# UNIVERSITY OF DUBLIN



## TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

**JF Maths/TP/TSM** **Trinity Term 2014**

MA1111 — LINEAR ALGEBRA I

Wednesday, May 7 EXAM HALL 14:00 — 16:00

Dr. Paschalis Karageorgis

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.

1. Find a quadratic polynomial $f(x)$ such that $f(1) = 3$, $f(2) = 5$ and $f(3) = 13$.

2. Find a basis for both the null space and the column space of

$$A = \begin{bmatrix} 1 & 1 & 1 & 3 & 8 \\ 2 & 1 & 4 & 2 & 7 \\ 4 & 3 & 6 & 1 & 9 \end{bmatrix}.$$

3. Determine $T(v)$ when $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation and

$$T\left(\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 7 \end{bmatrix}, \quad T\left(\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 8 \end{bmatrix}, \quad v = \begin{bmatrix} 8 \\ 8 \\ 7 \end{bmatrix}.$$

4. Suppose that $A$ is a $3 \times 3$ matrix whose third row is the sum of its first two rows. Show that $A$ is not invertible and find a vector $y \in \mathbb{R}^3$ such that $Ax = y$ has no solutions.

5. Let $A$ be a real, nonzero $m \times n$ matrix. Show that the trace of $AA^t$ is positive.

6. Suppose $v_1, v_2, v_3$ are linearly independent vectors of a vector space $V$ and let

$$w_1 = v_1 + av_2, \qquad w_2 = v_2 + av_3, \qquad w_3 = v_3 + av_1$$

for some $a \in \mathbb{R}$. For which values of $a$ are the vectors $w_1, w_2, w_3$ linearly independent?