Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF PHYSICS

Junior Freshman
Annual Examination

Trinity Term 2016

Physics, Paper 2

(Science (Physics), Nanoscience Physics and Chemistry of Advanced Materials, Chemistry with Molecular Modelling and Theoretical Physics)

Thursday 12 May 2016 RDS 9:30 – 12:30

Professors M. Moebius, J. Pethica and Dr. T. Hallam

ALL QUESTIONS CARRY EQUAL MARKS

USE SEPARATE ANSWER BOOKS FOR EACH SECTION

Booklets of Formulae and Tables are available from the invigilator for all students who require them. Graph paper is also available.

Non-programmable calculators are permitted for this examination – please indicate the make and model of your calculator on each answer book used.

All Students

Answer ***SIX*** questions, AT LEAST **ONE** from Section A, AT LEAST **TWO** from Section B, AT LEAST **TWO** from Section C AND **ONE** OTHER, in 3 hours.

EQUATION LIST:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}, \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}, \quad \vec{\tau} = \vec{p}\times\vec{E}, \quad \Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \quad V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi\, dl$$

$$C = \frac{Q}{V_{ab}} = \varepsilon_0\frac{A}{d}, \quad C = \frac{Q}{V_{ab}} = \varepsilon_0\frac{A}{d}$$

$$I = \frac{dQ}{dt} = n|q|v_d A, \quad R = \frac{\rho L}{A}, \quad P = V_{ab}I = I^2 R = \frac{V_{ab}^{\ 2}}{R}$$

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

$$\vec{F} = q\vec{v}\times\vec{B}, \quad \Phi_B = \int B_\perp\, dA = \int B\cos\phi\, dA = \int \vec{B}\cdot d\vec{A}, \quad \vec{F} = I\vec{l}\times\vec{B}, \quad \vec{\tau} = \vec{\mu}\times\vec{B}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}, \quad d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l}\times\hat{r}}{r^2}, \quad B_x = \frac{\mu_0 NI}{2a}, \quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \quad \mathcal{E} = \oint(\vec{v}\times\vec{B})\cdot d\vec{l}, \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

SECTION A

1. Let x be a continuous random variable governed by the following probability density function (PDF).

$$f(x) = \begin{cases} C(x^2 - 1) \; for \; 1 \leq x \leq 3 \\ 0 \quad\quad elsewhere \end{cases}$$

(a) Use the normalisation condition to find the value of C.

[2 Marks]

(b) Find the mean of $f(x)$.

[3 Marks]

(c) Find the second moment of $f(x)$.

[3 Marks]]

(d) Use the definition of the variance to show, that in general, it is given by $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$. Use this formula to calculate the variance of $f(x)$.

[2 Marks]

2. A student monitors the activity of a radioactive source with a detector. The number of counts measured in a given time window obey the Poisson distribution $P_\mu(x) = e^{-\mu}\frac{\mu^x}{x!}$.

(a) Show explicitly that the mean of this distribution is μ . [Hint: Start with the normalisation condition.]

[4 Marks]

(b) In the course of 10 minutes, the detector registers a total of 2540 counts. What is the corresponding rate R_{tot} (in counts per minute) and its uncertainty?

[2 Marks]

(c) The student removes the source in order to record counts due to the background radiation. The detector registers 95 counts in the 3 minutes. What is the corresponding rate of the background, R_{bkg} , and its uncertainty ?

[2 Marks]

(d) What are the rate and its uncertainty due to the radioactive source alone?

[2 Marks]

SECTION B

4. Describe the experimental evidence which suggests that the energy levels in an atom have discrete values. What principle did Bohr introduce to the model of the hydrogen atom to explain these experimental results?.

[2 Marks]

Outline the derivation of the Bohr expression for the energy levels E_n of the hydrogen atom

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right) = E_1\left(\frac{1}{n^2}\right)$$

where m is the mass of the electron and h is Planck's constant

[3 Marks]

An emission line of hydrogen is observed at a wavelength of 598 nm. Between which values of n does this represent a transition of an electron?

[2 Marks]

Briefly describe some of the additional features of electron orbitals, beyond the Bohr model, which are introduced by the full model of the hydrogen atom.

[3 Marks]

5. Give a description of the interaction of electromagnetic waves with matter by

(a) the Photoelectric effect and

(b) Compton scattering

and show how they demonstrate that electromagnetic waves are photons of defined energy and momentum.

[3 Marks]

Outline the steps used in deriving the Compton formula

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\phi)$$

relating change in wavelength to angle through which the wave is scattered

[2 Marks]

An X-Ray photon having energy 200keV collides with a stationary electron and is scattered through an angle of 90 degrees. Calculate its new wavelength.

[2 Marks]

Calculate the minimum energy of a photon required to create an electron-positron pair. What is required to conserve overall momentum?

