UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Maths/TP
SF TSM

Trinity Term 2013

MA1212 — LINEAR ALGEBRA II

Wednesday, May 1 LUCE UPPER 09.30 — 11.30

Dr. Paschalis Karageorgis

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.

1. Let $x_0 = 1$ and $y_0 = 7$. Suppose the sequences x_n, y_n are such that

$$x_n = 3x_{n-1} + 4y_{n-1}, \qquad y_n = 2x_{n-1} + 5y_{n-1}$$

for each $n \geq 1$. Determine each of x_n and y_n explicitly in terms of n.

2. Find the Jordan form and a Jordan basis for the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 1 & 2 \\ -4 & 2 & 1 \end{bmatrix}.$$

3. A square matrix A has characteristic polynomial $\lambda^3(\lambda - 1)$ and its column space is three-dimensional. Find the dimension of the column space of A^2.

4. Let Q be the quadratic form defined by the formula

$$Q(x, y, z) = x^2 + (a + 3)y^2 + (a + 2)z^2 + 4axy + 2(a - 1)yz.$$

Find the values of the real parameter a for which the form is positive definite.

5. Show that A and A^tA have the same null space for each $m \times n$ matrix A.

6. Suppose that A is a real, positive definite symmetric matrix. Show that there exists a matrix B such that $B^2 = A$.

© UNIVERSITY OF DUBLIN 2013