3

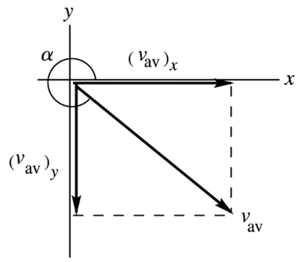
MOTION IN TWO OR THREE DIMENSIONS

3.1. IDENTIFY and SET UP: Use $\vec{v}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.

EXECUTE: (a) $v_{av\text{-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5.3\text{ m} - 1.1\text{ m}}{3.0\text{ s} - 0} = 1.4\text{ m/s}$

$v_{av\text{-}y} = \dfrac{\Delta y}{\Delta t} = \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{-0.5\text{ m} - 3.4\text{ m}}{3.0\text{ s} - 0} = -1.3\text{ m/s}$

(b)



$\tan\alpha = \dfrac{(v_{av})_y}{(v_{av})_x} = \dfrac{-1.3\text{ m/s}}{1.4\text{ m/s}} = -0.9286$

$\alpha = 360° - 42.9° = 317°$

$v_{av} = \sqrt{(v_{av})_x^2 + (v_{av})_y^2}$

$v_{av} = \sqrt{(1.4\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 1.9\text{ m/s}$

Figure 3.1

EVALUATE: Our calculation gives that $\vec{v}_{av}$ is in the 4th quadrant. This corresponds to increasing x and decreasing y.

3.2. IDENTIFY: Use $\vec{v}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form. The distance from the origin is the magnitude of $\vec{r}$.

SET UP: At time t_1, $x_1 = y_1 = 0$.

EXECUTE: (a) $x = (v_{av\text{-}x})\Delta t = (-3.8\text{ m/s})(12.0\text{ s}) = -45.6\text{ m}$ and $y = (v_{av\text{-}y})\Delta t = (4.9\text{ m/s})(12.0\text{ s}) = 58.8\text{ m}$.

(b) $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\text{ m})^2 + (58.8\text{ m})^2} = 74.4\text{ m}$.

EVALUATE: $\Delta\vec{r}$ is in the direction of $\vec{v}_{av}$. Therefore, Δx is negative since $v_{av\text{-}x}$ is negative and Δy is positive since $v_{av\text{-}y}$ is positive.

3.3. (a) IDENTIFY and SET UP: From $\vec{r}$ we can calculate x and y for any t.

Then use $\vec{v}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.

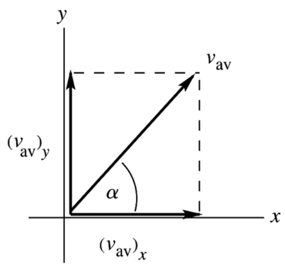
EXECUTE: $\vec{r} = [4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2]\hat{i} + (5.0\text{ cm/s})t\hat{j}$

At $t = 0$, $\vec{r} = (4.0\text{ cm})\,\hat{i}$.

At $t = 2.0\text{ s}$, $\vec{r} = (14.0\text{ cm})\,\hat{i} + (10.0\text{ cm})\,\hat{j}$.

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$

$$v_{\text{av-}y} = \frac{\Delta y}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$



$$v_{\text{av}} = \sqrt{(v_{\text{av}})_x^2 + (v_{\text{av}})_y^2} = 7.1\text{ cm/s}$$

$$\tan\alpha = \frac{(v_{\text{av}})_y}{(v_{\text{av}})_x} = 1.00$$

$$\theta = 45°.$$

Figure 3.3a

EVALUATE: Both x and y increase, so $\vec{v}_{\text{av}}$ is in the 1st quadrant.

(b) IDENTIFY and **SET UP:** Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

EXECUTE: $\vec{v} = \dfrac{d\vec{r}}{dt} = ([5.0\text{ cm/s}^2]t)\hat{i} + (5.0\text{ cm/s})\hat{j}$

$\underline{t=0}$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$\underline{t = 1.0\text{ s}}$: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$\underline{t = 2.0\text{ s}}$: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

(c) The trajectory is a graph of y versus x.

$x = 4.0\text{ cm} + (2.5\text{ cm/s}^2)\,t^2$, $y = (5.0\text{ cm/s})t$

For values of t between 0 and 2.0 s, calculate x and y and plot y versus x.

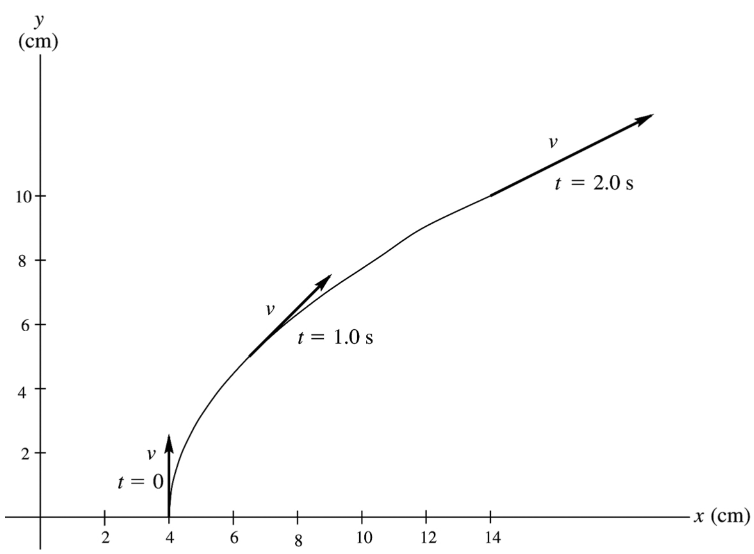


Figure 3.3b

EVALUATE: The sketch shows that the instantaneous velocity at any t is tangent to the trajectory.

3.4. IDENTIFY: Given the position vector of a squirrel, find its velocity components in general, and at a specific time find its velocity components and the magnitude and direction of its position vector and velocity.

SET UP: $v_x = dx/dt$ and $v_y = dy/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

EXECUTE: **(a)** Taking the derivatives gives $v_x(t) = 0.280\text{ m/s} + (0.0720\text{ m/s}^2)t$ and $v_y(t) = (0.0570\text{ m/s}^3)t^2$.

(b) Evaluating the position vector at $t = 5.00\text{ s}$ gives $x = 2.30\text{ m}$ and $y = 2.375\text{ m}$, which gives $r = 3.31\text{ m}$.

(c) At $t = 5.00\text{ s}$, $v_x = +0.64\text{ m/s}$, $v_y = 1.425\text{ m/s}$, which gives $v = 1.56\text{ m/s}$ and $\tan\theta = \dfrac{1.425}{0.64}$ so the direction is $\theta = 65.8^\text{o}$ (counterclockwise from +*x*-axis)

EVALUATE: The acceleration is not constant, so we cannot use the standard kinematics formulas.

3.5. **IDENTIFY** and **SET UP:** Use Eq. $\vec{a}_\text{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form to calculate $a_{\text{av-}x}$ and $a_{\text{av-}y}$.

EXECUTE: **(a)** The velocity vectors at $t_1 = 0$ and $t_2 = 30.0\text{ s}$ are shown in Figure 3.5a.

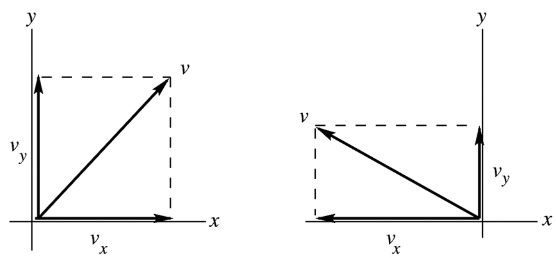
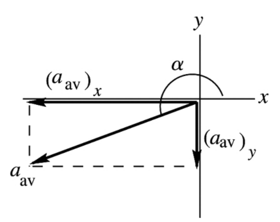


Figure 3.5a

(b) $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_{2x} - v_{1x}}{t_2 - t_1} = \dfrac{-170\text{ m/s} - 90\text{ m/s}}{30.0\text{ s}} = -8.67\text{ m/s}^2$

$a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t} = \dfrac{v_{2y} - v_{1y}}{t_2 - t_1} = \dfrac{40\text{ m/s} - 110\text{ m/s}}{30.0\text{ s}} = -2.33\text{ m/s}^2$

(c)



$a = \sqrt{(a_{\text{av-}x})^2 + (a_{\text{av-}y})^2} = 8.98\text{ m/s}^2$

$\tan\alpha = \dfrac{a_{\text{av-}y}}{a_{\text{av-}x}} = \dfrac{-2.33\text{ m/s}^2}{-8.67\text{ m/s}^2} = 0.269$

$\alpha = 15° + 180° = 195°$

Figure 3.5b

EVALUATE: The changes in v_x and v_y are both in the negative *x* or *y* direction, so both components of $\vec{a}_\text{av}$ are in the 3rd quadrant.

3.6. **IDENTIFY:** Use $\vec{a}_\text{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form.

SET UP: $a_x = (0.45\text{ m/s}^2)\cos 31.0° = 0.39\text{ m/s}^2$, $a_y = (0.45\text{ m/s}^2)\sin 31.0° = 0.23\text{ m/s}^2$

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$ and $v_x = 2.6\text{ m/s} + (0.39\text{ m/s}^2)(10.0\text{ s}) = 6.5\text{ m/s}$. $a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t}$ and $v_y = -1.8\text{ m/s} + (0.23\text{ m/s}^2)(10.0\text{ s}) = 0.52\text{ m/s}$.

(b) $v=\sqrt{(6.5\text{ m/s})^2+(0.52\text{ m/s})^2}=6.52\text{ m/s},$ at an angle of $\arctan\left(\dfrac{0.52}{6.5}\right)=4.6°$ counterclockwise from the +*x*-axis.

(c) The velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are sketched in Figure 3.6. The two velocity vectors differ in magnitude and direction.

EVALUATE: $\vec{v}_1$ is at an angle of $35°$ below the +*x*-axis and has magnitude $v_1=3.2$ m/s, so $v_2>v_1$ and the direction of $\vec{v}_2$ is rotated counterclockwise from the direction of $\vec{v}_1$.

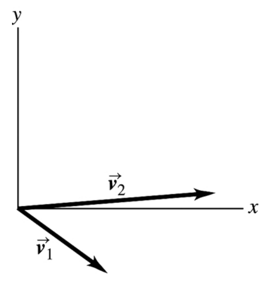


Figure 3.6

3.7. **IDENTIFY** and **SET UP:** Use $\vec{v}=\dfrac{d\vec{r}}{dt}$ and $\vec{a}=\dfrac{d\vec{v}}{dt}$ to find v_x, v_y, a_x, and a_y as functions of time. The magnitude and direction of $\vec{r}$ and $\vec{a}$ can be found once we know their components.

EXECUTE: **(a)** Calculate *x* and *y* for *t* values in the range 0 to 2.0 s and plot *y* versus *x*. The results are given in Figure 3.7a.

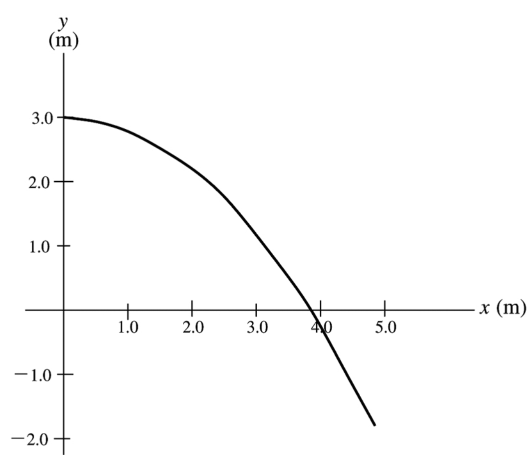


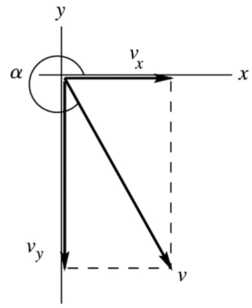
Figure 3.7a

(b) $v_x=\dfrac{dx}{dt}=\alpha \quad v_y=\dfrac{dy}{dt}=-2\beta t$

$a_x=\dfrac{dv_x}{dt}=0 \quad a_y=\dfrac{dv_y}{dt}=-2\beta$

Thus $\vec{v}=\alpha\hat{i}-2\beta t\hat{j}$, $\vec{a}=-2\beta\hat{j}$

(c) <u>velocity:</u> At $t=2.0$ s, $v_x=2.4$ m/s, $v_y=-2(1.2\text{ m/s}^2)(2.0\text{ s})=-4.8$ m/s



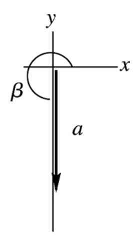
$$v = \sqrt{v_x^2 + v_y^2} = 5.4 \text{ m/s}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-4.8 \text{ m/s}}{2.4 \text{ m/s}} = -2.00$$

$$\alpha = -63.4° + 360° = 297°$$

Figure 3.7b

<u>acceleration:</u> At $t = 2.0$ s, $a_x = 0$, $a_y = -2\,(1.2 \text{ m/s}^2) = -2.4 \text{ m/s}^2$



$$a = \sqrt{a_x^2 + a_y^2} = 2.4 \text{ m/s}^2$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{-2.4 \text{ m/s}^2}{0} = -\infty$$

$$\beta = 270°$$

Figure 3.7c

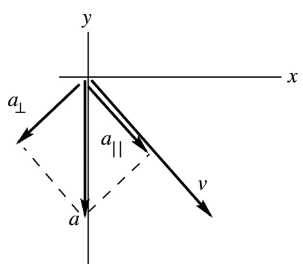


Figure 3.7d

EVALUATE: **(d)** $\vec{a}$ has a component $a_{||}$ in the same direction as $\vec{v}$, so we know that v is increasing (the bird is speeding up). $\vec{a}$ also has a component $a_\perp$ perpendicular to $\vec{v}$, so that the direction of $\vec{v}$ is changing; the bird is turning toward the $-y$-direction (toward the right)

$\vec{v}$ is always tangent to the path; $\vec{v}$ at $t = 2.0$ s shown in part (c) is tangent to the path at this t, conforming to this general rule. $\vec{a}$ is constant and in the $-y$-direction; the direction of $\vec{v}$ is turning toward the $-y$-direction.

3.8. **IDENTIFY:** Use the velocity components of a car (given as a function of time) to find the acceleration of the car as a function of time and to find the magnitude and direction of the car's velocity and acceleration at a specific time.

SET UP: $a_x = dv_x/dt$ and $a_y = dv_y/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

EXECUTE: **(a)** Taking the derivatives gives $a_x(t) = (-0.0360 \text{ m/s}^3)t$ and $a_y(t) = 0.550 \text{ m/s}^2$.

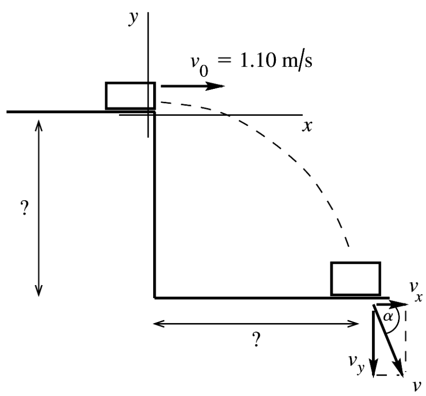
(b) Evaluating the velocity components at $t = 8.00$ s gives $v_x = 3.848$ m/s and $v_y = 6.40$ m/s, which gives $v = 7.47$ m/s. The direction is $\tan\theta = \dfrac{6.40}{3.848}$ so $\theta = 59.0°$ (counterclockwise from +x-axis).

(c) Evaluating the acceleration components at $t = 8.00\text{ s}$ gives $a_x = -0.288\text{ m/s}^2$ and $a_y = 0.550\text{ m/s}^2$, which gives $a = 0.621\text{ m/s}^2$. The angle with the $+y$ axis is given by $\tan\theta = \dfrac{0.288}{0.550}$, so $\theta = 27.6^\circ$. The direction is therefore 118° counterclockwise from $+x$-axis.

EVALUATE: The acceleration is not constant, so we cannot use the standard kinematics formulas.

3.9. IDENTIFY: The book moves in projectile motion once it leaves the tabletop. Its initial velocity is horizontal.

SET UP: Take the positive y-direction to be upward. Take the origin of coordinates at the initial position of the book, at the point where it leaves the table top.



<u>x-component</u>:
$a_x = 0$, $v_{0x} = 1.10\text{ m/s}$,
$t = 0.480\text{ s}$
<u>y-component</u>:
$a_y = -9.80\text{ m/s}^2$,
$v_{0y} = 0$,
$t = 0.480\text{ s}$

Figure 3.9a

Use constant acceleration equations for the x and y components of the motion, with $a_x = 0$ and $a_y = -g$.

