7.18

Moment of inertia of the rod

$$I_r = \int_0^l x^2 \, dm$$

$$= \int_0^l x^2 \frac{m}{l} dx = \frac{m}{l}\frac{1}{3} l^3 = \frac{1}{3} m l^2$$

Moment of inertia of the disk (parallel axis thm)

$$I_d = Ml^2 + \frac{1}{2} MR^2$$

Period

$$T = 2\pi \sqrt{\frac{I_d + I_r}{(M+m)gL}}$$

where L is the distance ~~from~~ between the axis of rotation ~~to~~ and the center of mass.

$$T = 2\pi\sqrt{\frac{I_d + I_r}{g(ml/2 + Ml)}} \Leftarrow L = \frac{m}{M+m} l/2 + \frac{Ml}{M+m}$$

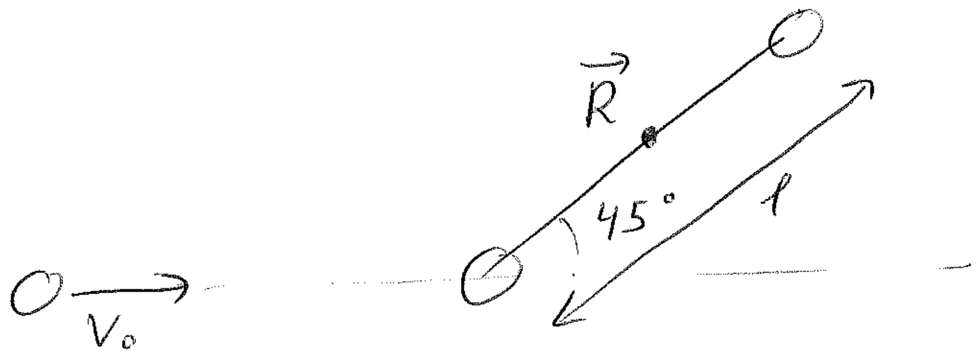
If the disk is free to spin, it keeps its original orientation. The contribution for spinning around its axis therefore disappears $\Longrightarrow$ $I_d = Ml^2$

The period decreases.

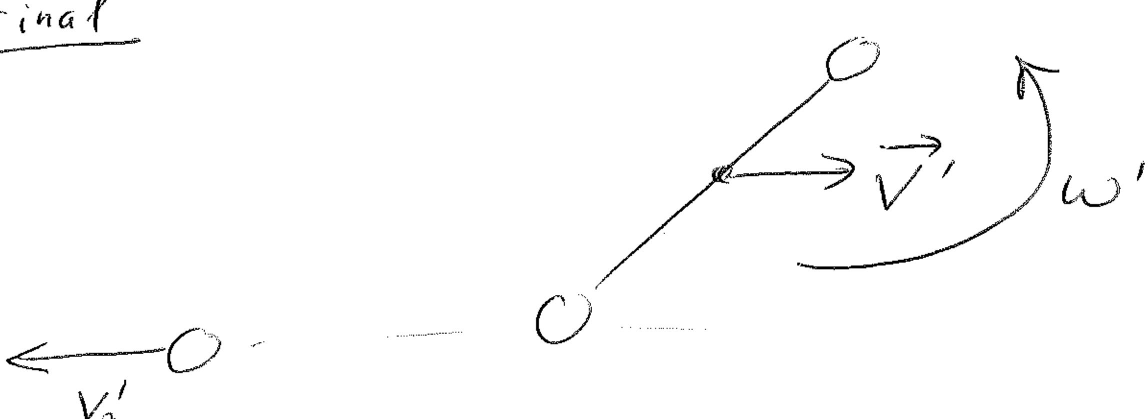
$$T = 2\pi \sqrt{\frac{\frac{1}{3}ml^2 + Ml^2}{(ml/2 + Ml)g}}$$

7.38

Initial



Final



Conservation of Momentum

$$mV_0 = -mV_0' + 2mV' \Rightarrow V' = \frac{1}{2}(V_0 + V_0')$$

The impulse of the collision, gives the angular momentum

$$\int |\vec{\tau}|\,dt = m(V_0 + V_0')\,\frac{1}{2}\ell\,\sin(45°)$$
$$= I\omega' = 2m\left(\tfrac{1}{2}\ell\right)^2\omega' = \frac{1}{2}m\ell^2\omega'$$
$$\Rightarrow \omega' = \frac{\sqrt{2}}{2}\,\frac{V_0+V_0'}{\ell} = \frac{\sqrt{2}\,V'}{\ell}$$

Conservation of Energy

$$\frac{1}{2}mV_0^2 = \frac{1}{2}m(V_0')^2 + \frac{1}{2}2m(V')^2 + \frac{1}{2}I(\omega')^2$$
$$= \frac{1}{2}m(\sqrt{2}\ell\omega' - V_0)^2 + \frac{1}{2}m\ell^2(\omega')^2 + \frac{1}{4}m\ell^2(\omega')^2$$
$$\Rightarrow 0 = \frac{7}{4}m\ell^2(\omega')^2 - \sqrt{2}\,m\ell\omega' V_0$$
$$\Rightarrow V_0 = \frac{7}{4\sqrt{2}}\omega'\ell \Rightarrow \boxed{\omega' = \frac{4\sqrt{2}}{7\ell}V_0}$$

We have moreover $V_0' = \frac{V_0}{7}$ and $V' = \frac{4}{7}V_0$.