Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF PHYSICS

Junior Freshman
Annual Examination

Trinity Term 2017

X-PY1P20-1

Physics, Paper 2

(Science (Physics), Nanoscience Physics and Chemistry of Advanced Materials, Chemistry with Molecular Modelling and Theoretical Physics)

Wednesday 17 May 2017 Sports Centre 14:00 – 17:00

Professors P. Gallagher, Graham Cross and J. Pethica

ALL QUESTIONS CARRY EQUAL MARKS

USE SEPARATE ANSWER BOOKS FOR EACH SECTION

Booklets of Formulae and Tables are available from the invigilator for all students who require them. Graph paper is also available.

Non-programmable calculators are permitted for this examination – please indicate the make and model of your calculator on each answer book used.

All Students

Answer ***SIX*** questions, AT LEAST **ONE** from Section A, AT LEAST **TWO** from Section B, AT LEAST **TWO** from Section C AND **ONE** OTHER, in 3 hours.

EQUATION LIST:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}, \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}, \;\; \vec{\tau} = \vec{p}\times\vec{E}, \; \Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \;\; V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi \; dl$$

$$C = \frac{Q}{V_{ab}} = \varepsilon_0\frac{A}{d}, \;\; C = \frac{Q}{V_{ab}} = \varepsilon_0\frac{A}{d}$$

$$I = \frac{dQ}{dt} = n|q|v_d A, \;\; R = \frac{\rho L}{A}, \;\; P = V_{ab}I = I^2 R = \frac{V_{ab}^{\;2}}{R}$$

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

$$\vec{F} = q\vec{v}\times\vec{B}, \; \Phi_B = \int B_\perp \, dA = \int B\cos\phi \, dA = \int \vec{B}\cdot d\vec{A}, \; \vec{F} = I\vec{l}\times\vec{B}, \; \vec{\tau} = \vec{\mu}\times\vec{B}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}, \; d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}, \; B_x = \frac{\mu_0 NI}{2a}, \; \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \;\; \mathcal{E} = \oint(\vec{v}\times\vec{B})\cdot d\vec{l}, \;\; \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

SECTION A

1.

(a) Describe the main components of the Milky Way Galaxy using a diagram to illustrate your answer.

[3 marks]

(b) Galaxies do not rotate as solid disks. Explain this statement using a galactic rotation curve to justify your answer. How does this provide evidence for dark matter?

[3 marks]

(c) The galaxy M33 is 71 arcminutes across and lies at a distance of 2.494 million light years (1 light year = 9.461×10^{15} m). Calculate its radius in metres. If the outer portions of the galaxy are in centripetal motion about the galactic centre at 1.58×10^{8} m/s, calculate the mass of the galactic centre in solar masses.

[4 marks]

2.

(a) Describe the classifications of stars based on their spectra and how this leads to the Hertzprung-Russell (H-R) diagram. Sketch the H-R diagram, labelling the main sequence and the positions of the Sun, red supergiants and white dwarfs.

[2 marks]

(b) Wien's Law ($T = 2.987 \times 10^{-3} / \lambda_{max}$) can be used to estimate the temperature of a star's surface. Explain this relation using a sketch of black body spectra for various temperatures. Use examples of stars or differing colours to illustrate your answer.

[2 marks]

(c) Given the temperature and radius of a star, its luminosity can be calculated using the Stefan-Boltzmann Law:

$$L = A \sigma T^4$$

where A is the area, σ is the Stefan-Boltzmann constant and T is the temperature.

i. Calculate the luminosity (in W) of the Sun.

ii. Calculate the flux (W/m^2) at the orbit of the Earth.

Comment on this.

[6 marks]

SECTION B

3. Cylindrical Capacitor

(a) State Gauss's Law in words and formula.

[1 mark]

(b) Two concentric cylindrical conducting shells of length L are separated by a vacuum. The inner shell has surface charge density $+\sigma$ and radius r_a. The outer shell has radius r_b. Using Gauss' Law, as a function of radius r find: The direction and magnitude of electric field inside and outside the shells. Be sure to clearly state the Gaussian surfaces that you are using. Find an expression for the voltage between the shells.

