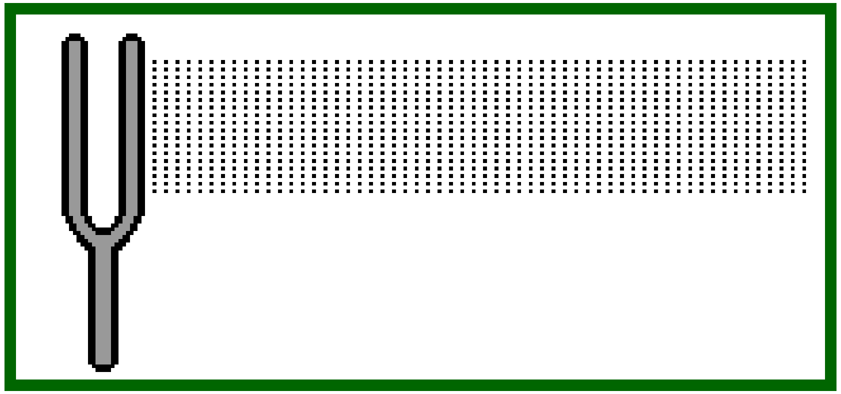
Hearing & Seeing

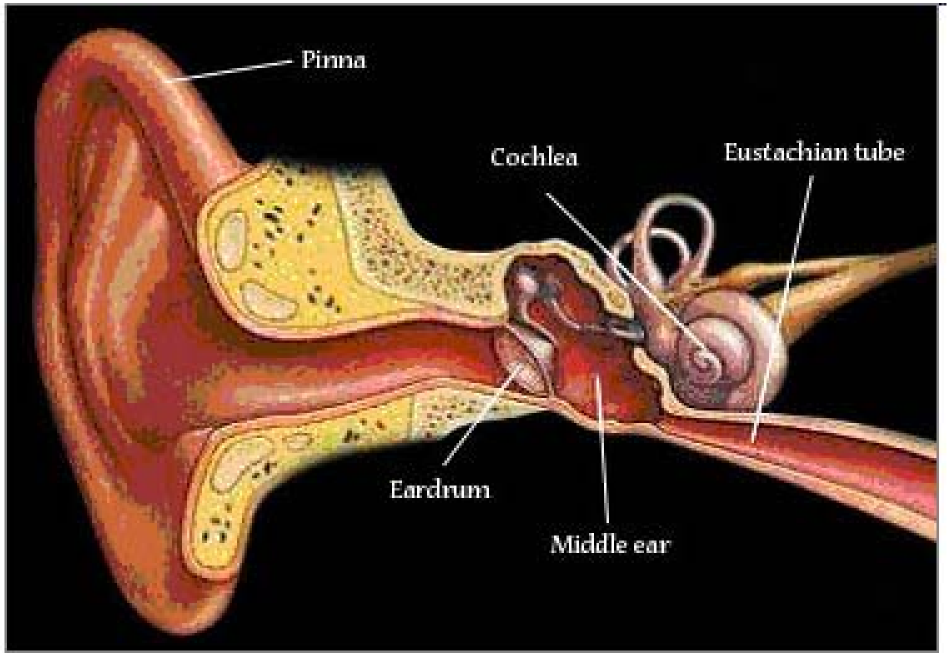
Part Two : Sound

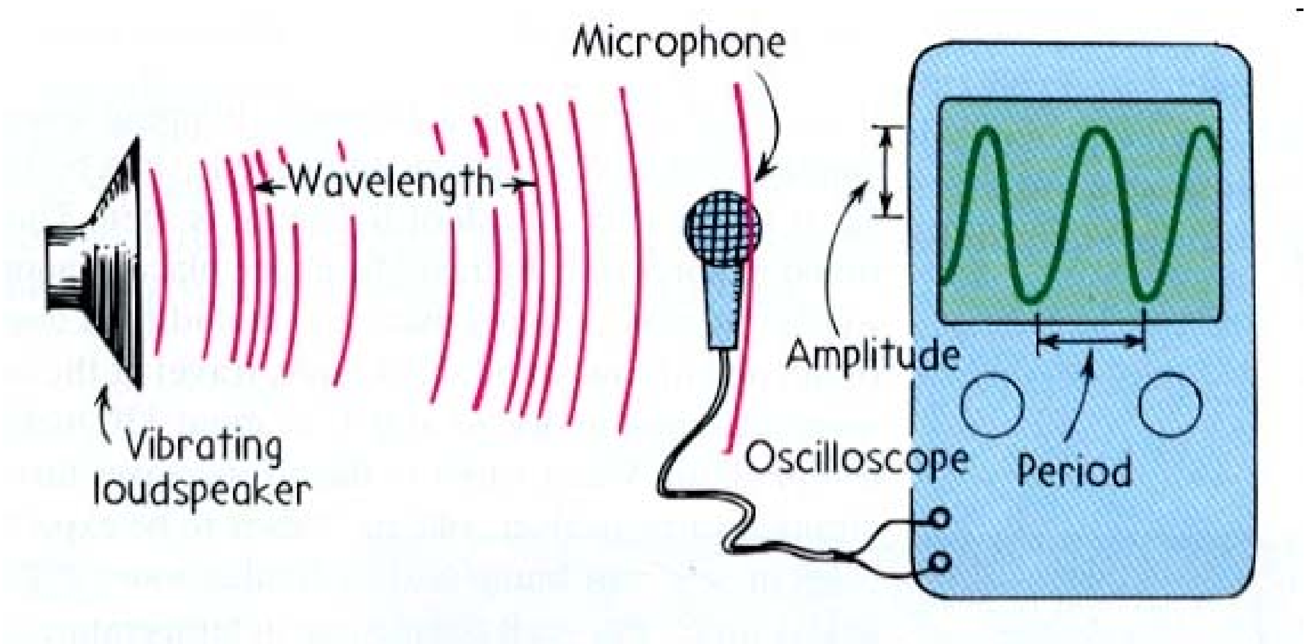
Young and Freedman
Chapter 16
Sound
Read Sections 16.1

- How to describe a sound wave in terms of either particle displacement or pressure fluctuations

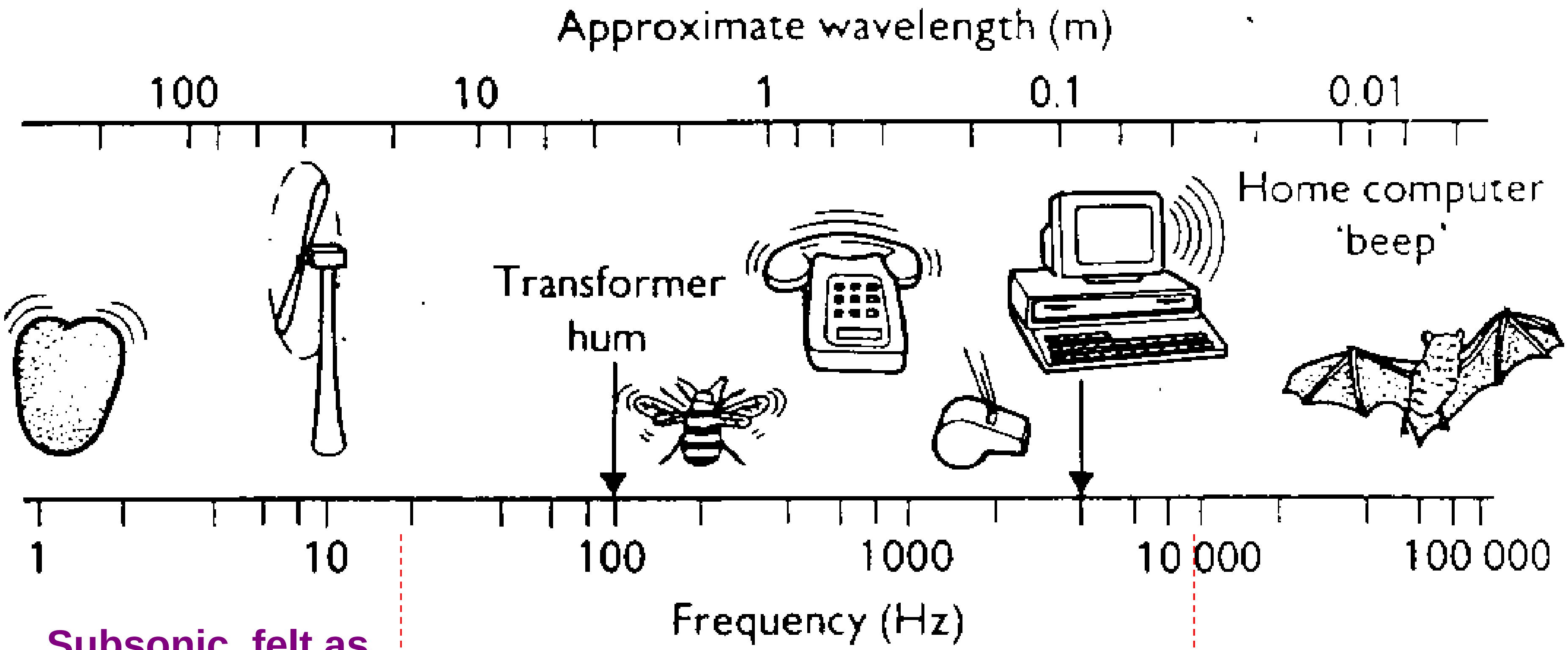


Longitudinal wave





Frequency Range of Sound



Subsonic, felt as Vibrations or Shock Waves e.g. earthquakes

Ultrasonic
10^6 to 10^7 Hz
Used in medical diagnostics

Speed of Sound Waves

We already talked about the speed of a transverse wave on a string

$$v_W = \sqrt{\frac{T}{\mu}}$$

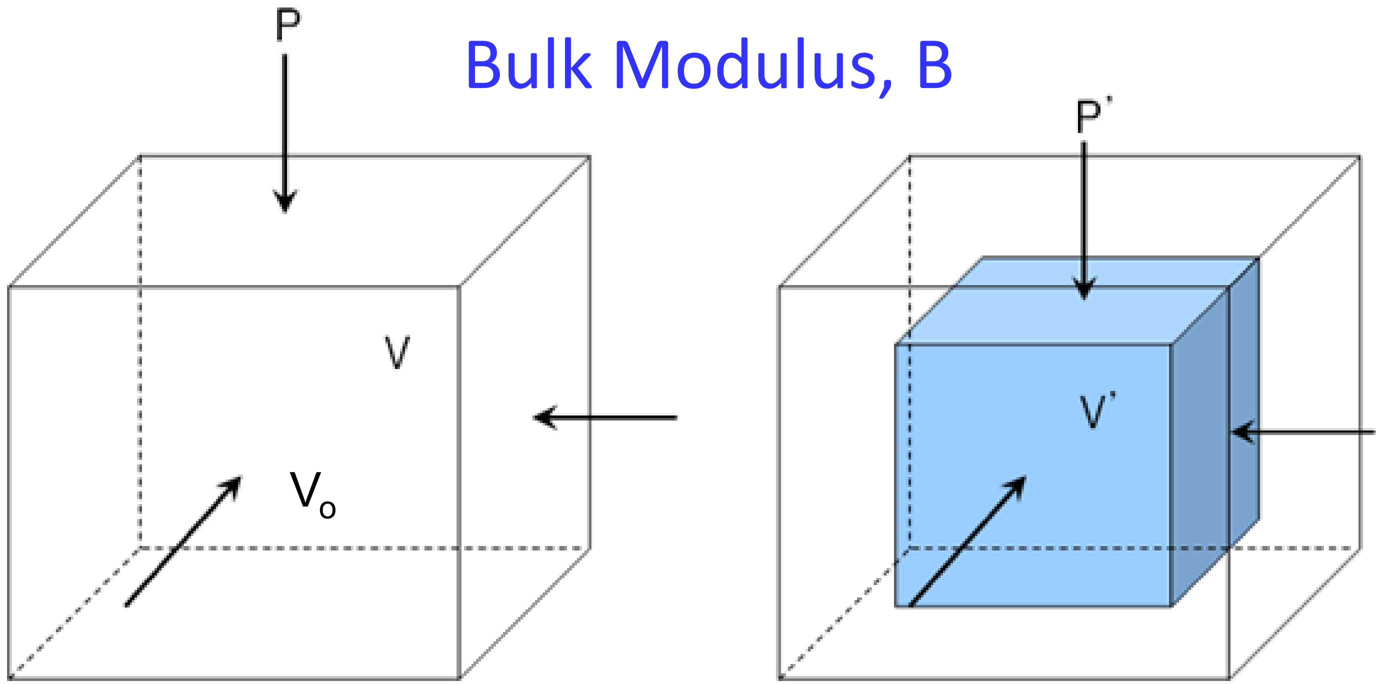
In general for mechanical waves, the speed depends on the properties of the medium

