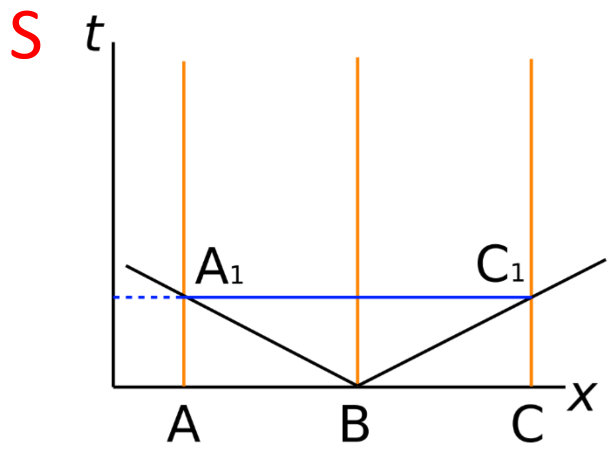
JF PY1T10: Special Relativity

Lecture 4:

- Continuing with Lorentz – Einstein Transformations

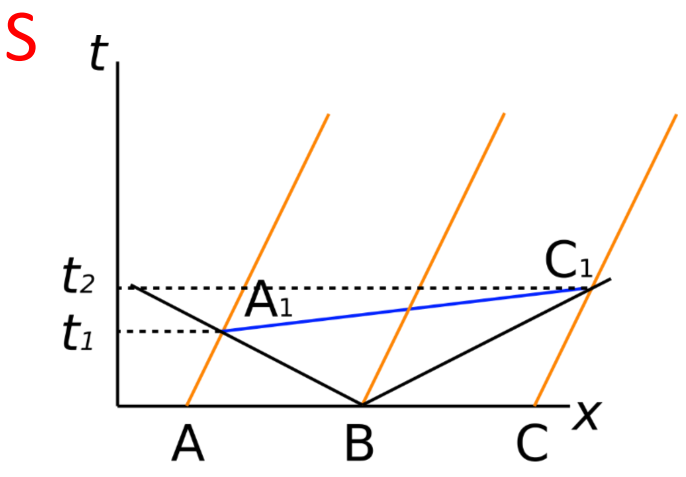
Summary of Lecture 3

Relativity of Simultaneity:



A, B, C at rest:

Light reaches A and C simultaneously, according to observer at rest in *S*.



A, B, C moving with velocity, v:

Light reaches A first (at t_1) and then C (at t_2), according to observer at rest in *S*.

But:

(i) All inertial frames are equivalent.

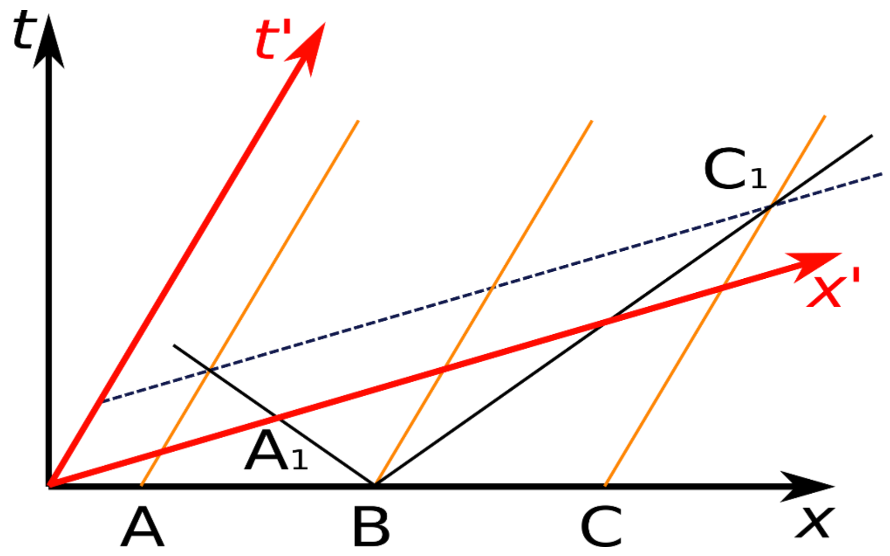
(ii) *c* is universal.

