end. The emfs in each half of the rod thus oppose each other and there is no net emf between the ends of the rod.

**EVALUATE:** $\omega$ and $B$ are the same as in part (a) but $L$ of each half is $\tfrac{1}{2}L$ for the whole rod. $\mathcal{E}$ is proportional to $L^2$, so is smaller by a factor of $\tfrac{1}{4}$.

**29.60.** **IDENTIFY:** Since the bar is straight and the magnetic field is uniform, integrating $d\mathcal{E} = \vec{v}\times\vec{B}\cdot d\vec{l}$ along the length of the bar gives $\mathcal{E} = (\vec{v}\times\vec{B})\cdot\vec{L}$.

**SET UP:** $\vec{v} = (6.80\ \text{m/s})\hat{i}$. $\vec{L} = (0.250\ \text{m})(\cos 36.9°\hat{i} + \sin 36.9°\hat{j})$.

**EXECUTE:** **(a)** $\mathcal{E} = (\vec{v}\times\vec{B})\cdot\vec{L} = (6.80\ \text{m/s})\hat{i}\times\left[(0.120\ \text{T})\hat{i} - (0.220\ \text{T})\hat{j} - (0.0900\ \text{T})\hat{k}\right]\cdot\vec{L}$.

$\mathcal{E} = \left[(0.612\ \text{V/m})\hat{j} - (1.496\ \text{V/m})\hat{k}\right]\cdot\left[(0.250\ \text{m})(\cos 36.9°\hat{i} + \sin 36.9°\hat{j})\right]$.

$\mathcal{E} = (0.612\ \text{V/m})(0.250\ \text{m})\sin 36.9° = 0.0919\ \text{V} = 91.9\ \text{mV}$.

**(b)** The higher potential end is the end to which positive charges in the rod are pushed by the magnetic force. $\vec{v}\times\vec{B}$ has a positive $y$-component, so the end of the rod marked + in Figure 29.60 is at higher potential.

**EVALUATE:** Since $\vec{v}\times\vec{B}$ has nonzero $\hat{j}$- and $\hat{k}$-components, and $\vec{L}$- has nonzero $\hat{i}$- and $\hat{j}$-components, only the $\hat{k}$-component of $\vec{B}$ contributes to $\varepsilon$. In fact, $|\varepsilon|=|v_x B_z L_y|(6.80\text{ m/s})(0.0900\text{ T})(0.250\text{ m})\sin 36.9° = 0.0919\text{ V} = 91.9\text{ mV}.$

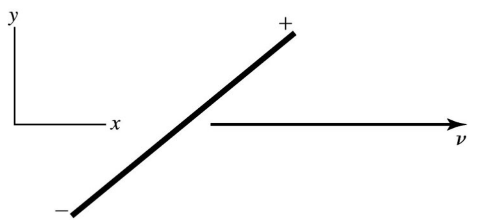



**Figure 29.60**

**29.61. (a) IDENTIFY:** Use Faraday's law to calculate the induced emf, Ohm's law to calculate $I$, and $\vec{F}=I\vec{l}\times\vec{B}$ to calculate the force on the rod due to the induced current.
**SET UP:** The force on the wire is shown in Figure 29.61.

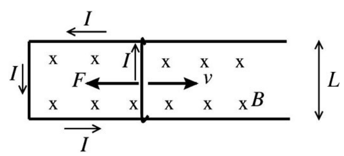



**Figure 29.61**

**EXECUTE:** When the wire has speed $v$ the induced emf is $\varepsilon = BvL$ and the induced current is $I=\varepsilon/R=\dfrac{BvL}{R}$.

