ELECTRO-MAGNETISM

JOSE GROH

ELECTRIC CHARGE 21.1

Charge: when plastic is rubbed w/ wool
- the plastic becomes negatively charged
- the wool becomes positively charged
- opposite charges attract, like charges repel

ELECTROSTATICS
Study of the interactions of charges at rest

Emperically: We observe conservation of charge
- in any single charge transfer, the total charge remains the same

PRINCIPLE OF CONSERVATION OF CHARGE
The Algebraic sum of all the electric charges in any closed system is constant

Atoms — ELECTRICALLY NEUTRAL (SAME NUMBER OF PROTONS AS NEUTRONS)
- Electron Removed → Positive Ion
- Electron Added → Negative Ion

GEORGE JOHNSTONE STONEY (1826 - 1911)
- BORN IN DUNLAOIGHRE, DIED IN LONDON, TC Dundargval
- proposed the word "electron"

CONDUCTOR - charge almost completely free to move around

SEMICONDUCTOR - control mobility of charge by temp changes/doping

INSULATOR - charge cannot move around

CHARGING BY INDUCTION
- CONDUCTING METAL INITIALLY ISOLATED + UNCHARGED
- NEGATIVE CHARGE BROUGHT NEAR IT, electrons repelled far side then EARTHED FOR SHORT TIME
- Electrons flow away into ground
- NET POSITIVE CHARGE REMAINS
- ROD REMOVED, STILL A POSITIVE CHARGE

Coulomb's Law + Electric Field

Emperical Observations: $F \propto \frac{1}{r^2}$ $F \propto |q_1 q_2|$

Coulomb's Law: $\vec{F} = \frac{k q_1 q_2}{r^2}\hat{r}$ F - force (N), r - distance (m), q - charge (C), k - constant (Nm^2C^{-2})

Unit of Charge = Coulomb (C) defined in terms of Electric current

$k = 10^{-7}c^2$ or $\frac{1}{4\pi\varepsilon_0} \Rightarrow (\varepsilon_0 = 8.854\times10^{-12}) \Rightarrow k \cong 8.94\times10^9$

- Fundamental Unit of Charge: electron (e)
  $e = 1.602\times10^{-19}C$

Charles Augustin de Coulomb

France (1736, 1806)
- trained to be Engineer in Mezieres
- main contributions on FRICTION
- 1st took advantage of Torsion Balance

$$\vec{F} = \frac{k q_1 q_2}{r^2}\hat{r}$$

Comparing Electrostatic/Gravitational

- Let us consider 2 α particles of
  - mass $m = 6.64\times10^{-27}kg$ o charge $q = 2e = 3.2\times10^{-19}C$
  - $F_e = \frac{q^2}{4\pi\varepsilon r^2}$ o $F_g = G\frac{m^2}{r^2}$ $\Rightarrow \frac{F_e}{F_g} = \frac{1}{4\pi\varepsilon_0 G}\cdot\frac{q^2}{r^2} = \boxed{3.1\times10^{35}}$ V BIG

Fundamental Forces

	Strength	Distance
Strong	1	~1 fm
Electromagnetic	$\frac{1}{137}$	$1/r^2$ long
Weak	10^{-9}	~.001 fm
Gravitational	10^{-38}	$1/r^2$ long

Principle of Superposition

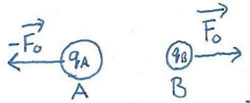
the Total Force on a charge is the Vector Sum of the individual forces between that charge and all others around it

Electric Field

Electric Field created by A at point B is given by $\vec{E} = \vec{F_0}/q_0$ $\boxed{\text{vector}}$

$\boxed{\text{UNITS: } NC^{-1}}$

- As the force exerted by B might cause A to shift, A stricter definition is $\vec{E} = \lim_{q_0\to0} \vec{F_0}/q_0$



$$\vec{E} = \frac{\vec{F_0}}{q_0} = \lim_{q_0\to0}\frac{\vec{F_0}}{q_0}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{r^2}\hat{r}$$

- A point charge creates an electric field at all points

Force:
- Positive Charge experiences force in same direction as $\vec{E}$
- Negative charge experiences force in opposite direction

- For extended objects, Total Field $\vec{E} = \vec{E_1} + \vec{E_2} + \dots$
- USE SUPERPOSITION TO EXTEND CONTINUOUS ~~distributions~~ distributions

Cases of interest: linear density : λ Cm^{-1}; surface density : σ Cm^{-2}; Volume density : ρ Cm^{-3} $\rightarrow$ $\boxed{\text{INTEGRATE}}$

Electric Field Examples / Lines

Ex Point charge next to a charged wire

$\boxed{\lambda = \frac{Q}{2a}}$

$\boxed{\Delta q = \lambda\Delta y}$

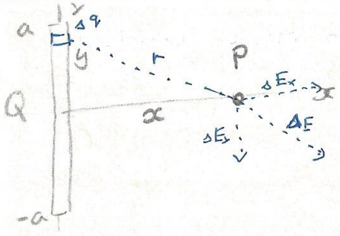
$$\Delta E = \frac{1}{4\pi\varepsilon}\frac{\Delta q}{r^2} = \frac{1}{4\pi\varepsilon}\frac{\lambda\Delta y}{(x^2+y^2)}\ (\lim \Delta y\to0)$$

$$\int_0^E dE = \int_{-a}^{a}\frac{\lambda}{4\pi\varepsilon\sqrt{(x^2+y^2)}}\,dy$$

~~$y = x\tan\theta$~~ $u^2 = x^2 + y^2$

d

$E =$



- So we found $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{i}$

we can rewrite this as by taking an x^2 out of the square root $\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2\sqrt{1+\left(\frac{a}{x}\right)^2}}\hat{i}$

- THIS means when far away ($x >> a$), it approximates a point source

$$\lim_{\frac{a}{x}\to0} \to \frac{Q}{4\pi\varepsilon x^2}\hat{i}$$

- ALSO, SUBSTITUTING $Q = 2a\lambda$ WE CAN REARRANGE THE ORIGINAL EXPRESSION TO BE

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x/a\sqrt{x^2+a^2}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x\sqrt{\left(\frac{x^2}{a^2}\right)+1^2}}$$

- Now getting really close s.t. $x << a$

$$\lim_{\frac{x}{a}\to0}:\quad E = \frac{\lambda}{2\pi\varepsilon_0 x}\hat{i}$$

- THIS MEANS THE DEPENDANCE ON x becomes LINEAR

Electric Field Lines

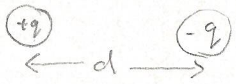
Electric Field Lines:

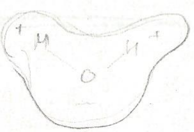
- Imaginary lines s.t. the direction of the field line is tangent to the electric field line
- Orientation given by direction of electric field ($\hat{E}$)
- Has only 1 direction (Fields Never Cross)
- Lines end up at charge or at infinity
- Lines drawn closer together where field is stronger
- Field lines are Not trajectories.

