JF PY1T10 Special Relativity

Lecture 11:

Collisions & Scattering

Summary of Lecture 10

Pair Production:

Electromagnetic radiation can be converted into matter.

Cannot occur in empty space.

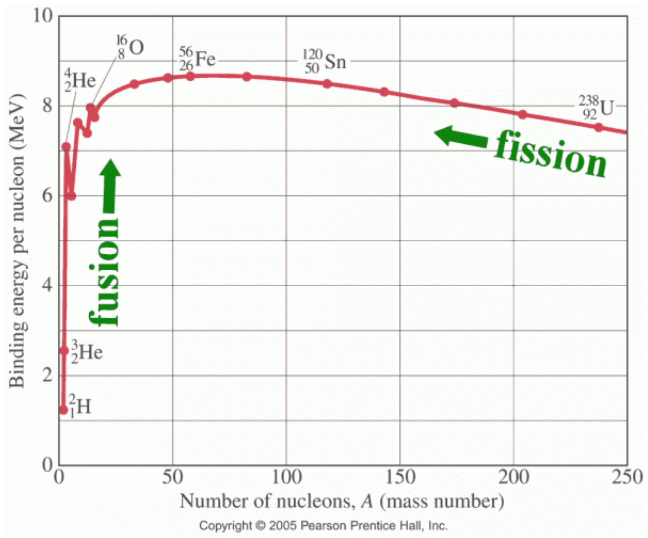
Need to conserve energy, charge, momentum.

Rest mass of an electron is 0.51 MeV. Hence, pair production requires at least 1.02 MeV. Any addition energy becomes the kinetic energy of the particle pair.

Nuclear Fission & Fusion:

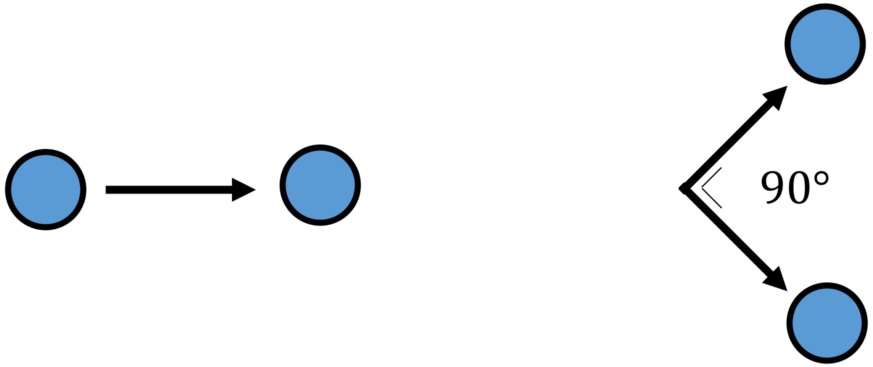
The curve reaches a peak of about 8.8 MeV/nucleon at A=62 (corresponding to the element Nickel).

When you break up a heavy nucleus or put light nuclei together, energy is released.



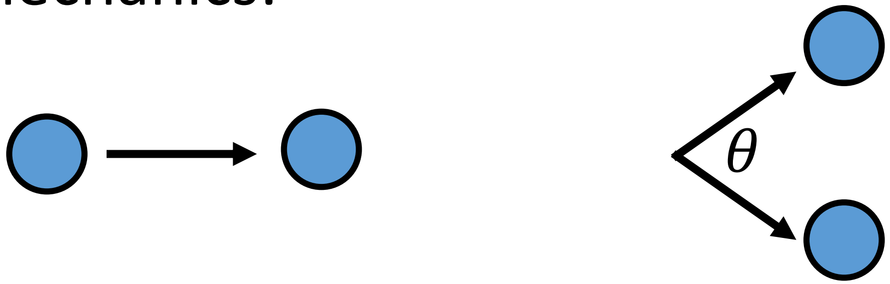
Elastic scattering of identical particles

In Newtonian mechanics (non head-on collision):



Prove this.

