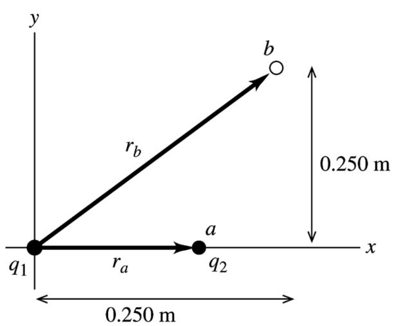
23

ELECTRIC POTENTIAL

23.1. IDENTIFY: Apply $W_{a\to b} = U_a - U_b$ to calculate the work. The electric potential energy of a pair of point charges is given by $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

SET UP: Let the initial position of q_2 be point a and the final position be point b, as shown in Figure 23.1.



$r_a = 0.150$ m.

$r_b = \sqrt{(0.250\text{ m})^2 + (0.250\text{ m})^2}$.

$r_b = 0.3536$ m.

Figure 23.1

EXECUTE: $W_{a\to b} = U_a - U_b$.

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.150\text{ m}}.$$

$U_a = -0.6184$ J.

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.3536\text{ m}}.$$

$U_b = -0.2623$ J.

$$W_{a\to b} = U_a - U_b = -0.6184\text{ J} - (-0.2623\text{ J}) = -0.356\text{ J}.$$

EVALUATE: The attractive force on q_2 is toward the origin, so it does negative work on q_2 when q_2 moves to larger r.

23.2. IDENTIFY: Apply $W_{a\to b} = U_a - U_b$.

SET UP: $U_a = +5.4\times10^{-8}$ J. Solve for U_b.

EXECUTE: $W_{a\to b} = -1.9\times10^{-8}\text{ J} = U_a - U_b$. $U_b = U_a - W_{a\to b} = +5.4\times10^{-8}\text{ J} - (-1.9\times10^{-8}\text{ J}) = 7.3\times10^{-8}$ J.

EVALUATE: When the electric force does negative work the electrical potential energy increases.

23.3. IDENTIFY: The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

SET UP: The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is e and the charges are equidistant from each other, so the total potential energy is $U = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r}+\dfrac{e^2}{r}+\dfrac{e^2}{r}\right) = \dfrac{3e^2}{4\pi\epsilon_0 r}$.

EXECUTE: Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60\times10^{-19}\ \text{C})^2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{2.00\times10^{-15}\ \text{m}} = 3.46\times10^{-13}\ \text{J} = 2.16\ \text{MeV}.$$

EVALUATE: This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

23.4. IDENTIFY: The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

(a) SET UP: Using the potential energy of a pair of point charges relative to infinity, $U = (1/4\pi\epsilon_0)(qq_0/r)$, we have $W = \Delta U = U_2 - U_1 = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r_2}-\dfrac{e^2}{r_1}\right)$.

EXECUTE: Factoring out the e^2 and substituting numbers gives

$$W = (9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2\left(\frac{1}{3.00\times10^{-15}\ \text{m}} - \frac{1}{2.00\times10^{-10}\ \text{m}}\right) = 7.68\times10^{-14}\ \text{J}$$

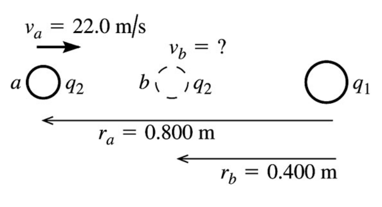
(b) SET UP: The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U = K_1 + K_2 = 2K = 2\left(\dfrac{1}{2}mv^2\right) = mv^2$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{\Delta U}{m}} = \sqrt{\dfrac{7.68\times10^{-14}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = 6.78\times10^6\ \text{m/s}$.

EVALUATE: The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

23.5. (a) IDENTIFY: Use conservation of energy: $K_a + U_a + W_{\text{other}} = K_b + U_b$. U for the pair of point charges is given by $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

SET UP:



Let point a be where q_2 is 0.800 m from q_1 and point b be where q_2 is 0.400 m from q_1, as shown in Figure 23.5a.

Figure 23.5a

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

$$K_a = \tfrac{1}{2}mv_a^2 = \tfrac{1}{2}(1.50\times10^{-3}\ \text{kg})(22.0\ \text{m/s})^2 = 0.3630\ \text{J}.$$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_a} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.800\ \text{m}} = +0.2454\ \text{J}.$$

$$K_b = \tfrac{1}{2}mv_b^2.$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.400\ \text{m}} = +0.4907\ \text{J}.$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$.

$$\tfrac{1}{2}mv_b^2 = +0.3630\ \text{J} + (0.2454\ \text{J} - 0.4907\ \text{J}) = 0.1177\ \text{J}.$$

$$v_b = \sqrt{\frac{2(0.1177\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 12.5\ \text{m/s}.$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) IDENTIFY: Let point c be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points a and c: $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points a and c are shown in Figure 23.5b.

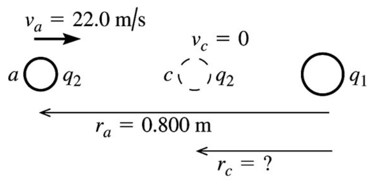


Figure 23.5b

EXECUTE: $K_a = +0.3630\ \text{J}$ (from part (a)).
$U_a = +0.2454\ \text{J}$ (from part (a)).

$K_c = 0$ (at distance of closest approach the speed is zero).

$$U_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_c}.$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r_c} = +0.3630\ \text{J} + 0.2454\ \text{J} = 0.6084\ \text{J}.$

$$r_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{0.6084\ \text{J}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{+0.6084\ \text{J}} = 0.323\ \text{m}.$$

EVALUATE: $U \to \infty$ as $r \to 0$ so q_2 will stop no matter what its initial speed is.

23.6. IDENTIFY: The total potential energy is the scalar sum of the individual potential energies of each pair of charges.

SET UP: For a pair of point charges the electrical potential energy is $U = k\dfrac{qq'}{r}$. In the O-H-N combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. U is positive for like charges and negative for unlike charges.

EXECUTE: (a) O-H-N:

$$O^-\text{-}H^+: U = -(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.170\times10^{-9}\ \text{m}} = -1.35\times10^{-18}\ \text{J}.$$

$$O^-\text{-}N^-: U = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.280\times10^{-9}\ \text{m}} = +8.22\times10^{-19}\ \text{J}.$$

N-H-N:

$$N^-\text{-}H^+: U = -(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.190\times10^{-9}\ \text{m}} = -1.21\times10^{-18}\ \text{J}.$$

$$\text{N}^-\text{-N}^-: U = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.300\times10^{-9}\ \text{m}} = +7.67\times10^{-19}\ \text{J}.$$

The total potential energy is

$U_{\text{tot}} = -1.35\times10^{-18}\ \text{J} + 8.22\times10^{-19}\ \text{J} - 1.21\times10^{-18}\ \text{J} + 7.67\times10^{-19}\ \text{J} = -9.71\times10^{-19}\ \text{J}.$

(b) In the hydrogen atom the electron is 0.0529 nm from the proton.

$$U = -(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.0529\times10^{-9}\ \text{m}} = -4.35\times10^{-18}\ \text{J}.$$

EVALUATE: The magnitude of the potential energy in the hydrogen atom is about a factor of 4 larger than what it is for the adenine-thymine bond.

23.7. **IDENTIFY:** Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach r_b. The maximum force is at the distance of closest approach, $F = k\frac{|q_1q_2|}{r_b^2}$.

SET UP: $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass 1.67×10^{-27} kg and charge $q = +e = +1.60\times10^{-19}$ C.

EXECUTE: $K_a = U_b$. $2(\frac{1}{2}mv_a^2) = k\frac{q_1q_2}{r_b}$. $mv_a^2 = k\frac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.67\times10^{-27}\ \text{kg})(2.00\times10^5\ \text{m/s})^2} = 3.45\times10^{-12}\ \text{m}.$$

$$F = k\frac{e^2}{r_b^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(3.445\times10^{-12}\ \text{m})^2} = 1.94\times10^{-5}\ \text{N}.$$

EVALUATE: The acceleration $a = F/m$ of each proton produced by this force is extremely large.

23.8. **IDENTIFY:** Call the three charges 1, 2, and 3. $U = U_{12} + U_{13} + U_{23}$.

SET UP: $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.400 m.

EXECUTE: $U = \frac{3kq^2}{0.400\ \text{m}} = \frac{3k(1.2\times10^{-6}\ \text{C})^2}{0.400\ \text{m}} = 0.0971\ \text{J}.$

EVALUATE: When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

23.9. **IDENTIFY:** The protons repel each other and therefore accelerate away from one another. As they get farther and farther away, their kinetic energy gets greater and greater but their acceleration keeps decreasing. Conservation of energy and Newton's laws apply to these protons.

SET UP: Let *a* be the point when they are 0.750 nm apart and *b* be the point when they are very far apart. A proton has charge $+e$ and mass 1.67×10^{-27} kg. As they move apart the protons have equal kinetic energies and speeds. Their potential energy is $U = ke^2/r$ and $K = \frac{1}{2}mv^2$. $K_a + U_a = K_b + U_b$.

EXECUTE: **(a)** They have maximum speed when they are far apart and all their initial electrical potential energy has been converted to kinetic energy. $K_a + U_a = K_b + U_b$.

$K_a = 0$ and $U_b = 0$, so

$$K_b = U_a = k\frac{e^2}{r_a} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.750\times10^{-9}\ \text{m}} = 3.07\times10^{-19}\ \text{J}.$$

$K_b = \frac{1}{2}mv_b^2 + \frac{1}{2}mv_b^2$, so $K_b = mv_b^2$ and $v_b = \sqrt{\frac{K_b}{m}} = \sqrt{\frac{3.07\times10^{-19}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = 1.36\times10^4\ \text{m/s}.$

(b) Their acceleration is largest when the force between them is largest and this occurs at $r = 0.750$ nm, when they are closest.

$$F = k\frac{e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{1.60\times10^{-19}\ \text{C}}{0.750\times10^{-9}\ \text{m}}\right)^2 = 4.09\times10^{-10}\ \text{N}.$$

$$a = \frac{F}{m} = \frac{4.09\times10^{-10}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 2.45\times10^{17}\ \text{m/s}^2.$$

EVALUATE: The acceleration of the protons decreases as they move farther apart, but the force between them is repulsive so they continue to increase their speeds and hence their kinetic energies.

23.10. IDENTIFY: The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

SET UP: We apply the formula $W_{a\to b} = U_a - U_b$. In this case, a is the center of the square and b is the midpoint of one of the sides. Therefore $W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}}$ is the work done by the Coulomb force. There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}$ nm from each electron. At the midpoint of the side, the alpha is a distance $r_2 = 5.00$ nm from the two nearest electrons and a distance $r_3 = \sqrt{125}$ nm from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\varepsilon_0)(qq_0/r)$, the total work done by the Coulomb force is

$$W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}} = 4\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\text{e}}{r_1} - \left(2\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\text{e}}{r_2} + 2\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\text{e}}{r_3}\right).$$

Substituting $q_\text{e} = -e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center}\to\text{side}} = -4e^2\frac{1}{4\pi\varepsilon_0}\left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3}\right)\right].$$

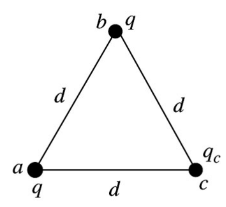
EXECUTE: Substituting the numerical values into the equation for the work gives

$$W = -4(1.60\times10^{-19}\ \text{C})^2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[\frac{2}{\sqrt{50}\ \text{nm}} - \left(\frac{1}{5.00\ \text{nm}} + \frac{1}{\sqrt{125}\ \text{nm}}\right)\right] = 6.08\times10^{-21}\ \text{J}.$$

EVALUATE: Since the work done by the Coulomb force is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side. To move the alpha particle to the midpoint of a side and leave it there at rest an external force must do -6.08×10^{-21} J of work.

23.11. IDENTIFY: Apply $W_{a\to b} = U_a - U_b$. The net work to bring the charges in from infinity is equal to the change in potential energy. The total potential energy is the sum of the potential energies of each pair of charges, calculated from $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$.

SET UP: Let 1 be where all the charges are infinitely far apart. Let 2 be where the charges are at the corners of the triangle, as shown in Figure 23.11.



Let q_c be the third, unknown charge.

Figure 23.11

EXECUTE: $W = -\Delta U = -(U_2 - U_1)$, where W is the work done by the Coulomb force.

$$U_1 = 0$$

$$U_2 = U_{ab} + U_{ac} + U_{bc} = \frac{1}{4\pi\varepsilon_0 d}(q^2 + 2qq_c).$$

We want $W = 0$, so $W = -(U_2 - U_1)$ gives $0 = -U_2$.

$$0 = \frac{1}{4\pi\varepsilon_0 d}(q^2 + 2qq_c).$$

$q^2 + 2qq_c = 0$ and $q_c = -q/2$.

EVALUATE: The potential energy for the two charges q is positive and for each q with q_c it is negative. There are two of the q, q_c terms so must have $q_c < q$.

23.12. IDENTIFY: Work is done on the object by the electric field, and this changes its kinetic energy, so we can use the work-energy theorem.

SET UP: $W_{A\to B} = \Delta K$ and $W_{A\to B} = q(V_A - V_B)$.

EXECUTE: (a) Applying the two equations above gives $W_{A\to B} = q(V_A - V_B) = K_B - 0 = K_B$.

$V_B = V_A - K_B/q = 30.0 \text{ V} - (3.00\times10^{-7}\text{ J})/(-6.00\times10^{-9}\text{ C}) = 80.0 \text{ V}$.

(b) The negative charge accelerates from A to B, so the electric field must point from B toward A. Since the field is uniform, we have $E = \dfrac{\Delta V}{\Delta x}$ = (50.0 V)/(0.500 m) = 100 V/m.

EVALUATE: A positive charge is accelerated from high to low potential, but a negative charge (as we have here) is accelerated from low to high potential.

23.13. IDENTIFY and SET UP: Apply conservation of energy to points A and B.

EXECUTE: $K_A + U_A = K_B + U_B$.

$U = qV$, so $K_A + qV_A = K_B + qV_B$.

$K_B = K_A + q(V_A - V_B) = 0.00250 \text{ J} + (-5.00\times10^{-6} \text{ C})(200 \text{ V} - 800 \text{ V}) = 0.00550 \text{ J}$.

$v_B = \sqrt{2K_B/m} = 7.42 \text{ m/s}$.

EVALUATE: It is faster at B; a negative charge gains speed when it moves to higher potential.

23.14. IDENTIFY: The work-energy theorem says $W_{a\to b} = K_b - K_a$. $\dfrac{W_{a\to b}}{q} = V_a - V_b$.

SET UP: Point a is the starting point and point b is the ending point. Since the field is uniform, $W_{a\to b} = Fs\cos\phi = E|q|s\cos\phi$. The field is to the left so the force on the positive charge is to the left. The particle moves to the left so $\phi = 0°$ and the work $W_{a\to b}$ is positive.

EXECUTE: (a) $W_{a\to b} = K_b - K_a = 2.20\times10^{-6} \text{ J} - 0 = 2.20\times10^{-6}$ J.

(b) $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{2.20\times10^{-6}\text{ J}}{4.20\times10^{-9}\text{ C}} = 524 \text{ V}$. Point a is at higher potential than point b.

(c) $E|q|s = W_{a\to b}$, so $E = \dfrac{W_{a\to b}}{|q|s} = \dfrac{V_a - V_b}{s} = \dfrac{524 \text{ V}}{6.00\times10^{-2}\text{ m}} = 8.73\times10^3 \text{ V/m}$.

