JF PY1T10 Special Relativity

Lecture 7:

- Doppler Effect for Light

Doppler Effect

Doppler effect: phenomena where the measured frequency f (or wavelength λ) of a wave is changed by relative motion of source and observer.

Acoustic Doppler effect:

- The medium supporting the wave is air
- Sound has well defined velocity relative to medium
- [See University Physics 16.8]
- Source moving, detector stationary *w.r.t.* medium
- Source stationary, detector moving *w.r.t.* medium
- Source and detector moving *w.r.t.* medium.

Relativistic Doppler Effect for Light

Consider longitudinal case: relative motion of source and observer is along line joining them.

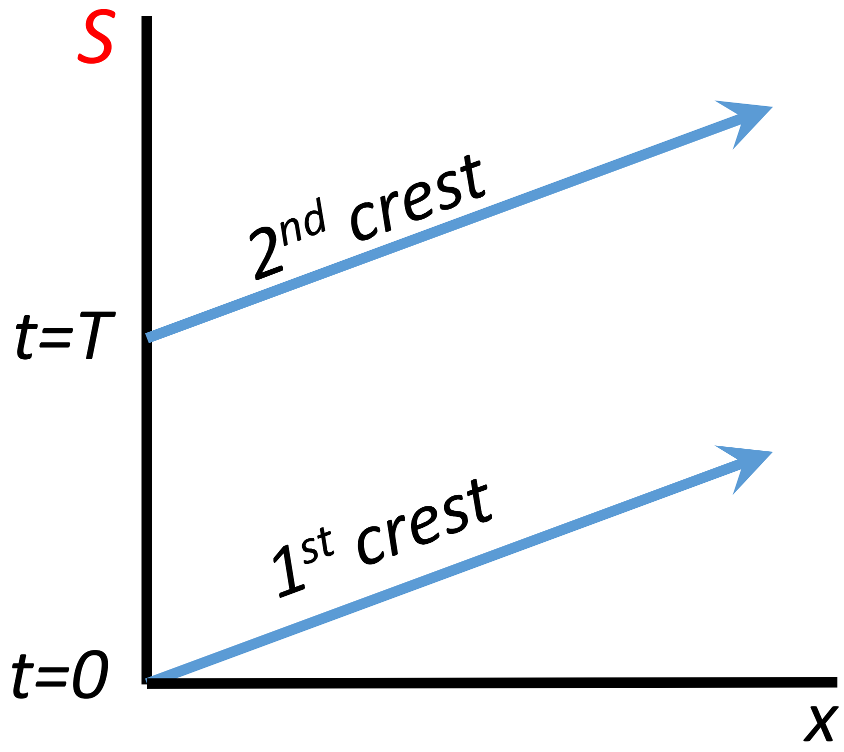
Source at rest at $x = 0$ on S.

Emits 1st crest at $t = 0$,

Emits 2nd crest at $t = T$.

