Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JS & SS Mathematics **Trinity Term 2018**
JS Theoretical Physics
Moderatorship

MA1242 — Mechanics II

Tuesday, May 15 **Regent House** **14:00 — 16:00**

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.

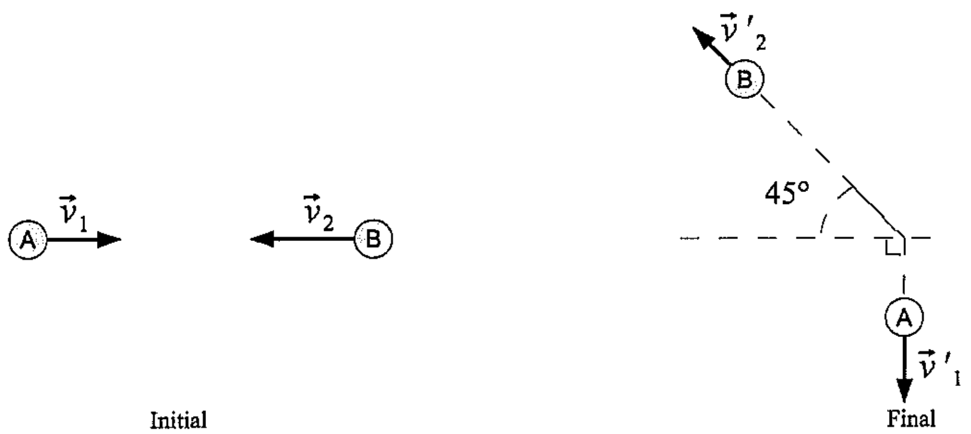
All questions have equal weight.

'Formulae & Tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination - please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Particle A of mass m and initial velocity $\vec{v}_1$ (magnitude v_1) collides elastically with particle B of unknown mass M and with initial velocity $\vec{v}_2$, oppositely directed to $\vec{v}_1$ but with *unknown* magnitude v_2. After the collision, A has final velocity $\vec{v}_1'$, orthogonal to its original velocity $\vec{v}_1$ and with magnitude $2v_1$. The direction of the final velocity of particle B, $\vec{v}_2{}'$, is as in the sketch.

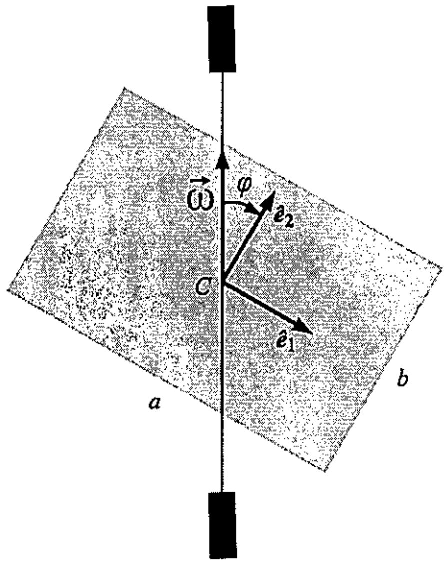


   (a) Which physical quantities are conserved during the collision?

   (b) Determine the initial speed v_2 of B.

   (c) Determine the ratio $\frac{M}{m}$.

2. We consider a thin, homogeneous, square plate with sides a and $b = a/\sqrt{3}$ and mass m. The plate rotates with angular velocity $\vec{\omega}$ around a fixed axis, which is in the plane of the plate and goes through the plate's center of mass C. The angle φ between the rotation axis and the principal axis $\hat{e}_2$ is $\pi/3$. The axis $\hat{e}_1$ in the figure is also a principal axis.



(a) Determine the moments of inertia with respect to the principal axes, I_1, I_2 and I_3.

(b) Determine the components, L_1, L_2 and L_3, of the angular momentum $\vec{L}$ with respect to C.

(c) Determine the angle γ between $\vec{\omega}$ and $\vec{L}$.

(d) Determine the magnitude of the torque $\vec{\tau}$.

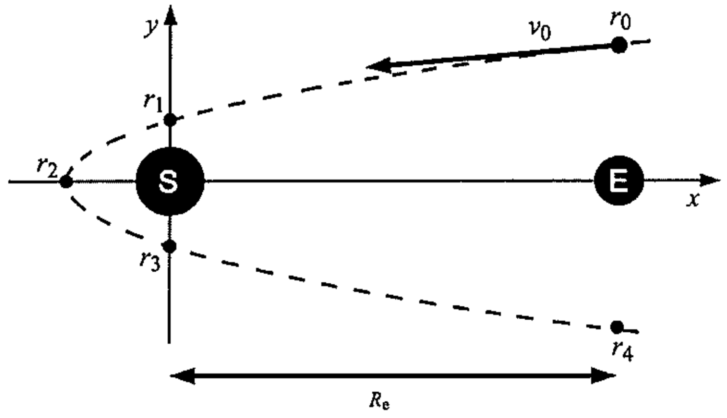


Figure 1: Diagram of the comet's trajectory with the positions r_j, $j = 0, \ldots, 4$.

3. A comet with mass m moves under the gravitational attraction of the sun, $\vec{F} = -(GM_s m/r^2)\hat{r}$. The comet can be observed from the Earth, which we assume to have a fixed position at distance R_e from the centre of the Sun. See the diagram. The turning point of the comet (r_2 in the diagram), the centre of the Sun, and the centre of the Earth, all lie on the x-axis.

   (a) The trajectory of the comet is a parabola, which can be written explicitly as

   $$y^2 - 2R_0 x = R_0^2.$$

   Let $\theta = \arctan(y/x)$. Derive that the distance $r(\theta)$ from the centre of the Sun can be expressed as

   $$r(\theta) = \frac{R_0}{1 - \cos(\theta)}.$$

   (b) Determine the speed of the comet at r_0 and r_2, and the magnitude of the angular momentum ℓ.

   (c) Recall Kepler's second law:

   $$\frac{dA}{dt} = \frac{\ell}{2m}.$$

   Determine the time intervals T_{13} between the comet passing r_1 and r_3, and T_{04} between the comet passing r_0 and r_4, in terms of R_0, R_e and GM_s.