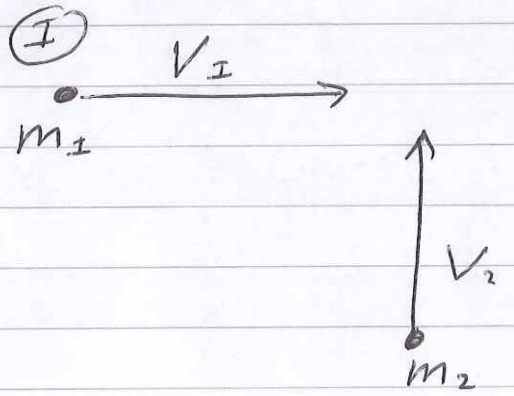
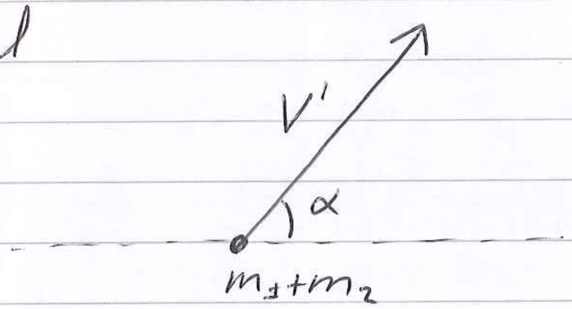
Problem 2.

(I)



Final



Cons. of mom.

10 $$m_1\vec{V_1} + m_2\vec{V_2} = (m_1+m_2)\vec{V}'$$

$$\begin{pmatrix} m_1 V_1 \\ m_2 V_2 \end{pmatrix} = (m_1+m_2)\vec{V}'$$

$$\Longrightarrow \tan\alpha = \frac{m_2 V_2}{m_1 V_1} = \frac{180}{240} = \frac{3}{4}$$

10 $$\Longrightarrow \alpha \approx 0.64 \text{ rad} \approx 37^\circ$$

$$|\vec{V}'| = \frac{1}{m_1+m_2}\left(m_1^2 V_1^2 + m_2^2 V_2^2\right)^{\frac{1}{2}}$$

20 $$= \frac{60}{12}\left(4^2+3^2\right)^{\frac{1}{2}} = 25 \text{ m/s}$$

Loss in kinetic energy

$$\tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2 - \tfrac{1}{2}(m_1+m_2)(V')^2$$

10 $$= 12270 \text{ J}.$$

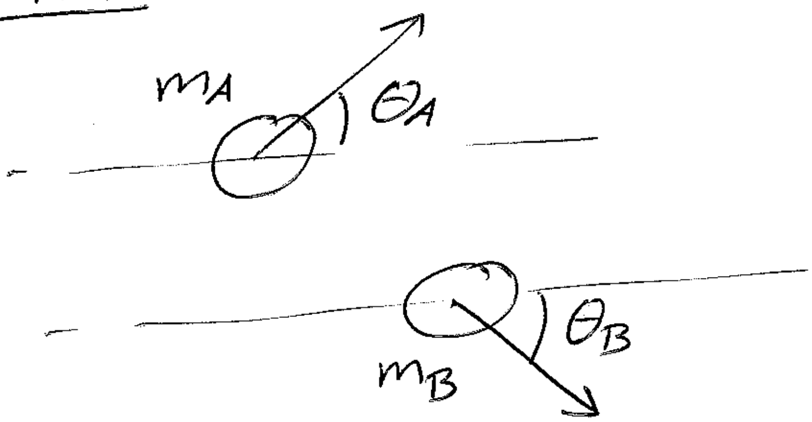
Problem

1. Initial

$m_A = m_B$

$\vec{V}_A = (2+\sqrt{3})\vec{V}_B$ (*)

Final



$\theta_A = \theta_B = \frac{\pi}{6}$

Conservation of momentum:

Horizontal:

$$V_A + V_B = \frac{1}{2}\sqrt{3}\,(V_A' + V_B')$$

Vertical: $\frac{1}{2} V_A' - \frac{1}{2} V_B' = 0$

Combine with (*)

$$\Longrightarrow \quad (3+\sqrt{3})V_B = \sqrt{3}\,V_A'$$

Conservation of energy:

$$V_A^2 + V_B^2 = (4+3+4\sqrt{3}+1)V_B^2 = 4(2+\sqrt{3})V_B^2$$

$$(V_A')^2 + (V_B')^2 = 2(1+\sqrt{3})^2 V_B^2 = 2(1+2\sqrt{3}+3)V_B^2$$

2. Center of Mass velocity:

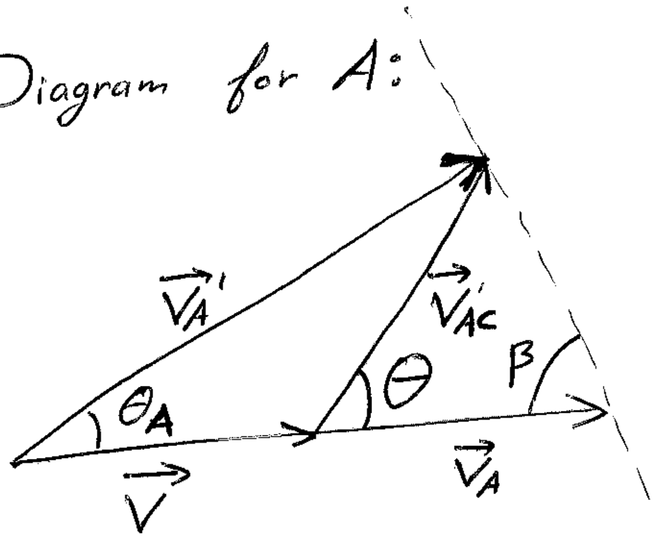
$$\vec{V} = \frac{1}{2}(3+\sqrt{3})\vec{V_B}$$

Velocities of A and B in center of mass frame:

$$\vec{V_{AC}} = \frac{1}{2}(1+\sqrt{3})\vec{V_B}$$

$$\vec{V_{BC}} = -\frac{1}{2}(1+\sqrt{3})\vec{V_B}$$

Diagram for A:



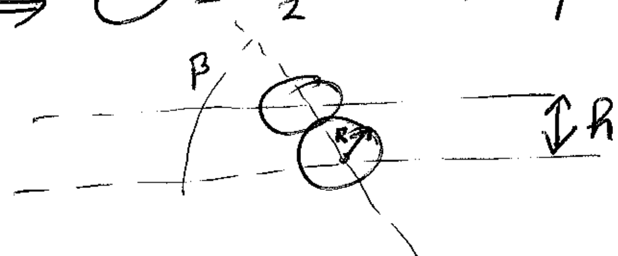
We see from the diagram that

$$V_A' \sin\theta_A = V_{AC}' \sin\Theta$$

$$\Longrightarrow \sin\Theta = \frac{V_A'}{V_{AC}'}\sin\theta_A$$

$$= \underbrace{\frac{(1+\sqrt{3})V_B}{\frac{1}{2}(1+\sqrt{3})V_B}}_{\text{since } V_{AC}' = V_{AC}} \frac{1}{2} = 1$$

$$\Longrightarrow \Theta = \frac{\pi}{2} \Longrightarrow \beta = \frac{\pi}{4}$$



$$\Longrightarrow h = 2R\sin\beta$$
$$= \sqrt{2}R$$