For an observer at rest w.r.t. A, B and C (i.e. in S') light reaches A and C simultaneously.

Simultaneity is relative, not absolute

Summary of Lecture 3

Derivation of Lorentz-Einstein Transformations:



Draw in axes of S':

t': This is the *x'*=0 line. (Remember S' has a velocity v w.r.t. S.)

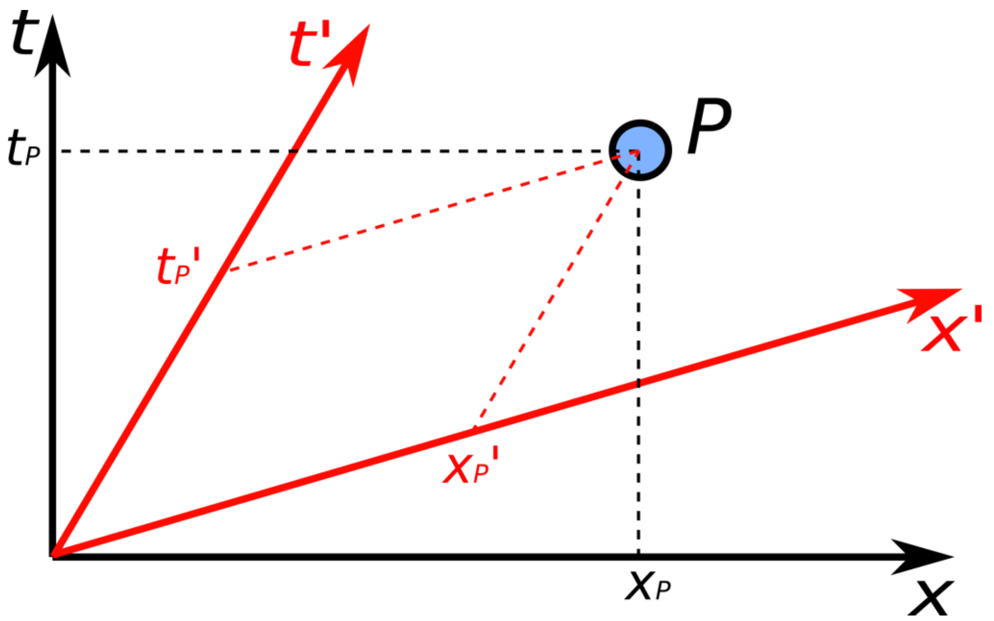
x': This is a line that connects all points corresponding to t'=0. It will be parallel to a line connecting all points corresponding to t'=constant.

The line A_1' and C_1' is such a line (as these events are simultaneous in S').

So draw the *x'* axis parallel to A_1' C_1' and going through the origin.

Summary of Lecture 3

Derivation of Lorentz-Einstein Transformations:



P = point event.
P can be characterised by points $\{x, t\}$ or $\{x', t'\}$.
x and *t* should be linear functions of x' and t'.
x' and t' should be linear functions of x and t.
(a non-linear transformation would predict acceleration in one system, even if the velocity were constant in the other. This would contradict the definition of inertial frames.)

