

SOURCES OF MAGNETIC FIELD

28.1. IDENTIFY and SET UP: Use $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$ to calculate \vec{B} at each point.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}, \text{ since } \hat{r} = \frac{\vec{r}}{r}.$$

$\vec{v} = (8.00 \times 10^6 \text{ m/s})\hat{j}$ and \vec{r} is the vector from the charge to the point where the field is calculated.

EXECUTE: (a) $\vec{r} = (0.500 \text{ m})\hat{i}$, $r = 0.500 \text{ m}$.

$$\vec{v} \times \vec{r} = vr\hat{j} \times \hat{i} = -vr\hat{k}.$$

$$\vec{B} = -\frac{\mu_0}{4\pi} \frac{qv}{r^2} \hat{k} = -(1 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(6.00 \times 10^{-6} \text{ C})(8.00 \times 10^6 \text{ m/s})}{(0.500 \text{ m})^2} \hat{k}.$$

$$\vec{B} = -(1.92 \times 10^{-5} \text{ T})\hat{k}.$$

(b) $\vec{r} = -(0.500 \text{ m})\hat{j}$, $r = 0.500 \text{ m}$.

$$\vec{v} \times \vec{r} = -vr\hat{j} \times \hat{j} = 0 \text{ and } \vec{B} = 0.$$

(c) $\vec{r} = (0.500 \text{ m})\hat{k}$, $r = 0.500 \text{ m}$.

$$\vec{v} \times \vec{r} = vr\hat{j} \times \hat{k} = vr\hat{i}.$$

$$\vec{B} = (1 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(6.00 \times 10^{-6} \text{ C})(8.00 \times 10^6 \text{ m/s})}{(0.500 \text{ m})^2} \hat{i} = +(1.92 \times 10^{-5} \text{ T})\hat{i}.$$

(d) $\vec{r} = -(0.500 \text{ m})\hat{j} + (0.500 \text{ m})\hat{k}$, $r = \sqrt{(0.500 \text{ m})^2 + (0.500 \text{ m})^2} = 0.7071 \text{ m}$.

$$\vec{v} \times \vec{r} = v(0.500 \text{ m})(-\hat{j} \times \hat{j} + \hat{j} \times \hat{k}) = (4.00 \times 10^6 \text{ m}^2/\text{s})\hat{i}.$$

$$\vec{B} = (1 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(6.00 \times 10^{-6} \text{ C})(4.00 \times 10^6 \text{ m}^2/\text{s})}{(0.7071 \text{ m})^3} \hat{i} = +(6.79 \times 10^{-6} \text{ T})\hat{i}.$$

EVALUATE: At each point \vec{B} is perpendicular to both \vec{v} and \vec{r} . $B = 0$ along the direction of \vec{v} .

28.2. IDENTIFY: A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$, and its electric field is

$$E = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2} \text{ since } q = e.$$

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(1.60 \times 10^{-19} \text{ C})(2.2 \times 10^6 \text{ m/s}) \sin 90^\circ}{(5.3 \times 10^{-11} \text{ m})^2} = 13 \text{ T, out of the page.}$$

$$E = \frac{1}{4\pi_0} \frac{e}{r^2} = \frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})}{(5.3 \times 10^{-11} \text{ m})^2} = 5.1 \times 10^{11} \text{ N/C, toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.3. IDENTIFY: A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

$$(a) B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} (1.60 \times 10^{-19} \text{ C})(3.0 \times 10^7 \text{ m/s}) \sin 30^\circ}{(2.00 \times 10^{-6} \text{ m})^2}.$$

$B = 6.00 \times 10^{-8} \text{ T}$, out of the paper, and it is the same at point B .

$$(b) B = (1.00 \times 10^{-7} \text{ T} \cdot \text{m/A})(1.60 \times 10^{-19} \text{ C})(3.00 \times 10^7 \text{ m/s}) / (2.00 \times 10^{-6} \text{ m})^2.$$

$B = 1.20 \times 10^{-7} \text{ T}$, out of the page.

$$(c) B = 0 \text{ T since } \sin(180^\circ) = 0.$$

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the earth's magnetic field.

28.4. IDENTIFY: Both moving charges produce magnetic fields, and the net field is the vector sum of the two fields.

SET UP: Both fields point out of the paper, so their magnitudes add, giving

$$B = B_{\text{alpha}} + B_{\text{el}} = \frac{\mu_0 v}{4\pi r^2} (e \sin 40^\circ + 2e \sin 140^\circ).$$

EXECUTE: Factoring out an e and putting in the numbers gives

$$B = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} (1.60 \times 10^{-19} \text{ C})(2.50 \times 10^5 \text{ m/s})}{(8.65 \times 10^{-9} \text{ m})^2} (\sin 40^\circ + 2 \sin 140^\circ).$$

$$B = 1.03 \times 10^{-4} \text{ T} = 0.103 \text{ mT, out of the page.}$$

EVALUATE: At distances very close to the charges, the magnetic field is strong enough to be important.

28.5. IDENTIFY: Apply $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$.

SET UP: Since the charge is at the origin, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

EXECUTE: (a) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{i}$; $\vec{v} \times \vec{r} = 0$, $B = 0$.

(b) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{j}$; $\vec{v} \times \vec{r} = vr\hat{k}$, $r = 0.500 \text{ m}$.

$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{|q|v}{r^2} = \frac{(1.0 \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2)(4.80 \times 10^{-6} \text{ C})(6.80 \times 10^5 \text{ m/s})}{(0.500 \text{ m})^2} = 1.31 \times 10^{-6} \text{ T}.$$

q is negative, so $\vec{B} = -(1.31 \times 10^{-6} \text{ T})\hat{k}$.

(c) $\vec{v} = v\hat{i}$, $\vec{r} = (0.500 \text{ m})(\hat{i} + \hat{j})$; $\vec{v} \times \vec{r} = (0.500 \text{ m})v\hat{k}$, $r = 0.7071 \text{ m}$.

$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{(|q||\vec{v} \times \vec{r}|/r^3)}{r^3} = \frac{(1.0 \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2)(4.80 \times 10^{-6} \text{ C})(0.500 \text{ m})(6.80 \times 10^5 \text{ m/s})}{(0.7071 \text{ m})^3}.$$

$$B = 4.62 \times 10^{-7} \text{ T. } \vec{B} = -(4.62 \times 10^{-7} \text{ T})\hat{k}.$$

(d) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{k}$; $\vec{v} \times \vec{r} = -vr\hat{j}$, $r = 0.500 \text{ m}$.

$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{|q|v}{r^2} = \frac{(1.0 \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2)(4.80 \times 10^{-6} \text{ C})(6.80 \times 10^5 \text{ m/s})}{(0.500 \text{ m})^2} = 1.31 \times 10^{-6} \text{ T}.$$

$$\vec{B} = (1.31 \times 10^{-6} \text{ T}) \hat{j}.$$

EVALUATE: In each case, \vec{B} is perpendicular to both \vec{r} and \vec{v} .

- 28.6. IDENTIFY:** Apply $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$. For the magnetic force, apply the results of Example 28.1, except here the two charges and velocities are different.

SET UP: In part (a), $r = d$ and \vec{r} is perpendicular to \vec{v} in each case, so $\frac{|\vec{v} \times \vec{r}|}{r^3} = \frac{v}{d^2}$. For calculating the force between the charges, $r = 2d$.

EXECUTE: (a) $B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi} \left(\frac{qv}{d^2} + \frac{q'v'}{d^2} \right).$

$$B = \frac{\mu_0}{4\pi} \left(\frac{(8.0 \times 10^{-6} \text{ C})(4.5 \times 10^6 \text{ m/s})}{(0.120 \text{ m})^2} + \frac{(3.0 \times 10^{-6} \text{ C})(9.0 \times 10^6 \text{ m/s})}{(0.120 \text{ m})^2} \right) = 4.38 \times 10^{-4} \text{ T}.$$

The direction of \vec{B} is into the page.

(b) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi} \frac{qq'vv'}{r^2} = (10^{-7} \text{ T} \cdot \text{m/A}) \frac{(8.00 \times 10^{-6} \text{ C})(3.00 \times 10^{-6} \text{ C})(4.50 \times 10^6 \text{ m/s})(9.00 \times 10^6 \text{ m/s})}{(0.240 \text{ m})^2}.$$

$F_B = 1.69 \times 10^{-3} \text{ N}$. The force on the upper charge points up and the force on the lower charge points down. The Coulomb force between the charges is

$$F_C = k \frac{q_1 q_2}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(8.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})}{(0.240 \text{ m})^2} = 3.75 \text{ N}.$$

The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb force to the magnetic force is $\frac{F_C}{F_B} = \frac{c^2}{v_1 v_2} = \frac{3.75 \text{ N}}{1.69 \times 10^{-3} \text{ N}} = 2.22 \times 10^3$; the Coulomb force is much larger.

(c) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

EVALUATE: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

- 28.7. IDENTIFY:** A moving charge creates a magnetic field.

SET UP: Apply $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$. $\vec{r} = (0.200 \text{ m})\hat{i} + (-0.300 \text{ m})\hat{j}$, and $r = 0.3606 \text{ m}$.

EXECUTE: $\vec{v} \times \vec{r} = [(7.50 \times 10^4 \text{ m/s})\hat{i} + (-4.90 \times 10^4 \text{ m/s})\hat{j}] \times [(0.200 \text{ m})\hat{i} + (-0.300 \text{ m})\hat{j}]$, which simplifies to $\vec{v} \times \vec{r} = (-2.25 \times 10^4 \text{ m}^2/\text{s})\hat{k} + (9.80 \times 10^3 \text{ m}^2/\text{s})\hat{k} = (-1.27 \times 10^4 \text{ m}^2/\text{s})\hat{k}$.

$$\vec{B} = (1.00 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(-3.00 \times 10^{-6} \text{ C})(-1.27 \times 10^4 \text{ m}^2/\text{s})}{(0.3606 \text{ m})^3} \hat{k} = (9.75 \times 10^{-8} \text{ T})\hat{k}.$$

EVALUATE: We can check the direction of the magnetic field using the right-hand rule, which shows that the field points in the $+z$ -direction.

- 28.8. IDENTIFY:** Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB \sin \phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$.

EXECUTE: (a) Both fields are into the page, so their magnitudes add, giving

$$B = B_e + B_p = \frac{\mu_0}{4\pi} \left(\frac{ev}{r_e^2} + \frac{ev}{r_p^2} \right) \sin 90^\circ.$$

$$B = \frac{\mu_0}{4\pi} (1.60 \times 10^{-19} \text{ C})(735,000 \text{ m/s}) \left[\frac{1}{(5.00 \times 10^{-9} \text{ m})^2} + \frac{1}{(4.00 \times 10^{-9} \text{ m})^2} \right].$$

$$B = 1.21 \times 10^{-3} \text{ T} = 1.21 \text{ mT, into the page.}$$

(b) Using $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$, where $r = \sqrt{41} \text{ nm}$ and $\phi = 180^\circ - \arctan(5/4) = 128.7^\circ$, we get

$$B = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(1.6 \times 10^{-19} \text{ C})(735,000 \text{ m/s}) \sin 128.7^\circ}{(\sqrt{41} \times 10^{-9} \text{ m})^2} = 2.24 \times 10^{-4} \text{ T, into the page.}$$

(c) $F_{\text{mag}} = qvB \sin 90^\circ = (1.60 \times 10^{-19} \text{ C})(735,000 \text{ m/s})(2.24 \times 10^{-4} \text{ T}) = 2.63 \times 10^{-17} \text{ N}$, in the +x-direction.

$$F_{\text{elec}} = (1/4\pi\epsilon_0)e^2/r^2 = \frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(\sqrt{41} \times 10^{-9} \text{ m})^2} = 5.62 \times 10^{-12} \text{ N, at } 129^\circ$$

counterclockwise from the +x-axis.

EVALUATE: The electric force is over 200,000 times as strong as the magnetic force.

28.9. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$.

EXECUTE: Applying the law of Biot and Savart gives

$$(a) \quad dB = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(10.0 \text{ A})(0.00110 \text{ m}) \sin 90^\circ}{(0.0500 \text{ m})^2} = 4.40 \times 10^{-7} \text{ T, out of the paper.}$$

(b) The same as above, except $r = \sqrt{(5.00 \text{ cm})^2 + (14.0 \text{ cm})^2}$ and $\phi = \arctan(5/14) = 19.65^\circ$, giving $dB = 1.67 \times 10^{-8} \text{ T, out of the page.}$

(c) $dB = 0$ since $\phi = 0^\circ$.

EVALUATE: This is a very small field, but it comes from a very small segment of current.

28.10. IDENTIFY: Apply the Biot-Savart law.

SET UP: Apply $d\vec{B} = \frac{\mu_0}{4\pi} \frac{q d\vec{l} \times \vec{r}}{r^3}$. $r = \sqrt{(-0.730 \text{ m})^2 + (0.390 \text{ m})^2} = 0.8276 \text{ m}$.

EXECUTE:

$$d\vec{l} \times \vec{r} = [0.500 \times 10^{-3} \text{ m}] \hat{j} \times [(-0.730 \text{ m}) \hat{i} + (0.390 \text{ m}) \hat{k}] = (+3.65 \times 10^{-4} \text{ m}^2) \hat{k} + (+1.95 \times 10^{-4} \text{ m}^2) \hat{i}.$$

$$d\vec{B} = (1.00 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{5.40 \text{ A}}{(0.8276 \text{ m})^3} [(3.65 \times 10^{-4} \text{ m}^2) \hat{k} + (1.95 \times 10^{-4} \text{ m}^2) \hat{i}].$$

$$d\vec{B} = (1.86 \times 10^{-10} \text{ T}) \hat{i} + (3.48 \times 10^{-10} \text{ T}) \hat{k}.$$

EVALUATE: The magnetic field lies in the xz-plane.