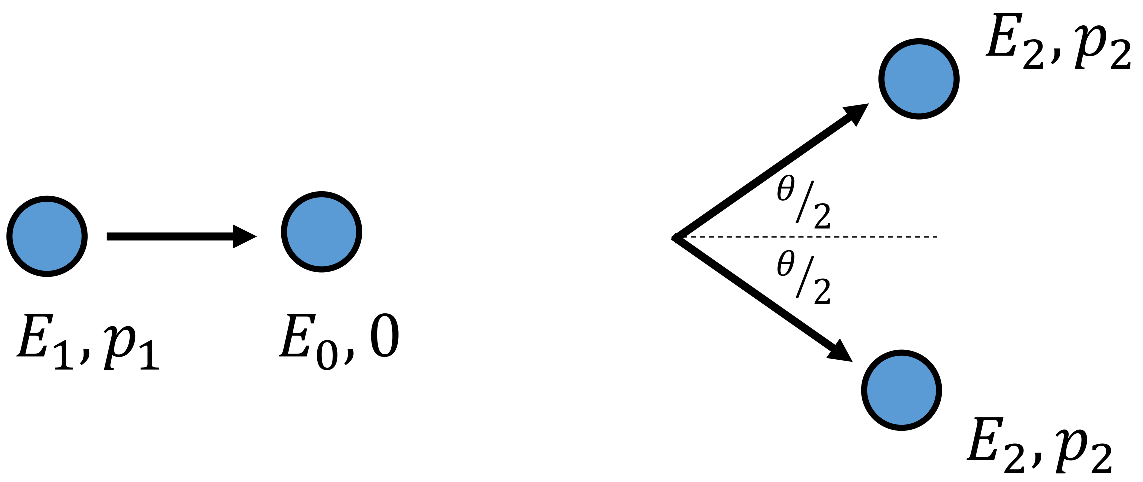
But in relativistic mechanics:



The angle is $< \ 90°$ due to increase of mass with v.

i.e., a squeezing in forward direction.

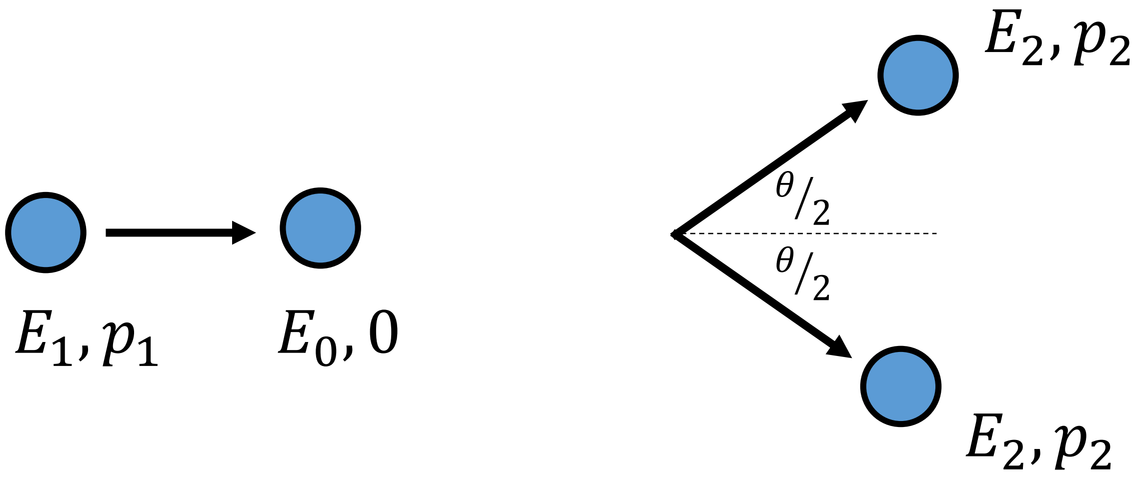
Elastic scattering of identical particles



Elastic Collision: Particles have the same energy, $E_0 = mc^2$ after the collision.

Consider the special case, where after collision the particles travel at equal angles to direction of incident particle (viewed in lab frame)

Elastic scattering of identical particles



Conservation of Energy:

$$E_1 + E_0 = 2E_2 \quad ①$$

Conservation of Momentum: ②

$$p_1 = 2p_2 \cos\frac{\theta}{2}$$

Introduce kinetic energy of incident particle:

$$E_1 = E_0 + K_1 \quad ④$$

Using

$$E^2 - p^2c^2 = m_0^2c^4 = E_0^2:$$

$$\left.\begin{aligned} E_1^2 - E_0^2 &= p_1^2c^2 \\ E_2^2 - E_0^2 &= p_2^2c^2 \end{aligned}\right\} \quad ③$$

