PYU11P/N/T20
Electromagnetic Interactions

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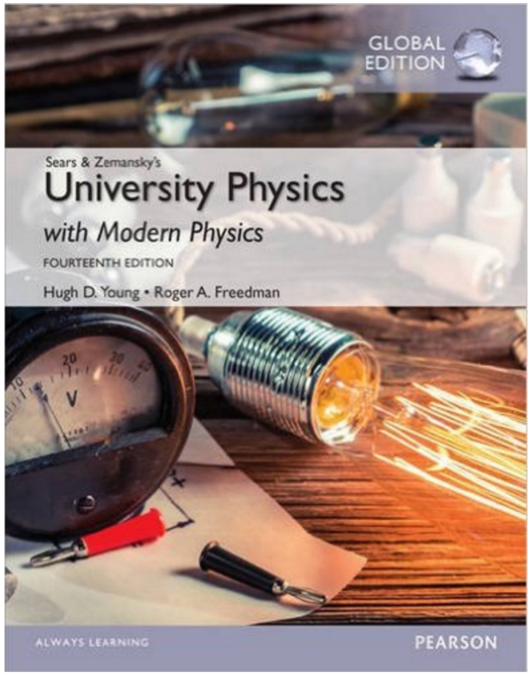
Introduction

- ***Textbook:*** ***University Physics***
  - by Young and Freedman (14th edition Addison-Wesley)
  - Pearson Higher Education: http://www.pearsonhighered.com/
- http://www.pearsonhighered.com/educator/product/University-Physics-Plus-Modern-Physics-Plus-MasteringPhysics-with-eText-Access-Card-Package-13E/9780321675460.page

- ***Lectures***: 20 hours total this semester
  (Lecture notes will be available on Blackboard)

- ***Method***:
  - You read relevant sections before lecture
  - Bring the book to the lecture
  - I review the material covered
  - Crucial to attend lectures
  - I will do some demonstrations
  - You do concept tests
  - Make your own summary of each topic (see book)
  - You do on-line problems
  - http://session.masteringphysics.com



DO AS MANY PROBLEMS AS POSSIBLE!!

Outline of Course Content

Electrostatics:

– Electric charge
– Coulomb's law
– Electric field
– Electric dipoles
– Gauss's law
– Electric potential energy
– Voltage
– Electric polarization
– Capacitance

Electricity:

– Current
– Resistance
– Ohm's law
– Electromotive force
– Power in electric circuits
– Kirchoff's laws
– RC circuits

Electromagnetism:

– Magnetic pole
– Magnetostatic forces
– Magnetic fields
– Force on a moving charge
– Electric motors
– Magnetic torque
– Field due to a moving charge
– Force between conductors
– Ampere's Law
– Magnetic materials
– Faraday's law
– Lenz's law

Mathematics used in this course

- Calculus of vectors in 3D
- **Dot product** and **vector (cross) product**
- (Simplified) Derivatives of vectors including **divergence** and **curl**
- (Simplified) 3D derivatives of **scalar fields** including **gradient**

➡ **All very useful tools.**

Mastering Physics
Online Assignments

There will be online assignments.

The assignments and due dates will be setup in the next days.

Electric charge

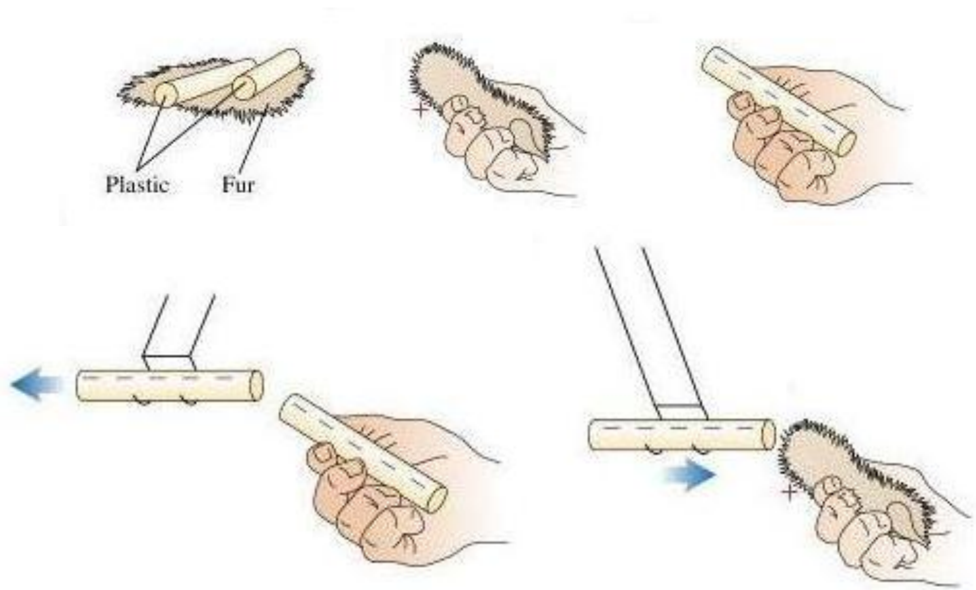
Read sections 21.1, 21.2

21.1 Electric charge

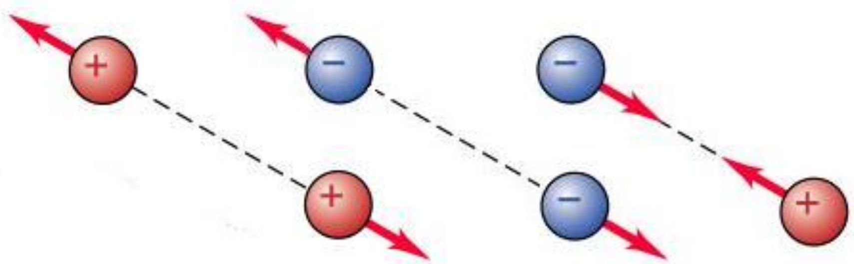
- Charge:

  - When plastic is rubbed with fur:

  The plastic becomes **negatively** charged and the fur **positively** charged.



  - The plastic is repelled by other similarly charged objects, but attracted to the fur.



Like charges repel
Opposite charges attract

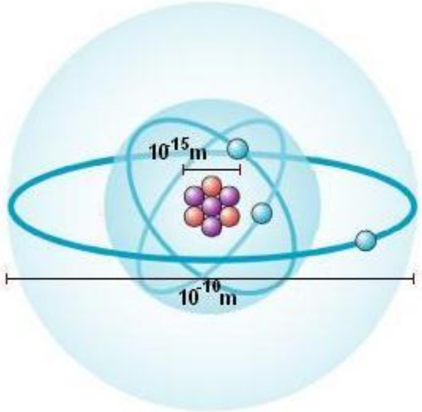
21.1 Electric charge

- Empirically: We observe conservation of charge
  - In any single charge transfer the total charge remains the same.

- **Principle of conservation of charge**:
  - The algebraic sum of all the electric charges in any **closed system** is constant.

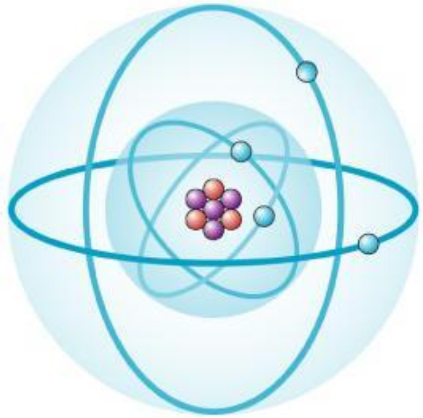
- **Electrostatics**
  - Study of the interactions of charges at rest

- Atoms and Ions:
  - Atoms are electrically neutral.
  - When an electron is removed it becomes a positive ion.
  - When an extra electron is added it becomes a negative ion.



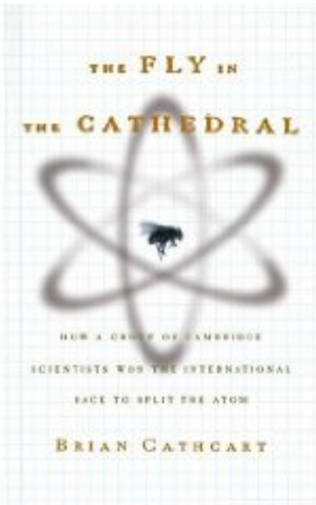
(a) Neutral lithium atom (Li): Nucleus has three protons (red) and four neutrons (purple); three electrons (blue) orbit nucleus

(b) Positive lithium ion (Li^+): Made by removing an electron from neutral lithium atom



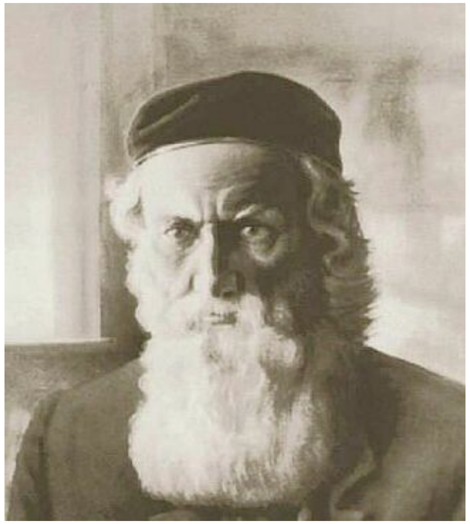
(c) Negative lithium ion (Li^-): Made by adding an electron to neutral lithium atom

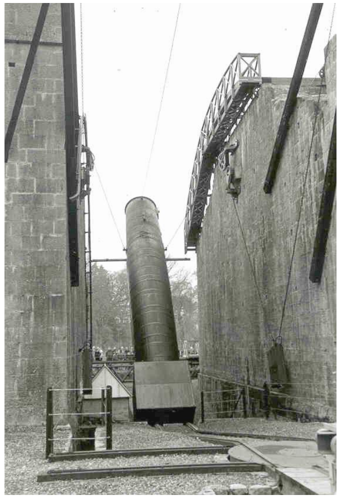
Ernest Walton



21.1 Electric charge

- Dr. George Johnstone **Stoney**:
  - The word "electron" was first proposed by the Irish physicist Dr. George Johnstone Stoney (1891).
  - Dr. Stoney was born in Dun Laoghaire in 1826.
  - He was an undergraduate at Trinity College Dublin.
  - He spent some time working under Lord Rosse at the Birr Observatory which at the time housed the largest telescope in the world.
  - He afterwards worked in Queen's University Galway and was Secretary to Queen's University in Dublin.
  - He died in London in 1911





- Conductors:
  - Substances within which charge is (almost) completely free to move around.
- Insulators:
  - Substances within which charge cannot move around.
- Semiconductors:
  - Control the **mobility** of charge by changing temperature and by "doping"

21.2 Motion of Electric Charge in Materials

- Charge by **induction**:

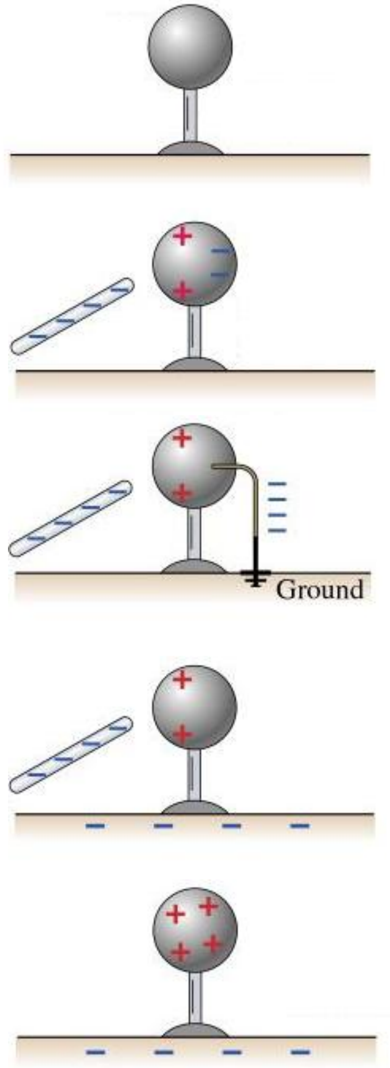
  – A **conducting** metal object isolated by an **insulating** stand is initially *uncharged*

  – A negatively charged rod is brought near to it. The free electrons are repelled and move to the other side.

  – That side is then earthed for a short time, the electrons flow to ground.

  – A net positive charge is left on the sphere.

  – The rod is removed, the positive charge remains



Online quiz

responseware.eu

or download app

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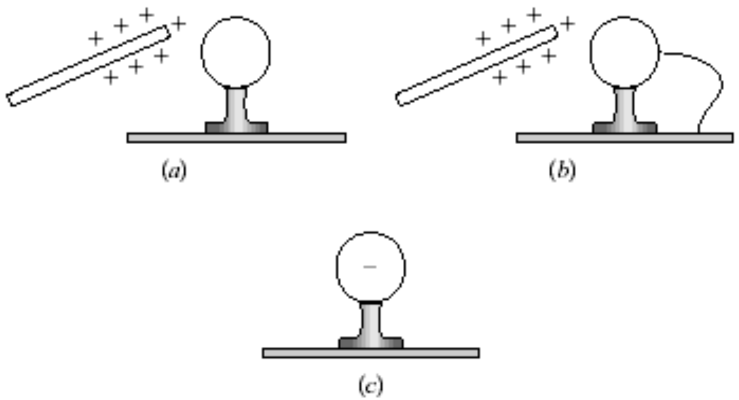
Concept Test #1

A positively charged object is placed close to a conducting object attached to an insulating glass pedestal (*a*). After the opposite side of the conductor is grounded for a short time interval (*b*), the conductor becomes negatively charged (*c*). Based on this information, we can conclude that within the conductor



1. both positive and negative charges move freely.
2. only negative charges move freely.
3. only positive charges move freely.
4. We can't really conclude anything.

Concept Test #2

Three pithballs are suspended from thin threads. Various objects are then rubbed against other objects (nylon against silk, glass against polyester, etc.) and each of the pithballs is charged by touching them with one of these objects. It is found that pithballs 1 and 2 repel each other and that pithballs 2 and 3 repel each other. From this we can conclude that

1. 1 and 3 carry charges of opposite sign.
2. 1 and 3 carry charges of equal sign.
3. all three carry the charges of the same sign.
4. one of the objects carries no charge.
5. we need to do more experiments to determine the sign of the charges.

Concept Test #3

Three pithballs are suspended from thin threads. Various objects are then rubbed against other objects (nylon against silk, glass against polyester, etc.) and each of the pithballs is charged by touching them with one of these objects. It is found that pithballs 1 and 2 ~~repel~~ each other and that pithballs 2 and 3 repel each other. From this we can conclude that

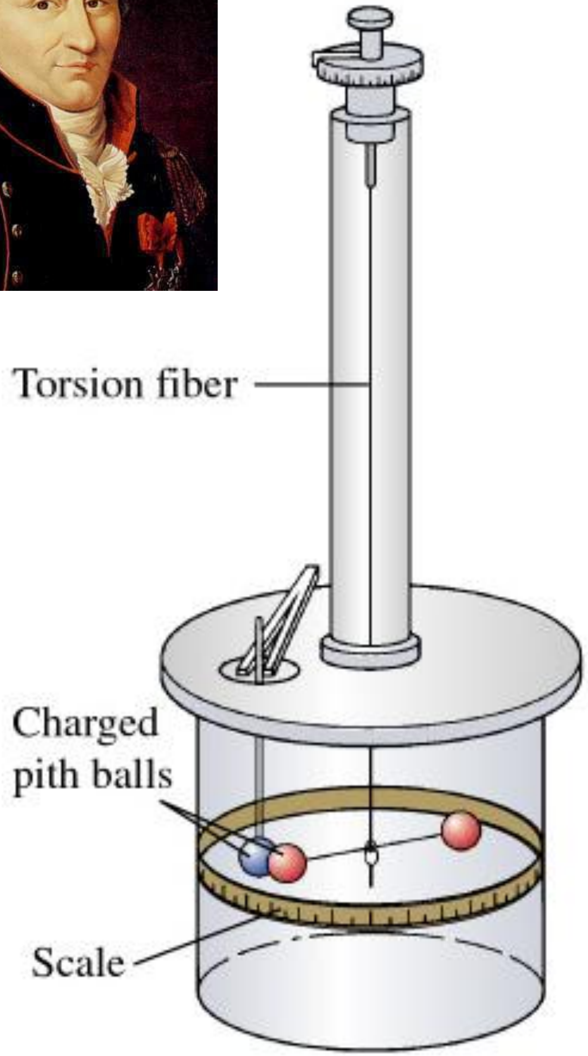
attract

1. 1 and 3 carry charges of opposite sign.
2. 1 and 3 carry charges of equal sign.
3. all three carry the charges of the same sign.
4. one of the objects carries no charge.
5. we need to do more experiments to determine the sign of the charges.

Coulomb's law

Read section 21.3

- Coulomb:
  - Charles Augustin de Coulomb was born in Angouleme, France in 1736.
  - He trained to be an engineer in Mezieres.
  - His main contributions were in the science of **friction**.
  - He was also the first to take proper advantage of the torsion balance.
  - He died in Paris in 1806.

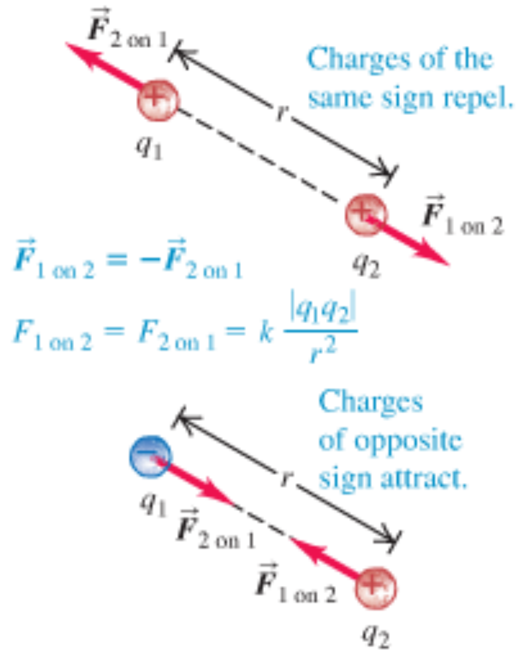


- It was observed experimentally ("*empirically*") for the **electric force between two point charges**
  - The magnitude of the force between two point charges (F) is inversely proportional to the square of the distance between them (r):
  $$F \propto \frac{1}{r^2}$$
  - It is also proportional to the product of the individual charges of the particles (q):
  $$F \propto |q_1 q_2|$$

Coulomb's law:

- The **magnitude** of the electric force between two point charges is given by:
$$F = \frac{k|q_1 q_2|}{r^2}$$
- **Direction** of force lies on a line drawn between the forces, pointing towards or away from other charge depending on signs

Vector properties:



21.3 Coulomb's Law

- What about the proportionality constant k?

$$F = \frac{k|q_1 q_2|}{r^2}$$

- Units
  - Force: newton (N) (or kg m / s^2)
  - Distance: metre (m)
  - Charge: coulomb (C)

  - Units of k must be $N\ m^2\ C^{-2}$ or $kg\ m^3\ C^{-2}\ s^{-2}$

- Coulomb?
  - Defined in terms of electric current: A certain amount of charge that flows in a certain unit of time
  - 1 ampere (amp) = 1 coulomb per sec.
  - Amp is defined in terms of magnetic force between two parallel wires each with current of 1 amp

- Study of electromagnetic radiation (light) relates *k* to velocity of light *c*
  - $k = 10^{-7}\,(N\,s^2\,C^{-2})\,c^2$,
  - Where $c = 2.99792458 \times 10^8\ m\,s^{-1}$ (by definition)

$$k \cong 8.94 \times 10^9 \frac{Nm^2}{C^2}$$

  - In SI units to simplify later formula we write *k* in terms of the

Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}$

$$k = \frac{1}{4\pi\varepsilon_0} \quad \text{so} \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

The coulomb:

- Two charges of one coulomb each separated by $1\ m$ would exert forces on each other of magnitude $9 \times 10^9\ N$. This is obviously a very large force (how big?)

- So one coulomb is a very large quantity of charge and rarely encountered.

- We will mostly deal with charges of the order of $1\ nC = 10^{-9}\ C$ or $1\ \mu C = 10^{-6}\ C$

In terms of the fundamental charge e (as found on the electron, proton, positron, anti-proton):

$$e = 1.60217653(14) \times 10^{-19}\ C$$

- Comparing the electrostatic and gravitational forces:
  - Let's take two alpha particles of mass: $m = 6.64 \times 10^{-27} kg$
  - And charge: $q = +2e = 3.2 \times 10^{-19}\mathrm{C}$
  - The magnitude of the electric force between these particles is: $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$
  - The magnitude of the gravitational force is: $F_g = G\frac{m^2}{r^2}$
  - The ratio of these two values is therefore: $\frac{F_e}{F_g} = \frac{1}{4\pi\varepsilon_0 G}\frac{q^2}{m^2} = 3.1 \times 10^{35}$

Electric force is ***much*** bigger then gravitational force.

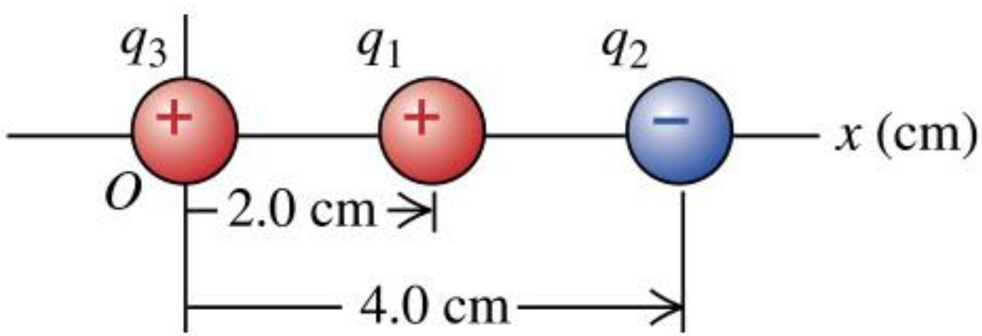
Four Fundamental Interactions

Interaction	Relative Strength	Range	Mediating Particle: Name	Mass	Charge	Spin
Strong	1	Short ($\sim$1 fm)	Gluon	0	0	1
Electromagnetic	$\frac{1}{137}$	Long ($1/r^2$)	Photon	0	0	1
Weak	10^{-9}	Short ($\sim$0.001 fm)	$W^{\pm}$, Z^0	80.4, 91.2 GeV/c^2	$\pm e$, 0	1
Gravitational	10^{-38}	Long ($1/r^2$)	Graviton	0	0	2

21.3 Coulomb's Law: Principle of Superposition

- We've stated Coulomb's Law to calculate the force between two (2) point charges.
- It's more general than that though: it can be applied to any collection of charges
- There is an additional property of electrostatics called the **Principle of Superposition** that allows this
- It says that the total force acting on a charge is the **vector sum** of the individual forces between that charge and all others around it

- Problem
  - Three point charges are positioned as shown.
  - Their respective charges are: $q_1 = 2nC$ $q_2 = -3nC$ $q_3 = 5nC$
  - What is the value of the total force on q_3?



- Solution:
  - The force due to q_1 is in the $-x$ direction and is of magnitude:

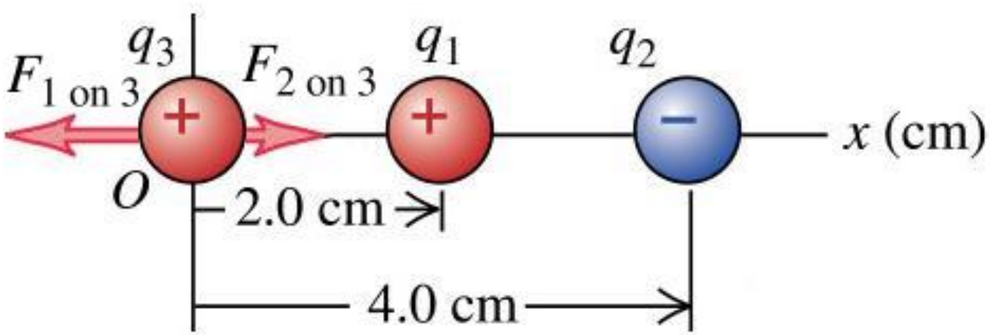
$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_3|}{(r_{13})^2} = 2.25\times10^{-4}\,N$$

  - The force due to q_2 is in the $+x$ direction and is of magnitude:

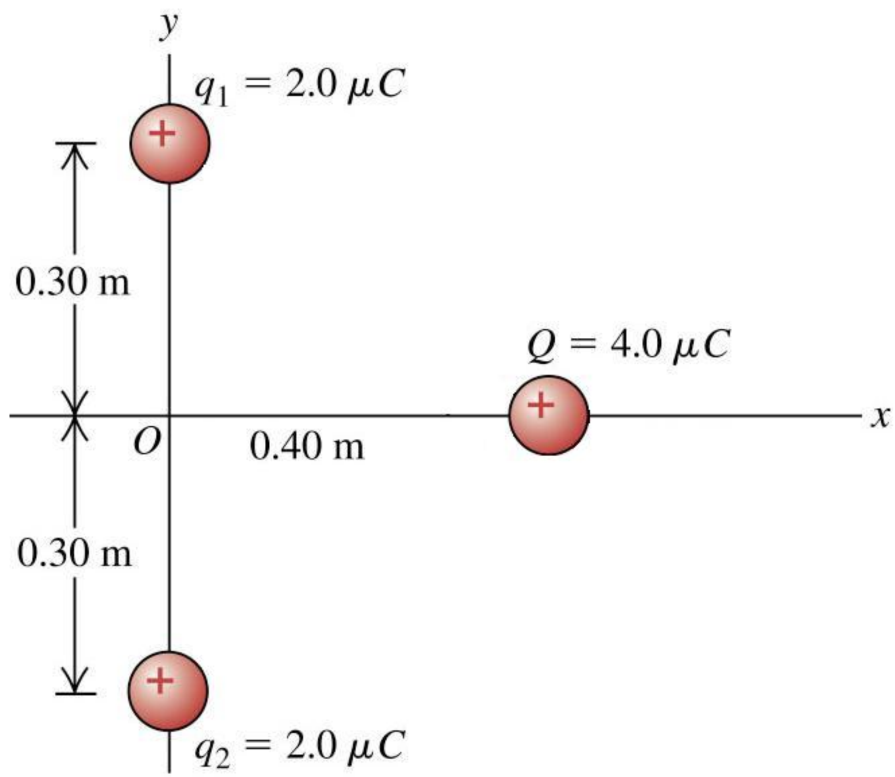
$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_2 q_3|}{(r_{23})^2} = 0.84\times10^{-4}\,N$$

  - The total force on q_3 is in the $-x$ direction and is of magnitude:

$$F = |F_2 - F_1| = \left|(0.84 - 2.25)\times10^{-4}\right| = 1.4\times10^{-4}\,\mathrm{N}$$



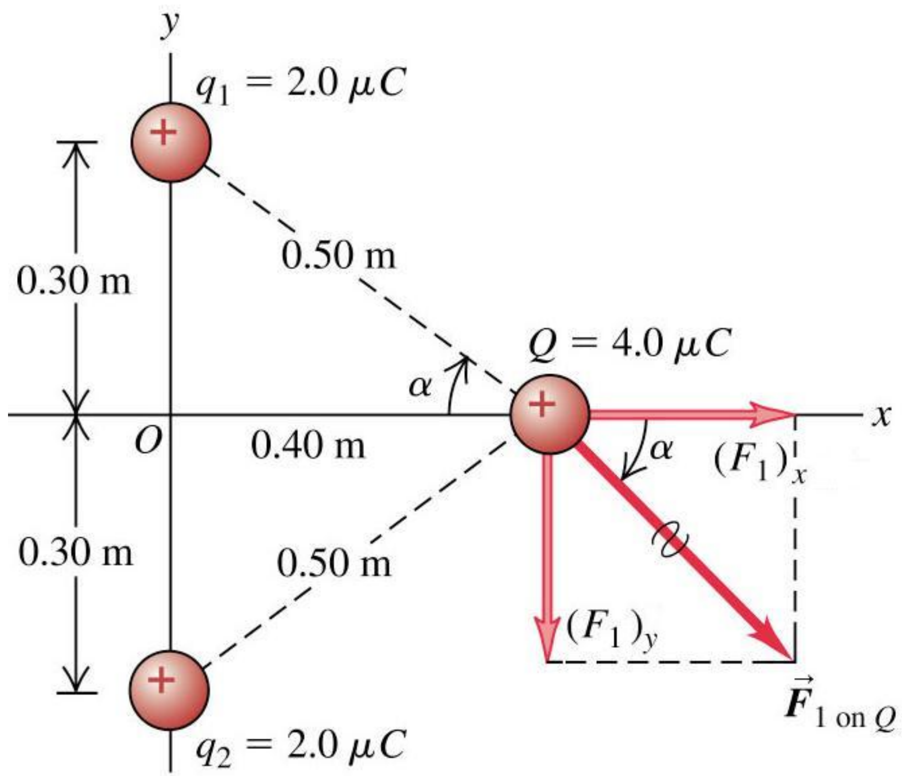
- Problem
  - Three point charges are positioned as shown with charge strengths as shown.
  - Find the magnitude and direction of the total net force on Q.



- Solution:
  - The force on Q from q_1 is shown.
  - It is of magnitude:

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 Q|}{r_1^2} = 0.29N$$

  - F_1 makes an angle $-\alpha$ with the $+x$ axis so its components are:



$$\left(F_1\right)_x = F_1\cos(-\alpha) = 0.29N\left(\frac{0.4m}{0.5m}\right) = 0.23N$$

$$\left(F_1\right)_y = F_1\sin(-\alpha) = 0.29N\left(\frac{-0.3m}{0.5m}\right) = -0.17N$$

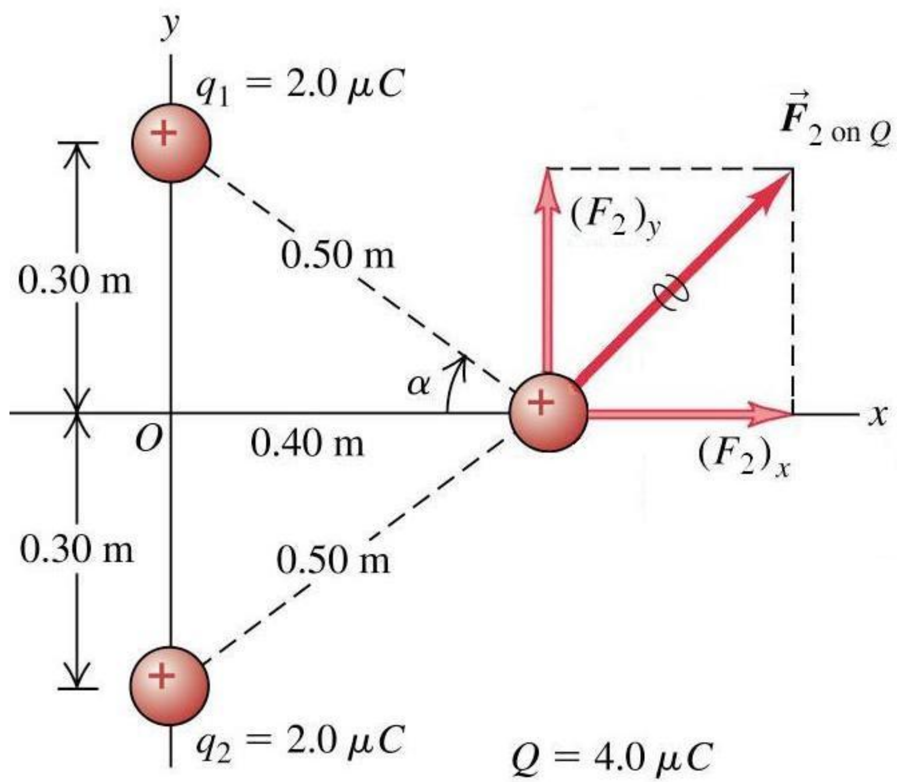
- By symmetry we see that the force on Q from q_2 is equal to that from q_1 except that it makes an angle α with the $+x$ axis.

- Therefore:

$$(F_2)_x = (F_1)_x$$
$$(F_2)_y = -(F_1)_y$$
$$F_x = (F_1)_x + (F_2)_x = 2(F_1)_x$$
$$F_y = (F_1)_y + (F_2)_y = 0$$

- So the total force on Q is in the $+x$ direction and has magnitude:

$$F = 2(0.23N) = 0.46N$$



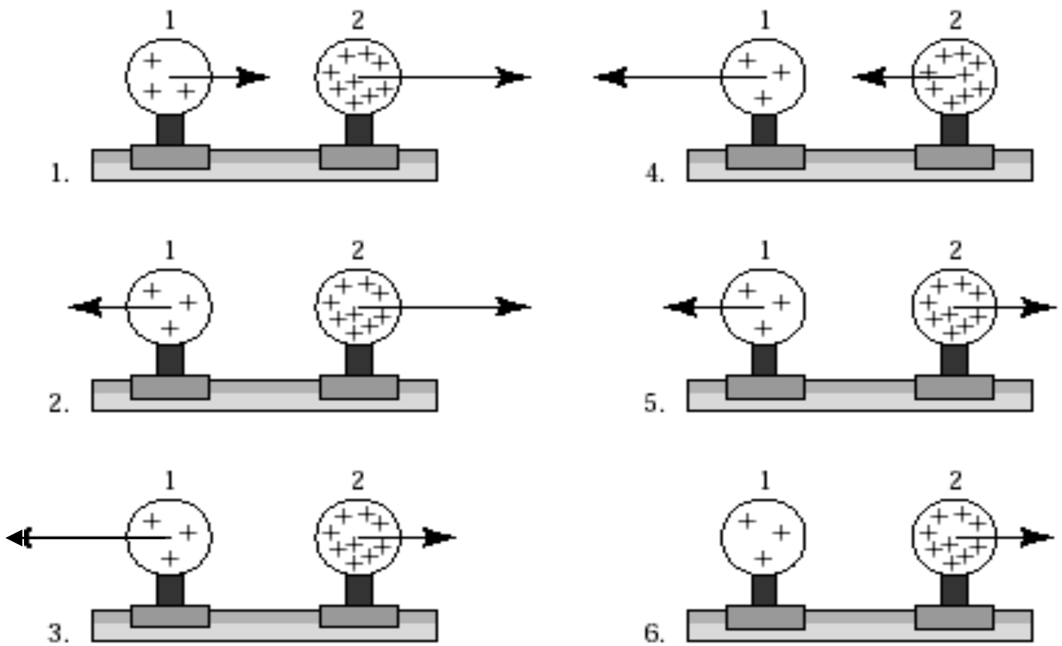
Concept Test #1

A hydrogen atom is composed of a nucleus containing a single proton, about which a single electron orbits. The electric force between the two particles is 2.3×10^{39} greater than the gravitational force! If we can adjust the distance between the two particles, can we find a separation at which the electric and gravitational forces are equal?

1. Yes, we must move the particles farther apart.
2. Yes, we must move the particles closer together.
3. no, at any distance

Concept Test #2

Two uniformly charged spheres are firmly fastened to and electrically insulated from frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces:



7. none of the above

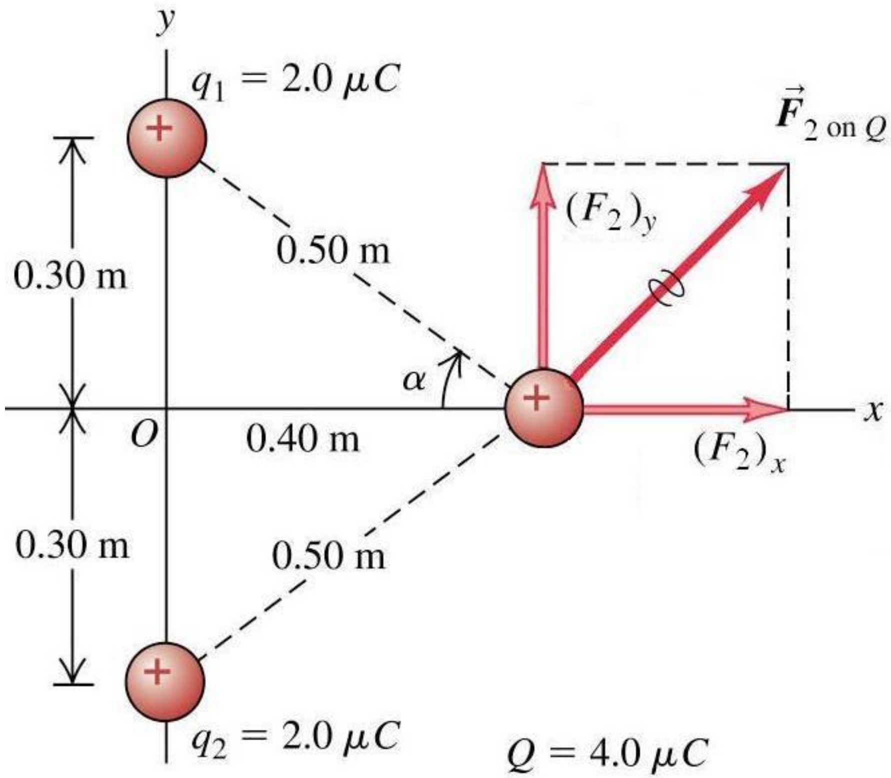
- By symmetry we see that the force on Q from q_2 is equal to that from q_1 except that it makes an angle α with the $+x$ axis.

- Therefore:

$$(F_2)_x = (F_1)_x$$
$$(F_2)_y = -(F_1)_y$$
$$F_x = (F_1)_x + (F_2)_x = 2(F_1)_x$$
$$F_y = (F_1)_y + (F_2)_y = 0$$

- So the total force on Q is in the $+x$ direction and has magnitude:

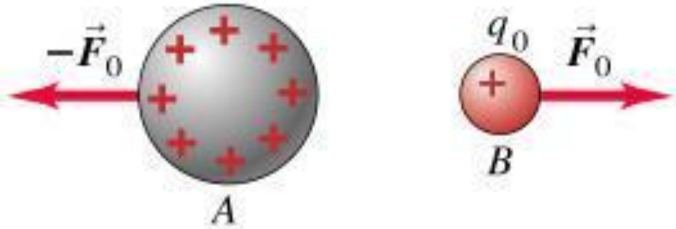
$$F = 2(0.23N) = 0.46N$$



Electric field

Read sections 21.4, 21.5, 21.6

21.4 Electric field



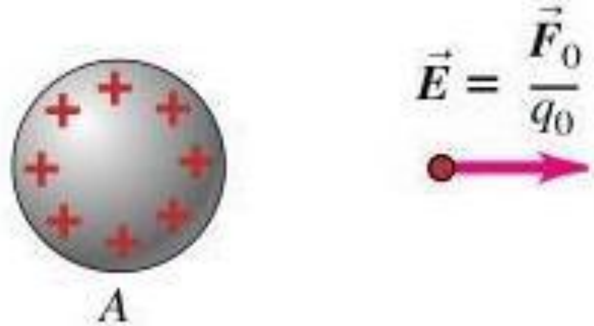
In the space around A, B will experience a force

- The **electric field** created by *A* at point *B* is given by:

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

- Units (Newtons per Coulomb): NC^{-1}

- It is a **vector** quantity



The force exerted by *B* may cause the charge in *A* to shift around. A stricter definition of the electric field is:

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}_0}{q_0}$$

21.4 Electric field of a point charge

Writing Vector Equations:

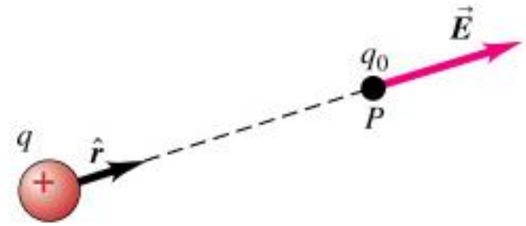
Define a unit vector $\hat{\boldsymbol{r}}$ pointing from charge $\boldsymbol{q}$ along point $\boldsymbol{S}$ to point $\boldsymbol{P}$.

The force $\vec{F}_P$ is given by Coulomb's Law.

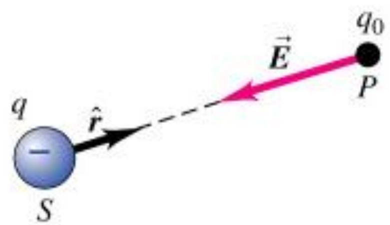
$$\vec{F}_P = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\boldsymbol{r}}$$

We now write a vector equation for the electric field:

$$\vec{E}_P = \frac{\vec{F}_P}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$$



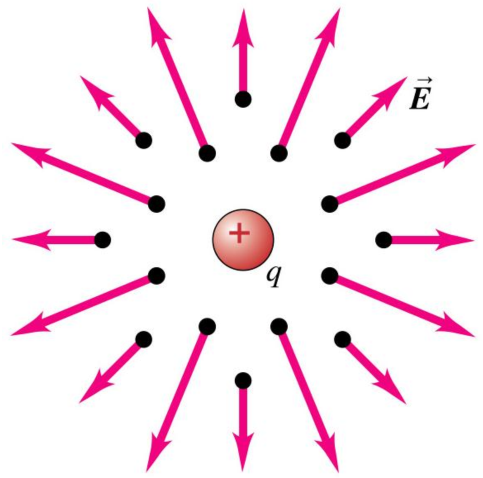
The electric field due to a positive charge points directly away from it.



The electric field due to a negative charge points directly towards it.

21.4 Features of the electric field

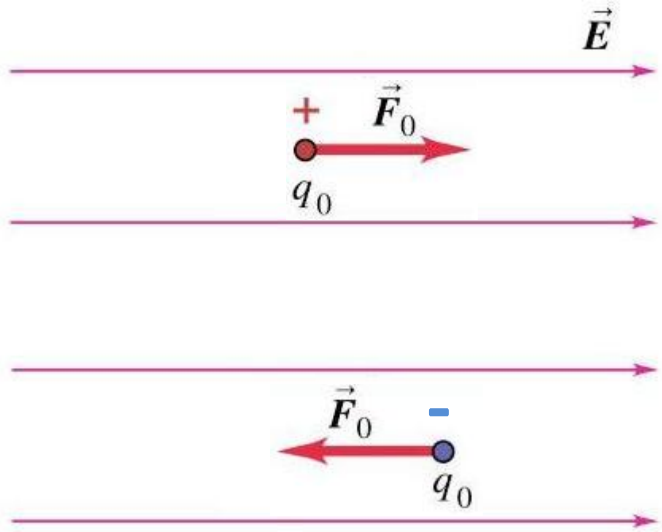
– A point charge creates an electric field at all points.



- Force on a charge in electric field:

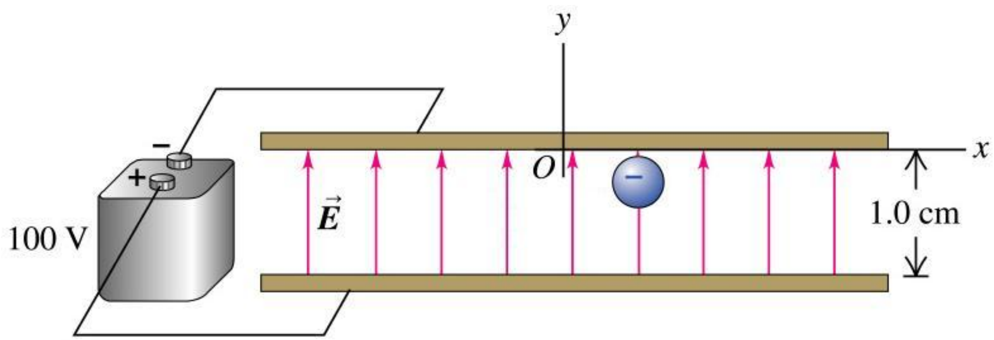
  – A positive charge experiences a force parallel to the electric field.

  – A negative charge experiences a force antiparallel to the electric field.



21.4 Electric field

- Problem
  - Consider an electron in the uniform electric field ($10^4\ N\ C^{-1}$) shown below.
    - If it is released from rest at the upper plate, what is its acceleration?
    - What are its final speed and kinetic energy?
    - After what time does it reach the lower plate?



21.4 Electric field

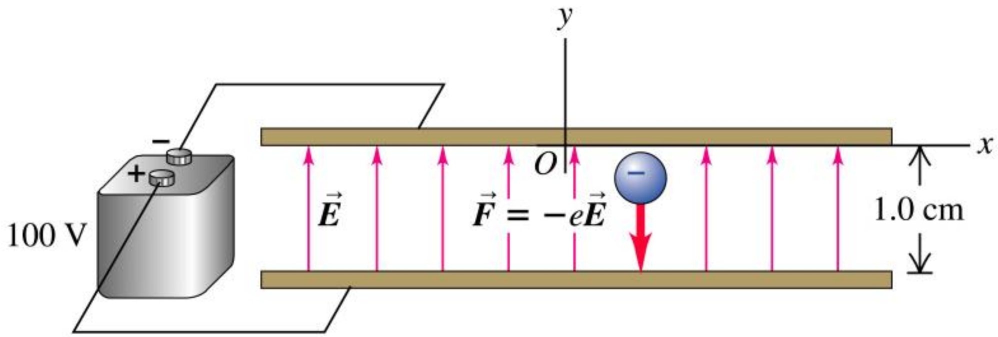
- Solution:
  - The field is uniform so the force exerted on the electron is constant and downward.
  - By Newton's Law and definition of electric field:

$$a_y = \frac{F_y}{m_e} = \frac{-eE}{m_e} = \frac{\left(-1.6\times10^{-19}C\right)\left(1\times10^{4}NC^{-1}\right)}{9.11\times10^{-31}kg} = -1.76\times10^{15}ms^{-2}$$

  - We obtain the velocity by (kinematics of constant acceleration):

$$v_y^2 = 2a_y y \qquad \text{From:} \quad d = \frac{1}{2}at^2 \ \text{ and } \quad v = at$$

$$\Rightarrow \left|v_y\right| = \sqrt{2a_y y} = \sqrt{2\left(-1.76\times10^{15}ms^{-2}\right)\left(-1\times10^{-2}m\right)} = 5.9\times10^{6}ms^{-1}$$



21.4 Electric field

– The final kinetic energy is:

$$K = \frac{1}{2}mv^2 = 0.5\left(9.11\times10^{-31}kg\right)\left(5.9\times10^{6}ms^{-1}\right)^2 = 1.6\times10^{-17}J$$

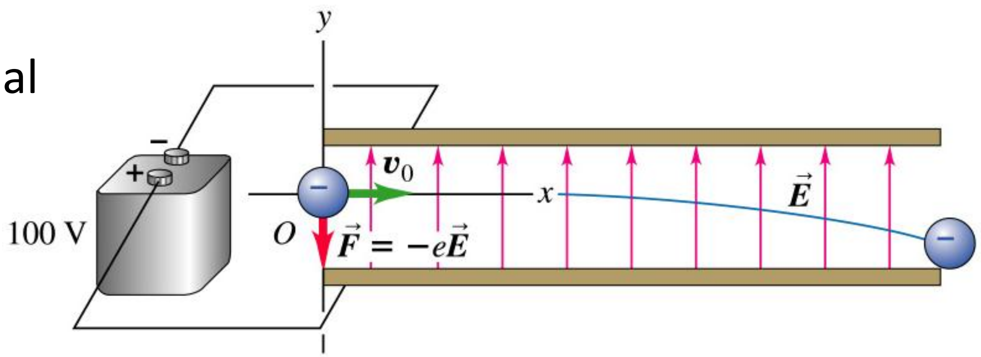
– The time taken is given by:

$$v_y = a_y t$$

$$\Rightarrow t = \frac{v_y}{a_y} = \frac{\left(-5.9\times10^{6}ms^{-1}\right)}{-1.76\times10^{15}ms^{-2}} = 3.4\times10^{-9}s$$

21.4 Electric field

Problem: If the electron were given an initial horizontal velocity, what would be its trajectory? Ie. What is $y=y(x)$?



Solution:

From the previous example: $$a_y = \frac{-eE}{m}$$

Now: $$x = v_o t$$

While in the y direction: $$y = \frac{1}{2}a_y t^2 = -\frac{1}{2}\frac{eE}{m}t^2$$

By eliminating t we obtain the equation of a parabola: $$y = -\frac{1}{2}\frac{eE}{mv_0^2}x^2$$

21.5 Electric field of many charges

- What is the field of an extended charge distribution?
  - Made up of point charges: $q_1, q_2, q_3, \ldots\ldots$
  - Each producing its own electric field: $\vec{E}_1, \vec{E}_2, \vec{E}_3 \ldots\ldots$
  - Total field is given by vector sum: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots$
- Use superposition to extend to continuous distributions of charge..

  Eg. Cases of interest:
  - Linear charge density - λ C/m.
  - Surface charge density - σ C/m^2.
  - Volume charge density - ρ C/m^3.

21.5 Electric field

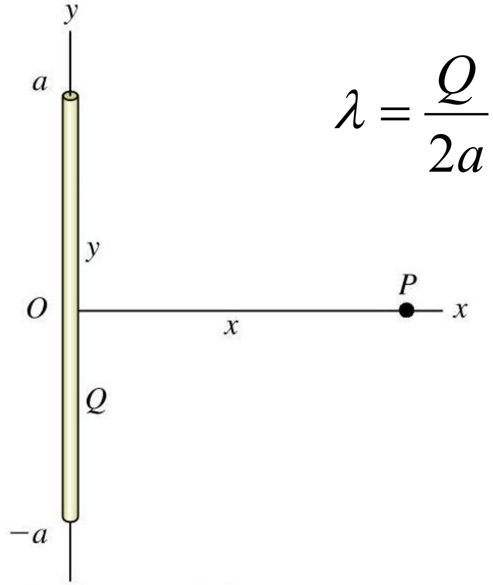
- Sample calculation:
  - What is the electric field at P induced by a line of charge with charge density λ.
- Solution:
  - Divide the line into infinitesimal segments of charge:

$$dQ = \lambda dy = \frac{Q dy}{2a}$$

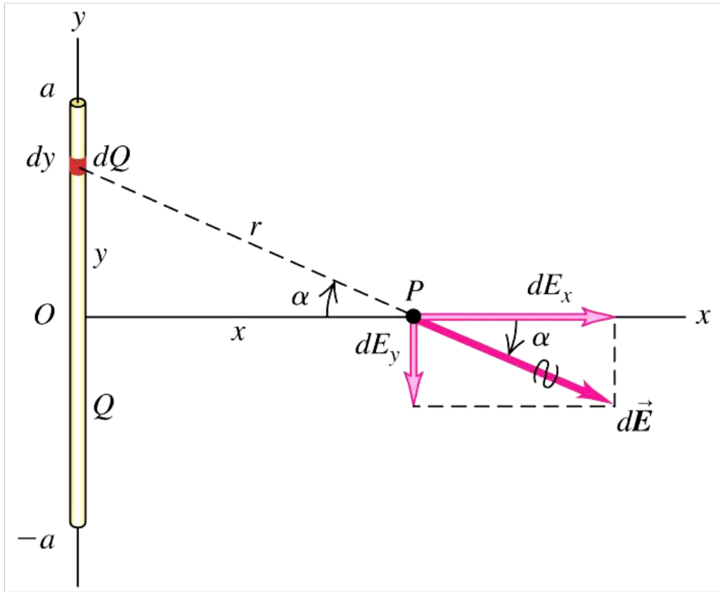
  - Each segment acts as a point charge:

$$\left|d\vec{E}\right| = dE = \frac{Q}{8\pi\varepsilon_0 a} \frac{dy}{r^2}$$

(Coulomb's Law)

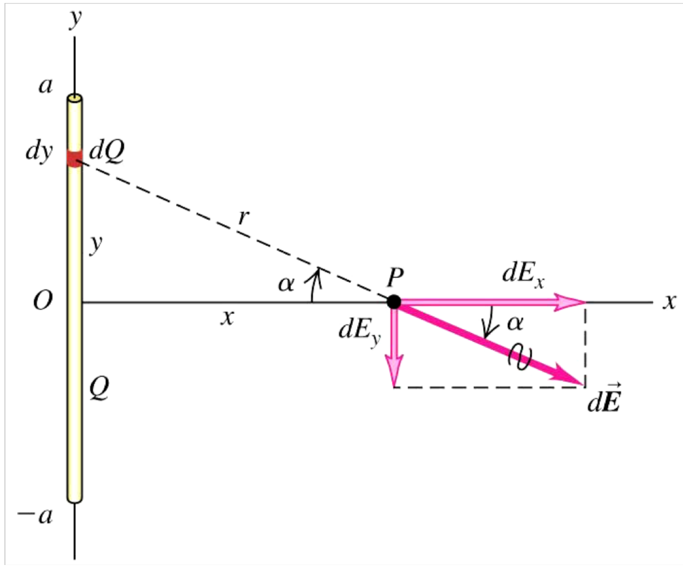


$$\lambda = \frac{Q}{2a}$$



21.5 Electric field

Resolving into components:



$$dE_x = \frac{Q}{8\pi\varepsilon_0 a}\frac{dy\cos\alpha}{r^2} \qquad dE_y = -\frac{Q}{8\pi\varepsilon_0}\frac{dy\sin\alpha}{r^2}$$

Convert α and r to x and y with some simple trigonometry:

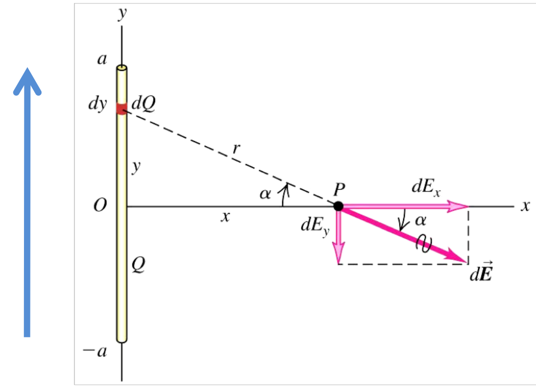
$$\sin\alpha = \frac{y}{r} \qquad \cos\alpha = \frac{x}{r} \qquad r = \sqrt{x^2 + y^2}$$

Giving:

$$dE_x = \frac{Q}{8\pi\varepsilon_0 a}\frac{xdy}{\left(x^2 + y^2\right)^{\frac{3}{2}}} \qquad dE_y = \frac{Q}{8\pi\varepsilon_0 a}\frac{ydy}{\left(x^2 + y^2\right)^{\frac{3}{2}}}$$

21.5 Electric field

Now: Integrate **along y** of the source line of charge to find the components of the total field...



