Lecture 5: Our planetary system

Read: Ch. 22.1, 26 of “Astronomy: a Physical Perspective” (M. Kutner)
Ch. S1 of “The Cosmic Perspective” (Bennett et al.)

Prof Aline Vidotto

Quick recap of last lecture

Kepler’s 3rd+Newton Gravitational

Used to derive the mass of astronomical objects

$$(M_1 + M_2) = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

If $M_2 \ll M_1$, then $M_1+M_2 \approx M_1$ and:

$$M_1 = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

- knowing the orbital period and semi-major axis of an orbiting planet, you derive the mass of the central star.
- knowing the orbital period and semi-major axis of an orbiting moon, you derive the mass of the central planet. etc

Useful trick: normalise general form of Kepler’s 3rd law in more useful units:

For the solar system:

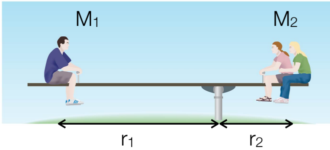
$$\left(\frac{P}{1\text{yr}}\right)^2 = \left(\frac{a}{1\text{AU}}\right)^3$$

In general:

$$\frac{(M_1 + M_2)}{M_\odot} = \frac{(a/1\text{AU})^3}{(P/1\text{yr})^2}$$

Centre of mass: $M_1 r_1 = M_2 r_2$

Unequal masses:



Energies of orbits:

$$E_{\text{circular}} = -\frac{GmM}{2r}$$

$$E_{\text{elliptical}} = -\frac{GmM}{2a}$$

The total orbital energy is **conserved** as long as no other object causes the planet (or other object in orbit) to gain or lose orbital energy

What we will cover today…

Goal: learn how to derive physical data of the planets and a general overview of the solar system

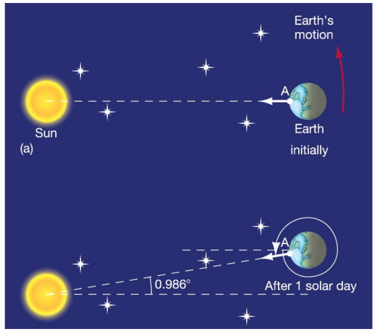
1. How to measure the period of the planets?
2. How to measure the orbital distances of the planets?
3. Angular momentum in the solar system
4. Solar system inventory

1. How to measure the period of the planets?

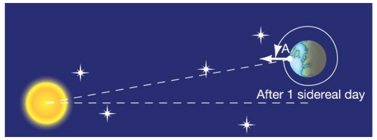
The Length of the Day (recap)

- Solar day
  - The amount of time time takes the Earth to spin once with respect to the Sun.
  - 24 hours
- Earth rotates through slightly more than 360º for the Sun to return to the same apparent location in the sky. This additional angle is

$$\frac{360^o}{365 \text{ days}} = 0.986^o/\text{day}$$



- Sidereal day
  - The amount of time it takes the Earth to spin once with respect to the distant stars.
  - 23 hours 56 minutes

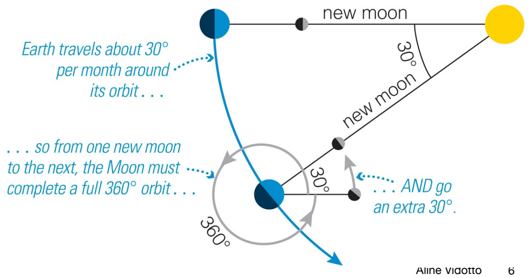


The Length of the Month

- Just like the solar day is not Earth's true rotation period, a synodic month is not the Moon's true orbital period
- Synodic month
  - The amount of time it takes the Moon to repeat its phase (i.e., new to the next new): 29.5 days
- moon orbits slightly more than 360º for the stars to return to the same phase. This additional angle is

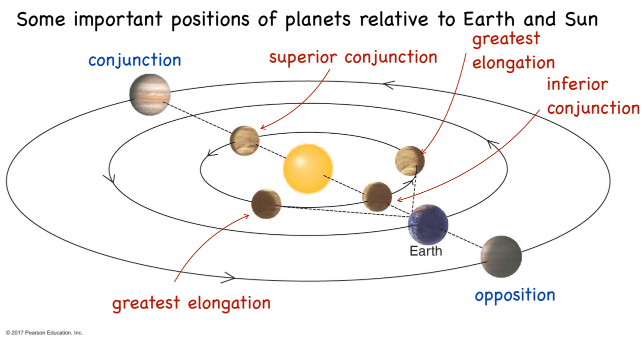
$$\frac{360^o}{12\text{months}} = 30^o/\text{month}$$