Elastic scattering of identical particles

③ & ④:

$$\boxed{p_1^2c^2 = (E_0 + K_1)^2 - E_0^2 = K_1(2E_0 + K_1)}$$

③,④ & ①:

$$p_2^2c^2 = E_2^2 - E_0^2 = \left[\frac{1}{2}(E_1 + E_0)\right]^2 - E_0^2$$
$$= \frac{1}{4}(E_0 + K_1 + E_0)^2 - E_0^2$$
$$= \frac{1}{4}(2E_0 + K_1)^2 - E_0^2$$
$$= \frac{1}{4}(4E_0^2 + 4E_0K_1 + K_1^2) - E_0^2$$
$$= E_1K_1 + \frac{1}{4}K_1^2$$

$$\boxed{p_2^2c^2 = K_1\left(E_0 + \frac{K_1}{4}\right)}$$

Elastic scattering of identical particles

Using ②:

$$\left(2p_2\cos\frac{\theta}{2}\right)^2 c^2 = K_1(2E_0+K_1)$$

$$4p_2^2\cos^2\frac{\theta}{2}c^2 = K_1(2E_0+K_1)$$

$$\cos^2\frac{\theta}{2} = \frac{2E_0+K_1}{4p_2^2c^2\frac{1}{K_1}} = \frac{2E_0+K_1}{4K_1\left(E_0+\frac{K_1}{4}\right)\frac{1}{K_1}}$$

$$\cos^2\frac{\theta}{2} = \frac{2E_0+K_1}{4E_0+K_1}$$

Elastic scattering of identical particles

Using $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$ so that $\cos^2\frac{\theta}{2} = \frac{1}{2}(\cos\theta + 1)$:

$$\frac{1}{2}(\cos\theta + 1) = \frac{2E_0 + K_1}{4E_0 + K_1}$$

$$\cos\theta = \frac{4E_0 + 2K_1}{4E_0 + K_1} - 1$$

$$= \frac{4E_0 + 2K_1 - 4E_0 - K_1}{4E_0 + K_1}$$

$$= \frac{K_1}{4E_0 + K_1}$$

Elastic scattering of identical particles

$$\boxed{\theta = \cos^{-1}\left(\frac{K_1}{4E_0 + K_1}\right)}$$

In the Newtonian case:

$$K_1 \ll E_0 \;\therefore\; \theta = \frac{\pi}{2} = 90°$$

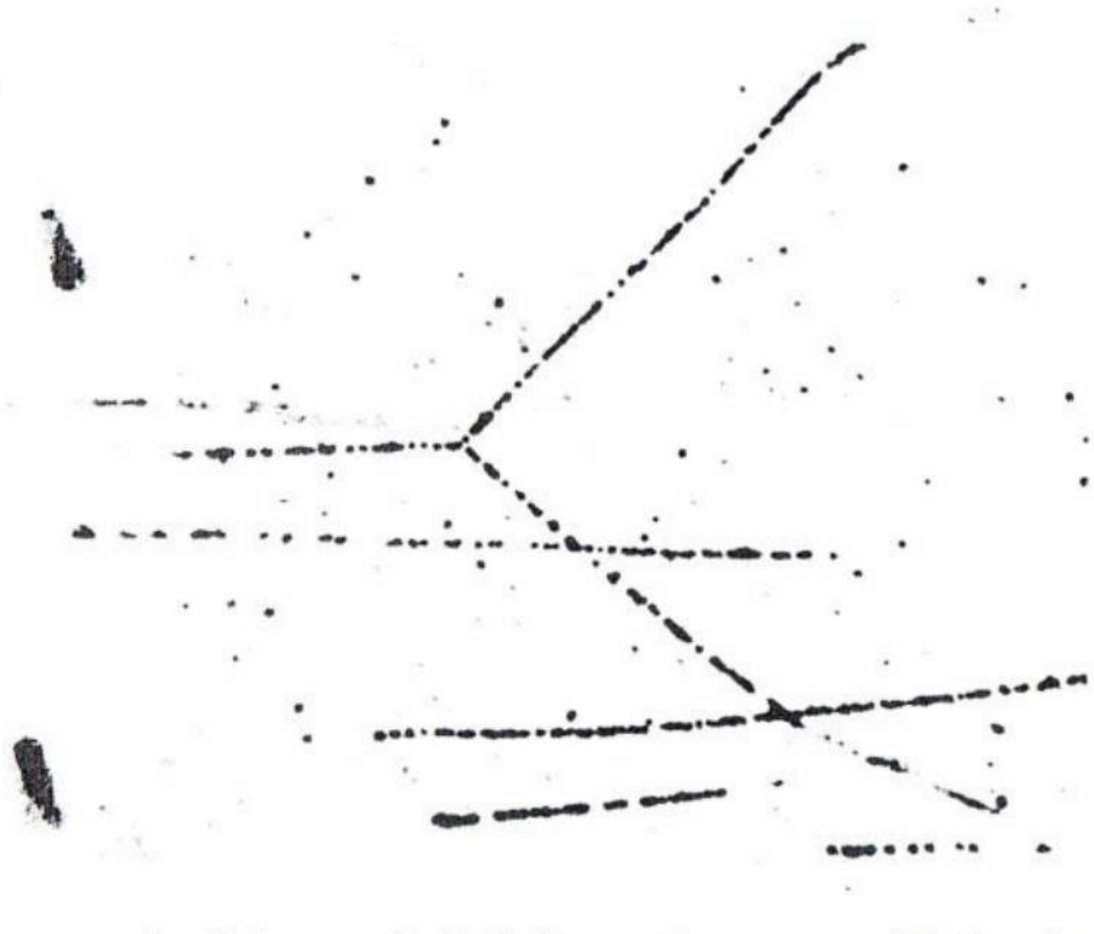
In the Relativistic case:

$$\theta < 90°$$

Note: For a proton $E_0 = m_0 c^2 = 900\ \mathrm{MeV}$

Elastic scattering of identical particles

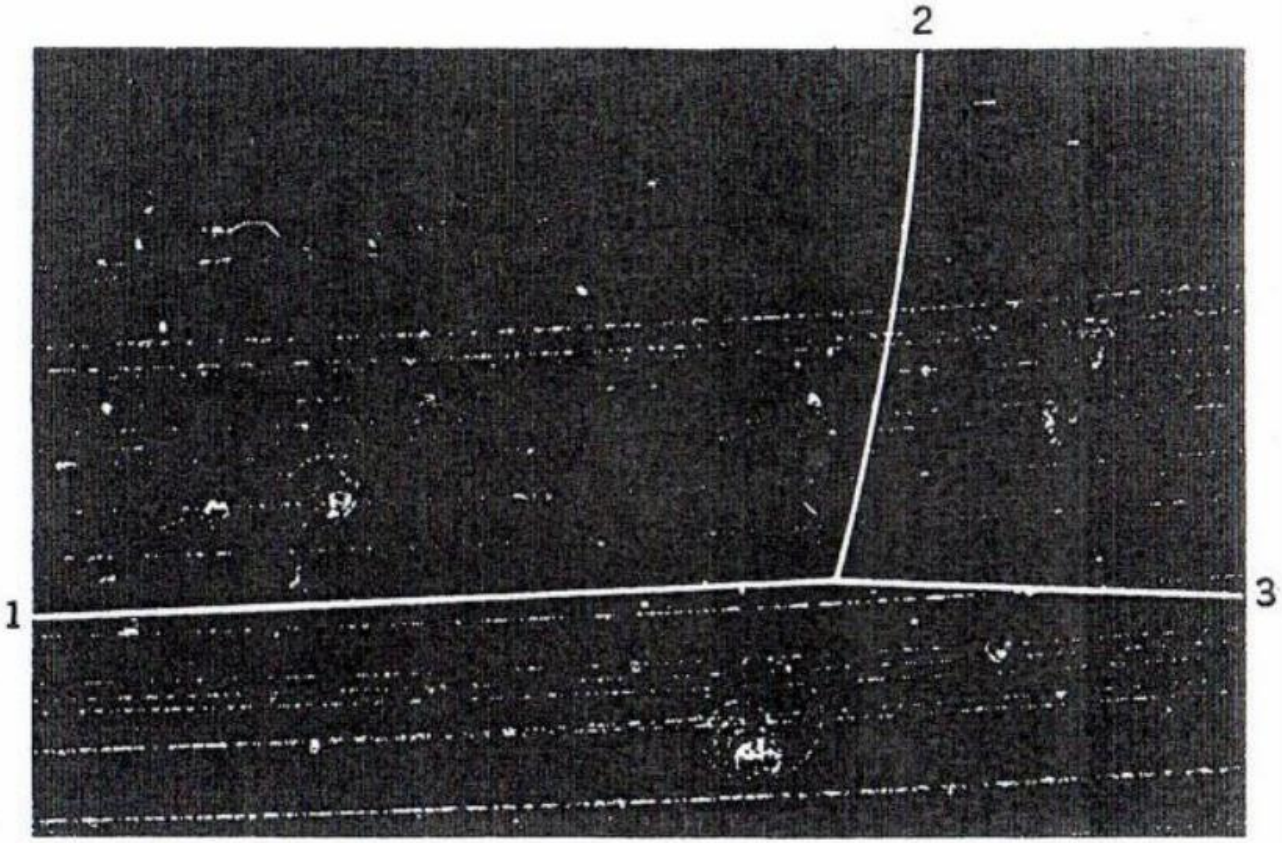
Fig. 6–8 Elastic scattering of an incident proton of about 5 MeV by an initially stationary proton in a photographic emulsion. The collision is "non-relativistic" ($(K/m_0c^2 \ll 1)$) with a 90° angle between the tracks of the protons after collision. (From C. F. Powell and G. P. S. Occhialini, Nuclear Physics in Photographs, Oxford Univ. Press, New York.)