EVALUATE: A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

23.15. IDENTIFY: Apply $W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l}$. Use coordinates where $+y$ is upward and $+x$ is to the right. Then $\vec{E} = E\hat{j}$ with $E = 4.00\times10^4$ N/C.

SET UP: (a) The path is sketched in Figure 23.15a.

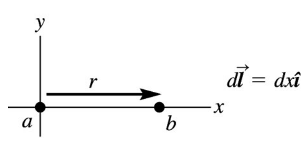
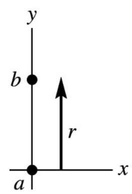


Figure 23.15a

EXECUTE: $\vec{E}\cdot d\vec{l}=(E\hat{j})\cdot(dx\hat{i})=0$ so $W_{a\to b}=q'\int_a^b \vec{E}\cdot d\vec{l}=0.$

EVALUATE: The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

(b) SET UP: The path is sketched in Figure 23.15b.



$d\vec{l}=dy\hat{j}.$

Figure 23.15b

EXECUTE: $\vec{E}\cdot d\vec{l}=(E\hat{j})\cdot(dy\hat{j})=E\,dy.$

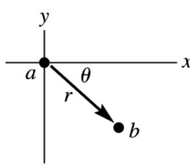
$$W_{a\to b}=q'\int_a^b \vec{E}\cdot d\vec{l}=q'E\int_a^b dy=q'E(y_b-y_a).$$

$y_b-y_a=+0.670$ m; it is positive since the displacement is upward and we have taken $+y$ to be upward.

$$W_{a\to b}=q'E(y_b-y_a)=(+28.0\times10^{-9}\text{ C})(4.00\times10^{4}\text{ N/C})(+0.670\text{ m})=+7.50\times10^{-4}\text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

(c) SET UP: The path is sketched in Figure 23.15c.



$y_a=0.$

$y_b=-r\sin\theta=-(2.60\text{ m})\sin45^\circ=-1.838$ m.

The vertical component of the 2.60 m displacement is 1.838 m downward.

Figure 23.15c

EXECUTE: $d\vec{l}=dx\hat{i}+dy\hat{j}$ (The displacement has both horizontal and vertical components.)

$\vec{E}\cdot d\vec{l}=(E\hat{j})\cdot(dx\hat{i}+dy\hat{j})=E\,dy$ (Only the vertical component of the displacement contributes to the work.)

$$W_{a\to b}=q'\int_a^b \vec{E}\cdot d\vec{l}=q'E\int_a^b dy=q'E(y_b-y_a).$$

$$W_{a\to b}=q'E(y_b-y_a)=(+28.0\times10^{-9}\text{C})(4.00\times10^{4}\text{ N/C})(-1.838\text{ m})=-2.06\times10^{-3}\text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

23.16. IDENTIFY: Apply $K_a+U_a=K_b+U_b$.

SET UP: Let $q_1=+3.00$ nC and $q_2=+2.00$ nC. At point *a*, $r_{1a}=r_{2a}=0.250$ m. At point *b*, $r_{1b}=0.100$ m and $r_{2b}=0.400$ m. The electron has $q=-e$ and $m_e=9.11\times10^{-31}$ kg. $K_a=0$ since the electron is released from rest.

EXECUTE: $-\dfrac{keq_1}{r_{1a}}-\dfrac{keq_2}{r_{2a}}=-\dfrac{keq_1}{r_{1b}}-\dfrac{keq_2}{r_{2b}}+\dfrac{1}{2}m_e v_b^2.$

$$E_a = K_a + U_a = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.250\text{ m}}+\frac{(2.00\times10^{-9}\text{ C})}{0.250\text{ m}}\right) = -2.88\times10^{-17}\text{ J.}$$

$$E_b = K_b + U_b = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.100\text{ m}}+\frac{(2.00\times10^{-9}\text{ C})}{0.400\text{ m}}\right)+\frac{1}{2}m_e v_b^2 = -5.04\times10^{-17}\text{ J}+\frac{1}{2}m_e v_b^2.$$

Setting $E_a = E_b$ gives $v_b = \sqrt{\dfrac{2}{9.11\times10^{-31}\text{ kg}}(5.04\times10^{-17}\text{ J}-2.88\times10^{-17}\text{ J})} = 6.89\times10^{6}\text{ m/s.}$

EVALUATE: $V_a = V_{1a}+V_{2a} = 180\text{ V}$. $V_b = V_{1b}+V_{2b} = 315\text{ V}$. $V_b > V_a$. The negatively charged electron gains kinetic energy when it moves to higher potential.

23.17. IDENTIFY: The potential at any point is the scalar sum of the potentials due to individual charges.

SET UP: $V = kq/r$ and $W_{ab} = q(V_a - V_b)$.

EXECUTE: **(a)** $r_{a1} = r_{a2} = \dfrac{1}{2}\sqrt{(0.0300\text{ m})^2+(0.0300\text{ m})^2} = 0.0212\text{ m}$. $V_a = k\left(\dfrac{q_1}{r_{a1}}+\dfrac{q_2}{r_{a2}}\right) = 0.$

(b) $r_{b1} = 0.0424\text{ m}$, $r_{b2} = 0.0300\text{ m}$.

$$V_b = k\left(\frac{q_1}{r_{b1}}+\frac{q_2}{r_{b2}}\right) = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.00\times10^{-6}\text{ C}}{0.0424\text{ m}}+\frac{-2.00\times10^{-6}\text{ C}}{0.0300\text{ m}}\right) = -1.75\times10^{5}\text{ V.}$$

(c) $W_{ab} = q_3(V_a - V_b) = (-5.00\times10^{-6}\text{ C})[0-(-1.75\times10^{5}\text{ V})] = -0.875\text{ J.}$

EVALUATE: Since $V_b < V_a$, a positive charge would be pulled by the existing charges from *a* to *b*, so they would do positive work on this charge. But they would repel a negative charge and hence do negative work on it, as we found in part (c).

23.18. IDENTIFY: The total potential is the *scalar* sum of the individual potentials, but the net electric field is the *vector* sum of the two fields.

SET UP: The net potential can only be zero if one charge is positive and the other is negative, since it is a scalar. The electric field can only be zero if the two fields point in opposite directions.

EXECUTE: **(a)** (i) Since both charges have the same sign, there are no points for which the potential is zero.
(ii) The two electric fields are in opposite directions only between the two charges, and midway between them the fields have equal magnitudes. So $E = 0$ midway between the charges, but *V* is never zero.
(b) (i) The two potentials have equal magnitude but opposite sign midway between the charges, so $V = 0$ midway between the charges, but $E \neq 0$ there since the fields point in the same direction.
(ii) Between the two charges, the fields point in the same direction, so *E* cannot be zero there. In the other two regions, the field due to the nearer charge is always greater than the field due to the more distant charge, so they cannot cancel. Hence *E* is not zero anywhere.

EVALUATE: It does *not* follow that the electric field is zero where the potential is zero, or that the potential is zero where the electric field is zero.

23.19. IDENTIFY: Apply $V = \dfrac{1}{4\pi\epsilon_0}\sum_i \dfrac{q_i}{r_i}$.

SET UP: The locations of the charges and points *A* and *B* are sketched in Figure 23.19.

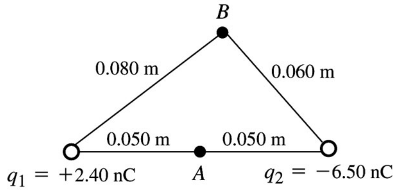


Figure 23.19

EXECUTE: **(a)** $V_A = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{A1}} + \dfrac{q_2}{r_{A2}}\right)$.

$$V_A = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times10^{-9}\ \text{C}}{0.050\ \text{m}} + \frac{-6.50\times10^{-9}\ \text{C}}{0.050\ \text{m}}\right) = -737\ \text{V}.$$

(b) $V_B = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{B1}} + \dfrac{q_2}{r_{B2}}\right)$.

$$V_B = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times10^{-9}\ \text{C}}{0.080\ \text{m}} + \frac{-6.50\times10^{-9}\ \text{C}}{0.060\ \text{m}}\right) = -704\ \text{V}.$$

(c) IDENTIFY and **SET UP:** Use $W_{a\to b} = q(V_a - V_b)$ and the results of parts (a) and (b) to calculate W.

EXECUTE: $W_{B\to A} = q(V_B - V_A) = (2.50\times10^{-9}\ \text{C})[-704\ \text{V} - (-737\ \text{V})] = +8.2\times10^{-8}\ \text{J}$.

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point B) to lower potential (point A).

23.20. **IDENTIFY** and **SET UP:** Apply conservation of energy: $K_a + U_a = K_b + U_b$. Use $V = U/q_0$ to express U in terms of V.

(a) EXECUTE: $K_1 + qV_1 = K_2 + qV_2$, $q(V_2 - V_1) = K_1 - K_2$; $q = -1.602\times10^{-19}$ C.

$K_1 = \frac{1}{2}m_e v_1^2 = 4.099\times10^{-18}$ J; $K_2 = \frac{1}{2}m_e v_2^2 = 2.915\times10^{-17}$ J. $\Delta V = V_2 - V_1 = \dfrac{K_1 - K_2}{q} = 156$ V.

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) EXECUTE: Now $K_1 = 2.915\times10^{-17}$ J, $K_2 = 0$. $V_2 - V_1 = \dfrac{K_1 - K_2}{q} = -182$ V.

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

23.21. **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: **(a)** The positions of the two charges are shown in Figure 23.21a.

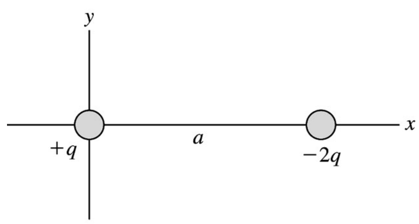


Figure 23.21a

(b) $x > a: V = \dfrac{kq}{x} - \dfrac{2kq}{x-a} = \dfrac{-kq(x+a)}{x(x-a)}$. $0 < x < a: V = \dfrac{kq}{x} - \dfrac{2kq}{a-x} = \dfrac{kq(3x-a)}{x(x-a)}$.

$x < 0: V = \dfrac{-kq}{x} + \dfrac{2kq}{x-a} = \dfrac{kq(x+a)}{x(x-a)}$. A general expression valid for any y is $V = k\left(\dfrac{q}{|x|} - \dfrac{2q}{|x-a|}\right)$.

(c) The potential is zero at $x = -a$ and $a/3$.

(d) The graph of V versus x is sketched in Figure 23.21b (next page).

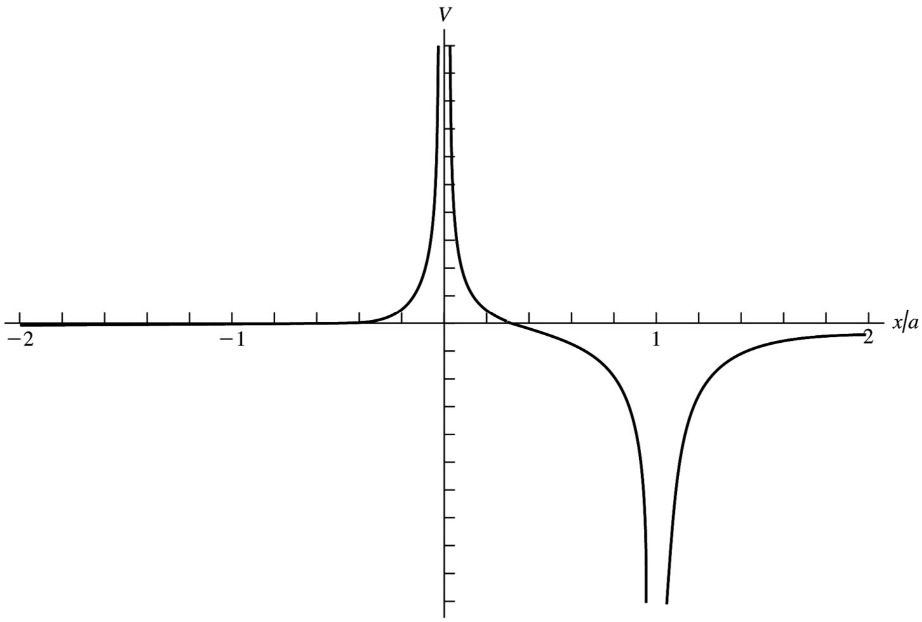


Figure 23.21b

EVALUATE: **(e)** For $x >> a: V \approx \dfrac{-kqx}{x^2} = \dfrac{-kq}{x}$, which is the same as the potential of a point charge $-q$. Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

23.22. **IDENTIFY:** For a point charge, $E = \dfrac{k|q|}{r^2}$ and $V = \dfrac{kq}{r}$.

SET UP: The electric field is directed toward a negative charge and away from a positive charge.

EXECUTE: **(a)** $V > 0$ so $q > 0$. $\dfrac{V}{E} = \dfrac{kq/r}{k|q|/r^2} = \left(\dfrac{kq}{r}\right)\left(\dfrac{r^2}{kq}\right) = r$. $r = \dfrac{4.98\text{ V}}{16.2\text{ V/m}} = 0.307\text{ m}$.

(b) $q = \dfrac{rV}{k} = \dfrac{(0.307\text{ m})(4.98\text{V})}{8.99 \times 10^9\text{ N} \cdot \text{m}^2/\text{C}^2} = 1.70 \times 10^{-10}\text{ C}$.

(c) $q > 0$, so the electric field is directed away from the charge.

EVALUATE: The ratio of V to E due to a point charge increases as the distance r from the charge increases, because E falls off as $1/r^2$ and V falls off as $1/r$.

23.23. **(a) IDENTIFY** and **EXECUTE:** The direction of $\vec{E}$ is always from high potential to low potential so point b is at higher potential.

(b) IDENTIFY and **SET UP:** Apply $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$ to relate $V_b - V_a$ to E.

EXECUTE: $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E\,dx = E(x_b - x_a)$.

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240\text{ V}}{0.90\text{ m} - 0.60\text{ m}} = 800\text{ V/m}$$

(c) SET UP and **EXECUTE:** $W_{b \to a} = q(V_b - V_a) = (-0.200 \times 10^{-6}\text{ C})(+240\text{ V}) = -4.80 \times 10^{-5}\text{ J}$.

EVALUATE: The electric force does negative work on a negative charge when the negative charge moves from high potential (point b) to low potential (point a).

24.24. IDENTIFY: For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges. For a point charge, $E = \dfrac{k|q|}{r^2}$. The net electric field is the vector sum of the electric fields of the two charges.

SET UP: $\vec{E}$ produced by a point charge is directed away from the point charge if it is positive and toward the charge if it is negative.

EXECUTE: **(a)** $V = V_Q + V_{2Q} > 0$, so V is zero nowhere except for infinitely far from the charges. The fields can cancel only between the charges, because only there are the fields of the two charges in opposite directions. Consider a point a distance x from Q and $d - x$ from $2Q$, as shown in Figure 23.24a.

$E_Q = E_{2Q} \rightarrow \dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d-x)^2} \rightarrow (d-x)^2 = 2x^2$. $x = \dfrac{d}{1+\sqrt{2}}$. The other root, $x = \dfrac{d}{1-\sqrt{2}}$, does not lie between the charges.

(b) V can be zero in 2 places, A and B, as shown in Figure 23.24b. Point A is a distance x from $-Q$ and $d - x$ from $2Q$. B is a distance y from $-Q$ and $d + y$ from $2Q$. At A: $\dfrac{k(-Q)}{x} + \dfrac{k(2Q)}{d-x} = 0 \rightarrow x = d/3$.

