8.11

$$I_{xx} = m(y^2+z^2) = 9m$$

$$I_{yy} = m(x^2+z^2) = 13m$$

$$I_{zz} = m(x^2+y^2) = 4m$$

$$I_{xy} = -mxy = 0$$

$$I_{xz} = -mxz = -6m$$

$$I_{yz} = -myz = 0$$

Rotation around z-axis by α.

$$x' = 2\cos\alpha \approx 2$$

$$y' = 2\sin\alpha \approx 2\alpha$$

$$z' = 3$$

$$I'_{xx} = m(4\alpha^2+9) \approx 9m$$

$$I'_{yy} = 13m$$

$$I'_{zz} = m(4+4\alpha^2) \approx 4m$$

$$I'_{xy} \approx -4m\alpha$$

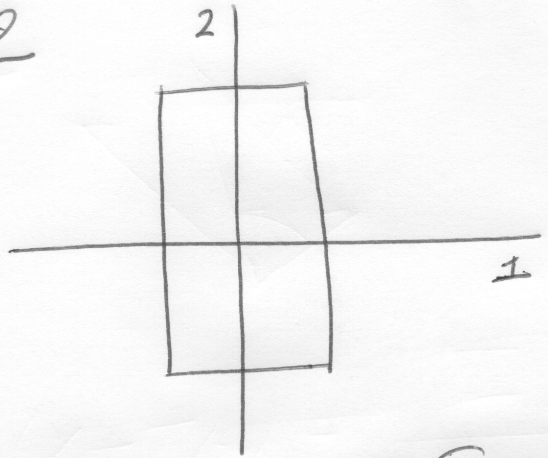
$$I'_{xz} = -6m$$

$$I'_{yz} = -6m\alpha$$

Thus I'_{xy} and I'_{yz} are changed to first order in α.

11.9

1.



$$I_1 = \int y^2 dm = \rho \int y^2 dx\, dy$$

$$= \rho \tfrac{1}{3} y^3 x \Big|_{-\frac{1}{2}a}^{\frac{1}{2}a} \Big|_{-a}^{a}$$

$$= \rho \tfrac{2}{3} a^4 = \frac{M}{2a^2} \tfrac{2}{3} a^4 = \frac{M}{3} a^2$$

$$I_2 = \int x^2 dm = \frac{1}{12} M a^2$$

$$I_3 = \int (x^2+y^2) dm = \frac{5}{12} M a^2$$

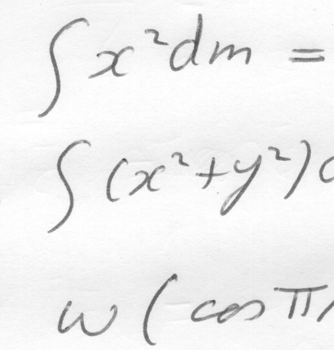
2. $\vec{\omega} = \omega(\cos\pi/4\, \vec{e}_2 - \cos\pi/4\, \vec{e}_1)$

$$= \tfrac{1}{2}\sqrt{2}\,\omega(\vec{e}_2 - \vec{e}_1)$$

$$\vec{L} = I\vec{\omega} = -\tfrac{1}{2}\sqrt{2}\,\omega I_1 \vec{e}_1 + \tfrac{1}{2}\sqrt{2}\,\omega I_2 \vec{e}_2$$

$$L_1 = -\frac{\omega I_1}{\sqrt{2}} \quad L_2 = \frac{\omega I_2}{\sqrt{2}} \quad L_3 = 0$$

3.



$$\tan(\gamma + \pi/4) = \frac{I_1}{I_2} = 4$$

$$\gamma = \arctan 4 - \frac{\pi}{4}$$

$$= 0.54 \text{ rad}$$

4. Let $\vec{e}_3 = \vec{e}_1 \times \vec{e}_2$

Then $\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{\omega} \times \vec{L} = (-\tfrac{1}{2}\omega^2 I_2 + \tfrac{1}{2}\omega^2 I_1)\vec{e}_3$

$$\Longrightarrow \tau = \tfrac{1}{8} M a^2 \omega^2$$