EXECUTE: **(a)** $y - y_0 = ?$

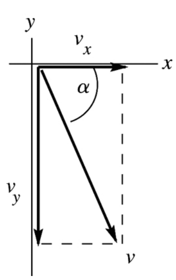
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80\text{ m/s}^2)(0.480\text{ s})^2 = -1.129\text{ m}$. The tabletop is therefore 1.13 m above the floor.

(b) $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10\text{ m/s})(0.480\text{ s}) + 0 = 0.528\text{ m}$.

(c) $v_x = v_{0x} + a_x t = 1.10\text{ m/s}$ (The x-component of the velocity is constant, since $a_x = 0$.)

$v_y = v_{0y} + a_y t = 0 + (-9.80\text{ m/s}^2)(0.480\text{ s}) = -4.704\text{ m/s}$



$v = \sqrt{v_x^2 + v_y^2} = 4.83\text{ m/s}$

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-4.704\text{ m/s}}{1.10\text{ m/s}} = -4.2764$

$\alpha = -76.8^\circ$

Direction of $\vec{v}$ is 76.8° below the horizontal

Figure 3.9b

(d) The graphs are given in Figure 3.9c.

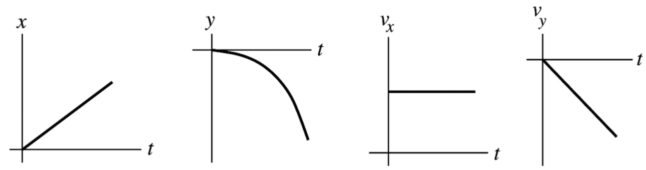


Figure 3.9c

EVALUATE: In the x-direction, $a_x = 0$ and v_x is constant. In the y-direction, $a_y = -9.80\ \text{m/s}^2$ and v_y is downward and increasing in magnitude since a_y and v_y are in the same directions. The x and y motions occur independently, connected only by the time. The time it takes the book to fall 1.13 m is the time it travels horizontally.

3.10. **IDENTIFY:** The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

SET UP: Take $+y$ downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Time to fall 9.00 m: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(9.00\ \text{m})}{9.80\ \text{m/s}^2}} = 1.36\ \text{s}$.

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives

$v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{1.75\ \text{m}}{1.36\ \text{s}} = 1.29\ \text{m/s}.$

EVALUATE: If she increases her initial speed she still takes 1.36 s to reach the level of the ledge, but has traveled horizontally farther than 1.75 m.

3.11. **IDENTIFY:** Each object moves in projectile motion.

SET UP: Take $+y$ to be downward. For each cricket, $a_x = 0$ and $a_y = +9.80\ \text{m/s}^2$. For Chirpy, $v_{0x} = v_{0y} = 0$. For Milada, $v_{0x} = 0.950\ \text{m/s}$, $v_{0y} = 0$.

EXECUTE: Milada's horizontal component of velocity has no effect on her vertical motion. She also reaches the ground in 2.70 s. $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2 = (0.950\ \text{m/s})(2.70\ \text{s}) = 2.57\ \text{m}$.

EVALUATE: The x and y components of motion are totally separate and are connected only by the fact that the time is the same for both.

3.12. **IDENTIFY:** The football moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. At the highest point in the trajectory, $v_y = 0$.

EXECUTE: **(a)** $v_y = v_{0y} + a_yt$. The time t is $\dfrac{v_{0y}}{g} = \dfrac{12.0\ \text{m/s}}{9.80\ \text{m/s}^2} = 1.224\ \text{s}$, which we round to 1.22 s.

(b) Different constant acceleration equations give different expressions but the same numerical result:

$\frac{1}{2}gt^2 = \frac{1}{2}v_{y0}t = \dfrac{v_{0y}^2}{2g} = 7.35\ \text{m}.$

(c) Regardless of how the algebra is done, the time will be twice that found in part (a), which is 2(1.224 s) = 2.45 s.

(d) $a_x = 0$, so $x - x_0 = v_{0x}t = (20.0\ \text{m/s})(2.45\ \text{s}) = 49.0\ \text{m}$.

(e) The graphs are sketched in Figure 3.12 (next page).

EVALUATE: When the football returns to its original level, $v_x = 20.0\text{ m/s}$ and $v_y = -12.0\text{ m/s}$.

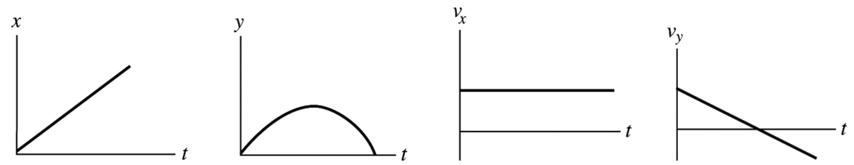


Figure 3.12

3.13. **IDENTIFY:** The car moves in projectile motion. The car travels $21.3\text{ m} - 1.80\text{ m} = 19.5\text{ m}$ downward during the time it travels 48.0 m horizontally.

SET UP: Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air:

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 \text{ gives } t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(19.5\text{ m})}{9.80\text{ m/s}^2}} = 1.995\text{ s}$$

Then $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{48.0\text{ m}}{1.995\text{ s}} = 24.1\text{ m/s}$.

(b) $v_x = 24.06\text{ m/s}$ since $a_x = 0$. $v_y = v_{0y} + a_y t = -19.55\text{ m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 31.0\text{ m/s}$.

EVALUATE: Note that the speed is considerably less than the algebraic sum of the x- and y-components of the velocity.

3.14. **IDENTIFY:** Knowing the maximum reached by the froghopper and its angle of takeoff, we want to find its takeoff speed and the horizontal distance it travels while in the air.

SET UP: Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. At the maximum height $v_y = 0$. The constant-acceleration formulas $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ apply.

EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y(y-y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.587\text{ m})} = 3.39\text{ m/s}$. $v_{0y} = v_0 \sin\theta_0$ so

$$v_0 = \frac{v_{0y}}{\sin\theta_0} = \frac{3.39\text{ m/s}}{\sin 58.0^\circ} = 4.00\text{ m/s}.$$

(b) Use the vertical motion to find the time in the air. When the froghopper has returned to the ground, $y - y_0 = 0$. $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(3.39\text{ m/s})}{-9.80\text{ m/s}^2} = 0.692\text{ s}$.

Then $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = (v_0 \cos\theta_0)t = (4.00\text{ m/s})(\cos 58.0^\circ)(0.692\text{ s}) = 1.47\text{ m}$.

EVALUATE: $v_y = 0$ when $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{3.39\text{ m/s}}{-9.80\text{ m/s}^2} = 0.346\text{ s}$. The total time in the air is twice this.

3.15. **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude v_0. The height h of the table and v_0 are the same; the acceleration due to gravity changes from $g_E = 9.80\text{ m/s}^2$ on earth to g_X on planet X.

SET UP: Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -g$, where g is either g_E or g_X.

EXECUTE: Use the vertical motion to find the time in the air: $y - y_0 = -h$. $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2h}{g}}$. Then $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $x - x_0 = v_{0x}t = v_0\sqrt{\dfrac{2h}{g}}$. $x - x_0 = D$ on earth and $2.76D$ on

Planet X. $(x-x_0)\sqrt{g}=v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_\mathrm{E}}=2.76D\sqrt{g_\mathrm{X}}$.

$$g_\mathrm{X}=\frac{g_\mathrm{E}}{(2.76)^2}=0.131g_\mathrm{E}=1.28\ \mathrm{m/s^2}.$$

EVALUATE: On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor and it travels farther horizontally.

3.16. **IDENTIFY:** The shell moves in projectile motion.

SET UP: Let $+x$ be horizontal, along the direction of the shell's motion, and let $+y$ be upward. $a_x=0$, $a_y=-9.80\ \mathrm{m/s^2}$.

EXECUTE: **(a)** $v_{0x}=v_0\cos\alpha_0=(40.0\ \mathrm{m/s})\cos 60.0°=20.0\ \mathrm{m/s}$,

$v_{0y}=v_0\sin\alpha_0=(40.0\ \mathrm{m/s})\sin 60.0°=34.6\ \mathrm{m/s}$.

(b) At the maximum height $v_y=0$. $v_y=v_{0y}+a_yt$ gives $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{0-34.6\ \mathrm{m/s}}{-9.80\ \mathrm{m/s^2}}=3.53\ \mathrm{s}$.

(c) $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(34.6\ \mathrm{m/s})^2}{2(-9.80\ \mathrm{m/s^2})}=61.2\ \mathrm{m}$.

(d) The total time in the air is twice the time to the maximum height, so

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(20.0\ \mathrm{m/s})(2)(3.53\ \mathrm{s})=141\ \mathrm{m}$.

(e) At the maximum height, $v_x=v_{0x}=20.0\ \mathrm{m/s}$ and $v_y=0$. At all points in the motion, $a_x=0$ and $a_y=-9.80\ \mathrm{m/s^2}$.

EVALUATE: The equation for the horizontal range R derived in the text is $R=\dfrac{v_0^2\sin 2\alpha_0}{g}$. This gives

$R=\dfrac{(40.0\ \mathrm{m/s})^2\sin(120.0°)}{9.80\ \mathrm{m/s^2}}=141\ \mathrm{m}$, which agrees with our result in part (d).

3.17. **IDENTIFY:** The baseball moves in projectile motion. In part (c) first calculate the components of the velocity at this point and then get the resultant velocity from its components.

SET UP: First find the x- and y-components of the initial velocity. Use coordinates where the $+y$-direction is upward, the $+x$-direction is to the right and the origin is at the point where the baseball leaves the bat.

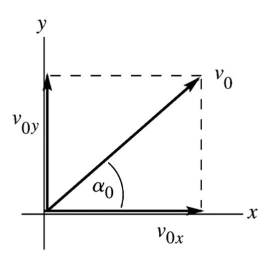


Figure 3.17a

$$v_{0x}=v_0\cos\alpha_0=(30.0\ \mathrm{m/s})\cos 36.9°=24.0\ \mathrm{m/s}$$
$$v_{0y}=v_0\sin\alpha_0=(30.0\ \mathrm{m/s})\sin 36.9°=18.0\ \mathrm{m/s}$$

Use constant acceleration equations for the x and y motions, with $a_x=0$ and $a_y=-g$.

EXECUTE: **(a)** <u>y-component</u> (vertical motion):

$y-y_0=+10.0\ \mathrm{m}$, $v_{0y}=18.0\ \mathrm{m/s}$, $a_y=-9.80\ \mathrm{m/s^2}$, $t=?$

$y-y_0=v_{0y}+\frac{1}{2}a_yt^2$

$10.0\ \mathrm{m}=(18.0\ \mathrm{m/s})t-(4.90\ \mathrm{m/s^2})t^2$

$(4.90\ \mathrm{m/s^2})t^2-(18.0\ \mathrm{m/s})t+10.0\ \mathrm{m}=0$

Apply the quadratic formula: $t = \frac{1}{9.80}\left[18.0 \pm \sqrt{(-18.0)^2 - 4\,(4.90)(10.0)}\right]\text{ s} = (1.837 \pm 1.154)\text{ s}$

The ball is at a height of 10.0 above the point where it left the bat at $t_1 = 0.683\text{ s}$ and at $t_2 = 2.99\text{ s}$. At the earlier time the ball passes through a height of 10.0 m as its way up and at the later time it passes through 10.0 m on its way down.

(b) $v_x = v_{0x} = +24.0\text{ m/s}$, at all times since $a_x = 0$.

$v_y = v_{0y} + a_y t$

$\underline{t_1 = 0.683\text{ s}}$: $v_y = +18.0\text{ m/s} + (-9.80\text{ m/s}^2)(0.683\text{ s}) = +11.3\text{ m/s}$. ($v_y$ is positive means that the ball is traveling upward at this point.)

$\underline{t_2 = 2.99\text{ s}}$: $v_y = +18.0\text{ m/s} + (-9.80\text{ m/s}^2)(2.99\text{ s}) = -11.3\text{ m/s}$. ($v_y$ is negative means that the ball is traveling downward at this point.)

(c) $v_x = v_{0x} = 24.0\text{ m/s}$

Solve for v_y:

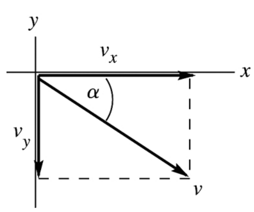
$v_y = ?$, $y - y_0 = 0$ (when ball returns to height where motion started),

$a_y = -9.80\text{ m/s}^2$, $v_{0y} = +18.0\text{ m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -v_{0y} = -18.0\text{ m/s}$ (negative, since the baseball must be traveling downward at this point)

Now solve for the magnitude and direction of $\vec{v}$.



$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{(24.0\text{ m/s})^2 + (-18.0\text{ m/s})^2} = 30.0\text{ m/s}$

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-18.0\text{ m/s}}{24.0\text{ m/s}}$

$\alpha = -36.9°$, $36.9°$ below the horizontal

Figure 3.17b

The velocity of the ball when it returns to the level where it left the bat has magnitude 30.0 m/s and is directed at an angle of $36.9°$ below the horizontal.

EVALUATE: The discussion in parts (a) and (b) explains the significance of two values of t for which $y - y_0 = +10.0\text{ m}$. When the ball returns to its initial height, our results give that its speed is the same as its initial speed and the angle of its velocity below the horizontal is equal to the angle of its initial velocity above the horizontal; both of these are general results.

3.18. IDENTIFY: The shot moves in projectile motion.

SET UP: Let $+y$ be upward.

EXECUTE: (a) If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80\text{ m/s}^2$ vertically downward.

(b) The x-component of velocity is constant at $v_x = (12.0\text{ m/s})\cos 51.0° = 7.55\text{ m/s}$. The y-component is $v_{0y} = (12.0\text{ m/s})\sin 51.0° = 9.32\text{ m/s}$ at release and

$v_y = v_{0y} - gt = (9.32\text{ m/s}) - (9.80\text{ m/s})(2.08\text{ s}) = -11.06\text{ m/s}$ when the shot hits.

(c) $x - x_0 = v_{0x}t = (7.55\text{ m/s})(2.08\text{ s}) = 15.7\text{ m}$.

(d) The initial and final heights are not the same.

(e) With $y = 0$ and v_{0y} as found above, the equation for $y - y_0$ as a function of time gives $y_0 = 1.81\text{m}$.

(f) The graphs are sketched in Figure 3.18.

EVALUATE: When the shot returns to its initial height, $v_y = -9.32$ m/s. The shot continues to accelerate downward as it travels downward 1.81 m to the ground and the magnitude of v_y at the ground is larger than 9.32 m/s.

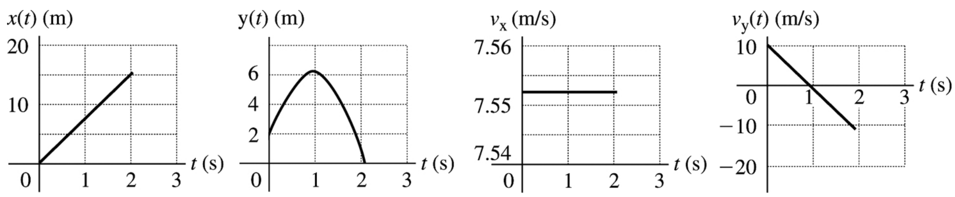
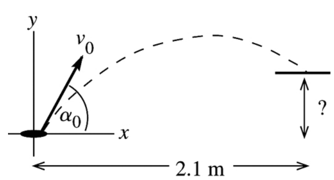


Figure 3.18

3.19. **IDENTIFY:** Take the origin of coordinates at the point where the quarter leaves your hand and take positive y to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.



$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s}) \cos 60°$

$v_{0x} = 3.20$ m/s

$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s}) \sin 60°$

$v_{0y} = 5.54$ m/s

Figure 3.19

(a) SET UP: Use the horizontal (x-component) of motion to solve for t, the time the quarter travels through the air:

$t = ?,\ x - x_0 = 2.1 \text{ m},\ v_{0x} = 3.2 \text{ m/s},\ a_x = 0$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t,$ since $a_x = 0$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1 \text{ m}}{3.2 \text{ m/s}} = 0.656 \text{ s}$

SET UP: Now find the vertical displacement of the quarter after this time:

$y - y_0 = ?,\ a_y = -9.80 \text{ m/s}^2,\ v_{0y} = +5.54 \text{ m/s},\ t = 0.656 \text{ s}$

$y - y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (5.54 \text{ m/s})(0.656 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.656 \text{ s})^2 = 3.63 \text{ m} - 2.11 \text{ m} = 1.5 \text{ m}.$

(b) SET UP: $v_y = ?,\ t = 0.656 \text{ s},\ a_y = -9.80 \text{ m/s}^2,\ v_{0y} = +5.54 \text{ m/s}\quad v_y = v_{0y} + a_y t$

EXECUTE: $v_y = 5.54 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.656 \text{ s}) = -0.89 \text{ m/s}.$

EVALUATE: The minus sign for v_y indicates that the y-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1$ m it is on its way down.

