JF PY1T10 Special Relativity

Lecture 10:

Relativistic Dynamics – Collisions and Conservation Laws

Summary of Lecture 9

The mass of a particle depends on its speed:

$$m(v) = \gamma m_0$$

Its momentum is given by:

$$p = m(v)v = \gamma m_0 v$$

The total energy of a particle is given by:

$$E = mc^2 = KE + m_0 c^2$$

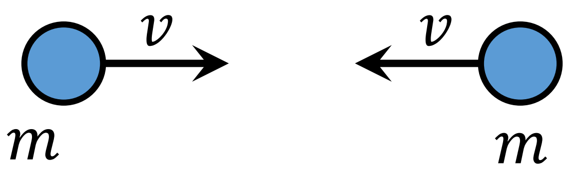
i.e., The total energy = kinetic energy + rest mass

$$E^2 - p^2 c^2 = m_0^2 c^4$$

For a photon: $E = pc = h\nu$

Inelastic Collisions

We will look at some examples of conservation of energy and mass, but *not* rest mass.



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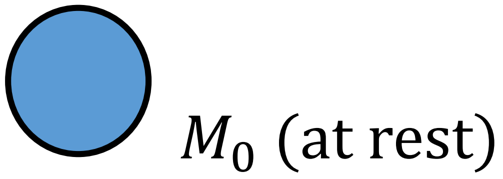


AFTER

The collision results in the formation of a stationary composite particle.

Conservation of Momentum:

$$mv + m(-v) = M_0(0)$$

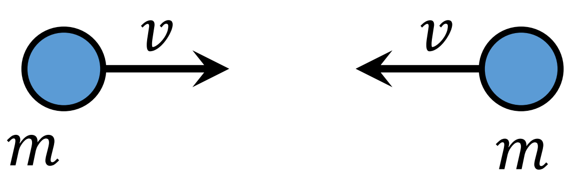
Conservation of Energy:

$$mc^2 + mc^2 = M_0 c^2$$

$$\therefore 2m = M_0 \text{ (mass conservation)}$$

Inelastic Collisions

We will look at some examples of conservation of energy and mass, but *not* rest mass.



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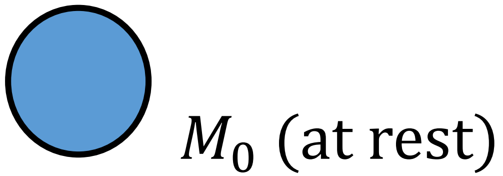

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But:

$$mc^2 = m_0c^2 + K.E.$$
$$\therefore 2(m_0c^2 + KE) = M_0c^2$$
$$\therefore 2(KE) = (M_0 - 2m_0)c^2 \neq 0$$
$$M_0 \neq 2m_0$$

Rather $M_0 > 2m_0$

Thus, the loss in KE after the collision $=$ (increase of rest mass)c^2

Creation of Particles

Mass-Energy equivalence suggests that it may be possible to create new particles if an adequate amount of energy is available.

To create a particle with mass m_0 we would need energy input $\geq m_0c^2$

In practice, we need more than m_0c^2:

Why?

- Other conservation laws may apply, e.g. angular momentum, electric charge, lepton number. Usually required to create both a particle and its anti-particle.
- Practically, particle creation occurs by energetically colliding already-existing particles. Significant amounts of energy are tied up in KE and is unavailable for conversion into rest mass.

Pair Production

The very first pair-production was observed by C. D. Anderson in 1932. He was awarded the Nobel prize in 1936 for this research.

$$\gamma \rightarrow e^{-} + e^{+}$$

i.e., a γ-ray photon creates an electron-positron pair.

