1

UNITS, PHYSICAL QUANTITIES, AND VECTORS

1.1. **IDENTIFY:** Convert units from mi to km and from km to ft.

SET UP: 1 in. = 2.54 cm, 1 km = 1000 m, 12 in. = 1 ft, 1 mi = 5280 ft.

EXECUTE: **(a)** $1.00\text{ mi} = (1.00\text{ mi})\left(\dfrac{5280\text{ ft}}{1\text{ mi}}\right)\left(\dfrac{12\text{ in.}}{1\text{ ft}}\right)\left(\dfrac{2.54\text{ cm}}{1\text{ in.}}\right)\left(\dfrac{1\text{ m}}{10^2\text{ cm}}\right)\left(\dfrac{1\text{ km}}{10^3\text{ m}}\right) = 1.61\text{ km}$

(b) $1.00\text{ km} = (1.00\text{ km})\left(\dfrac{10^3\text{ m}}{1\text{ km}}\right)\left(\dfrac{10^2\text{ cm}}{1\text{ m}}\right)\left(\dfrac{1\text{ in.}}{2.54\text{ cm}}\right)\left(\dfrac{1\text{ ft}}{12\text{ in.}}\right) = 3.28\times10^3\text{ ft}$

EVALUATE: A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

1.2. **IDENTIFY:** Convert volume units from L to in.^3.

SET UP: $1\text{ L} = 1000\text{ cm}^3$. 1 in. = 2.54 cm

EXECUTE: $0.473\text{ L}\times\left(\dfrac{1000\text{ cm}^3}{1\text{ L}}\right)\times\left(\dfrac{1\text{ in.}}{2.54\text{ cm}}\right)^3 = 28.9\text{ in.}^3$.

EVALUATE: 1 in.^3 is greater than 1 cm^3, so the volume in in.^3 is a smaller number than the volume in cm^3, which is 473 cm^3.

1.3. **IDENTIFY:** We know the speed of light in m/s. $t = d/v$. Convert 1.00 ft to m and t from s to ns.

SET UP: The speed of light is $v = 3.00\times10^8$ m/s. 1 ft = 0.3048 m. 1 s = 10^9 ns.

EXECUTE: $t = \dfrac{0.3048\text{ m}}{3.00\times10^8\text{ m/s}} = 1.02\times10^{-9}\text{ s} = 1.02\text{ ns}$

EVALUATE: In 1.00 s light travels $3.00\times10^8\text{ m} = 3.00\times10^5\text{ km} = 1.86\times10^5\text{ mi}$.

1.4. **IDENTIFY:** Convert the units from g to kg and from cm^3 to m^3.

SET UP: 1 kg = 1000 g. 1 m = 100 cm.

EXECUTE: $19.3\dfrac{\text{g}}{\text{cm}^3}\times\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)\times\left(\dfrac{100\text{ cm}}{1\text{ m}}\right)^3 = 1.93\times10^4\dfrac{\text{kg}}{\text{m}^3}$

EVALUATE: The ratio that converts cm to m is cubed, because we need to convert cm^3 to m^3.

1.5. **IDENTIFY:** Convert volume units from in.^3 to L.

SET UP: $1\text{ L} = 1000\text{ cm}^3$. 1 in. = 2.54 cm.

EXECUTE: $(327\text{ in.}^3)\times(2.54\text{ cm/in.})^3\times(1\text{ L}/1000\text{ cm}^3) = 5.36\text{ L}$

EVALUATE: The volume is 5360 cm^3. 1 cm^3 is less than 1 in.^3, so the volume in cm^3 is a larger number than the volume in in.^3.

1.6. **IDENTIFY:** Convert ft^2 to m^2 and then to hectares.

SET UP: 1.00 hectare $=1.00\times 10^4\ m^2$. 1 ft = 0.3048 m.

EXECUTE: The area is $(12.0\text{ acres})\left(\frac{43{,}600\ ft^2}{1\text{ acre}}\right)\left(\frac{0.3048\text{ m}}{1.00\text{ ft}}\right)^2\left(\frac{1.00\text{ hectare}}{1.00\times 10^4\ m^2}\right)=4.86$ hectares.

EVALUATE: Since 1 ft = 0.3048 m, $1\ ft^2=(0.3048)^2\ m^2$.

1.7. **IDENTIFY:** Convert seconds to years. 1 gigasecond is a billion seconds.

SET UP: 1 gigasecond $=1\times 10^9$ s. 1 day = 24 h. 1 h = 3600 s.

EXECUTE: 1.00 gigasecond $=(1.00\times 10^9\text{ s})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1\text{ day}}{24\text{ h}}\right)\left(\frac{1\text{ y}}{365\text{ days}}\right)=31.7$ y.

EVALUATE: The conversion $1\text{ y}=3.156\times 10^7$ s assumes 1 y = 365.24 d, which is the average for one extra day every four years, in leap years. The problem says instead to assume a 365-day year.

1.8. **IDENTIFY:** Apply the given conversion factors.

SET UP: 1 furlong = 0.1250 mi and 1 fortnight = 14 days. 1 day = 24 h.

EXECUTE: $(180{,}000\text{ furlongs/fortnight})\left(\frac{0.125\text{ mi}}{1\text{ furlong}}\right)\left(\frac{1\text{ fortnight}}{14\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ h}}\right)=67\text{ mi/h}$

EVALUATE: A furlong is less than a mile and a fortnight is many hours, so the speed limit in mph is a much smaller number.

1.9. **IDENTIFY:** Convert miles/gallon to km/L.

SET UP: 1 mi = 1.609 km. 1 gallon = 3.788 L.

EXECUTE: **(a)** 55.0 miles/gallon $=(55.0\text{ miles/gallon})\left(\frac{1.609\text{ km}}{1\text{ mi}}\right)\left(\frac{1\text{ gallon}}{3.788\text{ L}}\right)=23.4$ km/L.

(b) The volume of gas required is $\frac{1500\text{ km}}{23.4\text{ km/L}}=64.1$ L. $\frac{64.1\text{ L}}{45\text{ L/tank}}=1.4$ tanks.

EVALUATE: 1 mi/gal = 0.425 km/L. A km is very roughly half a mile and there are roughly 4 liters in a gallon, so 1 mi/gal $\sim\frac{2}{4}$ km/L, which is roughly our result.

1.10. **IDENTIFY:** Convert units.

SET UP: Use the unit conversions given in the problem. Also, 100 cm = 1 m and 1000 g = 1 kg.

EXECUTE: **(a)** $\left(60\frac{\text{mi}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)=88\frac{\text{ft}}{\text{s}}$

(b) $\left(32\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{30.48\text{ cm}}{1\text{ ft}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)=9.8\frac{\text{m}}{\text{s}^2}$

(c) $\left(1.0\frac{\text{g}}{\text{cm}^3}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3\left(\frac{1\text{ kg}}{1000\text{ g}}\right)=10^3\frac{\text{kg}}{\text{m}^3}$

EVALUATE: The relations 60 mi/h = 88 ft/s and $1\text{ g/cm}^3=10^3\text{ kg/m}^3$ are exact. The relation $32\text{ ft/s}^2=9.8\text{ m/s}^2$ is accurate to only two significant figures.

1.11. **IDENTIFY:** We know the density and mass; thus we can find the volume using the relation density = mass/volume = m/V. The radius is then found from the volume equation for a sphere and the result for the volume.

SET UP: Density $=19.5\text{ g/cm}^3$ and $m_{\text{critical}}=60.0$ kg. For a sphere $V=\frac{4}{3}\pi r^3$.

EXECUTE: $V=m_{\text{critical}}/\text{density}=\left(\frac{60.0\text{ kg}}{19.5\text{ g/cm}^3}\right)\left(\frac{1000\text{ g}}{1.0\text{ kg}}\right)=3080\text{ cm}^3$.

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3}{4\pi}(3080\ \text{cm}^3)} = 9.0\ \text{cm}.$$

EVALUATE: The density is very large, so the 130-pound sphere is small in size.

1.12. **IDENTIFY:** Convert units.

SET UP: We know the equalities $1\ \text{mg} = 10^{-3}\ \text{g}$, $1\ \mu\text{g}\ 10^{-6}\ \text{g}$, and $1\ \text{kg} = 10^3\ \text{g}$.

EXECUTE: **(a)** $(410\ \text{mg/day})\left(\frac{10^{-3}\ \text{g}}{1\ \text{mg}}\right)\left(\frac{1\ \mu\text{g}}{10^{-6}\ \text{g}}\right) = 4.10\times10^5\ \mu\text{g/day}.$

(b) $(12\ \text{mg/kg})(75\ \text{kg}) = (900\ \text{mg})\left(\frac{10^{-3}\ \text{g}}{1\ \text{mg}}\right) = 0.900\ \text{g}.$

(c) The mass of each tablet is $(2.0\ \text{mg})\left(\frac{10^{-3}\ \text{g}}{1\ \text{mg}}\right) = 2.0\times10^{-3}\ \text{g}$. The number of tablets required each day is the number of grams recommended per day divided by the number of grams per tablet:

$\frac{0.0030\ \text{g/day}}{2.0\times10^{-3}\ \text{g/tablet}} = 1.5\ \text{tablet/day}$. Take 2 tablets each day.

(d) $(0.000070\ \text{g/day})\left(\frac{1\ \text{mg}}{10^{-3}\ \text{g}}\right) = 0.070\ \text{mg/day}.$

EVALUATE: Quantities in medicine and nutrition are frequently expressed in a wide variety of units.

1.13. **IDENTIFY:** Model the bacteria as spheres. Use the diameter to find the radius, then find the volume and surface area using the radius.

SET UP: From Appendix B, the volume V of a sphere in terms of its radius is $V = \frac{4}{3}\pi r^3$ while its surface area A is $A = 4\pi r^2$. The radius is one-half the diameter or $r = d/2 = 1.0\ \mu\text{m}$. Finally, the necessary equalities for this problem are: $1\ \mu\text{m} = 10^{-6}\ \text{m}$; $1\ \text{cm} = 10^{-2}\ \text{m}$; and $1\ \text{mm} = 10^{-3}\ \text{m}$.

EXECUTE: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\ \mu\text{m})^3\left(\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}\right)^3\left(\frac{1\ \text{cm}}{10^{-2}\ \text{m}}\right)^3 = 4.2\times10^{-12}\ \text{cm}^3$ and

$$A = 4\pi r^2 = 4\pi(1.0\ \mu\text{m})^2\left(\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}\right)^2\left(\frac{1\ \text{mm}}{10^{-3}\ \text{m}}\right)^2 = 1.3\times10^{-5}\ \text{mm}^2$$

EVALUATE: On a human scale, the results are extremely small. This is reasonable because bacteria are not visible without a microscope.

1.14. **IDENTIFY:** When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

SET UP: 12 mm has two significant figures and 5.98 mm has three significant figures.

EXECUTE: **(a)** $(12\ \text{mm})\times(5.98\ \text{mm}) = 72\ \text{mm}^2$ (two significant figures)

(b) $\frac{5.98\ \text{mm}}{12\ \text{mm}} = 0.50$ (also two significant figures)

(c) 36 mm (to the nearest millimeter)

(d) 6 mm

(e) 2.0 (two significant figures)

EVALUATE: The length of the rectangle is known only to the nearest mm, so the answers in parts **(c)** and (d) are known only to the nearest mm.

1.15. **IDENTIFY:** Use your calculator to display $\pi\times10^7$. Compare that number to the number of seconds in a year.

SET UP: $1\ \text{yr} = 365.24\ \text{days}$, $1\ \text{day} = 24\ \text{h}$, and $1\ \text{h} = 3600\ \text{s}$.

EXECUTE: $(365.24 \text{ days/1 yr})\left(\dfrac{24 \text{ h}}{1 \text{ day}}\right)\left(\dfrac{3600 \text{ s}}{1 \text{ h}}\right) = 3.15567\ldots\times 10^7 \text{ s}$; $\pi\times 10^7 \text{ s} = 3.14159\ldots\times 10^7 \text{ s}$

The approximate expression is accurate to two significant figures. The percent error is 0.45%.

EVALUATE: The close agreement is a numerical accident.

1.16. **IDENTIFY:** To asses the accuracy of the approximations, we must convert them to decimals.

SET UP: Use a calculator to calculate the decimal equivalent of each fraction and then round the numeral to the specified number of significant figures. Compare to π rounded to the same number of significant figures.

EXECUTE: **(a)** 22/7 = 3.14286 **(b)** 355/113 = 3.14159 **(c)** The exact value of π rounded to six significant figures is 3.14159.

EVALUATE: We see that 355/113 is a much better approximation to π than is 22/7.

1.17. **IDENTIFY:** Express 200 kg in pounds. Express each of 200 m, 200 cm and 200 mm in inches. Express 200 months in years.

SET UP: A mass of 1 kg is equivalent to a weight of about 2.2 lbs. $1 \text{ in.} = 2.54 \text{ cm}$. $1 \text{ y} = 12 \text{ months}$.

EXECUTE: **(a)** 200 kg is a weight of 440 lb. This is much larger than the typical weight of a man.

(b) $200 \text{ m} = (2.00\times 10^4 \text{ cm})\left(\dfrac{1 \text{ in.}}{2.54 \text{ cm}}\right) = 7.9\times 10^3$ inches. This is much greater than the height of a person.

(c) $200 \text{ cm} = 2.00 \text{ m} = 79 \text{ inches} = 6.6 \text{ ft}$. Some people are this tall, but not an ordinary man.

(d) $200 \text{ mm} = 0.200 \text{ m} = 7.9 \text{ inches}$. This is much too short.

(e) $200 \text{ months} = (200 \text{ mon})\left(\dfrac{1 \text{ y}}{12 \text{ mon}}\right) = 17 \text{ y}$. This is the age of a teenager; a middle-aged man is much older than this.

EVALUATE: None are plausible. When specifying the value of a measured quantity it is essential to give the units in which it is being expressed.

1.18. **IDENTIFY:** Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.