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

Sound wave in a bulk medium

$$v = \sqrt{\frac{B}{\rho}}$$

Bulk Modulus, B



Pressure acting on a body results in volume deformation

$$\Delta P \propto -\frac{\Delta V}{V_0}$$

Hookes law: increase in pressure produces a proportional fractional change in volume

$$\Delta P = -\boxed{B}\frac{\Delta V}{V_0} \quad \text{or} \quad \mathrm{B} = -\frac{\Delta \mathrm{P}}{\Delta V / V_0}$$

B = constant of proportionality

SPEED OF SOUND

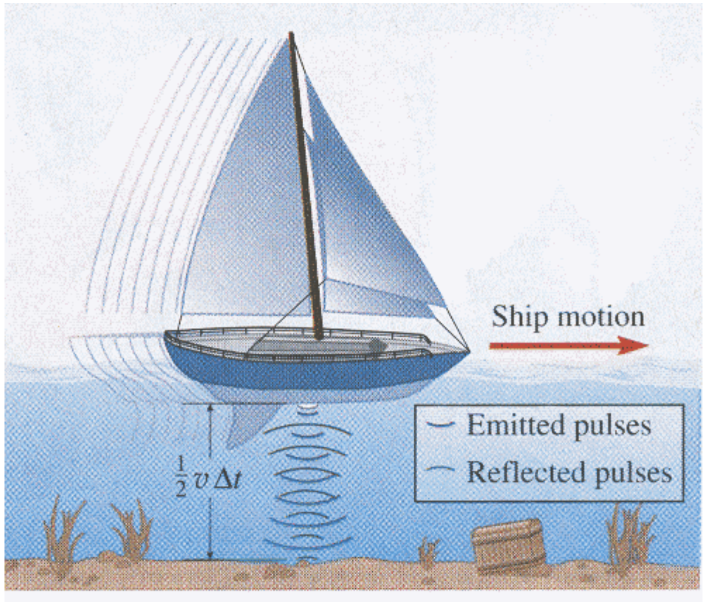
Material	Speed (m/s)
Air	340
Helium	965
Water	1450
Rubber	1800
Iron	5000
Glass	5000
Granite	6000

All other things being equal, sound will travel more slowly in denser materials, and faster in stiffer ones.

f ≈ 100,000 Hz, thus $\lambda_{Water} \approx$ 1.5cm – hunt objects this small

Echolocation

Sonar (<u>so</u>und <u>na</u>vigation and <u>r</u>anging)
Investigating the depths of the ocean using ultrasound. The instrument used is called a fathometer. It emits a pulse of ultrasound and detects the pulse reflected back off the ocean floor. The speed of ultrasound waves in water is known and therefore the distance between the bottom of the ship and the ocean floor can be determined hence avoiding reefs etc.



Auto-focusing cameras

Seismic P waves –sound waves generated by explosions or air guns, sent traveling through the earth to study the Earths interior structure or to find oil.

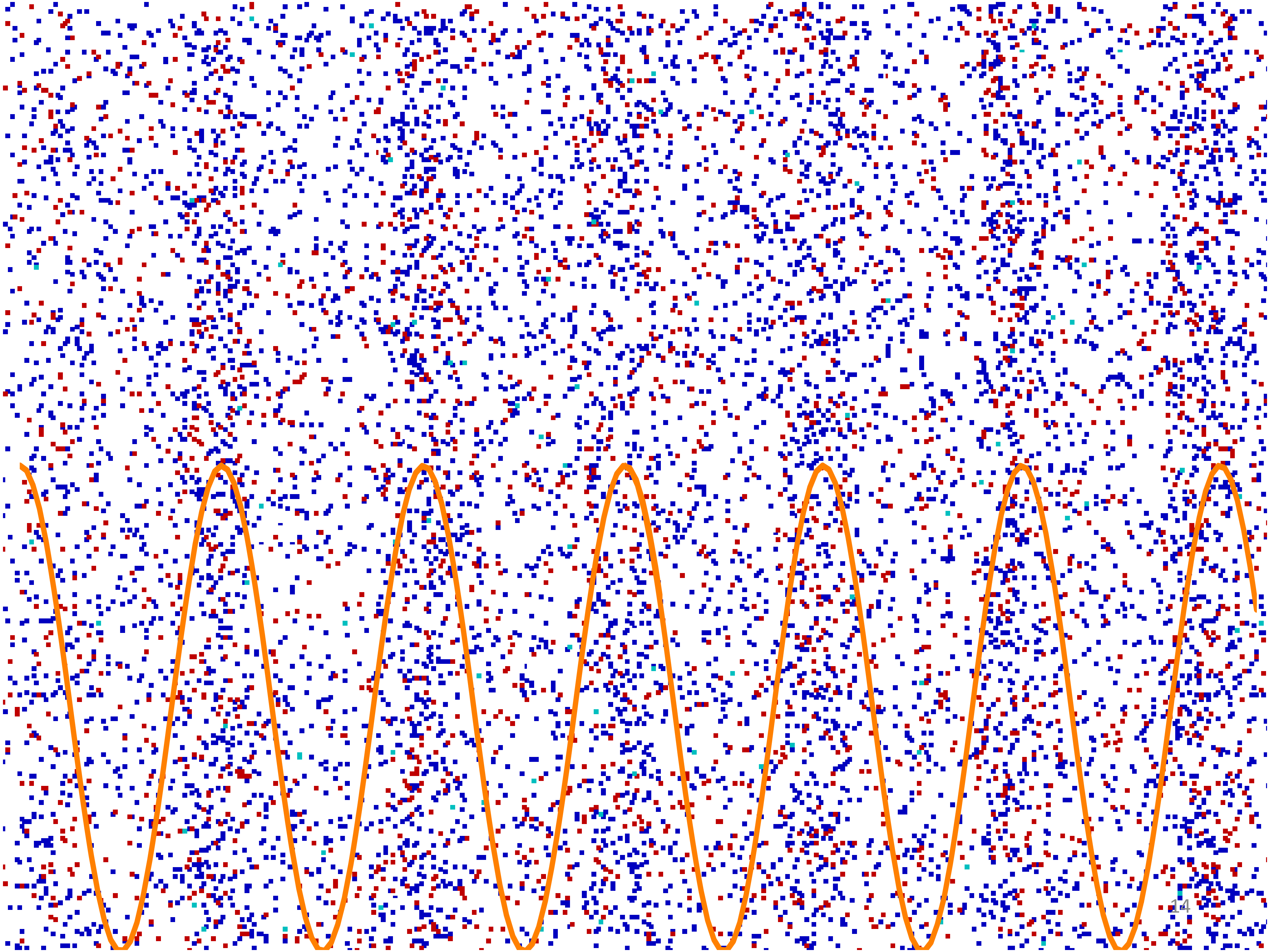
$f \approx 5\text{MHz}$,

thus $\lambda_{Water} \approx 0.3\text{mm}$

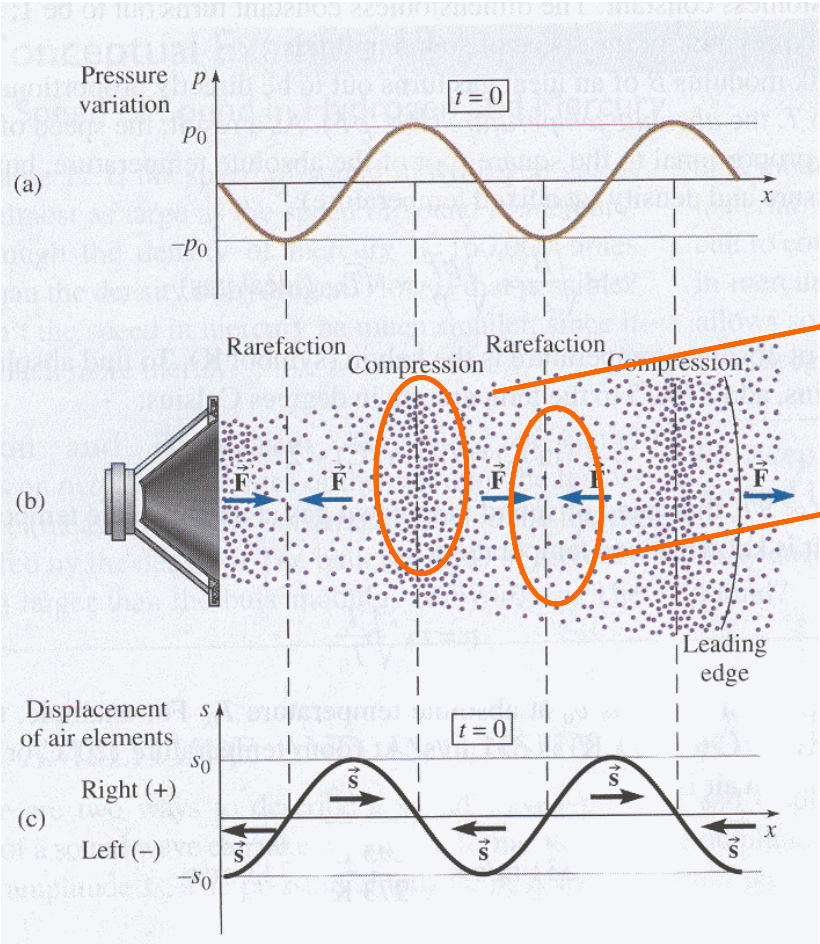
– We can resolve

objects this size

Mathematical Description of a Sound Wave



Mathematical Description of a Sound Wave



Sound can be described as displacement of the medium moving in simple harmonic motion relative to an equilibrium position

