9

ROTATION OF RIGID BODIES

9.1. IDENTIFY: $s = r\theta$, with θ in radians.

SET UP: π rad $= 180°$.

EXECUTE: **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50\text{ m}}{2.50\text{ m}} = 0.600\text{ rad} = 34.4°$

(b) $r = \dfrac{s}{\theta} = \dfrac{14.0\text{ cm}}{(128°)(\pi\text{ rad}/180°)} = 6.27\text{ cm}$

(c) $s = r\theta = (1.50\text{ m})(0.700\text{ rad}) = 1.05\text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate θ, the calculation gives θ in radians.

9.2. IDENTIFY: $\theta - \theta_0 = \omega t$, since the angular velocity is constant.

SET UP: $1\text{ rpm} = (2\pi/60)\text{ rad/s}$.

EXECUTE: **(a)** $\omega = (1900)(2\pi\text{ rad}/60\text{ s}) = 199\text{ rad/s}$

(b) $35° = (35°)(\pi/180°) = 0.611\text{ rad}$. $t = \dfrac{\theta - \theta_0}{\omega} = \dfrac{0.611\text{ rad}}{199\text{ rad/s}} = 3.1\times10^{-3}\text{ s}$

EVALUATE: In $t = \dfrac{\theta - \theta_0}{\omega}$ we must use the same angular measure (radians, degrees or revolutions) for both $\theta - \theta_0$ and ω.

9.3. IDENTIFY: $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Using $\omega_z = d\theta/dt$ gives $\theta - \theta_0 = \int_{t_1}^{t_2}\omega_z dt$.

SET UP: $\dfrac{d}{dt}t^n = nt^{n-1}$ and $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$

EXECUTE: **(a)** A must have units of rad/s and B must have units of rad/s^3.

(b) $\alpha_z(t) = 2Bt = (3.00\text{ rad/s}^3)t$. (i) For $t = 0$, $\alpha_z = 0$. (ii) For $t = 5.00$ s, $\alpha_z = 15.0\text{ rad/s}^2$.

(c) $\theta_2 - \theta_1 = \int_{t_1}^{t_2}(A + Bt^2)dt = A(t_2 - t_1) + \frac{1}{3}B(t_2^3 - t_1^3)$. For $t_1 = 0$ and $t_2 = 2.00$ s,

$\theta_2 - \theta_1 = (2.75\text{ rad/s})(2.00\text{ s}) + \frac{1}{3}(1.50\text{ rad/s}^3)(2.00\text{ s})^3 = 9.50\text{ rad}$.

EVALUATE: Both α_z and ω_z are positive and the angular speed is increasing.

9.4. IDENTIFY: $\alpha_z = d\omega_z/dt$. $\alpha_{\text{av-}z} = \dfrac{\Delta\omega_z}{\Delta t}$.

SET UP: $\dfrac{d}{dt}(t^2) = 2t$

EXECUTE: **(a)** $\alpha_z(t) = \dfrac{d\omega_z}{dt} = -2\beta t = (-1.60\text{ rad/s}^3)t$.

(b) $\alpha_z(3.0\text{ s}) = (-1.60\text{ rad/s}^3)(3.0\text{ s}) = -4.80\text{ rad/s}^2$.

$$\alpha_{\text{av-}z} = \frac{\omega_z(3.0\text{ s}) - \omega_z(0)}{3.0\text{ s}} = \frac{-2.20\text{ rad/s} - 5.00\text{ rad/s}}{3.0\text{ s}} = -2.40\text{ rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0$ s.

EVALUATE: $\alpha_z(t)$ increases linearly with time, so $\alpha_{\text{av-}z} = \dfrac{\alpha_z(0) + \alpha_z(3.0\text{ s})}{2}$. $\alpha_z(0) = 0$.

9.5. **IDENTIFY** and **SET UP:** Use $\omega_z = \dfrac{d\theta}{dt}$ to calculate the angular velocity and $\omega_{\text{av-}z} = \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$ to calculate the average angular velocity for the specified time interval.

EXECUTE: $\theta = \gamma t + \beta t^3$; $\gamma = 0.400\text{ rad/s}$, $\beta = 0.0120\text{ rad/s}^3$

(a) $\omega_z = \dfrac{d\theta}{dt} = \gamma + 3\beta t^2$

(b) At $t = 0$, $\omega_z = \gamma = 0.400\text{ rad/s}$

(c) At $t = 5.00$ s, $\omega_z = 0.400\text{ rad/s} + 3(0.0120\text{ rad/s}^3)(5.00\text{ s})^2 = 1.30\text{ rad/s}$

$$\omega_{\text{av-}z} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

For $t_1 = 0$, $\theta_1 = 0$.

For $t_2 = 5.00$ s, $\theta_2 = (0.400\text{ rad/s})(5.00\text{ s}) + (0.012\text{ rad/s}^3)(5.00\text{ s})^3 = 3.50\text{ rad}$

So $\omega_{\text{av-}z} = \dfrac{3.50\text{ rad} - 0}{5.00\text{ s} - 0} = 0.700\text{ rad/s}$.

EVALUATE: The average of the instantaneous angular velocities at the beginning and end of the time interval is $\frac{1}{2}(0.400\text{ rad/s} + 1.30\text{ rad/s}) = 0.850\text{ rad/s}$. This is larger than $\omega_{\text{av-}z}$, because $\omega_z(t)$ is increasing faster than linearly.

9.6. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. $\omega_{\text{av}-z} = \dfrac{\Delta\theta}{\Delta t}$.

SET UP: $\omega_z = (250\text{ rad/s}) - (40.0\text{ rad/s}^2)t - (4.50\text{ rad/s}^3)t^2$. $\alpha_z = -(40.0\text{ rad/s}^2) - (9.00\text{ rad/s}^3)t$.

EXECUTE: **(a)** Setting $\omega_z = 0$ results in a quadratic in t. The only positive root is $t = 4.23$ s.

(b) At $t = 4.23$ s, $\alpha_z = -78.1\text{ rad/s}^2$.

(c) At $t = 4.23$ s, $\theta = 586\text{ rad} = 93.3\text{ rev}$.

(d) At $t = 0$, $\omega_z = 250\text{ rad/s}$.

(e) $\omega_{\text{av-}z} = \dfrac{586\text{ rad}}{4.23\text{ s}} = 138\text{ rad/s}$.

EVALUATE: Between $t = 0$ and $t = 4.23$ s, ω_z decreases from 250 rad/s to zero. ω_z is not linear in t, so $\omega_{\text{av-}z}$ is not midway between the values of ω_z at the beginning and end of the interval.

9.7. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Use the values of θ and ω_z at $t = 0$ and α_z at 1.50 s to calculate a, b, and c.

SET UP: $\dfrac{d}{dt}t^n = nt^{n-1}$

EXECUTE: **(a)** $\omega_z(t) = b - 3ct^2$. $\alpha_z(t) = -6ct$. At $t = 0$, $\theta = a = \pi/4$ rad and $\omega_z = b = 2.00\text{ rad/s}$. At $t = 1.50$ s, $\alpha_z = -6c(1.50\text{ s}) = 1.25\text{ rad/s}^2$ and $c = -0.139\text{ rad/s}^3$.

(b) $\theta = \pi/4$ rad and $\alpha_z = 0$ at $t = 0$.

(c) $\alpha_z = 3.50\text{ rad/s}^2$ at $t = -\dfrac{\alpha_z}{6c} = -\dfrac{3.50\text{ rad/s}^2}{6(-0.139\text{ rad/s}^3)} = 4.20$ s. At $t = 4.20$ s,

$\theta = \frac{\pi}{4}\text{ rad} + (2.00\text{ rad/s})(4.20\text{ s}) - (-0.139\text{ rad/s}^3)(4.20\text{ s})^3 = 19.5\text{ rad}.$

$\omega_z = 2.00\text{ rad/s} - 3(-0.139\text{ rad/s}^3)(4.20\text{ s})^2 = 9.36\text{ rad/s}.$

EVALUATE: θ, ω_z, and α_z all increase as t increases.

9.8. **IDENTIFY:** $\alpha_z = \frac{d\omega_z}{dt}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$. When ω_z is linear in t, $\omega_{\text{av-}z}$ for the time interval t_1 to t_2 is $\omega_{\text{av-}z} = \frac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}$.

SET UP: From the information given, $\alpha_z = \frac{\Delta\omega}{\Delta t} = \frac{4.00\text{ rad/s} - (-6.00\text{ rad/s})}{7.00\text{ s}} = 1.429\text{ rad/s}^2$.

$\omega_z(t) = -6.00\text{ rad/s} + (1.429\text{ rad/s}^2)t.$

EXECUTE: **(a)** The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) It takes time $t = -\frac{\omega_{0z}}{\alpha_z} = -(-6.00\text{ rad/s})/(1.429\text{ rad/s}^2) = 4.20$ s for the wheel to stop $(\omega_z = 0)$. During this time its speed is decreasing. For the next 2.80 s its speed is increasing from 0 rad/s to $+4.00$ rad/s.

(c) The average angular velocity is $\frac{-6.00\text{ rad/s} + 4.00\text{ rad/s}}{2} = -1.00\text{ rad/s}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$ then leads to displacement of –7.00 rad after 7.00 s.

EVALUATE: When α_z and ω_z have the same sign, the angular speed is increasing; this is the case for $t = 4.20$ s to $t = 7.00$ s. When α_z and ω_z have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 4.20$ s.

9.9. **IDENTIFY:** Apply the constant angular acceleration equations.

SET UP: Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 1.50\text{ rad/s} + (0.200\text{ rad/s}^2)(2.50\text{ s}) = 2.00\text{ rad/s}.$

(b) $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (1.50\text{ rad/s})(2.50\text{ s}) + \frac{1}{2}(0.200\text{ rad/s}^2)(2.50\text{ s})^2 = 4.38\text{ rad}.$

EVALUATE: $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{1.50\text{ rad/s} + 2.00\text{ rad/s}}{2}\right)(2.50\text{ s}) = 4.38\text{ rad}$, the same as calculated with another equation in part (b).

9.10. **IDENTIFY:** Apply the constant angular acceleration equations to the motion of the fan.

(a) SET UP: $\omega_{0z} = (500\text{ rev/min})(1\text{ min}/60\text{ s}) = 8.333\text{ rev/s}$, $\omega_z = (200\text{ rev/min})(1\text{ min}/60\text{ s}) = 3.333\text{ rev/s}$, $t = 4.00$ s, $\alpha_z = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{3.333\text{ rev/s} - 8.333\text{ rev/s}}{4.00\text{ s}} = -1.25\text{ rev/s}^2$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (8.333\text{ rev/s})(4.00\text{ s}) + \frac{1}{2}(-1.25\text{ rev/s}^2)(4.00\text{ s})^2 = 23.3\text{ rev}$

(b) SET UP: $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333$ rev/s; $\alpha_z = -1.25\text{ rev/s}^2$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \frac{\omega_z - \omega_{0z}}{\alpha_z} = \frac{0 - 3.333\text{ rev/s}}{-1.25\text{ rev/s}^2} = 2.67\text{ s}$

EVALUATE: The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-}z}t$ gives $\theta - \theta_0 = 23.3$ rev, which checks.

9.11. **IDENTIFY:** Apply the constant angular acceleration equations to the motion. The target variables are t and $\theta-\theta_0$.

SET UP: **(a)** $\alpha_z = 1.50\ \text{rad/s}^2$; $\omega_{0z}=0$ (starts from rest); $\omega_z = 36.0\ \text{rad/s}$; $t=?$

$\omega_z = \omega_{0z}+\alpha_z t$

EXECUTE: $t=\dfrac{\omega_z-\omega_{0z}}{\alpha_z}=\dfrac{36.0\ \text{rad/s}-0}{1.50\ \text{rad/s}^2}=24.0\ \text{s}$

(b) $\theta-\theta_0=?$

$\theta-\theta_0=\omega_{0z}t+\tfrac{1}{2}\alpha_z t^2 = 0+\tfrac{1}{2}(1.50\ \text{rad/s}^2)(24.0\ \text{s})^2 = 432\ \text{rad}$

$\theta-\theta_0 = 432\ \text{rad}(1\ \text{rev}/2\pi\ \text{rad}) = 68.8\ \text{rev}$

EVALUATE: We could use $\theta-\theta_0=\tfrac{1}{2}(\omega_z+\omega_{0z})t$ to calculate $\theta-\theta_0=\tfrac{1}{2}(0+36.0\ \text{rad/s})(24.0\ \text{s}) = 432\ \text{rad}$, which checks.

9.12. **IDENTIFY:** In part (b) apply the equation derived in part (a).

SET UP: Let the direction the propeller is rotating be positive.

EXECUTE: **(a)** Solving $\omega_z=\omega_{0z}+\alpha_z t$ for t gives $t=\dfrac{\omega_z-\omega_{0z}}{\alpha_z}$. Rewriting $\theta-\theta_0=\omega_{0z}t+\tfrac{1}{2}\alpha_z t^2$ as

$\theta-\theta_0=t(\omega_{0z}+\tfrac{1}{2}\alpha_z t)$ and substituting for t gives

$$\theta-\theta_0=\left(\frac{\omega_z-\omega_{0z}}{\alpha_z}\right)(\omega_{0z}+\tfrac{1}{2}(\omega_z-\omega_{0z}))=\frac{1}{\alpha_z}(\omega_z-\omega_{0z})\left(\frac{\omega_z+\omega_{0z}}{2}\right)=\frac{1}{2\alpha_z}(\omega_z^2-\omega_{0z}^2),$$

which when rearranged gives $\omega_z^2=\omega_{z0}^2+2\alpha_z(\theta-\theta_0)$.

(b) $\alpha_z=\tfrac{1}{2}\left(\dfrac{1}{\theta-\theta_0}\right)(\omega_z^2-\omega_{0z}^2)=\tfrac{1}{2}\left(\dfrac{1}{7.00\ \text{rad}}\right)((16.0\ \text{rad/s})^2-(12.0\ \text{rad/s})^2) = 8.00\ \text{rad/s}^2$

EVALUATE: We could also use $\theta-\theta_0=\left(\dfrac{\omega_{0z}+\omega_z}{2}\right)t$ to calculate $t=0.500\ \text{s}$. Then $\omega_z=\omega_{0z}+\alpha_z t$ gives $\alpha_z = 8.00\ \text{rad/s}^2$, which agrees with our results in part (b).

9.13. **IDENTIFY:** Use a constant angular acceleration equation and solve for ω_{0z}.

SET UP: Let the direction of rotation of the flywheel be positive.

EXECUTE: $\theta-\theta_0=\omega_{0z}t+\tfrac{1}{2}\alpha_z t^2$ gives

$$\omega_{0z}=\frac{\theta-\theta_0}{t}-\tfrac{1}{2}\alpha_z\, t=\frac{30.0\ \text{rad}}{4.00\ \text{s}}-\tfrac{1}{2}(2.25\ \text{rad/s}^2)(4.00\ \text{s}) = 3.00\ \text{rad/s}.$$

EVALUATE: At the end of the 4.00 s interval, $\omega_z=\omega_{0z}+\alpha_z t = 12.0\ \text{rad/s}$.

$$\theta-\theta_0=\left(\frac{\omega_{0z}+\omega_z}{2}\right)t=\left(\frac{3.00\ \text{rad/s}+12.0\ \text{rad/s}}{2}\right)(4.00\ \text{s}) = 30.0\ \text{rad},\ \text{which checks.}$$

9.14. **IDENTIFY:** Apply the constant angular acceleration equations.