Electric Dipole

Electric Dipole: Pair of point charges, with equal magnetude and opposite sign

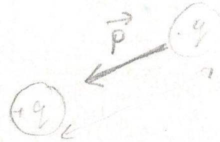
- many molecules behave like electric dipoles





Dipole Movement: Assume we have an electric dipole with charges q and $-q$ separated by a distance d, then, the electric dipole moment p is: $p = qd$

- the electric dipole moment vector $\vec{p}$ has magnitude p and points from negative to positive



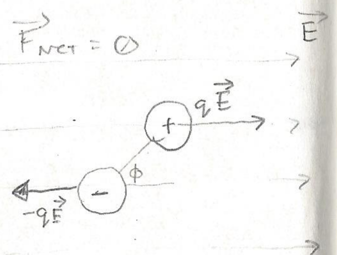
Dipole in Uniform Field of $\vec{E}$

- The total net force is Zero
- the torque of each force around the center of the dipole is: $\tau = \vec{r} \times \vec{F}$

$$|\vec{\tau}| = |qE||\tfrac{d}{2}|\sin\phi \overset{qd=p}{=\!=} \frac{pE}{2}\sin\phi$$

$$\Rightarrow \tau_{TOTAL} = \frac{qEd}{2}\sin\phi + \frac{(-q)Ed}{2}\sin(\phi+\pi) = \frac{2}{2}pE\sin\phi$$

Or simply: $\vec{\tau} = \vec{p} \times \vec{E}$ with $\phi = 0$ stable equilibrium, $\phi = \pi$ unstable equilibrium



$\vec{\tau} = \vec{p} \times \vec{E}$

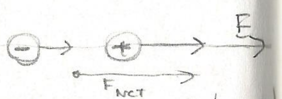
Dipole in Non-Uniform Field:

- Total net force can be non-zero (E.G: $p \parallel E$)

$$F_{NET} = F_+ - F_- = q(E + \Delta E) - qE = q\Delta E$$

For very small distances, $\Delta E \cong d \cdot \frac{dE}{dx}$

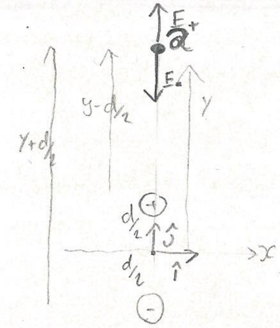
$$\Rightarrow F_{NET} = q\left(d\frac{dE}{dx}\right) = \boxed{p\frac{dE}{dx}}$$



$F = qd\frac{dE}{dx} = p\dot{E}$ (Not 0)

Electric Field due to a Dipole

- Electric field at a point a due to a dipole centered at the origin O.



- Consider the dipole oriented as shown
- The x-component of the electric field at point a is 0 for each charge
- y component is: $E_y = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(y-\frac{d}{2})^2} + \frac{-q}{(y+\frac{d}{2})^2}\right)$

– Combining the fractions, we get

$$E_y = \frac{q}{4\pi\varepsilon_0}\left[\frac{[y+\frac{d}{2}]^2 - [y-\frac{d}{2}]^2}{[(y-\frac{d}{2})(y+\frac{d}{2})]^2}\right] = \frac{q}{4\pi\varepsilon_0}\left(\frac{2(2\frac{d}{2}y)}{(y^2 - d^2/4)^2}\right)$$

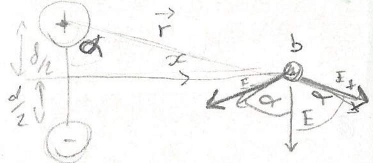
$$E_y = \frac{q}{2\pi\varepsilon_0}\left[\frac{y \cdot d}{y^4(1 - d^2/4y^2)^2}\right] = \frac{q}{2\pi\varepsilon_0}\left(\frac{d}{y^3(1-(\frac{d}{2y})^2)^2}\right)$$

$$E_y = \frac{\vec{p}}{2\pi\varepsilon_0} \cdot \frac{1}{[y^3(1-(\frac{d}{2y})^2)^2]}$$

- If $\frac{d}{2} \ll y$, we get $E_y^a \cong \frac{qd}{2\pi\varepsilon_0 y^3} = \frac{p}{2\pi\varepsilon_0 y^3}$

$$E_y \cong \frac{\vec{p}}{2\pi\varepsilon_0 y^3}$$

- Now consider another point of high symmetry, b (as shown):



Then $|E_1| = |E_2| = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{(x^2 + (\frac{d}{2})^2)}$

By symmetry, the x components cancel, so we look at the y components:

$$\boxed{E_y^b = 2 \cdot \frac{q}{4\pi\varepsilon_0} \cdot \frac{(-1)}{(x^2 + \frac{d^2}{4})}\cos\alpha}$$

But: $\cos\alpha = d/2 \div r = d/2 \div \sqrt{x^2 + (d/2)^2}$

$$E_y^b = -\frac{dq}{4\pi\varepsilon_0} \cdot \frac{1}{(x^2 + (\frac{d}{2})^2)^{3/2}} = \frac{-p}{4\pi\varepsilon_0 x^3(1 + (\frac{d}{2x})^2)^{3/2}}$$

$$E_y = \frac{-\vec{p}}{4\pi\varepsilon_0 x^3(1+(\frac{d}{2x})^2)^{3/2}}$$

- If $\frac{d}{2} \ll x$, we get $|E_y^b| \cong \frac{p}{4\pi\varepsilon_0 x^3} = \frac{1}{2}E_y^a$ when $x = y$

$$E_y \cong \frac{-\vec{p}}{4\pi\varepsilon_0 x^3}$$

- The full result for any general coordinate is given by

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon}\left(\frac{-\vec{p}}{r^3} + \frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5}\right)}$$

GAUSS' LAWS - ELECTRIC FLUX

ELECTRIC FLUX = a measure of the electric field passing at right angles through a surface area

$\overrightarrow{\text{ELECTRIC FIELD}}$ is analagous to the velocity field of a fluid

- IF fluid flows through a surface A at a volume V_{ol} per unit time, then

$$\text{Velocity flux } (\Phi_v) = \frac{d}{dt}(V_{ol}) = \frac{d}{dt}(Ax) = v \cdot A$$

- tilted at an angle ϕ, we get

$$\Phi = \frac{d}{dt}(V_{oL}) = vA_{\perp} = vA\cos\varphi$$

AREA VECTOR: magnitude A $\perp$ to surface

Then $\Phi_v = \frac{d}{dt}(V_{oL}) = \vec{v} \cdot \vec{A}$

- CONSIDER uniform electric field of $|E|$ $\perp$ to a surface (of area A)

THEN: $\Phi_E = EA$

- tilting by an angle of φ, we get:

$$\Phi = EA_{\perp} = E_{\perp}A = E\cos(\varphi)A$$

- now defining $\vec{A}$ to be the normal surface area vector again, we get

$$\Phi_E = \vec{E} \cdot \vec{A}$$

IRREGULAR SURFACES - ELECTRIC FLUX

- IF $\vec{E}$ NOT UNIFORM or surface is not regular, we must integrate over the surface.
  - CONSIDER AN element dA
  - We CALCULATE Φ_E USING THESE by Adding up

$$\rightarrow \boxed{\Phi_E = \int_{surface} E_{\perp} dA = \int \vec{E} \cdot d\vec{A}}$$