Compression

Rarefaction

OR
As a change in pressure from the equilibrium

To make a sound wave: Piston Oscillates Sinusoidally

Any small element moves with SHM

Position of small element relative to its eqm position =

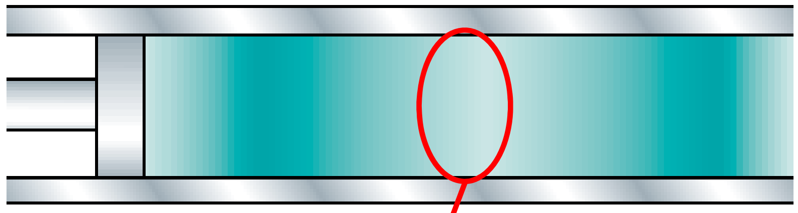
$$\Rightarrow s(x,t) = s_{\max} \cos(kx - \omega t)$$

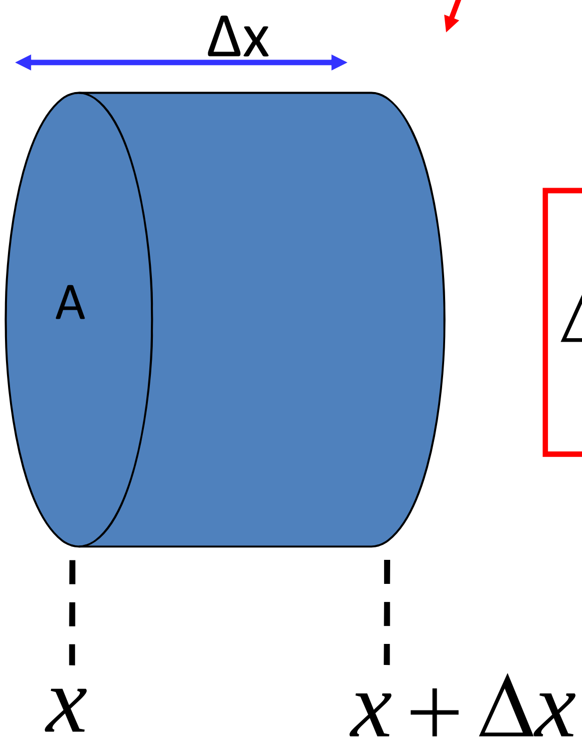
S_{max} = Max position of the element relative to equilibrium.

Variation in gas pressure, ΔP, from eqm p_o is also periodic

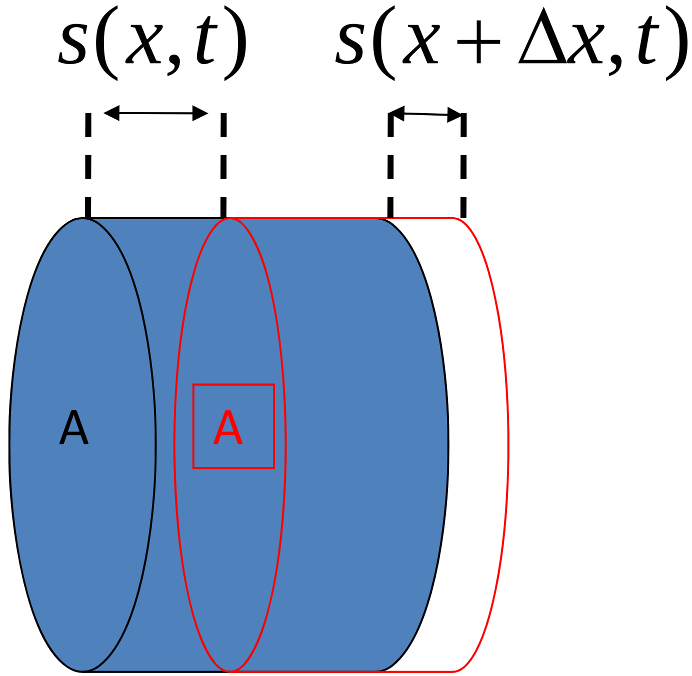
$$\boxed{\Delta P = ?}$$

The pressure fluctuation depends on the difference between the displacements of neighbouring points in the medium – remember they are not all in phase!





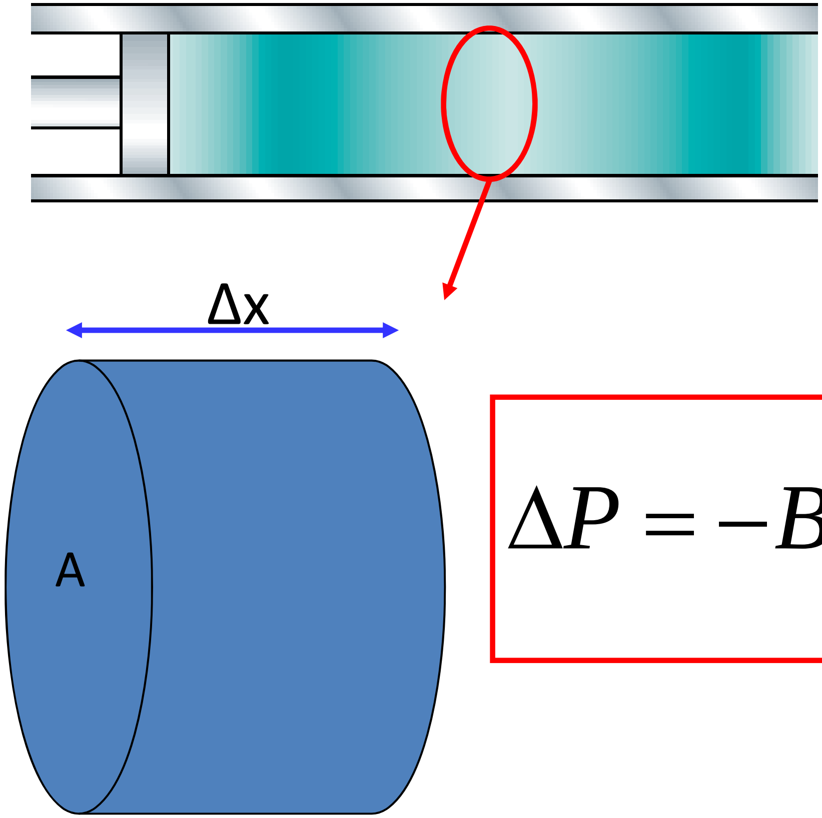
$$\Delta P = -B\frac{\Delta V}{V_0}$$



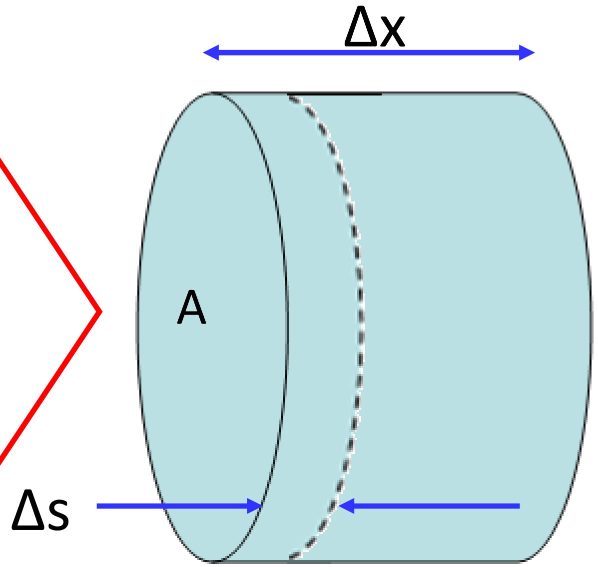
$$\boxed{V_0 = A\Delta x}$$

$$\boxed{\Delta V = A\left(s(x+\Delta x,t) - s(x,t)\right)}$$

$$\boxed{\Delta V = A\Delta s}$$



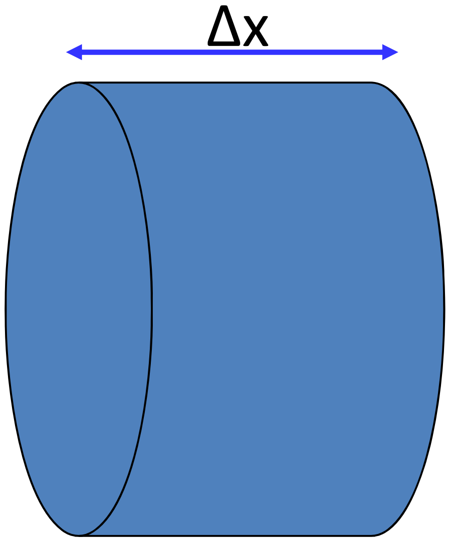
$$\Delta P = -B\frac{\Delta V}{V_0}$$



$$\boxed{V_0 = A\Delta x}$$

$$\boxed{\Delta V = A\Delta s}$$

$$\Delta P = -B\frac{\Delta V}{V_0} = -B\frac{\boxed{A\Delta s}}{\boxed{A\Delta x}} = -B\frac{\Delta s}{\Delta x}$$



$$\Delta P = -B\frac{\Delta s}{\Delta x}$$

$$\Delta P = -B\frac{\partial s}{\partial x}$$

$$\boxed{\Delta x \to 0}$$

$$\Delta P = -B\frac{\partial}{\partial x}\boxed{\left[s_{Max}\cos\left(kx-\omega t\right)\right]}$$

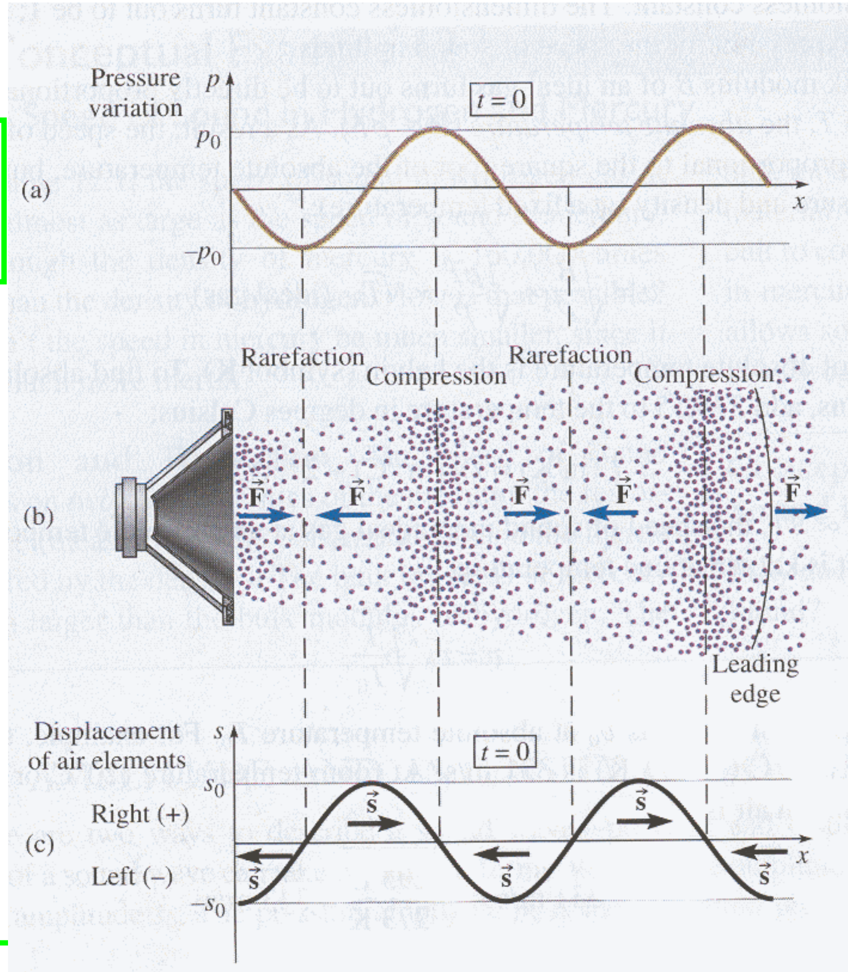
$$\Delta P = Bks_{Max}\sin\left(kx-\omega t\right)$$