SET UP: Let the direction of the rotation of the blade be positive. $\omega_{0z}=0$.

EXECUTE: $\omega_z=\omega_{0z}+\alpha_z t$ gives $\alpha_z=\dfrac{\omega_z-\omega_{0z}}{t}=\dfrac{140\ \text{rad/s}-0}{6.00\ \text{s}} = 23.3\ \text{rad/s}^2$.

$$(\theta-\theta_0)=\left(\frac{\omega_{0z}+\omega_z}{2}\right)t=\left(\frac{0+140\ \text{rad/s}}{2}\right)(6.00\ \text{s}) = 420\ \text{rad}$$

EVALUATE: We could also use $\theta-\theta_0=\omega_{0z}t+\tfrac{1}{2}\alpha_z t^2$. This equation gives

$\theta-\theta_0=\tfrac{1}{2}(23.3\ \text{rad/s}^2)(6.00\ \text{s})^2 = 419\ \text{rad}$, in agreement with the result obtained above.

9.15. **IDENTIFY:** Apply constant angular acceleration equations.

SET UP: Let the direction the flywheel is rotating be positive.

$\theta - \theta_0 = 200$ rev, $\omega_{0z} = 500$ rev/min $= 8.333$ rev/s, $t = 30.0$ s.

EXECUTE: **(a)** $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00$ rev/s $= 300$ rpm

(b) Use the information in part (a) to find α_z: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111\text{ rev/s}^2$. Then $\omega_z = 0$, $\alpha_z = -0.1111\text{ rev/s}^2$, $\omega_{0z} = 8.333$ rev/s in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0$ s and $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\theta - \theta_0 = 312$ rev.

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

9.16. **IDENTIFY:** Apply the constant angular acceleration equations separately to the time intervals 0 to 2.00 s and 2.00 s until the wheel stops.

(a) SET UP: Consider the motion from $t = 0$ to $t = 2.00$ s:

$\theta - \theta_0 = ?$; $\omega_{0z} = 24.0$ rad/s; $\alpha_z = 30.0\text{ rad/s}^2$; $t = 2.00$ s

EXECUTE: $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (24.0\text{ rad/s})(2.00\text{ s}) + \frac{1}{2}(30.0\text{ rad/s}^2)(2.00\text{ s})^2$

$\theta - \theta_0 = 48.0\text{ rad} + 60.0\text{ rad} = 108\text{ rad}$

Total angular displacement from $t = 0$ until stops: $108\text{ rad} + 432\text{ rad} = 540\text{ rad}$

Note: At $t = 2.00$ s, $\omega_z = \omega_{0z} + \alpha_z t = 24.0\text{ rad/s} + (30.0\text{ rad/s}^2)(2.00\text{ s}) = 84.0\text{ rad/s}$; angular speed when breaker trips.

(b) SET UP: Consider the motion from when the circuit breaker trips until the wheel stops. For this calculation let $t = 0$ when the breaker trips.

$t = ?$; $\theta - \theta_0 = 432$ rad; $\omega_z = 0$; $\omega_{0z} = 84.0$ rad/s (from part (a))

$\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$

EXECUTE: $t = \dfrac{2(\theta - \theta_0)}{\omega_{0z} + \omega_z} = \dfrac{2(432\text{ rad})}{84.0\text{ rad/s} + 0} = 10.3\text{ s}$

The wheel stops 10.3 s after the breaker trips so $2.00\text{ s} + 10.3\text{ s} = 12.3\text{ s}$ from the beginning.

(c) SET UP: $\alpha_z = ?$; consider the same motion as in part (b):

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{0 - 84.0\text{ rad/s}}{10.3\text{ s}} = -8.16\text{ rad/s}^2$

EVALUATE: The angular acceleration is positive while the wheel is speeding up and negative while it is slowing down. We could also use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to calculate

$\alpha_z = \dfrac{\omega_z^2 - \omega_{0z}^2}{2(\theta - \theta_0)} = \dfrac{0 - (84.0\text{ rad/s})^2}{2(432\text{ rad})} = -8.16\text{ rad/s}^2$ for the acceleration after the breaker trips.

9.17. **IDENTIFY:** Apply Eq. (9.12) to relate ω_z to $\theta - \theta_0$.

SET UP: Establish a proportionality.

EXECUTE: From $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta - \theta_0)$, with $\omega_{0z} = 0$, the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.00 rev.

EVALUATE: We don't have enough information to calculate α_z; all we need to know is that it is constant.

9.18. **IDENTIFY:** The linear distance the elevator travels, its speed and the magnitude of its acceleration are equal to the tangential displacement, speed and acceleration of a point on the rim of the disk. $s = r\theta$, $v = r\omega$ and $a = r\alpha$. In these equations the angular quantities must be in radians.

SET UP: $1\text{ rev} = 2\pi$ rad. $1\text{ rpm} = 0.1047$ rad/s. $\pi\text{ rad} = 180°$. For the disk, $r = 1.25$ m.

EXECUTE: **(a)** $v = 0.250 \text{ m/s}$ so $\omega = \dfrac{v}{r} = \dfrac{0.250 \text{ m/s}}{1.25 \text{ m}} = 0.200 \text{ rad/s} = 1.91 \text{ rpm}.$

(b) $a = \frac{1}{8}g = 1.225 \text{ m/s}^2.$ $\alpha = \dfrac{a}{r} = \dfrac{1.225 \text{ m/s}^2}{1.25 \text{ m}} = 0.980 \text{ rad/s}^2.$

(c) $s = 3.25 \text{ m}.$ $\theta = \dfrac{s}{r} = \dfrac{3.25 \text{ m}}{1.25 \text{ m}} = 2.60 \text{ rad} = 149°.$

EVALUATE: When we use $s = r\theta,$ $v = r\omega$ and $a_{\tan} = r\alpha$ to solve for $\theta,$ ω and $\alpha,$ the results are in rad, rad/s, and $\text{rad/s}^2.$

9.19. **IDENTIFY:** When the angular speed is constant, $\omega = \theta/t.$ $v_{\tan} = r\omega,$ $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2.$ In these equations radians must be used for the angular quantities.

SET UP: The radius of the earth is $R_{\text{E}} = 6.37 \times 10^6 \text{ m}$ and the earth rotates once in $1 \text{ day} = 86{,}400 \text{ s}.$ The orbit radius of the earth is $1.50 \times 10^{11} \text{ m}$ and the earth completes one orbit in $1 \text{ y} = 3.156 \times 10^7 \text{ s}.$ When ω is constant, $\omega = \theta/t.$

EXECUTE: **(a)** $\theta = 1 \text{ rev} = 2\pi \text{ rad}$ in $t = 3.156 \times 10^7 \text{ s}.$ $\omega = \dfrac{2\pi \text{ rad}}{3.156 \times 10^7 \text{ s}} = 1.99 \times 10^{-7} \text{ rad/s}.$

(b) $\theta = 1 \text{ rev} = 2\pi \text{ rad}$ in $t = 86{,}400 \text{ s}.$ $\omega = \dfrac{2\pi \text{ rad}}{86{,}400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$

(c) $v = r\omega = (1.50 \times 10^{11} \text{ m})(1.99 \times 10^{-7} \text{ rad/s}) = 2.98 \times 10^4 \text{ m/s}.$

(d) $v = r\omega = (6.37 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 463 \text{ m/s}.$

(e) $a_{\text{rad}} = r\omega^2 = (6.37 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s})^2 = 0.0337 \text{ m/s}^2.$ $a_{\tan} = r\alpha = 0.$ $\alpha = 0$ since the angular velocity is constant.

EVALUATE: The tangential speeds associated with these motions are large even though the angular speeds are very small, because the radius for the circular path in each case is quite large.

9.20. **IDENTIFY:** Linear and angular velocities are related by $v = r\omega.$ Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate $\alpha_z.$

SET UP: $\omega = v/r$ gives ω in rad/s.

EXECUTE: **(a)** $\dfrac{1.25 \text{ m/s}}{25.0 \times 10^{-3} \text{ m}} = 50.0 \text{ rad/s},$ $\dfrac{1.25 \text{ m/s}}{58.0 \times 10^{-3} \text{ m}} = 21.6 \text{ rad/s}.$

(b) $(1.25 \text{ m/s})(74.0 \text{ min})(60 \text{ s/min}) = 5.55 \text{ km}.$

(c) $\alpha_z = \dfrac{21.55 \text{ rad/s} - 50.0 \text{ rad/s}}{(74.0 \text{ min})(60 \text{ s/min})} = -6.41 \times 10^{-3} \text{ rad/s}^2.$

EVALUATE: The width of the tracks is very small, so the total track length on the disc is huge.

9.21. **IDENTIFY:** Use constant acceleration equations to calculate the angular velocity at the end of two revolutions. $v = r\omega.$

SET UP: $2 \text{ rev} = 4\pi \text{ rad}.$ $r = 0.200 \text{ m}.$

EXECUTE: **(a)** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0).$ $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(3.00 \text{ rad/s}^2)(4\pi \text{ rad})} = 8.68 \text{ rad/s}.$

$a_{\text{rad}} = r\omega^2 = (0.200 \text{ m})(8.68 \text{ rad/s})^2 = 15.1 \text{ m/s}^2.$

(b) $v = r\omega = (0.200 \text{ m})(8.68 \text{ rad/s}) = 1.74 \text{ m/s}.$ $a_{\text{rad}} = \dfrac{v^2}{r} = \dfrac{(1.74 \text{ m/s})^2}{0.200 \text{ m}} = 15.1 \text{ m/s}^2.$

EVALUATE: $r\omega^2$ and v^2/r are completely equivalent expressions for $a_{\text{rad}}.$

9.22. **IDENTIFY:** $v = r\omega$ and $a_{\tan} = r\alpha.$

SET UP: The linear acceleration of the bucket equals $a_{\tan}$ for a point on the rim of the axle.

EXECUTE: **(a)** $v = R\omega.$ $2.00 \text{ cm/s} = R\left(\dfrac{7.5 \text{ rev}}{\text{min}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)$ gives $R = 2.55 \text{ cm}.$

$D = 2R = 5.09 \text{ cm}.$

(b) $a_{tan} = R\alpha$. $\alpha = \dfrac{a_{tan}}{R} = \dfrac{0.400\ \text{m/s}^2}{0.0255\ \text{m}} = 15.7\ \text{rad/s}^2$.

EVALUATE: In $v = R\omega$ and $a_{tan} = R\alpha$, ω and α must be in radians.

9.23. **IDENTIFY** and **SET UP:** Use constant acceleration equations to find ω and α after each displacement. Use $a_{tan} = R\alpha$ and $a_{rad} = r\omega^2$ to find the components of the linear acceleration.

EXECUTE: **(a)** <u>at the start</u> $t = 0$

flywheel starts from rest so $\omega = \omega_{0z} = 0$

$a_{tan} = r\alpha = (0.300\ \text{m})(0.600\ \text{rad/s}^2) = 0.180\ \text{m/s}^2$

$a_{rad} = r\omega^2 = 0$

$a = \sqrt{a_{rad}^2 + a_{tan}^2} = 0.180\ \text{m/s}^2$

(b) $\underline{\theta - \theta_0 = 60^\circ}$

$a_{tan} = r\alpha = 0.180\ \text{m/s}^2$

Calculate ω:

$\theta - \theta_0 = 60^\circ(\pi\ \text{rad}/180^\circ) = 1.047\ \text{rad}$; $\omega_{0z} = 0$; $\alpha_z = 0.600\ \text{rad/s}^2$; $\omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\ \text{rad/s}^2)(1.047\ \text{rad})} = 1.121\ \text{rad/s}$ and $\omega = \omega_z$.

Then $a_{rad} = r\omega^2 = (0.300\ \text{m})(1.121\ \text{rad/s})^2 = 0.377\ \text{m/s}^2$.

$a = \sqrt{a_{rad}^2 + a_{tan}^2} = \sqrt{(0.377\ \text{m/s}^2)^2 + (0.180\ \text{m/s}^2)^2} = 0.418\ \text{m/s}^2$

(c) $\underline{\theta - \theta_0 = 120^\circ}$

$a_{tan} = r\alpha = 0.180\ \text{m/s}^2$

Calculate ω:

$\theta - \theta_0 = 120^\circ(\pi\ \text{rad}/180^\circ) = 2.094\ \text{rad}$; $\omega_{0z} = 0$; $\alpha_z = 0.600\ \text{rad/s}^2$; $\omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\ \text{rad/s}^2)(2.094\ \text{rad})} = 1.585\ \text{rad/s}$ and $\omega = \omega_z$.

Then $a_{rad} = r\omega^2 = (0.300\ \text{m})(1.585\ \text{rad/s})^2 = 0.754\ \text{m/s}^2$.

$a = \sqrt{a_{rad}^2 + a_{tan}^2} = \sqrt{(0.754\ \text{m/s}^2)^2 + (0.180\ \text{m/s}^2)^2} = 0.775\ \text{m/s}^2$.

EVALUATE: α is constant so α_{tan} is constant. ω increases so a_{rad} increases.

9.24. **IDENTIFY:** Apply constant angular acceleration equations. $v = r\omega$. A point on the rim has both tangential and radial components of acceleration.

SET UP: $a_{tan} = r\alpha$ and $a_{rad} = r\omega^2$.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 0.250\ \text{rev/s} + (0.900\ \text{rev/s}^2)(0.200\ \text{s}) = 0.430\ \text{rev/s}$

(Note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians).

(b) $\omega_{av\text{-}z}\Delta t = (0.340\ \text{rev/s})(0.2\ \text{s}) = 0.068\ \text{rev}$.

(c) Here, the conversion to radians must be made to use $v = r\omega$, and

$$v = r\omega = \left(\frac{0.750\ \text{m}}{2}\right)(0.430\ \text{rev/s})(2\pi\ \text{rad/rev}) = 1.01\ \text{m/s}.$$

(d) Combining $a_{\text{rad}} = r\omega^2$ and $a_{\tan} = R\alpha$,

$$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}.$$

$$a = \sqrt{\left[((0.430 \text{ rev/s})(2\pi \text{ rad/rev}))^2(0.375 \text{ m})\right]^2 + \left[(0.900 \text{ rev/s}^2)(2\pi \text{ rad/rev})(0.375 \text{ m})\right]^2}.$$

$$a = 3.46 \text{ m/s}^2.$$

EVALUATE: If the angular acceleration is constant, $a_{\tan}$ is constant but a_{rad} increases as ω increases.

9.25. **IDENTIFY:** Use $a_{\text{rad}} = r\omega^2$ and solve for r.

SET UP: $a_{\text{rad}} = r\omega^2$ so $r = a_{\text{rad}}/\omega^2$, where ω must be in rad/s

EXECUTE: $a_{\text{rad}} = 3000g = 3000(9.80 \text{ m/s}^2) = 29{,}400 \text{ m/s}^2$

$$\omega = (5000 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 523.6 \text{ rad/s}$$

Then $r = \dfrac{a_{\text{rad}}}{\omega^2} = \dfrac{29{,}400 \text{ m/s}^2}{(523.6 \text{ rad/s})^2} = 0.107 \text{ m}.$

EVALUATE: The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

9.26. **IDENTIFY:** $a_{\tan} = r\alpha$, $v = r\omega$ and $a_{\text{rad}} = v^2/r$. $\theta - \theta_0 = \omega_{\text{av-}z}t$.