3.20. **IDENTIFY:** Consider the horizontal and vertical components of the projectile motion. The water travels 45.0 m horizontally in 3.00 s.

SET UP: Let $+y$ be upward. $a_x = 0,\ a_y = -9.80 \text{ m/s}^2.\ v_{0x} = v_0 \cos\theta_0,\ v_{0y} = v_0 \sin\theta_0.$

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_0(\cos\theta_0)t$ and $\cos\theta_0 = \dfrac{45.0 \text{ m}}{(25.0 \text{ m/s})(3.00 \text{ s})} = 0.600;$

$\theta_0 = 53.1°$

(b) At the highest point $v_x = v_{0x} = (25.0\text{ m/s})\cos 53.1° = 15.0\text{ m/s}$, $v_y = 0$ and $v = \sqrt{v_x^2 + v_y^2} = 15.0\text{ m/s}$. At all points in the motion, $a = 9.80\text{ m/s}^2$ downward.

(c) Find $y - y_0$ when $t = 3.00\text{s}$:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (25.0\text{ m/s})(\sin 53.1°)(3.00\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.00\text{ s})^2 = 15.9\text{ m}$

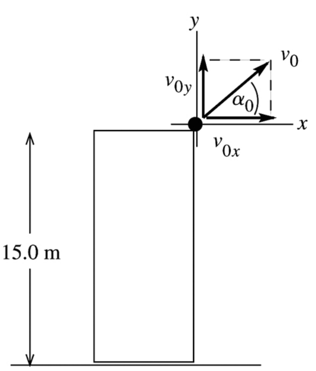
$v_x = v_{0x} = 15.0\text{ m/s}$, $v_y = v_{0y} + a_y t = (25.0\text{ m/s})(\sin 53.1°) - (9.80\text{m/s}^2)(3.00\text{ s}) = -9.41\text{ m/s}$, and

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.0\text{ m/s})^2 + (-9.41\text{ m/s})^2} = 17.7\text{ m/s}$

EVALUATE: The acceleration is the same at all points of the motion. It takes the water $t = -\frac{v_{0y}}{a_y} = -\frac{20.0\text{ m/s}}{-9.80\text{ m/s}^2} = 2.04\text{ s}$ to reach its maximum height. When the water reaches the building it has passed its maximum height and its vertical component of velocity is downward.

3.21. **IDENTIFY:** Take the origin of coordinates at the roof and let the $+y$-direction be upward. The rock moves in projectile motion, with $a_x = 0$ and $a_y = -g$. Apply constant acceleration equations for the x and y components of the motion.

SET UP:



$v_{0x} = v_0 \cos\alpha_0 = 25.2\text{ m/s}$

$v_{0y} = v_0 \sin\alpha_0 = 16.3\text{ m/s}$

Figure 3.21a

(a) At the maximum height $v_y = 0$.

$a_y = -9.80\text{ m/s}^2$, $v_y = 0$, $v_{0y} = +16.3\text{ m/s}$, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y\,(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (16.3\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = +13.6\text{ m}$

(b) SET UP: Find the velocity by solving for its x and y components.

$v_x = v_{0x} = 25.2\text{ m/s}$ (since $a_x = 0$)

$v_y = ?$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = -15.0\text{ m}$ (negative because at the ground the rock is below its initial position), $v_{0y} = 16.3\text{ m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ (v_y is negative because at the ground the rock is traveling downward.)

EXECUTE: $v_y = -\sqrt{(16.3\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-15.0\text{ m})} = -23.7\text{ m/s}$

Then $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2\text{ m/s})^2 + (-23.7\text{ m/s})^2} = 34.6\text{ m/s}$.

(c) SET UP: Use the vertical motion (y-component) to find the time the rock is in the air:
$t=?$, $v_y=-23.7\text{ m/s}$ (from part (b)), $a_y=-9.80\text{ m/s}^2$, $v_{0y}=+16.3\text{ m/s}$

EXECUTE: $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{-23.7\text{ m/s}-16.3\text{ m/s}}{-9.80\text{ m/s}^2}=+4.08\text{ s}$

SET UP: Can use this t to calculate the horizontal range:
$t=4.08\text{ s}$, $v_{0x}=25.2\text{ m/s}$, $a_x=0$, $x-x_0=?$

EXECUTE: $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(25.2\text{ m/s})(4.08\text{ s})+0=103\text{ m}$

(d) Graphs of x versus t, y versus t, v_x versus t and v_y versus t:

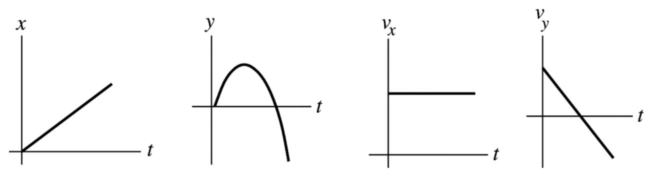


Figure 3.21b

EVALUATE: The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y}=+16.3\text{ m/s}$ the time it takes the rock to return to the level of the roof $(y=0)$ is $t=2v_{0y}/g=3.33\text{ s}$. The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

3.22. **IDENTIFY** and **SET UP:** The stone moves in projectile motion. Its initial velocity is the same as that of the balloon. Use constant acceleration equations for the x and y components of its motion. Take $+y$ to be downward.

EXECUTE: **(a)** Use the vertical motion of the rock to find the initial height.
$t=5.00\text{ s}$, $v_{0y}=+20.0\text{ m/s}$, $a_y=+9.80\text{ m/s}^2$, $y-y_0=?$

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $y-y_0=223\text{ m}$.

(b) In 5.00 s the balloon travels downward a distance $y-y_0=(20.0\text{ m/s})(5.00\text{ s})=100\text{ m}$. So, its height above ground when the rock hits is $223\text{ m}-100\text{ m}=123\text{ m}$.

(c) The horizontal distance the rock travels in 5.00 s is (15.0 m/s)(5.00 s) = 75.0 m. The vertical component of the distance between the rock and the basket is 123 m, so the rock is $\sqrt{(75\text{ m})^2+(123\text{ m})^2}=144\text{ m}$ from the basket when it hits the ground.

(d) (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the basket. Just before the rock hits the ground, its vertical component of velocity is $v_y=v_{0y}+a_yt=20.0\text{ m/s}+(9.80\text{ m/s}^2)(5.00\text{ s})=69.0\text{ m/s}$, downward, relative to the ground. The basket is moving downward at 20.0 m/s, so relative to the basket the rock has a downward component of velocity 49.0 m/s.
(ii) horizontal: 15.0 m/s; vertical: 69.0 m/s

EVALUATE: The rock has a constant horizontal velocity and accelerates downward.

3.23. **IDENTIFY:** Circular motion.

SET UP: Apply the equation $a_{\text{rad}}=4\pi^2R/T^2$, where $T=24\text{ h}$.

EXECUTE: **(a)** $a_{\text{rad}}=\dfrac{4\pi^2(6.38\times10^6\text{ m})}{[(24\text{ h})(3600\text{ s/h})]^2}=0.034\text{ m/s}^2=3.4\times10^{-3}g$.

(b) Solving the equation $a_{\text{rad}}=4\pi^2R/T^2$ for the period T with $a_{\text{rad}}=g$,

$$T=\sqrt{\frac{4\pi^2(6.38\times10^6\text{m})}{9.80\text{ m/s}^2}}=5070\text{ s}=1.4\text{ h}.$$

EVALUATE: a_{rad} is proportional to $1/T^2$, so to increase a_{rad} by a factor of $\dfrac{1}{3.4\times10^{-3}} = 294$ requires that T be multiplied by a factor of $\dfrac{1}{\sqrt{294}}$. $\dfrac{24\text{ h}}{\sqrt{294}} = 1.4\text{ h}$.

3.24. **IDENTIFY:** We want to find the acceleration of the inner ear of a dancer, knowing the rate at which she spins.

SET UP: $R = 0.070\text{ m}$. For 3.0 rev/s, the period T (time for one revolution) is $T = \dfrac{1.0\text{ s}}{3.0\text{ rev}} = 0.333\text{ s}$. The speed is $v = d/T = (2\pi R)/T$, and $a_{\text{rad}} = v^2/R$.

EXECUTE: $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(2\pi R/T)^2}{R} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(0.070\text{ m})}{(0.333\text{ s})^2} = 25\text{ m/s}^2 = 2.5g$.

EVALUATE: The acceleration is large and the force on the fluid must be 2.5 times its weight.

3.25. **IDENTIFY:** For the curved lowest part of the dive, the pilot's motion is approximately circular. We know the pilot's acceleration and the radius of curvature, and from this we want to find the pilot's speed.

SET UP: $a_{\text{rad}} = 5.5g = 53.9\text{ m/s}^2$. 1 mph = 0.4470 m/s. $a_{\text{rad}} = \dfrac{v^2}{R}$.

EXECUTE: $a_{\text{rad}} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(280\text{ m})(53.9\text{ m/s}^2)} = 122.8\text{ m/s} = 274.8\text{ mph}$. Rounding these answers to 2 significant figures (because of $5.5g$), gives $v = 120$ m/s = 270 mph.

EVALUATE: This speed is reasonable for the type of plane flown by a test pilot.

3.26. **IDENTIFY:** Each blade tip moves in a circle of radius $R = 3.40\text{ m}$ and therefore has radial acceleration $a_{\text{rad}} = v^2/R$.

SET UP: 550 rev/min = 9.17 rev/s, corresponding to a period of $T = \dfrac{1}{9.17\text{ rev/s}} = 0.109\text{ s}$.

EXECUTE: **(a)** $v = \dfrac{2\pi R}{T} = 196\text{ m/s}$.

(b) $a_{\text{rad}} = \dfrac{v^2}{R} = 1.13\times10^4\text{ m/s}^2 = 1.15\times10^3 g$.

EVALUATE: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives the same results for a_{rad} as in part (b).

3.27. **IDENTIFY:** Uniform circular motion.

SET UP: Since the magnitude of $\vec{v}$ is constant, $v_{\tan} = \dfrac{d|\vec{v}|}{dt} = 0$ and the resultant acceleration is equal to the radial component. At each point in the motion the radial component of the acceleration is directed in toward the center of the circular path and its magnitude is given by v^2/R.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(6.00\text{ m/s})^2}{14.0\text{ m}} = 2.57\text{ m/s}^2$, upward.

(b) The radial acceleration has the same magnitude as in part (a), but now the direction toward the center of the circle is downward. The acceleration at this point in the motion is 2.57 m/s^2, downward.

(c) SET UP: The time to make one rotation is the period T, and the speed v is the distance for one revolution divided by T.

EXECUTE: $v = \dfrac{2\pi R}{T}$ so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi\,(14.0\text{ m})}{6.00\text{ m/s}} = 14.7\text{ s}$.

EVALUATE: The radial acceleration is constant in magnitude since v is constant and is at every point in the motion directed toward the center of the circular path. The acceleration is perpendicular to $\vec{v}$ and is nonzero because the direction of $\vec{v}$ changes.

3.28. **IDENTIFY:** Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2/R$.

SET UP: The radius of the earth's orbit is $r = 1.50\times10^{11}$ m and its orbital period is $T = 365 \text{ days} = 3.16\times10^{7}$ s. For Mercury, $r = 5.79\times10^{10}$ m and $T = 88.0 \text{ days} = 7.60\times10^{6}$ s.

EXECUTE: **(a)** $v = \dfrac{2\pi r}{T} = 2.98\times10^{4}$ m/s

(b) $a_{\text{rad}} = \dfrac{v^2}{r} = 5.91\times10^{-3}\ \text{m/s}^2$.

(c) $v = 4.79\times10^{4}$ m/s, and $a_{\text{rad}} = 3.96\times10^{-2}\ \text{m/s}^2$.

EVALUATE: Mercury has a larger orbital velocity and a larger radial acceleration than earth.

3.29. **IDENTIFY:** Each part of his body moves in uniform circular motion, with $a_{\text{rad}} = \dfrac{v^2}{R}$. The speed in rev/s is $1/T$, where T is the period in seconds (time for 1 revolution). The speed v increases with R along the length of his body but all of him rotates with the same period T.

SET UP: For his head $R = 8.84$ m and for his feet $R = 6.84$ m.

EXECUTE: **(a)** $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(8.84\text{ m})(12.5)(9.80\text{ m/s}^2)} = 32.9$ m/s

(b) Use $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. Since his head has $a_{\text{rad}} = 12.5g$ and $R = 8.84$ m,

$T = 2\pi\sqrt{\dfrac{R}{a_{\text{rad}}}} = 2\pi\sqrt{\dfrac{8.84\text{m}}{12.5(9.80\text{m/s}^2)}} = 1.688\text{s}$. Then his feet have $a_{\text{rad}} = \dfrac{R}{T^2} = \dfrac{4\pi^2(6.84\text{m})}{(1.688\text{s})^2} = 94.8\text{m/s}^2 = 9.67$ g.

The difference between the acceleration of his head and his feet is $12.5g - 9.67g = 2.83g = 27.7\ \text{m/s}^2$.

(c) $\dfrac{1}{T} = \dfrac{1}{1.69\text{ s}} = 0.592\text{ rev/s} = 35.5\text{ rpm}$

EVALUATE: His feet have speed $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(6.84\text{ m})(94.8\text{ m/s}^2)} = 25.5$ m/s.

3.30. **IDENTIFY:** The relative velocities are $\vec{v}_{S/F}$, the velocity of the scooter relative to the flatcar, $\vec{v}_{S/G}$, the scooter relative to the ground and $\vec{v}_{F/G}$, the flatcar relative to the ground. $\vec{v}_{S/G} = \vec{v}_{S/F} + \vec{v}_{F/G}$. Carry out the vector addition by drawing a vector addition diagram.

SET UP: $\vec{v}_{S/F} = \vec{v}_{S/G} - \vec{v}_{F/G}$. $\vec{v}_{F/G}$ is to the right, so $-\vec{v}_{F/G}$ is to the left.

EXECUTE: In each case the vector addition diagram gives

(a) 5.0 m/s to the right

(b) 16.0 m/s to the left

(c) 13.0 m/s to the left.

EVALUATE: The scooter has the largest speed relative to the ground when it is moving to the right relative to the flatcar, since in that case the two velocities $\vec{v}_{S/F}$ and $\vec{v}_{F/G}$ are in the same direction and their magnitudes add.

3.31. **IDENTIFY:** Relative velocity problem. The time to walk the length of the moving sidewalk is the length divided by the velocity of the woman relative to the ground.

SET UP: Let W stand for the woman, G for the ground and S for the sidewalk. Take the positive direction to be the direction in which the sidewalk is moving.

The velocities are $v_{W/G}$ (woman relative to the ground), $v_{W/S}$ (woman relative to the sidewalk), and $v_{S/G}$ (sidewalk relative to the ground).

The equation for relative velocity becomes $v_{W/G} = v_{W/S} + v_{S/G}$.

The time to reach the other end is given by $t = \dfrac{\text{distance traveled relative to ground}}{v_{W/G}}$

EXECUTE: **(a)** $v_{S/G} = 1.0$ m/s

$v_{W/S} = +1.5$ m/s

$v_{W/G} = v_{W/S} + v_{S/G} = 1.5\text{ m/s} + 1.0\text{ m/s} = 2.5$ m/s.

$t = \dfrac{35.0\text{ m}}{v_{W/G}} = \dfrac{35.0\text{ m}}{2.5\text{ m/s}} = 14\text{ s.}$

(b) $v_{S/G} = 1.0\text{ m/s}$

$v_{W/S} = -1.5\text{ m/s}$

$v_{W/G} = v_{W/S} + v_{S/G} = -1.5\text{ m/s} + 1.0\text{ m/s} = -0.5\text{ m/s.}$ (Since $v_{W/G}$ now is negative, she must get on the moving sidewalk at the opposite end from in part (a).)

$t = \dfrac{-35.0\text{ m}}{v_{W/G}} = \dfrac{-35.0\text{ m}}{-0.5\text{ m/s}} = 70\text{ s.}$

EVALUATE: Her speed relative to the ground is much greater in part (a) when she walks with the motion of the sidewalk.

3.32. **IDENTIFY:** Calculate the rower's speed relative to the shore for each segment of the round trip.

SET UP: The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.

EXECUTE: The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).

The total time the rower takes is $\dfrac{1.5\text{ km}}{6.8\text{ km/h}} + \dfrac{1.5\text{ km}}{1.2\text{ km/h}} = 1.47\text{ h} = 88.2\text{ min.}$

EVALUATE: It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

3.33. **IDENTIFY:** Apply the relative velocity relation.

