JF PY1T10

Special Relativity

Special Relativity Dr. Nuala Caffrey

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- 16 Lectures (including Tutorial Classes)

- Textbooks:
  - **Special Relativity (A. P. French)**
    [Some copies in library, E-book also available via library website]
  - University Physics (Young & Freedman), Chapter 37

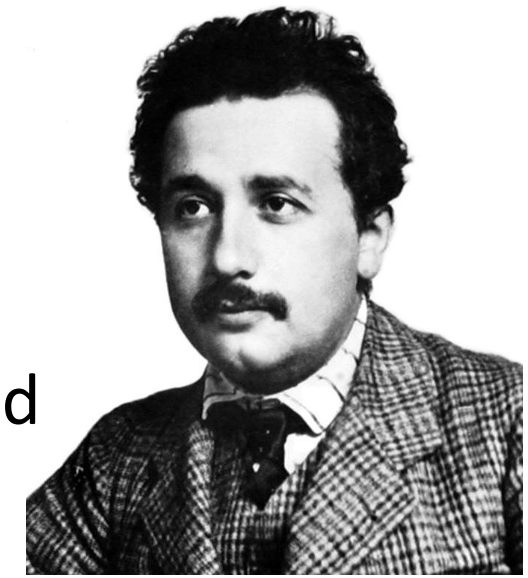
- Lecture notes will be available on Blackboard.

Introduction

1905: Albert Einstein published

"On the Electrodynamics of Moving Bodies"

He showed how measurements of space and time are affected by motion between an observer and what is been observed.



Relativity connects space and time, matter and energy, electricity and magnetism.

All of relativity's remarkable predictions have been confirmed by experiment.

Many of the conclusions of relativity can be reached with simple mathematics.

Introduction

In the limit of low velocity, v:

$$\text{Special Relativity} \rightarrow \text{Newtonian Mechanics}$$

But special relativity has many non-intuitive consequences when $v \approx c$.

For example:

- Events that are simultaneous for one observer may not be simultaneous for another
- When two observers moving relative to another measure a time interval or length they may not get the same result.

Introduction

Galilean Relativity

- Valid at low velocities
- Approximation of Special Relativity

Special Relativity

- Valid at velocities close to *c*
- Not valid in strong gravitational fields
- Approximation of General Relativity

General Relativity

- Valid at velocities close to *c*
- Describes gravity
- Still an open question: How to combine with Quantum Mechanics?

Course Outline

- Inertial frames and Galilean transformations
- Michelson-Morley experiment
- Einstein’s postulates, relativity of simultaneity
- Lorentz transformation
- Length contraction and time dilation
- Relativistic Doppler effect, moving clocks
- Transformation of velocities
- Relativistic dynamics: energy, mass, momentum
- Pair creation, fission, fusion
- Collisions, Compton effect
- Energy-momentum invariant

Inertial Frames of Reference

The motion of a body can only be described relative to something else – generally to a set of space-time coordinates – this is known as a **frame of reference**.

Choose an frame of reference in which the laws of mechanics take their simplest form:

An ‘inertial reference frame’ is a frame in which Newton’s First Law holds.

(i.e. a frame of reference in which a body with zero net force acting upon it is not accelerating).

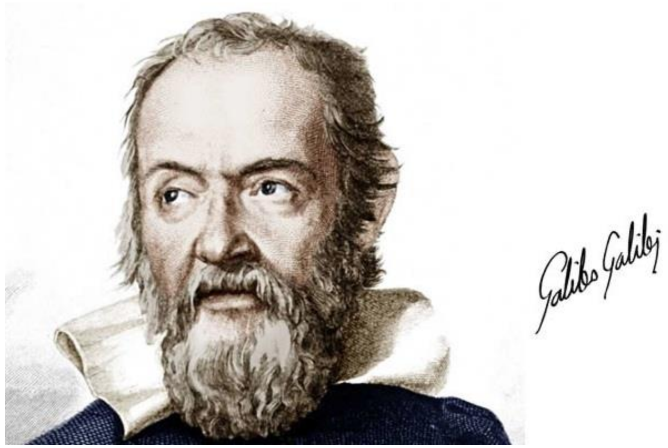
For most purposes, a frame fixed to the Earth’s surface (the “lab frame”) is a sufficient approximation to an inertial system. For some astronomical purposes, it may be necessary to use an inertial system fixed on distant galaxies.