28.11. IDENTIFY and SET UP: The magnetic field produced by an infinitesimal current element is given

$$\text{by } d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \hat{r}}{r^2}.$$

As in Example 28.2, use $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \hat{r}}{r^2}$ for the finite 0.500-mm segment of wire since the

$\Delta l = 0.500\text{-mm}$ length is much smaller than the distances to the field points.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{l} \times \vec{r}}{r^3}$$

I is in the +z-direction, so $\Delta \vec{l} = (0.500 \times 10^{-3} \text{ m}) \hat{k}$.

EXECUTE: (a) The field point is at $x = 2.00$ m, $y = 0$, $z = 0$ so the vector \vec{r} from the source point (at the origin) to the field point is $\vec{r} = (2.00 \text{ m})\hat{i}$.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times \hat{i} = +(1.00 \times 10^{-3} \text{ m}^2)\hat{j}.$$

$$\vec{B} = \frac{(1 \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})(1.00 \times 10^{-3} \text{ m}^2)}{(2.00 \text{ m})^3} \hat{j} = (5.00 \times 10^{-11} \text{ T})\hat{j}.$$

(b) $\vec{r} = (2.00 \text{ m})\hat{j}$, $r = 2.00$ m.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times \hat{j} = -(1.00 \times 10^{-3} \text{ m}^2)\hat{i}.$$

$$\vec{B} = \frac{(1 \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})(-1.00 \times 10^{-3} \text{ m}^2)}{(2.00 \text{ m})^3} \hat{i} = -(5.00 \times 10^{-11} \text{ T})\hat{i}.$$

(c) $\vec{r} = (2.00 \text{ m})(\hat{i} + \hat{j})$, $r = \sqrt{2}(2.00 \text{ m})$.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times (\hat{i} + \hat{j}) = (1.00 \times 10^{-3} \text{ m}^2)(\hat{j} - \hat{i}).$$

$$\vec{B} = \frac{(1 \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})(1.00 \times 10^{-3} \text{ m}^2)}{[\sqrt{2}(2.00 \text{ m})]^3} (\hat{j} - \hat{i}) = (-1.77 \times 10^{-11} \text{ T})(\hat{i} - \hat{j}).$$

(d) $\vec{r} = (2.00 \text{ m})\hat{k}$, $r = 2.00$ m.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times \hat{k} = 0; \vec{B} = 0.$$

EVALUATE: At each point \vec{B} is perpendicular to both \vec{r} and $\Delta\vec{l}$. $B = 0$ along the length of the wire.

28.12. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$.

Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the law of Biot and Savart for the 12.0-A current gives

$$dB = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(12.0 \text{ A})(0.00150 \text{ m}) \left(\frac{2.50 \text{ cm}}{8.00 \text{ cm}} \right)}{(0.0800 \text{ m})^2} = 8.79 \times 10^{-8} \text{ T}.$$

The field from the 24.0-A segment is twice this value, so the total field is $2.64 \times 10^{-7} \text{ T}$, into the page.

EVALUATE: The rest of each wire also produces field at P . We have calculated just the field from the two segments that are indicated in the problem.

28.13. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$. Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the Biot and Savart law, where $r = \frac{1}{2}\sqrt{(3.00 \text{ cm})^2 + (3.00 \text{ cm})^2} = 2.121 \text{ cm}$, we have

$$dB = 2 \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(28.0 \text{ A})(0.00200 \text{ m}) \sin 45.0^\circ}{(0.02121 \text{ m})^2} = 1.76 \times 10^{-5} \text{ T}, \text{ into the paper.}$$

EVALUATE: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

28.14. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$. All four fields are of equal magnitude and into the page, so their magnitudes add.

EXECUTE:
$$dB = 4 \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(8.00 \text{ A})(0.00120 \text{ m}) \sin 90^\circ}{(0.0500 \text{ m})^2} = 1.54 \times 10^{-6} \text{ T} = 1.54 \mu\text{T}, \text{ into}$$

the page.

EVALUATE: A small current element causes a small magnetic field.

- 28.15. IDENTIFY:** We can model the lightning bolt and the household current as very long current-carrying wires.

SET UP: The magnetic field produced by a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: Substituting the numerical values gives

(a)
$$B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(20,000 \text{ A})}{2\pi(5.0 \text{ m})} = 8 \times 10^{-4} \text{ T}.$$

(b)
$$B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(10 \text{ A})}{2\pi(0.050 \text{ m})} = 4.0 \times 10^{-5} \text{ T}.$$

EVALUATE: The field from the lightning bolt is about 20 times as strong as the field from the household current.

- 28.16. IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (8.20 \times 10^{18} \text{ el/s})(1.60 \times 10^{-19} \text{ C/el}) = 1.312 \text{ A}.$

Now find the magnetic field:
$$\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.312 \text{ A})}{2\pi(0.0400 \text{ m})} = 6.56 \times 10^{-6} \text{ T} = 6.56 \mu\text{T}.$$

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the earth, so any experiments involving such a current would have to be shielded from the earth's magnetic field, or at least would have to take it into consideration.

- 28.17. IDENTIFY:** We can model the current in the heart as that of a long straight wire. It produces a magnetic field around it.

SET UP: For a long straight wire, $B = \frac{\mu_0 I}{2\pi r}$. $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$. $1 \text{ gauss} = 10^{-4} \text{ T}.$

EXECUTE: Solving for the current gives

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.050 \text{ m})(1.0 \times 10^{-9} \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = 25 \times 10^{-5} \text{ A} = 250 \mu\text{A}.$$

EVALUATE: By household standards, this is a very small current. But the magnetic field around the heart ($\approx 10 \mu\text{G}$) is also very small.

- 28.18. IDENTIFY:** The current in the transmission line creates a magnetic field. If this field is greater than 5% of the earth's magnetic field, it will interfere with the navigation of the bacteria.

SET UP: $B = \frac{\mu_0 I}{2\pi r}$ due to a long straight wire.

EXECUTE: We know the field is $B = (0.05)(5 \times 10^{-5} \text{ T}) = 2.5 \times 10^{-6} \text{ T}$. Solving $B = \frac{\mu_0 I}{2\pi r}$ for r gives

$$r = \frac{\mu_0 I}{2\pi B} = \frac{(2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{100 \text{ A}}{2.5 \times 10^{-6} \text{ T}}}{2\pi} = 8 \text{ m}.$$

EVALUATE: If the bacteria are within 8 m ($\approx 25 \text{ ft}$) of the cable, its magnetic field may be strong enough to affect their navigation.

- 28.19. IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: First solve for the current, then substitute the numbers using the above equation.

(a) Solving for the current gives

$$I = 2\pi r B / \mu_0 = 2\pi(0.0200 \text{ m})(1.00 \times 10^{-4} \text{ T}) / (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) = 10.0 \text{ A}.$$

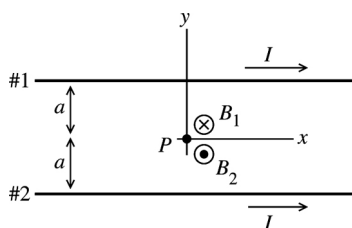
(b) The earth's horizontal field points northward, so at all points directly above the wire the field of the wire would point northward.

(c) At all points directly east of the wire, its field would point northward.

EVALUATE: Even though the earth's magnetic field is rather weak, it requires a fairly large current to cancel this field.

28.20. IDENTIFY: For each wire $B = \frac{\mu_0 I}{2\pi r}$, and the direction of \vec{B} is given by the right-hand rule (Figure 28.6 in the textbook). Add the field vectors for each wire to calculate the total field.

(a) **SET UP:** The two fields at this point have the directions shown in Figure 28.20a.

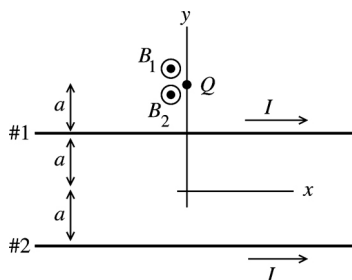


EXECUTE: At point P midway between the two wires the fields \vec{B}_1 and \vec{B}_2 due to the two currents are in opposite directions, so $B = B_2 - B_1$.

Figure 28.20a

But $B_1 = B_2 = \frac{\mu_0 I}{2\pi a}$, so $B = 0$.

(b) **SET UP:** The two fields at this point have the directions shown in Figure 28.20b.



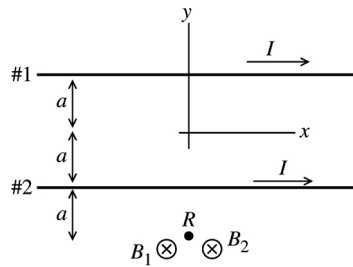
EXECUTE: At point Q above the upper wire \vec{B}_1 and \vec{B}_2 are both directed out of the page ($+z$ -direction), so $B = B_1 + B_2$.

Figure 28.20b

$$B_1 = \frac{\mu_0 I}{2\pi a}, B_2 = \frac{\mu_0 I}{2\pi(3a)}.$$

$$B = \frac{\mu_0 I}{2\pi a} \left(1 + \frac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a}; \vec{B} = \frac{2\mu_0 I}{3\pi a} \hat{k}.$$

(c) **SET UP:** The two fields at this point have the directions shown in Figure 28.20c (next page).



EXECUTE: At point R below the lower wire \vec{B}_1 and \vec{B}_2 are both directed into the page ($-z$ -direction), so $B = B_1 + B_2$.

Figure 28.20c

$$B_1 = \frac{\mu_0 I}{2\pi(3a)}, B_2 = \frac{\mu_0 I}{2\pi a}.$$

$$B_1 = \frac{\mu_0 I}{2\pi a} \left(1 + \frac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a}; \vec{B} = -\frac{2\mu_0 I}{3\pi a} \hat{k}.$$

EVALUATE: In the figures we have drawn, \vec{B} due to each wire is out of the page at points above the wire and into the page at points below the wire. If the two field vectors are in opposite directions the magnitudes subtract.

- 28.21. IDENTIFY:** The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

SET UP: For the wire, $B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$ and the direction of B_{wire} is given by the right-hand rule that is

illustrated in Figure 28.6 in the textbook. $\vec{B}_0 = (1.50 \times 10^{-6} \text{ T}) \hat{i}$.

EXECUTE: (a) At $(0, 0, 1 \text{ m})$, $\vec{B} = \vec{B}_0 - \frac{\mu_0 I}{2\pi r} \hat{i} = (1.50 \times 10^{-6} \text{ T}) \hat{i} - \frac{\mu_0(8.00 \text{ A})}{2\pi(1.00 \text{ m})} \hat{i} = -(1.0 \times 10^{-7} \text{ T}) \hat{i}$.

(b) At $(1 \text{ m}, 0, 0)$, $\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r} \hat{k} = (1.50 \times 10^{-6} \text{ T}) \hat{i} + \frac{\mu_0(8.00 \text{ A})}{2\pi(1.00 \text{ m})} \hat{k}$.

$\vec{B} = (1.50 \times 10^{-6} \text{ T}) \hat{i} + (1.6 \times 10^{-6} \text{ T}) \hat{k} = 2.19 \times 10^{-6} \text{ T}$, at $\theta = 46.8^\circ$ from x to z .

(c) At $(0, 0, -0.25 \text{ m})$, $\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r} \hat{i} = (1.50 \times 10^{-6} \text{ T}) \hat{i} + \frac{\mu_0(8.00 \text{ A})}{2\pi(0.25 \text{ m})} \hat{i} = (7.9 \times 10^{-6} \text{ T}) \hat{i}$.

EVALUATE: At point c the two fields are in the same direction and their magnitudes add. At point a they are in opposite directions and their magnitudes subtract. At point b the two fields are perpendicular.

- 28.22. IDENTIFY:** The magnetic field is that of a long current-carrying wire.

SET UP: $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: $B = \frac{\mu_0 I}{2\pi r} = \frac{(2.0 \times 10^{-7} \text{ T} \cdot \text{m/A})(150 \text{ A})}{8.0 \text{ m}} = 3.8 \times 10^{-6} \text{ T}$. This is 7.5% of the earth's field.

EVALUATE: Since this field is much smaller than the earth's magnetic field, it would be expected to have less effect than the earth's field.

- 28.23. IDENTIFY:** $B = \frac{\mu_0 I}{2\pi r}$. The direction of \vec{B} is given by the right-hand rule.

SET UP: Call the wires a and b , as indicated in Figure 28.23. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.23a. The fields at point 3 are shown in Figure 28.23b.

EXECUTE: (a) At P_1 , $B_a = B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a = \frac{\mu_0 I}{2\pi r_a}$. $B_b = \frac{\mu_0 I}{2\pi r_b}$. \vec{B}_a and \vec{B}_b are in the same direction so

$$B = B_a + B_b = \frac{\mu_0 I}{2\pi} \left(\frac{1}{r_a} + \frac{1}{r_b} \right) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})}{2\pi} \left[\frac{1}{0.300 \text{ m}} + \frac{1}{0.200 \text{ m}} \right] = 6.67 \times 10^{-6} \text{ T}.$$

\vec{B} has magnitude $6.67 \mu\text{T}$ and is directed toward the top of the page.

