Quantum Physics PY1T20/PYU11P20

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Lecture 2: Light as a particle

- Aims of this lecture:
- Understand two key experiments: the photoelectric effect and Compton scattering
- ... that show the energy and momentum of a light wave is transferred to matter only in certain, discrete amounts.
- And appreciate *how* these experiments show this.

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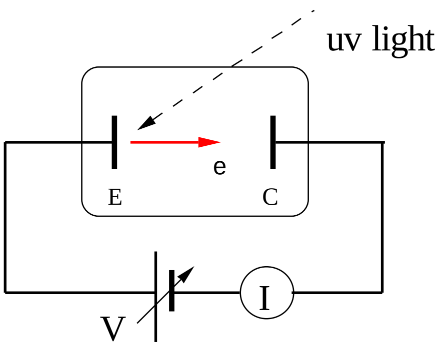
Photoelectric effect experiments



2 metal electrodes, the 'emitter' E and the 'collector' C

Light incident on emitter releases electrons

These (can) reach the collector and so a current *I* flows.

A voltage difference V between E and C allows us to accelerate/decelerate the electrons released from E.

Accelerates/decelerates depending on the sign of V

If we define sign of V such that V>0 means the potential accelerates the electron then

$$T_{\text{collector}} = T_{\text{emitter}} + |e|V$$

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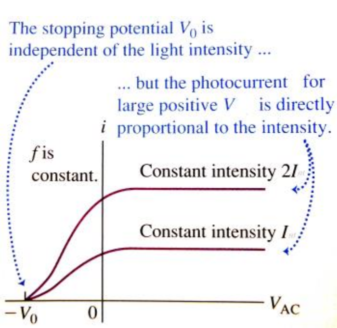
Photoelectric effect observations



Features of i-V characteristic, for fixed frequency of light :

- There is some $V=-V_0$ at which $i=0$, V_0='stopping potential'
- This implies a maximum kinetic energy for the electrons being emitted (such that V_0 is sufficient to stop all of them reaching the collector).
- The current is proportional to light intensity.
- But stopping potential/maximum electron k.e. is not.

ϕ

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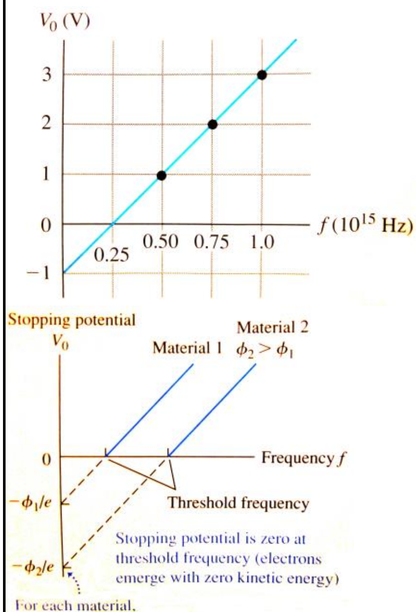
Photoelectric effect – frequency dependences



- The stopping potential is linear in frequency
- The slope is independent of the material.
- The intercept varies with material, so there is a 'threshold frequency' which must be exceeded for photoelectrons to appear.
- This behaviour is all inconsistent with a picture of light as a wave, which can continuously exchange energy with the electrons in the metal.
- It is, however, very easy to understand in terms of a particle picture of light.

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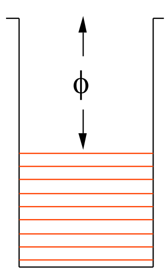
Photoelectric effect – classical view



- Electrons are held in the metal in some potential 'well', of depth ϕ
- ϕ is called the 'work function' of the metal and depends on the material.
- Classically, light is a wave and delivers a certain amount of energy/time to the electrons in the metal. They begin to oscillate as they absorb energy from the light, and these oscillations can build up enough that the electron is ripped out of the material.
- So we would expect :
  - The photocurrent would not depend of the light frequency, for the same intensity (because that means the light waves deliver the same power to the material).
  - The stopping potential would depend on the light intensity (because higher intensity = more power to sample = electron accumulates more energy before it escapes from the material)