The induced current flows upward in the wire as shown, so the force $\vec{F}=I\vec{l}\times\vec{B}$ exerted by the magnetic field on the induced current is to the left. $\vec{F}$ opposes the motion of the wire, as it must by Lenz's law. The magnitude of the force is $F=ILB=B^2L^2v/R$.
**(b) IDENTIFY and SET UP:** Apply $\sum\vec{F}=m\vec{a}$ to the wire. Take $+x$ to be toward the right and let the origin be at the location of the wire at $t=0$, so $x_0=0$.
**EXECUTE:** $\sum F_x = ma_x$ says $-F=ma_x$.

$$a_x=-\frac{F}{m}=-\frac{B^2L^2v}{mR}.$$

Use this expression to solve for $v(t)$:

$$a_x=\frac{dv}{dt}=-\frac{B^2L^2v}{mR}\text{ and }\frac{dv}{v}=-\frac{B^2L^2}{mR}dt.$$

$$\int_{v_0}^{v}\frac{dv'}{v'}=-\frac{B^2L^2}{mR}\int_0^t dt'.$$

$$\ln(v)-\ln(v_0)=-\frac{B^2L^2t}{mR}.$$

$$\ln\left(\frac{v}{v_0}\right)=-\frac{B^2L^2t}{mR}\text{ and }v=v_0e^{-B^2L^2t/mR}.$$

Note: At $t=0, v=v_0$ and $v\to 0$ when $t\to\infty$.

Now solve for $x(t)$:

$v = \dfrac{dx}{dt} = v_0 e^{-B^2L^2t/mR}$ so $dx = v_0 e^{-B^2L^2t/mR}dt.$

$\int_0^x dx' = \int_0^t v_0 e^{-B^2L^2t'/mR}dt'.$

$x = v_0\left(-\dfrac{mR}{B^2L^2}\right)\left[e^{-B^2L^2t'/mR}\right]_0^t = \dfrac{mRv_0}{B^2L^2}(1-e^{-B^2L^2t/mR}).$

Comes to rest implies $v = 0$. This happens when $t \to \infty$.

$t \to \infty$ gives $x = \dfrac{mRv_0}{B^2L^2}$. Thus this is the distance the wire travels before coming to rest.

**EVALUATE:** The motion of the slide wire causes an induced emf and current. The magnetic force on the induced current opposes the motion of the wire and eventually brings it to rest. The force and acceleration depend on $v$ and are constant. If the acceleration were constant, not changing from its initial value of $a_x = -B^2L^2v_0/mR$, then the stopping distance would be $x = -v_0^2/2a_x = mRv_0/2B^2L^2$. The actual stopping distance is twice this.

**29.62.** **IDENTIFY:** A bar moving in a magnetic field has an emf induced across its ends. The propeller acts as such a bar.
**SET UP:** Different parts of the propeller are moving at different speeds, so we must integrate to get the total induced emf. The potential induced across an element of length $dx$ is $d\varepsilon = vBdx$, where $B$ is uniform.
**EXECUTE:** **(a)** Call $x$ the distance from the center to an element of length $dx$, and $L$ the length of the propeller. The speed of $dx$ is $x\omega$, giving $d\varepsilon = vBdx = x\omega Bdx$. $\varepsilon = \int_0^{L/2} x\omega Bdx = \omega BL^2/8.$

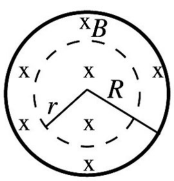
**(b)** The potential difference is zero since the potential is the same at both ends of the propeller.

**(c)** $\varepsilon = (2\pi \text{ rad/rev})\left(\dfrac{220 \text{ rev}}{60 \text{ s}}\right)(0.50\times10^{-4}\text{ T})\dfrac{(2.0\text{ m})^2}{8} = 5.8\times10^{-4}\text{ V} = 0.58\text{ mV}.$

**EVALUATE:** A potential difference of about $\frac{1}{2}$ mV is not large enough to be concerned about in a propeller.

**29.63.** **IDENTIFY:** Use $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to calculate the induced electric field at each point and then use $\vec{F} = q\vec{E}$.

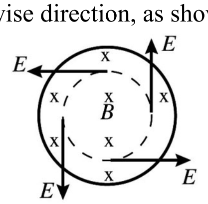
**SET UP:**




Apply $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to a concentric circle of radius $r$, as shown in Figure 29.63a. Take $\vec{A}$ to be into the page, in the direction of $\vec{B}$.