(c) In Figure 28.25b, \vec{B}_a is perpendicular to \vec{r}_a and \vec{B}_b is perpendicular to \vec{r}_b . $\tan \theta = \frac{5 \text{ cm}}{20 \text{ cm}}$ and

$$\theta = 14.04^\circ. \quad r_a = r_b = \sqrt{(0.200 \text{ m})^2 + (0.050 \text{ m})^2} = 0.206 \text{ m} \text{ and } B_a = B_b.$$

$$B = B_a \cos \theta + B_b \cos \theta = 2B_a \cos \theta = 2 \left(\frac{\mu_0 I}{2\pi r_a} \right) \cos \theta = \frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.0 \text{ A}) \cos 14.04^\circ}{2\pi(0.206 \text{ m})} = 7.54 \mu\text{T}$$

B has magnitude $7.53 \mu\text{T}$ and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

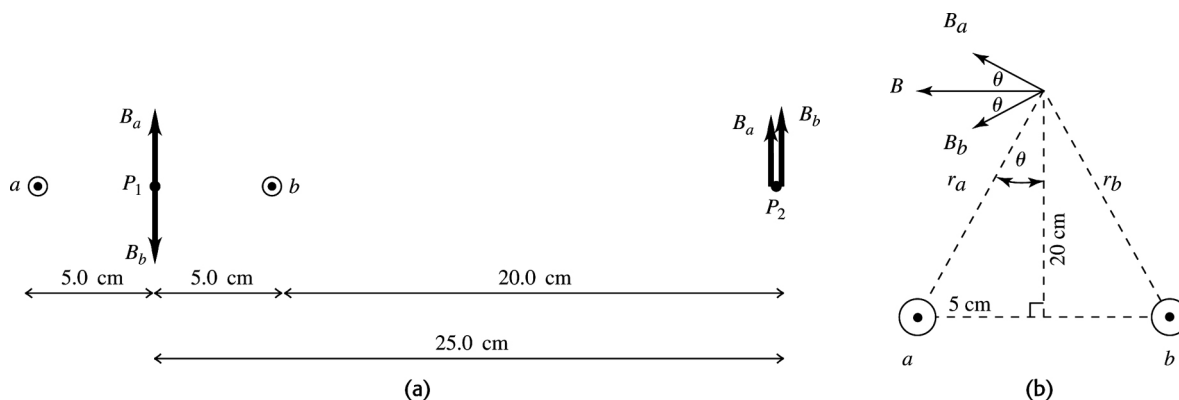


Figure 28.23

28.24. IDENTIFY: Each segment of the rectangular loop creates a magnetic field at the center of the loop, and all these fields are in the same direction.

SET UP: The field due to each segment is $B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}}$. \vec{B} is into paper so I is clockwise around the loop.

EXECUTE: Long sides: $a = 4.75 \text{ cm}$. $x = 2.10 \text{ cm}$. For the two long sides,

$$B = 2(1.00 \times 10^{-7} \text{ T} \cdot \text{m/A})I \frac{2(4.75 \times 10^{-2} \text{ m})}{(2.10 \times 10^{-2} \text{ m})\sqrt{(0.0210 \text{ m})^2 + (0.0475 \text{ m})^2}} = (1.742 \times 10^{-5} \text{ T/A})I.$$

Short sides: $a = 2.10 \text{ cm}$. $x = 4.75 \text{ cm}$. For the two short sides,

$$B = 2(1.00 \times 10^{-7} \text{ T} \cdot \text{m/A})I \frac{2(2.10 \times 10^{-2} \text{ m})}{(4.75 \times 10^{-2} \text{ m})\sqrt{(0.0475 \text{ m})^2 + (0.0210 \text{ m})^2}} = (3.405 \times 10^{-6} \text{ T/A})I.$$

Using the known field, we have $B = (2.082 \times 10^{-5} \text{ T/A})I = 5.50 \times 10^{-5} \text{ T}$, which gives $I = 2.64 \text{ A}$.

EVALUATE: This is a typical household current, yet it produces a magnetic field which is about the same as the earth's magnetic field.

28.25. IDENTIFY: The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of \vec{B} is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: (a) and (b) $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.25.

$$B = B_a \cos 45^\circ + B_b \cos 45^\circ + B_c \cos 45^\circ + B_d \cos 45^\circ = 4B_a \cos 45^\circ = 4 \left(\frac{\mu_0 I}{2\pi r} \right) \cos 45^\circ.$$

$$r = \sqrt{(10 \text{ cm})^2 + (10 \text{ cm})^2} = 10\sqrt{2} \text{ cm} = 0.10\sqrt{2} \text{ m, so}$$

$$B = 4 \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(100 \text{ A})}{2\pi(0.10\sqrt{2} \text{ m})} \cos 45^\circ = 4.0 \times 10^{-4} \text{ T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

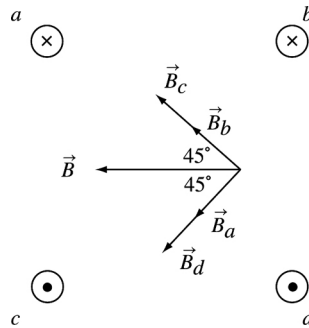
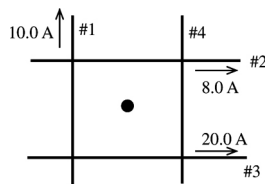


Figure 28.25

- 28.26. IDENTIFY:** Use $B = \frac{\mu_0 I}{2\pi r}$ and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.
SET UP: The three known currents are shown in Figure 28.26.



$$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$$

$$B = \frac{\mu_0 I}{2\pi r}; r = 0.200 \text{ m for each wire.}$$

Figure 28.26

EXECUTE: Let \odot be the positive z -direction. $I_1 = 10.0 \text{ A}$, $I_2 = 8.0 \text{ A}$, $I_3 = 20.0 \text{ A}$. Then

$$B_1 = 1.00 \times 10^{-5} \text{ T}, B_2 = 0.80 \times 10^{-5} \text{ T}, \text{ and } B_3 = 2.00 \times 10^{-5} \text{ T.}$$

$$B_{1z} = -1.00 \times 10^{-5} \text{ T}, B_{2z} = -0.80 \times 10^{-5} \text{ T}, B_{3z} = +2.00 \times 10^{-5} \text{ T.}$$

$$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0.$$

$$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0 \times 10^{-6} \text{ T.}$$

To give \vec{B}_4 in the \otimes direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200 \text{ m})(2.0 \times 10^{-6} \text{ T})}{(2 \times 10^{-7} \text{ T} \cdot \text{m/A})} = 2.0 \text{ A.}$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current $20.0 \text{ A} - 8.0 \text{ A} = 12.0 \text{ A}$ in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0 \text{ A} + I_4 = 12.0 \text{ A}$.

28.27. IDENTIFY: The net magnetic field at any point is the vector sum of the magnetic fields of the two wires.

SET UP: For each wire $B = \frac{\mu_0 I}{2\pi r}$ and the direction of \vec{B} is determined by the right-hand rule described in the text. Let the wire with 12.0 A be wire 1 and the wire with 10.0 A be wire 2.

EXECUTE: (a) Point Q: $B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(12.0 \text{ A})}{2\pi(0.15 \text{ m})} = 1.6 \times 10^{-5} \text{ T}$.

The direction of \vec{B}_1 is out of the page. $B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(10.0 \text{ A})}{2\pi(0.080 \text{ m})} = 2.5 \times 10^{-5} \text{ T}$.

The direction of \vec{B}_2 is out of the page. Since \vec{B}_1 and \vec{B}_2 are in the same direction,

$B = B_1 + B_2 = 4.1 \times 10^{-5} \text{ T}$ and \vec{B} is directed out of the page.

Point P: $B_1 = 1.6 \times 10^{-5} \text{ T}$, directed into the page. $B_2 = 2.5 \times 10^{-5} \text{ T}$, directed into the page.

$B = B_1 + B_2 = 4.1 \times 10^{-5} \text{ T}$ and \vec{B} is directed into the page.

(b) \vec{B}_1 is the same as in part (a), out of the page at Q and into the page at P. The direction of \vec{B}_2 is reversed from what it was in (a) so is into the page at Q and out of the page at P.

Point Q: \vec{B}_1 and \vec{B}_2 are in opposite directions so $B = B_2 - B_1 = 2.5 \times 10^{-5} \text{ T} - 1.6 \times 10^{-5} \text{ T} = 9.0 \times 10^{-6} \text{ T}$ and \vec{B} is directed into the page.

Point P: \vec{B}_1 and \vec{B}_2 are in opposite directions so $B = B_2 - B_1 = 9.0 \times 10^{-6} \text{ T}$ and \vec{B} is directed out of the page.

EVALUATE: Points P and Q are the same distances from the two wires. The only difference is that the fields point in either the same direction or in opposite directions.

28.28. IDENTIFY: Apply $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$ for the force from each wire.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: On the top wire $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi} \left(\frac{1}{d} - \frac{1}{2d} \right) = \frac{\mu_0 I^2}{4\pi d}$, upward. On the middle wire, the magnetic

forces cancel so the net force is zero. On the bottom wire $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi} \left(\frac{1}{d} - \frac{1}{2d} \right) = \frac{\mu_0 I^2}{4\pi d}$, downward.

EVALUATE: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

28.29. IDENTIFY: Apply $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: (a) $F = \frac{\mu_0 I_1 I_2 L}{2\pi r} = \frac{\mu_0 (5.00 \text{ A})(2.00 \text{ A})(1.20 \text{ m})}{2\pi(0.400 \text{ m})} = 6.00 \times 10^{-6} \text{ N}$, and the force is repulsive

since the currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to $F = 2.40 \times 10^{-5} \text{ N}$.

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

28.30. IDENTIFY: Apply $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: (a) $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$ gives $I_2 = \frac{F}{L} \frac{2\pi r}{\mu_0 I_1} = (4.0 \times 10^{-5} \text{ N/m}) \frac{2\pi(0.0250 \text{ m})}{\mu_0(0.60 \text{ A})} = 8.33 \text{ A}$.

(b) The two wires repel so the currents are in opposite directions.

EVALUATE: The force between the two wires is proportional to the product of the currents in the wires.

- 28.31. IDENTIFY:** The lamp cord wires are two parallel current-carrying wires, so they must exert a magnetic force on each other.

SET UP: First find the current in the cord. Since it is connected to a light bulb, the power consumed by the bulb is $P = IV$. Then find the force per unit length using $\frac{F}{L} = \frac{\mu_0 I'I}{2\pi r}$.

EXECUTE: For the light bulb, $100 \text{ W} = I(120 \text{ V})$ gives $I = 0.833 \text{ A}$. The force per unit length is

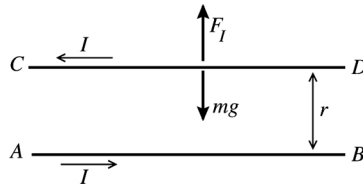
$$F/L = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} (0.833 \text{ A})^2}{2\pi (0.003 \text{ m})} = 4.6 \times 10^{-5} \text{ N/m}$$

Since the currents are in opposite directions, the force is repulsive.

EVALUATE: This force is too small to have an appreciable effect for an ordinary cord.

- 28.32. IDENTIFY:** The wire CD rises until the upward force F_I due to the currents balances the downward force of gravity.

SET UP: The forces on wire CD are shown in Figure 28.32.



Currents in opposite directions so the force is repulsive and F_I is upward, as shown.

Figure 28.32

$\frac{F}{L} = \frac{\mu_0 I'I}{2\pi r}$ says $F_I = \frac{\mu_0 I^2 L}{2\pi h}$ where L is the length of wire CD and h is the distance between the wires.

EXECUTE: $mg = \lambda Lg$.

Thus $F_I - mg = 0$ says $\frac{\mu_0 I^2 L}{2\pi h} = \lambda Lg$ and $h = \frac{\mu_0 I^2}{2\pi g \lambda}$.

EVALUATE: The larger I is or the smaller λ is, the larger h will be.

- 28.33. IDENTIFY:** We can model the current in the brain as a ring. Since we know the magnetic field at the center of the ring, we can calculate the current.

SET UP: At the center of a ring, $B = \frac{\mu_0 I}{2R}$. In this case, $R = 8 \text{ cm}$. $1 \text{ gauss} = 1 \times 10^{-4} \text{ T}$.

EXECUTE: Solving for I gives $I = \frac{2RB}{\mu_0} = \frac{2(8 \times 10^{-2} \text{ m})(3.0 \times 10^{-12} \text{ T})}{4 \times 10^{-7} \text{ T} \cdot \text{m/A}} = 3.8 \times 10^{-7} \text{ A}$.

EVALUATE: This current is about a third of a microamp, which is a very small current by household standards. However, the magnetic field in the brain is a very weak field, about a hundredth of the earth's magnetic field.

- 28.34. IDENTIFY:** The magnetic field at the center of a circular loop is $B = \frac{\mu_0 I}{2R}$. By symmetry each segment of

the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

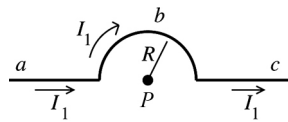
SET UP: Since the straight sections produce no field at P , the field at P is $B = \frac{\mu_0 I}{4R}$.

EXECUTE: $B = \frac{\mu_0 I}{4R}$. The direction of \vec{B} is given by the right-hand rule: \vec{B} is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B = \frac{\mu_0 I}{8R}$.

28.35. IDENTIFY: Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.35a.



Consider the three parts of this wire:

a : long straight section

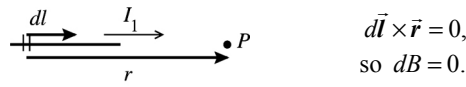
b : semicircle

c : long, straight section

Figure 28.35a

Apply the Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{r}}{r^3}$ to each piece.

EXECUTE: part a: See Figure 28.35b.

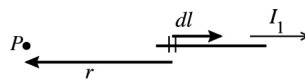


$$\begin{aligned} d\vec{l} \times \vec{r} &= 0, \\ \text{so } dB &= 0. \end{aligned}$$

Figure 28.35b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.

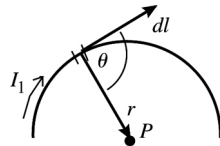
part c: See Figure 28.35c.



$$\begin{aligned} d\vec{l} \times \vec{r} &= 0, \\ \text{so } dB &= 0 \text{ and } B = 0 \text{ for this piece.} \end{aligned}$$

Figure 28.35c

part b: See Figure 28.35d.



