JF PY1T10: Special Relativity

Lecture 3:

- Relativity of Simultaneity
- Lorentz – Einstein Transformations

Summary of Lecture 2

Michelson – Morley Experiment:

No fringe shift observed when Michelson interferometer is rotated 90°.
Conclusion: We cannot detect any motion relative to ether (absolute space).

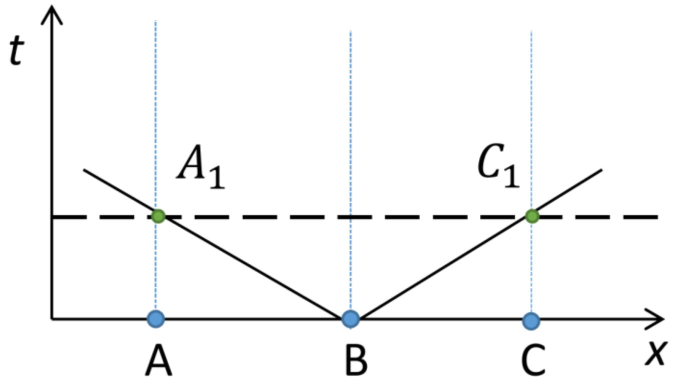
Einstein's Postulates of Special Relativity:

Postulate 1: All inertial frames are equivalent w.r.t. the laws of physics.
Postulate 2: The speed of light in empty space always has the same value, *i.e.*, the speed of light is independent of the motion of the source or receiver.

But: Galilean Transformations are not consistent will Postulate 2. Need to find new set of transformations.

Relativity of Simultaneity

If we use Einstein's method of synchronising clocks, we find that simultaneity is relative, not absolute. How?



World Lines
(graph of position vs. time)

Observation stations **A**, **B** and **C** equally spaced on x-axis in inertial frame in which they are at rest.

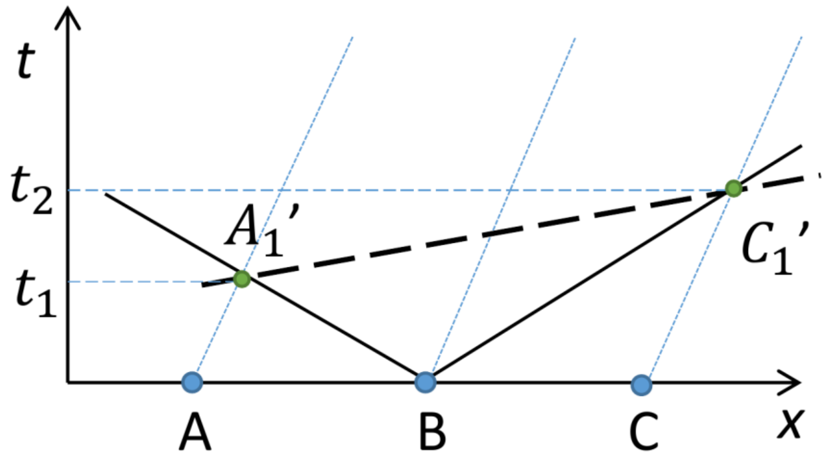
At $t = 0$, send out light signal from **B**. Light moves at speed *c* forward and backward, with $x = x_B \pm ct$.

Arrives at **A** and **C** given by events at the intersections $\mathbf{A_1}$, $\mathbf{C_1}$

Simultaneity at **A** and **C** defined by line $\mathbf{A_1}$ $\mathbf{C_1}$ || to *x*-axis which joins points possessing same value of t.

Relativity of Simultaneity

Now suppose **A**, **B** and **C** at rest in *S′* which is moving at speed *v* along x-axis:



World lines are now *inclined* in *S*.

Signal sent out from **B** at $t = 0$ is described in *S* by $x = x_B \pm ct$.

