Solutions to Homework 1

Problem 1

$$x(t) = \alpha t \cos(\omega t)$$

1. $[x] = L$

   $[t] = T$

   $[\cos(\omega t)] = 1 \qquad [\omega t] = 1$

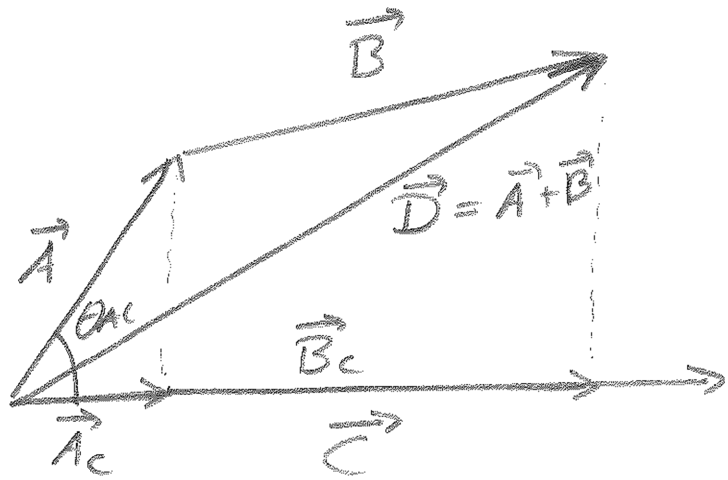
   $$\Longrightarrow [\alpha] = \frac{L}{T}, \quad [\omega] = \frac{1}{T}$$

2. $$v(t) = \frac{dx(t)}{dt}$$

   $$= \alpha \cos(\omega t) - \alpha \omega t \sin(\omega t)$$

   $$a(t) = -2\alpha \omega \sin(\omega t) - \alpha \omega^2 t \cos(\omega t)$$

Problem 2.



Short proof

We see from the picture that for the components of $\vec{A}$, $\vec{B}$ & $\vec{D}$ along $\vec{C}$ holds

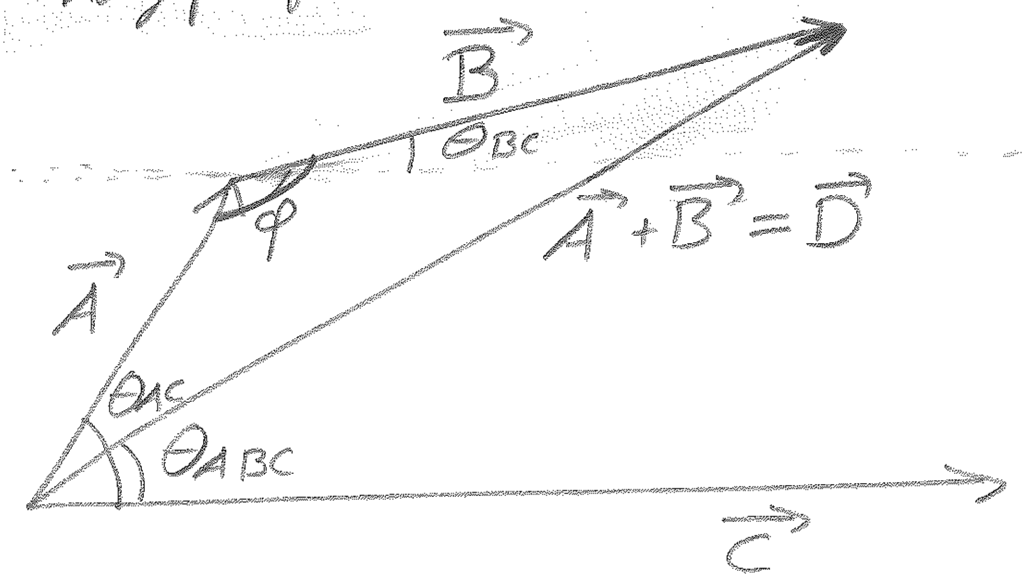
$$\vec{A}_c + \vec{B}_c = \vec{D}_c \Rightarrow |\vec{D}_c| = |\vec{A}_c| + |\vec{B}_c|$$

Since
$$\vec{A} \cdot \vec{C} = |\vec{A}||\vec{C}| \cos\theta_{AC}$$
$$= |\vec{A}_c||\vec{C}|$$

We have
$$(\vec{A} + \vec{B}) \cdot \vec{C} = |\vec{D}_c||\vec{C}|$$
$$= (|\vec{A}_c| + |\vec{B}_c|)|\vec{C}|$$
$$= \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C} \qquad \square$$

Problem 2

Long proof



We have

$$(\vec{A}+\vec{B})\cdot\vec{C} = |\vec{A}+\vec{B}||\vec{C}|\cos\theta_{ABC} \quad (1)$$

&

$$\vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{C} = |\vec{A}||\vec{C}|\cos\theta_{AC} + |\vec{B}||\vec{C}|\cos\theta_{BC} \quad (2)$$

Set $|\vec{A}|=A$, $|\vec{B}|=B$, $|\vec{D}|=D$

To show that (1) & (2) are equal, use law of cosines:

$$\begin{aligned} D^2 &= A^2+B^2-2AB\cos\varphi \\ &= A^2+B^2-2AB\cos(\pi-\theta_{AC}+\theta_{BC}) \\ &= A^2+B^2+2AB\cos(\theta_{BC}-\theta_{AC}) \end{aligned}$$

$$B^2 = A^2+D^2-2AD\cos(\theta_{AC}-\theta_{ABC})$$

Combining the last 2 identities

$$0 = 2A^2+2AB\cos(\theta_{BC}-\theta_{AC}) - 2AD\cos(\theta_{AC}-\theta_{ABC})$$

$$\Rightarrow$$

$$0 = A + B\cos(\theta_{BC} - \theta_{AC}) - D\cos\theta_{AC}$$

$$\Rightarrow A\cos\theta_{AC} + B\cos\theta_{AC}\cos(\theta_{BC} - \theta_{AC}) = D\cos\theta_{AC}\cos(\theta_{ABC} - \theta_{AC})$$

Use

$$\cos(\alpha+\beta) = \cos(\alpha)\cos\beta - \sin\alpha\sin\beta$$

$$\Rightarrow A\cos\theta_{AC} + B\cos\theta_{BC} + B\sin\theta_{AC}\sin(\theta_{BC} - \theta_{AC}) = D\cos\theta_{ABC} + D\sin\theta_{AC}\sin(\theta_{ABC} - \theta_{AC})$$

The law of sines gives

$$\frac{\sin(\pi - \theta_{AC} + \theta_{BC})}{D} = \frac{\sin(\theta_{AC} - \theta_{ABC})}{B}$$

This is equivalent to

$$\frac{\sin(\theta_{BC} - \theta_{AC})}{D} = \frac{\sin(\theta_{ABC} - \theta_{AC})}{B}$$

$$\Rightarrow A\cos\theta_{AC} + B\cos\theta_{BC} = D\cos\theta_{ABC}$$

which proves the distributivity