$d\vec{l} \times \vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so \vec{B} is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

Figure 28.35d

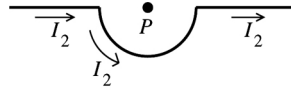
$|d\vec{l} \times \vec{r}| = rdl \sin \theta$. The angle θ between $d\vec{l}$ and \vec{r} is 90° and $r = R$, the radius of the semicircle. Thus

$$|d\vec{l} \times \vec{r}| = R dl.$$

$$dB = \frac{\mu_0}{4\pi} \frac{I |d\vec{l} \times \vec{r}|}{r^3} = \frac{\mu_0 I_1}{4\pi} \frac{R}{R^3} dl = \left(\frac{\mu_0 I_1}{4\pi R^2} \right) dl.$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2} \right) \int dl = \left(\frac{\mu_0 I_1}{4\pi R^2} \right) (\pi R) = \frac{\mu_0 I_1}{4R}.$$

(We used that $\int dl$ is equal to πR , the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to \vec{B} , so $B_1 = \mu_0 I_1 / 4R$ and is directed into the page.



For current in the direction shown in Figure 28.35e, a similar analysis gives $B_2 = \mu_0 I_2 / 4R$, out of the paper.

Figure 28.35e

\vec{B}_1 and \vec{B}_2 are in opposite directions, so the magnitude of the net field at P is $B = |B_1 - B_2| = \frac{\mu_0 |I_1 - I_2|}{4R}$.

EVALUATE: When $I_1 = I_2$, $B = 0$.

28.36. IDENTIFY: Apply $B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$.

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.0240 m.

EXECUTE: (a) $B_x = \mu_0 N I / 2a$, so $I = \frac{2a B_x}{\mu_0 N} = \frac{2(0.024 \text{ m})(0.0770 \text{ T})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(800)} = 3.68 \text{ A}$.

(b) At the center, $B_c = \mu_0 N I / 2a$. At a distance x from the center,

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} = \left(\frac{\mu_0 N I}{2a} \right) \left(\frac{a^3}{(x^2 + a^2)^{3/2}} \right) = B_c \left(\frac{a^3}{(x^2 + a^2)^{3/2}} \right). \quad B_x = \frac{1}{2} B_c \text{ says } \frac{a^3}{(x^2 + a^2)^{3/2}} = \frac{1}{2}, \text{ and}$$

$$(x^2 + a^2)^3 = 4a^6. \text{ Since } a = 0.024 \text{ m}, x = 0.0184 \text{ m} = 1.84 \text{ cm}.$$

EVALUATE: As shown in Figure 28.14 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.37. IDENTIFY: We use the equation for the magnetic field at the center of a single circular loop and then use the equation for the magnetic field inside a solenoid.

SET UP: The magnetic field at the center of a circular loop is $B_{\text{loop}} = \frac{\mu_0 I}{2R}$. The magnetic field at the

center of a solenoid is $B_{\text{solenoid}} = \mu_0 n I$, where $n = \frac{N}{L}$ is the number of turns per meter.

EXECUTE: (a) $B_{\text{loop}} = \frac{\mu_0 I}{2R} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.00 \text{ A})}{2(0.050 \text{ m})} = 2.51 \times 10^{-5} \text{ T}$.

(b) $n = \frac{N}{L} = \frac{1000}{5.00 \text{ m}} = 200 \text{ m}^{-1}$.

$B_{\text{solenoid}} = \mu_0 n I = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(200 \text{ m}^{-1})(2.00 \text{ A}) = 5.03 \times 10^{-4} \text{ T}$. $B_{\text{solenoid}} = 20 B_{\text{loop}}$. The field at the center of a circular loop depends on the radius of the loop. The field at the center of a solenoid depends on the length of the solenoid, not on its radius.

EVALUATE: The equation $B = \mu_0 n I$ for the field at the center of a solenoid is only correct for a very long solenoid, one whose length L is much greater than its radius R . We cannot consider the limit that L gets small and expect the expression for the solenoid to go over to the expression for N circular loops.

28.38. IDENTIFY and SET UP: The magnetic field at a point on the axis of N circular loops is given by

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}. \text{ Solve for } N \text{ and set } x = 0.0600 \text{ m}.$$

EXECUTE:
$$N = \frac{2B_x(x^2 + a^2)^{3/2}}{\mu_0 I a^2} = \frac{2(6.39 \times 10^{-4} \text{ T})[(0.0600 \text{ m})^2 + (0.0600 \text{ m})^2]^{3/2}}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.50 \text{ A})(0.0600 \text{ m})^2} = 69.$$

EVALUATE: At the center of the coil the field is $B_x = \frac{\mu_0 N I}{2a} = 1.8 \times 10^{-3} \text{ T}$. The field 6.00 cm from the center is a factor of $1/2^{3/2}$ times smaller.

28.39. IDENTIFY: The field at the center of the loops is the vector sum of the field due to each loop. They must be in opposite directions in order to add to zero.

SET UP: Let wire 1 be the inner wire with diameter 20.0 cm and let wire 2 be the outer wire with diameter 30.0 cm. To produce zero net field, the fields \vec{B}_1 and \vec{B}_2 of the two wires must have equal magnitudes

and opposite directions. At the center of a wire loop $B = \frac{\mu_0 I}{2R}$. The direction of \vec{B} is given by the right-hand rule applied to the current direction.

EXECUTE: $B_1 = \frac{\mu_0 I}{2R_1}$, $B_2 = \frac{\mu_0 I}{2R_2}$. $B_1 = B_2$ gives $\frac{\mu_0 I_1}{2R_1} = \frac{\mu_0 I_2}{2R_2}$. Solving for I_2 gives

$$I_2 = \left(\frac{R_2}{R_1} \right) I_1 = \left(\frac{15.0 \text{ cm}}{10.0 \text{ cm}} \right) (12.0 \text{ A}) = 18.0 \text{ A. The directions of } I_1 \text{ and of its field are shown in Figure 28.39.}$$

Since \vec{B}_1 is directed into the page, \vec{B}_2 must be directed out of the page and I_2 is counterclockwise.

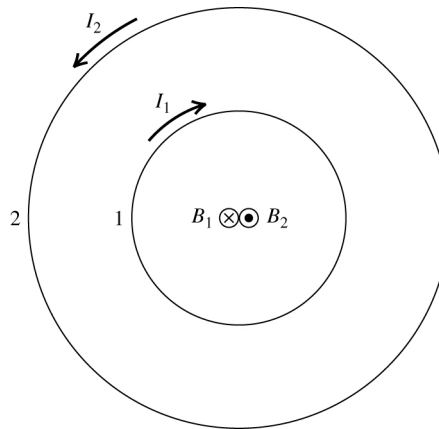


Figure 28.39

EVALUATE: The outer current, I_2 , must be larger than the inner current, I_1 , because the outer ring is larger than the inner ring, which makes the outer current farther from the center than the inner current is.

28.40. IDENTIFY: Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

EXECUTE: Path a: $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B} \cdot d\vec{l} = 0.$

Path b: $I_{\text{encl}} = -I_1 = -4.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = -\mu_0(4.0 \text{ A}) = -5.03 \times 10^{-6} \text{ T} \cdot \text{m}.$

Path c: $I_{\text{encl}} = -I_1 + I_2 = -4.0 \text{ A} + 6.0 \text{ A} = 2.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0(2.0 \text{ A}) = 2.51 \times 10^{-6} \text{ T} \cdot \text{m}.$

Path d: $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = +\mu_0(4.0 \text{ A}) = 5.03 \times 10^{-6} \text{ T} \cdot \text{m}.$

EVALUATE: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

28.41. IDENTIFY: Apply Ampere's law.

SET UP: $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}.$

EXECUTE: (a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83 \times 10^{-4} \text{ T} \cdot \text{m}$ and $I_{\text{encl}} = 305 \text{ A}$.

(b) $-3.83 \times 10^{-4} \text{ T} \cdot \text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

EVALUATE: The line integral $\oint \vec{B} \cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

28.42. IDENTIFY and SET UP: At the center of a long solenoid $B = \mu_0 n I = \mu_0 \frac{N}{L} I$.

EXECUTE: $I = \frac{BL}{\mu_0 N} = \frac{(0.150 \text{ T})(0.550 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4000)} = 16.4 \text{ A}$.

EVALUATE: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.

28.43. IDENTIFY: Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r . By symmetry, $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ for such a path.

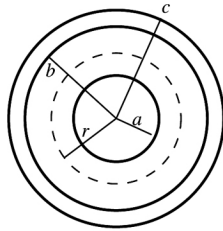
EXECUTE: (a) For $a < r < b$, $I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$.

(b) For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

EVALUATE: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

28.44. IDENTIFY: Apply Ampere's law to calculate \vec{B} .

(a) **SET UP:** For $a < r < b$ the end view is shown in Figure 28.44a.



Apply Ampere's law to a circle of radius r , where $a < r < b$. Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

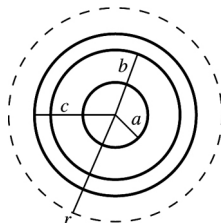
Figure 28.44a

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$, $I_{\text{encl}} = I_1$.

Thus $B(2\pi r) = \mu_0 I_1$ and $B = \frac{\mu_0 I_1}{2\pi r}$.

(b) **SET UP:** $r > c$: See Figure 28.44b.



Apply Ampere's law to a circle of radius r , where $r > c$. Both currents are in the positive direction.

Figure 28.44b

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1 + I_2.$$

Thus $B(2\pi r) = \mu_0(I_1 + I_2)$ and $B = \frac{\mu_0(I_1 + I_2)}{2\pi r}$.

EVALUATE: For $a < r < b$ the field is due only to the current in the central conductor. For $r > c$ both currents contribute to the total field.

28.45. IDENTIFY: We treat the solenoid as being ideal.

SET UP: At the center of an ideal solenoid, $B_{\text{solenoid}} = \mu_0 nI = \mu_0 \frac{N}{L} I$. A distance r from a long straight

wire, $B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: (a) $B_{\text{solenoid}} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{450}{0.35 \text{ m}} \right) (1.75 \text{ A}) = 2.83 \times 10^{-3} \text{ T}.$

(b) $B_{\text{wire}} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.75 \text{ A})}{2\pi(1.0 \times 10^{-2} \text{ m})} = 3.50 \times 10^{-5} \text{ T}.$

EVALUATE: The magnetic field due to the wire is much less than the field at the center of the solenoid. For the solenoid, the fields of all the wires add to give a much larger field.

28.46. IDENTIFY: $B = \mu_0 nI = \frac{\mu_0 NI}{L}$.

SET UP: $L = 0.150 \text{ m}.$

EXECUTE: $B = \frac{\mu_0 (600)(8.00 \text{ A})}{(0.150 \text{ m})} = 0.0402 \text{ T}.$

EVALUATE: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length, as it is here.

28.47. IDENTIFY and SET UP: The magnetic field near the center of a long solenoid is given by $B = \mu_0 nI$.

EXECUTE: (a) Turns per unit length $n = \frac{B}{\mu_0 I} = \frac{0.0270 \text{ T}}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(12.0 \text{ A})} = 1790 \text{ turns/m}.$

(b) $N = nL = (1790 \text{ turns/m})(0.400 \text{ m}) = 716 \text{ turns}.$

Each turn of radius R has a length $2\pi R$ of wire. The total length of wire required is

$$N(2\pi R) = (716)(2\pi)(1.40 \times 10^{-2} \text{ m}) = 63.0 \text{ m}.$$

EVALUATE: A large length of wire is required. Due to the length of wire the solenoid will have appreciable resistance.

28.48. IDENTIFY: Knowing the magnetic field at the center of the toroidal solenoid, we can find the current causing that field.

SET UP: $B = \frac{\mu_0 NI}{2\pi r}$. $r = 0.140 \text{ m}$ is the distance from the center of the torus to the point where B is to be calculated. This point must be between the inner and outer radii of the solenoid, but otherwise the field doesn't depend on those radii.

EXECUTE: Solving for N gives $N = \frac{2\pi r B}{\mu_0 I} = \frac{2\pi(0.140 \text{ m})(3.75 \times 10^{-3} \text{ T})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.50 \text{ A})} = 1750 \text{ turns}.$

EVALUATE: With an outer radius of 15 cm, the outer circumference of the toroid is about 100 cm, or about a meter. It is reasonable that the toroid could have 1750 turns spread over a circumference of one meter.

28.49. IDENTIFY and SET UP: Use the appropriate expression for the magnetic field produced by each current configuration.

EXECUTE: (a) $B = \frac{\mu_0 I}{2\pi r}$ so $I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(2.00 \times 10^{-2} \text{ m})(37.2 \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = 3.72 \times 10^6 \text{ A} = 3.72 \text{ MA}.$

$$(b) B = \frac{N\mu_0 I}{2R} \text{ so } I = \frac{2RB}{N\mu_0} = \frac{2(0.420 \text{ m})(37.2 \text{ T})}{(100)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 2.49 \times 10^5 \text{ A} = 249 \text{ kA}.$$

$$(c) B = \mu_0 \frac{N}{L} I \text{ so } I = \frac{BL}{\mu_0 N} = \frac{(37.2 \text{ T})(0.320 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(40,000)} = 237 \text{ A}.$$

EVALUATE: Much less current is needed for the solenoid, because of its large number of turns per unit length.

- 28.50. IDENTIFY:** Outside an ideal toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B = \frac{\mu_0 NI}{2\pi r}$.

SET UP: The torus extends from $r_1 = 15.0 \text{ cm}$ to $r_2 = 18.0 \text{ cm}$.

EXECUTE: (a) $r = 0.12 \text{ m}$, which is outside the torus, so $B = 0$.

$$(b) r = 0.16 \text{ m, so } B = \frac{\mu_0 NI}{2\pi r} = \frac{\mu_0(250)(8.50 \text{ A})}{2\pi(0.160 \text{ m})} = 2.66 \times 10^{-3} \text{ T}.$$

(c) $r = 0.20 \text{ m}$, which is outside the torus, so $B = 0$.

