Homework 5: Non-Inertial Systems and Fictitious Forces

Due: Tuesday March 12th, 13:00

Policy: Collaboration is allowed, but every student is required to hand in his/her own version of the solutions. Please include your name and student number on the solutions.

Problem 1. We consider a charged particle with mass m and charge $q > 0$ moving in the vicinity of a fixed particle with charge $-q' < 0$ and under the influence of a magnetic field $\vec{B}$. The Lorentz force on q exerted by q' and the magnetic field is

$$\vec{F}_{\mathrm{L}} = -\frac{k_e q q'}{r^2}\hat{r} + q\,\vec{v} \times \vec{B}, \tag{1}$$

where $\hat{r}$ is the unit vector pointing from q' to q.

1. (20%) Newton's 2nd Law for the particle with mass m is

   $$\vec{F}_{\mathrm{L}} = m\,\vec{a}_{\mathrm{in}},$$

   with respect to an inertial system S. Write this equation with respect to a rotating coordinate system R, which rotates with respect to S with an arbitrary angular velocity $\vec{\omega}$.

2. (20%) Choose $\vec{\omega}$ such that the acceleration in the rotating frame R, $\vec{a}_{\mathrm{rot}}$, can be written as a sum of two terms,

   $$\vec{a}_{\mathrm{rot}} = -\frac{k_e q q'}{m\,r^2}\hat{r} + \left(\frac{q}{2m}\right)^2 \vec{B} \times (\vec{B} \times \vec{r}).$$

3. (10%) Assume that $\vec{B}$ is sufficiently small such that the term quadratic in $\vec{B}$ can be neglected compared to the term independent of $\vec{B}$. Then the motion of q in the rotating frame satisfies Kepler's area law. Assume that the trajectory lies in a plane orthogonal to $\vec{B}$, and is closed but non-circular in the rotating frame. Make a sketch of trajectory of q in the inertial system.

The angular velocity ω above is called the Larmor frequency, named after the Irish physicist Joseph Larmor (1857-1942). The Larmor effect is relevant for modern technologies such as Magnetic Resonance Imaging.

Problem 2. One throws at the equator a particle vertically upwards with initial velocity v_0. The velocity v_0 is small such that the maximal height h of the particle is very small compared to the radius R_e of the earth.

1. (20%) The particle will hit the ground at a distance d from the point it departed. Explain whether this point will be to the east or to the west from the point of departure.

2. (30%) Let Ω be the angular velocity of the earth. Prove using the approximation $h \ll R_e$, that $d = \frac{4\Omega v_0^3}{3\,g^2}$.