- Sidereal month
  - The amount of time time takes the Moon to complete one orbit with respect to the background stars: 27.3 days



Planetary Periods

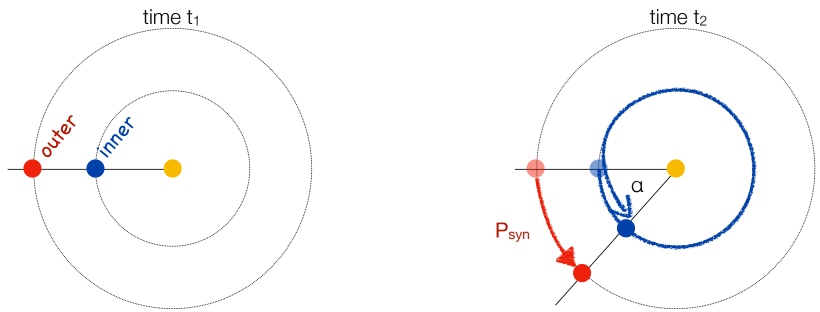
- Sidereal period: The amount of time it takes a planet to orbit the Sun, measured with respect to the background stars.
  - Hard to observe – Earth is not stationary
- Synodic period: The time from when a planet is aligned with the Sun in our sky until it is again aligned similarly.
  - Easy to observe



- Planets beyond Earth's orbit are called superior planets.
- Planets below Earth's orbit are called inferior planets

How to measure the period of the planets?

- We cannot directly measure a planet's orbital period, because we look at the planet from different points in our orbit at different times.
- However, we can measure synoptic periods simply by seeing how much time passes between one alignment and the next (e.g., between two conjunctions)



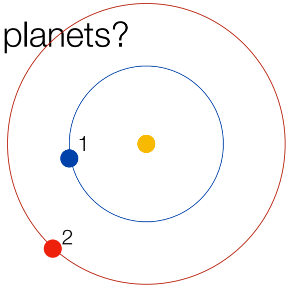
How to measure the period of the planets?

- Consider 2 planets (#1 and #2). Their relative (or synoptic) angular speed is

$$\Omega_{\rm rel} := \Omega_{\rm syn} = \Omega_1 - \Omega_2$$

$$\frac{\cancel{2\pi}}{P_{\rm syn}} = \frac{\cancel{2\pi}}{P_1} - \frac{\cancel{2\pi}}{P_2}$$

$$\frac{1}{P_{\rm syn}} = \frac{1}{P_1} - \frac{1}{P_2}$$



- If planet #1 is Earth (P_1=1year), then for the superior planet (planet #2), we have:

$$\frac{1}{P_{\rm syn}} = 1 - \frac{1}{P^{\rm sup}_{\rm orb}}$$

- If planet #2 is Earth (P_2=1year), then for the inferior planet (planet #1), we have:

$$\frac{1}{P_{\rm syn}} = \frac{1}{P^{\rm inf}_{\rm orb}} - 1$$

Example: measuring the period of planets

- Venus synoptic period is 583.9 days or 1.599 years. What is its actual orbital period (sidereal period)?

$$\frac{1}{P_{\rm syn}} = \frac{1}{P^{\rm inf}_{\rm orb}} - 1 \quad \rightarrow \quad P^{\rm inf}_{\rm orb} = \frac{P_{\rm syn}}{P_{\rm syn} + 1}$$

$$P^{\rm inf}_{\rm orb} = \frac{1.599}{1.5995 + 1} = 0.61 \ {\rm yr}$$

- We measure the time between two oppositions of Jupiter to 398.9 days or 1.092 yr. What is its actual orbital period (sidereal period)?

$$\frac{1}{P_{\rm syn}} = 1 - \frac{1}{P^{\rm sup}_{\rm orb}} \quad \rightarrow \quad P^{\rm sup}_{\rm orb} = \frac{P_{\rm syn}}{P_{\rm syn} - 1}$$

$$P^{\rm sup}_{\rm orb} = \frac{1.092}{1.092 - 1} = 11.87 \ {\rm yr}$$

	Synodic period (yr)	Sidereal period (yr)
Mercury	0.317	0.241
Venus	1.599	0.615
Mars	2.135	1.881
Jupiter	1.092	11.87
Saturn	1.035	29.46
Uranus	1.012	84.01
Neptune	1.006	164.8

Why is the synodic period of outermost planets only slightly larger than 1 year?

