Advanced Calculus
MA1132
Tutorial Exercises 6

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To be completed before and during tutorials of Friday, 15. March

1. Given that the functions $u = u(x, y, z)$, $v = v(x, y, z)$, $w = w(x, y, z)$ and $f(u, v, w)$ are all differentiable, show that if we regard f as a function of x, y and z, then
$$\nabla f = \frac{\partial f}{\partial u}\nabla u + \frac{\partial f}{\partial v}\nabla v + \frac{\partial f}{\partial w}\nabla w.$$

2. Find the maximum and minimum values of the function $f(x, y) = x^3+x^2-x-y^3-y^2+y$ on and inside the rectangle bounded by the lines $x = -1$, $x = 1$, $y = -2$ and $y = 2$.

3. Use the method of Lagrange Multipliers to find the maximum and minimum values of the function $f(x, y) = (x - 1)^2 + y^2$, subject to the constraint $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$.

4. Consider the function
$$f(x, y) = x^4 - x^2y + y^2 - 3y + 4$$
Locate all relative maxima, relative minima, and saddle points, if any.

5. Find the distance from the point (x_0, y_0, z_0) to the plane
$$ax + by + cz + d = 0\,.$$

6. What is the volume of the largest n-dimensional box with edges parallel to the coordinate axes that fits inside the n-dimensional ellipsoid
$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \cdots + \frac{x_n^2}{a_n^2} = 1\,. \tag{1}$$