Hypothesis of Galilean Invariance

Basic laws of physics are the same in all inertial frames.

First described by Galileo Galilei in 1632.

Imagine you are below the deck of a boat which can travel smoothly through water.
There is <u>no experiment</u> you can do to tell if the boat is moving or not.



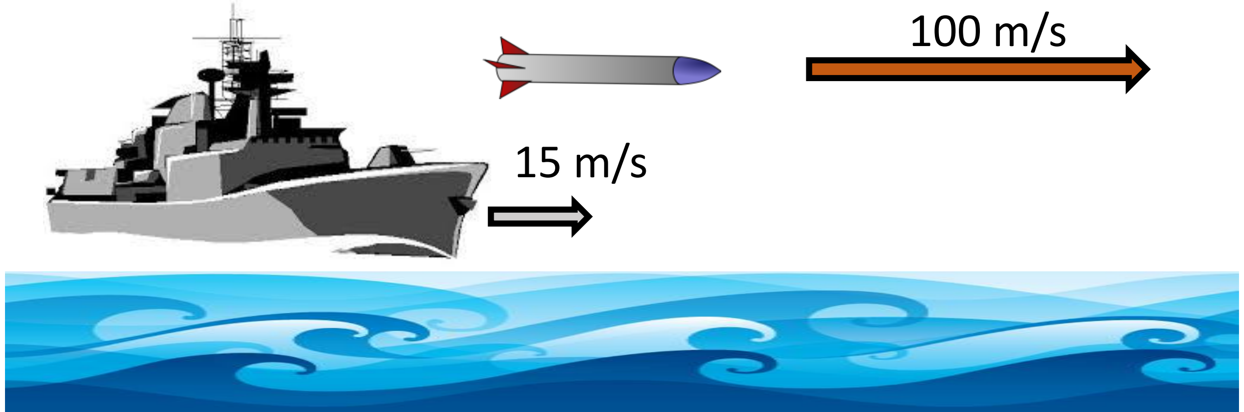
If one coordinate system is moving at a constant velocity with respect to an inertial frame, that coordinate system is also an inertial frame.

Galilean Transformations – 1

Aim: Transform coordinates of an event (x, y, z, t) between inertial frames moving at a speed relative to each other.

A warship is travelling at 15 m/s. It fires a missile from its deck at a speed of 100 m/s relative to the boat. How fast does the missile travel relative to the water?

2 inertial frames: Boat & Water



From inertial frame of water, velocity of missile = 100 m/s + 15 m/s = 115 m/s

Galilean Transformations – 2

Two trains are travelling on straight parallel tracks.

The constant speeds of train **A** and train **B** relative to a stationary observer **O** standing next to the track are 200 km/h and 150 km/h, respectively.

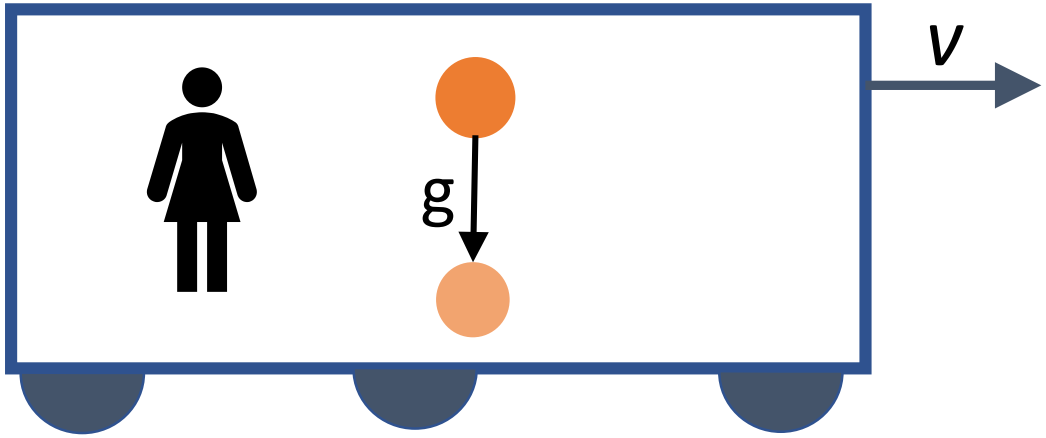
1. If the two trains are travelling in opposite directions, what is the speed of train **B** relative to train **A**?
2. What is the speed of train **A** in the rest frame of train **B**?

