Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JS & SS Mathematics
JS Theoretical Physics
Moderatorship

Trinity Term 2017

MA1241 — Mechanics I

Friday, May 12 **RDS** **09:30 — 11:30**

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.

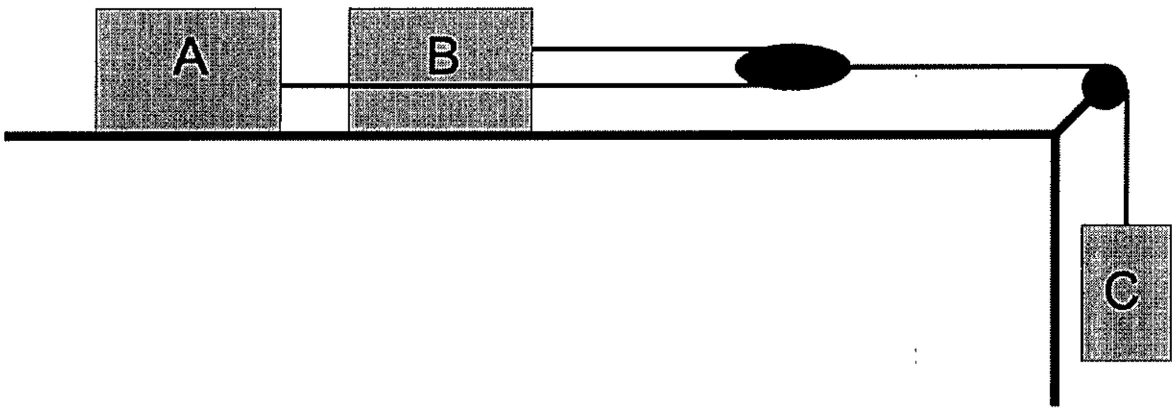
All questions have equal weight.

'Formulae & Tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination - please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Two masses, A and B, lie on a frictionless table. See the figure below. They are attached to either end of a rope with fixed length l_1, which passes around a pulley. This pulley can move without friction over the table, and is attached to a second rope with fixed length l_2. The second rope passes over a fixed pulley and is attached to a hanging mass C on the other end. Assume that the masses of the ropes and the pulleys are negligible.



   (a) Draw the force diagrams and give the equations of motions (of the center of masses) of the masses and the pulleys.

   (b) Determine the tension in the two ropes.

   (c) Determine the acceleration (magnitude and direction) of the masses.

2. We consider a door of length L and a block of mass m placed adjacent to the door at a distance r_0 from the pivot point. See the figure. We will consider the movement of the block while the door is being closed. We assume all friction forces can be neglected, and that the block is well approximated by a point particle of mass m.

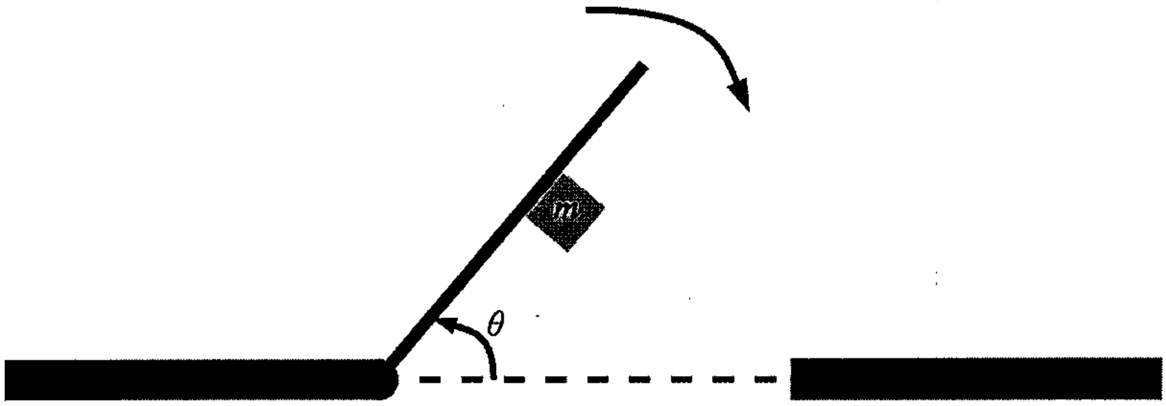


Figure 1: Door and block viewed from above.

(a) Give the force diagram of the block, and its equations of motion in a cylindrical coordinate system. Take the pivot point of the door as origin of the coordinate system, and the angular coordinate as shown in the figure.

(b) Assume that the door is being closed with constant angular velocity $\omega < 0$. At time $t = 0$, the initial conditions for the block are $r(0) = r_0$, $\dot{r}(0) = 0$ and $\theta(0) = \theta_0$. Show that the radial coordinate $r(t)$ of the block is given by

$$r(t) = r_0 \cosh(\omega t).$$

(c) Assume that the block reaches the end of the door before the door is closed. Show that the kinetic energy K of the block is given by

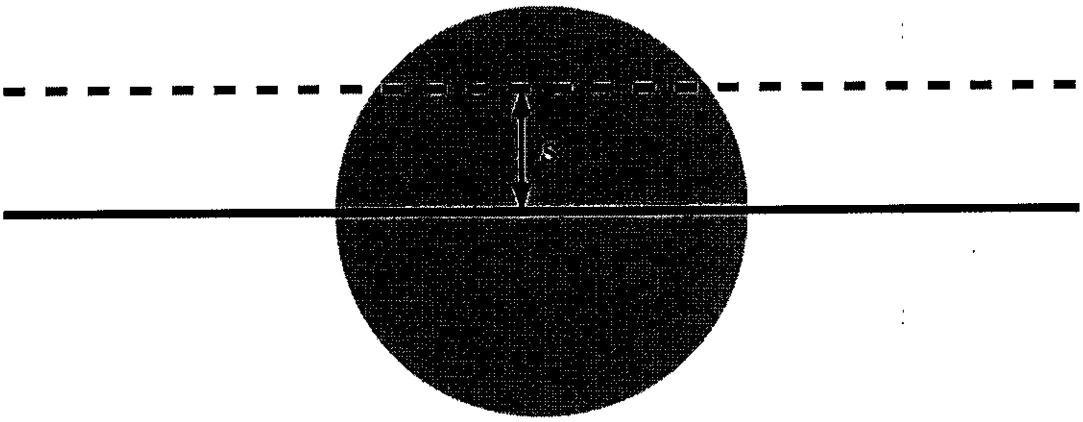
$$K = m\omega^2(L^2 - r_0^2),$$

after the block looses contact with the door.

Hint: The acceleration in polar coordinates is given by:

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}.$$

3. A thin disk with mass m and radius R can rotate around an axis through the center of the disk, which lies in the plane of the disk. (The solid line in the Figure below.)



(a) Show that the moment of inertia with respect to the solid axis is $\frac{1}{4}mR^2$.

(b) Assume now that the axis does not go through the center, but does lie in the plane of disk at a distance s from the previous axis. (The dashed line in the figure.) The disk can swing periodically around this axis. Determine the period T of this pendulum motion for small amplitudes.

(c) Determine the value of s for which the period is minimized.