EVALUATE: The magnetic field inside the torus is proportional to $1/r$, so it varies somewhat over the cross-section of the torus.

- 28.51. IDENTIFY:** Inside an ideal toroidal solenoid, $B = \frac{\mu_0 NI}{2\pi r}$.

SET UP: $r = 0.070 \text{ m}$.

$$\text{EXECUTE: } B = \frac{\mu_0 NI}{2\pi r} = \frac{\mu_0(600)(0.650 \text{ A})}{2\pi(0.070 \text{ m})} = 1.11 \times 10^{-3} \text{ T}.$$

EVALUATE: If the radial thickness of the torus is small compared to its mean diameter, B is approximately uniform inside its windings.

- 28.52. IDENTIFY:** Use $B = \frac{\mu_0 NI}{2\pi r}$, with μ_0 replaced by $\mu = K_m \mu_0$, with $K_m = 80$.

SET UP: The contribution from atomic currents is the difference between B calculated with μ and B calculated with μ_0 .

$$\text{EXECUTE: (a) } B = \frac{\mu NI}{2\pi r} = \frac{K_m \mu_0 NI}{2\pi r} = \frac{\mu_0(80)(400)(0.25 \text{ A})}{2\pi(0.060 \text{ m})} = 0.0267 \text{ T}.$$

$$(b) \text{ The amount due to atomic currents is } B' = \frac{79}{80} B = \frac{79}{80}(0.0267 \text{ T}) = 0.0263 \text{ T}.$$

EVALUATE: The presence of the core greatly enhances the magnetic field produced by the solenoid.

- 28.53. IDENTIFY:** The magnetic field from the solenoid alone is $B_0 = \mu_0 nI$. The total magnetic field is

$$B = K_m B_0. \text{ } M \text{ is given by } \vec{B} = \vec{B}_0 + \mu_0 \vec{M}.$$

SET UP: $n = 6000 \text{ turns/m}$.

$$\text{EXECUTE: (a) (i) } B_0 = \mu_0 nI = \mu_0(6000 \text{ m}^{-1})(0.15 \text{ A}) = 1.13 \times 10^{-3} \text{ T}.$$

$$(ii) M = \frac{K_m - 1}{\mu_0} B_0 = \frac{5199}{\mu_0}(1.13 \times 10^{-3} \text{ T}) = 4.68 \times 10^6 \text{ A/m}.$$

$$(iii) B = K_m B_0 = (5200)(1.13 \times 10^{-3} \text{ T}) = 5.88 \text{ T}.$$

(b) The directions of \vec{B} , \vec{B}_0 and \vec{M} are shown in Figure 28.53. Silicon steel is paramagnetic and \vec{B}_0 and \vec{M} are in the same direction.

EVALUATE: The total magnetic field is much larger than the field due to the solenoid current alone.

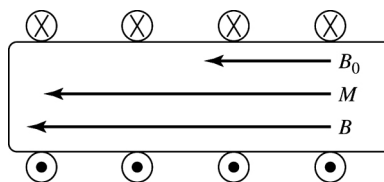


Figure 28.53

28.54. IDENTIFY: Apply $B = \frac{K_m \mu_0 NI}{2\pi r}$.

SET UP: K_m is the relative permeability and $\chi_m = K_m - 1$ is the magnetic susceptibility.

EXECUTE: (a) $K_m = \frac{2\pi r B}{\mu_0 NI} = \frac{2\pi(0.2500 \text{ m})(1.940 \text{ T})}{\mu_0(500)(2.400 \text{ A})} = 2021$.

(b) $\chi_m = K_m - 1 = 2020$.

EVALUATE: Without the magnetic material the magnetic field inside the windings would be $B/2021 = 9.6 \times 10^{-4} \text{ T}$. The presence of the magnetic material greatly enhances the magnetic field inside the windings.

28.55. IDENTIFY: Moving charges create magnetic fields. The net field is the vector sum of the two fields. A charge moving in an external magnetic field feels a force.

(a) **SET UP:** The magnitude of the magnetic field due to a moving charge is $B = \frac{\mu_0 |q| v \sin \phi}{4\pi r^2}$. Both fields

are into the paper, so their magnitudes add, giving $B_{\text{net}} = B + B' = \frac{\mu_0}{4\pi} \left(\frac{|q| v \sin \phi}{r^2} + \frac{|q'| v' \sin \phi'}{r'^2} \right)$.

EXECUTE: Substituting numbers gives

$$B_{\text{net}} = \frac{\mu_0}{4\pi} \left[\frac{(8.00 \mu\text{C})(9.00 \times 10^4 \text{ m/s}) \sin 90^\circ}{(0.300 \text{ m})^2} + \frac{(5.00 \mu\text{C})(6.50 \times 10^4 \text{ m/s}) \sin 90^\circ}{(0.400 \text{ m})^2} \right].$$

$B_{\text{net}} = 1.00 \times 10^{-6} \text{ T} = 1.00 \mu\text{T}$, into the paper.

(b) **SET UP:** The magnetic force on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$, and the magnetic field of charge q' at the location of charge q is into the page. The force on q is

$$\vec{F} = q\vec{v} \times \vec{B}' = (qv)\hat{i} \times \frac{\mu_0}{4\pi} \frac{q\vec{v}' \times \hat{r}}{r^2} = (qv)\hat{i} \times \left(\frac{\mu_0}{4\pi} \frac{qv' \sin \phi}{r^2} \right) (-\hat{k}) = \left(\frac{\mu_0}{4\pi} \frac{qq'vv' \sin \phi}{r^2} \right) \hat{j}$$

where ϕ is the angle between \vec{v}' and \hat{r} .

EXECUTE: Substituting numbers gives

$$\vec{F} = \frac{\mu_0}{4\pi} \left[\frac{(8.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})(9.00 \times 10^4 \text{ m/s})(6.50 \times 10^4 \text{ m/s}) \left(\frac{0.400}{0.500} \right)}{(0.500 \text{ m})^2} \right] \hat{j}.$$

$$\vec{F} = (7.49 \times 10^{-8} \text{ N}) \hat{j}.$$

EVALUATE: These are small fields and small forces, but if the charge has small mass, the force can affect its motion.

28.56. IDENTIFY: Charge q_1 creates a magnetic field due to its motion. This field exerts a magnetic force on q_2 , which is moving in that field.

SET UP: Find \vec{B}_1 , the field produced by q_1 at the location of q_2 . $\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{q_1 \vec{v}_1 \times \vec{r}_{1 \rightarrow 2}}{r_{1 \rightarrow 2}^3}$, since $\hat{r} = \vec{r}/r$.

EXECUTE: $\vec{r}_{1 \rightarrow 2} = (0.150 \text{ m})\hat{i} + (-0.250 \text{ m})\hat{j}$, so $r_{1 \rightarrow 2} = 0.2915 \text{ m}$.

$$\vec{v}_1 \times \vec{r}_{1 \rightarrow 2} = [(9.20 \times 10^5 \text{ m/s})\hat{i}] \times [(0.150 \text{ m})\hat{i} + (-0.250 \text{ m})\hat{j}] = (9.20 \times 10^5 \text{ m/s})(-0.250 \text{ m})\hat{k}.$$

$$\vec{B}_1 = (1.00 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(4.80 \times 10^{-6} \text{ C})(9.20 \times 10^5 \text{ m/s})(-0.250 \text{ m})}{(0.2915 \text{ m})^3} \hat{k} = -(4.457 \times 10^{-6} \text{ T})\hat{k}.$$

The force that \vec{B}_1 exerts on q_2 is

$$F_2 = q_2 \vec{v}_2 \times \vec{B}_1 = (-2.90 \times 10^{-6} \text{ C})(-5.30 \times 10^5 \text{ m/s})(-4.457 \times 10^{-6} \text{ T})\hat{j} \times \hat{k} = -(6.85 \times 10^{-6} \text{ N})\hat{i}.$$

EVALUATE: If we think of the moving charge q_1 as a current, we can use the right-hand rule for the direction of the magnetic field due to a current to find the direction of the magnetic field it creates in the vicinity of q_2 . Then we can use the cross product right-hand rule to find the direction of the force this field exerts on q_2 , which is in the $-x$ -direction, in agreement with our result.

28.57. IDENTIFY: Use $B = \frac{\mu_0 I}{2\pi r}$ and the right-hand rule to determine points where the fields of the two wires cancel.

(a) SET UP: The only place where the magnetic fields of the two wires are in opposite directions is between the wires, in the plane of the wires. Consider a point a distance x from the wire carrying $I_2 = 75.0 \text{ A}$. B_{tot} will be zero where $B_1 = B_2$.

$$\text{EXECUTE: } \frac{\mu_0 I_1}{2\pi(0.400 \text{ m} - x)} = \frac{\mu_0 I_2}{2\pi x}.$$

$$I_2(0.400 \text{ m} - x) = I_1 x; I_1 = 25.0 \text{ A}, I_2 = 75.0 \text{ A}.$$

$x = 0.300 \text{ m}$; $B_{\text{tot}} = 0$ along a line 0.300 m from the wire carrying 75.0 A and 0.100 m from the wire carrying current 25.0 A .

(b) SET UP: Let the wire with $I_1 = 25.0 \text{ A}$ be 0.400 m above the wire with $I_2 = 75.0 \text{ A}$. The magnetic fields of the two wires are in opposite directions in the plane of the wires and at points above both wires or below both wires. But to have $B_1 = B_2$ must be closer to wire #1 since $I_1 < I_2$, so can have $B_{\text{tot}} = 0$ only at points above both wires. Consider a point a distance x from the wire carrying $I_1 = 25.0 \text{ A}$. B_{tot} will be zero where $B_1 = B_2$.

$$\text{EXECUTE: } \frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(0.400 \text{ m} + x)}.$$

$$I_2 x = I_1(0.400 \text{ m} + x); x = 0.200 \text{ m}.$$

$B_{\text{tot}} = 0$ along a line 0.200 m from the wire carrying current 25.0 A and 0.600 m from the wire carrying current $I_2 = 75.0 \text{ A}$.

EVALUATE: For parts (a) and (b) the locations of zero field are in different regions. In each case the points of zero field are closer to the wire that has the smaller current.

28.58. IDENTIFY: The wire creates a magnetic field near it, and the moving electron feels a force due to this field.

SET UP: The magnetic field due to the wire is $B = \frac{\mu_0 I}{2\pi r}$, and the force on a moving charge is

$$F = |q|vB \sin \phi.$$

EXECUTE: $F = |q|vB \sin \phi = (ev\mu_0 I \sin \phi)/2\pi r$. Substituting numbers gives

$$F = (1.60 \times 10^{-19} \text{ C})(6.00 \times 10^4 \text{ m/s})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(8.60 \text{ A})(\sin 90^\circ)/[2\pi(0.0450 \text{ m})].$$

$F = 3.67 \times 10^{-19} \text{ N}$. From the right-hand rule for the cross product, the direction of $\vec{v} \times \vec{B}$ is opposite to the current, but since the electron is negative, the force is in the same direction as the current.

EVALUATE: This force is small at an everyday level, but it would give the electron an acceleration of over 10^{11} m/s^2 .

29.59. IDENTIFY: Find the force that the magnetic field of the wire exerts on the electron.

SET UP: The force on a moving charge has magnitude $F = |q|vB \sin \phi$ and direction given by the right-

hand rule. For a long straight wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of \vec{B} is given by the right-hand rule.

EXECUTE: (a) $a = \frac{F}{m} = \frac{|q|vB \sin \phi}{m} = \frac{ev}{m} \left(\frac{\mu_0 I}{2\pi r} \right)$. Substituting numbers gives

$$a = \frac{(1.6 \times 10^{-19} \text{ C})(2.50 \times 10^5 \text{ m/s})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(13.0 \text{ A})}{(9.11 \times 10^{-31} \text{ kg})(2\pi)(0.0200 \text{ m})} = 5.7 \times 10^{12} \text{ m/s}^2, \text{ away from the wire.}$$

(b) The electric force must balance the magnetic force. $eE = evB$, and

$$E = vB = v \frac{\mu_0 I}{2\pi r} = \frac{(250,000 \text{ m/s})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(13.0 \text{ A})}{2\pi(0.0200 \text{ m})} = 32.5 \text{ N/C.}$$

The magnetic force is directed away from the wire so the force from the electric field must be toward the wire. Since the charge of the electron is negative, the electric field must be directed away from the wire to produce a force in the desired direction.

EVALUATE: (c) $mg = (9.11 \times 10^{-31} \text{ kg})(9.8 \text{ m/s}^2) \approx 10^{-29} \text{ N}$.

$$F_{\text{el}} = eE = (1.6 \times 10^{-19} \text{ C})(32.5 \text{ N/C}) \approx 5 \times 10^{-18} \text{ N. } F_{\text{el}} \approx 5 \times 10^{11} F_{\text{grav}}, \text{ so we can neglect gravity.}$$

28.60. IDENTIFY: The current in the wire creates a magnetic field, and that field exerts a force on the moving electron.

SET UP: The magnetic field due to the current in the wire is $B = \frac{\mu_0 I}{2\pi r}$. The force the field exerts on the

electron is $\vec{F} = q\vec{v} \times \vec{B}$, where $q = -e$. The magnitude of a vector is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$. The electron is on the $+y$ -axis. The current is in the $-x$ -direction so, by the right-hand rule, the magnetic field it produces at

the location of the electron is in the $-z$ -direction, so $\vec{B} = -\frac{\mu_0 I}{2\pi r} \hat{k}$.

