Hearing and Seeing; Tutorial 4 - Answers

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Example 37.1 Measuring the Wavelength of a Light Source

Interactive

A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe ($m = 2$) is 4.5 cm from the center line.

(A) Determine the wavelength of the light.

Solution We can use Equation 37.5, with $m = 2$, $y_{\text{bright}} = 4.5 \times 10^{-2}$ m, $L = 1.2$ m, and $d = 3.0 \times 10^{-5}$ m:

$$\lambda = \frac{y_{\text{bright}} d}{mL} = \frac{(4.5 \times 10^{-2}\ \text{m})(3.0 \times 10^{-5}\ \text{m})}{2(1.2\ \text{m})}$$

$$= 5.6 \times 10^{-7}\ \text{m} = \boxed{560\ \text{nm}}$$

which is in the green range of visible light.

(B) Calculate the distance between adjacent bright fringes.

Solution From Equation 37.5 and the results of part (A), we obtain

$$y_{m+1} - y_m = \frac{\lambda L}{d}(m+1) - \frac{\lambda L}{d} m$$

$$= \frac{\lambda L}{d} = \frac{(5.6 \times 10^{-7}\ \text{m})(1.2\ \text{m})}{3.0 \times 10^{-5}\ \text{m}}$$

$$= 2.2 \times 10^{-2}\ \text{m} = \boxed{2.2\ \text{cm}}$$

Example 37.3 Interference in a Soap Film

Calculate the minimum thickness of a soap-bubble film that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm.

Solution The minimum film thickness for constructive interference in the reflected light corresponds to $m = 0$ in Equation 37.16. This gives $2nt = \lambda/2$, or

$$t = \frac{\lambda}{4n} = \frac{600 \text{ nm}}{4(1.33)} = \boxed{113 \text{ nm}}$$

What If? **What if the film is twice as thick? Does this situation produce constructive interference?**

Answer Using Equation 37.16, we can solve for the thicknesses at which constructive interference will occur:

$$t = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2n} = (2m + 1)\frac{\lambda}{4n} \qquad (m = 0, 1, 2, \ldots)$$

The allowed values of m show that constructive interference will occur for *odd* multiples of the thickness corresponding to $m = 0$, $t = 113$ nm. Thus, constructive interference will *not* occur for a film that is twice as thick.

1. What is the necessary condition on the path length difference between two waves that interfere (a) constructively and (b) destructively?
2. Explain why two flashlights held close together do not produce an interference pattern on a distant screen.
3. If Young's double-slit experiment were performed under water, how would the observed interference pattern be affected?
4. In Young's double-slit experiment, why do we use monochromatic light? If white light is used, how would the pattern change?

Q37.1 (a) Two waves interfere constructively if their path difference is zero, or an integral multiple of the wavelength, according to $\delta = m\lambda$, with $m = 0, 1, 2, 3, \ldots$.

(b) Two waves interfere destructively if their path difference is a half wavelength, or an odd multiple of $\frac{\lambda}{2}$, described by $\delta = \left(m + \frac{1}{2}\right)\lambda$, with $m = 0, 1, 2, 3, \ldots$.

Q37.2 The light from the flashlights consists of many different wavelengths (that's why it's white) with random time differences between the light waves. There is no *coherence* between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

Q37.3 Underwater, the wavelength of the light would decrease, $\lambda_{water} = \frac{\lambda_{air}}{n_{water}}$. Since the positions of light and dark bands are proportional to λ, (according to Equations 37.2 and 37.3), the underwater fringe separations will decrease.

Q37.4 Every color produces its own pattern, with a spacing between the maxima that is characteristic of the wavelength. With several colors, the patterns are superimposed and it can be difficult to pick out a single maximum. Using monochromatic light can eliminate this problem.

Example 38.1 Where Are the Dark Fringes? **Interactive**

Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Solution The problem statement cues us to conceptualize a single-slit diffraction pattern similar to that in Figure 38.6. We categorize this as a straightforward application of our discussion of single-slit diffraction patterns. To analyze the problem, note that the two dark fringes that flank the central bright fringe correspond to $m = \pm 1$ in Equation 38.1. Hence, we find that

$$\sin \theta_{\text{dark}} = \pm \frac{\lambda}{a} = \pm \frac{5.80 \times 10^{-7}\text{ m}}{0.300 \times 10^{-3}\text{ m}} = \pm 1.933 \times 10^{-3}$$

From the triangle in Figure 38.6, note that $\tan \theta_{\text{dark}} = y_1/L$. Because θ_{dark} is very small, we can use the approximation $\sin \theta_{\text{dark}} \approx \tan \theta_{\text{dark}}$; thus, $\sin \theta_{\text{dark}} \approx y_1/L$. Therefore, the positions of the first minima measured from the central axis are given by

$$\begin{aligned} y_1 \approx L \sin \theta_{\text{dark}} &= (2.00\text{ m})(\pm 1.933 \times 10^{-3}) \\ &= \boxed{\pm 3.87 \times 10^{-3}\text{ m}} \end{aligned}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to $2|y_1| = 7.74 \times 10^{-3}\text{ m} = \boxed{7.74\text{ mm.}}$ To finalize this problem, note that this value is much greater than the width of the slit. We finalize further by exploring what happens if we change the slit width.