$$(m_0)_{e^-} = (m_0)_{e^+}$$

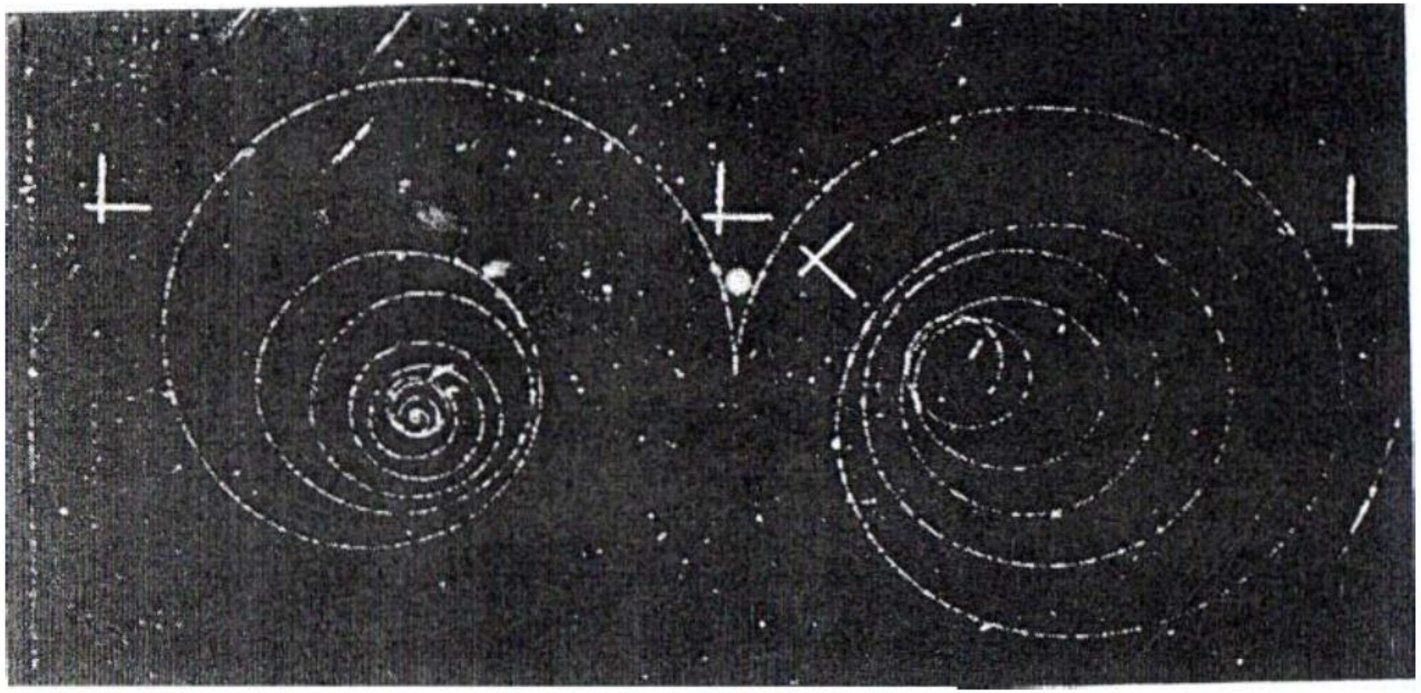
$$E_{min} = 2(m_0)_e c^2$$

$$= 2\ (9.1 \text{ x } 10^{-31}\ \text{kg})(3\ x\ 10^{8}\ \text{m/s})^2$$

$$= 1.64 \text{ x } 10^{-13}\text{J}$$

$$(\text{divide by } 1.6 \text{ x } 10^{-19} \text{ to convert to eV}) = 1.02 \text{ x } 10^{6}\ \text{eV} = 1.02\ \text{MeV}$$

Pair Production



Production of an electron-positron pair in a liquid hydrogen bubble chamber.

Bubble chamber: Charged particles create an ionization track, around which the liquid vaporizes, forming microscopic bubbles that can be photographed.

The photon that created the electron-positron pair came in from the bottom of the picture.

But this is not the whole story…

Pair Production

Is momentum conserved?

View from frame in which the center of mass of e^- and e^+ is at rest.

$$p_{e^+} + p_{e^-} = 0$$

But $p_\gamma \neq 0$ as there is no frame in which a photon is at rest.