Conceptual question

The synodic period of a planet is the time during which a body in the solar system makes one orbit of the Sun relative to the Earth, i.e., returns to the same elongation. For a superior planet to the Earth, a larger synodic period means that its sidereal period is

(a) smaller
(b) larger
(c) the same

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if $P_{\rm syn}$ gets larger, then 1-1/$P_{\rm syn}$ gets larger. Thus $P_{\rm orb}$ must get smaller

$$\frac{1}{P_{\rm syn}} = 1 - \frac{1}{P^{\rm sup}_{\rm orb}} \quad \longrightarrow \quad \frac{1}{P^{\rm sup}_{\rm orb}} = 1 - \frac{1}{P_{\rm syn}}$$

	Synodic period (yr)	Sidereal period (yr)
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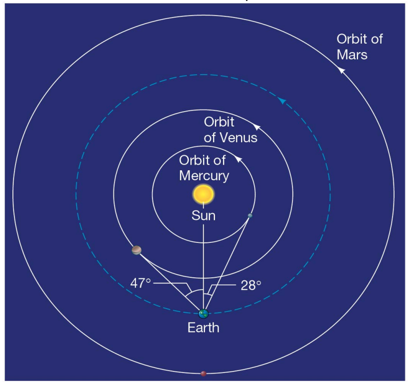
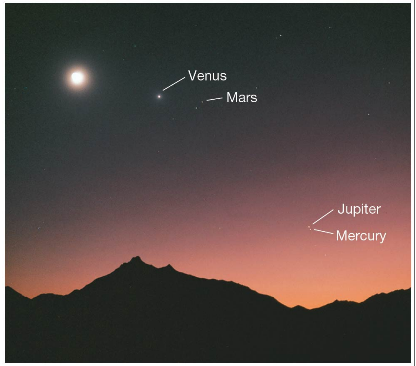
2. How to measure the orbital distances of the planets?

Greatest elongation: greatest angular distances between inferior planets & Sun



(a)

Planets & moon in the evening sky

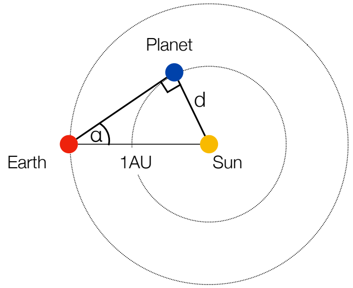


Case 1: inferior planets

- At greatest elongation, the planet appears farthest from the sun

$$d = (1\mathrm{AU})\ \sin\alpha$$

- By measuring the angle α between the Sun and the planet in the sky, we are then able to find the orbital radius d.
- This is given in AU
  - I.e., we can only find *relative* orbital radius
  - This means that we need a different method to find how much one AU is…



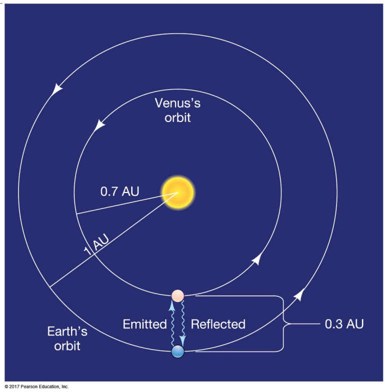
How to measure the Sun-Earth distance?

- We use radar ranging
  - Radio waves are transmitted towards Venus
  - We measure how long it takes for its reflected echo to arrive back at us
  - Multiply (half) the round-trip travel time of the radar signal by the speed of light (≈300,000 km/s)

1. It takes 300s for a radar signal to complete its round-trip when Venus is at closest approach from the Earth. In km, this is distance is

$$\frac{300\mathrm{s}}{2} \times c = 45{,}000{,}000\ \mathrm{km}$$

2. We know that Venus is at 0.7AU from the Sun (technique shown in previous slide). Therefore, at closest approach, it is at (1AU-0.7AU)=0.3AU from us.



- From 1 & 2: Since 0.3AU is 45x10⁶km, we know that 1AU = 1.5x10⁸km.