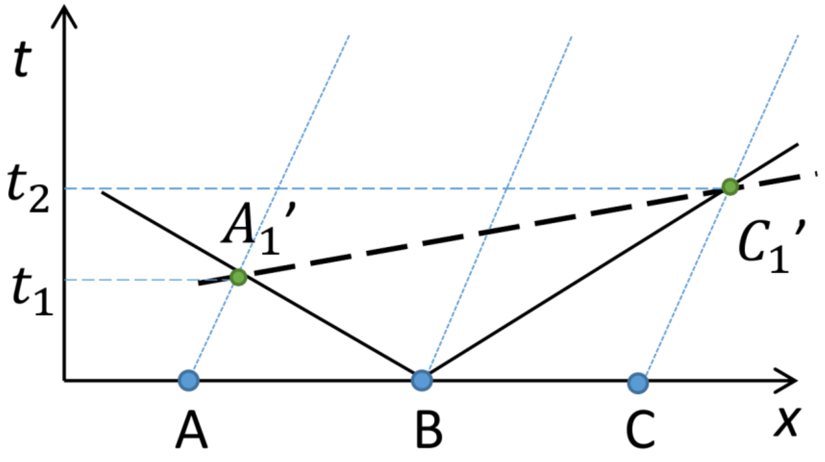
Arrival at **A** and **C** given by intersections **A_1'** and **C_1'**. These arrivals are <u>not</u> simultaneous in *S*.

Signal reaches **A** before **C**, because **A** is running to meet **B**, while **C** is running away.

Signal reaches **A** and **C** at different times in *S*; t_1 and t_2 respectively.

Relativity of Simultaneity

But **B** is midway between **A** and **C**.



If we accept universality of *c* and equivalence of inertial forms...

A' and **C'** must represent simultaneous events in *S'*.

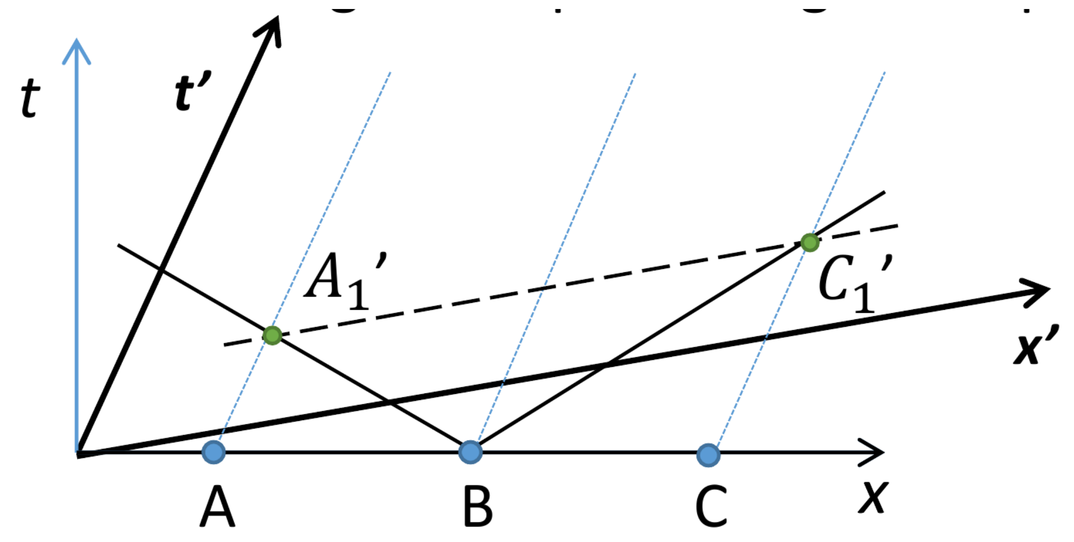
Line $\mathbf{A_1}'$ $\mathbf{C_1}'$ represents events which are simultaneous in *S'*. (An event is characterised by space and time coordinates in a given frame).

Our judgement of simultaneity depends on the particular frame of reference we use.

See University Physics, Section 37.2.