GAUSS' LAW:

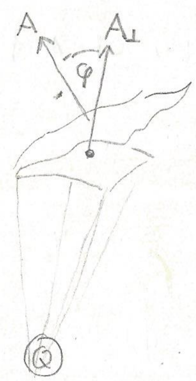
- CONSIDER an imaginary sphere of radius r surrounding a point charge Q

$$\Phi_E = \underset{\text{CLOSED SURFACE}}{\oint} \vec{E} \cdot d\vec{A} \overset{\text{All } \perp \text{ points}}{=} EA = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\cdot(4\pi r^2)$$

THUS: GAUSS'S LAW: THE FLUX THROUGH A SPHERE CONCENTRIC W/ A POINT CHARGE IS INDEPENDANT OF THE RADIUS

$$\boxed{\Phi_E = \frac{q}{\varepsilon_0}}$$

THUS: FLUX IS THE SAME FOR ALL CONTAINING CLOSED SURFACES



- For IRREGULAR closed surfaces:
  - SPLIT UP INTO SMALL PIECES
  - FOR PIECES, IS SAME AS HAVING PIECE OF SPHERE OF the SAME RADIUS (SHOWN)
  - All the PIECES become pieces of a sphere
  - CALCULATION STILL HOLDS

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{ENCLOSED}}{\varepsilon_0}$$

- IF SURFACE HAS MANY CHARGES enclosed, then just sum them all up

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{\Sigma q}{\varepsilon_0}$$

Gauss' Law - Applications

- Consider a charged object (conductor), that is in electrostatic condition (charges are not moving)

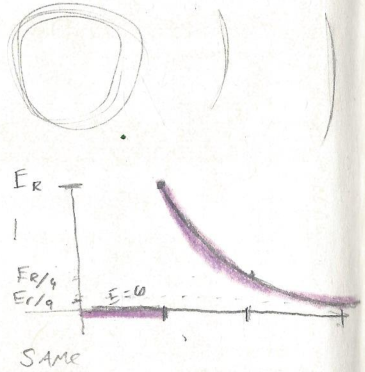
→ Construct Gaussean surface inside object

→ No charges are moving (Electrostatic)
 ⇒ $\vec{E}$ must be zero
 ⇒ By Gauss' Law, Electric flux must be zero
 ⇒ $\boxed{\text{Net charge inside surface is zero}}$

Field of a Charged, Conducting Sphere

- Consider a sphere of radius R, charge q
  - Inside: Electric field of conductor is 0
  - Outside: Construct Gaussean surface of radius r
    - By symmetry, $\vec{E}$ must be radial, so it is perpendicular to all points of the surface
    - Flux integral reduces to:

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = EA = E(4\pi r^2) = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$



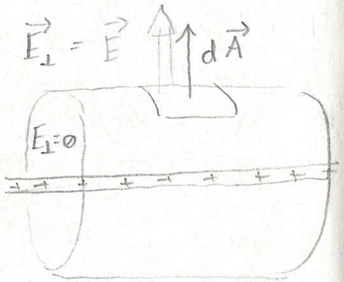
Field of a Line of Charge

- Consider an infinite line of charge density λ
- Construct Gaussean cylinder of radius r, length l

⇒ Enclosed charge $Q = \lambda l$

- By symmetry, the field must be pointed AWAY from the line at all points

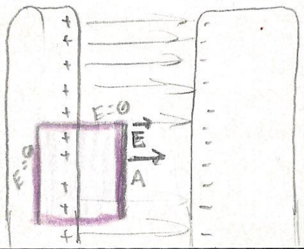
$$\Rightarrow \Phi_E = \oint \vec{E}\cdot d\vec{A} = E(2\pi r \cancel{l}) = \frac{\lambda \cancel{l}}{\varepsilon_0} \Rightarrow E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r} \quad (p3)$$



Field of Oppositely Charged Conducting Plates

- Consider 2 parallel plates of charge σ and $-\sigma$
- Gaussean cylinder, ends of area A as shown
  - Field must be ⊥ to plates, 0 inside conductor

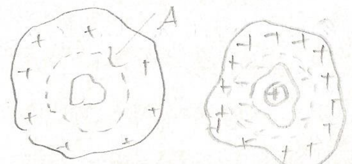
$$\Rightarrow \Phi_E = \oint \vec{E}\cdot dA = EA = \frac{Q}{\varepsilon_0} = \frac{A\sigma}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$



Gauss' Law - Applications + Verification

Charged Conductor w/ Cavity

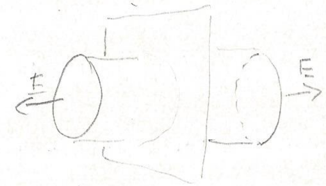
- E inside a conductor is zero, but positive charge is placed inside ⇒ negative charges move in, positive charges move out



Infinite Charged Sheet E Field

- Surface charge density σ, cylinder surface

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = 2EA = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{\cancel{A}\sigma}{2\cancel{A}\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$



Faraday's Ice Pail Experiment

- Positively charged ball lowered into a metal bucket-like container
  - → Induces charge on container
  - → Charge leaves ball on contact
  - → Remove ball → no longer charged
  - → Verifies Gauss' Law

Electric Potential

- work done by conservative force: $\int_a^b \vec{F}\cdot d\vec{r}$
- Potential Energy: defined s.t. total energy conserved.

  Difference in P.E: $U_a - U_b = \int_a^b \vec{F}\cdot dr$
- Electric Potential Energy: $\Delta U = \int \vec{F}\cdot d\vec{r} = \int q_0 \vec{E}\cdot d\vec{r}$
- Electric Potential: $\frac{\Delta U}{q} = \Delta V = \int_a^b \vec{E}\cdot d\vec{r}$

Potential due to point charge q:

$$V_{ab} = \int_a^b \vec{E}\cdot d\vec{r} = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

If $V=0$ when $r=\infty$, then: $\boxed{V = V_{r\to\infty} = \frac{q}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)}$

Potential due to many point charges: $\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}}$

Potential due to a charged sphere:

$\vec{E}$ outside is the same as point; $\vec{E}$ inside is 0

$$\int \vec{E}\cdot d\vec{r} = \underbrace{\int \vec{E}\cdot dr}_{\text{inside}} + \underbrace{\int \vec{E}\cdot dr}_{\text{outside}} = \boxed{V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}}$$

On surface: $\boxed{V = R\vec{E}}$

Dielectric Strength

- Maximum potential limited by air conducting

Dielectric Strength $E_{max} \approx 3\times10^6$ N/C (air ionised)

For spherical conductor: $V_{max} = R\,E_{max}$

→ larger radius means higher voltages

→ smaller radius means stronger E field

Van De Graaf Generator

- Conducting sphere on insulating stand/belt
- Bring charge up inside sphere
  - → moves to surface
  - → charge up again

Electrostatic Shielding

- Conducting box in uniform $\vec{E}$ field
- Induced box charge + E field

⇒ zero total field inside box

Equipotential Surfaces

- Equipotential surfaces = level curves / of constant $\vec{E}$
  - $\vec{E}$ is always ⊥ to equipotential surfaces
  - No work done along ∥ surfaces

