Advanced Calculus
MA1132
Homework Assignment 3

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To be completed and handed in AT THE BEGINNING of tutorial on Friday, 29. March
NO LATE ASSIGNMENTS WILL BE ACCEPTED.

IF YOU CANNOT ATTEND TUTORIALS, PLEASE MAKE ARRANGEMENTS TO EMAIL YOUR SOLUTIONS TO YOUR TUTOR

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You may use Mathematica to sketch the integration regions and solids, and to check the results of integration.

1. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = (x-1)^2 + y^2$ subject to the constraint $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$.

2. Consider the intersection of the surfaces
$$z = \sqrt{a^2 - x^2 - y^2}\,, \quad \text{and} \quad \frac{x^2}{b^2} + \frac{y^2}{c^2} = 1\,, \quad a > b > c\,.$$
   (a) What is the surface $z = \sqrt{a^2 - x^2 - y^2}$? Sketch the surface $z = \sqrt{a^2 - x^2 - y^2}$ and its projection onto the xy plane for $a = 3$.
   (b) What is the surface $\frac{x^2}{b^2} + \frac{y^2}{c^2} = 1$? Sketch the surface $\frac{x^2}{b^2} + \frac{y^2}{c^2} = 1$ and its projection onto the xy plane for $b = 2$, $c = 1$.
   (c) Use Lagrange multipliers to find the coordinates of the points on the intersection which have the maximum z-coordinate and the minimum z-coordinate.

3. Show that a triangle with fixed area has minimum perimeter if it is equilateral.

4. What is the volume of the largest n-dimensional box with edges parallel to the coordinate axes that fits inside the n-dimensional ellipsoid
$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \cdots + \frac{x_n^2}{a_n^2} = 1\,. \tag{1}$$

5. Find the integral of the function $f(x, y) = 4xye^{x^2+y^2}$ over the rectangle
$$\{(x, y) \in \mathbb{R}^2 \colon 0 \leqslant x \leqslant 2, 0 \leqslant y \leqslant 3\}.$$

6. Sketch the integration region R and reverse the order of integration

   (a)
$$\int_{-1/2}^{7/2} \int_{2-\sqrt{7+12y-4y^2}}^{2+\sqrt{7+12y-4y^2}} f(x,y)dxdy \tag{2}$$

   (b)
$$\int_0^1 \int_{x^2/2}^{\sqrt{3-x^2}} f(x,y)dydx \tag{3}$$

7. Prove the Dirichlet formula
$$\int_a^b \int_a^x f(x,y)dydx = \int_a^b \int_y^b f(x,y)dxdy\,, \tag{4}$$
   and use it to prove that
$$\int_a^b \int_a^s (s-t)^{n-1} f(t)dtds = \frac{1}{n} \int_a^b (b-t)^n f(t)dt\,. \tag{5}$$

8. Find the volume V of the solid bounded by

   (a) the planes $x = 1$, $z = 0$, the parabolic cylinder $x - y^2 = 0$, and the paraboloid $z = x^2 + y^2$.

   (b) the planes $x = 0$, $y = 0$, $z = 0$, the cylinders $az = x^2$, $a > 0$, $x^2 + y^2 = b^2$, and located in the first octant $x \geq 0, y \geq 0, z \geq 0$.