Lorentz – Einstein Transformation 1

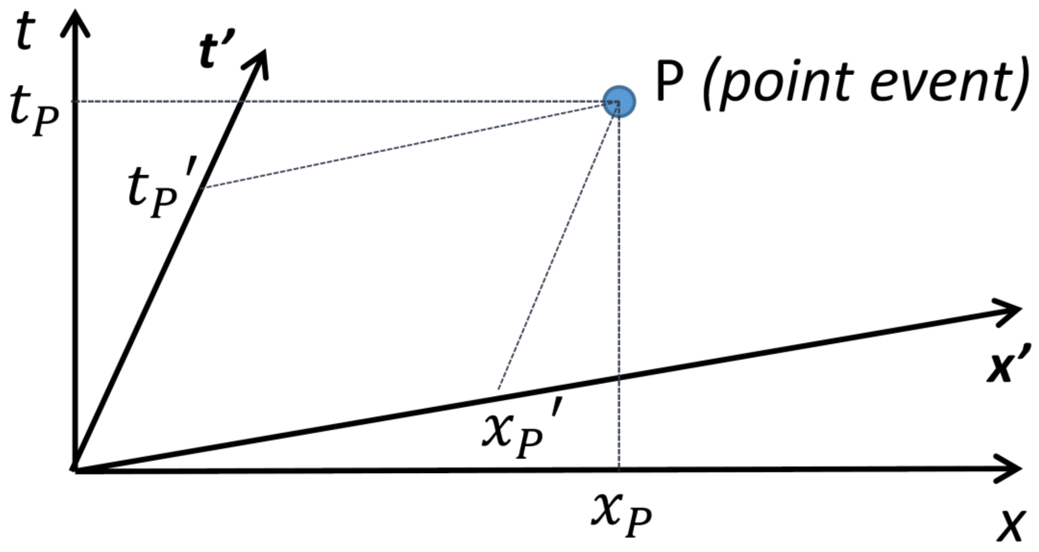
Look for a transformation that gives a speed of light independent of speed of source or receiver.



- Draw t' axis parallel to inclined world lines, as **A**, **B** and **C** will be stationary in S'. It coincides with $t = 0$ at $t' = 0$.
- Line **A1' C1'** (dotted line) defines simultaneity at stations **A** and **C** in S'. Add another coordinate axis x' parallel to this line, crossing $t' = 0$ at $x' = 0$. This is because positions along a line defining simultaneity have the same time value. The x'-axis is given by all points along line of simultaneity where $t' = 0$.
- Position of origin of S' given by $x = vt$ in S if origins coincide at $t = 0$.

Lorentz – Einstein Transformation 2

This kind of diagram shows us the kinematic transformations of SR:



Point event is given by (x, t) or (x', t')

x' is a linear function of x, likewise for t' and t.

Lorentz – Einstein Transformation 3

Take the most general transformation relating the coordinates of a given event in the two systems (it must be linear):

$$x = a\,x' + b\,t' \qquad ①$$
$$x' = a\,x - b\,t \qquad ②$$

Origin of S moves at velocity $-v$ in S', also given by putting $x = 0$ in ① :

$$0 = a\,x' + b\,t'$$
$$\frac{x'}{t'} = -\,\frac{b}{a} = -v$$

Origin of S' moves at $+v$ in S; put $x' = 0$ in ② :

$$0 = a\,x - b\,t$$
$$\frac{x}{t} = +\,\frac{b}{a} = +v$$

Lorentz – Einstein Transformation 4

Consider light signal moving in positive *x*-direction in *S* and *S'*, originating at $x, x' = 0$:

$$x_l = ct, \qquad x_l' = ct'$$

Sub for x and x' in ① and ② :

$$c\,t = a\,c + b\,t'$$
$$c\,t' = a\,c - b\,t$$

Eliminate *t* and *t'*:

$$c^2 = a^2\,c^2 - b^2$$
$$c^2 = a^2(c^2 - v^2)$$

$$a = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Define $a \equiv \gamma$, and note that $\gamma \geq 1$

Lorentz – Einstein Transformation 5

From ①:

$$x = a\,x' + b\,t'$$

$$x = \gamma(x' + v\,t') \qquad ③$$

From ②:

$$x' = \gamma(x - v\,t) \qquad ④$$

These differ from Galilean Transformations (GT) by having a factor of $\gamma \geq 1$.

