

GRAVITATION

13.1. IDENTIFY and SET UP: Use the law of gravitation, $F_g = \frac{Gm_1m_2}{r^2}$, to determine F_g .

EXECUTE: $F_{S \text{ on } M} = G \frac{m_S m_M}{r_{SM}^2}$ (S = sun, M = moon); $F_{E \text{ on } M} = G \frac{m_E m_M}{r_{EM}^2}$ (E = earth)

$$\frac{F_{S \text{ on } M}}{F_{E \text{ on } M}} = \left(G \frac{m_S m_M}{r_{SM}^2} \right) \left(\frac{r_{EM}^2}{G m_E m_M} \right) = \frac{m_S}{m_E} \left(\frac{r_{EM}}{r_{SM}} \right)^2$$

r_{EM} , the radius of the moon's orbit around the earth is given in Appendix F as 3.84×10^8 m. The moon is much closer to the earth than it is to the sun, so take the distance r_{SM} of the moon from the sun to be r_{SE} , the radius of the earth's orbit around the sun.

$$\frac{F_{S \text{ on } M}}{F_{E \text{ on } M}} = \left(\frac{1.99 \times 10^{30} \text{ kg}}{5.98 \times 10^{24} \text{ kg}} \right) \left(\frac{3.84 \times 10^8 \text{ m}}{1.50 \times 10^{11} \text{ m}} \right)^2 = 2.18.$$

EVALUATE: The force exerted by the sun is larger than the force exerted by the earth. The moon's motion is a combination of orbiting the sun and orbiting the earth.

13.2. IDENTIFY: The gravity force between spherically symmetric spheres is $F_g = \frac{Gm_1m_2}{r^2}$, where r is the separation between their centers.

SET UP: $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. The moment arm for the torque due to each force is 0.150 m.

EXECUTE: (a) For each pair of spheres, $F_g = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.10 \text{ kg})(25.0 \text{ kg})}{(0.120 \text{ m})^2} = 1.27 \times 10^{-7} \text{ N}$.

From Figure 13.4 in the textbook we see that the forces for each pair are in opposite directions, so $F_{\text{net}} = 0$.

(b) The net torque is $\tau_{\text{net}} = 2F_g l = 2(1.27 \times 10^{-7} \text{ N})(0.150 \text{ m}) = 3.81 \times 10^{-8} \text{ N} \cdot \text{m}$.

(c) The torque is very small and the apparatus must be very sensitive. The torque could be increased by increasing the mass of the spheres or by decreasing their separation.

EVALUATE: The quartz fiber must twist through a measurable angle when a small torque is applied to it.

13.3. IDENTIFY: The gravitational attraction of the astronauts on each other causes them to accelerate toward each other, so Newton's second law of motion applies to their motion.

SET UP: The net force on each astronaut is the gravity force exerted by the other astronaut. Call the astronauts A and B , where $m_A = 65 \text{ kg}$ and $m_B = 72 \text{ kg}$. $F_g = Gm_1m_2/r^2$ and $\Sigma F = ma$.

EXECUTE: (a) The free-body diagram for astronaut A is given in Figure 13.3a and for astronaut B in Figure 13.3b (next page).

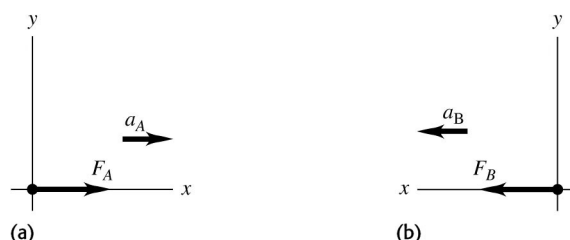


Figure 13.3

$\Sigma F_x = ma_x$ for A gives $F_A = m_A a_A$ and $a_A = \frac{F_A}{m_A}$. And for B , $a_B = \frac{F_B}{m_B}$.