SET UP: When α_z is constant, $\omega_{\text{av-}z} = \dfrac{\omega_{0z} + \omega_z}{2}$. Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\alpha = \dfrac{a_{\tan}}{r} = \dfrac{-10.0 \text{ m/s}^2}{0.200 \text{ m}} = -50.0 \text{ rad/s}^2$

(b) At $t = 3.00$ s, $v = 50.0$ m/s and $\omega = \dfrac{v}{r} = \dfrac{50.0 \text{ m/s}}{0.200 \text{ m}} = 250 \text{ rad/s}$ and at $t = 0$,

$v = 50.0 \text{ m/s} + (-10.0 \text{ m/s}^2)(0 - 3.00 \text{ s}) = 80.0 \text{ m/s}$, $\omega = 400 \text{ rad/s}$.

(c) $\omega_{\text{av-}z}t = (325 \text{ rad/s})(3.00 \text{ s}) = 975 \text{ rad} = 155 \text{ rev}$.

(d) $v = \sqrt{a_{\text{rad}}r} = \sqrt{(9.80 \text{ m/s}^2)(0.200 \text{ m})} = 1.40 \text{ m/s}$. This speed will be reached at time $\dfrac{50.0 \text{ m/s} - 1.40 \text{ m/s}}{10.0 \text{ m/s}^2} = 4.86 \text{ s}$ after $t = 3.00$ s, or at $t = 7.86$ s. (There are many equivalent ways to do this calculation.)

EVALUATE: At $t = 0$, $a_{\text{rad}} = r\omega^2 = 3.20\times 10^4 \text{ m/s}^2$. At $t = 3.00$ s, $a_{\text{rad}} = 1.25\times 10^4 \text{ m/s}^2$. For $a_{\text{rad}} = g$ the wheel must be rotating more slowly than at 3.00 s so it occurs some time after 3.00 s.

9.27. **IDENTIFY:** $v = r\omega$ and $a_{\text{rad}} = r\omega^2 = v^2/r$.

SET UP: 2π rad $= 1$ rev, so π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $\omega r = (1250 \text{ rev/min})\left(\dfrac{\pi \text{ rad/s}}{30 \text{ rev/min}}\right)\left(\dfrac{12.7\times 10^{-3} \text{ m}}{2}\right) = 0.831 \text{ m/s}.$

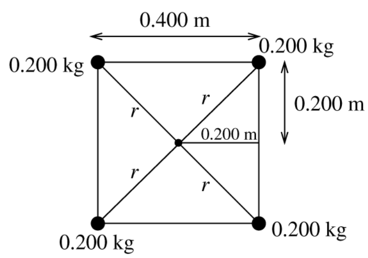
(b) $\dfrac{v^2}{r} = \dfrac{(0.831 \text{ m/s})^2}{(12.7\times 10^{-3} \text{ m})/2} = 109 \text{ m/s}^2.$

EVALUATE: In $v = r\omega$, ω must be in rad/s.

9.28. **IDENTIFY** and **SET UP:** Use $I = \sum m_i r_i^2$. Treat the spheres as point masses and ignore I of the light rods.

EXECUTE: The object is shown in Figure 9.28a.

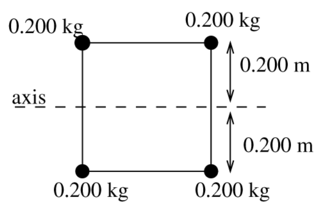
(a)



$$r = \sqrt{(0.200 \text{ m})^2 + (0.200 \text{ m})^2} = 0.2828 \text{ m}$$
$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.2828 \text{ m})^2$$
$$I = 0.0640 \text{ kg} \cdot \text{m}^2$$

Figure 9.28a

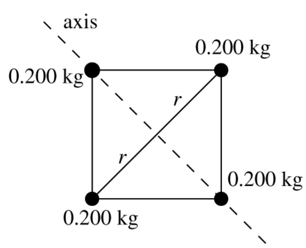
(b) The object is shown in Figure 9.28b.



$$r = 0.200 \text{ m}$$
$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.200 \text{ m})^2$$
$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

Figure 9.28b

(c) The object is shown in Figure 9.28c.



$$r = 0.2828 \text{ m}$$
$$I = \sum m_i r_i^2 = 2(0.200 \text{ kg})(0.2828 \text{ m})^2$$
$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

Figure 9.28c

EVALUATE: In general I depends on the axis and our answer for part (a) is larger than for parts (b) and (c). It just happens that I is the same in parts (b) and (c).

9.29. **IDENTIFY:** Use Table 9.2. The correct expression to use in each case depends on the shape of the object and the location of the axis.

SET UP: In each case express the mass in kg and the length in m, so the moment of inertia will be in $\text{kg} \cdot \text{m}^2$.

EXECUTE: **(a)** (i) $I = \frac{1}{3}ML^2 = \frac{1}{3}(2.50 \text{ kg})(0.750 \text{ m})^2 = 0.469 \text{ kg} \cdot \text{m}^2$.

(ii) $I = \frac{1}{12}ML^2 = \frac{1}{4}(0.469 \text{ kg} \cdot \text{m}^2) = 0.117 \text{ kg} \cdot \text{m}^2$. (iii) For a very thin rod, all of the mass is at the axis and $I = 0$.

(b) (i) $I = \frac{2}{5}MR^2 = \frac{2}{5}(3.00 \text{ kg})(0.190 \text{ m})^2 = 0.0433 \text{ kg} \cdot \text{m}^2$.

(ii) $I = \frac{2}{3}MR^2 = \frac{5}{3}(0.0433 \text{ kg} \cdot \text{m}^2) = 0.0722 \text{ kg} \cdot \text{m}^2$.

(c) (i) $I = MR^2 = (8.00 \text{ kg})(0.0600 \text{ m})^2 = 0.0288 \text{ kg} \cdot \text{m}^2$.

(ii) $I = \frac{1}{2}MR^2 = \frac{1}{2}(8.00 \text{ kg})(0.0600 \text{ m})^2 = 0.0144 \text{ kg} \cdot \text{m}^2$.

EVALUATE: I depends on how the mass of the object is distributed relative to the axis.

9.30. **IDENTIFY:** Treat each block as a point mass, so for each block $I = mr^2$, where r is the distance of the block from the axis. The total I for the object is the sum of the I for each of its pieces.

SET UP: In part (a) two blocks are a distance $L/2$ from the axis and the third block is on the axis. In part (b) two blocks are a distance $L/4$ from the axis and one is a distance $3L/4$ from the axis.

EXECUTE: **(a)** $I = 2m(L/2)^2 = \frac{1}{2}mL^2$.

(b) $I = 2m(L/4)^2 + m(3L/4)^2 = \dfrac{1}{16}mL^2(2+9) = \dfrac{11}{16}mL^2$.

EVALUATE: For the same object I is in general different for different axes.

9.31. **IDENTIFY:** I for the object is the sum of the values of I for each part.

SET UP: For the bar, for an axis perpendicular to the bar, use the appropriate expression from Table 9.2. For a point mass, $I = mr^2$, where r is the distance of the mass from the axis.

EXECUTE: **(a)** $I = I_{\text{bar}} + I_{\text{balls}} = \dfrac{1}{12}M_{\text{bar}}L^2 + 2m_{\text{balls}}\left(\dfrac{L}{2}\right)^2$.

$$I = \frac{1}{12}(4.00\text{ kg})(2.00\text{ m})^2 + 2(0.300\text{ kg})(1.00\text{ m})^2 = 1.93\text{ kg}\cdot\text{m}^2$$

(b) $I = \dfrac{1}{3}m_{\text{bar}}L^2 + m_{\text{ball}}L^2 = \dfrac{1}{3}(4.00\text{ kg})(2.00\text{ m})^2 + (0.300\text{ kg})(2.00\text{ m})^2 = 6.53\text{ kg}\cdot\text{m}^2$

(c) $I = 0$ because all masses are on the axis.

(d) All the mass is a distance $d = 0.500$ m from the axis and

$I = m_{\text{bar}}d^2 + 2m_{\text{ball}}d^2 = M_{\text{Total}}d^2 = (4.60\text{ kg})(0.500\text{ m})^2 = 1.15\text{ kg}\cdot\text{m}^2$.

EVALUATE: I for an object depends on the location and direction of the axis.

9.32. **IDENTIFY:** Moment of inertia of a bar.

SET UP: $I_{\text{end}} = \dfrac{1}{3}ML^2$, $I_{\text{center}} = \dfrac{1}{12}ML^2$

EXECUTE: **(a)** $\dfrac{1}{12}ML^2 = (0.400\text{ kg})(0.600\text{ m})^2/12 = 0.0120\ \text{kg}\cdot\text{m}^2$.

(b) Now we want the moment of inertia of two bars about their ends. Each has mass M/2 and length L/2.

$\dfrac{1}{3}ML^2 = \dfrac{1}{3}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2 + \dfrac{1}{3}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2 = \dfrac{1}{12}ML^2 = 0.0120\ \text{kg}\cdot\text{m}^2$.

EVALUATE: Neither the bend nor the 60° angle affects the moment of inertia. In (a) and (b), we can think of the rod as two 0.200-kg rods, each 0.300 m long, with the moment of inertia calculated about one end.

9.33. **IDENTIFY** and **SET UP:** $I = \sum m_i r_i^2$ implies $I = I_{\text{rim}} + I_{\text{spokes}}$

EXECUTE: $I_{\text{rim}} = MR^2 = (1.40\text{ kg})(0.300\text{ m})^2 = 0.126\text{ kg}\cdot\text{m}^2$

Each spoke can be treated as a slender rod with the axis through one end, so

$I_{\text{spokes}} = 8(\frac{1}{3}ML^2) = \frac{8}{3}(0.280\text{ kg})(0.300\text{ m})^2 = 0.0672\text{ kg}\cdot\text{m}^2$

$I = I_{\text{rim}} + I_{\text{spokes}} = 0.126\text{ kg}\cdot\text{m}^2 + 0.0672\text{ kg}\cdot\text{m}^2 = 0.193\text{ kg}\cdot\text{m}^2$

EVALUATE: Our result is smaller than $m_{\text{tot}}R^2 = (3.64\text{ kg})(0.300\text{ m})^2 = 0.328\text{ kg}\cdot\text{m}^2$, since the mass of each spoke is distributed between $r = 0$ and $r = R$.

9.34. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to calculate I.

SET UP: $I = \frac{1}{12}ML^2$. 1 rpm = 0.1047 rad/s

EXECUTE: **(a)** $I = \frac{1}{12}(117\text{ kg})(2.08\text{ m})^2 = 42.2\text{ kg}\cdot\text{m}^2$. $\omega = (2400\text{ rev/min})\left(\dfrac{0.1047\text{ rad/s}}{1\text{ rev/min}}\right) = 251\text{ rad/s}$.

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\text{ kg}\cdot\text{m}^2)(251\text{ rad/s})^2 = 1.33\times10^6\text{ J}$.

(b) $K_1 = \frac{1}{12}M_1L_1^2\omega_1^2$, $K_2 = \frac{1}{12}M_2L_2^2\omega_2^2$. $L_1 = L_2$ and $K_1 = K_2$, so $M_1\omega_1^2 = M_2\omega_2^2$.

$$\omega_2 = \omega_1\sqrt{\frac{M_1}{M_2}} = (2400\ \text{rpm})\sqrt{\frac{M_1}{0.750M_1}} = 2770\ \text{rpm}$$

EVALUATE: The rotational kinetic energy is proportional to the square of the angular speed and directly proportional to the mass of the object.

9.35. **IDENTIFY:** I for the compound disk is the sum of I of the solid disk and of the ring.

SET UP: For the solid disk, $I = \frac{1}{2}m_\text{d}r_\text{d}^2$. For the ring, $I_\text{r} = \frac{1}{2}m_\text{r}(r_1^2 + r_2^2)$, where $r_1 = 50.0$ cm, $r_2 = 70.0$ cm. The mass of the disk and ring is their area times their area density.

EXECUTE: $I = I_\text{d} + I_\text{r}$.

Disk: $m_\text{d} = (3.00\ \text{g/cm}^2)\pi r_\text{d}^2 = 23.56$ kg. $I_\text{d} = \dfrac{1}{2}m_\text{d}r_\text{d}^2 = 2.945\ \text{kg}\cdot\text{m}^2$.

Ring: $m_\text{r} = (2.00\ \text{g/cm}^2)\pi(r_2^2 - r_1^2) = 15.08$ kg. $I_\text{r} = \dfrac{1}{2}m_\text{r}(r_1^2 + r_2^2) = 5.580\ \text{kg}\cdot\text{m}^2$.

$I = I_\text{d} + I_\text{r} = 8.52\ \text{kg}\cdot\text{m}^2$.

EVALUATE: Even though $m_\text{r} < m_\text{d}$, $I_\text{r} > I_\text{d}$ since the mass of the ring is farther from the axis.

9.36. **IDENTIFY:** We can use angular kinematics (for constant angular acceleration) to find the angular velocity of the wheel. Then knowing its kinetic energy, we can find its moment of inertia, which is the target variable.

SET UP: $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ and $K = \dfrac{1}{2}I\omega^2$.

EXECUTE: Converting the angle to radians gives $\theta - \theta_0 = (8.20\ \text{rev})(2\pi\ \text{rad/1 rev}) = 51.52$ rad.

$\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = \dfrac{2(\theta - \theta_0)}{t} = \dfrac{2(51.52\ \text{rad})}{12.0\ \text{s}} = 8.587$ rad/s. Solving $K = \dfrac{1}{2}I\omega^2$ for I gives

$$I = \frac{2K}{\omega^2} = \frac{2(36.0\ \text{J})}{(8.587\ \text{rad/s})^2} = 0.976\ \text{kg}\cdot\text{m}^2.$$

EVALUATE: The angular velocity must be in radians to use the formula $K = \dfrac{1}{2}I\omega^2$.

9.37. **IDENTIFY:** Knowing the kinetic energy, mass and radius of the sphere, we can find its angular velocity. From this we can find the tangential velocity (the target variable) of a point on the rim.

SET UP: $K = \frac{1}{2}I\omega^2$ and $I = \frac{2}{5}MR^2$ for a solid uniform sphere. The tagential velocity is $v = r\omega$.

EXECUTE: $I = \frac{2}{5}MR^2 = \frac{2}{5}(28.0\ \text{kg})(0.380\ \text{m})^2 = 1.617\ \text{kg}\cdot\text{m}^2$. $K = \frac{1}{2}I\omega^2$ so

$$\omega = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(236\ \text{J})}{1.617\ \text{kg}\cdot\text{m}^2}} = 17.085\ \text{rad/s}.$$

$v = r\omega = (0.380\ \text{m})(17.085\ \text{rad/s}) = 6.49$ m/s.

EVALUATE: This is the speed of a point on the surface of the sphere that is farthest from the axis of rotation (the "equator" of the sphere). Points off the "equator" would have smaller tangential velocity but the same angular velocity.

9.38. **IDENTIFY:** Knowing the angular acceleration of the sphere, we can use angular kinematics (with constant angular acceleration) to find its angular velocity. Then using its mass and radius, we can find its kinetic energy, the target variable.

SET UP: $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, $K = \frac{1}{2}I\omega^2$, and $I = \frac{2}{3}MR^2$ for a uniform hollow spherical shell.

