Hearing and Seeing; Tutorial 1 – Answers

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Quick Quiz 16.1 In a long line of people waiting to buy tickets, the first person leaves and a pulse of motion occurs as people step forward to fill the gap. As each person steps forward, the gap moves through the line. Is the propagation of this gap (a) transverse (b) longitudinal?

Quick Quiz 16.2 Consider the "wave" at a baseball game: people stand up and shout as the wave arrives at their location, and the resultant pulse moves around the stadium. Is this wave (a) transverse (b) longitudinal?

Quick Quiz 16.3 A sinusoidal wave of frequency f is traveling along a stretched string. The string is brought to rest, and a second traveling wave of frequency $2f$ is established on the string. The wave speed of the second wave is (a) twice that of the first wave (b) half that of the first wave (c) the same as that of the first wave (d) impossible to determine.

Quick Quiz 16.4 Consider the waves in Quick Quiz 16.3 again. The wavelength of the second wave is (a) twice that of the first wave (b) half that of the first wave (c) the same as that of the first wave (d) impossible to determine.

Questions 1

1. Why is a pulse on a string considered to be transverse?
2. How would you create a longitudinal wave in a stretched spring? Would it be possible to create a transverse wave in a spring?
3. By what factor would you have to multiply the tension in a stretched string in order to double the wave speed?

6. If you shake one end of a taut rope steadily three times each second, what would be the period of the sinusoidal wave set up in the rope?

Answers 1

Q16.1 As the pulse moves down the string, the particles of the string itself move side to side. Since the medium—here, the string—moves perpendicular to the direction of wave propagation, the wave is transverse by definition.

Q16.2 To use a slinky to create a longitudinal wave, pull a few coils back and release. For a transverse wave, jostle the end coil side to side.

Q16.3 From $v = \sqrt{\frac{T}{\mu}}$, we must increase the tension by a factor of 4.

Q16.6 Since the frequency is 3 cycles per second, the period is $\frac{1}{3}$ second = 333 ms.

Questions 2

8. Consider a wave traveling on a taut rope. What is the difference, if any, between the speed of the wave and the speed of a small segment of the rope?
9. If a long rope is hung from a ceiling and waves are sent up the rope from its lower end, they do not ascend with constant speed. Explain.
10. How do transverse waves differ from longitudinal waves?
11. When all the strings on a guitar are stretched to the same tension, will the speed of a wave along the most massive bass string be faster, slower, or the same as the speed of a wave on the lighter strings?
12. If one end of a heavy rope is attached to one end of a light rope, the speed of a wave will change as the wave goes from the heavy rope to the light one. Will it increase or decrease? What happens to the frequency? To the wavelength?

Answers 2

Q16.8 The section of rope moves up and down in SHM. Its speed is always changing. The wave continues on with constant speed in one direction, setting further sections of the rope into up-and-down motion.

Q16.9 Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v = \sqrt{\frac{T}{\mu}}$ increases with height.

Q16.10 The difference is in the direction of motion of the elements of the medium. In longitudinal waves, the medium moves back and forth parallel to the direction of wave motion. In transverse waves, the medium moves perpendicular to the direction of wave motion.

Q16.11 Slower. Wave speed is inversely proportional to the square root of linear density.

Q16.12 As the wave passes from the massive string to the less massive string, the wave speed will increase according to $v = \sqrt{\frac{T}{\mu}}$. The frequency will remain unchanged. Since $v = f\lambda$, the wavelength must increase.

Example 16.3 A Sinusoidally Driven String

The string shown in Figure 16.10 is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Determine the angular frequency ω and wave number k for this wave, and write an expression for the wave function.

Solution Using Equations 16.3, 16.9, and 16.11, we find that

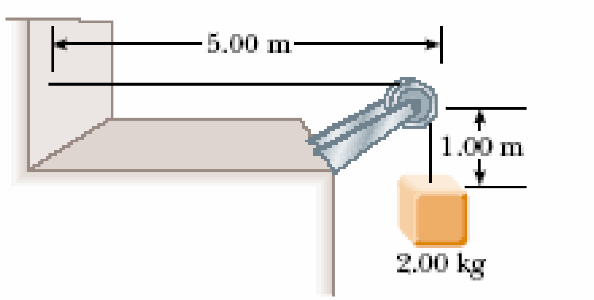
$$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00 \text{ Hz}) = \boxed{31.4 \text{ rad/s}}$$

$$k = \frac{\omega}{v} = \frac{31.4 \text{ rad/s}}{20.0 \text{ m/s}} = \boxed{1.57 \text{ rad/m}}$$