$$F_A = F_B = G \frac{m_A m_B}{r^2} = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(65 \text{ kg})(72 \text{ kg})}{(20.0 \text{ m})^2} = 7.807 \times 10^{-10} \text{ N} \text{ so}$$

$$a_A = \frac{7.807 \times 10^{-10} \text{ N}}{65 \text{ kg}} = 1.2 \times 10^{-11} \text{ m/s}^2 \text{ and } a_B = \frac{7.807 \times 10^{-10} \text{ N}}{72 \text{ kg}} = 1.1 \times 10^{-11} \text{ m/s}^2.$$

(b) Using constant-acceleration kinematics, we have $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, which gives $x_A = \frac{1}{2}a_A t^2$ and $x_B = \frac{1}{2}a_B t^2$. $x_A + x_B = 20.0 \text{ m}$, so $20.0 \text{ m} = \frac{1}{2}(a_A + a_B)t^2$ and

$$t = \sqrt{\frac{2(20.0 \text{ m})}{1.2 \times 10^{-11} \text{ m/s}^2 + 1.1 \times 10^{-11} \text{ m/s}^2}} = 1.32 \times 10^6 \text{ s} = 15 \text{ days}.$$

(c) Their accelerations would increase as they moved closer and the gravitational attraction between them increased.

EVALUATE: Even though the gravitational attraction of the astronauts is much weaker than ordinary forces on earth, if it were the only force acting on the astronauts, it would produce noticeable effects.

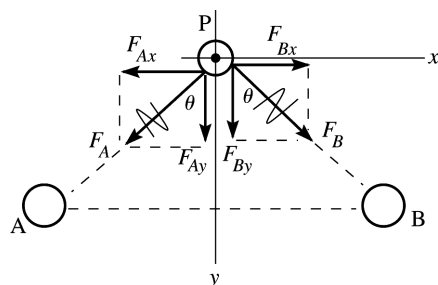
- 13.4. IDENTIFY:** Apply $F_g = \frac{Gm_1 m_2}{r^2}$, generalized to any pair of spherically symmetric objects.

SET UP: The separation of the centers of the spheres is $2R$.

EXECUTE: The magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

EVALUATE: The formula $F_g = \frac{Gm_1 m_2}{r^2}$ applies to any pair of spherically symmetric objects; one of the objects doesn't have to be the earth.

- 13.5. IDENTIFY:** Use $F_g = \frac{Gm_1 m_2}{r^2}$ to find the force exerted by each large sphere. Add these forces as vectors to get the net force and then use Newton's second law to calculate the acceleration.
SET UP: The forces are shown in Figure 13.5.



$$\sin \theta = 0.80$$

$$\cos \theta = 0.60$$

Take the origin of coordinate at point P.

Figure 13.5

EXECUTE: $F_A = G \frac{m_A m}{r^2} = G \frac{(0.26 \text{ kg})(0.010 \text{ kg})}{(0.100 \text{ m})^2} = 1.735 \times 10^{-11} \text{ N}$

$$F_B = G \frac{m_B m}{r^2} = 1.735 \times 10^{-11} \text{ N}$$

$$F_{Ax} = -F_A \sin \theta = -(1.735 \times 10^{-11} \text{ N})(0.80) = -1.39 \times 10^{-11} \text{ N}$$

$$F_{Ay} = +F_A \cos \theta = +(1.735 \times 10^{-11} \text{ N})(0.60) = +1.04 \times 10^{-11} \text{ N}$$

$$F_{Bx} = +F_B \sin \theta = +1.39 \times 10^{-11} \text{ N}$$

$$F_{By} = +F_B \cos \theta = +1.04 \times 10^{-11} \text{ N}$$

$$\Sigma F_x = ma_x \text{ gives } F_{Ax} + F_{Bx} = ma_x$$

$$0 = ma_x \text{ so } a_x = 0$$

$$\Sigma F_y = ma_y \text{ gives } F_{Ay} + F_{By} = ma_y$$

$$2(1.04 \times 10^{-11} \text{ N}) = (0.010 \text{ kg})a_y$$

$$a_y = 2.1 \times 10^{-9} \text{ m/s}^2, \text{ directed downward midway between } A \text{ and } B$$

EVALUATE: For ordinary size objects the gravitational force is very small, so the initial acceleration is very small. By symmetry there is no x -component of net force and the y -component is in the direction of the two large spheres, since they attract the small sphere.