Surface of a conductor

- is an equipotential surface by conservation of energy

Potential Gradient

$$\vec{E} = -\vec{\nabla}V = \begin{pmatrix} \frac{dV}{dx} \\ \frac{dV}{dy} \\ \frac{dV}{dz} \end{pmatrix}$$

$$\vec{E} = -\vec{\nabla}V$$

Millikan Oil Drop Experiment

$$F_E = F_G \Rightarrow qE = mg \Rightarrow q\frac{dV}{dx} = \rho_{oil}\cdot \text{Volume}\cdot g$$

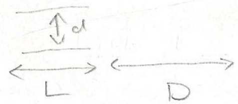
$$\Rightarrow q\frac{V}{d} = \rho_{oil}\left(\tfrac{4}{3}\pi r^3\right)g$$

From an experiment with terminal velocity we find $mg = 6\pi\eta r v_t$

$$\Rightarrow q = 18\pi\frac{d}{V_{ab}}\sqrt{\frac{\eta^3 v_t^3}{2\rho_{oil}\, g}} = 1.602\times10^{-19}\,C$$

Cathode Ray Tube (C.R.T.)

Heated filament → e → deflection plates → e → fluorescent screen



Speed: $\frac{1}{2}mv^2 = eV_{Acc} \Rightarrow v_L = \sqrt{\frac{2eV_L}{m}}$

Y-axis: $F = ma \Rightarrow \frac{eE}{m} = a = \frac{eV_y}{md}$

$$v_y = at = \frac{eV_y}{md}\cdot\frac{L}{v_L}$$

$$\tan\theta = \frac{v_y}{v_L} = \frac{eV_yL}{mdv_L}\cdot\frac{1}{v_L}$$

$$y = D\tan\theta = D\frac{eV_yL}{md}\cdot\frac{m}{2eV_L} = \frac{LD}{2d}\frac{V_y}{V_{Acc}}$$

Capacitor: Device that stores electric potential energy

→ Two oppositely charged plates separated from each other form a capacitor

$E \propto Q \Rightarrow V \propto Q$

$$C = \frac{Q}{V} \quad \text{(Farads)}$$

CAPACITANCE

CAPACITANCE: $C = Q/V$

- $\vec{E}$ FIELD BETWEEN CAPACITOR PLATES:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA = A\frac{\sigma}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}}$$

- VOLTAGE: $V_{ab} = \int_a^b \vec{E} \cdot dr = \frac{\sigma}{\varepsilon_0} d = \boxed{\frac{Qd}{A\varepsilon_0}}$

$\Rightarrow$ CAPACITANCE $C = \frac{Q}{V_{ab}} = \boxed{\frac{A\varepsilon_0}{d}}$

STORED ENERGY: WORK $= \int V dq = \int \frac{q}{C} dq$

$\Rightarrow W = \frac{1}{C}\int q\,dq = \frac{q^2}{2C}$ BUT $Q = CV$

$\Rightarrow U = \frac{C^2V^2}{2C} = \boxed{\frac{1}{2}CV^2}$

ENERGY DENSITY $u = \frac{U}{\text{volume}} = \frac{\frac{1}{2}CV^2}{Ad}$

$$u = \frac{CV^2}{2Ad} \xrightarrow[V = E \cdot d]{C = \frac{\varepsilon_0 A}{d}} \frac{1}{2}\frac{\varepsilon_0 A}{d} \cdot E^2 \cancel{d^2} \cdot \frac{1}{\cancel{Ad}} = \boxed{\frac{1}{2}\varepsilon_0 E^2}$$

DIELECTRIC STRENGTH

- PLACE solid insulator - "dielectric" between plates

REASONS: $\rightarrow$ Prevent plates touching
$\rightarrow$ HIGHER DIELECTRIC STRENGTH
$\rightarrow$ HIGHER $\varepsilon \Rightarrow$ HIGHER C

- DIELECTRIC CONSTANT

$\boxed{\varepsilon = k\varepsilon_0} \rightarrow C = kC_0 \rightarrow E = \frac{E_0}{K}$

PERMEATIVITY

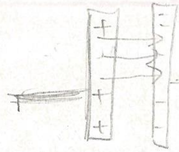
POLARISATION

= redistribution of positive & negative charges within a dielectric material

- MANY MOLECULES ARE DIPOLES
$\rightarrow$ ELECTRIC FIELD CAUSES THEM TO ALIGN
$\rightarrow$ MATERIALS W/ NONPOLAR MOLECULES CAN BE POLARISED W/ MAGNETIC FIELD (INDUCTION)

- DIELECTRIC POLARISATION:

$$E = E_0 - E_{IND} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_i}{\varepsilon_0} \qquad E = \frac{E_0}{K} \Rightarrow \frac{\sigma}{K\cancel{\varepsilon_0}} = \frac{\sigma - \sigma_i}{\cancel{\varepsilon_0}} \Rightarrow \boxed{\sigma_i = \sigma\left(1 - \frac{1}{K}\right)}$$



$E = \frac{\sigma}{\varepsilon_0}$ $\quad V = \frac{Qd}{A\varepsilon_0}$

$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$

$U = \frac{1}{2}CV^2$

PERMEATIVITY (F/m)

$\varepsilon = k\varepsilon_0$

$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$

COMBINING CAPACITORS, CURRENT DENSITY

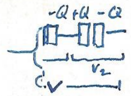
IN SERIES: $V_{AB} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$

BUT $Q = Q_1 = Q_2 \Rightarrow \frac{V_{AB}}{Q} = \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$

IN PARALLEL: $Q = Q_1 + Q_2 = C_1V_1 + C_2V_2$

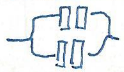
BUT $V = V_1 = V_2 \Rightarrow (C_T)V = (C_1 + C_2)V$

C SERIES:



$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$

IN PARALLEL:



$C_T = C_1 + C_2$

ELECTRIC CURRENT

ELECTRIC CURRENT = MOTION OF CHARGE FROM ONE REGION INTO ANOTHER

- When current moves around a closed loop, the path is called an electric circuit
  - ELECTRIC FIELD IS REQUIRED TO MAINTAIN A CURRENT
    - DOES WORK ON FREE CHARGE CARRIERS
    - MOST OF THIS ENERGY IS CONVERTED TO HEAT IN THEIR COLLISIONS
    - IN METALS, electrons are charge carriers
    - IN SEMICONDUCTORS: electrons and "positive holes"

ELECTRIC CURRENT $I = \frac{dQ}{dt}$ (IN Amperes (A))

- FREE ELECTRONS IN A CONDUCTOR HAVE RAPID RANDOM MOTION. ELECTRIC FIELD ADDS A GENERAL DRIFT SPEED V_d TO THIS

CURRENT DENSITY AT A POINT IN SPACE $J = I/A$

- IF WE CONSIDER A CURRENT W/ A DENSITY OF n free particles of charge q per m^3, w/ drift velocity $\vec{v}$ through an area A, we get

$$Q = n|q||v|A\,dt \Rightarrow \frac{dQ}{dt} = I = nA|v||q|$$

so $\vec{J} = nq\vec{v_d}$ (Always points w/ $\vec{E}$)

Current Density $\vec{J}$ and $\vec{E}$.