Because $A = 12.0$ cm $= 0.120$ m, we have

$$y = A \sin(kx - \omega t)$$
$$= \boxed{(0.120 \text{ m}) \sin(1.57x - 31.4t)}$$

A uniform cord has a mass of 0.300 kg and a length of 6.00 m (Fig. 16.12). The cord passes over a pulley and supports a 2.00-kg object. Find the speed of a pulse traveling along this cord.



Solution The tension T in the cord is equal to the weight of the suspended 2.00-kg object:

$$T = mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = 19.6 \text{ N}$$

(This calculation of the tension neglects the small mass of the cord. Strictly speaking, the cord can never be exactly horizontal, and therefore the tension is not uniform.) The mass per unit length μ of the cord is

$$\mu = \frac{m}{\ell} = \frac{0.300 \text{ kg}}{6.00 \text{ m}} = 0.050\,0 \text{ kg/m}$$

Therefore, the wave speed is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6 \text{ N}}{0.050\,0 \text{ kg/m}}} = \boxed{19.8 \text{ m/s}}$$

Example 16.6 Power Supplied to a Vibrating String

A taut string for which $\mu = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution The wave speed on the string is, from Equation 16.18,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80.0 \text{ N}}{5.00 \times 10^{-2} \text{ kg/m}}} = 40.0 \text{ m/s}$$

Because $f = 60.0$ Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0 \text{ Hz}) = 377 \text{ s}^{-1}$$

Using these values in Equation 16.21 for the power, with $A = 6.00 \times 10^{-2}$ m, we obtain

$$\begin{aligned}\mathcal{P} &= \tfrac{1}{2}\mu\omega^2A^2v \\ &= \tfrac{1}{2}(5.00 \times 10^{-2} \text{ kg/m})(377 \text{ s}^{-1})^2 \\ &\quad \times (6.00 \times 10^{-2} \text{ m})^2(40.0 \text{ m/s}) \\ &= \boxed{512 \text{ W}}\end{aligned}$$

13. A sinusoidal wave is described by

$$y = (0.25 \text{ m}) \sin(0.30x - 40t)$$

where x and y are in meters and t is in seconds. Determine for this wave the (a) amplitude, (b) angular frequency, (c) angular wave number, (d) wavelength, (e) wave speed, and (f) direction of motion.

P16.13 $y = 0.250 \sin(0.300x - 40.0t)$ m

Compare this with the general expression $y = A\sin(kx - \omega t)$

(a) $A = \boxed{0.250 \text{ m}}$

(b) $\omega = \boxed{40.0 \text{ rad/s}}$

(c) $k = \boxed{0.300 \text{ rad/m}}$

(d) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.300 \text{ rad/m}} = \boxed{20.9 \text{ m}}$

(e) $v = f\lambda = \left(\frac{\omega}{2\pi}\right)\lambda = \left(\frac{40.0 \text{ rad/s}}{2\pi}\right)(20.9 \text{ m}) = \boxed{133 \text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in } +x \text{ direction}}$.

1. Does the phenomenon of wave interference apply only to sinusoidal waves?

2. As oppositely moving pulses of the same shape (one upward, one downward) on a string pass through each other, there is one instant at which the string shows no displacement from the equilibrium position at any point. Has the energy carried by the pulses disappeared at this instant of time? If not, where is it?

3. Can two pulses traveling in opposite directions on the same string reflect from each other? Explain.

4. When two waves interfere, can the amplitude of the resultant wave be greater than either of the two original waves? Under what conditions?

9. Explain why your voice seems to sound better than usual when you sing in the shower.

10. What is the purpose of the slide on a trombone or of the valves on a trumpet?

11. Explain why all harmonics are present in an organ pipe open at both ends, but only odd harmonics are present in a pipe closed at one end.

20. If you inhale helium from a balloon and do your best to speak normally, your voice will have a comical quacky quality. Explain why this "Donald Duck effect" happens. *Caution*: Helium is an asphyxiating gas and asphyxiation can cause panic. Helium can contain poisonous contaminants.

Q18.1 No. Waves with other waveforms are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.

