

Advanced Calculus

MA1132

Exercises 1

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To be completed and handed in by the end of tutorial on Friday, 1. February

1. Find the domain of the function

$$\mathbf{r}(t) = (\sqrt{1-3t}, \tan^{-1}(-t), \ln(9-t^2)).$$

2. Show the graph of the function

$$\mathbf{r}(t) = t\mathbf{i} + \frac{3-2t}{t}\mathbf{j} + \frac{(t-3)^2}{t}\mathbf{k}$$

lies in the plane $x + 3y - z = 0$.

3. Describe what the graph of the function given by

$$\begin{aligned}x(t) &= t^2 - t \\ y(t) &= t - 2\end{aligned}$$

looks like.

4. Find a vector-valued function which has as its graph, the curve of intersection of the surfaces $x + y = 0$ and $z = \sqrt{2 - x^2 - y^2}$.

Geometrically, what does this curve look like?

5. Evaluate the limit

$$\lim_{t \rightarrow 1} \left(\frac{t^2 + t - 2}{t - 1}, \frac{\ln(t)}{t^3 - t^2 + t - 1}, t^2 - 1 \right)$$

For what values of a , b and c is the function

$$\mathbf{r}(t) = \begin{cases} \left(\frac{t^2 + t - 2}{t - 1}, \frac{\ln(t)}{t^3 - t^2 + t - 1}, t^2 - 1 \right) & \text{if } t \neq 1 \\ (a, b, c) & \text{if } t = 1 \end{cases}$$

continuous at $t = 1$?

6. Consider the vector-valued function (with values in \mathbb{R}^3)

$$\mathbf{r}(t) = \ln(3 - \sqrt{t})\mathbf{i} + (1 + \sqrt{t})\mathbf{j} + \frac{(3 - \sqrt{t})^2}{4}\mathbf{k} \quad (1)$$

- (a) Find the domain $\mathcal{D}(\mathbf{r})$ of the vector-valued function $\mathbf{r}(t)$.
- (b) Find the derivative $d\mathbf{r}/dt$.
- (c) Find the norm $|d\mathbf{r}/dt|$.
Simplify the expressions obtained.
- (d) Find the unit tangent vector \mathbf{T} for all values of t in $\mathcal{D}(\mathbf{r})$.
- (e) Find the vector equation of the line tangent to the graph of $\mathbf{r}(t)$ at the point $P_0(0, 3, \frac{1}{4})$ on the curve.

7. Find the indefinite integral

$$\int \ln(t)\mathbf{i} + \sin(t)e^{\cos(t)}\mathbf{j} + \frac{3t+2}{t^2-4}\mathbf{k} \, dt.$$