A traveler on train **A** fires a gun out of the window with a speed 360 km/h towards the rear of the train & parallel to the track.

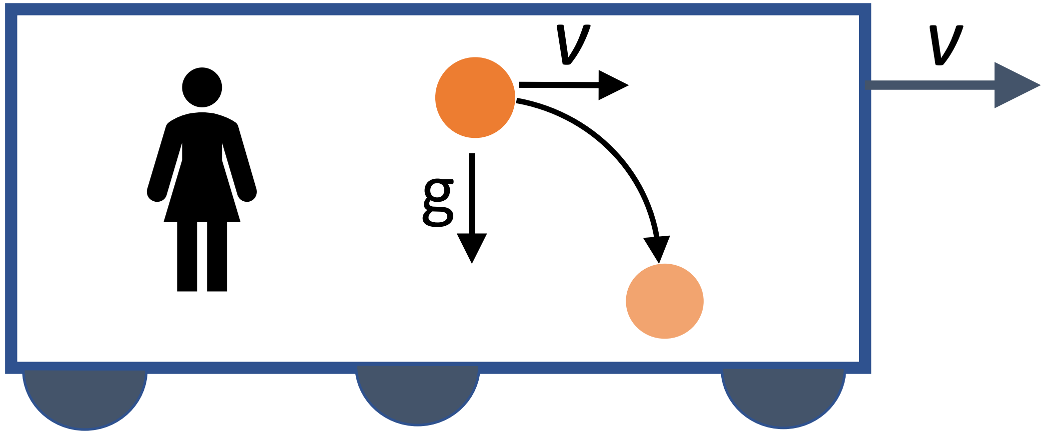
1. What is the speed of the bullet as measured by the observer **O**?
2. If the gun was fired perpendicular to the track, what would **O** observe the horizonal speed of the bullet to be?

Inertial Frames of Reference

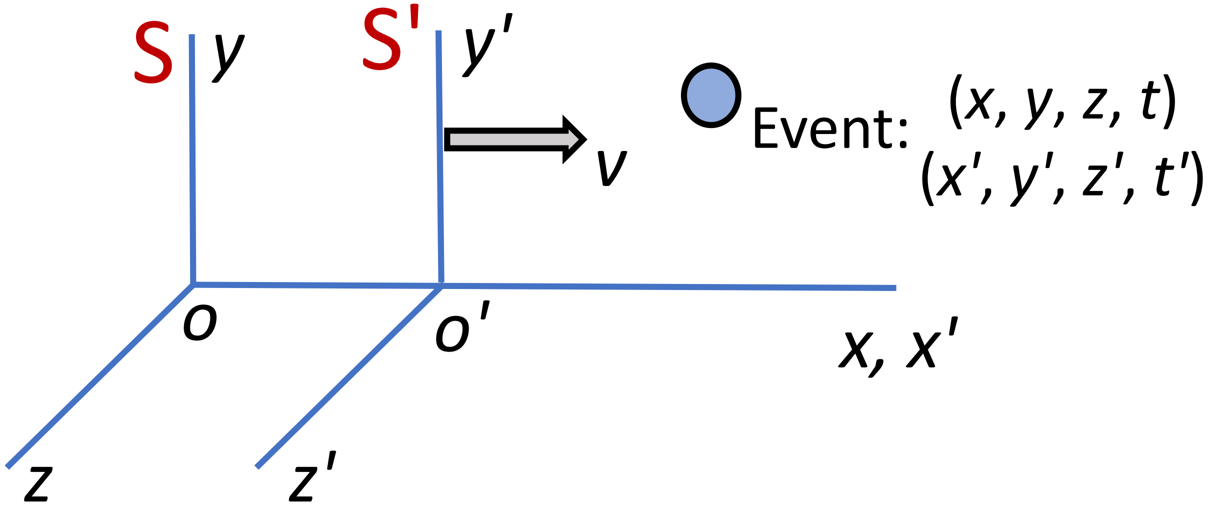
Drop a ball in a moving train (*S′*) with velocity *v*:



Observe from platform (*S*):



Galilean Transformations – 3



Inertial frame S' moves with velocity v relative to inertial frame S, along the x-axis. Origins coincide at t = 0 and t' = 0.

$$x = x' + vt, \qquad y = y', \qquad z = z', \qquad t = t'$$

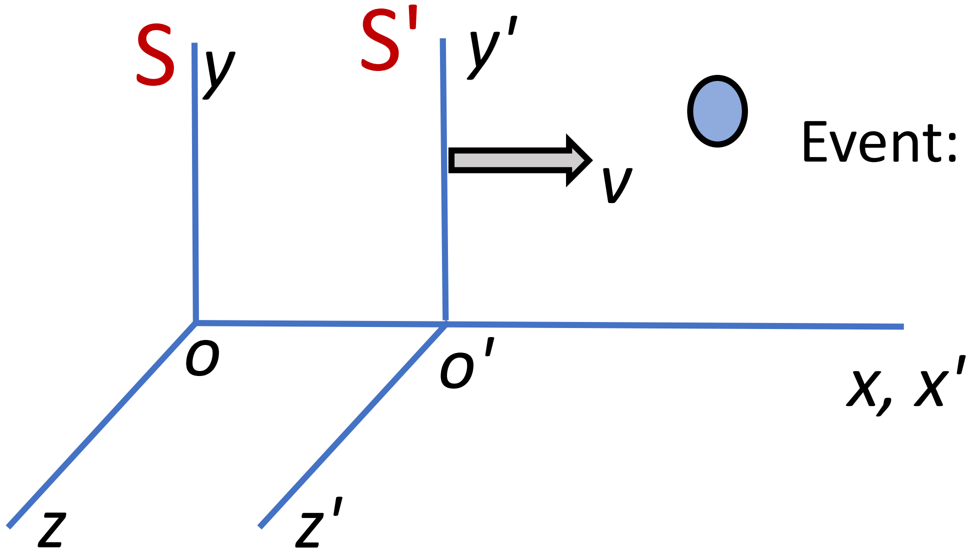
$$dx = dx' + vdt, \qquad dy = dy', \qquad dz = dz', \qquad dt = dt'$$

$$\frac{dx}{dt} = \frac{dx'}{dt} + v\frac{dt}{dt'}, \qquad \frac{dy}{dt} = \frac{dy'}{dt}, \qquad \frac{dz}{dt} = \frac{dz'}{dt}$$

$$u_x = u_x{}' + v, \qquad u_y = u_y{}', \qquad u_z = u_z{}'$$

$$\therefore \boldsymbol{u} = \boldsymbol{u}' + \boldsymbol{v}$$

Galilean Transformations – 4



Event: (x, y, z, t)
(x', y', z', t')

$$\boldsymbol{u} = \boldsymbol{u}' + \boldsymbol{v}$$

$$\boldsymbol{a} = \frac{d\boldsymbol{u}}{dt} = \frac{d\boldsymbol{u}'}{dt} = \boldsymbol{a}'$$

$$\boldsymbol{F} = m\boldsymbol{a}, \qquad \boldsymbol{F}' = m'\boldsymbol{a}'$$

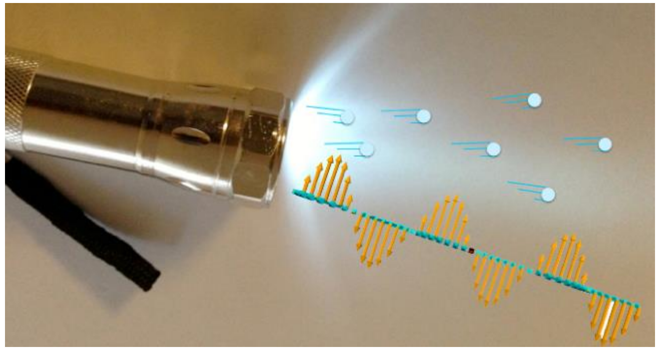
But:

$$m = m', \qquad \boldsymbol{a} = \boldsymbol{a}'$$

$$\therefore \boldsymbol{F} = \boldsymbol{F}'$$

Observers in both frames agree on magnitude and direction of force.

Theories of Light: Particle or Wave?



Pythagoras (6c. B.C) – Light is a stream of *particles*

- It travels in straight lines and can travel through a vacuum.

Robert Hooke & later Huygens (1667) – Light is a *wave*.

- Required to explain some phenomena e.g. light in region of geometric shadow.

Strong evidence for wave theory: <u>Polarization</u>, <u>Interference</u>, <u>Diffraction</u>

All waves they were familiar with requires a *medium*: e.g. Sound can be transmitted in solid, liquid or gas, but not a vacuum.

Search for the medium for light waves.... *The Luminiferous Ether*

[Read: Chapter 2 in "Special Relativity" by A.P. French]

The Ether

Expect light to propagate at a constant velocity *w.r.t.* the ether.

The velocity of light measured by an observer will depend on his motion *w.r.t.* the ether.

If we measure velocity of light in different directions, we should be able to detect our motion through the ether:

- Same in both directions – we are stationary
- Not the same in both directions – we are moving.

Mitchelson & Morley tried to measure this effect.

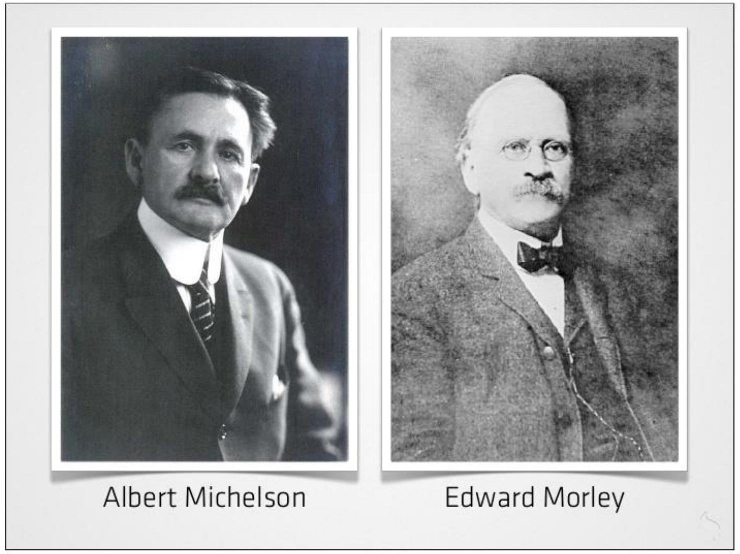
Earth moves around the Sun at 3×10^4 m/s

Velocity of light: $c = 3 \times 10^8$ m/s $\quad \therefore$ Need to measure a very small effect.

Michelson–Morley Experiment – 1

Prevailing theory: The ether formed an absolute reference frame with respect to which the rest of the universe is stationary.

The ether should appear to be moving from the perspective of an observer on the sun-orbiting Earth.