T is the period as measured in S

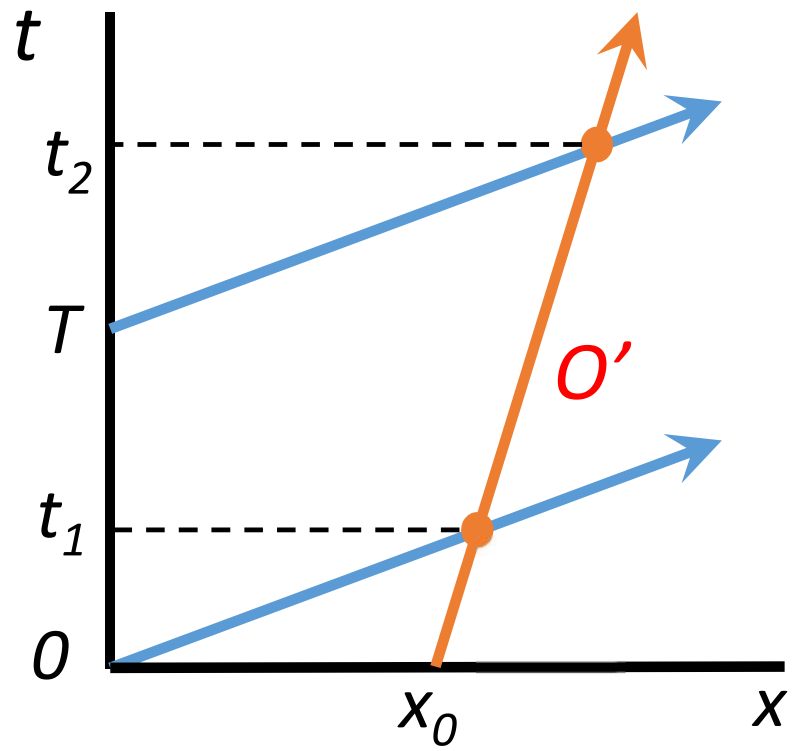
f is the frequency, $T = \frac{1}{f}$



Relativistic Doppler Effect for Light

Now consider observer, *O′*, moving relative to S with velocity *v′*, in frame *S′* (i.e. *O′* receding from light-source at $x = 0$).

What frequency, *f′*, does he measure?



Suppose *O′* is at $x = x_0$ at $t = 0$, when first crest is emitted at $x = 0$.

2nd crest has further to travel.

$$\therefore T \uparrow, f \downarrow$$

Relativistic Doppler Effect for Light

How much do *T* and *f* change? How do we proceed?

Step 1: Find times and positions of 1st and 2nd crests at *O'* as measured in *S*.

Step 2: Use LT:

$$t' = \gamma(t - \frac{vx}{c^2})$$

Arrival at *O'* given by $(x_1, t_2), (x_2, t_2)$ as measured in *S*.

Relativistic Doppler Effect for Light

$$x_1 = ct_1 = x_0 + vt_1 \qquad ①$$
$$x_2 = c(t_2 - T) = x_0 + vt_2 \qquad ②$$

①, ② $\Rightarrow \quad x_0 = ct_1 - vt_1 = c(t_2 - T) - vt_2$

$$\Rightarrow t_2 - t_1 = \frac{cT}{c-v} \qquad ③$$

Also using ①,② $x_2 - x_1 = v(t_2 - t_1) = \frac{vcT}{c-v}$ ④

Relativistic Doppler Effect for Light

But we need to find time difference <u>as measured in *S′*</u>, i.e. $t_2{}' - t_1{}'$.

Use LT:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$t_2{}' - t_1{}' = \gamma\left[(t_2 - t_1) - \frac{v(x_2 - x_1)}{c^2}\right]$$

Substitute in ③,④:

$$t_2{}' - t_1{}' = \gamma\left[\frac{cT}{c-v} - \frac{v}{c^2} . \frac{vcT}{c-v}\right]$$

$$= \frac{\gamma cT}{c-v}\left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \; t_2{}' - t_1{}' = \frac{1}{\gamma}(t_2 - t_1)$$

Relativistic Doppler Effect for Light

$$t_2' - t_1' = \frac{1}{\gamma}(t_2 - t_1)$$

Note that $(t_2' - t_1')$ is the *proper time interval* measured in *S'*, but $(t_2 - t_1)$ measured in *S* is not.

Put $\beta = \frac{v}{c}$. Then the period as measured in *S'* is:

$$T' = t_2' - t_1'$$

$$= \gamma \frac{1 - \beta^2}{1 - \beta} T$$

$$= \gamma(1 + \beta)T$$

But $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$

$$T' = \left[\frac{1 + \beta}{1 - \beta}\right]^{\frac{1}{2}} T$$

Relativistic Doppler Effect for Light

$$T' = \left[\frac{1+\beta}{1-\beta}\right]^{\frac{1}{2}} T$$

Time difference between *reception* of successive pulses.