Case 2: superior planets

- We have to measure planet positions at two times in the orbit (t_1 and t_2)

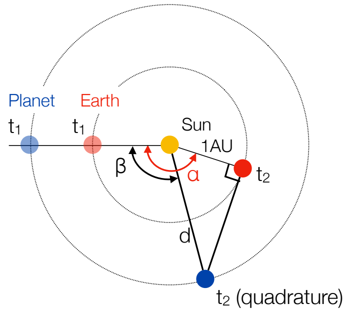
$$\Delta t = t_2 - t_1$$

- The angle α can be related to the time between alignment (opposition at t_1) and quadrature (at t_2). During this interval, Earth will sweep out an angle

$$\alpha = \Delta t \frac{360^o}{1\mathrm{yr}}$$

- If $P_{\mathrm{orb}}^{\mathrm{sup}}$ is the sidereal period of the superior planet, than the angle β is

$$\beta = \Delta t \frac{360^o}{P_{\mathrm{orb}}^{\mathrm{sup}}}$$



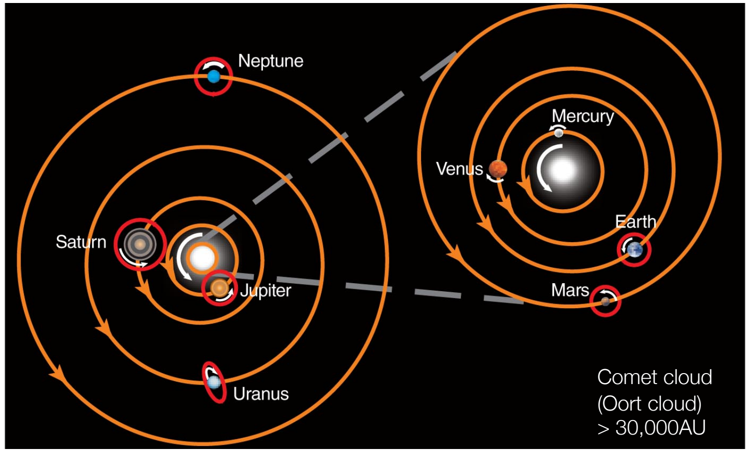
- From trigonometry, the distance is

$$\cos(\alpha - \beta) = \frac{(1\mathrm{AU})}{d} \longrightarrow d = \frac{1}{\cos(\alpha-\beta)}\mathrm{AU}$$

Distance scales

Jovian planets:
Jupiter (5AU), Saturn (10AU),
Uranus (20AU), Neptune (30AU)

Terrestrial planets:
Mercury, Venus, Earth,
Mars < 1.5AU



Conceptual question

Mercury is very hard to observe from Earth because

(a) it always appears only half lit.
(b) it is never more than 28º from the Sun.
(c) its elliptical orbit causes it to change speed unpredictably.
(d) its surface reflects too little sunlight.
(e) its surface does not allow radar to bounce back to Earth.

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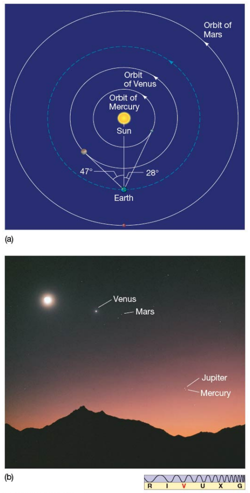
Explanation: Mercury's inner orbit keeps it close to the Sun, visible only for an hour or two before sunrise or after sunset.



3. Angular momentum in the solar system

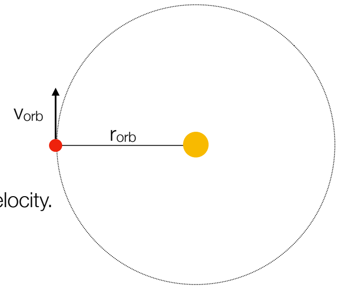
Angular momentum of the orbit

Consider a particle rotating relative to a centre:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$L = mrv_{\perp}$$



For a planet in circular motion, $v_{\perp}$ is the orbital velocity.

$$L_{\rm orb} = M_p r_{\rm orb} v_{\rm orb}$$

$$L_{\rm orb} = M_p r_{\rm orb} \frac{2\pi r_{\rm orb}}{P_{\rm orb}} = M_p r_{\rm orb}^2 \frac{2\pi}{P_{\rm orb}}$$

$P_{\rm orb}$ is the length of a planet-year.

Angular momentum of an object's own rotation

Consider a rigid body rotating about its own axis:

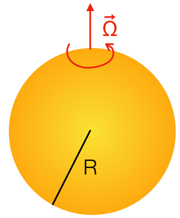
$$\vec{L} = I\vec{\Omega}$$

$$L = \frac{2}{5} MR^2 \Omega$$

$$L = \frac{2}{5} MR^2 \frac{2\pi}{P_{\rm rot}}$$

$$I = \frac{2}{5} MR^2$$

rigid sphere of radius R and mass M



For a planet, $P_{\rm rot}$ is the length of a "planet-day".

