Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF PHYSICS

Junior Freshman — Trinity Term 2018

Annual Examination

X-PY1P10-1

Physics, Paper 1

(Science (Physics), Nanoscience Physics and Chemistry of Advanced Materials, Chemistry with Molecular Modelling and Theoretical Physics)

Wednesday 09 May 2018 — RDS — 09.30 – 12:30

Professors M. Ferreira, L. Bradley, M. Moebuis and J. Lunney

ALL QUESTIONS CARRY EQUAL MARKS.

USE SEPARATE ANSWER BOOKS FOR EACH SECTION

Booklets of Formulae and Tables are available from the invigilator for all students who require them. Graph paper is also available.

Non-programmable calculators are permitted for this examination – please indicate the make and model of your calculator on each answer book used.

Science (Physics), Chemistry with Molecular Modelling, Nanoscience Physics and Chemistry of Advanced Materials Students

Answer *SIX* questions, AT LEAST **TWO** from Section A, AT LEAST **TWO** from Section B, AT LEAST **ONE** from Section C AND **ONE** OTHER from these Sections in 3 hours.

Theoretical Physics Students

Answer *SIX* questions, AT LEAST **TWO** from Section B, AT LEAST **ONE** from Section C, AT LEAST **TWO** from Section D AND **ONE** OTHER from these Sections in 3 hours.

SECTION A

1. Consider two spheres of respective masses M and m, where $M>m$. The spheres are stacked on top of one another as shown in the figure below. The pair is released simultaneously from a height H (measured from the bottom of the lower sphere). Assume that the centres of the spheres are vertically aligned and that there is no horizontal motion. Ignore air resistance and assume perfectly elastic collisions to answer the following questions:

(a) How long time does it take for the bottom sphere (of mass M) to hit the ground? Give your answers in terms of H and the gravitational acceleration g.

[1 mark]

(b) Draw graphs showing how the position and the velocity of the centre of mass of the pair change with time. In addition, indicate the maximum speed reached by the spheres before hitting the ground.

[1 mark]

(c) What is the linear momentum of each sphere just before hitting the ground ?

[2 marks]

(d) After bouncing off the ground, what is the bouncing height the upper sphere will reach if $M=2m$?

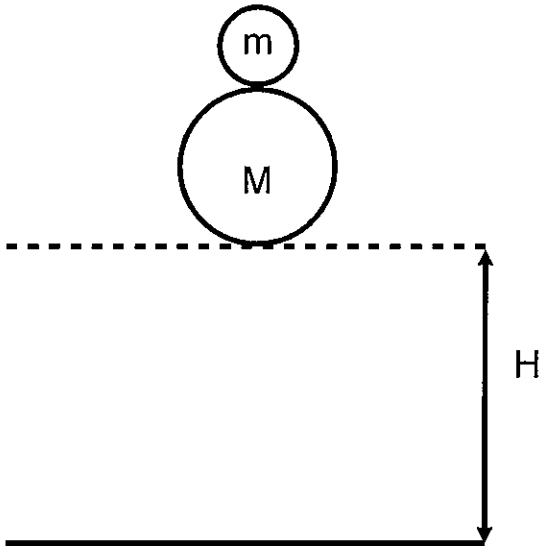
[3 marks]

Question **1(e)** continued on next page

(e) How does the bouncing-height depend on the ratio *M/m* ? What is the maximum bouncing height one can get in the limit of m being very much smaller than *M* (*m* << *M*)?

[3 marks]

Hint: *To work out the velocity of the bouncing ball, assume an infinitesimally small separation between the spheres and consider first the collision of the bottom sphere with the ground, followed by a second collision between the top and bottom spheres.*



2. A homogeneous ladder of length L leans against a frictionless vertical wall forming an angle α with the vertical surface (see diagram below). The friction coefficient between the ladder and the horizontal surface is μ_h.

(a) Draw a free-body diagram with all the forces acting on the ladder.

[1 mark]

(b) Calculate the minimum value of μ_h that keeps the ladder in equilibrium.

