Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JS & SS Mathematics
JS Theoretical Physics
Moderatorship

Trinity Term 2016

MA1242 — Mechanics II

Thursday, May 12 RDS 14:00 — 16:00

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.

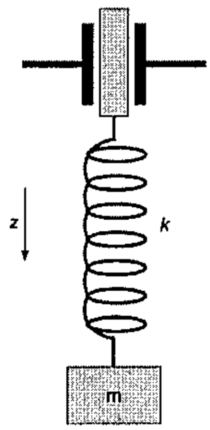
All questions have equal weight.

'Formulae & Tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination - please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. A spring is attached to a rod, which can move vertically. The spring constant is k. The mass of the spring is negligible. Attached to the spring is a mass m. See the figure below. The gravitational acceleration is $\vec{g}$, and is directed vertically downwards. The quality factor of the (weakly) damped system is Q.



(a) Show that the damping coefficient b is given by

$$b = \frac{m\omega_0}{\sqrt{Q^2 + \frac{1}{4}}}$$

(b) For $t < 0$, the system is at rest. At time $t = 0$, the rod is moved vertically upward over a distance A during a negligibly short time interval. Determine the work exerted by the rod on the spring + mass.

(c) For $t > 0$, the system is oscillating. How much energy is dissipated, after the oscillation is fully damped (for $t \to \infty$)?

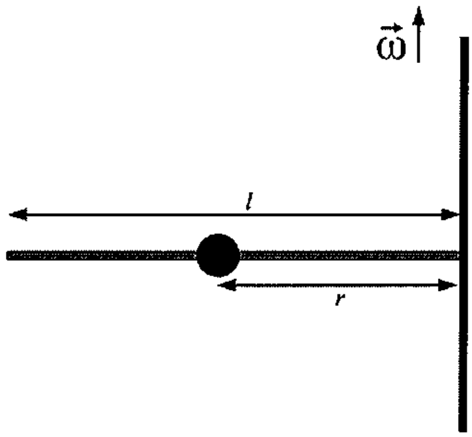
(d) Let z_0 be the new equilibrium position, and $u(t) = z(t) - z_0$ the displacement. Then $u(t)$ is given by

$$u(t) = u_{\max} e^{-\gamma t/2} \cos(\omega_1 t + \varphi).$$

Show

$$\varphi = -\arctan\left(\frac{1}{2Q}\right), \text{ and } u_{\max} = A\sqrt{1 + \frac{1}{4Q^2}}.$$

2. A bead of mass m slides without friction on a horizontal wire of length l and mass M. The wire is uniform and rigid and rotates at constant angular speed ω. The distance of the bead to the vertical axis of rotation is denoted by r. See the figure below. Neglect gravity.



   (a) Give the force diagram of the bead, and the corresponding equations of motion.

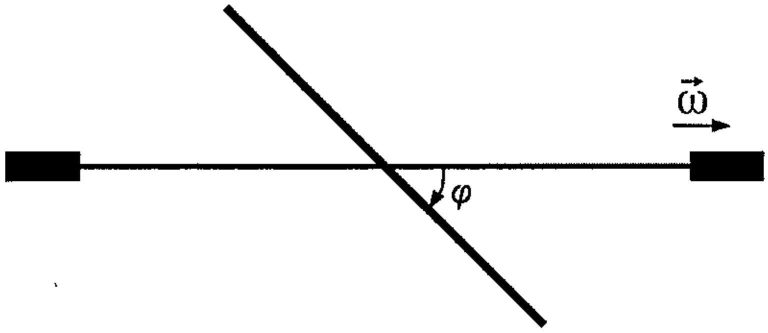
   (b) Show that the general solution of $r(t)$ is:

   $$r(t) = A\,e^{\omega t} + B\,e^{-\omega t},$$

   where A and B are two constants determined by the initial conditions $r(0)$ and $\dot{r}(0)$. Let $r(0) = r_0$ with $0 < r_0 < l$. Determine $\dot{r}(0)$, such that the bead does not reach the pivot point nor the other end point of the wire for $t < \infty$.

   (c) Determine the torque which needs to be exerted on the wire at the pivot point if the bead moves according to $r(t) = r_0 \cosh(\omega t)$.

3. A thin, homogeneous rod of length ℓ and mass m is mounted on an axle through its center. The rod makes an angle φ with the axle. See the figure below. The axle rotates together with the rod with angular velocity $\vec{\omega}$.



(a) Determine the angle between the angular momentum $\vec{L}$ and the angular velocity $\vec{\omega}$.

(b) Determine the torque $\vec{\tau}$ with respect to the center of the rod, which the axle needs to exert on the rod for the described motion.

(c) One places two point masses (each with mass $\frac{1}{2}m$) on a massless rod oriented orthogonally to the first rod through the center. See the figure below. The distance between each point mass and the rod is s. Determine s such that the system is dynamically balanced, i.e. such that $\vec{L}$ and $\vec{\omega}$ are parallel.