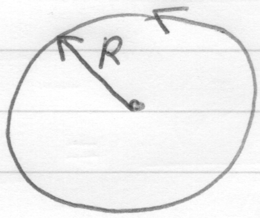
2.9

a)



$R = 25\ m$

$l(t) = 5t^2 - 5t$

$$v(t) = \frac{dl(t)}{dt} = 10t - 5$$

$$\omega(t) = \frac{d\theta}{dt} = \frac{1}{R}\frac{dl(t)}{dt} = \frac{10t-5}{25}$$

$$\alpha(t) = \frac{d}{dt}\omega(t) = \frac{10}{25} = 0,4\ \frac{rad}{s^2}$$

$$\vec{a}(t) = (R\ddot{\theta} + 2\dot{R}\dot{\theta})\hat{\theta} + (\ddot{R} - R\dot{\theta}^2)\hat{r}$$

$$= R\,\alpha(t)\,\hat{\theta} - R\,\omega(t)^2\,\hat{r}$$

$$\Rightarrow\ a(t) = |\vec{a}(t)| = R\sqrt{\alpha(t)^2 + \omega(t)^4}$$

At $t = 2\,s$:

$v(2) = 15\ m/s$

$\omega(2) = 0,6\ rad/s$

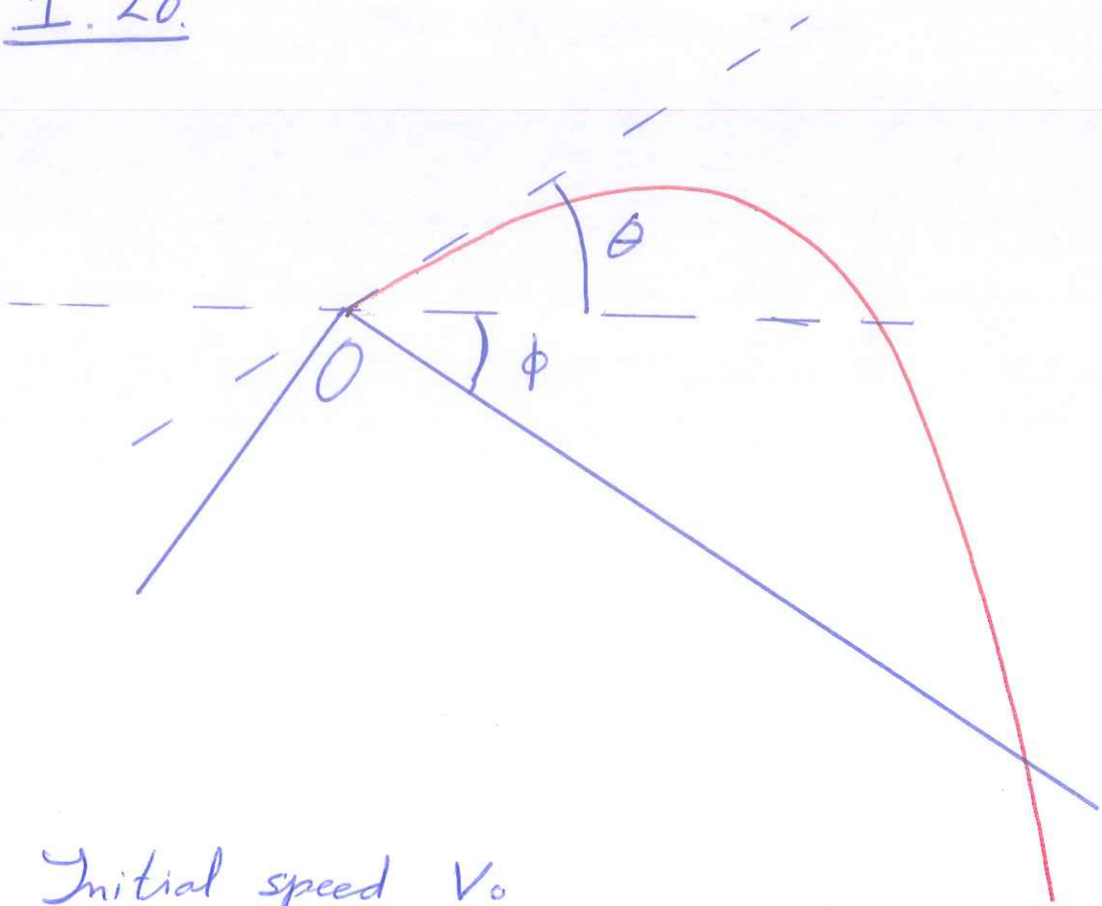
$\alpha(2) = 0,4\ rad/s$

$a(t) = 25\sqrt{0,16 + 0,6^4} \approx 13,5\ m/s^2$

b) $\vec{a}\cdot\vec{r} = -R^2\omega(t)^2 = |\vec{a}||\vec{r}|\cos\varphi$

$$\Rightarrow\ \varphi = \arccos\left(-\frac{0.36\cdot 25^2}{25\cdot 13,5}\right) = 2.3038\ rad.$$

1.26



Initial speed V_0

Choose Cartesian coordinate system with origin at the peak of the hill.

$$V_x = V_0 \cos\theta = \text{constant}$$

$$V_y(0) = V_0 \sin\theta$$

$$V_y(t) = V_0 \sin\theta - g t$$

We assume $t=0$ when the rock is thrown.

Coordinates of the rock are

$$(x(t), y(t)) = (V_0 t \cos\theta, V_0 t \sin\theta - \tfrac{1}{2} g t^2)$$

The coordinates of the slope are

$$(x_{slope}, y_{slope}) = (x_{slope}, - x_{slope} \tan\phi)$$

The rock hits the ground when

$$(x(t), y(t)) = (x_{slope}, y_{slope})$$

$\Longrightarrow$ 2 equations for 2 unknowns, x & t

Solving the system of equations, we find

$$t = \frac{x}{V_0 \cos\theta}$$

and
$$x = \frac{2\cos^2\theta \, V_0^2}{g}(\tan\phi + \tan\theta)$$

Maximize x as function of θ

$$\Rightarrow \frac{dx}{d\theta} = 0$$

$$\Updownarrow$$

$$-2\cos\theta\sin\theta(\tan\phi + \tan\theta) + 1 = 0$$

From this an equation for $\tan\theta$ can be derived

$$\tan\theta = -\tan\phi + \sqrt{\tan^2\phi + 1}$$
$$= \frac{1 - \sin\phi}{\cos\phi} \qquad \phi \in [0, \pi/2]$$