Take the most general transformation relating the coordination of event **P**:

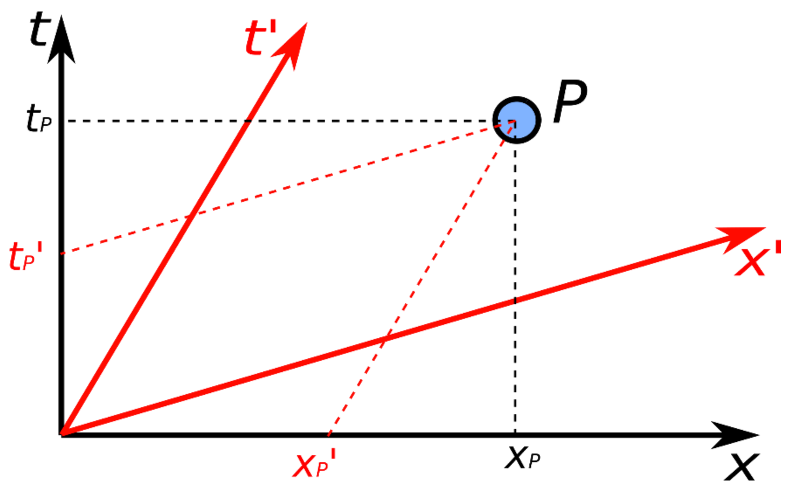
$$x = a\, x' + b\, t'.$$

$$x' = a\, x \; - b\, t$$

Remember: *S'* is moving along the positive *x*-axis wr.t. *S.*

Summary of Lecture 3

Derivation of Lorentz-Einstein Transformations:



Step 1: Consider the motion of the origin in both reference frames, i.e., put $x = 0$ and $x' = 0$ in these equations.

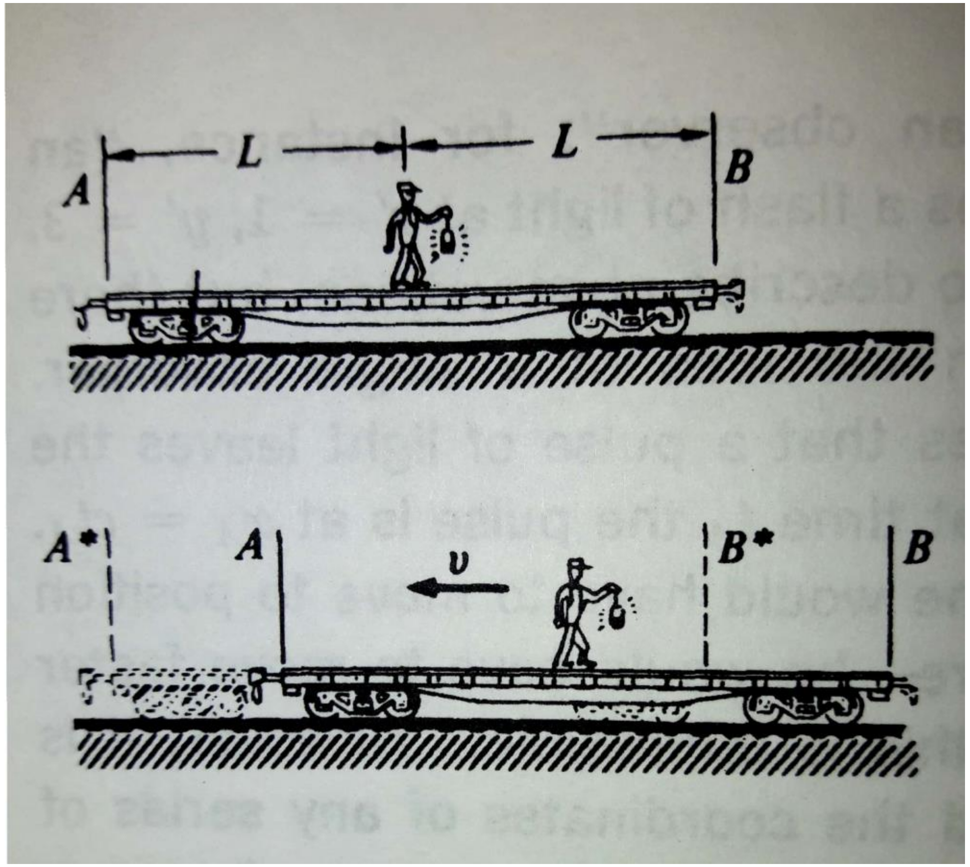
Step 2: Consider a light signal originating at the origin and travelling in the positive x-direction:
It can be described by:
$x = ct$ and $x' = ct$ (as the speed of light is constant).

