Module MA2341 (Frolov), Advanced Mechanics I
Homework Sheet 2
Each set of homework questions is worth 100 marks

Problem 1. Consider a particle of mass m moving on the surface

$$z = k\sqrt{x^2 + y^2 + c^2}\,, \quad k > 0\,,\ c > 0$$

in a uniform gravitational field $\vec{F} = \{0, 0, -mg\}$.

(a) What is the surface $z = k\sqrt{x^2 + y^2 + c^2}$, $k > 0$?
Use Mathematica to plot the surface for $k = c = 1$.

(b) Find the Lagrangian of the particle by using the polar coordinates r, ϕ.

(c) Find the equations of motion of the particle.

Problem 2. Consider a particle of mass m moving on the surface

$$x^2 + y^2 + \frac{z^2}{\kappa^2} = a^2\,, \quad a > 0\,,\ \kappa > 0$$

in a uniform gravitational field $\vec{F} = \{0, 0, -mg\}$.

(a) What is the surface $x^2 + y^2 + \frac{z^2}{\kappa^2} = a^2\,, \quad a > 0\,,\ \kappa > 0$?
Use Mathematica to plot the surface for $a = 2\,,\ \kappa = 1/2$.

(b) Introduce the spherical coordinates by using the physics conventions $(r\,, \theta\,, \varphi)$ (radial, polar, azimuthal), and draw the corresponding picture.

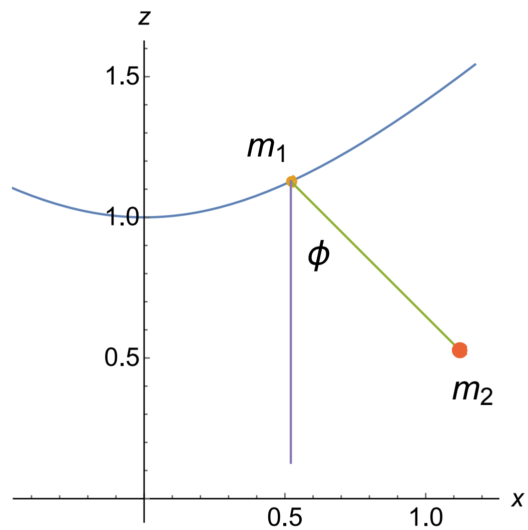
(c) Introduce coordinates $(\rho\,, \theta\,, \varphi)$ similar to the spherical ones so that the equation of the surface $x^2 + y^2 + \frac{z^2}{\kappa^2} = a^2\,, \quad a > 0\,,\ \kappa > 0$ in terms of these coordinates becomes $\rho = a$, and derive an expression for the Lagrangian of the particle in term of these coordinates.

(d) Find the equations of motion of the particle.

Problem 3. Consider a pendulum of mass m_2, with a mass m_1 at the point of support which can move on a curve in the vertical xz-plane defined parametrically by the equations $x = f(q)\,,\ z = h(q)$, where q is a parameter of the curve. Assume that the motion takes place only in the vertical xz-plane. The potential energy of the system is

$$U = m_1 g z_1 + m_2 g z_2\,,$$

where x_1 and x_2 are the coordinates of the particles.



(a) Find the Lagrangian of the system.

(b) Assume that $q = s$ where s is an arc length parameter, simplify the Lagrangian and find the eom.

(c) Let the curve be a hyperbola:

$$-\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1\,, \quad z > 0\,.$$

Introduce any parametrisation of the hyperbola, identify $f(q)$ and $g(q)$, and write the Lagrangian.

Use Mathematica to find an arc length parameter of the hyperbola as a function of your parameter.