**Figure 29.63a**

**EXECUTE:** $B$ increasing then gives $\dfrac{d\Phi_B}{dt} > 0$, so $\oint \vec{E}\cdot d\vec{l}$ is negative. This means that $E$ is tangent to the circle in the counterclockwise direction, as shown in Figure 29.63b.



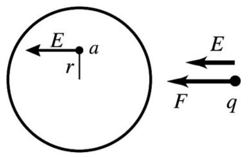

$\oint \vec{E}\cdot d\vec{l} = -E(2\pi r)$

$\dfrac{d\Phi_B}{dt} = \pi r^2 \dfrac{dB}{dt}$

**Figure 29.63b**

$-E(2\pi r) = -\pi r^2 \dfrac{dB}{dt}$ so $E = \frac{1}{2} r \dfrac{dB}{dt}$.

Point $a$: The induced electric field and the force on $q$ are shown in Figure 29.63c.

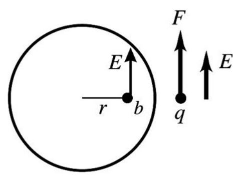



$F = qE = \frac{1}{2} qr \dfrac{dB}{dt}$.

$\vec{F}$ is to the left ($\vec{F}$ is in the same direction as $\vec{E}$ since $q$ is positive).

**Figure 29.63c**

Point $b$: The induced electric field and the force on $q$ are shown in Figure 29.63d.




$F = qE = \frac{1}{2} qr \dfrac{dB}{dt}$.

$\vec{F}$ is toward the top of the page.

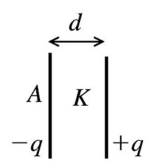
**Figure 29.63d**

Point $c$: $r = 0$ here, so $E = 0$ and $F = 0$.

**EVALUATE:** If there were a concentric conducting ring of radius $r$ in the magnetic field region, Lenz's law tells us that the increasing magnetic field would induce a counterclockwise current in the ring. This agrees with the direction of the force we calculated for the individual positive point charges.

**29.64. IDENTIFY** and **SET UP:** Apply Ohm's law to the dielectric to relate the current in the dielectric to the charge on the plates. Use $i = \dfrac{dq}{dt}$ for the current and obtain a differential equation for $q(t)$. Integrate this equation to obtain $q(t)$ and $i(t)$. Use $E = \dfrac{q}{\varepsilon A}$ and $j_D = \varepsilon \dfrac{dE}{dt}$ to calculate $j_D$.

**EXECUTE:** **(a)** Apply Ohm's law to the dielectric: The capacitor is sketched in Figure 29.64.




$i(t) = \dfrac{v(t)}{R}$.

$v(t) = \dfrac{q(t)}{C}$ and $C = K\dfrac{\varepsilon_0 A}{d}$.

**Figure 29.64**

$$v(t) = \left(\frac{d}{K\varepsilon_0 A}\right) q(t).$$

The resistance $R$ of the dielectric slab is $R = \rho d/A$. Thus $i(t) = \dfrac{v(t)}{R} = \left(\dfrac{q(t)d}{K\varepsilon_0 A}\right)\left(\dfrac{A}{\rho d}\right) = \dfrac{q(t)}{K\varepsilon_0 \rho}$. But the current $i(t)$ in the dielectric is related to the rate of change $dq/dt$ of the charge $q(t)$ on the plates by $i(t) = -dq/dt$ (a positive $i$ in the direction from the + to the – plate of the capacitor corresponds to a decrease in the charge). Using this in the above gives $-\dfrac{dq}{dt} = \left(\dfrac{1}{K\rho\varepsilon_0}\right) q(t)$. $\dfrac{dq}{q} = -\dfrac{dt}{K\rho\varepsilon_0}$. Integrate both sides of this equation from $t = 0$, where $q = Q_0$, to a later time $t$ when the charge is $q(t)$.

