JF PY1T10 Special Relativity

Lecture 8

Transformation of Velocity

Transformation of Velocity

We will find that c is an upper limit to the velocity of any material object, and to the transmission of information.

Also, **P2**: c same for all observers

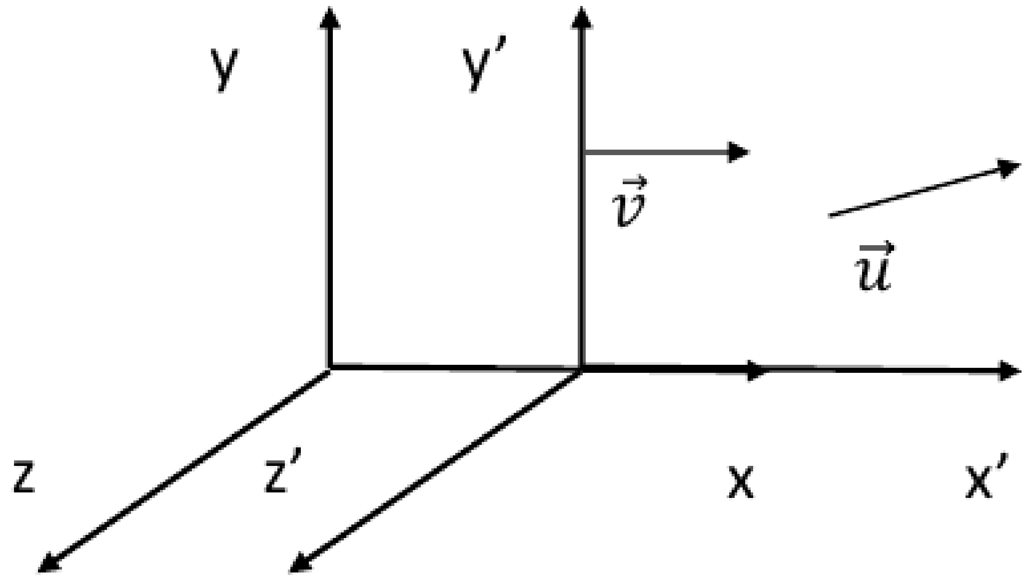
These ideas are contrary to our intuition

We are only familiar with behaviour for $v \ll c$, where “strange” effects are small, and LT reduces to GT.

Clearly results are not consistent with the Galilean transform, we need a new one!

Need to find the transformation to relate measurement of velocity in different inertial frames.

Transformation of Velocity – 2



Let us confine motion to the *xy*-plane.

Object with velocity components u_x', u_y', u_z' as measured in *S'*:

$$u_x' = \frac{dx'}{dt'}, \qquad u_y' = \frac{dy'}{dt'}$$

What are u_x and u_y?

Transformation of Velocity – 3

$$x = \gamma(x' + vt')$$

$$\Rightarrow \frac{dx}{dt'} = \gamma\left(\frac{dx'}{dt'} + v\frac{dt'}{dt'}\right)$$

$$\Rightarrow dx = \gamma(u_x{}' + v)dt'$$

$$\frac{dy}{dt'} = \frac{dy'}{dt'} = u_y{}'$$

$$\therefore dy = u_y{}'dt'$$

$$dt = \gamma\left(1 + \frac{vu_x{}'}{c^2}\right)dt'$$

$$\therefore u_x = \frac{dx}{dt} = \frac{u_x{}' + v}{1 + \frac{vu_x{}'}{c^2}}, \qquad \therefore u_y = \frac{dy}{dt} = \frac{u_y{}'/\gamma}{1 + vu_x{}'/c^2}$$

$$\boxed{\begin{aligned} \text{LT:}\quad & x = \gamma(x' + vt') \\ & y = y', \qquad z = z' \\ & t = \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned}}$$

Transformation of Velocity – 4

Similarly,

$$u_x{}' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

$$u_y{}' = \frac{u_y/\gamma}{1 - \frac{v u_x}{c^2}}$$

Example 1: Addition of Velocities

Suppose a particle is moving at ${u_x}' = 0.5c$ in *S'*, and *S'* moves at $v = 0.5c$ *w.r.t. S*.

