MA1125 – Calculus
Tutorial problems #10

1. Test each of the following series for convergence.

$$\sum_{n=1}^{\infty} \frac{2+\sin n}{n}, \qquad \sum_{n=1}^{\infty} \frac{2+\sin n}{n^2}.$$

2. Test each of the following series for convergence.

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}, \qquad \sum_{n=1}^{\infty} \frac{ne^{1/n}}{n^3+1}.$$

3. Test each of the following series for convergence.

$$\sum_{n=1}^{\infty} \frac{\ln n}{n}, \qquad \sum_{n=1}^{\infty} \frac{\ln n}{n!}.$$

4. Find the radius of convergence for each of the following power series.

$$\sum_{n=0}^{\infty} \frac{nx^n}{3^n}, \qquad \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} \cdot x^n.$$

5. Assuming that $|x| < 1$, use the formula for a geometric series to show that

$$\sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}.$$

6. Find the radius of convergence for each of the following power series.

$$\sum_{n=0}^{\infty} \frac{nx^{2n}}{4^n}, \qquad \sum_{n=0}^{\infty} \frac{3^n x^n}{2n+1}.$$

7. Use differentiation to show that the following power series is equal to $\ln(1+x)$.

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}, \qquad |x| < 1.$$

8. Use differentiation to show that the following power series is equal to $\tan^{-1} x$.

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \qquad |x| < 1.$$

9. Let $a \in \mathbb{R}$ be a given number. Find the radius of convergence for the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{a(a-1)(a-2)\cdots(a-n+1)}{n!} \cdot x^n.$$

10. Show that $\sum_{n=0}^{\infty} a_n y^n$ converges absolutely, if $\sum_{n=0}^{\infty} a_n x^n$ converges and $|y| < |x|$.