Variation in gas pressure, ΔP, from eqm p_o is also periodic

$$\Delta P = \Delta P_{Max} \boxed{\sin}(kx - \omega t)$$

$$\Delta P_{Max} = ?$$

Note that ΔP is at a max when the displacement from eqm is 0



$$s(x,t) = s_{\max} \boxed{\cos}(kx - \omega t)$$

$$\Delta P = \boxed{B} k s_{Max} \sin(kx - \omega t)$$

$$v = \sqrt{\frac{B}{\rho}}\ ,\ \therefore \boxed{B = v^2 \rho}$$

$$\Delta P = \boxed{v^2 \rho}\boxed{k}\, s_{Max} \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}\ \ \&\ \ \omega = 2\pi f = \frac{2\pi v}{\lambda}\left(\therefore \lambda = \frac{2\pi v}{\omega}\right)$$

$$\therefore k = \frac{2\pi\omega}{2\pi v} = \boxed{\frac{\omega}{v}}$$

$$\Delta P = v^2 \rho \frac{\omega}{v} s_{Max} \sin(kx - \omega t)$$

$$\Delta P = v \rho \, \omega \, s_{Max} \boxed{\sin(kx - \omega t)}$$

ΔP is maximised when this is = 1

$$\Delta P_{Max} = v \rho \, \omega \, s_{Max}$$

A loud factory machine produces sound having a displacement amplitude of 1.00μm, but the frequency of this sound can be adjusted. In order to prevent ear damage to the workers , the maximum pressure amplitude of the sound waves is limited to 10.0 Pa. In this factory the bulk modulus of air is 1.42×10^5 Pa. What is the highest frequency sound that the machine can be adjusted to without exceeding the prescribed limit?
Is this frequency audible to the workers?

Sound Intensity

Young and Freedman
Chapter 16
Sound
Read Section 16.3

- How to calculate the intensity of a sound wave

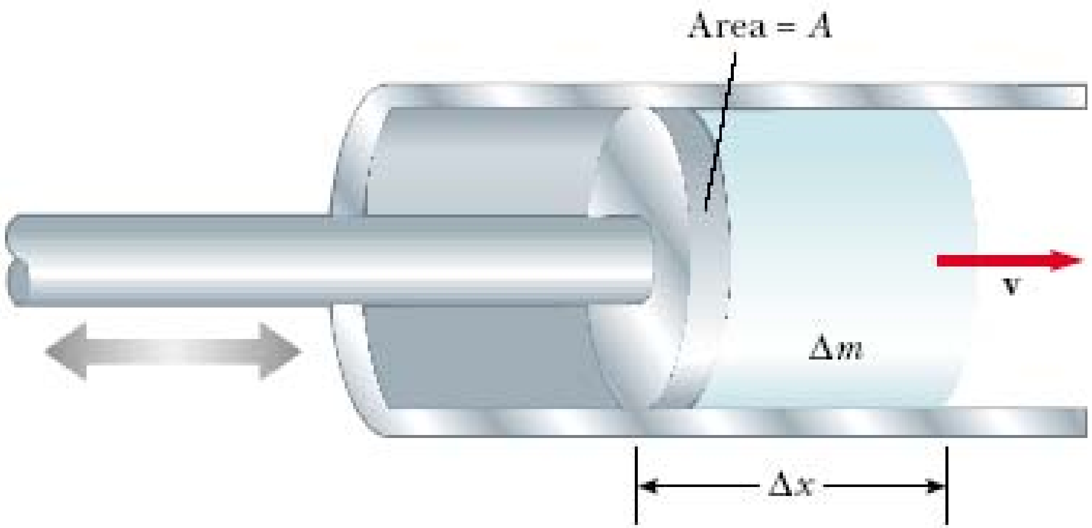
What quantity do we usually use to describe sound magnitude?

Sound Intensity

The time averaged rate at which energy is transferred per unit area across a surface perpendicular to the direction of propagation

Rate at which energy is transferred = Power

What's the rate of energy transfer?



Can use the same approach as for the transverse wave on a string

Kinetic energy of this element:

$$\boxed{E_K = \frac{1}{2}\Delta m v^2}$$

Etc….

$$\Delta E_K = \frac{1}{2}\boxed{\Delta m}\boxed{v^2}$$

$$v(x,t) = \frac{\partial}{\partial t} s(x,t) = \frac{\partial}{\partial t}\left[s_{\max}\cos(kx - \omega t)\right]$$

$$\therefore\ v^2 = \left(\omega s_{\max}\sin(kx - \omega t)\right)^2$$

$$\rho = \frac{\Delta m}{\Delta V} = \frac{\Delta m}{A\Delta x}$$

$$\therefore\ \Delta m = \rho A \Delta x$$

Approach 1

$$\boxed{\Delta E_K = \frac{1}{2}\boxed{\Delta m}\boxed{v^2}}$$

$$\therefore\ \Delta E_K = \frac{1}{2}\boxed{\rho A \Delta x}\boxed{\left(\omega s_{\max}\right)^2\left(\sin(kx-\omega t)\right)^2}$$

Take a snap shot of the energy the wave has at t = 0;

$$\Rightarrow \Delta E_K = \frac{1}{2}\rho A \Delta x\left(\omega s_{\max}\right)^2 \sin^2(kx)$$

Approach 1

$$\text{As } \Delta x \to 0$$

$$\Rightarrow dE_K = \frac{1}{2}\rho A(\omega s_{\max})^2 \sin^2(kx)\,dx$$

Integrate over one λ
to get total $E_{K(\lambda)}$

$$\Rightarrow E_{K_\lambda} = \int dE_K = \int_0^\lambda \frac{1}{2}\rho A(\omega s_{\max})^2 \sin^2(kx)\,dx$$

$$= \frac{1}{2}\rho A(\omega s_{\max})^2 \int_0^\lambda \sin^2(kx)\,dx$$

Approach 1

$$E_{K_\lambda} = \frac{1}{2}\rho A\left(\omega s_{\max}\right)^2\left[\frac{1}{2}\lambda\right] = \boxed{\frac{1}{4}\rho A\left(\omega s_{\max}\right)^2\lambda}$$

And don't forget the potential energy (worked out similarly)

$$E_{P_\lambda} = \boxed{\frac{1}{4}\rho A\left(\omega s_{\max}\right)^2\lambda}$$

$$\therefore E_{Total_\lambda} = \boxed{E_K} + \boxed{E_P} = \frac{1}{2}\rho A\left(\omega s_{\max}\right)^2\lambda$$

Approach 1

Power of the sound wave, P

$$\text{Power} = \frac{\Delta E}{\Delta t} = \frac{E_{Tot_\lambda}}{T}$$

$$P = \frac{E_{Tot_\lambda}}{T} = \frac{\frac{1}{2}\rho A(\omega s_{max})^2 \lambda}{T} = \frac{1}{2}\rho A(\omega s_{max})^2 \boxed{\left(\frac{\lambda}{T}\right)}$$

$$P = \frac{1}{2}\rho A(\omega s_{\max})^2 v$$

Approach 1

Sound Intensity is the time averaged rate at which energy is transferred per unit area across a surface perpendicular to the direction of propagation

$$P = \frac{1}{2}\rho A\left(\omega s_{\max}\right)^2 v$$

$$\boxed{I = \frac{P}{A} = \frac{1}{2}\rho\left(\omega s_{\max}\right)^2 v}$$

OR

Sound Intensity

time averaged power per unit area

$$\frac{\text{Power}}{\text{Area}} = \frac{\text{Work}}{\text{Area} \times \text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Time}}$$

?

$$\frac{\text{Power}}{\text{Area}} = \text{Pressure} \times \text{Velocity}$$

Approach 2

Already had

$$\boxed{s(x,t) = s_{\max} \cos(kx - \omega t)}$$

$$\boxed{v(x,t) = \frac{\partial}{\partial t} s(x,t) = \frac{\partial}{\partial t}\left[s_{\max} \cos(kx - \omega t)\right]}$$

$$\boxed{\therefore\ v(x,t) = \omega s_{\max} \sin(kx - \omega t)}$$

$$\boxed{\Delta P = Bks_{Max} \sin(kx - \omega t)}$$

$$\frac{\textbf{Power}}{\textbf{Area}} = Bk\omega s_{\max}^2 \sin^2(kx - \omega t)$$

Approach 2

Sound Intensity

time averaged power per unit area

$$\frac{\textbf{Power}}{\textbf{Area}} = Bk\omega s_{\max}^{2}\sin^{2}(kx-\omega t)$$

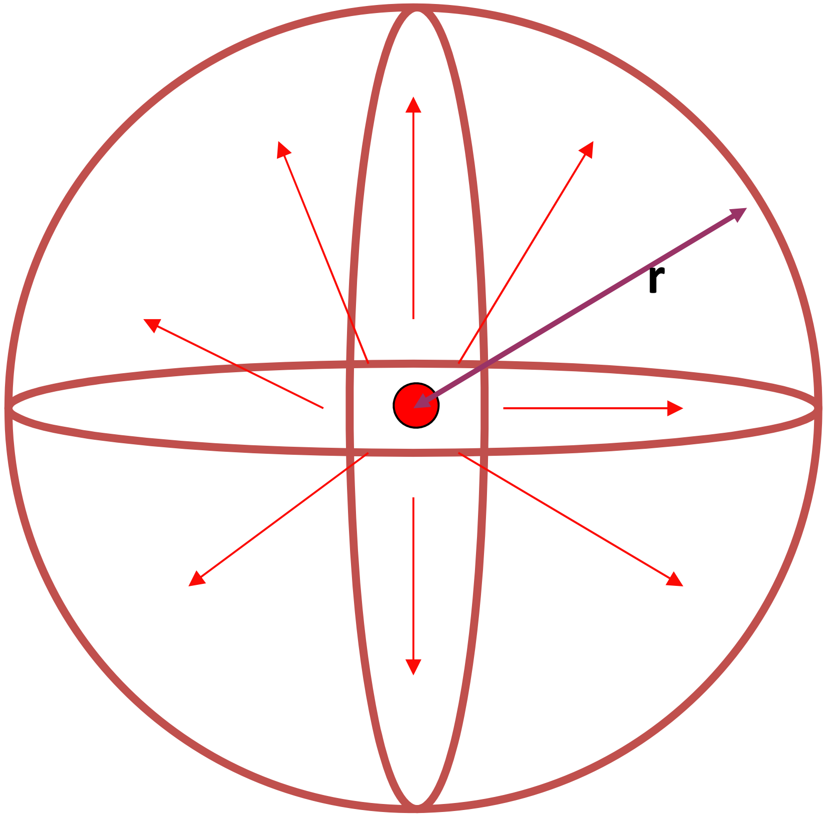
$$\textbf{Intensity} \equiv \frac{\textbf{Power}}{\textbf{Area}} = \frac{1}{2}Bk\omega s_{\max}^{2}$$

$$\boxed{I \equiv \frac{P}{A} = \frac{1}{2}\rho(\omega s_{\max})^{2}v}$$

Electromagnetic waves from the sun bring enormous quantities of energy. By comparison the energy in sound waves is extremely small. 10 million people talking at the same time would only produce enough energy to light a flash-lamp. Hearing is only possible because of the remarkable sensitivity of our ears.

