Advanced Calculus
MA1132
Homework Assignment 4

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To be completed and handed in AT THE BEGINNING of tutorial on Friday, 5. April
NO LATE ASSIGNMENTS WILL BE ACCEPTED.

IF YOU CANNOT ATTEND TUTORIALS, PLEASE MAKE ARRANGEMENTS TO EMAIL YOUR SOLUTIONS TO YOUR TUTOR

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You may use Mathematica to sketch the integration regions and solids, and to check the results of integration.

1. Find the area of the portion of the elliptic paraboloid $z = c - \frac{x^2}{2a} - \frac{y^2}{2b}$ that is inside the cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = c^2$. *Hint: choose a parameterization of the surface such that the region R over which we integrate is a disc.*

2. Consider the solid G bounded below by the surface $z = r^\alpha$, $\alpha > 0$, $r = \sqrt{x^2 + y^2}$ and above by the plane $z = 1$. Note that the surface $z = r$ is a cone, and $z = r^2$ is a paraboloid.

   (a) Sketch the surface $z = r^\alpha$ for $\alpha = 1/2, \alpha = 1, \alpha = 2$, and the projection of the solid G onto the xy-plane.

   (b) Find the volume V of the solid G, and its limit as $\alpha \to \infty$.

   (c) Explain the limiting values obtained in (b).

   Show the details of your work.

3. Find $\iiint_G \cos\left(\frac{z}{y}\right)\, dV$, where G is the solid defined by the inequalities $\frac{\pi}{6} \leqslant y \leqslant \frac{\pi}{2}$, $y \leqslant x \leqslant \frac{\pi}{2}$ and $0 \leqslant z \leqslant xy$.

4. Find the mass of a cylinder centered at the z-axis which has height h, radius a, and density $\delta(x, y) = 4x^2 + 4y^2$.