UNIVERSITY OF DUBLIN

MA1123-1

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS AND SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics **Trinity Term 2013**
JF Theoretical Physics
JF Two Subject Mod

MA1123 — ANALYSIS I

Tuesday, April 30 RDS 14.00 – 17.00

Prof. D. O'Donovan

ANSWER ALL QUESTIONS:

All questions carry equal marks.

Formulae & Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. (a) Define function, and state the vertical and horizontal line tests, if $f : \mathbb{R} \to \mathbb{R}$

   (b) Define what it means for f(x) :$\mathbb{R} \to \mathbb{R}$ to be continuous at x = a. What is a jump discontinuity?

   (c) Use the definition of limit to show $\lim_{x\to 2} x^2 = 4$

   (d) Let $f(x) = |x|$, find $f'(0)$ or show that it does not exist.

2. (a) Find the quadratic approximation to $\sqrt{8.9}$

   (b) Find $\frac{dy}{dx}$ if

      i. $y = \ln \cos^2(x^3 + 2)$

      ii. $y = x^3 \ln x \cos x \exp x$

      iii. $x^2y + y^3x + \cos(xy) = xy$

      iv. $x = \ln t^2, y = \cos(t^3 + t)$

   (c) Let $f(x) = x^3 - 2x^2 - 4x + 8$. Find where $f(x)$ is increasing, decreasing, concave up, concave down, has local extrema, and points of inflection. Use this information to sketch the function.

3. (a) State Rolle's Theorem and the Mean Value Theorem.

   (b) Use the Mean Value Theorem to prove that if $f'(x) = g'(x)$, for all x, then $f(x) = g(x)$ + constant

   (c) A circular swimming pool has a twelve foot radius, and can be filled to a maximum depth of 4 feet. Water enters the pool at a rate of 9 gallons per minute. How quickly is the depth of the water in the pool changing? How long does it take to fill the pool?

4. (a) State how $\int_a^b f(x)dx$ is defined in terms of Riemann Sums

   (b) Integrate the following.

      i. $\int \ln x dx$

      ii. $\int x \cos x^2 dx$

      iii. $\int x \cos x dx$

iv. $\int \frac{1}{x^2+x+1}dx$

v. $\int \frac{x}{(x-1)(x-2)}dx$

vi. $\int \frac{1}{(x-1)^2(x^2+1)}dx$

5. (a) Find the area of the region bounded by $y = \sin x$, $y = \cos x$ between $x = 0$ and $x = \pi$.

   (b) What is an improper integral?

   (c) Find the volume of the solid of revolution gotten by revolving the region bounded by $y = x^3, y = 0$, and $x = 1$ about the x-axis, first by the method of disks, and then by the method of cylindrical shells.

6. (a) Define $\lim_{n\to\infty} a_n = L$, and $\sum_{n=1}^{\infty} a_n = S$.

   (b) Prove that if $\lim_{n\to\infty} a_n = L_1$ and $\lim_{n\to\infty} b_n = L_2$, then $\lim_{n\to\infty}(a_n + b_n) = L_1 + L_2$.

   (c) Do the following series converge or diverge? Give reasons.

   i. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

   ii. $\sum_{n=1}^{\infty} \frac{n-1}{n^2+2n-1}$

   iii. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

   iv. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$