Intensity of the sound wave, I (W/m²)

$$I \equiv \frac{P}{A} = \frac{1}{2}\rho\left(\omega s_{\max}\right)^2 v$$



$$I \equiv \frac{P}{A} = \frac{P}{4\pi r^2}$$

The Decibel Scale

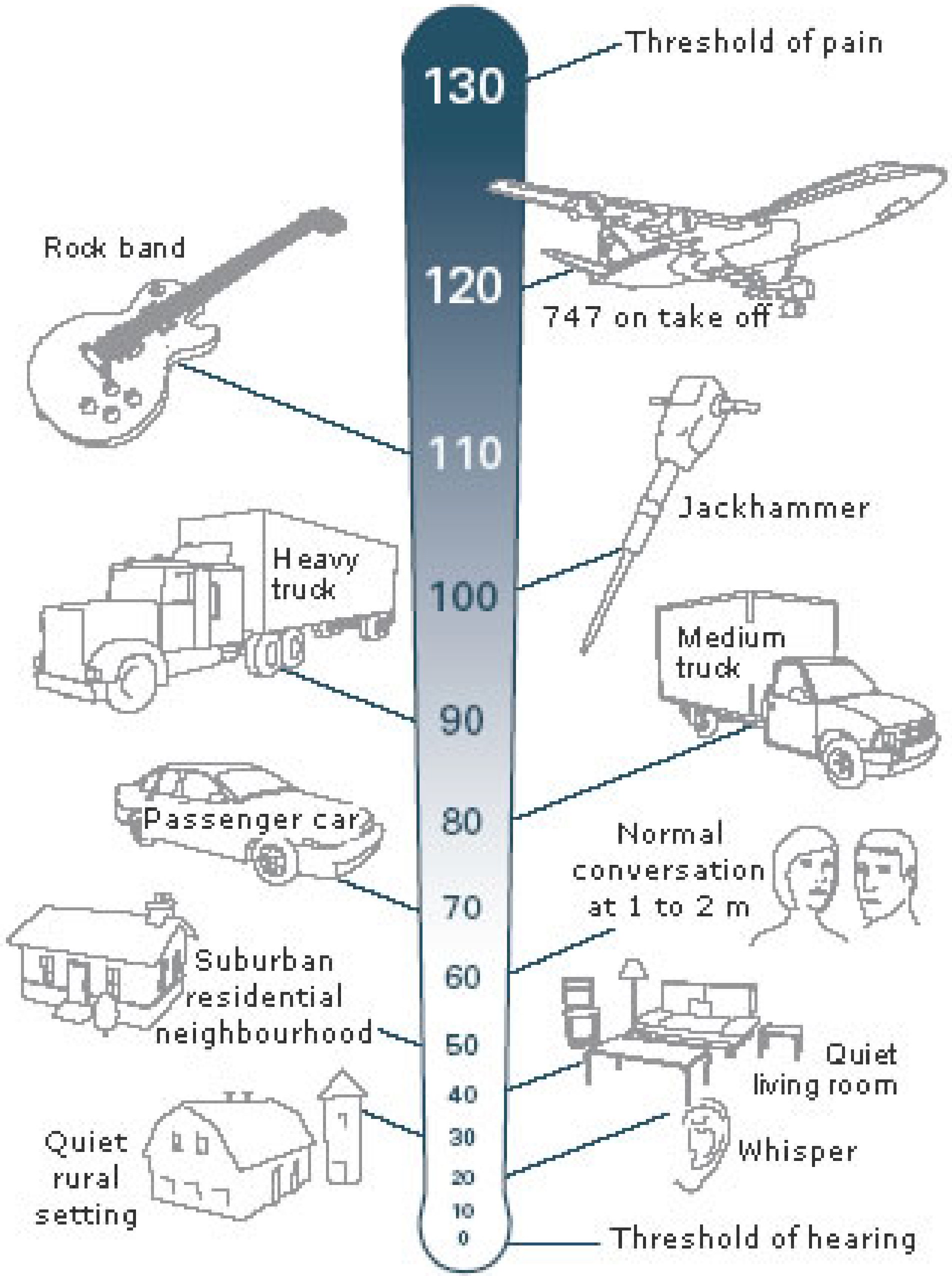
$P_{Whisper} = P_{Jet\ Engine} / 1{,}000{,}000{,}000{,}000$

$$\beta = (10dB)\log\frac{I}{I_0}$$

I = Intensity of the sound,

I_0 = Threshold of hearing = 10^{-12} W/m^2

DECIBEL SCALE (dBA)



How sensitive is our hearing?

What displacement of air is required for us to be able to hear a sound?

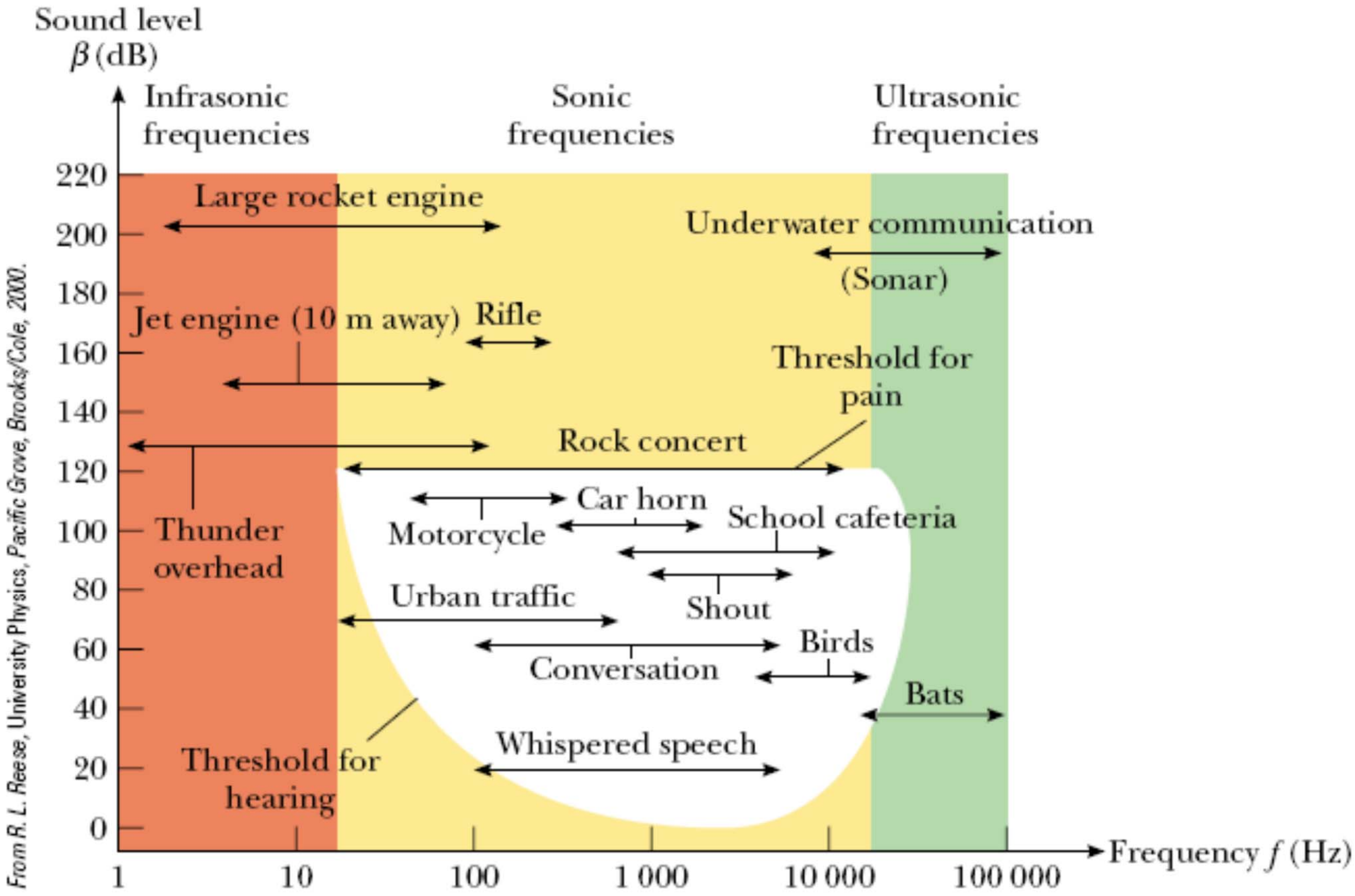
Intensity $= \dfrac{1}{2} Bk\omega s_{\max}^{2} = \dfrac{B2\pi^{2} f^{2} s_{\max}^{2}}{v}$

I_0 is the approximate threshold of human hearing at 1000 Hz

$I_0 = 10^{-12}\ \mathrm{Wm^{-2}}$ B = 1.42 x 10^5 Pa v = 330 m/s

$$s_{\max} = \sqrt{\frac{I_0 v}{B2\pi^{2} f^{2}}} \approx 10^{-11} m$$ less than the size of an atom

$$\Delta P_{Max} = 10^{-5} \quad Pa$$



Human Hearing Range 20 Hz to 20 kHz

Loudness

The intensity required to produce the same perceived loudness depends on frequency as we saw a moment ago and can vary significantly from person to person. Loudness is a physiological sensation.

The ear is most sensitive at 3 kHz

Any idea why?