After some elementary algebra (see last lecture):

Lorentz Transformations

$x' = \gamma(x - vt)$	$y' = y$	$z' = z$	$t' = \gamma\left(t - \frac{vx}{c^2}\right)$
$x = \gamma(x' + vt')$	$y = y'$	$z = z'$	$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

Relativity of Simultaneity



Consider a railwayman standing at the middle of a freight car of length $2L$. He flicks on his lantern and a light pulse travels out in all directions with velocity c.

Light arrives at the two ends of the car after a time interval L/c. In this system, the freight car's rest system, the light arrives simultaneously at **A** and **B**.

Now consider a second frame, moving to the right with velocity *v*. (In this frame the freight car is moving to the left with velocity *v)*.

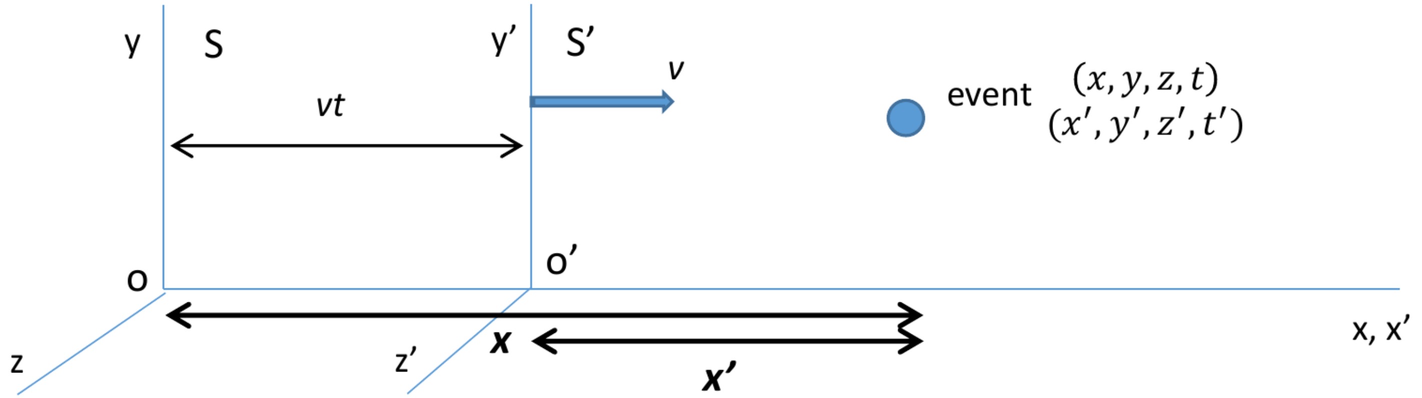
Light still has a velocity c, according to the second postulate of relativity. However, during the transit time, **A** moves to **A*** and **B** moves to **B***. The pulse therefore arrives at **B*** before **A***; the events are not simultaneous in this frame.

Problem Solving with the Lorentz Transformations

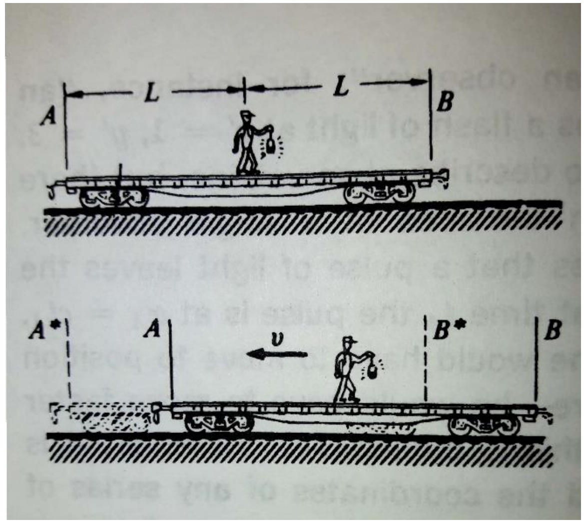
- Identify the target variable(s).
- Define the two inertial frames *S* and *S′*. Remember that *S′* moves relative to *S* at a constant velocity *v* in the +*x*-direction.
- Make a list of all the space-time coordinates in the two frames, e.g.:

$$
\begin{aligned}
x_1 &= \_\_\_\_\_ \\
t_1 &= \_\_\_\_\_ \\
x_1' &= \_\_\_\_\_ \\
t_1' &= \_\_\_\_\_
\end{aligned}
$$

- Fill in the values you know. Use the Lorentz Transformations to solve for the ones you don't.



Relativity of Simultaneity



What is the time of arrive of the light pulse at each end of the freight car?

Rest frame: Trivial.

Take the origin of coordinates at the centre of the car. Set $t = 0$ at the instant the lantern flashes.

$$x_A = -L \qquad x_B = L$$

$$t_A = L/c = T \qquad t_B = L/c = T$$

Moving frame:
Use the Lorentz Transformations:

$$t_A{}' = \gamma\left(t_A - \frac{vx_A}{c^2}\right) \qquad t_B{}' = \gamma\left(t_B - \frac{vx_B}{c^2}\right)$$

$$t_A{}' = \gamma\left(T - \frac{vL}{c^2}\right) = \frac{1}{\sqrt{1 - {v^2}/{c^2}}}\left(T + \frac{v}{c}T\right) = T\sqrt{\frac{1 + {v}/{c}}{1 - {v}/{c}}}$$

$$t_B{}' = \gamma\left(T - \frac{vL}{c^2}\right) = \frac{1}{\sqrt{1 - {v^2}/{c^2}}}\left(T - \frac{v}{c}T\right) = T\sqrt{\frac{1 - {v}/{c}}{1 + {v}/{c}}}$$

$t_B{}' < t_A{}'$. The pulse arrives at **B** earlier than it arrives at **A** when measured from the moving frame.

Concept Questions

on Simultaneity

Concept Question 1

As a high-speed spaceship flies past you at $0.5c$, it fires a strobe lamp. As measured by an observer on board the spaceship, a spherical wave front of light spreads away from the strobe lamp with the same speed c in all directions. The wave front that *you* measure

A. is spherical and remains centered on the spaceship as it moves.

B. is spherical and is centered on the point where the spaceship was when the strobe was fired.

C. is not spherical but remains centered on the spaceship as it moves.

D. is not spherical but is centered on the point where the spaceship was when the strobe was fired.

E. is none of the above.

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D. is not spherical but is centered on the point where the spaceship was when the strobe was fired.

E. is none of the above.

Concept Question 2

As a high-speed spaceship flies past you at $0.5c$, a strobe lamp fires at the center of a room aboard the spaceship. As measured by you, the light from the strobe:

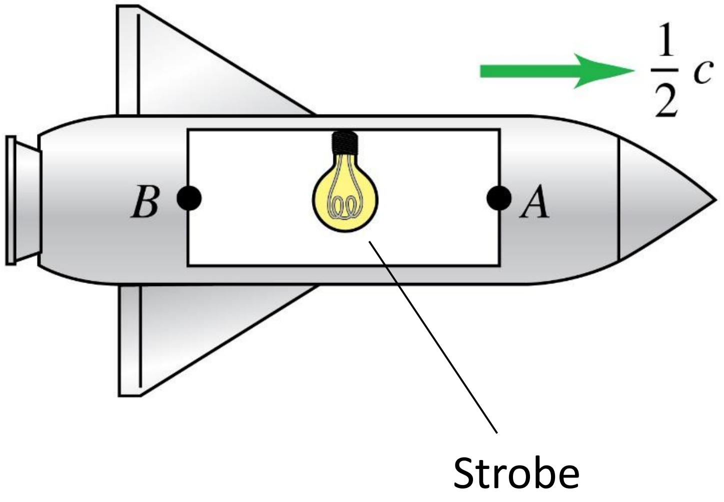
A. reaches point *A* before it reaches point *B.*