At B: $\dfrac{k(-Q)}{y} + \dfrac{k(2Q)}{d+y} = 0 \rightarrow y = d$.

The two electric fields are in opposite directions to the left of $-Q$ or to the right of $2Q$ in Figure 23.24c. But for the magnitudes to be equal, the point must be closer to the charge with smaller magnitude of charge. This can be the case only in the region to the left of $-Q$. $E_Q = E_{2Q}$ gives $\dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d+x)^2}$ and

$x = \dfrac{d}{\sqrt{2}-1}$.

EVALUATE: **(d)** E and V are not zero at the same places. $\vec{E}$ is a vector and V is a scalar. E is proportional to $1/r^2$ and V is proportional to $1/r$. $\vec{E}$ is related to the force on a test charge and ΔV is related to the work done on a test charge when it moves from one point to another.

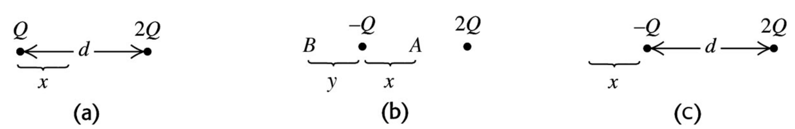


Figure 23.24

23.25. IDENTIFY: The potential at any point is the scalar sum of the potential due to each shell.

SET UP: $V = \dfrac{kq}{R}$ for $r \le R$ and $V = \dfrac{kq}{r}$ for $r > R$.

EXECUTE: **(a)** (i) $r = 0$. This point is inside both shells so

$$V = k\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\ \text{C}}{0.0300\ \text{m}} + \frac{-9.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}}\right).$$

$V = +1.798\times10^3\ \text{V} + (-1.618\times10^3\ \text{V}) = 180\ \text{V}$.

(ii) $r = 4.00$ cm. This point is outside shell 1 and inside shell 2.

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\ \text{C}}{0.0400\ \text{m}} + \frac{-9.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}}\right).$$

$V = +1.348\times10^3\ \text{V} + (-1.618\times10^3\ \text{V}) = -270\ \text{V}$.

(iii) $r = 6.00$ cm. This point is outside both shells.

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{r}\right) = \frac{k}{r}(q_1 + q_2) = \frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{0.0600\ \text{m}}\left[6.00\times10^{-9}\ \text{C} + (-9.00\times10^{-9}\ \text{C})\right].\quad V = -450\ \text{V}.$$

(b) At the surface of the inner shell, $r = R_1 = 3.00$ cm. This point is inside the larger shell,

so $V_1 = k\left(\dfrac{q_1}{R_1} + \dfrac{q_2}{R_2}\right) = 180$ V. At the surface of the outer shell, $r = R_2 = 5.00$ cm. This point is outside the smaller shell, so

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}} + \frac{-9.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}}\right).$$

$V_2 = +1.079\times10^3\ \text{V} + (-1.618\times10^3\ \text{V}) = -539$ V. The potential difference is $V_1 - V_2 = 719$ V. The inner shell is at higher potential. The potential difference is due entirely to the charge on the inner shell.

EVALUATE: Inside a uniform spherical shell, the electric field is zero so the potential is constant (but *not* necessarily zero).

23.26. IDENTIFY and SET UP: Outside a solid conducting sphere $V = k\dfrac{q}{r}$. Inside the sphere the potential is constant because $E = 0$, and it has the same value as at the surface of the sphere.

EXECUTE: **(a)** This is outside the sphere, so $V = \dfrac{kq}{r} = \dfrac{k(3.50\times10^{-9}\ \text{C})}{0.480\ \text{m}} = 65.6\ \text{V}.$

(b) This is at the surface of the sphere, so $V = \dfrac{k(3.50\times10^{-9}\ \text{C})}{0.240\ \text{m}} = 131\ \text{V}.$

(c) This is inside the sphere. The potential has the same value as at the surface, 131 V.

EVALUATE: All points of a conductor are at the same potential.

23.27. (a) IDENTIFY and SET UP: The electric field on the ring's axis is given by $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2 + a^2)^{3/2}}$. The magnitude of the force on the electron exerted by this field is given by $F = eE$.

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with a at the initial position of the electron and b at the center of the ring.

From Example 23.11, $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{\sqrt{x^2 + a^2}}$, where a is the radius of the ring.

EXECUTE: $x_a = 30.0$ cm, $x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$.

Thus $\frac{1}{2}mv^2 = U_a - U_b$.

And $U = qV = -eV$ so $v = \sqrt{\dfrac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_a^2 + a^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{\sqrt{(0.300\ \text{m})^2 + (0.150\ \text{m})^2}}.$$

$$V_a = 643\ \text{V}.$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_b^2 + a^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{0.150\ \text{m}} = 1438\ \text{V}.$$

$$v=\sqrt{\frac{2e(V_b-V_a)}{m}}=\sqrt{\frac{2(1.602\times10^{-19}\ \text{C})(1438\ \text{V}-643\ \text{V})}{9.109\times10^{-31}\ \text{kg}}}=1.67\times10^{7}\ \text{m/s}.$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

23.28. **IDENTIFY:** For an isolated conducting sphere, all the excess charge is on its outer surface. For points outside the sphere, it behaves like a point-charge at its center, and the electric field is zero inside the sphere.

SET UP: Use V at 1.20 m to find V at the surface. $V=k\frac{q}{r}$. We don't know the charge on the sphere, but we know the potential 1.20 m from its center.

EXECUTE: Take the ratio of the potentials: $\frac{V_{\text{surface}}}{V_{1.20\ \text{m}}}=\frac{kq/(0.400\ \text{m})}{kq(1.20\ \text{m})}=\frac{1.20}{0.400}=3.00,$ so $V_{\text{surface}}=(3.00)(24.0\ \text{V})=72.0\ \text{V}$.

The electric field is zero inside the sphere, so the potential inside is constant and equal to the potential at the surface. So at the center V = 72.0 V.

EVALUATE: An alternative approach would be to use the given information to find the charge on the sphere. Then use that charge to calculate the potential at the surface. The potential is 72.0 V at *all* points inside the sphere, not just at the center. Careful! Just because the electric field inside the sphere is zero, it does not follow that the potential is zero there.

23.29. **IDENTIFY:** If the small sphere is to have its *minimum* speed, it must just stop at 8.00 cm from the surface of the large sphere. In that case, the initial kinetic energy of the small sphere is all converted to electrical potential energy at its point of closest approach.

SET UP: $K_1+U_1=K_2+U_2$. $K_2=0$. $U_1=0$. Therefore, $K_1=U_2$. Outside a spherical charge distribution the potential is the same as for a point charge at the location of the center of the sphere, so $U=kqQ/r$. $K=\frac{1}{2}mv^2$.

EXECUTE: $U_2=\frac{kqQ}{r_2}$, with $r_2=12.0\ \text{cm}+8.0\ \text{cm}=0.200\ \text{m}$. $\frac{1}{2}mv_1^2=\frac{kqQ}{r_2}$.

$$v_1=\sqrt{\frac{2kqQ}{mr_2}}=\sqrt{\frac{2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\ \text{C})(5.00\times10^{-6}\ \text{C})}{(6.00\times10^{-5}\ \text{kg})(0.200\ \text{m})}}=150\ \text{m/s}.$$

EVALUATE: If the small sphere had enough initial speed to actually penetrate the surface of the large sphere, we could no longer treat the large sphere as a point charge once the small sphere was inside.

23.30. **IDENTIFY:** For a line of charge, $V_a-V_b=\frac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Apply conservation of energy to the motion of the proton.

SET UP: Let point *a* be 18.0 cm from the line and let point *b* be at the distance of closest approach, where $K_b=0$.

EXECUTE: **(a)** $K_a=\frac{1}{2}mv^2=\frac{1}{2}(1.67\times10^{-27}\ \text{kg})(3.50\times10^{3}\ \text{m/s})^2=1.02\times10^{-20}\ \text{J}$.

(b) $K_a+qV_a=K_b+qV_b$. $V_a-V_b=\frac{K_b-K_a}{q}=\frac{-1.02\times10^{-20}\ \text{J}}{1.60\times10^{-19}\ \text{C}}=-0.06397\ \text{V}$.

$$\ln(r_b/r_a)=\left(\frac{2\pi\epsilon_0}{\lambda}\right)(-0.06397\ \text{V}).$$

$$r_b=r_a\exp\left(\frac{2\pi\epsilon_0(-0.06397\ \text{V})}{\lambda}\right)=(0.180\ \text{m})\exp\left(-\frac{2\pi\epsilon_0(0.06397\ \text{V})}{5.00\times10^{-12}\ \text{C/m}}\right)=0.0883\ \text{m}=8.83\ \text{cm}.$$

EVALUATE: The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

23.31. IDENTIFY: The voltmeter measures the potential difference between the two points. We must relate this quantity to the linear charge density on the wire.

SET UP: For a very long (infinite) wire, the potential difference between two points is given by $\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a)$.

EXECUTE: **(a)** Solving for λ gives

$$\lambda = \frac{(\Delta V)2\pi\varepsilon_0}{\ln(r_b/r_a)} = \frac{575\text{ V}}{(18\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\ln\left(\dfrac{3.50\text{ cm}}{2.50\text{ cm}}\right)} = 9.49\times10^{-8}\text{ C/m}.$$

(b) The meter will read less than 575 V because the electric field is weaker over this 1.00-cm distance than it was over the 1.00-cm distance in part (a).

(c) The potential difference is zero because both probes are at the same distance from the wire, and hence at the same potential.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV, even though that is not stated explicitly in the problem.

23.32. IDENTIFY: The voltmeter reads the potential difference between the two points where the probes are placed. Therefore we must relate the potential difference to the distances of these points from the center of the cylinder. For points outside the cylinder, its electric field behaves like that of a line of charge.

SET UP: Using $\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a)$ and solving for r_b, we have $r_b = r_a e^{2\pi\varepsilon_0\Delta V/\lambda}$.

EXECUTE: The exponent is $\dfrac{\left(\dfrac{1}{2\times8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}\right)(175\text{ V})}{15.0\times10^{-9}\text{ C/m}} = 0.648$, which gives

$$r_b = (2.50\text{ cm})\,e^{0.648} = 4.78\text{ cm}.$$

The distance above the *surface* is $4.78\text{ cm} - 2.50\text{ cm} = 2.28\text{ cm}$.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV, even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each r is the distance from the *center* of the cylinder, not from the surface.

23.33. IDENTIFY: For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

SET UP: The potential difference is $\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a)$.

EXECUTE: **(a)** Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a) = (8.50\times10^{-6}\text{ C/m})(2\times9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\ln\left(\frac{10.0\text{ cm}}{6.00\text{ cm}}\right).$$

$$\Delta V = 7.82\times10^4\text{ V} = 78{,}200\text{ V} = 78.2\text{ kV}.$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.

EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

23.34. IDENTIFY: The work required is equal to the change in the electrical potential energy of the charge-ring system. We need only look at the beginning and ending points, since the potential difference is independent of path for a conservative field.

SET UP: **(a)** $W = \Delta U = q\Delta V = q(V_{\text{center}} - V_\infty) = q\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a} - 0\right)$.

EXECUTE: Substituting numbers gives

$$\Delta U = (3.00\times10^{-6}\text{ C})(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})/(0.0400\text{ m}) = 3.38\text{ J}.$$

(b) We can take any path since the potential is independent of path.

(c) SET UP: The net force is away from the ring, so the ball will accelerate away. Energy conservation gives $U_0 = K_{\max} = \frac{1}{2}mv^2$.

EXECUTE: Solving for v gives

$$v = \sqrt{\frac{2U_0}{m}} = \sqrt{\frac{2(3.38\text{ J})}{0.00150\text{ kg}}} = 67.1\text{ m/s}.$$

EVALUATE: Direct calculation of the work from the electric field would be extremely difficult, and we would need to know the path followed by the charge. But, since the electric field is conservative, we can bypass all this calculation just by looking at the end points (infinity and the center of the ring) using the potential.

23.35. IDENTIFY: The electric field of the line of charge does work on the sphere, increasing its kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$ and $K_1 = 0$. $U = qV$ so $qV_1 = K_2 + qV_2$. $V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r}\right)$.

EXECUTE: $V_1 = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r_1}\right)$. $V_2 = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r_2}\right)$.

$$K_2 = q(V_1 - V_2) = \frac{q\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{r_0}{r_1}\right) - \ln\left(\frac{r_0}{r_2}\right)\right) = \frac{\lambda q}{2\pi\varepsilon_0}(\ln r_2 - \ln r_1) = \frac{\lambda q}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right).$$

$$K_2 = \frac{(3.00\times10^{-6}\text{ C/m})(8.00\times10^{-6}\text{ C})}{2\pi(8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2))}\ln\left(\frac{4.50}{1.50}\right) = 0.474\text{ J}.$$

EVALUATE: The potential due to the line of charge does *not* go to zero at infinity but is defined to be zero at an arbitrary distance r_0 from the line.

23.36. IDENTIFY and **SET UP:** For oppositely charged parallel plates, $E = \sigma/\varepsilon_0$ between the plates and the potential difference between the plates is $V = Ed$.

EXECUTE: **(a)** $E = \frac{\sigma}{\varepsilon_0} = \frac{47.0\times10^{-9}\text{ C/m}^2}{\varepsilon_0} = 5310\text{ N/C}.$

(b) $V = Ed = (5310\text{ N/C})(0.0220\text{ m}) = 117\text{ V}.$

(c) The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

EVALUATE: The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles, the work, which is force times distance, doubles and the potential difference doubles.

23.37. IDENTIFY and **SET UP:** Use $\Delta V = Ed$ to relate the electric field between the plates to the potential difference between them and their separation. The magnitude of the force this field exerts on the particle is given by $F = qE$. Use $W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$ to calculate the work.

EXECUTE: **(a)** Using $\Delta V = Ed$ gives $E = \frac{V_{ab}}{d} = \frac{360\text{ V}}{0.0450\text{ m}} = 8000\text{ V/m}.$

(b) $F = |q|E = (2.40\times10^{-9}\text{ C})(8000\text{ V/m}) = +1.92\times10^{-5}\text{ N}.$

(c) The electric field between the plates is shown in Figure 23.37.

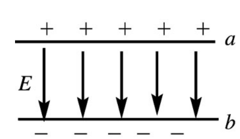


Figure 22.37

The plate with positive charge (plate a) is at higher potential. The electric field is directed from high potential toward low potential (or, $\vec{E}$ is from + charge toward − charge), so $\vec{E}$ points from a to b. Hence the force that $\vec{E}$ exerts on the positive charge is from a to b, so it does positive work.

$W = \int_a^b \vec{F} \cdot d\vec{l} = Fd$, where d is the separation between the plates.

$W = Fd = (1.92\times10^{-5}\ \text{N})(0.0450\ \text{m}) = +8.64\times10^{-7}\ \text{J}.$

(d) $V_a - V_b = +360\ \text{V}$ (plate a is at higher potential).

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40\times10^{-9}\ \text{C})(-360\ \text{V}) = -8.64\times10^{-7}\ \text{J}.$

EVALUATE: We see that $W_{a\to b} = -(U_b - U_a) = U_a - U_b$.

23.38. IDENTIFY and **SET UP:** $V_{ab} = Ed$ for parallel plates.

EXECUTE: $d = \dfrac{V_{ab}}{E} = \dfrac{1.5\ \text{V}}{1.0\times10^{-6}\ \text{V/m}} = 1.5\times10^{6}\ \text{m} = 1.5\times10^{3}\ \text{km}.$

EVALUATE: The plates would have to be nearly a thousand miles apart with only a AA battery across them! This is a small field!