- No electric field: electrons have velocity $\vec{v}$, but $[\vec{v}_d]_{Avg} = 0$

In electric field: electron is accelerated

$$\vec{v} = \vec{v}_0 + \vec{a}t = \boxed{\vec{v}_0 - \frac{e\vec{E}t}{m}}$$

t seconds after last collision

$\vec{v}_0$ is velocity after collision

Thus: $\vec{v}_d = [\vec{v}]_{av} = \left[\vec{v}_0 - \frac{e\vec{E}t}{m}\right]_{AV} = 0 - \frac{e\vec{E}}{m}[t]_{av} = \boxed{-\frac{e\tau\vec{E}}{m}}$

where τ is the average time between collisions

USING $\vec{v}_d = -\frac{e\tau}{m}\vec{E}$ & $\vec{J} = -ne\vec{v}_d \Rightarrow \boxed{\vec{J} = \frac{ne^2\tau}{m}\vec{E}}$

OHM'S LAW: $J \propto E$ or $J = \frac{1}{\rho}E$

- where $\rho = \frac{E}{J} = \frac{m}{ne^2\tau}$ is the resistivity ($V\,A^{-1}m$)
- CONDUCTIVITY IS DEFINED AS $1/\rho$
- FOR MOST MATERIALS, RESISTIVITY DEPENDS STRONGLY ON TEMP

FOR METALS $\rho(T) \approx \rho_0[1 + \alpha(T - T_0)]$

FOR SEMICONDUCTOR: RESISTIVITY DROPS w/ T

FOR SUPERCONDUCTORS: ZERO RESISTIVITY below some T

RESISTANCE R

- CONSIDER A CONDUCTING ROD SUBJECT TO UNIFORM $\vec{E}$

$\vec{E} = \rho\vec{J}$

BY DEFINITION, WE KNOW $J = \frac{I}{A}$ $\Big\}$ $\boxed{\vec{E} = \frac{\rho I}{A}}$

- But we know that potential difference between ends is:

$V = EL = \int \vec{E} \cdot d\vec{l}$

$\Rightarrow$ THIS GIVES US THAT $\frac{V}{L} = \frac{\rho I}{A} \Rightarrow V = \frac{\rho L}{A} I$

RESISTANCE: IS DEFINED BY: $R = \frac{V}{I}$

FROM OUR LAST EXAMPLE, we get $R = \frac{\rho L}{A}$

- RESISTIVITY IS A FUNCTION OF TEMPERATURE, WHICH MEANS THAT SO IS RESISTANCE:
  - IN METALS: $R(T) = R_0(1 + \alpha(T - T_0))$
  - IN VACUUM DIODE, ONLY ABOVE A CERTAIN V
  - IN SEMICONDUCTORS - COMPLEX DEPENDANCE

Electromotive Force - Circuits

For steady current: Need closed circuit

Charge moves in direction of decreasing U

must have same potential at start & end

$\rightarrow$ INCREASES somewhere in the circuit

($\mathcal{E}$) ELECTROMOTIVE FORCE: something that forces charge to move from lower to higher potential

SOURCES: BATTERY, SOLAR CELL, MAGNETIC DYNAMO GENERATOR...

WORK DONE FROM a to b: $W_n = q\mathcal{E} \Leftrightarrow$ CHANGE IN POTENTIAL ENERGY: $\Delta U = qV_{ab}$

- WHEN TERMINALS ARE CONNECTED, CURRENT FLOWS:

- potential increase = potential drop $\Rightarrow$ $\mathcal{E} = V_{ab} = IR$ (IN IDEAL SITUATIONS)
- charge forced to follow wire by equal but opposite charges

- FOR REAL SOURCES, CURRENT PASSES A RESISTANCE
  - INTERNAL RESISTANCE OF BATTERY IS r

$\Rightarrow V_{ab} = \mathcal{E} - Ir = IR \Rightarrow \boxed{I = \frac{\mathcal{E}}{r+R}}$

- ENERGY & POWER

$\frac{dQ}{dt} = I$, $\boxed{U_{ab} = V_{ab}dQ = V_{ab}I\,dt}$ potential energy

Power $= \frac{dU}{dt} \Rightarrow \boxed{P = V_{ab}I}$ POWER

OHMIC RESISTOR: $P = IV = I(IR) = I^2R = V^2/R$

DISSIPATED AS HEAT

SOURCE OUTPUT: $P = IV = (\mathcal{E} - Ir)I = \mathcal{E}I - I^2r$

- FIRST TERM: Non-electric sources
- 2^{nd} TERM: Power loss

Resistors in ~~Parallel~~ Series

$V_1 = IR_1$

$V_2 = IR_2$

$V_T = IR_T$

$V_T = V_1 + V_2 = IR_1 + IR_2$

$I R_T = I(R_1 + R_2)$

$\boxed{R_T = R_1 + R_2 = \sum_i R_i}$

Resistors in Parallel

$I_1 = \frac{V_T}{R_1}$

$I_2 = \frac{V_T}{R_2}$

$I_T = \frac{V_T}{R_T}$

$I_T = I_1 + I_2$

$\frac{V_T}{R_T} = \frac{V_T}{R_1} + \frac{V_T}{R_2} \Rightarrow \boxed{\frac{1}{R} = \sum_i \frac{1}{R_i}}$

Kirchoff's Rules for Complex Circuits

- **Junction Rule:** $\sum I = 0$

  By conservation of charge, sum of currents into junction $= 0$

- **Loop Rule:** $\sum V = 0$

  Electrostatic force = conservative ⇒ sum of p.d. is zero

  Sign convention: Low → High potential – positive

  High → Low potential – negative

Electrical Measuring Equipment

- **D'Arsonval Galvanometer:** (small current & voltage)
  - Current flows through coil in magnet
  - magnetic field exerts force on coil $\propto$ current
  - spring exerts counter force $\propto$ displacement
  - displacement $\propto$ current (and V if ohmic)

- **Ammeter:** (measure current)
  - connected in series
  - (usually) neglegeble resistance R_c (for I_{fs}) "full scale"
  - measure current outside range w/ Shunt Resistor

  Wish to read current $I_{extreme}$: $R_{shunt} = \frac{I_{fs}}{(I_{extr} - I_{fs})} \cdot R_c$

- **Voltmeter** (measure potential difference)
  - connected in parallel
  - Ideally infinite resistance of R_c
  - For $V_{full\ scale}$, need series resistor $V_{fs} = I_{fs}(R_c + R_{sh})$

Resistor-Capacitor Circuits

R-C circuits = resistor + capacitor in series

- Example with battery on right.