What is the velocity of the particle as measured in *S*?

$$u_x = \frac{{u_x}' + v}{1 + \frac{v{u_x}'}{c^2}}$$

$$= \frac{0.5c + 0.5c}{1 + 0.5c^2}$$

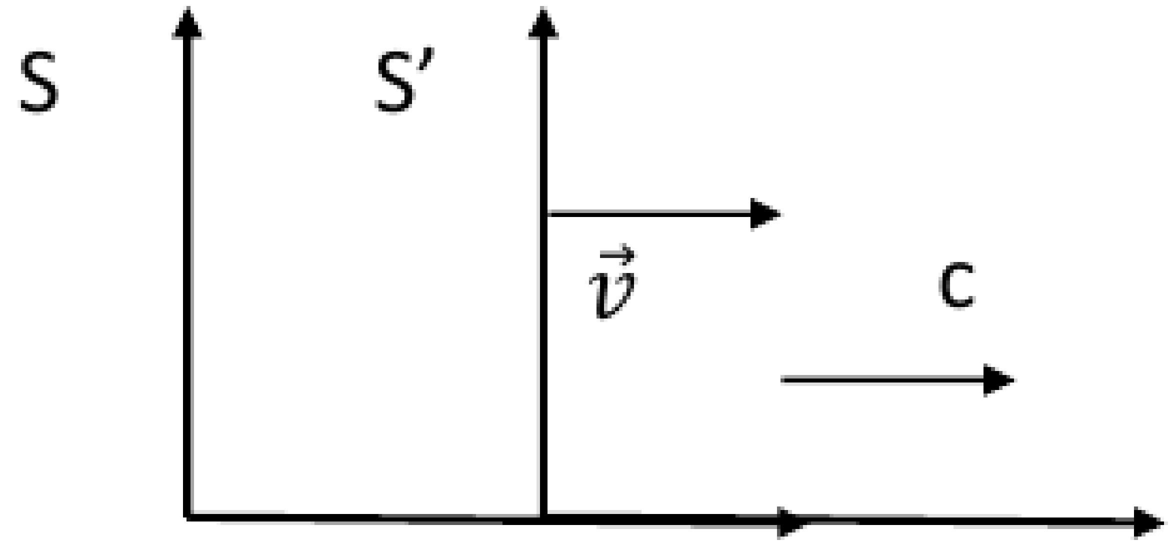
$$= 0.8c$$

Example 2: Addition of Velocities

A photon (velocity = *c*) is measured in *S'.*

S' moves at *v* relative to *S.*

What is the photon's velocity in *S*?



Example 2: Addition of Velocities

$$u_x = \frac{c+v}{1+\frac{vc}{c^2}}$$

$$= \frac{c+v}{c+v}c$$

$$= c$$

Measured in *S* or *S'*, the photon moves at *c.*

This is what we expect from LT, but still strange!

The velocity of light from moving source is always *c.*

Confirmed by experiment.

Velocity of Light from Moving Source – Experiment

Volume 12, number 3 PHYSICS LETTERS 1 October 1964

TEST OF THE SECOND POSTULATE OF SPECIAL RELATIVITY IN THE GeV REGION

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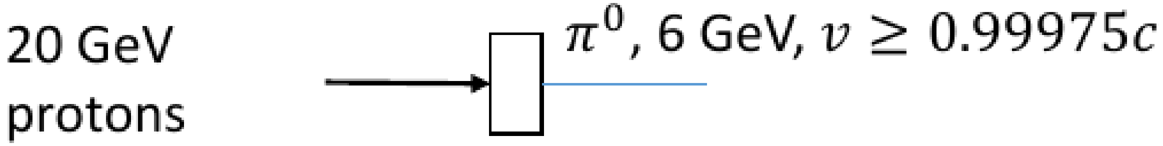
Received 20 August 1964

Velocity of Light from Moving Source – Experiment

Measure the velocity of light emitted in the decay of neutral π^0 mesons (pions) in flight



$$\pi^0 \rightarrow \gamma_1 + \gamma_2$$

Mean lifetime approx. 2×10^{-16} s – can only travel a very small distance before decaying.

Proton beam and π^0 production pulsed, move γ detector to measure velocity of γ.