23.39. IDENTIFY: The potential of a solid conducting sphere is the same at every point inside the sphere because $E = 0$ inside, and this potential has the value $V = q/4\pi\epsilon_0 R$ at the surface. Use the given value of E to find q.

SET UP: For negative charge the electric field is directed toward the charge.
For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

EXECUTE: $E = \dfrac{|q|}{4\pi\epsilon_0 r^2}$ and $|q| = 4\pi\epsilon_0 E r^2 = \dfrac{(3800\ \text{N/C})(0.200\ \text{m})^2}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.69\times10^{-8}\ \text{C}.$

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is $V = \dfrac{q}{4\pi\epsilon_0 r} = \dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.69\times10^{-8}\ \text{C})}{0.200\ \text{m}} = -760\ \text{V}$ at the surface of the sphere.

Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be –760 V.

EVALUATE: Inside the sphere the electric field is zero and the potential is constant.

23.40. IDENTIFY: The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

SET UP: At the surface, and hence also at the center of the sphere, the potential is that of a point-charge, $V = Q/(4\pi\epsilon_0 R)$.

EXECUTE: **(a)** Solving for Q and substituting the numbers gives

$$Q = 4\pi\epsilon_0 RV = (0.125\ \text{m})(3750\ \text{V})/(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2) = 5.21\times10^{-8}\ \text{C} = 52.1\ \text{nC}.$$

(b) Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 3.75 kV.

EVALUATE: The electric field inside the sphere is zero, so the potential is constant but is not zero.

23.41. IDENTIFY and **SET UP:** For a solid metal sphere or for a spherical shell, $V = \dfrac{kq}{r}$ outside the sphere and $V = \dfrac{kq}{R}$ at all points inside the sphere, where R is the radius of the sphere. When the electric field is radial, $E = -\dfrac{\partial V}{\partial r}$.

EXECUTE: **(a)** (i) $r < r_a$: This region is inside both spheres. $V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right).$

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell. $V = \frac{kq}{r} - \frac{kq}{r_b} = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$.

(iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for a point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

(b) $V_a = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$ and $V_b = 0$, so $V_{ab} = \frac{1}{4\pi\varepsilon_0} q\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$.

(c) Between the spheres $r_a < r < r_b$ and $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$.

$$E_r = -\frac{\partial V}{\partial r} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{r_b}\right) = +\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{V_{ab}}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}\frac{1}{r^2}.$$

(d) Since $E_r = -\frac{\partial V}{\partial r}$, $E = 0$, since V is constant (zero) outside the spheres.

(e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change. However, now that the potential does vary outside the spheres, there is an electric field there:

$$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{kq}{r} + \frac{-kQ}{r}\right) = \frac{kq}{r^2}\left(1 - \frac{Q}{q}\right) = \frac{k}{r^2}(q-Q).$$

EVALUATE: In part (a) the potential is greater than zero for all $r < r_b$.

23.42. **IDENTIFY:** By the definition of electric potential, if a positive charge gains potential along a path, then the potential along that path must have increased. The electric field produced by a very large sheet of charge is uniform and is independent of the distance from the sheet.

(a) SET UP: No matter what the reference point, we must do work on a positive charge to move it away from the negative sheet.

EXECUTE: Since we must do work on the positive charge, it gains potential energy, so the potential increases.

(b) SET UP: Since the electric field is uniform and is equal to $\sigma/2\varepsilon_0$, we have $\Delta V = Ed = \frac{\sigma}{2\varepsilon_0}d$.

EXECUTE: Solving for d gives

$$d = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.00\ \text{V})}{6.00\times10^{-9}\ \text{C/m}^2} = 0.00295\ \text{m} = 2.95\ \text{mm}.$$

EVALUATE: Since the spacing of the equipotential surfaces (d = 2.95 mm) is independent of the distance from the sheet, the equipotential surfaces are planes parallel to the sheet and spaced 2.95 mm apart.

23.43. **IDENTIFY** and **SET UP:** Use $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and $E_z = \frac{\partial V}{\partial z}$ to calculate the components of $\vec{E}$.

EXECUTE: $V = Axy - Bx^2 + Cy$.

(a) $E_x = -\frac{\partial V}{\partial x} = -Ay + 2Bx$.

$E_y = -\frac{\partial V}{\partial y} = -Ax - C$.

$E_z = \dfrac{\partial V}{\partial z} = 0.$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0$.

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A$.

And E_x is also equal to zero for this x, any value of z and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: V doesn't depend on z so $E_z = 0$ everywhere.

23.44. IDENTIFY: Apply $E_x = -\dfrac{\partial V}{\partial x}$ and $E_y = -\dfrac{\partial V}{\partial y}$ to find the components of $\vec{E}$, then use them to find its magnitude and direction. $V(x, y) = Ax^2y - Bxy^2$.

SET UP: $E = \sqrt{E_x^2 + E_y^2}$ and $\tan\theta = E_y/E_x$.

EXECUTE: First find the components of $\vec{E}$: $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}(Ax^2y - Bxy^2) = -(2Axy - By^2)$.

Now evaluate this result at the point x = 2.00 m, y = 0.400 m using the given values for A and B.

$E_x = -[2(5.00\ \text{V/m}^3)(2.00\ \text{m})(0.400\ \text{m}) - (8.00\ \text{V/m}^3)(0.400\ \text{m})^2] = -6.72\ \text{V/m}$.

$E_y = -\dfrac{\partial V}{\partial y} = -\dfrac{\partial}{\partial y}(Ax^2y - Bxy^2) = -(Ax^2 - 2Bxy)$. At the point (2.00 m, 0.400 m), this is

$E_y = -[(5.00\ \text{V/m}^3)(2.00\ \text{m})^2 - 2(8.00\ \text{V/m}^3)(2.00\ \text{m})(0.400\ \text{m})] = -7.20\ \text{V/m}$.

Now use the components to find the magnitude and direction of $\vec{E}$.

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-6.72\ \text{V/m})^2 + (-7.20\ \text{V/m})^2} = 9.85\ \text{V/m}$.

$\tan\theta = E_y/E_x$ = (–7.20 V/m)/(–6.72 V/m), which gives θ = 47.0°. Since both components are negative, the vector lies in the third quadrant in the xy-plane and makes an angle of 47.0° + 180.0° = 227.0° with the +x-axis.

EVALUATE: V is a scalar but $\vec{E}$ is a vector and has components.

23.45. IDENTIFY: Exercise 23.41 shows that $V = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$ for $r < r_a$, $V = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$ for $r_a < r < r_b$ and

$V_{ab} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

SET UP: $E = \dfrac{kq}{r^2}$, radially outward, for $r_a \le r \le r_b$.

EXECUTE: **(a)** $V_{ab} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right) = 500\ \text{V}$ gives $q = \dfrac{500\ \text{V}}{k\left(\dfrac{1}{0.012\ \text{m}} - \dfrac{1}{0.096\ \text{m}}\right)} = 7.62\times10^{-10}\ \text{C} = 0.762\ \text{nC}$.

(b) $V_b = 0$ so $V_a = 500$ V. The inner metal sphere is an equipotential with $V = 500$ V. $\dfrac{1}{r} = \dfrac{1}{r_a} + \dfrac{V}{kq}$.

$V = 400$ V at $r = 1.45$ cm, $V = 300$ V at $r = 1.85$ cm, $V = 200$ V at $r = 2.53$ cm, $V = 100$ V at $r = 4.00$ cm, $V = 0$ at $r = 9.60$ cm. The equipotential surfaces are sketched in Figure 23.45.

EVALUATE: **(c)** The equipotential surfaces are concentric spheres and the electric field lines are radial, so the field lines and equipotential surfaces are mutually perpendicular. The equipotentials are closest at smaller r, where the electric field is largest.

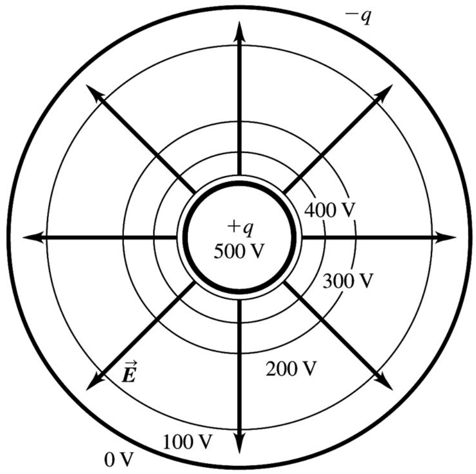


Figure 23.45

23.46. **IDENTIFY:** As the sphere approaches the point charge, the speed of the sphere decreases because it loses kinetic energy, but its acceleration increases because the electric force on it increases. Its mechanical energy is conserved during the motion, and Newton's second law and Coulomb's law both apply.

SET UP: $K_a + U_a = K_b + U_b$, $K = \frac{1}{2}mv^2$, $U = kq_1q_2/r$, $F = kq_1q_2/r^2$, and $F = ma$.

EXECUTE: Find the distance between the two charges when $v_2 = 25.0$ m/s.

$K_a + U_a = K_b + U_b$.

$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(40.0\text{ m/s})^2 = 3.20\text{ J}.$

$K_b = \frac{1}{2}mv_b^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(25.0\text{ m/s})^2 = 1.25\text{ J}.$

$$U_a = k\frac{q_1q_2}{r_a} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{0.0600\text{ m}} = 1.498\text{ J}.$$

$U_b = K_a + U_a - K_b = 3.20\text{ J} + 1.498\text{ J} - 1.25\text{ J} = 3.448\text{ J}.$ $U_b = k\dfrac{q_1q_2}{r_b}$ and

$$r_b = \frac{kq_1q_2}{U_b} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{3.448\text{ J}} = 0.02607\text{ m}.$$

$$F_b = \frac{kq_1q_2}{r_b^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{(0.02607\text{ m})^2} = 132.3\text{ N}.$$

$$a = \frac{F}{m} = \frac{132.3\text{ N}}{4.00\times10^{-3}\text{ kg}} = 3.31\times10^4\text{ m/s}^2.$$

EVALUATE: As the sphere approaches the point charge, its speed decreases but its acceleration keeps increasing because the electric force on it keeps increasing.

23.47. **IDENTIFY:** $U = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{r_{13}} + \dfrac{q_2q_3}{r_{23}}\right)$.

SET UP: In part (a), $r_{12} = 0.200$ m, $r_{23} = 0.100$ m and $r_{13} = 0.100$ m. In part (b) let particle 3 have coordinate *x*, so $r_{12} = 0.200$ m, $r_{13} = x$ and $r_{23} = 0.200\text{ m} - x$.

EXECUTE: **(a)** $U = k\left(\dfrac{(4.00\text{ nC})(-3.00\text{ nC})}{(0.200\text{ m})} + \dfrac{(4.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})} + \dfrac{(-3.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})}\right) = -3.60\times10^{-7}\text{ J}.$

(b) If $U = 0$, then $0 = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{x} + \dfrac{q_2q_3}{r_{12} - x}\right)$. Solving for x we find:

$0 = -60 + \dfrac{8}{x} - \dfrac{6}{0.2 - x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074\text{ m}, 0.360\text{ m}$. Therefore, $x = 0.074\text{ m}$ since it is the only value between the two charges.

EVALUATE: U_{13} is positive and both U_{23} and U_{12} are negative. If $U = 0$, then $|U_{13}| = |U_{23}| + |U_{12}|$. For $x = 0.074\text{ m}$, $U_{13} = +9.7\times10^{-7}\text{ J}$, $U_{23} = -4.3\times10^{-7}\text{ J}$ and $U_{12} = -5.4\times10^{-7}\text{ J}$. It is true that $U = 0$ at this x.

23.48. IDENTIFY: The electric field of the fixed charge does work on the charged object and therefore changes it kinetic energy. We apply the work-energy theorem.

SET UP: $W_{a\to b} = \Delta K$ and $W_{a\to b} = q(V_a - V_b)$, $V = k\dfrac{q}{r}$.

EXECUTE: $W_{a\to b} = \Delta K = K_b - K_a = q_2(V_a - V_b)$, which gives $K_b = K_a + q_2(V_a - V_b)$.

$$K_b = \frac{1}{2}mv_a^2 + q_2\left(\frac{kq_1}{r_a} - \frac{kq_1}{r_b}\right) = \frac{1}{2}mv_a^2 + kq_1q_2\left(\frac{1}{r_a} - \frac{1}{r_b}\right).$$

Putting in the numbers gives

$$K_b = \frac{1}{2}(0.00400\text{ kg})(800\text{ m/s})^2 + (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-4}\text{ C})(-3.00\times10^{-4}\text{ C})\times [1/(0.400\text{ m}) - 1/(0.200\text{ m})].$$

$K_b = 4651\text{ J}$.

$v_b = (2K_b/m)^{1/2} = [2(4651\text{ J})/(0.00400\text{ kg})]^{1/2} = 1520\text{ m/s}$.

EVALUATE: The negatively charged small object gains kinetic energy because it is attracted by the positive charge q_1, which does positive work on the object, so $v_b > v_a$.

23.49. IDENTIFY and SET UP: Treat the gold nucleus as a point charge so that $V = k\dfrac{q}{r}$. According to conservation of energy we have $K_1 + U_1 = K_2 + U_2$, where $U = qV$.

EXECUTE: Assume that the alpha particle is at rest before it is accelerated and that it momentarily stops when it arrives at its closest approach to the surface of the gold nucleus. Thus we have $K_1 = K_2 = 0$, which implies that $U_1 = U_2$. Since $U = qV$ we conclude that the accelerating voltage must be equal to the voltage at its point of closest approach to the surface of the gold nucleus. Therefore

$$V_a = V_b = k\frac{q}{r} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{79(1.60\times10^{-19}\text{ C})}{(7.3\times10^{-15}\text{ m} + 2.0\times10^{-14}\text{ m})} = 4.2\times10^6\text{ V}.$$

EVALUATE: Although the alpha particle has kinetic energy as it approaches the gold nucleus this is irrelevant to our solution since energy is conserved for the whole process.

23.50. IDENTIFY: Two forces do work on the sphere as it falls: gravity and the electrical force due to the sheet. The energy of the sphere is conserved.

SET UP: The gravity force is mg, downward. The electric field of the sheet is $E = \dfrac{\sigma}{2\varepsilon_0}$ upward, and the force it exerts on the sphere is $F = qE$. The sphere gains kinetic energy $K = \dfrac{1}{2}mv^2$ as it falls.

EXECUTE: $mg = 4.90\times10^{-6}\text{ N}$. $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{8.00\times10^{-12}\text{ C/m}^2}{2(8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2))} = 0.4518\text{ N/C}$. The electric force is $qE = (7.00\times10^{-6}\text{ C})(0.4518\text{ N/C}) = 3.1626\times10^{-6}\text{ N}$, upward. The net force is downward, so the sphere moves downward when released. Let $y = 0$ at the sheet. $U_{\text{grav}} = mgy$. For the electric force, $\dfrac{W_{a\to b}}{q} = V_a - V_b$. Let point a be at the sheet and let point b be a distance y above the sheet. Take $V_a = 0$.