From Special Relativity by A. P. French

Elastic scattering of identical particles

Fig. 6–10 Elastic proton-proton collision in a liquid-hydrogen bubble chamber, using incident protons of about 3 Gev. The incident proton enters at 1, and the two recoiling protons leave at 2 and 3. One cannot tell which of the latter was the incident proton. Relevant tracks emphasized. (Brookhaven National Laboratory.)

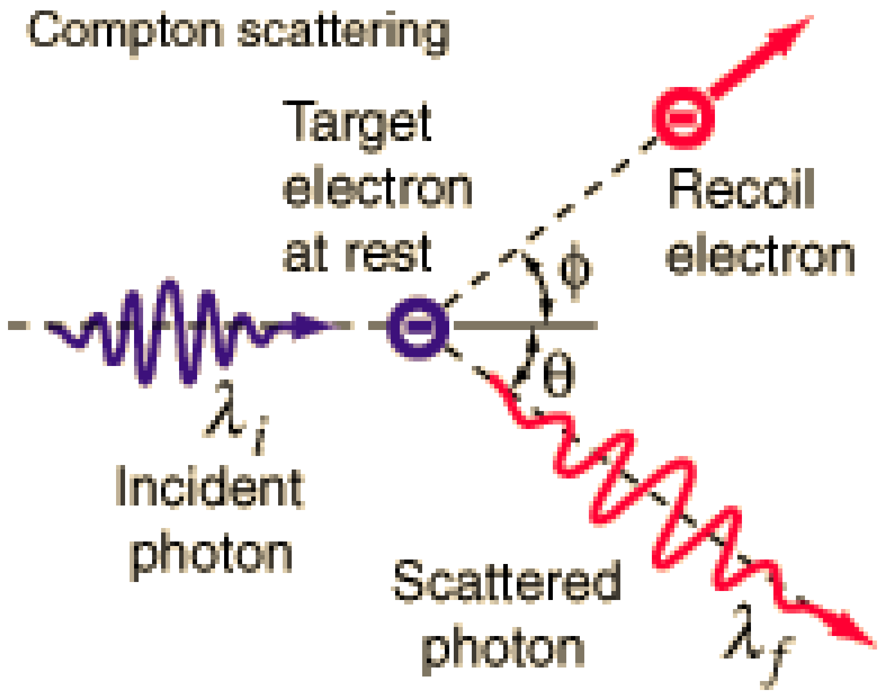


From Special Relativity by A. P. French

Compton Effect

Confirmation that light behaves as a particle

Collision of photon (γ) with a free electron (or nearly free – e.g. loosely bound to an atom.



The collision is elastic, but energy is transferred from γ to e.