. [2 marks]

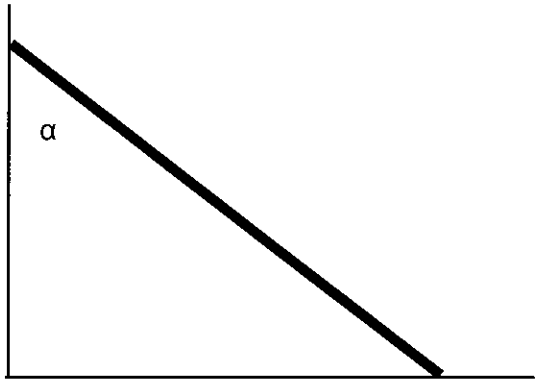
(c) At time t=0 a child of mass M starts climbing the ladder (from the bottom) at a constant speed u. How long time will it take until the ladder is no longer in equilibrium and starts collapsing to ground? Assume the mass of the ladder to be much smaller than the mass of the child, so that the former can be disregarded in this part of the calculation.

[3 marks]

(d) Determine the minimum friction coefficient necessary for the child to climb all the way to the top of the ladder.

[2 marks]

(d) Now consider that the ladder has friction with both surfaces, μ_v and μ_h being the friction coefficients with the vertical and horizontal surfaces, respectively. Find how high the child can climb before the ladder collapses in this case.



[2 marks]

3. Two cannons C_1 and C_2 can fire cannonballs at speeds V_1 and V_2, respectively. They are placed a distance D apart and aimed at respective angles θ_1 and θ_2 with the horizontal such that they land at each other's launching point. Ignore air resistance when answering the following questions:

(a) How are the launching angles and launching velocities related ?

[2 marks]

(b) Write the maximum heights reached by the cannonballs. If both cannons are fired simultaneously, establish the conditions that determine which cannonball will land first?

[2 marks]

(c) Derive an equation describing the trajectory followed by the cannonballs. Express this equation in terms of the maximum vertical and horizontal distances reached by the cannonball.

[2 marks]

(d) How are the launching angles θ_1 and θ_2 related if the relationship between the maximum heights H_1 and H_2 is $H1 = 3\ H_2$?

[2 marks]

(e) Assume that $\theta_1 = \pi/6$ and $\theta_2 = \pi/4$ to answer this part of the question. With cannonballs being fired simultaneously and landing on each other's launching position, find the time t after launch when the distance between both cannonballs is minimum.

[2 marks]

SECTION B

4.

(a)

(i) Derive the equation for a transverse wave with displacement $y(x,t)$ propagating in the +*x* direction as a function of time.

(ii) Demonstrate that this wave is a solution of the wave equation

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

[4 marks]

(b)

(i) For a string fixed at both ends sketch the first 4 resonant modes.

[2 marks]

(ii) A string is fixed at one end with the other end oscillating. It can oscillate with a range of frequencies. Find the frequencies for which a point 45.0 cm from one end of the string does not move. The speed of waves on the string is 0.720 m/s.

[4 marks]

5.

(a) Three polarizers are aligned such that there is an angle of 25^0 between the polarising axes of the first and second polarizer and an angle of 35^0 between the polarising axes of the first and third polarizer. If 28 mW/m^2 is transmitted through the three polarizer stack, determine the intensity of the incident unpolarized light.

[5 marks]

(b) Light incident on an air-glass boundary.

(i) Sketch a plot of the reflectance as a function angle of incidence for each of the two orthogonal states of polarization of the incident light, where the polarization states are defined with respect to plane of incidence.

(ii) Define Brewster's angle and indicate it on the plot in part **(i).**

(iii) Derive the expression for Brewster's angle as function of the refractive indices of the two media. Illustrate using a clear diagram.

[5 marks]

6.

(a) The angular radius of the first dark ring of the diffraction pattern for a circular aperture is given by

$$\sin\theta_1 = 1.22\frac{\lambda}{D}.$$

Sketch an example of diffraction for a circular aperture, showing D, θ_1 and the diffraction pattern on the diagram.

[2 marks]

(b) Derive an expression for the minimum spot size to which a lens can focus light, in terms of the lens diameter, focal distance and wavelength of the light.

[3 marks]

(c) You are asked to design a space telescope for earth orbit. When Jupiter is 5.93×10^8 km away (its closest approach to earth), the telescope is to resolve, by Rayleigh's criterion, features on Jupiter that are 250 km apart.

(i) State Rayleigh's criterion.

(ii) What minimum diameter mirror is required? Assume a wavelength of 500 nm

[5 marks]

SECTION C

7. Consider a particle moving in one dimension only. Let its energy E be a *continuous* random variable governed by the following probability density function: $P(E) = Ce^{-\beta E}\ for\ E \geq 0$, where C and β are constants.

(a) Use the normalisation condition to find the value of C in terms of β.

[3 marks]

(b) Find the mean, $\langle E \rangle$, of $P(E)$ (Hint: Use integration by parts).

