TRINITY COLLEGE DUBLIN

THE UNIVERSITY OF DUBLIN

School of Mathematics

JF Mathematics
JF Theoretical Physics

Trinity Term 2015

MA1241 — Mechanics I

Tuesday, May 5th **Luce Lower** **09.30 — 11.30**

Dr. J. Manschot

Instructions to Candidates:

Credit will be given for the best 2 questions.

All questions have equal weight.

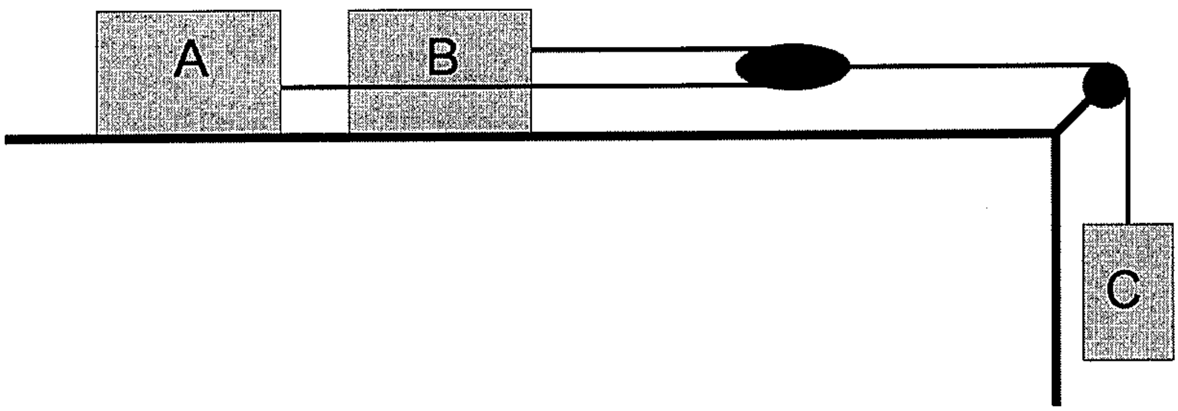
Materials Permitted for this Examination:

Formulae and Tables tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Two masses, A and B, lie on a frictionless table. See the figure below. They are attached to either end of a rope with fixed length l_1, which passes around a pulley. This pulley can move without friction over the table, and is attached to a second rope with fixed length l_2. The second rope passes over a fixed pulley and on the other end is attached to a hanging mass C. Assume that the masses of the ropes and the pulleys are negligible.



(a) Draw the force diagrams and give the equations of motions (of the center of masses) of the masses and the pulleys.

(b) Determine the tension in the two ropes.

(c) Determine the acceleration (magnitude and direction) of the masses.

2. A rocket of mass M departs vertically from the surface of the earth. Let $R_e = 6371\,\mathrm{km}$ and $g = 9.81\,\mathrm{m/s}$.

   (a) Derive the escape velocity v_e of the rocket from the surface of the earth, if the rocket does not burn any fuel after take off.

   (b) We will now consider a more realistic situation, in which the rocket is loaded with an amount of fuel of mass m_0. The initial velocity $v(0)$ of the rocket is 0 m/s, and fuel is expelled at a rate $r = 0.05\,m_0/s$. The velocity u of the expelled gas with respect to the rocket is constant, and equal to $u = 8000$ m/s. Prove that the velocity $v(t)$ of the rocket as function of time t is given by:

$$v(t) = -gt + u\ln\left(\frac{M+m_0}{M+m_0-rt}\right), \qquad t \in \left[0, \frac{m_0}{r}\right].$$

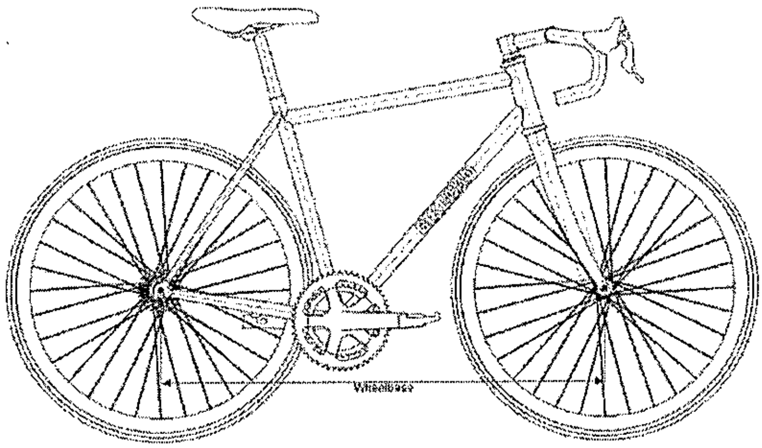
   (c) Let $M = \frac{1}{4}m_0$, such that the above formula reduces to

$$v(t) = -gt - u\ln(1-0.04\,t).$$

   Determine the speed $v(20)$ and the height $h(20)$ of the rocket after 20 s. Did the rocket already reach the escape velocity v_e? Can one neglect the height $h(20)$ compared to R_e?

   Hint: $\int \ln x\,dx = x\ln x - x$.

3. We consider a road bike. The wheelbase, the distance between the points where the wheels touch the road, is W. The total mass of bike and rider is m, and their center of mass is located in the middle of the wheelbase and a height h over the road surface. The bike accelerates with a constant acceleration a_C forward.



(a) The surface of the road is such that the wheels do not slip. Assume that the wheels are well approximated by a uniform thin hoop of radius R_w. Express the angular momentum of a wheel (with respect to its center) in terms of its mass m_w, radius R_w and the velocity of the center of mass v_C of the bike + rider.

(b) Determine the (instantaneous) total angular momentum $L(t)$ and the total torque $\tau(t)$ of the accelerating bike + rider. Take as origin of the coordinate system, the point where the rare wheel touches the road. Argue by choosing some representative values that the angular momentum of the wheels is small ($< 10\%$) compared to the angular momentum of the center of mass.

(c) Explain why the front of the bike tends to move upward during the acceleration. For which value of the acceleration does the wheel come loose from the road surface?