Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

Trinity Term 2016

MA1111: Linear Algebra I

Tuesday, May 10 RDS 14:00 — 16:00

Prof. V. Dotsenko

Instructions to Candidates:

ATTEMPT **ALL** QUESTIONS

The number of marks you can get for a complete solution to any individual question is written next to the question. A complete solution to a question includes coherent explanations of answers you give.

Unless otherwise specified, you may use all statements proved in class without proof; when using some statement, you should formulate it clearly, e.g. "in class, we proved that a square matrix A is invertible if and only if $\det(A) \neq 0$".

Non-programmable calculators are permitted for this examination.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Denote by A the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ and by b the vector $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

   (a) (15 points) Show how to compute the matrix A^{-1} using elementary row operations, and use the matrix A^{-1} to solve the system $Ax = b$.

   (b) (10 points) Show how to use Cramer's rule to solve the system $Ax = b$.

2. (a) (10 points) Outline the proof from class of the fact the determinant of a matrix does not change if we add to one of its rows a multiple of another row.

   (b) (15 points) For the matrix $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & 4 & -3 \\ -3 & -4 & 1 & 2 \\ -4 & 3 & -2 & 1 \end{pmatrix}$, compute the matrix product of A and its transpose matrix, and explain how to use your result to calculate the determinant of A.

3. (a) (12 points) State the definition of a basis of a vector space. Show that the vectors $f_1 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $f_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ form a basis of $\mathbb{R}^2$.

   (b) (13 points) Suppose that the matrix of a linear transformation of $\mathbb{R}^2$ relative to the basis f_1, f_2 from (b) is $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Compute the matrix of the same linear transformation relative to the basis of standard unit vectors of $\mathbb{R}^2$.

4. (a) (5 points) State the definition of an eigenvalue and of an eigenvector of a linear transformation of a vector space V.

   (b) (15 points) What are the eigenvalues of the linear transformation T of the four-dimensional space of 2×2-matrices which sends every matrix X to $AX - XA$, where $A = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$?

   (c) (5 points) Does the linear transformation T from (b) have a basis of eigenvectors?