SET UP: Estimate 3×10^8 people, so 2×10^8 cars.

EXECUTE: (Number of cars × miles/car day)/(mi/gal) = gallons/day

$$(2\times 10^8 \text{ cars}\times 10000 \text{ mi/yr/car}\times 1 \text{ yr/365 days})/(20 \text{ mi/gal}) = 3\times 10^8 \text{ gal/day}$$

EVALUATE: The number of gallons of gas used each day approximately equals the population of the U.S.

1.19. **IDENTIFY:** Estimate the number of blinks per minute. Convert minutes to years. Estimate the typical lifetime in years.

SET UP: Estimate that we blink 10 times per minute. $1 \text{ y} = 365 \text{ days}$. $1 \text{ day} = 24 \text{ h}$, $1 \text{ h} = 60 \text{ min}$. Use 80 years for the lifetime.

EXECUTE: The number of blinks is $(10 \text{ per min})\left(\dfrac{60 \text{ min}}{1 \text{ h}}\right)\left(\dfrac{24 \text{ h}}{1 \text{ day}}\right)\left(\dfrac{365 \text{ days}}{1 \text{ y}}\right)(80 \text{ y/lifetime}) = 4\times 10^8$

EVALUATE: Our estimate of the number of blinks per minute can be off by a factor of two but our calculation is surely accurate to a power of 10.

1.20. **IDENTIFY:** Approximate the number of breaths per minute. Convert minutes to years and cm^3 to m^3 to find the volume in m^3 breathed in a year.

SET UP: Assume 10 breaths/min. $1 \text{ y} = (365 \text{ d})\left(\dfrac{24 \text{ h}}{1 \text{ d}}\right)\left(\dfrac{60 \text{ min}}{1 \text{ h}}\right) = 5.3\times 10^5 \text{ min}$. $10^2 \text{ cm} = 1 \text{ m}$ so $10^6 \text{ cm}^3 = 1 \text{ m}^3$. The volume of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$, where r is the radius and d is the diameter. Don't forget to account for four astronauts.

EXECUTE: **(a)** The volume is $(4)(10 \text{ breaths/min})(500\times 10^{-6} \text{ m}^3)\left(\dfrac{5.3\times 10^5 \text{ min}}{1 \text{ y}}\right) = 1\times 10^4 \text{ m}^3\text{/yr}$.

(b) $d = \left(\dfrac{6V}{\pi}\right)^{1/3} = \left(\dfrac{6[1\times 10^4 \text{ m}^3]}{\pi}\right)^{1/3} = 27 \text{ m}$

EVALUATE: Our estimate assumes that each cm^3 of air is breathed in only once, where in reality not all the oxygen is absorbed from the air in each breath. Therefore, a somewhat smaller volume would actually be required.

1.21. **IDENTIFY:** Estimation problem.

SET UP: Estimate that the pile is $18 \text{ in.}\times 18 \text{ in.}\times 5 \text{ ft } 8 \text{ in.}$. Use the density of gold to calculate the mass of gold in the pile and from this calculate the dollar value.

EXECUTE: The volume of gold in the pile is $V = 18 \text{ in.}\times 18 \text{ in.}\times 68 \text{ in.} = 22{,}000 \text{ in.}^3$. Convert to cm^3:

$$V = 22{,}000 \text{ in.}^3(1000 \text{ cm}^3/61.02 \text{ in.}^3) = 3.6\times 10^5 \text{ cm}^3.$$

The density of gold is 19.3 g/cm^3, so the mass of this volume of gold is

$$m = (19.3 \text{ g/cm}^3)(3.6\times 10^5 \text{ cm}^3) = 7\times 10^6 \text{ g}.$$

The monetary value of one gram is \$10, so the gold has a value of $(\$10/\text{gram})(7\times 10^6 \text{ grams}) = \$7\times 10^7$, or about $\$100\times 10^6$ (one hundred million dollars).

EVALUATE: This is quite a large pile of gold, so such a large monetary value is reasonable.

1.22. **IDENTIFY:** Estimate the number of beats per minute and the duration of a lifetime. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

SET UP: An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years.

EXECUTE: $N_{\text{beats}} = (75 \text{ beats/min})\left(\dfrac{60 \text{ min}}{1 \text{ h}}\right)\left(\dfrac{24 \text{ h}}{1 \text{ day}}\right)\left(\dfrac{365 \text{ days}}{\text{yr}}\right)\left(\dfrac{80 \text{ yr}}{\text{lifespan}}\right) = 3\times 10^9 \text{ beats/lifespan}$

$V_{\text{blood}} = (50 \text{ cm}^3/\text{beat})\left(\dfrac{1 \text{ L}}{1000 \text{ cm}^3}\right)\left(\dfrac{1 \text{ gal}}{3.788 \text{ L}}\right)\left(\dfrac{3\times 10^9 \text{ beats}}{\text{lifespan}}\right) = 4\times 10^7 \text{ gal/lifespan}$

EVALUATE: This is a very large volume.

1.23. **IDENTIFY:** Estimate the diameter of a drop and from that calculate the volume of a drop, in m^3. Convert m^3 to L.

SET UP: Estimate the diameter of a drop to be $d = 2 \text{ mm}$. The volume of a spherical drop is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$. $10^3 \text{ cm}^3 = 1 \text{ L}$.

EXECUTE: $V = \frac{1}{6}\pi(0.2 \text{ cm})^3 = 4\times 10^{-3} \text{ cm}^3$. The number of drops in 1.0 L is $\dfrac{1000 \text{ cm}^3}{4\times 10^{-3} \text{ cm}^3} = 2\times 10^5$

EVALUATE: Since $V \sim d^3$, if our estimate of the diameter of a drop is off by a factor of 2 then our estimate of the number of drops is off by a factor of 8.

1.24. **IDENTIFY:** Draw the vector addition diagram to scale.

SET UP: The two vectors $\vec{A}$ and $\vec{B}$ are specified in the figure that accompanies the problem.

EXECUTE: **(a)** The diagram for $\vec{R} = \vec{A} + \vec{B}$ is given in Figure 1.24a. Measuring the length and angle of $\vec{R}$ gives $R = 9.0 \text{ m}$ and an angle of $\theta = 34°$.

(b) The diagram for $\vec{E} = \vec{A} - \vec{B}$ is given in Figure 1.24b. Measuring the length and angle of $\vec{E}$ gives $D = 22 \text{ m}$ and an angle of $\theta = 250°$.

(c) $-\vec{A} - \vec{B} = -(\boldsymbol{A} + \boldsymbol{B})$, so $-\vec{A} - \vec{B}$ has a magnitude of 9.0 m (the same as $\vec{A} + \vec{B}$) and an angle with the $+x$ axis of $214°$ (opposite to the direction of $\vec{A} + \vec{B}$).

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has a magnitude of 22 m and an angle with the $+x$ axis of $70°$ (opposite to the direction of $\vec{A} - \vec{B}$).

EVALUATE: The vector $-\vec{A}$ is equal in magnitude and opposite in direction to the vector $\vec{A}$.

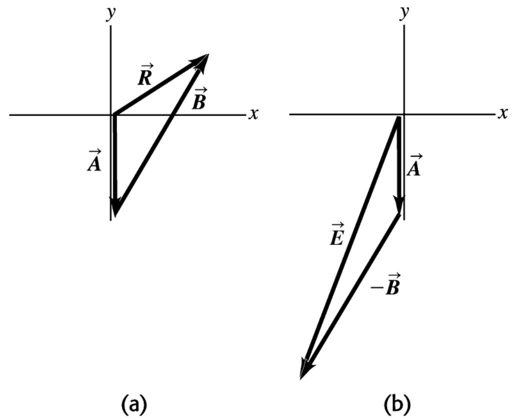


Figure 1.24

1.25. **IDENTIFY:** Draw each subsequent displacement tail to head with the previous displacement. The resultant displacement is the single vector that points from the starting point to the stopping point.

SET UP: Call the three displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$. The resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

EXECUTE: The vector addition diagram is given in Figure 1.25. Careful measurement gives that $\vec{R}$ is 7.8 km, 38° north of east.

EVALUATE: The magnitude of the resultant displacement, 7.8 km, is less than the sum of the magnitudes of the individual displacements, $2.6\text{ km} + 4.0\text{ km} + 3.1\text{ km}$.

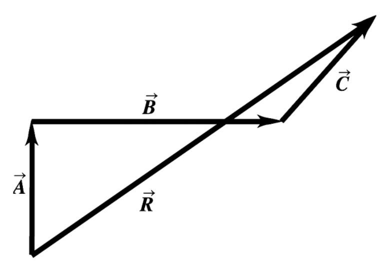


Figure 1.25

1.26. **IDENTIFY:** Since she returns to the starting point, the vector sum of the four displacements must be zero.

SET UP: Call the three given displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$, and call the fourth displacement $\vec{D}$. $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$.

EXECUTE: The vector addition diagram is sketched in Figure 1.26. Careful measurement gives that $\vec{D}$ is 144 m, 41° south of west.

EVALUATE: $\vec{D}$ is equal in magnitude and opposite in direction to the sum $\vec{A} + \vec{B} + \vec{C}$.

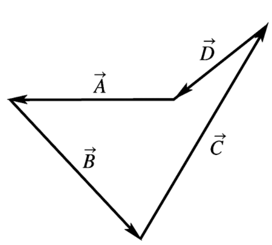


Figure 1.26

1.27. **IDENTIFY:** For each vector $\vec{V}$, use that $V_x = V\cos\theta$ and $V_y = V\sin\theta$, when θ is the angle $\vec{V}$ makes with the $+x$ axis, measured counterclockwise from the axis.

SET UP: For $\vec{A}$, $\theta = 270.0°$. For $\vec{B}$, $\theta = 60.0°$. For $\vec{C}$, $\theta = 205.0°$. For $\vec{D}$, $\theta = 143.0°$.

EXECUTE: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

EVALUATE: The signs of the components correspond to the quadrant in which the vector lies.

1.28. **IDENTIFY:** $\tan\theta = \dfrac{A_y}{A_x}$, for θ measured counterclockwise from the $+x$-axis.

SET UP: A sketch of A_x, A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE:

(a) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{2.00\text{ m}} = -0.500.\ \theta = \tan^{-1}(-0.500) = 360° - 26.6° = 333°.$

(b) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{2.00\text{ m}} = 0.500.\ \theta = \tan^{-1}(0.500) = 26.6°.$

(c) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{-2.00\text{ m}} = -0.500.\ \theta = \tan^{-1}(-0.500) = 180° - 26.6° = 153°.$

(d) $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{-2.00\text{ m}} = 0.500.\ \theta = \tan^{-1}(0.500) = 180° + 26.6° = 207°$

EVALUATE: The angles 26.6° and 207° have the same tangent. Our sketch tells us which is the correct value of θ.

1.29. **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: **(a)** $\tan 32.0° = \dfrac{|A_x|}{|A_y|}$

$|A_x| = (9.60\text{ m})\tan 32.0° = 6.00\text{ m}.\ A_x = -6.00\text{ m}.$

(b) $A = \sqrt{A_x^2 + A_y^2} = 11.3\text{ m}.$

EVALUATE: The magnitude is greater than either of the components.

1.30. **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: **(a)** $\tan 34.0° = \dfrac{|A_x|}{|A_y|}$

$$|A_y| = \frac{|A_x|}{\tan 34.0°} = \frac{16.0\text{ m}}{\tan 34.0°} = 23.72\text{ m}$$

$A_y = -23.7$ m.

(b) $A = \sqrt{A_x^2 + A_y^2} = 28.6\text{ m}.$

EVALUATE: The magnitude is greater than either of the components.

1.31. **IDENTIFY:** If $\vec{C} = \vec{A} + \vec{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Use C_x and C_y to find the magnitude and direction of $\vec{C}$.

SET UP: From Figure E1.24 in the textbook, $A_x = 0$, $A_y = -8.00$ m and $B_x = +B\sin 30.0° = 7.50$ m, $B_y = +B\cos 30.0° = 13.0$ m.

EXECUTE: **(a)** $\vec{C} = \vec{A} + \vec{B}$ so $C_x = A_x + B_x = 7.50$ m and $C_y = A_y + B_y = +5.00$ m. $C = 9.01$ m.

$\tan\theta = \dfrac{C_y}{C_x} = \dfrac{5.00 \text{ m}}{7.50 \text{ m}}$ and $\theta = 33.7°$.

(b) $\vec{B} + \vec{A} = \vec{A} + \vec{B}$, so $\vec{B} + \vec{A}$ has magnitude 9.01 m and direction specified by $33.7°$.

(c) $\vec{D} = \vec{A} - \vec{B}$ so $D_x = A_x - B_x = -7.50$ m and $D_y = A_y - B_y = -21.0$ m. $D = 22.3$ m.

$\tan\phi = \dfrac{D_y}{D_x} = \dfrac{-21.0 \text{ m}}{-7.50 \text{ m}}$ and $\phi = 70.3°$. $\vec{D}$ is in the 3^{rd} quadrant and the angle θ counterclockwise from the $+x$ axis is $180° + 70.3° = 250.3°$.

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has magnitude 22.3 m and direction specified by $\theta = 70.3°$.

EVALUATE: These results agree with those calculated from a scale drawing in Problem 1.24.

1.32. **IDENTIFY:** Find the vector sum of the three given displacements.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The driver's vector displacements are:

$\vec{A} = 2.6$ km, 0° of north; $\vec{B} = 4.0$ km, 0° of east; $\vec{C} = 3.1$ km, 45° north of east.

EXECUTE: $R_x = A_x + B_x + C_x = 0 + 4.0 \text{ km} + (3.1 \text{ km})\cos(45°) = 6.2$ km; $R_y = A_y + B_y + C_y =$ $2.6 \text{ km} + 0 + (3.1 \text{ km})(\sin 45°) = 4.8$ km; $R = \sqrt{R_x^2 + R_y^2} = 7.8$ km; $\theta = \tan^{-1}[(4.8 \text{ km})/(6.2 \text{ km})] = 38°$;

$\vec{R} = 7.8$ km, 38° north of east. This result is confirmed by the sketch in Figure 1.32.