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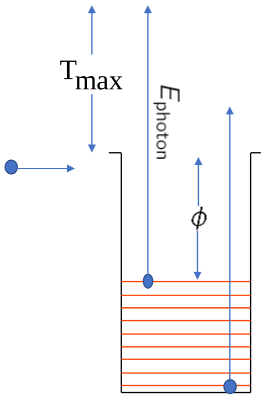
Photoelectric effect – quantum picture



- Instead of the wave transferring energy to the electrons in any amount, suppose it can only transfer in discrete units – particles of light, which we now call 'photons'.
- A photon in the incoming light can be absorbed by the electron, and transfer its energy E_{photon} to that electron.
- Suppose it is absorbed by one of the highest-energy electrons in the material. If the photon energy exceeds the work function this electron will be freed from the material, and be left with a kinetic energy $T_{\max} = E_{\text{photon}} - \phi$
- One of the lower-energy electrons could also absorb a photon, and then come off with a lower kinetic energy (or possibly not escape at all).
- Experimental results show we should take $E_{\text{photon}} \propto f = hf$
- Stopping potential is $eV_0 = T_{\max} \Rightarrow eV_0 = hf - \phi \Rightarrow V_0 = \frac{h}{e}f - \frac{\phi}{e} = \frac{h}{e}(f - f_0)$
- Rate of electrons emitted is proportional to rate of photons arriving = intensity / (hf)
- So photocurrent is proportional to the intensity (at fixed frequency), as seen.

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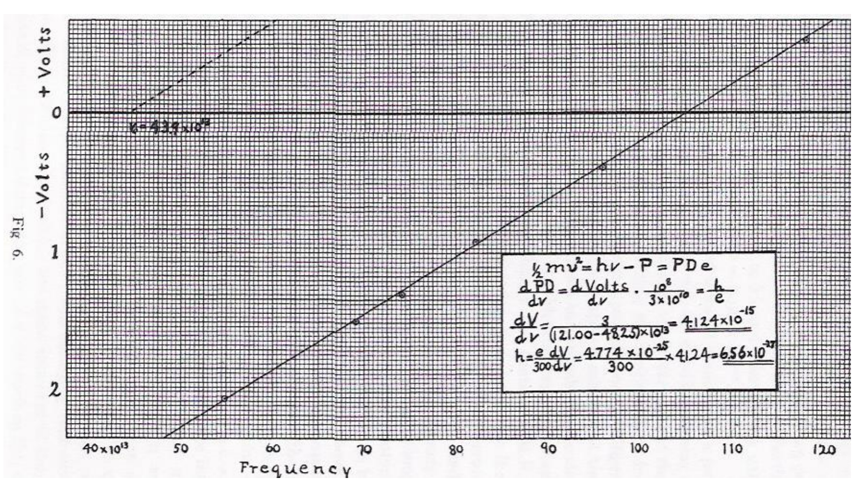

"Despite then the apparently complete success of the Einstein equation, the physical theory of which it was designed to be the symbolic expression is found so untenable *that Einstein himself, I believe, no longer holds to it……"*

(Millikan)

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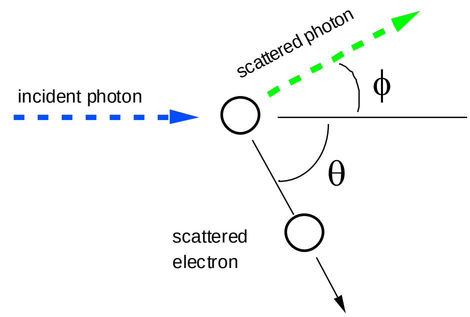
Compton Effect

- Another phenomenon which reveals the relationship between photon energy and light frequency, $E_{\text{photon}} \propto f = hf$

- Suppose light scatters off a (stationary) electron

- Electron scatters through angle θ and gains some kinetic energy.