$\int_{Q_0}^{q}\frac{dq}{q}=-\left(\frac{1}{K\rho\varepsilon_0}\right)\int_0^t dt.$ $\ln\left(\frac{q}{Q_0}\right)=-\frac{t}{K\rho\varepsilon_0}$ and $q(t)=Q_0e^{-t/K\rho\varepsilon_0}$. Then $i(t)=-\frac{dq}{dt}=\left(\frac{Q_0}{K\rho\varepsilon_0}\right)e^{-t/K\rho\varepsilon_0}$ and $j_{\text{C}}=\frac{i(t)}{A}=\left(\frac{Q_0}{AK\rho\varepsilon_0}\right)e^{-t/K\rho\varepsilon_0}$. The conduction current flows from the positive to the negative plate of the capacitor.

**(b)** $E(t)=\frac{q(t)}{\varepsilon A}=\frac{q(t)}{K\varepsilon_0 A}$.

$j_{\text{D}}(t)=\varepsilon\frac{dE}{dt}=K\varepsilon_0\frac{dE}{dt}=K\varepsilon_0\frac{dq(t)/dt}{K\varepsilon_0 A}=-\frac{i_{\text{C}}(t)}{A}=-j_{\text{C}}(t).$

The minus sign means that $j_{\text{D}}(t)$ is directed from the negative to the positive plate. $\vec{E}$ is from + to – but $dE/dt$ is negative ($E$ decreases) so $j_{\text{D}}(t)$ is from – to +.

**EVALUATE:** There is no conduction current to and from the plates so the concept of displacement current, with $\vec{j}_{\text{D}}=-\vec{j}_{\text{C}}$ in the dielectric, allows the current to be continuous at the capacitor.

**29.65. IDENTIFY:** Apply $i_{\text{D}}=\varepsilon\frac{d\Phi_E}{dt}$.

**SET UP:** $\varepsilon=3.5\times10^{-11}$ F/m.

**EXECUTE:** $i_{\text{D}}=\varepsilon\frac{d\Phi_E}{dt}=(3.5\times10^{-11}\text{ F/m})(24.0\times10^{3}\text{ V}\cdot\text{m/s}^3)t^2$. $i_{\text{D}}=21\times10^{-6}$ A gives $t=5.0$ s.

**EVALUATE:** $i_{\text{D}}$ depends on the rate at which $\Phi_E$ is changing.

**29.66. IDENTIFY:** Faraday's law and Ohm's law both apply. The flux change is due to the changing magnetic field.

**SET UP:** $\varepsilon=\left|\frac{d\Phi_B}{dt}\right|$ and $V = IR$, where $V = \varepsilon$ since it is caused by the changing flux. Since the flux change is due only to the change in $B$, we have $\varepsilon=\left|\frac{d\Phi_B}{dt}\right| = AN\left|\frac{dB}{dt}\right|$, where $N$ is the number of turns.

**EXECUTE:** **(a)** Combining Ohm's law and Faraday's law and dropping the absolute value signs gives
$\frac{dB}{dt}=\frac{\varepsilon}{AN}=\frac{RI}{AN} \rightarrow dB=\frac{RI}{AN}dt.$

Integrating gives $\Delta B_{0\to2}=\frac{R}{AN}\int_0^{2.00\text{ s}}Idt$. The integral is the area under the curve in the $i$-versus-$t$ graph shown with the problem. We can get that using simple geometry on the graph.
area = integral = (1/2)(2.00 s)(3.00 mA) = 0.00300 A · s.
The field starts out with zero magnitude, so at 2.00 s it is
$B = R$(integral)$/AN$ = (0.250 Ω)(0.00300 A · s )/[$\pi$(0.00800 m)$^2$(4)] = 0.9325 T, which rounds to 0.933 T.

**(b)** We use the same geometric approach as in part (a).
$\Delta B_{2\to5} = R$(area from 2.00 s to 5.00 s)$/AN$ = (0.250 Ω)(3.00 mA)(3.0 s)/[$\pi$(0.00800 m)$^2$(4)] = 2.798 T.

$B_5 = B_2 + \Delta B_{2\to5}$ = 0.9325 T + 2.798 T = 3.73 T.