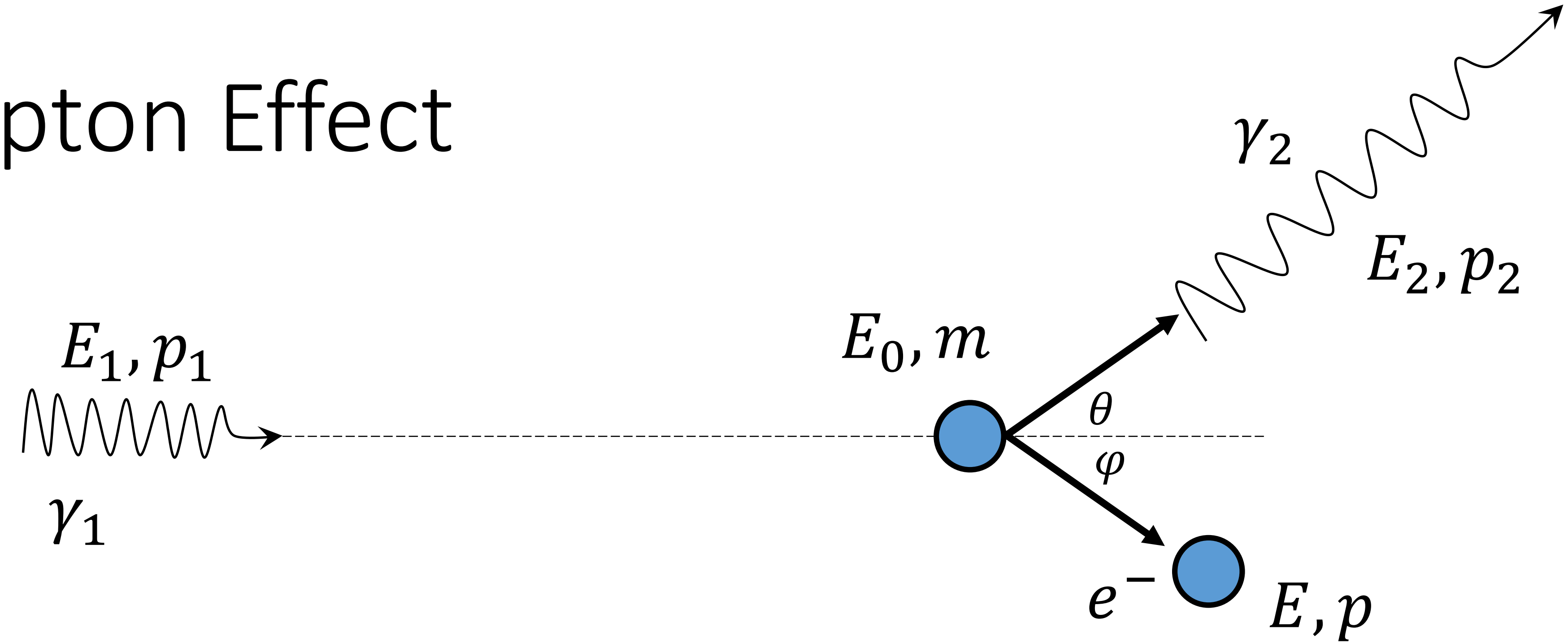
As

$$h\nu \downarrow \quad \therefore \quad \uparrow$$

That is, the scattered photon has a longer wavelength.

A. H. Compton studied this effect, and won a Nobel prize in 1927 for his efforts.

Compton Effect



Photon of energy E_1 strikes a stationary electron.

<u>Conservation of Energy:</u>

$$E_1 + E_0 = E + E_2$$
$$E_1 + m_0 c^2 = E + E_2$$

Compton Effect

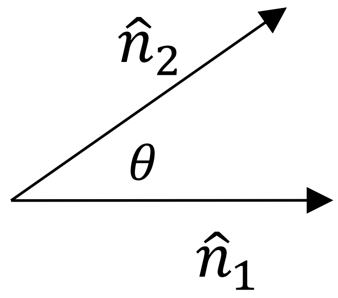
<u>Conservation of Momentum:</u>

Remember $p = mv = \frac{E}{c^2}v$ and $v = c$ for photons:

$$\vec{p}_1 = \frac{E_1}{c}\hat{n}_1$$

$$\vec{p}_2 = \frac{E_2}{c}\hat{n}_2$$

$$\therefore\ \frac{E_1}{c}\hat{n}_1 = \frac{E_2}{c}\hat{n}_2 + \vec{p}$$



$\hat{n}_1$ and $\hat{n}_2$ are unit vectors

$\underline{E^2 - p^2c^2 = m_0^2c^4}$

Compton Effect

Then:

$$[E_1 - E_2 + m_0c^2]^2 - [E_1\hat{n}_1 - E_2\hat{n}_2]^2 = m_0^2c^4$$

$$E_1^2 - 2E_1E_2 + E_2^2 + 2m_0c^2(E_1 - E_2) + m_0^2c^4 - E_1^2 + 2E_1E_2\cos\theta - E_2^2 = m_0^2c^4$$