The result is:

$$E_x = \int dE_x = \frac{Qx}{8\pi\varepsilon_0 a}\int_{-a}^{a}\frac{dy}{\left(x^2+y^2\right)^{\frac{3}{2}}} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{x\sqrt{x^2+a^2}}$$

$$y = x\tan\theta$$

(Substitution to solve integral)

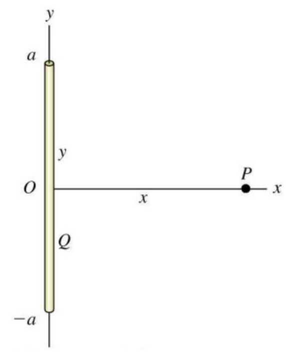
$$E_y = \int dE_y = \frac{Q}{8\pi\varepsilon_0 a}\int_{-a}^{a}\frac{y\,dy}{\left(x^2+y^2\right)^{\frac{3}{2}}} = 0$$

By symmetry of problem

Or in vector form:

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{\boldsymbol{i}}$$

21.5 Electric field: Limiting conditions



For the line charge we found: $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{i}$$

- We can rewrite this:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2\sqrt{1+\left(\frac{a}{x}\right)^2}}\hat{i} \Rightarrow \lim_{x>>a}\vec{E} = \frac{Q}{4\pi\varepsilon_0 x^2}\hat{i}$$

- This means that the electric field **far away** from the wire (ie. for $x >> a$) is that of a **point source**.

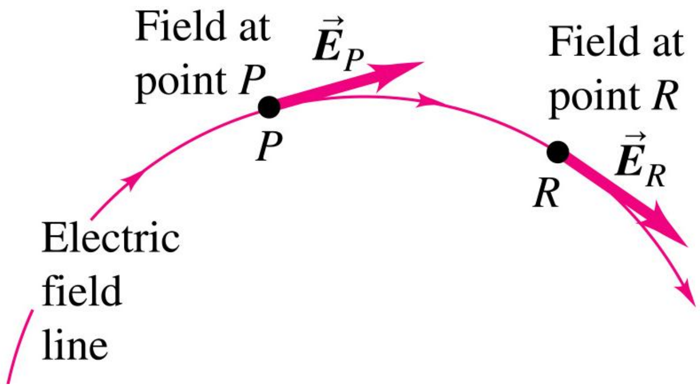
- Also, substituting: $Q = 2a\lambda$ $\Rightarrow$ $$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\frac{x}{a}\sqrt{x^2+a^2}}\hat{i} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x\sqrt{\left(\frac{x}{a}\right)^2+1}}\hat{i}$$

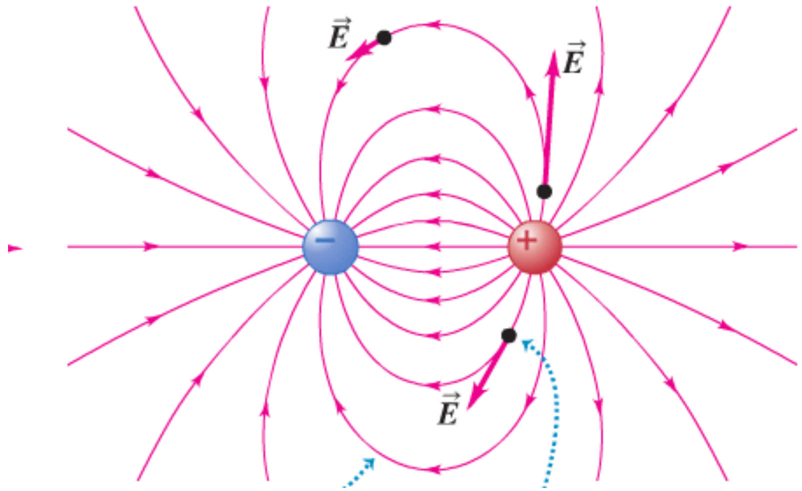
- Then another interesting limit is: $$\lim_{a\to\infty}\vec{E} = \lim_{x<<a}\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 x}\hat{i}$$

- This means that if *a* is very large compared to *x* (ie. you are very **close to** the wire) then the electric field varies linearly with $1/x$.

21.6 Electric field lines

- **Electric field lines**:
  - These are imaginary lines such that the direction of the electric field is a tangent to the electric field line.
  - Field line orientation is given by the electric field direction
  - The electric field always has only one direction so field lines never cross.
  - Field lines always end up at a charge or at infinity…
  - Field lines are drawn closer together where the field is stronger.
  - Field lines are NOT trajectories of (charged) particles!!





Online quiz

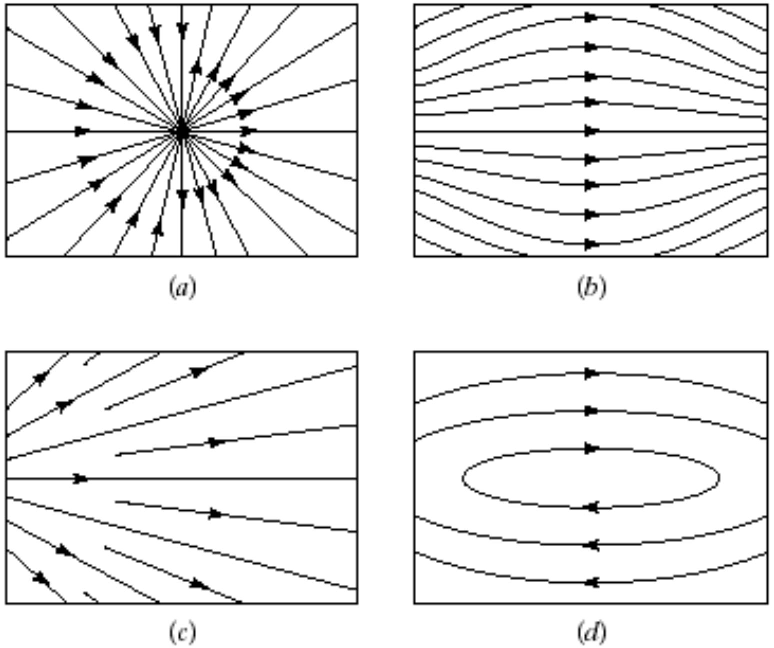
responseware.eu

or download app

session ID: em18

Concept test

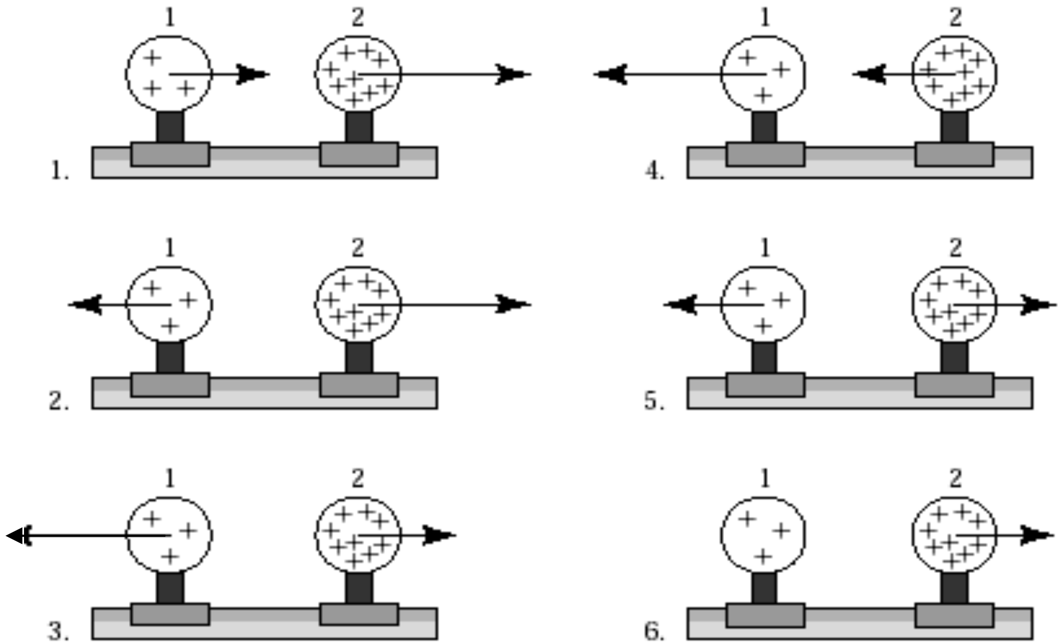
Consider the four field patterns shown. Assuming there are no charges in the regions shown, which of the patterns represent(s) a possible electrostatic field:



1. (a)
2. (b)
3. (b) and (d)
4. (a) and (c)
5. (b) and (c)
6. some other combination
7. None of the above.

Concept Test #2

Two uniformly charged spheres are firmly fastened to and electrically insulated from frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces:



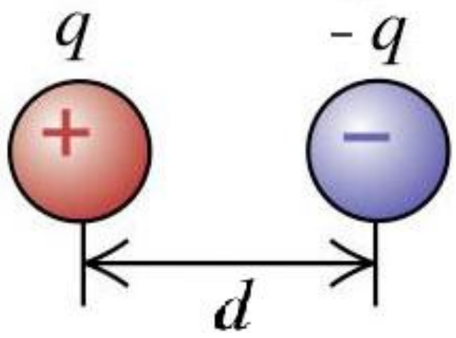
7. none of the above

Electric dipole

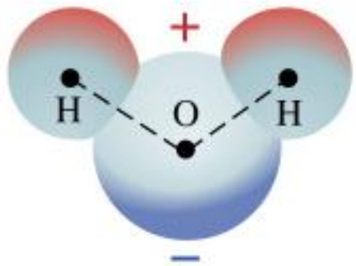
Read section 21.7

21.7 Electric dipole

– An electric dipole is a pair of point charges with equal magnitude and opposite sign.



Many molecules behave like electric dipoles.



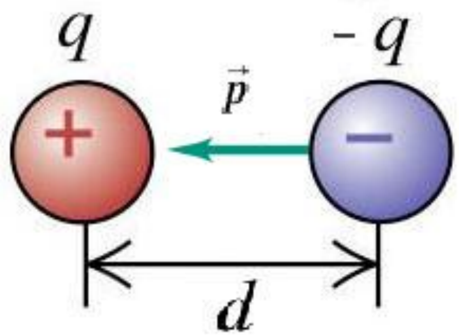
So do antennas on wireless devices...

21.7 Electric dipole

- Dipole moment:
  - Assume we have an electric dipole with charges q and $-q$ and separated by a distance d.
  - The **electric dipole moment** p is defined:

$$p = qd$$

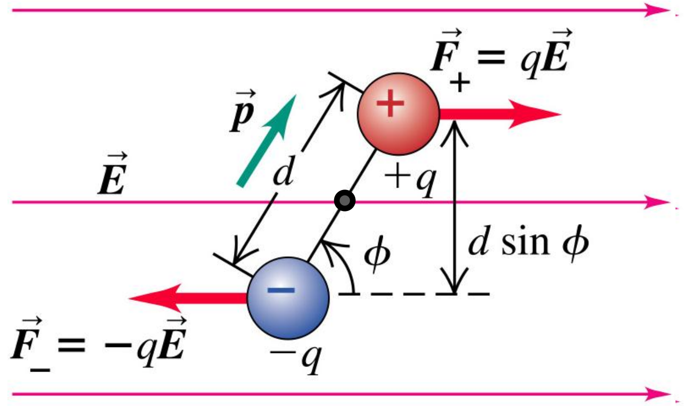
  - The **electric dipole moment vector** $\vec{\mathrm{p}}$ is defined to have magnitude p and direction from the negative charge to the positive charge.



21.7 Electric dipole

- **Dipole in *uniform* E-field**:

$$p = qd$$



  - The total net force on the dipole is zero.
  - The torque of each force around the **centre** of the dipole is:

$$qE\frac{d}{2}\sin\varphi = \frac{pE}{2}\sin\varphi$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \text{Eg.} \quad \vec{\tau}_+ = \frac{\vec{d}}{2} \times q\vec{E}$$

  - So the total torque is:

$$\tau_{total} = qE\frac{d}{2}\sin\varphi + (-q)E\frac{d}{2}\sin(\varphi+\pi) = qEd\sin\varphi = pE\sin\varphi$$

  - Which can be written simply in vector form:

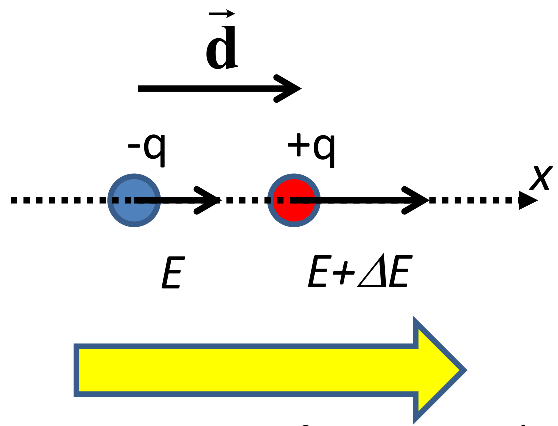
$$\vec{\tau} = \vec{p} \times \vec{E}$$

$\varphi = 0$ Stable equilibrium

$\varphi = \pi$ Unstable equilibrium

21.7 Electric dipole

- **Dipole in non-uniform field:**
  - When a dipole is in a non uniform field, the net force can be non-zero.
  - For instance, lets look at a case where: $\vec{p}\,\|\,\vec{E}$
  - The total force is:
    $$F = F_+ - F_-$$
    $$= q\left(E + \Delta E\right) - qE$$
    $$= q\Delta E$$



  - Over a very short distance : $\Delta E \cong d\,\dfrac{dE}{dx}$

Direction of increasing (ie. nonuniform) electric field

  - We then have:
    $$F = qd\,\frac{dE}{dx}$$
    $$= p\,\frac{dE}{dx}$$

Ie. “the dipole moment times the field gradient”

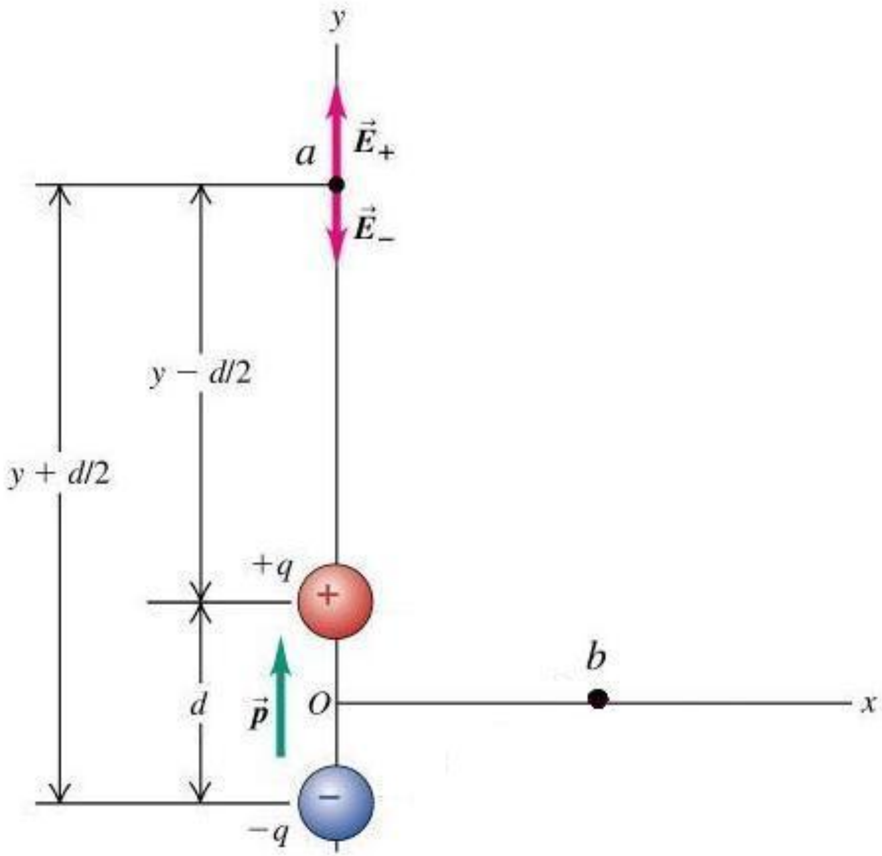
21.7 Electric dipole

- **Electric field at point *a* due to a dipole at centred at the origin *O*:**
  - Consider a dipole oriented as shown.
  - The x-component of the electric field at point ***a*** is 0 for each charge.
  - The y-component of the electric field at location ***a*** is:

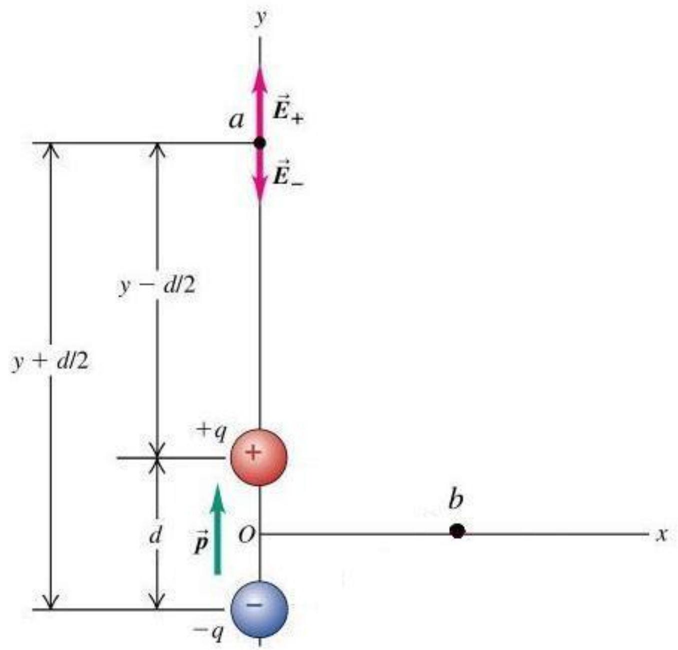
+ve $\vec{E}_+$ -ve $\vec{E}_-$

$$E_y^a = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\left(y-\frac{d}{2}\right)^2} + \frac{-q}{\left(y+\frac{d}{2}\right)^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{\left(y+\frac{d}{2}\right)^2 - \left(y-\frac{d}{2}\right)^2}{\left(y^2-\left(\frac{d}{2}\right)^2\right)^2}$$



21.7 Electric dipole



$$E_y^a = \frac{q}{4\pi\varepsilon_0} \frac{\left(y+\frac{d}{2}\right)^2 - \left(y-\frac{d}{2}\right)^2}{\left(y^2 - \left(\frac{d}{2}\right)^2\right)^2}$$

$$E_y^a = \frac{q}{4\pi\varepsilon_0} \frac{\left(y^2+\frac{2yd}{2}+\frac{d^2}{4}\right) - \left(y^2-\frac{2yd}{2}+\frac{d^2}{4}\right)}{y^4\left(1 - \left(\frac{d}{2y}\right)^2\right)^2}$$

$$E_y^a = \frac{q}{4\pi\varepsilon_0} \frac{2yd}{y^4\left(1 - \left(\frac{d}{2y}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0} \frac{d}{y^3\left(1 - \left(\frac{d}{2y}\right)^2\right)^2}$$

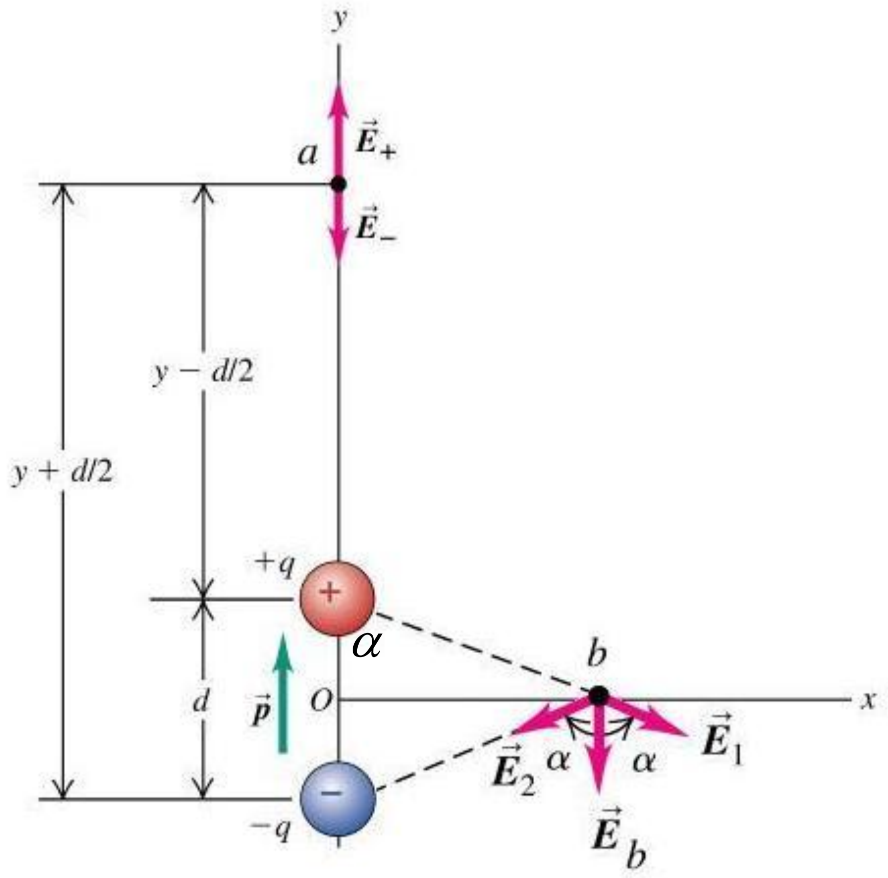
Dipole moment

$$p = qd$$

– If $\frac{d}{2} << y$: $E_y^a \cong \frac{q}{2\pi\varepsilon_0}\frac{d}{y^3}$ ⇒ Result: $E_y^a \cong \frac{p}{2\pi\varepsilon_0 y^3}$

– Another point of **high symmetry** is location *b:*

$$E_1 = E_2 = \frac{q}{4\pi\varepsilon_0}\frac{1}{\left(x^2+\left(\frac{d}{2}\right)^2\right)}$$



– By symmetry, the x-components cancel

– And so we are left with the sum of two y components:

$$E_y^b = 2\times -\left(\frac{q}{4\pi\varepsilon_0}\frac{1}{\left(x^2+\left(\frac{d}{2}\right)^2\right)}\cos\alpha\right)$$

–But:

$$\cos\alpha = \frac{\frac{d}{2}}{\sqrt{\left(x^2+\left(\frac{d}{2}\right)^2\right)}}$$

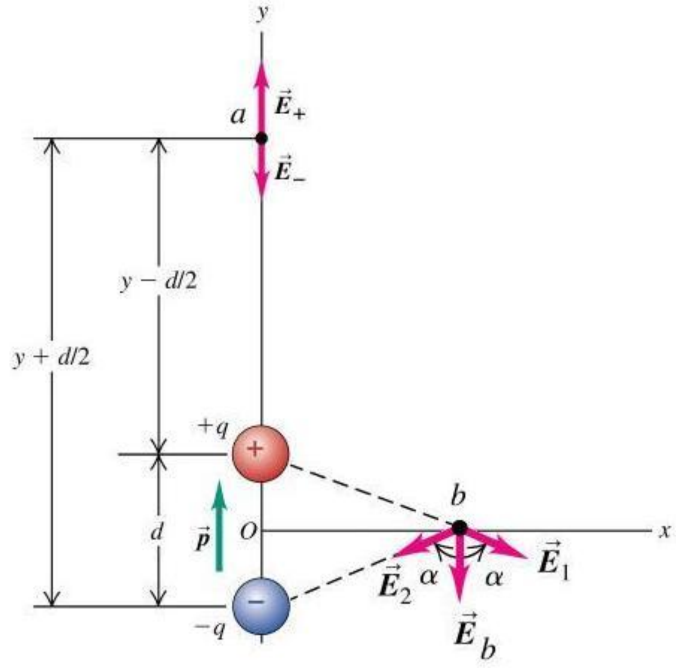
21.7 Electric dipole

So from

$$E_y^b = -\frac{q}{2\pi\varepsilon_0}\frac{1}{\left(x^2+\left(\frac{d}{2}\right)^2\right)}\cos\alpha$$

and

$$\cos\alpha = \frac{\frac{d}{2}}{\sqrt{\left(x^2+\left(\frac{d}{2}\right)^2\right)}}$$



$$E_y^b = -\frac{q}{4\pi\varepsilon_0}\frac{d}{\left(x^2+\left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

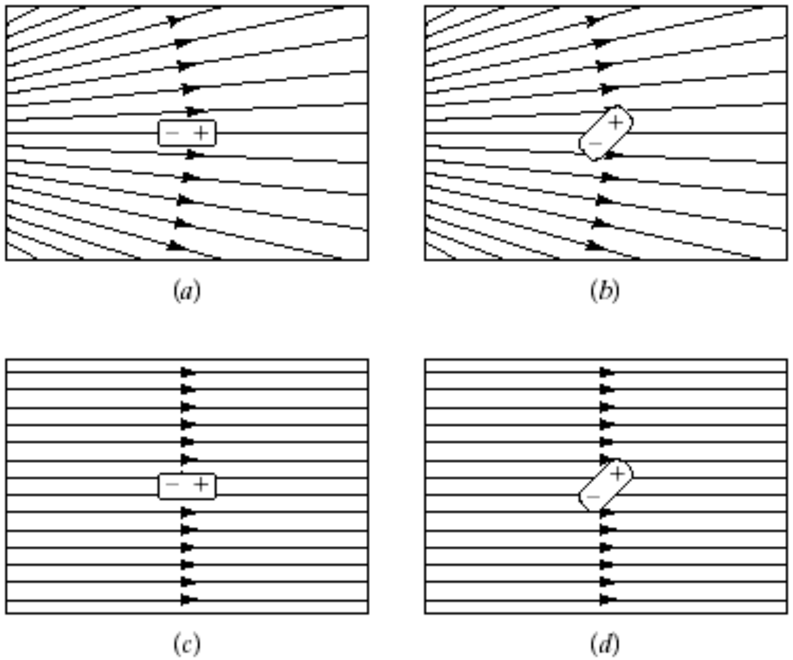
$$E_y^b = -\frac{q}{4\pi\varepsilon_0}\frac{d}{x^3\left(1+\left(\frac{d}{2x}\right)^2\right)^{\frac{3}{2}}}$$

– Taking: $\frac{d}{2} << x$ ⇒ Result: $\left|E_y^b\right| \cong \frac{p}{4\pi\varepsilon_0 x^3} = \frac{1}{2}E_y^a$ When $x = y$

The full result for a general coordinate is: $\vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-\vec{\boldsymbol{p}}}{r^3}+\frac{3\left(\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{r}}\right)\vec{\boldsymbol{r}}}{r^5}\right)$

Concept test

An electrically neutral dipole is placed in an external field. In which situation(s) is the net force on the dipole zero?



1. (a)
2. (c)
3. (b) and (d)
4. (a) and (c)
5. (c) and (d)
6. some other combination
7. none of the above

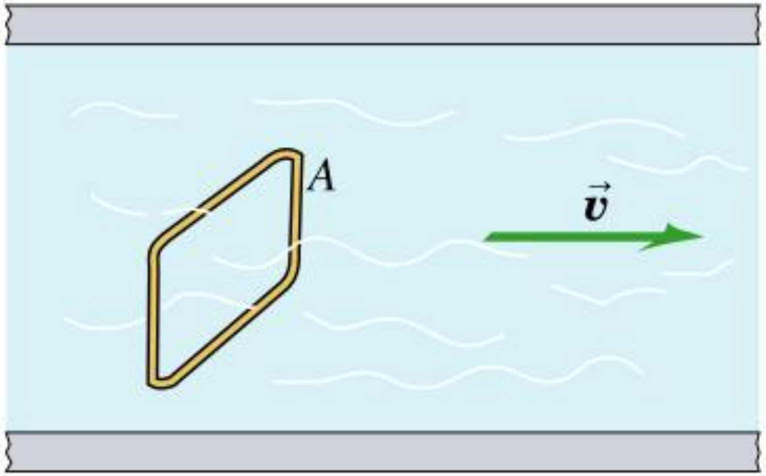
Introduction to Gauss's Law

Read sections 22.1, 22.2, 22.3

22.1 Flux of fluid flow

Properties of a flux:

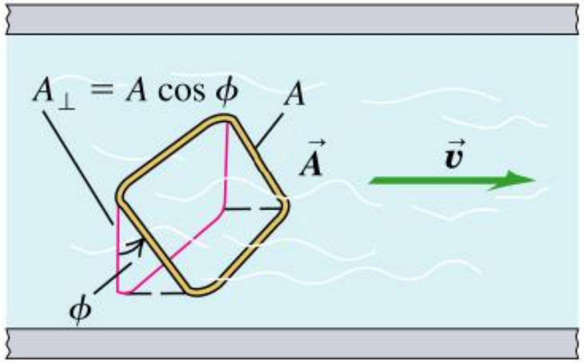
- **Electric flux** is a measure of the electric field passing *at right angles* through a surface area.
- The electric field (a **vector field**) has some similarity to the velocity field of a fluid.



- If fluid flows through a surface (area A) with a volume Vol per unit time, the resulting **velocity flux** is:

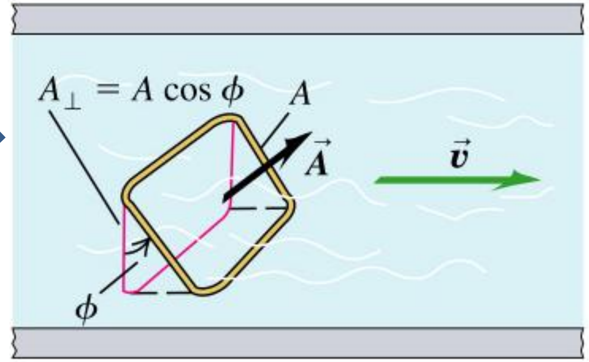
$$\Phi_v = \frac{d\left(Vol\right)}{dt} = \frac{d\left(Ax\right)}{dt} = vA$$

- If we tilt the surface an angle ϕ the flux becomes:



$$\Phi_v = \frac{d(Vol)}{dt} = vA_\perp = vA\cos\varphi$$

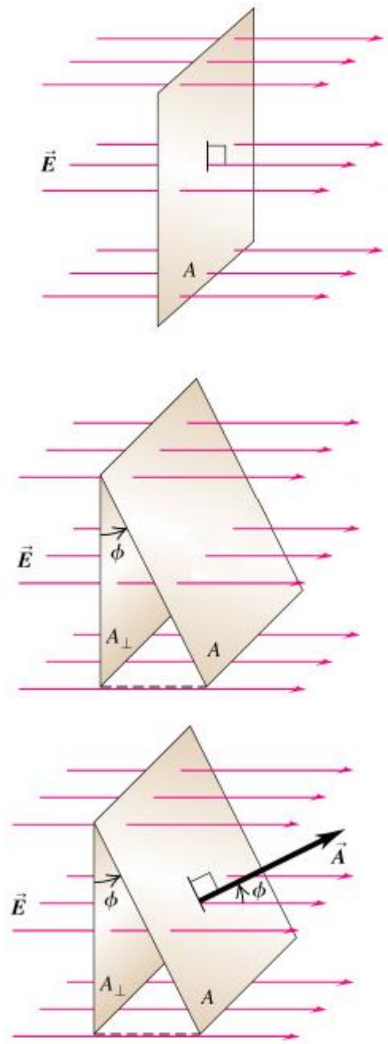
- We can define an **area vector** of magnitude A direction normal to the surface.
- The flux can then be written as the scalar product:



$$\Phi_v = \frac{d(Vol)}{dt} = \vec{v}\cdot\vec{A}$$

22.1,2 Electric Flux

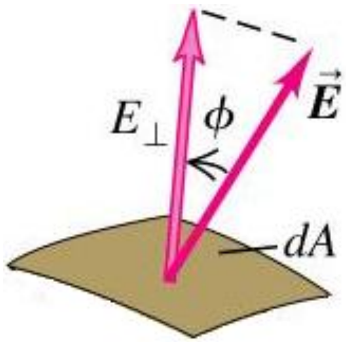
- **Electric flux:**
  - Consider a uniform electric field (magnitude E) **perpendicular** to a surface (area A).
  - The electric flux through that surface is: $\Phi_E = EA$
  - Now tilt the surface an angle ϕ.
  - The new electric flux is: $\Phi_E = EA_\perp = E_\perp A$ $= E\cos\varphi A$
  - Again define an area vector of magnitude *A* and direction normal to the surface.
  - The flux can now be written as: $\Phi_E = \vec{\mathrm{E}} \cdot \vec{\mathrm{A}}$



22.2 Electric flux at irregular surfaces

- Electric flux:
  - If the electric field is not uniform or the surface is not regular, we must **integrate** over the surface.
  - Consider a differential element of the surface with area dA.
  - We calculate the electric flux through all such elements and add.
  - This is equivalent to a **surface integral**:

$$\Phi_E = \int_{surface} E_\perp \, dA = \int_{surface} \vec{E} \cdot d\vec{A}$$



- The electric flux through a surface is given by:

$$\Phi_E = \int_{surface} E_\perp \, dA = \int_{surface} \vec{E} \cdot d\vec{A}$$

- Units: Nm^2 / C

22.3 Gauss' law for spherical surface

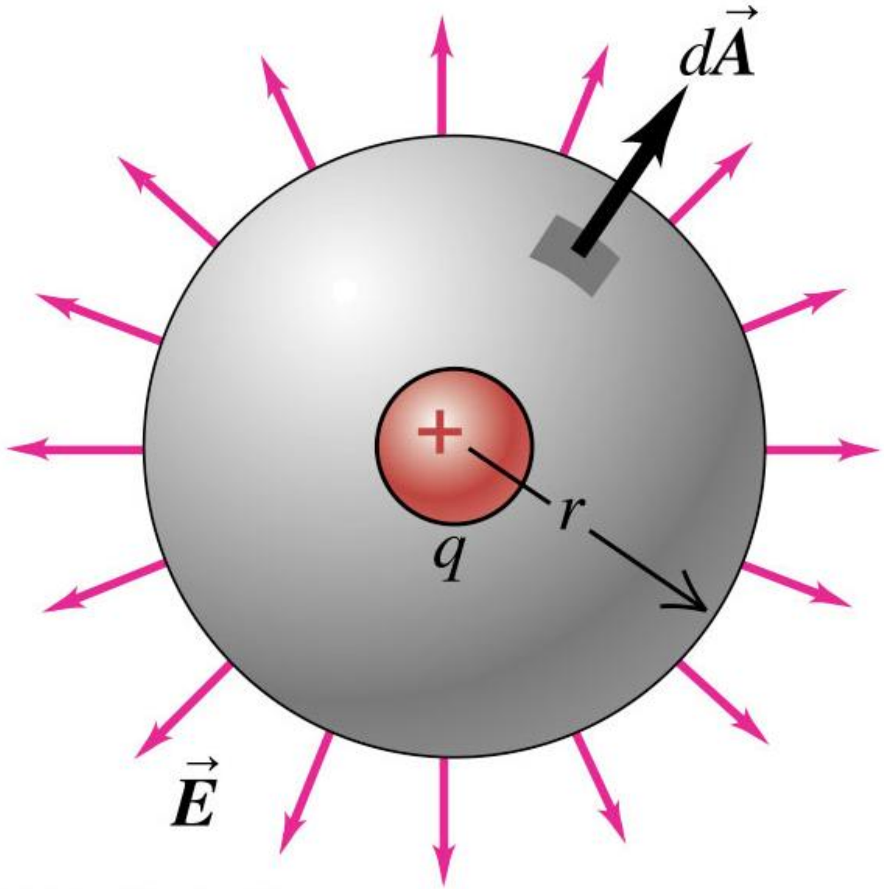
- **Gauss' law**:
  - Consider an imaginary sphere of radius *r* surrounding a point charge *q*.
  - Because the electric field is perpendicular to the surface at all points the flux integral simplifies to :

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA$$

Closed surface

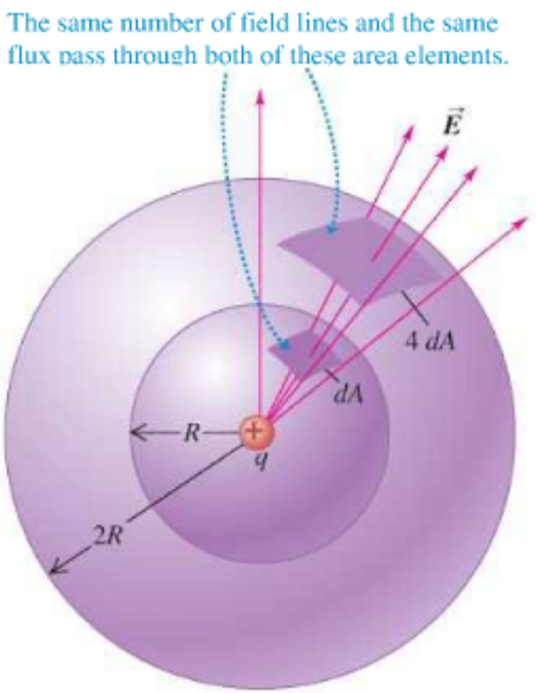
$$= \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \right) \left(4\pi r^2 \right) = \frac{q}{\varepsilon_0}$$

  - **The flux through a sphere concentric with a point of charge is independent of the radius.**

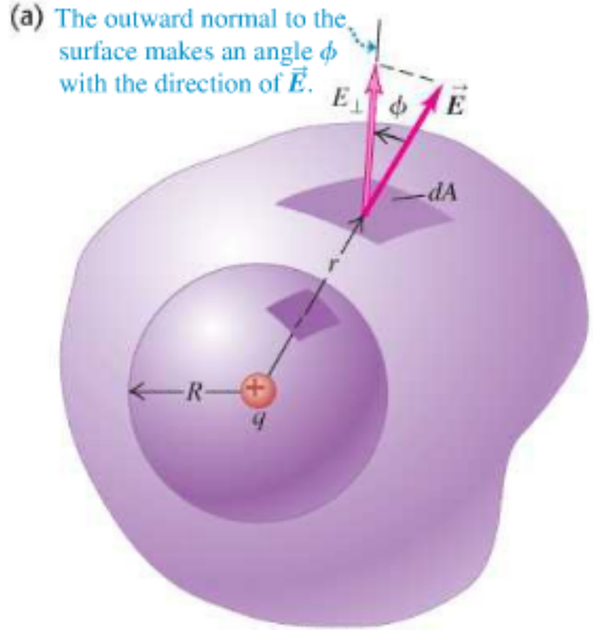


22.3 Flux: Containing surfaces

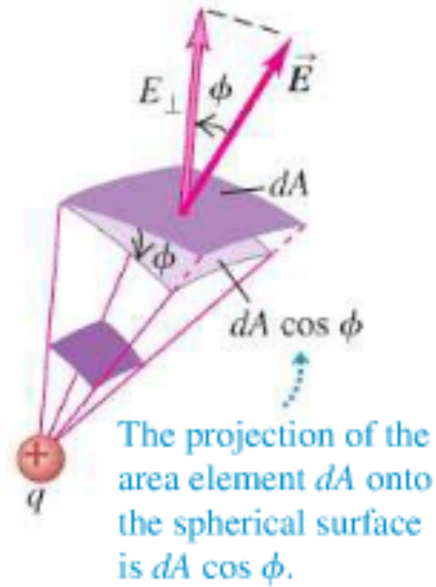
Concentric spherical surfaces



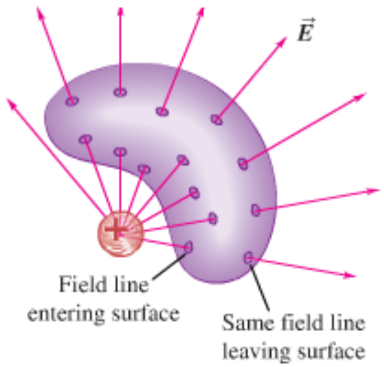
Irregular surface containing spherical surface



$$d\Phi = EdA\cos\varphi$$



Flux is the same for all containing, closed surface cases!



Zero flux

Applications of Gauss's Law

Read Chapter 22.4, 22.5

22.3 Gauss' law: General surface

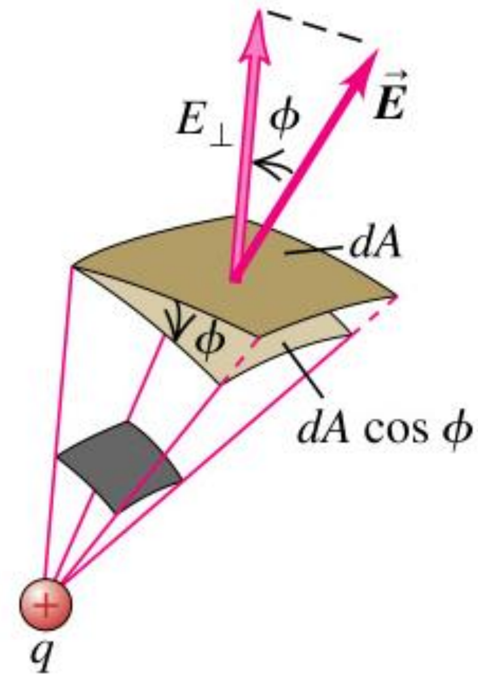
- Gauss' law
  - Surround the charge with any irregular, closed surface.
  - The flux through each area element is equal to the flux through an area element of a sphere at the same radius.
  - So the calculation still holds:

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q}{\varepsilon_0}$$

Closed surface

  - If the surface encloses many charges $q_1, q_2, q_3, \ldots$ then we just sum:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_1 + q_2 + q_3 \ldots\ldots}{\varepsilon_0}$$

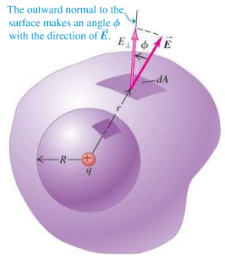


- Gauss' law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

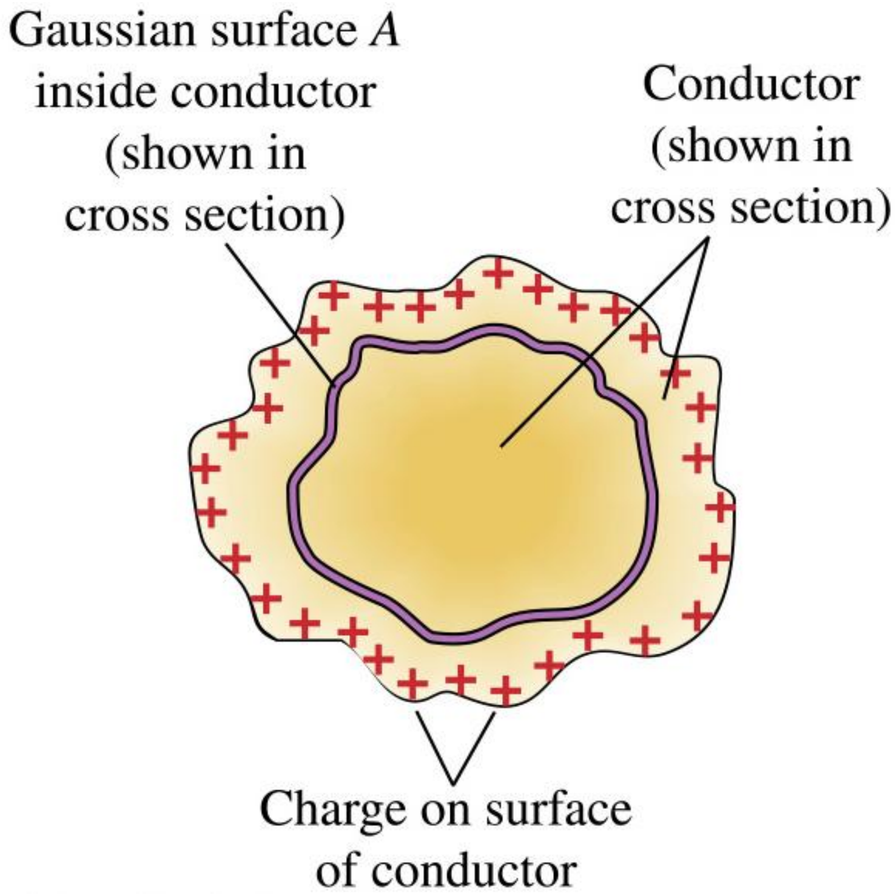
22.4 Gauss' Law: Distribution of charge inside a conductor

- **Applications of Gauss' law:**
  - Consider a **charged** conducting object, in an **electrostatic** condition.
  - Construct a Gaussian surface inside object.
  - Since no charges are moving (electrostatic condition), the electric field inside the conductor must be zero.
  - So by Gauss' law the electric flux through our surface must be zero.
  - So the net charge inside the surface is also zero.
  - Therefore no excess charge can exist within the object. It must all be on the surface.



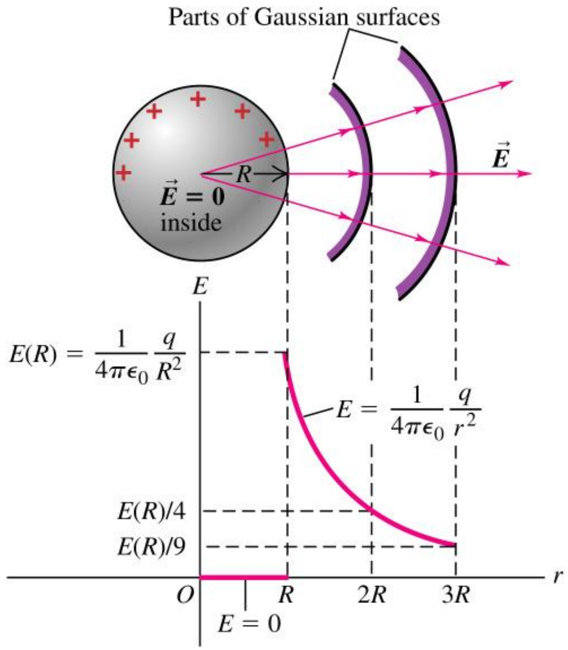
- Gauss' law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$



22.4 Gauss' law: Conducting sphere

- **Field of a charged conducting sphere:**
  - Consider a sphere of radius *R* with a charge *q*.
  - The electric field within a conductor is zero.
  - Construct a Gaussian surface outside the sphere of radius *r*.
  - By symmetry the electric field must be radial so it is perpendicular to all points of the surface.
  - So the flux integral reduces to:



$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = EA = E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- So the field **outside** a conducting sphere is the same as for a **point charge**.

22.4 Gauss' law: Line of charge

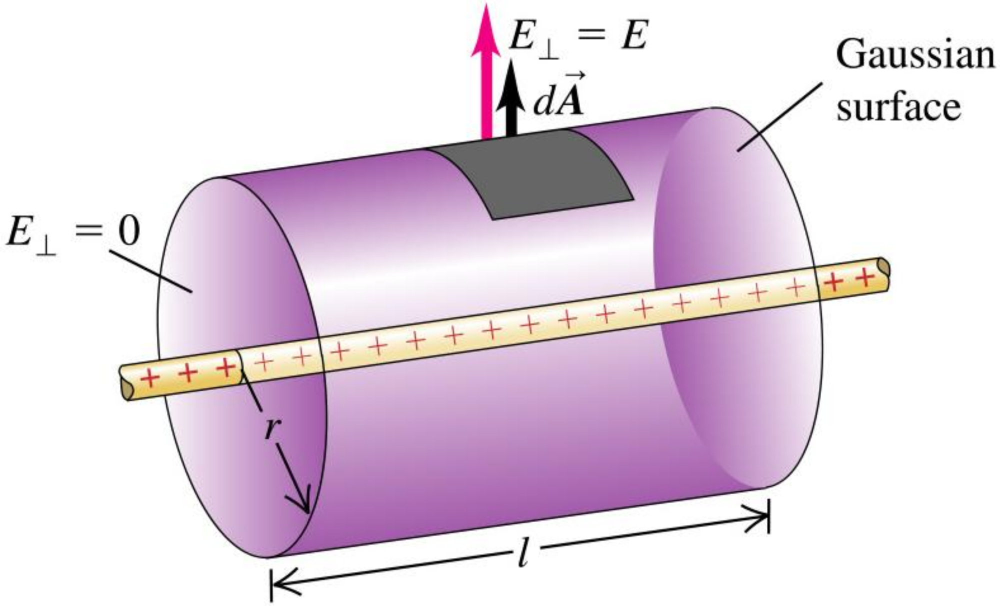
- **Field of a line of charge**:
  - Consider an infinite line of charge with charge density λ.
  - Construct a Gaussian cylinder of radius *r* and length *l* around it as shown.
  - Note that the enclosed charge $Q = \lambda l$
  - By symmetry the field must be directed directly away from the wire at all points.
  - The flux integral simplifies to:

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = E\left(2\pi r l\right) = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

Line of charge, slide 47:

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{\boldsymbol{i}}$$

$$\lim_{a\to\infty}\vec{\boldsymbol{E}} = \lim_{x<<a}\vec{\boldsymbol{E}} = \frac{\lambda}{2\pi\varepsilon_0 x}\hat{\boldsymbol{i}}$$



1.5 Gauss' law: Capacitor

- **Field of oppositely charged conducting plates:**

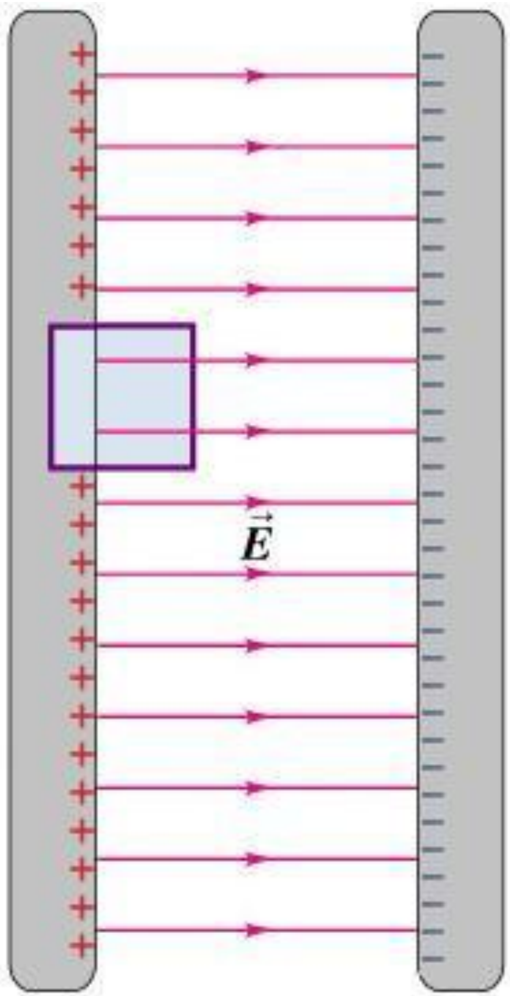
  – Consider two parallel conducting plates with charge densities σ and $-\sigma$ respectively.

  – Construct a Gaussian cylinder through one of the plates as shown with **ends** of area *A*.

  – The field must be directed perpendicular to the plates.

  – The field is zero inside the conductor.

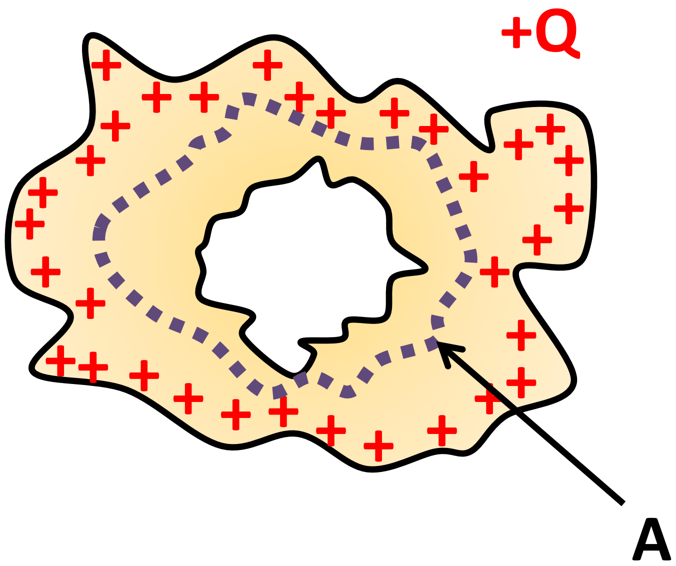
  – The flux integral reduces to:

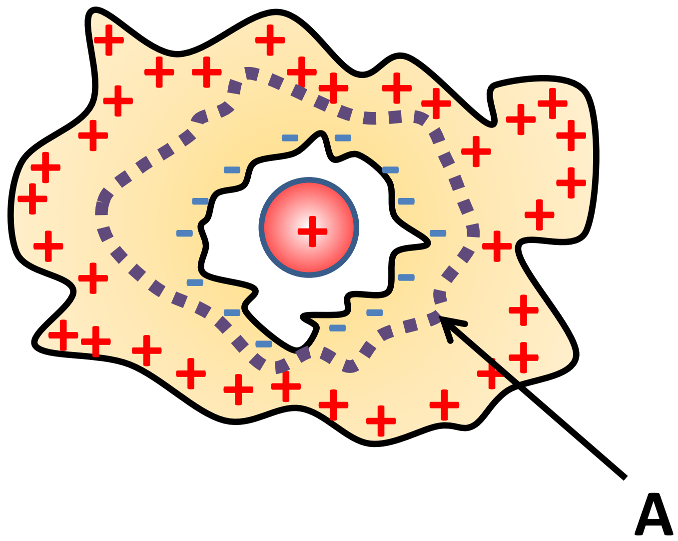


$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = EA = \frac{Q}{\varepsilon_0} = \frac{A\sigma}{\varepsilon_0} \qquad \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

22.5 Gauss' law: Charged conductor with cavity

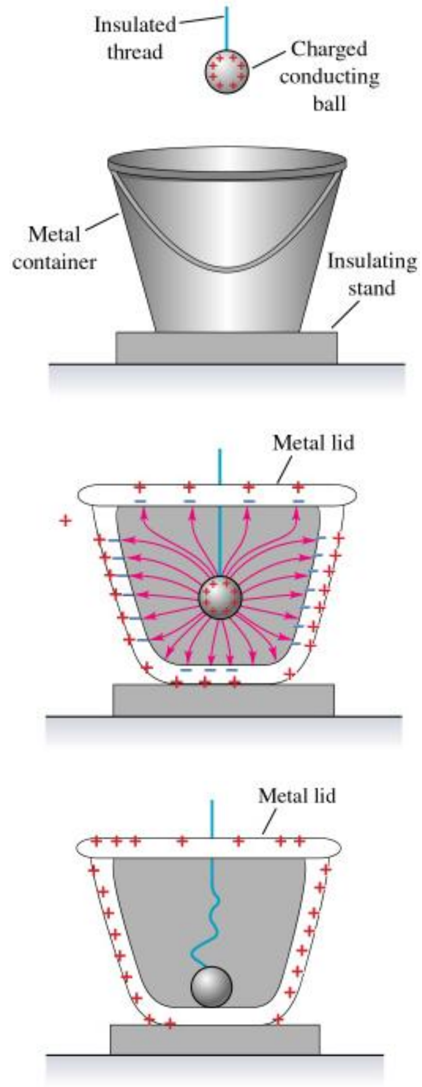
- **Where is the charge in this case?**
  - Consider a (positively) charged conducting object with a cavity. Construct a Gaussian surface A inside the conductor, surrounding the cavity
  - Field within conductor is zero so flux through A is zero:
    - there is no charge inside A,
    - and the postive charge must exist on **outer** surface.
  - Further step: Put point charge in cavity.
  - Flux through surface A must still be zero as there is no electric field , so there must be a negative charge on interior surface to balance the total charge
  - It is the **excess** positive charge that moves to the exterior surface.