[5 marks]

(c) What is the capacitance of the cylindrical capacitor?

[2 marks]

(d) The capacitor of part (b) is wired in series to a resistor R and allowed to discharge. If r_b = *2 mm*, r_a = *20 mm*, $L = 100\,mm$, σ = *20 nC* m^{-2}, and R = *1000* Ω, how long does it take for the voltage on the capacitor to reach half the original value?

[2 marks]

4. Circuits

(a) Discuss the difference between the drift velocity in a current carrying wire and individual electron velocity, and why this difference exists.

[1 mark]

(b) You are to construct a circular capacitor with a dielectric sheet of thickness *1 mm* and dielectric constant K=2.2. For a 12 pF capacitor, what radius of dielectric sheet is required?

[4 marks]

(c) The capacitor you have made is charged to 5 μC. What is the voltage across the plates?

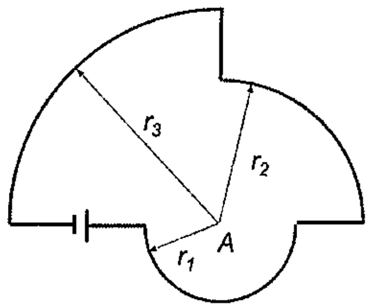
[2 marks]

(d) Now you discharge the capacitor through a resistor. If it takes 200 ns to discharge to one half the voltage, what is the value of the resistor?

[3 marks]

5.

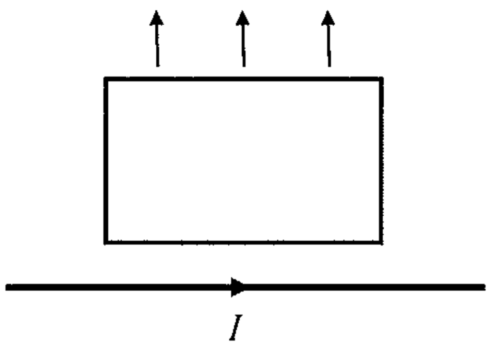
(a) The circuit pictured below consists of three circular arcs with radii $r_3 = 3r_1$, $r_2 = 2r_1$, and r_1 connected by radial paths all concentric to point A. If the total resistance of the wire is $R = 5\,\Omega$ and an EMF source of voltage $V = 100V$ is connected as shown, what is the magnitude and direction of the magnetic field at A if $r_1 = 5\,m$?



[7 marks]

(b) A long, straight wire carries a steady current *I*. A rectangular conducting loop lies in a plane shared by the wire, with two sides parallel to the wire and two sides perpendicular. If the loop is moved **away from** the wire as shown, what is the direction of the induced current in the loop? Explain your answer.

[3 marks]



SECTION C

6.

(a) What is the Photoelectric effect?

Light of wavelength 250nm causes electrons of maximum energy of 1eV to be emitted from a metal surface. What light wavelength is required to cause emitted electrons from the same metal surface to have a maximum energy of 2eV?

Briefly explain why there is a range of emitted electron energies below these maxima.

[3 marks]

(b) What is meant by the Uncertainty Principle? Give the formulae for the cases of momentum and of energy.

A particle of mass m is confined within a rigid walled 1-dimensional box of length L. Use the Uncertainty Principle to estimate the energy of the ground state of the particle.

Derive the full expression for the energy levels of the particle in the box and compare the ground state energy with that estimated from the Uncertainty Principle.

[5 marks]

(c) What is the de Broglie relation? A beam of electrons of energy 40eV is incident perpendicularly on a 2-D crystalline sheet of atoms. At what angle will they be diffracted back from the sheet if the lattice parameter of the sheet is 0.5nm?

[2 marks]

7.

(a) What conditions are placed on the functional form of the wavefunction ψ in the steady state Schroedinger equation?

