

UNIVERSITY OF DUBLIN

MA1123-1

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Trinity Term 2014

MA1123 — ANALYSIS ON THE REAL LINE I

Monday, April 28

LUCE UPPER

09:30 — 12:30

Prof. D. O'Donovan

ANSWER ALL QUESTIONS

All questions carry equal marks.

Formulae & Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. (a) Define what it means for a function $f : \mathbb{R} \rightarrow \mathbb{R}$ to be continuous at a point a .
(b) Prove from the definition of a limit that $f(x) = x^3 + 2$ is continuous at $x = 2$.
(c) Prove that the left hand and right hand limit of $f(x)$ at $x = a$ both exist and are equal, if the limit of $f(x)$ at $x = a$ exists. Is the converse true?
(d) Let $f(x) = |x - 2|$, find $f'(2)$ or show that it does not exist.
(e) Prove that the limit of a function is unique.

2. (a) Explain the quadratic approximation formula

$$f(x + h) \sim f(x) + f'(x)h + \frac{1}{2}f''(x)h^2.$$

- (b) Find $\frac{dy}{dx}$ if

- i. $y = e^{\sin(\sqrt{x^3+2})}$

- ii. $xy^2 + e^{x^2y} = \sin(xy)$

- iii. $y = e^{x^2}(\sin x)(\ln x)(\cos x)$

- (c) Let $f(x) = \frac{(x+1)^2}{x^2+1}$. Find where $f(x)$ is increasing, decreasing, concave up, concave down, has local extrema, has points of inflection, and find any asymptotes. Use this information to sketch the function.

3. (a) State and prove Rolle's Theorem.
(b) Use the Mean Value Theorem to prove that if $f'(x) = g'(x)$, for all x , then $f(x) = g(x) + \text{constant}$
(c) A right angled triangle whose hypotenuse is $\sqrt{3}$ metres long is revolved about one of its legs to generate a circular cone. Find the radius, height, and volume of the largest cone that can be generated in this way.

4. (a) Give an example of a function on $[0,1]$ that has no Riemann Integral. And show clearly why it hasn't.

(b) Integrate the following.

i. $\int x^2 e^x dx$

ii. $\int x e^{x^2} dx$

iii. $\int (\sin^2 x)(\cos^2 x) dx$

iv. $\int \frac{x^3 + x + 1}{x^2 + x + 1} dx$

v. $\int \frac{x}{(x^2 + 2)(x + 2)(x - 2)} dx$

5. (a) Find the area of the region bounded by $y = x$, $y = -x + 2\pi$ and $y = \sin x$.

(b) What is an improper integral?

(c) Find the volume of the solid of revolution gotten by revolving the region bounded by $y = x^2$, $y = 0$, $x = -1$ and $x = 1$ about the x -axis, first by the method of disks, and then by the method of cylindrical shells.

6. (a) Define $\lim_{n \rightarrow \infty} a_n = L$, and $\sum_{n=1}^{\infty} a_n = S$.

(b) Do the following series converge or diverge? Give reasons.

i. $\sum_{n=1}^{\infty} \frac{n^2 + n + 1}{n^3 + 2}$

ii. $\sum_{n=2}^{\infty} \frac{n}{\ln n}$

iii. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(c) Prove that if $\sum_{n=1}^{\infty} a_n = S$ then $\lim_{n \rightarrow \infty} a_n = 0$.

(d) Find for what values of x the following power series converges absolutely, conditionally, or diverges:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}.$$