  By loop rule: $\mathcal{E}_{emf} - iR - \frac{q}{c} = 0$

$$\frac{dq}{dt} = i = \frac{\mathcal{E}_{emf}}{R} - \frac{q}{RC}$$

$$\int dq' = \int \frac{\mathcal{E}_{emf} C - q}{RC} dt$$

$$\int \frac{1}{-C\mathcal{E}_{emf} + q'} dq' = \frac{-1}{RC} \int dt$$

$$\ln\left|-C\mathcal{E}_{emf} + q\right|_{q=0}^{q} = -\frac{t}{RC} = \ln\left|\frac{-C\mathcal{E}_{emf} + q}{-C\mathcal{E}_{emf}}\right|$$

- But $Q_{final} = C\mathcal{E}_{emf}$ (CV)

$$\Rightarrow e^{-\frac{t}{RC}} = \frac{q - Q_F}{-Q_F} \Rightarrow q = Q_F(1 - e^{-\frac{t}{RC}})$$

- **Time Constant** $\tau = RC$
  - charge is $(1 - \frac{1}{e})$ of final
  - current is $\frac{1}{e}$ of initial value

- **Capacitor Discharge:**

$$\frac{dq}{dt} = i = \frac{-q}{RC} \quad \text{(from above)}$$

$$\int \frac{dq'}{q'} = \int \frac{-1}{RC} dt$$

$$\ln q' \Big|_{q'=Q_0}^{q} = -\frac{t'}{RC}\Big|_{t'=0}^{t}$$

$$\ln q - \ln Q_0 = \ln\left(\frac{q}{Q_0}\right) = \frac{-t}{RC}$$

$$\Rightarrow \boxed{q = Q_0 e^{-\frac{t}{RC}}} \quad \boxed{\frac{dq}{dt} = \frac{-Q_0}{RC} e^{-\frac{t}{RC}}}$$

- **Power/Energy:**

$$P = IV = i\mathcal{E}_{emf} = i(iR + \frac{q}{c})$$

Energy supplied $= \int P\,dt = Q\mathcal{E}_{emf}$

Energy stored in capacitor $= \frac{1}{2}CV^2 = \frac{Q\mathcal{E}_{emf}}{2}$

Energy lost in resistor is half

MAGNETISM

- MAGNETIC EFFECTS INITIALLY EXPLAINED IN TERMS OF MAGNETIC POLES.
  - IF FREE TO ROTATE, NORTH POLE WILL POINT NORTH
  - OPPOSITE POLES ATTRACT, LIKE POLES REPEL
  - BOTH ATTRACT UNMAGNETISED OBJECTS w/ Fe, Ni, Co
  - ISOLATED MAGNETIC MONOPOLES ~~[illegible]~~ DON'T EXIST IRL (AFAWK)

MAGNETIC FIELD: explained by saying magnetised objects create a magnetic field in the space around them, & other objects react

MAGNETIC FIELD LINE: Lines that show where a compass would point at any point

- Magnetic FIELD FROM A CURRENT:
  - OERSTED 1819: CURRENT DEFLECTS A NEARBY COMPASS
  - FARADAY: Moving a magnet causes current to appear in a loop
  - CHANGING CURRENT IN 1 loop → INDUCE current in other loop

$\Rightarrow$ Close link between electricity & magnetism

MAGNETIC FORCE

- MAGNETIC FIELD exerts force on moving particles

$$\vec{F} = q\vec{v}\times\vec{B}$$

PROPORTIONAL TO CHARGE q
PROPORTIONAL TO STRENGTH OF MAG FIELD $\vec{B}$
PROPORTIONAL TO $\perp$ VELOCITY

ACTS ON DIRECTION $\perp$ TO BOTH $\vec{v}$ AND $\vec{B}$ (RHand Rule)

$[B] = N\,(C\,ms^{-1})^{-1} = NA^{-1}m^{-1} = T$ (TESLAS)

- MAGNETIC FIELD GENERATED BY:
  - BAR MAGNETS
  - CURRENT CARRYING WIRE
  - CURRENT CARRYING LOOPS / COILS / SOLENOIDS

MAGNETIC FLUX

$$\Phi_B = \int \vec{B}\cdot d\vec{A}$$

- MAGNETIC MONOPOLES DO NOT EXIST

$\Rightarrow$ GAUSS' LAW: $\oint \vec{B}\cdot dA = 0$

PARTICLE MOTION IN MAGNETIC FIELDS

- CONSIDER A CHARGED PARTICLE MOVING PERPENDICULAR TO A UNIFORM MAGNETIC FIELD:

experiences a force $\perp$ to velocity
$\Rightarrow$ magnitude of velocity must stay the same
$\Rightarrow$ particle moves in a circle

$F = q\,v\times B \qquad |F| = |q|vB$ all constant
$\Rightarrow$ CIRCULAR MOTION

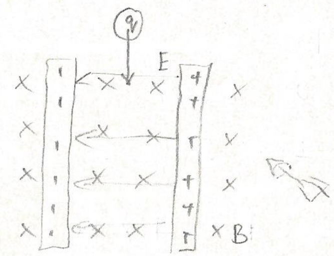
$|q|vB = \dfrac{mv^2}{R} \Rightarrow \boxed{R = \dfrac{mv}{|q|B}}$ RADIUS OF CIRCLE

$\dfrac{v}{R} = \boxed{\omega = \dfrac{|q|B}{m}}$ ANGULAR VELOCITY

$\dfrac{\omega}{2\pi} = \boxed{f = \dfrac{|q|B}{2\pi m}}$ "CYCLOTRON" FREQUENCY

- CYCLOTRON:
  - PARTICLE ACCELERATORS
  - MAGNETIC FIELD CONFINES PARTICLE IN CIRCLE
  - ELECTRIC FIELD ACCELERATES PARTICLES

- VELOCITY SELECTOR:
  - charge moving through a tube
  - horizontal electric field
  - vertical magnetic field



LEFT FORCE DUE TO $\vec{E}$: $F_E = qE$

RIGHT FORCE DUE TO $\vec{B}$: $F_B = qvB$

GOES STRAIGHT IF WE HAVE: $F_E = F_B \Rightarrow qE = qvB \Rightarrow \boxed{v = \dfrac{E}{B}}$

- THOMPSON $e : m_e$ ratio experiment:
  - FIRST ACCELERATE electron through a voltage V

  ENERGY: $\frac{1}{2}m_e v_e^2 = eV \Rightarrow \boxed{v_e = \sqrt{\dfrac{2eV}{m_e}}}$

  - ADJUST E/B s.t. electrons PASS THROUGH

$\Rightarrow v_e = \sqrt{\dfrac{2eV}{m_e}} = \dfrac{E}{B} \Rightarrow \dfrac{2eV}{m_e} = \dfrac{E^2}{B^2} \Rightarrow \dfrac{e}{m_e} = \dfrac{E^2}{B^2}\dfrac{1}{2V}$

- MASS SPECTROMETRY: $R = mv/|q|B$

Mass Spectrometer:

- velocity selectors $\overset{\text{THEN}}{\Longrightarrow}$ $R = \frac{mv}{|q|B}$
  - Particles of same velocity but different masses travel in circles of different radii in a magnetic field:

$\Rightarrow$ Isotopes can be separated

Magnetic Force on a Current-Carrying Conductor

- Consider current-carrying wire, $\perp$ to uniform magnetic field:
- Average "Drift Velocity" $\vec{v}_d$ of charges
- Average force on charges:

$$\vec{F} = q\vec{v}_d \times \vec{B} \Rightarrow q v_d B = F$$

- suppose n charges per unit ~~length~~ volume

$$F = n \cdot V_{olume} \cdot q v_d B$$

$$F = n \cdot AL \cdot q v_d B$$

$|\vec{J}| = n$ charges per unit volume $\cdot\, q$ charge $\cdot\, v_d$ charge speed