B. reaches point *B* before it reaches point *A.*

C. reaches points *A* and *B* simultaneously.

D. never reaches point *A*.

E. Not enough information is given to decide.



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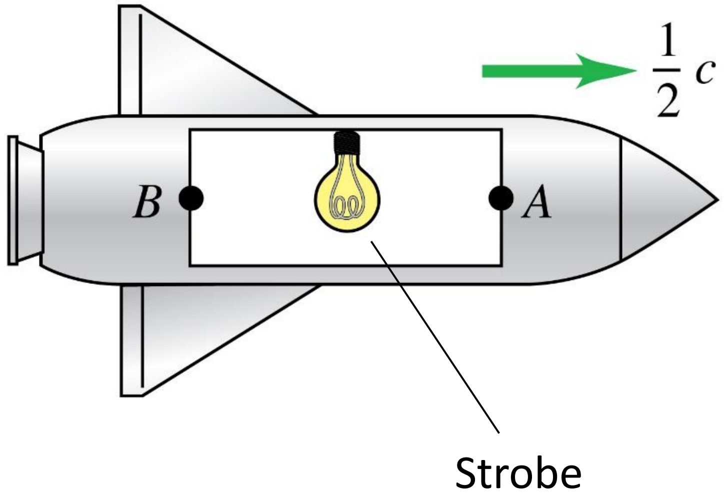

Concept Question 3

Beth is sitting at the midpoint of a moving train. Alan is at rest on the platform. Just as Beth passes Alan, he observes two sparks flash at the two end points of the train at the same time. The sparks emit light which radiates outwards in two wavefronts (W_f and W_b).

In Alan's frame, does the wavefront from the front of the train, W_f, hit him:

a. at the same time as W_b

b. before W_b

c. after W_b

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In Alan's frame, does the wavefront from the front of the train, W_f, hit him:

a. at the same time as W_b
b. before W_b
c. after W_b

Concept Question 3

In Alan's frame, does W_f reach Beth:

a. before
b. after
c. at the same instant

as W_b?

Concept Question 3

In Alan's frame, does W_f reach Beth:

a. before

b. after

c. at the same instant

as W_b?

A music device sits at Beth's feet. It works as follows:

When a wavefront hits this device, it starts to play music. When a second wavefront hits the device, the music stops. If both wavefronts hit the device at the same time, it remains silent.

Concept Question 3

Does the music play:

a. In Beth’s frame?

b. In Alan’s frame?

c. Both?

Concept Question 3

Does the music play:

a. In Beth's frame?
b. In Alan's frame?
c. Both?

In Beth's frame, does W_f reach Beth:

a. before
b. after
c. at the same instant

as W_b?

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Does the music play:

a. In Beth's frame?

b. In Alan's frame?

c. Both?

In Beth's frame, does W_f reach Beth:

a. before

b. after

c. at the same instant

as W_b?

In Beth's frame, do the sparks occur

a. At the same instant

b. The spark at the front of the train happens first

c. The spark at the front of the train happens second

Concept Question 3

Does the music play:

a. In Beth's frame?
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a. At the same instant
b. The spark at the front of the train happens first
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Concept Question 4

A high-speed train is traveling at a constant velocity along a straight-line track from Galway to Dublin. According to an observer at rest on the ground, the clocks at the railroad stations in Galway and Dublin both strike noon at the same time. According to a passenger on the train, when the Galway clock strikes noon, what time is it in Dublin?