The force on q is qE, upward, so $\dfrac{W_{a\to b}}{q} = Ey$ and $V_b = -Ey$. $U_b = -Eyq$. $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$.
$y_1 = 0.400$ m, $y_2 = 0.100$ m. $K_2 = U_1 - U_2 = mg(y_1 - y_2) - E(y_1 - y_2)q$.
$K_2 = (5.00\times10^{-7}\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) - (0.4518\text{ N/C})(0.300\text{ m})(7.00\times10^{-6}\text{ C})$.
$K_2 = 1.470\times10^{-6}\text{ J} - 0.94878\times10^{-6}\text{ J} = 0.52122\times10^{-6}\text{ J}$. $K_2 = \frac{1}{2}mv_2^2$ so

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.52122\times10^{-6}\text{ J})}{5.00\times10^{-7}\text{ kg}}} = 1.44\text{ m/s}.$$

EVALUATE: Because the weight is greater than the electric force, the sphere will accelerate downward, but if it were light enough the electric force would exceed the weight. In that case it would never get closer to the sheet after being released. We could also solve this problem using Newton's second law and the constant-acceleration kinematics formulas. $a = F/m = (mg - qE)/m$ gives the acceleration. Then we use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v_{0x} = 0$ to find v.

23.51. IDENTIFY: The remaining nucleus (radium minus the ejected alpha particle) repels the alpha particle, giving it 4.79 MeV of kinetic energy when it is far from the nucleus. The mechanical energy of the system is conserved.

SET UP: $U = k\dfrac{q_1q_2}{r}$. $U_a + K_a = U_b + K_b$. The charge of the alpha particle is $+2e$ and the charge of the radon nucleus is $+86e$.

EXECUTE: **(a)** The final energy of the alpha particle, 4.79 MeV, equals the electrical potential energy of the alpha-radon combination just before the decay. $U = 4.79\text{ MeV} = 7.66\times10^{-13}$ J.

(b) $r = \dfrac{kq_1q_2}{U} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(86)(1.60\times10^{-19}\text{ C})^2}{7.66\times10^{-13}\text{ J}} = 5.17\times10^{-14}$ m.

EVALUATE: Although we have made some simplifying assumptions (such as treating the atomic nucleus as a spherically symmetric charge, even when very close to it), this result gives a fairly reasonable estimate for the size of a nucleus.

23.52. IDENTIFY: The charged particles repel each other and therefore accelerate away from one another, causing their speeds and kinetic energies to continue to increase. They do not have equal speeds because they have different masses. The mechanical energy and momentum of the system are conserved.

SET UP: The proton has charge $q_\text{p} = +e$ and mass $m_\text{p} = 1.67\times10^{-27}$ kg. The alpha particle has charge $q_\text{a} = +2e$ and mass $m_\text{a} = 4m_\text{p} = 6.68\times10^{-27}$ kg. We can apply both conservation of energy and conservation of linear momentum to the system. $a = \dfrac{F}{m}$, where $F = k\dfrac{|q_1q_2|}{r^2}$.

EXECUTE: *Acceleration:* The maximum force and hence the maximum acceleration occurs just after they are released, when $r = 0.225$ nm. $F = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(1.60\times10^{-19}\text{ C})^2}{(0.225\times10^{-9}\text{ m})^2} = 9.09\times10^{-9}$ N.

$a_\text{p} = \dfrac{F}{m_\text{p}} = \dfrac{9.09\times10^{-9}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 5.44\times10^{18}\text{ m/s}^2$; $a_\text{a} = \dfrac{F}{m_\text{a}} = \dfrac{9.09\times10^{-9}\text{ N}}{6.68\times10^{-27}\text{ kg}} = 1.36\times10^{18}\text{ m/s}^2$. The acceleration of the proton is larger by a factor of m_a/m_p.

Speed: Conservation of energy says $U_1 + K_1 = U_2 + K_2$. $K_1 = 0$ and $U_2 = 0$, so $K_2 = U_1$.

$U_1 = k\dfrac{q_1q_2}{r} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(1.60\times10^{-19}\text{ C})^2}{0.225\times10^{-9}\text{ m}} = 2.05\times10^{-18}$ J, so the total kinetic energy of the two particles when they are far apart is $K_2 = 2.05\times10^{-18}$ J. Conservation of linear momentum says how this energy is divided between the proton and alpha particle. $p_1 = p_2$. $0 = m_\text{p}v_\text{p} - m_\text{a}v_\text{a}$ so $v_\text{a} = \left(\dfrac{m_\text{p}}{m_\text{a}}\right)v_\text{p}$.

$$K_2 = \tfrac{1}{2}m_\text{p}v_\text{p}^2 + \tfrac{1}{2}m_\text{a}v_\text{a}^2 = \tfrac{1}{2}m_\text{p}v_\text{p}^2 + \tfrac{1}{2}m_\text{a}\left(\frac{m_\text{p}}{m_\text{a}}\right)^2 v_\text{p}^2 = \tfrac{1}{2}m_\text{p}v_\text{p}^2\left(1+\frac{m_\text{p}}{m_\text{a}}\right).$$

$$v_\text{p} = \sqrt{\frac{2K_2}{m_\text{p}(1+(m_\text{p}/m_\text{a}))}} = \sqrt{\frac{2(2.05\times10^{-18}\text{ J})}{(1.67\times10^{-27}\text{ kg})(1+\frac{1}{4})}} = 4.43\times10^4 \text{ m/s}.$$

$v_\text{a} = \left(\dfrac{m_\text{p}}{m_\text{a}}\right)v_\text{p} = \tfrac{1}{4}(4.43\times10^4 \text{ m/s}) = 1.11\times10^4$ m/s. The maximum acceleration occurs just after they are released. The maximum speed occurs after a long time.

EVALUATE: The proton and alpha particle have equal momenum, but proton has a greater acceleration and more kinetic energy.

23.53. **(a) IDENTIFY:** Apply the work-energy theorem.

SET UP: Points *a* and *b* are shown in Figure 23.53a.

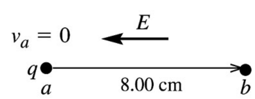


Figure 23.53a

EXECUTE: $W_\text{tot} = \Delta K = K_b - K_a = K_b = 4.35\times10^{-5}$ J.

The electric force F_E and the additional force F both do work, so that $W_\text{tot} = W_{F_E} + W_F$.

$$W_{F_E} = W_\text{tot} - W_F = 4.35\times10^{-5}\text{ J} - 6.50\times10^{-5}\text{ J} = -2.15\times10^{-5}\text{ J}.$$

EVALUATE: The forces on the charged particle are shown in Figure 23.53b.

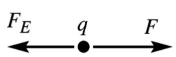


Figure 23.53b

The electric force is to the left (in the direction of the electric field since the particle has positive charge). The displacement is to the right, so the electric force does negative work. The additional force F is in the direction of the displacement, so it does positive work.

(b) IDENTIFY and SET UP: For the work done by the electric force, $W_{a\to b} = q(V_a - V_b)$.

EXECUTE: $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{-2.15\times10^{-5}\text{ J}}{7.60\times10^{-9}\text{ C}} = -2.83\times10^3$ V.

EVALUATE The starting point (point *a*) is at 2.83×10^3 V lower potential than the ending point (point *b*). We know that $V_b > V_a$ because the electric field always points from high potential toward low potential.

(c) IDENTIFY: Calculate *E* from $V_a - V_b$ and the separation *d* between the two points.

SET UP: Since the electric field is uniform and directed opposite to the displacement $W_{a\to b} = -F_E d = -qEd$, where $d = 8.00$ cm is the displacement of the particle.

EXECUTE: $E = -\dfrac{W_{a\to b}}{qd} = -\dfrac{V_a - V_b}{d} = -\dfrac{-2.83\times10^3\text{ V}}{0.0800\text{ m}} = 3.54\times10^4$ V/m.

EVALUATE: In part (a), W_tot is the total work done by both forces. In parts (b) and (c) $W_{a\to b}$ is the work done just by the electric force.

23.54. **IDENTIFY:** The net force on q_0 is the vector sum of the forces due to the two charges. Coulomb's law applies.

SET UP: $F = k\dfrac{|q_1 q_2|}{r^2}$, $W_{a\to b} = q(V_a - V_b)$, $V = k\dfrac{q}{r}$.

EXECUTE: **(a)** The magnitude of the force on q_0 due to each of the two charges at opposite corners of the square is $F = k\dfrac{|q_1 q_2|}{r^2}$ = $k(5.00\ \mu\text{C})(3.00\ \mu\text{C})/(0.0800\ \text{m})^2$ = 21.07 N. Adding the two forces vectorially gives the net force F_{net} = (21.07 N) $\sqrt{2}$ = 29.8 N. The direction is from A to B since both charges attract q_0. Figure 23.54 shows this force.

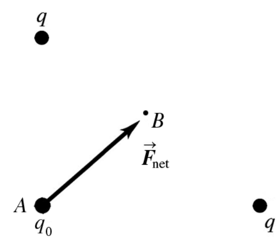


Figure 23.54

(b) At point B the two forces on q_0 are in opposite directions and have equal magnitudes, so they add to zero: $F_{\text{net}} = 0$.

(c) For each charge, $W_{A\to B} = q(V_A - V_B)$, so for both we must double this. Using $V = k\dfrac{q}{r}$ and simplifying we get $W_{A\to B} = 2q(V_A - V_B) = 2kqq_0\left(\dfrac{1}{r_A} - \dfrac{1}{r_B}\right)$. Putting in $q_0 = -3.00\ \mu\text{C}$, $q = 5.00\ \mu\text{C}$, $r_A = 0.0800$ m, and $r_b = 0.0400\sqrt{2}$ m, we get $W_{A\to B} = +1.40$ J. The work done on q_0 by the electric field is positive since it this charge moves from A to B in the direction of the force. The charge loses potential energy as it gains kinetic energy. But since q_0 is negative, it moves to a point of higher potential.

EVALUATE: Positive charges accelerate toward lower potential, but negative charges accelerate toward higher potential.

23.55. **IDENTIFY** and **SET UP:** Calculate the components of $\vec{E}$ using $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, and $E_z = -\dfrac{\partial V}{\partial z}$, and use $\vec{F} = q\vec{E}$.

EXECUTE: **(a)** $V = Cx^{4/3}$.

$$C = V/x^{4/3} = 240\ \text{V}/(13.0\times10^{-3}\ \text{m})^{4/3} = 7.85\times10^{4}\ \text{V/m}^{4/3}.$$

(b) $E_x(x) = -\dfrac{\partial V}{\partial x} = -\dfrac{4}{3}Cx^{1/3} = -(1.05\times10^{5}\ \text{V/m}^{4/3})x^{1/3}$.

The minus sign means that E_x is in the $-x$-direction, which says that $\vec{E}$ points from the positive anode toward the negative cathode.

(c) $\vec{F} = q\vec{E}$ so $F_x = -eE_x = \frac{4}{3}eCx^{1/3}$.

Halfway between the electrodes means $x = 6.50\times10^{-3}$ m.

$$F_x = \tfrac{4}{3}(1.602\times10^{-19}\ \text{C})(7.85\times10^{4}\ \text{V/m}^{4/3})(6.50\times10^{-3}\ \text{m})^{1/3} = 3.13\times10^{-15}\ \text{N}.$$

F_x is positive, so the force is directed toward the positive anode.

EVALUATE: V depends only on x, so $E_y = E_z = 0$. $\vec{E}$ is directed from high potential (anode) to low potential (cathode). The electron has negative charge, so the force on it is directed opposite to the electric field.

23.56. IDENTIFY: At each point (a and b), the potential is the sum of the potentials due to *both* spheres. The voltmeter reads the difference between these two potentials. The spheres behave like point charges since the meter is connected to the surface of each one.

SET UP: **(a)** Call a the point on the surface of one sphere and b the point on the surface of the other sphere, call r the radius of each sphere and call d the center-to-center distance between the spheres. The potential difference V_{ba} between points a and b is then

$$V_b - V_a = V_{ba} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{r} + \frac{q}{d-r} - \left(\frac{q}{r} + \frac{-q}{d-r}\right)\right] = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{d-r} - \frac{1}{r}\right).$$

EXECUTE: Substituting the numbers gives

$V_b - V_a = 2(250\ \mu\text{C})\ (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\dfrac{1}{0.750\ \text{m}} - \dfrac{1}{0.250\ \text{m}}\right) = -12.0\times10^6\ \text{V} = -12.0\ \text{MV}.$ The meter reads 12.0 MV.

(b) Since $V_b - V_a$ is negative, $V_a > V_b$, so point a is at the higher potential.

EVALUATE: An easy way to see that the potential at a is higher than the potential at b is that it would require positive work to move a positive test charge from b to a since this charge would be attracted by the negative sphere and repelled by the positive sphere.

23.57. IDENTIFY: $U = \dfrac{kq_1q_2}{r}$.

SET UP: Eight charges means there are $8(8-1)/2 = 28$ pairs. There are 12 pairs of q and $-q$ separated by d, 12 pairs of equal charges separated by $\sqrt{2}d$ and 4 pairs of q and $-q$ separated by $\sqrt{3}d$.

EXECUTE: **(a)** $U = kq^2\left(-\dfrac{12}{d} + \dfrac{12}{\sqrt{2}d} - \dfrac{4}{\sqrt{3}d}\right) = -\dfrac{12kq^2}{d}\left(1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{3\sqrt{3}}\right) = -1.46q^2/\pi\varepsilon_0 d.$

EVALUATE: **(b)** The fact that the electric potential energy is less than zero means that it is energetically favorable for the crystal ions to be together.

23.58. IDENTIFY: For two small spheres, $U = \dfrac{kq_1q_2}{r}$. For part (b) apply conservation of energy.

SET UP: Let $q_1 = 2.00\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. Let $r_a = 0.180\ \text{m}$ and $r_b \to \infty$.

EXECUTE: **(a)** $U = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\ \text{C})(-3.50\times10^{-6}\ \text{C})}{0.180\ \text{m}} = -0.350\ \text{J}.$

(b) $K_b = 0$. $U_b = 0$. $U_a = -0.350$ J. $K_a + U_a = K_b + U_b$ gives $K_a = 0.350$ J. $K_a = \frac{1}{2}mv_a^2$, so

$$v_a = \sqrt{\frac{2K_a}{m}} = \sqrt{\frac{2(0.350\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 21.6\ \text{m/s}.$$

EVALUATE: As the sphere moves away, the attractive electrical force exerted by the other sphere does negative work and removes all the kinetic energy it initially had.

23.59. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_\text{e} = qE$. The potential difference between the plates is $V = Ed$.

SET UP: The free-body diagram for the sphere is given in Figure 23.59.

EXECUTE: $T\cos\theta = mg$ and $T\sin\theta = F_\text{e}$ gives

$$F_\text{e} = mg\tan\theta = (1.50\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan(30°) = 0.0085\ \text{N}.$$

$F_\text{e} = Eq = \dfrac{Vq}{d}$ and $V = \dfrac{Fd}{q} = \dfrac{(0.0085\ \text{N})(0.0500\ \text{m})}{8.90\times10^{-6}\ \text{C}} = 47.8\ \text{V}.$

EVALUATE: $E = V/d = 956$ V/m. $E = \sigma/\varepsilon_0$ and $\sigma = E\varepsilon_0 = 8.46\times10^{-9}\ \text{C/m}^2$.

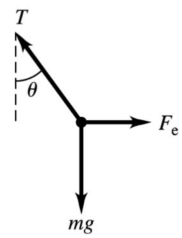


Figure 23.59

23.60. **IDENTIFY:** Outside a uniform spherical shell of charge, the electric field and potential are the same as for a point-charge at the center. Inside the shell, the electric field is zero so the potential is constant and equal to its value at the surface of the shell. The net potential is the scalar sum of the individual potentials.

SET UP: $V = k\frac{q}{r}$. Call V_1 the potential due to the inner shell and V_2 the potential due to the outer shell. $V_{net} = V_1 + V_2$.