EVALUATE: Both R_x and R_y are positive and $\vec{R}$ is in the first quadrant.

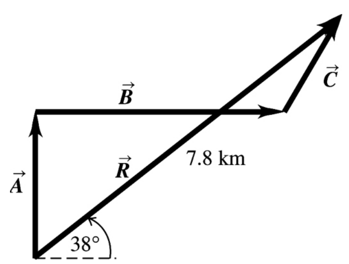
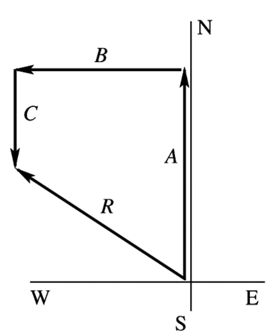


Figure 1.32

1.33. **IDENTIFY:** Vector addition problem. We are given the magnitude and direction of three vectors and are asked to find their sum.

SET UP:



$A = 3.25$ km

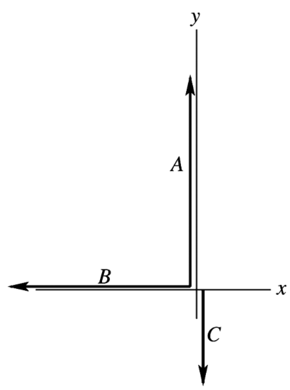
$B = 2.20$ km

$C = 1.50$ km

Figure 1.33a

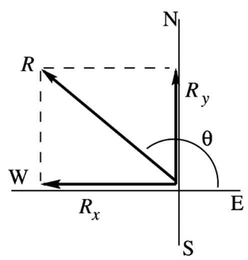
Select a coordinate system where $+x$ is east and $+y$ is north. Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements of the professor. Then the resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. By the method of components, $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$. Find the x and y components of each vector; add them to find the components of the resultant. Then the magnitude and direction of the resultant can be found from its x and y components that we have calculated. As always it is essential to draw a sketch.

EXECUTE:



$$A_x = 0,\ A_y = +3.25\text{ km}$$
$$B_x = -2.20\text{ km},\ B_y = 0$$
$$C_x = 0,\ C_y = -1.50\text{ km}$$
$$R_x = A_x + B_x + C_x$$
$$R_x = 0 - 2.20\text{ km} + 0 = -2.20\text{ km}$$
$$R_y = A_y + B_y + C_y$$
$$R_y = 3.25\text{ km} + 0 - 1.50\text{ km} = 1.75\text{ km}$$

Figure 1.33b



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20\text{ km})^2 + (1.75\text{ km})^2}$$
$$R = 2.81\text{ km}$$
$$\tan\theta = \frac{R_y}{R_x} = \frac{1.75\text{ km}}{-2.20\text{ km}} = -0.800$$
$$\theta = 141.5°$$

Figure 1.33c

The angle θ measured counterclockwise from the $+x$-axis. In terms of compass directions, the resultant displacement is $38.5°$ N of W.

EVALUATE: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant. This agrees with the vector addition diagram.

1.34. **IDENTIFY:** Use $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \dfrac{A_y}{A_x}$ to calculate the magnitude and direction of each of the given vectors.

SET UP: A sketch of A_x, A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE: **(a)** $\sqrt{(-8.60\text{ cm})^2 + (5.20\text{ cm})^2} = 10.0\text{ cm}$, $\arctan\left(\dfrac{5.20}{-8.60}\right) = 148.8°$ (which is $180° - 31.2°$).

(b) $\sqrt{(-9.7\text{ m})^2 + (-2.45\text{ m})^2} = 10.0\text{ m}$, $\arctan\left(\dfrac{-2.45}{-9.7}\right) = 14° + 180° = 194°$.

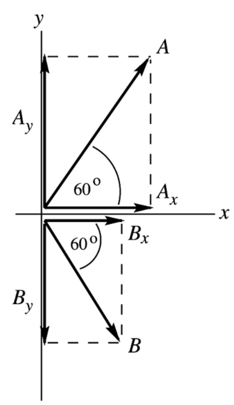
(c) $\sqrt{(7.75\text{ km})^2 + (-2.70\text{ km})^2} = 8.21\text{ km}$, $\arctan\left(\dfrac{-2.7}{7.75}\right) = 340.8°$ (which is $360° - 19.2°$).

EVALUATE: In each case the angle is measured counterclockwise from the $+x$ axis. Our results for θ agree with our sketches.

1.35. **IDENTIFY:** Vector addition problem. $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$.

SET UP: Find the x- and y-components of $\vec{A}$ and $\vec{B}$. Then the x- and y-components of the vector sum are calculated from the x- and y-components of $\vec{A}$ and $\vec{B}$.

EXECUTE:



$A_x = A\cos(60.0°)$
$A_x = (2.80\text{ cm})\cos(60.0°) = +1.40\text{ cm}$
$A_y = A\sin(60.0°)$
$A_y = (2.80\text{ cm})\sin(60.0°) = +2.425\text{ cm}$
$B_x = B\cos(-60.0°)$
$B_x = (1.90\text{ cm})\cos(-60.0°) = +0.95\text{ cm}$
$B_y = B\sin(-60.0°)$
$B_y = (1.90\text{ cm})\sin(-60.0°) = -1.645\text{ cm}$

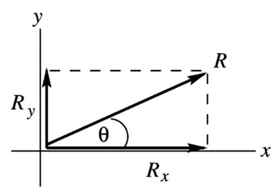
Note that the signs of the components correspond to the directions of the component vectors.

Figure 1.35a

(a) Now let $\vec{R}=\vec{A}+\vec{B}$.

$R_x = A_x + B_x = +1.40\text{ cm} + 0.95\text{ cm} = +2.35\text{ cm}.$

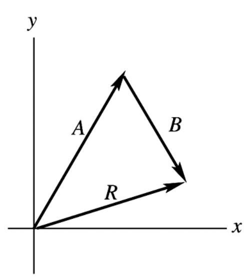
$R_y = A_y + B_y = +2.425\text{ cm} - 1.645\text{ cm} = +0.78\text{ cm}.$



$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35\text{ cm})^2 + (0.78\text{ cm})^2}$
$R = 2.48\text{ cm}$
$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{+0.78\text{ cm}}{+2.35\text{ cm}} = +0.3319$
$\theta = 18.4°$

Figure 1.35b

EVALUATE: The vector addition diagram for $\vec{R}=\vec{A}+\vec{B}$ is



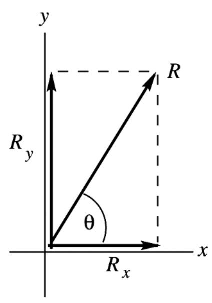
$\vec{R}$ is in the 1st quadrant, with $|R_y| < |R_x|$, in agreement with our calculation.

Figure 1.35c

(b) EXECUTE: Now let $\vec{R} = \vec{A} - \vec{B}$.

$R_x = A_x - B_x = +1.40 \text{ cm} - 0.95 \text{ cm} = +0.45 \text{ cm}.$

$R_y = A_y - B_y = +2.425 \text{ cm} + 1.645 \text{ cm} = +4.070 \text{ cm}.$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45 \text{ cm})^2 + (4.070 \text{ cm})^2}$$

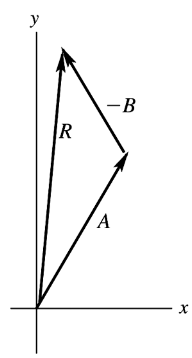
$$R = 4.09 \text{ cm}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{4.070 \text{ cm}}{0.45 \text{ cm}} = +9.044$$

$$\theta = 83.7°$$

Figure 1.35d

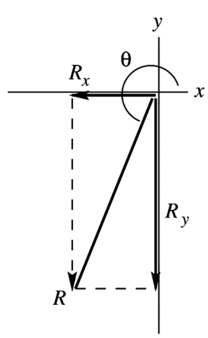
EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + (-\vec{B})$ is



$\vec{R}$ is in the 1st quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.35e

(c) EXECUTE:



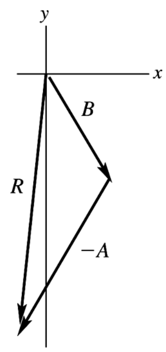
$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$

$\vec{B} - \vec{A}$ and $\vec{A} - \vec{B}$ are equal in magnitude and opposite in direction.

$R = 4.09 \text{ cm}$ and $\theta = 83.7° + 180° = 264°$

Figure 1.35f

EVALUATE: The vector addition diagram for $\vec{R} = \vec{B} + (-\vec{A})$ is



$\vec{R}$ is in the 3rd quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.35g

1.36. **IDENTIFY:** The general expression for a vector written in terms of components and unit vectors is $\vec{A} = A_x\hat{i} + A_y\hat{j}$.

SET UP: $5.0\vec{B} = 5.0(4\hat{i} - 6\hat{j}) = 20\vec{i} - 30\vec{j}$

EXECUTE: **(a)** $A_x = 5.0$, $A_y = -6.3$ **(b)** $A_x = 11.2$, $A_y = -9.91$ **(c)** $A_x = -15.0$, $A_y = 22.4$
(d) $A_x = 20$, $A_y = -30$

EVALUATE: The components are signed scalars.

1.37. **IDENTIFY:** Find the components of each vector and then use the general equation $\vec{A} = A_x\hat{i} + A_y\hat{j}$ for a vector in terms of its components and unit vectors.

SET UP: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

EXECUTE: $\vec{A} = (-8.00 \text{ m})\hat{j}$; $\vec{B} = (7.50 \text{ m})\hat{i} + (13.0 \text{ m})\hat{j}$; $\vec{C} = (-10.9 \text{ m})\hat{i} + (-5.07 \text{ m})\hat{j}$;
$\vec{D} = (-7.99 \text{ m})\hat{i} + (6.02 \text{ m})\hat{j}$.

EVALUATE: All these vectors lie in the *xy*-plane and have no *z*-component.

1.38. **IDENTIFY:** Find *A* and *B*. Find the vector difference using components.

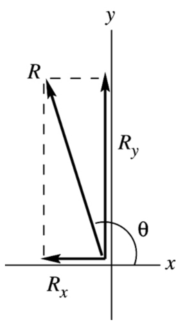
SET UP: Identify the *x*- and *y*-components and use $A = \sqrt{A_x^2 + A_y^2}$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$; $A_x = +4.00$; $A_y = +7.00$.
$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (7.00)^2} = 8.06$. $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $B_x = +5.00$; $B_y = -2.00$;
$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$.

EVALUATE: Note that the magnitudes of $\vec{A}$ and $\vec{B}$ are each larger than either of their components.

EXECUTE: **(b)** $\vec{A} - \vec{B} = 4.00\hat{i} + 7.00\hat{j} - (5.00\hat{i} - 2.00\hat{j}) = (4.00 - 5.00)\hat{i} + (7.00 + 2.00)\hat{j}$.
$\vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$

(c) Let $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$. Then $R_x = -1.00$, $R_y = 9.00$.



$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(-1.00)^2 + (9.00)^2} = 9.06.$

$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{9.00}{-1.00} = -9.00$

$\theta = -83.6° + 180° = 96.3°.$

Figure 1.38

EVALUATE: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant.

1.39. **IDENTIFY:** Use trigonometry to find the components of each vector. Use $R_x = A_x + B_x + \cdots$ and $R_y = A_y + B_y + \cdots$ to find the components of the vector sum. The equation $\vec{A} = A_x\hat{i} + A_y\hat{j}$ expresses a vector in terms of its components.

SET UP: Use the coordinates in the figure that accompanies the problem.

EXECUTE: **(a)** $\vec{A} = (3.60 \text{ m})\cos 70.0°\hat{i} + (3.60 \text{ m})\sin 70.0°\hat{j} = (1.23 \text{ m})\hat{i} + (3.38 \text{ m})\hat{j}$

$\vec{B} = -(2.40 \text{ m})\cos 30.0°\hat{i} - (2.40 \text{ m})\sin 30.0°\hat{j} = (-2.08 \text{ m})\hat{i} + (-1.20 \text{ m})\hat{j}$

(b) $\vec{C} = (3.00)\,\vec{A} - (4.00)\,\vec{B} = (3.00)(1.23 \text{ m})\hat{i} + (3.00)(3.38 \text{ m})\hat{j} - (4.00)(-2.08 \text{ m})\hat{i} - (4.00)(-1.20 \text{ m})\hat{j}$

$\vec{C} = (12.01 \text{ m})\hat{i} + (14.94 \text{ m})\hat{j}$

(c) From $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \dfrac{A_y}{A_x}$,

$C = \sqrt{(12.01 \text{ m})^2 + (14.94 \text{ m})^2} = 19.17 \text{ m}, \arctan\left(\dfrac{14.94 \text{ m}}{12.01 \text{ m}}\right) = 51.2°$

EVALUATE: C_x and C_y are both positive, so θ is in the first quadrant.

1.40. **IDENTIFY:** We use the vector components and trigonometry to find the angles.

SET UP: Use the fact that $\tan\theta = A_y / A_x$.

EXECUTE: **(a)** $\tan\theta = A_y / A_x = \dfrac{6.00}{-3.00}$. $\theta = 117°$ with the $+x$-axis.

(b) $\tan\theta = B_y / B_x = \dfrac{2.00}{7.00}$. $\theta = 15.9°$.

(c) First find the components of $\vec{C}$. $C_x = A_x + B_x = -3.00 + 7.00 = 4.00$,
$C_y = A_y + B_y = 6.00 + 2.00 = 8.00$

$\tan\theta = C_y / C_x = \dfrac{8.00}{4.00} = 2.00$. $\theta = 63.4°$

EVALUATE: Sketching each of the three vectors to scale will show that the answers are reasonable.

1.41. **IDENTIFY:** $\vec{A}$ and $\vec{B}$ are given in unit vector form. Find A, B and the vector difference $\vec{A} - \vec{B}$.