**(c)** The area under the curve from 5.00 s to 6.00 s is half the area from 0.00 s to 2.00 s, so
$\Delta B_{5\to6}$ = ½ $\Delta B_{0\to2}$ = (0.9325 T)/2 = 0.46625 T.

$B_6 = B_5 + \Delta B_{5\to6}$ = 3.73 T + 0.46625 T = 4.20 T.

**EVALUATE:** Careful! Just because the current $i$ is constant between 2.0 s and 5.0 s does *not* mean that $B$ is constant since $i$ is induced by a changing $B$. A constant $i$ just means that $B$ is changing at a constant rate.

**29.67. IDENTIFY:** An emf is induced across the moving metal bar, which causes current to flow in the circuit. The magnetic field exerts a force on the moving bar due to the current in it, which causes acceleration of the bar. Newton's second law applies to the accelerating bar. Ohm's law applies to the resistor in the circuit.

**SET UP:** The induced potential across the moving bar is $\varepsilon = vBL$, the magnetic force on the bar is $F_{mag} = ILB$, and Ohm's law is $\varepsilon = IR$. Newton's second law is $\Sigma\vec{F} = m\vec{a}$, and $a_x = dv_x/dt$. The flux through the loop is increasing, so the induced current is counterclockwise. Alternatively, the magnetic force $\vec{F} = q\vec{v}\times\vec{B}$ on positive charge in the moving bar is upward, by the right-hand rule, which also gives a counterclockwise current. So the magnetic force on the bar is to the left, opposite to the velocity of the bar.

**EXECUTE:** **(a)** Combining the equations discussed in the set up, the magnetic force on the moving bar is $F_{mag} = ILB = (\varepsilon/R)LB = (vBL/R)LB = v(BL)^2/R$.

Newton's second law gives

$F_{mag} = ma.$

$ma = v(BL)^2/R.$

$$a = \frac{(BL)^2}{mR}v.$$

A graph of $a$ versus $v$ should be a straight line having slope equal to $(BL)^2/mR$. The graph of $a$ versus $v$ is shown in Figure 29.67. The best-fit slope of this graph is 0.3071 $s^{-1}$.

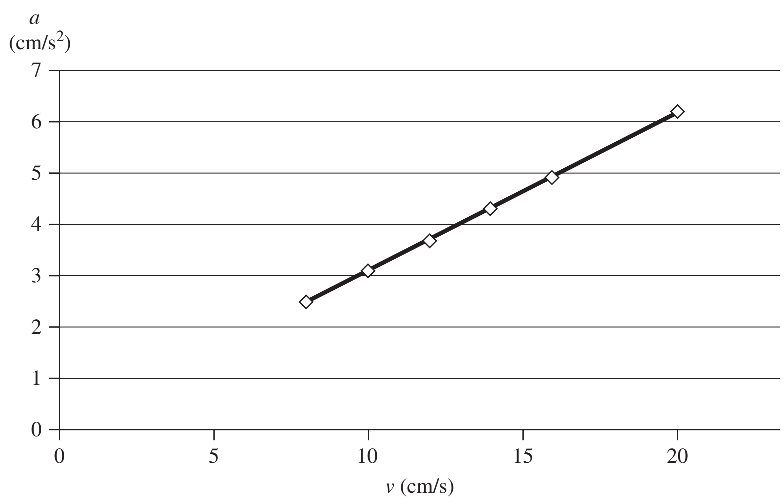



**Figure 29.67**

**(b)** $(BL)^2/mR$ = slope, so $B = \sqrt{\frac{(\text{slope})mR}{L^2}} = \sqrt{\frac{(0.3071\ \text{s}^{-1})(0.200\ \text{kg})(0.800\ \Omega)}{(0.0600\ \text{m})^2}} = 3.69\ \text{T}.$

**(c)** The current flows in a counterclockwise direction in the circuit. Therefore the charges lose potential energy as they pass through the resistor $R$ from $a$ to $b$, which makes point $a$ at a higher potential than $b$.