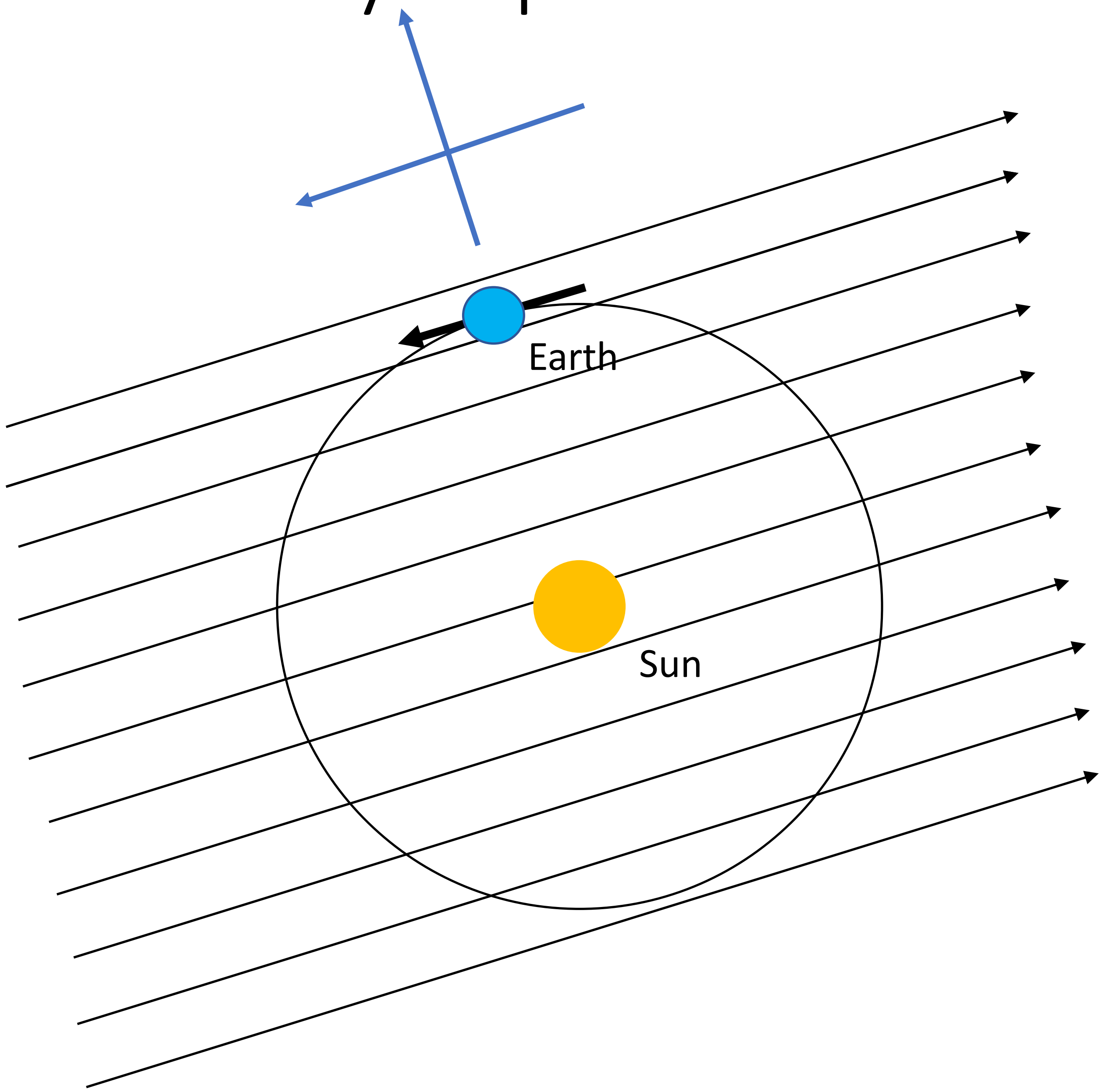


1887, Cleveland, Ohio

As a result, light would sometimes travel in the same direction as the ether, and others times in the opposite direction.

Aim: measure the speed of light in different directions in order to measure speed of the ether relative to Earth, thus establishing its existence.

Michelson–Morley Experiment – 2



Michelson–Morley Experiment – 3

P = glass plate with semi-transparent metal coating on its front face. At 45° to the source beam. Splits the light beam in two.

$\mathbf{M_1}$, $\mathbf{M_2}$ = Two mirrors.

C = Compensating plate to ensure both beams travel through the same amount of glass.

See interference fringes in telescope. (If M_1 is not exactly perpendicular to M_2, the fringes are parallel lines.

Used multiple reflection to extend the optical path.

For rotation, the whole interferometer was floated in a bath of mercury.