- 13.6. IDENTIFY:** The net force on A is the vector sum of the force due to B and the force due to C . In part (a), the two forces are in the same direction, but in (b) they are in opposite directions.

SET UP: Use coordinates where $+x$ is to the right. Each gravitational force is attractive, so is toward the mass exerting it. Treat the masses as uniform spheres, so the gravitational force is the same as for point masses with the same center-to-center distances. The free-body diagrams for (a) and (b) are given in Figures 13.6a and 13.6b. The gravitational force is $F_g = Gm_1m_2/r^2$.

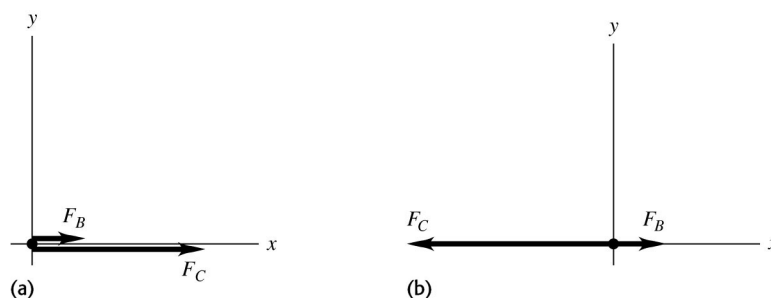


Figure 13.6

EXECUTE: (a) Calling F_B the force due to mass B and likewise for C , we have

$$F_B = G \frac{m_A m_B}{r_{AB}^2} = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(2.00 \text{ kg})^2}{(0.50 \text{ m})^2} = 1.069 \times 10^{-9} \text{ N} \text{ and}$$

$$F_C = G \frac{m_A m_C}{r_{AC}^2} = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(2.00 \text{ kg})^2}{(0.10 \text{ m})^2} = 2.669 \times 10^{-8} \text{ N. The net force is}$$

$$F_{\text{net}, x} = F_{Bx} + F_{Cx} = 1.069 \times 10^{-9} \text{ N} + 2.669 \times 10^{-8} \text{ N} = 2.8 \times 10^{-8} \text{ N to the right.}$$

(b) Following the same procedure as in (a), we have

$$F_B = G \frac{m_A m_B}{r_{AB}^2} = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(2.00 \text{ kg})^2}{(0.40 \text{ m})^2} = 1.668 \times 10^{-9} \text{ N}$$

$$F_C = G \frac{m_A m_C}{r_{AC}^2} = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(2.00 \text{ kg})^2}{(0.10 \text{ m})^2} = 2.669 \times 10^{-8} \text{ N}$$

$$F_{\text{net},x} = F_{Bx} + F_{Cx} = 1.668 \times 10^{-9} \text{ N} - 2.669 \times 10^{-8} \text{ N} = -2.5 \times 10^{-8} \text{ N}$$

The net force on A is $2.5 \times 10^{-8} \text{ N}$, to the left.

EVALUATE: As with any force, the gravitational force is a vector and must be treated like all other vectors. The formula $F_g = Gm_1m_2/r^2$ only gives the magnitude of this force.

- 13.7. IDENTIFY:** The force exerted by the moon is the gravitational force, $F_g = \frac{Gm_M m}{r^2}$. The force exerted on the person by the earth is $w = mg$.