EXECUTE: **(a)** At $r = 2.50$ cm, we are inside both shells. V_1 is the potential at the surface of the inner shell, so $V_1 = kq_1/R_1$; and V_2 is the potential at the surface of the outer shell, so $V_2 = kq_2/R_2$. The net potential is

$V_{net} = kq_1/R_1 + kq_2/R_2 = k(q_1/R_1 + q_2/R_2)$.

$V_{net} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)[(3.00\mu\text{C})/(0.0500\ \text{m}) + (-5.00\ \mu\text{C})/(0.150\ \text{m})] = 2.40\times10^5\ \text{V} = 240\ \text{kV}$.

(b) At $r = 10.0$ cm, we are outside the inner shell but still inside the outer shell. The inner shell now is equivalent to a point-charge at its center, so the net potential is

$V_{net} = kq_1/r + kq_2/R_2 = k(q_1/r + q_2/R_2)$.

$V_{net} = k[(3.00\ \mu\text{C})/(0.100\ \text{m}) + (-5.00\ \mu\text{C})/(0.150\ \text{m})] = -30.0$ kV.

(c) At $r = 20.0$ cm, we are outside both shells, so both are equivalent to point-charges at their center. So $V_{net} = kq_1/r + kq_2/r = k(q_1 + q_2)/r = k(-2.00\ \mu\text{C})/(0.200\ \text{m}) = -89.9$ kV.

EVALUATE: $E = 0$ inside a spherically symmetric shell, but that does not necessarily mean that $V = 0$ there. It only means that $V_a - V_b = 0$ for any two points in side the shell, so V is constant.

23.61. **(a) IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.

SET UP: For a conducting cylinder with charge per unit length λ the potential outside the cylinder is given by $V = (\lambda/2\pi\varepsilon_0)\ln(r_0/r)$ where r is the distance from the cylinder axis and r_0 is the distance from the axis for which we take $V = 0$. Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for V applies to both. This problem says to take $r_0 = b$.

EXECUTE: For the hollow tube of radius b and charge per unit length $-\lambda$: outside $V = -(\lambda/2\pi\varepsilon_0)\ln(b/r)$; inside $V = 0$ since $V = 0$ at $r = b$.

For the metal cylinder of radius a and charge per unit length λ:

outside $V = (\lambda/2\pi\varepsilon_0)\ln(b/r)$, inside $V = (\lambda/2\pi\varepsilon_0)\ln(b/a)$, the value at $r = a$.

(i) $r < a$; inside both $V = (\lambda/2\pi\varepsilon_0)\ln(b/a)$.

(ii) $a < r < b$; outside cylinder, inside tube $V = (\lambda/2\pi\varepsilon_0)\ln(b/r)$.

(iii) $r > b$; outside both the potentials are equal in magnitude and opposite in sign so $V = 0$.

(b) For $r = a$, $V_a = (\lambda/2\pi\varepsilon_0)\ln(b/a)$.

For $r = b$, $V_b = 0$.

Thus $V_{ab} = V_a - V_b = (\lambda/2\pi\varepsilon_0)\ln(b/a)$.

(c) IDENTIFY and **SET UP:** Use $E_r = -\frac{\partial V}{\partial r}$ to calculate E.

EXECUTE: $E=-\dfrac{\partial V}{\partial r}=-\dfrac{\lambda}{2\pi\epsilon_0}\dfrac{\partial}{\partial r}\ln\left(\dfrac{b}{r}\right)=-\dfrac{\lambda}{2\pi\epsilon_0}\left(\dfrac{r}{b}\right)\left(-\dfrac{b}{r^2}\right)=\dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}.$

(**d**) The electric field between the cylinders is due only to the inner cylinder, so V_{ab} is not changed, $V_{ab}=(\lambda/2\pi\epsilon_0)\ln(b/a)$.

EVALUATE: The electric field is not uniform between the cylinders, so $V_{ab}\neq E(b-a)$.

23.62. **IDENTIFY:** The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives $E(r)=\dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$.

SET UP: $a=145\times10^{-6}$ m, $b=0.0180$ m.

EXECUTE: $E=\dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$ and

$V_{ab}=E\ln(b/a)r=(2.00\times10^4 \text{ N/C})(\ln(0.018 \text{ m}/145\times10^{-6} \text{ m}))0.012 \text{ m}=1157 \text{ V}.$

EVALUATE: The electric field at any *r* is directly proportional to the potential difference between the wire and the cylinder.

23.63. **IDENTIFY** and **SET UP:** Use $\vec{F}=q\vec{E}$ to calculate $\vec{F}$ and then $\vec{F}=m\vec{a}$ gives $\vec{a}$. $E=V/d$.

EXECUTE: **(a)** $\vec{F}_E=q\vec{E}$. Since $q=-e$ is negative $\vec{F}_E$ and $\vec{E}$ are in opposite directions; $\vec{E}$ is upward so $\vec{F}_E$ is downward. The magnitude of *E* is $E=\dfrac{V}{d}=\dfrac{22.0 \text{ V}}{0.0200 \text{ m}}=1.10\times10^3 \text{ V/m}=1.10\times10^3 \text{ N/C}$. The magnitude of F_E is $F_E=|q|E=eE=(1.602\times10^{-19} \text{ C})(1.10\times10^3 \text{ N/C})=1.76\times10^{-16} \text{ N}$.

(b) Calculate the acceleration of the electron produced by the electric force:

$a=\dfrac{F}{m}=\dfrac{1.76\times10^{-16} \text{ N}}{9.109\times10^{-31} \text{ kg}}=1.93\times10^{14} \text{ m/s}^2.$

EVALUATE: This acceleration is much larger than $g=9.80 \text{ m/s}^2$, so the gravity force on the electron can be neglected. $\vec{F}_E$ is downward, so $\vec{a}$ is downward.

(c) IDENTIFY and **SET UP:** The acceleration is constant and downward, so the motion is like that of a projectile. Use the horizontal motion to find the time and then use the time to find the vertical displacement.

EXECUTE: x-component: $v_{0x}=6.50\times10^6$ m/s; $a_x=0$; $x-x_0=0.060$ m; $t=?$

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ and the a_x term is zero, so $t=\dfrac{x-x_0}{v_{0x}}=\dfrac{0.060 \text{ m}}{6.50\times10^6 \text{ m/s}}=9.231\times10^{-9}$ s.

y-component: $v_{0y}=0$; $a_y=1.93\times10^{14} \text{ m/s}^2$; $t=9.231\times10^{-9}$ m/s; $y-y_0=?$

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$. $y-y_0=\frac{1}{2}(1.93\times10^{14} \text{ m/s}^2)(9.231\times10^{-9} \text{ s})^2=0.00822 \text{ m}=0.822 \text{ cm}.$

(d) IDENTIFY and **SET UP:** The velocity and its components as the electron leaves the plates are sketched in Figure 23.63.

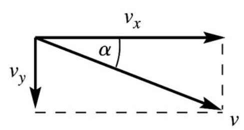


Figure 23.63

EXECUTE:

$v_x=v_{0x}=6.50\times10^6$ m/s (since $a_x=0$).

$v_y=v_{0y}+a_yt.$

$v_y=0+(1.93\times10^{14} \text{ m/s}^2)(9.231\times10^{-9} \text{ s}).$

$v_y=1.782\times10^6$ m/s.

$\tan\alpha=\dfrac{v_y}{v_x}=\dfrac{1.782\times10^6 \text{ m/s}}{6.50\times10^6 \text{ m/s}}=0.2742$ so $\alpha=15.3°$.

EVALUATE: The greater the electric field or the smaller the initial speed the greater the downward deflection.
(e) IDENTIFY and **SET UP:** Consider the motion of the electron after it leaves the region between the plates. Outside the plates there is no electric field, so $a = 0$. (Gravity can still be neglected since the electron is traveling at such high speed and the times are small.) Use the horizontal motion to find the time it takes the electron to travel 0.120 m horizontally to the screen. From this time find the distance downward that the electron travels.

EXECUTE: <u>x-component</u>: $v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.120$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the a_x term is term is zero, so $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{0.120\text{ m}}{6.50\times10^6\text{ m/s}} = 1.846\times10^{-8}$ s.

<u>y-component</u>: $v_{0y} = 1.782\times10^6$ m/s (from part (b)); $a_y = 0$; $t = 1.846\times10^{-8}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.782\times10^6\text{ m/s})(1.846\times10^{-8}\text{ s}) = 0.0329\text{ m} = 3.29$ cm.

EVALUATE: The electron travels downward a distance 0.822 cm while it is between the plates and a distance 3.29 cm while traveling from the edge of the plates to the screen. The total downward deflection is 0.822 cm + 3.29 cm = 4.11 cm. The horizontal distance between the plates is half the horizontal distance the electron travels after it leaves the plates. And the vertical velocity of the electron increases as it travels between the plates, so it makes sense for it to have greater downward displacement during the motion after it leaves the plates.

23.64. IDENTIFY: The charge on the plates and the electric field between them depend on the potential difference across the plates.

SET UP: For two parallel plates, the potential difference between them is $V = Ed = \dfrac{\sigma}{\varepsilon_0}d = \dfrac{Qd}{\varepsilon_0 A}$.

EXECUTE: **(a)** Solving for Q gives $Q = \varepsilon_0 AV/d = \dfrac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.030\text{ m})^2(25.0\text{ V})}{0.0050\text{ m}}$.

$Q = 3.98\times10^{-11}\text{C} = 39.8$ pC.

(b) $E = V/d = (25.0\text{ V})/(0.0050\text{ m}) = 5.00\times10^3$ V/m.

(c) SET UP: Energy conservation gives $\frac{1}{2}mv^2 = eV$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{C})(25.0\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 2.96\times10^6$ m/s.

EVALUATE: Typical voltages in student laboratory work run up to around 25 V, so typical reasonable values for the charge on the plates is about 40 pC and a reasonable value for the electric field is about 5000 V/m, as we found here. The electron speed would be about 3 million m/s.

23.65. (a) IDENTIFY and **SET UP:** Problem 23.61 derived that $E = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$, where a is the radius of the inner cylinder (wire) and b is the radius of the outer hollow cylinder. The potential difference between the two cylinders is V_{ab}. Use this expression to calculate E at the specified r.

EXECUTE: Midway between the wire and the cylinder wall is at a radius of
$r = (a+b)/2 = (90.0\times10^{-6}\text{ m} + 0.140\text{ m})/2 = 0.07004$ m.

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} = \frac{50.0\times10^3\text{ V}}{\ln(0.140\text{ m}/90.0\times10^{-6}\text{ m})(0.07004\text{ m})} = 9.71\times10^4\text{ V/m}.$$

(b) IDENTIFY and **SET UP:** The magnitude of the electric force is given by $F = |q|E$. Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

EXECUTE: $F_E = 10mg$.

$$|q|E = 10mg \text{ and } |q| = \frac{10mg}{E} = \frac{10(30.0\times10^{-9}\text{ kg})(9.80\text{ m/s}^2)}{9.71\times10^4\text{ V/m}} = 3.03\times10^{-11}\text{ C}.$$

EVALUATE: It requires only this modest net charge for the electric force to be much larger than the weight.

23.66. (a) IDENTIFY: Calculate the potential due to each thin ring and integrate over the disk to find the potential. V is a scalar so no components are involved.

SET UP: Consider a thin ring of radius y and width dy. The ring has area $2\pi y\,dy$ so the charge on the ring is $dq = \sigma(2\pi y\,dy)$.

EXECUTE: The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance x from the ring is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}.$$

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\varepsilon_0}(\sqrt{x^2+R^2}-x).$$

EVALUATE: For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2+R^2} = x(1+R^2/x^2)^{1/2} \approx x(1+R^2/2x^2) \text{ so } \sqrt{x^2+R^2}-x \approx R^2/2x.$$

Then $V \approx \dfrac{\sigma}{2\varepsilon_0}\dfrac{R^2}{2x} = \dfrac{\sigma\pi R^2}{4\pi\varepsilon_0 x} = \dfrac{Q}{4\pi\varepsilon_0 x}$, as expected.

(b) IDENTIFY and SET UP: Use $E_x = -\dfrac{\partial V}{\partial x}$ to calculate E_x.

EXECUTE: $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{x}{\sqrt{x^2+R^2}}-1\right) = \dfrac{\sigma x}{2\varepsilon_0}\left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x^2+R^2}}\right).$

EVALUATE: Our result agrees with the results of Example 21.11.

23.67. IDENTIFY: We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

SET UP: If ρ is the uniform volume charge density, the charge of a spherical shell of radius r and thickness dr is $dq = \rho 4\pi r^2\,dr$, and $\rho = Q/(4/3\ \pi R^3)$. The charge already present in a sphere of radius r is $q = \rho(4/3\ \pi r^3)$. The energy to bring the charge dq to the surface of the charge q is Vdq, where V is the potential due to q, which is $q/4\pi\varepsilon_0 r$.

EXECUTE: The total energy to assemble the entire sphere of radius R and charge Q is sum (integral) of the tiny increments of energy.

$$U = \int V dq = \int \frac{q}{4\pi\varepsilon_0 r}\,dq = \int_0^R \frac{\rho\frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r}(\rho 4\pi r^2 dr) = \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}\right)$$

where we have substituted $\rho = Q/(4/3\ \pi R^3)$ and simplified the result.

EVALUATE: For a point charge, $R \to 0$ so $U \to \infty$, which means that a point charge should have infinite self-energy. This suggests that either point charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

23.68. IDENTIFY: Divide the rod into infinitesimal segments with charge dq. The potential dV due to the segment is $dV = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r}$. Integrate over the rod to find the total potential.

SET UP: $dq = \lambda dl$, with $\lambda = Q/\pi a$ and $dl = a\,d\theta$.

EXECUTE: $dV = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\,dl}{a} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\pi a}\dfrac{dl}{a} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q\,d\theta}{\pi a}$. $V = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_0^\pi \dfrac{Q\,d\theta}{\pi a} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a}.$

EVALUATE: All the charge of the ring is the same distance a from the center of curvature.

23.69. IDENTIFY and SET UP: The sphere no longer behaves as a point charge because we are inside of it. We know how the electric field varies with distance from the center of the sphere and want to use this to find

the potential difference between the center and surface, which requires integration. $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$.
The electric field is radially outward, so $\vec{E} \cdot d\vec{l} = E\,dr$.

EXECUTE: For $r < R: E = \dfrac{kQr}{R^3}$. Integrating gives

$V = -\int_\infty^R \vec{E} \cdot d\vec{r}' - \int_R^r \vec{E} \cdot d\vec{r}' = \dfrac{kQ}{R} - \dfrac{kQ}{R^3}\int_R^r r'dr' = \dfrac{kQ}{R} - \dfrac{kQ}{R^3}\dfrac{1}{2}r'^2\Big|_R^r = \dfrac{kQ}{R} + \dfrac{kQ}{2R} - \dfrac{kQr^2}{2R^3} = \dfrac{kQ}{2R}\left[3 - \dfrac{r^2}{R^2}\right]$. At the center of the sphere, $r = 0$ and $V_1 = \dfrac{3kQ}{2R}$. At the surface of the sphere, $r = R$ and $V_2 = \dfrac{kQ}{R}$. The potential difference is $V_1 - V_2 = \dfrac{kQ}{2R} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.00\times10^{-6}\ \text{C})}{2(0.0500\ \text{m})} = 3.60\times10^5\ \text{V}$.

EVALUATE: To check our answer, we could actually do the integration. We can use the fact that $E = \dfrac{kQr}{R^3}$ so $V_1 - V_2 = \int_0^R E dr = \dfrac{kQ}{R^3}\int_0^R r dr = \dfrac{kQ}{R^3}\left(\dfrac{R^2}{2}\right) = \dfrac{kQ}{2R}$.