As ${}^{v}/_{c} \rightarrow 0, \gamma \rightarrow 1$

As $v \rightarrow c, \gamma \rightarrow \infty$

Given ③ and ④, we can obtain:

$$t = \gamma\left(t' + \frac{vx^2}{c^2}\right)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

Lorentz – Einstein Transformation 6

What about y and z transverse to the direction of relative motion?

If space is isotropic (same in all directions), all displacements transverse to the direction defined by relative motion are equivalent.

We can conclude that the appropriate transformations are:

$$y = y'$$
$$z = z'$$

If this were not true, we would have a way to detect absolute displacements and motions. But, this would violate the essential idea of relativity.

The measure of transverse distance (y or z) is the same for all inertial systems that are in relative motion along x.

Lorentz – Einstein Transformation – 7

Lorentz Transformations

$\boldsymbol{x' = \gamma(x - vt)}$	$\boldsymbol{y' = y}$	$\boldsymbol{z' = z}$	$\boldsymbol{t' = \gamma\left(t - \frac{vx}{c^2}\right)}$
$\boldsymbol{x = \gamma(x' + vt')}$	$\boldsymbol{y = y'}$	$\boldsymbol{z = z'}$	$\boldsymbol{t = \gamma\left(t' + \frac{vx'}{c^2}\right)}$

where v is the velocity of S' as measured in S

Note that Lorentz Transformations → Galilean Transformations as $v/_c \rightarrow 0$.

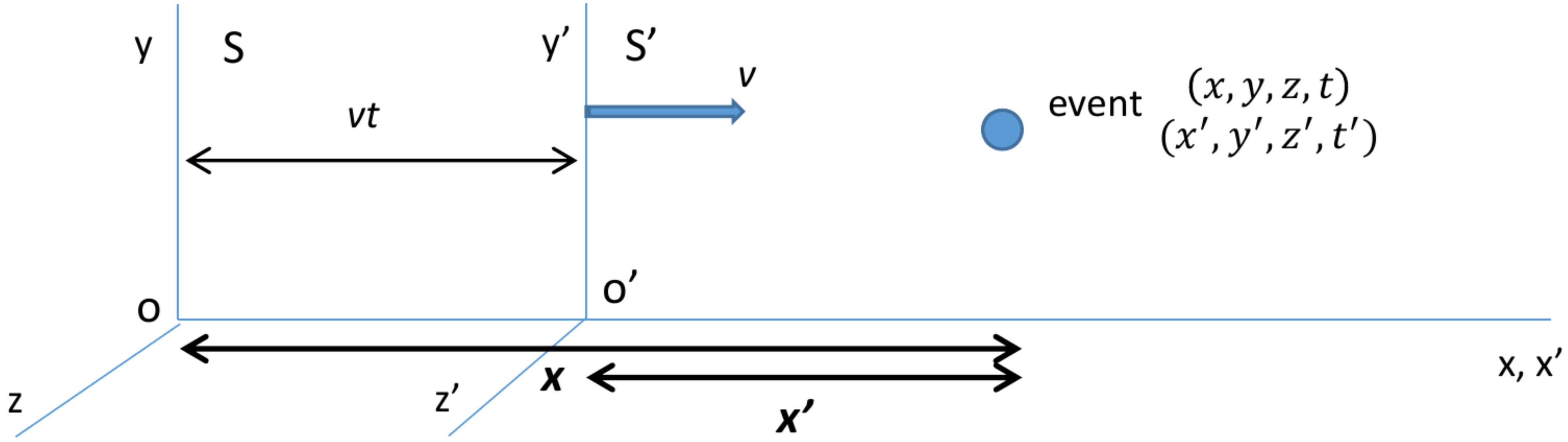
Introduced by H.A. Lorentz to account for the null result in the Michelson-Morley experiment, but assuming the existence of a unique inertial frame provide by the ether.

