UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Theoretical Physics
JF Mathematics
SF Two Subject Moderatorship

Trinity Term 2014

MA1242 — CLASSICAL MECHANICS II

Thursday, May 15 LUCE UPPER 14:00 — 16:00

Dr. S. Kovacs

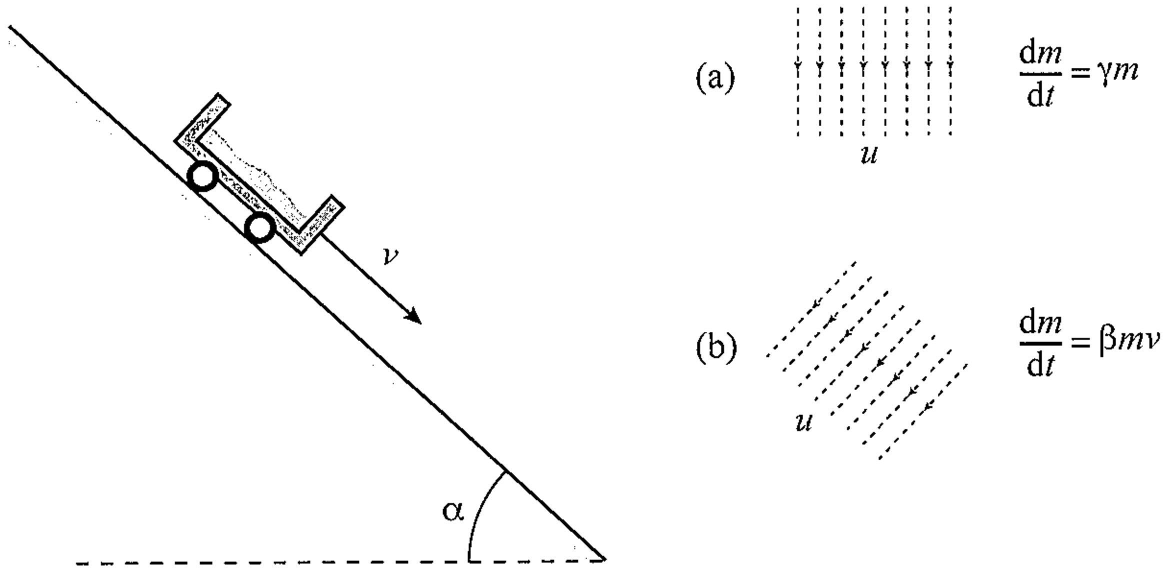
Credit will be given for the best 2 answers.

All questions have equal weight.

'Formulae & tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. A cart, initially at rest, rolls down a plane inclined at an angle α without friction. Rain falls with constant speed u and water gets collected in the cart.



Compute the velocity of the cart as a function of time in the following two cases.

(a) The rain falls vertically and gets collected in the cart at a rate $\mathrm{d}m/\mathrm{d}t = \gamma m$, where γ is a positive constant and m is the total mass of the cart at time t.

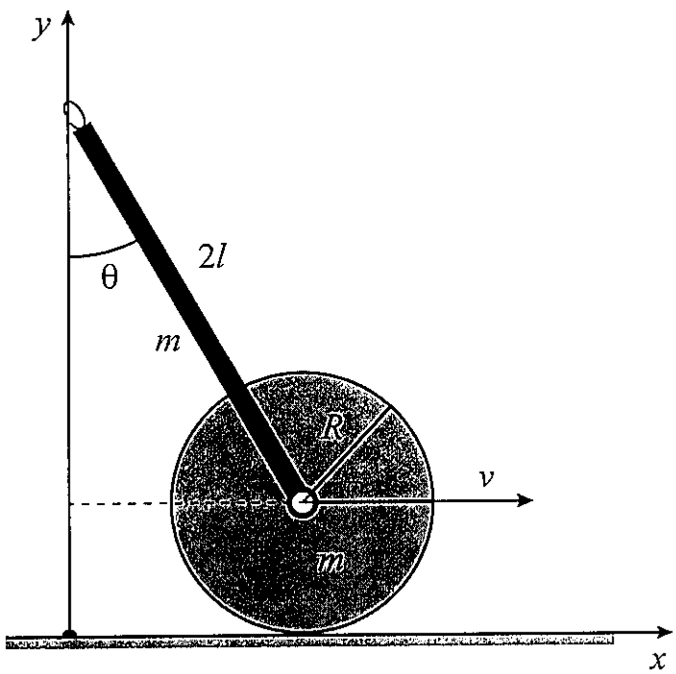
(b) The rain falls perpendicularly to the plane and gets collected in the cart at a rate $\mathrm{d}m/\mathrm{d}t = \beta m v$, where β is a positive constant and m is the mass of the cart at time t.