$$p_\gamma \neq p_{e^+} + p_{e^-} = 0$$

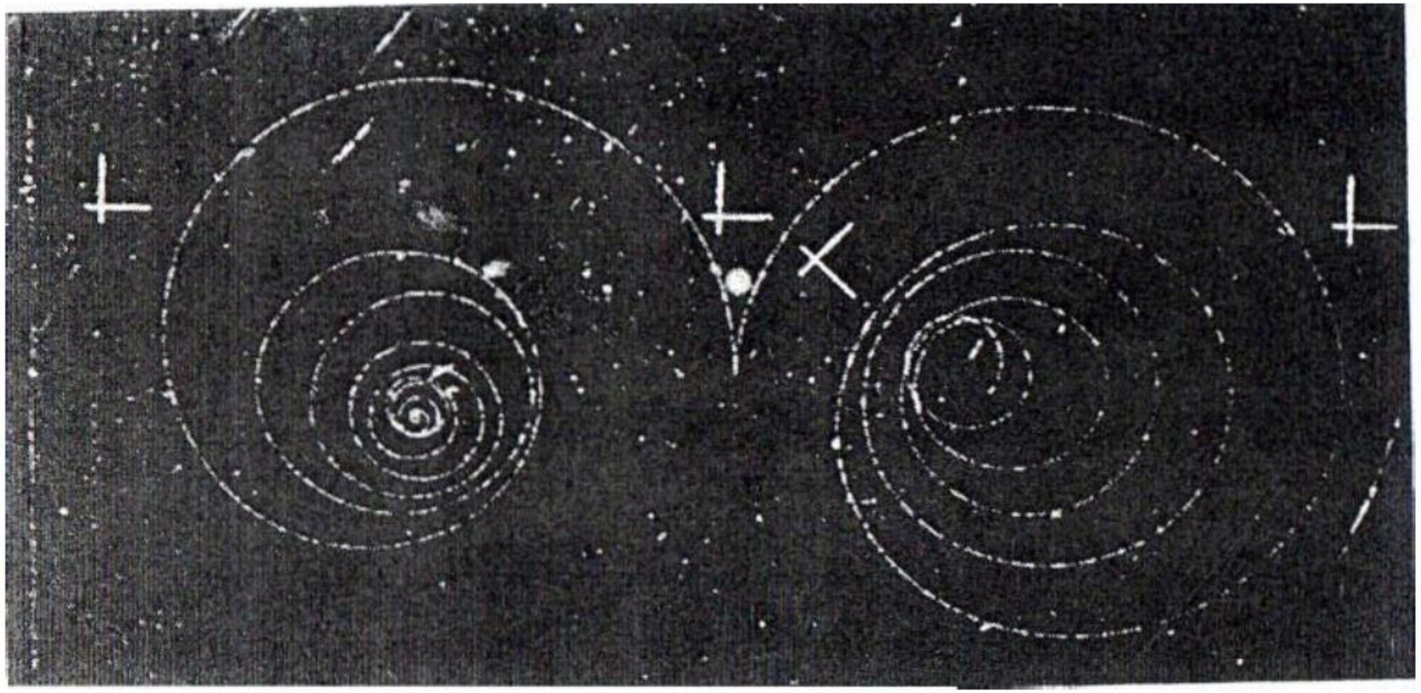
So momentum is not conserved in the center-of-mass frame!?

∴ This process cannot occur in free space, ***but*** it can take place in the vicinity of another particle, e.g. a nucleus.

$$p_\gamma + p_{nuc} = p_{e^+} + p_{e^-} + p'_{nuc}$$

i.e., the momentum of the nucleus is changed.

