# Quantum Physics PY1T20/PYU11P20

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## Course overview: what is quantum mechanics

Quantum mechanics/physics = our best description of the physics of everything.

Absolutely necessary to use it for small stuff, e.g. to describe and understand the properties of atoms, molecules, electrons, but almost certain to apply at all levels.

Developed since the early 1900s and radically different from the ‘classical’ physics that preceded it.

In this course you will learn what quantum mechanics is, how it describes our world, some of the things it explains, and how it explains them.

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# Introduction: waves and superposition

- This is an introductory lecture. Before the next lecture you should have :

- Reviewed the key concepts of waves and oscillations.
- Learnt about the principle of superposition, through waves and oscillations.
- Seen some new mathematics ('partial differential equations') we'll need later in the course.



# Simple harmonic motion (Uni. Phys. Ch 14) & ordinary differential equations

- e.g. mass on a spring

$$F = ma \Rightarrow m\frac{d^2x}{dt^2} = -kx$$

$$F = -\mu x$$

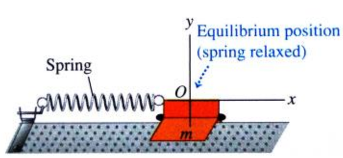
Restoring force

Displacement from equilibrium

"x(t) is a function, which if I differentiate twice, gives the same function times (-k/m)."

What functions can you think of that do this?

**14.1** A system that can have periodic motion.




This equation, and the physics it describes, is 'linear'.

In consequence :

- (constant) times (solution) = also a solution
- (solution) plus (solution) = also a solution

This adding together of valid solutions/physical situations to get another valid solution/physical scenario is called 'superposition'.

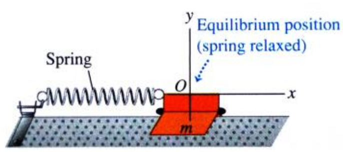
# Simple harmonic motion (Uni. Phys. Ch 14) & ordinary differential equations

- e.g. mass on a spring

$$F = -\mu x$$

Restoring force    Displacement from equilibrium

**14.1** A system that can have periodic motion.




$$F = ma \Rightarrow m\frac{d^2x}{dt^2} = -kx$$

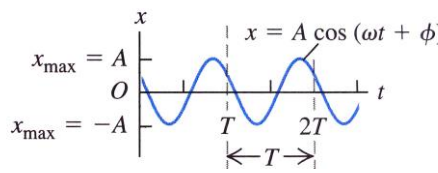
If we superpose solutions we build up a general solution :

$$x(t) = C\cos(\omega t) + D\sin(\omega t)$$

$$\omega = \sqrt{k/m}$$

$$x(t) = A\cos(\omega t + \phi)$$

An oscillation with amplitude A, frequency $\omega$, phase (offset) $\phi$.




Constants A, $\phi$ (or C, D) are determined by the initial conditions – position and velocity at (say) t=0.



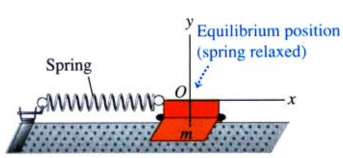
# Simple harmonic motion (Uni. Phys. Ch 14) & complex numbers

- e.g. mass on a spring

$$F = -\mu x$$

Restoring force    Displacement from equilibrium

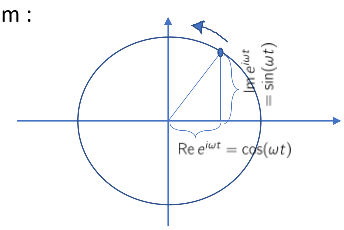
**14.1** A system that can have periodic motion.




Another very useful form is :

$$x(t) = \mathrm{Re}(De^{\pm i\omega t})$$

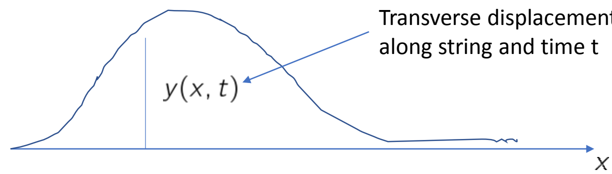
We can usually* leave taking the real part to the end and think of x as complex – allows us to interpret the oscillation as circular motion on an Argand diagram :



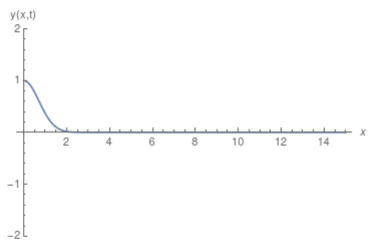

* Provided we're only interested in working out answers which are linear in x(t), e.g. 3x, dx/dt, but not e.g. $x^2$ – $\mathrm{Re}\, x^2 \neq (\mathrm{Re}\, x)^2$

