Module MA2341 (Frolov), Advanced Mechanics I
Homework Sheet 4
Each set of homework questions is worth 100 marks

You may use Mathematica to answer the questions but you have to be able to compute all the necessary integrals by hand.

Problem 1

Consider the one-dimensional particle motion in the Poschl-Teller potential

$$U(x) = -\frac{V}{\cosh^2(a\,x)}\,, \quad V > 0\,,$$

representing an atomic well.

1. Consider the finite motion
   (a) Determine for which values of E the motion is finite, and for which it is infinite.
   (b) Find the period of oscillations as a function of the particle's mass m, energy E, and parameters of the potential a and V, and explain the results obtained.
   (c) Find x as an explicit function of t.
   (d) Plot the graph of x for three representative values of E (of your choice).
2. Consider the infinite motion
   (a) Find the time delay in its motion from minus to plus infinity compared to a free moving particle.
   (b) Expand the time delay for small and large E, and explain the results obtained.
   (c) Find x as an explicit function of t.
   (d) Plot the graph of x for three representative values of E (of your choice).

Problem 2

Consider a particle moving with the energy $E > V$ in the Poschl-Teller potential

$$U(x) = \frac{V}{\cosh^2(a\,x)}\,, \quad V > 0\,,$$

representing an atomic barrier.

1. Find the time delay in its motion from minus to plus infinity compared to a free moving particle.
2. Expand the time delay for small and large E, and explain the results obtained.
3. Find x as an explicit function of t.
4. Plot the graph of x for three representative values of E (of your choice).

Problem 3

Consider the one-dimensional particle motion in the trigonometric potential

$$U(x) = V \tan^2(a\,x)\,, \quad V > 0\,.$$

1. Find the period of oscillations as a function of the particle's mass m, energy E, and parameters of the potential a and V.
2. Expand the period for small and large E, and explain the results obtained.
3. Find x as an explicit function of t.
4. Plot the graph of x for three representative values of E (of your choice).

Bonus question (each bonus question is worth extra 25 marks)

Find the one-dimensional particle motion in the Morse potential

$$U(x) = A\left(1 - e^{-a\,x}\right)^2\,.$$

The Morse potential, named after physicist Philip M. Morse, is a convenient model for the potential energy of a diatomic molecule.