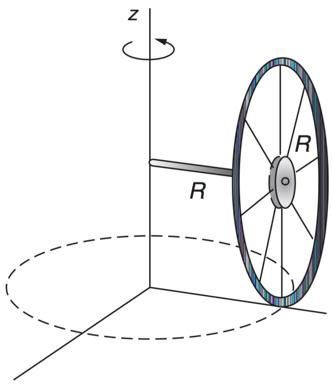
Tutorial 3

Problem 1. K. & K. 8.1
A thin hoop of mass M and radius R rolls without slipping about the z axis. It is supported by an axle of length R through its center, as shown. The hoop circles around the z axis with angular speed Ω.



1. What is the instantaneous angular velocity $\vec{\omega}$ of the hoop?
2. What is the angular velocity $\vec{L}$ of the hoop? Is $\vec{L}$ parallel to $\vec{\omega}$? Is $\vec{L}$ parallel to $\vec{\omega}$? (The moment of inertia of a hoop for an axis along its diameter is $\frac{1}{2}MR^2$.)

Problem 2. (modified version of Kleppner & Kolenkow 8.9)
This problem involves investigating the effect of the angular momentum of a bicycle's wheels on the stability of the bicyle and the rider. Assume that the center of mass of the bike and rider is height 2ℓ above the ground. Each wheel has mass m, radius ℓ, and moment of inertia $m\ell^2$. The bicycle moves with velocity V in a circular path of radius R.

1. Give the diagram with the center of mass angular momentum $\vec{L}_R$, angular momentum of the wheels $\vec{L}_s$ and the total angular momentum $\vec{L}$.
2. Give the force diagram with the external forces acting on the bike and the rider.
3. Derive that the bicyle leans through an angle ϕ given by:

$$\tan\phi = \frac{V^2}{Rg}\left(1 + \frac{m}{M}\right),$$

where M is the total mass.

4. What would the angle be without angular momentum of the wheels? How does this show that the rotation of the wheels makes the bike more stable?