SET UP: $\vec{A} = -2.00\vec{i} + 3.00\vec{j} + 4.00\vec{k}$, $\vec{B} = 3.00\vec{i} + 1.00\vec{j} - 3.00\vec{k}$

Use $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ to find the magnitudes of the vectors.

EXECUTE: **(a)** $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = 5.38$

$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = 4.36$

(b) $\vec{A} - \vec{B} = (-2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}) - (3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k})$

$\vec{A} - \vec{B} = (-2.00 - 3.00)\hat{i} + (3.00 - 1.00)\hat{j} + (4.00 - (-3.00))\hat{k} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}.$

(c) Let $\vec{C} = \vec{A} - \vec{B}$, so $C_x = -5.00$, $C_y = +2.00$, $C_z = +7.00$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83$$

$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$ have the same magnitude but opposite directions.

EVALUATE: A, B, and C are each larger than any of their components.

1.42. **IDENTIFY:** Target variables are $\vec{A} \cdot \vec{B}$ and the angle ϕ between the two vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the scalar product using $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$. The angle ϕ can then be found from $\vec{A} \cdot \vec{B} = AB\cos\phi$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $A = 8.06$, $B = 5.39$.

$\vec{A} \cdot \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \cdot (5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (7.00)(-2.00) = 20.0 - 14.0 = +6.00.$

(b) $\cos\phi = \dfrac{\vec{A} \cdot \vec{B}}{AB} = \dfrac{6.00}{(8.06)(5.39)} = 0.1382$; $\phi = 82.1°$.

EVALUATE: The component of $\vec{B}$ along $\vec{A}$ is in the same direction as $\vec{A}$, so the scalar product is positive and the angle ϕ is less than $90°$.

1.43. **IDENTIFY:** $\vec{A} \cdot \vec{B} = AB\cos\phi$

SET UP: For $\vec{A}$ and $\vec{B}$, $\phi = 150.0°$. For $\vec{B}$ and $\vec{C}$, $\phi = 145.0°$. For $\vec{A}$ and $\vec{C}$, $\phi = 65.0°$.

EXECUTE: **(a)** $\vec{A} \cdot \vec{B} = (8.00\text{ m})(15.0\text{ m})\cos 150.0° = -104\text{ m}^2$

(b) $\vec{B} \cdot \vec{C} = (15.0\text{ m})(12.0\text{ m})\cos 145.0° = -148\text{ m}^2$

(c) $\vec{A} \cdot \vec{C} = (8.00\text{ m})(12.0\text{ m})\cos 65.0° = 40.6\text{ m}^2$

EVALUATE: When $\phi < 90°$ the scalar product is positive and when $\phi > 90°$ the scalar product is negative.

1.44. **IDENTIFY:** Target variable is the vector $\vec{A} \times \vec{B}$ expressed in terms of unit vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the vector product using $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, and $\hat{j} \times \hat{i} = -\hat{k}$.

EXECUTE: $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$.

$\vec{A} \times \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \times (5.00\hat{i} - 2.00\hat{j}) = 20.0\hat{i} \times \hat{i} - 8.00\hat{i} \times \hat{j} + 35.0\hat{j} \times \hat{i} - 14.0\hat{j} \times \hat{j}$. But $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$ and $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$, so $\vec{A} \times \vec{B} = -8.00\hat{k} + 35.0(-\hat{k}) = -43.0\hat{k}$. The magnitude of $\vec{A} \times \vec{B}$ is 43.0.

EVALUATE: Sketch the vectors $\vec{A}$ and $\vec{B}$ in a coordinate system where the xy-plane is in the plane of the paper and the z-axis is directed out toward you. By the right-hand rule $\vec{A} \times \vec{B}$ is directed into the plane of the paper, in the $-z$-direction. This agrees with the above calculation that used unit vectors.

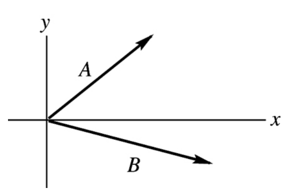


Figure 1.44

1.45. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining $\vec{A}\cdot\vec{B}=AB\cos\phi$ and $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$, to give the angle ϕ as $\phi=\arccos\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right)=\arccos\left(\dfrac{A_xB_x+A_yB_y}{AB}\right)$.

SET UP: $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$ shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B}=-22,\ A=\sqrt{40},\ B=\sqrt{13},$ and so $\phi=\arccos\left(\dfrac{-22}{\sqrt{40}\sqrt{13}}\right)=165°.$

(b) $\vec{A}\cdot\vec{B}=60,\ A=\sqrt{34},\ B=\sqrt{136},\ \ \phi=\arccos\left(\dfrac{60}{\sqrt{34}\sqrt{136}}\right)=28°.$

(c) $\vec{A}\cdot\vec{B}=0$ and $\phi=90°.$

EVALUATE: If $\vec{A}\cdot\vec{B}>0,\ 0\le\phi<90°$. If $\vec{A}\cdot\vec{B}<0,\ 90°<\phi\le180°$. If $\vec{A}\cdot\vec{B}=0,\ \phi=90°$ and the two vectors are perpendicular.

1.46. **IDENTIFY:** The right-hand rule gives the direction and $|\vec{A}\times\vec{B}|=AB\sin\phi$ gives the magnitude.

SET UP: $\phi=120.0°.$

EXECUTE: **(a)** The direction of $\vec{A}\times\vec{B}$ is into the page (the $-z$-direction). The magnitude of the vector product is $AB\sin\phi=(2.80\text{ cm})(1.90\text{ cm})\sin120°=4.61\text{ cm}^2.$

(b) Rather than repeat the calculations, $\vec{B}\times\vec{A}=-\vec{A}\times\vec{B}$ may be used to see that $\vec{B}\times\vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$-direction (out of the page).

EVALUATE: For part (a) we could use the components of the cross product and note that the only non-vanishing component is $C_z=A_xB_y-A_yB_x=(2.80\text{ cm})\cos60.0°(-1.90\text{ cm})\sin60°$

$$-(2.80\text{ cm})\sin60.0°(1.90\text{ cm})\cos60.0°=-4.61\text{ cm}^2.$$

This gives the same result.

1.47. **IDENTIFY:** $\vec{A}\times\vec{D}$ has magnitude $AD\sin\phi$. Its direction is given by the right-hand rule.

SET UP: $\phi=180°-53°=127°$

EXECUTE: **(a)** $|\vec{A}\times\vec{D}|=(8.00\text{ m})(10.0\text{ m})\sin127°=63.9\text{ m}^2$. The right-hand rule says $\vec{A}\times\vec{D}$ is in the $-z$-direction (into the page).

(b) $\vec{D}\times\vec{A}$ has the same magnitude as $\vec{A}\times\vec{D}$ and is in the opposite direction.

EVALUATE: The component of $\vec{D}$ perpendicular to $\vec{A}$ is $D_\perp=D\sin53.0°=7.99$ m.

$|\vec{A}\times\vec{D}|=AD_\perp=63.9\text{ m}^2$, which agrees with our previous result.

1.48. **IDENTIFY:** Apply Eqs. (1.16) and (1.20).

SET UP: The angle between the vectors is $20°+90°+30°=140°.$

EXECUTE: **(a)** $\vec{A}\cdot\vec{B}=AB\cos\phi$ gives $\vec{A}\cdot\vec{B}=(3.60\text{ m})(2.40\text{ m})\cos140°=-6.62\text{ m}^2.$

(b) From $|\vec{A}\times\vec{B}|=AB\sin\phi$, the magnitude of the cross product is $(3.60\text{ m})(2.40\text{ m})\sin140°=5.55\text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$-direction).

EVALUATE: We could also use $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$ and the cross product, with the components of $\vec{A}$ and $\vec{B}$.

1.49. **IDENTIFY:** We model the earth, white dwarf, and neutron star as spheres. Density is mass divided by volume.

SET UP: We know that density = mass/volume = m/V where $V=\frac{4}{3}\pi r^3$ for a sphere. From Appendix B, the earth has mass of $m=5.97\times10^{24}$ kg and a radius of $r=6.37\times10^6$ m whereas for the sun at the end of its lifetime, $m=1.99\times10^{30}$ kg and r = 7500 km = 7.5×10^6 m. The star possesses a radius of r = 10 km = 1.0×10^4 m and a mass of $m=1.99\times10^{30}$ kg.

EXECUTE: **(a)** The earth has volume $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.37\times10^6\ \text{m})^3 = 1.0827\times10^{21}\ \text{m}^3$. Its density is

$$\text{density} = \frac{m}{V} = \frac{5.97\times10^{24}\ \text{kg}}{1.0827\times10^{21}\ \text{m}^3} = (5.51\times10^3\ \text{kg/m}^3)\left(\frac{10^3\ \text{g}}{1\ \text{kg}}\right)\left(\frac{1\ \text{m}}{10^2\ \text{cm}}\right)^3 = 5.51\ \text{g/cm}^3$$

(b) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7.5\times10^6\ \text{m})^3 = 1.77\times10^{21}\ \text{m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99\times10^{30}\ \text{kg}}{1.77\times10^{21}\ \text{m}^3} = (1.1\times10^9\ \text{kg/m}^3)\left(\frac{1\ \text{g/cm}^3}{1000\ \text{kg/m}^3}\right) = 1.1\times10^6\ \text{g/cm}^3$$

(c) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\times10^4\ \text{m})^3 = 4.19\times10^{12}\ \text{m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99\times10^{30}\ \text{kg}}{4.19\times10^{12}\ \text{m}^3} = (4.7\times10^{17}\ \text{kg/m}^3)\left(\frac{1\ \text{g/cm}^3}{1000\ \text{kg/m}^3}\right) = 4.7\times10^{14}\ \text{g/cm}^3$$

EVALUATE: For a fixed mass, the density scales as $1/r^3$. Thus, the answer to (c) can also be obtained from (b) as

$$(1.1\times10^6\ \text{g/cm}^3)\left(\frac{7.50\times10^6\ \text{m}}{1.0\times10^4\ \text{m}}\right)^3 = 4.7\times10^{14}\ \text{g/cm}^3.$$

1.50. **IDENTIFY:** Area is length times width. Do unit conversions.

SET UP: 1 mi = 5280 ft. 1 ft^3 = 7.477 gal.

EXECUTE: **(a)** The area of one acre is $\frac{1}{8}\ \text{mi}\times\frac{1}{80}\ \text{mi} = \frac{1}{640}\ \text{mi}^2$, so there are 640 acres to a square mile.

(b) $(1\ \text{acre})\times\left(\frac{1\ \text{mi}^2}{640\ \text{acre}}\right)\times\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right)^2 = 43{,}560\ \text{ft}^2$

(all of the above conversions are exact).

(c) $(1\ \text{acre-foot}) = (43{,}560\ \text{ft}^3)\times\left(\frac{7.477\ \text{gal}}{1\ \text{ft}^3}\right) = 3.26\times10^5$ gal, which is rounded to three significant figures.

EVALUATE: An acre is much larger than a square foot but less than a square mile. A volume of 1 acre-foot is much larger than a gallon.

1.51. **IDENTIFY:** The density relates mass and volume. Use the given mass and density to find the volume and from this the radius.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg and radius $r_E = 6.37\times10^6$ m. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. $\rho = 1.76\ \text{g/cm}^3 = 1760\ \text{km/m}^3$.

EXECUTE: **(a)** The planet has mass $m = 5.5m_E = 3.28\times10^{25}$ kg. $V = \frac{m}{\rho} = \frac{3.28\times10^{25}\ \text{kg}}{1760\ \text{kg/m}^3} = 1.86\times10^{22}\ \text{m}^3$.

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3[1.86\times10^{22}\ \text{m}^3]}{4\pi}\right)^{1/3} = 1.64\times10^7\ \text{m} = 1.64\times10^4\ \text{km}$$

(b) $r = 2.57r_E$

EVALUATE: Volume V is proportional to mass and radius r is proportional to $V^{1/3}$, so r is proportional to $m^{1/3}$. If the planet and earth had the same density its radius would be $(5.5)^{1/3}r_E = 1.8r_E$. The radius of the planet is greater than this, so its density must be less than that of the earth.

1.52. **IDENTIFY** and **SET UP:** Unit conversion.

EXECUTE: **(a)** $f = 1.420\times10^9$ cycles/s, so $\frac{1}{1.420\times10^9}\ \text{s} = 7.04\times10^{-10}$ s for one cycle.

(b) $\frac{3600\ \text{s/h}}{7.04\times10^{-10}\ \text{s/cycle}} = 5.11\times10^{12}$ cycles/h

(c) Calculate the number of seconds in 4600 million $\text{years} = 4.6\times10^9\ \text{y}$ and divide by the time for 1 cycle:

$$\frac{(4.6\times10^9\ \text{y})(3.156\times10^7\ \text{s/y})}{7.04\times10^{-10}\ \text{s/cycle}} = 2.1\times10^{26}\ \text{cycles}$$

(d) The clock is off by 1 s in $100{,}000\ \text{y} = 1\times10^5\ \text{y}$, so in $4.60\times10^9\ \text{y}$ it is off by

$(1\ \text{s})\left(\dfrac{4.60\times10^9}{1\times10^5}\right) = 4.6\times10^4\ \text{s}$ (about 13 h).

EVALUATE: In each case the units in the calculation combine algebraically to give the correct units for the answer.

1.53. **IDENTIFY:** Using the density of the oxygen and volume of a breath, we want the mass of oxygen (the target variable in part (a)) breathed in per day and the dimensions of the tank in which it is stored.

SET UP: The mass is the density times the volume. Estimate 12 breaths per minute. We know 1 day = 24 h, 1 h = 60 min and 1000 L = 1 m^3. The volume of a cube having faces of length l is $V = l^3$.