$$L_{\rm rot} = M_p R_p^2 \frac{2\pi}{P_{\rm rot}}$$

Total angular momentum of solar system objects

- The total angular momentum is the sum of different forms of angular momenta. Example, for a planet rotating about its own axis and rotating around the Sun, we have:

$$L_p = L_{\rm rot} + L_{\rm orb}$$

- For the solar system, the total angular momentum is the sum of all angular momenta of all the objects in the solar system:

$$L_{\rm tot} = L_{\rm sun} + L_{\rm planets}$$

$$L_{\rm tot} = L_{\rm rot,sun} + \sum_{planets} L_{\rm orb,p}$$

For a planet:

$$L_{\rm rot} = M_p R_p^2 \frac{2\pi}{P_{\rm rot}}$$

$$L_{\rm orb} = M_p r_{\rm orb}^2 \frac{2\pi}{P_{\rm orb}}$$

$$\frac{L_{\rm rot}}{L_{\rm orb}} = \frac{R_p^2}{r_{\rm orb}^2} \frac{P_{\rm orb}}{P_{\rm rot}}$$

Because $r_{\rm orb} \gg R_p$

$$\frac{L_{\rm rot}}{L_{\rm orb}} \ll 1$$

$$L_p \simeq L_{\rm orb}$$

Total angular momentum of solar system objects

$$L_{\rm tot} = L_{\rm rot,sun} + \sum_{planets} L_{\rm orb,p}$$

$$L_{\rm tot} = M_\odot R_\odot^2 \frac{2\pi}{P_{\rm rot,\odot}} + \sum_{planets} M_p r_{\rm orb,p}^2 \frac{2\pi}{P_{\rm orb,p}}$$

Where is most of the angular momentum concentrated in the solar system?

(a) In the sun, because it is a lot more massive than any planet

(b) In the terrestrial planets, because their orbital periods are a lot smaller than the Jovian planets

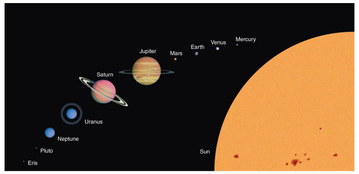
(c) In the Jovian planets, because their orbital radii are a lot larger than any other orbital radius

The "paradox" of the angular momentum

<u>Homework</u>: using a table of data of solar system, calculate the angular momentum of the planets (orbital AM) and of the Sun (rotation AM) and demonstrate that most of the angular momentum in the solar system is in the planets.

- the planets are only ~1% of the total mass of the solar system, and yet they represent 97% of the total angular momentum.

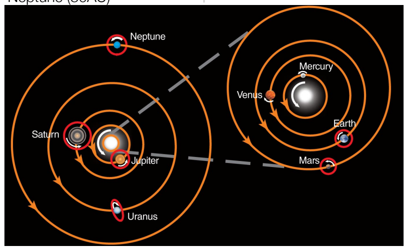
Mass distribution of the solar system



<u>Object</u>	<u>% of total mass</u>
Sun	99.85
Planets	0.135 of which: Jupiter 71% Saturn 21%

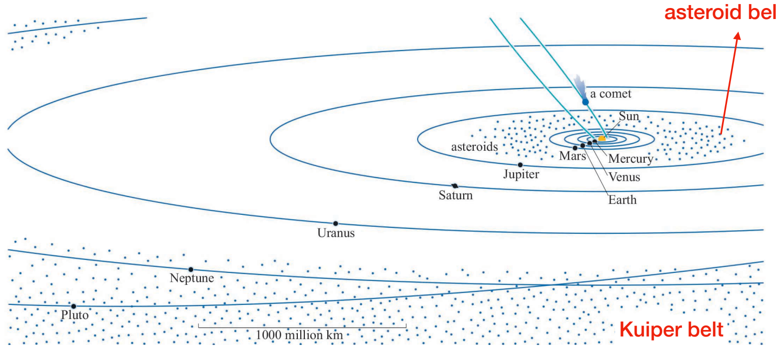
Jovian planets: Jupiter (5AU), Saturn (10AU), Uranus (20AU), Neptune (30AU)

Terrestrial planets: Mercury, Venus, Earth, Mars < 1.5AU



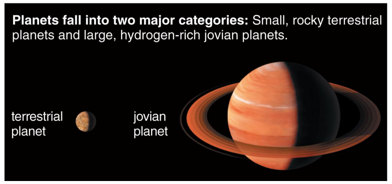
4. Solar system inventory

- There are eight major planets with nearly circular orbits.
  - ✦Planets all orbit in same direction and nearly in same plane.
  - ✦Planets are very tiny compared to distances between them.
- Dwarf planets (like Pluto) are smaller than the major planets and some have quite elliptical orbits.