22.5 Gauss' law: Experimental Test

- Faraday's ice pail experiment:
  - A positively charged metal ball is lowered into a metal container.
  - The ball induces charge on the container.
  - When the ball touches the metal the charge quickly moves and this positive charge is left on the outside of the container.
  - When the ball is removed, it is found that **it is no longer charged!!**
  - This shows that charge always ends up on the the **outside** of conductors in the electrostatic condition.



➡ Verifies Gauss' Law…and therefore Coulomb's Law

22.5 Gauss' law

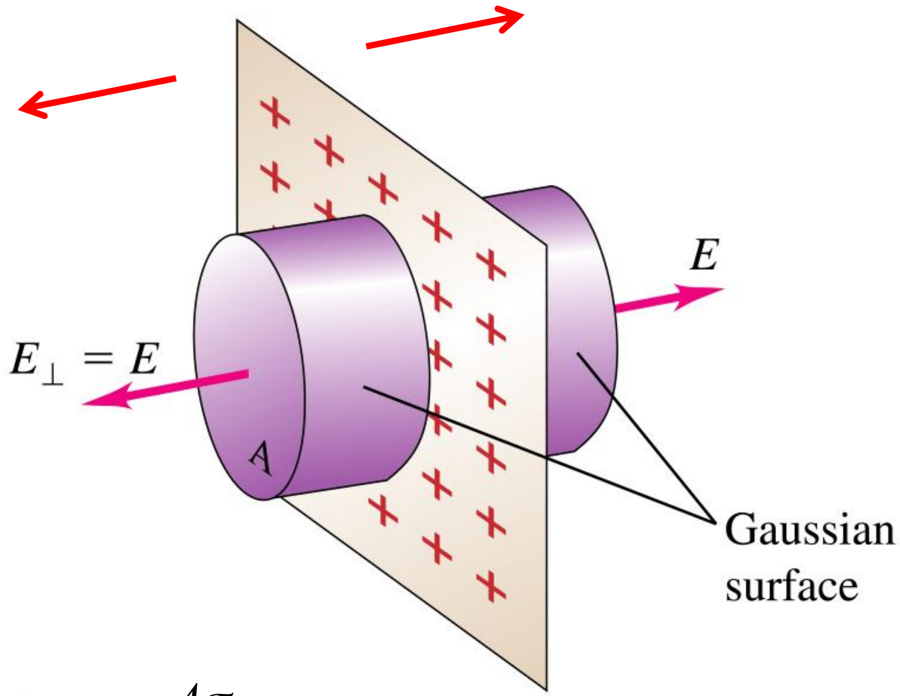
- **Field of an infinite charged sheet**:

  - Consider an infinite charged sheet with surface charge density σ.

  - The field must be directed away form the sheet by symmetry.

  - Construct a Gaussian cylinder around it as shown.

  - The flux integral simplifies to:

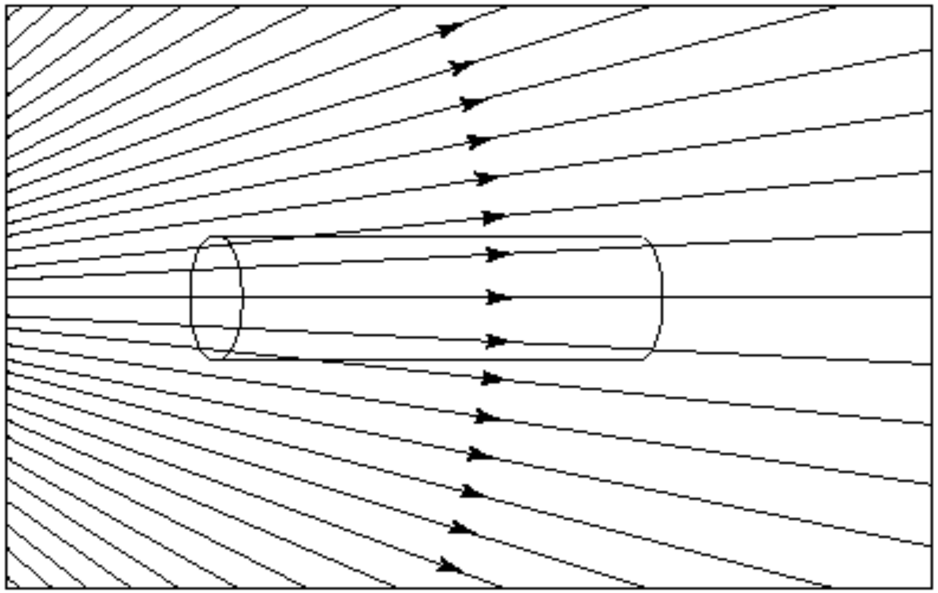


$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = 2EA = \frac{A\sigma}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Concept test

A cylindrical piece of insulating material is placed in an external electric field, as shown. The net electric flux passing through the surface of the cylinder is



1. positive.
2. negative.
3. zero.

Recommended Extra Problems for Gauss's Law

See the problem solving strategy 22.1 (p. 737 text) for Gauss's Law

(Textbook Edition 13)

Flux: 22.4, 22.5 , 22.6

Gauss's Law: 22.9, 22.11, 22.12, 22.13

Applications of Gauss's Law: 22.17, 22.20-23, 22.30-33

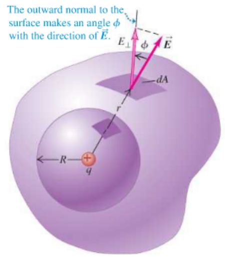
Then try some problems: 22.37, 22.39, 22.46, 22.47

Applications of Gauss's Law

Read Chapter 22.4, 22.5

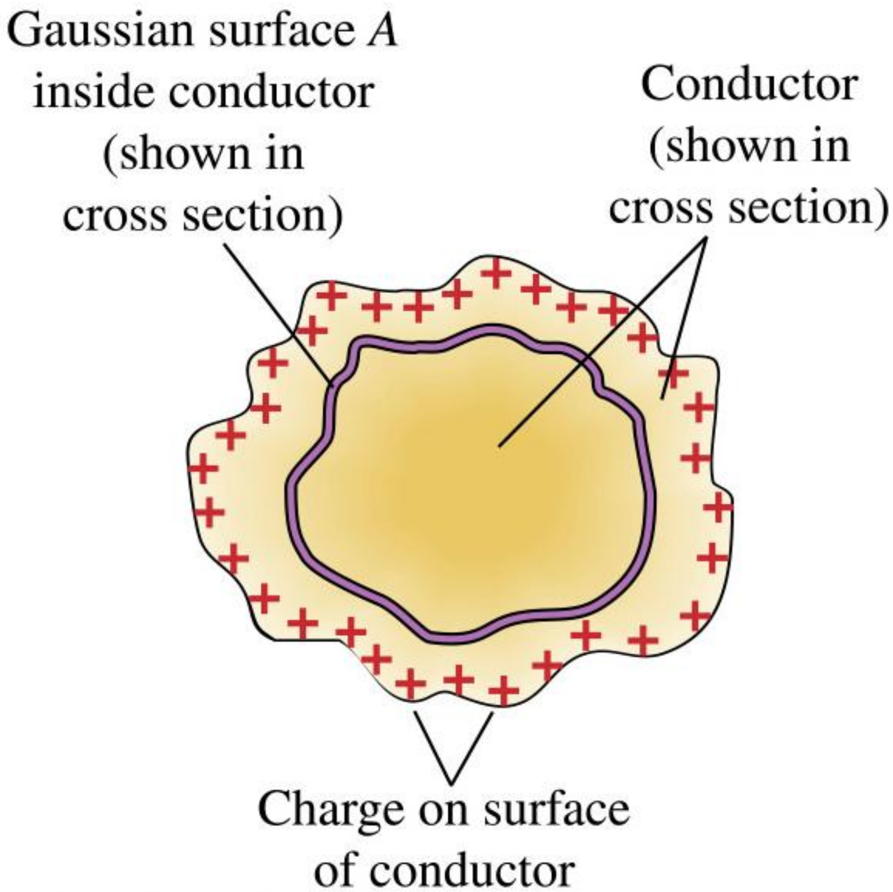
22.4 Gauss' Law: Distribution of charge inside a conductor

- **Applications of Gauss' law:**
  - Consider a **charged** conducting object, in an **electrostatic** condition.
  - Construct a Gaussian surface inside object.
  - Since no charges are moving (electrostatic condition), the electric field inside the conductor must be zero.
  - So by Gauss' law the electric flux through our surface must be zero.
  - So the net charge inside the surface is also zero.
  - Therefore no excess charge can exist within the object. It must all be on the surface.



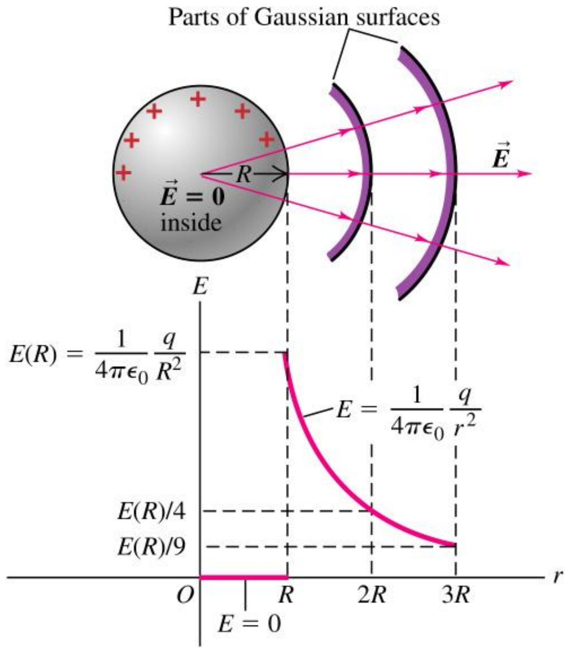
- Gauss' law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$



22.4 Gauss' law: Conducting sphere

- **Field of a charged conducting sphere:**
  - Consider a sphere of radius *R* with a charge *q*.
  - The electric field within a conductor is zero.
  - Construct a Gaussian surface outside the sphere of radius *r*.
  - By symmetry the electric field must be radial so it is perpendicular to all points of the surface.
  - So the flux integral reduces to:



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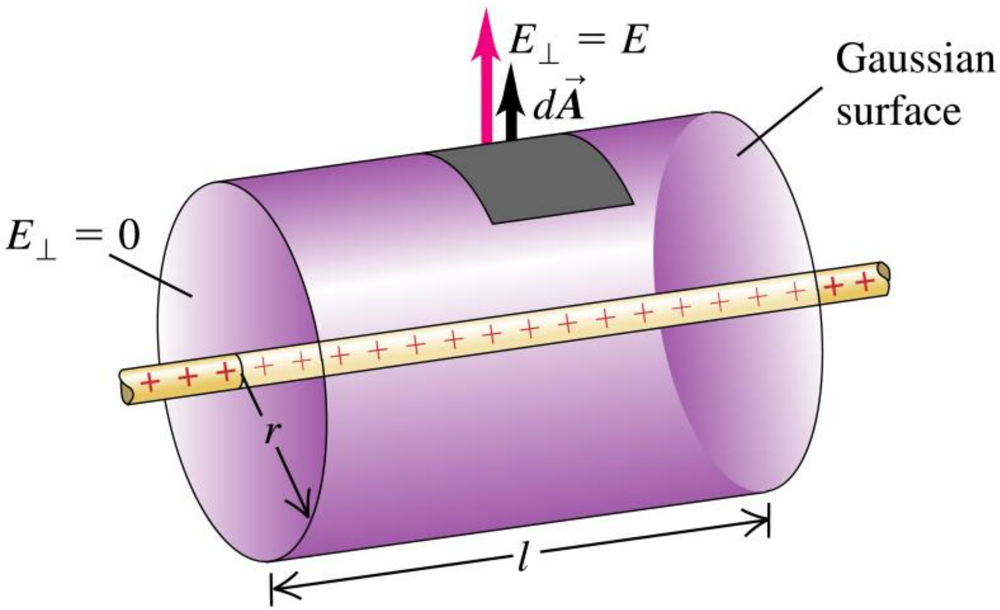

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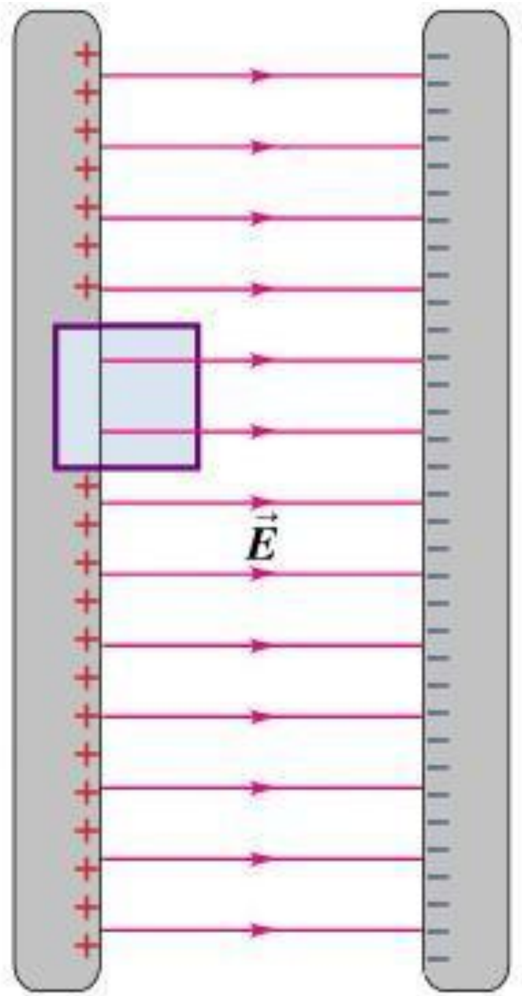

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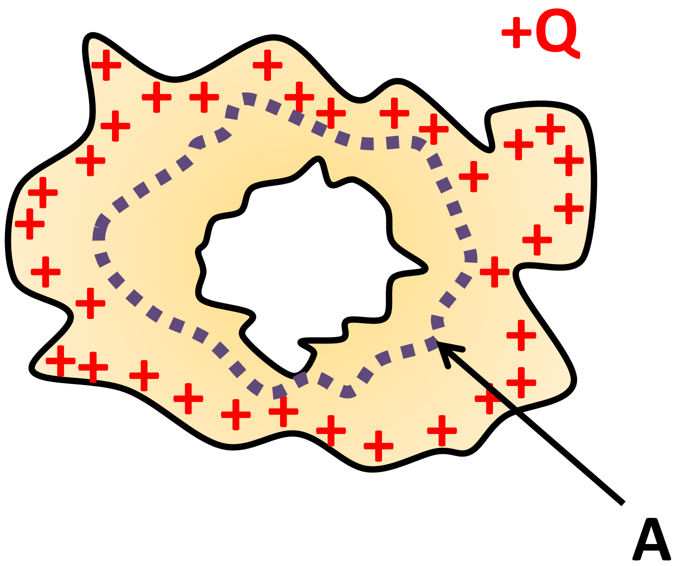

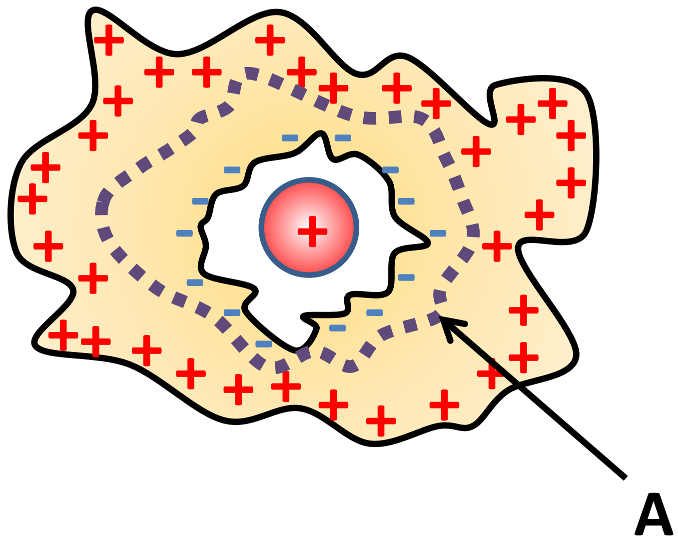

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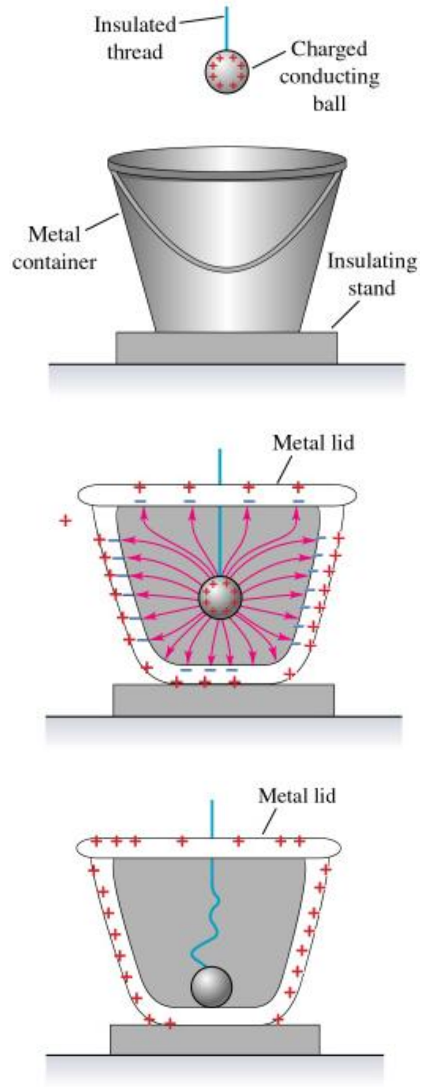

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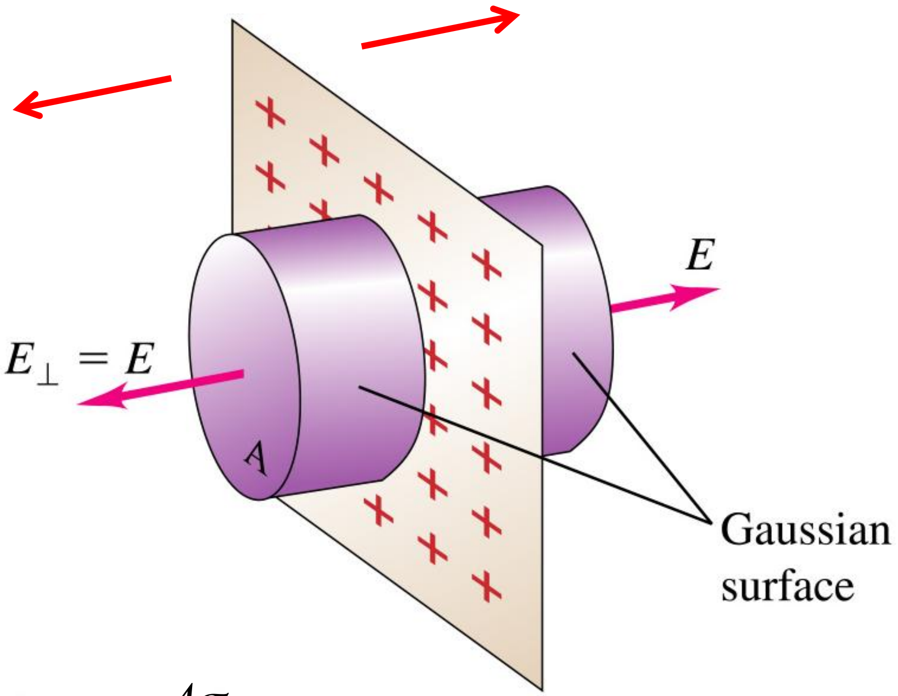

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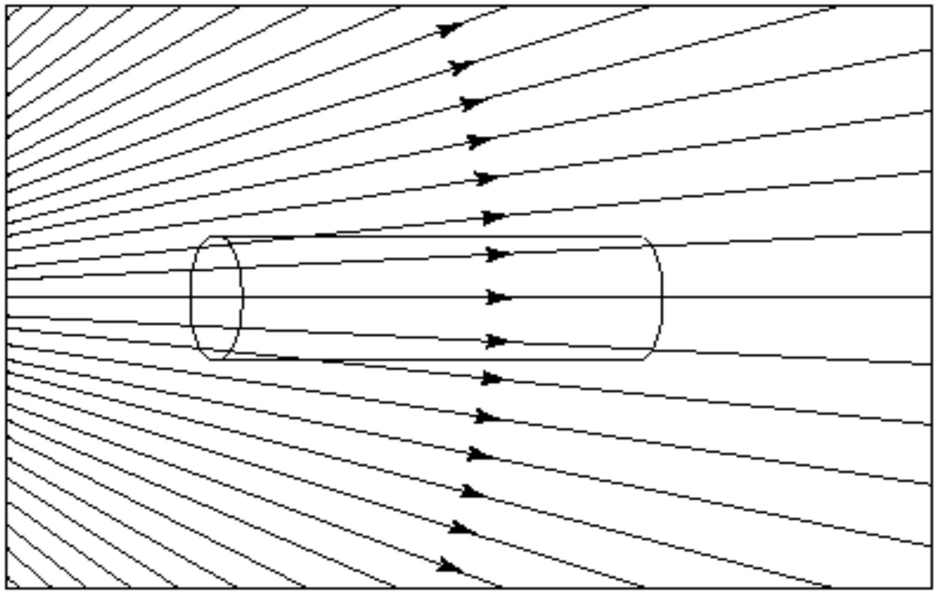

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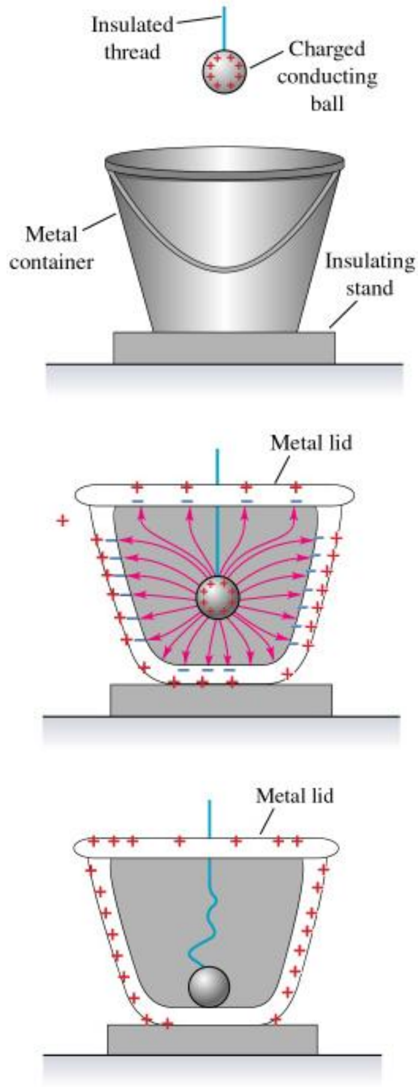

Verifies Gauss' Law…and therefore Coulomb's Law

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  – When the ball is removed, it is found that **it** **charged!!**

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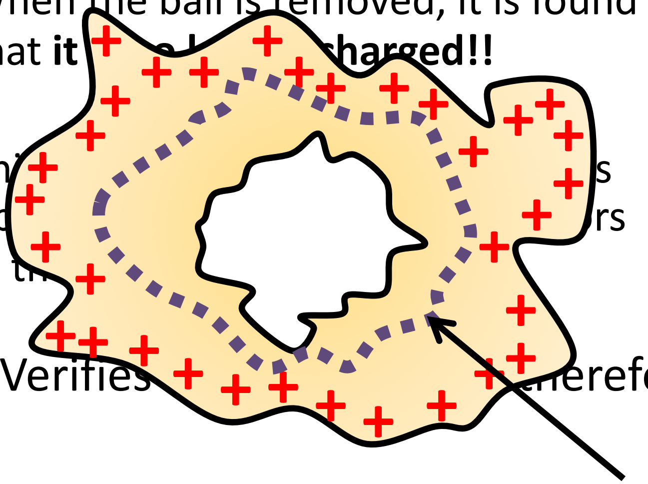
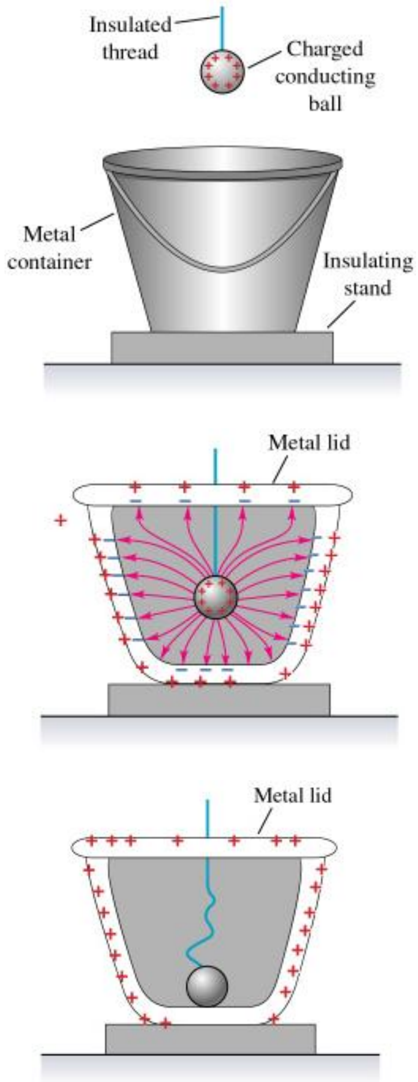




Verifies erefore Coulomb’s Law

Electric potential

Read Chapter 23

23.1 Potential energy

- **Review of potential energy:**
  - The work done by a **conservative** force pushing on a mass in space is given by:

$$W_{a\to b} = \int_a^b \vec{F} \bullet d\vec{r}$$

$$= K_b - K_a \qquad \text{(Change in kinetic energy)}$$

  - The **potential energy** associated with this conservative force is defined as a function $U(\vec{r})$ such that that the total energy is conserved:

$$K_a + U_a = K_b + U_b = E$$

  - Thus: $K_b - K_a = U_a - U_b = W_{a\to b}$

  - Gain of KE = loss of PE

  - The difference in potential energy between *a* and *b* is:

$$U_a - U_b = \int_a^b \vec{F} \bullet d\vec{r}$$

  - Units: joule (*J*)

23.2 Electric potential

- **Potential**:

  – Consider a **positive** test charge q_0 in an electric field with potential energy U.

  – A change in **electric potential energy** is given by:

  $$U_a - U_b = \int_a^b \vec{F} \bullet d\vec{r} = \int_a^b q_0 \vec{E} \bullet d\vec{r}$$

  – Dividing by q_0 we get the change in **electric potential**:

  $$\frac{U_a}{q_0} - \frac{U_b}{q_0} = V_a - V_b = \int_a^b \vec{E} \bullet d\vec{r}$$

  – The electric potential at q_0 is given by:

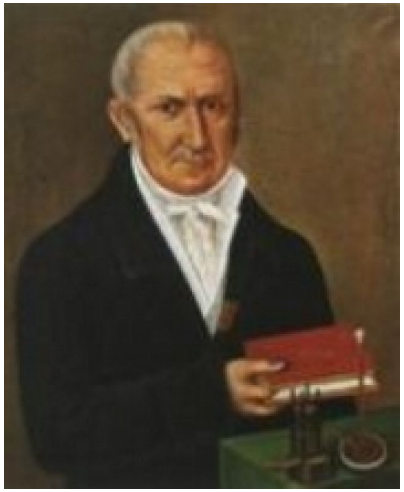
  $$V \equiv \frac{U}{q_0}$$

  – "Potential energy per unit charge"

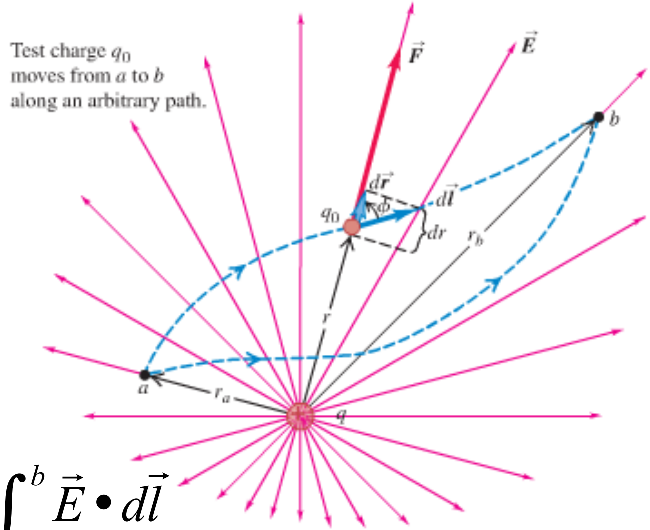
  – Units: J / C or volt (V)

23.2 Electric potential

- **Volta:**
  - Alessandro Volta was born in 1745 in Como, Italy.
  - He discovered methane and also invented the first electrical battery.
  - He was awarded the Legion of Honor by Napoleon in recognition of his work.
  - He died in 1827.



23.2 Electric potential



- **Potential due to a point charge q**:
  - Consider a test charge q_0 being moved from **a** to **b**
  - By the definition of potential difference: $V_{ab} = V_a - V_b = \int_a^b \vec{E} \bullet d\vec{l}$
  - As the field is radial: $V_{ab} = \int_a^b E\,dr = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$ (general)
  - Normally define V relative to some point where we set $V = 0$
  - Set $V=0$ when $r = \infty$. Then: $V = V_{r\,to\,\infty} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
  - Place a charge q' at r.
  - Then its electric potential energy is: $U = q'V = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$
  - The potential due to many point charges q_i is: $V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

23.2 Electric potential

- **Sample problem**:

  – A proton moves $0.50\ m$ from a to b in a uniform electric field of magnitude $E = 1.5\times10^7 V/m = 1.5\times10^7 N/C$ pointing from a to b.

  – Determine:

    - The force on the proton;
    - The work done on it by the field;
    - The potential difference $V_a - V_b$

- **Solution:**

  - The force on the proton is in the direction from *a* to *b* and is of magnitude:

$$F = qE = \left(1.6\times10^{-19}C\right)\left(1.5\times10^{7}N/C\right) = 2.4\times10^{-12}N$$

  - The work done on the particle is:

$$W_{a\to b} = \int_a^b \vec{F}\bullet d\vec{r} = Fd = \left(2.4\times10^{-12}N\right)\left(0.5m\right) = 1.2\times10^{-12}J$$

  - The potential difference between *a* and *b* is:

$$V_a - V_b = \frac{W_{a\to b}}{q} = \frac{1.2\times10^{-12}J}{1.6\times10^{-19}C} = 7.5\times10^{6}V = 7.5MV$$

  - Or:

$$V_a - V_b = \int_a^b \vec{E}\bullet d\vec{r}$$

Concept test

An electron is pushed into an electric field where it acquires a 1-V electrical potential. Suppose instead that two electrons are pushed the same distance into the same electric field. The electrical potential of the two electrons is

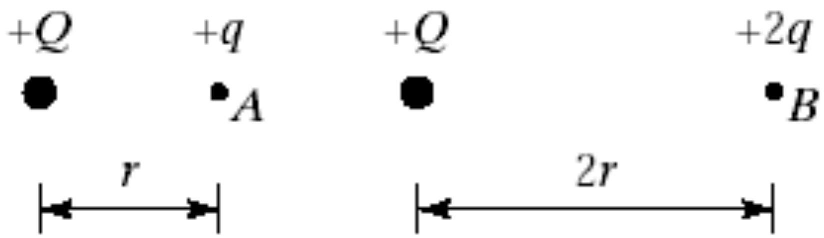
1. 0.25 V.
2. 0.5 V.
3. 1 V.
4. 2 V.
5. 4 V.

(More) Electric potential

Read Chapter 23

Concept test

Two test charges are brought separately into the vicinity of a charge $+Q$. First, test charge $+q$ is brought to point A a distance r from $+Q$. Next, $+q$ is removed and a test charge $+2q$ is brought to point B a distance $2r$ from $+Q$. Compared with the electrostatic potential of the charge at A, that of the charge at B is

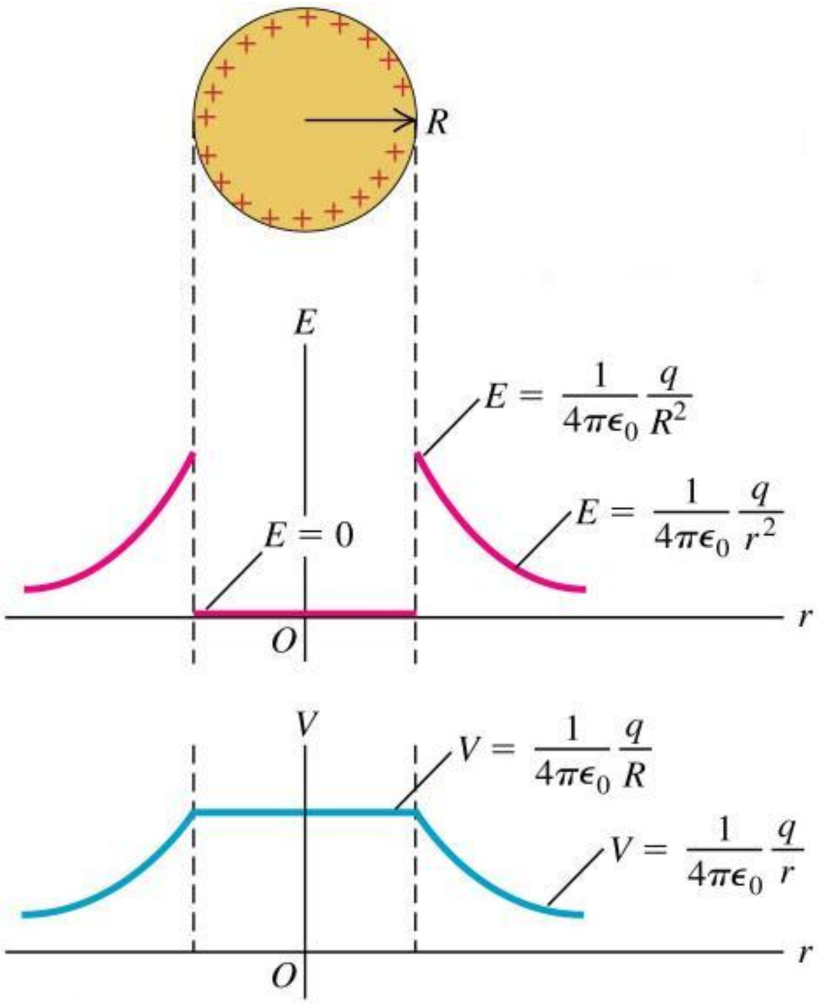


1. greater.
2. smaller.
3. the same.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

23.3 Electric potential

- **Potential due to a charged sphere**:
  - Electric field outside sphere is the same as for a point charge:
    $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$
  - So the potential is also the same:
    $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$
  - Conducting body so *E=0* inside.
  - No work done on particles inside so potential same as at surface.
  - At the surface: $V = RE$



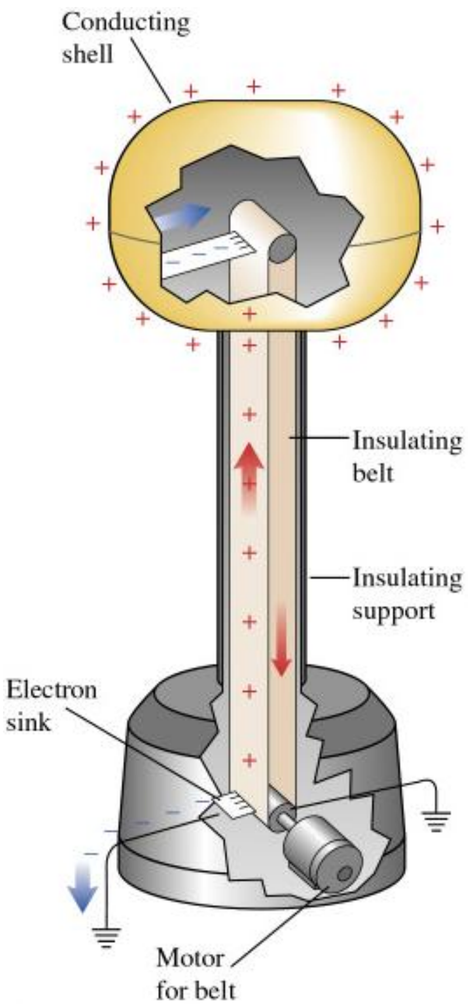
23.3 Electric potential

- **Dielectric strength**
  - Maximum potential for a conductor in air is limited since at high electric field the air molecules become ionised and the air conducts.
  - The electric field strength at which air becomes ionised (dielectric strength) is:

$$E_{\max} \approx 3\times10^6\, N/C$$

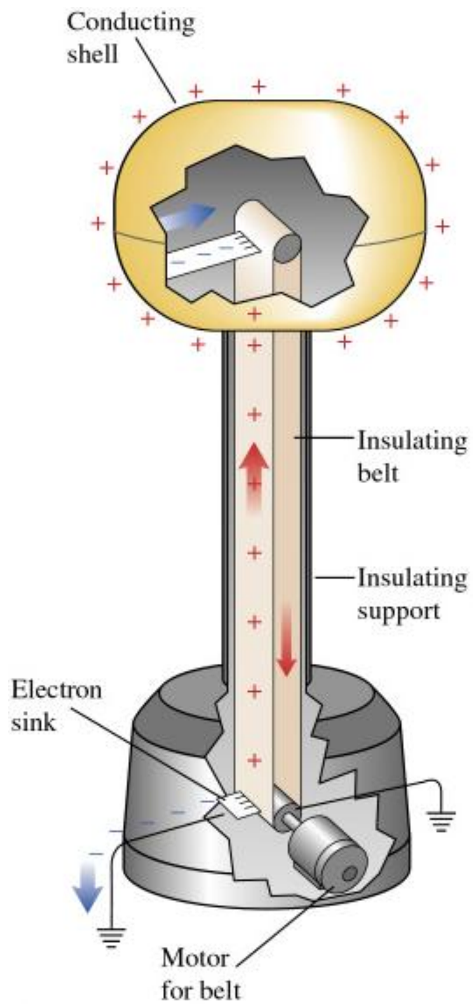
  - The maximum voltage to which a spherical conductor radius *R* can be raised is: $V_{\max} = RE_{\max}$
  - For $R = 10\ cm$ this gives: $V_{\max} \approx \left(3\times10^6\, N/C\right)\left(0.1m\right) = 3\times10^5\, V$
  - Large spherical terminals are used on machines like the Van de Graaff generator to ensure high voltages.
  - Conversely: $E_{\max} = \dfrac{V_{\max}}{R}$
  - So for sharp objects, small voltages can produce strong fields.
  - If the air becomes ionised, charge will dissipate through it producing a glow called a corona.

- **Van de Graaff generator:**
  - Hollow metal sphere on insulating support
  - Insulating belt brings positive charge to inside shell.
  - This positive charge moves to outside of shell.
  - Bottom roller – neg.
  - Top roller - pos.



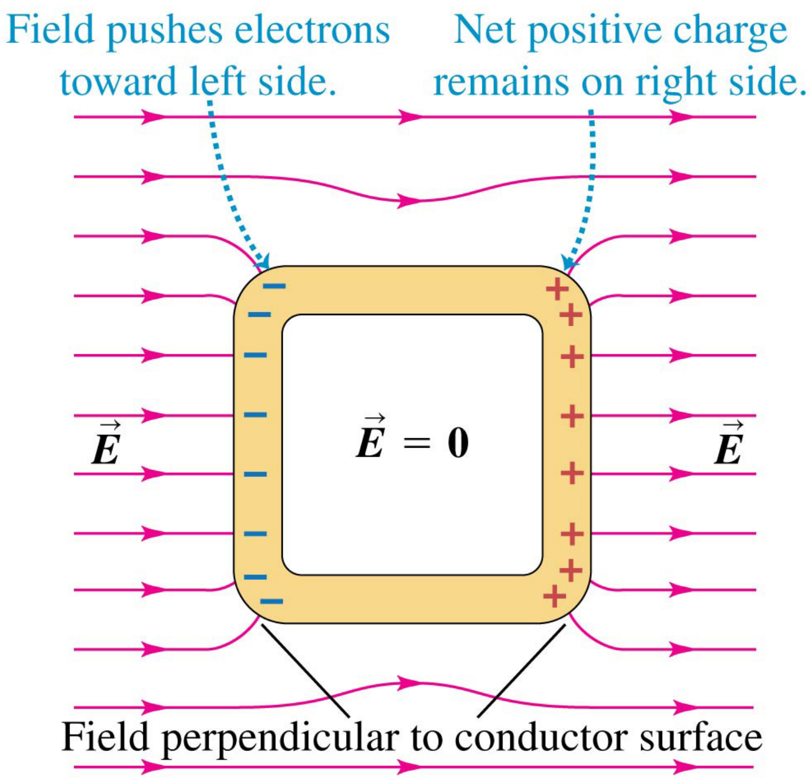
22.5 Gauss' law

- **Van de Graaff generator:**
  - Work done in moving pos. charge against field
  - Charge builds up until E-field at surface equals dielectric strength of air
  - Used for accelerating charged particles to high energy for collision experiments



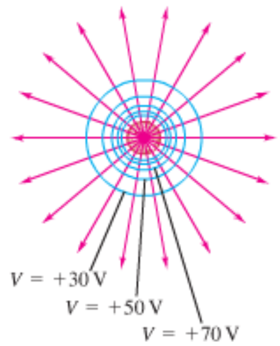
Electrostatic shielding

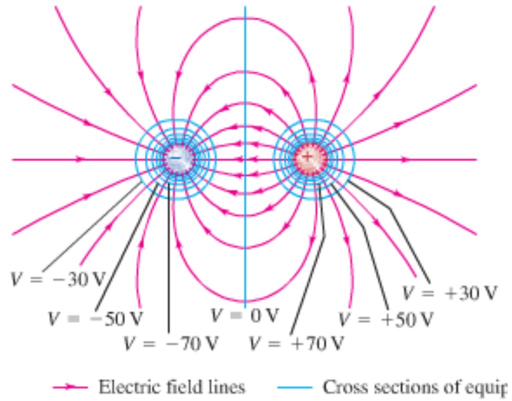
- A conducting box is immersed in a uniform electric field.
- The field of the induced charges on the box combines with the uniform field to give *zero* total field inside the box.

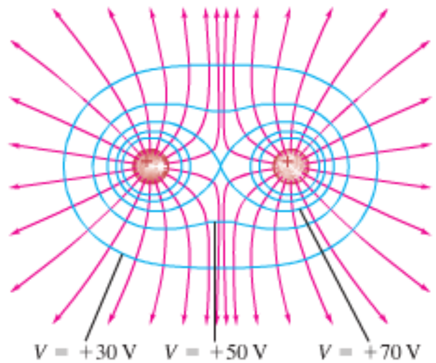


23.4 Equipotential Surfaces

- **Equipotential surfaces**:
  - Electric field lines are used to visualize the electric field.
  - Equipotential surfaces serve a similar purpose.
  - They are surfaces on which the potential is constant.
  - An electric field does no work on a test charge as it is moved over an equipotential surface.
  - So there is no component of the field parallel to the surface.
  - So the electric field is always **perpendicular** to equipotential surfaces.

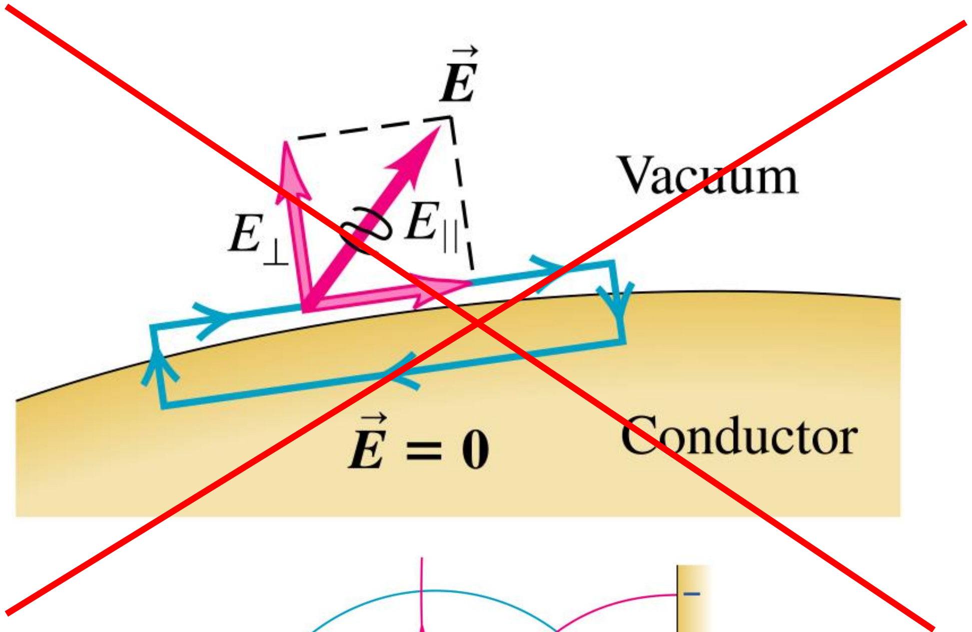


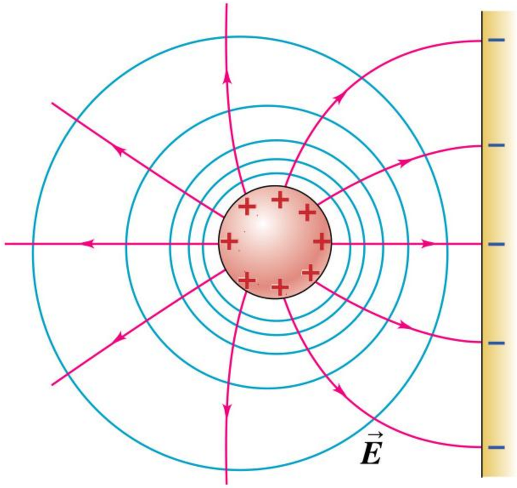




23.4 Equipotential Surface of Conductors

- **Surface of a conductor:**
  - The electric field just inside the surface of a conductor is zero.
  - Just outside the surface the field must be perpendicular to the surface, otherwise a particle could travel around a closed loop with a total net amount of work done on it.
  - So the surface of a conducting body is an equipotential surface.



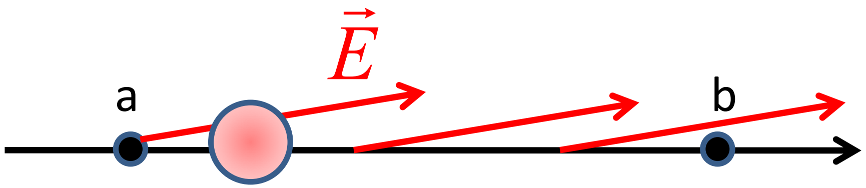


23.5 Electric potential gradient

- **Relationship of electric field and potential: The potential gradient:**

  – Consider a test charge moving on the *x*-axis:

$$V_{ab} \equiv V_a - V_b = \int_a^b \vec{E} \bullet d\vec{r}$$



$$-\left(V_b - V_a\right) = \int_a^b E_x\, dx$$

$$\Rightarrow -\int_a^b dV = \int_a^b E_x\, dx$$

  – Integrals are equal so integrands are equal:

$$-dV = E_x dx$$

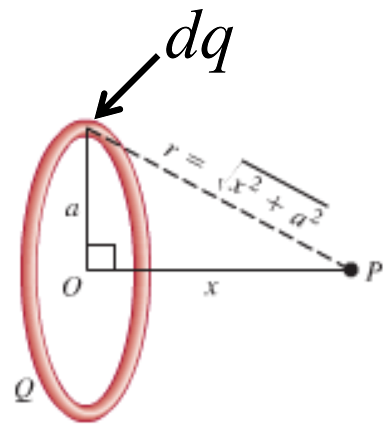
$$E_x = -\frac{dV}{dx}$$

  – In general for 3D:

$$\vec{E} = -\left(\frac{dV}{dx}\hat{i} + \frac{dV}{dy}\hat{j} + \frac{dV}{dz}\hat{k}\right) = -\vec{\nabla} V$$

Gradient or "Grad" operator, "Del"

23.5 Calculating electric potential



- **Electric field from potential:**
  - The potential for a point on the axis of a charged ring is:

$$V(x) = \int \frac{1}{4\pi\varepsilon_0} \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2 + a^2}} \int dq = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$$

  - From this we easily obtain the electric field:

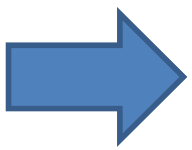
$$E_x = -\frac{dV}{dx} = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{\left(x^2 + a^2\right)^{\frac{3}{2}}}$$

NB By symmetry of problem:

$$E_y = E_z = 0$$

OR, using the gradient operator:

$$\vec{E} = -\vec{\nabla} V = -\left(\frac{dV}{dx}\hat{i} + \frac{dV}{dy}\hat{j} + \frac{dV}{dz}\hat{k} \right) = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{\left(x^2 + a^2\right)^{\frac{3}{2}}} \hat{i}$$



Electric potential is very useful as well as theoretically important

Electric potential and capacitance

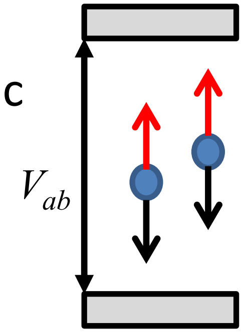
Read Chapter 23 and 24

24. Millikan Oil Drop Experiment

- **Millikan's oil drop experiment:**
  - Small oil droplets of density ρ are sprayed into region of uniform electric field E, created by potential difference V_{ab} across distance d.

$$E = \left| -\frac{dV}{dx} \right| = \frac{V_{ab}}{d}$$

  - Some droplets spontaneously obtain charge from the air or ionizing radiation.
  - These droplets can be held **motionless** against gravity by varying the electric field (ie. using the voltage) :



$$F_e = F_g \quad \Rightarrow \quad qE = mg \quad \Rightarrow \quad q = mg\frac{1}{E} = \frac{4\pi\rho_{oil}r^3}{3} g \frac{d}{V_{ab}}$$

  - Measure *r* by observing, in a separate experiment, the **terminal velocity** v_t at which uncharged droplets fall through the air:

$$F_{gravity} = F_{drag}$$

Stokes drag force on sphere, η = viscosity of air

$$\frac{4\pi\rho_{oil}r^3}{3} g = 6\pi\eta r v_t \Rightarrow r = 3\sqrt{\frac{\eta v_t}{2\rho_{oil}g}} \quad \Rightarrow \quad q = 18\pi\frac{d}{V_{ab}}\sqrt{\frac{\eta^3 v_t^3}{2\rho_{oil}g}}$$

  - Values observed were integer multiples of: $1.602 \times 10^{-19} C$
  - This was deduced to be the value of the fundamental carrier of charge the electron.