EXECUTE: **(a)** $(12\ \text{breaths/min})\left(\dfrac{60\ \text{min}}{1\ \text{h}}\right)\left(\dfrac{24\ \text{h}}{1\ \text{day}}\right) = 17{,}280\ \text{breaths/day}.$ The volume of air breathed in one day is $(\frac{1}{2}\ \text{L/breath})(17{,}280\ \text{breaths/day}) = 8640\ \text{L} = 8.64\ \text{m}^3.$ The mass of air breathed in one day is the density of air times the volume of air breathed: $m = (1.29\ \text{kg/m}^3)(8.64\ \text{m}^3) = 11.1\ \text{kg}.$ As 20% of this quantity is oxygen, the mass of oxygen breathed in 1 day is $(0.20)(11.1\ \text{kg}) = 2.2\ \text{kg} = 2200\ \text{g}.$

(b) $V = 8.64\ \text{m}^3$ and $V = l^3$, so $l = V^{1/3} = 2.1\ \text{m}.$

EVALUATE: A person could not survive one day in a closed tank of this size because the exhaled air is breathed back into the tank and thus reduces the percent of oxygen in the air in the tank. That is, a person cannot extract all of the oxygen from the air in an enclosed space.

1.54. **IDENTIFY:** Use the extreme values in the piece's length and width to find the uncertainty in the area.

SET UP: The length could be as large as 7.61 cm and the width could be as large as 1.91 cm.

EXECUTE: **(a)** The area is $14.44 \pm 0.095\ \text{cm}^2$.

(b) The fractional uncertainty in the area is $\dfrac{0.095\ \text{cm}^2}{14.44\ \text{cm}^2} = 0.66\%,$ and the fractional uncertainties in the length and width are $\dfrac{0.01\ \text{cm}}{7.61\ \text{cm}} = 0.13\%$ and $\dfrac{0.01\ \text{cm}}{1.9\ \text{cm}} = 0.53\%.$ The sum of these fractional uncertainties is $0.13\% + 0.53\% = 0.66\%,$ in agreement with the fractional uncertainty in the area.

EVALUATE: The fractional uncertainty in a product of numbers is greater than the fractional uncertainty in any of the individual numbers.

1.55. **IDENTIFY:** Calculate the average volume and diameter and the uncertainty in these quantities.

SET UP: Using the extreme values of the input data gives us the largest and smallest values of the target variables and from these we get the uncertainty.

EXECUTE: **(a)** The volume of a disk of diameter d and thickness t is $V = \pi(d/2)^2 t$.

The average volume is $V = \pi(8.50\ \text{cm}/2)^2(0.050\ \text{cm}) = 2.837\ \text{cm}^3.$ But t is given to only two significant figures so the answer should be expressed to two significant figures: $V = 2.8\ \text{cm}^3$.

We can find the uncertainty in the volume as follows. The volume could be as large as $V = \pi(8.52\ \text{cm}/2)^2(0.055\ \text{cm}) = 3.1\ \text{cm}^3,$ which is $0.3\ \text{cm}^3$ larger than the average value. The volume could be as small as $V = \pi(8.48\ \text{cm}/2)^2(0.045\ \text{cm}) = 2.5\ \text{cm}^3,$ which is $0.3\ \text{cm}^3$ smaller than the average value. The uncertainty is $\pm0.3\ \text{cm}^3,$ and we express the volume as $V = 2.8 \pm 0.3\ \text{cm}^3$.

(b) The ratio of the average diameter to the average thickness is $8.50 \text{ cm}/0.050 \text{ cm} = 170$. By taking the largest possible value of the diameter and the smallest possible thickness we get the largest possible value for this ratio: $8.52 \text{ cm}/0.045 \text{ cm} = 190$. The smallest possible value of the ratio is $8.48/0.055 = 150$. Thus the uncertainty is ± 20 and we write the ratio as 170 ± 20.

EVALUATE: The thickness is uncertain by 10% and the percentage uncertainty in the diameter is much less, so the percentage uncertainty in the volume and in the ratio should be about 10%.

1.56. **IDENTIFY:** Estimate the volume of each object. The mass m is the density times the volume.

SET UP: The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. The volume of a cylinder of radius r and length l is $V = \pi r^2 l$. The density of water is 1000 kg/m^3.

EXECUTE: **(a)** Estimate the volume as that of a sphere of diameter 10 cm: $V = 5.2\times10^{-4} \text{ m}^3$.

$m = (0.98)(1000 \text{ kg / m}^3)(5.2\times10^{-4} \text{ m}^3) = 0.5 \text{ kg}$.

(b) Approximate as a sphere of radius $r = 0.25 \mu\text{m}$ (probably an overestimate): $V = 6.5\times10^{-20} \text{ m}^3$.

$m = (0.98)(1000 \text{ kg/m}^3)(6.5\times10^{-20} \text{ m}^3) = 6\times10^{-17} \text{ kg} = 6\times10^{-14} \text{ g}$.

(c) Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm: $V = \pi r^2 l = 2.8\times10^{-7} \text{ m}^3$.

$m = (0.98)(1000 \text{ kg/m}^3)(2.8\times10^{-7} \text{ m}^3) = 3\times10^{-4} \text{ kg} = 0.3 \text{ g}$.

EVALUATE: The mass is directly proportional to the volume.

1.57. **IDENTIFY:** The number of atoms is your mass divided by the mass of one atom.

SET UP: Assume a 70-kg person and that the human body is mostly water. Use Appendix D to find the mass of one H_2O molecule: $18.015 \text{ u}\times1.661\times10^{-27} \text{ kg/u} = 2.992\times10^{-26} \text{ kg/molecule}$.

EXECUTE: $(70 \text{ kg})/(2.992\times10^{-26} \text{ kg/molecule}) = 2.34\times10^{27}$ molecules. Each H_2O molecule has 3 atoms, so there are about 6×10^{27} atoms.

EVALUATE: Assuming carbon to be the most common atom gives 3×10^{27} molecules, which is a result of the same order of magnitude.

1.58. **IDENTIFY:** We know the vector sum and want to find the magnitude of the vectors. Use the method of components.

SET UP: The two vectors $\vec{A}$ and $\vec{B}$ and their resultant $\vec{C}$ are shown in Figure 1.58. Let $+y$ be in the direction of the resultant. $A = B$.

EXECUTE: $C_y = A_y + B_y$. $372 \text{ N} = 2A\cos 36.0°$ gives $A = 230 \text{ N}$.

EVALUATE: The sum of the magnitudes of the two forces exceeds the magnitude of the resultant force because only a component of each force is upward.

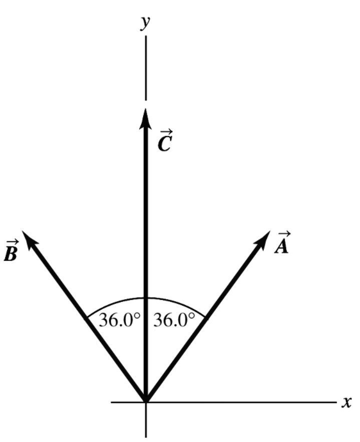


Figure 1.58

1.59. **IDENTIFY:** We know the magnitude and direction of the sum of the two vector pulls and the direction of one pull. We also know that one pull has twice the magnitude of the other. There are two unknowns, the magnitude of the smaller pull and its direction. $A_x + B_x = C_x$ and $A_y + B_y = C_y$ give two equations for these two unknowns.

SET UP: Let the smaller pull be $\vec{A}$ and the larger pull be $\vec{B}$. $B = 2A$. $\vec{C} = \vec{A} + \vec{B}$ has magnitude 460.0 N and is northward. Let $+x$ be east and $+y$ be north. $B_x = -B\sin 21.0°$ and $B_y = B\cos 21.0°$. $C_x = 0$, $C_y = 460.0$ N. $\vec{A}$ must have an eastward component to cancel the westward component of $\vec{B}$. There are then two possibilities, as sketched in Figures 1.59 a and b. $\vec{A}$ can have a northward component or $\vec{A}$ can have a southward component.

EXECUTE: In either Figure 1.59 a or b, $A_x + B_x = C_x$ and $B = 2A$ gives $(2A)\sin 21.0° = A\sin\phi$ and $\phi = 45.79°$. In Figure 1.59a, $A_y + B_y = C_y$ gives $2A\cos 21.0° + A\cos 45.79° = 460.0$ N, so $A = 179.4$ N. In Figure 1.59b, $2A\cos 21.0° - A\cos 45.79° = 460.0$ N and $A = 393$ N. One solution is for the smaller pull to be $45.8°$ east of north. In this case, the smaller pull is 179 N and the larger pull is 358 N. The other solution is for the smaller pull to be 45.8° south of east. In this case the smaller pull is 393 N and the larger pull is 786 N.

EVALUATE: For the first solution, with $\vec{A}$ east of north, each worker has to exert less force to produce the given resultant force and this is the sensible direction for the worker to pull.

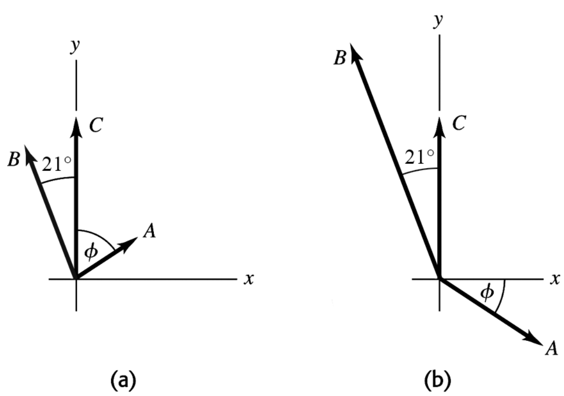


Figure 1.59

1.60. **IDENTIFY:** Let $\vec{D}$ be the fourth force. Find $\vec{D}$ such that $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$.

SET UP: Use components and solve for the components D_x and D_y of $\vec{D}$.

EXECUTE: $A_x = +A\cos 30.0° = +86.6\,\text{N}$, $A_y = +A\sin 30.0° = +50.00\,\text{N}$.

$B_x = -B\sin 30.0° = -40.00\,\text{N}$, $B_y = +B\cos 30.0° = +69.28\,\text{N}$.

$C_x = -C\cos 53.0° = -24.07\,\text{N}$, $C_y = -C\sin 53.0° = -31.90\,\text{N}$.

Then $D_x = -22.53$ N, $D_y = -87.34$ N and $D = \sqrt{D_x^2 + D_y^2} = 90.2$ N. $\tan\alpha = |D_y/D_x| = 87.34/22.53$.

$\alpha = 75.54°$. $\phi = 180° + \alpha = 256°$, counterclockwise from the $+x$-axis.

EVALUATE: As shown in Figure 1.60, since D_x and D_y are both negative, $\vec{D}$ must lie in the third quadrant.

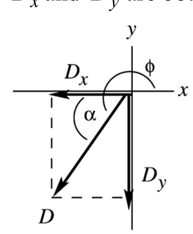


Figure 1.60

1.61. **IDENTIFY:** Vector addition. Target variable is the 4th displacement.

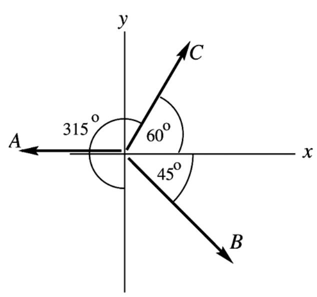
SET UP: Use a coordinate system where east is in the $+x$-direction and north is in the $+y$-direction.

Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements that are given and let $\vec{D}$ be the fourth unmeasured displacement. Then the resultant displacement is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. And since she ends up back where she started, $\vec{R} = 0$.

$0 = \vec{A} + \vec{B} + \vec{C} + \vec{D}$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$

$D_x = -(A_x + B_x + C_x)$ and $D_y = -(A_y + B_y + C_y)$

EXECUTE:



$A_x = -180 \text{ m}, \quad A_y = 0$

$B_x = B\cos 315° = (210 \text{ m})\cos 315° = +148.5 \text{ m}$

$B_y = B\sin 315° = (210 \text{ m})\sin 315° = -148.5 \text{ m}$

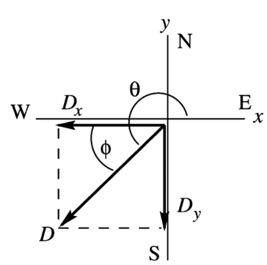
$C_x = C\cos 60° = (280 \text{ m})\cos 60° = +140 \text{ m}$

$C_y = C\sin 60° = (280 \text{ m})\sin 60° = +242.5 \text{ m}$

Figure 1.61a

$D_x = -(A_x + B_x + C_x) = -(-180 \text{ m} + 148.5 \text{ m} + 140 \text{ m}) = -108.5 \text{ m}$

$D_y = -(A_y + B_y + C_y) = -(0 - 148.5 \text{ m} + 242.5 \text{ m}) = -94.0 \text{ m}$



$D = \sqrt{D_x^2 + D_y^2}$

$D = \sqrt{(-108.5 \text{ m})^2 + (-94.0 \text{ m})^2} = 144 \text{ m}$

$\tan\theta = \dfrac{D_y}{D_x} = \dfrac{-94.0 \text{ m}}{-108.5 \text{ m}} = 0.8664$

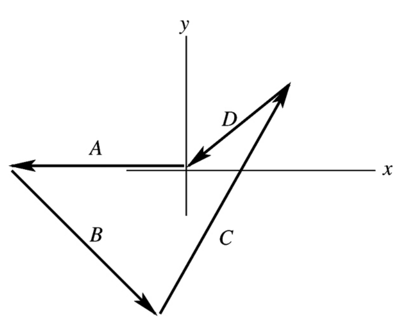
$\theta = 180° + 40.9° = 220.9°$

($\vec{D}$ is in the third quadrant since both D_x and D_y are negative.)

Figure 1.61b

The direction of $\vec{D}$ can also be specified in terms of $\phi = \theta - 180° = 40.9°$; $\vec{D}$ is $41°$ south of west.

EVALUATE: The vector addition diagram, approximately to scale, is



Vector $\vec{D}$ in this diagram agrees qualitatively with our calculation using components.

Figure 1.61c

1.62. **IDENTIFY:** Find the vector sum of the two displacements.