SET UP: The relative velocities are $\vec{v}_{C/E}$, the canoe relative to the earth, $\vec{v}_{R/E}$, the velocity of the river relative to the earth and $\vec{v}_{C/R}$, the velocity of the canoe relative to the river.

EXECUTE: $\vec{v}_{C/E} = \vec{v}_{C/R} + \vec{v}_{R/E}$ and therefore $\vec{v}_{C/R} = \vec{v}_{C/E} - \vec{v}_{R/E}$. The velocity components of $\vec{v}_{C/R}$ are $-0.50\text{ m/s} + (0.40\text{ m/s})/\sqrt{2}$, east and $(0.40\text{ m/s})/\sqrt{2}$, south, for a velocity relative to the river of 0.36 m/s, at $52.5°$ south of west.

EVALUATE: The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.

3.34. **IDENTIFY:** Relative velocity problem in two dimensions.

(a) SET UP: $\vec{v}_{P/A}$ is the velocity of the plane relative to the air. The problem states that $\vec{v}_{P/A}$ has magnitude 35 m/s and direction south.

$\vec{v}_{A/E}$ is the velocity of the air relative to the earth. The problem states that $\vec{v}_{A/E}$ is to the southwest ($45°$ S of W) and has magnitude 10 m/s.

The relative velocity equation is $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

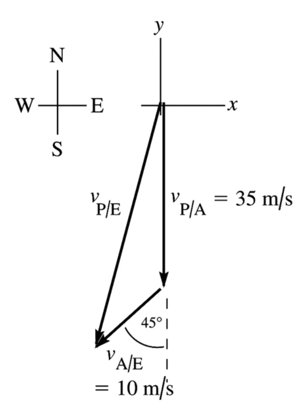


Figure 3.34a

EXECUTE: **(b)** $(v_{P/A})_x = 0,\ (v_{P/A})_y = -35\text{ m/s}$

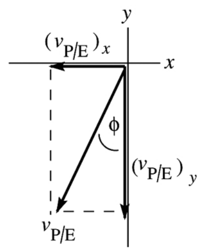
$(v_{\text{A/E}})_x = -(10\text{ m/s})\cos 45° = -7.07\text{ m/s},$

$(v_{\text{A/E}})_y = -(10\text{ m/s})\sin 45° = -7.07\text{ m/s}$

$(v_{\text{P/E}})_x = (v_{\text{P/A}})_x + (v_{\text{A/E}})_x = 0 - 7.07\text{ m/s} = -7.1\text{ m/s}$

$(v_{\text{P/E}})_y = (v_{\text{P/A}})_y + (v_{\text{A/E}})_y = -35\text{ m/s} - 7.07\text{ m/s} = -42\text{ m/s}$

(c)



$v_{\text{P/E}} = \sqrt{(v_{\text{P/E}})_x^2 + (v_{\text{P/E}})_y^2}$

$v_{\text{P/E}} = \sqrt{(-7.1\text{ m/s})^2 + (-42\text{ m/s})^2} = 43\text{ m/s}$

$\tan\phi = \dfrac{(v_{\text{P/E}})_x}{(v_{\text{P/E}})_y} = \dfrac{-7.1}{-42} = 0.169$

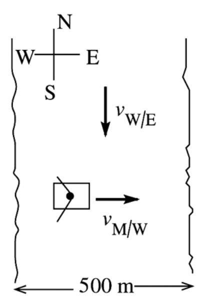
$\phi = 9.6°$; ($9.6°$ west of south)

Figure 3.34b

EVALUATE: The relative velocity addition diagram does not form a right triangle so the vector addition must be done using components. The wind adds both southward and westward components to the velocity of the plane relative to the ground.

3.35. **IDENTIFY:** Relative velocity problem in two dimensions. His motion relative to the earth (time displacement) depends on his velocity relative to the earth so we must solve for this velocity.

(a) SET UP: View the motion from above.



The velocity vectors in the problem are:

$\vec{v}_{\text{M/E}}$, the velocity of the man relative to the earth

$\vec{v}_{\text{W/E}}$, the velocity of the water relative to the earth

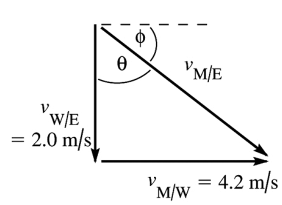
$\vec{v}_{\text{M/W}}$, the velocity of the man relative to the water

The rule for adding these velocities is

$\vec{v}_{\text{M/E}} = \vec{v}_{\text{M/W}} + \vec{v}_{\text{W/E}}$

Figure 3.35a

The problem tells us that $\vec{v}_{\text{W/E}}$ has magnitude 2.0 m/s and direction due south. It also tells us that $\vec{v}_{\text{M/W}}$ has magnitude 4.2 m/s and direction due east. The vector addition diagram is then as shown in Figure 3.35b.



This diagram shows the vector addition

$\vec{v}_{\text{M/E}} = \vec{v}_{\text{M/W}} + \vec{v}_{\text{W/E}}$

and also has $\vec{v}_{\text{M/W}}$ and $\vec{v}_{\text{W/E}}$ in their specified directions. Note that the vector diagram forms a right triangle.

Figure 3.35b

The Pythagorean theorem applied to the vector addition diagram gives $v_{\text{M/E}}^2 = v_{\text{M/W}}^2 + v_{\text{W/E}}^2$.

EXECUTE: $v_{M/E} = \sqrt{v_{M/W}^2 + v_{W/E}^2} = \sqrt{(4.2\text{ m/s})^2 + (2.0\text{ m/s})^2} = 4.7\text{ m/s}$; $\tan\theta = \dfrac{v_{M/W}}{v_{W/E}} = \dfrac{4.2\text{ m/s}}{2.0\text{ m/s}} = 2.10$;

$\theta = 65°$; or $\phi = 90° - \theta = 25°$. The velocity of the man relative to the earth has magnitude 4.7 m/s and direction 25° S of E.

(b) This requires careful thought. To cross the river the man must travel 500 m due east relative to the earth. The man's velocity relative to the earth is $\vec{v}_{M/E}$. But, from the vector addition diagram the eastward component of $v_{M/E}$ equals $v_{M/W} = 4.2$ m/s.

Thus $t = \dfrac{x - x_0}{v_x} = \dfrac{500\text{ m}}{4.2\text{ m/s}} = 119$ s, which we round to 120 s.

(c) The southward component of $\vec{v}_{M/E}$ equals $v_{W/E} = 2.0$ m/s. Therefore, in the 120 s it takes him to cross the river, the distance south the man travels relative to the earth is

$$y - y_0 = v_y t = (2.0\text{ m/s})(119\text{ s}) = 240\text{ m}.$$

EVALUATE: If there were no current he would cross in the same time, $(500\text{ m})/(4.2\text{ m/s}) = 120$ s. The current carries him downstream but doesn't affect his motion in the perpendicular direction, from bank to bank.

3.36. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the water relative to the earth, $\vec{v}_{W/E}$, the boat relative to the water, $\vec{v}_{B/W}$, and the boat relative to the earth, $\vec{v}_{B/E}$. $\vec{v}_{B/E}$ is due east, $\vec{v}_{W/E}$ is due south and has magnitude 2.0 m/s. $v_{B/W} = 4.2$ m/s. $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$. The velocity addition diagram is given in Figure 3.36.

EXECUTE: **(a)** Find the direction of $\vec{v}_{B/W}$. $\sin\theta = \dfrac{v_{W/E}}{v_{B/W}} = \dfrac{2.0\text{ m/s}}{4.2\text{ m/s}}$. $\theta = 28.4°$, north of east.

(b) $v_{B/E} = \sqrt{v_{B/W}^2 - v_{W/E}^2} = \sqrt{(4.2\text{ m/s})^2 - (2.0\text{ m/s})^2} = 3.7\text{ m/s}$

(c) $t = \dfrac{800\text{ m}}{v_{B/E}} = \dfrac{800\text{ m}}{3.7\text{ m/s}} = 216$ s.

EVALUATE: It takes longer to cross the river in this problem than it did in Problem 3.35. In the direction straight across the river (east) the component of his velocity relative to the earth is lass than 4.2 m/s.

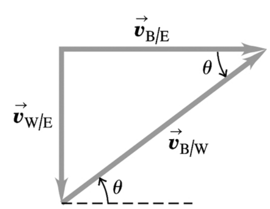


Figure 3.36

3.37. **IDENTIFY:** The resultant velocity, relative to the ground, is directly southward. This velocity is the sum of the velocity of the bird relative to the air and the velocity of the air relative to the ground.

SET UP: $v_{B/A} = 100$ km/h. $\vec{v}_{A/G} = 40$ km/h, east. $\vec{v}_{B/G} = \vec{v}_{B/A} + \vec{v}_{A/G}$.

EXECUTE: We want $\vec{v}_{B/G}$ to be due south. The relative velocity addition diagram is shown in Figure 3.37.

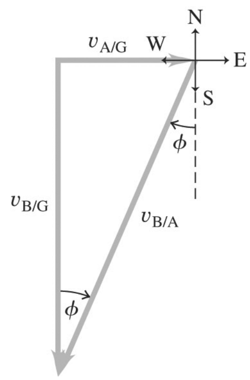


Figure 3.37

(a) $\sin\phi = \dfrac{v_{A/G}}{v_{B/A}} = \dfrac{40 \text{ km/h}}{100 \text{ km/h}}$, $\phi = 24°$, west of south.

(b) $v_{B/G} = \sqrt{v_{B/A}{}^2 - v_{A/G}{}^2} = 91.7$ km/h. $t = \dfrac{d}{v_{B/G}} = \dfrac{500 \text{ km}}{91.7 \text{ km/h}} = 5.5$ h.

EVALUATE: The speed of the bird relative to the ground is less than its speed relative to the air. Part of its velocity relative to the air is directed to oppose the effect of the wind.

3.38. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the velocity of the plane relative to the ground, $\vec{v}_{P/G}$, the velocity of the plane relative to the air, $\vec{v}_{P/A}$, and the velocity of the air relative to the ground, $\vec{v}_{A/G}$. $\vec{v}_{P/G}$ must be due west and $\vec{v}_{A/G}$ must be south. $v_{A/G} = 80$ km/h and $v_{P/A} = 320$ km/h. $\vec{v}_{P/G} = \vec{v}_{P/A} + \vec{v}_{A/G}$. The relative velocity addition diagram is given in Figure 3.38.

EXECUTE: **(a)** $\sin\theta = \dfrac{v_{A/G}}{v_{P/A}} = \dfrac{80 \text{ km/h}}{320 \text{ km/h}}$ and $\theta = 14°$, north of west.

(b) $v_{P/G} = \sqrt{v_{P/A}^2 - v_{A/G}^2} = \sqrt{(320 \text{ km/h})^2 - (80.0 \text{ km/h})^2} = 310$ km/h.

EVALUATE: To travel due west the velocity of the plane relative to the air must have a westward component and also a component that is northward, opposite to the wind direction.

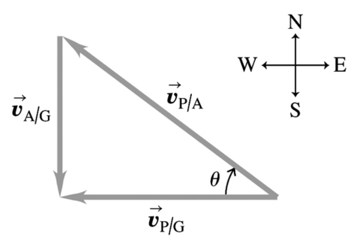


Figure 3.38

3.39. **IDENTIFY:** $\vec{v} = \dfrac{d\vec{r}}{dt}$ and $\vec{a} = \dfrac{d\vec{v}}{dt}$

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$. At $t = 1.00$ s, $a_x = 4.00 \text{ m/s}^2$ and $a_y = 3.00 \text{ m/s}^2$. At $t = 0$, $x = 0$ and $y = 50.0$ m.

EXECUTE: **(a)** $v_x = \frac{dx}{dt} = 2Bt$. $a_x = \frac{dv_x}{dt} = 2B$, which is independent of t. $a_x = 4.00\text{ m/s}^2$ gives $B = 2.00\text{ m/s}^2$. $v_y = \frac{dy}{dt} = 3Dt^2$. $a_y = \frac{dv_y}{dt} = 6Dt$. $a_y = 3.00\text{ m/s}^2$ gives $D = 0.500\text{ m/s}^3$. $x = 0$ at $t = 0$ gives $A = 0$. $y = 50.0$ m at $t = 0$ gives $C = 50.0$ m.

(b) At $t = 0$, $v_x = 0$ and $v_y = 0$, so $\vec{v} = 0$. At $t = 0$, $a_x = 2B = 4.00\text{ m/s}^2$ and $a_y = 0$, so $\vec{a} = (4.00\text{ m/s}^2)\hat{i}$.

(c) At $t = 10.0$ s, $v_x = 2\,(2.00\text{ m/s}^2)(10.0\text{ s}) = 40.0$ m/s and $v_y = 3(0.500\text{ m/s}^3)(10.0\text{ s})^2 = 150$ m/s. $v = \sqrt{v_x^2 + v_y^2} = 155$ m/s.

(d) $x = (2.00\text{ m/s}^2)(10.0\text{ s})^2 = 200$ m, $y = 50.0\text{ m} + (0.500\text{ m/s}^3)(10.0\text{ s})^3 = 550$ m. $\vec{r} = (200\text{ m})\hat{i} + (550\text{ m})\hat{j}$.

EVALUATE: The velocity and acceleration vectors as functions of time are $\vec{v}(t) = (2Bt)\hat{i} + (3Dt^2)\hat{j}$ and $\vec{a}(t) = (2B)\hat{i} + (6Dt)\hat{j}$. The acceleration is not constant.

3.40. **IDENTIFY:** The acceleration is not constant but is known as a function of time.

SET UP: Integrate the acceleration to get the velocity and the velocity to get the position. At the maximum height $v_y = 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \frac{\alpha}{3}t^3$, $v_y = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, and $x = v_{0x}t + \frac{\alpha}{12}t^4$, $y = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$.

(b) Setting $v_y = 0$ yields a quadratic in t, $0 = v_{0y} + \beta t - \frac{\gamma}{2}t^2$. Using the numerical values given in the problem, this equation has as the positive solution $t = \frac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_{0y}\gamma}\right] = 13.59$ s. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

(c) $y = 0$ gives $0 = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$ and $\frac{\gamma}{6}t^2 - \frac{\beta}{2}t - v_{0y} = 0$. Using the numbers given in the problem, the positive solution is $t = 20.73$ s. For this t, $x = 3.85\times10^4$ m.

EVALUATE: We cannot use the constant-acceleration kinematics formulas, but calculus provides the solution.

3.41. **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a $45°$ angle with both axes when its x- and y-components are equal.

SET UP: $\frac{d(t^n)}{dt} = nt^{n-1}$.

EXECUTE: $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

EVALUATE: Both components of $\vec{v}$ change with t.

3.42. **IDENTIFY:** Use the position vector of a dragonfly to determine information about its velocity vector and acceleration vector.

SET UP: Use the definitions $v_x = dx/dt$, $v_y = dy/dt$, $a_x = dv_x/dt$, and $a_y = dv_y/dt$.

EXECUTE: **(a)** Taking derivatives of the position vector gives the components of the velocity vector: $v_x(t) = (0.180\text{ m/s}^2)t$, $v_y(t) = (-0.0450\text{ m/s}^3)t^2$. Use these components and the given direction:

$$\tan 30.0^\circ = \frac{(0.0450\text{ m/s}^3)t^2}{(0.180\text{ m/s}^2)t},$$ which gives $t = 2.31$ s.

(b) Taking derivatives of the velocity components gives the acceleration components: $a_x = 0.180\text{ m/s}^2$, $a_y(t) = -(0.0900\text{ m/s}^3)t$. At $t = 2.31\text{ s}$, $a_x = 0.180\text{ m/s}^2$ and $a_y = -0.208\text{ m/s}^2$, giving $a = 0.275\text{ m/s}^2$. The direction is $\tan\theta = \dfrac{0.208}{0.180}$, so $\theta = 49.1^\circ$ clockwise from +x-axis.

EVALUATE: The acceleration is not constant, so we cannot use the standard kinematics formulas.

3.43. **IDENTIFY:** Once the rocket leaves the incline it moves in projectile motion. The acceleration along the incline determines the initial velocity and initial position for the projectile motion.

SET UP: For motion along the incline let $+x$ be directed up the incline. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = \sqrt{2(1.90\text{ m/s}^2)(200\text{ m})} = 27.57\text{ m/s}$. When the projectile motion begins the rocket has $v_0 = 27.57\text{ m/s}$ at 35.0° above the horizontal and is at a vertical height of $(200.0\text{ m})\sin 35.0^\circ = 114.7\text{ m}$. For the projectile motion let $+x$ be horizontal to the right and let $+y$ be upward. Let $y = 0$ at the ground. Then $y_0 = 114.7\text{ m}$, $v_{0x} = v_0\cos 35.0^\circ = 22.57\text{ m/s}$, $v_{0y} = v_0 \sin 35.0^\circ = 15.81\text{ m/s}$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. Let $x = 0$ at point *A*, so $x_0 = (200.0\text{ m})\cos 35.0^\circ = 163.8\text{ m}$.