Einstein discovered the equations independently through different approach (no ether).

Lorentz – Einstein Transformation 8

Transformations give the relation between (x, y, z, t) of an event in S to (x', y', z', t') of the same event measured in S'.

Note that there is *no universal time* between frames.



Lorentz – Einstein Transformation 9

Relativity predicts some strange effects; strange because they appear different from everyday experience.

(but we do not experience speeds ≥ 0.000001 c)

- Moving clocks appear to run slow, yet they keep time as accurately as any other clocks.
- Objects moving at high speeds seem to contract along the direction of motion, without losing any physical matter.
- Objects seem to gain mass at high speeds, without an increase in physical matter.

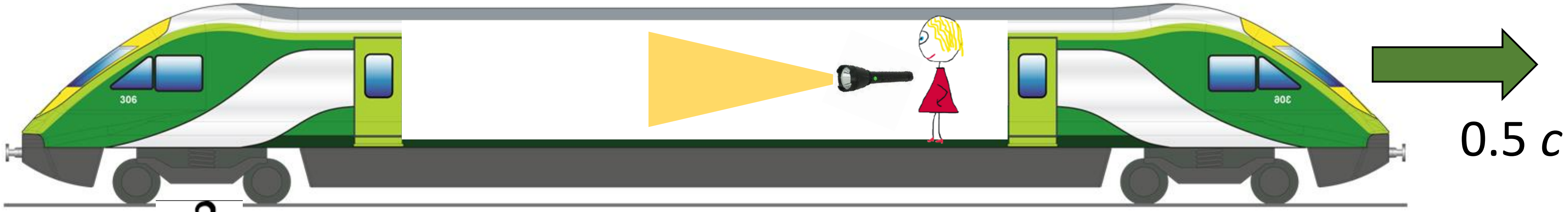
These strange results follows from **P2**; "*c* is the same for all observers".

Concept Questions

On Simultaneity

1. Motion on a Train

While John is standing on a station platform he observes a train going from left to right at a speed of 0.5c. Mary is standing at the front of the train and is sending pulses of light towards the back of the train (i.e. opposite to the direction of motion).



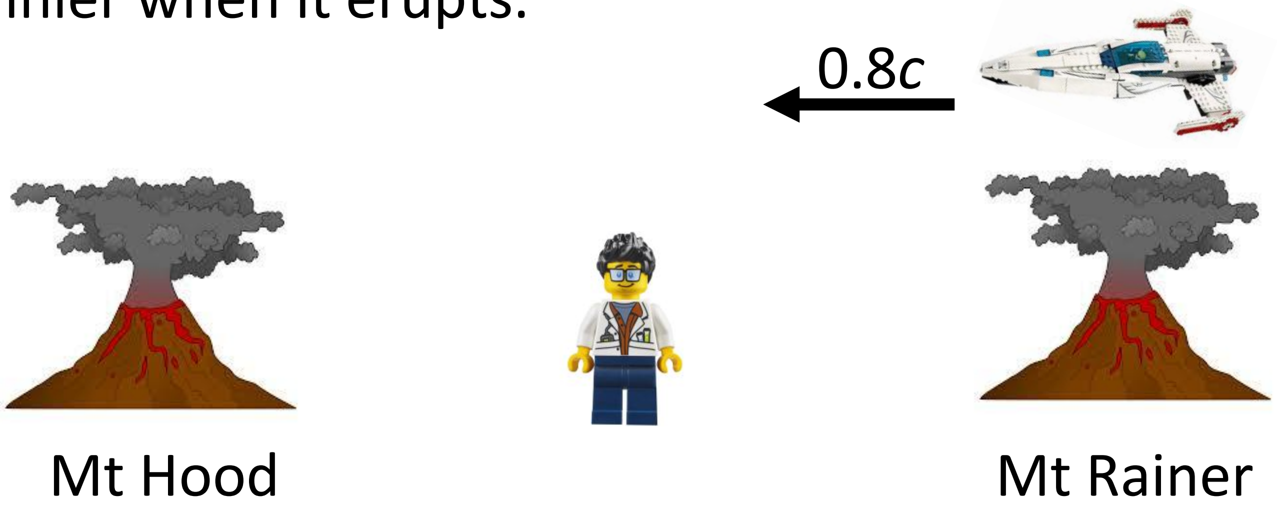
What is the velocity the light pulses as measured by John?

