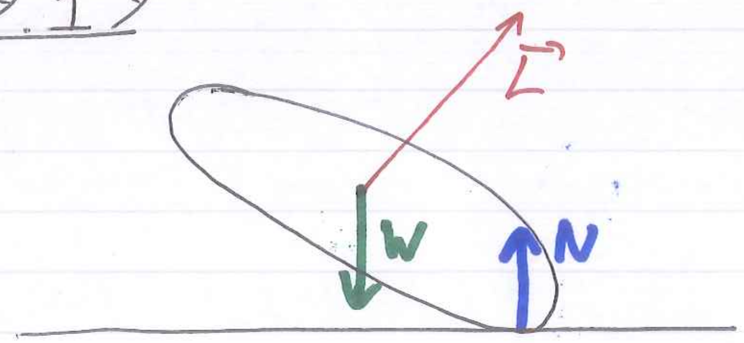
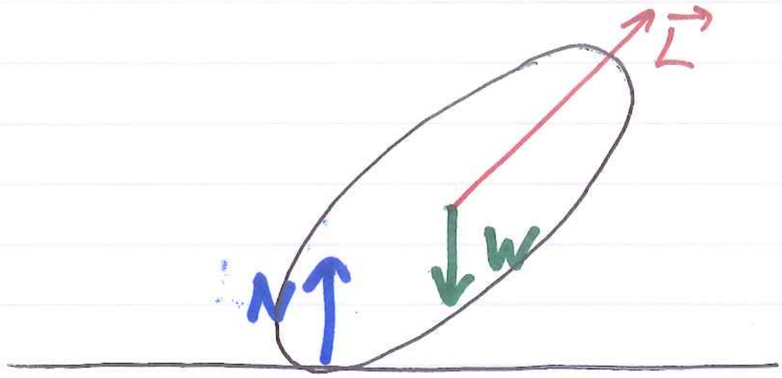
9.19

Oblate





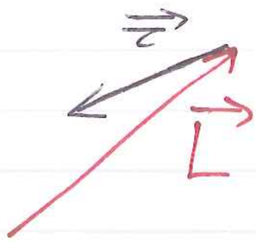
Torque of normal force w.r.t. c.o.m is
1) for oblate coming out of paper
2) for prolate going in to paper.

$\Rightarrow$ precession of $\vec{L}$ for oblate 

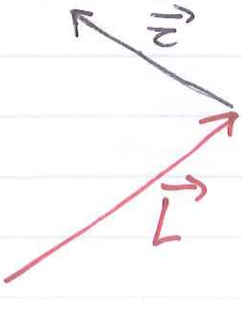
precession of $\vec{L}$ for prolate 

The friction force is for both case coming out of the paper.

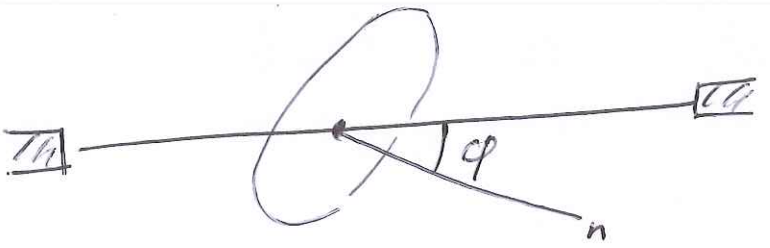
Therefore for oblate



for prolate



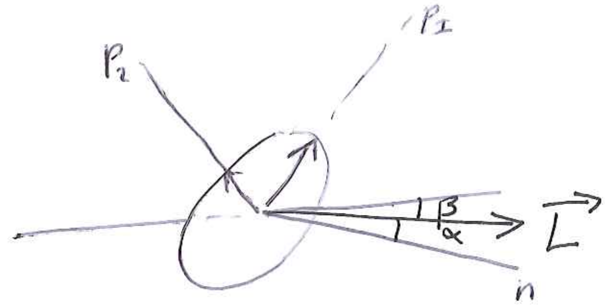
Problem 2.



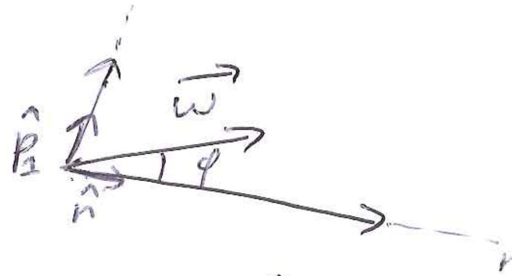
$I_n = \frac{1}{2} m R^2$ $\qquad \varphi = \frac{\pi}{6}$

$I_p = \frac{1}{4} m R^2$

I.



$\Rightarrow$



$\vec{\omega} = \omega \cos\varphi \, \hat{n} + \omega \sin\varphi \, \hat{p}$ $\Leftarrow$ at same point in time

$= \omega \, \frac{1}{2}\sqrt{3} \, \hat{n} + \omega \, \frac{1}{2} \hat{p}_1$

20 $\vec{L} = \begin{pmatrix} I_n & & 0 \\ & I_p & \\ 0 & & I_p \end{pmatrix} \vec{\omega} = \frac{\omega}{4} \sqrt{3} \, m R^2 \hat{n}$

$+ \frac{\omega}{8} m R^2 \hat{p}_1$

$\tan\alpha = \frac{\frac{1}{8}}{\frac{1}{4}\sqrt{3}} = \frac{\sqrt{3}}{6} \Rightarrow \alpha = 0.281$ rad

$\Rightarrow$ angle between $\vec{L}$ & $\vec{\omega}$ is $\frac{\pi}{6} - 0.281$

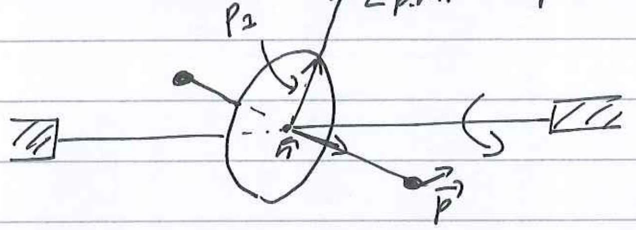
$= 0.243$ rad $= \beta$

2. $\left|\frac{d\vec{L}}{dt}\right| = L \sin\beta\, \omega$

$$= \sin\beta \left(\frac{3}{16} + \frac{1}{64}\right)^{\frac{1}{2}} m R^2 \omega^2$$

10 $$= 0.108\, m R^2 \omega^2$$

3. $\vec{L}_{p.m.}$ in plane of disk orthogonal to $\vec{p}$



$$\vec{L}_{p.m.} = 2 s \hat{n} \times \vec{p}$$

$$= 2 s \hat{n} \times (\vec{\omega} \times s \hat{n}) \tfrac{1}{2} m$$

20 $$= m \omega s^2 \sin\alpha\, \hat{p}_1 = \tfrac{1}{2} m \omega s^2 \hat{p}_1$$

$\vec{L}_{disk} + \vec{L}_{p.m.}$ must be parallel to $\vec{\omega}$

$$= \frac{\omega}{4} \sqrt{3}\, m R^2 \hat{n} + \frac{\omega}{8} m R^2 \hat{p}_1$$

$$+ \tfrac{1}{2} \omega m s^2 \hat{p}_1$$

$$\Longrightarrow \frac{1}{8} R^2 + \frac{1}{2} s^2 = \frac{1}{4} R^2$$

$$\Longrightarrow \boxed{s = \tfrac{1}{2} R}$$