Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Maths/TP/TSM — Trinity Term 2018

MA1212 — Linear Algebra II

Monday, May 21 — Sports Centre — 14:00 — 16:00

Paschalis Karageorgis

Instructions to Candidates:

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.
Formulae and tables are available from the invigilators, if needed.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Let $x_0 = 1$ and $y_0 = 4$. Suppose the sequences x_n, y_n are such that

$$x_n = 9x_{n-1} - 2y_{n-1}, \qquad y_n = 4x_{n-1} + 3y_{n-1}$$

for each integer $n \geq 1$. Determine each of x_n and y_n explicitly in terms of n.

2. Find the Jordan form and a Jordan basis for the matrix

$$A = \begin{bmatrix} -3 & 1 & 7 \\ -4 & 2 & 3 \\ -1 & 1 & 5 \end{bmatrix}.$$

3. The following matrix has eigenvalues $\lambda = 0, 1, 1$. Use this fact to find its Jordan form, its minimal polynomial and also its power A^{2018}.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & -1 \\ 2 & 2 & -1 \end{bmatrix}.$$

4. Define a bilinear form on the space M_{22} of all 2×2 real matrices by setting

$$\langle A, B \rangle = \mathrm{tr}(A^t B) \text{ for all } 2 \times 2 \text{ real matrices } A, B.$$

Express this equation in terms of the entries of A, B. Is the form positive definite?

5. Let $v \in \mathbb{R}^n$ be a given vector and let I_n denote the $n \times n$ identity matrix. Show that the matrix $A = I_n + vv^t$ is positive definite symmetric and determine its inverse.

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