$\Rightarrow$Photon, scattered through angle ϕ must lose energy



- So we expect the scattered photon to have a lower frequency ($\Rightarrow$longer wavelength)

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Relativistic mechanics primer/revision

- Newtonian mechanics is not (all) correct when things move at/near to the speed of light.
- Instead you need to use relativistic mechanics.
- $E = mc^2$: mass is not a constant but depends on the energy something has.
- In relativistic mechanics, moving things are harder to deflect than you'd expect from Newton's equations
- e.g. moving electrons being deflected by electric field – the deflection, for a given electric field, is less than you would expect if you thought their mass was fixed at $m_0 = 9.11 \times 10^{-31}$kg
- If an object is at rest it has no kinetic energy, and $E = m_0 c^2$ where m_0 is called the 'rest mass'.
- For a moving object, with momentum p, energy is higher than the 'rest energy' $m_0 c^2$. The relation is:

$$E^2 = p^2 c^2 + m_0^2 c^4$$

- Photons travel at the speed of light. This means they must have zero rest mass so $E = pc$

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Energy-wavelength relationship, h and h-bar

- Since photons have

$$E = pc = hf \Rightarrow p = h/\lambda$$

- Wavelength is related to momentum!
- This is what is revealed in the Compton effect.

- The constant h in the above expression is 'Planck's constant' $h = 6.63 \times 10^{-34}$ Js
- We've seen it in the photoelectric effect but it's everywhere in quantum mechanics – it is the fundamental constant which sets the scale of quantum effects.
- The above expressions is written in terms of energy, frequency, and wavelength.
- Some people prefer to work with angular frequency $\omega = 2\pi f$, and wavevector $k = 2\pi/\lambda$
- They like to write the above $E = \hbar\omega$ and $p = \hbar k$, where $\hbar = h/(2\pi) = 1.054 \times 10^{-34}$ Js
- $\hbar$ in writing is called 'the reduced Planck constant' or 'Dirac's constant' but everyone calls it 'h bar'

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Compton Effect in Detail

- Another phenomenon which reveals the relationship between photon energy and light frequency, $E_{\text{photon}} \propto f = hf$

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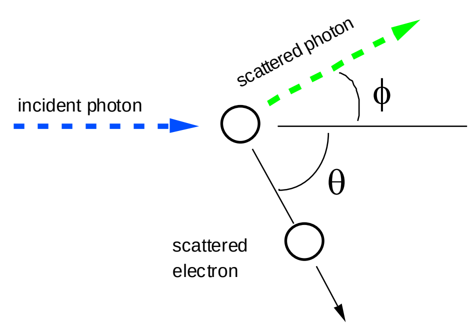
$\Rightarrow$Photon, scattered through angle ϕ must lose energy



- So we expect the scattered photon to have a lower frequency ($\Rightarrow$longer wavelength)

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Compton Effect in Detail

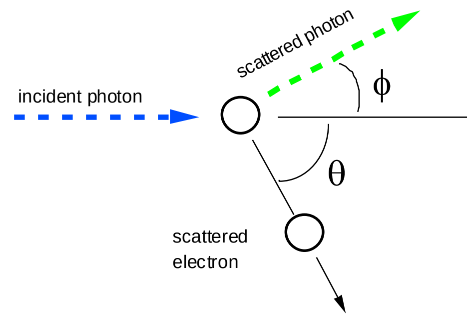
- Wavelength shift when a photon scatters from an electron. What is it?
- Use conservation of momentum and $p = hf$ for the photon (f_i, f_s for 'incident' and 'scattered' photon)

Horizontal momentum

$$\frac{hf_i}{c} + 0 = \frac{hf_s}{c}\cos(\phi) + p_e\cos(\theta)$$
$$\Rightarrow p_e c\cos(\theta) = h(f_i - f_s)\cos(\phi)$$