EXECUTE: $I = \frac{2}{3}MR^2 = \frac{2}{3}(8.20\ \text{kg})(0.220\ \text{m})^2 = 0.2646\ \text{kg}\cdot\text{m}^2$. Converting the angle to radians gives $\theta - \theta_0 = (6.00\ \text{rev})(2\pi\ \text{rad/1 rev}) = 37.70$ rad. The angular velocity is $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, which gives

$$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.890\ \text{rad/s}^2)(37.70\ \text{rad})} = 8.192\ \text{rad/s}.$$

$K = \frac{1}{2}(0.2646 \text{ kg} \cdot \text{m}^2)(8.192 \text{ rad/s})^2 = 8.88 \text{ J}.$

EVALUATE: The angular velocity must be in radians to use the formula $K = \frac{1}{2}I\omega^2$.

9.39. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$, with ω in rad/s. Solve for I.

SET UP: 1 rev/min $= (2\pi/60)$ rad/s. $\Delta K = -500$ J

EXECUTE: $\omega_i = 650 \text{ rev/min} = 68.1 \text{ rad/s}$. $\omega_f = 520 \text{ rev/min} = 54.5 \text{ rad/s}$. $\Delta K = K_f - K_i = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$

and $I = \dfrac{2(\Delta K)}{\omega_f^2 - \omega_i^2} = \dfrac{2(-500 \text{ J})}{(54.5 \text{ rad/s})^2 - (68.1 \text{ rad/s})^2} = 0.600 \text{ kg} \cdot \text{m}^2.$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s.

9.40. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to relate I to the mass M of the disk.

SET UP: 45.0 rpm $= 4.71$ rad/s. For a uniform solid disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** $I = \dfrac{2K}{\omega^2} = \dfrac{2(0.250 \text{ J})}{(4.71 \text{ rad/s})^2} = 0.0225 \text{ kg} \cdot \text{m}^2.$

(b) $I = \frac{1}{2}MR^2$ and $M = \dfrac{2I}{R^2} = \dfrac{2(0.0225 \text{ kg} \cdot \text{m}^2)}{(0.300 \text{ m})^2} = 0.500 \text{ kg}.$

EVALUATE: No matter what the shape is, the rotational kinetic energy is proportional to the mass of the object.

9.41. **IDENTIFY** and **SET UP:** Combine $K = \frac{1}{2}I\omega^2$ and $a_{rad} = r\omega^2$ to solve for K. Use Table 9.2 to get I.

EXECUTE: $K = \frac{1}{2}I\omega^2$

$a_{rad} = R\omega^2$, so $\omega = \sqrt{a_{rad}/R} = \sqrt{(3500 \text{ m/s}^2)/1.20 \text{ m}} = 54.0 \text{ rad/s}$

For a disk, $I = \frac{1}{2}MR^2 = \frac{1}{2}(70.0 \text{ kg})(1.20 \text{ m})^2 = 50.4 \text{ kg} \cdot \text{m}^2$

Thus $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(50.4 \text{ kg} \cdot \text{m}^2)(54.0 \text{ rad/s})^2 = 7.35 \times 10^4 \text{ J}$

EVALUATE: The limit on a_{rad} limits ω which in turn limits K.

9.42. **IDENTIFY:** The work done on the cylinder equals its gain in kinetic energy.

SET UP: The work done on the cylinder is PL, where L is the length of the rope. $K_1 = 0$. $K_2 = \frac{1}{2}I\omega^2$.

$I = mr^2 = \left(\dfrac{w}{g}\right)r^2.$

EXECUTE: $PL = \dfrac{1}{2}\dfrac{w}{g}v^2$, or $P = \dfrac{1}{2}\dfrac{w}{g}\dfrac{v^2}{L} = \dfrac{(40.0 \text{ N})(6.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(5.00 \text{ m})} = 14.7 \text{ N}.$

EVALUATE: The linear speed v of the end of the rope equals the tangential speed of a point on the rim of the cylinder. When K is expressed in terms of v, the radius r of the cylinder doesn't appear.

9.43. **IDENTIFY:** Apply conservation of energy to the system of stone plus pulley. $v = r\omega$ relates the motion of the stone to the rotation of the pulley.

SET UP: For a uniform solid disk, $I = \frac{1}{2}MR^2$. Let point 1 be when the stone is at its initial position and point 2 be when it has descended the desired distance. Let $+y$ be upward and take $y = 0$ at the initial position of the stone, so $y_1 = 0$ and $y_2 = -h$, where h is the distance the stone descends.

EXECUTE: **(a)** $K_p = \frac{1}{2}I_p\omega^2$. $I_p = \frac{1}{2}M_pR^2 = \frac{1}{2}(2.50 \text{ kg})(0.200 \text{ m})^2 = 0.0500 \text{ kg} \cdot \text{m}^2$.

$\omega = \sqrt{\dfrac{2K_p}{I_p}} = \sqrt{\dfrac{2(4.50 \text{ J})}{0.0500 \text{ kg} \cdot \text{m}^2}} = 13.4$ rad/s. The stone has speed $v = R\omega = (0.200 \text{ m})(13.4 \text{ rad/s}) = 2.68 \text{ m/s}.$

The stone has kinetic energy $K_s = \frac{1}{2}mv^2 = \frac{1}{2}(1.50\text{ kg})(2.68\text{ m/s})^2 = 5.39\text{ J}$. $K_1 + U_1 = K_2 + U_2$ gives $0 = K_2 + U_2$. $0 = 4.50\text{ J} + 5.39\text{ J} + mg(-h)$. $h = \dfrac{9.89\text{ J}}{(1.50\text{ kg})(9.80\text{ m/s}^2)} = 0.673\text{ m}$.

(b) $K_{tot} = K_p + K_s = 9.89\text{ J}$. $\dfrac{K_p}{K_{tot}} = \dfrac{4.50\text{ J}}{9.89\text{ J}} = 45.5\%$.

EVALUATE: The gravitational potential energy of the pulley doesn't change as it rotates. The tension in the wire does positive work on the pulley and negative work of the same magnitude on the stone, so no net work on the system.

9.44. **IDENTIFY:** $K_p = \frac{1}{2}I\omega^2$ for the pulley and $K_b = \frac{1}{2}mv^2$ for the bucket. The speed of the bucket and the rotational speed of the pulley are related by $v = R\omega$.

SET UP: $K_p = \frac{1}{2}K_b$

EXECUTE: $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mv^2) = \frac{1}{4}mR^2\omega^2$. $I = \frac{1}{2}mR^2$.

EVALUATE: The result is independent of the rotational speed of the pulley and the linear speed of the mass.

9.45. **IDENTIFY:** With constant acceleration, we can use kinematics to find the speed of the falling object. Then we can apply the work-energy expression to the entire system and find the moment of inertia of the wheel. Finally, using its radius we can find its mass, the target variable.

SET UP: With constant acceleration, $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$. The angular velocity of the wheel is related to the linear velocity of the falling mass by $\omega_z = \dfrac{v_y}{R}$. The work-energy theorem is $K_1 + U_1 + W_{other} = K_2 + U_2$, and the moment of inertia of a uniform disk is $I = \frac{1}{2}MR^2$.

EXECUTE: Find v_y, the velocity of the block after it has descended 3.00 m. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = \dfrac{2(y - y_0)}{t} = \dfrac{2(3.00\text{ m})}{2.00\text{ s}} = 3.00\text{ m/s}$. For the wheel, $\omega_z = \dfrac{v_y}{R} = \dfrac{3.00\text{ m/s}}{0.280\text{ m}} = 10.71\text{ rad/s}$. Apply the work-energy expression: $K_1 + U_1 + W_{other} = K_2 + U_2$, giving $mg(3.00\text{ m}) = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$. Solving for I gives

$$I = \frac{2}{\omega^2}\left[mg(3.00\text{ m}) - \frac{1}{2}mv^2\right].$$

$I = \dfrac{2}{(10.71\text{ rad/s})^2}\left[(4.20\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m}) - \dfrac{1}{2}(4.20\text{ kg})(3.00\text{ m/s})^2\right]$. $I = 1.824\text{ kg}\cdot\text{m}^2$. For a solid disk, $I = \frac{1}{2}MR^2$ gives $M = \dfrac{2I}{R^2} = \dfrac{2(1.824\text{ kg}\cdot\text{m}^2)}{(0.280\text{ m})^2} = 46.5\text{ kg}$.

EVALUATE: The gravitational potential of the falling object is converted into the kinetic energy of that object and the rotational kinetic energy of the wheel.

9.46. **IDENTIFY:** The work the person does is the negative of the work done by gravity. $W_{grav} = U_{grav,1} - U_{grav,2}$. $U_{grav} = Mgy_{cm}$.

SET UP: The center of mass of the ladder is at its center, 1.00 m from each end. $y_{cm,1} = (1.00\text{ m})\sin 53.0° = 0.799\text{ m}$. $y_{cm,2} = 1.00\text{ m}$.

EXECUTE: $W_{grav} = (9.00\text{ kg})(9.80\text{ m/s}^2)(0.799\text{ m} - 1.00\text{ m}) = -17.7\text{ J}$. The work done by the person is 17.7 J. The increase in gravitational potential energy of the ladder is $U_{grav,1} - U_{grav,2} = -W_{grav} = +17.7\text{ J}$.

EVALUATE: The gravity force is downward and the center of mass of the ladder moves upward, so gravity does negative work. The person pushes upward and does positive work.

9.47. **IDENTIFY:** The general expression for I is $I = \sum m_i r_i^2$. $K = \frac{1}{2}I\omega^2$.

SET UP: R will be multiplied by f.

EXECUTE: **(a)** In the equation $I = \sum m_i r_i^2$, each term will have the mass multiplied by f^3 and the distance multiplied by f, and so the moment of inertia is multiplied by $f^3(f)^2 = f^5$.

(b) $(2.5\text{ J})(48)^5 = 6.37\times 10^8$ J.

EVALUATE: Mass and volume are proportional to each other so both scale by the same factor.

9.48. **IDENTIFY:** Apply the parallel-axis theorem.

SET UP: The center of mass of the hoop is at its geometrical center.

EXECUTE: In the parallel-axis theorem, $I_{\text{cm}} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.

EVALUATE: I is larger for an axis at the edge than for an axis at the center. Some mass is closer than distance R from the axis but some is also farther away. Since I for each piece of the hoop is proportional to the square of the distance from the axis, the increase in distance has a larger effect.

9.49. **IDENTIFY:** Use the parallel-axis theorem to relate I for the wood sphere about the desired axis to I for an axis along a diameter.

SET UP: For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.

For a solid sphere with mass M and radius R, $I_{\text{cm}} = \frac{2}{5}MR^2$, for an axis along a diameter.

EXECUTE: Find d such that $I_P = I_{\text{cm}} + Md^2$ with $I_P = \frac{2}{3}MR^2$:

$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$

The factors of M divide out and the equation becomes $(\frac{2}{3} - \frac{2}{5})R^2 = d^2$

$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R$.

The axis is parallel to a diameter and is $0.516R$ from the center.

EVALUATE: $I_{\text{cm}}(\text{lead}) > I_{\text{cm}}(\text{wood})$ even though M and R are the same since for a hollow sphere all the mass is a distance R from the axis. The parallel-axis theorem says $I_P > I_{\text{cm}}$, so there must be a d where $I_P(\text{wood}) = I_{\text{cm}}(\text{lead})$.

9.50. **IDENTIFY:** Consider the plate as made of slender rods placed side-by-side.

SET UP: The expression in Table 9.2 gives I for a rod and an axis through the center of the rod.

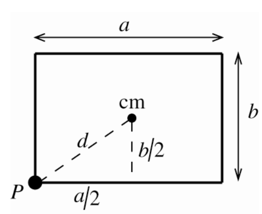
EXECUTE: **(a)** I is the same as for a rod with length a: $I = \frac{1}{12}Ma^2$.

(b) I is the same as for a rod with length b: $I = \frac{1}{12}Mb^2$.

EVALUATE: I is smaller when the axis is through the center of the plate than when it is along one edge.

9.51. **IDENTIFY** and **SET UP:** Use the parallel-axis theorem. The cm of the sheet is at its geometrical center. The object is sketched in Figure 9.51.

EXECUTE: $I_P = I_{\text{cm}} + Md^2$.



From Table 9.2, $I_{\text{cm}} = \frac{1}{12}M(a^2 + b^2)$.

The distance d of P from the cm is $d = \sqrt{(a/2)^2 + (b/2)^2}$.

Figure 9.51

Thus $I_P = I_{\text{cm}} + Md^2 = \frac{1}{12}M(a^2+b^2) + M(\frac{1}{4}a^2 + \frac{1}{4}b^2) = (\frac{1}{12} + \frac{1}{4})M(a^2+b^2) = \frac{1}{3}M(a^2+b^2)$

EVALUATE: $I_P = 4I_{\text{cm}}$. For an axis through P mass is farther from the axis.

9.52. **IDENTIFY:** Use the equations in Table 9.2. I for the rod is the sum of I for each segment. The parallel-axis theorem says $I_p = I_{cm} + Md^2$.

SET UP: The bent rod and axes a and b are shown in Figure 9.52. Each segment has length $L/2$ and mass $M/2$.

EXECUTE: **(a)** For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis through one end. For one segment, $I_s = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}ML^2$. For the rod, $I_a = 2I_s = \frac{1}{12}ML^2$.

(b) The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each segment, $I_{cm} = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{96}ML^2$. Axis b is a distance $L/4$ from the cm of each segment, so for each segment the parallel axis theorem gives I for axis b to be $I_s = \frac{1}{96}ML^2 + \frac{M}{2}\left(\frac{L}{4}\right)^2 = \frac{1}{24}ML^2$ and $I_b = 2I_s = \frac{1}{12}ML^2$.

EVALUATE: I for these two axes are the same.

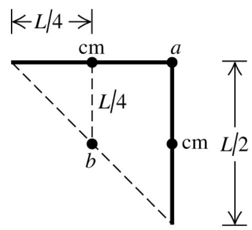


Figure 9.52

9.53. **IDENTIFY:** Apply $I = \int r^2\, dm$.

SET UP: $dm = \rho dV = \rho(2\pi rL\, dr)$, where L is the thickness of the disk. $M = \pi L\rho R^2$.

EXECUTE: The analysis is identical to that of Example 9.10, with the lower limit in the integral being zero and the upper limit being R. The result is $I = \frac{1}{2}MR^2$.

9.54. **IDENTIFY:** Use $I = \int r^2\, dm$.

SET UP:

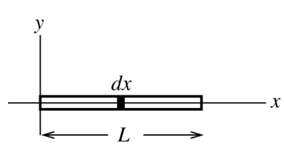


Figure 9.54

Take the x-axis to lie along the rod, with the origin at the left end. Consider a thin slice at coordinate x and width dx, as shown in Figure 9.54. The mass per unit length for this rod is M/L, so the mass of this slice is $dm = (M/L)\, dx$.

EXECUTE: $I = \int_0^L x^2 (M/L)\, dx = (M/L)\int_0^L x^2\, dx = (M/L)(L^3/3) = \frac{1}{3}ML^2$

EVALUATE: This result agrees with the table in the text.