EXECUTE: **(a)** At the maximum height $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (15.81\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 12.77\text{ m}$$ and $y = 114.7\text{ m} + 12.77\text{ m} = 128\text{ m}$. The maximum height above ground is 128 m.

(b) The time in the air can be calculated from the vertical component of the projectile motion: $y - y_0 = -114.7\text{ m}$, $v_{0y} = 15.81\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $(4.90\text{ m/s}^2)t^2 - (15.81\text{ m/s})t - 114.7\text{ m}$. The quadratic formula gives $t = 6.713\text{ s}$ for the positive root. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (22.57\text{ m/s})(6.713\text{ s}) = 151.6\text{ m}$ and $x = 163.8\text{ m} + 151.6\text{ m} = 315\text{ m}$. The horizontal range of the rocket is 315 m.

EVALUATE: The expressions for *h* and *R* derived in the range formula do not apply here. They are only for a projectile fired on level ground.

3.44. **IDENTIFY:** $\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t)dt$ and $\boldsymbol{a} = \dfrac{d\vec{v}}{dt}$.

SET UP: At $t = 0$, $x_0 = 0$ and $y_0 = 0$.

EXECUTE: **(a)** Integrating, $\vec{r} = \left(\alpha t - \dfrac{\beta}{3}t^3\right)\hat{i} + \left(\dfrac{\gamma}{2}t^2\right)\hat{j}$. Differentiating, $\vec{a} = (-2\beta t)\hat{i} + \gamma\hat{j}$.

(b) The positive time at which $x = 0$ is given by $t^2 = 3\alpha/\beta$. At this time, the y-coordinate is

$$y = \frac{\gamma}{2}t^2 = \frac{3\alpha\gamma}{2\beta} = \frac{3(2.4\text{ m/s})(4.0\text{ m/s}^2)}{2(1.6\text{ m/s}^3)} = 9.0\text{ m}.$$

EVALUATE: The acceleration is not constant.

3.45. **IDENTIFY:** Take $+y$ to be downward. Both objects have the same vertical motion, with v_{0y} and $a_y = +g$. Use constant acceleration equations for the *x* and *y* components of the motion.

SET UP: Use the vertical motion to find the time in the air:

$v_{0y} = 0$, $a_y = 9.80\text{ m/s}^2$, $y - y_0 = 25\text{ m}$, $t = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.259\text{ s}$.

During this time the dart must travel 90 m, so the horizontal component of its velocity must be

$$v_{0x} = \frac{x - x_0}{t} = \frac{70\text{ m}}{2.259\text{ s}} = 31\text{ m/s}.$$

EVALUATE: Both objects hit the ground at the same time. The dart hits the monkey for any muzzle velocity greater than 31 m/s.

3.46. **IDENTIFY:** The velocity has a horizontal tangential component and a vertical component. The vertical component of acceleration is zero and the horizontal component is $a_{\text{rad}} = \dfrac{v_x^2}{R}.$

SET UP: Let $+y$ be upward and $+x$ be in the direction of the tangential velocity at the instant we are considering.

EXECUTE: **(a)** The bird's tangential velocity can be found from

$$v_x = \frac{\text{circumference}}{\text{time of rotation}} = \frac{2\pi(6.00\text{ m})}{5.00\text{ s}} = 7.54\text{ m/s}.$$

Thus its velocity consists of the components $v_x = 7.54$ m/s and $v_y = 3.00$ m/s. The speed relative to the ground is then $v = \sqrt{v_x^2 + v_y^2} = 8.11$ m/s.

(b) The bird's speed is constant, so its acceleration is strictly centripetal—entirely in the horizontal direction, toward the center of its spiral path—and has magnitude $a_{\text{rad}} = \dfrac{v_x^2}{r} = \dfrac{(7.54\text{ m/s})^2}{6.00\text{ m}} = 9.48\text{ m/s}^2.$

(c) Using the vertical and horizontal velocity components $\theta = \tan^{-1}\dfrac{3.00\text{ m/s}}{7.54\text{ m/s}} = 21.7°.$

EVALUATE: The angle between the bird's velocity and the horizontal remains constant as the bird rises.

3.47. **IDENTIFY:** The cannister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the x and y components of motion.

SET UP:

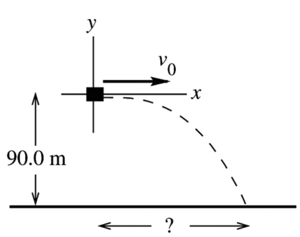


Figure 3.47

Take the origin of coordinates at the point where the cannister is released. Take $+y$ to be upward. The initial velocity of the cannister is the velocity of the plane, 64.0 m/s in the $+x$-direction.

Use the vertical motion to find the time of fall:

$t = ?,\ \ v_{0y} = 0,\ \ a_y = -9.80\text{ m/s}^2,\ \ y - y_0 = -90.0\text{ m}$ (When the cannister reaches the ground it is 90.0 m <u>below</u> the origin.)

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

EXECUTE: Since $v_{0y} = 0,\ \ t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(-90.0\text{ m})}{-9.80\text{ m/s}^2}} = 4.286\text{ s}.$

SET UP: Then use the horizontal component of the motion to calculate how far the cannister falls in this time:

$x - x_0 = ?,\ \ a_x - 0,\ \ v_{0x} = 64.0\text{ m/s}$

EXECUTE: $x - x_0 = v_0 t + \tfrac{1}{2}at^2 = (64.0\text{ m/s})(4.286\text{ s}) + 0 = 274\text{ m}.$

EVALUATE: The time it takes the cannister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

3.48. **IDENTIFY:** The person moves in projectile motion. Her vertical motion determines her time in the air.

SET UP: Take $+y$ upward. $v_{0x} = 15.0$ m/s, $v_{0y} = +10.0$ m/s, $a_x = 0$, $a_y = -9.80\text{ m/s}^2.$

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ with $y - y_0 = -30.0$ m gives $-30.0\text{ m} = (10.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$ The quadratic formula gives

$t = \frac{1}{2(4.9)}\left(+10.0 \pm \sqrt{(-10.0)^2 - 4(4.9)(-30)}\right)$ s. The positive solution is $t = 3.70$ s. During this time she travels a horizontal distance $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (15.0 \text{ m/s})(3.70 \text{ s}) = 55.5$ m. She will land 55.5 m south of the point where she drops from the helicopter and this is where the mats should have been placed.

(b) The *x*-*t*, *y*-*t*, v_x -*t* and v_y -*t* graphs are sketched in Figure 3.48.

EVALUATE: If she had dropped from rest at a height of 30.0 m it would have taken her $t = \sqrt{\frac{2(30.0 \text{ m})}{9.80 \text{ m/s}^2}} = 2.47$ s. She is in the air longer than this because she has an initial vertical component of velocity that is upward.

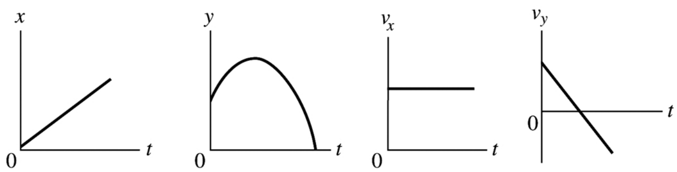


Figure 3.48

3.49. **IDENTIFY:** The suitcase moves in projectile motion. The initial velocity of the suitcase equals the velocity of the airplane.

SET UP: Take $+y$ to be upward. $a_x = 0$, $a_y = -g$.

EXECUTE: Use the vertical motion to find the time it takes the suitcase to reach the ground: $v_{0y} = v_0 \sin 23°$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -114$ m, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 9.60$ s.

The distance the suitcase travels horizontally is $x - x_0 = v_{0x} = (v_0 \cos 23.0°)t = 795$ m.

EVALUATE: An object released from rest at a height of 114 m strikes the ground at $t = \sqrt{\frac{2(y - y_0)}{-g}} = 4.82$ s. The suitcase is in the air much longer than this since it initially has an upward component of velocity.

3.50. **IDENTIFY:** The shell moves as a projectile. To just clear the top of the cliff, the shell must have $y - y_0 = 25.0$ m when it has $x - x_0 = 60.0$ m.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. $v_{0x} = v_0 \cos 43°$, $v_{0y} = v_0 \sin 43°$.

EXECUTE: **(a)** horizontal motion: $x - x_0 = v_{0x}t$ so $t = \frac{60.0 \text{ m}}{(v_0 \cos 43°)}$.

vertical motion: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $25.0 \text{ m} = (v_0 \sin 43.0°)\, t + \frac{1}{2}(-9.80 \text{ m/s}^2)\, t^2$.

Solving these two simultaneous equations for v_0 and *t* gives $v_0 = 32.6$ m/s and $t = 2.51$ s.

(b) v_y when shell reaches cliff:

$$v_y = v_{0y} + a_y t = (32.6 \text{ m/s}) \sin 43.0° - (9.80 \text{ m/s}^2)(2.51 \text{ s}) = -2.4 \text{ m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

EVALUATE: The shell reaches its maximum height at $t = -\frac{v_{0y}}{a_y} = 2.27$ s, which confirms that at $t = 2.51$ s it has passed its maximum height and is on its way down when it strikes the edge of the cliff.

3.51. **IDENTIFY:** Find the horizontal distance a rocket moves if it has a non-constant horizontal acceleration but a constant vertical acceleration of *g* downward.

SET UP: The vertical motion is *g* downward, so we can use the constant acceleration formulas for that component of the motion. We must use integration for the horizontal motion because the acceleration is not constant. Solving for t in the kinematics formula for y gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}$. In the horizontal direction we must use $v_x(t)=v_{0x}+\int_0^t a_x(t')dt'$ and $x-x_0=\int_0^t v_x(t')dt'$.

EXECUTE: Use vertical motion to find t. $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(30.0\text{ m})}{9.80\text{ m/s}^2}}=2.474\text{ s}.$

In the horizontal direction we have

$v_x(t)=v_{0x}+\int_0^t a_x(t')dt'=v_{0x}+(0.800\text{ m/s}^3)t^2=12.0\text{ m/s}+(0.800\text{ m/s}^3)t^2$. Integrating $v_x(t)$ gives $x-x_0=(12.0\text{ m/s})t+(0.2667\text{ m/s}^3)t^3$. At $t=2.474$ s, $x-x_0=29.69\text{ m}+4.04\text{ m}=33.7\text{ m}.$

EVALUATE: The vertical part of the motion is familiar projectile motion, but the horizontal part is not.

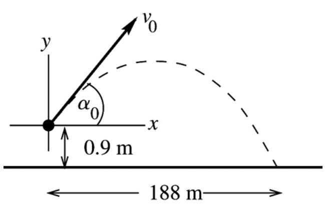
3.52. IDENTIFY: The equipment moves in projectile motion. The distance D is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

SET UP: For the motion of the equipment take $+x$ to be to the right and $+y$ to be upward. Then $a_x=0$, $a_y=-9.80\text{ m/s}^2$, $v_{0x}=v_0\cos\alpha_0=7.50\text{ m/s}$ and $v_{0y}=v_0\sin\alpha_0=13.0\text{ m/s}$. When the equipment lands in the front of the ship, $y-y_0=-8.75\text{ m}$.

EXECUTE: Use the vertical motion of the equipment to find its time in the air: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\dfrac{1}{9.80}\left(13.0\pm\sqrt{(-13.0)^2+4(4.90)(8.75)}\right)$ s. The positive root is $t=3.21$ s. The horizontal range of the equipment is $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(7.50\text{ m/s})(3.21\text{ s})=24.1\text{ m}$. In 3.21 s the ship moves a horizontal distance $(0.450\text{ m/s})(3.21\text{ s})=1.44\text{ m}$, so $D=24.1\text{ m}+1.44\text{ m}=25.5\text{ m}$.

EVALUATE: The range equation $R=\dfrac{v_0^2\sin 2\alpha_0}{g}$ cannot be used here because the starting and ending points of the projectile motion are at different heights.

3.53. IDENTIFY: Projectile motion problem.



Take the origin of coordinates at the point where the ball leaves the bat, and take $+y$ to be upward.

$v_{0x}=v_0\cos\alpha_0$

$v_{0y}=v_0\sin\alpha_0$,

but we don't know v_0.

Figure 3.53

Write down the equation for the horizontal displacement when the ball hits the ground and the corresponding equation for the vertical displacement. The time t is the same for both components, so this will give us two equations in two unknowns (v_0 and t).

(a) SET UP: <u>y-component:</u>

$a_y=-9.80\text{ m/s}^2$, $y-y_0=-0.9\text{ m}$, $v_{0y}=v_0\sin 45^\circ$

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$

EXECUTE: $-0.9\text{ m}=(v_0\sin 45^\circ)t+\frac{1}{2}(-9.80\text{ m/s}^2)t^2$

SET UP: <u>x-component:</u>

$a_x=0$, $x-x_0=188\text{ m}$, $v_{0x}=v_0\cos 45^\circ$

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{188\text{ m}}{v_0 \cos 45°}$

Put the expression for t from the x-component motion into the y-component equation and solve for v_0. (Note that $\sin 45° = \cos 45°$.)

$$-0.9\text{ m} = (v_0 \sin 45°)\left(\frac{188\text{ m}}{v_0 \cos 45°}\right) - (4.90\text{ m/s}^2)\left(\frac{188\text{ m}}{v_0 \cos 45°}\right)^2$$

$$4.90\text{ m/s}^2 \left(\frac{188\text{ m}}{v_0 \cos 45°}\right)^2 = 188\text{ m} + 0.9\text{ m} = 188.9\text{ m}$$

$$\left(\frac{v_0 \cos 45°}{188\text{ m}}\right)^2 = \frac{4.90\text{ m/s}^2}{188.9\text{ m}},\quad v_0 = \left(\frac{188\text{ m}}{\cos 45°}\right)\sqrt{\frac{4.90\text{ m/s}^2}{188.9\text{ m}}} = 42.8\text{ m/s}$$

(b) Use the horizontal motion to find the time it takes the ball to reach the fence:

SET UP: _x-component:_

$x - x_0 = 116\text{ m},\ \ a_x = 0,\ \ v_{0x} = v_0 \cos 45° = (42.8\text{ m/s}) \cos 45° = 30.3\text{ m/s},\ \ t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{116\text{ m}}{30.3\text{ m/s}} = 3.83\text{ s}$

SET UP: Find the vertical displacement of the ball at this t:

y-component:

$y - y_0 = ?,\ \ a_y = -9.80\text{ m/s}^2,\ \ v_{0y} = v_0 \sin 45° = 30.3\text{ m/s},\ \ t = 3.83\text{ s}$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (30.3\text{ s})(3.83\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.83\text{ s})^2$

$y - y_0 = 116.0\text{ m} - 71.9\text{ m} = +44.1\text{ m}$, above the point where the ball was hit. The height of the ball above the ground is $44.1\text{ m} + 0.90\text{ m} = 45.0\text{ m}$. Its height then above the top of the fence is $45.0\text{ m} - 3.0\text{ m} = 42.0\text{ m}$.

EVALUATE: With $v_0 = 42.8\text{ m/s}$, $v_{0y} = 30.3\text{ m/s}$ and it takes the ball 6.18 s to return to the height where it was hit and only slightly longer to reach a point 0.9 m below this height. $t = (188\text{ m})/(v_0 \cos 45°)$ gives $t = 6.21\text{ s}$, which agrees with this estimate. The ball reaches its maximum height approximately $(188\text{ m})/2 = 94\text{ m}$ from home plate, so at the fence the ball is not far past its maximum height of 47.6 m, so a height of 45.0 m at the fence is reasonable.

3.54. **IDENTIFY:** While the hay falls 150 m with an initial upward velocity and with a downward acceleration of g, it must travel a horizontal distance (the target variable) with constant horizontal velocity.

SET UP: Use coordinates with $+y$ upward and $+x$ horizontal. The bale has initial velocity components $v_{0x} = v_0 \cos\alpha_0 = (75\text{ m/s})\cos 55° = 43.0\text{ m/s}$ and $v_{0y} = v_0 \sin\alpha_0 = (75\text{ m/s})\sin 55° = 61.4\text{ m/s}$. $y_0 = 150\text{ m}$ and $y = 0$. The equation $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applies to the vertical motion and a similar equation to the horizontal motion.