Q18.2 The energy has not disappeared, but is still carried by the wave pulses. Each particle of the string still has kinetic energy. This is similar to the motion of a simple pendulum. The pendulum does not stop at its equilibrium position during oscillation—likewise the particles of the string do not stop at the equilibrium position of the string when these two waves superimpose.

Q18.3 No. A wave is not a solid object, but a chain of disturbance. As described by the principle of superposition, the waves move through each other.

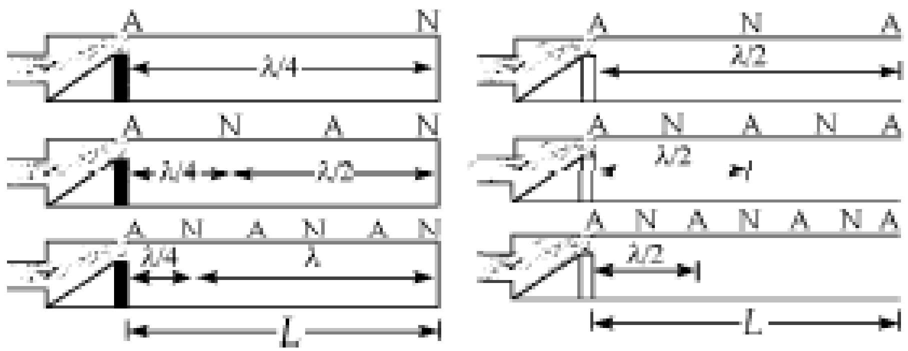
Q18.4 They can, wherever the two waves are nearly enough in phase that their displacements will add to create a total displacement greater than the amplitude of either of the two original waves.

When two one-dimensional sinusoidal waves of the same amplitude interfere, this condition is satisfied whenever the absolute value of the phase difference between the two waves is less than 120°.

Q18.9 The air in the shower stall can vibrate in standing wave patterns to intensify those frequencies in your voice which correspond to its free vibrations. The hard walls of the bathroom reflect sound very well to make your voice more intense at all frequencies, giving the room a longer reverberation time. The reverberant sound may help you to stay on key.

Q18.10 The trombone slide and trumpet valves change the length of the air column inside the instrument, to change its resonant frequencies.

Q18.11 The vibration of the air must have zero amplitude at the closed end. For air in a pipe closed at one end, the diagrams show how resonance vibrations have NA distances that are odd integer submultiples of the NA distance in the fundamental vibration. If the pipe is open, resonance vibrations have NA distances that are all integer submultiples of the NA distance in the fundamental.



Q18.20 Helium is less dense than air. It carries sound at higher speed. Each cavity in your vocal apparatus has a standing-wave resonance frequency, and each of these frequencies is shifted to a higher value. Your vocal chords can vibrate at the same fundamental frequency, but your vocal tract amplifies by resonance a different set of higher frequencies. Then your voice has a different quacky quality.

Example 18.2 Formation of a Standing Wave

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = (4.0 \text{ cm}) \sin(3.0x - 2.0t)$$
$$y_2 = (4.0 \text{ cm}) \sin(3.0x + 2.0t)$$

where x and y are measured in centimeters.

(A) Find the amplitude of the simple harmonic motion of the element of the medium located at $x = 2.3$ cm.

Solution The standing wave is described by Equation 18.3; in this problem, we have $A = 4.0$ cm, $k = 3.0$ rad/cm, and $\omega = 2.0$ rad/s. Thus,

$$y = (2A \sin kx) \cos \omega t = [(8.0 \text{ cm})\sin 3.0x] \cos 2.0t$$

Thus, we obtain the amplitude of the simple harmonic motion of the element at the position $x = 2.3$ cm by evaluating the coefficient of the cosine function at this position:

$$y_{max} = (8.0 \text{ cm})\sin 3.0x|_{x=2.3}$$
$$= (8.0 \text{ cm}) \sin (6.9 \text{ rad}) = \boxed{4.6 \text{ cm}}$$

(B) Find the positions of the nodes and antinodes if one end of the string is at $x = 0$.