A. 0.5c to his left
B. 0.5c to his right
C. c to his left
D. c to his right
E. 1.5c to his left

2. Erupting Volcanos

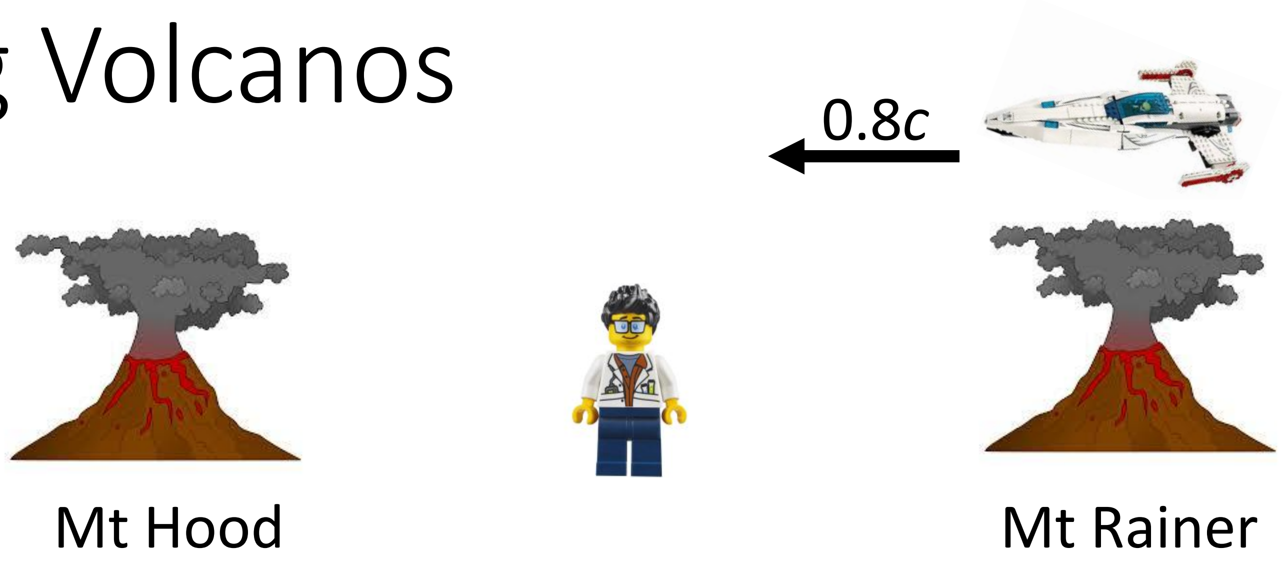
Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes.

A fast spacecraft flying with constant speed v = 0.8*c* from Rainier towards Hood is directly over Mt. Rainier when it erupts.



The seismologist and the observer in the spacecraft are intelligent observers, i.e., they correct for signal travel time to determine the time of events in their reference frame. Each observer has clocks which are synchronised with all other observers in their reference frame.

2. Erupting Volcanos



Which of the following statements regarding the timing of the eruptions is correct?

A. In both the seismologist’s and the spacecraft’s reference frame both volcanos erupt simultaneously.

B. In the seismologist’s reference frame the eruptions are simultaneous, while in the spacecraft’s reference frame Mt Rainer erupts before Mt Hood.

C. In the seismologist’s reference frame the eruptions are simultaneous, while in the spacecraft’s reference frame Mt Hood erupts before Mt Rainier.

D. In the spacecraft’s reference frame the eruptions are simultaneous, while in the seismologist's reference frame Mt Hood erupts before Mt Rainier.