Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Trinity Term 2018

MA1132: Advanced Calculus

Saturday, May 19 Exam Hall 09:30 — 11:30

Prof. Anthony Brown

Instructions to Candidates:

Students need to complete all questions to obtain full marks.

Each question is worth 25 marks and the part marks are shown in square brackets on the right.

Materials Permitted for this Examination:

Formulae and Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

This is a closed-book exam, so no notes or other study materials are allowed

You may not start this examination until you are instructed to do so by the Invigilator.

1. *(a)* Find an arc length parametrization of the curve

$$\mathbf{r}(t) = \left(\frac{\cos^{-1}(2t) - 2t\sqrt{1-4t^2}}{2}, \frac{1-4t^2}{2}, 0\right), \quad t \in \left[0, \frac{1}{2}\right].$$

[6]

(b) Consider the curve

$$\mathbf{r}(t) = 3\sin(t)\mathbf{i} + 4t\mathbf{j} + 3\cos(t)\mathbf{k}, \quad t \in \mathbb{R},$$

(i) Find $\mathbf{T}(t)$, $\mathbf{N}(t)$ and $\mathbf{B}(t)$. **[8]**

(ii) Find the curvature and torsion, and hence show that the Frenet-Serret equations hold. **[11]**

2. *(a)* Consider the function $f(x, y) = \sqrt{x^2 + y^2 - 1}$.

(i) Write down the domain of f in set notation and sketch the set. **[3]**

(ii) Find and sketch the level curves of f for $c = 0$, $c = 1$, $c = 2$ and $c = 3$. **[4]**

(iii) Describe what the graph of f looks like. **[3]**

(b) Either find the limit

$$\lim_{(x,y)\to(2,1)} \frac{x^3 - 2x^2 - xy^2 + 2y^2}{x^3 - 2x^2 + xy^2 - 2y^2 + x - 2}$$

or show that it does not exist. **[5]**

(c) Find the total differential of the function $f(x, y) = \sin(xy^2)$. **[4]**

(d) Use the chain rule to find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ if

$$f(x, y) = e^{x^2 y}, \quad x(u, v) = \sqrt{uv} \quad \text{and} \quad y(u, v) = \frac{1}{v}.$$

[6]

3. *(a)* Find the directional derivative of the function $f(x, y, z) = \tan^{-1}\left(\frac{x}{y+z}\right)$ at the point $(4, 2, 2)$ in the direction $(2, 3, 6)$. **[6]**

(b) Find an equation for the tangent plane and a parametric equation for the normal line to the level surface $xz - yz^3 + yz^2 = 2$ at the point $(2, -1, 1)$. **[4]**

(c) Find and classify the critical points of the function $f(x, y) = xye^{-(x^2+y^2)/2}$. **[8]**

(d) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 4x^2 - 4xy + y^2$ subject to the constraint $x^2 + y^2 = 1$. **[7]**

4. *(a)* Find the integral of the function $f(x, y) = xye^{x^2 y}$ over the rectangle

$$\{(x, y) \in \mathbb{R}^2 \colon 0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant \ln(2)\}.$$

[5]

(b) Find the integral of the function $f(x, y) = \sin(y^3)$ over the set bounded by the curves $y = \sqrt{x}$, $y = 2$ and $x = 0$. **[6]**

(c) Use a double integral in polar coordinates to find the area enclosed by the cardioid $r = 1 - \cos(\theta)$. **[6]**

(d) Use spherical polar coordinates to integrate the function $f(x, y, z) = xyz$ over the solid in the first octant bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the coordinate planes. **[8]**