Useful formulae:

$$\int \frac{1}{a - bx}\,\mathrm{d}x = -\frac{1}{b}\log(a - bx)\,,$$

$$\int \frac{\mathrm{d}x}{a - bx^2} = \frac{1}{\sqrt{ab}}\tanh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right).$$

2. The system in the figure below consists of a rod of mass m and length $2l$ with a disk of radius R and mass m (equal to that of the rod) mounted at one end point. The disk can rotate without friction about its axis and it rolls without slipping on a horizontal plane. A small ring constrains the other end point of the rod to move along a fixed vertical wire. The small ring slides without friction on the wire. The rod is initially in a vertical position and a small perturbation causes it to fall under gravity, pushing the disk along the plane.



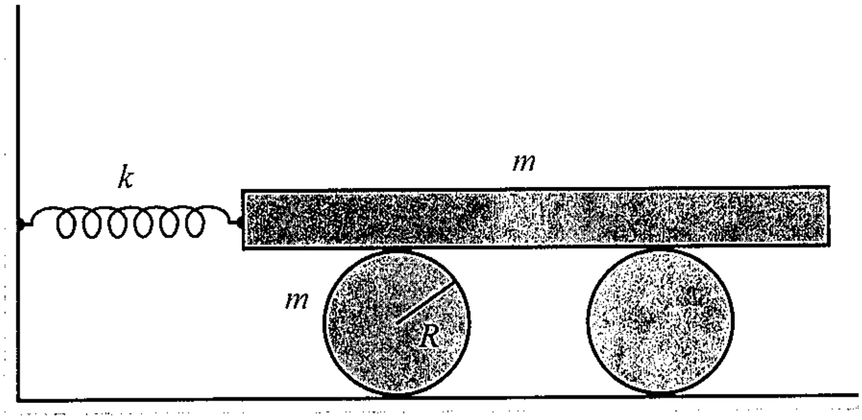
(a) Use the constraints described above to express the positions and velocities of the centres of mass of rod and disk and the angular velocity of the disk about its axis in terms of the angle θ in the figure and its derivative, $\dot{\theta}$.

(b) Use energy conservation to compute the velocity of the centre of mass of the disk as a function of the angle θ.

Useful formulae:

The moment of inertia of a disk of mass m and radius R with respect to a perpendicular axis through its centre is $I_{\mathrm{d}} = \frac{1}{2}mR^2$. The moment of inertia of a rod of mass m and length d with respect to a perpendicular axis through its midpoint is $I_{\mathrm{r}} = \frac{1}{12}md^2$.

3. A board of mass m can roll without slipping on two identical cylinders of mass m and radius R. The cylinders lie on a horizontal plane on which they also roll without slipping. The board is connected to a wall by a horizontal spring of spring constant k.



The board is pulled away from the wall and released.

(a) Find the constraints relating the acceleration of the board, the acceleration of the centres of mass of the two cylinders and their angular acceleration.

(b) Compute the period of the oscillations of the board. (Assume that the amplitude of the oscillations is such that the board does not hit the wall or fall off the cylinders.)

Useful formulae:

The moment of inertia of a cylinder of mass m and radius R about its axis is $I = \frac{1}{2}mR^2$.

Hint:

For the cylinders to roll without slipping and the board to not slip on the cylinders, there must be friction forces at the contact points. These are to be treated as unknowns. At the contact points between board and cylinders the friction forces exerted by the board on the cylinders and by the cylinders on the board satisfy Newton's third law.