23.5 The electron volt

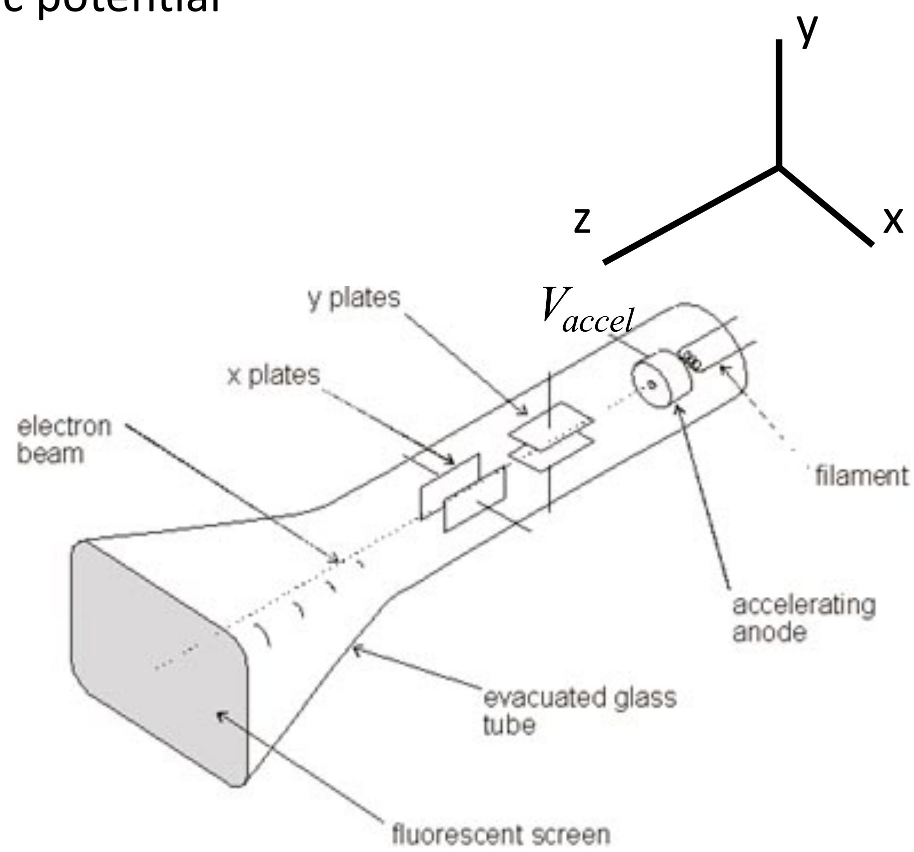
- **The electron-volt (eV)**
  - Consider an electron moving across a *1V* potential.
  - The change in potential energy is:

$$\Delta U = eV_{ab} = \left(1.6\times10^{-19}C\right)\left(1V\right) = 1.6\times10^{-19}J$$

  - This defines a new unit of energy, the electron-volt:

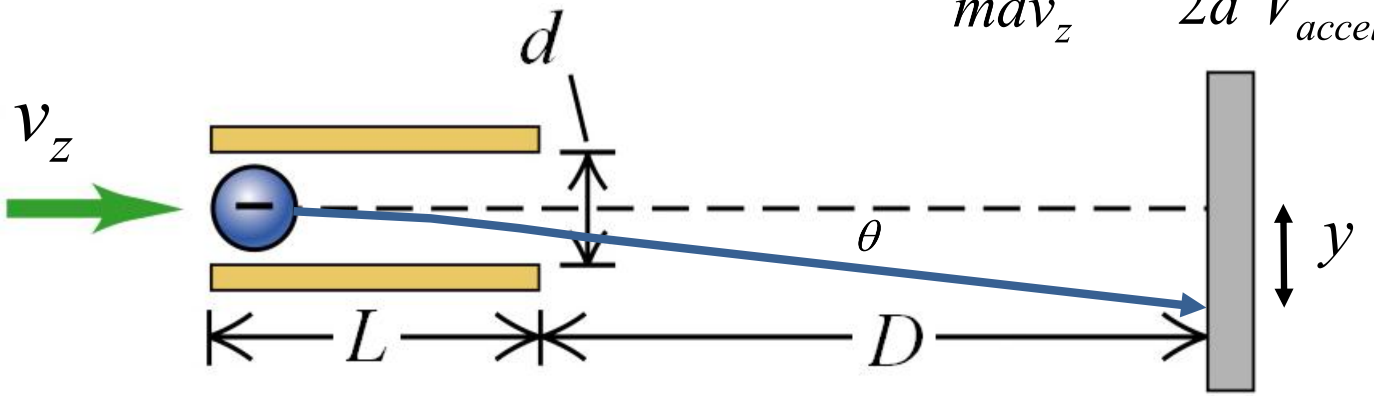
$$1\,eV = 1.6\times10^{-19}J$$

- **Cathode ray tube:**
  - This device is used in television screens and monitors.
  - Electrons are emitted from a heated filament and then are accelerated towards the anode.
  - A narrow beam passes through the anode and can be deflected in both directions by the plates.
  - It then strikes and illuminates a fluorescent screen.



Operation of the Cathode Ray Tube

- The speed of the electrons on reaching the anode is: $\frac{1}{2}mv_z^2 = eV_{accel} \Rightarrow v_z = \sqrt{\frac{2eV_{accel}}{m}}$
- Apply voltage to y-plates: $a_y = \frac{eE_y}{m} = \frac{eV_y}{md}$ (acceleration in y direction)
- Time spent between plates is: $t = \frac{L}{v_z}$
- Velocity in the y-direction on leaving the plates is: $v_y = a_y t = \frac{eV_y L}{mdv_z}$
- Exit trajectory makes angle θ with z-axis: $\tan\theta = \frac{v_y}{v_z} = \frac{eV_y L}{mdv_z^2}$
- Vertical position on reaching screen is: $y = D\tan\theta = \frac{DeV_y L}{mdv_z^2} = \frac{LD}{2d}\frac{V_y}{V_{accel}}$



Concept test

A solid spherical conductor is given a net nonzero charge. The electrostatic potential of the conductor is

1. largest at the center.
2. largest on the surface.
3. largest somewhere between center and surface.
4. constant throughout the volume.

Concept test

Consider two isolated spherical conductors each having net charge Q. The spheres have radii a and b, where $b > a$. Which sphere has the higher potential?

1. the sphere of radius a
2. the sphere of radius b
3. They have the same potential.

Capacitance

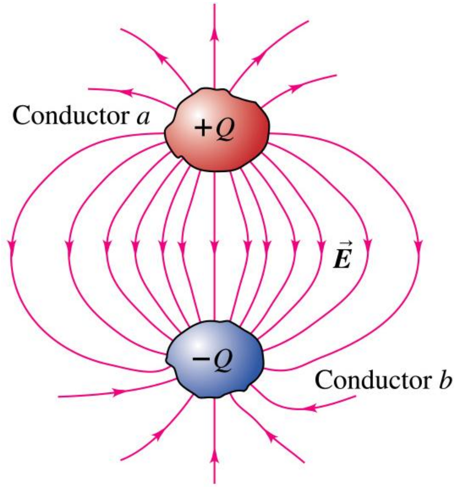
Read Chap 24

Capacitance

- **Capacitor:**
  - A **capacitor** is a device that stores electric potential energy.
  - Any two oppositely charged conductors insulated from each other form a capacitor.
  - At any point:

$$E \propto Q$$

$$\therefore V_{ab} \propto Q$$



  - The **capacitance** of a capacitor is defined as:

$$C = \frac{Q}{V_{ab}}$$

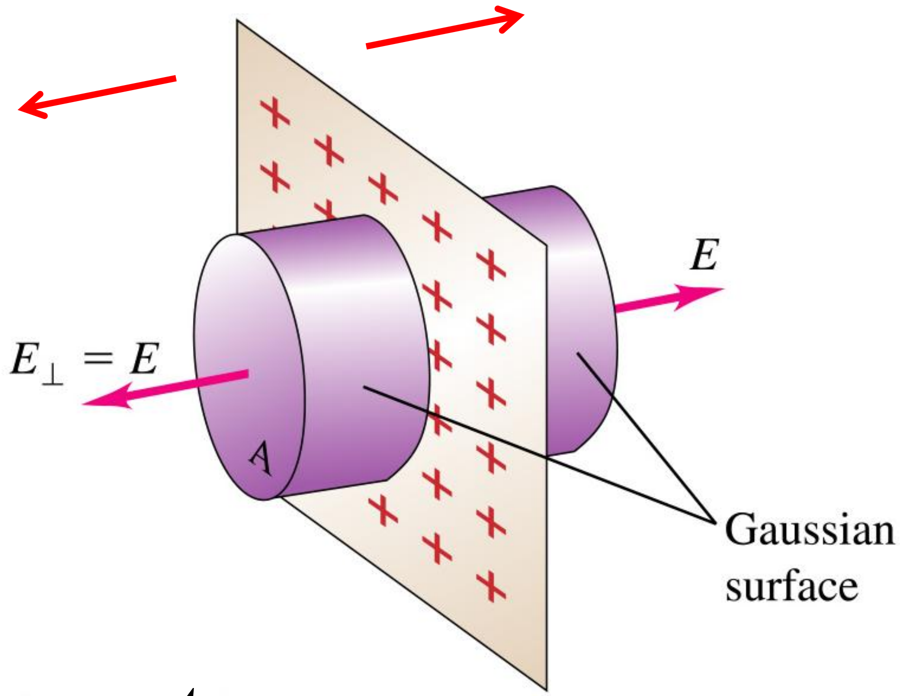
  - Units: farad (F)

$$F = \frac{C}{V} = \frac{C}{J/C} = \frac{C^2}{J} = \frac{C^2\, s^2}{kg\, m^2}$$

$$\Rightarrow \quad Q = CV_{ab}$$

Review: Gauss' law for infinite sheet

- **Field of an infinite charged sheet**:
  - Consider an infinite charged sheet with surface charge density σ.
  - The field must be directed away form the sheet by symmetry.
  - Construct a Gaussian cylinder around it as shown.
  - The flux integral simplifies to:



$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = 2EA = \frac{A\sigma}{\varepsilon_0}$$

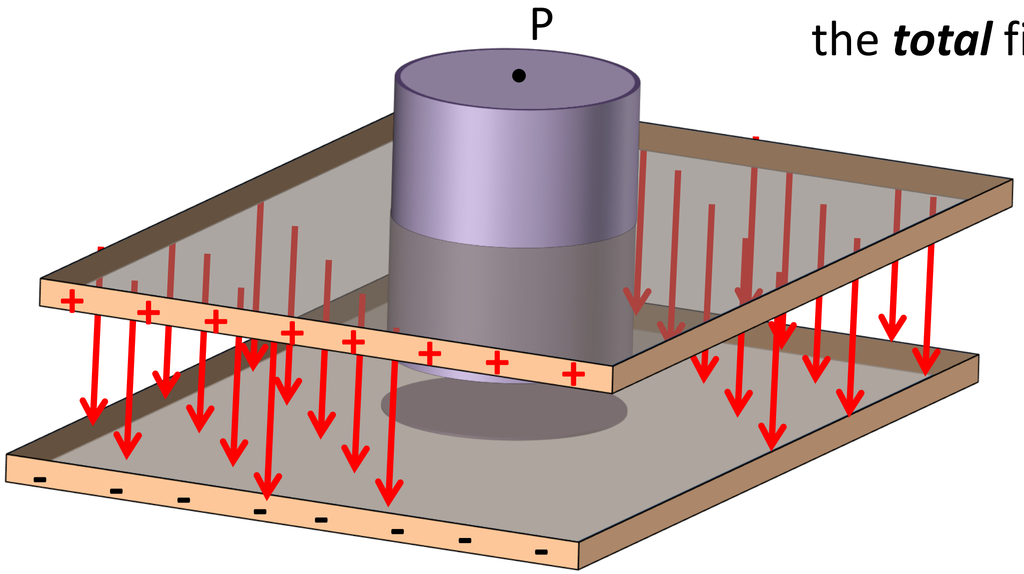
$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

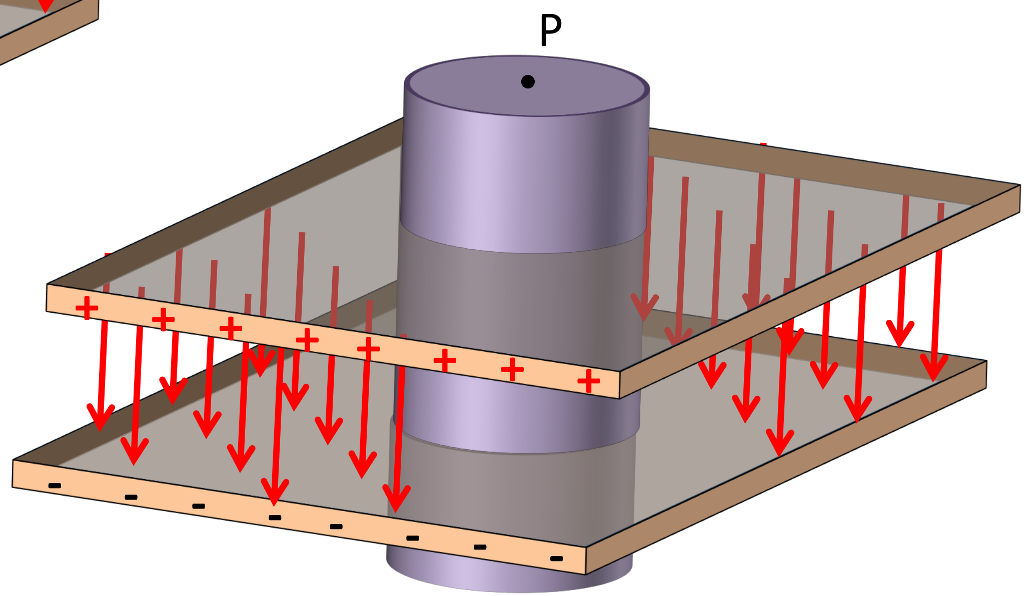
No dependence on position!

Review: Gaussian Surface Analysis of Field Outside of a Plate Capacitor

Field at P?

This Gaussian Surface gives no conclusion about the ***total*** field at P due to charge on both plates..



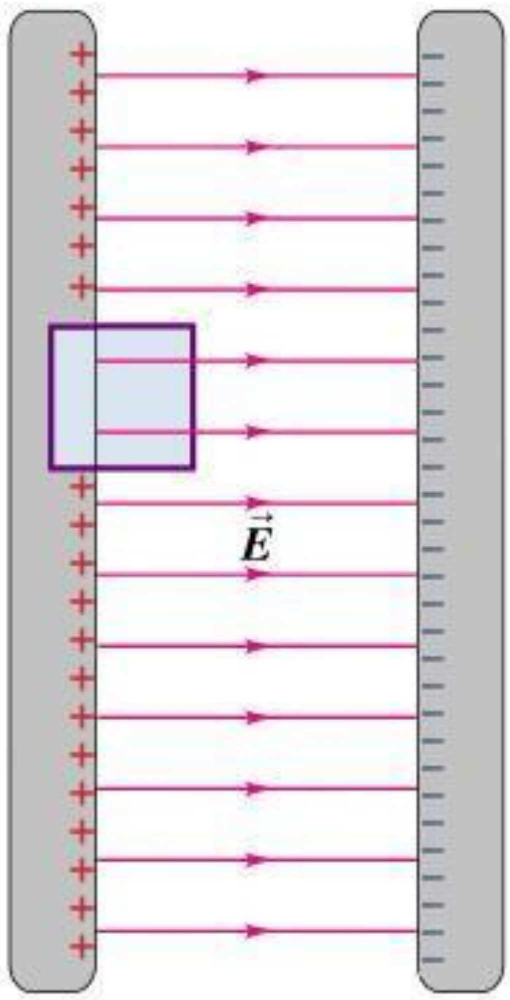


Field at P must be zero!

Review: Gaussian Surface for Field Inside a Plate Capacitor

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = EA = \frac{A\sigma}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$



Capacitance

Read Chap 24

Concept test

A solid spherical conductor is given a net nonzero charge. The electrostatic potential of the conductor is

1. largest at the center.
2. largest on the surface.
3. largest somewhere between center and surface.
4. constant throughout the volume.

Concept test

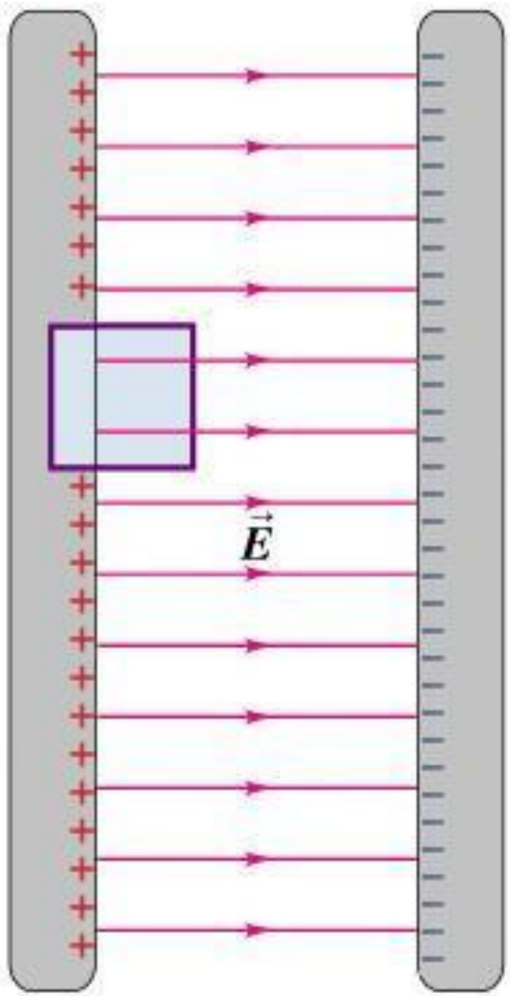
Consider two isolated spherical conductors each having net charge Q. The spheres have radii a and b, where $b > a$. Which sphere has the higher potential?

1. the sphere of radius a
2. the sphere of radius b
3. They have the same potential.

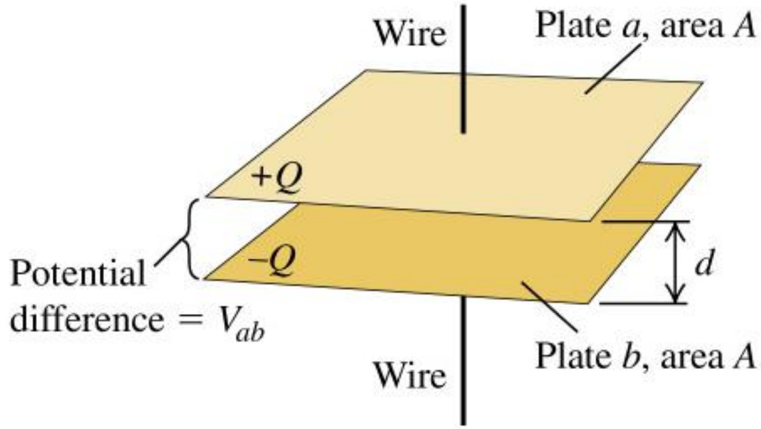
Review: Gaussian Surface for Field Inside a Plate Capacitor

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = EA = \frac{A\sigma}{\varepsilon_0}$$

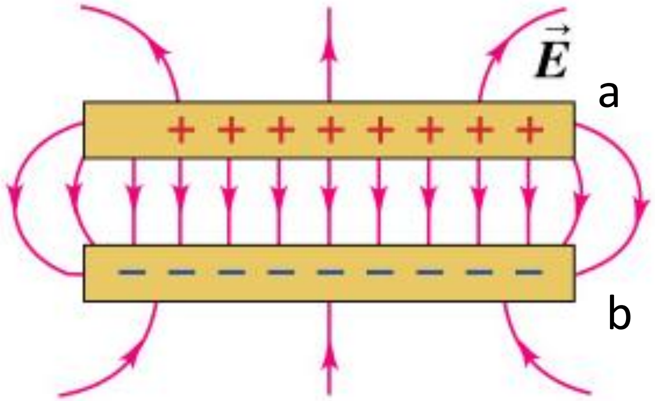
$$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$



- **Parallel plate capacitor**:
  - The simplest capacitor consists of two parallel plates.
  - The electric field is concentrated between the plates and the charge distributions are uniform
  - The uniform field between the plates is:



(eg. Gauss Law)



  - The voltage is:

$$V_{ab} = \int_a^b \vec{E} \bullet d\vec{r} = Ed = \frac{1}{\varepsilon_0}\frac{Qd}{A}$$

  - But:

$$C \equiv \frac{Q}{V_{ab}} = \frac{\varepsilon_0 A}{d}$$

24.1 Capacitance

- **Problem**:
  - Two parallel plates of area *2* m^2 are separated by *5.0 mm*.
  - A voltage of *10,000 V* is applied across them in air.
  - What is the
    - Capacitance of the plates,
    - The charge on each plate,
    - The electric field between them.

24. 1 Capacitance

- **Solution:**
  - Capacitance: $C = \varepsilon_0 \dfrac{A}{d} = 8.85\times 10^{-12}\,\dfrac{C^2}{Nm^2}\cdot\dfrac{2\,m^2}{0.005\,m}$

    $= 3.54\times 10^{-9}\,\dfrac{C^2}{N\,m} = 3.54\times 10^{-9}\,\dfrac{C}{J\,/\,C} = 3.54\,nF$
  - Charge: $Q = CV_{ab} = 35.4\mu C$
  - Field: $E = \dfrac{Q}{\varepsilon_0 A} = 2\times 10^{6}\,NC^{-1}$
  - Or: $E = \dfrac{V_{ab}}{d} = 2\times 10^{6}\,Vm^{-1}$

24.3 Stored energy in capacitor

- **Stored energy**:
  - Energy stored by a capacitor is equal to the work done in charging it.
  - Let q be the charge and V the potential difference at an intermediate stage in the charging process.
  - At this stage, the work required to transfer an additional element of charge dq is:

$$dW = V dq = \frac{q dq}{C}$$

  - The total work done in charging is then:

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

  - If the potential energy of an uncharged capacitor is zero then the potential energy of a capacitor is given by:

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$ Using $Q = CV$

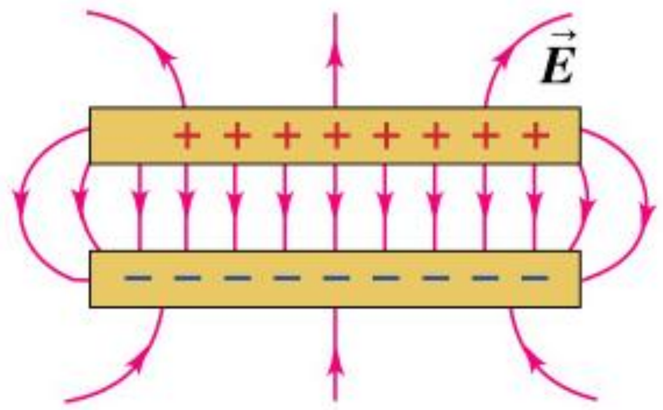
24.3 Energy storage in plate capacitor

- **Energy density in E-field**
  - The energy is stored (mostly) in the space between the plates.
  - The volume of this space is Ad.
  - The total energy is: $U = \frac{1}{2}CV^2$
  - So the **energy density** in the gap is: $u = \frac{CV^2}{2Ad}$
  - The capacitance is given by: $C = \varepsilon_0 \frac{A}{d}$
  - And the potential difference: $V = Ed$
  - Using these we obtain:

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Note all geometric terms drop out!

  - It turns out this is valid for ***any*** E-field in a vacuum.



24. 4 Dielectrics in Capacitors

- **Dielectrics**:
  - A solid insulator is placed in the gap between the plates of capacitors.
  - This material is called a solid dielectric.
  - It serves several purposes:
    - Prevent plates from touching (simple mechanical support)
    - For all materials there is a voltage at which they start to conduct: the **dielectric strength**.
    - For many materials the dielectric strength is higher than that of air. This allows capacitors to hold more energy and charge.
    - Capacitance is generally increased on the insertion of a dielectric (**most important**).

24.4 Dielectric in a Capacitor

- **Dielectric constant**:
  - The capacitance of a vacuum gap cap is given by:

$$C_0 = \frac{Q}{V_0}$$

  - On the insertion of the dielectric as shown, the voltage is observed to drop. $V < V_0$
  - The charge remains the same, so the capacitance is now increased to:

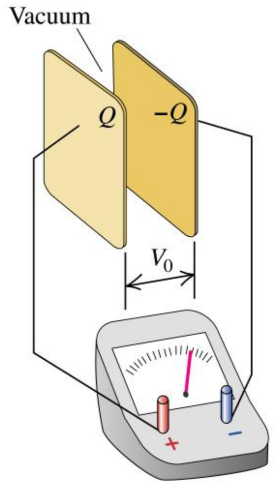
$$C = \frac{Q}{V}$$

  - Note that if the voltage is reduced by a factor *K* the field must be too:

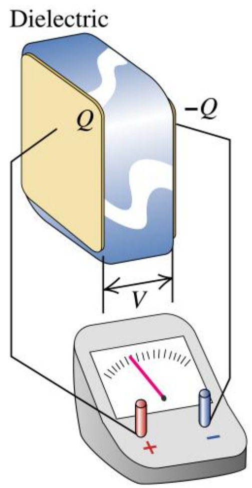
$$E = \frac{E_0}{K}$$

  - The **dielectric constant** K of this material is defined to be:

$$K = \frac{C}{C_0}$$



Voltmeter measures V_0

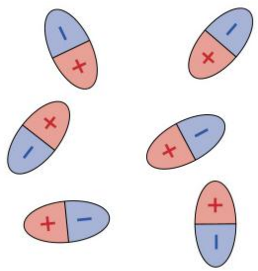


Voltmeter now measures

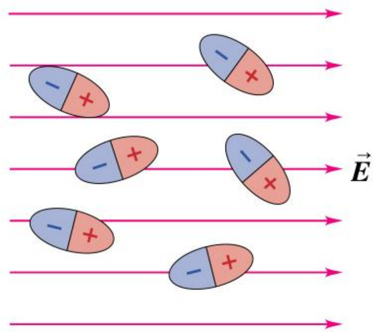
$$V < V_0$$

24.4 Polarization

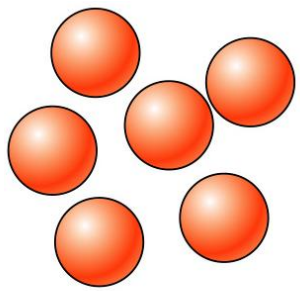
- **Polarization:**
  - Redistribution of positive and negative charges within a dieletric material

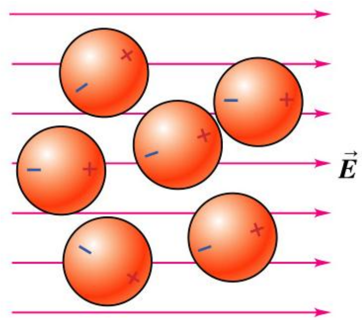


  - Many molecules are natural electric dipoles.

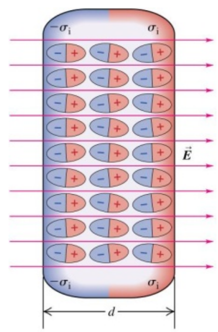


  - When an electric field is applied to these molecules they align in the direction of the field.





  - Furthermore, materials with nonpolar molecules can be polarized by an electric field, ie. the process of **induction**



  - Dielectrics in capacitors become polarized.

24.1 Induced surface charge

- **Polarization of dielectric**:
  - Without the dielectric in place the field strength is:

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

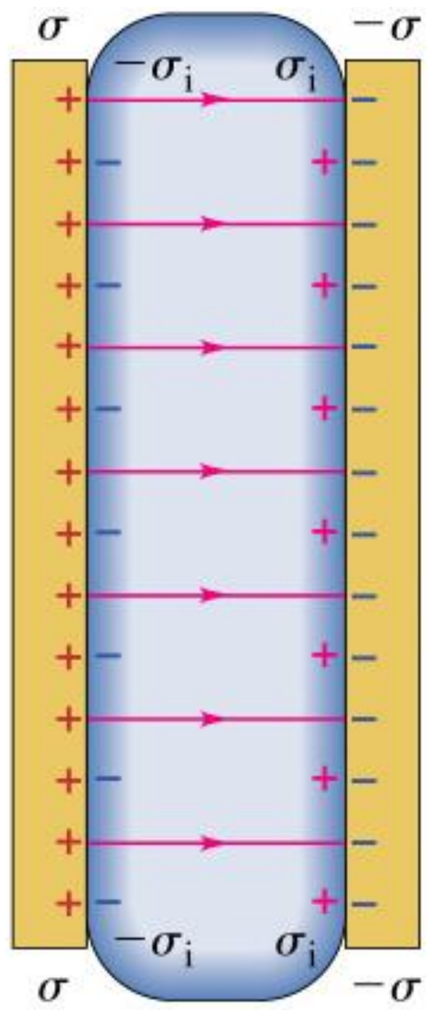
  - The polarization within the dielectric induces an opposite surface charge σ_i at each interface, reducing the total electric field:

$$E = E_0 - E_i = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

  - Combining expressions from here and slide 114:

$$E = \frac{E_0}{K} = \frac{\sigma}{K\varepsilon_0} = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

$$\Rightarrow \quad \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$



24.4 Dielectrics

Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Dielectric Constant, K	Dielectric Strength, E_{max} (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

24.4 Permittivity

- **Permittivity:**
  - The **permittivity of a substance** is defined to be:
  $$\varepsilon = K\varepsilon_0$$
  - Units: *F/m*

  - The electric field in the dielectric becomes: $$E = \frac{\sigma}{\varepsilon}$$
  - The capacitance becomes: $$C = KC_0 = K\varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$
  - The energy density becomes: $$u = \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2$$

Capacitance, Electric Current, and Resistance

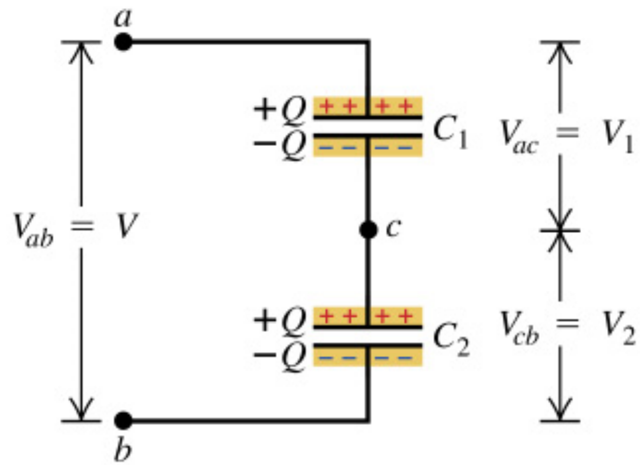
Read Chap 24

24. 2 Combining capacitors

In series

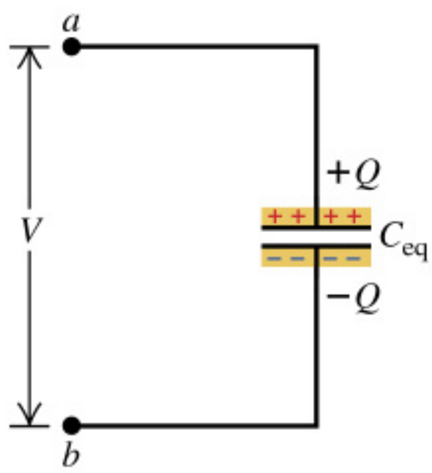
$$V_{ab} = V = V_{ac} + V_{cb} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2}$$



(a)

$$Q = Q_1 = Q_2$$



(b)

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}$$

Extend to any number of capacitors in series

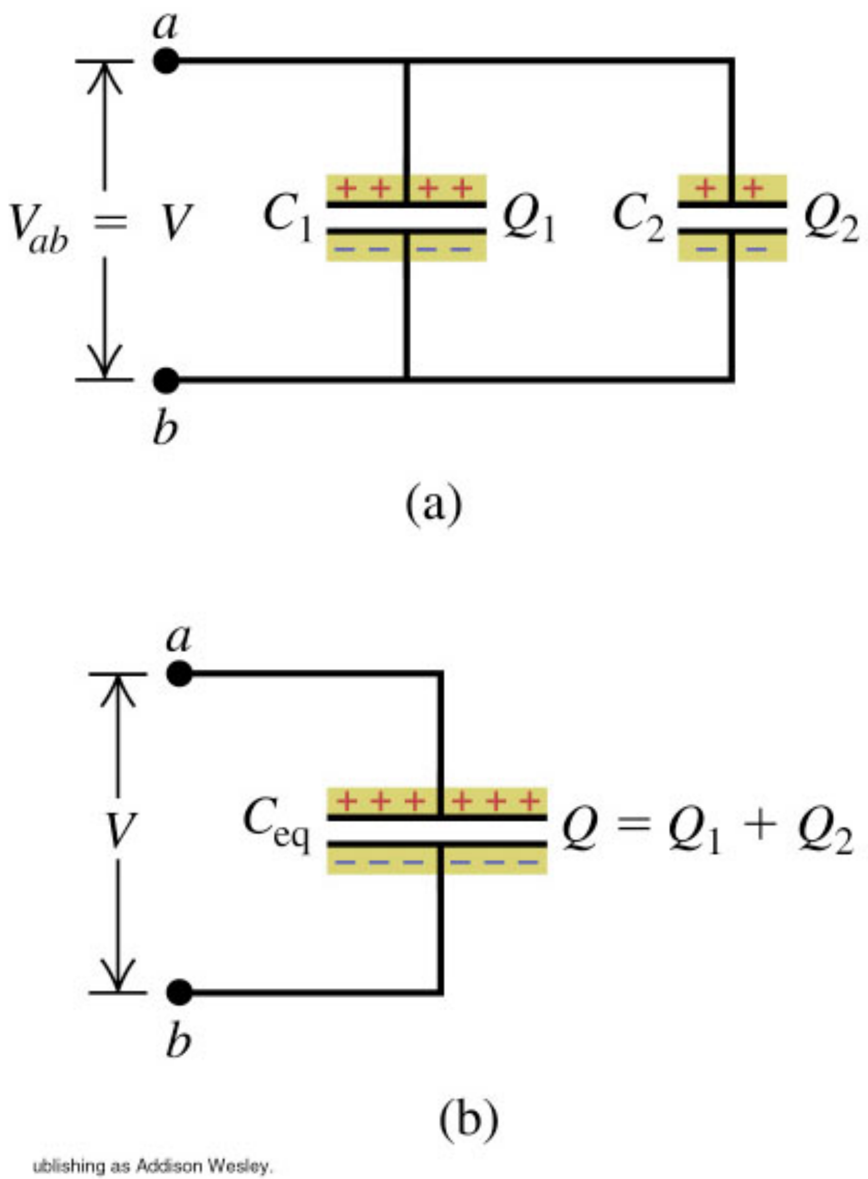
24. 2 Combining capacitors

In parallel

$$Q = Q_1 + Q_2 = VC_1 + VC_2$$
$$= V\left(C_1 + C_2\right)$$

$$\frac{Q}{V} = C_{eq.} = C_1 + C_2$$

Extend to any number of capacitors in series



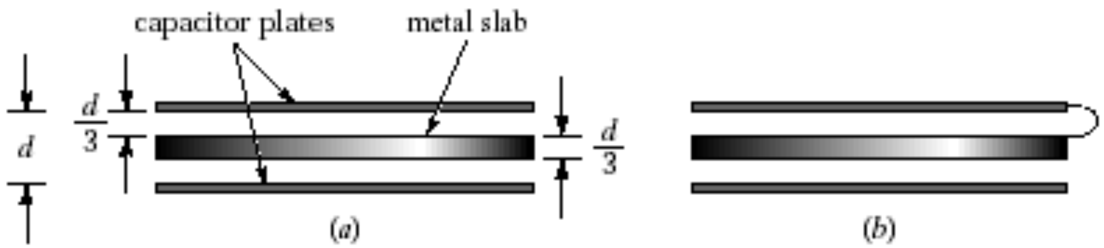
Concept test

Consider a capacitor made of two parallel metallic plates separated by a distance d. The top plate has a surface charge density $+\sigma$, the bottom plate $-\sigma$. A slab of metal of thickness $l < d$ is inserted between the plates, not connected to either one. Upon insertion of the metal slab, the potential difference between the plates

1. increases.
2. decreases.
3. remains the same.

Concept test

Consider two capacitors, each having plate separation d. In each case, a slab of metal of thickness $d/3$ is inserted between the plates. In case (a), the slab is not connected to either plate. In case (b), it is connected to the upper plate. The capacitance is higher for



1. case (a).
2. case (b).
3. The two capacitances are equal.

Concept test

Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d. Suppose the plates are pulled apart until they are separated by a distance $D > d$. The electrostatic energy stored in the capacitor is

1. greater than
2. the same as
3. smaller than

before the plates were pulled apart.

Concept test

A dielectric is inserted between the plates of a capacitor. The system is then charged and the dielectric is removed. The electrostatic energy stored in the capacitor is

1. greater than
2. the same as
3. smaller than

it would have been if the dielectric were left in place.

Concept test

A parallel-plate capacitor is attached to a battery that maintains a constant potential difference V between the plates. While the battery is still connected, a glass slab is inserted so as to just fill the space between the plates. The stored energy

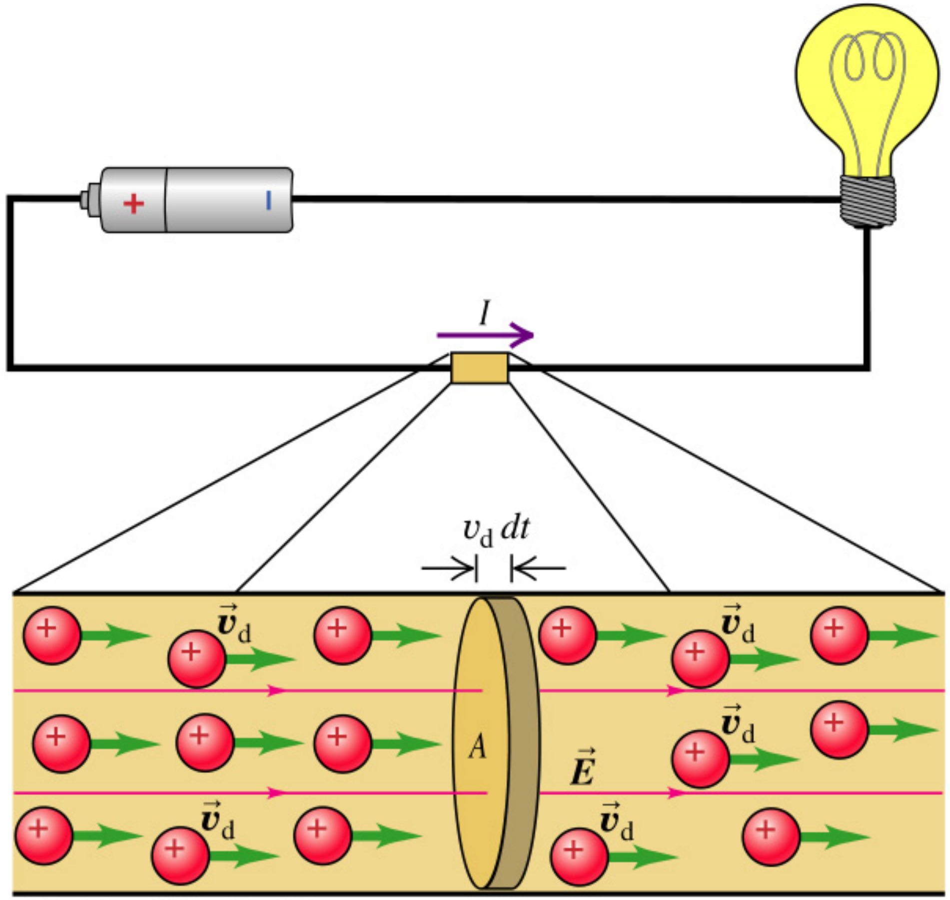
1. increases.
2. decreases.
3. remains the same.

25.1 Electric current

– Electric current is motion of charge from one region to another

– When current moves around a closed loop the path is called an electric circuit.

– Electric field is required to maintain a current



4 Pearson Education, Inc., publishing as Addison Wesley.

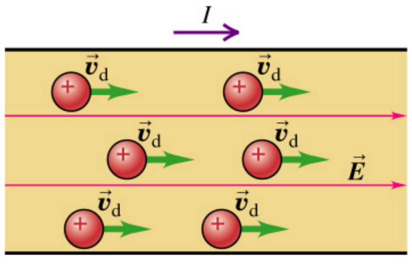
25.1 Electric current

- The electric field does work on free **charge carriers**.
- Most of this energy is converted to heat in their collisions with atoms.
- In metals, the charge carriers are electrons.
- In semiconductors charge carriers are electrons and “holes”, positively charged vacancies in a sea of electrons.

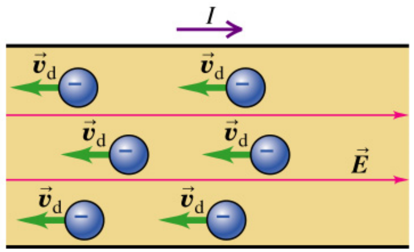
- The **electric current** through an area is defined by:

$$I = \frac{dQ}{dt}$$

- Units: ampere (*A*).
- Note I is defined in direction that positive charge would move
- Electrons move in opposite direction to I



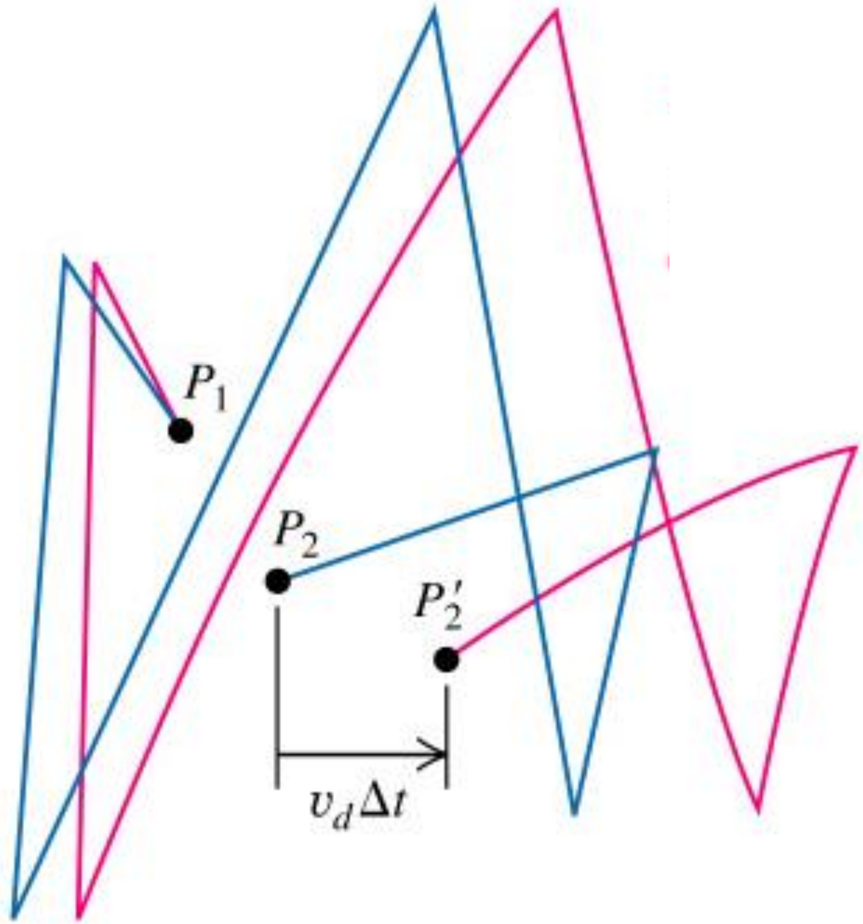
(a)



(b)

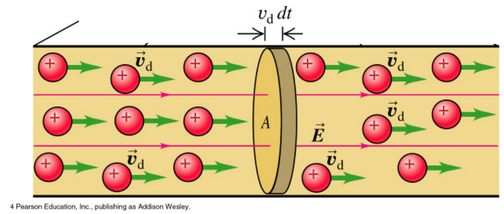
- **Electric current**
  - Electric field is required to maintain a current
  - Free electrons in a **conductor** have rapid random motion (~10^6 m s^{-1}) and undergo many collisions with the ion cores
  - The electric field adds a general **drift** to this motion.
  - This causes an overall movement of charge at drift speed:

$$v_d$$



25.1 Current Density

- **Current density**:
  - Consider a current with a density of n (free) particles of charge q per m^3 moving with a drift velocity v_d through an area A.
  - In time dt, a charge $Q = n|q||v_d|A\,dt$ moves by.



  - The current is given by:

$$I = \frac{dQ}{dt} = n|q||v_d|A$$

NB: Current IS NOT defined as a vector quantity!

  - The **current density** at a point in space is defined by:

$$J = \frac{I}{A}$$

  - Units: A/m^2

  - Also given by: $J = \frac{I}{A} = n|q||v_d|$ (magnitude)
  - In general, current density **is** a vector: $\vec{J} = nq\vec{v}_d$ $\vec{J}$ always points with $\vec{E}$

- **Problem**:
  - A $1.02\ mm$ diameter wire carries $1.67\ A$ of current to a $200\ W$ lamp.
  - The density n of free electrons is 8.5×10^{28} electrons per cubic metre.
  - Find the magnitudes of
    - The current density
    - The drift velocity

25.1 Electric current

- **Solution:**
  - The cross sectional area is: $A = \pi \left(\frac{d}{2} \right)^2 = \frac{\pi \left(1.02 \times 10^{-3} m \right)^2}{4} = 8.17 \times 10^{-7} m^2$
  - Current density: $J = \frac{I}{A} = \frac{1.67 A}{8.17 \times 10^{-7}} = 2.04 \times 10^6 Am^{-2}$
  - Drift velocity:

$$v_d = \frac{J}{n|q|} = \frac{2.04 \times 10^6 Am^{-2}}{\left(8.5 \times 10^{28} m^{-3} \right) \left| -1.60 \times 10^{-19} C \right|} = 1.5 \times 10^{-4} m\, s^{-1} = 0.15\, mm\, s^{-1}$$

Compare this to individual electron speed of > 10^6 m/s!

Resistance, electromotive force – emf ,DC circuits

Read Chap 25 and 26.1

Concept test

Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d. Suppose the plates are pulled apart until they are separated by a distance $D > d$. The electrostatic energy stored in the capacitor is

1. greater than
2. the same as
3. smaller than

before the plates were pulled apart.

25.2 Resistivity

- **Relating current density J to electric field E:**
  - Without electric field, typical electrons have a velocity $\vec{v}$, but $\left[\vec{v}\right]_{av} = 0$
  - If we add an electric field, the **electron** will also be **accelerated:**
  - If t is the time since the last collision with an atom $$\vec{v} = \vec{v}_0 + \vec{a}t = \vec{v}_0 - \frac{e\vec{E}t}{m}$$

    where $\vec{v}_0$ is the velocity immediately after the last collision (randomly distributed)
  - Thus: $$\vec{v}_d = \left[\vec{v}\right]_{av} = \left[\vec{v}_0 - \frac{e\vec{E}t}{m}\right]_{av} = \left[\vec{v}_0\right]_{av} - \left[\frac{e\vec{E}t}{m}\right]_{av} = 0 - \frac{e\vec{E}}{m}\left[t\right]_{av} = -\frac{e\tau}{m}\vec{E}$$

    where τ is the average time between collisions
  - And so the current density is: $$\vec{J} = \frac{ne^2\tau}{m}\vec{E}$$ since $\vec{J} = -ne\vec{v}_d$

    (again for case of electrons)

25.2 Ohm's Law and Resistivity

– Ohm's law: $J \propto E$ or $J = \dfrac{1}{\rho}E$

"Proportional"

– Where $\rho = \dfrac{E}{J} = \dfrac{m}{ne^2\tau}$ is the **resistivity** ($V A^{-1} m$), an intrinsic material property

– Shortly we will see: 1 $V A^{-1}$ = 1 ohm = $1\ \Omega$, thus unit of resistivity is the $\Omega\, m$

– Examples of resistivity:
  - Cu (RT): $1.7 \times 10^{-8}\ \Omega$ m metal
  - Glass: 10^{10} - $10^{14}\ \Omega$ m insulator

– Conductivity = $\dfrac{1}{\rho}$

Substance		$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

25.2 Resistance

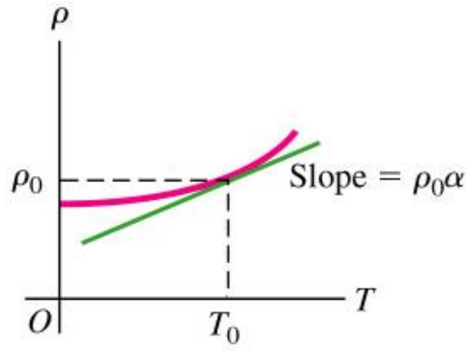
- Ohm:
  - George Simon Ohm was born in 1789 in Erlangen, Germany.
  - He was schooled at home by his father.
  - He was unrecognised as a valuable contributor to physics for most of his life.
  - He is remembered for his discovery of the proportionality between voltage and current.

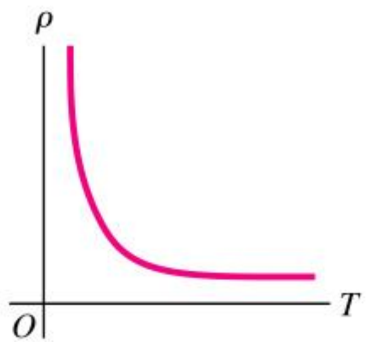
25.2 Resistivity

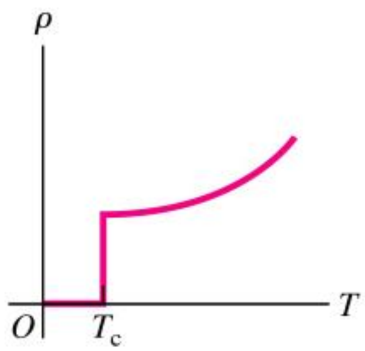
- Temperature dependence of resistivity
  - For most materials resistivity depends strongly on temperature.
  - The temperature dependence of the resistivity of metals is approximately described by:

$$\rho(T) = \rho_0\left[1+\alpha(T-T_0)\right]$$

  - The resistivity of semiconductors decreases with temperature.
  - Materials called superconductors display zero resistivity below a certain critical temperature.







25.3 Resistance

- Resistance:
  - Consider a conducting rod subject to a uniform electric field: $\vec{E} = \rho\vec{J}$
  - By definition: $J = \dfrac{I}{A}$

$$\Rightarrow \quad E = \frac{\rho I}{A}$$

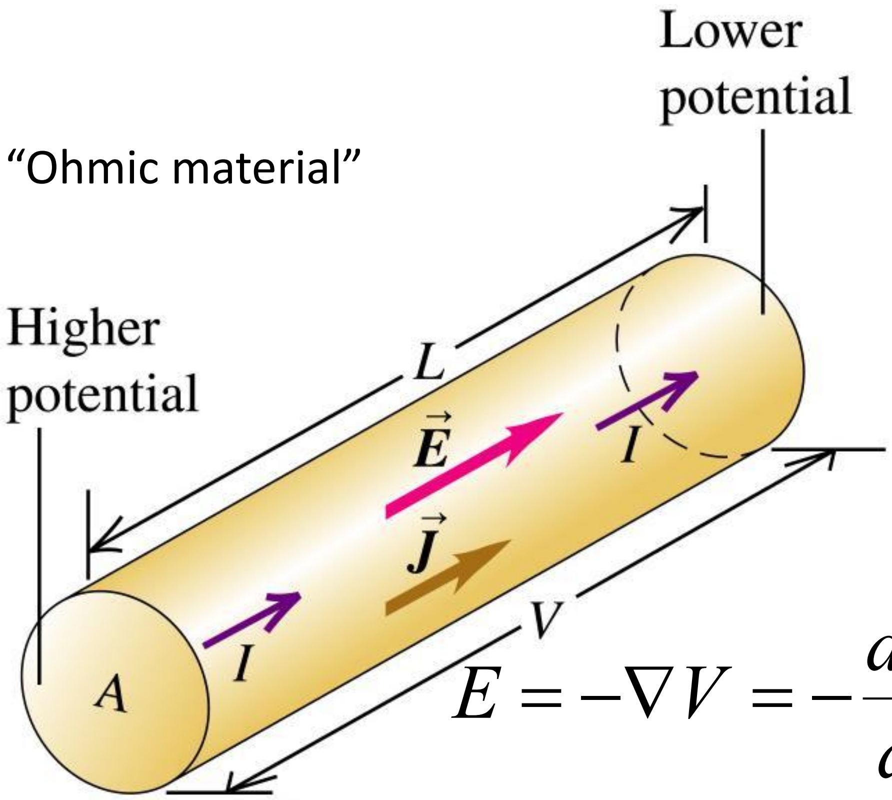
  - The potential difference between the ends is: $V = EL$

(why ?)

$$\Rightarrow \quad \frac{V}{L} = E = \frac{\rho I}{A}$$

$$V = \frac{\rho L}{A} I$$

"Ohmic material"



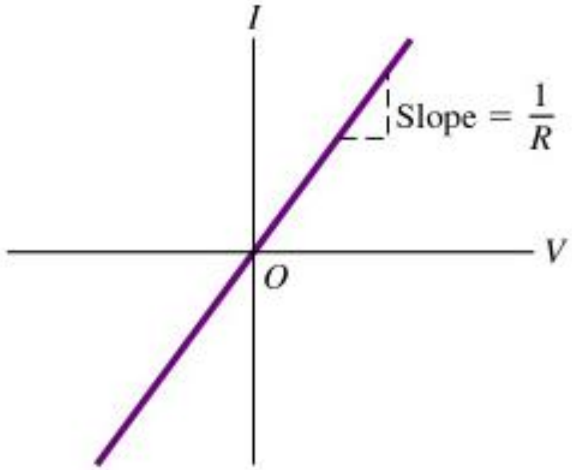
$$E = -\nabla V = -\frac{dV}{dx}$$

25.3 Resistance

- The resistance of a conducting body is defined by:

$$R = \frac{V}{I}$$

- Units: ohms (Ω), or Volts per Ampere (V/A)



- For our last example , given $V = \frac{\rho L}{A} I$
- Then resistance is given by: $R = \frac{\rho L}{A}$
- Current is given by: $I = \frac{1}{R} V$
- Because resisitivity is a function of temperature, so is resistance:

$$R(T) = R_0 \left(1 + \alpha (T - T_0)\right)$$

- A vacuum diode will only conduct above a certain voltage (not Ohmic).
- The resistance of semiconductors has a complex dependence on voltage.