SET UP: Call the two displacements $\vec{A}$ and $\vec{B}$, where $A = 170$ km and $B = 230$ km. $\vec{A} + \vec{B} = \vec{R}$. $\vec{A}$ and $\vec{B}$ are as shown in Figure 1.62.

EXECUTE: $R_x = A_x + B_x = (170 \text{ km})\sin 68° + (230 \text{ km})\cos 36° = 343.7$ km.

$R_y = A_y + B_y = (170 \text{ km})\cos 68° - (230 \text{ km})\sin 36° = -71.5$ km.

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(343.7 \text{ km})^2 + (-71.5 \text{ km})^2} = 351$ km. $\tan\theta_R = |\frac{R_y}{R_x}| = \frac{71.5 \text{ km}}{343.7 \text{ km}} = 0.208.$

$\theta_R = 11.8°$ south of east.

EVALUATE: Our calculation using components agrees with $\vec{R}$ shown in the vector addition diagram, Figure 1.62.

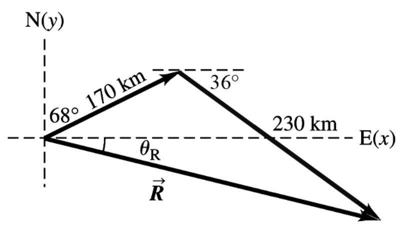


Figure 1.62

1.63. **IDENTIFY:** We know the resultant of two forces of known equal magnitudes and want to find that magnitude (the target variable).

SET UP: Use coordinates having a horizontal $+x$ axis and an upward $+y$ axis. Then $A_x + B_x = R_x$ and $R_x = 12.8$ N.

SOLVE: $A_x + B_x = R_x$ and $A\cos 32° + B\sin 32° = R_x$. Since $A = B$,

$2A\cos 32° = R_x$, so $A = \frac{R_x}{(2)(\cos 32°)} = 7.55$ N.

EVALUATE: The magnitude of the x component of each pull is 6.40 N, so the magnitude of each pull (7.55 N) is greater than its x component, as it should be.

1.64. **IDENTIFY:** Solve for one of the vectors in the vector sum. Use components.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 2.00$ km, 0°of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

EXECUTE: $C_x = R_x - A_x - B_x = 5.80 \text{ km} - (2.00 \text{ km}) - (3.50 \text{ km})(\cos 45°) = 1.33$ km; $C_y = R_y - A_y - B_y$ $= 0 \text{ km} - 0 \text{ km} - (-3.50 \text{ km})(\sin 45°) = 2.47$ km; $C = \sqrt{(1.33 \text{ km})^2 + (2.47 \text{ km})^2} = 2.81$ km;

$\theta = \tan^{-1}[(2.47 \text{ km})/(1.33 \text{ km})] = 61.7°$ north of east. The vector addition diagram in Figure 1.64 shows good qualitative agreement with these values.

EVALUATE: The third leg lies in the first quadrant since its x and y components are both positive.

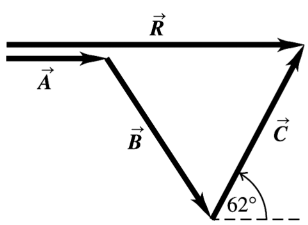


Figure 1.64

1.65. **IDENTIFY:** We have two known vectors and a third unknown vector, and we know the resultant of these three vectors.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 23.0$ km at 34.0° south of east; $\vec{B} = 46.0$ km due north; $\vec{R} = 32.0$ km due west ; $\vec{C}$ is unknown.

EXECUTE: $C_x = R_x - A_x - B_x = -32.0 \text{ km} - (23.0 \text{ km})\cos 34.0° - 0 = -51.07 \text{ km}$;

$C_y = R_y - A_y - B_y = 0 - (-23.0 \text{ km})\sin 34.0° - 46.0 \text{ km} = -33.14 \text{ km}$;

$C = \sqrt{C_x^2 + C_y^2} = 60.9 \text{ km}$

Calling θ the angle that $\vec{C}$ makes with the $-x$-axis (the westward direction), we have

$\tan\theta = C_y / C_x = \dfrac{33.14}{51.07}$; $\theta = 33.0°$ south of west.

EVALUATE: A graphical vector sum will confirm this result.

1.66. **IDENTIFY:** The four displacements return her to her starting point, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$, where $\vec{A}$, $\vec{B}$, and $\vec{C}$ are in the three given displacements and $\vec{D}$ is the displacement for her return.

SET UP: Let $+x$ be east and $+y$ be north.

EXECUTE: **(a)** $D_x = -[(147 \text{ km})\sin 85° + (106 \text{ km})\sin 167° + (166 \text{ km})\sin 235°] = -34.3 \text{ km}$.

$D_y = -[(147 \text{ km})\cos 85° + (106 \text{ km})\cos 167° + (166 \text{ km})\cos 235°] = +185.7 \text{ km}$.

$D = \sqrt{(-34.3 \text{ km})^2 + (185.7 \text{ km})^2} = 189 \text{ km}$.

(b) The direction relative to north is $\phi = \arctan\left(\dfrac{34.3 \text{ km}}{185.7 \text{ km}}\right) = 10.5°$. Since $D_x < 0$ and $D_y > 0$, the direction of $\vec{D}$ is $10.5°$ west of north.

EVALUATE: The four displacements add to zero.

1.67. **IDENTIFY:** We want to find the resultant of three known displacement vectors: $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

SET UP: Let $+x$ be east and $+y$ be north and find the components of the vectors.

EXECUTE: The magnitudes are $A = 20.8$ m, $B = 38.0$ m, $C = 18.0$ m. The components are

$A_x = 0$, $A_y = 28.0$ m, $B_x = 38.0$ m, $B_y = 0$,

$C_x = -(18.0 \text{ m})(\sin 33.0°) = -9.804$ m, $C_y = -(18.0 \text{ m})(\cos 33.0°) = -15.10$ m

$R_x = A_x + B_x + C_x = 0 + 38.0 \text{ m} + (-9.80 \text{ m}) = 28.2$ m

$R_y = A_y + B_y + C_y = 20.8 \text{ m} + 0 + (-15.10 \text{ m}) = 5.70$ m

$R = \sqrt{R_x^2 + R_y^2} = 28.8$ m is the distance you must run. Calling θ_R the angle the resultant makes with the $+x$-axis (the easterly direction), we have

$\tan\theta_R = R_y/R_x = (5.70 \text{ km})/(28.2 \text{ km})$; $\theta_R = 11.4°$ north of east.

EVALUATE: A graphical sketch will confirm this result.

1.68. **IDENTIFY:** Let the three given displacements be $\vec{A}$, $\vec{B}$ and $\vec{C}$, where $A = 40$ steps, $B = 80$ steps and $C = 50$ steps. $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. The displacement $\vec{C}$ that will return him to his hut is $-\vec{R}$.

SET UP: Let the east direction be the $+x$-direction and the north direction be the $+y$-direction.

EXECUTE: **(a)** The three displacements and their resultant are sketched in Figure 1.68.

(b) $R_x = (40)\cos 45° - (80)\cos 60° = -11.7$ and $R_y = (40)\sin 45° + (80)\sin 60° - 50 = 47.6$.

The magnitude and direction of the resultant are $\sqrt{(-11.7)^2 + (47.6)^2} = 49$, acrtan$\left(\dfrac{47.6}{11.7}\right) = 76°$, north of west. We know that $\vec{R}$ is in the second quadrant because $R_x < 0$, $R_y > 0$. To return to the hut, the explorer must take 49 steps in a direction $76°$ south of east, which is $14°$ east of south.

EVALUATE: It is useful to show R_x, R_y, and $\vec{R}$ on a sketch, so we can specify what angle we are computing.

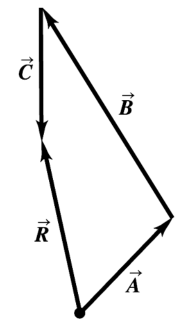


Figure 1.68

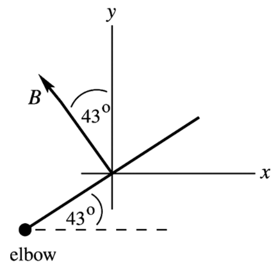
1.69. **IDENTIFY:** We know the resultant of two vectors and one of the vectors, and we want to find the second vector.
SET UP: Let the westerly direction be the $+x$-direction and the northerly direction be the $+y$-direction. We also know that $\vec{R} = \vec{A} + \vec{B}$ where $\vec{R}$ is the vector from you to the truck. Your GPS tells you that you are 122.0 m from the truck in a direction of 58.0° east of south, so a vector from the truck to you is 122.0 m at 58.0° east of south. Therefore the vector from you to the truck is 122.0 m at 58.0° west of north. Thus $\vec{R}$ = 122.0 m at 58.0° west of north and $\vec{A}$ is 72.0 m due west. We want to find the magnitude and direction of vector $\vec{B}$.
EXECUTE: $B_x = R_x - A_x = (122.0\text{ m})(\sin 58.0°) - 72.0\text{ m} = 31.462\text{ m}$
$B_y = R_y - A_y = (122.0\text{ m})(\cos 58.0°) - 0 = 64.450\text{ m}$; $B = \sqrt{B_x^2 + B_y^2} = 71.9\text{ m}$.

$$\tan\theta_B = B_y / B_x = \frac{64.650\text{ m}}{31.462\text{ m}} = 2.05486\text{ ; } \theta_B = 64.1°\text{ north of west.}$$

EVALUATE: A graphical sum will show that the results are reasonable.

1.70. **IDENTIFY:** We use vector addition. One vector and the sum are given; find the magnitude and direction of the second vector.
SET UP: Let $+x$ be east and $+y$ be north. Let $\vec{A}$ be the displacement 285 km at 62.0° north of west and let $\vec{B}$ be the unknown displacement.
$\vec{A} + \vec{B} = \vec{R}$ where $\vec{R} = 115\text{ km, east}$
$\vec{B} = \vec{R} - \vec{A}$
$B_x = R_x - A_x$, $B_y = R_y - A_y$
EXECUTE: $A_x = -A\cos 62.0° = -133.8\text{ km}$, $A_y = +A\sin 62.0° = +251.6\text{ km}$
$R_x = 115\text{ km}$, $R_y = 0$
$B_x = R_x - A_x = 115\text{ km} - (-133.8\text{ km}) = 248.8\text{ km}$
$B_y = R_y - A_y = 0 - 251.6\text{ km} = -251.6\text{ km}$
$B = \sqrt{B_x^2 + B_y^2} = 354\text{ km}$. Since $\vec{B}$ has a positive x component and a negative y component, it must lie in the fourth quadrant. Its angle with the $+x$-axis is given by $\tan\alpha = |B_y/B_x| = (251.6\text{ km})/(248.8\text{ km})$, so $\alpha = 45.3°$ south of east.
EVALUATE: A graphical vector sum will confirm these results.

1.71. **IDENTIFY:** Vector addition. One force and the vector sum are given; find the second force.
SET UP: Use components. Let $+y$ be upward.



$\vec{B}$ is the force the biceps exerts.

Figure 1.71a

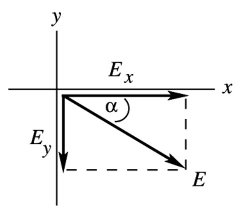
$\vec{E}$ is the force the elbow exerts. $\vec{E}+\vec{B}=\vec{R}$, where $R = 132.5\text{ N}$ and is upward.

$E_x = R_x - B_x$, $E_y = R_y - B_y$

EXECUTE: $B_x = -B\sin 43° = -158.2\text{ N}$, $B_y = +B\cos 43° = +169.7\text{ N}$, $R_x = 0$, $R_y = +132.5\text{ N}$

Then $E_x = +158.2\text{ N}$, $E_y = -37.2\text{ N}$.

$E = \sqrt{E_x^2 + E_y^2} = 160\text{ N}$;



$\tan\alpha = |E_y/E_x| = 37.2/158.2$
$\alpha = 13°$, below horizontal

Figure 1.71b

EVALUATE: The x-component of $\vec{E}$ cancels the x-component of $\vec{B}$. The resultant upward force is less than the upward component of $\vec{B}$, so E_y must be downward.

1.72. **IDENTIFY:** Find the vector sum of the four displacements.
SET UP: Take the beginning of the journey as the origin, with north being the y-direction, east the x-direction, and the z-axis vertical. The first displacement is then $(-30\text{ m})\hat{k}$, the second is $(-15\text{ m})\hat{j}$, the third is $(200\text{ m})\hat{i}$, and the fourth is $(100\text{ m})\hat{j}$.

EXECUTE: **(a)** Adding the four displacements gives

$(-30\text{ m})\hat{k} + (-15\text{ m})\hat{j} + (200\text{ m})\hat{i} + (100\text{ m})\hat{j} = (200\text{ m})\hat{i} + (85\text{ m})\hat{j} - (30\text{ m})\hat{k}$.

(b) The total distance traveled is the sum of the distances of the individual segments:
$30\text{ m} + 15\text{ m} + 200\text{ m} + 100\text{ m} = 345\text{ m}$. The magnitude of the total displacement is:

$$D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(200\text{ m})^2 + (85\text{ m})^2 + (-30\text{ m})^2} = 219\text{ m}.$$

EVALUATE: The magnitude of the displacement is much less than the distance traveled along the path.

1.73. **IDENTIFY:** The sum of the four displacements must be zero. Use components.
SET UP: Call the displacements $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, where $\vec{D}$ is the final unknown displacement for the return from the treasure to the oak tree. Vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are sketched in Figure 1.73a.

$\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$ says $A_x + B_x + C_x + D_x = 0$ and $A_y + B_y + C_y + D_y = 0$. $A = 825\text{ m}$, $B = 1250\text{ m}$, and $C = 1000\text{ m}$. Let $+x$ be eastward and $+y$ be north.