Solution With $k = 2\pi/\lambda = 3.0$ rad/cm, we see that the wavelength is $\lambda = (2\pi/3.0)$ cm. Therefore, from Equation 18.4 we find that the nodes are located at

$$x = n\frac{\lambda}{2} = n\left(\frac{\pi}{3}\right) \text{cm} \qquad n = \boxed{0, 1, 2, 3, \ldots}$$

and from Equation 18.5 we find that the antinodes are located at

$$x = n\frac{\lambda}{4} = n\left(\frac{\pi}{6}\right) \text{cm} \qquad n = \boxed{1, 3, 5, \ldots}$$

(C) What is the maximum value of the position in the simple harmonic motion of an element located at an antinode?

Solution According to Equation 18.3, the maximum position of an element at an antinode is the amplitude of the standing wave, which is twice the amplitude of the individual traveling waves:

$$y_{max} = 2A(\sin kx)_{max} = 2(4.0 \text{ cm})(\pm 1) = \boxed{\pm 8.0 \text{ cm}}$$

where we have used the fact that the maximum value of $\sin kx$ is ± 1. Let us check this result by evaluating the coefficient of our standing-wave function at the positions we found for the antinodes:

$$y_{max} = (8.0 \text{ cm})\sin 3.0x|_{x=n(\pi/6)}$$
$$= (8.0 \text{ cm}) \sin \left[3.0n\left(\frac{\pi}{6}\right) \text{rad}\right]$$
$$= (8.0 \text{ cm}) \sin \left[n\left(\frac{\pi}{2}\right) \text{rad}\right] = \pm 8.0 \text{ cm}$$

In evaluating this expression, we have used the fact that n is an odd integer; thus, the sine function is equal to ± 1, depending on the value of n.

Example 18.6 Wind in a Culvert

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows.

(A) Determine the frequencies of the first three harmonics of the culvert if it is cylindrical in shape and open at both ends. Take $v = 343$ m/s as the speed of sound in air.

Solution The frequency of the first harmonic of a pipe open at both ends is

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = \boxed{139 \text{ Hz}}$$

Because both ends are open, all harmonics are present; thus,

$$f_2 = 2f_1 = \boxed{278 \text{ Hz}} \quad \text{and} \quad f_3 = 3f_1 = \boxed{417 \text{ Hz}}$$

(B) What are the three lowest natural frequencies of the culvert if it is blocked at one end?

Solution The fundamental frequency of a pipe closed at one end is

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = \boxed{69.7 \text{ Hz}}$$

In this case, only odd harmonics are present; hence, the next two harmonics have frequencies $f_3 = 3f_1 = \boxed{209 \text{ Hz}}$ and $f_5 = 5f_1 = \boxed{349 \text{ Hz.}}$

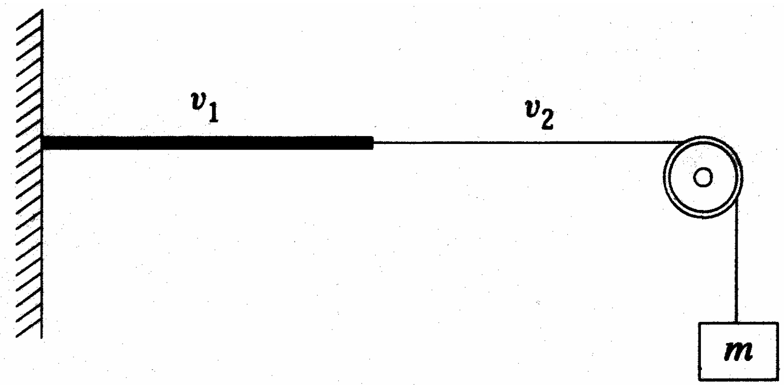
(C) For the culvert open at both ends, how many of the harmonics present fall within the normal human hearing range (20 to 20 000 Hz)?

Solution Because all harmonics are present for a pipe open at both ends, we can express the frequency of the highest harmonic heard as $f_n = nf_1$ where n is the number of harmonics that we can hear. For $f_n = 20\,000$ Hz, we find that the number of harmonics present in the audible range is

$$n = \frac{20\,000 \text{ Hz}}{139 \text{ Hz}} = \boxed{143}$$

Only the first few harmonics are of sufficient amplitude to be heard.

A weight is hung over a pulley and attached to a string composed of two parts, each made of the same material but one having four times the diameter of the other. The string is plucked so that a pulse moves along it, moving at speed v_1 in the thick part and at speed v_2 in the thin part. What is v_1/v_2?



1. 1
2. 2
3. 1/2
4. 1/4