13.5

1. $m\ddot{x} + b\dot{x} + kx = 0$

$$m = \frac{4}{3}\pi R^3 \rho$$

$$\omega_0 = \frac{2\pi}{T_0} = \sqrt{\frac{k}{m}}$$

$$\omega_1 = \sqrt{\omega_0^2 - \gamma^2/4} \quad \text{with } \gamma = \frac{b}{m}$$

2. We have

$$b = 6\pi\eta R$$

$$\Rightarrow \eta = \frac{b}{6\pi R} = \frac{m\gamma}{6\pi R}$$

$$\omega_1 = 0.9998 \cdot \omega_0 = \alpha\omega_0$$

$$\Rightarrow \gamma = 2\omega_0\sqrt{1-\alpha^2} = 0.04\,\omega_0$$

$$\eta = \frac{\frac{4}{3}\pi R^3 \rho \cdot 0.04 \cdot \frac{2\pi}{T_0}}{6\pi R}$$

$$= \frac{4R^2\rho \cdot 0.04 \cdot \pi}{9 \cdot T_0} = \frac{4 \times 9 \times 10^{-4} \times 10^5 \cdot 0.2 \cdot \pi}{\cancel{9} \cdot 12\pi \cdot \cancel{\pi}}$$

$$= \frac{2}{3\pi} \approx 0.21 \text{ kg s}^{-1}\text{m}^{-1}$$

3. The amplitude is reduced by the factor

$$e^{-\frac{\gamma}{2}\cdot 10 \cdot T_0} = e^{-0.2\cdot 2\pi} \approx 0.285$$

Homework 1.

Problem 1.

Q is large $\Rightarrow$ $\omega_1 = \sqrt{\omega_0^2 - \gamma^2/4} \approx \omega_0$

The damped solution is well approximated by

$x(t) = A e^{-\gamma t/2} \sin(\omega_0 t)$

It has a maximum for $\frac{dx}{dt} = 0$

20 $\Rightarrow$ $-\frac{\gamma}{2} \sin(\omega_0 t_m) + \omega_0 \cos(\omega_0 t_m) = 0$

At the maximum $\sin(\omega_0 t_m) \approx 1$ &

$\cos(\omega_0 t_m) \approx \frac{\pi}{2} - \omega_0 t_m$

10

$\Rightarrow$ $-\frac{\gamma}{2} + \omega_0 \left(\frac{\pi}{2} - \omega_0 t_m\right) = 0$

Which gives $\omega_0 t_m = \frac{\pi}{2} - \frac{\gamma}{2\omega_0}$

10 $\Rightarrow$ The difference is $\frac{\gamma}{2\omega_0} = \frac{1}{2Q}$

b) $x(0) = A = 0$

$$\dot{x}(0) = A(-\gamma/2) + B = B$$

$$\Longrightarrow \quad B = I/m$$

$$x(t) = \frac{I}{m} t e^{-\gamma t/2}$$

Velocity starts to decrease when

$\gamma^2/4 \, t - \gamma \leq 0$ is at $t \in [0, 4/\gamma]$

Then acceleration is positive, but oscillator is still slowing down