SET UP: The mass of the moon is $m_M = 7.35 \times 10^{22} \text{ kg}$. $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

EXECUTE: (a) $F_{\text{moon}} = F_g = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(7.35 \times 10^{22} \text{ kg})(70 \text{ kg})}{(3.78 \times 10^8 \text{ m})^2} = 2.4 \times 10^{-3} \text{ N}$.

(b) $F_{\text{earth}} = w = (70 \text{ kg})(9.80 \text{ m/s}^2) = 690 \text{ N}$. $F_{\text{moon}}/F_{\text{earth}} = 3.5 \times 10^{-6}$.

EVALUATE: The force exerted by the earth is much greater than the force exerted by the moon. The mass of the moon is less than the mass of the earth and the center of the earth is much closer to the person than is the center of the moon.

- 13.8. IDENTIFY:** Use $F_g = Gm_1m_2/r^2$ to find the force each point mass exerts on the particle, find the net force, and use Newton's second law to calculate the acceleration.

SET UP: Each force is attractive. The particle (mass m) is a distance $r_1 = 0.200 \text{ m}$ from $m_1 = 8.00 \text{ kg}$ and therefore a distance $r_2 = 0.300 \text{ m}$ from $m_2 = 12.0 \text{ kg}$. Let $+x$ be toward the 12.0-kg mass.

EXECUTE: $F_1 = \frac{Gm_1m}{r_1^2} = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(8.00 \text{ kg})m}{(0.200 \text{ m})^2} = (1.334 \times 10^{-8} \text{ N/kg})m$, in the

$-x$ -direction. $F_2 = \frac{Gm_2m}{r_2^2} = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(12.0 \text{ kg})m}{(0.300 \text{ m})^2} = (8.893 \times 10^{-9} \text{ N/kg})m$, in the

$+x$ -direction. The net force is

$$F_x = F_{1x} + F_{2x} = (-1.334 \times 10^{-8} \text{ N/kg} + 8.893 \times 10^{-9} \text{ N/kg})m = (-4.447 \times 10^{-9} \text{ N/kg})m.$$

$a_x = \frac{F_x}{m} = -4.45 \times 10^{-9} \text{ m/s}^2$. The acceleration is $4.45 \times 10^{-9} \text{ m/s}^2$, toward the 8.00 kg mass.

EVALUATE: The smaller mass exerts the greater force, because the particle is closer to the smaller mass.

- 13.9. IDENTIFY:** Use $F_g = Gm_1m_2/r^2$ to calculate the gravitational force each particle exerts on the third mass.

The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.

SET UP: For the net force to be zero, the two forces on M must be in opposite directions. This is the case only when M is on the line connecting the two particles and between them. The free-body diagram for M is given in Figure 13.9. $m_1 = 3m$ and $m_2 = m$. If M is a distance x from m_1 , it is a distance $1.00 \text{ m} - x$ from m_2 .

EXECUTE: (a) $F_x = F_{1x} + F_{2x} = -G \frac{3mM}{x^2} + G \frac{mM}{(1.00 \text{ m} - x)^2} = 0$. Cancelling and simplifying gives

$$3(1.00 \text{ m} - x)^2 = x^2. \text{ Taking square roots gives } 1.00 \text{ m} - x = \pm x/\sqrt{3}. \text{ Since } M \text{ is between the two particles, } x \text{ must be less than } 1.00 \text{ m and } x = \frac{1.00 \text{ m}}{1 + 1/\sqrt{3}} = 0.634 \text{ m. } M \text{ must be placed at a point that is } 0.634 \text{ m from the}$$

particle of mass $3m$ and 0.366 m from the particle of mass m .

(b) (i) If M is displaced slightly to the right in Figure 13.9, the attractive force from m is larger than the force from $3m$ and the net force is to the right. If M is displaced slightly to the left in Figure 13.9, the

attractive force from $3m$ is larger than the force from m and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.

(ii) If M is displaced a very small distance along the y -axis in Figure 13.9, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.

EVALUATE: The point where the net force on M is zero is closer to the smaller mass.