9.55. **IDENTIFY:** Apply $I=\int r^2\,dm$ and $M=\int dm.$

SET UP: For this case, $dm=\gamma x\,dx.$

EXECUTE: **(a)** $M=\int dm=\int_0^L \gamma x\,dx=\gamma\dfrac{x^2}{2}\Big|_0^L=\dfrac{\gamma L^2}{2}$

(b) $I=\int_0^L x^2(\gamma x)dx=\gamma\dfrac{x^4}{4}\Big|_0^L=\dfrac{\gamma L^4}{4}=\dfrac{M}{2}L^2.$ This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater farther away from the axis than nearer the axis.

(c) $I=\int_0^L (L-x)^2\gamma x dx=\gamma\int_0^L (L^2x-2Lx^2+x^3)dx=\gamma\left(L^2\dfrac{x^2}{2}-2L\dfrac{x^3}{3}+\dfrac{x^4}{4}\right)\Big|_0^L=\gamma\dfrac{L^4}{12}=\dfrac{M}{6}L^2.$

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

EVALUATE: For a uniform rod with an axis at one end, $I=\frac{1}{3}ML^2.$ The result in (b) is larger than this and the result in (c) is smaller than this.

9.56. **IDENTIFY:** Using the equation for the angle as a function of time, we can find the angular acceleration of the disk at a given time and use this to find the linear acceleration of a point on the rim (the target variable).

SET UP: We can use the definitions of the angular velocity and the angular acceleration: $\omega_z(t)=\dfrac{d\theta}{dt}$ and $\alpha_z(t)=\dfrac{d\omega_z}{dt}.$ The acceleration components are $a_{\text{rad}}=R\omega^2$ and $a_{\text{tan}}=R\alpha,$ and the magnitude of the acceleration is $a=\sqrt{a_{\text{rad}}^2+a_{\text{tan}}^2}.$

EXECUTE: $\omega_z(t)=\dfrac{d\theta}{dt}=1.10\text{ rad/s}+(12.6\text{ rad/s}^2)t.$ $\alpha_z(t)=\dfrac{d\omega_z}{dt}=12.6\text{ rad/s}^2$ (constant).

$\theta=0.100\text{ rev}=0.6283\text{ rad}$ gives $6.30t^2+1.10t-0.6283=0,$ so $t=0.2403\text{ s},$ using the positive root. At this t, $\omega_z(t)=4.1278\text{ rad/s}$ and $\alpha_z(t)=12.6\text{ rad/s}^2.$ For a point on the rim, $a_{\text{rad}}=R\omega^2=6.815\text{ m/s}^2$ and $a_{\text{tan}}=R\alpha=5.04\text{ m/s}^2,$ so $a=\sqrt{a_{\text{rad}}^2+a_{\text{tan}}^2}=8.48\text{ m/s}^2.$

EVALUATE: Since the angular acceleration is constant, we could use the constant acceleration formulas as a check. For example, the coefficient of t^2 is $\frac{1}{2}\alpha_z=6.30\text{ rad/s}^2$ gives $\alpha_z=12.6\text{ rad/s}^2.$

9.57. **IDENTIFY:** The target variable is the horizontal distance the piece travels before hitting the floor. Using the angular acceleration of the blade, we can find its angular velocity when the piece breaks off. This will give us the linear horizontal speed of the piece. It is then in free fall, so we can use the linear kinematics equations.

SET UP: $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0)$ for the blade, and $v=r\omega$ is the horizontal velocity of the piece.

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ for the falling piece.

EXECUTE: Find the initial horizontal velocity of the piece just after it breaks off.

$\theta-\theta_0=(155\text{ rev})(2\pi\text{ rad/1 rev})=973.9\text{ rad}.$

$\alpha_z=(2.00\text{ rev/s}^2)(2\pi\text{ rad/1 rev})=12.566\text{ rad/s}^2.$ $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0).$

$\omega_z=\sqrt{2\alpha_z(\theta-\theta_0)}=\sqrt{2(12.566\text{ rad/s}^2)(973.9\text{ rad})}=156.45\text{ rad/s}.$ The horizontal velocity of the piece is $v=r\omega=(0.120\text{ m})(156.45\text{ rad/s})=18.774\text{ m/s}.$ Now consider the projectile motion of the piece. Take

$+y$ downward and use the vertical motion to find t. Solving $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ for t gives

$t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(0.820\text{ m})}{9.8\text{ m/s}^2}} = 0.4091\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (18.774\text{ m/s})(0.4091\text{ s}) = 7.68\text{ m}$.

EVALUATE: Once the piece is free of the blade, the only force acting on it is gravity so its acceleration is g downward.

9.58. **IDENTIFY** and **SET UP:** Use $\omega_z = \dfrac{d\theta}{dt}$ and $\alpha_z = \dfrac{d\omega_z}{dt}$. As long as $\alpha_z > 0$, ω_z increases. At the t when $\alpha_z = 0$, ω_z is at its maximum positive value and then starts to decrease when α_z becomes negative. $\theta(t) = \gamma t^2 - \beta t^3$; $\gamma = 3.20\text{ rad/s}^2$, $\beta = 0.500\text{ rad/s}^3$

EXECUTE: **(a)** $\omega_z(t) = \dfrac{d\theta}{dt} = \dfrac{d(\gamma t^2 - \beta t^3)}{dt} = 2\gamma t - 3\beta t^2$

(b) $\alpha_z(t) = \dfrac{d\omega_z}{dt} = \dfrac{d(2\gamma t - 3\beta t^2)}{dt} = 2\gamma - 6\beta t$

(c) The maximum angular velocity occurs when $\alpha_z = 0$.

$2\gamma - 6\beta t = 0$ implies $t = \dfrac{2\gamma}{6\beta} = \dfrac{\gamma}{3\beta} = \dfrac{3.20\text{ rad/s}^2}{3(0.500\text{ rad/s}^3)} = 2.133\text{ s}$

At this t, $\omega_z = 2\gamma t - 3\beta t^2 = 2(3.20\text{ rad/s}^2)(2.133\text{ s}) - 3(0.500\text{ rad/s}^3)(2.133\text{ s})^2 = 6.83\text{ rad/s}$

The maximum positive angular velocity is 6.83 rad/s and it occurs at 2.13 s.

EVALUATE: For large t both ω_z and α_z are negative and ω_z increases in magnitude. In fact, $\omega_z \to -\infty$ at $t \to \infty$. So the answer in (c) is not the largest angular speed, just the largest positive angular velocity.

9.59. **IDENTIFY:** The angular acceleration α of the disk is related to the linear acceleration a of the ball by $a = R\alpha$. Since the acceleration is not constant, use $\omega_z - \omega_{0z} = \int_0^t \alpha_z dt$ and $\theta - \theta_0 = \int_0^t \omega_z dt$ to relate θ, ω_z, α_z, and t for the disk. $\omega_{0z} = 0$.

SET UP: $\int t^n dt = \dfrac{1}{n+1} t^{n+1}$. In $a = R\alpha$, α is in rad/s^2.

EXECUTE: **(a)** $A = \dfrac{a}{t} = \dfrac{1.80\text{ m/s}^2}{3.00\text{ s}} = 0.600\text{ m/s}^3$

(b) $\alpha = \dfrac{a}{R} = \dfrac{(0.600\text{ m/s}^3)t}{0.250\text{ m}} = (2.40\text{ rad/s}^3)t$

(c) $\omega_z = \int_0^t (2.40\text{ rad/s}^3)t\,dt = (1.20\text{ rad/s}^3)t^2$. $\omega_z = 15.0\text{ rad/s}$ for $t = \sqrt{\dfrac{15.0\text{ rad/s}}{1.20\text{ rad/s}^3}} = 3.54\text{ s}$.

(d) $\theta - \theta_0 = \int_0^t \omega_z dt = \int_0^t (1.20\text{ rad/s}^3)t^2 dt = (0.400\text{ rad/s}^3)t^3$. For $t = 3.54\text{ s}$, $\theta - \theta_0 = 17.7\text{ rad}$.

EVALUATE: If the disk had turned at a constant angular velocity of 15.0 rad/s for 3.54 s it would have turned through an angle of 53.1 rad in 3.54 s. It actually turns through less than half this because the angular velocity is increasing in time and is less than 15.0 rad/s at all but the end of the interval.

9.60. **IDENTIFY:** The flywheel gains rotational kinetic energy as it spins. This kinetic energy depends on the flywheel's rate of spin but also on its moment of inertia. The angular acceleration is constant.

SET UP: $K = \frac{1}{2}I\omega^2$, $I = \frac{1}{2}mR^2$, $\omega = \omega_0 + \alpha t$, $m = \rho V = \rho\pi R^2 h$.

EXECUTE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mR^2)(\omega_0 + \alpha t)^2 = \frac{1}{4}\left[(\rho\pi R^2 h)R^2\right](0 + \alpha t)^2$. Solving for h gives

$h = \dfrac{4K}{\rho\pi R^4(\alpha t)^2} = 4(800\text{ J})/[\pi(8600\text{ kg/m}^3)(0.250\text{ m})^4(3.00\text{ rad/s}^2)^2(8.00\text{ s})^2] = 0.0526\text{ m} = 5.26\text{ cm}$.

EVALUATE: If we could turn the disk into a thin-walled cylinder of the same mass and radius, the moment of inertia would be twice as great, so we could store twice as much energy as for the given disk.

9.61. **IDENTIFY:** As it turns, the wheel gives kinetic energy to the marble, and this energy is converted into gravitational potential energy as the marble reaches its highest point in the air.

SET UP: The marble starts from rest at point A at the same level as the center of the wheel and after 20.0 revolutions it leaves the rim of the wheel at point A. $K_1 + U_1 = K_2 + U_2$ applies once the marble has left the cup. While the marble is turning with the wheel, $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ applies.

EXECUTE: Applying $K_1 + U_1 = K_2 + U_2$ gives $v_A = \sqrt{2gh}$. The marble is at the rim of the wheel, so $v_A = R\omega_A$. Using this formula in the angular velocity formula gives $(v_A/R)^2 = 0 + 2\alpha(\theta - \theta_0)$. The marble turns through 20.0 rev = 40.0π rad, $R = 0.260$ m, and $h = 12.0$ m. Solving the previous equation for α gives $\alpha = gh/40\pi R^2 = (9.80\ \text{m/s}^2)(12.0\ \text{m})/[40\pi(0.260\ \text{m})^2] = 13.8\ \text{rad/s}^2$.

EVALUATE: The marble has a tangential acceleration $a_{\text{tang}} = R\alpha = (0.260\ \text{m})(13.8\ \text{rad/s}^2) = 3.59\ \text{m/s}^2$ upward just before it leaves the cup. But this acceleration ends the instant the marble leaves the cup, and after that its acceleration is 9.80 m/s^2 downward due to gravity.

9.62. **IDENTIFY:** Apply conservation of energy to the system of drum plus falling mass, and compare the results for earth and for Mars.

SET UP: $K_{\text{drum}} = \frac{1}{2}I\omega^2$. $K_{\text{mass}} = \frac{1}{2}mv^2$. $v = R\omega$ so if K_{drum} is the same, ω is the same and v is the same on both planets. Therefore, K_{mass} is the same. Let $y = 0$ at the initial height of the mass and take $+y$ upward. Configuration 1 is when the mass is at its initial position and 2 is when the mass has descended 5.00 m, so $y_1 = 0$ and $y_2 = -h$, where h is the height the mass descends.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$ gives $0 = K_{\text{drum}} + K_{\text{mass}} - mgh$. $K_{\text{drum}} + K_{\text{mass}}$ are the same on both planets, so $mg_{\text{E}}h_{\text{E}} = mg_{\text{M}}h_{\text{M}}$. $h_{\text{M}} = h_{\text{E}}\left(\dfrac{g_{\text{E}}}{g_{\text{M}}}\right) = (5.00\ \text{m})\left(\dfrac{9.80\ \text{m/s}^2}{3.71\ \text{m/s}^2}\right) = 13.2\ \text{m}$.

(b) $mg_{\text{M}}h_{\text{M}} = K_{\text{drum}} + K_{\text{mass}}$. $\frac{1}{2}mv^2 = mg_{\text{M}}h_{\text{M}} - K_{\text{drum}}$ and

$$v = \sqrt{2g_{\text{M}}h_{\text{M}} - \frac{2K_{\text{drum}}}{m}} = \sqrt{2(3.71\ \text{m/s}^2)(13.2\ \text{m}) - \frac{2(250.0\ \text{J})}{15.0\ \text{kg}}} = 8.04\ \text{m/s}$$

EVALUATE: We did the calculations without knowing the moment of inertia I of the drum, or the mass and radius of the drum.

9.63. **IDENTIFY** and **SET UP:** All points on the belt move with the same speed. Since the belt doesn't slip, the speed of the belt is the same as the speed of a point on the rim of the shaft and on the rim of the wheel, and these speeds are related to the angular speed of each circular object by $v = r\omega$.

EXECUTE:

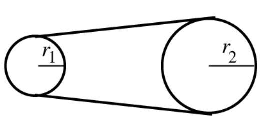


Figure 9.63

(a) $v_1 = r_1\omega_1$

$\omega_1 = (60.0\ \text{rev/s})(2\pi\ \text{rad/1 rev}) = 377\ \text{rad/s}$

$v_1 = r_1\omega_1 = (0.45\times10^{-2}\ \text{m})(377\ \text{rad/s}) = 1.70\ \text{m/s}$

(b) $v_1 = v_2$

$r_1\omega_1 = r_2\omega_2$

$\omega_2 = (r_1/r_2)\omega_1 = (0.45\ \text{cm}/1.80\ \text{cm})(377\ \text{rad/s}) = 94.2\ \text{rad/s}$

EVALUATE: The wheel has a larger radius than the shaft so turns slower to have the same tangential speed for points on the rim.

9.64. **IDENTIFY:** The speed of all points on the belt is the same, so $r_1\omega_1 = r_2\omega_2$ applies to the two pulleys.

SET UP: The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade. π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $v_2 = (2(3450\text{ rev/min}))\left(\dfrac{\pi \quad \text{rad/s}}{30\text{ rev/min}}\right)\left(\dfrac{0.208\text{ m}}{2}\right) = 75.1\text{ m/s}.$

(b) $a_{\text{rad}} = \omega^2 r = \left(2(3450\text{ rev/min})\left(\dfrac{\pi \quad \text{rad/s}}{30\text{ rev/min}}\right)\right)^2\left(\dfrac{0.208\text{ m}}{2}\right) = 5.43\times10^4\text{ m/s}^2,$

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

EVALUATE: In $v = r\omega$ and $a_{\text{rad}} = r\omega^2$, ω must be in rad/s.

9.65. **IDENTIFY:** Apply $v = r\omega$.

SET UP: Points on the chain all move at the same speed, so $r_{\text{r}}\omega_{\text{r}} = r_{\text{f}}\omega_{\text{f}}$.

EXECUTE: The angular velocity of the rear wheel is $\omega_{\text{r}} = \dfrac{v_{\text{r}}}{r_{\text{r}}} = \dfrac{5.00\text{ m/s}}{0.330\text{ m}} = 15.15\text{ rad/s}.$

The angular velocity of the front wheel is $\omega_{\text{f}} = 0.600\text{ rev/s} = 3.77\text{ rad/s}$. $r_{\text{r}} = r_{\text{f}}(\omega_{\text{f}}/\omega_{\text{r}}) = 2.99\text{ cm}.$

EVALUATE: The rear sprocket and wheel have the same angular velocity and the front sprocket and wheel have the same angular velocity. $r\omega$ is the same for both, so the rear sprocket has a smaller radius since it has a larger angular velocity. The speed of a point on the chain is $v = r_{\text{r}}\omega_{\text{r}} = (2.99\times10^{-2}\text{ m})(15.15\text{ rad/s}) =$ 0.453 m/s. The linear speed of the bicycle is 5.00 m/s.