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - U\right)\psi = 0$$

How is physical meaning assigned to ψ? What is meant by normalisation? Give the general form of solutions for ψ when E > U and for when U > E and show that they satisfy the Schroedinger equation.

[3 marks]

(b) Particles of kinetic energy E are incident on a potential barrier of width *d* and of height U such that U > E. What is the form of the wavefunction ψ inside the potential barrier? Find a simplified expression for the probability of particles being transmitted through the barrier by considering the value of the wavefunction inside the barrier at entry compared to its value at exit. A full calculation with boundary matching is not required.

[3 marks]

(c) Two pieces of metal having workfunctions of 4eV are brought to within 0.5nm of each other. Using your expression derived above, estimate the probability of an electron arriving at the surface in one metal tunneling into the other.

[2 marks]

(d) Briefly describe two other physical phenomena involving tunneling through a potential barrier.

[2 marks]

8.

(a) What experimental observations show that a purely classical model of the hydrogen atom is inadequate?

[2 marks]

(b) Explaining the concepts required, outline the derivation of the Bohr expression for the energy levels of the hydrogen atom

$$E_n = -\frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n^2}\right) = E_1\left(\frac{1}{n^2}\right)$$

[3 marks]

(c) What is meant by the Pauli Exclusion Principle, and to what types of particle does it apply?

[2 marks]

(d) Outline the additional features introduced by the full 3-D solution of Schroedinger's equation for the atom, and how, with the Pauli Principle, they lead to the structure of the periodic table of the elements.

[3 marks]

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Electron rest mass	m_e	9.11 x 10^{-31} kg
Proton rest mass	m_p	1.67 x 10^{-27} kg
Electronic charge	e	1.60 x 10^{-19} C
Speed of light in free space	c	3.00 x 10^{8} m s^{-1}
Planck's constant	h	6.63 x 10^{-34} J s
	$\hbar = h/2\pi$	1.05 x 10^{-34} J s
Boltzmann's constant	k	1.38 x 10^{-23} J K^{-1}
Molar gas constant	R	8.31 J K^{-1} mol^{-1}
Avogadro's number	N_A	6.02 x 10^{23} mol^{-1}
Standard molar volume		22.4 x 10^{-3} m^3
Bohr magneton	μ_B	9.27 x 10^{-24} A m^2 <u>OR</u> J T^{-1}
Nuclear magneton	μ_N	5.05 x 10^{-27} A m^2 <u>OR</u> J T^{-1}
Bohr radius	a_o	5.29 x 10^{-11} m
Fine structure constant	$e^2/(4\pi\varepsilon_0\hbar c)$	$(137)^{-1}$
Rydberg's constant	R_∞	1.10 x 10^{7} m^{-1}
Stefan's constant	σ	5.67 x 10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67 x 10^{-11} N m^2 kg^{-2}
Proton magnetic moment	μ_p	2.79 μ_N
Neutron magnetic moment	μ_n	-1.91 μ_N
Permeability of free space	μ_o	4π x 10^{-7} H m^{-1}
Permittivity of free space	ε_o	8.85 x 10^{-12} F m^{-1}
1 electron volt (1 eV)		1.60 x 10^{-19} J
1 unified atomic mass unit (^{12}C scale)		1.66 x 10^{-27} kg = 931 MeV/c^2
Wavelength of 1 eV photon		1.24 x 10^{-6} m
1 atmosphere		1.01 x 10^{5} N m^{-2}
Acceleration due to gravity	g	9.8 m s^{-2}
Free space impedance	Z_0	377 Ω

Astronomical unit (1 au)		1.50×10^{11} m
Parsec (1 pc)		3.09×10^{16} m
Solar radius	$R_\odot$	6.96×10^{8} m
Solar mass	$M_\odot$	1.99×10^{30} kg
Solar luminosity	$L_\odot$	3.85×10^{26} W
Earth mass	$M_\oplus$	5.97×10^{24} kg
Earth radius (equatorial)	$R_\oplus$	6.378×10^{6} m