

Advanced Calculus

MA1132

Tutorial Exercises 7

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To be completed before and during tutorials of Friday, 22. March

1. Find the integral of the function $f(x, y) = 4xye^{x^2+y^2}$ over the rectangle

$$\{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2, 0 \leq y \leq 3\}.$$

2. Find the integral of the function $f(x, y) = 4xy^2 + 4x^3 + \frac{28}{3}$ over the rectangle

$$\{(x, y) \in \mathbb{R}^2 : -2 \leq x \leq 0, 0 \leq y \leq 1\}.$$

What does the result tell us about the signed volume of the region bounded between the rectangle and $f(x, y)$? What does it tell us about the unsigned volume (meaning the absolute volume of this region, ignoring that part of it might be below $z = 0$).

3. Evaluate

$$\int_0^1 \int_{\sqrt[4]{x}}^1 \sqrt{1-y^5} dy dx.$$

4. Evaluate the double integral

$$I = \iint_R \frac{1}{x+y} dx dy,$$

where R is the region enclosed by the lines $y = 2$, $y = x$, and the hyperbola $xy = 1$.

5. Evaluate the double integral

$$I = \iint_R \sqrt{4x^2 - y^2} dx dy,$$

where R is the region enclosed by the lines $y = 0$, $y = x$, and $x = 1$.

6. Reverse the order of integration

(a)

$$\int_0^4 \int_{3x^2}^{12x} f(x, y) dy dx \tag{1}$$

(b)

$$\int_{-7}^1 \int_{2-\sqrt{7-6y-y^2}}^{2+\sqrt{7-6y-y^2}} f(x, y) dx dy \tag{2}$$

(c)

$$\int_0^1 \int_{2x}^{3x} f(x, y) dy dx \tag{3}$$

(d)

$$\int_0^1 \int_{y^2/2}^{\sqrt{3-y^2}} f(x, y) dx dy \tag{4}$$