MA1125 – Calculus
Homework #3
due Thursday, Oct. 4

1. Show that there exists a real number $0 < x < \pi$ that satisfies the equation

$$x^2 = \frac{x^2+1}{2+\sin x} + 4.$$

2. For which values of a, b is the function f continuous at the point $x = 2$? Explain.

$$f(x) = \left\{ \begin{array}{cc} 2x^3 - ax^2 + bx & \text{if } x < 2 \\ a^2 + b & \text{if } x = 2 \\ 2x^2 + bx - a & \text{if } x > 2 \end{array} \right\}.$$

3. Show that $f(x) = x^5 - x^2 - 3x + 1$ has three roots in the interval $(-2, 2)$. Hint: you need only consider the values that are attained by f at the points ± 2, ± 1 and 0.

4. Compute each of the following limits.

$$L = \lim_{x \to +\infty} \frac{3x^3 - 2x + 4}{5x^3 - x^2 + 7}, \qquad M = \lim_{x \to 2^-} \frac{x^3 + 5x^2 - 4}{3x^3 - 16x + 8}.$$

5. Use the definition of the derivative to compute $f'(x_0)$ in each of the following cases.

$$f(x) = x^3, \qquad f(x) = 1/x^2, \qquad f(x) = (3x+4)^2.$$

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and course (Maths, TP, TSM) on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.