**(d)** We know that $a_x = dv_x/dt$, and in part (a) we found that the magnitude of the acceleration is $a = \frac{(BL)^2}{mR}v$. We also saw that $a$ is opposite to $v$, so $a_x = -\frac{(BL)^2}{mR}v$. Therefore $\frac{dv}{dt} = -\frac{(BL)^2}{mR}v$. Separating variables and integrating gives

$$\int_{20.0\ \text{cm/s}}^{10.0\ \text{cm/s}} \frac{dv}{v} = -\int_0^t \frac{(BL)^2}{mR}dt'.$$

$$\ln\left(\frac{10}{20}\right) = -\frac{(BL)^2}{mR}t.$$

$$t = -\frac{mR}{(BL)^2}\ln(1/2) = -(0.200\ \text{kg})(0.800\ \Omega)(\ln ½)/[(3.69\ \text{T})(0.0600\ \text{m})]^2 = 2.26\ \text{s}.$$

**EVALUATE:** We cannot use the standard kinematics formulas because the acceleration is not constant.

**29.68.** **IDENTIFY:** The 8.00-cm long left side of the loop is a bar moving in a magnetic field, so an emf is induced across its ends. This emf causes current to flow through the loop, and the external magnetic field exerts a force on this bar due to the current in it. Ohm's law applies to the circuit and Newton's second law applies to the loop.

**SET UP:** The induced potential across the left-end side is $\varepsilon = vBL$, the magnetic force on the 8.00-cm bar is $F_{\text{mag}} = ILB$, and Ohm's law is $\varepsilon = IR$. Newton's second law is $\Sigma\vec{F} = m\vec{a}$. The flux through the loop is decreasing, so the induced current is counterclockwise to oppose this decrease. Alternatively, the magnetic force on positive charge in the moving left-end segment is downward, by the right-hand rule, which also gives a counterclockwise current. Therefore the magnetic force on the 8.00-cm segment is to the left, opposite to the velocity and the external $\vec{F}$. Since the speed of the loop is constant, the external force is equal in magnitude to the magnetic force, so $F_{\text{mag}} = F$.

**EXECUTE:** **(a)** Combining the equations discussed in the set up, the magnetic force on the 8.00-cm bar (and on the loop) is $F = F_{\text{mag}} = ILB = (\varepsilon/R)LB = (vBL/R)LB = v(BL)^2/R$, so $F = v(BL)^2/R$. Therefore a graph of $F$ versus $v$ should be a straight line having slope equal to $(BL)^2/R$. Figure 29.68 shows a graph of $F$ versus $v$. The best-fit slope of the line in this graph is 0.0520 N/(cm/s) = 5.20 N·s/m.

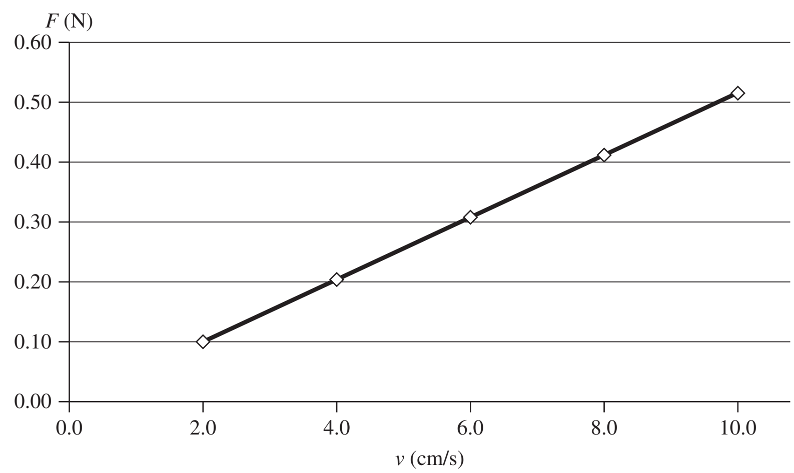



**Figure 29.68**

**(b)** Since $(BL)^2/R$ = slope, we solve for $B$ and have

$$B = \sqrt{\frac{R(\text{slope})}{L^2}} = \sqrt{\frac{(0.00400\ \Omega)(5.20\ \text{N}\cdot\text{s/m})}{(0.0800\ \text{m})^2}} = 1.80\ \text{T}.$$

**(c)** The magnetic flux is decreasing through the loop, so the induced current must flow counterclockwise to oppose the decrease.