$|\vec{J}| = nqv_d \qquad m^{-3} \cdot C \cdot ms^{-1} = m^{-2}A$ current density

$I = J \cdot A = nqv_d A$

$$\Rightarrow \boxed{F = IlB}$$

If not perpendicular, we get:

$$\boxed{\vec{F} = I\vec{L} \times \vec{B}} \boxed{= \int I\, d\vec{L} \times B}$$

Torque on a Current Loop

- Rectangular loop w/ current in magnetic field
  - Net Force $= \emptyset$
  - Torque $\vec{\tau} = \vec{r} \times \vec{F}$
    - But $F = Il \times B$
    - But $r_\perp = \frac{h}{2}\sin\varphi$

$$\Rightarrow |\tau| = F_1 \frac{h}{2}\sin\varphi - (-F_1 \frac{h}{2}\sin\varphi) = Fh\sin\varphi$$

$$\Rightarrow |\tau| = IlB\, h\sin\varphi \qquad \text{but } lh = A$$

$$|\tau| = IAB\sin\varphi$$

- We define $\vec{\mu}$ to be $I\vec{A}$
  - called the "magnetic moment vector"
  - s.t. right hand rule has current (shown)

- Thus, we get an expression for τ:

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

- Maximum torque when $\varphi = \pm\frac{\pi}{2}$
- Zero at $\varphi = \emptyset$ (stable equilibrium) & π (unstable equilibrium)
- Any body which experiences a magnetic torque as above is called a *magnetic dipole*
- A magnetic field tends to *line up with the magnetic field*

Potential Energy of Magnetic Dipole

$$dW = F\,dr = \tau\, d\varphi \quad \text{work done through } \varphi$$

$$W = \int_{\varphi_1}^{\varphi_2} dW = \int_{\varphi_1}^{\varphi_2} \tau\, d\varphi = \int_{\varphi_1}^{\varphi_2} \mu B \sin\varphi = \mu B(\cos\varphi_2 - \cos\varphi_1)$$

Potential Energy: $U(\varphi) = -\mu B\cos\varphi = -\vec{\mu}\cdot\vec{B}$

Torque on a Solenoid

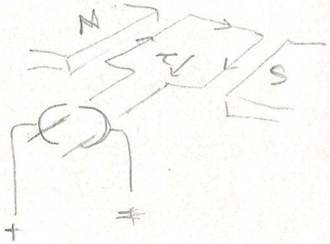
SOLENOID = "helenoid" WINDED UP WIRE

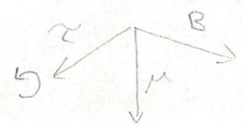
- APPROXIMATES A NUMBER o CIRCULAR LOOPS

$\vec{\mu} = NI\vec{A}$

$\vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow NIAB\sin\varphi$

DC Motor





Magnetic Field of Moving Charge

- Consider a charge q moving w/ constant $\vec{v}$

(look at special relativity for more information)

IT IS FOUND THAT: experimentally

$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v} \times \hat{r}}{r^2}}$$

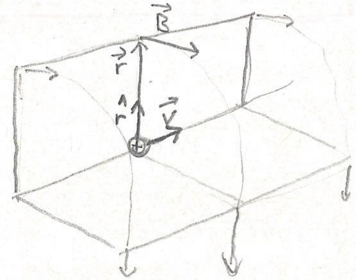
where $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{m \cdot T \cdot A^{-1}}$ Permeability of freespace

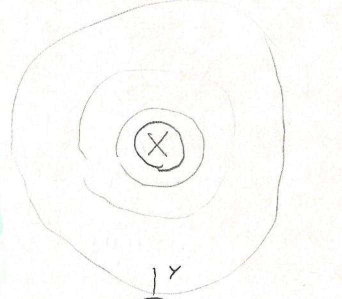
q = charge of source

v = velocity of source

r = distance to source

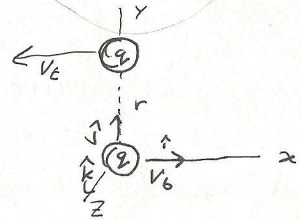
- The resulting field encircles the path of charge





Ex:

- 2 protons passing w/ opposite velocities V_t, V_b as shown.
  - Find electric & magnetic forces on upper proton
  - Find ratio between these forces



$$\boxed{F_E = \frac{1}{4\pi\varepsilon} \frac{q_t q_b}{r^2}\hat{j}} \qquad F_B = qv \times B$$

$$B = \frac{\mu_0}{4\pi\varepsilon} \cdot \frac{q\vec{v}_b \times \vec{r}}{r^2} = k \cdot \frac{q}{r^2} v_b(\hat{i} \times \hat{j}) = \frac{\mu_0 q_b v_b}{4\pi r^2}\hat{k}$$

$$\Rightarrow F_B = q_t \vec{V}_t \times \left(\frac{\mu_0 q_b v_b}{r^2}\right)\hat{k} = \boxed{\frac{q_t q_b V_t V_b \mu_0}{4\pi r^2}\hat{j}}$$

- BOTH FORCES ACT IN THE Y DIRECTION ($\hat{j}$)

$$\frac{F_B}{F_E} = \mu_0 \varepsilon_0 V_t V_b \xrightarrow{\text{IF } V_t = V_b} \frac{F_B}{F_e} = \mu_0 \varepsilon_0 V^2$$

- FROM ELECTROMAGNETIC WAVES: $\mu_0 \varepsilon_0 = \frac{1}{c^2}$

$$\Rightarrow \frac{F_B}{F_e} = \frac{V^2}{C^2}$$ Relativistic Correction to Electrostatic force

Magnetic Field of a Current

- We know magnetic field of a single moving charge
- We can figure it out for many charges at once

$\Rightarrow \; dQ = nqA\,dl$ — n = charges per unit volume, A = cross sectional Area, q = size of each charge

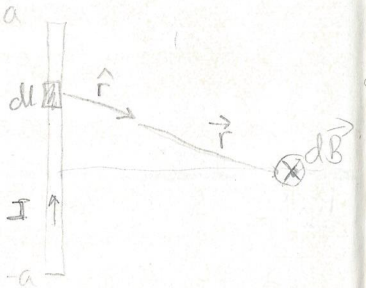
For point charges: $|\vec{B}| = \left|\frac{\mu_0}{4\pi r^2}\right| qv\sin\varphi$

$$\Rightarrow dB = \frac{\mu_0}{4\pi r^2} dQ\cdot v\sin\varphi = \frac{\mu_0}{4\pi r^2}(n|q|v_d A)dl\sin\varphi$$

$$= \frac{\mu_0}{4\pi r^2} \underline{nqA}\,dl\cdot\underline{v}\sin\varphi$$

$$= \frac{\mu_0}{4\pi r^2} I\sin\varphi\, dl$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}\times\hat{r}}{r^2}}$$ Biot & Savart Law