A. noon

B. before noon

C. after noon

D. The answer depends on the speed of the train.

E. The answer depends on the speed of the train and the distance between Galway and Dublin.

Concept Question 4

A high-speed train is traveling at a constant velocity along a straight-line track from Galway to Dublin. According to an observer at rest on the ground, the clocks at the railroad stations in Galway and Dublin both strike noon at the same time. According to a passenger on the train, when the Galway clock strikes noon, what time is it in Dublin?

A. noon

B. before noon

C. after noon

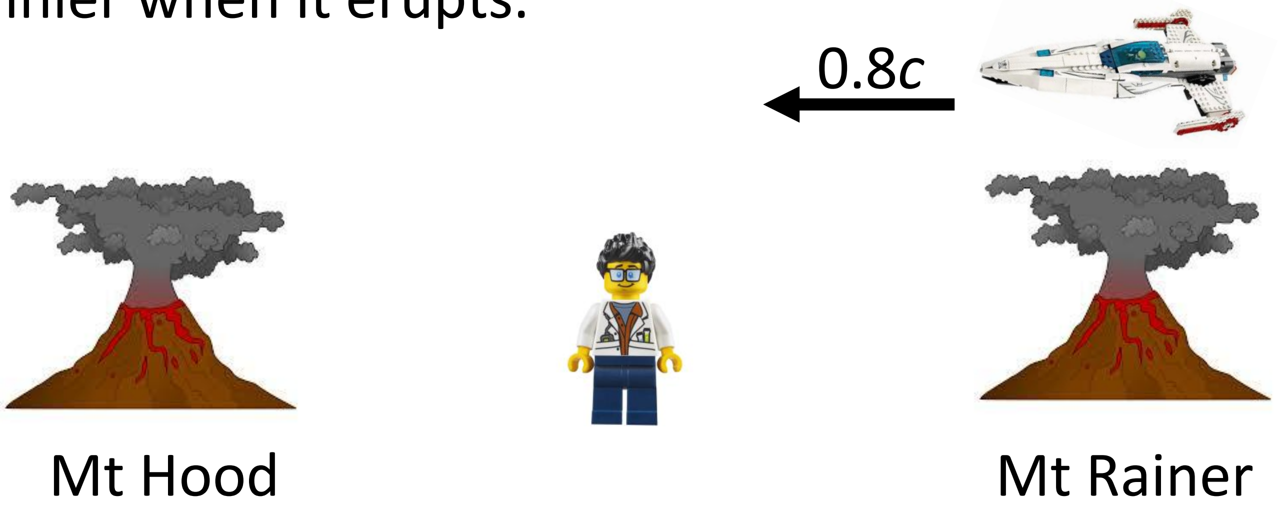
D. The answer depends on the speed of the train.

E. The answer depends on the speed of the train and the distance between Galway and Dublin.

Concept Question 5a

Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes.

A fast spacecraft flying with constant speed v = 0.8*c* from Rainier towards Hood is directly over Mt. Rainier when it erupts.



The seismologist and the observer in the spacecraft are intelligent observers, i.e., they correct for signal travel time to determine the time of events in their reference frame. Each observer has clocks which are synchronised with all other observers in their reference frame.

Concept Question 5a

Which of the following statements regarding the timing of the eruptions is correct?