Result: 2.9977×10^8 ms^{-1}

Source velocity $\approx c$ but photon velocity still c

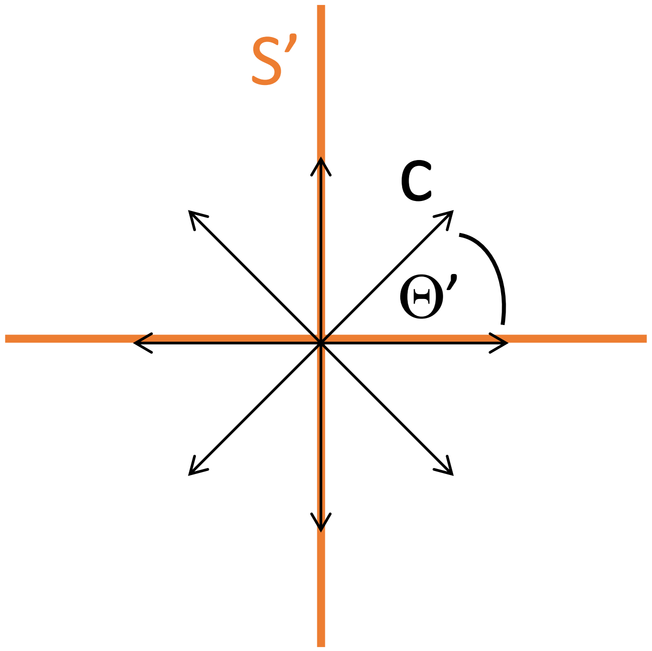
“Headlight effect”

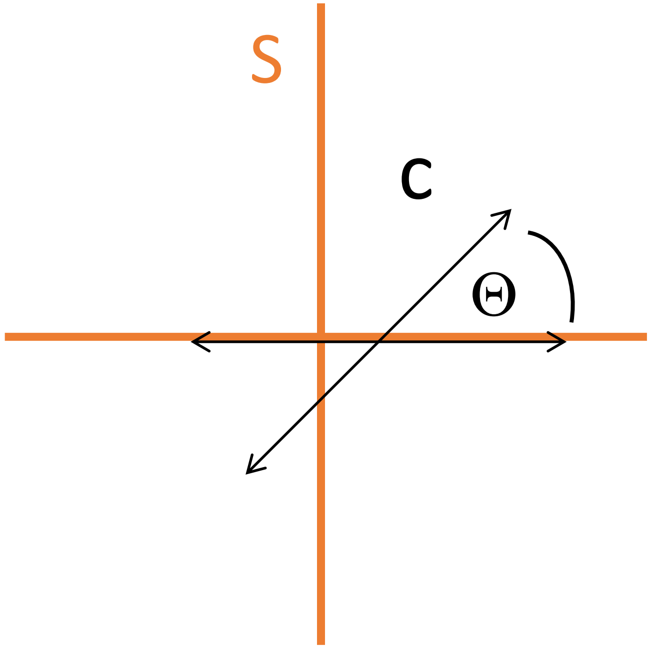
Source of light is travelling at velocity v parallel to x relative to an observer in frame S.

Suppose that frame in which source is at rest is S’.

Suppose that it is a point source – radiating uniformly in all directions in S’.

In what direction is this photon travelling as measured in S?





$$v_x = c\cos(\theta)$$

“Headlight effect”

Use L.T.:

$$v_x = \frac{u_x{}' + v}{1 + {}^{v u_x{}'}\!/_{c^2}}$$

For the photon:

$$c\ \cos\theta = \frac{c\ \cos\theta' + v}{1 + {}^{vc\cos\theta'}\!/_{c^2}}$$

$$\cos\theta = \frac{\cos\theta' + \ \beta}{1 + \beta\cos\theta'}$$

"Headlight effect"

Suppose $\beta = 0.9$

For $\theta' = 0, \theta = 0$
For $\theta' = 180°, \theta = 180°$
For $\theta' = \frac{\pi}{2}, \cos\theta = 0.9, \theta = 25°$

Radiation is concentrated in the forward direction.

Electron Synchrotron – get very intense x-ray pulses.

Concept Question

Santiago stands on the ground as Miriam flies directly toward him in her spaceship at 0.5*c*. She fires a small rocket directly toward Santiago that flies at a speed of 0.8*c* relative to her spaceship. According to Santiago, the speed of the rocket is

A. 1.3 *c*

B. Faster than *c* but slower than 1.3 *c*

C. *c*

D. Faster than 0.8 c but slower than *c*

E. 0.8 *c*

Concept Question

Two spaceships, **A** and **B**, approach Earth at 0.5 *c* from opposite directions as measured by an observer on Earth. What is the speed of **B** as measured by an observer on **A**?

A. 1.0 *c*

B. 0.6 *c*

C. 0.8 *c*

D. 0.25 *c*

Problem Question (37.23)

An Imperial spaceship, moving at a high speed relative to the planet Arrakis, fires a rocket towards the planet with a speed of 0.92 *c* relative to the spaceship.

An observer on Arrakis measures that the rocket is approaching with a speed of 0.36 *c*.

Q1: What is the speed of the spaceship relative to Arrakis?

Q2: Is the spaceship moving towards or away from Arrakis?