- Suppose wire is of length $2a$, point shown

$$B = \frac{\mu_0}{4\pi}\int_{-a}^{a}\frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{I\mu_0}{4\pi}\int_{-a}^{a}\frac{\sin\varphi}{r^2}(-\hat{\imath})$$

$$= \frac{-I\mu_0}{4\pi}\int_{-a}^{a}\frac{1}{x^2+y^2}\cdot\frac{x}{\sqrt{x^2+y^2}}\,\hat{\imath} =$$

$$B = \frac{-\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}\,\hat{\imath}$$

- For a long wire / short x, $\sqrt{x^2+y^2} \sim a$

$$\lim_{a\to\infty}|\vec{B}| = \frac{\mu_0 I}{2\pi x}$$

- Force between parallel conductors

- suppose separated by r

$$F = I'\vec{L}\times\vec{B} = I'L\frac{\mu_0 I}{2\pi r}$$

Force per unit length: $\frac{F}{L} = \frac{\mu_0}{2\pi r} II'$

Ampere's Law

Line Integral $\oint \vec{B}\cdot dl$

- we consider circular line integral shown

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{on the circle}$$

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \int B\,dl = \frac{\mu_0 I}{2\pi r}\oint dl = \frac{\mu_0 I}{2\pi r}(2\pi r) = \boxed{\mu_0 I}$$

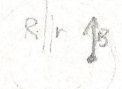
- only circumferential sectors contribute
- consider another path in general

○ for any piece:

$$\vec{B}\cdot d\vec{l} = B\,dl\cos\varphi = \boxed{B\,r\,d\theta}$$

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \oint B\,r\,d\theta = \oint \frac{\mu_0 I}{2\pi r} r\,d\theta = \mu_0 I$$

Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$



- Magnetic Field of a Wire:

Inside the wire

$$\oint \vec{B}\cdot d\vec{l} = B\oint d\vec{l} = \boxed{2\pi r B = \mu_0 I_{enc}}$$

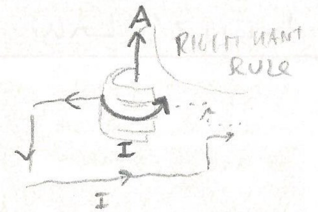
but $I_{enc} = I\frac{\pi r^2}{\pi R^2} = \boxed{I\frac{r^2}{R^2}}$

$$\Rightarrow \boxed{B = \frac{\mu_0}{2\pi} I \frac{r}{R^2}} \quad \text{inside the wire}$$

outside the wire, $I_{enc} = I$

$$\Rightarrow \boxed{B = \frac{\mu_0}{2\pi r} I} \quad \text{outside the wire}$$

Electromagnetic Induction

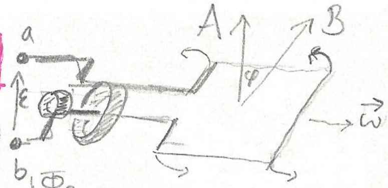


Magnetic Flux: $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\varphi$

Faraday's Law: induced EMF $\mathcal{E} = -\frac{d\Phi_B}{dt}$

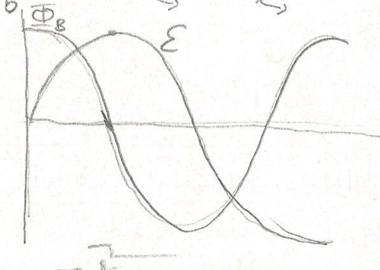
- the induced EMF in a closed loop equals to the negative of the time rate of change of magnetic flux

Dynamo - AC Generator ("Alternator")



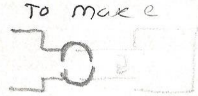
$$\Phi_B = BA\cos\phi = BA\cos\omega t$$

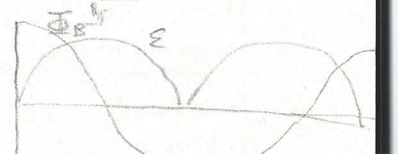
$$\frac{d\Phi_B}{dt} = -\frac{d}{dt}BA\cos\omega t = \boxed{\omega BA\sin\omega t = \mathcal{E}}$$



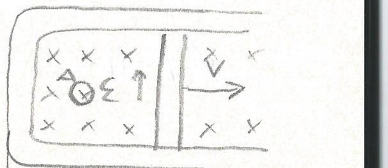
- this gives the graph on the right because of the slope $\mathcal{E} = -\frac{d\Phi_B}{dt}$

- one could devide the connection using a devided commutator to make a D.C. dynamo





Motional EMF



- can also cause emf by moving the rod shown

– consider magnetic forces on charges in the conductor

$$\vec{F}_m = q\vec{v}\times\vec{B}$$

- a positive charge gets pushed up
- a negative charge gets pushed down

→ causes electric field to form building up to $qE = qvB$

→ top has higher potential differences

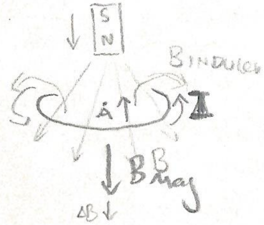
$$V = \frac{F\times d}{q} \Rightarrow V = EL = vBL$$

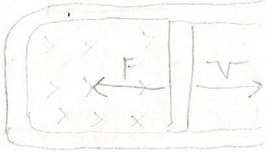
$$\text{but } \mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d}{dt}(L\cdot vt) = \boxed{-BLv}$$

- Faradays law answer agrees.

LENZ' LAW

LENZ'S LAW: THE DIRECTION OF ANY MAGNETIC INDUCTION EFFECT IS SUCH AS TO OPPOSE THE EFFECT PRODUCING IT (CAN BE DERIVED FROM FARADAY'S LAW)



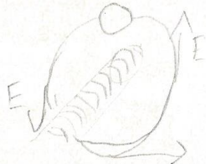


INDUCED ELECTRIC FIELDS

- LONG THIN SOLENOID
- CIRCULAR CONDUCTING LOOP AROUND IT

→ SOLENOID CURRENT CHANGES
→ MAGNETIC FLUX ALSO CHANGES
→ INDUCED EMF CAN BE WRITTEN IN TERMS OF INDUCED ELECTRIC FIELD

$$\oint \vec{E} \cdot dl = -\frac{d\Phi_B}{dt}$$



SUMMARY OF LAWS

COULOMB'S LAW

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{r}$$

GAUSS' LAW

$$\Phi_E = \oint \vec{E} \cdot dA = \frac{q}{\varepsilon_0}$$

OHM'S LAW

$$J = \frac{E}{\rho} \quad \left(\rho = \frac{m}{ne^2\tau}\right)$$

$$\rightarrow V = IR$$

GAUSS' LAW ~~FOR M~~ FOR MAGNETISM

$$\Phi_B = \oint \vec{B} \cdot dA = 0$$

BIOT-SAVART LAW

$$B = \frac{\mu_0}{4\pi}\int \frac{I\, dl \times \hat{r}}{r^2}$$

AMPERE'S LAW

LINE INTEGRAL

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

FARADAY'S LAW

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

AMPERE LINE INTEGRAL

$$\oint \vec{E} \cdot dr = \Delta V_{AB}$$

LENZ' LAW

DIRECTION OF INDUCED B-FIELD OPPOSES THE CAUSING FIELD