9.66. **IDENTIFY:** Use the constant angular acceleration equations, applied to the first revolution and to the first two revolutions.

SET UP: Let the direction the disk is rotating be positive. 1 rev $= 2\pi$ rad. Let t be the time for the first revolution. The time for the first two revolutions is $t + 0.0865$ s.

EXECUTE: **(a)** $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ applied to the first revolution and then to the first two revolutions gives $2\pi\text{ rad} = \frac{1}{2}\alpha_z t^2$ and $4\pi\text{ rad} = \frac{1}{2}\alpha_z (t + 0.0865\text{ s})^2$. Eliminating α_z between these equations gives

$4\pi\text{ rad} = \dfrac{2\pi\text{ rad}}{t^2}(t + 0.0865\text{ s})^2$. $2t^2 = (t + 0.0865\text{ s})^2$. $\sqrt{2}t = \pm(t + 0.0865\text{ s})$. The positive root is

$t = \dfrac{0.0865\text{ s}}{\sqrt{2} - 1} = 0.209\text{ s}.$

(b) $2\pi\text{ rad} = \frac{1}{2}\alpha_z t^2$ and $t = 0.209$ s gives $\alpha_z = 288\text{ rad/s}^2$

EVALUATE: At the start of the second revolution, $\omega_{0z} = (288\text{ rad/s}^2)(0.209\text{ s}) = 60.19\text{ rad/s}$. The distance the disk rotates in the next 0.0865 s is

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (60.19\text{ rad/s})(0.0865\text{ s}) + \frac{1}{2}(288\text{ rad/s}^2)(0.0865\text{ s})^2 = 6.28\text{ rad},$ which is two revolutions.

9.67. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. $a_{\text{rad}} = r\omega^2$. $m = \rho V$.

SET UP: For a disk with the axis at the center, $I = \frac{1}{2}mR^2$. $V = t\pi R^2$, where $t = 0.100$ m is the thickness of the flywheel. $\rho = 7800\text{ kg/m}^3$ is the density of the iron.

EXECUTE: **(a)** $\omega = 90.0\text{ rpm} = 9.425\text{ rad/s}$. $I = \dfrac{2K}{\omega^2} = \dfrac{2(10.0\times10^6\text{ J})}{(9.425\text{ rad/s})^2} = 2.252\times10^5\text{ kg}\cdot\text{m}^2.$

$m = \rho V = \rho\pi R^2 t$. $I = \frac{1}{2}mR^2 = \frac{1}{2}\rho\pi t R^4$. This gives $R = (2I/\rho\pi t)^{1/4} = 3.68$ m and the diameter is 7.36 m.

(b) $a_{\text{rad}} = R\omega^2 = 327\text{ m/s}^2$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s. a_{rad} is about $33g$; the flywheel material must have large cohesive strength to prevent the flywheel from flying apart.

9.68. **IDENTIFY:** The moment of inertia of the section that is removed must be one-half the moment of inertia of the original disk.

SET UP: For a solid disk, $I = \frac{1}{2}mR^2$. Call m the mass of the removed piece and R its radius. $I_m = \frac{1}{2}I_{M0}$.

EXECUTE: $I_m = \frac{1}{2}I_{M0}$ gives $\frac{1}{2}mR^2 = \frac{1}{2}(\frac{1}{2}M_0R_0^2)$. We need to find m. Since the disk is uniform, the mass of a given segment will be proportional to the area of that segment. In this case, the segment is the piece cut out of the center. So $\frac{m}{M_0} = \frac{A_R}{A_{R_0}} = \frac{\pi R^2}{\pi R_0^2} = \frac{R^2}{R_0^2}$, which gives $m = M_0\left(\frac{R^2}{R_0^2}\right)$. Combining the two results gives $\frac{1}{2}M_0\left(\frac{R^2}{R_0^2}\right)R^2 = \frac{1}{2}(\frac{1}{2}M_0R_0^2)$, from which we get $R = \frac{R_0}{2^{1/4}} = 0.841R_0$.

EVALUATE: Notice that the piece that is removed does not have one-half the mass of the original disk, nor it its radius one-half the original radius.

9.69. **IDENTIFY:** The falling wood accelerates downward as the wheel undergoes angular acceleration. Newton's second law applies to the wood and the wheel, and the linear kinematics formulas apply to the wood because it has constant acceleration.

SET UP: $\Sigma\vec{F} = m\vec{a}$, $\tau = I\alpha$, $a_{\text{tan}} = R\alpha$, $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$.

EXECUTE: First use $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ to find the downward acceleration of the wood. With $v_0 = 0$, we have $a_y = 2(y - y_0)/t^2 = 2(12.0\text{ m})/(4.00\text{ s})^2 = 1.50\text{ m/s}^2$. Now apply Newton's second to the wood to find the tension in the rope. $\Sigma\vec{F} = m\vec{a}$ gives $mg - T = ma$, $T = m(g - a)$, which gives $T = (8.20\text{ kg})(9.80\text{ m/s}^2 - 1.50\text{ m/s}^2) = 68.06\text{ N}$. Now use $a_{\text{tan}} = R\alpha$ and apply Newton's second law (in its rotational form) to the wheel. $\tau = I\alpha$ gives $TR = I\alpha$, $I = TR/\alpha = TR/(a/R) = TR^2/a$

$I = (68.06\text{ N})(0.320\text{ m})^2/(1.50\text{ m/s}^2) = 4.65\text{ kg}\cdot\text{m}^2$.

EVALUATE: The tension in the rope affects the acceleration of the wood and causes the angular acceleration of the wheel.

9.70. **IDENTIFY:** Using energy considerations, the system gains as kinetic energy the lost potential energy, mgR.

SET UP: The kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, with $I = \frac{1}{2}mR^2$ for the disk. $v = R\omega$.

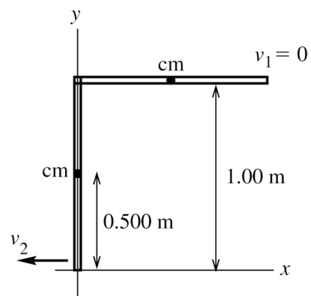
EXECUTE: $K = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)\omega^2$. Using $I = \frac{1}{2}mR^2$ and solving for ω, $\omega^2 = \frac{4}{3}\frac{g}{R}$ and

$$\omega = \sqrt{\frac{4}{3}\frac{g}{R}}.$$

EVALUATE: The small object has speed $v = \sqrt{\frac{2}{3}}\sqrt{2gR}$. If it was not attached to the disk and was dropped from a height h, it would attain a speed $\sqrt{2gR}$. Being attached to the disk reduces its final speed by a factor of $\sqrt{\frac{2}{3}}$.

9.71. **IDENTIFY:** Use conservation of energy. The stick rotates about a fixed axis so $K = \frac{1}{2}I\omega^2$. Once we have ω use $v = r\omega$ to calculate v for the end of the stick.

SET UP: The object is sketched in Figure 9.71.



Take the origin of coordinates at the lowest point reached by the stick and take the positive y-direction to be upward.

Figure 9.71

EXECUTE: **(a)** Use $U = Mgy_{cm}$. $\Delta U = U_2 - U_1 = Mg(y_{cm2} - y_{cm1})$. The center of mass of the meter stick is at its geometrical center, so $y_{cm1} = 1.00\text{ m}$ and $y_{cm2} = 0.50\text{ m}$. Then

$\Delta U = (0.180\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m} - 1.00\text{ m}) = -0.882\text{ J}$.

(b) Use conservation of energy: $K_1 + U_1 + W_{other} = K_2 + U_2$. Gravity is the only force that does work on the meter stick, so $W_{other} = 0$. $K_1 = 0$. Thus $K_2 = U_1 - U_2 = -\Delta U$, where ΔU was calculated in part (a). $K_2 = \frac{1}{2}I\omega_2^2$ so $\frac{1}{2}I\omega_2^2 = -\Delta U$ and $\omega_2 = \sqrt{2(-\Delta U)/I}$. For stick pivoted about one end, $I = \frac{1}{3}ML^2$ where $L = 1.00\text{ m}$, so $\omega_2 = \sqrt{\dfrac{6(-\Delta U)}{ML^2}} = \sqrt{\dfrac{6(0.882\text{ J})}{(0.180\text{ kg})(1.00\text{ m})^2}} = 5.42\text{ rad/s}$.

(c) $v = r\omega = (1.00\text{ m})(5.42\text{ rad/s}) = 5.42\text{ m/s}$.

(d) For a particle in free fall, with $+y$ upward, $v_{0y} = 0$; $y - y_0 = -1.00\text{ m}$; $a_y = -9.80\text{ m/s}^2$; and $v_y = ?$ Solving the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for v_y gives

$v_y = -\sqrt{2a_y(y - y_0)} = -\sqrt{2(-9.80\text{ m/s}^2)(-1.00\text{ m})} = -4.43\text{ m/s}$.

EVALUATE: The magnitude of the answer in part (c) is larger. $U_{1,grav}$ is the same for the stick as for a particle falling from a height of 1.00 m. For the stick $K = \frac{1}{2}I\omega_2^2 = \frac{1}{2}(\frac{1}{3}ML^2)(v/L)^2 = \frac{1}{6}Mv^2$. For the stick and for the particle, K_2 is the same but the same K gives a larger v for the end of the stick than for the particle. The reason is that all the other points along the stick are moving slower than the end opposite the axis.

9.72. **IDENTIFY:** The student accelerates downward and causes the wheel to turn. Newton's second law applies to the student and to the wheel. The acceleration is constant so the kinematics formulas apply.

SET UP: $\Sigma\tau = I\alpha$, $\Sigma\vec{F} = m\vec{a}$, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$.

EXECUTE: Apply $\Sigma\tau = I\alpha$ to the wheel: $TR = I\alpha = I(\alpha/R)$, so $T = I\alpha/R^2$.

Apply $\Sigma\vec{F} = m\vec{a}$ to the student: $mg - T = ma$, so $T = m(g - a)$.

Equating these two expressions for T and solving for the acceleration gives $a = \dfrac{mg}{m + I/R^2}$. Now apply kinematics for $y - y_0$ to the student, using $v_{0y} = 0$, and solve for t. $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(y - y_0)(m + I/R^2)}{mg}}$.

Putting in $y - y_0 = 12.0\text{ m}$, $m = 43.0\text{ kg}$, $I = 9.60\text{ kg}\cdot\text{m}^2$, and $R = 0.300\text{ m}$, we get $t = 2.92\text{ s}$.

Now use $v_y = v_{0y} + a_y t$ to get v_y, where $a = \dfrac{mg}{m + I/R^2}$. Putting in the numbers listed above, the result is $v_y = 8.22\text{ m/s}$.

EVALUATE: If the wheel were massless, her speed would simply be $v = \sqrt{2gy} = 15.3\text{ m/s}$, so the effect of the massive wheel reduces her speed by nearly half.

9.73. **IDENTIFY:** Mechanical energy is conserved since there is no friction.

SET UP: $K_1 + U_1 = K_2 + U_2$, $K = \frac{1}{2}I\omega^2$ (for rotational motion), $K = \frac{1}{2}mv^2$ (for linear motion),

$I = \dfrac{1}{12}ML^2$ for a slender rod.

EXECUTE: Take the initial position with the rod horizontal, and the final position with the rod vertical. The heavier sphere will be at the bottom and the lighter one at the top. Call the gravitational potential energy zero with the rod horizontal, which makes the initial potential energy zero. The initial kinetic energy is also zero. Applying $K_1 + U_1 = K_2 + U_2$ and calling A and B the spheres gives

$0 = K_A + K_B + K_{rod} + U_A + U_B + U_{rod}$. $U_{rod} = 0$ in the final position since its center of mass has not moved.

Therefore $0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}I\omega^2 + m_A g\dfrac{L}{2} - m_B g\dfrac{L}{2}$. We also know that $v_A = v_B = (L/2)\,\omega$.

Calling v the speed of the spheres, we get $0 = \frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2 + \frac{1}{2}(\frac{1}{12})(ML^2)(2v/L)^2 + m_A g\frac{L}{2} - m_B g\frac{L}{2}$

Putting in $m_A = 0.0200$ kg, $m_B = 0.0500$ kg, $M = 0.120$ kg, and $L = 800$ m, we get $v = 1.46$ m/s.

EVALUATE: As the rod turns, the heavier sphere loses potential energy but the lighter one gains potential energy.

9.74. **IDENTIFY:** Apply conservation of energy to the system of cylinder and rope.

SET UP: Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle).When the rope has unwound, its center of mass is a distance πR below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R$, and when the rope has unwound, and the cylinder has angular speed ω, the speed of the rope is ωR (the upper end of the rope has the same tangential speed at the edge of the cylinder). $I = (1/2)MR^2$ for a uniform cylinder.

EXECUTE: $K_1 = K_2 + U_2$. $\left(\dfrac{M}{4} + \dfrac{m}{2}\right)R^2\omega_0^2 = \left(\dfrac{M}{4} + \dfrac{m}{2}\right)R^2\omega^2 - mg\pi R$. Solving for ω gives

$\omega = \sqrt{\omega_0^2 + \dfrac{(4\pi mg/R)}{(M + 2m)}}$, and the speed of any part of the rope is $v = \omega R$.

EVALUATE: When $m \to 0$, $\omega \to \omega_0$, When $m >> M$, $\omega = \sqrt{\omega_0^2 + \dfrac{2\pi g}{R}}$ and $v = \sqrt{v_0^2 + 2\pi gR}$. This is the final speed when an object with initial speed v_0 descends a distance πR.

9.75. **IDENTIFY:** Apply conservation of energy to the system consisting of blocks A and B and the pulley.

SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.75.

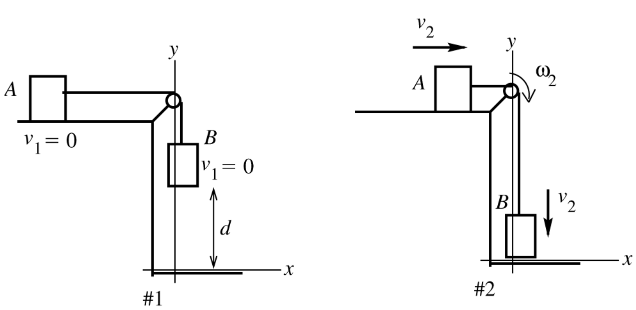


Figure 9.75

Use the work-energy relation $K_1 + U_1 + W_{other} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block B after it has descended. The tension in the rope does positive work on block A and negative work of the same magnitude on block B, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

EXECUTE: Gravity does work on block B and kinetic friction does work on block A. Therefore $W_{\text{other}} = W_f = -\mu_k m_A g d.$

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B g d;\; U_2 = m_B g y_{B2} = 0$

$K_2 = \frac{1}{2} m_A v_2^2 + \frac{1}{2} m_B v_2^2 + \frac{1}{2} I \omega_2^2.$

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2/R$ and

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2} I (v_2/R)^2 = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

Putting all this into the work-energy relation gives

$m_B g d - \mu_k m_A g d = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

$(m_A + m_B + I/R^2)v_2^2 = 2gd(m_B - \mu_k m_A)$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$$

EVALUATE: If $m_B >> m_A$ and I/R^2, then $v_2 = \sqrt{2gd}$; block B falls freely. If I is very large, v_2 is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of B will be larger than the friction force on A. I/R^2 has units of mass and is in a sense the "effective mass" of the pulley.