Ear canal acts as a closed pipe 2.5 cm long

Closed pipe resonant frequencies: $f_n = \frac{nv}{4L}$, n odd

Velocity of sound, v = 344 ms-1

Fundmental $f_1 = \frac{344\ ms^{-1}}{4(0.025\ m)} \approx 3\ kHz$

Sounds also louder for n=3, approx 10 kHz

Dogs, bats – shorter ear canals, can hear higher frequencies than humans

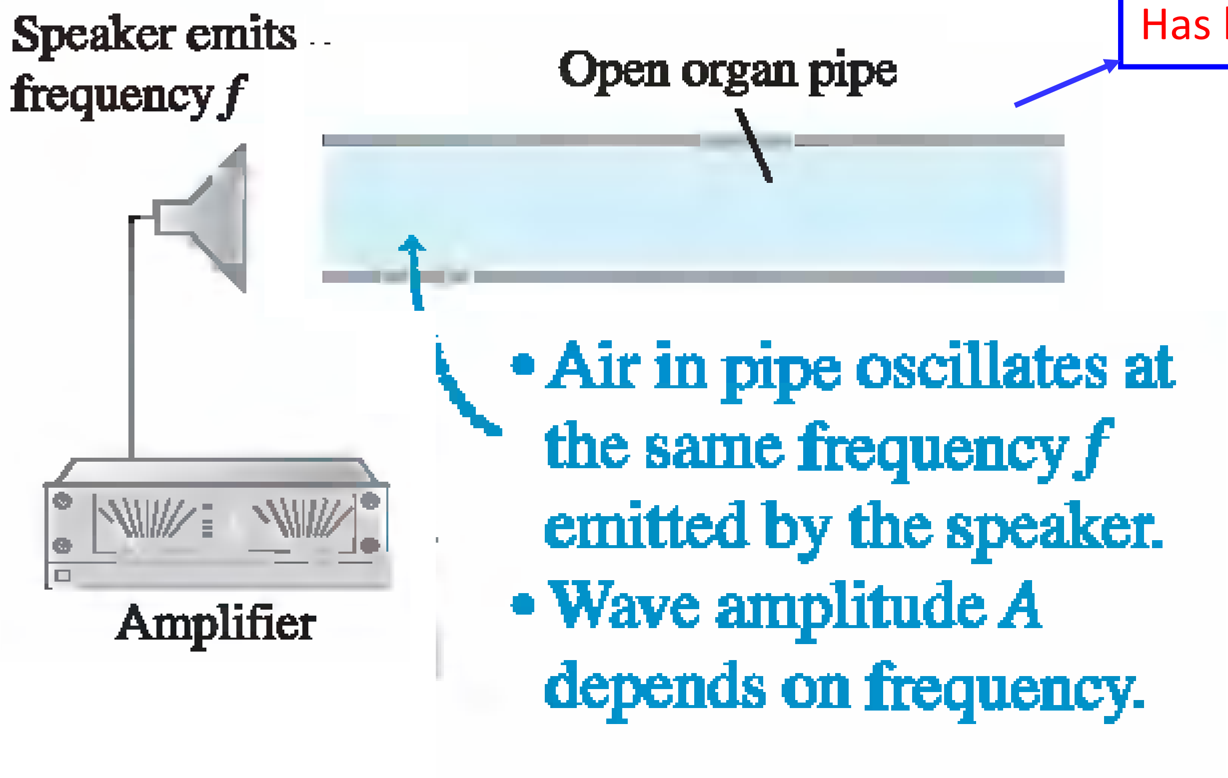
Determine the amplitude of the displacement for sound with an intensity of 120 dB, which corresponds to the threshold of pain for the human ear. The sound has a frequency of 1 kHz. The bulk modulus of air is 1.42×10^5 Pa and the speed of sound in air is 340 m/s.

Resonance

Young and Freedman
Chapter 16
Sound
Read Section 16.5

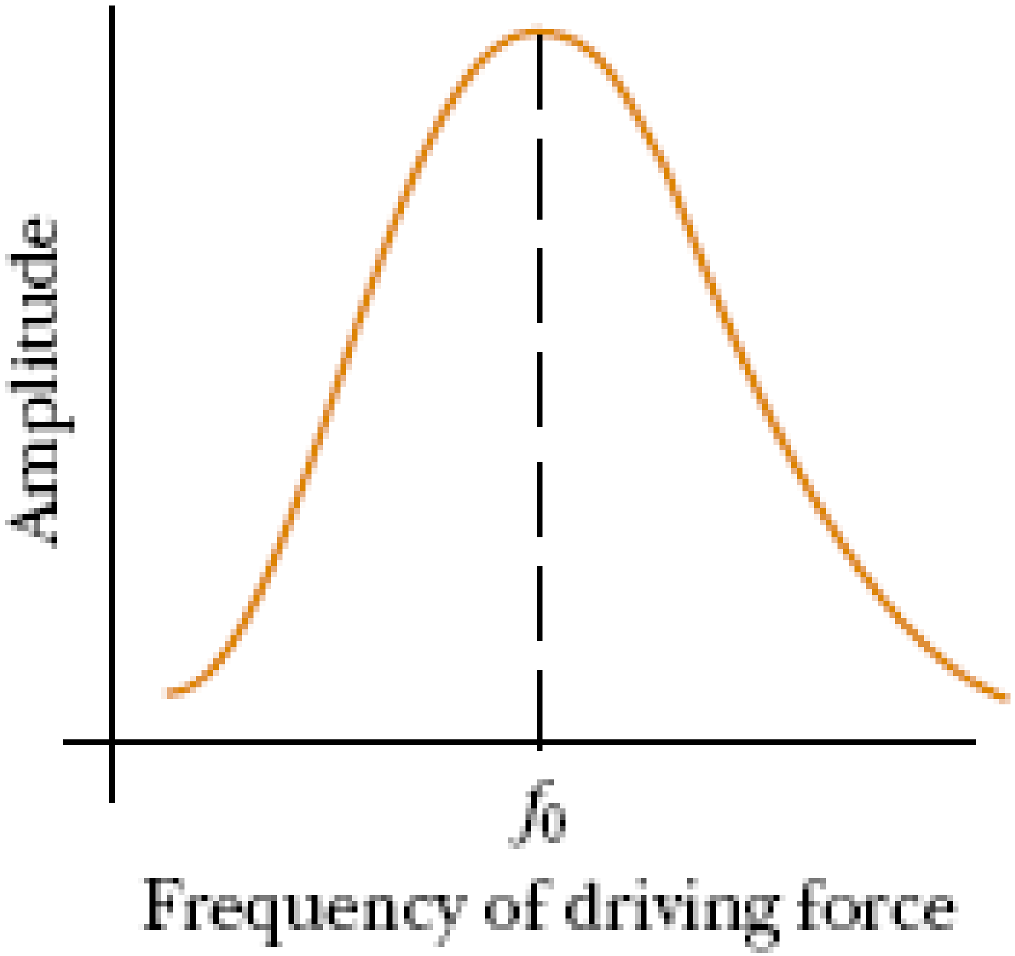
- Describe how resonance occurs

Resonance



Has harmonics

Resonance occurs when the frequency of the driving force matches the natural frequency of oscillation, f_0



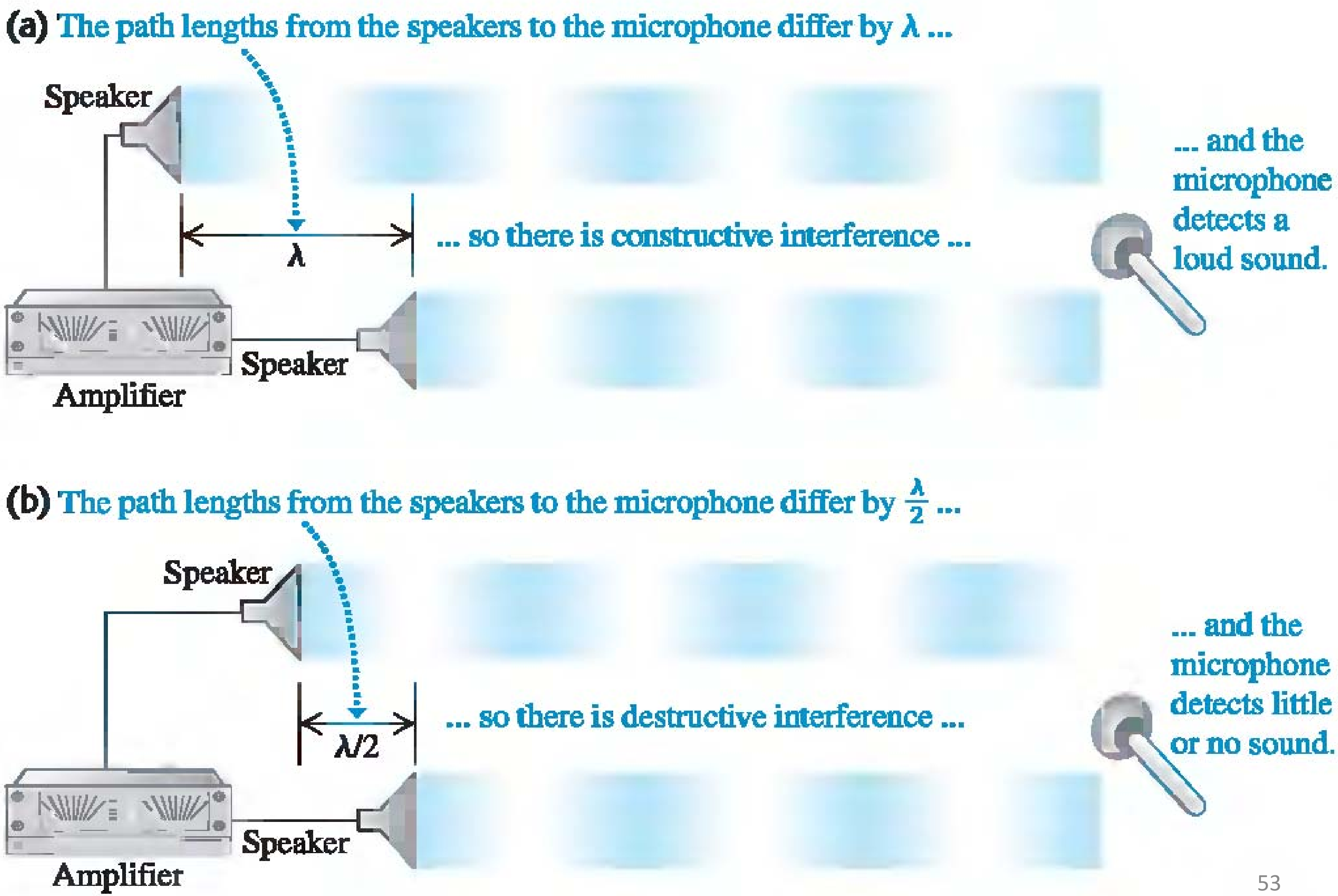
http://www.walter-fendt.de/ph14e/resonance.htm

Interference

Young and Freedman
Chapter 16
Sound
Read Sections 16.6 and 16.7

- Determine what happens when sound waves from different sources overlap

Interference (Superposition) of Sound Waves



Hear the beat?