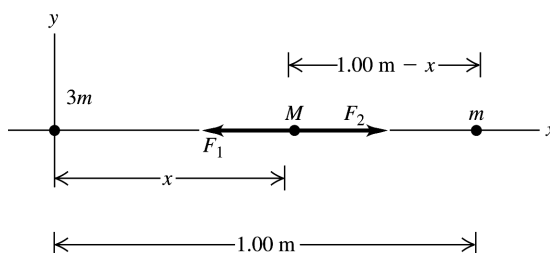


Figure 13.9

13.10. IDENTIFY: The force \vec{F}_1 exerted by m on M and the force \vec{F}_2 exerted by $2m$ on M are each given by

$F_g = Gm_1m_2/r^2$ and the net force is the vector sum of these two forces.

SET UP: Each force is attractive. The forces on M in each region are sketched in Figure 13.10a. Let M be at coordinate x on the x -axis.

EXECUTE: (a) For the net force to be zero, \vec{F}_1 and \vec{F}_2 must be in opposite directions and this is the case

only for $0 < x < L$. $\vec{F}_1 + \vec{F}_2 = 0$ then requires $F_1 = F_2$. $\frac{GmM}{x^2} = \frac{G(2m)M}{(L-x)^2}$. $2x^2 = (L-x)^2$ and

$L-x = \pm\sqrt{2}x$. x must be less than L , so $x = \frac{L}{1+\sqrt{2}} = 0.414L$.

(b) For $x < 0$, $F_x > 0$. $F_x \rightarrow 0$ as $x \rightarrow -\infty$ and $F_x \rightarrow +\infty$ as $x \rightarrow 0$. For $x > L$, $F_x < 0$. $F_x \rightarrow 0$ as $x \rightarrow \infty$ and $F_x \rightarrow -\infty$ as $x \rightarrow L$. For $0 < x < 0.414L$, $F_x < 0$ and F_x increases from $-\infty$ to 0 as x goes from 0 to $0.414L$. For $0.414L < x < L$, $F_x > 0$ and F_x increases from 0 to $+\infty$ as x goes from $0.414L$ to L . The graph of F_x versus x is sketched in Figure 13.10b (next page).

EVALUATE: Any real object is not exactly a point so it is not possible to have both m and M exactly at $x = 0$ or $2m$ and M both exactly at $x = L$. But the magnitude of the gravitational force between two objects approaches infinity as the objects get very close together.

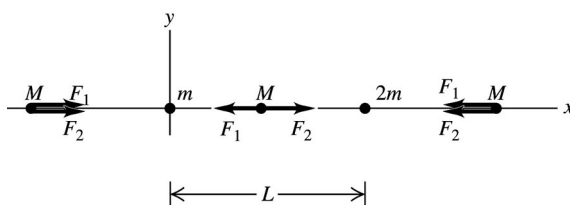


Figure 13.10a

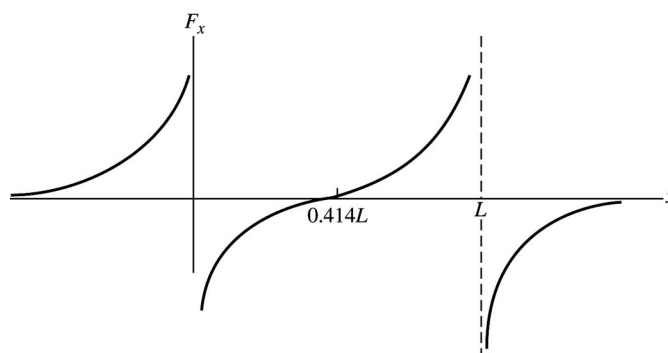


Figure 13.10b

- 13.11. IDENTIFY:** $F_g = G \frac{mm_E}{r^2} = mg$, so $g = G \frac{m_E}{r^2}$, where r is the distance of the object from the center of the earth.

SET UP: $r = h + R_E$, where h is the distance of the object above the surface of the earth and

$R_E = 6.37 \times 10^6$ m is the radius of the earth.