EXECUTE: The magnitude of the magnetic field is $B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0(9.00 \text{ A})}{2\pi(0.200 \text{ m})} = 9.00 \times 10^{-6} \text{ T}$, so

$$\vec{B} = -9.00 \times 10^{-6} \text{ T } \hat{k}. \text{ The force on the electron is } \vec{F} = q\vec{v} \times \vec{B}, \text{ so}$$

$$\vec{F} = q\vec{v} \times \vec{B} = -e(5.00 \times 10^4 \text{ m/s } \hat{i} - 3.00 \times 10^4 \text{ m/s } \hat{j}) \times (-9.00 \times 10^{-6} \text{ T } \hat{k}).$$

Taking out common factors gives $\vec{F} = (9 \times 10^{-2} e \text{ T} \cdot \text{m/s})(5\hat{i} - 3\hat{j}) \times \hat{k}$. Using the fact that $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$, we get $\vec{F} = (9 \times 10^{-2} e \text{ T} \cdot \text{m/s})(-5\hat{j} - 3\hat{i})$. Using $e = 1.60 \times 10^{-19} \text{ C}$ gives

$$\vec{F} = -4.32 \times 10^{-20} \text{ N } \hat{i} - 7.20 \times 10^{-20} \text{ N } \hat{j}.$$

The magnitude of this force is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(-4.32 \times 10^{-20} \text{ N})^2 + (-7.20 \times 10^{-20} \text{ N})^2} = 8.40 \times 10^{-20} \text{ N}.$$

EVALUATE: This is a small force on an everyday scale, but it would give the electron an acceleration of $a = F/m = (8.40 \times 10^{-20} \text{ N})/(9.11 \times 10^{-31} \text{ kg}) \approx 9 \times 10^{10} \text{ m/s}^2$.

28.61. IDENTIFY and SET UP: The power input of the motor is 65 hp. We know that 1 hp = 746 W. The relation between power, voltage, and current is $P = VI$. The attractive force between two parallel wires is

$$F = \frac{\mu_0 L I_1 I_2}{2\pi r}.$$

EXECUTE: (a) We find the current from $I = \frac{P}{V} = \frac{(65 \text{ hp})(746 \text{ W/hp})}{600 \text{ V}} = 80.8 \text{ A}$, which rounds to 81 A.

(b) The attractive force between the wires per unit length is

$$F/L = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(80.8 \text{ A})^2}{2\pi(0.55 \text{ m})} = 2.4 \times 10^{-3} \text{ N/m}.$$

EVALUATE: If the current from the cables is in the same direction, the force will be attractive; however, if the current runs in opposite directions the force will be repulsive.

28.62. IDENTIFY: Find the vector sum of the magnetic fields due to each wire.

SET UP: For a long straight wire $B = \frac{\mu_0 I}{2\pi r}$. The direction of \vec{B} is given by the right-hand rule and is perpendicular to the line from the wire to the point where the field is calculated.

EXECUTE: (a) The magnetic field vectors are shown in Figure 28.62a.

(b) At a position on the x -axis $B_{\text{net}} = 2 \frac{\mu_0 I}{2\pi r} \sin \theta = \frac{\mu_0 I}{\pi \sqrt{x^2 + a^2}} \frac{a}{\sqrt{x^2 + a^2}} = \frac{\mu_0 I a}{\pi (x^2 + a^2)}$, in the positive x -direction.

(c) The graph of B versus x/a is given in Figure 28.62b.

EVALUATE: (d) The magnetic field is a maximum at the origin, $x = 0$.

(e) When $x \gg a$, $B \approx \frac{\mu_0 I a}{\pi x^2}$.

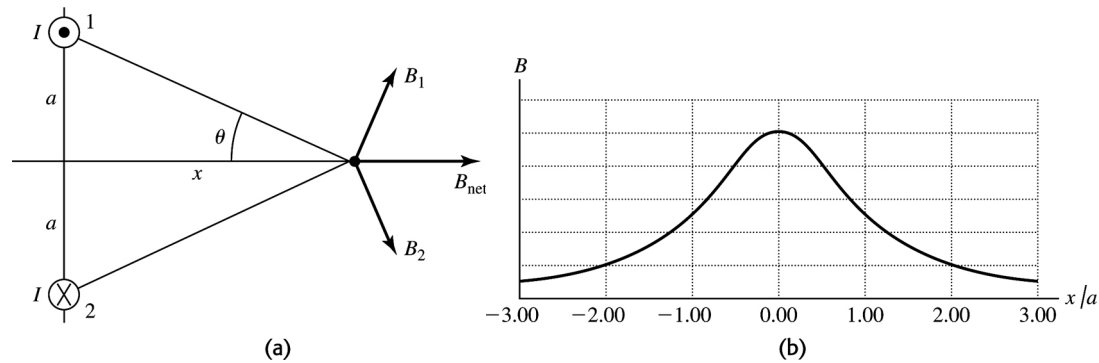
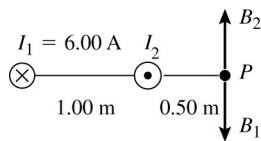


Figure 28.62

28.63. IDENTIFY: Use $B = \frac{\mu_0 I}{2\pi r}$ and the right-hand rule to calculate the magnitude and direction of the magnetic field at P produced by each wire. Add these two field vectors to find the net field.

(a) SET UP: The directions of the fields at point P due to the two wires are sketched in Figure 28.63a.



EXECUTE: \vec{B}_1 and \vec{B}_2 must be equal and opposite for the resultant field at P to be zero. \vec{B}_2 is to the upward so I_2 is out of the page.

Figure 28.63a

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi} \left(\frac{6.00 \text{ A}}{1.50 \text{ m}} \right) \quad B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi} \left(\frac{I_2}{0.50 \text{ m}} \right).$$

$$B_1 = B_2 \text{ says } \frac{\mu_0}{2\pi} \left(\frac{6.00 \text{ A}}{1.50 \text{ m}} \right) = \frac{\mu_0}{2\pi} \left(\frac{I_2}{0.50 \text{ m}} \right).$$

$$I_2 = \left(\frac{0.50 \text{ m}}{1.50 \text{ m}} \right) (6.00 \text{ A}) = 2.00 \text{ A}.$$

(b) SET UP: The directions of the fields at point Q are sketched in Figure 28.63b.

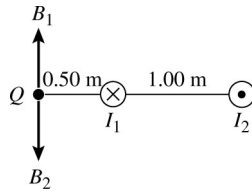


Figure 28.63b

$$\text{EXECUTE: } B_1 = \frac{\mu_0 I_1}{2\pi r_1}.$$

$$B_1 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{6.00 \text{ A}}{0.50 \text{ m}} \right) = 2.40 \times 10^{-6} \text{ T}.$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{2.00 \text{ A}}{1.50 \text{ m}} \right) = 2.67 \times 10^{-7} \text{ T}.$$

\vec{B}_1 and \vec{B}_2 are in opposite directions and $B_1 > B_2$ so

$$B = B_1 - B_2 = 2.40 \times 10^{-6} \text{ T} - 2.67 \times 10^{-7} \text{ T} = 2.13 \times 10^{-6} \text{ T}, \text{ and } \vec{B} \text{ is upward.}$$

(c) **SET UP:** The directions of the fields at point S are sketched in Figure 28.63c.

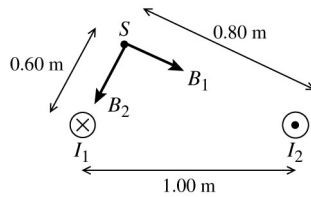


Figure 28.63c

$$\text{EXECUTE: } B_1 = \frac{\mu_0 I_1}{2\pi r_1}.$$

$$B_1 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{6.00 \text{ A}}{0.60 \text{ m}} \right) = 2.00 \times 10^{-6} \text{ T}.$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{2.00 \text{ A}}{0.80 \text{ m}} \right) = 5.00 \times 10^{-7} \text{ T}.$$

\vec{B}_1 and \vec{B}_2 are right angles to each other, so the magnitude of their resultant is given by

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2.00 \times 10^{-6} \text{ T})^2 + (5.00 \times 10^{-7} \text{ T})^2} = 2.06 \times 10^{-6} \text{ T}.$$

EVALUATE: The magnetic field lines for a long, straight wire are concentric circles with the wire at the center. The magnetic field at each point is tangent to the field line, so \vec{B} is perpendicular to the line from the wire to the point where the field is calculated.

28.64. IDENTIFY: Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

$$\text{EXECUTE: } F = F_t - F_b = \left(\frac{\mu_0 I_{\text{wire}}}{2\pi} \right) \left(\frac{Il}{r_t} - \frac{Il}{r_b} \right) = \frac{\mu_0 I I_{\text{wire}}}{2\pi} \left(\frac{1}{r_t} - \frac{1}{r_b} \right).$$

$$F = \frac{\mu_0 (5.00 \text{ A})(0.200 \text{ m})(14.0 \text{ A})}{2\pi} \left(-\frac{1}{0.100 \text{ m}} + \frac{1}{0.026 \text{ m}} \right) = 7.97 \times 10^{-5} \text{ N. The force on the top segment is}$$

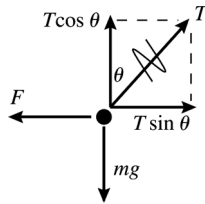
toward the wire, so the net force is toward the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform; it is stronger closer to the wire.

28.65. IDENTIFY: Apply $\sum \vec{F} = 0$ to one of the wires. The force one wire exerts on the other depends on I so

$\sum \vec{F} = 0$ gives two equations for the two unknowns T and I .

SET UP: The force diagram for one of the wires is given in Figure 28.65 (next page).



The force one wire exerts on the other is $F = \left(\frac{\mu_0 I^2}{2\pi r} \right) L$, where $r = 2(0.040 \text{ m}) \sin \theta = 8.362 \times 10^{-3} \text{ m}$ is the distance between the two wires.

Figure 28.65

EXECUTE: $\sum F_y = 0$ gives $T \cos \theta = mg$ and $T = mg / \cos \theta$.

$\sum F_x = 0$ gives $F = T \sin \theta = (mg / \cos \theta) \sin \theta = mg \tan \theta$.

And $m = \lambda L$, so $F = \lambda L g \tan \theta$.

$$\left(\frac{\mu_0 I^2}{2\pi r} \right) L = \lambda L g \tan \theta.$$

$$I = \sqrt{\frac{\lambda g r \tan \theta}{(\mu_0 / 2\pi)}}.$$

$$I = \sqrt{\frac{(0.0125 \text{ kg/m})(9.80 \text{ m/s}^2)(\tan 6.00^\circ)(8.362 \times 10^{-3} \text{ m})}{2 \times 10^{-7} \text{ T} \cdot \text{m/A}}} = 23.2 \text{ A}.$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

29.66. IDENTIFY: Apply $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P . The field at the center of a circular loop of radius R is $B = \frac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B = \frac{\mu_0 I}{4R}$.

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \frac{1}{2} \left(\frac{\mu_0 I}{2} \right) \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{\mu_0 I}{4a} \left(1 - \frac{a}{b} \right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.67. IDENTIFY: Find the vector sum of the fields due to each loop.

SET UP: For a single loop $B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring

the field along the x -axis from between them means that the “ x ” in the formula is different for each case.

EXECUTE: (a)

$$\text{Left coil: } x \rightarrow x + \frac{a}{2} \Rightarrow B_l = \frac{\mu_0 N I a^2}{2[(x + a/2)^2 + a^2]^{3/2}}.$$

$$\text{Right coil: } x \rightarrow x - \frac{a}{2} \Rightarrow B_r = \frac{\mu_0 N I a^2}{2[(x - a/2)^2 + a^2]^{3/2}}.$$

So, the total field at a point a distance x from the point between them is

$$B = \frac{\mu_0 N I a^2}{2} \left(\frac{1}{[(x + a/2)^2 + a^2]^{3/2}} + \frac{1}{[(x - a/2)^2 + a^2]^{3/2}} \right).$$

(b) B versus x is graphed in Figure 28.67. Figure 28.67a is the total field and Figure 28.67b is the field from the right-hand coil.

(c) At point P , $x = 0$ and $B = \frac{\mu_0 N I a^2}{2} \left(\frac{1}{[(a/2)^2 + a^2]^{3/2}} + \frac{1}{[(-a/2)^2 + a^2]^{3/2}} \right) = \frac{\mu_0 N I a^2}{(5a^2/4)^{3/2}} = \left(\frac{4}{5} \right)^{3/2} \frac{\mu_0 N I}{a}$.

$$(d) B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 NI}{a} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 (300)(6.00 \text{ A})}{(0.080 \text{ m})} = 0.0202 \text{ T}.$$

$$(e) \frac{dB}{dx} = \frac{\mu_0 NI a^2}{2} \left(\frac{-3(x+a/2)}{[(x+a/2)^2 + a^2]^{5/2}} + \frac{-3(x-a/2)}{[(x-a/2)^2 + a^2]^{5/2}} \right). \text{ At } x=0,$$

$$\left. \frac{dB}{dx} \right|_{x=0} = \frac{\mu_0 NI a^2}{2} \left(\frac{-3(a/2)}{[(a/2)^2 + a^2]^{5/2}} + \frac{-3(-a/2)}{[(-a/2)^2 + a^2]^{5/2}} \right) = 0.$$

$$\frac{d^2B}{dx^2} = \frac{\mu_0 NI a^2}{2} \left(\frac{-3}{[(x+a/2)^2 + a^2]^{5/2}} + \frac{6(x+a/2)^2(5/2)}{[(x+a/2)^2 + a^2]^{7/2}} + \frac{-3}{[(x-a/2)^2 + a^2]^{5/2}} + \frac{6(x-a/2)^2(5/2)}{[(x-a/2)^2 + a^2]^{7/2}} \right).$$

At $x=0$,

$$\left. \frac{d^2B}{dx^2} \right|_{x=0} = \frac{\mu_0 NI a^2}{2} \left(\frac{-3}{[(a/2)^2 + a^2]^{5/2}} + \frac{6(a/2)^2(5/2)}{[(a/2)^2 + a^2]^{7/2}} + \frac{-3}{[(a/2)^2 + a^2]^{5/2}} + \frac{6(-a/2)^2(5/2)}{[(a/2)^2 + a^2]^{7/2}} \right) = 0.$$

EVALUATE: Since both first and second derivatives are zero, the field can only be changing very slowly.

