10.4

$$U(r) = -\frac{A}{r^n}, \quad A > 0$$

For a stable circular orbit, the effective potential $U_{eff}(r) = \frac{\ell^2}{2\mu r^2} - \frac{A}{r^n}$ must have a minimum.

$$\Rightarrow \quad U'_{eff}(r_*) = 0 \quad \text{and} \quad U''_{eff}(r_*) > 0$$

$$\Rightarrow \quad \frac{\ell^2}{2\mu r^3}(-2) - \frac{A}{r^{n+1}}(-n) = 0$$

$$\Rightarrow \quad r_*^{n-2} = \frac{\mu n A}{\ell^2} \quad \Longrightarrow \quad n > 0$$

$$U''_{eff}(r) = \frac{3\ell^2}{\mu r^4} - \frac{n(n+1)A}{r^{n+2}}$$

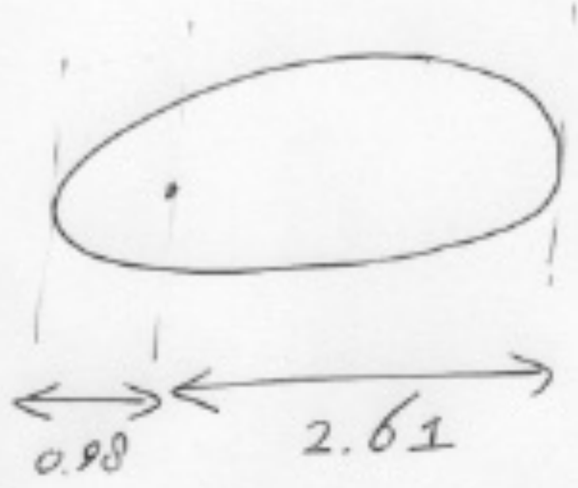
$$U''_{eff}(r_*) = \frac{1}{r_*^4}\left(\frac{3\ell^2}{\mu} - \frac{n(n+1)A\,\ell^2}{\mu n A}\right)$$

$$= \frac{\ell^2}{\mu r_*^4}\left(3 - (n+1)\right) > 0$$

$\Rightarrow$ For a stable orbit $\quad 0 < n < 2.$

Problem Tesla Roadster Orbit

1.



$$E = -\frac{C}{2A} \quad \text{with} \quad 2A = 0.98 + 2.61\ AU = 3.59\ AU$$

$$E = \tfrac{1}{2} m v^2 - \frac{C}{r}$$

$$\Rightarrow v = \sqrt{\frac{2}{m}\left(E + \frac{C}{r}\right)} = \sqrt{2 M_s G\left(\frac{1}{r} - \frac{1}{2A}\right)}$$

Perihelion: $r = 0.98\ AU \Rightarrow v = 36.3$ km/s

Aphelion: $r = 2.61\ AU \Rightarrow v = 13.6$ km/s

2. $$T^2 = \frac{4\pi^2}{GM} A^3$$

$$\Rightarrow T = 7.59 \times 10^7\ s$$

$$= 878 \text{ days}$$