EXECUTE: To decrease the acceleration due to gravity by one-tenth, the distance from the center of the earth must be increased by a factor of $\sqrt{10}$, and so the distance above the surface of the earth is

$$(\sqrt{10} - 1)R_E = 1.38 \times 10^7 \text{ m.}$$

EVALUATE: This height is about twice the radius of the earth.

- 13.12. IDENTIFY:** Apply $g = G \frac{m_E}{r^2}$ to the earth and to Venus. $w = mg$.

SET UP: $g = \frac{Gm_E}{R_E^2} = 9.80 \text{ m/s}^2$. $m_V = 0.815m_E$ and $R_V = 0.949R_E$. $w_E = mg_E = 75.0 \text{ N}$.

EXECUTE: (a) $g_V = \frac{Gm_V}{R_V^2} = \frac{G(0.815m_E)}{(0.949R_E)^2} = 0.905 \frac{Gm_E}{R_E^2} = 0.905g_E$.

(b) $w_V = mg_V = 0.905mg_E = (0.905)(75.0 \text{ N}) = 67.9 \text{ N}$.

EVALUATE: The mass of the rock is independent of its location but its weight equals the gravitational force on it and that depends on its location.

- 13.13. (a) IDENTIFY and SET UP:** Apply $g = G \frac{m_E}{r^2}$ to the earth and to Titania. The acceleration due to gravity at

the surface of Titania is given by $g_T = Gm_T/R_T^2$, where m_T is its mass and R_T is its radius.

For the earth, $g_E = Gm_E/R_E^2$.

EXECUTE: For Titania, $m_T = m_E/1700$ and $R_T = R_E/8$, so

$$g_T = \frac{Gm_T}{R_T^2} = \frac{G(m_E/1700)}{(R_E/8)^2} = \left(\frac{64}{1700} \right) \frac{Gm_E}{R_E^2} = 0.0377g_E.$$

Since $g_E = 9.80 \text{ m/s}^2$, $g_T = (0.0377)(9.80 \text{ m/s}^2) = 0.37 \text{ m/s}^2$.

EVALUATE: g on Titania is much smaller than on earth. The smaller mass reduces g and is a greater effect than the smaller radius, which increases g .

(b) IDENTIFY and SET UP: Use density = mass/volume. Assume Titania is a sphere.

EXECUTE: From Section 13.2 we know that the average density of the earth is 5500 kg/m^3 . For Titania

$$\rho_T = \frac{m_T}{\frac{4}{3}\pi R_T^3} = \frac{m_E/1700}{\frac{4}{3}\pi (R_E/8)^3} = \frac{512}{1700} \rho_E = \frac{512}{1700} (5500 \text{ kg/m}^3) = 1700 \text{ kg/m}^3.$$

EVALUATE: The average density of Titania is about a factor of 3 smaller than for earth. We can write

$$a_g = G \frac{m_E}{r^2} \text{ for Titania as } g_T = \frac{4}{3} \pi G R_T \rho_T. \quad g_T < g_E \text{ both because } \rho_T < \rho_E \text{ and } R_T < R_E.$$

- 13.14. IDENTIFY:** Apply $g = G \frac{m_E}{r^2}$ to Rhea.

SET UP: $\rho = m/V$. The volume of a sphere is $V = \frac{4}{3} \pi R^3$.

$$\text{EXECUTE: } M = \frac{gR^2}{G} = 2.32 \times 10^{21} \text{ kg and } \rho = \frac{M}{(4\pi/3)R^3} = 1.24 \times 10^3 \text{ kg/m}^3.$$

EVALUATE: The average density of Rhea is a bit less than one-fourth that of the earth.

- 13.15. IDENTIFY:** Apply $F_g = Gm_1m_2/r^2$ to the astronaut.

SET UP: $m_E = 5.97 \times 10^{24} \text{ kg}$ and $R_E = 6.37 \times 10^6 \text{ m}$.

