UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Maths/TP/TSM **Trinity Term 2013**

MA1111 — LINEAR ALGEBRA I

Monday, April 29 RDS 14.00 — 16.00

Dr. Paschalis Karageorgis

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.

1. Express $\boldsymbol{w}$ as a linear combination of $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ when

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \qquad \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \qquad \boldsymbol{v}_3 = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \qquad \boldsymbol{w} = \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix}.$$

2. Find a basis for both the null space and the column space of

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 & 3 \\ 2 & 1 & 5 & 6 & 9 \\ 2 & 3 & 7 & 2 & 7 \end{bmatrix}.$$

3. Find a linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ such that

$$T\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 5 \end{bmatrix}.$$

4. Let A_n be the $n \times n$ matrix whose diagonal entries are equal to 2 and all other entries are equal to 1. Compute the determinant of A_n for each positive integer n.

5. Suppose that the vectors $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ form a complete set of a vector space V. Show that the vectors $\boldsymbol{u}+\boldsymbol{v}$, $\boldsymbol{u}+\boldsymbol{w}$, $\boldsymbol{v}+\boldsymbol{w}$ form a complete set as well.

6. Suppose that AB is invertible for some $m \times n$ matrix A and some $n \times m$ matrix B. Show that the columns of B are linearly independent.