Two Major Planet Types

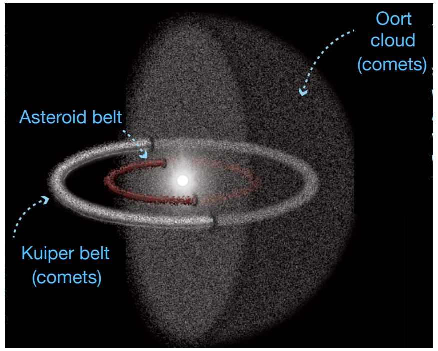
1. Terrestrial planets are rocky, relatively small, and close to the Sun.
2. Jovian planets are gaseous, larger, and farther from the Sun.



Terrestrial Planets	Jovian Planets
Smaller size and mass	Larger size and mass
Higher average density	Lower average density
Made mostly of rocks and metals	Made mostly of hydrogen, helium, and hydrogen compounds
Solid surface	No solid surface
Few (if any) moons and no rings	Rings and many moons
Closer to the Sun (and closer together), with warmer surfaces	Farther from the Sun (and farther apart), with cool temperatures at cloud tops

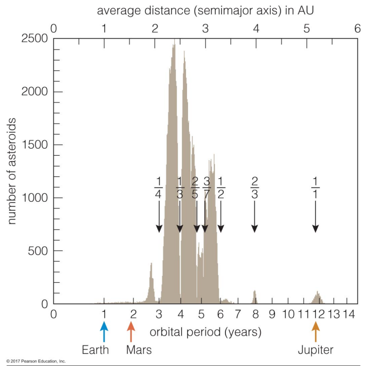
Smaller bodies: rocky asteroids and icy comets

- found throughout the solar system, but are concentrated in 3 distinct regions:

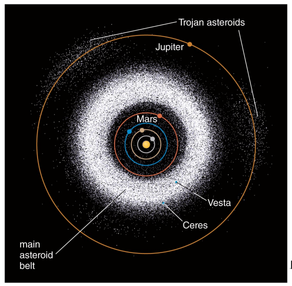


1. Asteroid belt: between the orbit of Mars and Jupiter; where most asteroids lie
2. Oort cloud: spherical shell around the solar system at about 50,000AU from the sun
   - est. 10^{12} - 10^{13} comets in the cloud [total mass: 1-10 M_{earth}]
3. Kuiper belt: icy objects found between orbits of Neptune [30AU] out to 50AU
   - est. 70,000 objects >100 km across. More confined to the ecliptic, contrary to the spherical shell shape of Oort cloud. Likely source of short period comets.

Asteroids

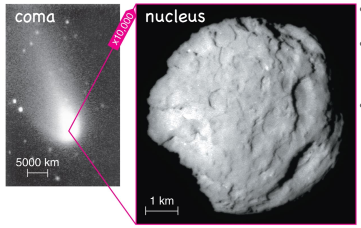


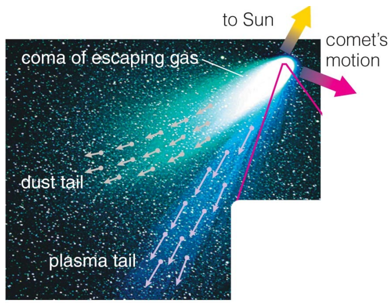
- Asteroids are **rocky** leftovers of planet formation.
- 150,000 in catalogs, and probably over a million with diameter >1 kilometre.
- Small asteroids are more common than large asteroids.
- All the asteroids in the solar system would not add up to even a small terrestrial planet.



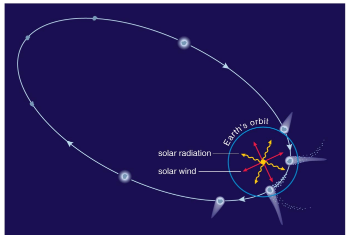
Gaps in asteroid distribution are due to orbital resonances with Jupiter: gaps occur where asteroids would have orbital periods that are simple fractions of Jupiter's 12-year period (e.g., 1/4, 1/3, 1/2, etc)