Time difference at source of *emission* of successive pulses.

Note that $T' \neq \gamma T$, since we don't measure the time interval between the same pair of events.

Also: $f = \frac{1}{T} \quad \Rightarrow \quad f' = \left[\frac{1-\beta}{1+\beta}\right]^{\frac{1}{2}} f$

Relativistic Doppler Effect for Light

If $\beta \ll 1$, we can use the Binomial Expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2}x^2 + \cdots$$

$$\left[\frac{1+\beta}{1-\beta}\right]^{\frac{1}{2}} \cong 1+\beta \qquad T' \cong (1+\beta)T$$

$$\left[\frac{1-\beta}{1+\beta}\right]^{\frac{1}{2}} \cong 1-\beta \qquad \Rightarrow f' \cong (1-\beta)f$$

Relativistic Doppler Effect for Light

But $\lambda = {}^{c}/_{f} = c\,T$, so λ changes also, proportionally to T:

$$\lambda' = \left[\frac{1+\beta}{1-\beta}\right]^{\frac{1}{2}} \lambda$$

$$\therefore\ \lambda' \cong (1+\beta)\lambda$$

If S' is receding from S:
v is positive, β is positive,
you get a red shift:
$v' < v, \qquad \lambda' > \lambda$

If S' is approaching S:
v is negative, β is negative,
you get a blue shift:
$v' > v, \qquad \lambda' < \lambda$

Another Viewpoint

Two observers:

- O stationary in *S*
- O' and source moving at *v w.r.t. S*

	Emission of pulse 1	Emission of pulse 2
Measured by O’	(t_1', x')	(t_2', x')
Measured by O	(t_1, x_1)	(t_2, x_2)

N.B. These are not the same events as earlier.

$$(t_2 - t_1) = \gamma(t_2' - t_1') = \gamma\tau_0$$

$$(t_2 - t_1) > (t_2' - t_1')$$

(The moving clock runs slow)

Another Viewpoint

	Emission of pulse 1	Emission of pulse 2
Measured by O’	(t_1', x')	(t_2', x')
Measured by O	(t_1, x_1)	(t_2, x_2)

	Reception of pulse 1	Reception of pulse 2
Measured by O	(t_3, x)	(t_4, x)

$$t_4 - t3 = \tau \text{ (Period seen by O)}$$

$$\tau = \left[\frac{1+\beta}{1-\beta}\right]^{\frac{1}{2}} \tau_0$$

Also, since the second pulse has further to travel than the first: $t_4 - t_3 > t_2 - t_1$

Doppler effect is an example of **looking** at a moving clock.

Recessional Red Shift

A famous example of the Doppler effect is the red shift of light from distant galaxies.

Atoms at rest emit narrow spectral lines, e.g. H 1s-2p : λ_0 = 121.6nm (Lyman α)

But spectral lines from distant galaxies are shifted to longer λ, "red shifted".