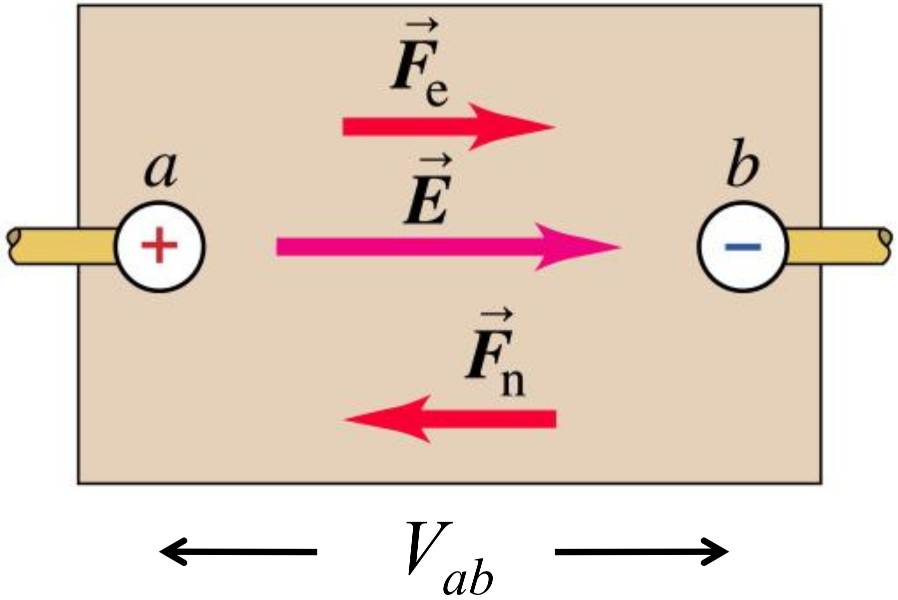
25.4 Electromotive force

- **Circuits:**
  - For a steady current to flow we need a **closed circuit**.
  - Charge moves in direction of decreasing potential.
  - Potential must be equal at start and end in closed loop.
  - So potential must increase *somewhere* in the circuit.
  - Something must force charge to move from lower to higher potential.
  - The thing that causes this effect is called an **electromotive force**.

25.4 Electromotive force

- Sources of emf:
  - A **source of emf** exerts an non-electric force against the electric force on any charge within it. Emf is characterized by a quantity ε measured in energy per unit charge (its not really force.)
  - The work done by the nonelectric force on a positive charge moving from **terminals** *b* to *a* is: $$W_n = q\varepsilon$$
  - The change in potential energy of the charge is: $$qV_{ab}$$
  - In an ideal source of emf the electric and nonelectric forces are just equal so the potential energy change is equal to the work done:

Battery, magnetic dynamo generator, van der Graff generator, fuel cell, solar cell, etc.



$$q\varepsilon = qV_{ab}$$
$$\Rightarrow \varepsilon = V_{ab}$$

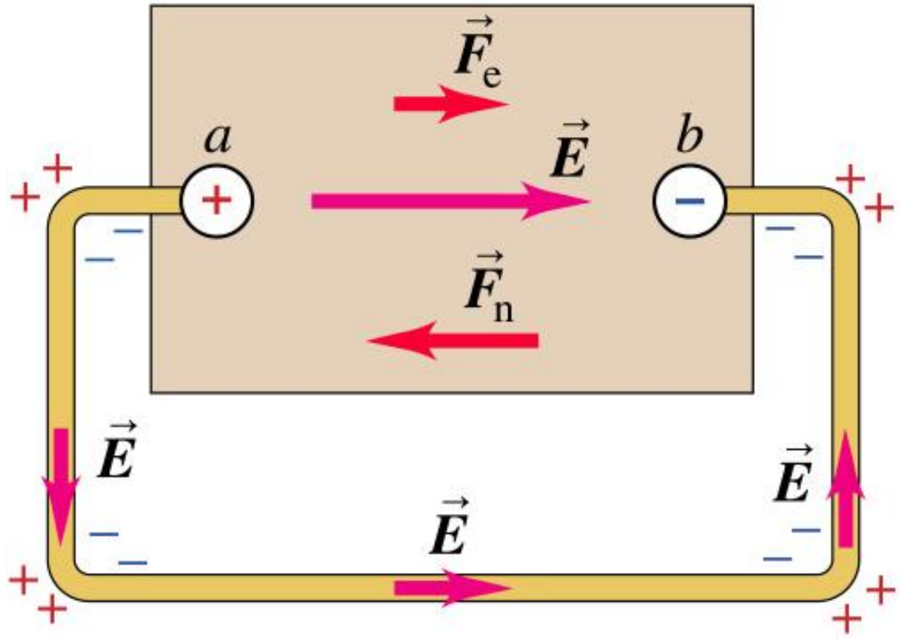
25.4 Electromotive force

– When the terminals of the source of emf are connected, current flows.

– The potential increase in the source must equal the potential drop in the circuit:

$$\varepsilon = V_{ab} = IR \qquad \text{(ideal)}$$

– The charge is forced to follow the wire by equal but opposite charges that persist on the "inside" and "outside" of bends.



Note that current is not used up in a circuit, its just recycled!

- **Internal resistance**:
  - On passing through a **real source of emf** (ie. not ideal), current must pass through a resistance.
  - For a battery, this is called the internal resistance r.
  - As the current moves through r it experiences a drop in potential difference Ir.
  - So when current flows across the terminals of a source the potential difference between the terminals is: $V_{ab} = \varepsilon - Ir$
  - Combining this with our previous result: $\varepsilon - Ir = IR$

$$\Rightarrow I = \frac{\varepsilon}{R + r}$$

Outline of Course Content

Electrostatics:

- Electric charge
- Coulomb’s law
- Electric field
- Electric dipoles
- Gauss’s law
- Electric potential energy
- Voltage
- Electric polarization
- Capacitance

Electricity:

- Current
- Resistance
- Ohm’s law
- Electromotive force
- Power in electric circuits
- Kirchoff’s laws
- RC circuits

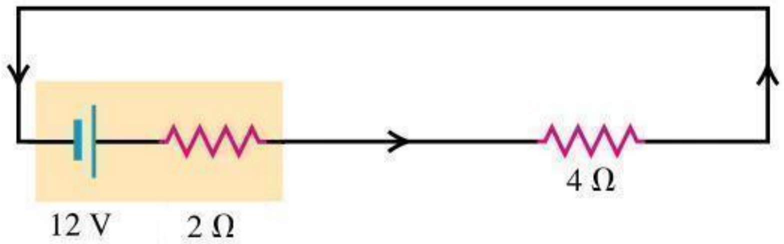
Electromagnetism:

- Magnetic pole
- Magnetostatic forces
- Magnetic fields
- Force on a moving charge
- Electric motors
- Magnetic torque
- Field due to a moving charge
- Force between conductors
- Ampere’s Law
- Magnetic materials
- Faraday’s law
- Lenz’s law

Circuits

Read sections:26.2 to 26.5

- Sample problem:
  - Consider the circuit diagram shown.
  - What
    - Is the current through the circuit?
    - The voltage at each point in the circuit?



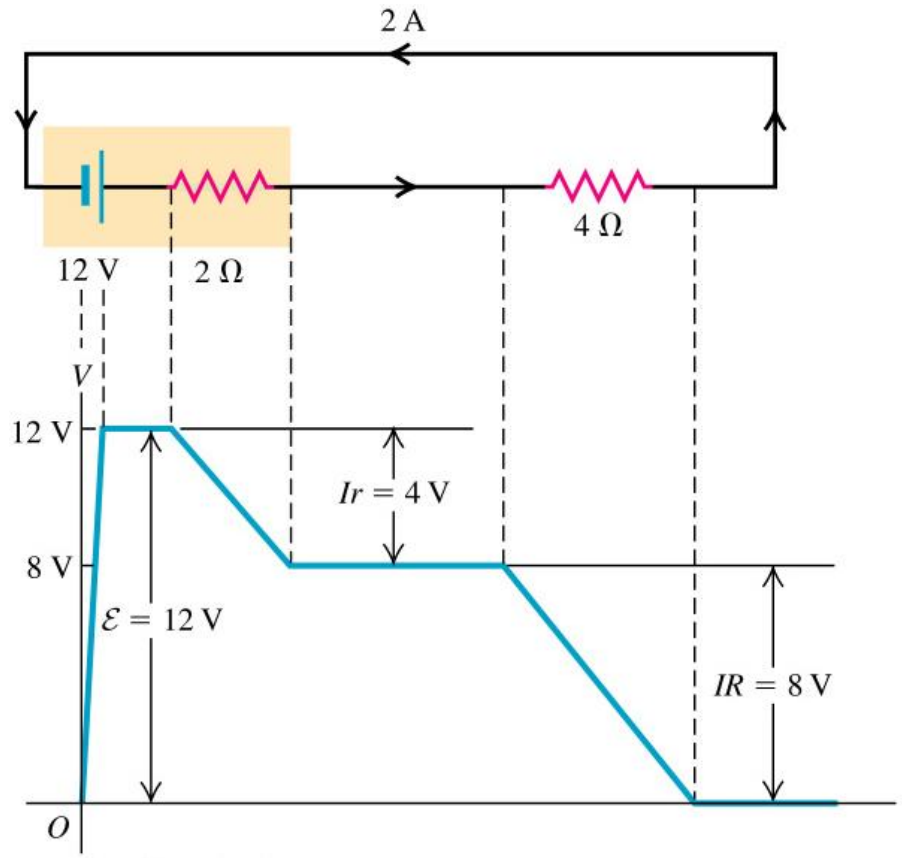
Real 12 V battery with internal resistance 2 Ω

25.4 Circuits

- Solution:
  - The current is given by:

$$I = \frac{\varepsilon}{R+r} = \frac{12V}{(4+2)\Omega} = 2A$$

  - The potential is zero at the negative terminal of the battery, say.
  - The potential rises due to the emf of the battery.
  - It falls because of the internal resistance.
  - The wire has negligible resistance so potential is constant.
  - Through the resistor the potential drops back to zero.



25.5 Energy and Power in Circuits

- **Energy and power**.
  - Consider any element of an electric circuit.
  - In a time interval *dt* the charge amount of charge passing through the circuit element is:

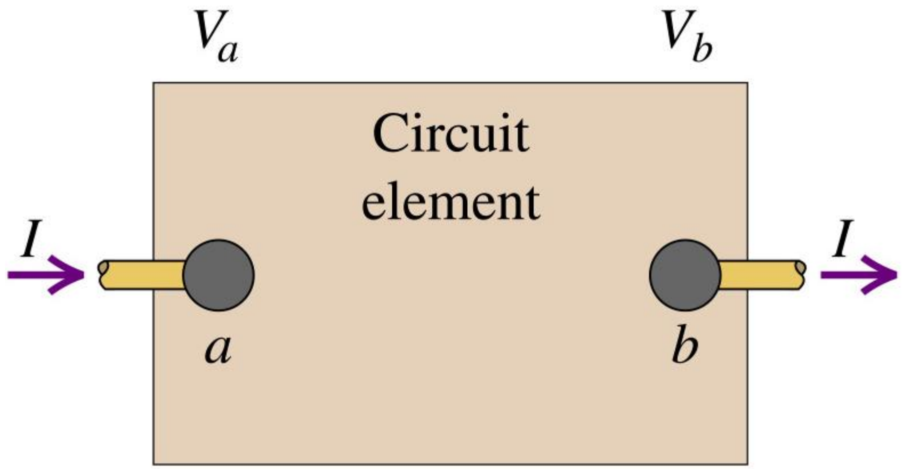
$$dQ = Idt$$

  - The **potential energy change** for this is:

$$V_{ab}dQ = V_{ab}Idt$$

  - So the **rate** at which energy is transferred into or out of the element (power) is:

$$P = V_{ab}I$$



25.5 Power

- **Resistor**:

  - If the element is an (Ohmic) resistor then the electrical power consumed by it is:

$$P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R} \qquad\qquad V = IR$$

  - This power is dissipated in the resistor as heat.

- **Output of source:**

  - If the element is a source of emf, the power delivered by it to the circuit is:

$$P = V_{ab} I = \left(\varepsilon - Ir\right) I = \varepsilon I - I^2 R$$

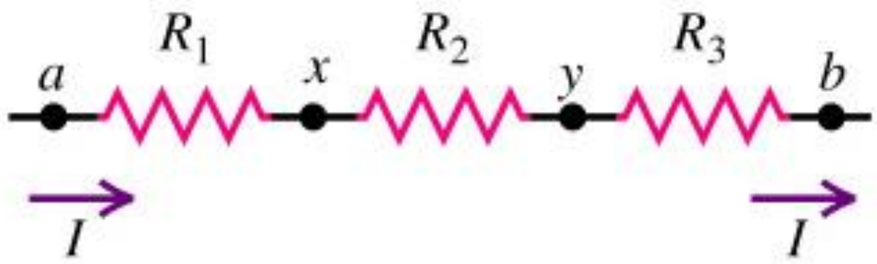
  - The first term here represents the power contributed by the nonelectric forces in the source.

  - The second term represents the power that dissipates in the internal resistance of the source.

26.1 Resistor Networks

- **Resistors in series**:

  – Components are said to be in series when they are connected in sequence as shown.



  – The voltages across the resistors are:

$$V_{ax} = IR_1 \quad V_{xy} = IR_2 \quad V_{yb} = IR_3$$

  – The total voltage is:

$$V_{ab} = V_{ax} + V_{xy} + V_{yb} = I\left(R_1 + R_2 + R_3\right)$$

  – So the resistance of the combined components is:

$$\frac{V_{ab}}{I} = R_1 + R_2 + R_3$$

  – The resistance of resistors combined in series is:

$$R = \sum_i R_i$$

- **Resistors in parallel**:

  – Components are said to in parallel when they are connected as shown.

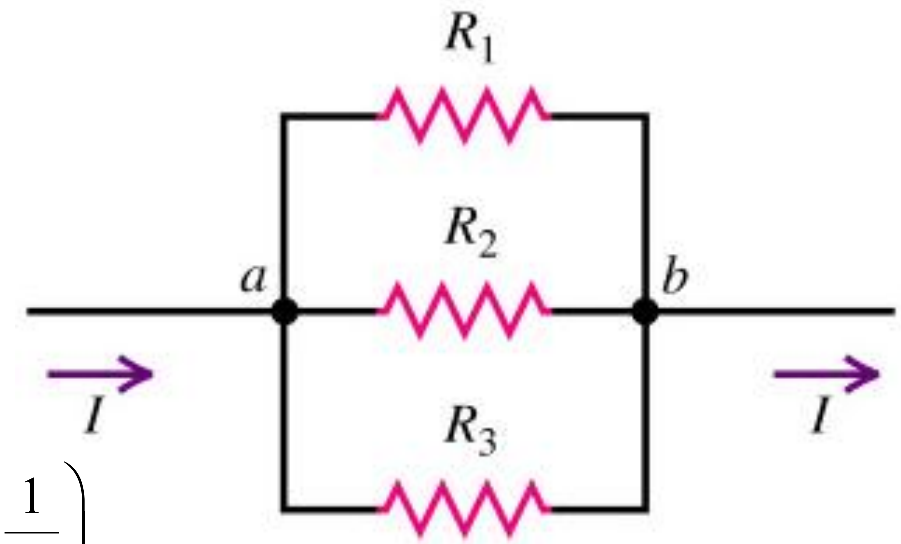
  – The currents through the resistors are:

  $$I_1 = \frac{V_{ab}}{R_1} \quad I_2 = \frac{V_{ab}}{R_2} \quad I_3 = \frac{V_{ab}}{R_3}$$

  – The total current through the combination is:

  $$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

  – The resistance of the combined coimponents is given by:



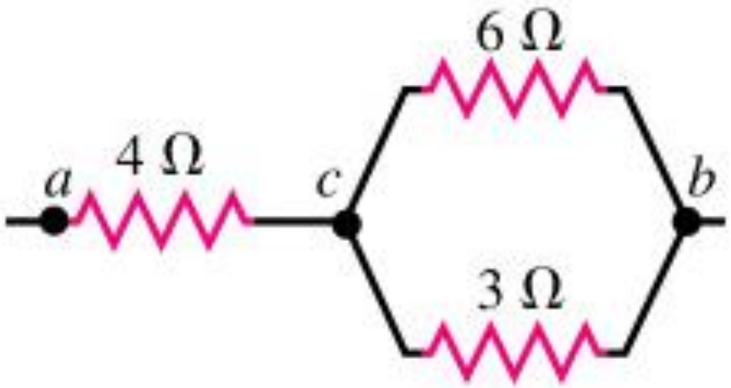
$$\frac{1}{R} = \frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

– The resistance of a number of resistors in parallel is given by:

$$\frac{1}{R} = \sum_i \frac{1}{R_i}$$

26.1 Resistor Networks

- Sample problem:
  - What is the equivalent resistance of the configuration shown below?

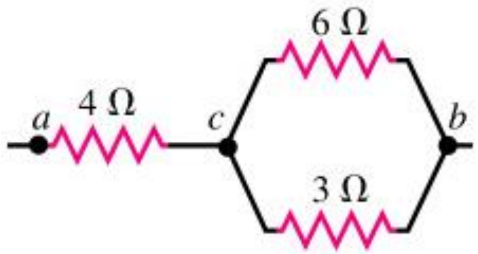


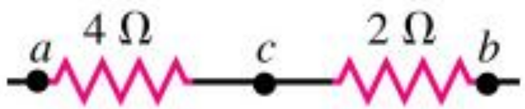
26.1 Resistor Networks

- Solution:

  - The resistance between *c* and *b* is obtained by:

$$\frac{1}{R} = \frac{1}{6\Omega} + \frac{1}{3\Omega} = \frac{1}{2\Omega}$$
$$\Rightarrow R = 2\Omega$$



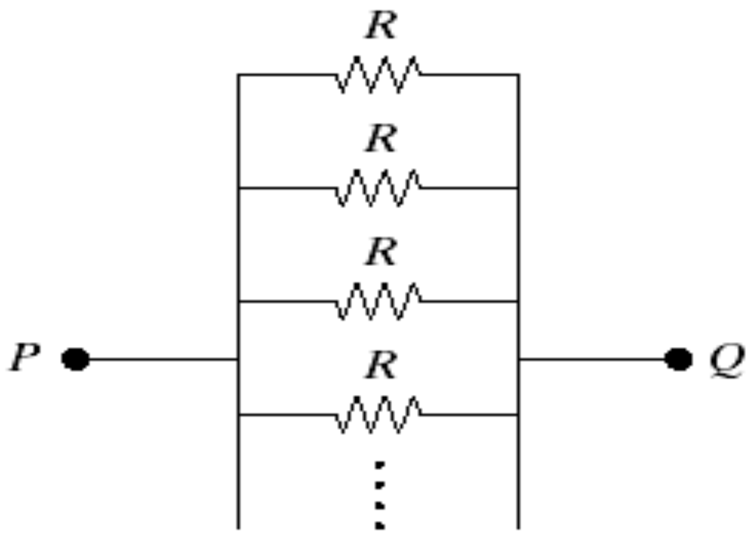


  - So the total resistance is:

$$R = 4\Omega + 2\Omega = 6\Omega$$

Concept test

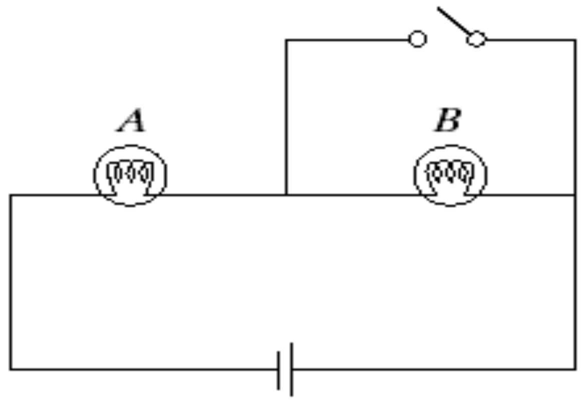
As more identical resistors R are added to the parallel circuit shown here, the total resistance between points P and Q



1. increases.
2. remains the same.
3. decreases.

Concept test

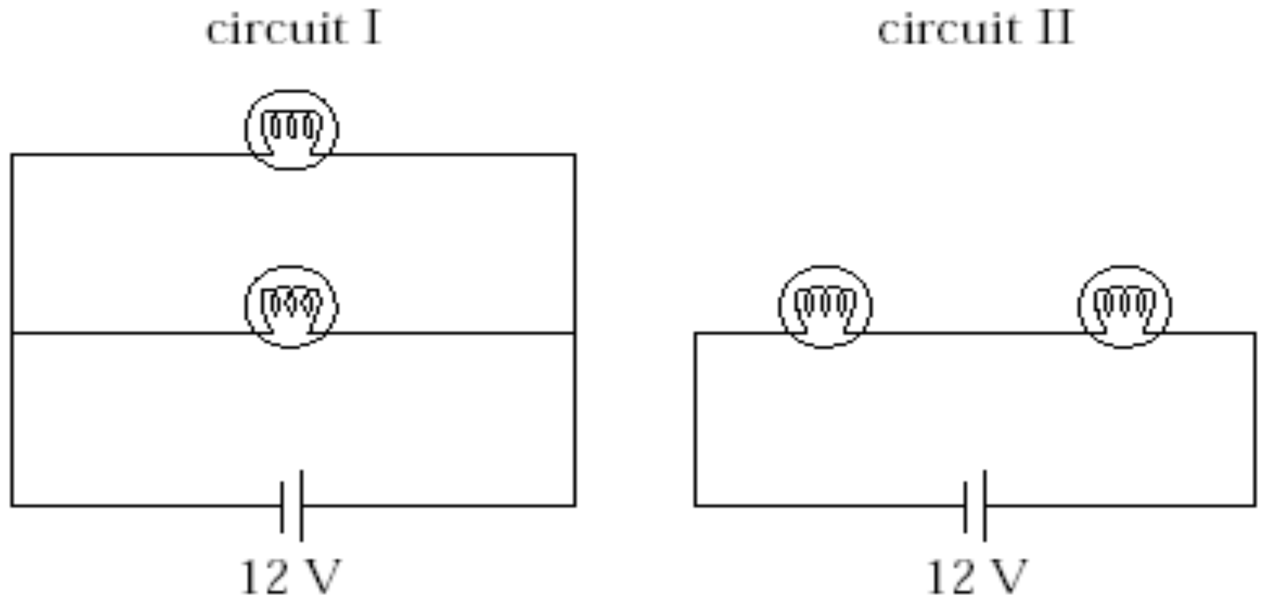
The circuit below consists of two identical light bulbs burning with equal brightness and a single 12 V battery. When the switch is closed, the brightness of bulb *A*



1. increases.
2. remains unchanged.
3. decreases.

Concept test

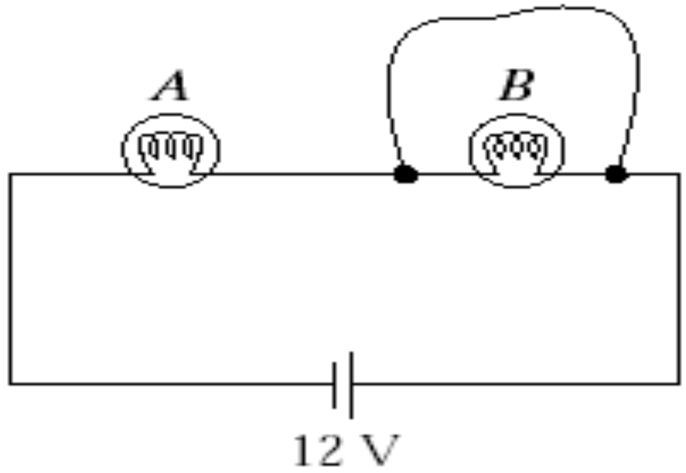
If the four light bulbs in the figure are identical, which circuit puts out more light?



1. I.
2. The two emit the same amount of light.
3. II.

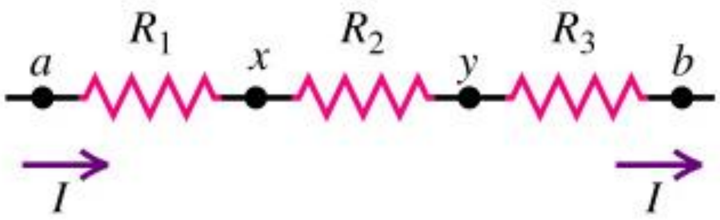
Concept test

Two light bulbs A and B are connected in series to a constant voltage source. When a wire is connected across B as shown, bulb A

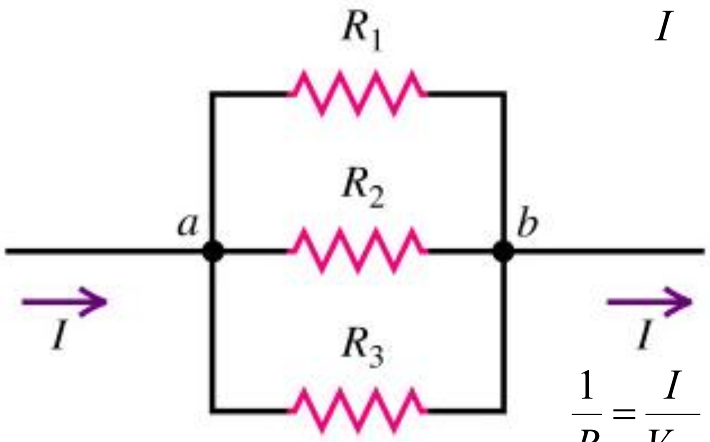


1. burns more brightly.
2. burns as brightly.
3. burns more dimly.
4. goes out.

26.2 Circuits



$$\frac{V_{ab}}{I} = R_1 + R_2 + R_3$$



$$\frac{1}{R} = \frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

– The resistance of resistors combined in series is:

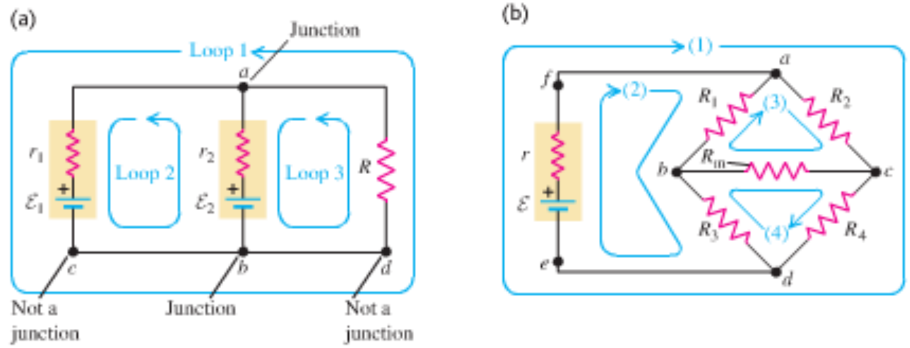
$$R = \sum_i R_i$$

– The resistance of a number of resistors in parallel is given by:

$$\frac{1}{R} = \sum_i \frac{1}{R_i}$$

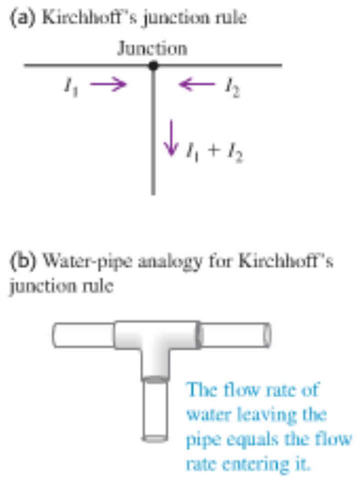
What about more complex circuits?

These cannot be reduced to simple series or parallel equivalent networks:



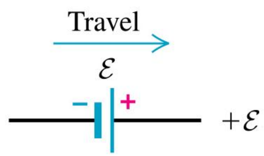
26.2 Kirchoff's Rules for Complex Circuits

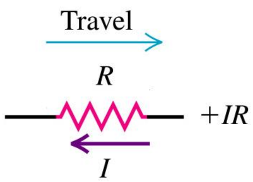
- **Kirchhoff's junction rule**: $$\sum I = 0$$
  - The algebraic sum of the currents into any junction is zero.
  - This is an expression of the principle of conservation of charge.



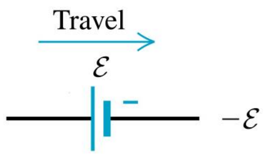
- **Kirchhoff's loop rule**: $$\sum V = 0$$
  - The algebraic sum of the potential differences in any loop, must equal zero.
  - This expresses the fact that the electrostatic force is conservative.

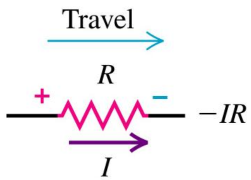
- Sign convention:
  - To execute Kirchhoff's **loop rule** we must add the voltages in a circuit in a certain order and give each a correct sign.
  - We add the voltages in the order that they appear on traveling around the loop either clockwise or anticlockwise.
  - If we go from a lower to a higher potential the difference is given a positive sign:





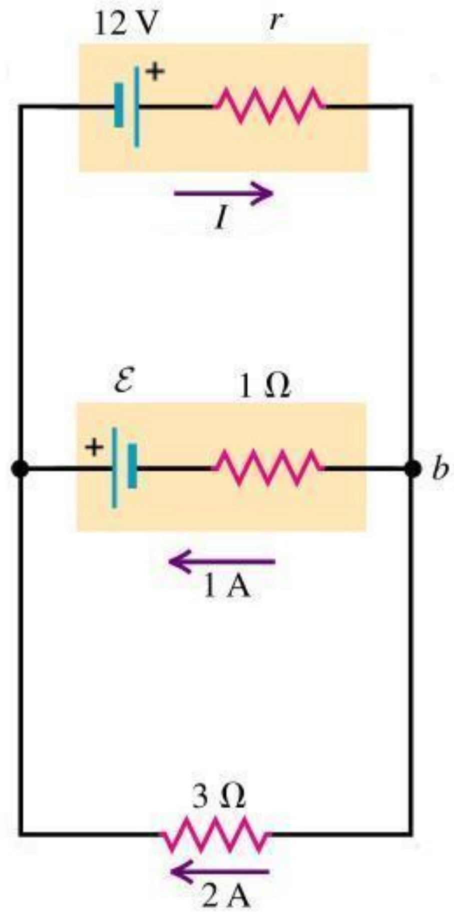
  - If we go from a higher to a lower potential, it is given a negative sign:





26.2 Circuits

- Sample problem:
  - Consider the circuit shown.
  - Find
    - The unknown current *I*,
    - The internal resistance *r*,
    - The emf ε.



- Solution:
  - First we apply the junction rule at *b*:

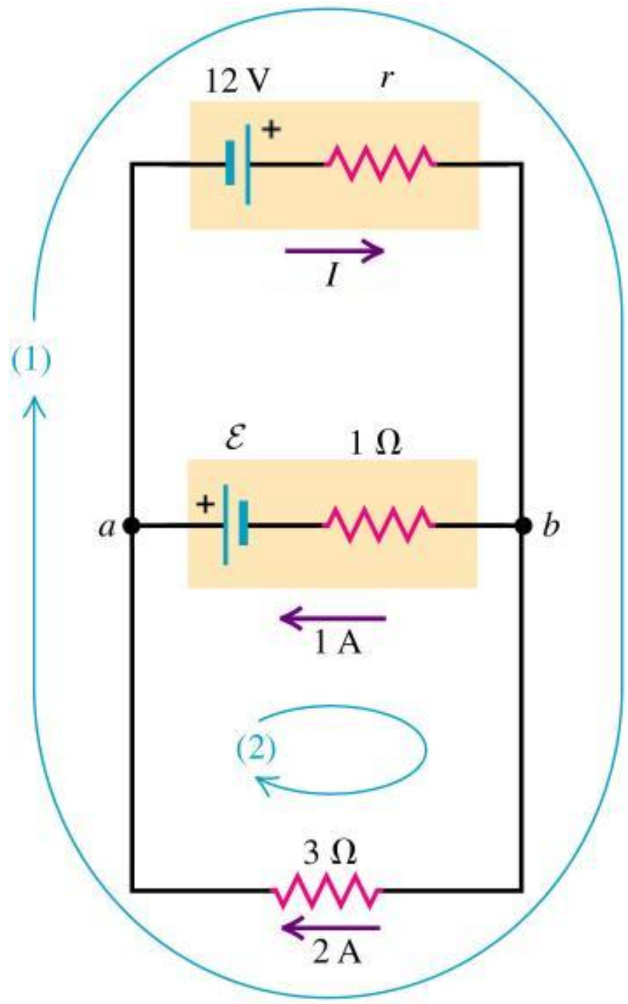
$$I - 1A - 2A = 0$$
$$\Rightarrow I = 3A$$

  - Then, the loop rule to the outside loop:

$$12V - (3A)r - (2A)(3\Omega) = 0$$
$$\Rightarrow r = 2\Omega$$

  - Lastly, the bottom loop:

$$-\varepsilon + (1A)(1\Omega) - (2A)(3\Omega) = 0$$
$$\Rightarrow \varepsilon = -5V$$

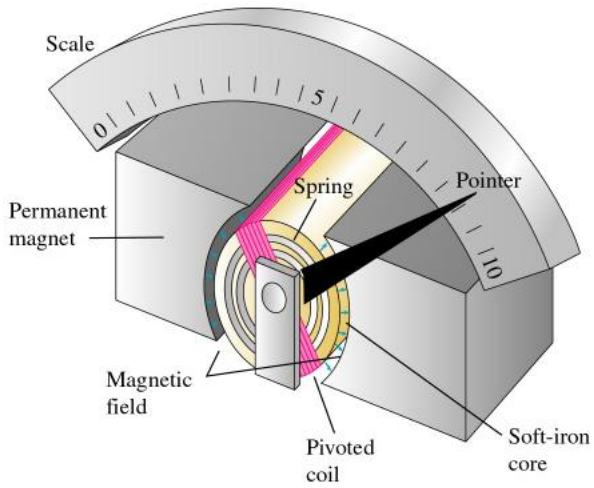


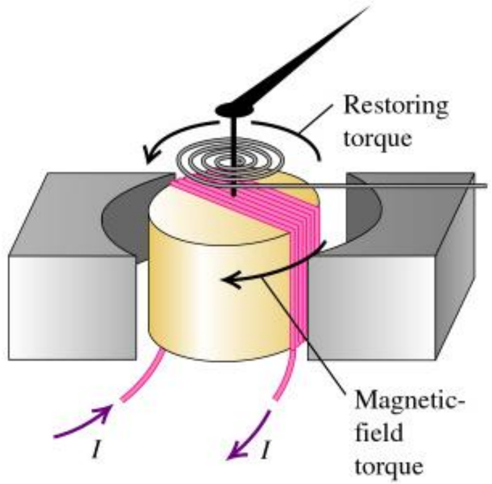
Note:

- The direction you decide to travel in your loop may not always point with the current
- For unknown currents, we simply assume a direction to start and see what sign we get at the end. In the above example, if we had found I to be -3A, then it would the flow in the opposite direction to that we originally indicated in the diagram

26.3 Electrical Measuring Equipment

- D'Arsonval Galvanometer:
  - This is a device used for measuring small currents and voltages.
  - A current flows through the coil.
  - The magnetic field exerts a force on the coil that is proportional to **current**.
  - The spring exerts a force that is proportional to the angular displacement.
  - The pointer indicates the current.
  - If the coil obeys Ohm's law the deflection is also proportional to the **potential difference**.



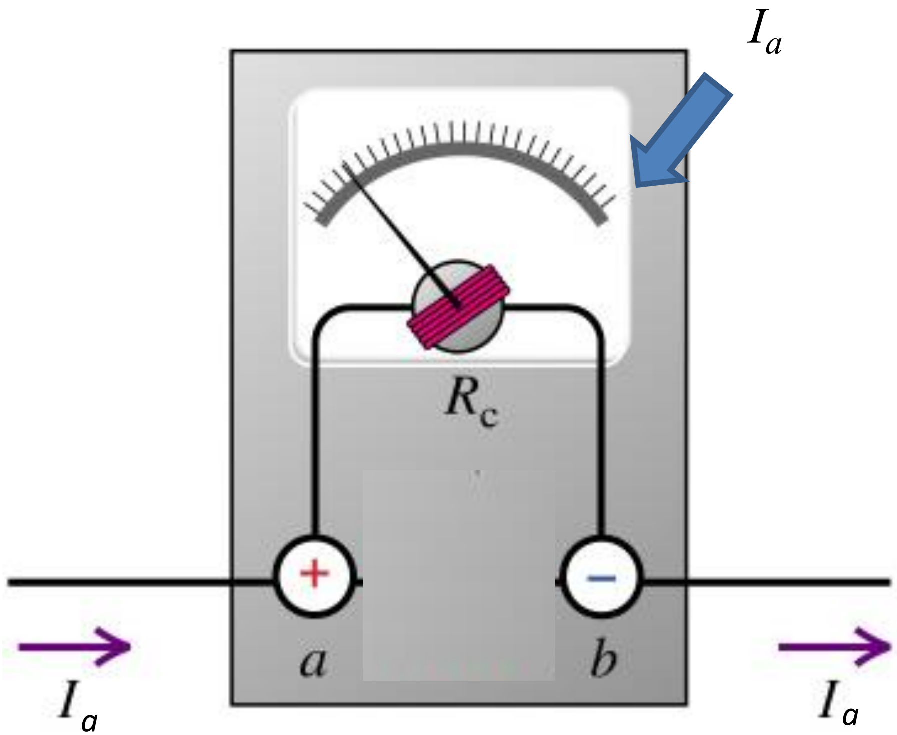


Circuits, Magnetism, magnetic force

Read 27.1 to 27.5

26.3 Electrical Measuring Equipment

- Ammeter:
  - Ammeters measure current.
  - They are connected in series. Ideally they have zero resistance. In practice a coil resistance of R_c gives a full-scale deflection for current I_{fs}.
  - We can adapt an ammeter to measure currents outside its range by combining it in parallel with a shunt resistor R_{sh}.
  - By shunting, we wish to achieve a full-scale current I_a given the ammeter with coil resistance R_c and unshunted full-scale reading of I_{fs}.
  - Parallel wiring means the potential drop is the same for both paths so:



$$I_{fs}R_c = \left(I_a - I_{fs}\right)R_{sh} \quad \Rightarrow \quad R_{sh} = \frac{I_{fs}R_c}{\left(I_a - I_{fs}\right)}$$

26.3 Electrical Measuring Equipment

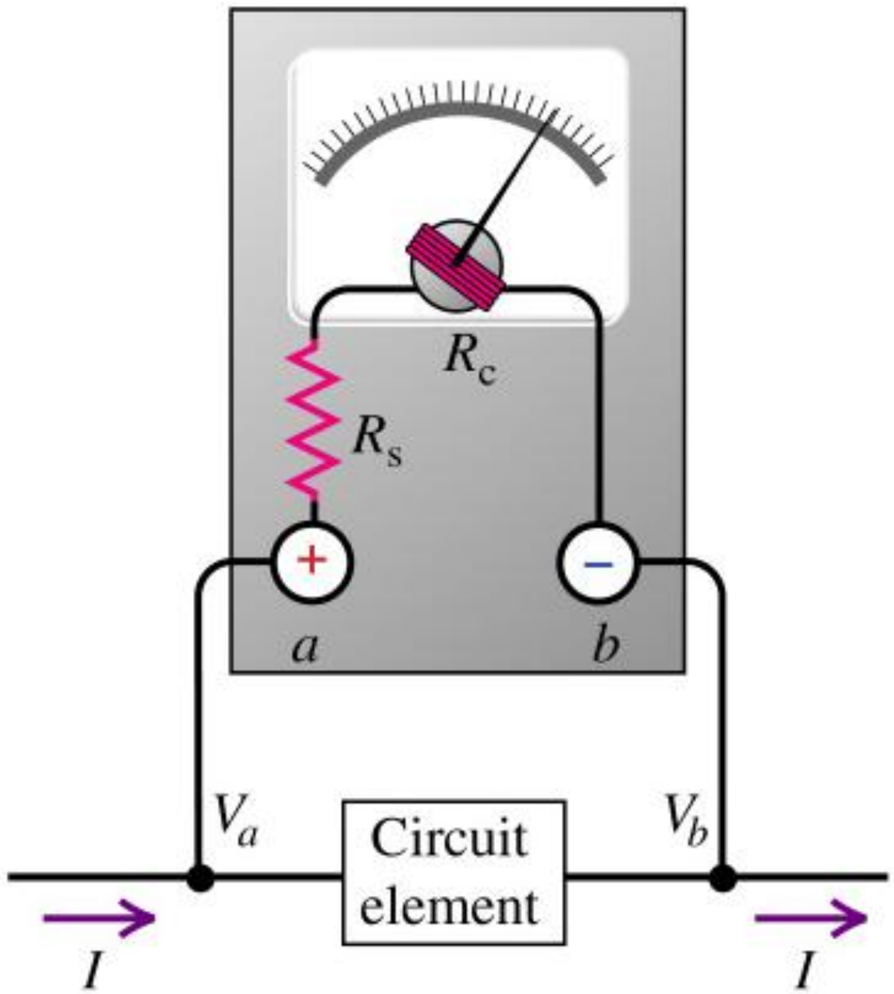
- Sample problem:
  - A galvanometer has a full-scale deflection of *1.00mA* and a coil resistance of *20.0Ω.*
  - What shunt resistance is required to make an ammeter of full-scale deflection *50.0mA*?
- Solution:
  - We know: $I_{fs}R_c = \left(I_a - I_{fs}\right)R_{sh}$

$$\Rightarrow R_{sh} = \frac{I_{fs}R_c}{I_a - I_{fs}} = \frac{\left(1.00\times10^{-3}A\right)\left(20.0\Omega\right)}{50.0\times10^{-3} - 1.00\times10^{-3}A} = 0.408\Omega$$

26.3 Electrical Measuring Equipment

- Voltmeter:
  - Voltmeters measure potential differences.
  - They are connected in parallel.
  - Ideally they have infinite resistance.
  - We can adapt a voltmeter to measure higher potential differences above their range by combining them in series with a resistor R_s
  - For a voltmeter with full-scale reading V_V, we need a series resistor such that

$$V_V = I_{fs}\left(R_c + R_s\right)$$



26.3 Electrical Measuring Equipment

- **Sample problem**:
  - A galvanometer has a full-scale deflection of *1.00mA* and a coil resistance of *20.0Ω.*
  - What series resistance is required to make a voltmeter with a maximum range of *10V*.
- **Solution:**
  - We know: $V_V = I_{fs}\left(R_c + R_s\right)$

$$\Rightarrow R_s = \frac{V_V}{I_{fs}} - R_c = \frac{10.0V}{0.00100A} - 20\Omega = 9980\Omega$$

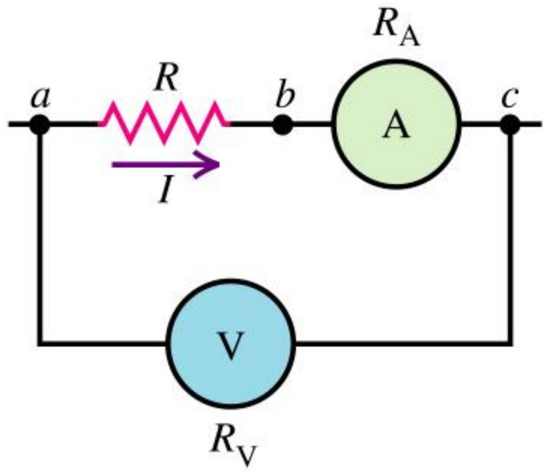
26.3 Electrical Measuring Equipment

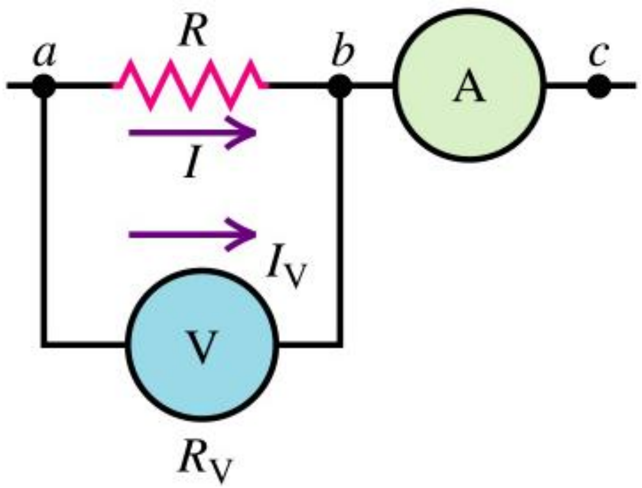
- Meters in combination:

  – Ammeters and voltmeters can be combined to measure resistance and power.

  – If the voltmeter is connected across the resistor and the ammeter: The readings have to be corrected for the voltage across the ammeter.

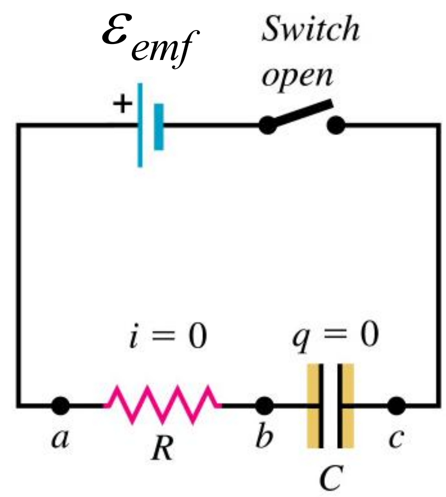
  – If the voltmeter is connected across the resistor only, the readings have to be corrected for the current through the voltmeter.

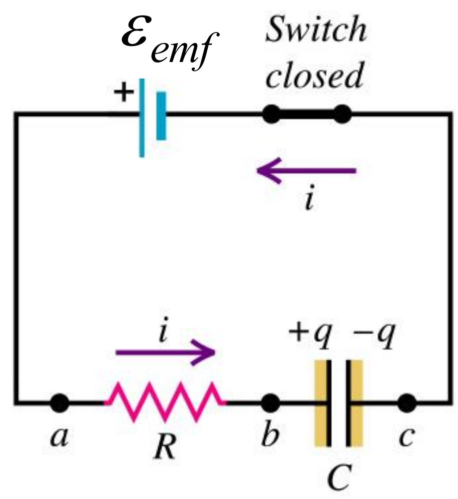




26.4 RC Circuits

- **R-C Circuits**:
  - Circuits with a resistor and capacitor in series are called *R-C* Circuits.
- **Charging a capacitor:**
  - Shown is a suitable circuit for charging a capacitor.
  - The switch is closed at $t=0$ and current flows.
  - Using the **loop rule** we find: $\varepsilon_{emf} - iR - \dfrac{q}{C} = 0$





$$\Rightarrow i = \frac{\varepsilon_{emf}}{R} - \frac{q}{RC}$$

True at any instant of time!

26.4 RC Circuits

– But: $i = \dfrac{dq}{dt}$, giving a DE for q: $\dfrac{dq}{dt} = \dfrac{\varepsilon_{emf}}{R} - \dfrac{q}{RC} = \dfrac{C\varepsilon_{emf} - q}{RC}$

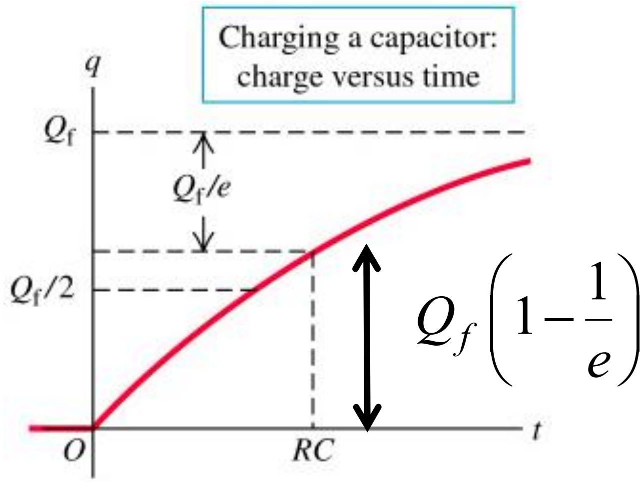
– Rearrange and integrate:

$$\int_0^q \frac{dq'}{q' - C\varepsilon_{emf}} = -\frac{1}{RC}\int_0^t dt'$$

Use primed (‘) dummy integration variables here

$$\Rightarrow \ln\left(q - C\varepsilon_{emf}\right) - \ln\left(-C\varepsilon_{emf}\right) = \ln\left(\frac{q - C\varepsilon_{emf}}{-C\varepsilon_{emf}}\right)$$

$$= \frac{-t}{RC}$$

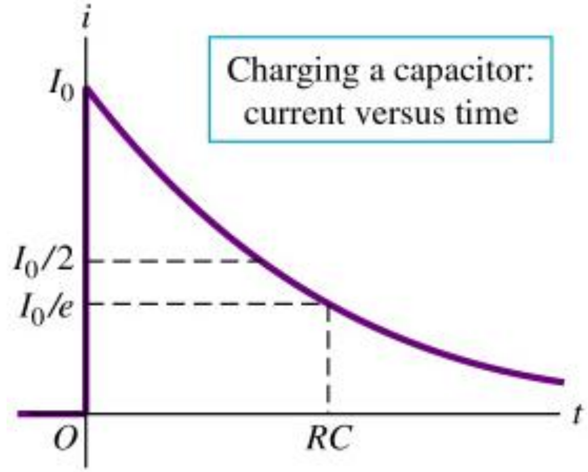


– So, since the final charge is: $Q_f = CV = C\varepsilon_{emf}$

$$q = C\varepsilon_{emf}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

– And:

$$i = \frac{dq}{dt} = \frac{\varepsilon_{emf}}{R}e^{-t/RC} = I_0 e^{-t/RC}$$



26.4 RC Circuits

- Time constant:
  - Note when $\tau = RC$, the **current** is reduced to $1/e$ of its initial value.
  - At this time, the **charge** on the capacitor is $(1-1/e)$ of its final value.

  - The time constant of an *R-C* circuits is defined as:

$$\tau = RC$$

  - Units: seconds (*s*)

26.4 RC Circuits

- Discharging a capacitor:
  - Shown is a suitable circuit for discharging a capacitor.
  - At $t=0$, $q=Q_0$ and the switch is closed.
  - From slide 170, using the loop rule: $i = \dfrac{\varepsilon_{emf}}{R} - \dfrac{q}{RC}$

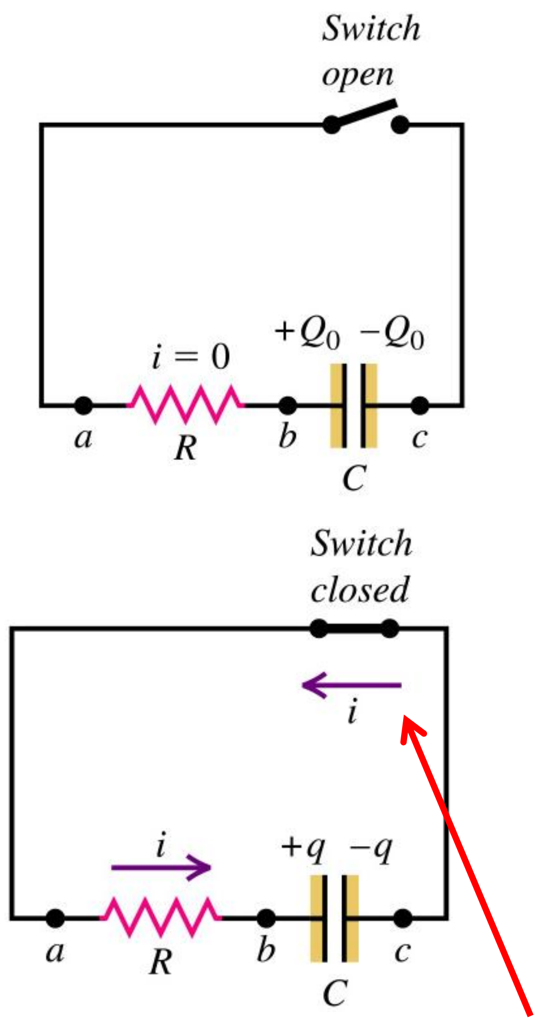
$$\varepsilon_{emf} = 0$$

$$i = \frac{-q}{RC} \qquad \Rightarrow \frac{dq}{dt} = \frac{-q}{RC}$$

  - Integrating:

$$\int \frac{dq}{q} = -\frac{1}{RC} \int dt$$

$$\Rightarrow \ln q = -\frac{t}{RC} + k$$



Note this is actually a negative current!

Fig. 26.23, p. 898

26.4 RC Circuits

– At $t = 0$ we have charge Q_0:

$$\ln Q_0 = k$$

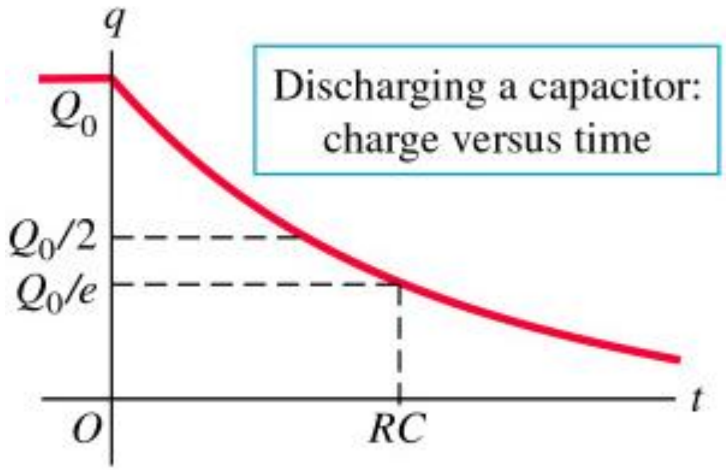
$$\Rightarrow \ln \frac{q}{Q_0} = -\frac{t}{RC}$$

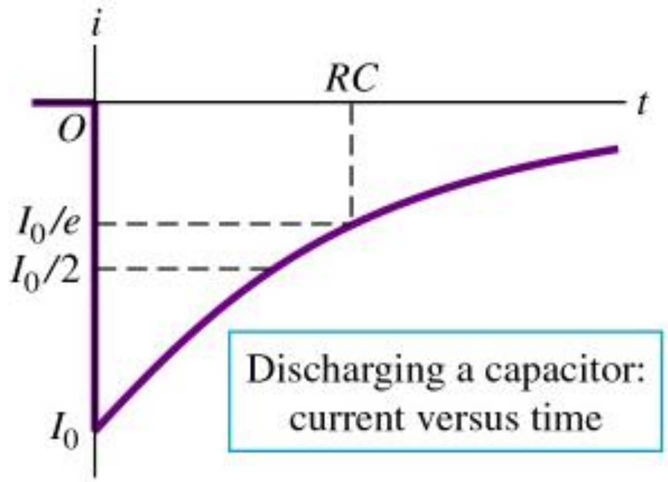
– So:

$$q = Q_0 e^{-t/RC}$$

– And:

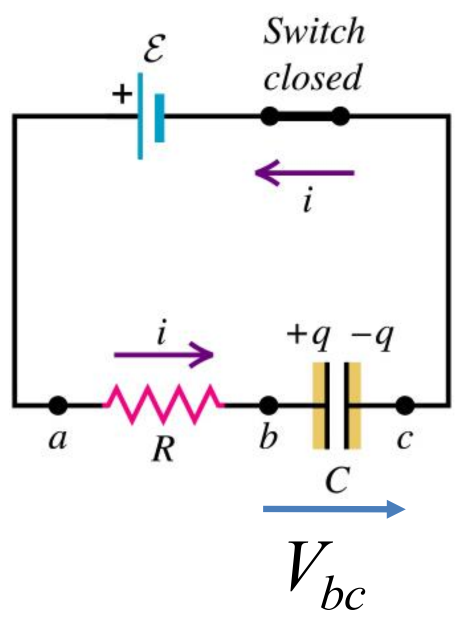
$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$





Note that i is negative!!

