Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Mathematics **Trinity Term 2017**
JF Theoretical Physics
JF Two Subject Mod

MA1132: Advanced Calculus

Wednesday, May 24 Goldsmith Hall 14:00 — 16:00

Prof. Larry Rolen

Instructions to Candidates:

Attempt all questions. All questions will be weighted equally.

Materials Permitted for this Examination:

Formulae and Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used. This is a closed-book exam, so no notes or other study materials are allowed.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Suppose a curve is given by the parametric equations

$$\begin{cases} x = t^4 \\ y = \log t \\ z = \sqrt{2} \cdot t^2, \end{cases}$$

for $t > 0$ (recall that $\log$ denotes the natural logarithm).

(a) Find the arc length along the curve from the point when $t = 1$ to the point where $t = 2$.

(b) Find the unit tangent vector $T(t)$ to the curve as a vector-valued function of t.

(c) Show that if $f(t)\colon \mathbb{R} \to \mathbb{R}^3$ is a vector-valued function with constant norm (length) $|f(t)| = c$, then $f'(t)$ is always perpendicular to $f(t)$. (Hint: What is another way to express the norm of a vector?)

(d) The calculation from (c) was our basis for the formula for the unit normal vector $N(t)$ of a curve. Find the unit normal vector at $t = 1$, $N(1)$.

2. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = 5x^2y - xy^3$.

(a) Compute the partial derivatives f_x and f_y.

(b) Find the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(2, 2)$.

(c) Find parametric equations for the normal line to the tangent plane in part (b) through the point $(2, 2, f(2, 2))$.

(d) Use your answer from part (b) to give the linear approximation for $f(1.9, 2.1)$.

3. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^2 + 2x + 3y^2$.

(a) Find all critical points of f on the interior of the disc $x^2 + y^2 \leq 4$, that is, at points with $x^2 + y^2 < 4$.

(b) Classify all of the critical points from (a) using the second derivative test as local minima, local maxima, or saddle points (or state that the test gives no information).

(c) Find the maximum and minimum values of f on the circle $x^2 + y^2 = 4$ by using the method of Lagrange multipliers.

(d) Find the absolute minimum and maximum values of f inside the closed disk where $x^2 + y^2 \leq 4$, which are guaranteed to exist by the Extreme Value Theorem, by comparing the values at the critical points you found in (a) with the extreme values on the boundary you found in (c).

4. (a) Evaluate the double integral $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx dy$ by switching the order of integration; that is, by writing this as an integral over a region in the plane and then integrating it in the order $dydx$.

(b) Evaluate the integral $\iint_R \sin\left(\frac{x^2}{9} + \frac{y^2}{25}\right) dA$ where R is the elliptical region $\frac{x^2}{9} + \frac{y^2}{25} \leq 1$ by making the change of variables $x = 3u$, $y = 5v$.