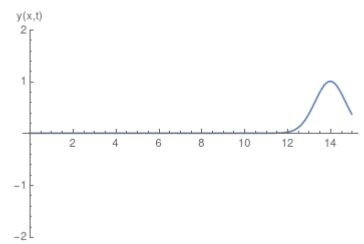
# Superposition & waves

- Waves are a moving pattern of some displacement of a physical quantity from equilibrium.
- E.g. a string in 1D:




- Wave 'moves' on a string at velocity $c = \sqrt{T/\rho}$ :

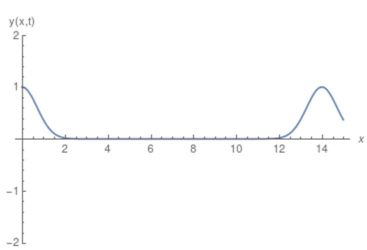


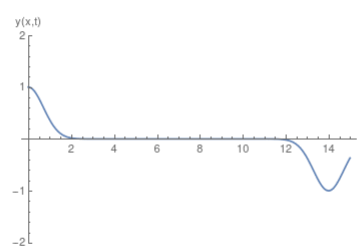






# Superposition & waves

- These motions – waves on strings – are 'superposable' :

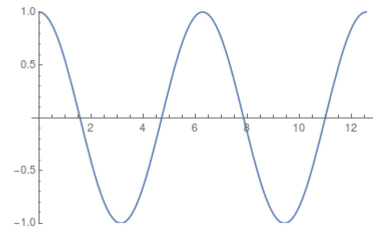



- Notice also: sometimes waves add to reinforce one another, sometimes they cancel one another out = 'Constructive' and 'destructive' interference.

- Caveat: the superposition principle is only approximately valid for mechanical waves. It holds when the displacements are small – specifically, small enough for the restoring force to be proportional to the displacement, like in the simple harmonic motion example.

# Harmonic waves




- * Special form of wave in which every point in the medium executes simple harmonic motion.
- Basic solutions in 1D:

'Angular frequency' $\omega = 2\pi f = 2\pi / T$

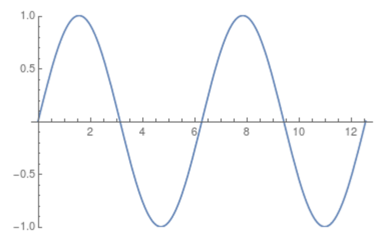
'Wavenumber' $k = 2\pi/\lambda$

'Amplitude'

$$y_{\pm}(x,t) = A\cos(\omega t \pm kx + \phi)$$

Right/left moving

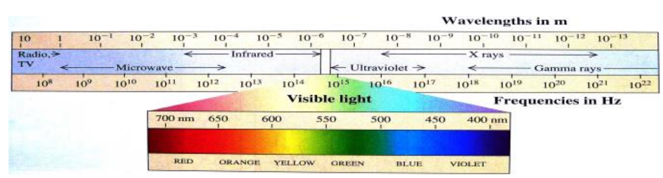
- General solutions can be made by superposing many such solutions.
- E.g. standing waves by superposing right- and left- moving waves :




- These are still harmonic waves (both components had the same frequency, so * is still true).
- But if we superpose components of different frequencies we can make more complicated, non-harmonic waves -- like the pulse before.



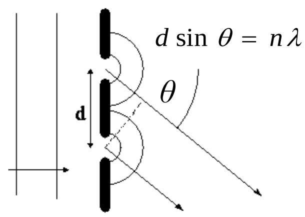
# Waves and dispersion

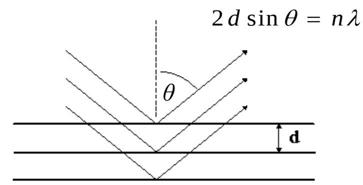



- 1D wave in complex notation : $y(x,t) = Ae^{i(kx-\omega t)}$
- Waves carry energy; the energy flux or 'intensity' is proportional to $A^2$
- The velocity of a point in a harmonic wave is $v = \omega/k$
- For some waves (light, small displacements on a string) v is a constant, independent of k.
  - Frequency is linear in wavenumber: 'a linear dispersion relation' $\omega(k) = vk$
  - Important implication for wave-groups, where we add together harmonic waves to make some wave packet. At a later time, all the harmonic waves in the group will have moved the same distance. So the wave packet will have moved as a whole, at speed *v,* and not changed in shape.
- For others, e.g. water waves, v depends on k. Frequency is not linear in wavenumber, e.g. $\omega = bk^2$
  - We can still add together harmonic waves to make wave-packets. But at a later time, these harmonic waves will have moved different distances to one another.
  - The wave-packet will have moved, but not at the velocity $\omega/k$. Instead, at the group velocity $v_g = d\omega/dk$
  - The wave-packet will change shape as it moves.