**(d)** $P = Fv = \dfrac{(BL)^2 v}{R} v = \dfrac{(BLv)^2}{R} = [(1.80\ \text{T})(0.0800\ \text{m})(0.0500\ \text{m/s})]^2/(0.00400\ \Omega) = 0.0130\ \text{W} = 13.0\ \text{mW}.$

**EVALUATE:** For (d) we could use $P = I^2R = (vBL/R)^2/R = (vBL)^2/R$, the same result we got.

**29.69.** **IDENTIFY:** The motion of the bar produces an induced current and that results in a magnetic force on the bar.

**SET UP:** $\vec{F}_B$ is perpendicular to $\vec{B}$, so is horizontal. The vertical component of the normal force equals $mg\cos\phi$, so the horizontal component of the normal force equals $mg\tan\phi$.

**EXECUTE:** **(a)** As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from $a$ to $b$.

$$F_B = iLB = \frac{LB}{R}\varepsilon = \frac{LB}{R}\frac{d\Phi_B}{dt} = \frac{LB}{R}(B\cos\phi)\frac{dA}{dt} = \frac{LB^2}{R}(vL\cos\phi) = \frac{vL^2B^2}{R}\cos\phi.$$

**(b)** At the terminal speed the horizontal forces balance, so $mg\tan\phi = \frac{v_t L^2 B^2}{R}\cos\phi$ and $v_t = \frac{Rmg\tan\phi}{L^2B^2\cos\phi}$.

**(c)** $i = \frac{\varepsilon}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{1}{R}(B\cos\phi)\frac{dA}{dt} = \frac{B}{R}(v_t L\cos\phi) = \frac{v_t LB\cos\phi}{R} = \frac{mg\tan\phi}{LB}$.

**(d)** $P = i^2R = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}$.

**(e)** $P_g = Fv_t\cos(90° - \phi) = mg\left(\frac{Rmg\tan\phi}{L^2B^2\cos\phi}\right)\sin\phi$ and $P_g = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}$.

**EVALUATE:** The power in part (e) equals that in part (d), as is required by conservation of energy.

**29.70.** **IDENTIFY:** A current is induced in the loop because of its motion and because of this current the magnetic field exerts a torque on the loop.

**SET UP:** Each side of the loop has mass $m/4$ and the center of mass of each side is at the center of each side. The flux through the loop is $\Phi_B = BA\cos\phi$.

**EXECUTE:** **(a)** $\vec{\tau}_g = \sum \vec{r}_{cm} \times m\vec{g}$ summed over each leg.

$$\tau_g = \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + (L)\left(\frac{m}{4}\right)g\sin(90° - \phi).$$

$\tau_g = \frac{mgL}{2}\cos\phi$ (clockwise).

$\tau_B = \left|\vec{\tau}\times\vec{B}\right| = IAB\sin\phi$ (counterclockwise).

$I = \frac{\varepsilon}{R} = -\frac{BA}{R}\frac{d}{dt}\cos\phi = \frac{BA}{R}\frac{d\phi}{dt}\sin\phi = \frac{BA\omega}{R}\sin\phi$. The current is going counterclockwise looking to the $-\hat{k}$-direction. Therefore, $\tau_B = \frac{B^2A^2\omega}{R}\sin^2\phi = \frac{B^2L^4\omega}{R}\sin^2\phi$. The net torque is

$\tau = \frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi$, opposite to the direction of the rotation.