$$2(E_1 - E_2)m_0c^2 - 2E_1E_2(1 - \cos\theta) = 0$$

$$2(E_1 - E_2)m_0c^2 = 2E_1E_2(1 - \cos\theta)$$

$$\frac{E_1 - E_2}{E_1E_2} = \frac{1 - \cos\theta}{m_0c^2}$$

$$\frac{1}{E_1} - \frac{1}{E_2} = \frac{1 - \cos\theta}{m_0c^2}$$

Compton Effect

$$\frac{1}{E_1} - \frac{1}{E_2} = \frac{1 - \cos\theta}{m_0 c^2}$$

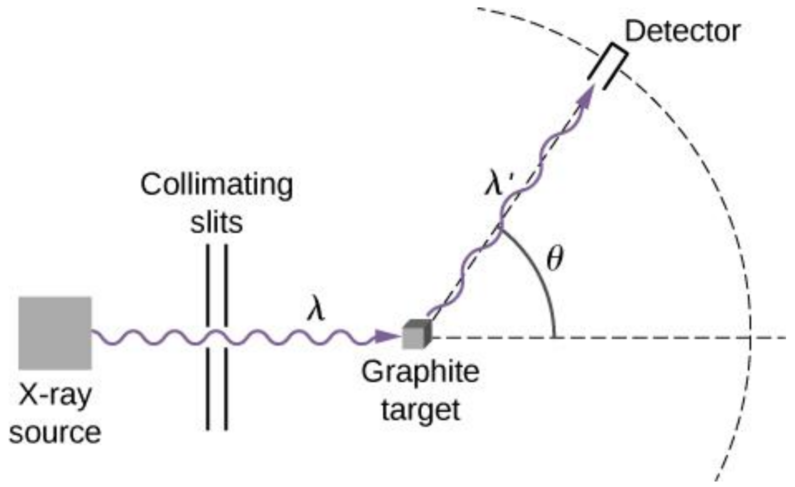
But: $E_1 = h\nu_1 = \frac{hc}{\lambda_1}$ and $E_2 = h\nu_2 = \frac{hc}{\lambda_2}$

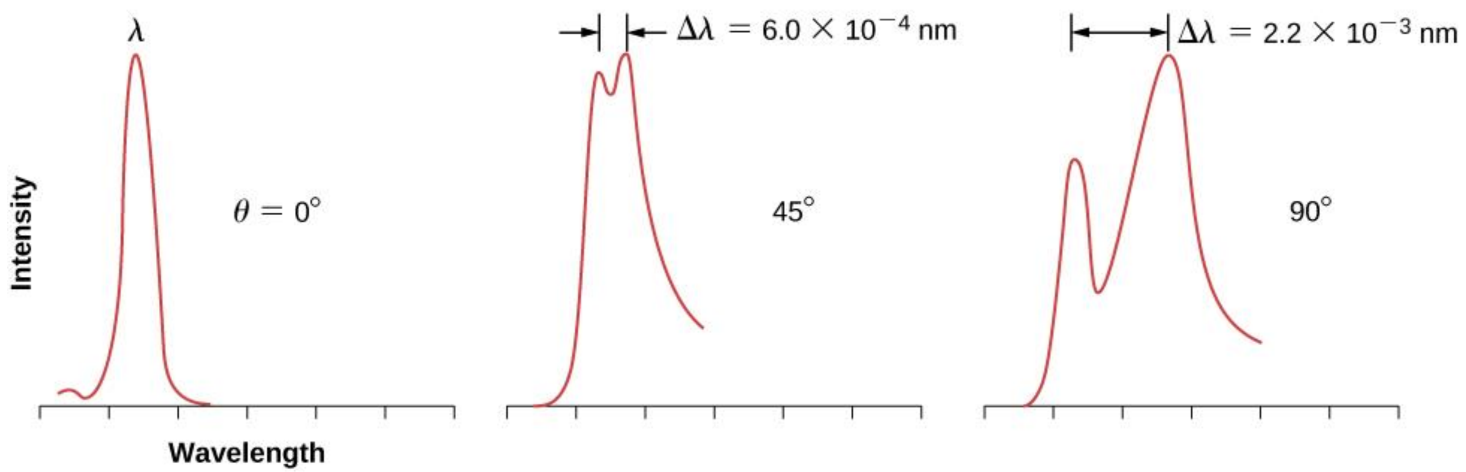
$$\boxed{\lambda_2 - \lambda_1 = \frac{h(1 - \cos\theta)}{m_0 c}}$$