400 Hz & 400.5 Hz

400 Hz & 403 Hz

400 Hz & 410 Hz

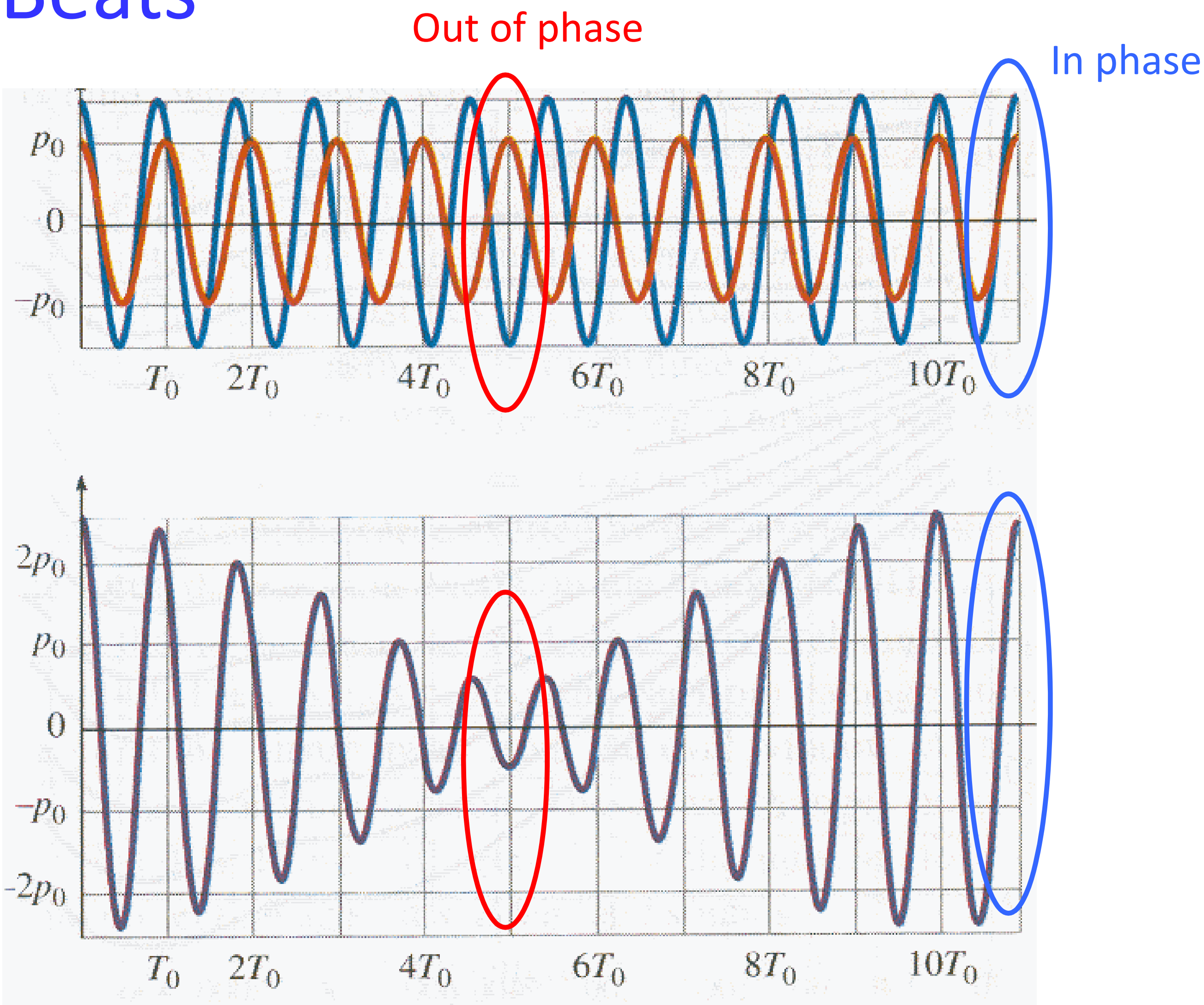
400 Hz & 440 Hz

400 Hz & 500 Hz(3rd)

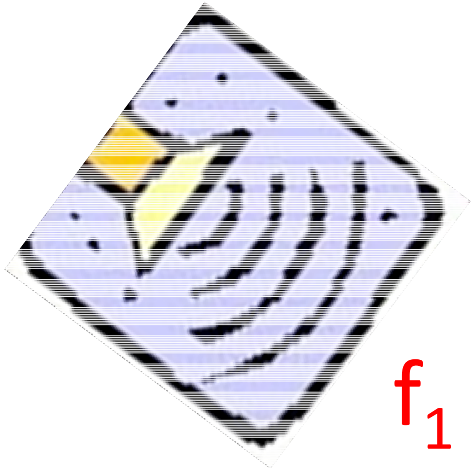
400 Hz & 600 Hz(5th)

400 Hz & 800 Hz(octave)

Beats



http://surendranath.tripod.com/Applets/Waves/Beats/BeatsApplet.html



f_1

f_2

$$\boxed{y_1 = A\cos(\omega_1 t) = A\cos(2\pi f_1 t)} \quad + \quad \boxed{y_2 = A\cos(2\pi f_2 t)}$$

$$\boxed{y = y_1 + y_2 = A[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]}$$

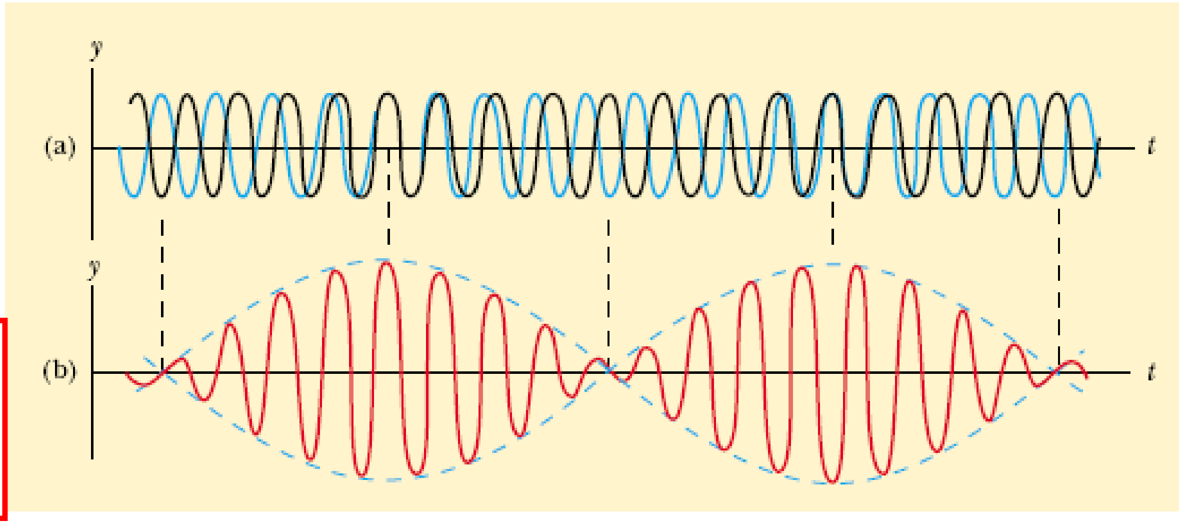
$$\cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

$$\boxed{y = \left[2A\cos\left(\left(\frac{f_1 - f_2}{2}\right)2\pi t\right)\right]\left[\cos\left(\left(\frac{f_1 + f_2}{2}\right)2\pi t\right)\right]}$$

$$y = \left[2A\boxed{\cos\left(\left(\frac{f_1 - f_2}{2}\right)2\pi t\right)}\right]\left[\cos\left(\left(\frac{f_1 + f_2}{2}\right)2\pi t\right)\right]$$

Two interfering waves

Combined wave (amp is blue dotted line)



$$A_{\text{Res}} = \left[2A\cos\left(\left(\frac{f_1 - f_2}{2}\right)2\pi t\right)\right]$$

$$\boxed{A_{\mathrm{Res}} = \left[2A\cos\left(\left(\frac{f_1 - f_2}{2} \right) 2\pi t \right) \right]}$$

This amplitude factor is maximised, when cos term = ± 1

The intensity the ear hears is proportional to the square of the amplitude

Thus, we have two maxima per period

The beat freq is given by:

$$f_{Beat} = \left| f_1 - f_2 \right|$$

E.g. 16 Hz and 18 Hz

- Ear detects two maxima and two minima per period, so the beat frequency we hear is twice the frequency of the amplitude factor

  Beat frequency = f_1-f_2
- Outer envelope – amplitude variation – slowly varying component - 2 Hz
- Inside envelope - 17 Hz

While attempting to tune to a note of 523 Hz, a piano tuner hears 2.00 beats/s between a reference oscillator and the string.

(i) What are the possible frequencies of the string?

(ii) When the string is tightened slightly the piano tuner now hears 3.00 beats/s. What is the new frequency of the string?

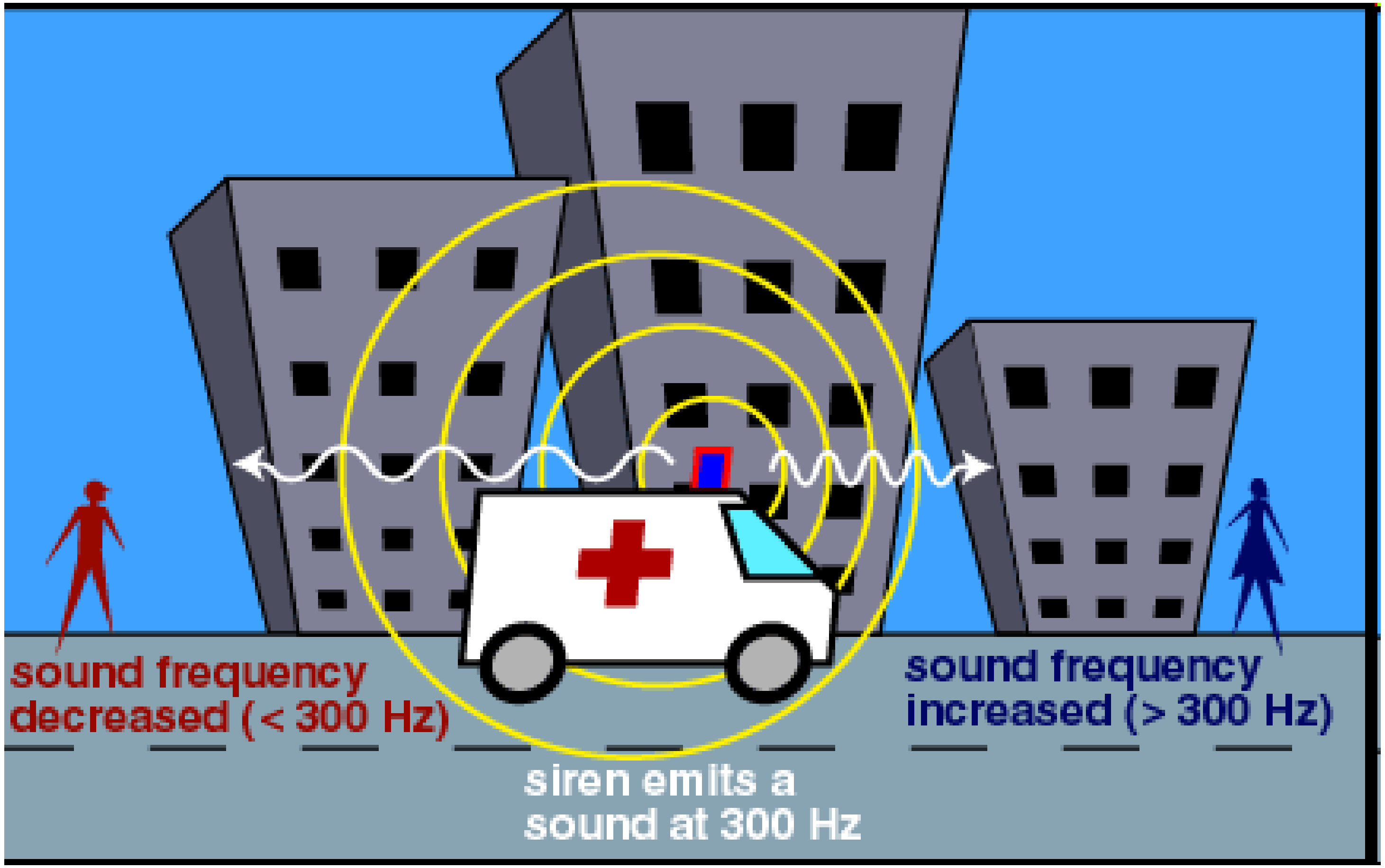
(iii) By what percentage must the tension of the string be adjusted to bring it to the correct frequency of 523 Hz?