9.76. **IDENTIFY:** Apply conservation of energy to the system of two blocks and the pulley.

SET UP: Let the potential energy of each block be zero at its initial position. The kinetic energy of the system is the sum of the kinetic energies of each object. $v = R\omega$, where v is the common speed of the blocks and ω is the angular velocity of the pulley.

EXECUTE: The amount of gravitational potential energy which has become kinetic energy is $K = (4.00\text{ kg} - 2.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 98.0\text{ J}$. In terms of the common speed v of the blocks, the kinetic energy of the system is $K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2} I \left(\frac{v}{R}\right)^2$.

$K = v^2 \frac{1}{2}\left(4.00\text{ kg} + 2.00\text{ kg} + \frac{(0.380\text{ kg}\cdot\text{m}^2)}{(0.160\text{ m})^2}\right) = v^2(10.422\text{ kg})$. Solving for v gives

$$v = \sqrt{\frac{98.0\text{ J}}{10.422\text{ kg}}} = 3.07\text{ m/s}.$$

EVALUATE: If the pulley is massless, $98.0\text{ J} = \frac{1}{2}(4.00\text{ kg} + 2.00\text{ kg})v^2$ and $v = 5.72\text{ m/s}$. The moment of inertia of the pulley reduces the final speed of the blocks.

9.77. **IDENTIFY:** $I = I_1 + I_2$. Apply conservation of energy to the system. The calculation is similar to Example 9.8.

SET UP: $\omega = \dfrac{v}{R_1}$ for part (b) and $\omega = \dfrac{v}{R_2}$ for part (c).

EXECUTE: **(a)** $I = \frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2 = \frac{1}{2}((0.80\text{ kg})(2.50\times10^{-2}\text{ m})^2 + (1.60\text{ kg})(5.00\times10^{-2}\text{ m})^2)$

$I = 2.25\times10^{-3}\text{ kg}\cdot\text{m}^2.$

(b) The method of Example 9.8 yields $v = \sqrt{\dfrac{2gh}{1 + (I/mR_1^2)}}$.

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})}{(1 + ((2.25\times10^{-3}\text{ kg}\cdot\text{m}^2)/(1.50\text{ kg})(0.025\text{ m})^2))}} = 3.40\text{ m/s}.$$

(c) The same calculation, with R_2 instead of R_1 gives $v = 4.95\text{ m/s}$.

EVALUATE: The final speed of the block is greater when the string is wrapped around the larger disk. $v = R\omega$, so when $R = R_2$ the factor that relates v to ω is larger. For $R = R_2$ a larger fraction of the total kinetic energy resides with the block. The total kinetic energy is the same in both cases (equal to mgh), so when $R = R_2$ the kinetic energy and speed of the block are greater.

9.78. **IDENTIFY:** The potential energy of the falling block is transformed into kinetic energy of the block and kinetic energy of the turning wheel, but some of it is lost to the work by friction. Energy conservation applies, with the target variable being the angular velocity of the wheel when the block has fallen a given distance.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $K = \frac{1}{2}mv^2$, $U = mgh$, and W_{other} is the work done by friction.

EXECUTE: Energy conservation gives $mgh + (-9.00\text{ J}) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $v = R\omega$, so $\frac{1}{2}mv^2 = \frac{1}{2}mR^2\omega^2$ and $mgh + (-9.00\text{ J}) = \frac{1}{2}(mR^2 + I)\omega^2$. Solving for ω gives

$$\omega = \sqrt{\frac{2[mgh + (-9.00\text{ J})]}{mR^2 + I}} = \sqrt{\frac{2[(0.340\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m}) - 9.00\text{ J}]}{(0.340\text{ kg})(0.180\text{ m})^2 + 0.480\text{ kg}\cdot\text{m}^2}} = 2.01\text{ rad/s}.$$

EVALUATE: Friction does negative work because it opposes the turning of the wheel.

9.79. **IDENTIFY:** Apply conservation of energy to relate the height of the mass to the kinetic energy of the cylinder.

SET UP: First use $K(\text{cylinder}) = 480\text{ J}$ to find ω for the cylinder and v for the mass.

EXECUTE: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.150\text{ m})^2 = 0.1125\text{ kg}\cdot\text{m}^2$. $K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{2K/I} = 92.38\text{ rad/s}$. $v = R\omega = 13.86\text{ m/s}$.

SET UP: Use conservation of energy $K_1 + U_1 = K_2 + U_2$ to solve for the distance the mass descends. Take $y = 0$ at lowest point of the mass, so $y_2 = 0$ and $y_1 = h$, the distance the mass descends.

EXECUTE: $K_1 = U_2 = 0$ so $U_1 = K_2$. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where $m = 12.0\text{ kg}$. For the cylinder, $I = \frac{1}{2}MR^2$ and $\omega = v/R$, so $\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$. Solving $mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$ for h gives

$$h = \frac{v^2}{2g}\left(1 + \frac{M}{2m}\right) = 13.9\text{ m}.$$

EVALUATE: For the cylinder $K_{\text{cyl}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}MR^2)(v/R)^2 = \frac{1}{4}Mv^2$. $K_{\text{mass}} = \frac{1}{2}mv^2$, so $K_{\text{mass}} = (2m/M)K_{\text{cyl}} = [2(12.0\text{ kg})/10.0\text{ kg}](480\text{ J}) = 1150\text{ J}$. The mass has 1150 J of kinetic energy when the cylinder has 480 J of kinetic energy and at this point the system has total energy 1630 J since $U_2 = 0$. Initially the total energy of the system is $U_1 = mgy_1 = mgh = 1630\text{ J}$, so the total energy is shown to be conserved.

9.80. **IDENTIFY:** Energy conservation: Loss of U of box equals gain in K of system. Both the cylinder and pulley have kinetic energy of the form $K = \frac{1}{2}I\omega^2$.

$$m_{\text{box}}gh = \frac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \frac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2 + \frac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2.$$

SET UP: $\omega_{\text{pulley}} = \dfrac{v_{\text{box}}}{r_{\text{pulley}}}$ and $\omega_{\text{cylinder}} = \dfrac{v_{\text{box}}}{r_{\text{cylinder}}}$.

Let B = box, P = pulley, and C = cylinder.

EXECUTE: $m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{2}\left(\frac{1}{2}m_{\text{P}}r_{\text{P}}^2\right)\left(\frac{v_{\text{B}}}{r_{\text{P}}}\right)^2 + \frac{1}{2}\left(\frac{1}{2}m_{\text{C}}r_{\text{C}}^2\right)\left(\frac{v_{\text{B}}}{r_{\text{C}}}\right)^2$. $m_{\text{B}}gh = \dfrac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \dfrac{1}{4}m_{\text{P}}v_{\text{B}}^2 + \dfrac{1}{4}m_{\text{C}}v_{\text{B}}^2$

and $v_{\text{B}} = \sqrt{\dfrac{m_{\text{B}}gh}{\frac{1}{2}m_{\text{B}} + \frac{1}{4}m_{\text{P}} + \frac{1}{4}m_{\text{C}}}} = \sqrt{\dfrac{(3.00\text{ kg})(9.80\text{ m/s}^2)(2.50\text{ m})}{1.50\text{ kg} + \frac{1}{4}(7.00\text{ kg})}} = 4.76\text{ m/s}.$

EVALUATE: If the box was disconnected from the rope and dropped from rest, after falling 2.50 m its speed would be $v = \sqrt{2g(2.50\text{ m})} = 7.00\text{ m/s}$. Since in the problem some of the energy of the system goes into kinetic energy of the cylinder and of the pulley, the final speed of the box is less than this.

9.81. **IDENTIFY:** The total kinetic energy of a walker is the sum of his translational kinetic energy plus the rotational kinetic of his arms and legs. We can model these parts of the body as uniform bars.

SET UP: For a uniform bar pivoted about one end, $I = \frac{1}{3}mL^2$. $v = 5.0\text{ km/h} = 1.4\text{ m/s}$.

$K_{\text{tran}} = \frac{1}{2}mv^2$ and $K_{\text{rot}} = \frac{1}{2}I\omega^2$.

EXECUTE: **(a)** $60° = (\frac{1}{3})$ rad. The average angular speed of each arm and leg is $\dfrac{\frac{1}{3}\text{ rad}}{1\text{ s}} = 1.05\text{ rad/s}$.

(b) Adding the moments of inertia gives

$I = \frac{1}{3}m_{\text{arm}}L_{\text{arm}}{}^2 + \frac{1}{3}m_{\text{leg}}L_{\text{leg}}{}^2 = \frac{1}{3}[(0.13)(75\text{ kg})(0.70\text{ m})^2 + (0.37)(75\text{ kg})(0.90\text{ m})^2]$. $I = 9.08\text{ kg}\cdot\text{m}^2$.

$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(9.08\text{ kg}\cdot\text{m}^2)(1.05\text{ rad/s})^2 = 5.0\text{ J}$.

(c) $K_{\text{tran}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})(1.4\text{ m/s})^2 = 73.5\text{ J}$ and $K_{\text{tot}} = K_{\text{tran}} + K_{\text{rot}} = 78.5\text{ J}$.

(d) $\dfrac{K_{\text{rot}}}{K_{\text{tran}}} = \dfrac{5.0\text{ J}}{78.5\text{ J}} = 6.4\%$.

EVALUATE: If you swing your arms more vigorously more of your energy input goes into the kinetic energy of walking and it is more effective exercise. Carrying weights in our hands would also be effective.

9.82. **IDENTIFY:** The total kinetic energy of a runner is the sum of his translational kinetic energy plus the rotational kinetic of his arms and legs. We can model these parts of the body as uniform bars.

SET UP: Now $v = 12\text{ km/h} = 3.33\text{ m/s}$. $I_{\text{tot}} = 9.08\text{ kg}\cdot\text{m}^2$ as in the previous problem.

EXECUTE: **(a)** $\omega_{\text{av}} = \dfrac{\pi/3\text{ rad}}{0.5\text{ s}} = 2.1\text{ rad/s}$.

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(9.08\text{ kg}\cdot\text{m}^2)(2.1\text{ rad/s})^2 = 20\text{ J}$.

(c) $K_{\text{tran}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})(3.33\text{ m/s})^2 = 416\text{ J}$. Therefore

$K_{\text{tot}} = K_{\text{tran}} + K_{\text{rot}} = 416\text{ J} + 20\text{ J} = 436\text{ J}$.

(d) $\dfrac{K_{\text{rot}}}{K_{\text{tot}}} = \dfrac{20\text{ J}}{436\text{ J}} = 0.046$, so K_{rot} is 4.6% of K_{tot}.

EVALUATE: The amount rotational energy depends on the geometry of the object.

9.83. **IDENTIFY:** We know (or can calculate) the masses and geometric measurements of the various parts of the body. We can model them as familiar objects, such as uniform spheres, rods, and cylinders, and calculate their moments of inertia and kinetic energies.

SET UP: My total mass is $m = 90$ kg. I model my head as a uniform sphere of radius 8 cm. I model my trunk and legs as a uniform solid cylinder of radius 12 cm. I model my arms as slender rods of length 60 cm. $\omega = 72\text{ rev/min} = 7.5\text{ rad/s}$. For a solid uniform sphere, $I = 2/5\, MR^2$, for a solid cylinder, $I = \frac{1}{2}MR^2$, and for a rod rotated about one end $I = 1/3\, ML^2$.

EXECUTE: **(a)** Using the formulas indicated above, we have $I_{\text{tot}} = I_{\text{head}} + I_{\text{trunk+legs}} + I_{\text{arms}}$, which gives $I_{\text{tot}} = \frac{2}{5}(0.070m)(0.080\text{ m})^2 + \frac{1}{2}(0.80m)(0.12\text{ m})^2 + 2\left(\frac{1}{3}\right)(0.13m)(0.60\text{ m})^2 = 3.3\text{ kg}\cdot\text{m}^2$ where we have used $m = 90$ kg.

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(3.3\text{ kg}\cdot\text{m}^2)(7.5\text{ rad/s})^2 = 93\text{ J}$.

EVALUATE: According to these estimates about 85% of the total I is due to the outstretched arms. If the initial translational kinetic energy $\frac{1}{2}mv^2$ of the skater is converted to this rotational kinetic energy as he goes into a spin, his initial speed must be 1.4 m/s.

9.84. **IDENTIFY:** Apply the parallel-axis theorem to each side of the square.

SET UP: Each side has length a and mass $M/4$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12}\left(\frac{1}{4}Ma^2\right)=\frac{1}{48}Ma^2$.

EXECUTE: The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\frac{Ma^2}{48}+\frac{M}{4}\left(\frac{a}{2}\right)^2=\frac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4\times\frac{Ma^2}{12}=\frac{Ma^2}{3}$.

EVALUATE: If all the mass of a side were at its center, a distance $a/2$ from the axis, we would have $I=4\left(\frac{M}{4}\right)\left(\frac{a}{2}\right)^2=\frac{1}{4}Ma^2$. If all the mass was divided equally among the four corners of the square, a distance $a/\sqrt{2}$ from the axis, we would have $I=4\left(\frac{M}{4}\right)\left(\frac{a}{\sqrt{2}}\right)^2=\frac{1}{2}Ma^2$. The actual I is between these two values.

9.85. **IDENTIFY:** The density depends on the distance from the center of the sphere, so it is a function of r. We need to integrate to find the mass and the moment of inertia.

SET UP: $M=\int dm=\int \rho dV$ and $I=\int dI$.

EXECUTE: **(a)** Divide the sphere into thin spherical shells of radius r and thickness dr. The volume of each shell is $dV=4\pi r^2dr$. $\rho(r)=a-br$, with $a=3.00\times10^3\ \text{kg/m}^3$ and $b=9.00\times10^3\ \text{kg/m}^4$. Integrating gives $M=\int dm=\int \rho dV=\int_0^R (a-br)4\pi r^2dr=\frac{4}{3}\pi R^3\left(a-\frac{3}{4}bR\right)$.

$$M=\frac{4}{3}\pi(0.200\ \text{m})^3\left(3.00\times10^3\ \text{kg/m}^3-\frac{3}{4}(9.00\times10^3\ \text{kg/m}^4)(0.200\ \text{m})\right)=55.3\ \text{kg}.$$

(b) The moment of inertia of each thin spherical shell is

$$dI=\frac{2}{3}r^2dm=\frac{2}{3}r^2\rho dV=\frac{2}{3}r^2(a-br)4\pi r^2dr=\frac{8\pi}{3}r^4(a-br)dr.$$

$$I=\int_0^R dI=\frac{8\pi}{3}\int_0^R r^4(a-br)dr=\frac{8\pi}{15}R^5\left(a-\frac{5b}{6}R\right).$$

$$I=\frac{8\pi}{15}(0.200\ \text{m})^5\left(3.00\times10^3\ \text{kg/m}^3-\frac{5}{6}(9.00\times10^3\ \text{kg/m}^4)(0.200\ \text{m})\right)=0.804\ \text{kg}\cdot\text{m}^2.$$

EVALUATE: We cannot use the formulas $M=\rho V$ and $I=\frac{1}{2}MR^2$ because this sphere is not uniform throughout. Its density increases toward the surface. For a uniform sphere with density $3.00\times10^3\ \text{kg/m}^3$, the mass is $\frac{4}{3}\pi R^3\rho=100.5\ \text{kg}$. The mass of the sphere in this problem is less than this. For a uniform sphere with mass 55.3 kg and $R=0.200$ m, $I=\frac{2}{5}MR^2=0.885\ \text{kg}\cdot\text{m}^2$. The moment of inertia for the sphere in this problem is less than this, since the density decreases with distance from the center of the sphere.

