Module MA2341 (Frolov), Advanced Mechanics I
Homework Sheet 8
Each set of homework questions is worth 100 marks

You may use Mathematica.

Problem 1

Find the equations of small oscillations under friction for a particle moving on the surface of a paraboloid $z = k(x^2 + y^2 + 4x - 6y)$ in a uniform gravitational field and being acted on by a force of friction $\vec{F}_{fr} = -\lambda\vec{v}$. Solve these equations, and determine the motion of the particle.

Problem 2

Determine the motion of an oscillator of mass $m = 1$ with frequency ω_0 due to an external force

$$F(t) = \begin{cases} F_0 e^{\alpha t} & t < 0 \\ F_0 e^{-\alpha t} & t > 0 \end{cases}, \quad F_0 > 0\,,\ \alpha > 0\,,$$

in the presence of the friction force $-2\lambda\dot{x}$. The initial energy as $t \to -\infty$ is $E_0 = 0$.

Use Mathematica to plot the solution for

$$\omega_0 = 1\,, \quad \lambda = 1.01\,, \quad F_0/m = 10\,, \quad \alpha = 1/1000\,,\ 1/32\,,\ 1/16\,,\ 1/8\,,\ 1/4\,,\ 1/2\,,\ 1\,,\ 2\,,\ 4\,,$$

and

$$\omega_0 = 1\,, \quad \lambda = .01\,, \quad F_0/m = 10\,, \quad \alpha = 1/1000\,,\ 1/32\,,\ 1/16\,,\ 1/8\,,\ 1/4\,,\ 1/2\,,\ 1\,,\ 2\,,\ 4\,.$$

Explain the results obtained.

Problem 3

The motion of a particle of mass m in the "Mexican hat" potential is described by the potential

$$U = \frac{k^2}{4g} - \frac{k}{2}x^2 + \frac{g}{4}x^4\,, \quad k > 0\,,\ g > 0\,.$$

Use Mathematica to plot the potential for $g = 1, k = 1, 2, 4$.

Find positions of stable equilibrium.

Find the equations of anharmonic oscillations in the vicinity of the equilibrium positions.

Find the fundamental frequencies up to the second order in the amplitude of the oscillations.