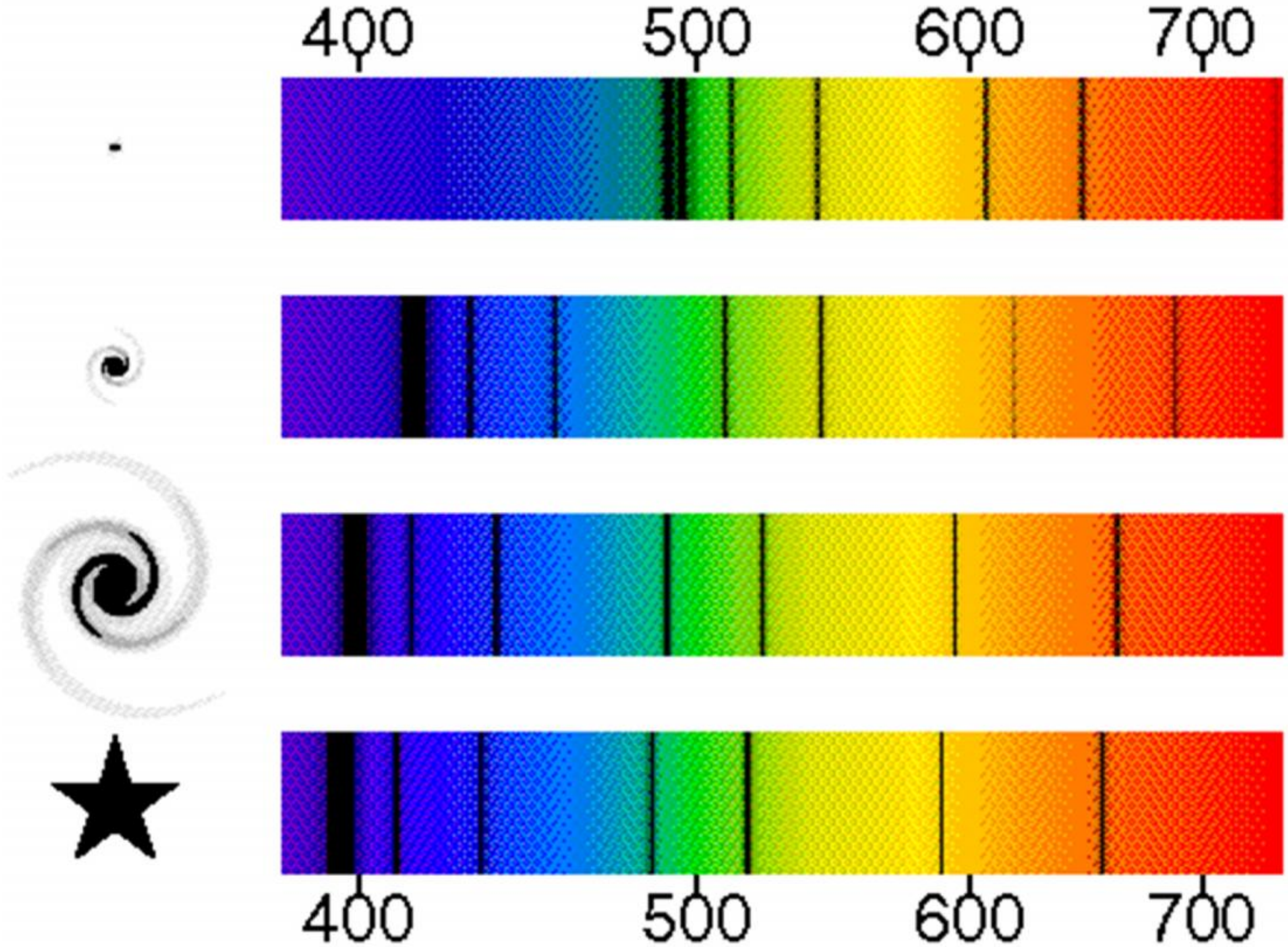
This implies those galaxies are receding from us!

In fact, the spectrum of light is almost continuous but with some weak absorption lines – produced when escaping radiation passes through a cooler region.

e.g. H and K lines in ionised Ca

$$\beta = \frac{\left(\frac{\lambda'}{\lambda}\right)^2 - 1}{\left(\frac{\lambda'}{\lambda}\right)^2 + 1}, \qquad \beta = \frac{v}{c}$$

Recessional Red Shift



Edwin Hubble (1889 – 1953)

From observations of the redshift of 46 galaxies, Hubble was able to conclude that the speed at which a galaxy recedes is proportional to its distance from us:

$$\boxed{v = H_0 d}$$

where $H_0 = 72 \pm 8$ km s^{-1}/ Mpc

[1pc = 3.26 light years]

v = the velocity of the receding galaxy

d = the distance from us, in megaparsecs

Redshift

Astronomers define redshift, 'z', as follows:

$$z = \frac{\lambda_{obs}\lambda_0}{\lambda_0}, \qquad (z+1)^2 = \frac{1+\beta}{1-\beta}$$

λ_0 = wavelength of emitted light

λ_{obs}= wavelength we record on Earth.