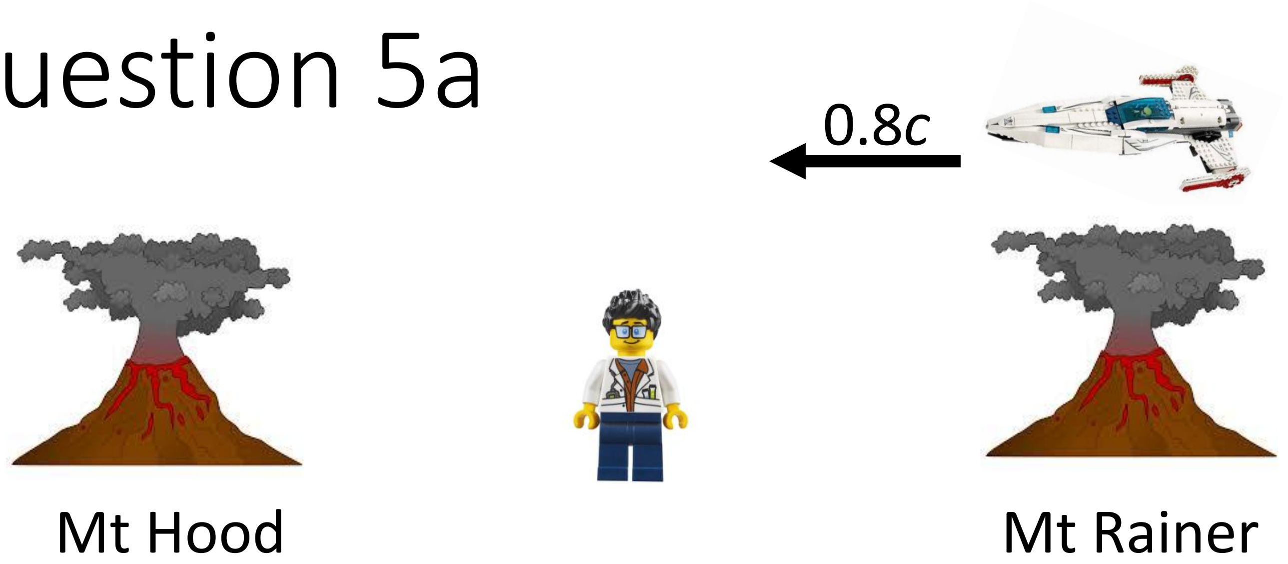
A. In both the seismologist’s and the spacecraft’s reference frame both volcanos erupt simultaneously.

B. In the seismologist’s reference frame the eruptions are simultaneous, while in the spacecraft’s reference frame Mt Rainer erupts before Mt Hood.

C. In the seismologist’s reference frame the eruptions are simultaneous, while in the spacecraft’s reference frame Mt Hood erupts before Mt Rainier.

D. In the spacecraft’s reference frame the eruptions are simultaneous, while in the seismologist's reference frame Mt Hood erupts before Mt Rainier.

Concept Question 5a



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C. In the seismologist's reference frame the eruptions are simultaneous, while in the spacecraft's reference frame Mt Hood erupts before Mt Rainier.

D. In the spacecraft's reference frame the eruptions are simultaneous, while in the seismologist's reference frame Mt Hood erupts before Mt Rainier.

Concept Question 5b

Mt Hood Mt Rainer

Again, the seismologist is standing, at rest, midway between the two volcanos. He observes them to suddenly erupt at the same time.

Now assume the seismologist has an assistant located near Mt Hood. In the reference frame of the assistant, which of the following statements regarding the timing of the eruptions is correct?

A. Mt Hood erupts before Mt Rainier.

B. Mt Rainer erupts before Mt Hood.

C. Both eruptions occur simultaneously.

Concept Question 5b

Again, the seismologist is standing, at rest, midway between the two volcanos. He observes them to suddenly erupt at the same time.

Now assume the seismologist has an assistant located near Mt Hood. In the reference frame of the assistant, which of the following statements regarding the timing of the eruptions is correct?

A. Mt Hood erupts before Mt Rainier.

B. Mt Rainer erupts before Mt Hood.

C. Both eruptions occur simultaneously.

Concept Question 6

Jim observes a large spacecraft travelling at a speed of $0.7c$ away from him. He observes (taking into account signal delay) that the tail light and nose light of the craft flash once and simultaneously in his reference frame.

$0.7\ c$

Jim

Which of the following statements about observations made in the reference frame of the spacecraft is/are true?

A. There is no way to be sure what is happening on the spacecraft.
B. The nose light flashes before the tail light.
C. The lights flash simultaneously.
D. The tail light flashes before the nose light

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