The Doppler Effect

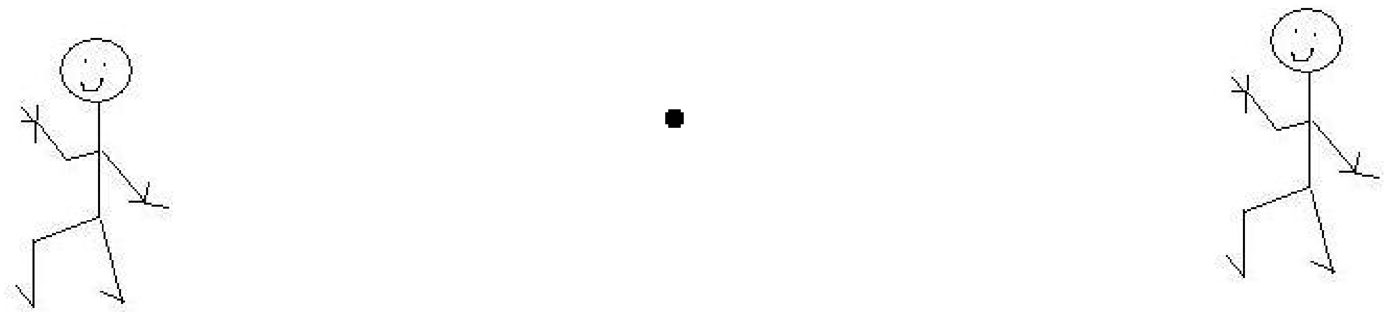
Young and Freedman
Chapter 16
Sound
Read Sections 16.8

- Determine what we hear as a sound source passes

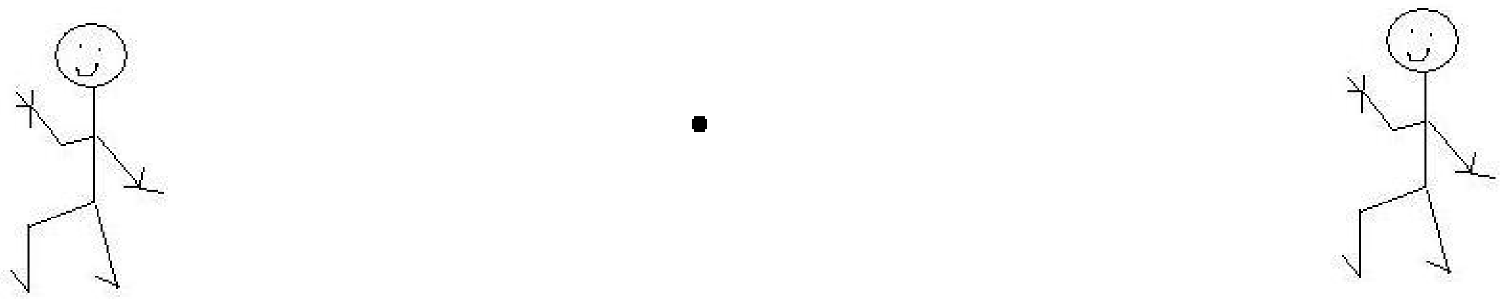
The Doppler Effect



Stationary Sound Source



Moving Sound Source



Source is moving with velocity v_s
Sound emitted has frequency f_s and travels with velocity v

Source moving toward the Listener

The wavelength at the Listener has decrease by the distance moved by the source before the emission of the next wavefront

$$\Delta\lambda = v_s \times T = \frac{v_s}{f_s}$$

Source moving toward the listener

$$\lambda_L = \lambda - \Delta\lambda = \frac{v}{f_s} - \frac{v_s}{f_s} = \frac{v - v_s}{f_s}$$

$$\boxed{f_L} = \frac{v}{\lambda_L} = \boxed{f_s\left[\frac{v}{v - v_s}\right]}$$

Apparent frequency is higher

Source moving away from the listener

$$\lambda_L = \lambda + \Delta\lambda = \frac{v + v_s}{f_s}$$

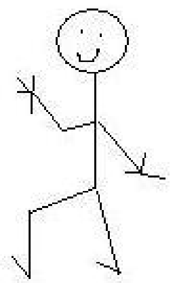
$$\boxed{f_L = f_s\left[\frac{v}{v + v_s}\right]}$$

Apparent frequency is lower

Note convention: Direction Listener to Source is positive

Stationary Source

Moving Listener



v_L

v

$$v = \lambda f_S$$

Stationary Source, Moving Listener

Wave fronts moving towards Listener have a relative velocity, v′ of

$$v' = v + v_L$$

So the freq with which the wave fronts arrive at the listeners position is

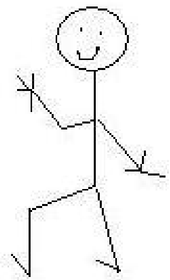
$$f_L = \frac{v + v_L}{\lambda}$$

$$\Rightarrow \boxed{f_L} = \frac{v + v_L}{v/f_s} = \left(\frac{v + v_L}{v}\right) f_s = \boxed{\left(1 + \frac{v_L}{v}\right) f_s}$$

$f_L > f_S$ when the observer is moving towards source

Stationary Source

Moving Listener



$v_{Listener}$

v

$$v = \lambda f_s$$

$$v' = v - v_L$$

Apparent frequency for the Listener is lower

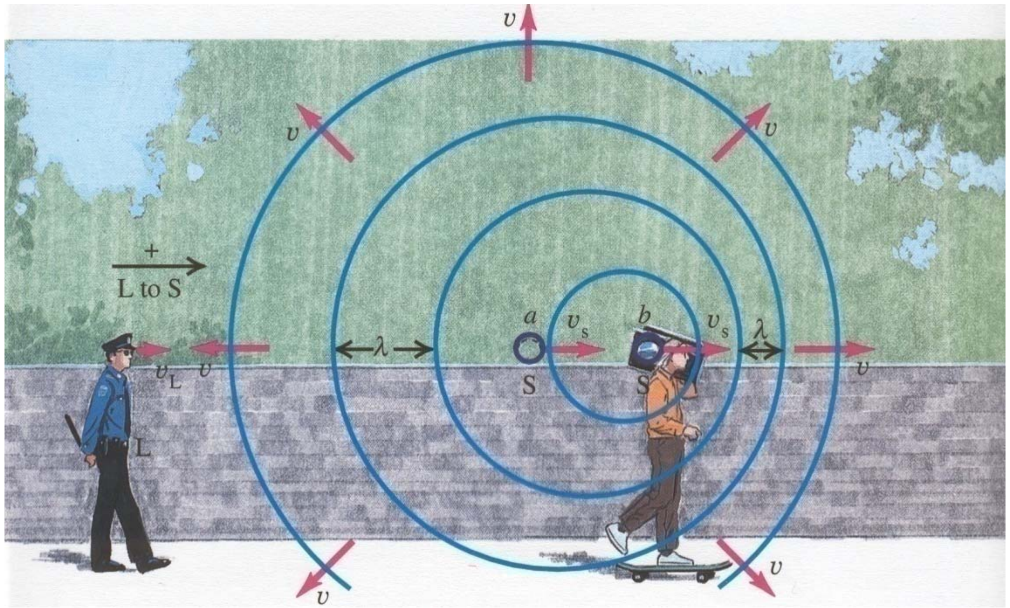
$$\boxed{f_L = \left(1 - \frac{v_L}{v}\right) f_s}$$

Moving Source, Moving Listener

What is the frequency heard by the listener coming behind?

$$f_L = \frac{v + v_L}{\lambda_L} = \frac{v + v_L}{(v + v_S)/f_S}$$

$$\Rightarrow f_L = f_S\left(\frac{v + v_L}{v + v_S}\right)$$



General Case

$$f_L = f_S\left(\frac{v \pm v_L}{v \pm v_S}\right)$$

Sign is determined by the convention

Listener to source is the positive direction

- Source and listener moving toward each other : (v_L>0, v_S<0) $f_L > f_S$
- Moving apart : $f_L < f_S$
- No relative motion => $f_L = f_S$

Bats use echolocation to catch insects. They emit a chirping sound at 15 kHz and detect the reflected sound. At what speed and in what direction relative to you must the bat be flying if you detect a chirp at 16.5 kHz? The speed of sound in air is 340 m/s.

Example 17.1 Speed of Sound in a Liquid Interactive

(A) Find the speed of sound in water, which has a bulk modulus of $2.1 \times 10^9\ \mathrm{N/m^2}$ at a temperature of $0°C$ and a density of $1.00 \times 10^3\ \mathrm{kg/m^3}$.

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(B) Dolphins use sound waves to locate food. Experiments have shown that a dolphin can detect a 7.5-cm target 110 m away, even in murky water. For a bit of "dinner" at that distance, how much time passes between the moment the dolphin emits a sound pulse and the moment the dolphin hears its reflection and thereby detects the distant target?

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1. Why are sound waves characterized as longitudinal?

$\boxed{2.}$ If an alarm clock is placed in a good vacuum and then activated, no sound is heard. Explain.

3. A flowerpot is knocked off a balcony 20.0 m above the sidewalk and falls toward an unsuspecting 1.75-m-tall man who is standing below. How close to the sidewalk can the flower pot fall before it is too late for a warning shouted from the balcony to reach the man in time? Assume that the man below requires 0.300 s to respond to the warning.

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Note: Use the following values as needed unless otherwise specified: the equilibrium density of air at 20°C is $\rho = 1.20\ \text{kg/m}^3$. The speed of sound in air is $v = 343\ \text{m/s}$. Pressure variations ΔP are measured relative to atmospheric pressure, $1.013 \times 10^5\ \text{N/m}^2$. Problem 70 in Chapter 2 can also be assigned with this section.

7. A bat (Fig. P17.7) can detect very small objects, such as an insect whose length is approximately equal to one wavelength of the sound the bat makes. If a bat emits chirps at a frequency of 60.0 kHz, and if the speed of sound in air is 340 m/s, what is the smallest insect the bat can detect?

10. A sound wave in air has a pressure amplitude equal to $4.00 \times 10^{-3}\ \text{N/m}^2$. Calculate the displacement amplitude of the wave at a frequency of 10.0 kHz.

11. A sinusoidal sound wave is described by the displacement wave function

$$s(x, t) = (2.00\ \mu\text{m})\cos[(15.7\ \text{m}^{-1})x - (858\ \text{s}^{-1})t]$$

(a) Find the amplitude, wavelength, and speed of this wave. (b) Determine the instantaneous displacement from equilibrium of the elements of air at the position $x = 0.050\,0$ m at $t = 3.00$ ms. (c) Determine the maximum speed of the element's oscillatory motion.

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18. The area of a typical eardrum is about 5.00×10^{-5} m^2. Calculate the sound power incident on an eardrum at (a) the threshold of hearing and (b) the threshold of pain.

$\boxed{\textbf{19.}}$ Calculate the sound level in decibels of a sound wave that has an intensity of 4.00 μW/m^2.

20. A vacuum cleaner produces sound with a measured sound level of 70.0 dB. (a) What is the intensity of this sound in W/m^2? (b) What is the pressure amplitude of the sound?

21. The intensity of a sound wave at a fixed distance from a speaker vibrating at 1.00 kHz is 0.600 W/m^2. (a) Determine the intensity if the frequency is increased to 2.50 kHz while a constant displacement amplitude is maintained. (b) Calculate the intensity if the frequency is reduced to 0.500 kHz and the displacement amplitude is doubled.

22. The intensity of a sound wave at a fixed distance from a speaker vibrating at a frequency f is I. (a) Determine the intensity if the frequency is increased to f' while a constant displacement amplitude is maintained. (b) Calculate the intensity if the frequency is reduced to $f/2$ and the displacement amplitude is doubled.

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Example 16.19

The police car with its 300-Hz siren is moving toward a warehouse at 30 m/s, intending to crash through the door. What frequency does the driver of the police car hear reflected from the warehouse?

16.42. In Example 16.19 (Section 16.8), suppose the police car is moving away from the warehouse at 20 m/s. What frequency does the driver of the police car hear reflected from the warehouse?

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