26.4 RC Circuits



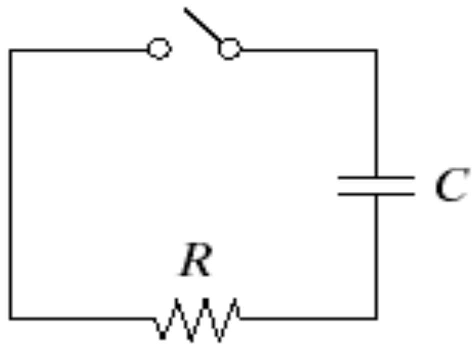
- **Energy:**
  - The power delivered to the circuit by the source is:

$$P = i\varepsilon_{emf} = i^2R + iV_{bc} = i^2R + i\frac{q}{C}$$

  - The total energy supplied during charging is: $\varepsilon_{emf}Q_f$
  - But recall the energy stored on the capacitor is: $\dfrac{Q_f\varepsilon_{emf}}{2}$
  - So half the energy is lost in the resistor (independent of R, C, or ε_{emf})!

Concept test

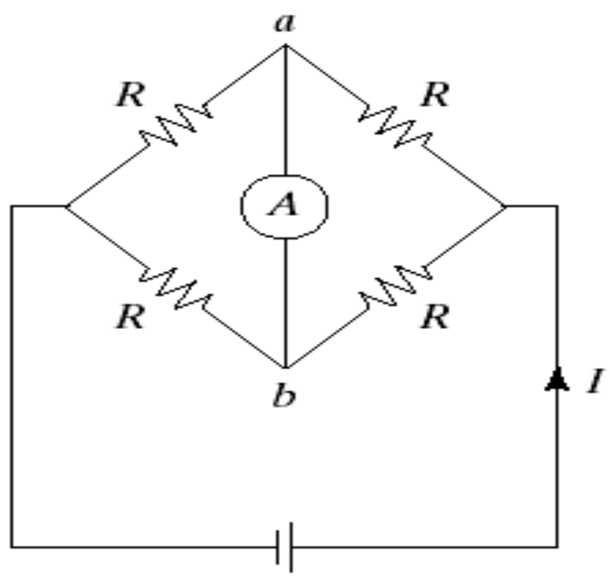
A simple circuit consists of a resistor R, a capacitor C charged to a potential V_o, and a switch that is initially open but then thrown closed. Immediately after the switch is thrown closed, the current in the circuit is



1. V_o/R.
2. zero.
3. need more information

Concept test

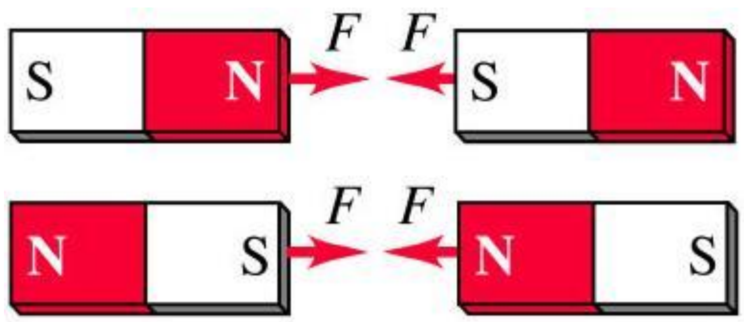
An ammeter A is connected between points a and b in the circuit below, in which the four resistors are identical. The current through the ammeter is

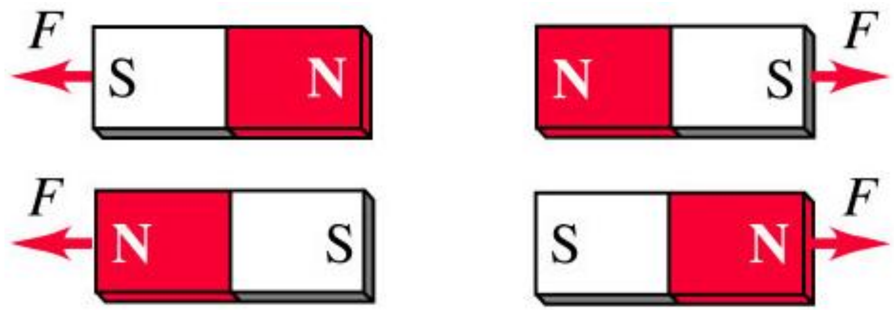


1. $I/2$.
2. $I/4$.
3. zero.
4. need more information

Magnetism, magnetic force

- Magnetic poles:
  - Before magnetism was explained in terms of moving charges, magnetic effects were explained in terms of **magnetic poles**.
  - If a bar magnet is free to rotate one end will point north.
  - This end is called the **north pole** and the other the **south pole** of the magnet.
  - Opposite poles attract each other.
  - Like poles repel.





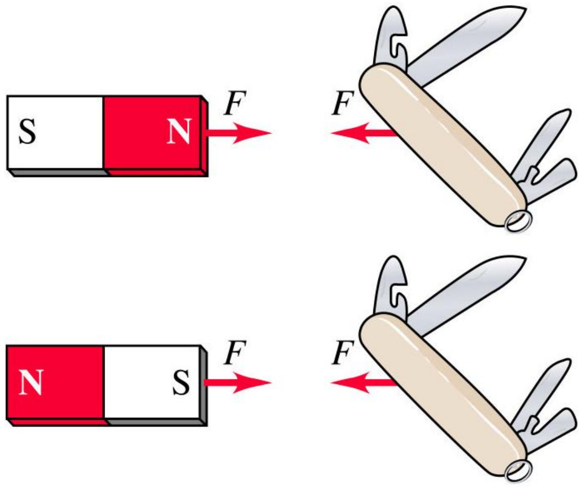
27.1 Magnetic pole

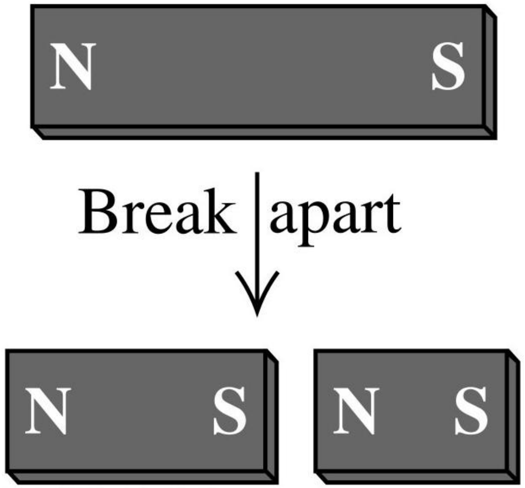
- Both types of magnetic pole attract unmagnetized objects containing iron.

- An isolated magnetic pole is called a **magnetic monopole**.

- Magnetic monopoles are theoretically predicted objects, but so far none has been found.

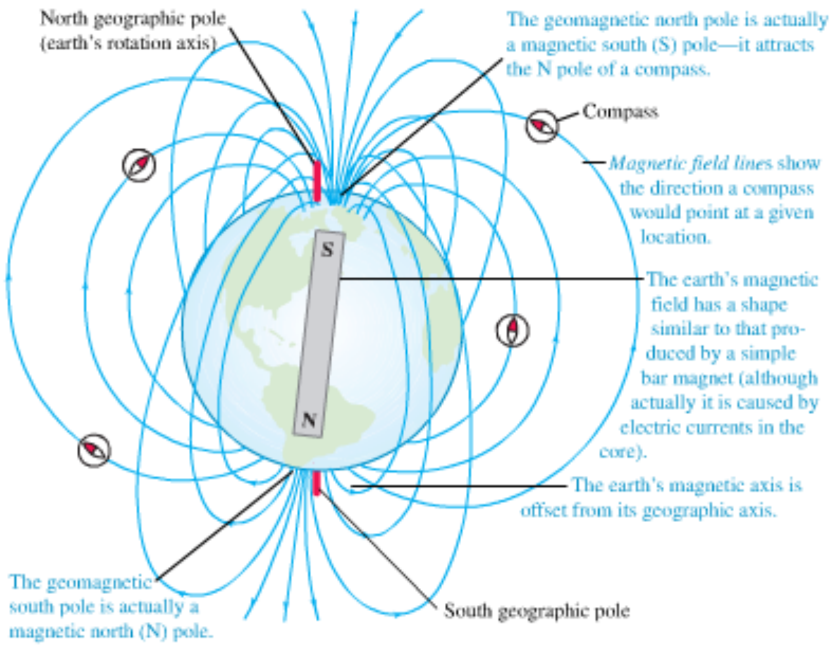
- If a bar magnet is broken, two weaker bar magnets are created.





27.2 Magnetic field

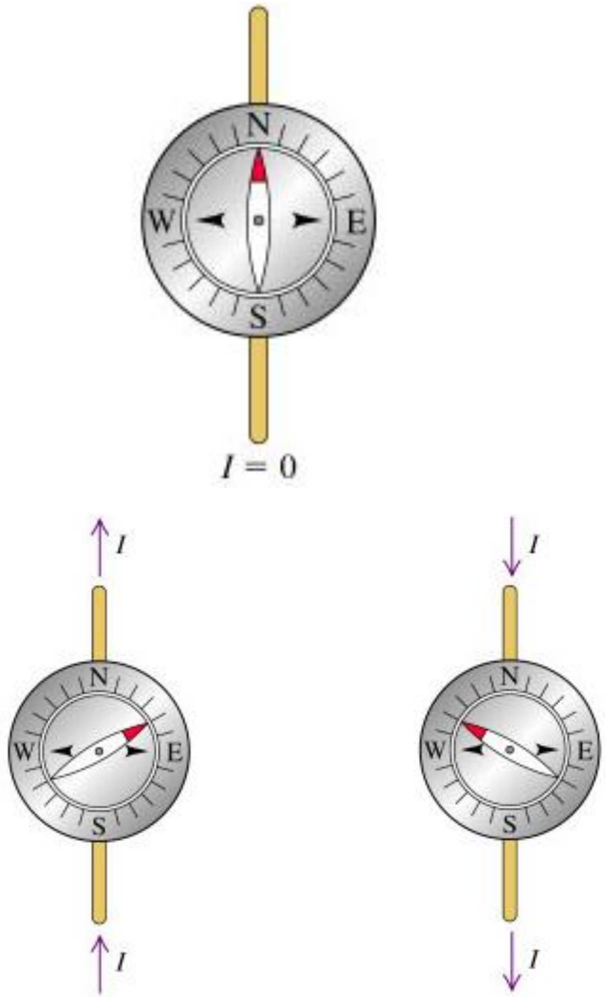
- Magnetic field:
  - Magnetic forces are explained by saying that magnetized objects create a magnetic field in the space around them and other objects react to this.
  - The earth has a magnetic field caused by the core.
  - The magnetic field is represented by magnetic field lines.

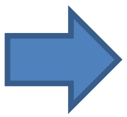


- The magnetic field line at any location points in the direction that a compass placed there would point.

27.2 Magnetic field

- **Magnetic field from current:**
  - Oersted 1819:
  - The first connection between magnetism and electricity to be discovered was the deflection of a compass by a current.
  - On the other hand, Faraday discovered that moving a magnet can cause a current to appear in a loop of conductor
  - Furthermore, changing current in one loop can induce a current in another loop



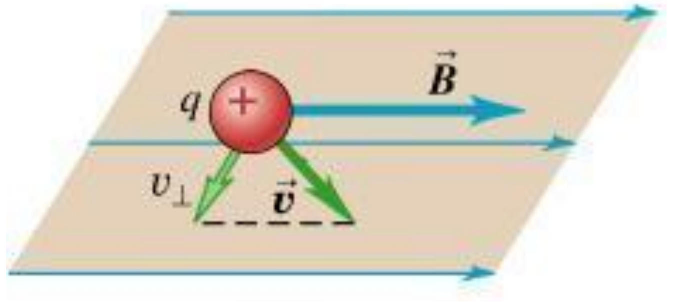


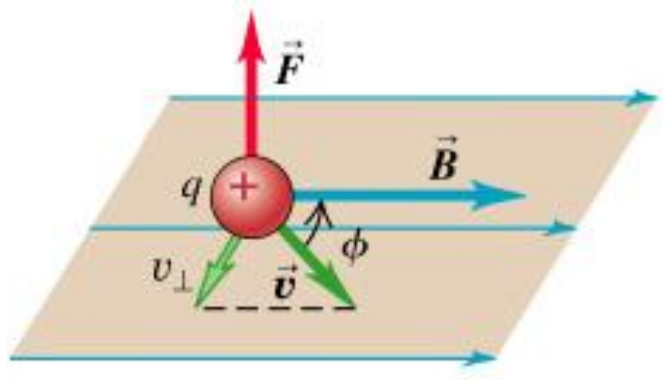
Close link between electric and magnetic interactions.

27.2 Magnetic Force

- Magnetic force:
  - A magnetic field exerts a force on a **moving** $(\vec{v})$ charged particle.
  - The magnitude of the force is proportional to the charge.
  - It is also proportional to the strength of the magnetic field $|\vec{B}|$
  - It is also proportional to the *component* of the velocity perpendicular to the magnetic field... !!!
  - The direction of the force is perpendicular to *both* the magnetic field and the direction of particle motion... !!!!!!!!!!

Uniform magnetic field



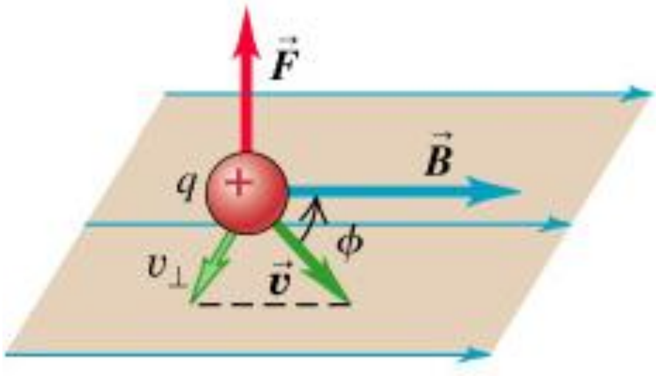


A very odd force

27.2 Magnetic Force

– Combining these we write:

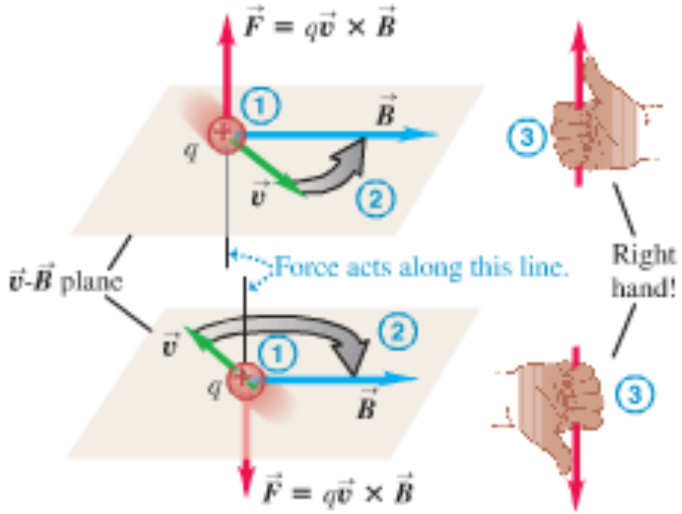
$$F = |q| v_{\perp} B = |q| vB \sin\phi$$



– The direction of the force is perpendicular to both the magnetic field and the velocity.

– In vector form the force is:

$$\vec{F} = q\vec{v} \times \vec{B}$$

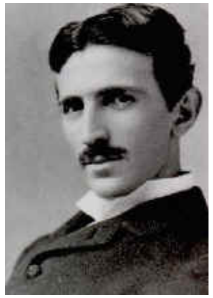


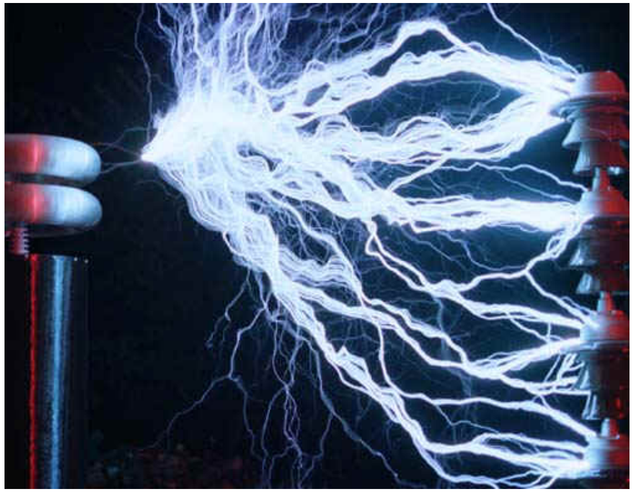
– Remember to use the **right-hand rule** to evaluate the direction of the vector product

– Units for the magnetic field: $B = \dfrac{F}{qv}$ $\quad [B] = \dfrac{Ns}{Cm} = NA^{-1}m^{-1} = T\ (tesla)$

27.2 Magnetic field

- Tesla:
  - Nikola Tesla was born in Croatia in 1856.
  - While working in Paris he developed his induction motor, the first step to useful alternating current.
  - He emigrated to America where he became involved in an industrial dispute between manufacturers of direct and alternating current components.
  - He also invented the **Tesla Coil**, a device used in radios and televisions to produce high voltages.
  - He died in 1943 in New York.





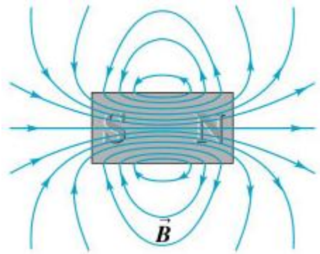
27.2 Typical values of magnetic field

- Magnetic field of earth: 10^{-4} T
- Lab. magnets for research (DC): 30 T
- Pulsed magnets (10^{-3} s): 100 T

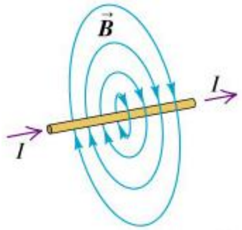
- cgs unit: 1 gauss = 1 G = 10^{-4} T
- gaussmeter

27.3 Magnetic field lines and flux

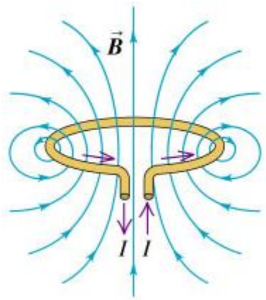
- **Various magnetic fields, generated by:**
  - A bar magnet



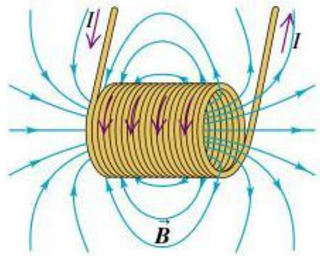
  - A current carrying wire



  - A current carrying loop



  - A current carrying coil (a solenoid).



$\boxed{\text{Magnetic field lines are NOT lines of force!}}$

27.3 Magnetic field lines and magnetic flux

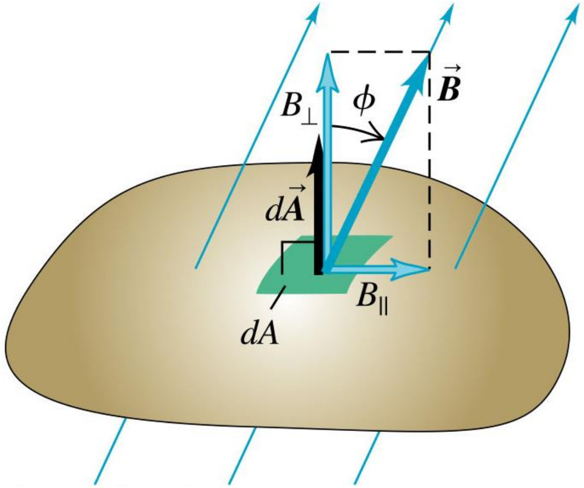
- **Magnetic flux:**

  - Magnetic flux is defined just as electric flux.

  - The flux through infinitesimal area dA is given:

$$d\Phi_B = B_\perp dA = B\cos\varphi dA = \vec{B}\bullet d\vec{A}$$

  - Integrate to get the total flux.



  - The magnetic flux through a surface A is:

$$\Phi_B = \int B_\perp dA = \int B\cos\varphi\, dA = \int \vec{B}\bullet d\vec{A}$$

  - Units: weber (Wb) or $T\,m^2$

27.3 Magnetic field lines and flux

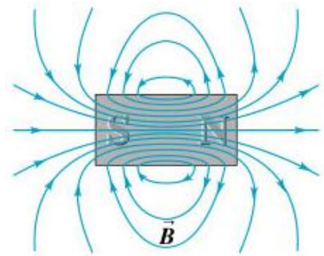
– Because of the non-existence of **magnetic monopoles**, the net magnetic flux through any *closed* surface is zero.

– This is often called Gauss' law for magnetism.

- Gauss' law for magnetism:

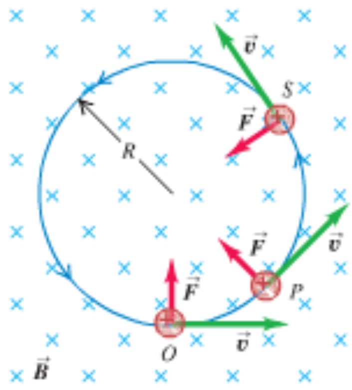
$$\oint \vec{B} \bullet d\vec{A} = 0$$

Magnetic field lines have no endpoints.



27.4 Particle motion in magnetic fields

- Motions due to magnetic field:
  - Consider a charged particle moving perpendicular to a uniform magnetic field.
  - It experiences a force perpendicular to its velocity.
  - The magnitude of its velocity must stay the same.
  - The particle moves in a circle.
  - The magnetic force is balanced by a centripetal force: $F = |q| vB = m\frac{v^2}{R}$
  - So the radius of the circle is: $R = \frac{mv}{|q| B}$



Magnetic field points into page

27.4 Cyclotron

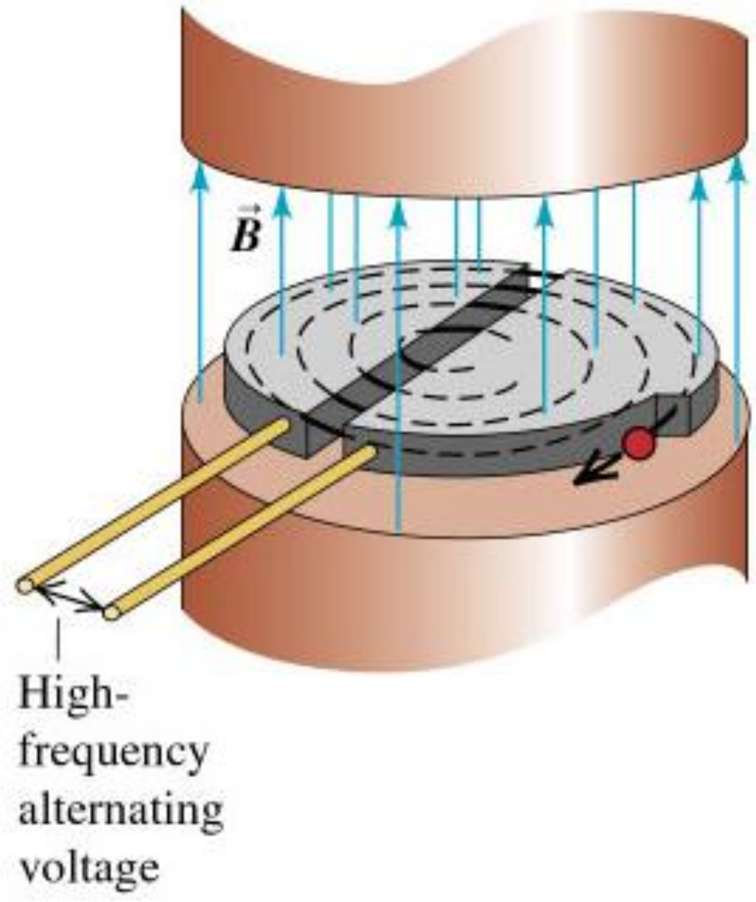
- The angular velocity is:

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}$$

- The orbital frequency is given by:

$$f = \frac{\omega}{2\pi} = \frac{|q|B}{2\pi m}$$

- This is called the **cyclotron frequency**.



- Cyclotron:
  - Cyclotrons are particle accelerators.
  - They use a magnetic field to confine the particle to a circle.
  - And an electric field to accelerate the particles through a gap.

Magnetic force on a current
Torque on a current loop
DC motor

Read Sections 27.6 to 27.8

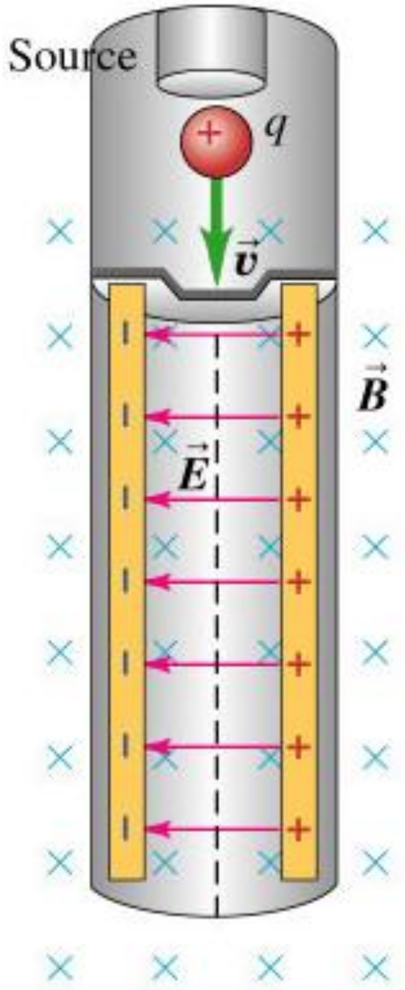
27.5 Applications of Charge Particle Motion

- Velocity selector:
  - Consider a charge moves through electric and magnetic fields as shown.
  - The electric force to the left is: $F_E = qE$
  - The magnetic force to the right is: $F_B = qvB$
  - The particle will be undeflected if:

$$F_E = F_B$$

$$\Rightarrow \quad qE = qvB$$

$$v = \frac{E}{B}$$

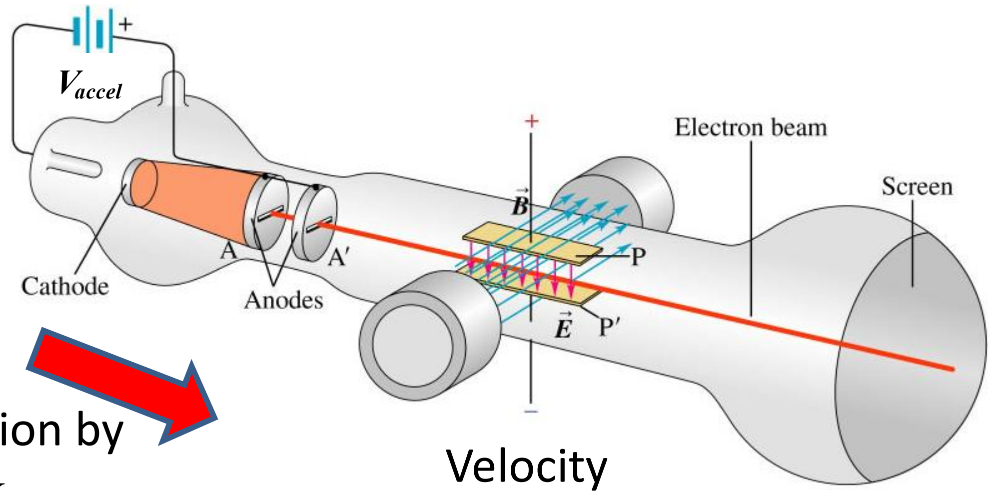


Magnetic field points into page

27.4 Thomson e/m experiment

- Thomson's experiment to determine the ratio of charge e to mass m of an electron:
  - Electrons are accelerated from the cathode to the anode.
  - The kinetic energy gained is: $KE = \frac{1}{2}mv^2 = e \cdot V_{accel}$ $\Rightarrow$ $v = \sqrt{\frac{2eV_{accel}}{m}}$
  - They then pass through a velocity selector.
  - They will be undeflected if they have velocity: $\frac{E}{B} = v = \sqrt{\frac{2eV_{accel}}{m}}$ $\Rightarrow$ $\frac{e}{m} = \frac{E^2}{2V_{accel}B^2}$

Vacuum tube:



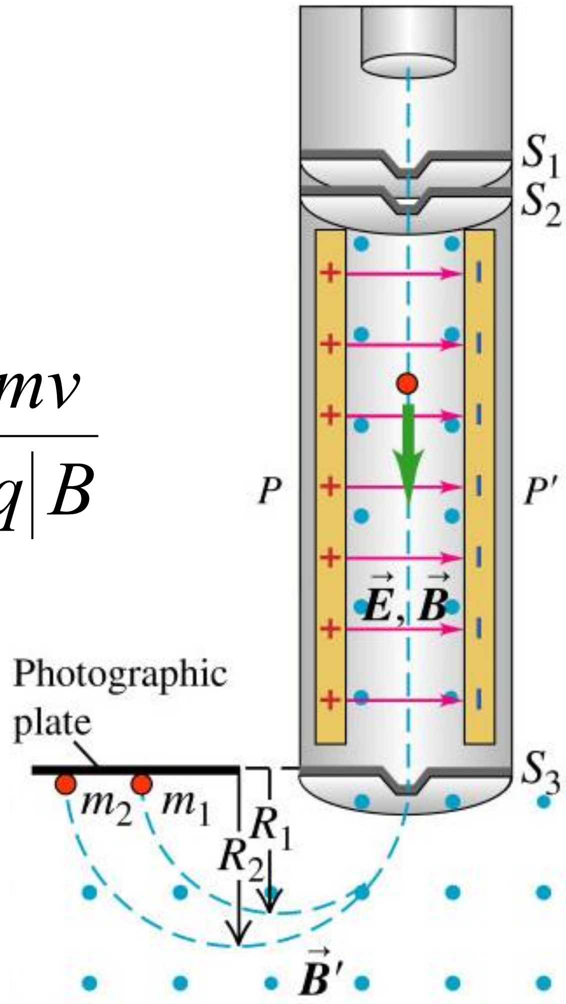
Acceleration by voltage V_{accel}

Velocity selector

27.5 Applications of Charge Particle Motion

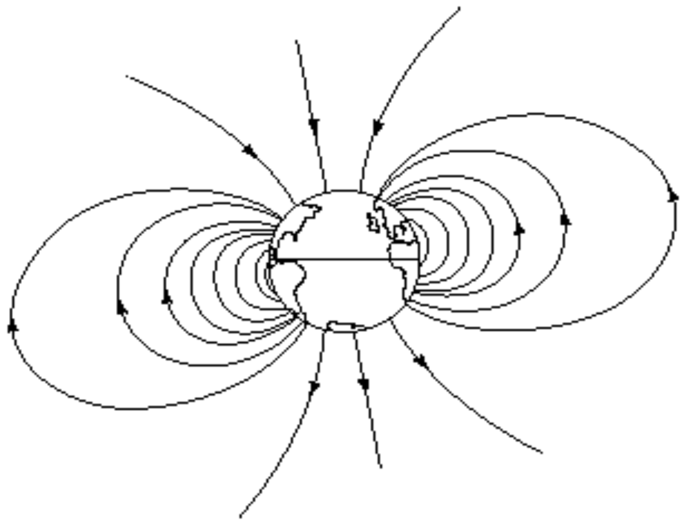
- Mass spectrometer:
  - Velocity selectors are also used in mass spectrometers.
  - Particles with the same velocity but different masses travel in circles with different radii in the magnetic field.
  - So different isotopes (which have same charge, but different mass due to number of neutrons in the nucleus) can be separated.

$$R = \frac{mv}{|q|B}$$



Concept test

Cosmic rays (atomic nuclei stripped bare of their electrons) would continuously bombard Earth's surface if most of them were not deflected by Earth's magnetic field. Given that Earth is, to an excellent approximation, a magnetic dipole, the intensity of cosmic rays bombarding its surface is greatest at the



1. poles.
2. mid-latitudes.
3. equator.

Magnetic force on a current
Torque on a current loop
DC motor

Read Sections 27.6 to 27.8

(review Sect 10.1 on torque)

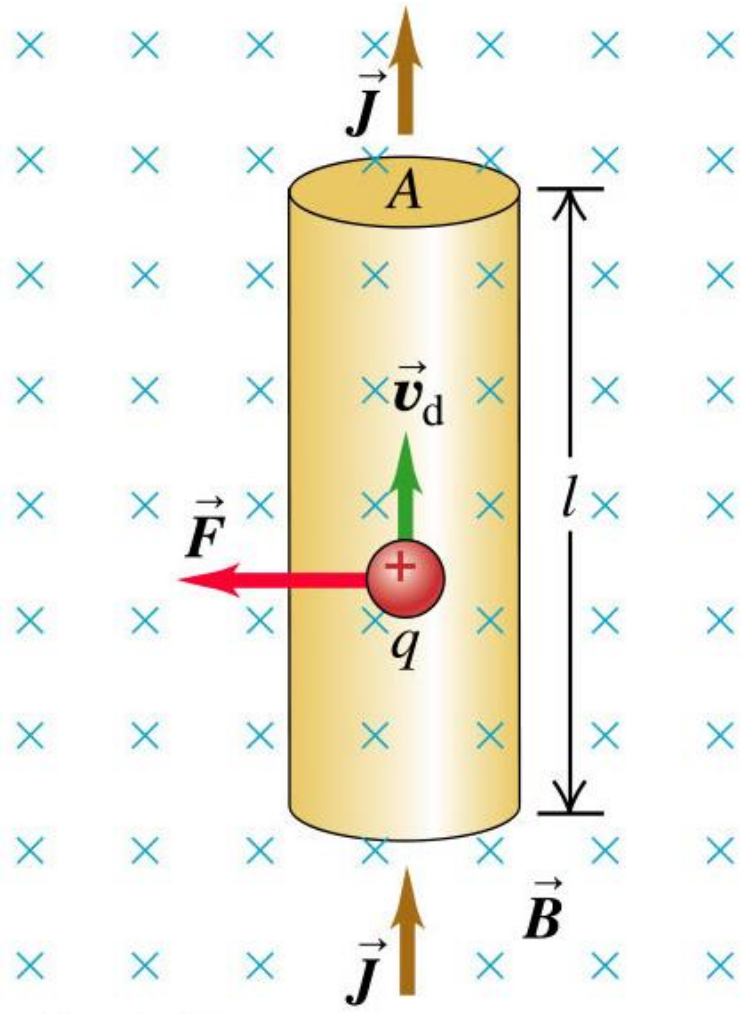
27.6 Magnetic Force on Current Carrying Conductor

- Force on a current:
  - Consider a current carrying wire in a uniform magnetic field at right angles to each other as shown
  - The drift velocity is in the direction of the current.
  - The average force on a charge is:

$$\vec{F} = q\vec{v}_d \times \vec{B}$$
$$F = qv_d B$$

  - The total force on a segment with n charges q per unit volume is:

$$F = (nAl)(qv_d B) = (nqv_d A)(lB)$$
$$= (JA)(lB) = IlB$$



$\vec{B}$ *points into the page*

$|\vec{J}| = nqv_d$ current density

27.6 Magnetic Force on Current Carrying Conductor

- If the magnetic field is not perpendicular to the wire, the magnetic force on the wire is:

$$F = IlB_{\perp} = IlB\sin\varphi$$

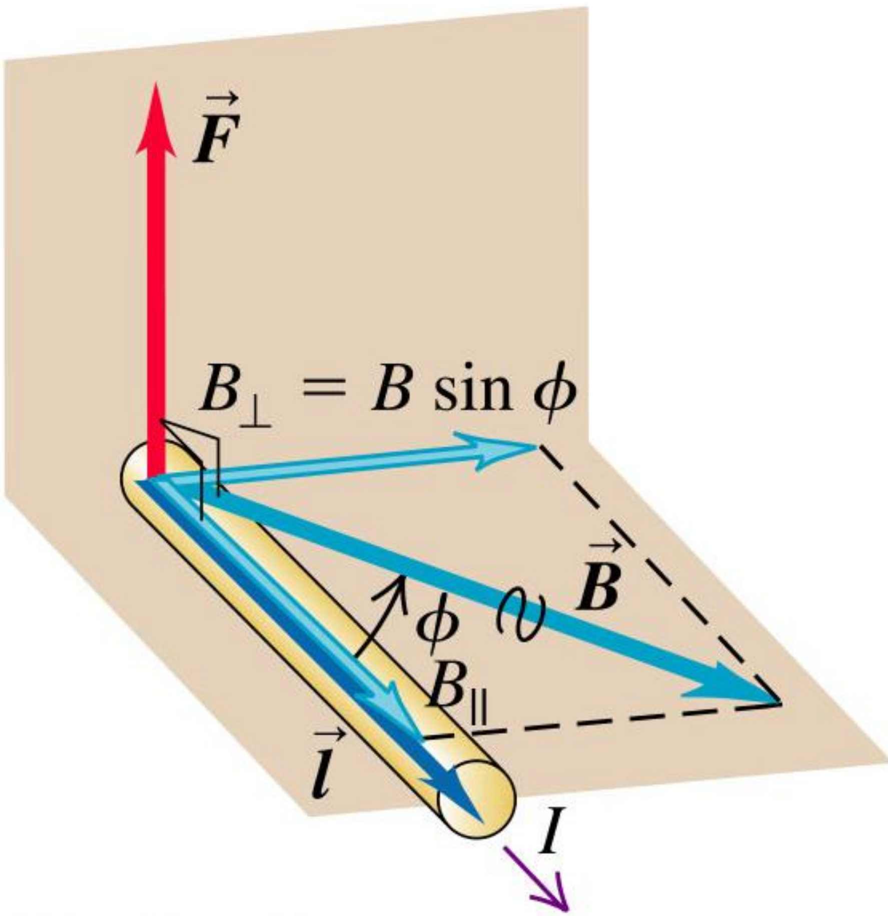
- Define a **length vector** of length l and in the direction of the current.

- The magnetic force on the wire becomes:

$$\vec{F} = I\vec{l} \times \vec{B}$$

- If the wire is not straight the force is an integral:

$$\vec{F} = \int I\, d\vec{l} \times \vec{B}$$



27.4 Thomson e/m experiment

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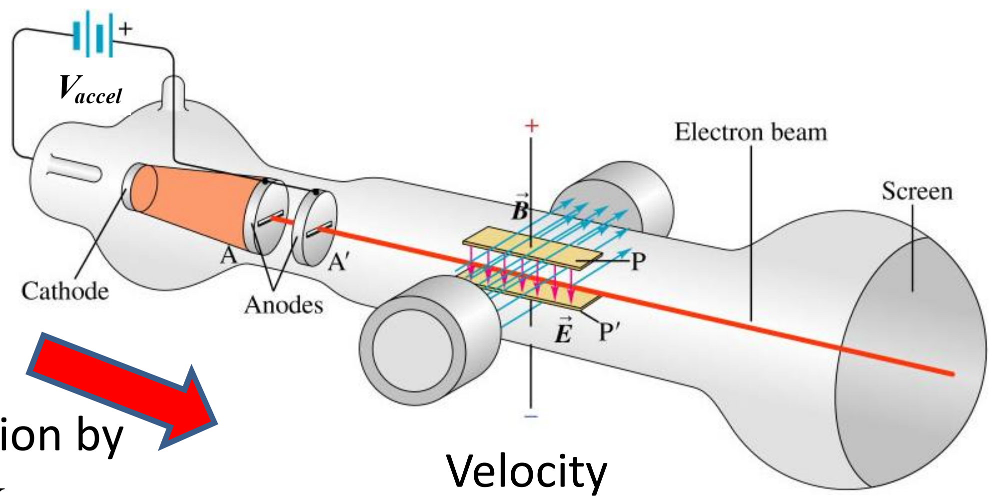
  - Electrons are accelerated from the cathode to the anode.
  - The kinetic energy gained is: $KE = \frac{1}{2}mv^2 = e \cdot V_{accel}$ ⇒ $v = \sqrt{\frac{2eV_{accel}}{m}}$
  - They then pass through a velocity selector.
  - They will be undeflected if they have velocity: $\frac{E}{B} = v = \sqrt{\frac{2eV_{accel}}{m}}$ ⇒ $\frac{e}{m} = \frac{E^2}{2V_{accel}B^2}$

Vacuum tube:



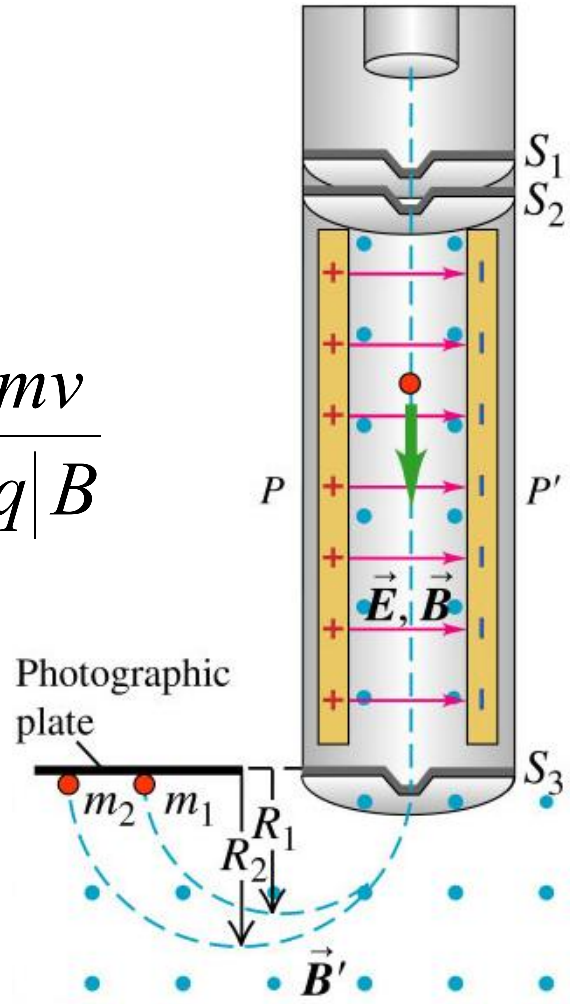
Acceleration by voltage V_{accel}

Velocity selector

27.5 Applications of Charge Particle Motion

- Mass spectrometer:
  - Velocity selectors are also used in mass spectrometers.
  - Particles with the same velocity but different masses travel in circles with different radii in the magnetic field.
  - So different isotopes (which have same charge, but different mass due to number of neutrons in the nucleus) can be separated.

$$R = \frac{mv}{|q|B}$$



Torque on a current loop
DC motor

Read Sections 27.6 to 27.8

(review Sect 10.1 on torque)

27.7 Magnetic Force and Torque on Current Loop

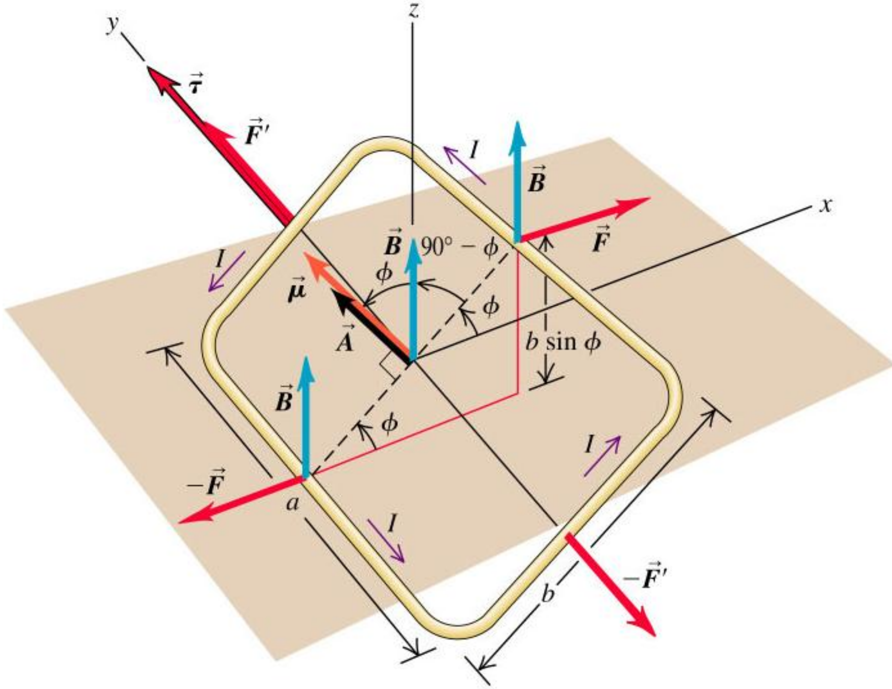
- Torque on a current loop:

$$\vec{F} = I\vec{l} \times \vec{B}$$

  - Consider a rectangular loop, around which a current flows, in a magnetic field as shown.
  - There is **no** net force on the loop as all the forces sum to zero.
  - But there is a net **torque**! $(\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}})$
  - Magnitude of the r vector perpendicular to F is $\frac{b}{2}\sin\varphi$
  - The magnitude of the torque is

$$\tau = F\frac{b}{2}\sin\varphi - F\left(-\frac{b}{2}\sin\varphi\right)$$

$$= Fb\sin\varphi = IaBb\sin\varphi = IAB\sin\varphi$$



27.7 Magnetic Force and Torque on Current Loop

- Torque on a current loop:

$$\vec{F} = I\vec{l} \times \vec{B}$$

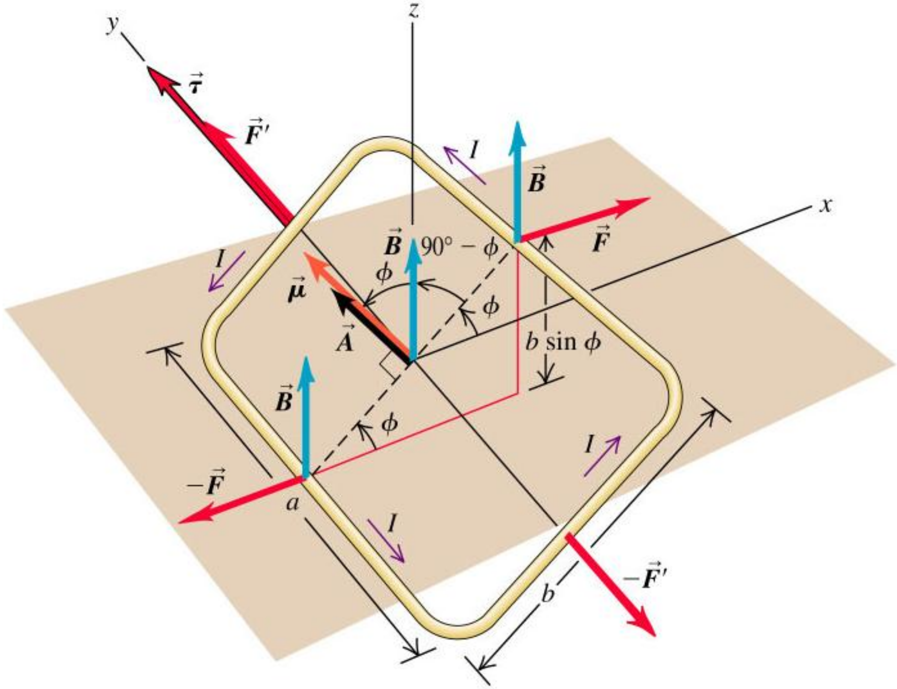
  – The magnitude of the torque on the longer edges is:

$$\tau = F\frac{b}{2}\sin\varphi - F\left(-\frac{b}{2}\sin\varphi\right)$$

$$= Fb\sin\varphi = IaBb\sin\varphi = IAB\sin\varphi$$

  – Define the **magnetic moment** of the loop as:

$$\mu = IA$$



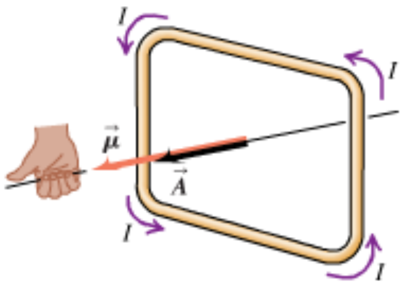
27.7 Magnetic Force and Torque on Current Loop

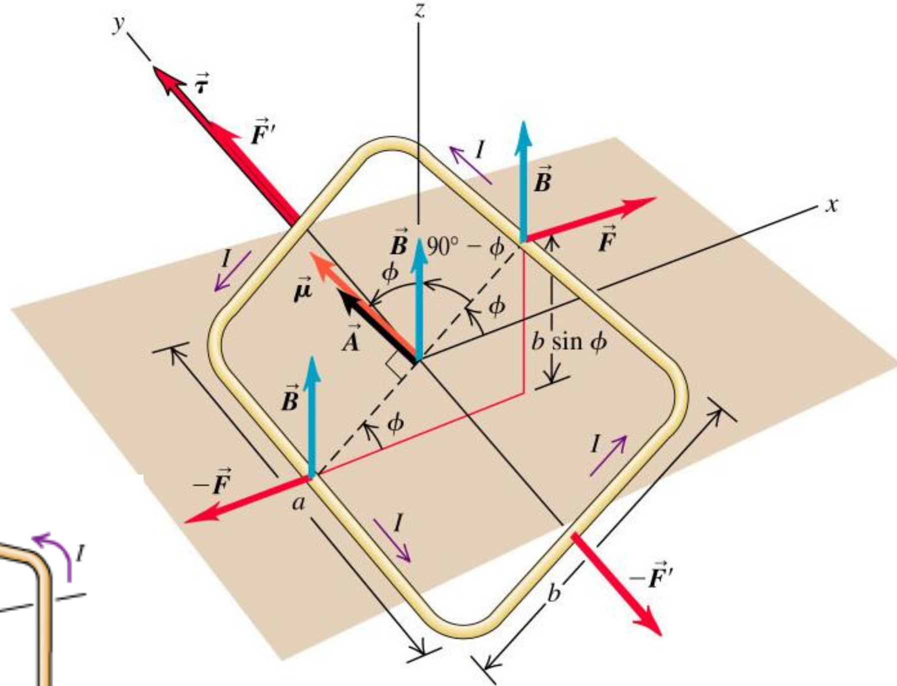
– The torque becomes:

$$\tau = \mu B \sin\varphi$$

– We can define a **magnetic moment vector** along the same axis as the area vector, with the orientation given by the right hand rule applied to the current circulation:

$$\vec{\mu} = I\vec{A}$$



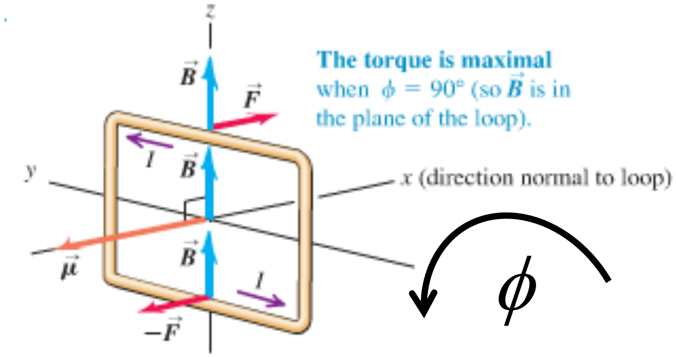


– The torque becomes:

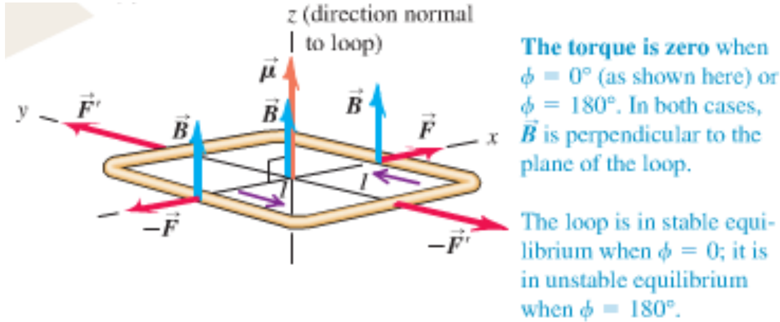
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

27.7 Magnetic Force and Torque on Current Loop

Since $\vec{\tau} = \vec{\mu} \times \vec{B}$



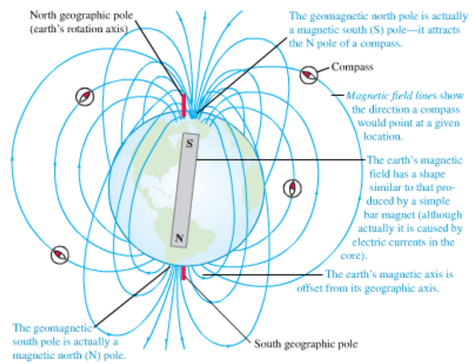
- Torque is a maximum when $\phi = \pi/2, -\pi/2$ (ie. when the magnetic moment vector is *perpendicular* to the magnetic field)



- And zero when $\phi = 0, \pi$.

- The equilibrium at $\phi=0$ is stable and that at $\phi=\pi$ is unstable.

- Any body which experiences a magnetic torque of this type is called a **magnetic dipole**.



- So a magnetic dipole tends to line up with the magnetic field.