Vertical momentum

$$0 = \frac{hf_s}{c}\sin(\phi) - p_e\sin(\theta)$$
$$\Rightarrow p_e c\sin(\theta) = hf_s\sin(\phi)$$



$$\Rightarrow p_e^2c^2 = (hf_i)^2 - 2(hf_i)(hf_s)\cos(\phi) + (hf_s)^2$$

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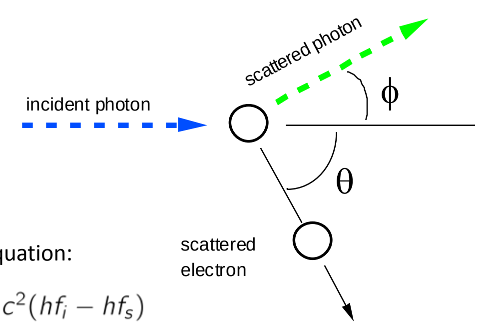
Compton Effect in Detail

- Momentum conservation gives $p_e^2c^2 = (hf_i)^2 - 2(hf_i)(hf_s)\cos(\phi) + (hf_s)^2$
- Now use energy conservation too. Note that we can write the relativistic energy of the electron in two ways

$$E = \sqrt{m_0^2c^4 + p_e^2c^2} = \mathrm{KE} + m_0c^2$$
$$\Rightarrow m_0^2c^4 + p_e^2c^2 = (\mathrm{KE} + m_0c^2)^2$$
$$\Rightarrow p_e^2c^2 = (\mathrm{KE})^2 + 2m_0c^2(\mathrm{KE})$$



- Kinetic energy gained by electron is energy lost by photon so
$$(\mathrm{KE}) = hf_i - hf_s$$
- Substitute this into energy equation and subtract from momentum equation:

$$\cancel{p_e^2c^2} = \cancel{(hf_i)^2} - 2(hf_i)(hf_s) + \cancel{(hf_s)^2} + 2m_0c^2(hf_i - hf_s)$$
$$\cancel{p_e^2c^2} = \cancel{(hf_i)^2} - 2(hf_i)(hf_s)\cos(\phi) + \cancel{(hf_s)^2}$$

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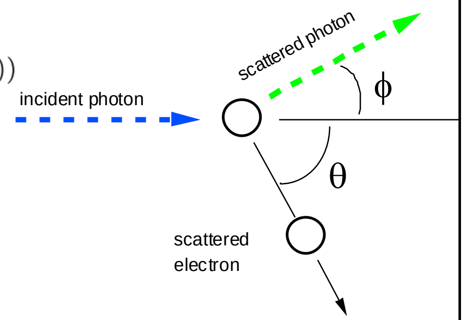
Compton Effect in Detail

- So we get

$$2m_0c^2(hf_i - hfs) = 2(hf_i)(hf_s)(1 - \cos(\phi))$$



- which is usually converted to express in terms of wavelength (exercise) :

$$\lambda_f - \lambda_i = \frac{h}{m_0 c}(1 - \cos\phi))$$

- Notice the interesting quantity with the units of wavelength, $\lambda_c = \frac{h}{m_0 c} = 2.4 \times 10^{-12}\text{m}$
- Called the 'Compton wavelength (of the electron)'; it is equal to the wavelength of a photon whose energy is the same as the (rest) mass of the electron.

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Light as a particle

- The photoelectric effect, and the Compton effect, and many other things, show that the transfer of energy from light to matter is not a continuous process – it occurs via the emission/absorption/scattering of discrete units of energy.
- This 'discrete unit of energy' is what is now called a photon – a particle of light.
- Monochromatic light is made of photons, all with energy $E = hf = \hbar\omega$
- And the momentum carried by each photon is $p = h/\lambda$, where λ is the wavelength of the light.
  - Follows from E=hf and relativity,   and the relationship is confirmed by the Compton effect.
- At the same time light is also a wave (e.g. Young's slits!).
- We will see in the next lectures how to reconcile these behaviours, also more about what 'a particle' is and isn't.

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