**(b)** $\tau = I\alpha$ ($I$ being the moment of inertia). About this axis $I = \frac{5}{12}mL^2$. Therefore,

$$\alpha = \frac{12}{5}\frac{1}{mL^2}\left[\frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi\right] = \frac{6g}{5L}\cos\phi - \frac{12B^2L^2\omega}{5mR}\sin^2\phi.$$

**EVALUATE:** **(c)** The magnetic torque slows down the fall (since it opposes the gravitational torque).
**(d)** Some energy is lost through heat from the resistance of the loop.

**29.71.** **IDENTIFY** and **SET UP:** Apply Lenz's law to determine the direction of the induced current. The figure shows the current pulse in the coil is in the counterclockwise direction as viewed from above. Also, the figure shows that direction-1 for the induced current is clockwise and direction-2 is counterclockwise.

**EXECUTE:** As the current pulse increases, it produces an increasing upward magnetic field in the brain. To oppose the increasing flux, the induced current must flow clockwise (direction-1). As the current pulse decreases its upward magnetic field decreases and the induced current must flow counterclockwise (direction-2) to oppose this. The correct choice is (c).

**EVALUATE:** Although the brain is made up of tissue, in some ways it behaves like a resistor and allows current to flow in it.

**29.72.** **IDENTIFY** and **SET UP:** Apply Faraday's law, $\varepsilon = \left|\frac{d\Phi_B}{dt}\right|$.

**EXECUTE:** $\varepsilon = \left|\frac{d\Phi_B}{dt}\right| = d(BA)/dt = A\,dB/dt$. The greater the area, the greater the flux and hence the greater the rate of change of the flux in a given time. Therefore the largest area will have the greatest induced emf, and this is the periphery of the dashed line, which is choice (b).

**EVALUATE:** Only the field is changing, but the flux depends on the field *and* the area.

**29.73.** **IDENTIFY** and **SET UP:** Faraday's law gives $\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right| = \frac{d(B_{\text{av}}A)}{dt} = A\frac{dB_{\text{av}}}{dt}$. $d(B_{\text{av}}A)/dt = A\ dB_{\text{av}}/dt$. The quantity $dB_{\text{av}}/dt$ is the slope in a $B$-versus-$t$ graph, so the induced emf is greatest when the slope is steepest. Ohm's law gives $\mathcal{E} = IR$, so the current will be greatest when $\mathcal{E}$ is the greatest, which is where the slope of the $B$-versus-$t$ graph is the greatest.

**EXECUTE:** We need to compare the slopes of graphs A and B with the slope of the graph in part (b) of the introduction to this set of passage problems. The graph in part (b) rises to 4 T in about 0.15 ms. In Figure P29.73, graph A rises to 4 T in less than 0.1 ms, and graph B also reaches 4 T in less than 0.1 ms. Therefore both graphs A and B have steeper slopes than the graph in part (b), so both of them would achieve a larger current than the process shown by the graph in part (b). This makes choice (c) correct.

**EVALUATE:** It is not the magnitude of the magnetic field that induces potential, but rather the *rate* at which the field changes.

**29.74.** **IDENTIFY** and **SET UP:** Faraday's law gives $\mathcal{E} = -\frac{d\Phi_B}{dt} - \frac{d(B_{\text{av}}A)}{dt} = -A\frac{dB_{\text{av}}}{dt}$. Ohm's law gives $\mathcal{E} = IR$, so the current is proportional to the rate at which the magnetic field is changing. That is, the current is proportional to the slope of the $B$-$t$ graph.

**EXECUTE:** From the graph in part (b) of the figure shown with the introduction to the passage problems, we see that the magnetic field first increases rapidly as the graph has a positive slope. It then reaches a maximum value at around 0.15 ms, and then gradually decreases and the graph has a negative slope that approaches zero. Since the current is proportional to the slope of the $B$-$t$ graph, the current is initially positive, then curves down to zero when the $B$-$t$ graph is a maximum and becomes negative as the slope becomes negative, and it then gradually approaches zero as the slope approaches zero. Graph C most closely describes this behavior, so (c) it is the best choice.

**EVALUATE:** From Faraday's law, we see that the current depends on the rate at which $B$ changes, not on the magnitude of $B$.