27.7 Potential energy for magnetic dipole

- Potential energy of magnetic dipoles:
  - When a magnetic dipole moves through an angle $d\phi$ the work done by the magnetic field is given by:

$$dW = \tau d\varphi$$

  - Integrating:

$$W = \int_{\varphi_1}^{\varphi_2} dW = \int_{\varphi_1}^{\varphi_2} \tau \, d\varphi = \int_{\varphi_1}^{\varphi_2} \mu B \sin\varphi = \mu B\left(\cos\varphi_2 - \cos\varphi_1\right)$$

  - So we can **define** a potential energy function:

$$U\left(\varphi\right) = -\mu B \cos\varphi = -\vec{\mu} \bullet \vec{B}$$

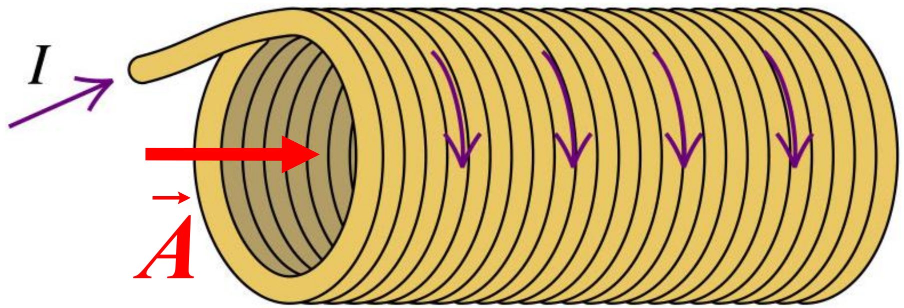
27.7 Torque on Solenoid

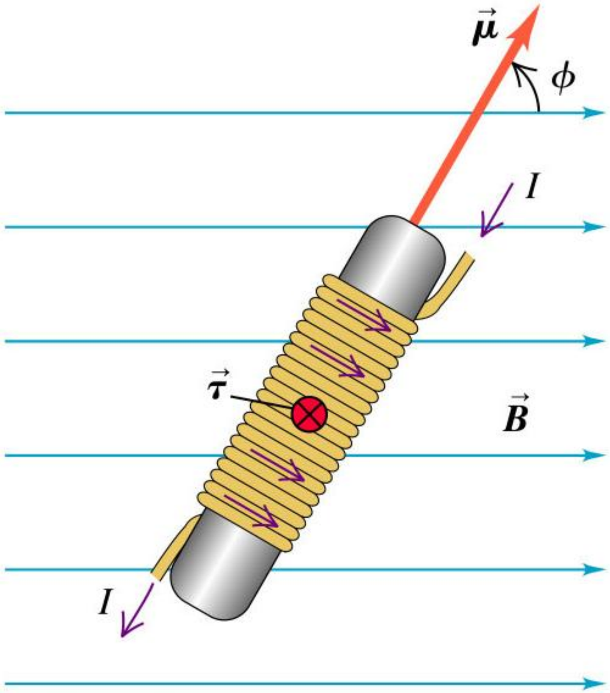
- Solenoids:
  - A solenoid is a helical winding of wire.
  - If the windings are closely spaced they can be approximated by a number of circular loops.
  - The magnetic moment of a solenoid with *N* turns is:

$$\vec{\mu} = NI\vec{A}$$

  - So the total torque on the solenoid is:

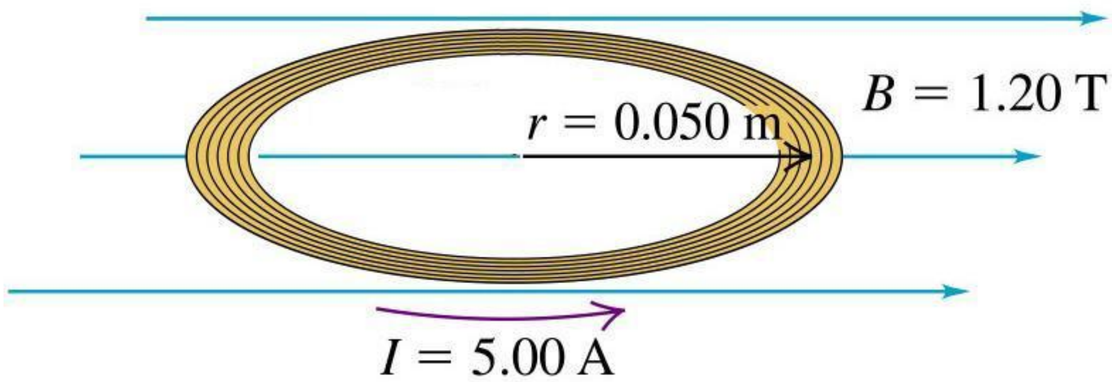
$$\tau = NIAB\sin\varphi$$





27.7 Torque on Solenoid

- Sample problem:
  - A circular coil of *30* turns has radius *0.05 m* and carries a current of *5 A* in a magnetic field of *1.2 T* as shown.
  - Find
    - The magnetic moment of the coil,
    - The torque on the coil



- Solution:

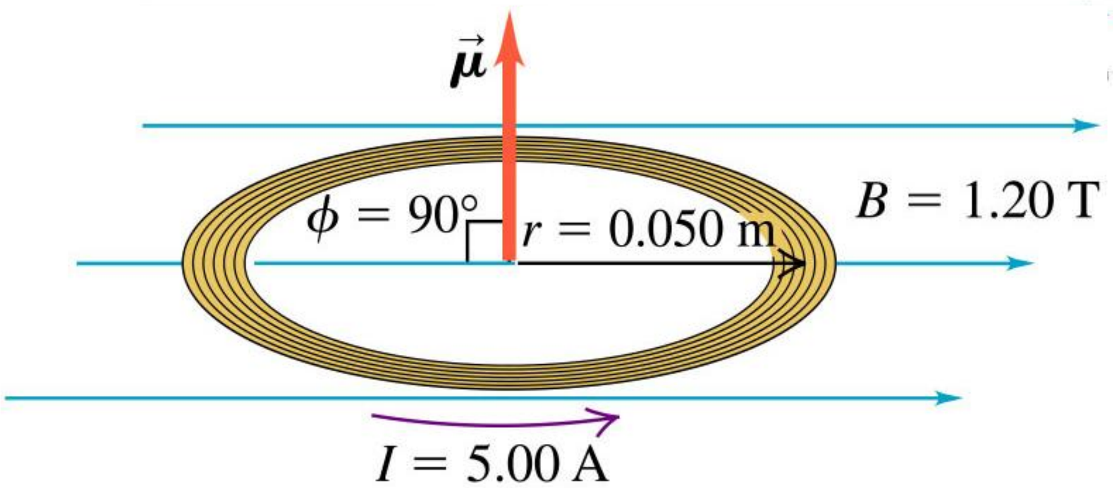
  - The magnetic moment vector points upward and has magnitude:

$$\mu = NIA = (30)(5A)\left(\pi(0.05m)^2\right) = 1.18Am^2$$

  - The torque on the coil is given by:

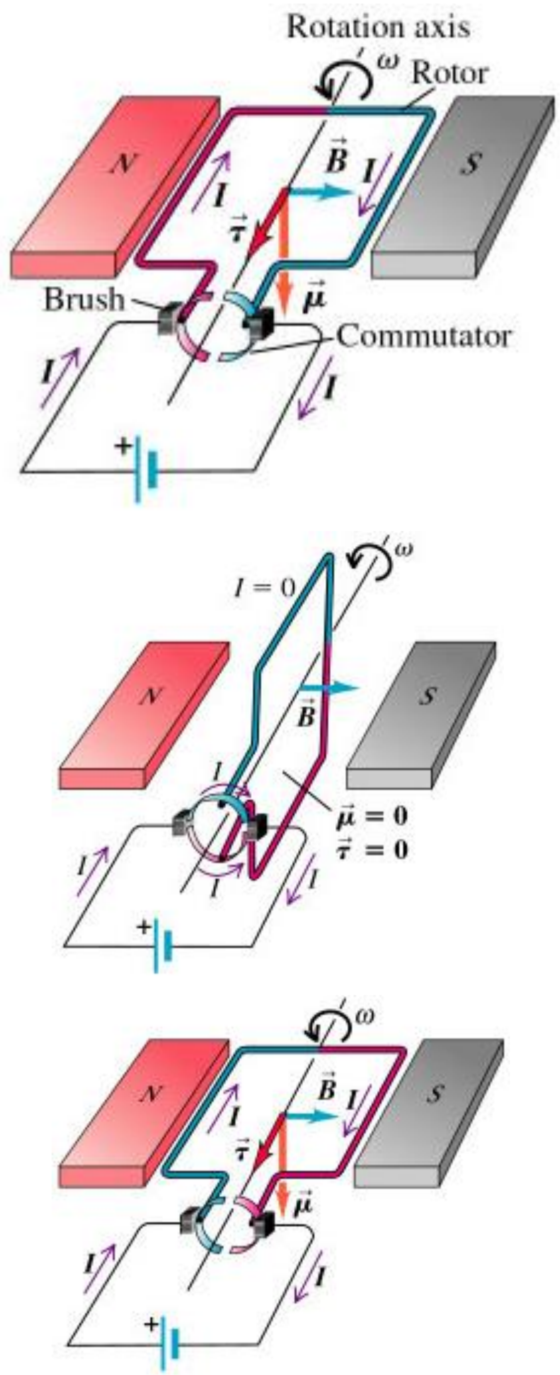
$$\tau = \mu B \sin\varphi = \left(1.18Am^2\right)(1.2T)\left(\sin\frac{\pi}{2}\right) = 1.41Nm$$

  - The coil rotates so that the magnetic moment lines up with the magnetic field.



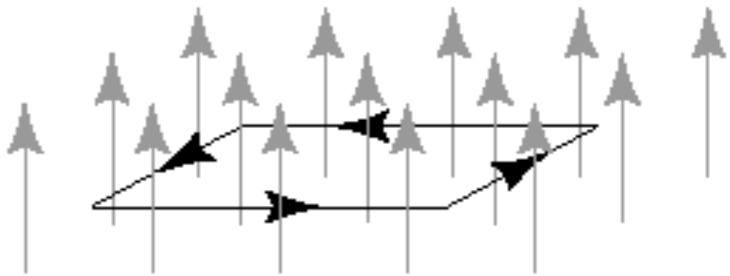
27.8 Direct Current Motor

- DC Motor: converts electric energy to mechanical energy
  - A simple **DC motor** consists of a loop of wire free to rotate (rotor) in a magnetic field.
  - The rotor is connected to a source of emf by **commutators** (conducting circular segments)
  - Initially, the current flows through the loop so the loop experiences a magnetic torque.
  - When the loop has gone through *90º* degrees the current **bypasses** the motor loop (commutators touch the brush) so there is no torque.
  - When the loop rotates beyond *90º*, the current passes through it again and the torque resumes in the same direction.



Concept test

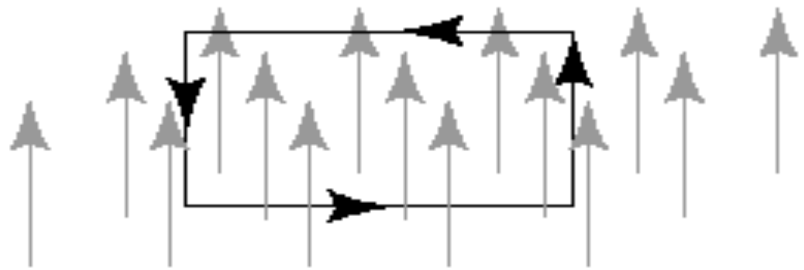
A rectangular loop is placed in a uniform magnetic field with the plane of the loop perpendicular to the direction of the field. If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



1. a net force.
2. a net torque.
3. a net force and a net torque.
4. neither a net force nor a net torque.

Concept test

A rectangular loop is placed in a uniform magnetic field with the plane of the loop parallel to the direction of the field. If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



1. a net force.
2. a net torque.
3. a net force and a net torque.
4. neither a net force nor a net torque.

Sources of magnetic field

Read sections 28.1 to 28.5

28.1 Magnetic field of moving charge

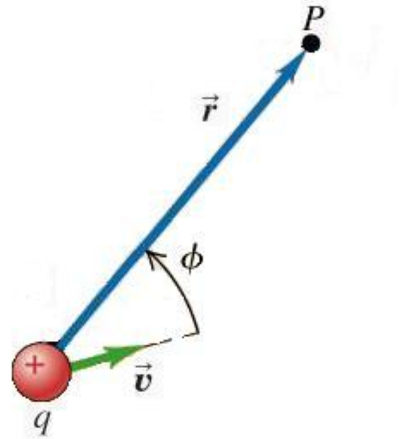
- **We now ask about the field *produced* by a moving charge**:

  – Consider a charge *q* moving with constant velocity *v*.

  – It is found that for the magnetic field produced at any point *P* that :

$$B \propto |q| \qquad B \propto v$$

$$B \propto \frac{1}{r^2} \qquad B \propto \sin\varphi$$



  – The direction of the field $\boldsymbol{B}$ is perpendicular to the plane defined by the velocity of the charge and the position vector of the point.

28.1 Magnetic field of moving charge

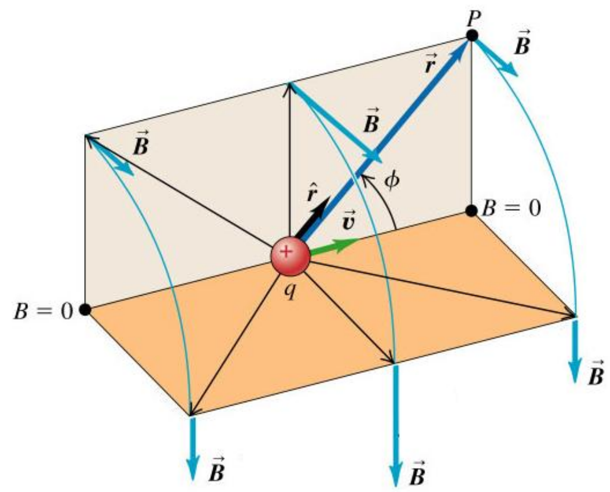
– Introduce a unit vector:

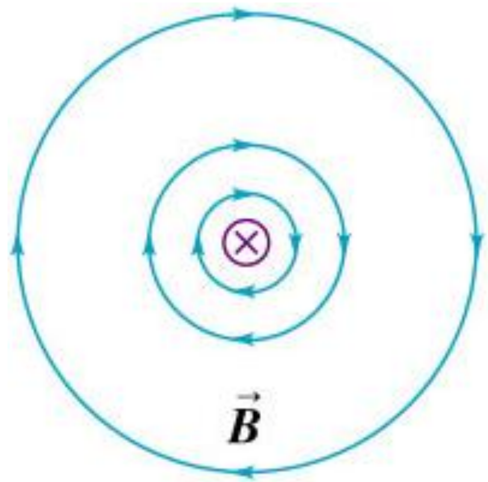
$$\hat{r} = \frac{\vec{r}}{r} \quad \Rightarrow \hat{r} \parallel \vec{r}$$

– The magnetic field can now be written as:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

– Resulting field encircles the path of the charge.

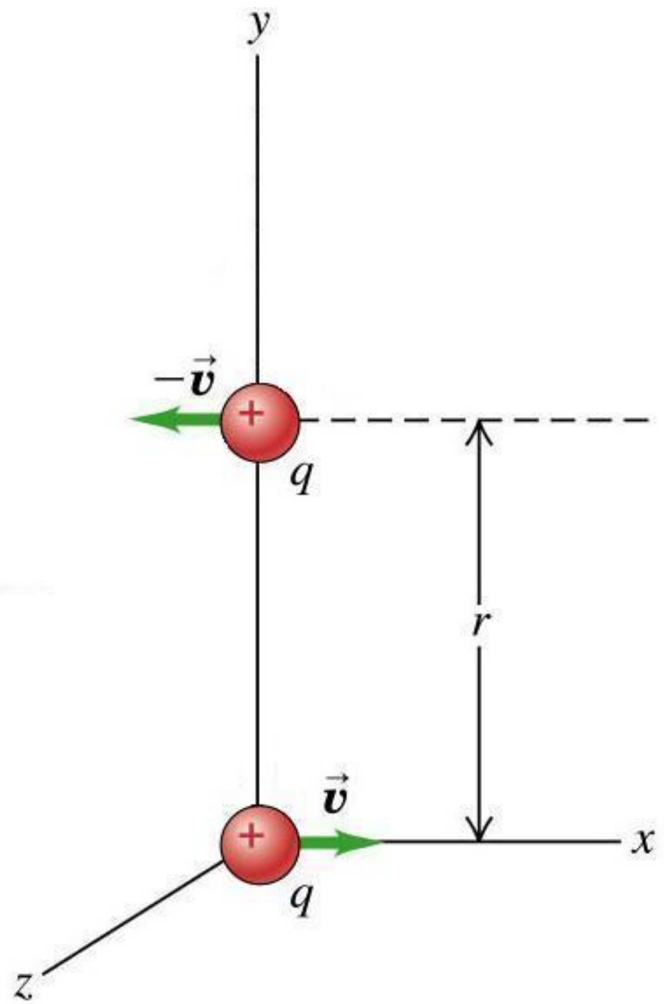




$$\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$$ Is a constant

Permeability of free space

- **Sample problem**:
  - Two protons pass each other moving with equal but opposite velocities.
  - Find the electric and magnetic forces on the upper proton and calculate their ratio.



- Solution:
  - The electric force is:

$$\vec{F}_E = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\hat{j}$$

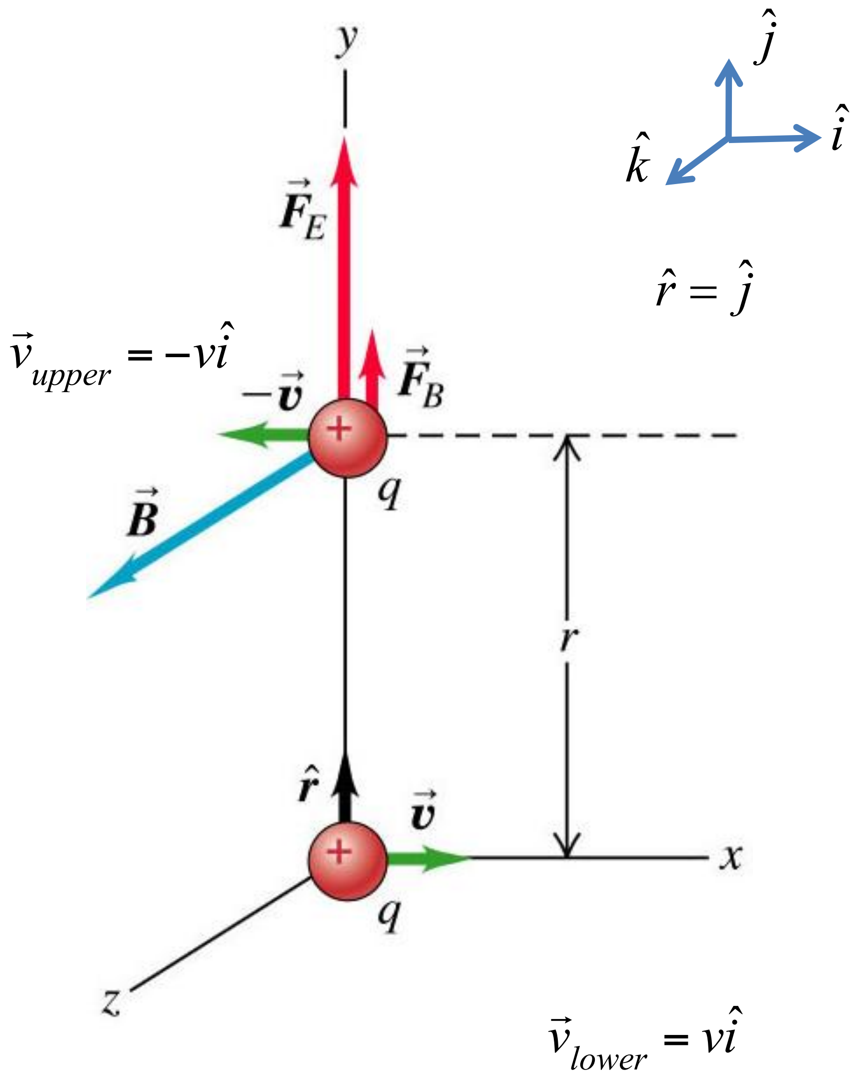
  - Then, we find the magnetic field at the upper charge, given by:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}_{lower}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q}{r^2}v\left(\hat{i}\times\hat{j}\right) = \frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k}$$

  - It follows that the magnetic force is given by:

$$\vec{F}_B = q\left(\vec{v}_{upper}\times\vec{B}\right) = q\left(-v\hat{i}\right)\times\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k}$$

$$= \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}\hat{j}$$ in the same direction as the electric force!



28.1 Magnetic field of moving charge

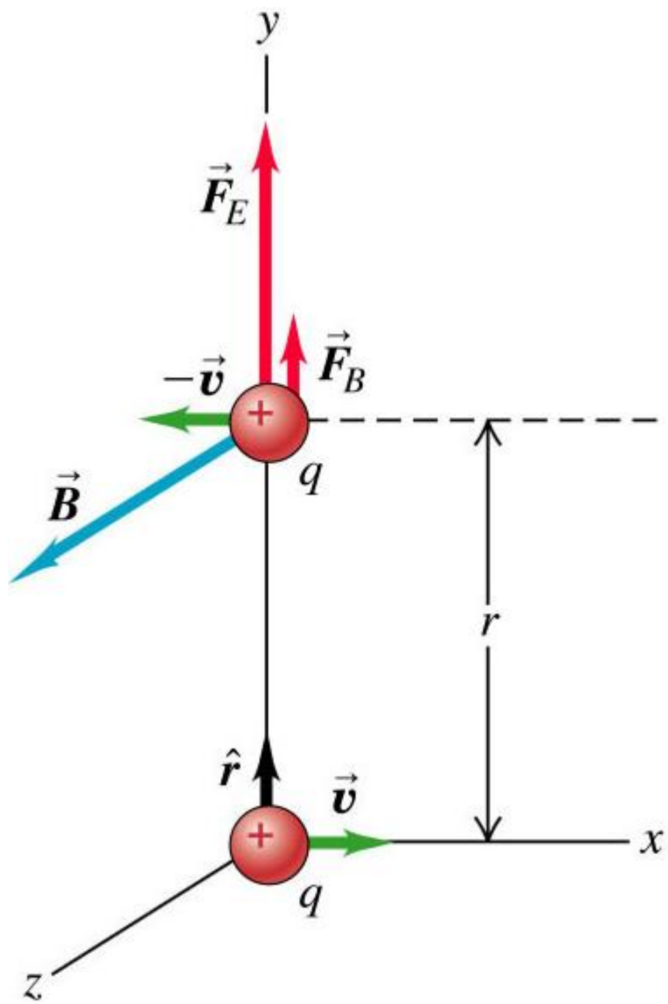
- So the forces are in the same direction and the ratio of their magnitudes is:

$$\frac{F_B}{F_E} = \frac{\mu_0 q^2 v^2}{4\pi r^2}\,\frac{4\pi\varepsilon_0 r^2}{q^2} = \varepsilon_0\mu_0 v^2$$

- From electromagnetic waves: $\varepsilon_0\mu_0 = \dfrac{1}{c^2}$

- And: $\dfrac{F_B}{F_E} = \dfrac{v^2}{c^2}$

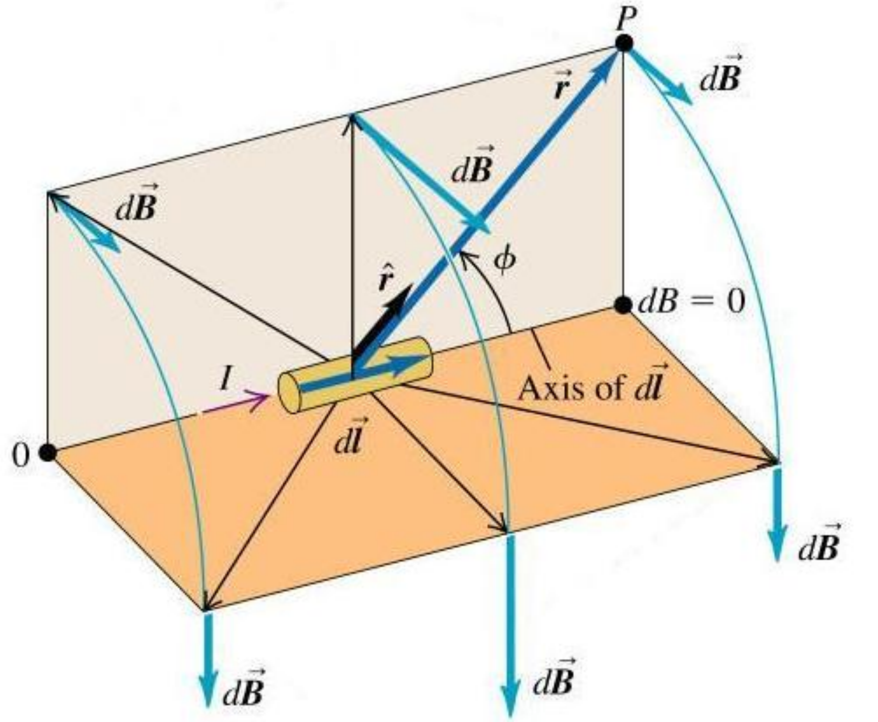
- Thus for velocities much less than the speed of light the magnetic force is small compared to the electric force.
- We can view the magnetic force as a relativistic correction to the electrostatic force



28.2 Magnetic field of current element

- **Magnetic field of a current**:

Point charge: $\left|\vec{B}\right| = \left|\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}\right| = \frac{\mu_0}{4\pi}\frac{qv\sin\varphi}{r^2}$



  - We know the magnetic field produced by a moving charge q
  - If we have *n* charges per unit volume moving at an average speed of v_d through an element of wire with cross-sectional area *A* then the charge moving in the element is:

$$dQ = nqAdl$$

  - The magnitude of the magnetic field at any point *P* is:

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d\sin\varphi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d A dl\sin\varphi}{r^2}$$

28.2 Magnetic field of current element

– But:

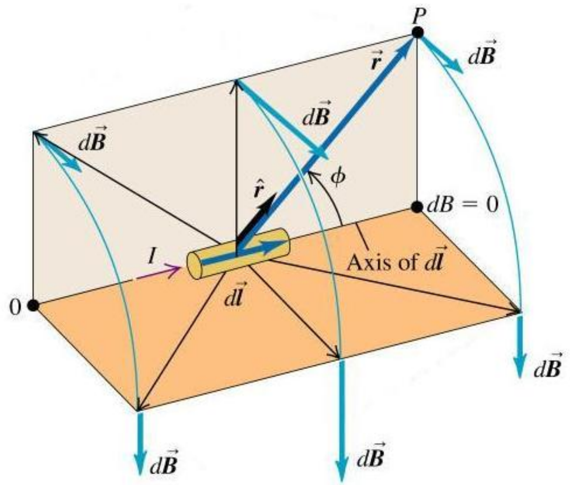
$$n|q|v_d A = I$$

– So:

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\varphi}{r^2}$$

– Or in vector form:

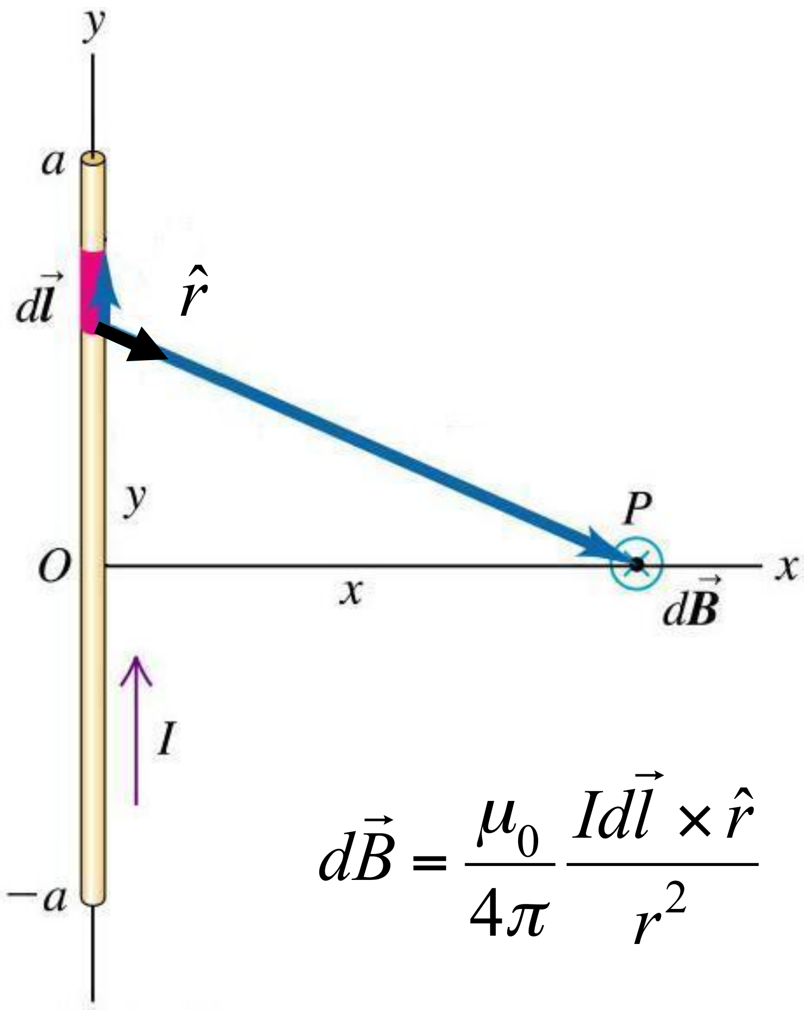
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$



- Law of Biot and Savart:
  - The magnetic field due to a current is given by the following line integral:

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{l} \times \hat{r}}{r^2}$$

- **Magnetic field of current carrying wire:**
  - Consider a wire of length $2a$ carrying a current I.
  - Divide the wire into elemental segments of length $dL=dy$.
  - Consider the magnetic field on the x-axis.
  - Assign an elemental length vector to each segment.
  - By the right-hand rule the field is directed into the page.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

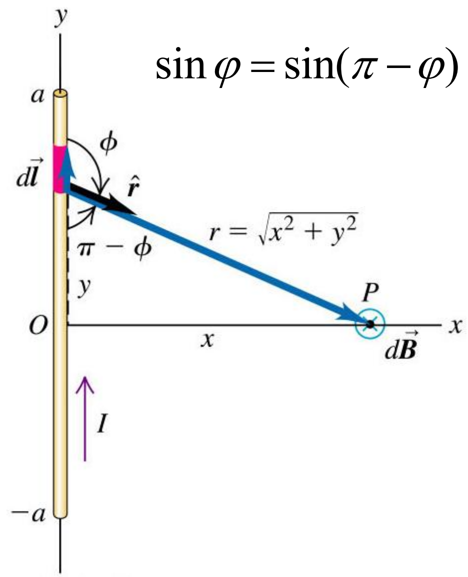
28.3 Magnetic field of long, straight wire

– Use the Biot-Savart line integral:

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int \frac{dl\sin\varphi}{r^2}(-\hat{k})$$

$$= -\frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{xdy}{\left(x^2+y^2\right)^{\frac{3}{2}}}\hat{k}$$

$$\sin\varphi = \sin(\pi-\varphi) = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$



– The result of this integral is:

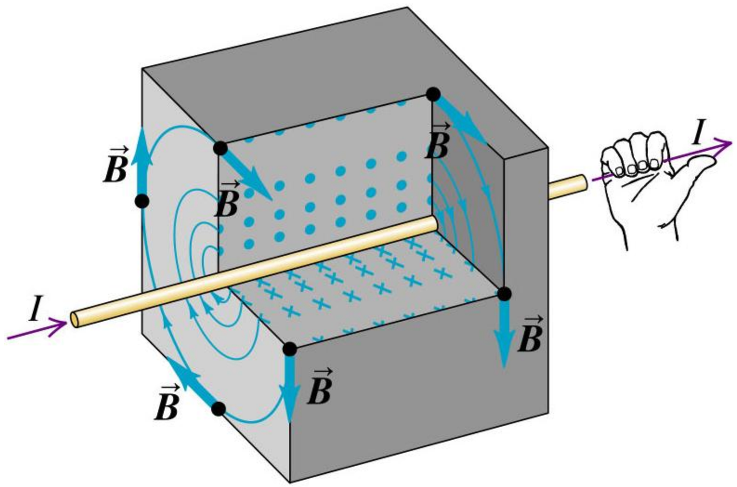
$$\vec{B} = -\frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}\hat{k}$$

See: “Line integral for lecture 14.pdf”

– If x is much less than a (eg. a long wire):

$$\lim_{a\to\infty}\left|\vec{B}\right| = \frac{\mu_0 I}{2\pi x}$$

– The resulting field encircles the wire.



- **Force between parallel conductors**:

  - Consider two parallel wires separated by a distance *r*.

  - The lower wire sets up a magnetic field as shown.

  - The force on a length *L* of the top wire is:

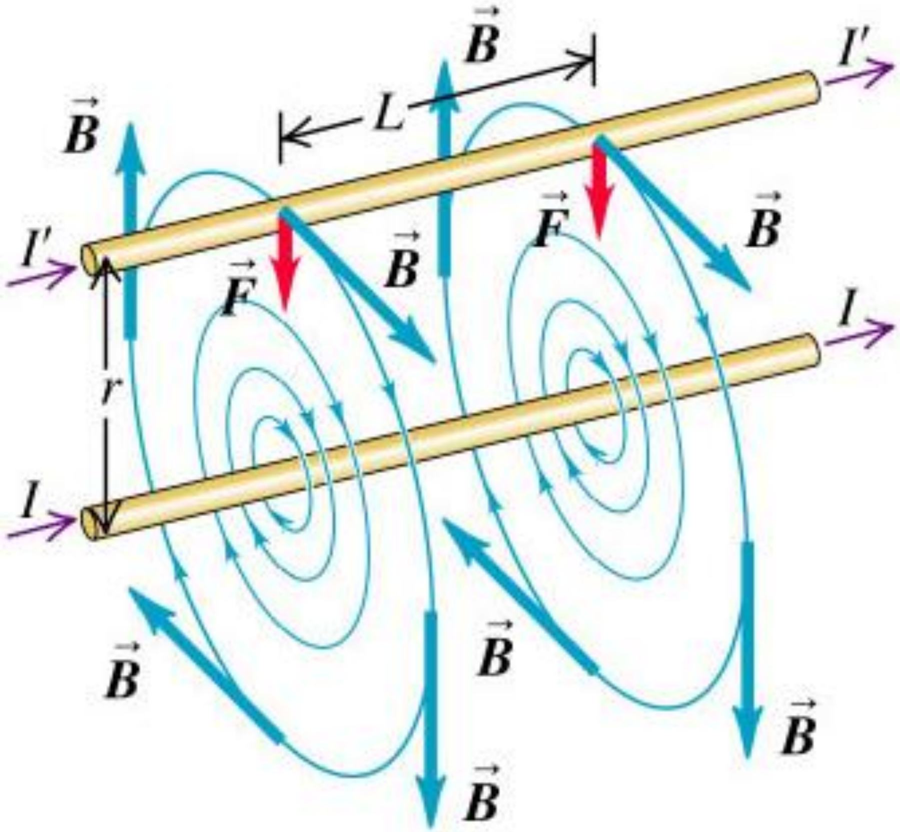
$$\vec{F} = I'\vec{L} \times \vec{B}$$

Slide 199

  - This is directed downwards. Using our calculation for the B field from the last slide, this force has magnitude

$$F = I'LB = \frac{\mu_0 II'L}{2\pi r}$$

  - So the force per unit length is: $\dfrac{F}{L} = \dfrac{\mu_0 II'}{2\pi r}$

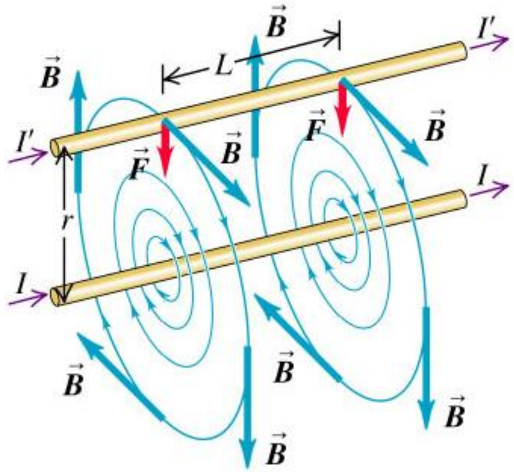


Sources of magnetic field and Ampere’s law

Read sections 28.1 to 28.5

Read sections 29.

28.4 Force between parallel conductors



– The current in the top wire also sets up a magnetic field (not shown).

– The lower wire experiences a force upwards as a result.

– This shows that parallel currents are attracted to each other.

– If one of the currents were reversed, both forces would be reversed also.

– So antiparallel currents repel each other.

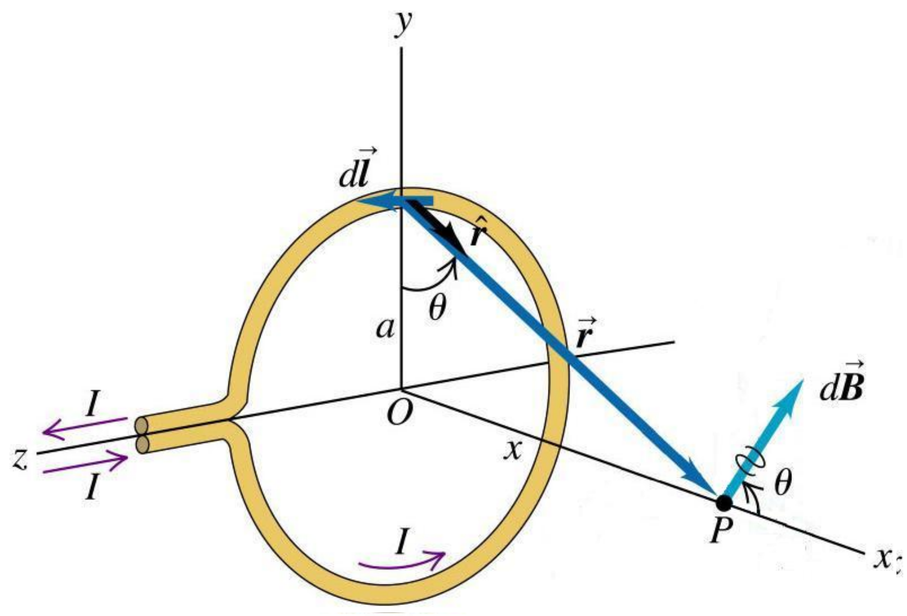
- Definition of the ampere:

  – Two wires of infinite length *1 m* apart in empty space with *1* ampere of current through them will experience a force per per metre of

$$2 \times 10^{-7}\, Nm^{-1}$$

28.5 Magnetic field of a circular loop

- Magnetic field of a circular loop:
  - Consider a loop of wire, radius a carrying a current I.
  - Divide the loop into elemental segments.
  - Consider the magnetic field from one segment at a point *P* on the *x*-axis.
  - The field direction is perpendicular to both $\vec{r}, d\vec{L}$ given by the right hand-rule.
  - Its magnitude is:



$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{r^2} = \frac{\mu_0 I}{4\pi}\frac{dl}{\left(x^2 + a^2\right)}$$

Biot-Savart

28.5 Magnetic field of a circular loop

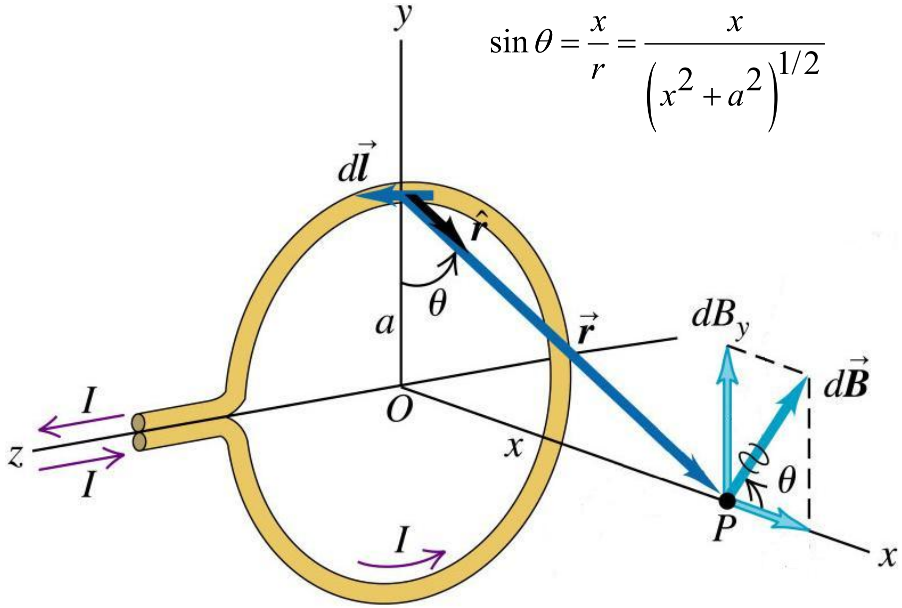
– Breaking into components:

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{adL}{\left(x^2+a^2\right)^{3/2}}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi}\frac{xdL}{\left(x^2+a^2\right)^{3/2}}$$

$$\cos\theta = \frac{a}{r} = \frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$\sin\theta = \frac{x}{r} = \frac{x}{\left(x^2+a^2\right)^{1/2}}$$



– By symmetry, all the *yz-plane* components must sum to zero.

– The total *x* component is:

$$B_x = \int \frac{\mu_0 I}{4\pi}\frac{adL}{\left(x^2+a^2\right)^{3/2}} = \frac{\mu_0 Ia}{4\pi\left(x^2+a^2\right)^{3/2}}\int dL$$

$$= \frac{\mu_0 Ia^2}{2\left(x^2+a^2\right)^{3/2}} \qquad \text{since} \qquad \int dL = 2\pi a$$

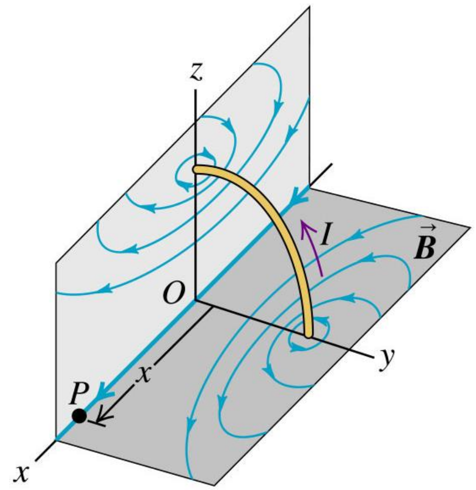
28.5 Magnetic field of a circular loop

– If we have a coil of *N* such loops the magnetic field becomes:

$$B_x = N\frac{\mu_0 I a^2}{2\left(x^2+a^2\right)^{3/2}}$$

– At the centre (x=0) of such a coil:

$$B_x = \frac{\mu_0 N I}{2a}$$



Ampere’s law

Read section 28.6 to 28.9

- **Ampere's law**

  - Ampere's law deals with closed-loop **line integrals**:

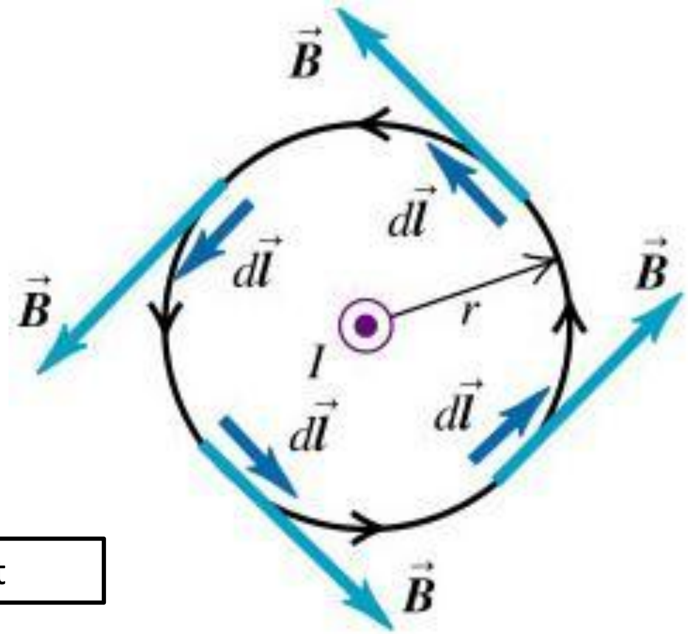
$$\oint \vec{B} \bullet d\vec{l}$$

  - Consider the circular line integral shown.

  - The magnetic field strength on the circle is:

$$B = \frac{\mu_0 I}{2\pi r}$$

Slide Biot Savart



  - At every point the path of integration is parallel to the field, so the integral reduces to:

$$\oint \vec{B} \bullet d\vec{l} = \oint B\,dl = \frac{\mu_0 I}{2\pi r}\oint dl$$

$$= \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

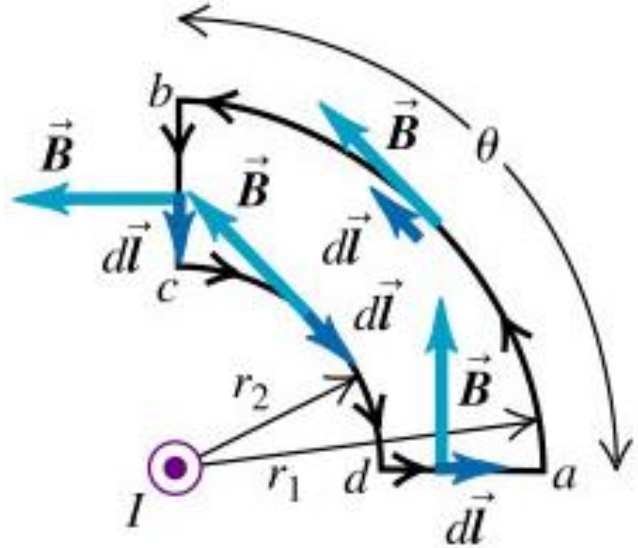
Consider another path integral for $\boldsymbol{B}$.

NB: Only circumferential and not radial sections will (individually) make a contribution!

$$\oint \vec{B} \bullet d\vec{l} = \int_a^b \vec{B} \bullet d\vec{l} + \int_c^d \vec{B} \bullet d\vec{l}$$

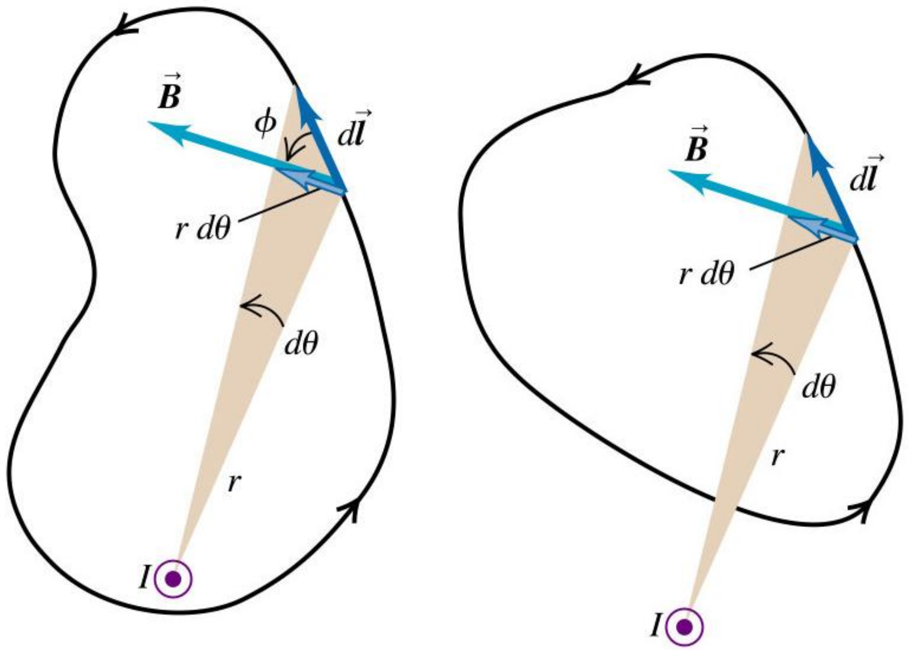
$$= \int_a^b B\,dl + \int_c^d -B\,dl$$

$$= \frac{\mu_0 I}{2\pi r_1}\left(r_1\theta\right) - \frac{\mu_0 I}{2\pi r_2}\left(r_2\theta\right) = 0$$



28.6 Ampere's law

- Consider the general integration paths shown.



- By the figure:

$$\vec{B}\bullet d\vec{l} = Bdl\cos\varphi = B\,r\,d\theta$$

- So:

$$\oint \vec{B}\bullet d\vec{l} = \oint B\,r\,d\theta = \oint \frac{\mu_0 I}{2\pi r} r\,d\theta$$

$$= \frac{\mu_0 I}{2\pi}\oint d\theta = \mu_0 I$$

- Ampere's law:

$$\oint \vec{B}\bullet d\vec{l} = \mu_0 I_{enclosed}$$

Compare to loop integral of $\vec{E}\bullet d\vec{l}$

28.7 Ampere's law to calculate B field

- **Magnetic field inside a wire**:
  - Consider a cylindrical conductor radius *R* carrying a **uniform** current *I*.
  - To find the field inside the wire we integrate around a circle radius *r*<*R* inside the wire.
  - The field is parallel to the path on the circle so the integral becomes:

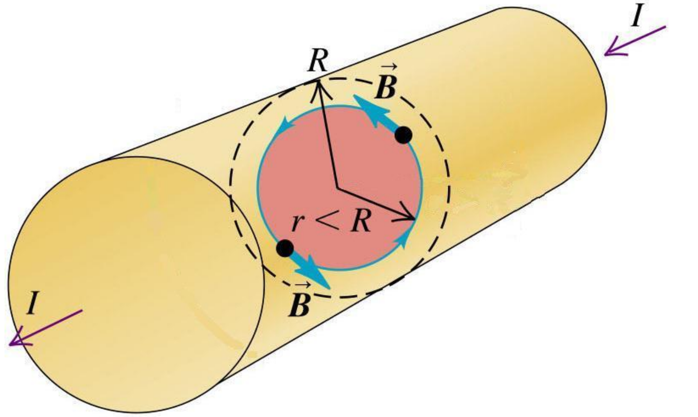
$$\oint \vec{B} \bullet d\vec{L} = B\oint dL = B\left(2\pi r\right) = \mu_0 I_{encl}$$

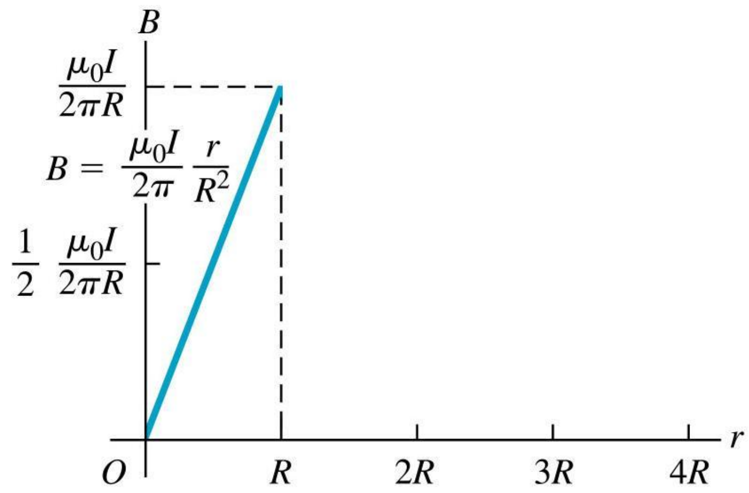
  - The current through the disc is:

$$I_{encl} = I\,\frac{\pi r^2}{\pi R^2} = \frac{Ir^2}{R^2}$$

  - So the magnetic field magnitude is:

$$B = \frac{\mu_0 I r}{2\pi R^2}$$





Ampere's Law and Electromagnetic induction

Read sections 29.1 to 29.6

28.7 Ampere's law to calculate B field

– To find the field strength outside the wire we integrate around a circle radius *r>R*.