EXECUTE: Use the vertical motion to find t: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-150\text{ m} = (61.4\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives $t = 6.27 \pm 8.36\text{ s}$. The physical value is the positive one, and $t = 14.6\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (43.0\text{ m/s})(14.6\text{ s}) = 630\text{ m}$.

EVALUATE: If the airplane maintains constant velocity after it releases the bales, it will also travel horizontally 630 m during the time it takes the bales to fall to the ground, so the airplane will be directly over the impact spot when the bales land.

3.55. **IDENTIFY:** Two-dimensional projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. With $x_0 = y_0 = 0$, algebraic manipulation of the equations for the horizontal and vertical motion shows that x and y are related by

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2.$$

$\theta_0 = 60.0°$. $y = 8.00\text{ m}$ when $x = 18.0\text{ m}$.

EXECUTE: **(a)** Solving for v_0 gives $v_0 = \sqrt{\dfrac{gx^2}{2(\cos^2\theta_0)(x\tan\theta_0 - y)}} = 16.6\text{ m/s}$.

(b) We find the horizontal and vertical velocity components:

$v_x = v_{0x} = v_0\cos\theta_0 = 8.3\text{ m/s}$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(v_0\sin\theta_0)^2 + 2a_y(y - y_0)} = -\sqrt{(14.4\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(8.00\text{ m})} = -7.1\text{ m/s}$

$v = \sqrt{v_x^2 + v_y^2} = 10.9\text{ m/s}$. $\tan\theta = \dfrac{|v_y|}{|v_x|} = \dfrac{7.1}{8.3}$ and $\theta = 40.5°$, below the horizontal.

EVALUATE: We can check our calculated v_0.

$$t = \frac{x - x_0}{v_{0x}} = \frac{18.0\text{ m}}{8.3\text{ m/s}} = 2.17\text{ s}.$$

Then $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = (14.4\text{ m/s})(2.17\text{ s}) - (4.9\text{ m/s}^2)(2.17\text{ s})^2 = 8\text{ m}$, which checks.

3.56. **IDENTIFY:** The water moves in projectile motion.

SET UP: Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0$, $a_y = -g$.

EXECUTE: The equations of motions are $y = (v_0\sin\alpha)t - \frac{1}{2}gt^2$ and $x = (v_0\cos\alpha)t$. When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin\alpha = \cos\alpha = \sqrt{2}/2$.

To reach the *minimum* distance: $6D = \dfrac{\sqrt{2}}{2}v_0t$, and $2D = \dfrac{\sqrt{2}}{2}v_0t - \frac{1}{2}gt^2$. Solving the first equation for t gives $t = \dfrac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \frac{1}{2}g\left(\dfrac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \dfrac{\sqrt{2}}{2}v_0t$, and $2D = \dfrac{\sqrt{2}}{2}v_0t - \frac{1}{2}gt^2$. Solving the first equation for t gives $t = \dfrac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\dfrac{7D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

EVALUATE: A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of $6D$. The minimum speed required is greater than this, because the water must be at a height of at least $2D$ when it reaches the front of the tank.

3.57. **IDENTIFY:** From the figure in the text, we can read off the maximum height and maximum horizontal distance reached by the grasshopper. Knowing its acceleration is g downward, we can find its initial speed and the height of the cliff (the target variables).

SET UP: Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. The constant-acceleration kinematics formulas $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ apply.

EXECUTE: **(a)** $v_y = 0$ when $y - y_0 = 0.0674$ m. $v_y^2 = v_{0y}^2 + 2a_y\,(y - y_0)$ gives $v_{0y} = \sqrt{-2a_y\,(y - y_0)} = \sqrt{-2\,(-9.80\text{ m/s}^2)(0.0674\text{ m})} = 1.15$ m/s. $v_{0y} = v_0 \sin\alpha_0$ so $v_0 = \dfrac{v_{0y}}{\sin\alpha_0} = \dfrac{1.15\text{ m/s}}{\sin 50.0°} = 1.50$ m/s.

(b) Use the horizontal motion to find the time in the air. The grasshopper travels horizontally $x - x_0 = 1.06$ m. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos 50.0°} = 1.10$ s. Find the vertical displacement of the grasshopper at $t = 1.10$ s:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.15\text{ m/s})(1.10\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(1.10\text{ s})^2 = -4.66$ m. The height of the cliff is 4.66 m.

EVALUATE: The grasshopper's maximum height (6.74 cm) is physically reasonable, so its takeoff speed of 1.50 m/s must also be reasonable. Note that the equation $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ does *not* apply here since the launch point is not at the same level as the landing point.

3.58. **IDENTIFY:** To clear the bar the ball must have a height of 10.0 ft when it has a horizontal displacement of 36.0 ft. The ball moves as a projectile. When v_0 is very large, the ball reaches the goal posts in a very short time and the acceleration due to gravity causes negligible downward displacement.

SET UP: 36.0 ft $= 10.97$ m; 10.0 ft $= 3.048$ m. Let $+x$ be to the right and $+y$ be upward, so $a_x = 0$, $a_y = -g$, $v_{0x} = v_0 \cos\alpha_0$ and $v_{0y} = v_0 \sin\alpha_0$.

EXECUTE: **(a)** The ball cannot be aimed lower than directly at the bar. $\tan\alpha_0 = \dfrac{10.0\text{ ft}}{36.0\text{ ft}}$ and $\alpha_0 = 15.5°$.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos\alpha_0}$. Then $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$y - y_0 = (v_0 \sin\alpha_0)\left(\frac{x - x_0}{v_0 \cos\alpha_0}\right) - \frac{1}{2}g\,\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0} = (x - x_0)\tan\alpha_0 - \frac{1}{2}g\,\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0}.$$

$$v_0 = \frac{(x - x_0)}{\cos\alpha_0}\sqrt{\frac{g}{2[(x - x_0)\tan\alpha_0 - (y - y_0)]}} = \frac{10.97\text{ m}}{\cos 45.0°}\sqrt{\frac{9.80\text{ m/s}^2}{2[10.97\text{ m} - 3.048\text{ m}]}} = 12.2\text{ m/s} = 43.9\text{ km/h}.$$

EVALUATE: With the v_0 and 45° launch angle in part (b), the horizontal range of the ball is $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ = 15.2 m = 49.9 ft. The ball reaches the highest point in its trajectory when $x - x_0 = R/2$, which is 25 ft, so when it reaches the goal posts it is on its way down.

3.59. **IDENTIFY:** The snowball moves in projectile motion. In part (a) the vertical motion determines the time in the air. In part (c), find the height of the snowball above the ground after it has traveled horizontally 4.0 m.

SET UP: Let $+y$ be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0 \cos\theta_0 = 5.36$ m/s, $v_{0y} = v_0 \sin\theta_0 = 4.50$ m/s.

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 14.0$ m gives $14.0\text{ m} = (4.50\text{ m/s})\,t + (4.9\text{ m/s}^2)\,t^2$. The quadratic formula gives $t = \dfrac{1}{2(4.9)}\left(-4.50 \pm \sqrt{(4.50)^2 - 4\,(4.9)(-14.0)}\right)$ s. The positive root is $t = 1.29$ s. Then

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (5.36\text{ m/s})(1.29\text{ s}) = 6.91$ m.

(b) The x-t, y-t, v_x-t and v_y-t graphs are sketched in Figure 3.59.

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{4.0\text{ m}}{5.36\text{ m/s}} = 0.746\text{ s}$. In this time the snowball travels downward a distance $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 6.08\text{ m}$ and is therefore $14.0\text{ m} - 6.08\text{ m} = 7.9\text{ m}$ above the ground. The snowball passes well above the man and doesn't hit him.

EVALUATE: If the snowball had been released from rest at a height of 14.0 m it would have reached the ground in $t = \sqrt{\dfrac{2(14.0\text{ m})}{9.80\text{ m/s}^2}} = 1.69\text{ s}$. The snowball reaches the ground in a shorter time than this because of its initial downward component of velocity.

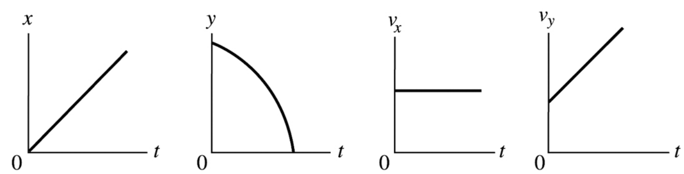


Figure 3.59

3.60. **IDENTIFY:** The dog runs horizontally at constant velocity, and the ball is in two-dimensional projectile motion. The ball starts out traveling only horizontally.

SET UP: Use coordinates with the origin at the boy and with $+y$ downward. For the ball $v_{0y} = 0$, $v_{0x} = 8.50\text{ m/s}$, $a_x = 0$ and $a_y = 9.80\text{ m/s}^2$.

EXECUTE: **(a)** The dog must travel horizontally the same distance the ball travels horizontally, so the dog must have speed 8.50 m/s.

(b) Use the vertical motion of the ball to find its time in the air. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(12.0\text{ m})}{9.80\text{ m/s}^2}} = 1.56\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (8.50\text{ m/s})(1.56\text{ s}) = 13.3\text{ m}$

EVALUATE: The dog is about 40 ft from the tree, which is not unreasonable since the tree is nearly 40 ft high.

3.61. **IDENTIFY:** The dog runs horizontally at constant velocity, and the ball is in two-dimensional projectile motion. But this time the ball has an upward component to its initial velocity.

SET UP: Use coordinates with the origin at the boy and with $+y$ upward. The ball has $v_{0x} = v_0 \cos\theta_0 = (8.50\text{ m/s})\cos 60.0^\circ = 4.25\text{ m/s}$, $v_{0y} = v_0 \sin\theta_0 = (8.50\text{ m/s})\sin 60.0^\circ = 7.36\text{ m/s}$, $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** The dog must travel horizontally the same distance the ball travels horizontally, so the dog must have speed 4.25 m/s.

(b) Use the vertical motion of the ball to find its time in the air. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-12.0\text{ m} = (7.36\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives $t = 0.751 \pm 1.74\text{ s}$. The negative value is not physical, so $t = 2.49\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (4.25\text{ m/s})(2.49\text{ s}) = 10.6\text{ m}$.

EVALUATE: The ball is in the air longer than when it is thrown horizontally (as we saw in the previous problem), but it doesn't travel as far horizontally. The dog doesn't have to run as far or as fast as when the ball is thrown horizontally.

3.62. **IDENTIFY:** The rock moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. Eqs. (3.21) and (3.22) give v_x and v_y.

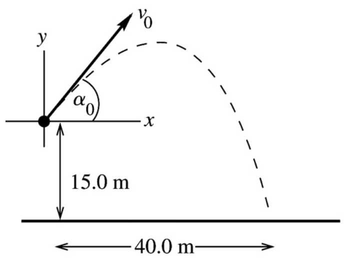
EXECUTE: Combining Eqs. 3.24, 3.21 and 3.22 gives

$v^2 = v_0^2 \cos^2\alpha_0 + (v_0 \sin\alpha_0 - gt)^2 = v_0^2(\sin^2\alpha_0 + \cos^2\alpha_0) - 2v_0 \sin\alpha_0 gt + (gt)^2$.

$v^2 = v_0^2 - 2g\left(v_0 \sin\alpha_0 t - \frac{1}{2}gt^2\right) = v_0^2 - 2gy,$ where Eq. (3.20) has been used to eliminate t in favor of y. For the case of a rock thrown from the roof of a building of height h, the speed at the ground is found by substituting $y = -h$ into the above expression, yielding $v = \sqrt{v_0^2 + 2gh},$ which is independent of α_0.

EVALUATE: This result, as will be seen in the chapter dealing with conservation of energy (Chapter 7), is valid for any y, positive, negative or zero, as long as $v_0^2 - 2gy > 0$.

3.63. **(a) IDENTIFY:** Projectile motion.



Take the origin of coordinates at the top of the ramp and take $+y$ to be upward.

The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

Figure 3.63

We don't know t, the time in the air, and we don't know v_0. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

SET UP: <u>y-component:</u>

$y - y_0 = -15.0 \text{ m}, \;\; a_y = -9.80 \text{ m/s}^2, \;\; v_{0y} = v_0 \sin 53.0°$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $-15.0 \text{ m} = (v_0 \sin 53.0°)\, t - (4.90 \text{ m/s}^2)\, t^2$

SET UP: <u>x-component:</u>

$x - x_0 = 40.0 \text{ m}, \;\; a_x = 0, \;\; v_{0x} = v_0 \cos 53.0°$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

EXECUTE: $40.0 \text{ m} = (v_0 t)\cos 53.0°$

The second equation says $v_0 t = \dfrac{40.0 \text{ m}}{\cos 53.0°} = 66.47 \text{ m}.$

Use this to replace $v_0 t$ in the first equation:

$-15.0 \text{ m} = (66.47 \text{ m}) \sin 53° - (4.90 \text{ m/s}^2)\, t^2$

$$t = \sqrt{\frac{(66.47 \text{ m})\sin 53° + 15.0 \text{ m}}{4.90 \text{ m/s}^2}} = \sqrt{\frac{68.08 \text{ m}}{4.90 \text{ m/s}^2}} = 3.727 \text{ s}.$$

Now that we have t we can use the x-component equation to solve for v_0:

$$v_0 = \frac{40.0 \text{ m}}{t \cos 53.0°} = \frac{40.0 \text{ m}}{(3.727 \text{ s}) \cos 53.0°} = 17.8 \text{ m/s}.$$

EVALUATE: Using these values of v_0 and t in the $y = y_0 = v_{0y} + \frac{1}{2}a_y t^2$ equation verifies that $y - y_0 = -15.0$ m.

(b) IDENTIFY: $v_0 = (17.8 \text{ m/s})/2 = 8.9 \text{ m/s}$

This is less than the speed required to make it to the other side, so he lands in the river.
Use the vertical motion to find the time it takes him to reach the water:

SET UP: $y - y_0 = -100 \text{ m}; \;\; v_{0y} = +v_0 \sin 53.0° = 7.11 \text{ m/s}; \;\; a_y = -9.80 \text{ m/s}^2$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-100 = 7.11t - 4.90t^2$

EXECUTE: $4.90t^2 - 7.11t - 100 = 0$ and $t = \frac{1}{9.80}\left(7.11 \pm \sqrt{(7.11)^2 - 4\,(4.90)(-100)}\right)$

$t = 0.726\ \text{s} \pm 4.57\ \text{s}$ so $t = 5.30\ \text{s}$.

The horizontal distance he travels in this time is

$$x - x_0 = v_{0x}t = (v_0\cos 53.0°)\,t = (5.36\ \text{m/s})(5.30\ \text{s}) = 28.4\ \text{m}.$$

He lands in the river a horizontal distance of 28.4 m from his launch point.

EVALUATE: He has half the minimum speed and makes it only about halfway across.

3.64. **IDENTIFY:** The ball moves in projectile motion.

SET UP: The woman and ball travel for the same time and must travel the same horizontal distance, so for the ball $v_{0x} = 6.00\ \text{m/s}$.

EXECUTE: **(a)** $v_{0x} = v_0\cos\theta_0$. $\cos\theta_0 = \dfrac{v_{0x}}{v_0} = \dfrac{6.00\ \text{m/s}}{20.0\ \text{m/s}}$ and $\theta_0 = 72.5°$. The ball is in the air for 5.55s and she runs a distance of $(6.00\ \text{m/s})(5.55\ \text{s}) = 33.3\ \text{m}$.

(b) Relative to the ground the ball moves in a parabola. The ball and the runner have the same horizontal component of velocity, so relative to the runner the ball has only vertical motion. The trajectories as seen by each observer are sketched in Figure 3.64.

EVALUATE: The ball could be thrown with a different speed, so long as the angle at which it was thrown was adjusted to keep $v_{0x} = 6.00\ \text{m/s}$.

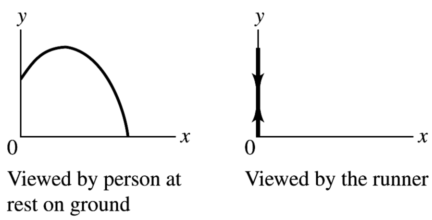


Figure 3.64

3.65. **IDENTIFY:** The boulder moves in projectile motion.

SET UP: Take $+y$ downward. $v_{0x} = v_0$, $a_x = 0$, $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$.

EXECUTE: **(a)** Use the vertical motion to find the time for the boulder to reach the level of the lake:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = +20\ \text{m}$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(20\ \text{m})}{9.80\ \text{m/s}^2}} = 2.02\ \text{s}$. The rock must travel horizontally 100 m during this time. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{100\ \text{m}}{2.02\ \text{s}} = 49.5\ \text{m/s}$

(b) In going from the edge of the cliff to the plain, the boulder travels downward a distance of

$y - y_0 = 45\ \text{m}$. $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(45\ \text{m})}{9.80\ \text{m/s}^2}} = 3.03\ \text{s}$ and $x - x_0 = v_{0x}t = (49.5\ \text{m/s})(3.03\ \text{s}) = 150\ \text{m}$.