23.70. IDENTIFY: For $r < c$, $E = 0$ and the potential is constant. For $r > c$, E is the same as for a point charge and $V = \dfrac{kq}{r}$.

SET UP: $V_\infty = 0$.

EXECUTE: **(a)** Points $a, b,$ and c are all at the same potential, so $V_a - V_b = V_b - V_c = V_a - V_c = 0$.

$$V_c - V_\infty = \frac{kq}{R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-6}\ \text{C})}{0.60\ \text{m}} = 2.25\times10^6\ \text{V}.$$

(b) They are all at the same potential.

(c) Only $V_c - V_\infty$ would change; it would be -2.25×10^6 V.

EVALUATE: The voltmeter reads the potential difference between the two points to which it is connected.

23.71. IDENTIFY: Apply Newton's second law to calculate the acceleration. Apply conservation of energy and conservation of momentum to the motions of the spheres.

SET UP: Since the spheres behave as though all the charge were at their centers, we have $F = k\dfrac{|q_1 q_2|}{r^2}$ and $U = \dfrac{kq_1q_2}{r}$, where q_1 and q_2 are the charges of the objects and r is the distance between their centers.

EXECUTE: Maximum speed occurs when the spheres are very far apart. Energy conservation gives $\dfrac{kq_1q_2}{r} = \dfrac{1}{2}m_{50}v_{50}^2 + \dfrac{1}{2}m_{150}v_{150}^2$. Momentum conservation gives $m_{50}v_{50} = m_{150}v_{150}$ and $v_{50} = 3v_{150}$. $r = 0.50$ m. Solve for v_{50} and v_{150}: $v_{50} = 12.7$ m/s, $v_{150} = 4.24$ m/s. Maximum acceleration occurs just after spheres are released. $\sum F = ma$ gives $\dfrac{kq_1q_2}{r^2} = m_{150}a_{150}$.

$\dfrac{(9\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(10^{-5}\ \text{C})(3\times10^{-5}\ \text{C})}{(0.50\ \text{m})^2} = (0.15\ \text{kg})a_{150}$. $a_{150} = 72.0\ \text{m/s}^2$ and $a_{50} = 3a_{150} = 216\ \text{m/s}^2$.

EVALUATE: The more massive sphere has a smaller acceleration and a smaller final speed.

23.72. IDENTIFY: The potential at the surface of a uniformly charged sphere is $V = \dfrac{kQ}{R}$.

SET UP: For a sphere, $V = \dfrac{4}{3}\pi R^3$. When the raindrops merge, the total charge and volume are conserved.

EXECUTE: **(a)** $V = \dfrac{kQ}{R} = \dfrac{k(-3.60\times10^{-12}\text{ C})}{6.50\times10^{-4}\text{ m}} = -49.8\text{ V}.$

(b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2}\,R = 8.19\times10^{-4}\text{ m}$ and the new charge is $Q_{\text{new}} = 2Q = -7.20\times10^{-12}\text{ C}$. The new potential is

$V_{\text{new}} = \dfrac{kQ_{\text{new}}}{R_{\text{new}}} = \dfrac{k(-7.20\times10^{-12}\text{ C})}{8.19\times10^{-4}\text{ m}} = -79.0\text{ V}.$

EVALUATE: The charge doubles but the radius also increases and the potential at the surface increases by only a factor of $\dfrac{2}{2^{1/3}} = 2^{2/3} \approx 1.6$.

23.73. **IDENTIFY:** Slice the rod into thin slices and use $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$ to calculate the potential due to each slice. Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point P are shown in Figure 23.73a.

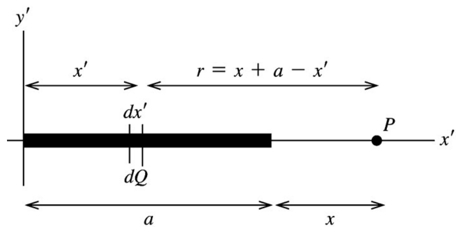


Figure 23.73a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance x given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx'. Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point P. The potential at P due to the small slice dQ is

$$dV = \frac{1}{4\pi\varepsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\left(\frac{dx'}{x+a-x'}\right).$$

Compute the total V at P due to the entire rod by integrating dV over the length of the rod $(x'=0$ to $x'=a)$:

$$V = \int dV = \frac{Q}{4\pi\varepsilon_0 a}\int_0^a \frac{dx'}{(x+a-x')} = \frac{Q}{4\pi\varepsilon_0 a}[-\ln(x+a-x')]_0^a = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{x+a}{x}\right).$$

EVALUATE: As $x \to \infty$, $V \to \dfrac{Q}{4\pi\varepsilon_0 a}\ln\left(\dfrac{x}{x}\right) = 0.$

(b) SET UP: An infinitesimal slice of the rod and its distance from point R are shown in Figure 23.73b.

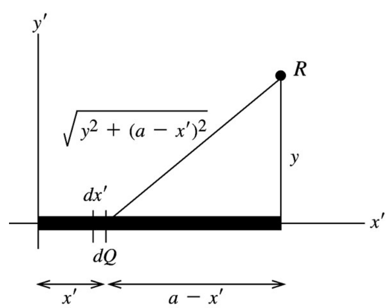


Figure 23.73b

$dQ = (Q/a)dx'$ as in part (a).

Each slice dQ is a distance $r = \sqrt{y^2 + (a - x')^2}$ from point *R*.

EXECUTE: The potential *dV* at *R* due to the small slice dQ is

$$dV = \frac{1}{4\pi\varepsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\varepsilon_0 a}\int_0^a \frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

In the integral make the change of variable $u = a - x'; du = -dx'$

$$V = -\frac{Q}{4\pi\varepsilon_0 a}\int_a^0 \frac{du}{\sqrt{y^2 + u^2}} = -\frac{Q}{4\pi\varepsilon_0 a}\left[\ln\left(u + \sqrt{y^2 + u^2}\right)\right]_a^0.$$

$$V = -\frac{Q}{4\pi\varepsilon_0 a}\left[\ln y - \ln(a + \sqrt{y^2 + a^2})\right] = \frac{Q}{4\pi\varepsilon_0 a}\left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right)\right].$$

(The expression for the integral was found in Appendix B.)

EVALUATE: As $y \to \infty, V \to \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{y}{y}\right) = 0.$

(c) SET UP: *part (a):* $V = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{x+a}{x}\right) = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(1 + \frac{a}{x}\right).$

From Appendix B, $\ln(1+u) = u - u^2/2\ldots$, so $\ln(1 + a/x) = a/x - a^2/2x^2$ and this becomes a/x when *x* is large.

EXECUTE: Thus $V \to \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{a}{x}\right) = \frac{Q}{4\pi\varepsilon_0 x}$. For large *x*, *V* becomes the potential of a point charge.

part (b): $V = \frac{Q}{4\pi\varepsilon_0 a}\left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right)\right] = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{a}{y} + \sqrt{1 + \frac{a^2}{y^2}}\right).$

From Appendix B, $\sqrt{1 + a^2/y^2} = (1 + a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \ldots$

Thus $a/y + \sqrt{1 + a^2/y^2} \to 1 + a/y + a^2/2y^2 + \ldots \to 1 + a/y$. And then using $\ln(1+u) \approx u$ gives

$$V \to \frac{Q}{4\pi\varepsilon_0 a}\ln(1 + a/y) \to \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{a}{y}\right) = \frac{Q}{4\pi\varepsilon_0 y}.$$

EVALUATE: For large *y*, *V* becomes the potential of a point charge.

23.74. IDENTIFY: Apply conservation of energy, $K_a + U_a = K_b + U_b$.

SET UP: Assume the particles initially are far apart, so $U_a = 0$. The alpha particle has zero speed at the distance of closest approach, so $K_b = 0$. $1\text{ eV} = 1.60 \times 10^{-19}$ J. The alpha particle has charge $+2e$ and the lead nucleus has charge $+82e$.

EXECUTE: Set the alpha particle's kinetic energy equal to its potential energy: $K_a = U_b$ gives

$$9.50\text{ MeV} = \frac{k(2e)(82e)}{r} \text{ and } r = \frac{k(164)(1.60 \times 10^{-19}\text{ C})^2}{(9.50 \times 10^6\text{ eV})(1.60 \times 10^{-19}\text{ J/eV})} = 2.48 \times 10^{-14}\text{ m}.$$

EVALUATE: The calculation assumes that at the distance of closest approach the alpha particle is outside the radius of the lead nucleus.

23.75. (a) IDENTIFY and SET UP: The potential at the surface of a charged conducting sphere is: $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R}$.

For spheres A and B this gives $V_A = \dfrac{Q_A}{4\pi\epsilon_0 R_A}$ and $V_B = \dfrac{Q_B}{4\pi\epsilon_0 R_B}$.

EXECUTE: $V_A = V_B$ gives $Q_A/4\pi\epsilon_0 R_A = Q_B/4\pi\epsilon_0 R_B$ and $Q_B/Q_A = R_B/R_A$. And then $R_A = 3R_B$ implies $Q_B/Q_A = 1/3$.

(b) IDENTIFY and SET UP: The electric field at the surface of a charged conducting sphere is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{R^2}$.

EXECUTE: For spheres A and B this gives $E_A = \dfrac{|Q_A|}{4\pi\epsilon_0 R_A^2}$ and $E_B = \dfrac{|Q_B|}{4\pi\epsilon_0 R_B^2}$.

$$\frac{E_B}{E_A} = \left(\frac{|Q_B|}{4\pi\epsilon_0 R_B^2}\right)\left(\frac{4\pi\epsilon_0 R_A^2}{|Q_A|}\right) = |Q_B/Q_A|(R_A/R_B)^2 = (1/3)(3)^2 = 3.$$

EVALUATE: The sphere with the larger radius needs more net charge to produce the same potential. We can write $E = V/R$ for a sphere, so with equal potentials the sphere with the smaller R has the larger E.

23.76. IDENTIFY and SET UP: For points outside of them, the spheres behave as though all the charge were concentrated at their centers. The charge initially on sphere 1 spreads between the two spheres such as to bring them to the same potential.

EXECUTE: (a) $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{R_1^2}$, $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{R_1} = R_1 E_1$.

(b) Two conditions must be met:

1) Let q_1 and q_2 be the final charges of each sphere. Then $q_1 + q_2 = Q_1$ (charge conservation).

2) Let V_1 and V_2 be the final potentials of each sphere. All points of a conductor are at the same potential, so $V_1 = V_2$.

$V_1 = V_2$ requires that $\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1}{R_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_2}{R_2}$ and then $q_1/R_1 = q_2/R_2$.

$$q_1 R_2 = q_2 R_1 = (Q_1 - q_1)R_1.$$

This gives $q_1 = (R_1/[R_1 + R_2])Q_1$ and $q_2 = Q_1 - q_1 = Q_1(1 - R_1/[R_1 + R_2]) = Q_1(R_2/[R_1 + R_2])$.

(c) $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1}{R_1} = \dfrac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$ and $V_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_2}{R_2} = \dfrac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$, which equals V_1 as it should.

(d) $E_1 = \dfrac{V_1}{R_1} = \dfrac{Q_1}{4\pi\epsilon_0 R_1(R_1 + R_2)}$. $E_2 = \dfrac{V_2}{R_2} = \dfrac{Q_1}{4\pi\epsilon_0 R_2(R_1 + R_2)}$.

EVALUATE: Part (a) says $q_2 = q_1(R_2/R_1)$. The sphere with the larger radius needs more charge to produce the same potential at its surface. When $R_1 = R_2$, $q_1 = q_2 = Q_1/2$. The sphere with the larger radius has the smaller electric field at its surface.

23.77. IDENTIFY: Apply conservation of energy: $E_1 = E_2$.

SET UP: In the collision the initial kinetic energy of the two particles is converted into potential energy at the distance of closest approach.

EXECUTE: (a) The two protons must approach to a distance of $2r_\text{p}$, where r_p is the radius of a proton.

$E_1 = E_2$ gives $2\left[\dfrac{1}{2}m_\text{p}v^2\right] = \dfrac{ke^2}{2r_\text{p}}$ and $v = \sqrt{\dfrac{k(1.60\times10^{-19}\text{ C})^2}{2(1.2\times10^{-15}\text{ m})(1.67\times10^{-27}\text{ kg})}} = 7.58\times10^6$ m/s.

(b) For a helium-helium collision, the charges and masses change from (a) and

$$v = \sqrt{\frac{k(2(1.60\times10^{-19}\ \text{C}))^2}{(3.5\times10^{-15}\ \text{m})(2.99)(1.67\times10^{-27}\ \text{kg})}} = 7.26\times10^{6}\ \text{m/s}.$$

(c) $K = \dfrac{3kT}{2} = \dfrac{mv^2}{2}$. $T_\text{p} = \dfrac{m_\text{p}v^2}{3k} = \dfrac{(1.67\times10^{-27}\,\text{kg})(7.58\times10^{6}\,\text{m/s})^2}{3(1.38\times10^{-23}\,\text{J/K})} = 2.3\times10^{9}\,\text{K}.$

$$T_\text{He} = \frac{m_\text{He}v^2}{3k} = \frac{(2.99)(1.67\times10^{-27}\ \text{kg})(7.26\times10^{6}\ \text{m/s})^2}{3(1.38\times10^{-23}\ \text{J/K})} = 6.4\times10^{9}\ \text{K}.$$

(d) These calculations were based on the particles' average speed. The distribution of speeds ensures that there is always a certain percentage with a speed greater than the average speed, and these particles can undergo the necessary reactions in the sun's core.

EVALUATE: The kinetic energies required for fusion correspond to very high temperatures.

23.78. IDENTIFY and **SET UP:** Apply $\vec{E} = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial y}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right)$. $\dfrac{W_{a\to b}}{q_0} = V_a - V_b$ and $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$.

EXECUTE: **(a)** $\vec{E} = -\dfrac{\partial V}{\partial x}\hat{i} - \dfrac{\partial V}{\partial y}\hat{j} - \dfrac{\partial V}{\partial z}\hat{k} = -2Ax\hat{i} + 6Ay\hat{j} - 2Az\hat{k}.$

(b) A charge is moved in along the z-axis. The work done is given by

$W = q\int_{z_0}^{0} \vec{E}\cdot\hat{k}dz = q\int_{z_0}^{0}(-2Az)dz = +(Aq)z_0^2$. Therefore, $A = \dfrac{W_{a\to b}}{qz_0^2} = \dfrac{6.00\times10^{-5}\ \text{J}}{(1.5\times10^{-6}\ \text{C})(0.250\ \text{m})^2} = 640\ \text{V/m}^2.$

(c) $\vec{E}(0,0,0.250) = -2(640\ \text{V/m}^2)(0.250\ \text{m})\hat{k} = -(320\ \text{V/m})\hat{k}.$

(d) In every plane parallel to the xz-plane, y is constant, so $V(x,y,z) = Ax^2 + Az^2 - C$, where $C = 3Ay^2$.

$x^2 + z^2 = \dfrac{V+C}{A} = R^2$, which is the equation for a circle since R is constant as long as we have constant potential on those planes.

(e) $V = 1280\ \text{V}$ and $y = 2.00\ \text{m}$, so $x^2 + z^2 = \dfrac{1280\ \text{V} + 3(640\ \text{V/m}^2)(2.00\ \text{m})^2}{640\ \text{V/m}^2} = 14.0\ \text{m}^2$ and the radius of the circle is $3.74\ \text{m}$.

