Homework 6: Central force motion

Due: Tuesday March 19th, 13:00

Policy: Collaboration is allowed, but every student is required to hand in his/her own version of the solutions. Please include your name and student number on the solutions.

Problem 1. (K. & K. Ex. 10.2) A particle of mass 50 g moves under an attractive force of magnitude $4\,r^3 \cdot 10^{-5}$ N. The angular momentum is equal to 1000 g cm^2/s.

1. Find the effective potential energy.
2. Indicate on a sketch of the effective potential the total energy for circular motion.
3. The radius of the particle's orbit varies between r_0 and $2r_0$. Find r_0.

Problem 2. A comet with mass m follows a trajectory in the gravitational field of the sun (mass M). The gravitational force experienced by the comet is:

$$\vec{F} = -\frac{C}{r^2}, \qquad C = GMm.$$

In the perihelion (the point closest to the sun), the distance r between the sun and the comet is r_-, and the speed of the comet is:

$$v_0 = \sqrt{\frac{1,8\,GM}{r_-}}.$$

1. Determine the magnitude of the angular momentum of the comet (with respect to the origin $r = 0$), expressed in M, m, G and r_-.
2. Determine the total mechanical energy E, expressed in M, m, G and r_-. Take the potential energy $U(r)$ such that $\lim_{r\to\infty} U(r) = 0$.
3. Determine the constants r_0 and ε in the equation of the trajectory:

$$r = \frac{r_0}{1 - \varepsilon\cos(\theta)}.$$