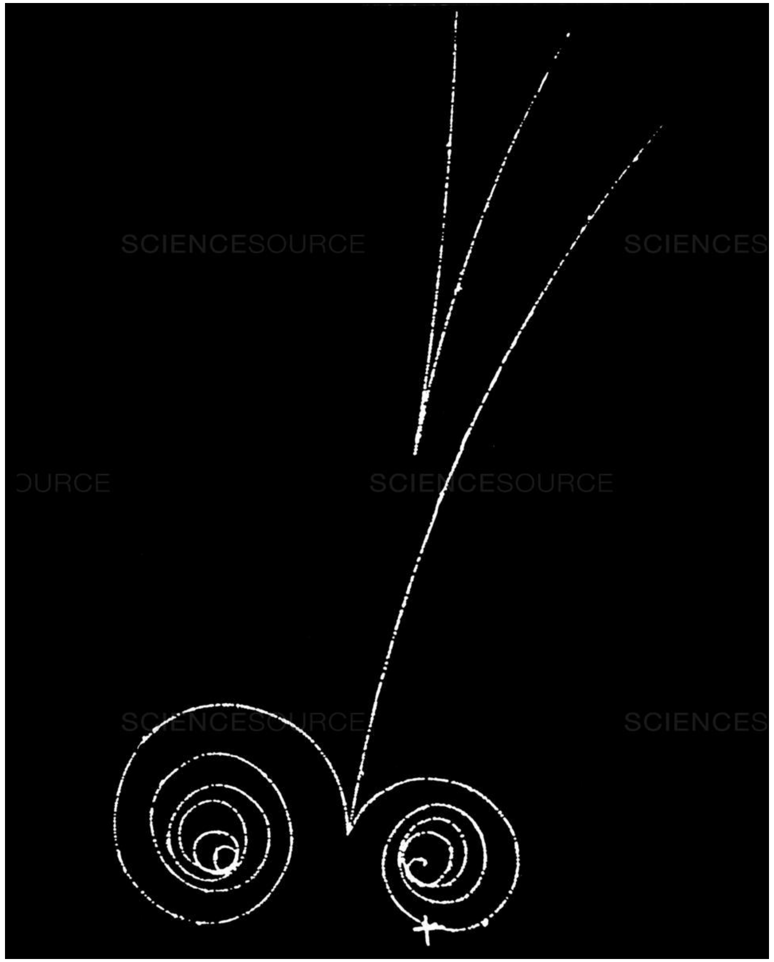
Pair Production



Production of an electron-positron pair in a liquid hydrogen bubble chamber.

Here, we see no track of a fourth particle → it must be uncharged.

Pair Production



Here, the track of a fourth charged particle can be seen.

Probably an proton from one of the hydrogen atoms.

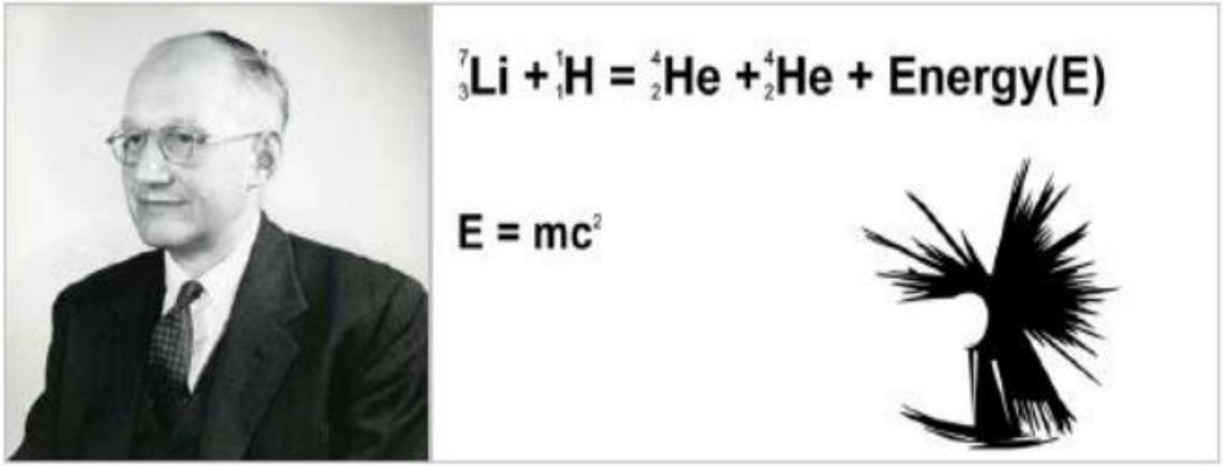
Creation of New Particles in Collisions

New particles may also be created by the collision of existing particles.

But if p before the collision is $\neq 0$ then p after the collision must be also $\neq 0$.

So some of the initial KE appears as the final KE of the particle. Therefore, not all of the initial energy is available for particle creation.

Creation of New Particles in Collisions



- Ernest Walton and Sir John D. Cockcroft were jointly awarded the Nobel Prize for Physics in 1951 for their pioneering work on the transmutation of atomic nuclei by artificially accelerated atomic particles.