What If? **What if the slit width is increased by an order of magnitude to 3.00 mm? What happens to the diffraction pattern?**

Answer Based on Equation 38.1, we expect that the angles at which the dark bands appear will decrease as a increases. Thus, the diffraction pattern narrows. For $a = 3.00$ mm, the sines of the angles θ_{dark} for the $m = \pm 1$ dark fringes are

$$\sin \theta_{\text{dark}} = \pm \frac{\lambda}{a} = \pm \frac{5.80 \times 10^{-7}\text{ m}}{3.00 \times 10^{-3}\text{ m}} = \pm 1.933 \times 10^{-4}$$

The positions of the first minima measured from the central axis are given by

$$\begin{aligned} y_1 \approx L \sin \theta_{\text{dark}} &= (2.00\text{ m})(\pm 1.933 \times 10^{-4}) \\ &= \pm 3.87 \times 10^{-4}\text{ m} \end{aligned}$$

and the width of the central bright fringe is equal to $2|y_1| = 7.74 \times 10^{-4}\text{ m} = 0.774$ mm. Notice that this is *smaller* than the width of the slit.

In general, for large values of a, the various maxima and minima are so closely spaced that only a large central bright area resembling the geometric image of the slit is observed. This is very important in the performance of optical instruments such as telescopes.

Example 38.7 The Orders of a Diffraction Grating

Interactive

Monochromatic light from a helium–neon laser (λ = 632.8 nm) is incident normally on a diffraction grating containing 6 000 grooves per centimeter. Find the angles at which the first- and second-order maxima are observed.

Solution First, we must calculate the slit separation, which is equal to the inverse of the number of grooves per centimeter:

$$d = \frac{1}{6\,000}\text{ cm} = 1.667 \times 10^{-4}\text{ cm} = 1\,667\text{ nm}$$

For the first-order maximum ($m = 1$), we obtain

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{632.8\text{ nm}}{1\,667\text{ nm}} = 0.379\,6$$

$$\theta_1 = \boxed{22.31°}$$

For the second-order maximum ($m = 2$), we find

$$\sin\theta_2 = \frac{2\lambda}{d} = \frac{2(632.8\text{ nm})}{1\,667\text{ nm}} = 0.759\,2$$

$$\theta_2 = \boxed{49.39°}$$

What If? **What if we look for the third-order maximum? Do we find it?**

Answer For $m = 3$, we find $\sin\theta_3 = 1.139$. Because $\sin\theta$ cannot exceed unity, this does not represent a realistic solution. Hence, only zeroth-, first-, and second-order maxima are observed for this situation.

2. Holding your hand at arm's length, you can readily block sunlight from reaching your eyes. Why can you not block sound from reaching your ears this way?

$\boxed{5.}$ Describe the change in width of the central maximum of the single-slit diffraction pattern as the width of the slit is made narrower.

Q38.2 The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around an obstacle the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

Q38.5 The intensity of the light coming through the slit decreases, as you would expect. The central maximum increases in width as the width of the slit decreases. In the condition $\sin\theta = \frac{\lambda}{a}$ for destructive interference on each side of the central maximum, θ increases as a decreases.

1. Helium–neon laser light (λ = 632.8 nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?

2. A beam of green light is diffracted by a slit of width 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 4.10 mm. Calculate the wavelength of the laser light.

P38.1 $\sin\theta = \dfrac{\lambda}{a} = \dfrac{6.328 \times 10^{-7}}{3.00 \times 10^{-4}} = 2.11 \times 10^{-3}$

$$\frac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta = \theta \quad (\text{for small } \theta)$$

$$2y = \boxed{4.22\text{ mm}}$$

P38.2 The positions of the first-order minima are $\dfrac{y}{L} \approx \sin\theta = \pm\dfrac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y = 2\left(\dfrac{\lambda}{a}\right)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10 \times 10^{-3}\text{ m}}{2}\right)\left(\frac{0.550 \times 10^{-3}\text{ m}}{2.06\text{ m}}\right) = \boxed{547\text{ nm}}.$$

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