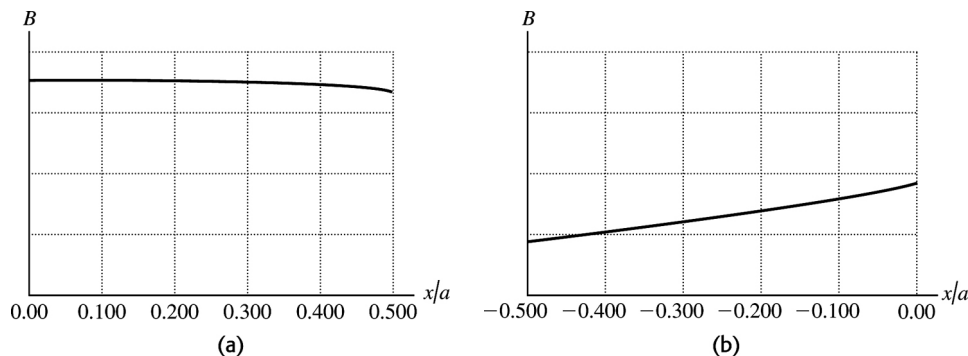


Figure 28.67

- 28.68. IDENTIFY:** Both arcs produce magnetic fields at point P perpendicular to the plane of the page. The field due to arc DA points into the page, and the field due to arc BC points out of the page. The field due to DA has a greater magnitude than the field due to arc BC . The net field is the sum of these two fields.

SET UP: The magnitude field at the center of a circular loop of radius a is $B = \frac{\mu_0 I}{2\pi a}$. Each arc is

$$120^\circ/360^\circ = 1/3 \text{ of a complete loop, so the field due to each of them is } B = \frac{1}{3} \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{6\pi a}.$$

EXECUTE: The net field is

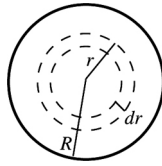
$$B_{\text{net}} = B_{20} - B_{30} = \frac{\mu_0 (12.0 \text{ A})}{6\pi} \left(\frac{1}{0.200 \text{ m}} - \frac{1}{0.300 \text{ m}} \right) = 4.19 \times 10^{-6} \text{ T} = 4.19 \mu\text{T}. \text{ Since } B_{20} > B_{30}, \text{ the net}$$

field points into the page at P .

EVALUATE: The current in segments CD and AB produces no magnetic field at P because its direction is directly toward (or away from) point P .

- 28.69. (a) IDENTIFY:** Consider current density J for a small concentric ring and integrate to find the total current in terms of α and R .

SET UP: We can't say $I = JA = J\pi R^2$, since J varies across the cross section.



To integrate J over the cross section of the wire, divide the wire cross section up into thin concentric rings of radius r and width dr , as shown in Figure 28.69.

Figure 28.69

EXECUTE: The area of such a ring is dA , and the current through it is $dI = J dA$; $dA = 2\pi r dr$ and $dI = J dA = \alpha r(2\pi r dr) = 2\pi\alpha r^2 dr$.

$$I = \int dI = 2\pi\alpha \int_0^R r^2 dr = 2\pi\alpha(R^3/3) \text{ so } \alpha = \frac{3I}{2\pi R^3}.$$

(b) IDENTIFY and SET UP: (i) $r \leq R$.

Apply Ampere's law to a circle of radius $r < R$. Use the method of part (a) to find the current enclosed by Ampere's law path.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B(2\pi r)$, by the symmetry and direction of \vec{B} . The current passing through the path is $I_{\text{encl}} = \int dI$, where the integration is from 0 to r .

$$I_{\text{encl}} = 2\pi\alpha \int_0^r r^2 dr = \frac{2\pi\alpha r^3}{3} = \frac{2\pi}{3} \left(\frac{3I}{2\pi R^3} \right) r^3 = \frac{Ir^3}{R^3}. \text{ Thus } \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \text{ gives}$$

$$B(2\pi r) = \mu_0 \left(\frac{Ir^3}{R^3} \right) \text{ and } B = \frac{\mu_0 Ir^2}{2\pi R^3}.$$

(ii) IDENTIFY and SET UP: $r \geq R$.

Apply Ampere's law to a circle of radius $r > R$.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B(2\pi r)$.

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi r) = \mu_0 I$ and $B = \frac{\mu_0 I}{2\pi r}$.

EVALUATE: Note that at $r = R$ the expression in (i) (for $r \leq R$) gives $B = \frac{\mu_0 I}{2\pi R}$. At $r = R$ the

expression in (ii) (for $r \geq R$) gives $B = \frac{\mu_0 I}{2\pi R}$, which is the same.

28.70. IDENTIFY: Apply $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$.

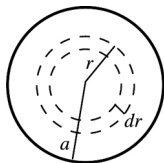
SET UP: The horizontal wire yields zero magnetic field since $d\vec{l} \times \hat{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2} \left(\frac{\mu_0 I}{2\pi a} \right) = \frac{\mu_0 I}{4\pi a}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq. (28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B .

28.71. IDENTIFY: Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross section. Then use Ampere's law to find \vec{B} at the specified distance from the center of the wire.

(a) SET UP:



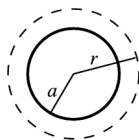
Divide the cross section of the cylinder into thin concentric rings of radius r and width dr , as shown in Figure 28.71a. The current through each ring is $dI = J dA = J 2\pi r dr$.

Figure 28.71a

EXECUTE: $dI = \frac{2I_0}{\pi a^2} [1 - (r/a)^2] 2\pi r dr = \frac{4I_0}{a^2} [1 - (r/a)^2] r dr$. The total current I is obtained by integrating

dI over the cross section $I = \int_0^a dI = \left(\frac{4I_0}{a^2} \right) \int_0^a (1 - r^2/a^2) r dr = \left(\frac{4I_0}{a^2} \right) \left[\frac{1}{2} r^2 - \frac{1}{4} r^4/a^2 \right]_0^a = I_0$, as was to be shown.

(b) SET UP: Apply Ampere's law to a path that is a circle of radius $r > a$, as shown in Figure 28.71b.



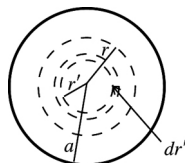
$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r).$$

$$I_{\text{encl}} = I_0 \text{ (the path encloses the entire cylinder).}$$

Figure 28.71b

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 I_0$ and $B = \frac{\mu_0 I_0}{2\pi r}$.

(c) SET UP:



Divide the cross section of the cylinder into concentric rings of radius r' and width dr' , as was done in part (a). See Figure 28.71c. The current

$$dI \text{ through each ring is } dI = \frac{4I_0}{a^2} \left[1 - \left(\frac{r'}{a} \right)^2 \right] r' dr'.$$

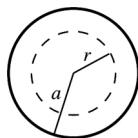
Figure 28.71c

EXECUTE: The current I is obtained by integrating dI from $r' = 0$ to $r' = r$:

$$I = \int dI = \frac{4I_0}{a^2} \int_0^r \left[1 - \left(\frac{r'}{a} \right)^2 \right] r' dr' = \frac{4I_0}{a^2} \left[\frac{1}{2} (r')^2 - \frac{1}{4} (r')^4/a^2 \right]_0^r.$$

$$I = \frac{4I_0}{a^2} (r^2/2 - r^4/4a^2) = \frac{I_0 r^2}{a^2} \left(2 - \frac{r^2}{a^2} \right).$$

(d) SET UP: Apply Ampere's law to a path that is a circle of radius $r < a$, as shown in Figure 28.71d.



$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r).$$

$$I_{\text{encl}} = \frac{I_0 r^2}{a^2} \left(2 - \frac{r^2}{a^2} \right) \text{ (from part (c)).}$$

Figure 28.71d

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 \frac{I_0 r^2}{a^2} (2 - r^2/a^2)$ and $B = \frac{\mu_0 I_0}{2\pi} \frac{r}{a^2} (2 - r^2/a^2)$.

EVALUATE: Result in part (b) evaluated at $r = a$: $B = \frac{\mu_0 I_0}{2\pi a}$. Result in part (d) evaluated at

$r = a$: $B = \frac{\mu_0 I_0}{2\pi} \frac{a}{a^2} (2 - a^2/a^2) = \frac{\mu_0 I_0}{2\pi a}$. The two results, one for $r > a$ and the other for $r < a$, agree at $r = a$.

28.72. IDENTIFY: The net field is the vector sum of the fields due to the circular loop and to the long straight wire.

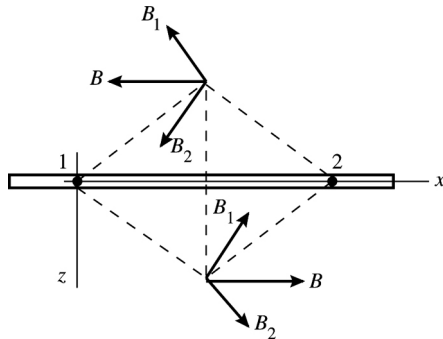
SET UP: For the long wire, $B = \frac{\mu_0 I_1}{2\pi D}$, and for the loop, $B = \frac{\mu_0 I_2}{2R}$.

EXECUTE: At the center of the circular loop the current I_2 generates a magnetic field that is into the page, so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude: $\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}$. Thus, $I_1 = \frac{\pi D}{R} I_2$.

EVALUATE: If I_1 is to the left the two fields add.

28.73. IDENTIFY: Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance a above and below the current sheet.

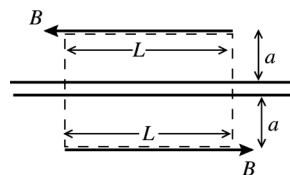
SET UP: Do parts (a) and (b) together.



Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where \vec{B} is being calculated. Figure 28.73a shows that for each pair the z -components cancel, and that above the sheet the field is in the $-x$ -direction and that below the sheet it is in the $+x$ -direction.

Figure 28.73a

Also, by symmetry the magnitude of \vec{B} a distance a above the sheet must equal the magnitude of \vec{B} a distance a below the sheet. Now that we have deduced the symmetry of \vec{B} , apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.73b.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

Figure 28.73b

I is directed out of the page, so for I to be positive the integral around the path is taken in the counterclockwise direction.

EXECUTE: Since \vec{B} is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B} \cdot d\vec{l} = 0$. On the long sides of length L , \vec{B} is parallel to the side, in the direction we are integrating around the path,

and has the same magnitude, B , on each side. Thus $\oint \vec{B} \cdot d\vec{l} = 2BL$. n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL$. Ampere's law then gives

$$2BL = \mu_0 InL \text{ and } B = \frac{1}{2} \mu_0 In.$$

EVALUATE: Note that B is independent of the distance a from the sheet. Compare this result to the electric field due to an infinite sheet of charge in Chapter 22.

28.74. IDENTIFY: Find the vector sum of the fields due to each sheet.

SET UP: Problem 28.73 shows that for an infinite sheet $B = \frac{1}{2} \mu_0 In$. If I is out of the page, \vec{B} is to the left

above the sheet and to the right below the sheet. If I is into the page, \vec{B} is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P , R and S are shown in Figure 28.74.

EXECUTE: (a) Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

(b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

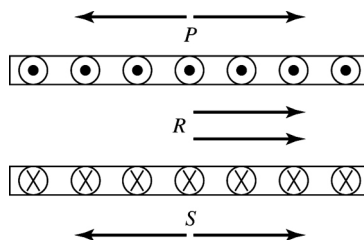


Figure 28.74

28.75. IDENTIFY: Apply Ampere's law to a circle of radius r .

SET UP: The current within a radius r is $I = \int \vec{J} \cdot d\vec{A}$, where the integration is over a disk of radius r .

EXECUTE: (a) $I_0 = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r} e^{(r-a)/\delta} \right) r dr d\theta = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b \delta e^{(r-a)/\delta} \Big|_0^a = 2\pi b \delta (1 - e^{-a/\delta})$.

$$I_0 = 2\pi(600 \text{ A/m})(0.025 \text{ m})(1 - e^{(0.050/0.025)}) = 81.5 \text{ A}.$$

(b) For $r \geq a$, $\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0$ and $B = \frac{\mu_0 I_0}{2\pi r}$.

(c) For $r \leq a$, $I(r) = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r'} e^{(r'-a)/\delta} \right) r' dr' d\theta = 2\pi b \int_0^r e^{(r'-a)/\delta} dr' = 2\pi b \delta e^{(r'-a)/\delta} \Big|_0^r$.

$$I(r) = 2\pi b \delta (e^{(r-a)/\delta} - e^{-a/\delta}) = 2\pi b \delta e^{-a/\delta} (e^{r/\delta} - 1) \text{ and } I(r) = I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}.$$

(d) For $r \leq a$, $\oint \vec{B} \cdot d\vec{l} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I(r)$ and $B = \frac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi r (e^{a/\delta} - 1)}$.

(e) At $r = \delta = 0.025 \text{ m}$, $B = \frac{\mu_0 I_0 (e - 1)}{2\pi \delta (e^{a/\delta} - 1)} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.025 \text{ m}) (e^{0.050/0.025} - 1)} = 1.75 \times 10^{-4} \text{ T}$.

At $r = a = 0.050 \text{ m}$, $B = \frac{\mu_0 I_0 (e^{a/\delta} - 1)}{2\pi a (e^{a/\delta} - 1)} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.050 \text{ m})} = 3.26 \times 10^{-4} \text{ T}$.

At $r = 2a = 0.100 \text{ m}$, $B = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.100 \text{ m})} = 1.63 \times 10^{-4} \text{ T}$.

EVALUATE: At points outside the cylinder, the magnetic field is the same as that due to a long wire running along the axis of the cylinder.