Current record: $z = 10$

H Lyman α = 121.6nm $\rightarrow$ 1337.6nm

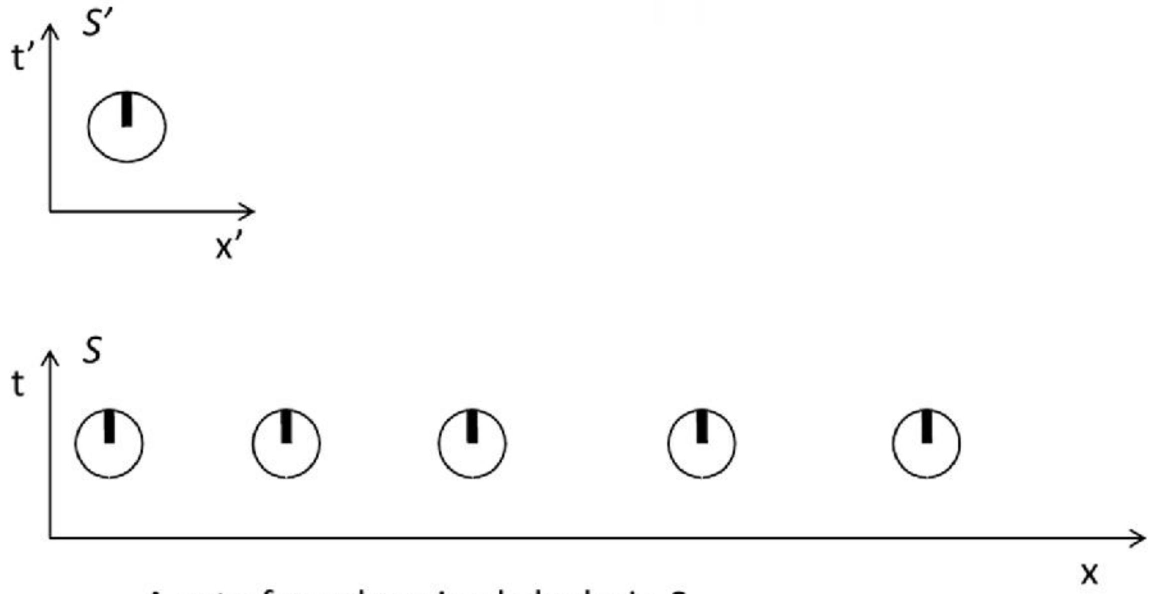
$$\boxed{\beta = 0.9836}$$

More about Moving Clocks

As the moving clock travels its reading is compared with a stationary clock at the same point.

Observer O, in *S*, measures the moving clock to be running slow by a factor:

$$\gamma = (1 - \beta^2)^{\frac{1}{2}}$$



A set of synchronised clocks in S

More about Moving Clocks

What happens if O *looks* at the moving clock?

What does O <u>see</u>?

Suppose the moving clock sends out light pulses at equal intervals τ_0 of its proper time.

What we see at time t (on our clock) was the reading on the moving clock at earlier time $t - {}^{r}/_{c}$, where *r* is the distance of the moving clock at that earlier time.

O sees signals at later time when they reach him – This is just the Doppler effect!

More about Moving Clocks

At some instant, we see the moving clock reading *t*.

At time τ' later (as measured by us!) we see the moving clock reading $t + \tau_0$

If clock is moving on straight line through our own position, then:

$$\tau' = \left[\frac{1+\beta}{1-\beta}\right]^{\frac{1}{2}} \tau_0$$

If clock is moving *towards* us, β is negative and clock will appear to be running fast, not slow! If the clock is a collection of moving atoms – blueshift!

<u>Moral</u>: Be specific about what event or process is being described.
Do not confuse *observing* with *seeing*
(seeing involves finite time for transit of light)