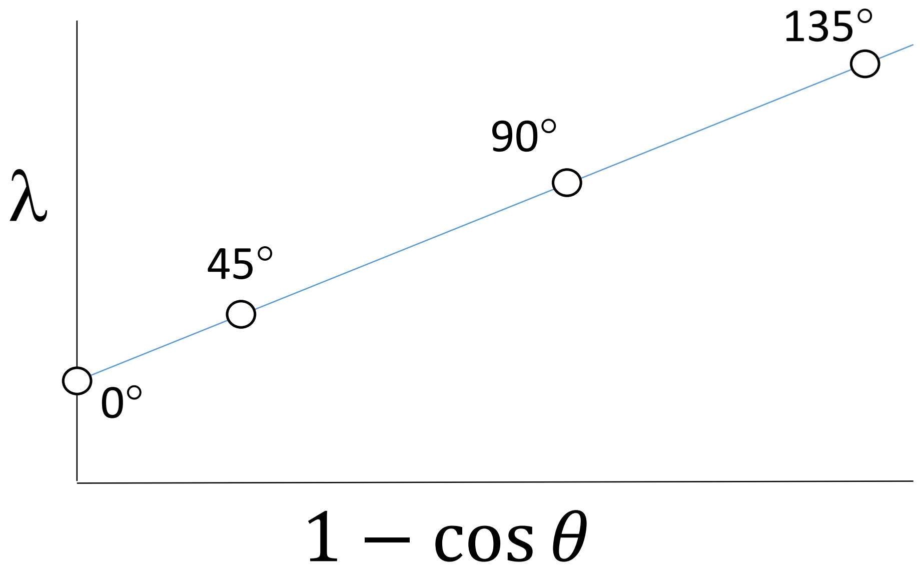
$$\frac{h}{m_0 c} \equiv \lambda_C = \text{Compton wavelength}$$

For e^-, $\lambda_C = 2.426 \text{ x } 10^{-12}\,\text{m} = 0.002426$ nm

Compton Effect







The nonshifted peaks are due to photon collisions with tightly bound inner electrons in the target material.

Compton Effect

Later, in 1950, Cross and Ramsey using γ-rays of 2.6 MeV also detected recoiling e.

They confirmed the angle between e and scattered photon is correct

Furthermore, the scattered photon and the recoiling e are always detected in coincidence.

These experiments are important because it demonstrates that light cannot be explained purely as a wave phenomenon. Compton's work convinced the scientific community that light can behave as a stream of particles (photons) whose energy is proportional to the frequency.

Compton Effect

A Quantum Theory of the Scattering of X-rays by Light Elements

Arthur H. Compton
Phys. Rev. **21**, 483 – Published 1 May 1923

Physics See Focus story: *Landmarks*: Photons are Real

Article | References | Citing Articles (312) | PDF | Export Citation

ABSTRACT

A quantum theory of the scattering of X-rays and γ-rays by light elements.—The hypothesis is suggested that when an X-ray quantum is scattered it spends all of its energy and momentum upon some particular electron. This electron in turn scatters the ray in some definite direction. The change in momentum of the X-ray quantum due to the change in its direction of propagation results in a recoil of the scattering electron. The energy in the scattered quantum is thus less than the energy in the primary quantum by the kinetic energy of recoil of the scattering electron. The corresponding *increase in the wave-length of the scattered beam* is $\lambda_\theta - \lambda_0 = \left(\frac{2h}{mc}\right) \sin^2 \frac{1}{2}\theta = 0.0484 \sin^2 \frac{1}{2}\theta$, where h is the Planck constant, m is the mass of the scattering electron, c is the velocity of light, and θ is the angle between the incident and the scattered ray. Hence the increase is independent of the wave-length. *The*

Concept Test

At what speed is a particle's kinetic energy equal to twice it rest energy?

A. 0.9c
B. 0.94c
C. 0.5c
D. 0.707c

Concept Test

According to the relativistic expression for momentum, if the speed of an object is doubled, the magnitude of its momentum:

A. Increases by a factor greater than 2.
B. Increases by a factor of 2.
C. Increases by a factor greater than 1 but less than 2.
D. Depends on the value of the initial speed.
E. Depends on the value of the initial speed and on the rest mass of the object