Nuclear Fission and Nuclear Fusion

Consider a nucleus:

-- Mass number A (= # protons + # neutrons)

-- Atomic number Z (= # protons)

The rest energy of nucleus = ${}^{A}_{Z}m_0 c^2$

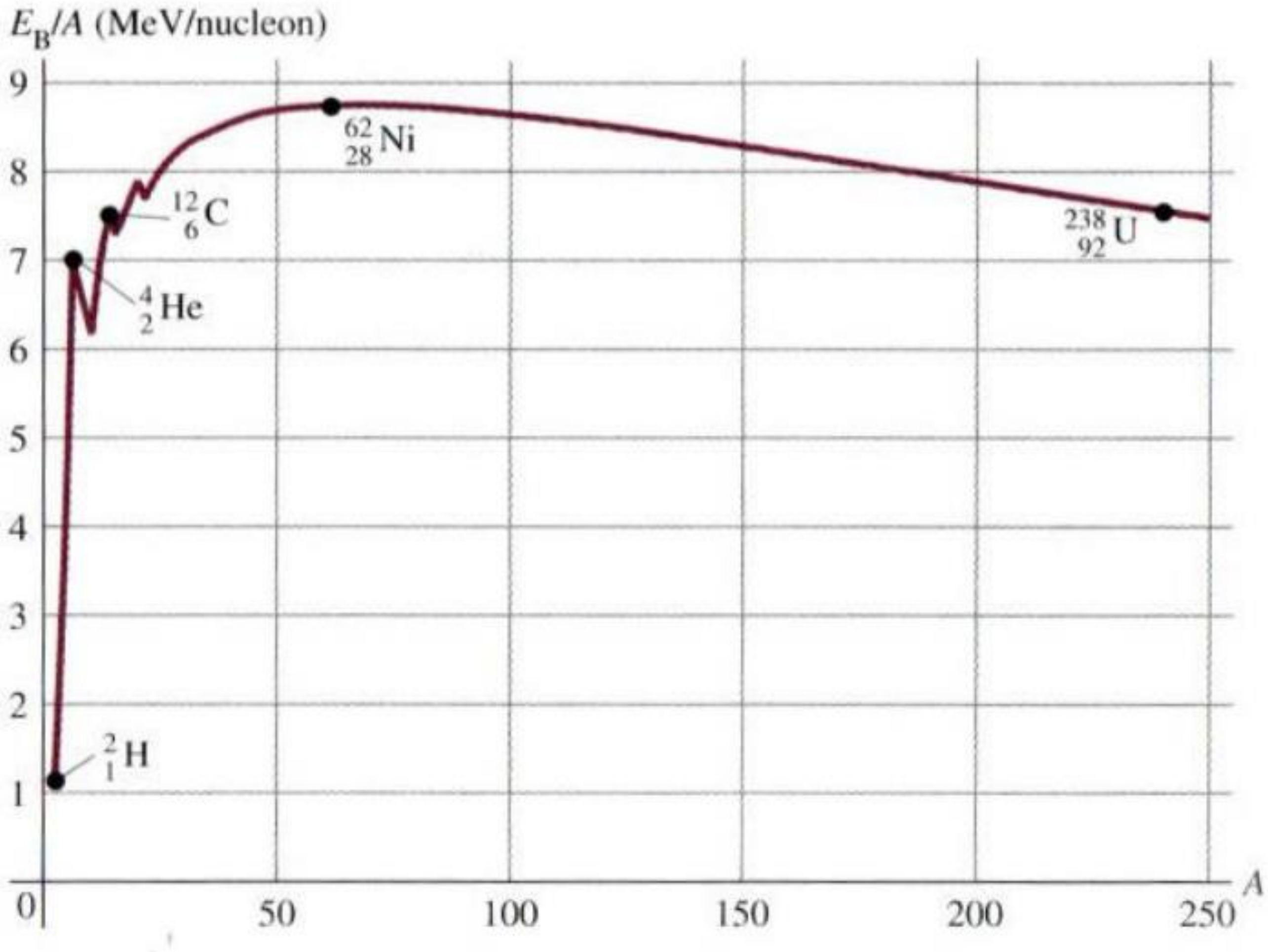
To break this nucleus up into protons and neutrons, we need to add an energy equal to the binding energy, E_b.

Then:

$${}^{A}_{Z}m_0 c^2 + E_b = Z m_{p,0} c^2 + (A - Z) m_{n,0} c^2$$

Nuclear Fission and Fusion

Measure E_b/A (the binding energy per nucleon)



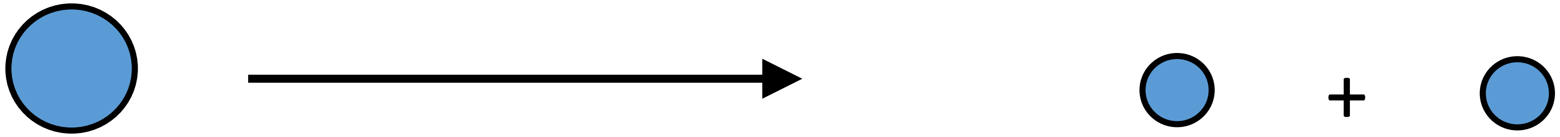
Approximate binding energy per nucleon as a function of mass number A for stable nuclides.

