Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Mathematics   Trinity Term 2018
JF Theoretical Physics
JF Two Subject Mod

MA1125 — Single Variable Calculus and Introductory Analysis

Monday, April 30   RDS   14:00 – 17:00

Prof. D. O'Donovan

Instructions to Candidates:

ANSWER ALL QUESTIONS:

All questions carry equal marks.

Formulae & Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. (a) Define function, and inverse function.

   (b) Use the definition of a limit to calculate $\lim_{x \to 2} 2x^3 + 3x^2 - 2$.

   (c) Define $\lim_{x \to a^+} f(x) = L$.

   (d) Prove that limits are unique.

2. (a) Derive the formula for the linear approximation to $f(x)$, and use it to find $\sqrt{47}$ to two decimal places.

   (b) Find $\frac{dy}{dx}$ if

   i. $y = \dfrac{\sin x \ln x \cos x}{x^2}$.

   ii. $x^2 y^3 + \sin(x^2 y) = \exp(xy^3)$

   iii. $y = \tan^{-1} x$.

   (c) Find where the function $x \exp x$ is increasing, decreasing, concave up, concave down, has local extrema, and points of inflection. Draw a rough sketch of the function.

3. (a) State The Generalised Mean Value Theorem. State the Extreme Value Theorem.

   (b) Solve the following

   i. $x\dfrac{dy}{dx} = y^2 + 1$, $y(1) = 0$.

   ii. $\dfrac{d^2 y}{dx^2} + 4\dfrac{dy}{dx} + 3y = 0$

   iii. $\dfrac{d^2 y}{dx^2} + 4\dfrac{dy}{dx} + 3y = x + \sin x$

   (c) Find the area of the largest rectangle that can be inscribed in a circle of radius r

4. (a) How is the Riemann Integral $\int_a^b f(x)dx$ defined?

   (b) Integrate the following.

   i. $\int \dfrac{1}{\sqrt{4-x^2}}dx$

   ii. $\int x \ln x\, dx$

   iii. $\int \sin^2 x \cos^3 x\, dx$

   iv. $\int \dfrac{3x^3 - x^2 + 6x - 4}{(x^2+1)(x^2+2)}$

5. (a) Find the length of the curve $y = \int_1^x \sqrt{\sqrt{t} - 1}\, dt$ from x=1 to x=16.

   (b) Find the area enclosed between $y = x^2$ and the line $y = x + 4$..

   (c) The bounded region between $y = x$, $x = 2$ and above the x-axis is rotated about the line $x = 3$. Find the volume first using washers and then using cylindrical shells.

6. (a) Define $\sum_{n=1}^{\infty} a_n = L$.

   (b) Do the following series converge or diverge? Give reasons.

   i. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

   ii. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$

   (c) Find the Taylor Series for $f(x) = \exp(x)$ about $x = 0$. How would you determine where this series converges to $\exp x$?

   (d) What is the integral test?