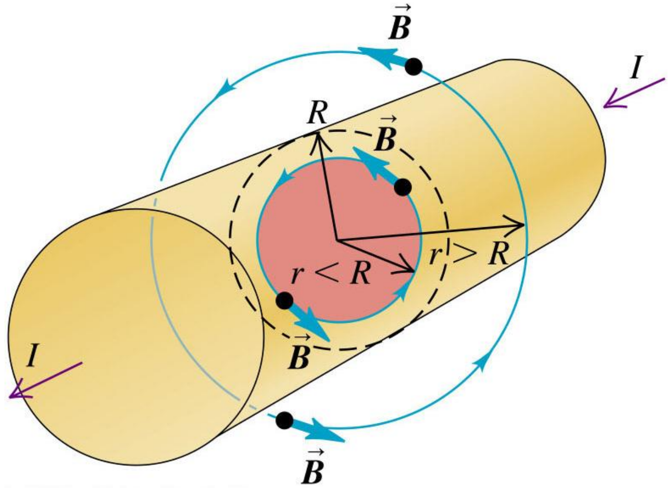
– The field is constant and parallel to the path on the circle so the integral is:

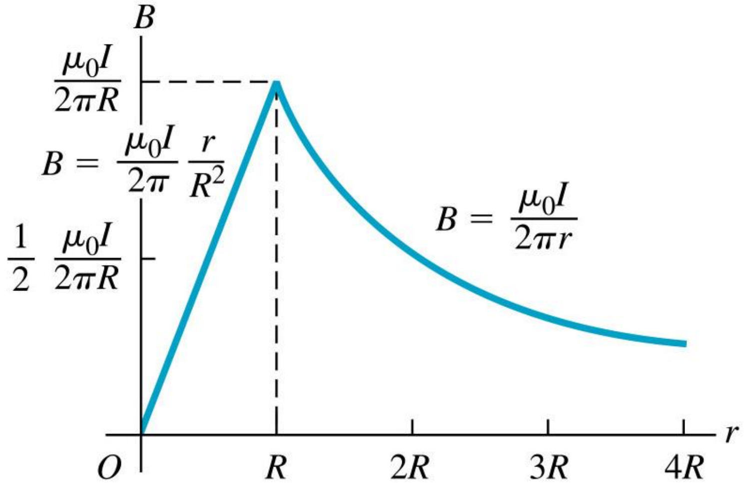
$$\oint \vec{B} \bullet d\vec{L} = B\oint dL = B\left(2\pi r\right)$$

– The current through the disc is the total current *I.*

– So the field strength is:

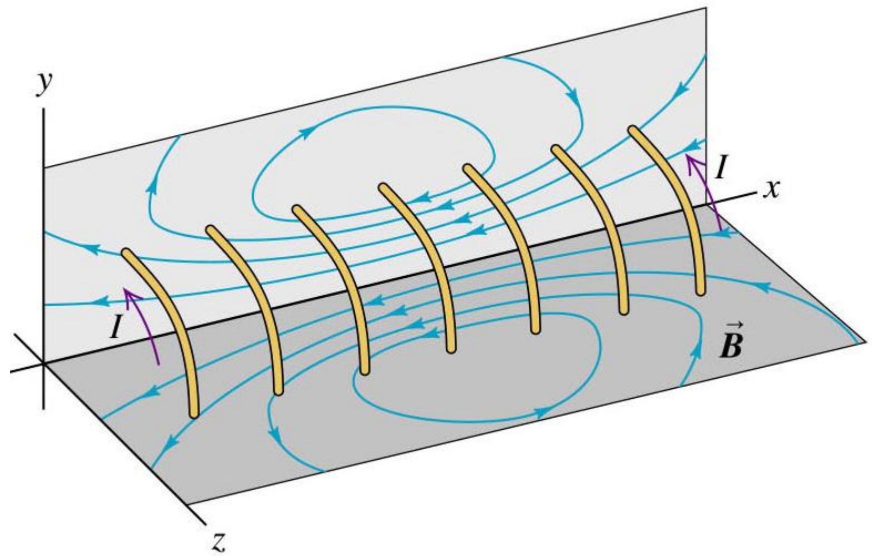
$$B = \frac{\mu_0 I}{2\pi r}$$





Magnetic field of solenoid

- Helical winding
- Same current in all turns
- Field at any point is vector sum of fields due to each turn
- Calculation shows that:
  - Half of field lines “leak” out sides
  - Half emerge from ends
  - Field at ends is half of value at centre
  - For length >> diameter field near centre is nearly uniform
  - Near centre, field outside coil is very weak



- Estimate field at centre?
  - Yes, use Ampere’s law

Magnetic field of solenoid

Solenoid of *n* turns per unit length

Using Ampere's law on loop abcd:

$$\oint \vec{B} \bullet d\vec{l} = BL = \mu_0 I_{encl} = \mu_0 nLI$$

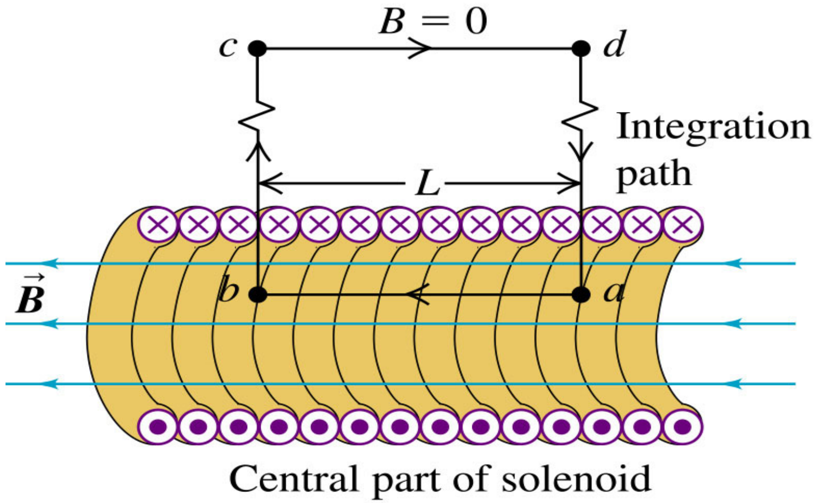
Ie. Paths bc and da are zero due to orthogonality of ***B*** and ***l***.

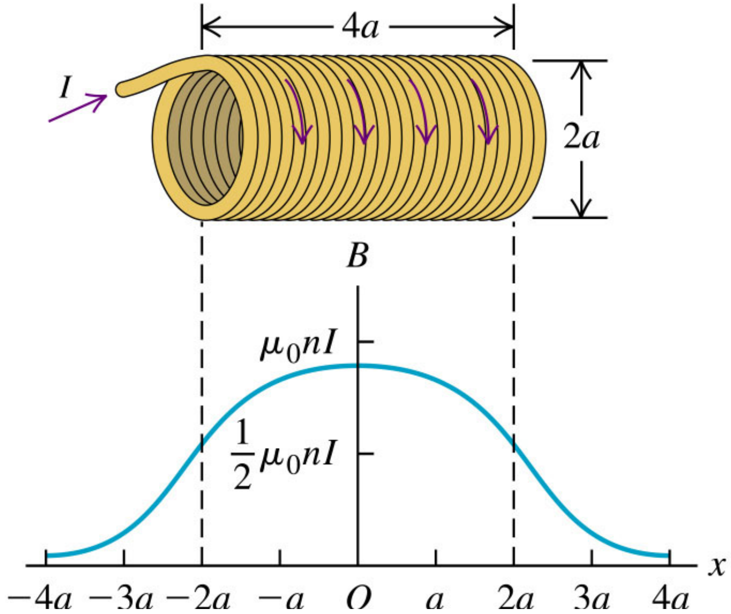
Path cd is zero since field is zero outside.

$$B = \mu_0 nI$$

Slide 224 direct calculation: $B_x = \dfrac{\mu_0 NI}{2a}$

More complete calculation shows:

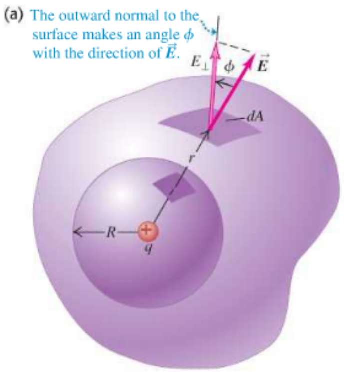


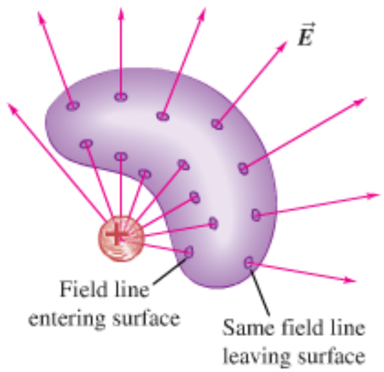


Gauss and Ampere Laws
Closed Surface and Line Integrals

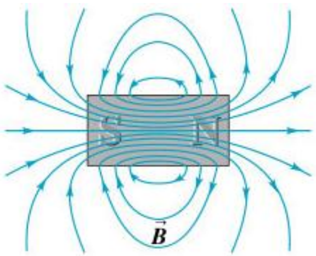
Gaussian surface integrals

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$



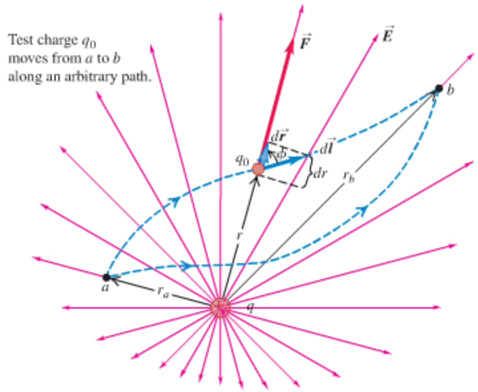


$$\Phi_B = \oint \vec{B} \bullet d\vec{A} = 0$$



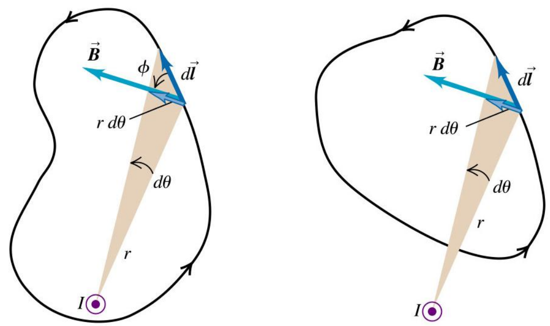
Amperian line integrals

$$\oint \vec{E} \bullet d\vec{l} = 0$$



$$V_{ab} = V_a - V_b = \int_a^b \vec{E} \bullet d\vec{l}$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I_{enclosed}$$



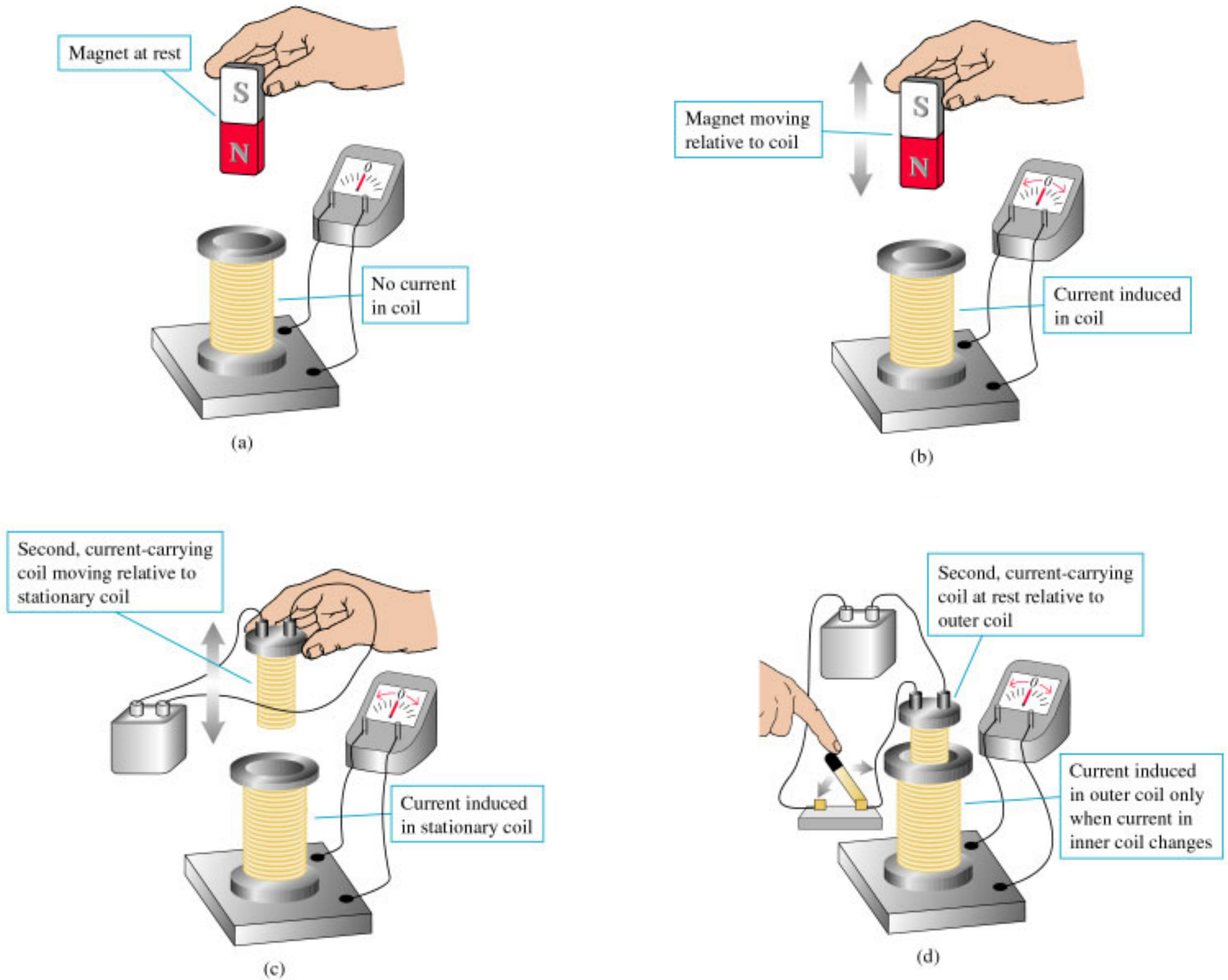
Electromagnetic induction

Read sections 29.1 to 29.6

29. Electromagnetic Induction

- Batteries produce **emf**, but this is not the most common source!
- Magnetic swipe card:
  - Information stored in 1 μm magnetised regions on card strip
  - These move past reading head when swiped
  - An **induced emf** is produced, which then is amplified
- Electromagnetic induction
  - Discovered by Michael Faraday (1830): **Faraday's Law**
  - Also contribution by Joseph Henry (USA)
  - Move magnet near coil connected to galvanometer -> induced current
  - Or replace magnet with coil connected to battery
  - Or keep both coils fixed and change current in one -> induced current in the other
  - Changing magnetic flux creates the emf: basis for **electric energy conversion** found in **motors**, **generators**, and **transformers**

Electromagnetic induction experiments



Faraday's Law

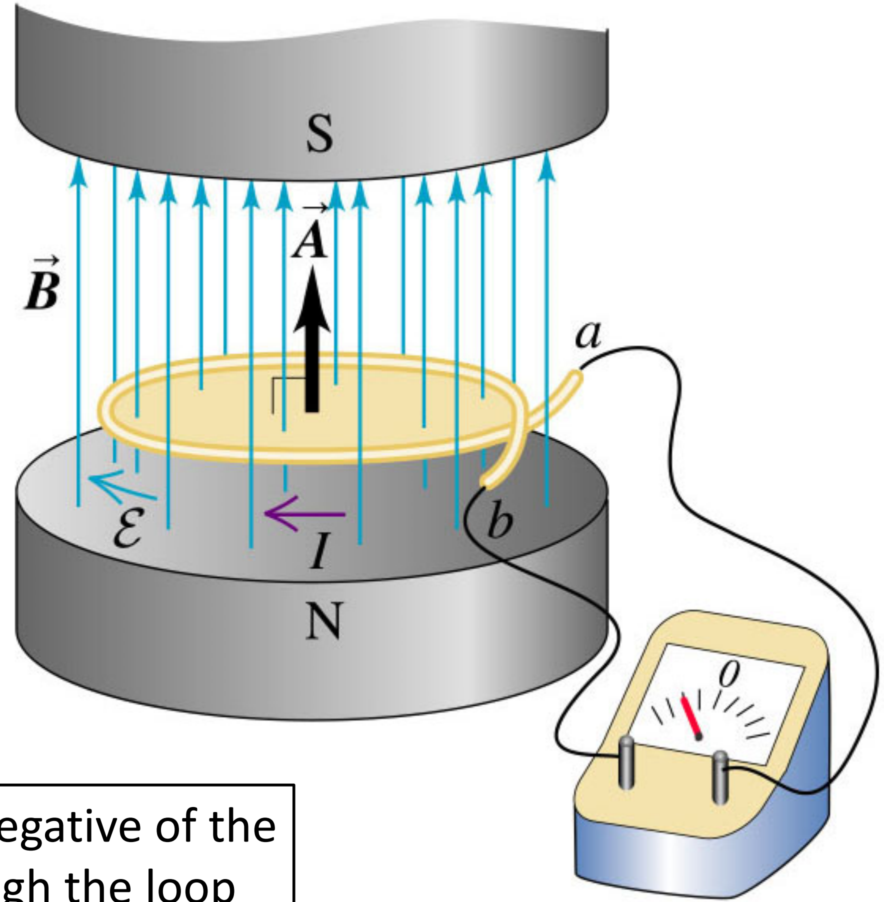
Magnetic flux:

$$\Phi_B = \vec{B} \bullet \vec{A} = BA\cos\phi$$

An emf is induced when a magnetic flux Φ_B through a coil is changing:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday's law of induction



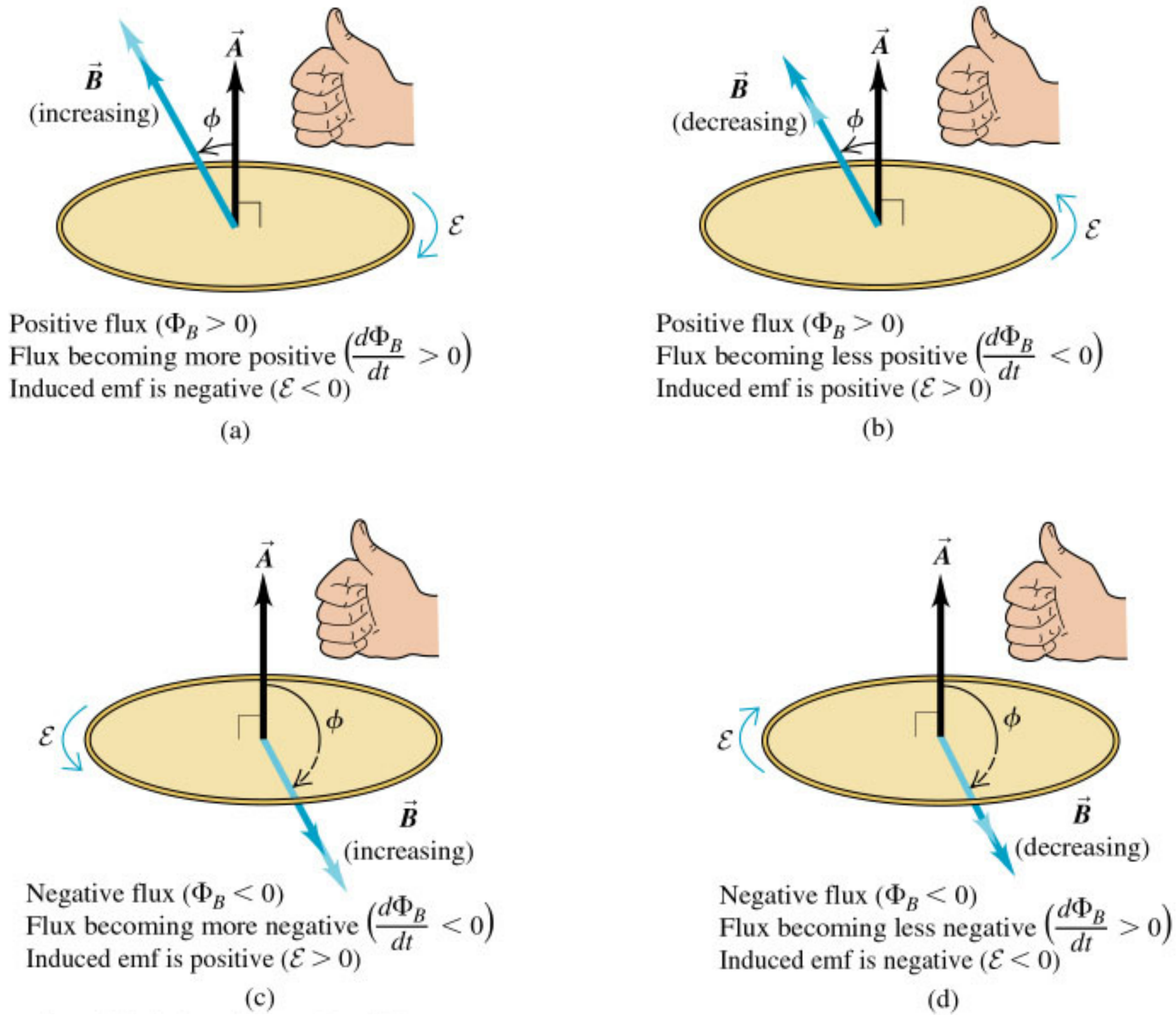
The induced emf in a closed loop equals the negative of the time rate of change of the magnetic flux through the loop

Direction of the emf in Faraday's Law

Emf induction is due to the **change** in magnetic flux, not just the flux alone!!

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

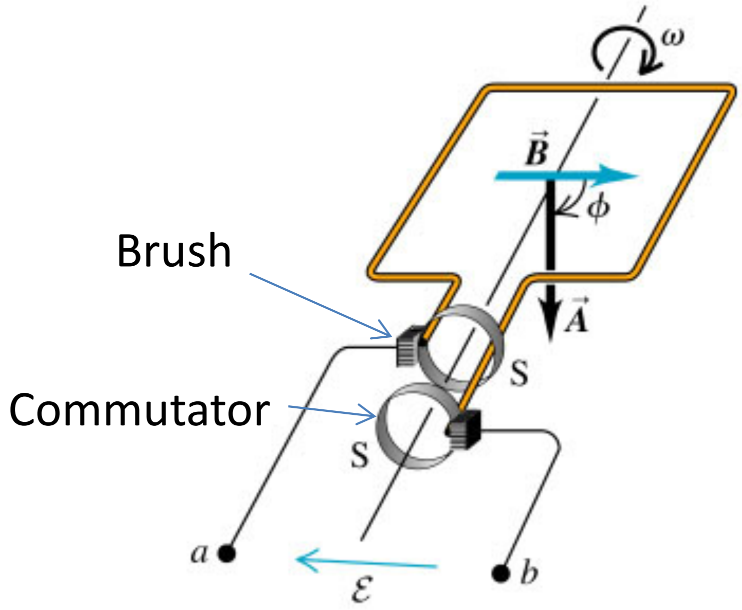
Use right hand rule to determine direction of emf: Thumb points in direction of area vector $\boldsymbol{A}$, fingers give sense of positive emf.



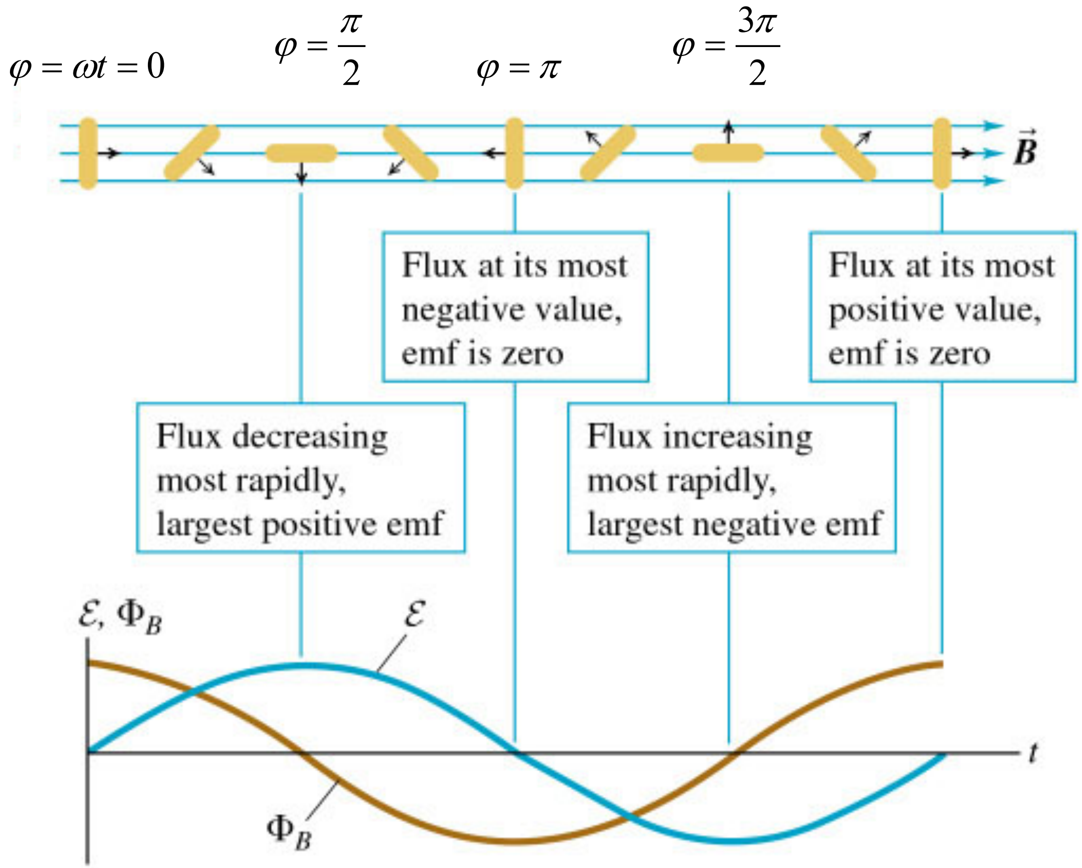
Electromagnetic induction

Read sections 29.1 to 29.6

Dynamo – AC generator (“alternator”)



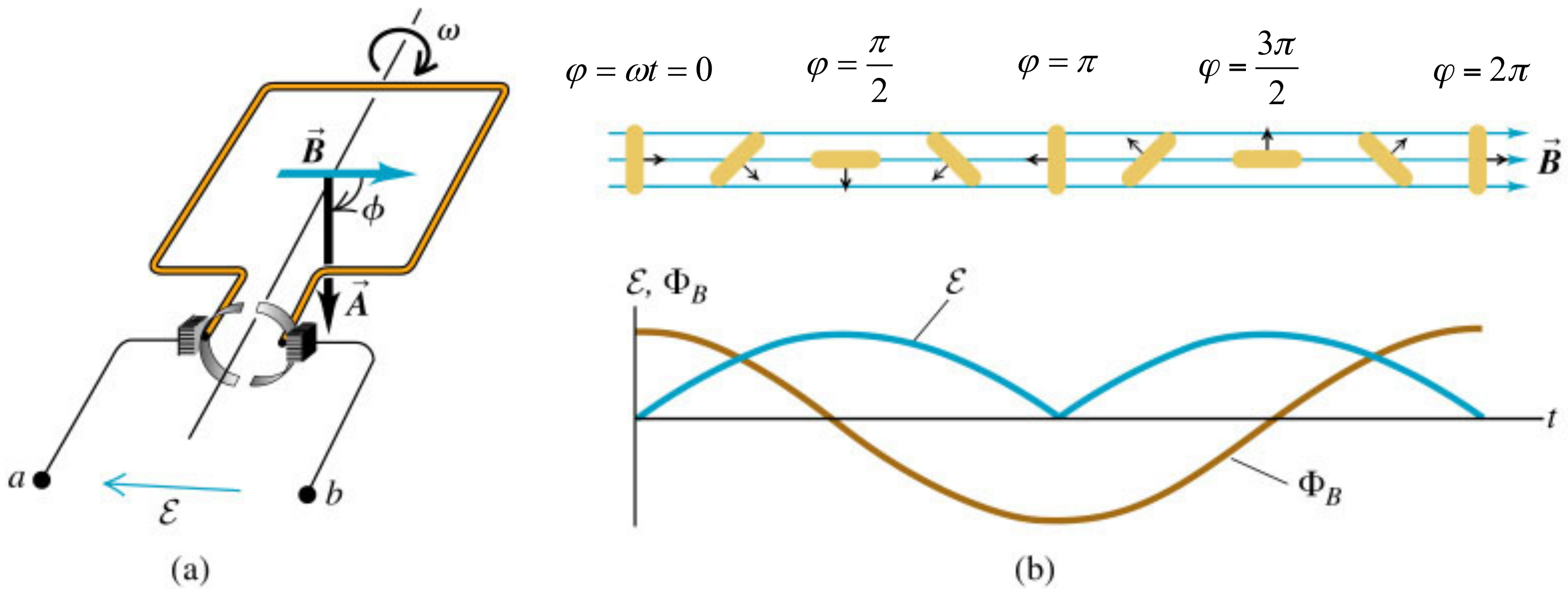
(a)



(b)

$$\Phi_B = BA\cos\phi = BA\cos\omega t \qquad \varepsilon = -\frac{d\Phi_B}{dt} = \omega BA\sin\omega t$$

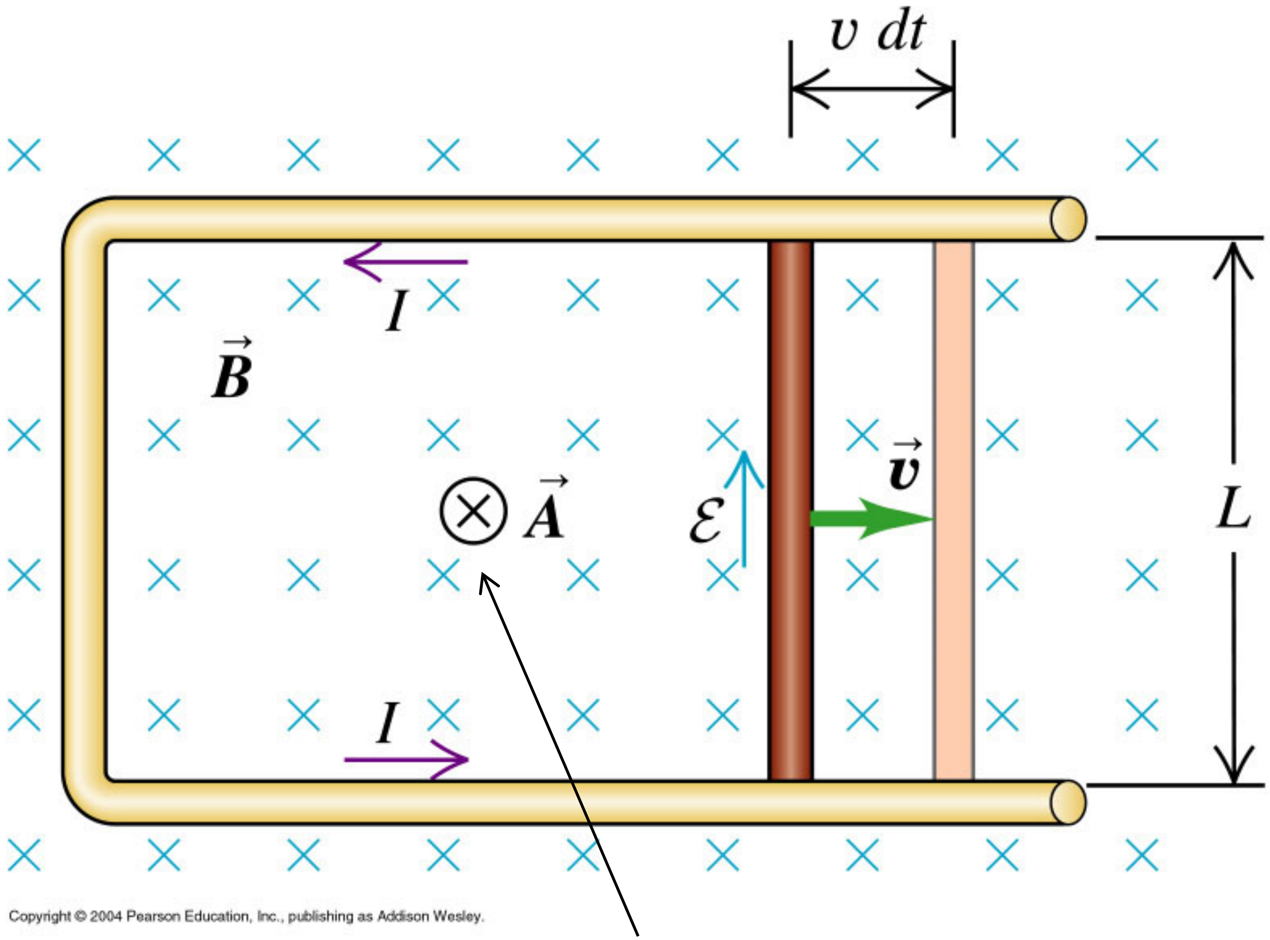
DC dynamo



Use a divided **commutator** to make a DC **dynamo.**

This **rectifies** the emf signal.

emf due to motion of conductor in B-field



Area vector pointing
into the screen

Motional emf

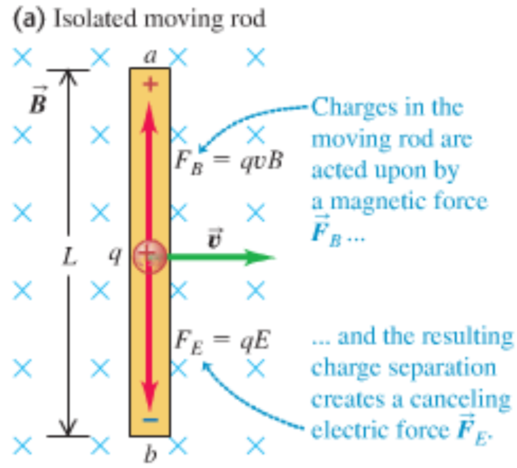
- Consider magnetic forces on charges in conductor

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

- Magnetic force on +ve charge drives it bottom to top
- Then: Net +ve charge at top, -ve charge at bottom

→ This begins to generate an E-field from top to bottom

- Charge builds up until $qE = qvB$
- Top at higher potential: $V = EL = vBL$



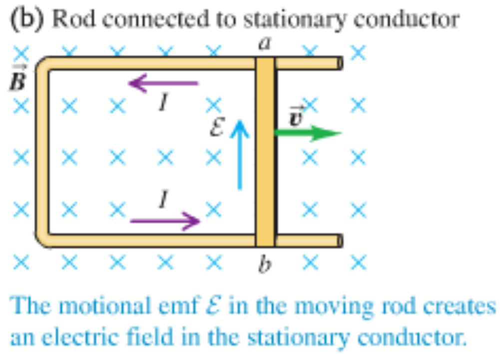
- If this rod is sliding on a U-shaped conductor, then anticlockwise current arises
- Conductor moving in B-field is source of **motional emf:** $\varepsilon = vBL$

- But

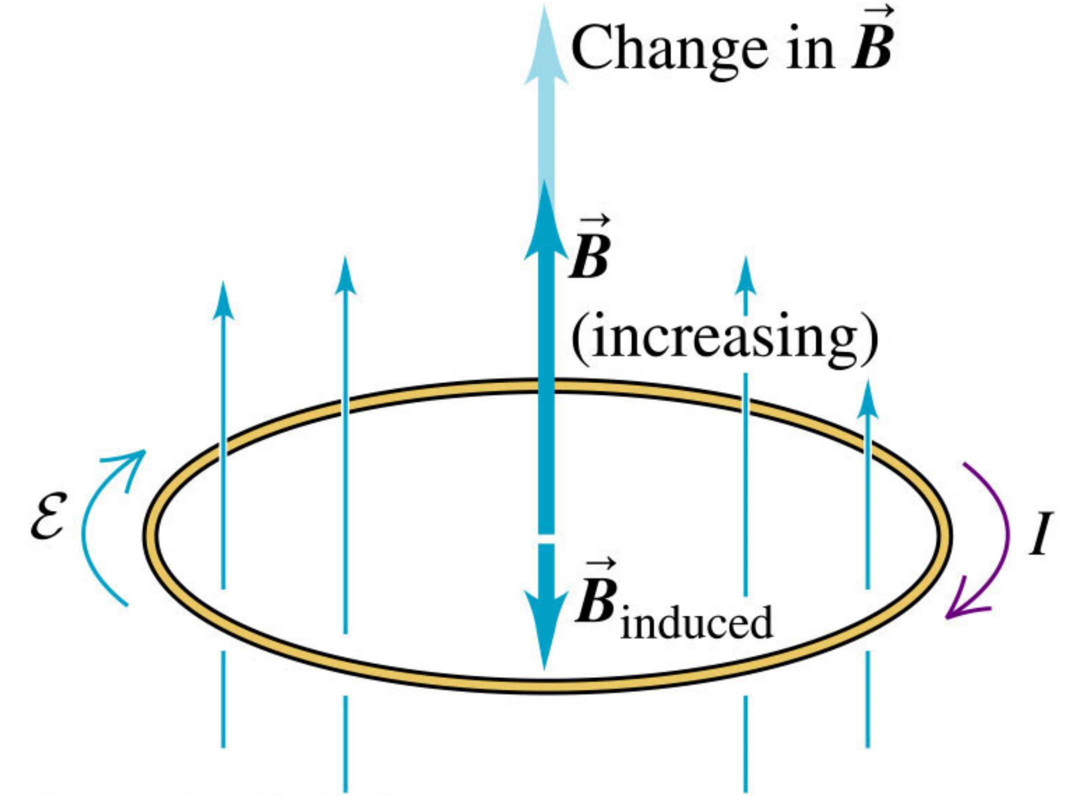
$$-\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -BLv$$

- So same result from Faraday's law

(area vector points *into* screen)



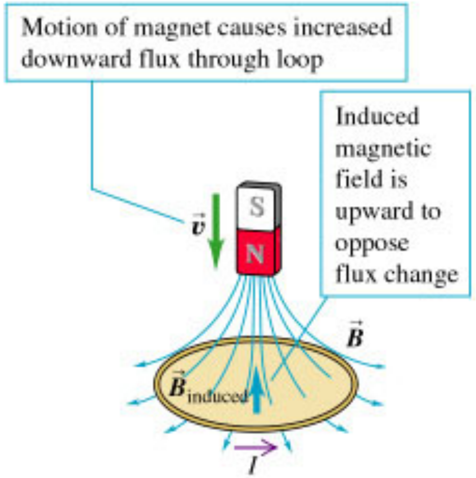
29.3 Lenz's law



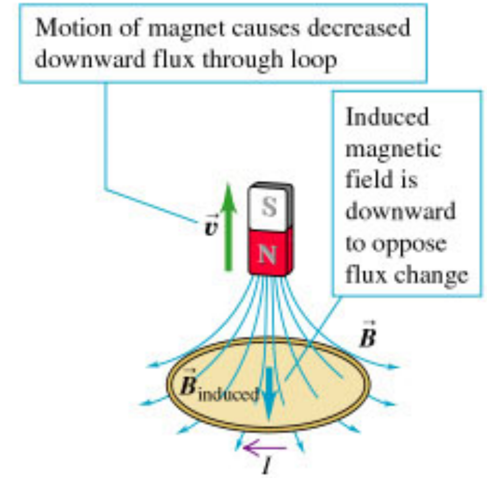
Lenz's law : the direction of any magnetic induction effect is such as to oppose the effect producing it

...can be derived from Faraday's Law

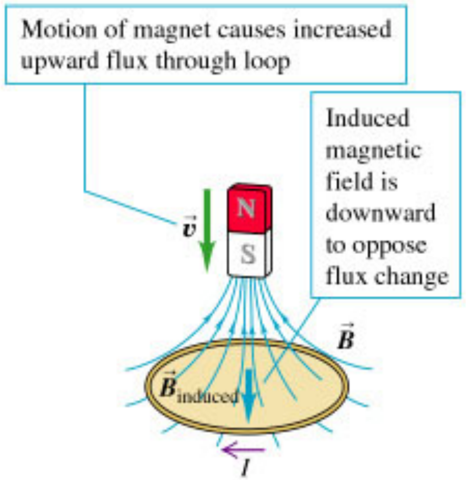
Lenz's law in practice



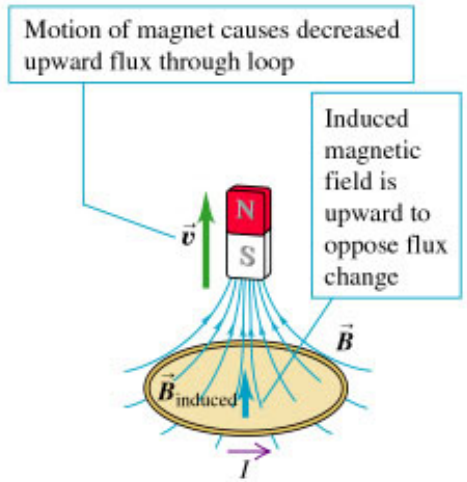
(a) To produce this induced field, induced current must be *counterclockwise* as seen from above loop



(b) To produce this induced field, induced current must be *clockwise* as seen from above loop

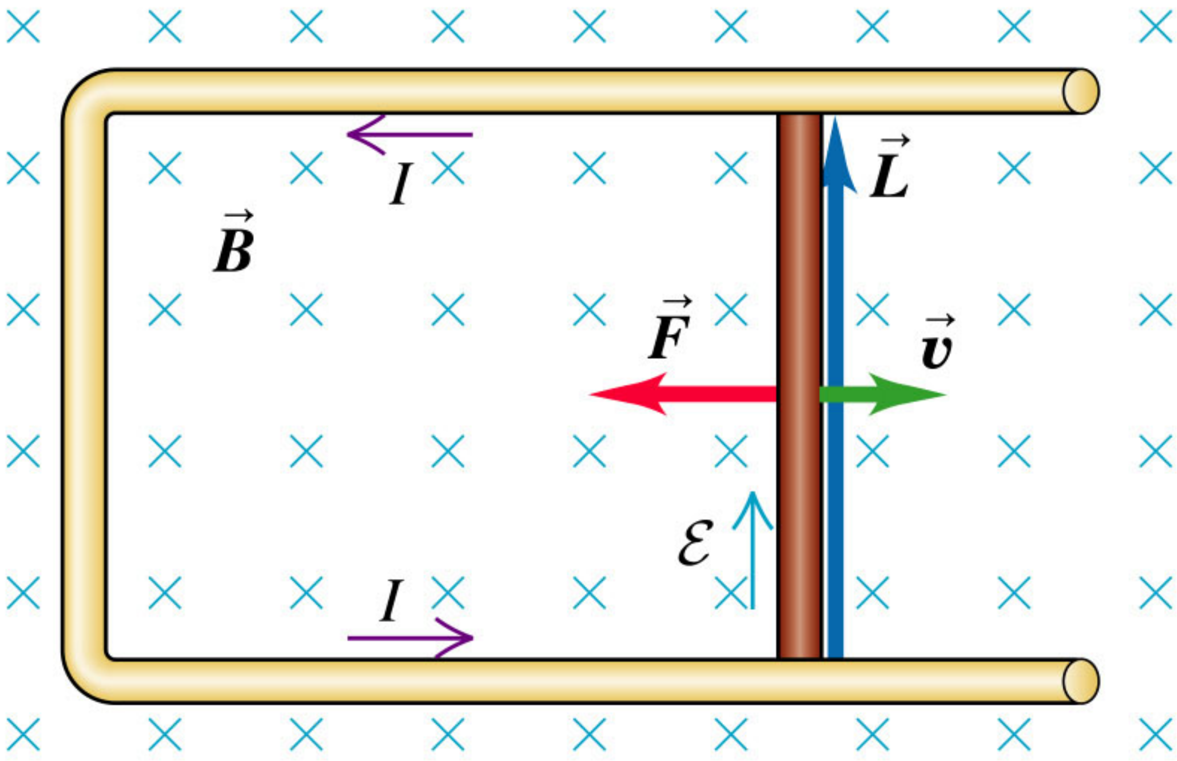


(c) To produce this induced field, induced current must be *clockwise* as seen from above loop



(d) To produce this induced field, induced current must be *counterclockwise* as seen from above loop

Lenz's law

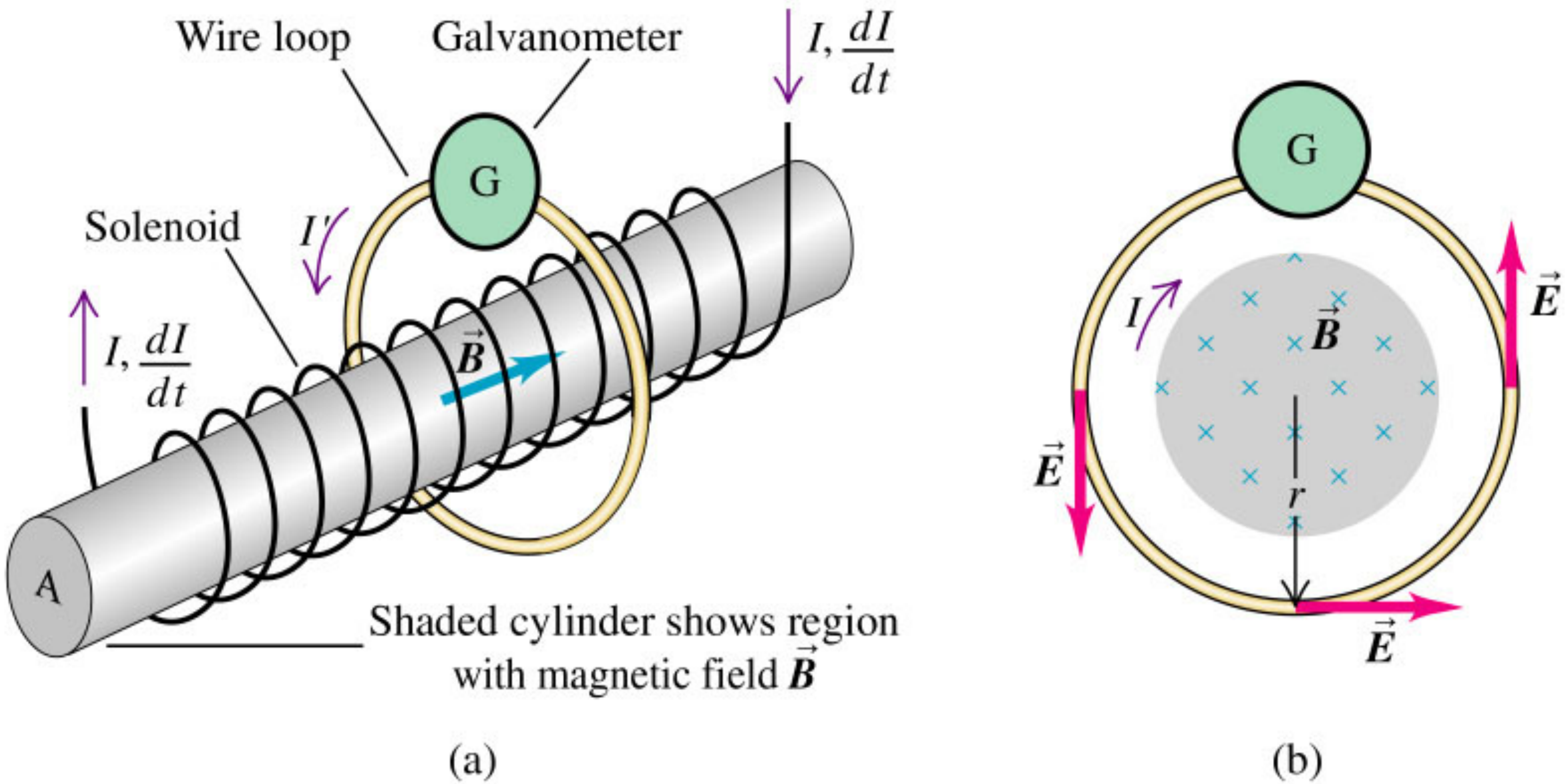


$$\vec{F} = I_{induced}\vec{L} \times \vec{B}$$

Induced current creates magnetic field to oppose increase in flux

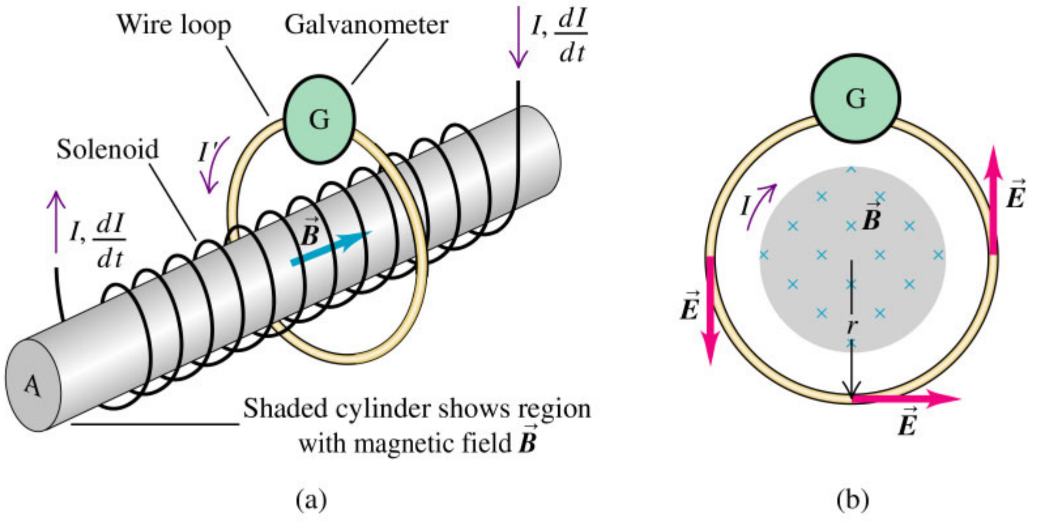
The current also causes opposing force on slider: it must! Why?

Induced electric field



Induced electric fields

- A long, thin solenoid is encircled by a circular conducting loop.
- Electric field in the loop is what must drive the current.
- When the solenoid current I changes with time, the magnetic flux also changes, and the induced emf can be written in terms of **induced electric field**:



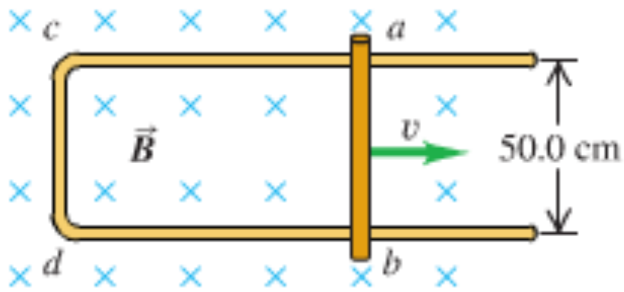
Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E}\cdot d\vec{l}}^{\text{Line integral of electric field around path}} = -\underbrace{\frac{d\Phi_B}{dt}}$$

Negative of the time rate of change of magnetic flux through path

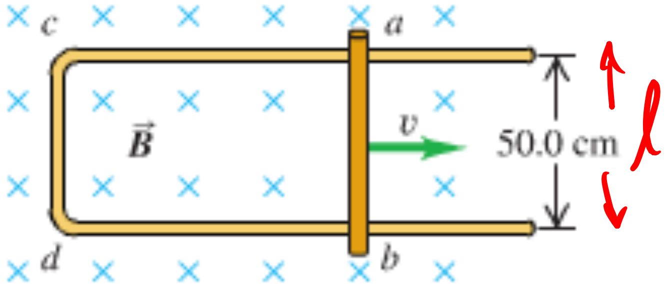
Slidewire motional emf

Figure 29.38 Exercise 29.25.



29.25. The conducting rod *ab* shown in Fig. 29.38 makes contact with metal rails *ca* and *db*. The apparatus is in a uniform magnetic field of 0.800 T, perpendicular to the plane of the figure (a) Find the magnitude of the emf induced in the rod when it is moving toward the right with a speed 7.50 m/s. (b) In what direction does the current flow in the rod? (c) If the resistance of the circuit *abdc* is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. You can ignore friction. (d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit $(I^2 R)$.

Slidewire motional emf



Faraday: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

$\Phi_B = \vec{B}\cdot\vec{A} = BA$ (Area vector into screen)

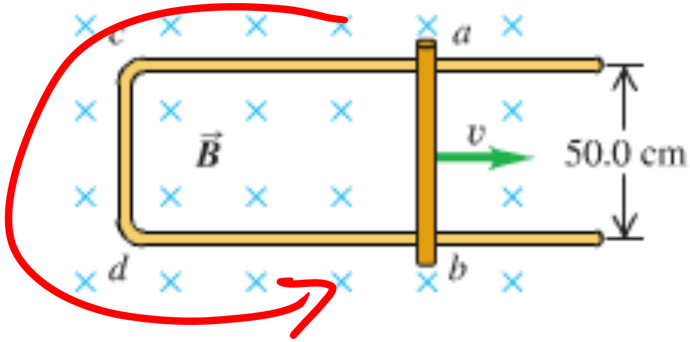
$$\frac{d\Phi_B}{dt} = \cancel{\frac{dB}{dt}}A + B\frac{dA}{dt} = B\frac{dA}{dt}$$

$$= B\,l\,\frac{dx}{dt}$$

$$= Blv$$

$$\mathcal{E} = -Blv = -0.8\,T \cdot 50\,cm \cdot 7.5\,m\,s^{-1} = -3\,V$$

Slidewire motional emf



$\mathcal{E} = -\frac{d\Phi_B}{dt}$

Lenz Law $\boxed{\checkmark}$

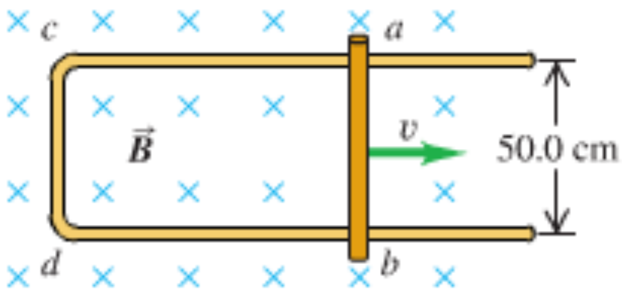
Force? maintain $v = 7.5\ ms^{-1}$

$R = 1.5\ \Omega$
$\mathcal{E} = -3\ V$ $\Big\}$ Ohm: $I = \frac{|\mathcal{E}|}{R} = \frac{3V}{1.5\Omega} = 2.0\ A$

$\vec{F} = I\vec{l} \times \vec{B}$

$= IlB = 2.0A \cdot 50cm \cdot 0.8T = 0.8N$

Slidewire motional emf



Power slider: $F \cdot v$

$= 0.8N \cdot 7.5ms^{-1}$

$= 6W$

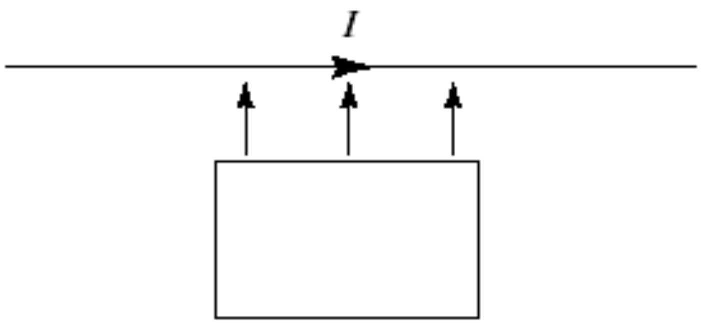
$P_{circuit} = I^2R$

$= (2A)^2 \cdot 1.5\Omega$

$= 6W$

Concept test

A long, straight wire carries a steady current I. A rectangular conducting loop lies in the same plane as the wire, with two sides parallel to the wire and two sides perpendicular. Suppose the loop is pushed toward the wire as shown. Given the direction of I, the induced current in the loop is



1. clockwise.
2. counterclockwise.
3. need more information