Comets: icy counterparts to asteroids



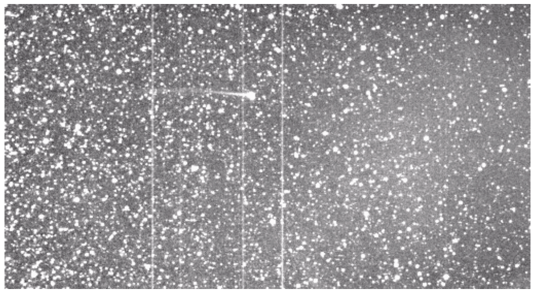


- Nucleus of comet is a *dirty snowball*
- Coma: extended region of gas and dust outside nucleus
- Only when close to the Sun that comets grow tails: comets have two tails
  - ion/plasma tail: blown straight away out by the interaction with the solar wind
  - dust tail: material left behind in the orbit; ejected by pressure from sunlight; dust continues in a Keplerian orbit



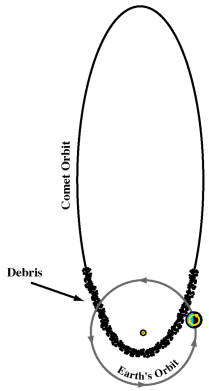
The interaction of a comet with the solar wind

Comet Encke (Vourlidas et al 2007)



- Solar wind: stream of charged particles (ions and electrons) flowing outward from the sun
- Note: all stars have winds. The solar wind is particularly "weak" compared to other types of stellar winds.

Meteoroids: small chunks of matter



- meteor: when a meteoroid falls through the Earth's atmosphere, it is heated by friction and glows
- meteorites: the meteors that don't burn completely and hit the ground
  - ➡ largest meteorites produce craters on Earth and they could have cause extinction of dinosaurs (could they?)
- Meteoroids that produce meteors are probably the debris of comet tails
  - when Earth crosses the comet's orbit, we see a large number of meteors (meteor shower)
  - showers occur at the same time of year, since they represent the passage of Earth through the orbit of the comet
  - most meteors seen after midnight, as after midnight, the observer is on the side of the Earth facing the direction of motion of the Earth

Conceptual question

Which of the following is not a characteristic of jovian planets?

(a) They all have many moons.
(b) They have a higher overall density than terrestrial planets.
(c) They are larger than terrestrial planets.
(d) They are farther from the Sun and farther apart from each other than the terrestrial planets.
(e) They have deep atmospheres made of hydrogen, helium, and hydrogen compounds.

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Conceptual question

How do comets differ from asteroids?

(a) They are mostly ices, not rock.
(b) Their orbits are usually much farther from the Sun.
(c) They are leftover pieces of a smashed planet.
(d) All of the above
(e) A and B

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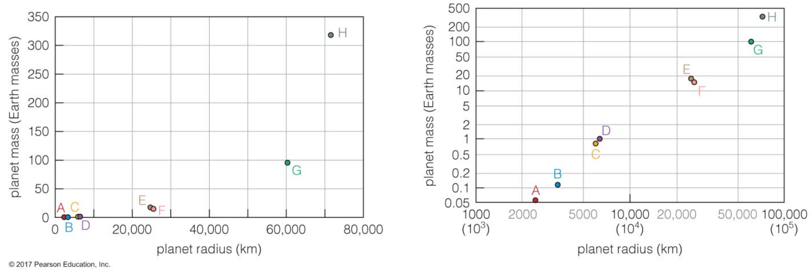
Extra slides - read at home

Conceptual question

<u>Homework</u>: The plots below show the masses of the eight major planets in the solar system on the y-axis and their radii on the x-axis. Left is a linear plot. Right is a log-log plot. Find out which planet corresponds to which dot. Then, notice how the eight planets group roughly into pairs on the graphs. Which planets are in each pair?



Physical data for the planets

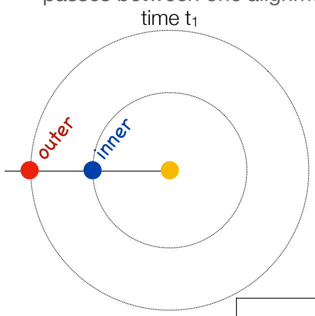
- Earth radius: 6400km
- Earth mass: 6.0×10^{24} kg
- Density: $<\rho> = M/(4/3\ \pi\ R^3)$

