Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JS & SS Mathematics
JS Theoretical Physics
Moderatorship

Trinity Term 2016

MA1241 — Mechanics I

Wednesday, May 4 **RDS** **09:30 — 11:30**

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.

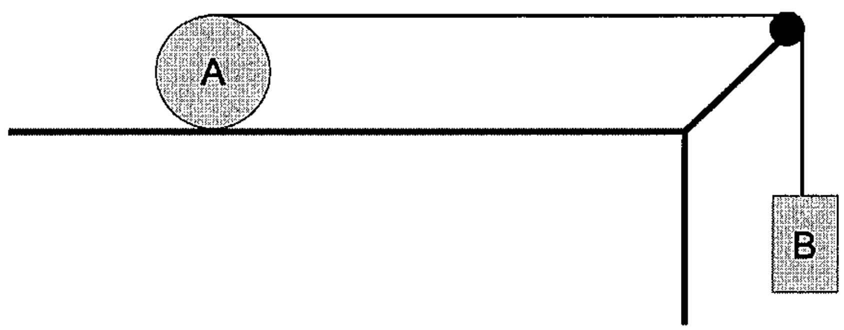
All questions have equal weight.

'Formulae & Tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination - please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. A thin cord is wound around a homogeneous cylinder with mass m_{A} and radius R. The cord extends horizontally from the cylinder to a pulley and then vertically downwards to a block B with mass m_{B}. See the figure below. The pulley is massless and frictionless. The gravitational acceleration is $\vec{g}$, and directed vertically downwards. The moment of inertia of the cylinder with respect to the axis through the center of the cylinder is $I_{\mathrm{A}} = \frac{1}{2} m_{\mathrm{A}} R^2$. The cylinder rolls without slipping.

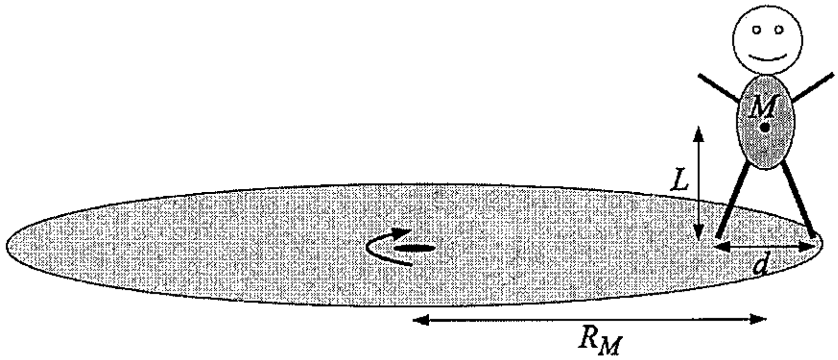


(a) Express the instantaneous velocity V_{A} of the center of mass of the cylinder in terms of its angular velocity ω_{A}. What is the relation between the velocity V_{A} of the center of mass of the cylinder, and the velocity of block B?

(b) Give the instantaneous kinetic and potential energy for this system.

(c) Determine the acceleration a_{A} of the center of mass of the cylinder and the tension in the cord.

2. A person of mass M stands on a disk of radius R which rotates with constant angular velocity. See the figure below. The feet of the person are a distance d apart. His center of mass is a height L above the disc and is equidistant to his feet. Let $R_M = R - \frac{1}{2}d$ be the radius of the circle along which the center of mass of the person moves. The center of mass moves with constant speed v.



   (a) Make a diagram of the person and draw the centripetal, friction, gravitational and normal forces.

   (b) We consider torques with respect to the center of the disk. Explain why the torque due to the centripetal force $\vec{\tau}_\mathrm{c}$ must equal

$$\vec{\tau}_\mathrm{c} = \vec{\tau}_\mathrm{g} + \vec{\tau}_\mathrm{n},$$

   where $\vec{\tau}_\mathrm{g}$ and $\vec{\tau}_\mathrm{n}$ are respectively the torques due to the gravitational and normal forces.

   (c) Determine the normal force acting on each foot.

3. A left alone little wagon, with mass m_0, rolls with velocity $\vec{v}_0$ on straight horizontal rails. Friction, air resistance and wheel mass can be neglected. Suddenly it starts to rain, such that every second r kg water falls into the wagon. The rain falls down vertically. We consider two cases:

   1. The rain remains in the wagon.
   2. The wagon contains a vertical drain pipe through which water leaves the wagon. The water leaves the wagon at the same rate r kg/s as rain falls in the wagon.

   (a) Let $\vec{v}(t)$ be the velocity of the wagon, t seconds after it starts raining. Derive for case 1., that the acceleration $\vec{a}(t) = d\vec{v}(t)/dt$ is given by

   $$\vec{a}(t) = -\frac{r\vec{v}(t)}{m_0 + rt},$$

   and for case 2.,

   $$\vec{a}(t) = -\frac{r\vec{v}(t)}{m_0}.$$

   (b) Determine for both cases the velocity $\vec{v}(t)$ of the wagon, t seconds after it starts raining.

   (c) After how much time is the velocity reduced to $\frac{1}{2}\vec{v}_0$ in each case? Give a qualitative explanation for the difference.