EXECUTE: (a) $A_x + B_x + C_x + D_x = 0$ gives
$D_x = -(A_x + B_x + C_x) = -[0 - (1250\ \text{m})\sin 30.0° + (1000\ \text{m})\cos 32.0°] = -223.0\ \text{m}$. $A_y + B_y + C_y + D_y = 0$ gives $D_y = -(A_y + B_y + C_y) = -[-825\ \text{m} + (1250\ \text{m})\cos 30.0° + (1000\ \text{m})\sin 32.0°] = -787.4\ \text{m}$. The fourth displacement $\vec{D}$ and its components are sketched in Figure 1.73b. $D = \sqrt{D_x^2 + D_y^2} = 818.4\ \text{m}$.
$\tan\phi = \dfrac{|D_x|}{|D_y|} = \dfrac{223.0\ \text{m}}{787.4\ \text{m}}$ and $\phi = 15.8°$. You should head 15.8° west of south and must walk 818 m.

(b) The vector diagram is sketched in Figure 1.73c. The final displacement $\vec{D}$ from this diagram agrees with the vector $\vec{D}$ calculated in part **(a)** using components.

EVALUATE: Note that $\vec{D}$ is the negative of the sum of $\vec{A}$, $\vec{B}$, and $\vec{C}$, as it should be.

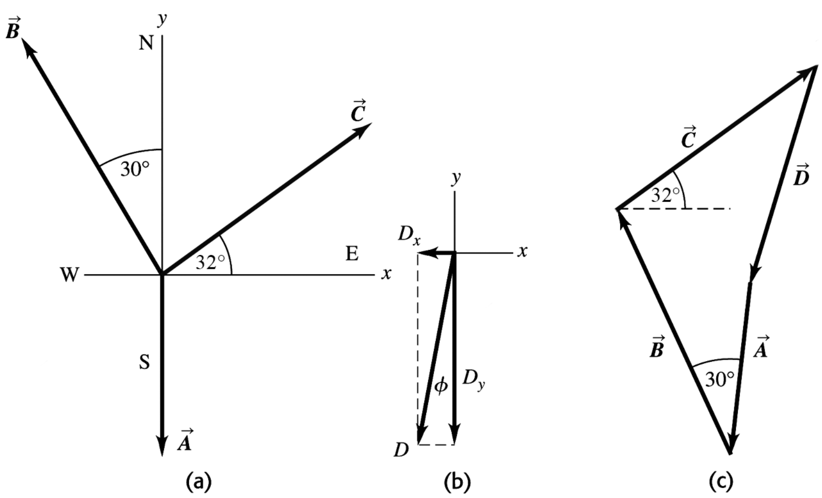


Figure 1.73

1.74. **IDENTIFY:** The displacements are vectors in which we want to find the magnitude of the resultant and know the other vectors.

SET UP: Calling $\vec{A}$ the vector from you to the first post, $\vec{B}$ the vector from you to the second post, and $\vec{C}$ the vector from the first post to the second post, we have $\vec{A} + \vec{C} = \vec{B}$. We want to find the magnitude of vector $\vec{B}$. We use components and the magnitude of $\vec{C}$. Let $+x$ be toward the east and $+y$ be toward the north.

EXECUTE: $B_x = 0$ and B_y is unknown. $C_x = -A_x = -(52.0\ \text{m})(\cos 37.0°) = -41.529\ \text{m}$ $A_x = 41.53\ \text{m}$
$C = 68.0\ \text{m}$, so $C_y = \pm\sqrt{C^2 - C_x^2} = -53.8455\ \text{m}$. We use the minus sign because the second post is south of the first post.
$B_y = A_y + C_y = (52.0\ \text{m})(\sin 37°) + (-53.8455\ \text{m}) = -22.551\ \text{m}$.
Therefore you are 22.6 m from the second post.

EVALUATE: B_y is negative since post is south of you (in the negative y direction), but the distance to you is positive.

1.75. **IDENTIFY:** We are given the resultant of three vectors, two of which we know, and want to find the magnitude and direction of the third vector.

SET UP: Calling $\vec{C}$ the unknown vector and $\vec{A}$ and $\vec{B}$ the known vectors, we have $\vec{A} + \vec{B} + \vec{C} = \vec{R}$. The components are $A_x + B_x + C_x = R_x$ and $A_y + B_y + C_y = R_y$.

EXECUTE: The components of the known vectors are $A_x = 12.0\text{ m}$, $A_y = 0$, $B_x = -B\sin 50.0° = -21.45\text{ m}$, $B_y = B\cos 50.0° = +18.00\text{ m}$, $R_x = 0$, and $R_y = -10.0\text{ m}$. Therefore the components of $\vec{C}$ are $C_x = R_x - A_x - B_x = 0 - 12.0\text{ m} - (-21.45\text{ m}) = 9.45\text{ m}$ and $C_y = R_y - A_y - B_y = -10.0\text{ m} - 0 - 18.0\text{ m} = -28.0\text{ m}$.

Using these components to find the magnitude and direction of $\vec{C}$ gives $C = 29.6\text{ m}$ and $\tan\theta = \dfrac{9.45}{28.0}$ and $\theta = 18.6°$ east of south.

EVALUATE: A graphical sketch shows that this answer is reasonable.

1.76. **IDENTIFY:** The displacements are vectors in which we know the magnitude of the resultant and want to find the magnitude of one of the other vectors.

SET UP: Calling $\vec{A}$ the vector of Ricardo's displacement from the tree, $\vec{B}$ the vector of Jane's displacement from the tree, and $\vec{C}$ the vector from Ricardo to Jane, we have $\vec{A} + \vec{C} = \vec{B}$. Let the $+x$-axis be to the east and the $+y$-axis be to the north. Solving using components we have $A_x + C_x = B_x$ and $A_y + C_y = B_y$.

EXECUTE: **(a)** The components of $\vec{A}$ and $\vec{B}$ are $A_x = -(26.0\text{ m})\sin 60.0° = -22.52\text{ m}$, $A_y = (26.0\text{ m})\cos 60.0° = +13.0\text{ m}$, $B_x = -(16.0\text{ m})\cos 30.0° = -13.86\text{ m}$, $B_y = -(16.0\text{ m})\sin 30.0° = -8.00\text{ m}$, $C_x = B_x - A_x = -13.86\text{ m} - (-22.52\text{ m}) = +8.66\text{ m}$, $C_y = B_y - A_y = -8.00\text{ m} - (13.0\text{ m}) = -21.0\text{ m}$

Finding the magnitude from the components gives $C = 22.7\text{ m}$.

(b) Finding the direction from the components gives $\tan\theta = \dfrac{8.66}{21.0}$ and $\theta = 22.4°$, east of south.

EVALUATE: A graphical sketch confirms that this answer is reasonable.

1.77. **IDENTIFY:** If the vector from your tent to Joe's is $\vec{A}$ and from your tent to Karl's is $\vec{B}$, then the vector from Karl's tent to Joe's tent is $\vec{A} - \vec{B}$.

SET UP: Take your tent's position as the origin. Let $+x$ be east and $+y$ be north.

EXECUTE: The position vector for Joe's tent is
$([21.0\text{ m}]\cos 23°)\hat{i} - ([21.0\text{ m}]\sin 23°)\hat{j} = (19.33\text{ m})\hat{i} - (8.205\text{ m})\hat{j}$.

The position vector for Karl's tent is $([32.0\text{ m}]\cos 37°)\hat{i} + ([32.0\text{ m}]\sin 37°)\hat{j} = (25.56\text{ m})\hat{i} + (19.26\text{ m})\hat{j}$.
The difference between the two positions is
$(19.33\text{ m} - 25.56\text{ m})\hat{i} + (-8.205\text{ m} - 19.25\text{ m})\hat{j} = -(6.23\text{ m})\hat{i} - (27.46\text{ m})\hat{j}$. The magnitude of this vector is the distance between the two tents: $D = \sqrt{(-6.23\text{ m})^2 + (-27.46\text{ m})^2} = 28.2\text{ m}$

EVALUATE: If both tents were due east of yours, the distance between them would be $32.0\text{ m} - 21.0\text{ m} = 11.0\text{ m}$. If Joe's was due north of yours and Karl's was due south of yours, then the distance between them would be $32.0\text{ m} + 21.0\text{ m} = 53.0\text{ m}$. The actual distance between them lies between these limiting values.

1.78. **IDENTIFY:** Calculate the scalar product and use Eq. (1.16) to determine ϕ.

SET UP: The unit vectors are perpendicular to each other.

EXECUTE: The direction vectors each have magnitude $\sqrt{3}$, and their scalar product is $(1)(1) + (1)(-1) + (1)(-1) = -1$, so from Eq. (1.16) the angle between the bonds is

$$\arccos\left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) = \arccos\left(-\frac{1}{3}\right) = 109°.$$

EVALUATE: The angle between the two vectors in the bond directions is greater than $90°$.

1.79. **IDENTIFY:** We know the scalar product and the magnitude of the vector product of two vectors and want to know the angle between them.

SET UP: The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\theta$ and the vector product is $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$.

EXECUTE: $\vec{A}\cdot\vec{B} = AB\cos\theta = -6.00$ and $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta = +9.00$. Taking the ratio gives $\tan\theta = \dfrac{9.00}{-6.00}$, so $\theta = 124°$.

EVALUATE: Since the scalar product is negative, the angle must be between 90° and 180°.

1.80. **IDENTIFY:** Find the angle between specified pairs of vectors.

SET UP: Use $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

EXECUTE: **(a)** $\vec{A} = \hat{k}$ (along line *ab*)

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$ (along line *ad*)

$A = 1,\ B = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\vec{A}\cdot\vec{B} = \hat{k}\cdot(\hat{i} + \hat{j} + \hat{k}) = 1$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = 1/\sqrt{3};\ \phi = 54.7°$

(b) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ (along line *ad*)

$\vec{B} = \hat{j} + \hat{k}$ (along line *ac*)

$A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3};\ \ B = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{A}\cdot\vec{B} = (\hat{i} + \hat{j} + \hat{k})\cdot(\hat{i} + \hat{j}) = 1 + 1 = 2$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{2}{\sqrt{3}\sqrt{2}} = \dfrac{2}{\sqrt{6}};\ \ \phi = 35.3°$

EVALUATE: Each angle is computed to be less than 90°, in agreement with what is deduced from the figure shown with this problem in the textbook.

1.81. **IDENTIFY:** We know the magnitude of two vectors and their scalar product and want to find the magnitude of their vector product.

SET UP: The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\phi$ and the vector product is $|\vec{A}\times\vec{B}| = AB\sin\phi$.

EXECUTE: $\vec{A}\cdot\vec{B} = AB\cos\phi = 90.0\ \text{m}^2$, which gives $\cos\phi = \dfrac{112.0\ \text{m}^2}{AB} = \dfrac{112.0\ \text{m}^2}{(12.0\ \text{m})(16.0\ \text{m})} = 0.5833$, so $\phi = 54.31°$. Therefore $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi = (12.0\ \text{m})(16.0\ \text{m})(\sin 54.31°) = 156\ \text{m}^2$.

EVALUATE: The magnitude of the vector product is greater than the scalar product because the angle between the vectors is greater than 45º.

1.82. **IDENTIFY:** The cross product $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

SET UP: Use Eq. (1.23) to calculate the components of $\vec{A}\times\vec{B}$.

EXECUTE: The cross product is

$(-13.00)\hat{i} + (6.00)\hat{j} + (-11.00)\hat{k} = 13\left[-(1.00)\hat{i} + \left(\dfrac{6.00}{13.00}\right)\hat{j} - \dfrac{11.00}{13.00}\hat{k}\right]$. The magnitude of the vector in square brackets is $\sqrt{1.93}$, and so a unit vector in this direction is

$$\left[\frac{-(1.00)\hat{i} + (6.00/13.00)\hat{j} - (11.00/13.00)\hat{k}}{\sqrt{1.93}}\right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{\boldsymbol{i}}-(6.00/13.00)\hat{\boldsymbol{j}}+(11.00/13.00)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right],$$

is also a unit vector perpendicular to $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

EVALUATE: Any two vectors that are not parallel or antiparallel form a plane and a vector perpendicular to both vectors is perpendicular to this plane.

1.83. **IDENTIFY:** We know the scalar product of two vectors, both their directions, and the magnitude of one of them, and we want to find the magnitude of the other vector.

SET UP: $\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{B}} = AB\cos\phi$. Since we know the direction of each vector, we can find the angle between them.

EXECUTE: The angle between the vectors is $\theta = 79.0°$. Since $\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{B}} = AB\cos\phi$, we have

$$B = \frac{\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{B}}}{A\cos\phi} = \frac{48.0\ \text{m}^2}{(9.00\ \text{m})\cos 79.0°} = 28.0\ \text{m}.$$

EVALUATE: Vector $\vec{\boldsymbol{B}}$ has the same units as vector $\vec{\boldsymbol{A}}$.

1.84. **IDENTIFY:** Calculate the magnitude of the vector product and then use $|\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}| = AB\sin\theta$.

SET UP: The magnitude of a vector is related to its components by Eq. (1.11).

EXECUTE: $|\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}| = AB\sin\theta$. $\sin\theta = \dfrac{|\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}|}{AB} = \dfrac{\sqrt{(-5.00)^2+(2.00)^2}}{(3.00)(3.00)} = 0.5984$ and

$\theta = \sin^{-1}(0.5984) = 36.8°$.

EVALUATE: We haven't found $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$, just the angle between them.

1.85. **IDENTIFY** and **SET UP:** The target variables are the components of $\vec{\boldsymbol{C}}$. We are given $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$. We also know $\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{C}}$ and $\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{C}}$, and this gives us two equations in the two unknowns C_x and C_y.

EXECUTE: $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{C}}$ are perpendicular, so $\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{C}} = 0$. $A_xC_x + A_yC_y = 0$, which gives $5.0C_x - 6.5C_y = 0$.

$\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{C}} = 15.0$, so $3.5C_x - 7.0C_y = 15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x = -8.0$ and $C_y = -6.1$.

