MA1125 – Calculus
Tutorial problems #9

1. Compute each of the following indefinite integrals.

$$\int \frac{x^3 - x}{x^2 + 5}\,dx, \qquad \int \frac{x^2 + 5}{x^3 - x}\,dx.$$

2. Compute each of the following indefinite integrals.

$$\int \frac{\sqrt{x}}{x + 1}\,dx, \qquad \int \frac{\sqrt{x}}{x - 1}\,dx.$$

3. Compute each of the following indefinite integrals.

$$\int e^{2x}\cos(e^x)\,dx, \qquad \int \frac{\sin^3 x}{\cos^6 x}\,dx.$$

4. Show that each of the following sequences converges.

$$a_n = \sqrt{\frac{n^2 + 1}{n^3 + 2}}, \qquad b_n = \frac{\sin n}{n^2}, \qquad c_n = n^{1/n}.$$

5. Define a sequence $\{a_n\}$ by setting $a_1 = 1$ and $a_{n+1} = \sqrt{6 + a_n}$ for each $n \geq 1$. Show that $1 \leq a_n \leq a_{n+1} \leq 3$ for each $n \geq 1$, use this fact to conclude that the sequence converges and then find its limit.

6. Use the formula for a geometric series to compute each of the following sums.

$$\sum_{n=0}^{\infty} \frac{2^n}{7^n}, \qquad \sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{3n+1}}, \qquad \sum_{n=2}^{\infty} \frac{3^{n+1}}{2^{4n-3}}.$$

7. Infinite sums of continuous functions need not be continuous. In fact, show that

$$f(x) = \sum_{n=0}^{\infty} \frac{x^2}{(1 + x^2)^n}$$

is a geometric series that satisfies $f(x) = 1 + x^2$ for all $x \neq 0$, while $f(0) = 0$.

8. Compute each of the following indefinite integrals.

$$\int \frac{x + 2}{x^2 + 4x + 8}\,dx, \qquad \int \frac{5x + 7}{x^2 + 4x + 8}\,dx.$$

9. Define a sequence $\{a_n\}$ by setting $a_1 = 1$ and $a_{n+1} = 3 + \sqrt{a_n}$ for each $n \geq 1$. Show that $1 \leq a_n \leq a_{n+1} \leq 9$ for each $n \geq 1$, use this fact to conclude that the sequence converges and then find its limit.

10. Suppose the series $\sum_{n=1}^{\infty} a_n$ converges. Show that the series $\sum_{n=1}^{\infty} \frac{1}{1+a_n}$ diverges.