MA1125 – Calculus
Homework #5
due Thursday, Oct. 18

1. Show that the polynomial $f(x) = x^3 - 4x^2 - 3x + 1$ has exactly one root in $(0, 2)$.

2. Suppose that $0 < a < b$. Use the mean value theorem to show that

$$1 - \frac{a}{b} < \ln b - \ln a < \frac{b}{a} - 1.$$

3. Compute each of the following limits.

$$L_1 = \lim_{x \to 3} \frac{2x^3 - 8x^2 + 7x - 3}{3x^3 - 8x^2 - x - 6}, \qquad L_2 = \lim_{x \to \infty} \frac{x^2}{e^x}, \qquad L_3 = \lim_{x \to 0} (e^x + x)^{1/x}.$$

4. On which intervals is f increasing? On which intervals is it concave up?

$$f(x) = \frac{x}{x^2 + 3}.$$

5. Find the intervals on which f is increasing/decreasing and the intervals on which f is concave up/down. Use this information to sketch the graph of f.

$$f(x) = \frac{(x-1)^2}{x^2 + 1}.$$

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and course (Maths, TP, TSM) on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.