Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Mathematics   Trinity Term 2016
JF Theoretical Physics
JF Two Subject Mod

MA1123 — Analysis I

Tuesday, May 3   RDS   14:00 – 17:00

Prof. D. O'Donovan

Instructions to Candidates:

ANSWER ALL QUESTIONS:

All questions carry equal marks.

Formulae & Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. (a) Define $f'(a)$ the derivative of $f(x)$ at $x = a$. Use this definition to find $f'(x)$ for $f(x) = \sin x$.

   (b) Use the definition of limit to evaluate $\lim_{x \to 1} 2x^3 + 3x^2 - 2$.

   (c) Prove that if $\lim_{x \to a+} f(x) = L$ and if $\lim_{x \to a-} f(x) = L$ then $\lim_{x \to a} f(x) = L$

   (d) Define $\lim_{x \to \infty} f(x) = L$ and prove that if it exists such a limit is unique.

2. (a) Find the quadratic approximation to $\sqrt{35}$

   (b) Find $\frac{dy}{dx}$ if

       i. $y = \cos\left(\sqrt{\sin \exp(x^3 + x)}\right)$

       ii. $xy^2 + x^3 \exp y = \ln(xy^3)$

   (c) Car A is driving due West at 90 Km/Hour. Car B is driving due North at 100 Km/Hour. Their paths intersect at right angles. At what rate is the distance beween the cars changing when Car A is 60 Km and car B 80 Km from the junction, and both are proceeding towards it?

   (d) Find where the function $\frac{x^2}{x^2 - 1}$ is increasing, decreasing, concave up, concave down, has local extrema, and points of inflection. Draw a rough sketch of the function.

3. (a) State the Mean Value Theorem. Prove Rolle's Theorem.

   (b) Solve the following

       i. $x\frac{dy}{dx} + y = y^2$.

       ii. $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12 = 0$

       iii. $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12 = 3\exp 2x$

   (c) A cylindrical can is to hold 1,000 cubic centimetres. Find the dimensions of the can that minimizes the amount of material used.

4. (a) How is the Riemann Integral $\int_a^b f(x)dx$ defined?

   (b) Integrate the following.

   i. $\int x \exp(x^2 + 3)dx$

   ii. $\int x \exp x \, dx$

   iii. $\int \sin^2 x \cos^2 x \, dx$

   iv. $\int \dfrac{x}{(x+2)(x+3)}$

   v. $\int \dfrac{x^2 + x + 1}{(x+1)^3} dx$

5. (a) Find the length of the curve $y = x^{\frac{3}{2}}$ between $x = 1$ and $x = 4$?

   (b) Find the area of the region bounded by $y = x^2$, $y = -x+6$, $y = x+6$, and $x = 0$.

   (c) The area between $y = x^2$, $x = 1$, and $y = 0$, in the first quadrant, is rotated about the x-axis. Find the volume by disks and then by cylindrical shells?

6. (a) Define $\sum_{n=1}^{\infty} a_n = L$.

   (b) Do the following series converge or diverge? Give reasons.

   i. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2 - 2n + 4}$

   ii. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{\exp n^2}$

   iii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   (c) What is the nth Taylor polynomial for a function $f(x)$?

   (d) Find for what values of x the following power series converges absolutely, conditionally, or diverges? What is the radius of convergence?

   $\displaystyle\sum_{n=1}^{\infty} \frac{(2x - 3)^n}{n}$