- 28.76. IDENTIFY and SET UP:** We assume that both solenoids are ideal, in which case the field due to each one is given by $B = \mu_0 nI = \mu_0 \frac{N}{L} I$. The net field inside is the sum of both the fields.

EXECUTE: (a) The net field is $B = \mu_0 \frac{N_1}{L} I_1 + \mu_0 \frac{N_2}{L} I_2 = \frac{\mu_0}{L} [N_1 I_1 + N_2 I_2]$. For the numbers in this

problem, we have $BL/\mu_0 = (0.00200 \text{ A})N_1 + N_2 I_2$. Therefore a graph of BL/μ_0 versus I_2 should be a straight line with slope equal to N_2 and y -intercept equal to $(0.00200 \text{ A})N_1$.

(b) Using the graph given with the problem, we calculate the slope using the points (5.00 mA, 16.00 A) and (2.00 mA, 8.00 A), which gives slope = $(16.00 \text{ A} - 8.00 \text{ A}) / (5.00 \text{ mA} - 2.00 \text{ mA}) = 2667$. Therefore $N_2 = 2667$ turns, which rounds to 2670 turns. To find the y -intercept, we use the point (5.00 mA, 16.00 A)

and the slope to deduce the equation of the line. This gives $\frac{y - 16.00 \text{ A}}{x - 0.00500 \text{ A}} = 2667$, which simplifies to

$y = 2667x + 2.67$. When $x = 0$, $y = 2.67 \text{ A}$. As we saw, the y -intercept is equal to $(0.00200 \text{ A})N_1$, so $N_1 = (2.67 \text{ A}) / (0.00200 \text{ A}) = 1335$ turns, which rounds to 1340 turns.

(c) Now the fields are in opposite directions, so $B = \mu_0 \frac{N_1}{L} I_1 - \mu_0 \frac{N_2}{L} I_2 = \frac{\mu_0}{L} [N_1 I_1 - N_2 I_2]$.

$B = [(\mu_0) / (0.400 \text{ m})][(0.00200 \text{ A})(1335) - (0.00500 \text{ A})(2667)] = -3.35 \times 10^{-5} \text{ T}$. The minus sign just tells us that the field due to I_2 is stronger than the field due to I_1 . So the magnitude of the net field is

$$B = 3.35 \times 10^{-5} \text{ T} = 33.5 \mu\text{T}.$$

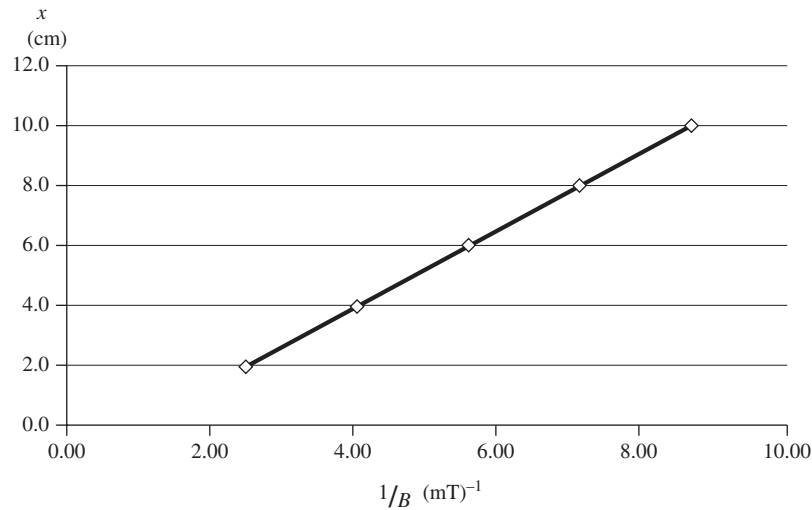
EVALUATE: As a check for N_1 in part (b), we could use a ruler to extrapolate the graph in the textbook back to its intersection with the y -axis to find the y -intercept. This method is not particularly accurate, but it should give reasonable agreement with the result for N_1 from part (b).

- 28.77. IDENTIFY and SET UP:** The magnitude of the magnetic a distance r from the center of a very long current-carrying wire is $B = \frac{\mu_0 I}{2\pi r}$. In this case, the measured quantity x is the distance from the *surface* of the cable, not from the center.

EXECUTE: (a) Multiplying the quantities given in the table in the problem, we get the following values for Bx in units of $\text{T} \cdot \text{cm}$, starting with the first pair: 0.812, 1.00, 1.09, 1.13, 1.16. As we can see, these values are not constant. However the last three values are nearly constant. Therefore Bx is not truly constant. The reason for this is that x is the distance from the *surface* of the cable, not from the center. In the formula $B = \frac{\mu_0 I}{2\pi r}$, r is the distance from the center of the cable. In that case, we would expect Br to be constant. For the last three points, it does appear that Bx is nearly constant. The reason for this is that the proper formula for the magnetic field for this cable is $B = \frac{\mu_0 I}{2\pi(R+x)}$, where R is the radius of the cable.

As x gets large compared to R , $r \approx x$ and the magnitude approaches $\frac{\mu_0 I}{2\pi r}$.

(b) Using the equation appropriate for the cable and solving for x gives $x = (\mu_0 I / 2\pi) \frac{1}{B} - R$. A graph of x versus $1/B$ should have a slope equal to $\mu_0 I / 2\pi$ and a y -intercept equal to $-R$. Figure 28.77 shows the graph of x versus $1/B$.

**Figure 28.77**

(c) The best-fit equation for this graph is $x = (1.2981 \text{ mT} \cdot \text{cm}) \frac{1}{B} - 1.1914 \text{ cm}$. The slope is

$1.2981 \text{ mT} \cdot \text{cm} = 1.2981 \times 10^{-5} \text{ T} \cdot \text{m}$. Since the slope is equal to $\mu_0 I / 2\pi$, we have

$\mu_0 I / 2\pi = \text{slope}$, which gives $I = 2\pi(\text{slope}) / \mu_0 = 2\pi(1.2981 \times 10^{-5} \text{ T} \cdot \text{m}) / \mu_0 = 64.9 \text{ A}$, which rounds to 65 A. The y-intercept is $-R$, so $R = -(-1.1914 \text{ cm}) = 1.2 \text{ cm}$.

EVALUATE: As we can see, the field within 2 cm or so of the surface of the cable would vary considerably from the value given by $B = \frac{\mu_0 I}{2\pi r}$.

- 28.78. IDENTIFY and SET UP:** The wires repel each other since they carry currents in opposite directions, so the wires will move away from each other until the magnetic force is just balanced by the force due to the spring. The force per unit length between two parallel current-carrying wires of equal length and separation r is $\frac{F}{L} = \frac{\mu_0}{2\pi} \frac{II}{r}$. In this case, the currents are the same and the distance between the wires is $l_0 + x$, where

x is the distance the spring stretches. Therefore the force is $F = \frac{\mu_0 I^2 L}{2\pi(l_0 + x)}$. The magnitude of the force

that each spring exerts is $F = kx$, by Hooke's law. On each wire, $F_{\text{spr}} = F_{\text{mag}}$, and there are *two* spring

forces on each wire. Therefore $\frac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx$.

EXECUTE: (a) We are given two cases with values for I and x , and each one leads to an equation involving l_0 and k . If we take the ratio of these two equations, common factors such as L will cancel. This gives us

$$\frac{(13.1 \text{ A})^2 (l_0 + 0.40 \text{ m})}{(8.05 \text{ A})^2 (l_0 + 0.80 \text{ m})} = \frac{0.80 \text{ cm}}{0.40 \text{ cm}} = 2.0. \text{ Solving for } l_0 \text{ gives } l_0 = 0.834 \text{ cm, which rounds to } 0.83 \text{ cm.}$$

Now we can solve for k using this value for l_0 using $\frac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx$.

$$\frac{\mu_0 (13.1 \text{ A})^2 (0.50 \text{ m})}{2\pi(0.0080 \text{ m} + 0.00834 \text{ m})} = 2k(0.0080 \text{ m}). \quad k = 0.0656 \text{ N/m, which rounds to } 0.066 \text{ N/m.}$$

(b) For a 12.0-A current, we have $\frac{\mu_0(12.0 \text{ A})^2(0.50 \text{ m})}{2\pi(x + 0.00834 \text{ m})} = 2(0.0656 \text{ N/m})x$. Carrying out the

multiplication and division and simplifying we get the quadratic equation

$$x^2 + (0.00834 \text{ m})x - 1.097 \times 10^{-4} \text{ m}^2 = 0.$$

Using the quadratic formula and taking the positive solution gives $x = 0.0071 \text{ m} = 0.71 \text{ cm}$.

(c) To stretch the spring by 1.00 cm, the current must satisfy the equation

$$\frac{\mu_0 I^2 (0.50 \text{ m})}{2\pi(0.0100 \text{ m} + 0.00834 \text{ m})} = 2(0.0656 \text{ N/m})(0.0100 \text{ m}). \text{ This gives } I = 15.5 \text{ A, which rounds to 16 A.}$$

EVALUATE: The spring force in part (c) is $kx = (0.0656 \text{ N/m})(0.0100 \text{ m}) = 6.56 \times 10^{-4} \text{ N}$. This is a very small force resulting from a rather large 16-A current. This tells us that magnetic forces between parallel wires, such as extension cords, are not very significant for typical household currents.

28.79. IDENTIFY: The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

SET UP: The magnetic force per unit length is $\frac{F}{L} = \frac{\mu_0}{2\pi} \frac{I^2}{d}$, and the acceleration obeys the equation

$$F/L = m/L a. \text{ The rms current over a short discharge time is } I_0/\sqrt{2}.$$

EXECUTE: (a) First get the force per unit length:

$$\frac{F}{L} = \frac{\mu_0}{2\pi} \frac{I^2}{d} = \frac{\mu_0}{2\pi d} \left(\frac{I_0}{\sqrt{2}} \right)^2 = \frac{\mu_0}{4\pi d} \left(\frac{V}{R} \right)^2 = \frac{\mu_0}{4\pi d} \left(\frac{Q_0}{RC} \right)^2.$$

Now apply Newton's second law using the result above: $\frac{F}{L} = \frac{m}{L} a = \lambda a = \frac{\mu_0}{4\pi d} \left(\frac{Q_0}{RC} \right)^2$. Solving for a gives

$$a = \frac{\mu_0 Q_0^2}{4\pi \lambda R^2 C^2 d}. \text{ From the kinematics equation } v_x = v_{0x} + a_x t, \text{ we have } v_0 = at = aRC = \frac{\mu_0 Q_0^2}{4\pi \lambda R C d}.$$

$$(b) \text{ Conservation of energy gives } \frac{1}{2} m v_0^2 = mgh \text{ and } h = \frac{v_0^2}{2g} = \frac{\left(\frac{\mu_0 Q_0^2}{4\pi \lambda R C d} \right)^2}{2g} = \frac{1}{2g} \left(\frac{\mu_0 Q_0^2}{4\pi \lambda R C d} \right)^2.$$

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

28.80. IDENTIFY: Approximate the moving belt as an infinite current sheet.

SET UP: Problem 28.73 shows that $B = \frac{1}{2} \mu_0 I n$ for an infinite current sheet. Let L be the width of the sheet, so $n = 1/L$.

EXECUTE: The amount of charge on a length Δx of the belt is $\Delta Q = L \Delta x \sigma$, so $I = \frac{\Delta Q}{\Delta t} = L \frac{\Delta x}{\Delta t} \sigma = L v \sigma$.

Approximating the belt as an infinite sheet $B = \frac{\mu_0 I}{2L} = \frac{\mu_0 v \sigma}{2}$. \vec{B} is directed out of the page, as shown in

Figure 28.80.

EVALUATE: The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.

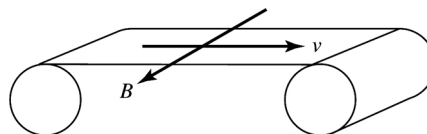


Figure 28.80

28.81. IDENTIFY and SET UP: This solenoid is not ideal since its width is fairly large compared to its length. But we can get a rough estimate using the ideal formula, $B = \mu_0 nI$.

EXECUTE: $B = \mu_0 nI = \mu_0 (1000 \text{ m}^{-1})I = 150 \times 10^{-6} \text{ T}$, which gives $I = 0.12 \text{ A}$, choice (b).

EVALUATE: This is a reasonable laboratory current of 120 mA.

28.82. IDENTIFY and SET UP: The magnetic field of an ideal solenoid is $B = \mu_0 nI$.

EXECUTE: Both solenoids have the same current, the same length, and the same number of turns, so the magnetic field inside both of them should be the same, which is choice (c).

EVALUATE: This answer is somewhat of an approximation. Even though both solenoids have the same current and same length and number of turns, the second (larger) solenoid is even farther from the ideal case than the first one. Therefore there would be some difference in the magnetic fields inside.

28.83. IDENTIFY and SET UP: The enclosure is no longer present to shield the solenoid from the earth's magnetic field of $50 \mu\text{T}$, so net field inside is a sum of the solenoid field and the earth's field. Whether the earth's field adds or subtracts from the solenoid's field depends on the orientation of the solenoid. The magnetic field due to the solenoid is $150 \mu\text{T}$.

EXECUTE: When the solenoid field is parallel to the earth's field, the net field is $150 \mu\text{T} + 50 \mu\text{T} = 200 \mu\text{T}$. When the field's are antiparallel (opposite), the net field is $150 \mu\text{T} - 50 \mu\text{T} = 100 \mu\text{T}$. So the field that the bacteria experience is between $100 \mu\text{T}$ and $200 \mu\text{T}$, which is choice (c).

EVALUATE: Since the earth's field is quite appreciable compared to the solenoid's field, it is important to shield the solenoid from external fields, such as that of the earth. The earth's field can make a difference of up to a factor of 2 in the field experienced by the bacteria.