

MA1125 – Calculus
Tutorial problems #7

1. Find the area of the region enclosed by the graphs of $f(x) = 3x^2$ and $g(x) = x + 2$.
2. Compute the volume of the solid that is obtained when the graph of $f(x) = x^2 + 3$ is rotated around the x -axis over the interval $[0, 2]$.
3. Compute the length of the graph of $f(x) = \frac{1}{3}(x^2 + 2)^{3/2}$ over the interval $[1, 3]$.
4. Find both the mass and the centre of mass for a thin rod whose density is given by

$$\delta(x) = x^2 + 4x + 1, \quad 0 \leq x \leq 2.$$

5. A chain that is 4m long has a uniform density of 3kg/m. If the chain is hanging from the top of a tall building, then how much work is needed to pull it up to the top?
6. Find the area of the region enclosed by the graphs of $f(x)$ and $g(x)$ in the case that

$$f(x) = \sin x, \quad g(x) = \cos x, \quad 0 \leq x \leq \pi/2.$$

7. The graph of $f(x) = 2e^{6x}$ is rotated around the x -axis over the interval $[0, a]$. If the volume of the resulting solid is equal to π , then what is the value of a ?
8. Compute the length of the graph of $f(x) = x^{3/2} - \frac{1}{3}x^{1/2}$ over the interval $[0, 2]$.
9. Show that the function f is integrable on $[0, 1]$ for any given constants a, b when

$$f(x) = \begin{cases} a & \text{if } x \neq 0 \\ b & \text{if } x = 0 \end{cases}.$$

10. Compute each of the following improper integrals.

$$I_1 = \int_2^\infty \frac{dx}{(x-1)^5}, \quad I_2 = \int_2^3 \frac{dx}{\sqrt[4]{x-2}}, \quad I_3 = \int_0^\infty \frac{dx}{x^2+1}.$$