The curve reaches a peak of about 8.8 MeV/nucleon at A=62 (corresponding to the element Nickel).

The spike at A=4 shows the unusual stability of the $^{4}_{2}\mathrm{He}$ structure.
This graph implies that when you break up a heavy nucleus or put light nuclei together, energy is released.

(From University Physics by Sears and Zemansky)

Fission – Imaginary Case



A = 240

$$\frac{E_b}{A} = 7.2\,\frac{\text{MeV}}{\text{nucleon}}$$

A = 120 + A = 120

$$\frac{E_b}{A} = 8.2\,\frac{\text{MeV}}{\text{nucleon}}$$

Total binding energy of:

- parent nuclei $= 7.2 \times 240 = 1728$ MeV
- daughter nuclei $= 2\,(120 \times 8.2) = 1968$ MeV

The daughter nuclei are more stable. This energy is released as kinetic energy.

The energy released is must be: $1968 - 1728 = 240$ MeV

Fission – Actual Case

Bombard ${}^{235}_{92}U$ with ${}^{1}_{0}n$

Strontium-90 10 neutrons

$$ {}^{235}_{92}U + {}^{1}_{0}n = {}^{90}_{38}Sr + {}^{138}_{54}Xe + 10\,{}^{1}_{0}n + 141MeV $$

Uranium-235 1 neutron Xenon-138 K.E.

Capture the KE to generate electricity

10 ${}^{1}_{0}n$ (i.e., neutrons) are released.

∴ a chain reaction is possible → A nuclear fission bomb.

Stellar Fusion

The sun burns H to make He:

$4\,{}_1^1\mathrm{H} \rightarrow {}_2^4\mathrm{He}$ (more complicated in reality)

$$m_{\mathrm{H}} = 1.007825\ \mathrm{a.m.u.}$$
$$m_{\ \mathrm{He}} = 4.00260\ \mathrm{a.m.u.}$$

where $1\ \mathrm{a.m.u.} = \dfrac{\mathrm{mass\ of\ }{}_6^{12}\mathrm{C\ atom}}{12} = 1.6605 \text{ x } 10^{-27}\mathrm{kg}$

$$\therefore\ \Delta m \approx 0.287\ \mathrm{a.m.u.}$$

and

$$\Delta mc^2 = 27\ \mathrm{MeV}$$

Stellar Fusion

For fusion, protons in the sun must get very close: $\sim 2 \times 10^{-15}$m

They must then overcome electrical repulsion at 2×10^{-15}m.

$$PE = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 1.2 \times 10^{-13}\text{J} = 0.7\text{ MeV}$$

Therefore the two protons must approach with comparable KE

This implies a very high temperature at the centre of the Sun $= 1.5 \times 10^{7}$ K.

($kT = 1$ eV when $T = 1.1 \times 10^{4}$ K)

Artificial Fusion

Fusion reactor:

$$\begin{array}{ccccccc} Deuterium & + & Tritium & \rightarrow & Helium & + & neutron \\ {}_{1}^{2}H & + & {}_{1}^{3}H & \rightarrow & {}_{2}^{4}He & + & {}_{0}^{1}n \end{array}$$

$$\Delta mc^2 \approx 27\ MeV$$

Need to make the gas mixture very hot.

⇘ It becomes ionized – a plasma.

Must confine the plasma in a magnetic field (ITER [France])

or

Heat and compress with laser (HiPER [EU], NIF [California])

Concept Test

At what speed is a particle's kinetic energy equal to twice it rest energy?

A. 0.9c
B. 0.94c
C. 0.5c
D. 0.707c

Housekeeping

Homework Set 1 due 30th October (for credit).

Next Lecture: 5th November (i.e. no lecture during Reading Week or following week)

For Revision:

There is a Tutorial Question Sheet on Blackboard (no credit).

Solutions given in Tutorial Class on 19th November.