Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Mathematics **Trinity Term 2017**
JF Theoretical Physics
JF Two Subject Mod

MA1125 — Single Variable Calculus and Introductory Analysis

Tuesday, May 9 **Sports Centre** **14:00 — 17:00**

Prof. D. O'Donovan

Instructions to Candidates:

ANSWER ALL QUESTIONS:

All questions carry equal marks.

Formulae & Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. (a) Use the ϵ, δ definition of limit to evaluate $\lim_{x \to 1} 2x^2 + 3x - 2$.

   (b) If $f(x) = \frac{9x - 3\sin 3x}{5x^2}$ for$x \neq 0$ and = c for x = 0. For what value of c is f(x) continuous at x = 0?

   (c) Find $\lim_{x \to 0^+} x \ln x$

   (d) State the squeezing theorem for limits and use it to evaluate $\lim_{x \to 0} \frac{\sin x}{x}$

2. (a) Derive the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for Newton's Method. Does Newton's Method always work?

   (b) Find $\frac{dy}{dx}$ if

      i. $y = \sqrt{\tan(\exp(x^3 + x))}$

      ii. $x^2y^2 + x^2 \exp y = \ln(xy^3)$

   (c) A girl flies a kite at a constant height of 30 metres. The wind carries the kite horizontally at 40 m/sec. How fast must she let the string out when the kite is 50 metres away from her?

   (d) Find where the function $y = x^4 - 4x^3$ is increasing, decreasing, concave up, concave down, has local extrema, and points of inflection. Draw a rough sketch of the function.

3. (a) State The Intermediate Value Theorem. State the Extreme Value Theorem.

   (b) Solve the following

      i. $x\frac{dy}{dx} + y = x^2$.

      ii. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6 = 0$

      iii. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6 = x^2$

   (c) You are planning to close off a corner of the first quadrant in the plane with a line segment 20 units long running from (a,0) to (0,b). What is the largest area possible?

4. (a) How is the Riemann Integral $\int_a^b f(x)dx$ defined?

   (b) Integrate the following.

      i. $\int x\ln(x^2+3)dx$

      ii. $\int x\sin x dx$

      iii. $\displaystyle\int \frac{x^2+2}{(x+2)(x+3)}$

      iv. $\displaystyle\int \frac{x}{x^2+x+1}dx$

5. (a) Derive the formula for the length of the curve y = f(x) between x = a and x = b.

   (b) Find the area enclosed between $y = x^3 - 3x^2$ and the x-axis.

   (c) The bounded region between $y = x$, $y = -x + 2$ and above the x-axis is rotated about the y-axis. Find the volume first using washers and then using cylindrical shells.

6. (a) Define $\sum_{n=1}^{\infty} a_n = L$.

   (b) Do the following series converge or diverge? Give reasons.

      i. $\displaystyle\sum_{n=1}^{\infty} \frac{n^7}{7^n}$

      ii. $\displaystyle\sum_{n=1}^{\infty} \frac{n-2}{n}$

      iii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

   (c) Find the Taylor Series for $f(x) = \exp(x)$ about x = 0.

   (d) Find for what values of x the following power series converges absolutely, conditionally, or diverges? What is the radius of convergence?
   $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n^3}$