Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Trinity Term 2017

MA1111: Linear Algebra I

Saturday, May 13 Goldsmith Hall 09:30 — 11:30

Prof. Larry Rolen

Instructions to Candidates:

Attempt all questions. All questions will be weighted equally.

Materials Permitted for this Examination:

Formulae and Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used. This is a closed-book exam, so no notes or other study materials are allowed.

You may not start this examination until you are instructed to do so by the Invigilator.

1. The solutions of the differential equation $y''' - 2y'' - y' + 2y = 0$ are those functions of the general form $y(x) = c_1 e^x + c_2 e^{2x} + c_3 e^{-x}$, where c_1, c_2, c_3 are arbitrary real constants. A particular solution can be specified by giving initial conditions.

   (a) Suppose $y(x)$ is a solution to the differential equation above, and satisfies the initial conditions $y(0) = 1$, $y'(0) = 3$, and $y''(0) = 2$. Give a system of linear equations for c_1, c_2, c_3 (plug in the initial conditions into the general form of $y(x)$).

   (b) Solve the system of equations from (a) to find c_1, c_2, c_3, and hence $y(x)$.

2. Consider the linear transformation $T\colon \mathcal{P}_{\leq 3}(\mathbb{R}) \to P_{\leq 3}(\mathbb{R})$, from the vector space of real-coefficient polynomials of degree at most 3 to itself, given by $T(f(x)) = f(x) + x^3 f\left(\frac{1}{x}\right)$.

   (a) Find a matrix representation of T with respect to the standard basis $\{1, x, x^2, x^3\}$.

   (b) Use (a) to find bases for the image and kernel of T.

3. (a) Use Laplace expansions to find the determinant $\det \begin{pmatrix} 3 & 1 & 1 & 0 & 4 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & -2 \end{pmatrix}$.

   (b) Explain why $\det \begin{pmatrix} 3 & 4 & 6 & 8 \\ 2 & 9 & 4 & 100 \\ 1 & 11 & 2 & -7 \\ 2 & -5 & 4 & \pi \end{pmatrix} = 0$.

4. Are the following sets bases for $\mathbb{R}^3$? If so, show it, and if not, explain why.

   (a) $\{(1,2,3), (-1,2,4), (0,5,-1)\}$.

   (b) $\{(1,2,3), (4,7,1), (0,0,1), (1,-2,3)\}$.

5. Suppose that $\{v_1, \ldots, v_k\}$ is a set of vectors in a vector space V and T is a linear transformation from V to another vector space W.

   (a) Show that if $\{T(v_1), \ldots, T(v_k)\}$ is a linearly independent set of vectors in W, then $\{v_1, \ldots, v_k\}$ is also linearly independent.

   (b) Show that if T is injective and $\{v_1, \ldots, v_k\}$ is linearly independent, then $\{T(v_1), \ldots, T(v_k)\}$ is also linearly independent.