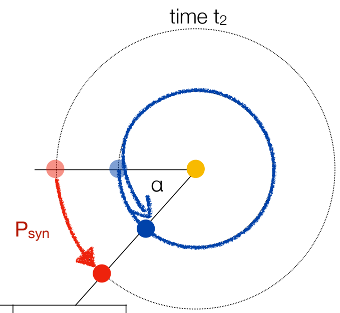
Photo	Planet	Relative Size	Average Distance from Sun (AU)	Average Equatorial Radius (km)	Mass (Earth = 1)	Average Density (g/cm^3)	Orbital Period	Rotation Period	Axis Tilt	Average Surface (or Cloud-Top) Temperature[b]	Composition	Known Moons (2015)	Rings?
	Mercury		0.387	2440	0.055	5.43	87.9 days	58.6 days	0.0°	700 K (day) 100 K (night)	Rocks, metals	0	No
	Venus		0.723	6051	0.82	5.24	225 days	243 days	177.3°	740 K	Rocks, metals	0	No
	Earth		1.00	6378	1.00	5.52	1.00 year	23.93 hours	23.5°	290 K	Rocks, metals	1	No
	Mars		1.52	3397	0.11	3.93	1.88 years	24.6 hours	25.2°	220 K	Rocks, metals	2	No
	Jupiter		5.20	71,492	318	1.33	11.9 years	9.93 hours	3.1°	125 K	H, He, hydrogen compounds[c]	67	Yes
	Saturn		9.54	60,268	95.2	0.70	29.5 years	10.6 hours	26.7°	95 K	H, He, hydrogen compounds[c]	62	Yes
	Uranus		19.2	25,559	14.5	1.32	83.8 years	17.2 hours	97.9°	60 K	H, He, hydrogen compounds[c]	27	Yes
	Neptune		30.1	24,764	17.1	1.64	165 years	16.1 hours	29.6°	60 K	H, He, hydrogen compounds[c]	14	Yes
	Pluto		39.5	1185	0.0022	1.9	248 years	6.39 days	112.5°	44 K	Ices, rock	5	No
	Eris		67.7	1168	0.0028	2.3	557 years	1.08 days	78°	43 K	Ices, rock	1	No

No need to write this down! It can be found in any textbook

How to measure the period of the planets?

- We cannot directly measure a planet's orbital period, because we look at the planet from different points in our orbit at different times.
- However, we can measure synoptic periods simply by seeing how much time passes between one alignment and the next





	Outer planet:	Inner planet:
time for one full revolution (360°)	P_{orb}^o	P_{orb}^i
time for swapping angle α	P_{syn}	$P_{syn} - P_{orb}^i$

$$\frac{P^o_{\rm orb}}{P_{\rm syn}} = \frac{P^i_{\rm orb}}{P_{\rm syn} - P^i_{\rm orb}}$$

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- If inner planet is the Earth, then $P_{orb}^i = 1$ year and the orbital period of the outer planet is (in years)

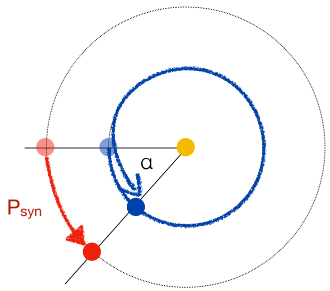
$$P^o_{\rm orb} = \frac{P_{\rm syn}}{P_{\rm syn} - 1}$$

Planets beyond Earth's orbit are called superior planets

- If the outer planet is the Earth, then $P_{orb}^o = 1$ year, and the orbital period of the inner planet is (in years)

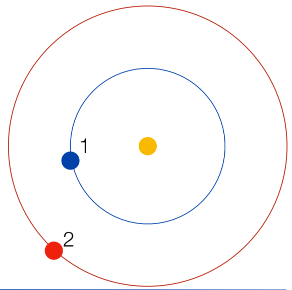
$$P^i_{\rm orb} = \frac{P_{\rm syn}}{P_{\rm syn} + 1}$$

Planets below Earth's orbit are called inferior planets



Another way to demonstrate this

$$P^o_{\rm orb} = \frac{P_{\rm syn}}{P_{\rm syn} - 1} \qquad P^i_{\rm orb} = \frac{P_{\rm syn}}{P_{\rm syn} + 1}$$



- Consider 2 planets (#1 and #2). Their relative (or synoptic) angular speed is

$$\Omega_{\rm rel} := \Omega_{\rm syn} = \Omega_1 - \Omega_2$$

$$\frac{\cancel{2\pi}}{P_{\rm syn}} = \frac{\cancel{2\pi}}{P_1} - \frac{\cancel{2\pi}}{P_2}$$

$$\frac{1}{P_{\rm syn}} = \frac{1}{P_1} - \frac{1}{P_2}$$

- If planet #1 is Earth (P_1=1year), then for the superior planet (planet #2), we have:

$$\frac{1}{P_{\rm syn}} = 1 - \frac{1}{P^o_{\rm orb}}$$

- If planet #2 is Earth (P_2=1year), then for the inferior planet (planet #1), we have:

$$\frac{1}{P_{\rm syn}} = \frac{1}{P^i_{\rm orb}} - 1$$