The rock lands $150\ \text{m} - 100\ \text{m} = 50\ \text{m}$ beyond the foot of the dam.

EVALUATE: The boulder passes over the dam 2.02 s after it leaves the cliff and then travels an additional 1.01 s before landing on the plain. If the boulder has an initial speed that is less than 49 m/s, then it lands in the lake.

3.66. **IDENTIFY:** The bagels move in projectile motion. Find Henrietta's location when the bagels reach the ground, and require the bagels to have this horizontal range.

SET UP: Let $+y$ be downward and let $x_0 = y_0 = 0$. $a_x = 0$, $a_y = +g$. When the bagels reach the ground, $y = 38.0\ \text{m}$.

EXECUTE: **(a)** When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 38.0 m from rest. Get the time to fall: $y = \dfrac{1}{2}gt^2$, $38.0\ \text{m} = \dfrac{1}{2}(9.80\ \text{m/s}^2)\,t^2$ and $t = 2.78\ \text{s}$.

So, she has been jogging for $9.00\ \text{s} + 2.78\ \text{s} = 11.78\ \text{s}$. During this time she has gone

$x = vt = (3.05\ \text{m/s})(11.78\ \text{s}) = 35.9\ \text{m}.$ Bruce must throw the bagels so they travel 35.9 m horizontally in 2.78 s. This gives $x = vt.$ $35.9\ \text{m} = v\,(2.78\ \text{s})$ and $v = 12.9\ \text{m/s}.$

(b) 35.9 m from the building.

EVALUATE: If $v > 12.9\ \text{m/s}$ the bagels land in front of her and if $v < 12.9\ \text{m/s}$ they land behind her. There is a range of velocities greater than 12.9 m/s for which she would catch the bagels in the air, at some height above the sidewalk.

3.67. **IDENTIFY:** The cart has a constant horizontal velocity, but the missile has horizontal and vertical motion once it has left the cart and is in free fall.

SET UP: Let $+y$ be upward and $+x$ be to the right. The missile has $\upsilon_{0x} = 30.0\ \text{m/s},$ $\upsilon_{0y} = 40.0\ \text{m/s},$ $a_x = 0$ and $a_y = -9.80\ \text{m/s}^2.$ The cart has $a_x = 0$ and $\upsilon_{0x} = 30.0\ \text{m/s}.$

EXECUTE: **(a)** At the missile's maximum height, $\upsilon_y = 0.$

$$\upsilon_y{}^2 = \upsilon_{0y}{}^2 + 2a_y(y - y_0) \text{ gives } y - y_0 = \frac{\upsilon_y{}^2 - \upsilon_{0y}{}^2}{2a_y} = \frac{0 - (40.0\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 81.6\ \text{m}$$

(b) Find t for $y - y_0 = 0$ (missile returns to initial level).

$$y - y_0 = \upsilon_{0y}t + \tfrac{1}{2}a_yt^2 \text{ gives } t = -\frac{2\upsilon_{0y}}{a_y} = -\frac{2(40.0\ \text{m/s})}{-9.80\ \text{m/s}^2} = 8.16\ \text{s}$$

Then $x - x_0 = \upsilon_{0x}t + \tfrac{1}{2}a_xt^2 = (30.0\ \text{m/s})(8.16\ \text{s}) = 245\ \text{m}.$

(c) The missile also travels horizontally 245 m so the missile lands in the cart.

EVALUATE: The vertical motion of the missile does not affect its horizontal motion, which is the same as that of the cart, so the missile is always directly above the cart throughout its motion.

3.68. **IDENTIFY:** The water follows a parabolic trajectory since it is affected only by gravity, so we apply the principles of projectile motion to it.

SET UP: Use coordinates with $+y$ upward. Once the water leaves the cannon it is in free-fall and has $a_x = 0$ and $a_y = -9.80\ \text{m/s}^2.$ The water has $\upsilon_{0x} = \upsilon_0 \cos\theta_0 = 15.0\ \text{m/s}$ and $\upsilon_{0y} = \upsilon_0 \sin\theta_0 = 20.0\ \text{m/s}.$

EXECUTE: Use the vertical motion to find t that gives $y - y_0 = 10.0\ \text{m}$: $y - y_0 = \upsilon_{0y}t + \tfrac{1}{2}a_yt^2$ gives

$$10.0\ \text{m} = (20.0\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2.$$

The quadratic formula gives $t = 2.04 \pm 1.45\ \text{s},$ and $t = 0.59\ \text{s}$ or $t = 3.49\ \text{s}.$ Both answers are physical.

$$\text{For } t = 0.59\ \text{s}, \quad x - x_0 = \upsilon_{0x}t = (15.0\ \text{m/s})(0.59\ \text{s}) = 8.8\ \text{m}.$$

$$\text{For } t = 3.49\ \text{s}, \quad x - x_0 = \upsilon_{0x}t = (15.0\ \text{m/s})(3.49\ \text{s}) = 52.4\ \text{m}.$$

When the cannon is 8.8 m from the building, the water hits this spot on the wall on its way up to its maximum height. When is it 52.4 m from the building it hits this spot after it has passed through its maximum height.

EVALUATE: The fact that we have two possible answers means that the firefighters have some choice on where to stand. If the fire is extremely fierce, they would no doubt prefer to stand at the more distant location.

3.69. **IDENTIFY:** The rock is in free fall once it is in the air, so it has only a downward acceleration of $9.80\ \text{m/s}^2$, and we apply the principles of two-dimensional projectile motion to it. The constant-acceleration kinematics formulas apply.

SET UP: The vertical displacement must be $\Delta y = y - y_0 = 5.00\ \text{m} - 1.60\ \text{m} = 3.40\ \text{m}$ at the instant that the horizontal displacement $\Delta x = x - x_0 = 14.0\ \text{m}$, and $a_y = -9.80\ \text{m/s}^2$ with $+y$ upward.

EXECUTE: **(a)** There is no horizontal acceleration, so $14.0\ \text{m} = v_0 \cos(56.0°)t$, which gives $t = \dfrac{14.0\ \text{m}}{v_0 \cos 56.0°}$. Putting this quantity, along with the numerical quantities, into the equation $y - y_0 = v_{0y}t + \tfrac{1}{2}a_yt^2$ and solving for v_0 we get $v_0 = 13.3\ \text{m/s}.$

(b) The initial horizontal velocity of the rock is (13.3 m/s)(cos 56.0°), and when it lands on the ground, $y - y_0 = -1.60$ m. Putting these quantities into the equation $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ leads to a quadratic equation in t. Using the positive square root, we get t = 2.388 s when the rock lands. The horizontal position at that instant is $x - x_0$ = (13.3 m/s)(cos 56.0°)(2.388 s) = 17.8 m from the launch point. So the distance beyond the fence is 17.8 m – 14.0 m = 3.8 m.

EVALUATE: We cannot use the range formula to find the distance in (b) because the rock's motion does not start and end at the same height.

3.70. **IDENTIFY:** The object moves with constant acceleration in both the horizontal and vertical directions.

SET UP: Let $+y$ be downward and let $+x$ be the direction in which the firecracker is thrown.

EXECUTE: The firecracker's falling time can be found from the vertical motion: $t = \sqrt{\dfrac{2h}{g}}$.

The firecracker's horizontal position at any time t (taking the student's position as $x = 0$) is $x = vt - \frac{1}{2}at^2$. $x = 0$ when cracker hits the ground, so $t = 2v/a$. Combining this with the expression for the falling time gives $\dfrac{2v}{a} = \sqrt{\dfrac{2h}{g}}$ and $h = \dfrac{2v^2 g}{a^2}$.

EVALUATE: When h is smaller, the time in the air is smaller and either v must be smaller or a must be larger.

3.71. **IDENTIFY:** Relative velocity problem. The plane's motion relative to the earth is determined by its velocity relative to the earth.

SET UP: Select a coordinate system where $+y$ is north and $+x$ is east.

The velocity vectors in the problem are:

$\vec{v}_{P/E}$, the velocity of the plane relative to the earth.

$\vec{v}_{P/A}$, the velocity of the plane relative to the air (the magnitude $v_{P/A}$ is the airspeed of the plane and the direction of $\vec{v}_{P/A}$ is the compass course set by the pilot).

$\vec{v}_{A/E}$, the velocity of the air relative to the earth (the wind velocity).

The rule for combining relative velocities gives $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

(a) We are given the following information about the relative velocities:

$\vec{v}_{P/A}$ has magnitude 220 km/h and its direction is west. In our coordinates it has components $(v_{P/A})_x = -220$ km/h and $(v_{P/A})_y = 0$.

From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{P/E}$ has components in our coordinate system of

$$(v_{P/E})_x = -\frac{120\text{ km}}{0.500\text{ h}} = -240\text{ km/h (west)}$$

$$(v_{P/E})_y = -\frac{20\text{ km}}{0.500\text{ h}} = -40\text{ km/h (south)}$$

With this information the diagram corresponding to the velocity addition equation is shown in Figure 3.71a.

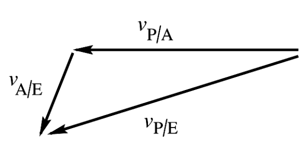


Figure 3.71a

We are asked to find $\vec{v}_{A/E}$, so solve for this vector:

$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ gives $\vec{v}_{A/E} = \vec{v}_{P/E} - \vec{v}_{P/A}$.

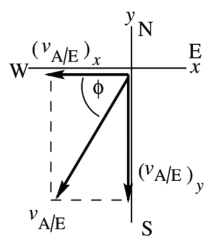
EXECUTE: The x-component of this equation gives

$(v_{A/E})_x = (v_{P/E})_x - (v_{P/A})_x = -240\text{ km/h} - (-220\text{ km/h}) = -20\text{ km/h}.$

The y-component of this equation gives

$(v_{A/E})_y = (v_{P/E})_y - (v_{P/A})_y = -40\text{ km/h}.$

Now that we have the components of $\vec{v}_{A/E}$ we can find its magnitude and direction.



$$v_{A/E} = \sqrt{(v_{A/E})_x^2 + (v_{A/E})_y^2}$$

$$v_{A/E} = \sqrt{(-20\text{ km/h})^2 + (-40\text{ km/h})^2} = 44.7\text{ km/h}$$

$$\tan\phi = \frac{40\text{ km/h}}{20\text{ km/h}} = 2.00;\quad \phi = 63.4°$$

The direction of the wind velocity is $63.4°$ S of W, or $26.6°$ W of S.

Figure 3.71b

EVALUATE: The plane heads west. It goes farther west than it would without wind and also travels south, so the wind velocity has components west and south.

(b) SET UP: The rule for combining the relative velocities is still $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$, but some of these velocities have different values than in part (a).

$\vec{v}_{P/A}$ has magnitude 220 km/h but its direction is to be found.

$\vec{v}_{A/E}$ has magnitude 40 km/h and its direction is due south.

The direction of $\vec{v}_{P/E}$ is west; its magnitude is not given.

The vector diagram for $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ and the specified directions for the vectors is shown in Figure 3.71c.

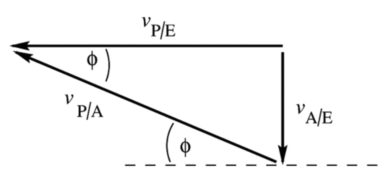


Figure 3.71c

The vector addition diagram forms a right triangle.

EXECUTE: $\sin\phi = \dfrac{v_{A/E}}{v_{P/A}} = \dfrac{40\text{ km/h}}{220\text{ km/h}} = 0.1818;\quad \phi = 10.5°.$

The pilot should set her course $10.5°$ north of west.

EVALUATE: The velocity of the plane relative to the air must have a northward component to counteract the wind and a westward component in order to travel west.

3.72. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the raindrop relative to the earth, $\vec{v}_{R/E}$, the raindrop relative to the train, $\vec{v}_{R/T}$, and the train relative to the earth, $\vec{v}_{T/E}$. $\vec{v}_{R/E} = \vec{v}_{R/T} + \vec{v}_{T/E}$. $\vec{v}_{T/E}$ is due east and has magnitude 12.0 m/s. $\vec{v}_{R/T}$ is 30.0° west of vertical. $\vec{v}_{R/E}$ is vertical. The relative velocity addition diagram is given in Figure 3.72.

EXECUTE: **(a)** $\vec{v}_{R/E}$ is vertical and has zero horizontal component. The horizontal component of $\vec{v}_{R/T}$ is $-\vec{v}_{T/E}$, so is 12.0 m/s westward.

(b) $v_{R/E} = \dfrac{v_{T/E}}{\tan 30.0°} = \dfrac{12.0 \text{ m/s}}{\tan 30.0°} = 20.8 \text{ m/s}$. $v_{R/T} = \dfrac{v_{T/E}}{\sin 30.0°} = \dfrac{12.0 \text{ m/s}}{\sin 30.0°} = 24.0 \text{ m/s}$.

EVALUATE: The speed of the raindrop relative to the train is greater than its speed relative to the earth, because of the motion of the train.

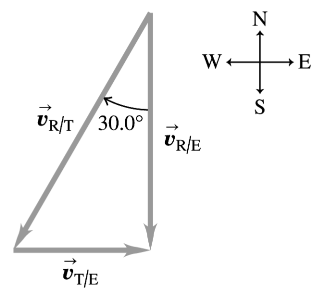


Figure 3.72

3.73. **IDENTIFY:** Relative velocity problem.

SET UP: The three relative velocities are:

$\vec{v}_{J/G}$, Juan relative to the ground. This velocity is due north and has magnitude $v_{J/G} = 8.00$ m/s.

$\vec{v}_{B/G}$, the ball relative to the ground. This vector is 37.0° east of north and has magnitude $v_{B/G} = 12.00$ m/s.

$\vec{v}_{B/J}$, the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{B/G} = \vec{v}_{B/J} + \vec{v}_{J/G}$, so $\vec{v}_{B/J} = \vec{v}_{B/G} - \vec{v}_{J/G}$.

The relative velocity addition diagram does not form a right triangle so we must do the vector addition using components.

Take $+y$ to be north and $+x$ to be east.

EXECUTE: $v_{B/Jx} = +v_{B/G} \sin 37.0° = 7.222$ m/s

$v_{B/Jy} = +v_{B/G} \cos 37.0° - v_{J/G} = 1.584$ m/s

These two components give $v_{B/J} = 7.39$ m/s at 12.4° north of east.

EVALUATE: Since Juan is running due north, the ball's eastward component of velocity relative to him is the same as its eastward component relative to the earth. The northward component of velocity for Juan and the ball are in the same direction, so the component for the ball relative to Juan is the difference in their components of velocity relative to the ground.

3.74. **IDENTIFY:** Both the bolt and the elevator move vertically with constant acceleration.

SET UP: Let $+y$ be upward and let $y = 0$ at the initial position of the floor of the elevator, so y_0 for the bolt is 3.00 m.

EXECUTE: **(a)** The position of the bolt is $3.00 \text{ m} + (2.50 \text{ m/s})\,t - (1/2)(9.80 \text{ m/s}^2)\,t^2$ and the position of the floor is (2.50 m/s)t. Equating the two, $3.00 \text{ m} = (4.90 \text{ m/s}^2)\,t^2$. Therefore, $t = 0.782$ s.

(b) The velocity of the bolt is $2.50 \text{ m/s} - (9.80 \text{ m/s}^2)(0.782 \text{ s}) = -5.17$ m/s relative to earth, therefore, relative to an observer in the elevator $v = -5.17 \text{ m/s} - 2.50 \text{ m/s} = -7.67$ m/s.

(c) As calculated in part (b), the speed relative to earth is 5.17 m/s.
(d) Relative to earth, the distance the bolt traveled is
$(2.50\text{ m/s})\,t-(1/2)(9.80\text{ m/s}^2)\,t^2=(2.50\text{ m/s})(0.782\text{ s})-(4.90\text{ m/s}^2)(0.782\text{ s})^2=-1.04\text{ m}.$

EVALUATE: As viewed by an observer in the elevator, the bolt has $v_{0y}=0$ and $a_y=-9.80\text{ m/s}^2$, so in 0.782 s it falls $-\frac{1}{2}(9.80\text{ m/s}^2)(0.782\text{ s})^2=-3.00\text{ m}.$

3.75. **IDENTIFY:** We need to use relative velocities.
SET UP: If B is moving relative to M and M is moving relative to E, the velocity of B relative to E is $\vec{v}_{\text{B/E}}=\vec{v}_{\text{B/M}}+\vec{v}_{\text{M/E}}.$
EXECUTE: Let $+x$ be east and $+y$ be north. We have $v_{\text{B/M},x}=2.50\text{ m/s}$, $v_{\text{B/M,y}}=-4.33\text{ m/s}$, $v_{\text{M/E},x}=0$, and $v_{\text{M/E,y}}=6.00\text{ m/s}$. Therefore $v_{\text{B/E},x}=v_{\text{B/M},x}+v_{\text{M/E},x}=2.50\text{ m/s}$ and $v_{\text{B/E,y}}=v_{\text{B/M,y}}+v_{\text{M/E,y}}=-4.33\text{ m/s}+6.00\text{ m/s}=+1.67\text{ m/s}.$ The magnitude is $v_{\text{B/E}}=\sqrt{(2.50\text{ m/s})^2+(1.67\text{ m/s})^2}=3.01\text{ m/s},$ and the direction is $\tan\theta=\dfrac{1.67}{2.50},$ which gives $\theta=33.7^\circ$ north of east.
EVALUATE: Since Mia is moving, the velocity of the ball relative to her is different from its velocity relative to the ground or relative to Alice.