[3 Marks]

6. Give the general form of the wavefunction ψ for the state of a particle of mass m confined in a one-dimensional box. The box has a potential U which is zero between $x = 0$ and $x = L$ and infinite elsewhere. Show that the solution satisfies the Schrödinger Equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - U)\psi = 0$$

[3 Marks]

Find an expression for the energy states E_n of the particle in this 1-D box

[2 Marks]

If the particle is instead confined in a 2-dimensional square box of side L in x and y directions, list the first few possible quantum states of the particle in terms of the quantum numbers n_x and n_y

[2 Marks]

What is meant by the Pauli exclusion principle? If 7 electrons are placed in the 2-D box, what are the lowest energy states they can occupy? What is the energy of the electron in the highest energy state?

[3 Marks]

SECTION C

7.

(a) State Gauss's Law in words and formula.

[2 marks]

(b) Two concentric conducting spheres of radius r_a and r_b are separated by vacuum. Using Gauss' Law, find the direction and magnitude of the electric field inside and outside the shells as a function of radius *r*. Be sure to clearly state the Gaussian surfaces that you are using. Find an expression for the voltage between the shells.

[3 marks]

(c) Determine the capacitance of the capacitor.

[2 marks]

(d) Determine the capacitance of a single isolated spherical conductor with radius R (Consider the opposing sphere far away).

[3 marks]

8.

The electric potential as a perpendicular distance from a long straight wire of cross sectional radius *a* is given by:

$$V(r) = -K \ln \frac{r}{a}$$

Where *K* is a constant.

(a) Calculate the electric field as a function of distance.

[3 marks]

(b) Using Gauss' law determine the charge *q* per unit length of the wire.

[3 marks]

(c) A second identical wire, carrying charge $-q$ per unit length is placed parallel to the first at a distance of *d* from it. Calculate the potential difference between the wires

[4 marks]

9.

(a) Two resistors R_1 and R_2 are wired up in Parallel over a voltage drop V_{ab}. Draw a diagram showing the current through the resistors and derive an expression for the equivalent combined resistance R_{eq}.

[2 marks]

(b) Two capacitors C_1 and C_2 are wired up in series over a voltage drop V_{ab}. Draw a diagram showing the charge on the capacitors and derive an expression for the equivalent combined capacitance C_{eq}.

[2 marks]

(c) Now consider an open circuit with 2 capacitors in series, C_1=C_2 and C_1 is charged to voltage V. Determine the difference in stored energy between this circuit when open and after the circuit is closed and has reached a steady state.

[3 Marks]

(d) A square parallel plate capacitor with plate area A, separated by distance d has a dielectric block of material with a dielectric constant K fully inserted between the plates as shown. Calculate the force required to remove the dielectric when a voltage V_{ab} is maintained across the plates.

(Hint: Consider potential energy to calculate the force).

[3 marks]

TRINITY COLLEGE
SCHOOL OF PHYSICS

Electron rest mass	m_e	9.11×10^{-31} kg
Proton rest mass	m_p	1.67×10^{-27} kg
Electronic charge	e	1.60×10^{-19} C
Speed of light in free space	c	3.00×10^{8} m s^{-1}
Planck's constant	h	6.63×10^{-34} J s
	$h/2\pi = \hbar$	1.05×10^{-34} J s
Boltzmann's constant	k	1.38×10^{-23} J K^{-1}
Molar gas constant	R	8.31×10^{3} JK^{-1}kmol^{-1}
Avogadro's number	N_A	6.02×10^{26} kmol^{-1} $= 6.02 \times 10^{23}$ mol^{-1}
Standard molar volume		22.4×10^{-3} m^3
Bohr magneton	μ_B	9.27×10^{-24} A m^2 <u>OR</u> J T^{-1}
Nuclear magneton	μ_N	5.05×10^{-27} A m^2 <u>OR</u> J T^{-1}
Bohr radius	a_o	5.29×10^{-11} m
Fine structure constant		
	$e^2/(4\pi\varepsilon_o\hbar c) = \alpha$	$(137)^{-1}$
Rydberg's constant	R_∞	1.10×10^{7} m^{-1}
Stefan's constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} N m^2 kg^{-2}
Proton magnetic moment	μ_p	$2.79\ \mu_N$
Neutron magnetic moment	μ_n	$-1.91\ \mu_N$
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of free space	ε_o	8.85×10^{-12} F m^{-1}
1 electron volt	eV	1.60×10^{-19} J

1 unified atomic mass unit (^{12}C scale)		1.66×10^{-27} kg = 931 MeV/c^2
Wavelength of 1 eV photon		1.24×10^{-6} m
1 atmosphere		1.01×10^{5} N m^{-2}
Standard acceleration due to gravity		10 m s^{-2}
Free space impedance	Z_0	377 Ω
Astronomical unit	au	1.50×10^{11} m
Parsec	pc	3.09×10^{16} m
Solar radius	$R_\odot$	6.96×10^{8} m
Solar mass	$M_\odot$	1.99×10^{30} kg
Solar luminosity	$L_\odot$	3.85×10^{26} W
Earth mass	$M_\oplus$	5.97×10^{24} kg
Earth radius (equatorial)	$R_\oplus$	6378 km