9.86. **IDENTIFY:** Write K in terms of the period T and take derivatives of both sides of this equation to relate dK/dt to dT/dt.

SET UP: $\omega=\frac{2\pi}{T}$ and $K=\frac{1}{2}I\omega^2$. The speed of light is $c=3.00\times10^8$ m/s.

EXECUTE: **(a)** $K=\frac{2\pi^2I}{T^2}$. $\frac{dK}{dt}=-\frac{4\pi^2I}{T^3}\frac{dT}{dt}$. The rate of energy loss is $\frac{4\pi^2I}{T^3}\frac{dT}{dt}$. Solving for the moment of inertia I in terms of the power P,

$$I = \frac{PT^3}{4\pi^2}\frac{1}{dT/dt} = \frac{(5\times10^{31}\ \text{W})(0.0331\ \text{s})^3}{4\pi^2}\frac{1\ \text{s}}{4.22\times10^{-13}\ \text{s}} = 1.09\times10^{38}\ \text{kg}\cdot\text{m}^2$$

(b) $R = \sqrt{\dfrac{5I}{2M}} = \sqrt{\dfrac{5(1.08\times10^{38}\ \text{kg}\cdot\text{m}^2)}{2(1.4)(1.99\times10^{30}\ \text{kg})}} = 9.9\times10^3\ \text{m}$, about 10 km.

(c) $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(9.9\times10^3\text{m})}{(0.0331\ \text{s})} = 1.9\times10^6\ \text{m/s} = 6.3\times10^{-3}c.$

(d) $\rho = \dfrac{M}{V} = \dfrac{M}{(4\pi/3)R^3} = 6.9\times10^{17}\,\text{kg/m}^3$, which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

EVALUATE: I is huge because M is huge. A small rate of change in the period corresponds to a large release of energy.

9.87. **IDENTIFY:** The graph with the problem in the text shows that the angular acceleration increases linearly with time and is therefore not constant.

SET UP: $\omega_z = d\theta/dt,\ \ \alpha_z = d\omega_z/dt.$

EXECUTE: **(a)** Since the angular acceleration is not constant, Eq. (9.11) cannot be used, so we must use $\alpha_z = d\omega_z/dt$ and $\omega_z = d\theta/dt$ and integrate to find the angle. The graph passes through the origin and has a constant positive slope of 6/5 rad/s^3, so the equation for α_z is $\alpha_z = (1.2\ \text{rad/s}^3)t$. Using $\alpha_z = d\omega_z/dt$ gives $\omega_z = \omega_{0z} + \int_0^t \alpha_z dt = 0 + \int_0^t (1.2\ \text{rad/s}^3)t\,dt = (0.60\ \text{rad/s}^3)t^2$. Now we must use $\omega_z = d\theta/dt$ and integrate again to get the angle.

$\theta_2 - \theta_1 = \int_0^t \omega_z dt = \int_0^t (0.60\ \text{rad/s}^3)t^2 dt = (0.20\ \text{rad/s}^3)t^3 = (0.20\ \text{rad/s}^3)(5.0\ \text{s})^3 = 25\ \text{rad}.$

(b) The result of our first integration gives $\omega_z = (0.60\ \text{rad/s}^3)(5.0\ \text{s})^2 = 15\ \text{rad/s}.$

(c) The result of our second integration gives $4\pi\ \text{rad} = (0.20\ \text{rad/s}^3)t^3$, so $t = 3.98$ s. Therefore $\omega_z = (0.60\ \text{rad/s}^3)(3.98\ \text{s})^2 = 9.48\ \text{rad/s}.$

EVALUATE: When the constant-acceleration angular kinematics formulas do not apply, we must go back to basic definitions.

9.88. **IDENTIFY** and **SET UP:** The graph of a^2 versus $(\theta-\theta_0)^2$ is shown in Figure 9.88. It is a straight line with a positive slope. The angular acceleration is constant.

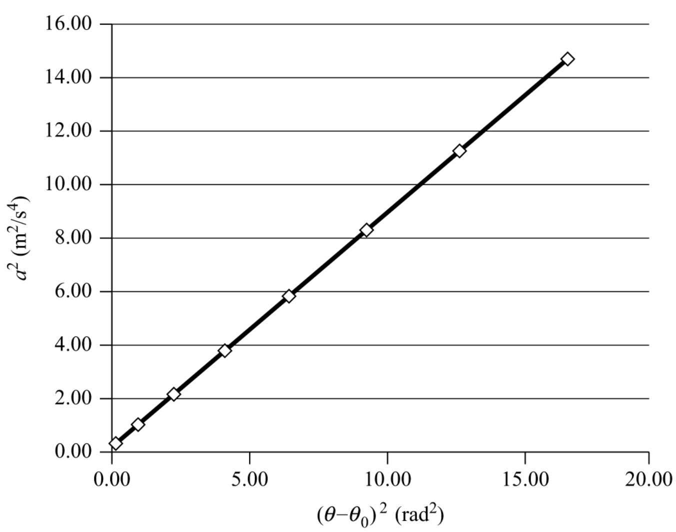


Figure 9.88

EXECUTE: **(a)** From graphing software, the slope is 0.921 m^2/s^4 and the y-intercept is 0.233 m^2/s^4.

(b) The resultant acceleration is $a^2 = a_{\tan}^2 + a_{\text{rad}}^2$. $a_{\tan} = r\alpha_z$ and $a_{\text{rad}} = r\omega_z^2$, where

$\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta - \theta_0) = 0 + 2\alpha_z(\theta - \theta_0)$. Therefore the resultant acceleration is

$a^2 = (r\alpha_z)^2 + [2r\alpha_z(\theta - \theta_0)]^2$

$a^2 = 4r^2\alpha_z^2(\theta - \theta_0)^2 + (r\alpha_z)^2$.

From this result, we see that the slope of the graph is $4r^2\alpha_z^2$, so $4r^2\alpha_z^2 = 0.921\ \text{m}^2/\text{s}^4$. Solving for α_z gives $\alpha_z = \sqrt{\dfrac{0.921\ \text{m}^2/\text{s}^4}{4(0.800\ \text{m})^2}} = 0.600\ \text{rad/s}^2$.

(c) Using $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega_z^2 = 0 + 2(0.600\ \text{rad/s}^2)(3\pi/4\ \text{rad})$, $\omega_z = 1.6815$ rad/s. The speed is $v = r\omega_z = (0.800\ \text{m})(1.6815\ \text{rad/s}) = 1.35$ m/s.

(d) Call ϕ the angle between the linear velocity and the resultant acceleration. The resultant velocity is tangent to the circle, so $\tan\phi = \dfrac{a_{\text{rad}}}{a_{\tan}} = \dfrac{r\omega_z^2}{r\alpha_z} = \dfrac{\omega_z^2}{\alpha_z}$. It is also the case that $\omega_z^2 = 2\alpha_z\Delta\theta$, so

$\tan\phi = \dfrac{2\alpha_z\Delta\theta}{\alpha_z} = 2\Delta\theta = 2(\pi/2) = \pi$. Thus $\phi = \arctan \pi = 72.3°$.

EVALUATE: According to the work in parts (a) and (b), the y-intercept of the graph is $(r\alpha_z)^2$ and is equal to 0.233 m^2/s^4. Solving for α_z gives $\alpha_z = \sqrt{\dfrac{0.233\ \text{m}^2/\text{s}^4}{(0.800\ \text{m})^2}} = 0.60\ \text{rad/s}^2$, as we found in part (b).

9.89. **IDENTIFY** and **SET UP:** The equation of the graph in the text is $d = (165\ \text{cm/s}^2)t^2$. For constant acceleration, the second time derivative of the position (d in this case) is a constant.

EXECUTE: **(a)** $\dfrac{d(d)}{dt} = (330\ \text{cm/s}^2)t$ and $\dfrac{d^2(d)}{dt^2} = 330\ \text{cm/s}^2$, which is a constant. Therefore the acceleration of the metal block is a constant 330 cm/s^2 = 3.30 m/s^2.

(b) $v = \dfrac{d(d)}{dt} = (330\ \text{cm/s}^2)t$. When d = 1.50 m = 150 cm, we have 150 cm = $(165\ \text{cm/s}^2)t^2$, which gives t = 0.9535 s. Thus v = 330 cm/s^2)(0.9535 s) = 315 cm/s = 3.15 m/s.

(c) Energy conservation $K_1 + U_1 = K_2 + U_2$ gives $mgd = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$. Using $\omega = v/r$, solving for I and putting in the numbers m = 5.60 kg, d = 1.50 m, r = 0.178 m, v = 3.15 m/s, we get $I = 0.348\ \text{kg}\cdot\text{m}^2$.

(d) Newton's second law gives $mg - T = ma$, $T = m(g - a) = (5.60\ \text{kg})(9.80\ \text{m/s}^2 - 3.30\ \text{m/s}^2) = 36.4$ N.

EVALUATE: When dealing with non-uniform objects, such as this flywheel, we cannot use the standard moment of inertia formulas and must resort to other ways.

9.90. **IDENTIFY:** Apply $I = \int r^2\, dm$.

SET UP: Let z be the coordinate along the vertical axis. $r(z) = \dfrac{zR}{h}$. $dm = \pi\rho\dfrac{R^2z^2}{h^2}$ and $dI = \dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}z^4\, dz$.

EXECUTE: $I = \int dI = \dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}\int_0^h z^4\, dz = \dfrac{\pi\rho}{10}\dfrac{R^4}{h^4}\Big[z^5\Big]_0^h = \dfrac{1}{10}\pi\rho R^4 h$. The volume of a right circular cone is $V = \frac{1}{3}\pi R^2 h$, the mass is $\frac{1}{3}\pi\rho R^2 h$ and so $I = \dfrac{3}{10}\left(\dfrac{\pi\rho R^2 h}{3}\right)R^2 = \dfrac{3}{10}MR^2$.

EVALUATE: For a uniform cylinder of radius R and for an axis through its center, $I = \frac{1}{2}MR^2$. I for the cone is less, as expected, since the cone is constructed from a series of parallel discs whose radii decrease from R to zero along the vertical axis of the cone.

9.91. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: $\omega_z = d\theta/dt$. $\alpha_z = d^2\omega_z/dt^2$.

EXECUTE: **(a)** $ds = r\,d\theta = r_0 d\theta + \beta\theta\,d\theta$ so $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$. θ must be in radians.

(b) Setting $s = vt = r_0\theta + \frac{\beta}{2}\theta^2$ gives a quadratic in θ. The positive solution is

$$\theta(t) = \frac{1}{\beta}\left[\sqrt{r_0^2 + 2\beta vt} - r_0\right].$$

(The negative solution would be going backwards, to values of r smaller than r_0.)

(c) Differentiating, $\omega_z(t) = \dfrac{d\theta}{dt} = \dfrac{v}{\sqrt{r_0^2 + 2\beta vt}}$, $\alpha_z = \dfrac{d\omega_z}{dt} = -\dfrac{\beta v^2}{(r_0^2 + 2\beta vt)^{3/2}}$. The angular acceleration α_z is not constant.

(d) $r_0 = 25.0$ mm. θ must be measured in radians, so $\beta = (1.55\,\mu\text{m/rev})(1\text{ rev}/2\pi\text{ rad}) = 0.247\,\mu\text{m/rad}$. Using $\theta(t)$ from part (b), the total angle turned in $74.0\text{ min} = 4440\text{ s}$ is

$$\theta = \frac{1}{2.47\times10^{-7}\text{ m/rad}}\left(\sqrt{2(2.47\times10^{-7}\text{ m/rad})(1.25\text{ m/s})(4440\text{ s}) + (25.0\times10^{-3}\text{ m})^2} - 25.0\times10^{-3}\text{ m}\right)$$

$\theta = 1.337\times10^5$ rad, which is 2.13×10^4 rev.

(e) The graphs are sketched in Figure 9.91.

EVALUATE: ω_z must decrease as r increases, to keep $v = r\omega$ constant. For ω_z to decrease in time, α_z must be negative.

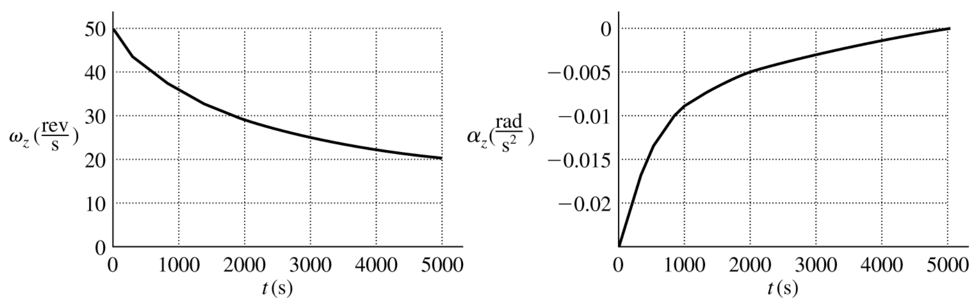


Figure 9.91

9.92. **IDENTIFY and SET UP:** For constant angular speed $\theta = \omega t$.

EXECUTE: **(a)** $\theta = \omega t$ = (14 rev/s)(2π rad/rev)(1/120 s) = 42°, which is choice (d).

EVALUATE: This is quite a large rotation in just one frame.

9.93. **IDENTIFY and SET UP:** The average angular acceleration is $\alpha_{\text{av}} = \dfrac{\omega - \omega_0}{t}$.

EXECUTE: **(a)** $\alpha_{\text{av}} = \dfrac{\omega - \omega_0}{t}$ = [8 rev/s – (–14 rev/s)]/(10 s) = (2.2 rev/s)(2π rad/rev) = 44π/10 rad/s^2 which is choice (d).

EVALUATE: This is nearly 14 rad/s^2.

9.94. **IDENTIFY and SET UP:** The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$ and the kinetic energy due to running is $K = \frac{1}{2}mv^2$.

EXECUTE: Equating the two kinetic energies gives $\frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$. Using $I = \frac{1}{2}mR^2$, we have $\frac{1}{2}(\frac{1}{2}mr^2)\omega^2 = \frac{1}{2}mv^2$, which gives $v = \frac{r\omega}{\sqrt{2}} = \frac{(0.05\text{ m})(14\text{ rev/s})(2\pi\text{ rad/rev})}{\sqrt{2}} = 3.11$ m/s, choice (c).

EVALUATE: This is about 3 times as fast as a human walks.

9.95. **IDENTIFY** and **SET UP:** $I = \frac{1}{2}mR^2$.

EXECUTE: **(a)** $I = \frac{1}{2}mR^2$, so if we double the radius but keep the mass fixed, the moment of inertia increases by a factor of 4, which is choice (d).

EVALUATE: The difference in length of the two eels plays no part in their moment of inertia if their mass is the same in both cases.