[3 marks]

(c) The energy of the particle can be expressed in terms of its mass m, which is constant, and its speed v ($v \geq 0$): $E = \frac{1}{2}mv^2$. Find the corresponding probability density function $P(v)$.

[4 marks]

8.

(a) Given two independent variables f_0 and v, with associated uncertainties Δf_0 and Δv, state the general error propagation formula that yields the uncertainty of a function $f(f_0, v)$.

[2 marks]

(b) A sound source with frequency f_0 is moving towards a stationary observer who hears the Doppler shifted frequency f:

$$f = \frac{c \cdot f_0}{c - v}$$

where c is the speed of sound, which can be considered a constant, i.e. $\Delta c = 0$. Derive the error in f in terms of the uncertainties in f_0 and c using the error propagation formula.

[5 marks]

(c) Calculate the frequency f and its uncertainty for the following parameters: $f_0 = (1000 \pm 3)\ Hz$, $v = (27.0 \pm 0.5)\ m/s$ and $c = 344\ m/s$.

[3 marks]

SECTION D

9.

(a) As a meter stick, moving along its length, flies past you at high speed, you simultaneously measure the positions of the two ends and conclude that the length L < 1m. Show that for an observer in the rest frame of the stick you did not make your measurements simultaneously, and which end did you measure first?

[5 marks]

(b) In an experiment an electron is fired to the right with a speed of 0.8c and a proton is fired to the left with a speed of 0.9c. What is the speed of the electron relative to proton?

[5 marks]

10. The refractive index of air for visible light is 1.000277.

(a) What would be the kinetic energy of an electron travelling through vacuum which would have the same speed as light pulse through air?

[4 marks]

(b) Through what voltage should the electron be accelerated to reach this speed?

[2 marks]

(c) What force would a beam of these electrons, equivalent to a current of 16 mA, exert on a metal block in which they are absorbed?

[4 marks]

(Rest mass energy of electron = 0.51 MeV)

11.

(a) Two electrons travelling in opposite directions with speeds of 0.9 c collide head-on and produce two gamma-ray photons. What are the energy and the wavelength of the photons?

[5 marks]

(b) At what speed is a particle's kinetic energy equal to twice its rest energy?

[5 marks]

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Electron rest mass	m_e	9.11×10^{-31} kg
Proton rest mass	m_p	1.67×10^{-27} kg
Electronic charge	e	1.60×10^{-19} C
Speed of light in free space	c	3.00×10^{8} m s^{-1}
Planck's constant	h	6.63×10^{-34} J s
	$\hbar = h/2\pi$	1.05×10^{-34} J s
Boltzmann's constant	k	1.38×10^{-23} J K^{-1}
Molar gas constant	R	8.31 J K^{-1}mol^{-1}
Avogadro's number	N_A	6.02×10^{23} mol^{-1}
Standard molar volume		22.4×10^{-3} m^3
Bohr magneton	μ_B	9.27×10^{-24} A m^2 OR J T^{-1}
Nuclear magneton	μ_N	5.05×10^{-27} A m^2 OR J T^{-1}
Bohr radius	a_o	5.29×10^{-11} m
Fine structure constant	$e^2/(4\pi\varepsilon_o\hbar c)$	$(137)^{-1}$
Rydberg's constant	R_∞	1.10×10^{7} m^{-1}
Stefan's constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} N m^2 kg^{-2}
Proton magnetic moment	μ_p	2.79 μ_N
Neutron magnetic moment	μ_n	-1.91 μ_N
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of free space	ε_o	8.85×10^{-12} F m^{-1}
1 electron volt (1 eV)		1.60×10^{-19} J
1 unified atomic mass unit (^{12}C scale)		1.66×10^{-27} kg = 931 MeV/c^2
Wavelength of 1 eV photon		1.24×10^{-6} m
1 atmosphere		1.01×10^{5} N m^{-2}
Acceleration due to gravity	g	9.8 m s^{-2}

Free space impedance	Z_0	377 Ω
Astronomical unit (1 au)		1.50×10^{11} m
Parsec (1 pc)		3.09×10^{16} m
Solar radius	$R_\odot$	6.96×10^{8} m
Solar mass	$M_\odot$	1.99×10^{30} kg
Solar luminosity	$L_\odot$	3.85×10^{26} W
Earth mass	$M_\oplus$	5.97×10^{24} kg
Earth radius (equatorial)	$R_\oplus$	6.378×10^{6} m