# Interference and superposition

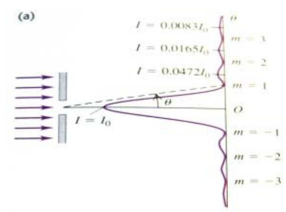
Maxima for path difference $= n\lambda \qquad n = 0,1,2,3,......$

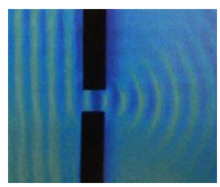


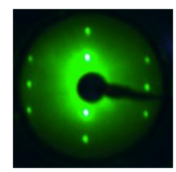




*Normal incidence on plane apertures*

*Scattering from multiple planes of atoms*









# The maths of wave motion : partial differential equations

- For a string, the displacement obeys

$$\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial x^2}$$

- $\frac{\partial y(x,t)}{\partial x}$ : differentiate y(x,t) with respect to x, treating t as a constant.
- Called the ‘partial derivative of y with respect to x’.
- $\frac{\partial^2 y}{\partial x^2}$ : differentiate twice with respect to x, treating t as a constant.

- Function of one variable z(x)=curve.
  - dz/dx=slope.
- Function of two variables z(x,y)=surface.
  - $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ are the slopes as you move parallel to the x, y axes.

# Solving the wave equation

- Guess! What function of x and t can you differentiate twice with respect to t, and get the same thing you would get if you differentiate twice with respect to x, and then multiply by a constant?
- $Ae^{i(kx \pm \omega t)}$ for any k, so long as $\omega = vk$
- This is a harmonic wave – our guess was special.
- The equation is linear so we can add solutions to make wave groups.
- And it is non-dispersive so those wave groups don’t spread.
- For interest: the usual way to solve this is a two-step process called ‘separation of variables’. You first guess the solution is a product of (function of x) x (function of t), which gives differential equations for each of these functions, and then guess the solutions to those.



# Quantum physics

$$\psi(x, y, z, t)$$

- Things are not the way you think. An electron, say, is not a thing with a position and a momentum. It is a ‘particle and a wave’, and everything you can possibly find out about it is encoded in its ‘wave function’
- ... which obeys a wave equation (‘Schrodinger’s equation), closely related to the one for waves on a string.
- Schrodinger’s equation is linear and so we can superpose solutions. So we get interference effects for electrons, just like classical waves.
- BUT the wavefunction tells us not what happens in an experiment, but what might happen (specifically, it tells you only probabilities of outcomes).

→In quantum physics *possibilities* interfere with one another.

So far as we know everything works like this: electrons, light, cabbages, professors...

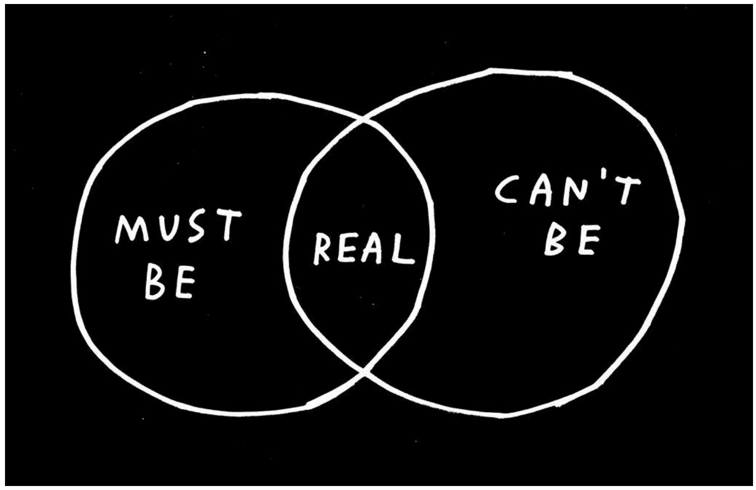



Image: Tucker Nichols, from New York Times 8th May 2018, Review of 'What is Real?', by James Gleick.