3.76. **IDENTIFY:** You have a graph showing the horizontal range of the rock as a function of the angle at which it was launched and want to find its initial velocity. Because air resistance is negligible, the rock is in free fall. The range formula applies since the rock rock was launced from the ground and lands at the ground.

SET UP: **(a)** The range formula is $R=\dfrac{v_0^2\sin(2\theta)}{g}$, so a plot of R versus $\sin(2\theta_0)$ will give a straight line having slope equal to $v_0^2/g.$ We can use that data in the graph in the problem to construct our graph by hand, or we can use graphing software. The resulting graph is shown in Figure 3.76.

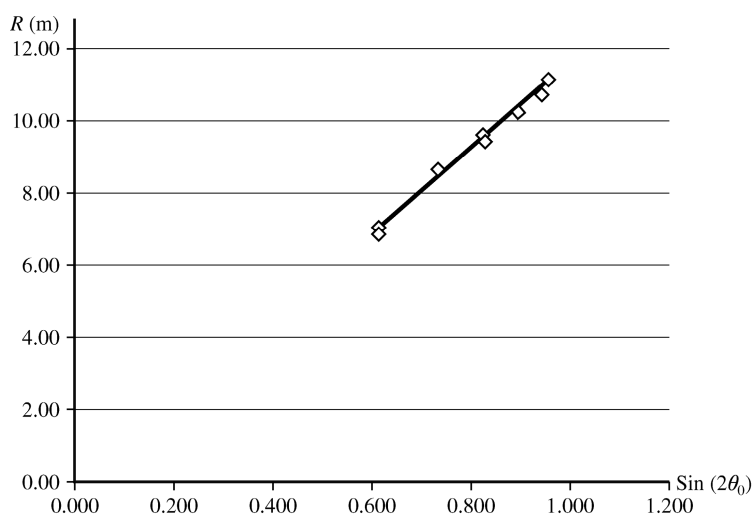


Figure 3.76

(b) The slope of the graph is 10.95 m, so 10.95 m = $v_0^2/g.$ Solving for v_0 we get v_0 = 10.4 m/s.

(c) Solving the formula $v_y^2=v_{0y}^2+2a_y(y-y_0)$ for $y-y_0$ with $v_y=0$ at the highest point, we get $y-y_0$ = 1.99 m.

EVALUATE: This approach to finding the launch speed v_0 requires only simple measurements: the range and the launch angle. It would be difficult and would require special equipment to measure v_0 directly.

3.77. **IDENTIFY:** The table gives data showing the horizontal range of the potato for various launch heights. You want to use this information to determine the launch speed of the potato, assuming negligible air resistance.

SET UP: The potatoes are launched horizontally, so $v_{0y} = 0$, and they are in free fall, so $a_y = 9.80\ \text{m/s}^2$ downward and $a_x = 0$. The time a potato is in the air is just the time it takes for it to fall vertically from the launch point to the ground, a distance h.

EXECUTE: **(a)** For the vertical motion of a potato, we have $h = ½\, gt^2$, so $t = \sqrt{2h/g}$. The horizontal range R is given by $R = v_0 t = v_0\sqrt{2h/g}$. Squaring gives $R^2 = \left(\dfrac{2v_0^2}{g}\right)h$. Graphing R^2 versus h will give a straight line with slope $2v_0^2/g$. We can graph the data from the table in the text by hand, or we could use graphing software. The result is shown in Figure 3.77.

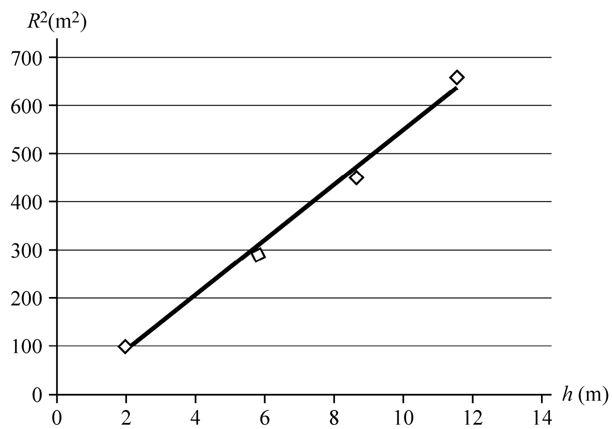


Figure 3.77

(b) The slope of the graph is 55.2 m, so $v_0 = \sqrt{\dfrac{(9.80\ \text{m/s}^2)(55.2\ \text{m})}{2}} = 16.4\ \text{m/s}.$

(c) In this case, the potatoes are launched and land at ground level, so we can use the range formula with $\theta = 30.0°$ and $v_0 = 16.4$ m/s. The result is $R = \dfrac{v_0^2 \sin(2\theta)}{g} = 23.8$ m.

EVALUATE: This approach to finding the launch speed v_0 requires only simple measurements: the range and the launch height. It would be difficult and would require special equipment to measure v_0 directly.

3.78. **IDENTIFY:** This is a vector addition problem. The boat moves relative to the water and the water moves relative to the earth. We know the speed of the boat relative to the water and the times for the boat to go directly across the river, and from these things we want to find out how fast the water is moving and the width of the river.

SET UP: For both trips of the boat, $\vec{v}_{\text{B/E}} = \vec{v}_{\text{B/W}} + \vec{v}_{\text{W/E}}$, where the subscripts refer to the boat, earth, and water. The speed of the boat relative to the earth is $v_{\text{B/E}} = d/t$, where d is the width of the river and t is the time to cross the river, which is different in the two crossings.

EXECUTE: Figure 3.78 shows a vector sum for the first trip and for the return trip.

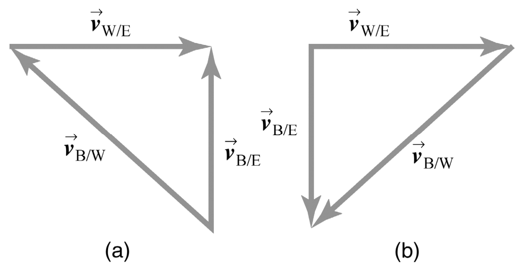


Figure 3.78a-b

(a) For both trips, the vectors in Figures 3.78 a & b form right triangles, so we can apply the Pythagorean theorem. $v_{B/E}^2 = v_{B/W}^2 - v_{W/E}^2$ and $v_{B/E} = d/t$. For the first trip, $v_{B/W} = 6.00$ m/s and $t = 20.1$ s, giving $d^2/(20.1\text{ s})^2 = (60.00\text{ m/s}^2) - (v_{W/E})^2$. For the return trip, $v_{B/W} = 9.0$ m/s and $t = 11.2$ s, which gives $d^2/(11.2\text{ s})^2 = (9.00\text{ m/s}^2) - (v_{W/E})^2$. Solving these two equations together gives $d = 90.48$ m, which rounds to 90.5 m (the width of the river) and $v_{W/E} = 3.967$ m/s which rounds to 3.97 m/s (the speed of the current).

(b) The shortest time is when the boat heads perpendicular to the current, which is due north. Figure 3.78c illustrates this situation. The time to cross is $t = d/v_{B/W} = (90.48\text{ m})/(6.00\text{ m/s}) = 15.1$ s. The distance x east (down river) that you travel is $x = v_{W/E}t = (3.967\text{ m/s})(15.1\text{ s}) = 59.9$ m east of your starting point.

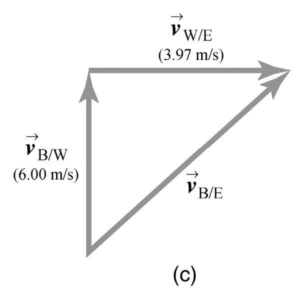


Figure 3.78c

EVALUATE: In part (a), the boat must have a velocity component up river to cancel out the current velocity. In part (b), velocity of the current has no effect on the crossing time, but it does affect the landing position of the boat.

3.79. **IDENTIFY:** Write an expression for the square of the distance (D^2) from the origin to the particle, expressed as a function of time. Then take the derivative of D^2 with respect to t, and solve for the value of t when this derivative is zero. If the discriminant is zero or negative, the distance D will never decrease.

SET UP: $D^2 = x^2 + y^2$, with $x(t)$ and $y(t)$ given by Eqs. (3.19) and (3.20).

EXECUTE: Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°$.

EVALUATE: We know that if the object is thrown straight up it moves away from P and then returns, so we are not surprised that the projectile angle must be less than some maximum value for the distance to always increase with time.

3.80. **IDENTIFY:** Apply the relative velocity relation.

SET UP: Let $v_{C/W}$ be the speed of the canoe relative to water and $v_{W/G}$ be the speed of the water relative to the ground.

EXECUTE: **(a)** Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{C/W} - v_{W/G} = 2$. We know that heading downstream for a time t, $(v_{C/W} + v_{W/G})t = 5$. We also know that for the bottle $v_{W/G}(t+1)=3$. Solving these three equations for $v_{W/G}=x$, $v_{C/W}=2+x$, therefore $(2+x+x)t=5$ or $(2+2x)t=5$. Also $t=3/x-1$, so $(2+2x)\left(\dfrac{3}{x}-1\right)=5$ or $2x^2+x-6=0$.

The positive solution is $x=v_{W/G}=1.5$ km/h.

(b) $v_{C/W}=2\text{ km/h}+v_{W/G}=3.5$ km/h.

EVALUATE: When they head upstream, their speed relative to the ground is $3.5\text{ km/h}-1.5\text{ km/h}=2.0$ km/h. When they head downstream, their speed relative to the ground is $3.5\text{ km/h}+1.5\text{ km/h}=5.0$ km/h. The bottle is moving downstream at 1.5 km/s relative to the earth, so they are able to overtake it.

3.81. **IDENTIFY:** The rocket has two periods of constant acceleration motion.

SET UP: Let $+y$ be upward. During the free-fall phase, $a_x=0$ and $a_y=-g$. After the engines turn on, $a_x=(3.00g)\cos 30.0°$ and $a_y=(3.00g)\sin 30.0°$. Let t be the total time since the rocket was dropped and let T be the time the rocket falls before the engine starts.

EXECUTE: (i) The diagram is given in Figure 3.81 a.

(ii) The x-position of the plane is $(236\text{ m/s})t$ and the x-position of the rocket is $(236\text{ m/s})t+(1/2)(3.00)(9.80\text{ m/s}^2)\cos 30°(t-T)^2$. The graphs of these two equations are sketched in Figure 3.81 b.

(iii) If we take $y=0$ to be the altitude of the airliner, then $y(t)=-1/2gT^2-gT(t-T)+1/2(3.00)(9.80\text{ m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. The airliner has constant y. The graphs are sketched in Figure 3.81b.

In each of the Figures 3.81a–c, the rocket is dropped at $t=0$ and the time T when the motor is turned on is indicated.

By setting $y=0$ for the rocket, we can solve for t in terms of T:

$0=-(4.90\text{ m/s}^2)T^2-(9.80\text{ m/s}^2)T(t-T)+(7.35\text{ m/s}^2)(t-T)^2$. Using the quadratic formula for the variable $x=t-T$ we find $x=t-T=\dfrac{(9.80\text{ m/s}^2)T+\sqrt{(9.80\text{ m/s}^2T)^2+(4)(7.35\text{ m/s}^2)(4.9)T^2}}{2(7.35\text{ m/s}^2)}$, or $t=2.72\,T$. Now, using the condition that $x_{\text{rocket}}-x_{\text{plane}}=1000$ m, we find $(236\text{ m/s})t+(12.7\text{ m/s}^2)(t-T)^2-(236\text{ m/s})t=1000$ m, or $(1.72T)^2=78.6\text{ s}^2$. Therefore $T=5.15$ s.

EVALUATE: During the free-fall phase the rocket and airliner have the same x coordinate but the rocket moves downward from the airliner. After the engines fire, the rocket starts to move upward and its horizontal component of velocity starts to exceed that of the airliner.

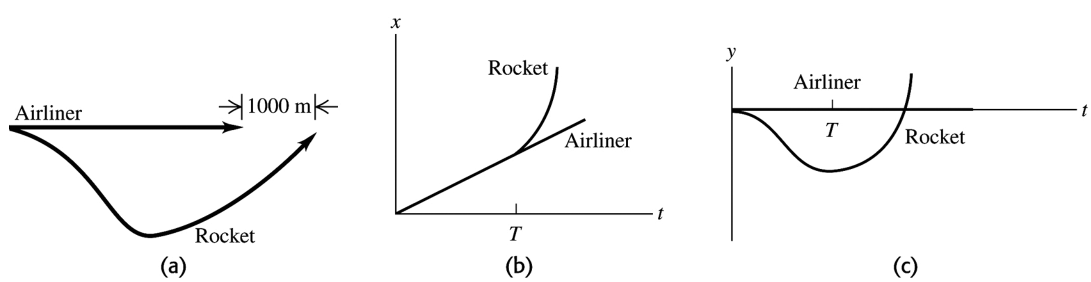


Figure 3. 81

3.82. **IDENTIFY:** We know the speed of the seeds and the distance they travel.

SET UP: We can treat the speed as constant over a very short distance, so $v = d/t$. The minimum frame rate is determined by the maximum speed of the seeds, so we use $v = 4.6$ m/s.

EXECUTE: Solving for t gives $t = d/v = (0.20 \times 10^{-3}\text{ s})/(4.6\text{ m/s}) = 4.3 \times 10^{-5}$ s per frame. The frame rate is $1/(4.3 \times 10^{-5}$ s per frame) = 23,000 frames/seconde. Choice (c) 25,000 frames per second is closest to this result, so choice (c) is the best one.

EVALUATE: This experiment would clearly require high-speed photography.

3.83. **IDENTIFY:** A seed launched at 90° goes straight up. Since we are ignoring air resistance, its acceleration is 9.80 m/s^2 downward.

SET UP: For the highest possible speed $v_{0y} = 4.6$ m/s, and $v_y = 0$ at the highest point.

EXECUTE: $v_y = v_{0y} - gt$ gives $t = v_{0y}/g = (4.6\text{ m/s})/(9.80\text{ m/s}^2) = 0.47$ s, which is choice (b).

EVALUATE: Seeds are rather light and 4.6 m/s is fairly fast, so it might not be such a good idea to ignore air resistance. But doing so is acceptable to get a first approximation to the time.

3.84. **IDENTIFY:** A seed launched at 0° starts out traveling horizontally from a height of 20 cm above the ground. Since we are ignoring air resistance, its acceleration is 9.80 m/s^2 downward.

SET UP: Its horizontal distance is determined by the time it takes the seed to fall 20 cm, starting from rest vertically.

EXECUTE: The time to fall 20 cm is $0.20\text{ m} = \frac{1}{2}gt^2$, which gives $t = 0.202$ s. The horizontal distance traveled during this time is $x = (4.6\text{ m/s})(0.202\text{ s}) = 0.93\text{ m} = 93$ cm, which is choice (b).

EVALUATE: In reality the seed would travel a bit less distance due to air resistance.

3.85. **IDENTIFY:** About 2/3 of the seeds are launched between 6° and 56° above the horizontal, and the average for all the seeds is 31°. So clearly most of the seeds are launched above the horizontal.

SET UP and **EXECUTE:** For choice (a) to be correct, the seeds would need to cluster around 90°, which they do not. For choice (b), most seeds would need to launch below the horizontal, which is not the case. For choice (c), the launch angle should be around +45°. Since 31° is not far from 45°, this is the best choice. For choice (d), the seeds should go straight downward. This would require a launch angle of –90°, which is not the case.

EVALUATE: Evolutionarily it would be an advantage for the seeds to get as far from the parent plant as possible so the young plants would not compete with the parent for water and soil nutrients, so 45° is a biologically plausible result. Natural selection would tend to favor plants that launched their seeds at this angle over those that did not.