EVALUATE: We can check that our result does give us a vector $\vec{\boldsymbol{C}}$ that satisfies the two equations $\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{C}} = 0$ and $\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{C}} = 15.0$.

1.86. **(a) IDENTIFY:** Prove that $\vec{\boldsymbol{A}}\cdot(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}}) = (\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})\cdot\vec{\boldsymbol{C}}$.

SET UP: Express the scalar and vector products in terms of components.

EXECUTE:

$$\vec{\boldsymbol{A}}\cdot(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}}) = A_x(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}})_x + A_y(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}})_y + A_z(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}})_z$$

$$\vec{\boldsymbol{A}}\cdot(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}}) = A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y - B_yC_x)$$

$$(\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})\cdot\vec{\boldsymbol{C}} = (\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})_xC_x + (\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})_yC_y + (\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})_zC_z$$

$$(\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})\cdot\vec{\boldsymbol{C}} = (A_yB_z - A_zB_y)C_x + (A_zB_x - A_xB_z)C_y + (A_xB_y - A_yB_x)C_z$$

Comparison of the expressions for $\vec{\boldsymbol{A}}\cdot(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}})$ and $(\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})\cdot\vec{\boldsymbol{C}}$ shows they contain the same terms, so $\vec{\boldsymbol{A}}\cdot(\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}}) = (\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})\cdot\vec{\boldsymbol{C}}$.

(b) IDENTIFY: Calculate $(\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})\cdot\vec{\boldsymbol{C}}$, given the magnitude and direction of $\vec{\boldsymbol{A}}$, $\vec{\boldsymbol{B}}$, and $\vec{\boldsymbol{C}}$.

SET UP: Use $|\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}| = AB\sin\phi$ to find the magnitude and direction of $\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}$. Then we know the components of $\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}$ and of $\vec{\boldsymbol{C}}$ and can use an expression like $\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{B}} = A_xB_x + A_yB_y + A_zB_z$ to find the scalar product in terms of components.

EXECUTE: $A = 5.00;\ \theta_A = 26.0°;\ B = 4.00,\ \theta_B = 63.0°$

$|\vec{A}\times\vec{B}| = AB\sin\phi.$

The angle ϕ between $\vec{A}$ and $\vec{B}$ is equal to $\phi = \theta_B - \theta_A = 63.0° - 26.0° = 37.0°$. So $|\vec{A}\times\vec{B}| = (5.00)(4.00)\sin 37.0° = 12.04$, and by the right hand-rule $\vec{A}\times\vec{B}$ is in the $+z$-direction. Thus $(\vec{A}\times\vec{B})\cdot\vec{C} = (12.04)(6.00) = 72.2$

EVALUATE: $\vec{A}\times\vec{B}$ is a vector, so taking its scalar product with $\vec{C}$ is a legitimate vector operation. $(\vec{A}\times\vec{B})\cdot\vec{C}$ is a scalar product between two vectors so the result is a scalar.

1.87. **IDENTIFY:** Express all the densities in the same units to make a comparison.

SET UP: Density ρ is mass divided by volume. Use the numbers given in the table in the problem and convert all the densities to kg/m^3.

EXECUTE: Sample A: $\rho_A = \dfrac{8.00\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.67\times10^{-6}\text{ m}^3} = 4790\text{ kg/m}^3$

Sample B: $\rho_B = \dfrac{6.00\times10^{-6}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{9.38\times10^{6}\ \mu\text{m}^3\left(\dfrac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)^3} = 640\text{ kg/m}^3$

Sample C: $\rho_C = \dfrac{8.00\times10^{-3}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{2.50\times10^{-3}\text{ cm}^3\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)^3} = 3200\text{ kg/m}^3$

Sample D: $\rho_D = \dfrac{9.00\times10^{-4}\text{ kg}}{2.81\times10^{3}\text{ mm}^3\left(\dfrac{1\text{ m}}{1000\text{ mm}}\right)^3} = 320\text{ kg/m}^3$

Sample E: $\rho_E = \dfrac{9.00\times10^{4}\text{ ng}\left(\dfrac{1\text{ g}}{10^9\text{ ng}}\right)\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.41\times10^{-2}\text{ mm}^3\left(\dfrac{1\text{ m}}{1000\text{ mm}}\right)^3} = 6380\text{ kg/m}^3$

Sample F: $\rho_F = \dfrac{6.00\times10^{-5}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.25\times10^{8}\ \mu\text{m}^3\left(\dfrac{1\text{ m}}{10^6\ \mu\text{m}}\right)^3} = 480\text{ kg/m}^3$

EVALUATE: In order of increasing density, the samples are D, F, B, C, A, E.

1.88. **IDENTIFY:** We know the magnitude of the resultant of two vectors at four known angles between them, and we want to find out the magnitude of each of these two vectors.

SET UP: Use the information in the table in the problem for θ = 0.0° and 90.0°. Call A and B the magnitudes of the vectors.

EXECUTE: **(a)** At 0°: The vectors point in the same direction, so $A + B = 8.00$ N.

At 90.0°: The vectors are perpendicular to each other, so $A^2 + B^2 = R^2 = (5.83\text{ N})^2 = 33.99\text{ N}^2$.

Solving these two equations simultaneously gives

$B = 8.00\text{ N} - A$

$A^2 + (8.00\text{ N} - A)^2 = 33.99\text{ N}^2$

$A^2 + 64.00\ \text{N}^2 - 16.00\ \text{N}\,A + A^2 = 33.99\ \text{N}^2$

The quadratic formula gives two solutions: $A = 5.00$ N and $B = 3.00$ N or $A = 3.00$ N and $B = 5.00$ N. In either case, the larger force has magnitude 5.00 N.

(b) Let $A = 5.00$ N and $B = 3.00$ N, with the larger vector along the x-axis and the smaller one making an angle of +30.0° with the +x-axis in the first quadrant. The components of the resultant are

$R_x = A_x + B_x = 5.00\ \text{N} + (3.00\ \text{N})(\cos 30.0°) = 7.598\ \text{N}$

$R_y = A_y + B_y = 0 + (3.00\ \text{N})(\sin 30.0°) = 1.500\ \text{N}$

$R = \sqrt{R_x^2 + R_y^2} = 7.74\ \text{N}$

EVALUATE: To check our answer, we could use the other resultants and angles given in the table with the problem.

1.89. **IDENTIFY:** Use the x and y coordinates for each object to find the vector from one object to the other; the distance between two objects is the magnitude of this vector. Use the scalar product to find the angle between two vectors.

SET UP: If object A has coordinates (x_A, y_A) and object B has coordinates (x_B, y_B), the vector $\vec{r}_{AB}$ from A to B has x-component $x_B - x_A$ and y-component $y_B - y_A$.

EXECUTE: **(a)** The diagram is sketched in Figure 1.89.

(b) (i) In AU, $\sqrt{(0.3182)^2 + (0.9329)^2} = 0.9857$.

(ii) In AU, $\sqrt{(1.3087)^2 + (-0.4423)^2 + (-0.0414)^2} = 1.3820$.

(iii) In AU, $\sqrt{(0.3182 - 1.3087)^2 + (0.9329 - (-0.4423))^2 + (0.0414)^2} = 1.695$.

(c) The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Eqs. (1.16) and (1.19),

$$\phi = \arccos\left(\frac{(-0.3182)(1.3087 - 0.3182) + (-0.9329)(-0.4423 - 0.9329) + (0)}{(0.9857)(1.695)}\right) = 54.6°.$$

(d) Mars could not have been visible at midnight, because the Sun-Mars angle is less than 90°.

EVALUATE: Our calculations correctly give that Mars is farther from the Sun than the earth is. Note that on this date Mars was farther from the earth than it is from the Sun.

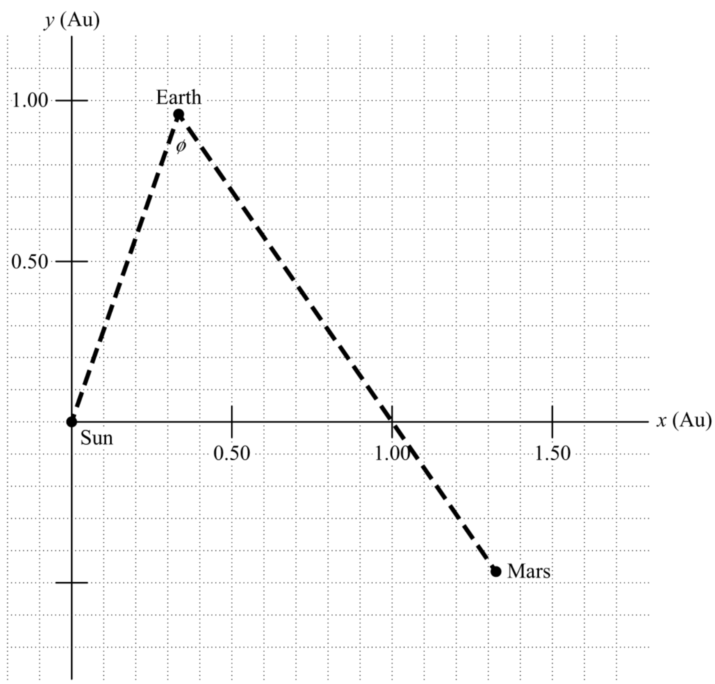


Figure 1.89

1.90. **IDENTIFY:** Add the vector displacements of the receiver and then find the vector from the quarterback to the receiver.
SET UP: Add the x-components and the y-components.
EXECUTE: The receiver's position is
$[(+1.0+9.0-6.0+12.0)\text{yd}]\hat{\boldsymbol{i}}+[(-5.0+11.0+4.0+18.0)\text{ yd}]\hat{\boldsymbol{j}}=(16.0\text{ yd})\hat{\boldsymbol{i}}+(28.0\text{ yd})\hat{\boldsymbol{j}}.$
The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0\text{ yd})\hat{\boldsymbol{i}}+(35.0\text{ yd})\hat{\boldsymbol{j}}$, a vector with magnitude $\sqrt{(16.0\text{ yd})^2+(35.0\text{ yd})^2}=38.5\text{ yd}$. The angle is $\arctan\left(\dfrac{16.0}{35.0}\right)=24.6°$ to the right of downfield.
EVALUATE: The vector from the quarterback to receiver has positive x-component and positive y-component.

1.91. **IDENTIFY:** Draw the vector addition diagram for the position vectors.
SET UP: Use coordinates in which the Sun to Merak line lies along the x-axis. Let $\vec{A}$ be the position vector of Alkaid relative to the Sun, $\vec{M}$ is the position vector of Merak relative to the Sun, and $\vec{R}$ is the position vector for Alkaid relative to Merak. $A=138$ ly and $M=77$ ly.
EXECUTE: The relative positions are shown in Figure 1.91. $\vec{M}+\vec{R}=\vec{A}$. $A_x=M_x+R_x$ so $R_x=A_x-M_x=(138\text{ ly})\cos 25.6°-77\text{ ly}=47.5\text{ ly}$. $R_y=A_y-M_y=(138\text{ ly})\sin 25.6°-0=59.6\text{ ly}$. $R=76.2$ ly is the distance between Alkaid and Merak.
(b) The angle is angle ϕ in Figure 1.91. $\cos\theta=\dfrac{R_x}{R}=\dfrac{47.5\text{ ly}}{76.2\text{ ly}}$ and $\theta=51.4°$. Then $\phi=180°-\theta=129°$.
EVALUATE: The concepts of vector addition and components make these calculations very simple.

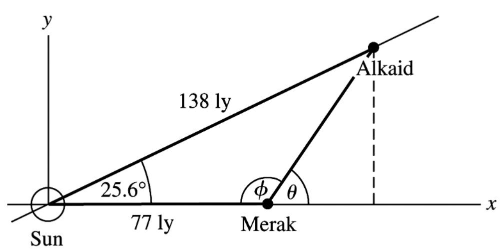


Figure 1.91

1.92. **IDENTIFY:** The total volume of the gas-exchanging region of the lungs must be at least as great as the total volume of all the alveoli, which is the product of the volume per alveoli times the number of alveoli.
SET UP: $V=NV_{alv}$, and we use the numbers given in the introduction to the problem.
EXECUTE: $V=NV_{alv}=(480\times 10^6)(4.2\times 10^6\ \mu\text{m}^3)=2.02\times 10^{15}\ \mu\text{m}^3$. Converting to liters gives

$$V=2.02\times 10^{15}\ \text{m}^3\left(\frac{1\text{ m}}{10^6\ \mu\text{m}}\right)^3=2.02\text{ L}\approx 2.0\text{ L}.$$ Therefore choice (c) is correct.

EVALUATE: A volume of 2 L is reasonable for the lungs.

1.93. **IDENTIFY:** We know the volume and want to find the diameter of a typical alveolus, assuming it to be a sphere.
SET UP: The volume of a sphere of radius r is $V=4/3\ \pi r^3$ and its diameter is $D=2r$.
EXECUTE: Solving for the radius in terms of the volume gives $r=(3V/4\pi)^{1/3}$, so the diameter is

$$D=2r=2(3V/4\pi)^{1/3}=2\left[\frac{3\left(4.2\times 10^6\ \mu\text{m}^3\right)}{4\pi}\right]^{1/3}=200\ \mu\text{m}.$$ Converting to mm gives

$D=(200\ \mu\text{m})[(1\text{ mm})/(1000\ \mu\text{m})]=0.20\text{ mm}$, so choice (a) is correct.
EVALUATE: A sphere that is 0.20 mm in diameter should be visible to the naked eye for someone with good eyesight.

1.94. **IDENTIFY:** Draw conclusions from a given graph.
SET UP: The dots lie more-or-less along a horizontal line, which means that the average alveolar volume does not vary significantly as the lung volume increases.
EXECUTE: The volume of individual alveoli does not vary (as stated in the introduction). The graph shows that the volume occupied by alveoli stays constant for higher and higher lung volumes, so there must be more of them, which makes choice (c) the correct one.
EVALUATE: It is reasonable that a large lung would need more alveoli than a small lung because a large lung probably belongs to a larger person than a small lung.