EVALUATE: In any plane parallel to the xz-plane, $\vec{E}$ projected onto the plane is radial and hence perpendicular to the equipotential circles.

23.79. IDENTIFY and **SET UP:** We know that the potential is of the mathematical form $V(x,y,z) = Ax^l + By^m + Cz^n + D$. We also know that $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, and $E_z = -\dfrac{\partial V}{\partial z}$. Various measurements are given in the table with the problem in the text.

EXECUTE: **(a)** First get A, B, C, and D using data from the table in the problem.

$V(0, 0, 0) = 10.0\ \text{V} = 0 + 0 + 0 + D$, so $D = 10.0\ \text{V}$.

$V(1.00, 0, 0) = A(1.00\ \text{m})^l + 0 + 0 + 10.0\ \text{V} = 4.00\ \text{V}$, so $A = -6.0\ \text{V}\cdot\text{m}^{-l}$.

$V(0, 1.00, 0) = B(1.00\ \text{m})^m + 10.0\ \text{V} = 6.0\ \text{V}$, so $B = -4.0\ \text{V}\cdot\text{m}^{-m}$.

$V(0, 0, 1.00\ \text{m}) = C(1.00\ \text{m})^n + 10.0\ \text{V} = 8.0\ \text{V}$, so $C = -2.0\ \text{V}\cdot\text{m}^{-n}$.

Now get l, m, and n.

$E_x = -\dfrac{\partial V}{\partial x} = -lAx^{l-1}$, and from the table we know that $E_x(1.00, 0, 0) = 12.0$ V/m. Therefore

$-l(-6.0\ \text{V}\cdot\text{m}^{-l})(1.00\ \text{m})^{l-1} = 12.0\ \text{V/m}$.

$l(6.0\ \text{V}\cdot\text{m}^{-1}) = 12.0\ \text{V/m}$.

$l = 2.0$.

$E_y = -\dfrac{\partial V}{\partial y} = -mBy^{m-1}$.

$E_y(0, 1.00, 0) = -m(-4.0\ \text{V}\cdot\text{m}^{-m})(1.00\ \text{m})^{m-1} = 12.0\ \text{V/m}$.

$m = 3.0$.

$E_z = -\dfrac{\partial V}{\partial z} = -nCz^{n-1}$.
$E_z(0, 0, 1.00) = -n(-2.0\ \text{V}\cdot\text{m}^{-n})(1.00\ \text{m})^{n-1} = 12.0$ V/m.
$n = 6.0$.
Now that we have l, m, and n, we see the units of A, B, and C, so
$A = -6.0\ \text{V/m}^2$.
$B = -4.0\ \text{V/m}^3$.
$C = -2.0\ \text{V/m}^6$.
Therefore the equation for $V(x,y,z)$ is
$V = (-6.0\ \text{V/m}^2)x^2 + (-4.0\ \text{V/m}^3)y^3 + (-2.0\ \text{V/m}^6)z^6 + 10.0$ V.
(b) At (0, 0, 0): $V = 0$ and $E = 0$ (from the table with the problem).
At (0.50 m, 0.50 m, 0.50 m):
$V = (-6.0\ \text{V/m}^2)(0.50\ \text{m})^2 + (-4.0\ \text{V/m}^3)(0.50\ \text{m})^3 + (-2.0\ \text{V/m}^6)(0.50\ \text{m})^6 + 10.0\ \text{V} = 8.0$ V.

$$E_x = -\frac{\partial V}{\partial x} = -(-12.0\ \text{V/m}^2)x = (12.0\ \text{V/m}^2)(0.50\ \text{m}) = 6.0\ \text{V/m}.$$

$$E_y = -\frac{\partial V}{\partial y} = -3(-4.0\ \text{V/m}^3)y^2 = (12\ \text{V/m}^3)(0.50\ \text{m})^2 = 3.0\ \text{V/m}.$$

$$E_z = -\frac{\partial V}{\partial z} = -(-12.0\ \text{V/m}^6)z^5 = (12.0\ \text{V/m}^6)(0.50\ \text{m})^5 = 0.375\ \text{V/m}.$$

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(6.0\ \text{V/m})^2 + (3.0\ \text{V/m})^2 + (0.375\ \text{V/m})^2} = 6.7\ \text{V/m}.$$

At (1.00 m, 1.00 m, 1.00 m):
Follow the same procedure as above. The results are $V = -2.0$ V, $E = 21$ V/m.
EVALUATE: We know that l, m, and n must be greater that 1 because the components of the electric field are all zero at (0, 0, 0).

23.80. IDENTIFY and SET UP: Energy is conserved and the potential energy is $U = k\dfrac{q_1q_2}{r}$. $K_1 + U_1 = K_2 + U_2$.

EXECUTE: **(a)** Energy conservation gives $K_1 + 0 = K_2 + U_2$.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + k\frac{qQ}{x} \quad \rightarrow \quad v^2 = v_0^2 - \frac{2kqQ}{m}\cdot\frac{1}{x}.$$

On a graph of v_2 versus $1/x$, the graph of this equation will be a straight line with y-intercept equal to v_0^2 and slope equal to $-\dfrac{2kqQ}{m}$.

(b) With the given equation of the line in the problem, we have $v^2 = 400\ \text{m}^2/\text{s}^2 - (15.75\ \text{m}^3/\text{s}^2)\dfrac{1}{x}$. As x gets very large, $1/x$ approaches zero, so $v_0 = \sqrt{400\,\text{m}^2/\text{s}^2} = 20$ m/s.

(c) The slope is $-\dfrac{2kqQ}{m} = -15.75\ \text{m}^3/\text{s}^2$, which gives

$Q = -m(\text{slope})/2kq = -(4.00\times10^{-4}\ \text{kg})(-15.75\ \text{m}^3/\text{s}^2)/[2k(5.00\times10^{-8}\ \text{C})] = +7.01\times10^{-6}\ \text{C} = +7.01\ \mu\text{C}$.
(d) The particle is closest when its speed is zero, so

$v^2 = 400\ \text{m}^2/\text{s}^2 - (15.75\ \text{m}^3/\text{s}^2)\dfrac{1}{x} = 0$, which gives $x = 3.94\times10^{-2}\ \text{m} = 3.94$ cm.

EVALUATE: From the graph in the problem, we see that v^2 decreases as $1/x$ increases, so v^2 decreases as x decreases. This means that the positively charged particle is slowing down as it gets closer to the sphere, so the sphere is repelling it. Therefore the sphere must be positively charged, as we found.

23.81. IDENTIFY: When the oil drop is at rest, the upward force $|q|E$ from the electric field equals the downward weight of the drop. When the drop is falling at its terminal speed, the upward viscous force equals the downward weight of the drop.

SET UP: The volume of the drop is related to its radius r by $V = \dfrac{4}{3}\pi r^3$.

EXECUTE: **(a)** $F_g = mg = \frac{4\pi r^3}{3}\rho g$. $F_e = |q|E = |q|V_{AB}/d$. $F_e = F_g$ gives $|q| = \frac{4\pi}{3}\frac{\rho r^3 gd}{V_{AB}}$.

(b) $\frac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t$ gives $r = \sqrt{\frac{9\eta v_t}{2\rho g}}$. Using this result to replace r in the expression in part (a) gives

$$|q| = \frac{4\pi}{3}\frac{\rho g d}{V_{AB}}\left[\sqrt{\frac{9\eta v_t}{2\rho g}}\right]^3 = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

(c) We use the values for V_{AB} and v_t given in the table in the problem and the formula $|q| = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}$ from (c). For example, for drop 1 we get

$|q| = 18\pi\frac{1.00\times10^{-3}\text{ m}}{9.16\text{ V}}\sqrt{\frac{(1.81\times10^{-5}\text{ N}\cdot\text{s/m}^2)^3(2.54\times10^{-5}\text{ m/s})^3}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 4.79\times10^{-19}\text{ C}$. Similar calculations for the remaining drops gives the following results:

Drop 1: 4.79×10^{-19} C
Drop 2: 1.59×10^{-19} C
Drop 3: 8.09×10^{-19} C
Drop 4: 3.23×10^{-19} C

(d) Use $n = q/e_2$ to find the number of excess electrons on each drop. Since all quantities have a power of 10^{-19} C, this factor will cancel, so all we need to do is divide the coefficients of 10^{-19} C. This gives
Drop 1: $n = q_1/q_2 = 4.79/1.59 = 3$ excess electrons
Drop 2: $n = q_2/q_2 = 1$ excess electron
Drop 3: $n = q_3/q_2 = 8.09/1.59 = 5$ excess electrons
Drop 4: $n = q_4/q_2 = 3.23/1.59 = 2$ excess electrons

(e) Using $q = -ne$ gives $e = -q/n$. All the charges are negative, so e will come out positive. Thus we get
Drop 1: $e_1 = q_1/n_1 = (4.79\times10^{-19}\text{ C})/3 = 1.60\times10^{-19}$ C
Drop 2: $e_2 = q_2/n_2 = (1.59\times10^{-19}\text{ C})/1 = 1.59\times10^{-19}$ C
Drop 3: $e_3 = q_3/n_3 = (8.09\times10^{-19}\text{ C})/5 = 1.62\times10^{-19}$ C
Drop 4: $e_4 = q_4/n_4 = (3.23\times10^{-19}\text{ C})/2 = 1.61\times10^{-19}$ C
The average is
$e_{av} = (e_1 + e_2 + e_3 + e_4)/4 = [(1.60 + 1.59 + 1.62 + 1.61)\times10^{-19}\text{ C}]/4 = 1.61\times10^{-19}$ C.

EVALUATE: The result $e = 1.61\times10^{-19}$ C is very close to the well-established value of 1.60×10^{-19} C.

23.82. **IDENTIFY:** Consider the potential due to an infinitesimal slice of the cylinder and integrate over the length of the cylinder to find the total potential. The electric field is along the axis of the tube and is given by $E = -\frac{\partial V}{\partial x}$.

SET UP: Use the expression from Example 23.11 for the potential due to each infinitesimal slice. Let the slice be at coordinate z along the x-axis, relative to the center of the tube.

EXECUTE: **(a)** For an infinitesimal slice of the finite cylinder, we have the potential

$dV = \frac{k\,dQ}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\frac{dz}{\sqrt{(x-z)^2 + R^2}}$. Integrating gives

$V = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dz}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\int_{-L/2-x}^{L/2-x}\frac{du}{\sqrt{u^2 + R^2}}$ where $u = x - z$. Therefore,

$V = \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2-x)^2 + R^2} + (L/2-x)}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right]$ on the cylinder axis.

(b) For $L << R$, $V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2-x)^2 + R^2} + L/2 - x}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right] \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{x^2 - xL + R^2} + L/2 - x}{\sqrt{x^2 + xL + R^2} - L/2 - x}\right]$.

$$V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{1-xL/(R^2+x^2)}+(L/2-x)/\sqrt{R^2+x^2}}{\sqrt{1+xL/(R^2+x^2)}+(-L/2-x)/\sqrt{R^2+x^2}}\right]=\frac{kQ}{L}\ln\left[\frac{1-xL/2(R^2+x^2)+(L/2-x)/\sqrt{R^2+x^2}}{1+xL/2(R^2+x^2)+(-L/2-x)/\sqrt{R^2+x^2}}\right].$$

$$V \approx \frac{kQ}{L}\ln\left[\frac{1+L/2\sqrt{R^2+x^2}}{1-L/2\sqrt{R^2+x^2}}\right]=\frac{kQ}{L}\left(\ln\left[1+\frac{L}{2\sqrt{R^2+x^2}}\right]-\ln\left[1-\frac{L}{2\sqrt{R^2+x^2}}\right]\right).$$

$V \approx \dfrac{kQ}{L}\dfrac{2L}{2\sqrt{x^2+R^2}}=\dfrac{kQ}{\sqrt{x^2+R^2}}$, which is the same as for a ring.

(c) $E_x=-\dfrac{\partial V}{\partial x}=\dfrac{2kQ\left(\sqrt{(L-2x)^2+4R^2}-\sqrt{(L+2x)^2+4R^2}\right)}{\sqrt{(L-2x)^2+4R^2}\sqrt{(L+2x)^2+4R^2}}.$

EVALUATE: For $L \ll R$ the expression for E_x reduces to that for a ring of charge, $E_x=\dfrac{kQx}{(x^2+a^2)^{3/2}}$, as shown in Example 23.14.

23.83. **IDENTIFY:** Angular momentum and energy must be conserved.

SET UP: At the distance of closest approach the speed is not zero. $E=K+U$. $q_1=2e$, $q_2=82e$.

EXECUTE: $mv_1b=mv_2r_2$. $E_1=E_2$ gives $E_1=\frac{1}{2}mv_2{}^2+\frac{kq_1q_2}{r_2}$. $E_1=11\text{ MeV}=1.76\times10^{-12}$ J. r_2 is the distance of closest approach. Substituting in for $v_2=v_1\left(\frac{b}{r_2}\right)$ we find $E_1=E_1\frac{b^2}{r_2{}^2}+\frac{kq_1q_2}{r_2}$.

$(E_1)r_2^2-(kq_1q_2)r_2-E_1b^2=0$. For $b=10^{-12}$ m, $r_2=1.01\times10^{-12}$ m. For $b=10^{-13}$ m, $r_2=1.11\times10^{-13}$ m. And for $b=10^{-14}$ m, $r_2=2.54\times10^{-14}$ m.

EVALUATE: As b decreases the collision is closer to being head-on and the distance of closest approach decreases. Problem 23.74 shows that the distance of closest approach is 2.48×10^{-14} m when $b=0$, which is very close to our value.

23.84. **IDENTIFY** and **SET UP:** The He ions are first accelerated toward the center and then accelerated away from the center, but always in the same direction. During the first acceleration, their charge is $-e$, and during the second acceleration it is $+2e$. The work-energy theorem gives $\Delta K=q\Delta V$. Call V the voltage at the center.

EXECUTE: **(a)** Toward the center: $\Delta K=q\Delta V=eV$.

Away from the center: $\Delta K=q\Delta V=2eV$.

The ions gain 3.0 MeV of kinetic energy, so $eV+2eV=3.0$ MeV.

$3eV=3.0$ MeV.

$V=+1.0$ MV, since the e cancels. This is choice (d).

EVALUATE: The negative He^- ions are accelerating to higher potential, and the positive He^{++} ions are accelerating toward lower potential.

23.85. **IDENTIFY** and **SET UP:** Conservation of energy gives $K=U_{\text{electric}}=k\dfrac{q_1q_2}{r}$.

EXECUTE: Solve for Q: $Q=rK/kq=(10\times10^{-15}\text{ m})(3.0\text{ MeV})/(2ek)=1.67\times10^{-18}$ C. In terms of e, this is $Q=(1.67\times10^{-18}\text{ C})/(1.60\times10^{-19}\text{ C})=10.4e\approx 11e$, so choice (b) is best.

EVALUATE: If $Q=11e$, the atom is sodium (Na), which has an atomic mass of 23, compared to 4 for He. So it is reasonable to assume that the nucleus does not move appreciably, since it is about 6 times more massive than the He.

23.86. **IDENTIFY** and **SET UP:** The potential changes by 6.0 MV over a distance of 12 m. $E_{\text{av}}=\dfrac{\Delta V}{\Delta x}$.

EXECUTE: $E_{\text{av}}=\dfrac{\Delta V}{\Delta x}=(6.0\text{ MV})/(12\text{ m})=0.50\times10^6\text{ V/m}=500{,}000\text{ V/m}$, which is choice (c).

EVALUATE: The actual variation of the field may be somewhat complicated, but the average value gives a good idea of a typical electric field in such apparatus.