EXECUTE: $F_g = G \frac{mm_E}{r^2}$. $r = 600 \times 10^3 \text{ m} + R_E$ so $F_g = 610 \text{ N}$. At the surface of the earth,

$w = mg = 735 \text{ N}$. The gravity force is not zero in orbit. The satellite and the astronaut have the same acceleration so the astronaut's apparent weight is zero.

EVALUATE: In $F_g = G \frac{mm_E}{r^2}$, r is the distance of the object from the center of the earth.

- 13.16. IDENTIFY:** The gravity of Io limits the height to which volcanic material will rise. The acceleration due to gravity at the surface of Io depends on its mass and radius.

SET UP: The radius of Io is $R = 1.821 \times 10^6 \text{ m}$. Use coordinates where $+y$ is upward. At the maximum height, $v_{0y} = 0$, $a_y = -g_{\text{Io}}$, which is assumed to be constant. Therefore the constant-acceleration kinematics formulas apply. The acceleration due to gravity at Io's surface is given by $g_{\text{Io}} = Gm/R^2$.

$$\text{EXECUTE: At the surface of Io, } g_{\text{Io}} = \frac{Gm}{R^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(8.93 \times 10^{22} \text{ kg})}{(1.821 \times 10^6 \text{ m})^2} = 1.797 \text{ m/s}^2. \text{ For}$$

constant acceleration (assumed), the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies, so

$$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-1.797 \text{ m/s}^2)(5.00 \times 10^5 \text{ m})} = 1.3405 \times 10^3 \text{ m/s. Now solve for } y - y_0 \text{ when}$$

$$v_{0y} = 1.3405 \times 10^3 \text{ m/s and } a_y = -9.80 \text{ m/s}^2. \text{ The equation } v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives}$$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{-(1.3405 \times 10^3 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 9.17 \times 10^4 \text{ m} = 91.7 \text{ km.}$$

EVALUATE: Even though the mass of Io is around 100 times smaller than that of the earth, the acceleration due to gravity at its surface is only about 1/6 of that of the earth because Io's radius is much smaller than earth's radius.

- 13.17. IDENTIFY:** The escape speed, as shown in Example 13.5, is $\sqrt{2GM/R}$.

SET UP: For Mars, $M = 6.42 \times 10^{23} \text{ kg}$ and $R = 3.39 \times 10^6 \text{ m}$. For Jupiter, $M = 1.90 \times 10^{27} \text{ kg}$ and $R = 6.99 \times 10^7 \text{ m}$.

$$\text{EXECUTE: (a) } v = \sqrt{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(6.42 \times 10^{23} \text{ kg})/(3.39 \times 10^6 \text{ m})} = 5.03 \times 10^3 \text{ m/s.}$$

$$\text{(b) } v = \sqrt{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.90 \times 10^{27} \text{ kg})/(6.99 \times 10^7 \text{ m})} = 6.02 \times 10^4 \text{ m/s.}$$

(c) Both the kinetic energy and the gravitational potential energy are proportional to the mass of the spacecraft.

EVALUATE: Example 13.5 calculates the escape speed for earth to be $1.12 \times 10^4 \text{ m/s}$. This is larger than our result for Mars and less than our result for Jupiter.

13.18. IDENTIFY: The kinetic energy is $K = \frac{1}{2}mv^2$ and the potential energy is $U = -\frac{GMm}{r}$.

SET UP: The mass of the earth is $M_E = 5.97 \times 10^{24}$ kg.

EXECUTE: (a) $K = \frac{1}{2}(629 \text{ kg})(3.33 \times 10^3 \text{ m/s})^2 = 3.49 \times 10^9 \text{ J}$

(b) $U = -\frac{GM_E m}{r} = -\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(629 \text{ kg})}{2.87 \times 10^9 \text{ m}} = -8.73 \times 10^7 \text{ J}.$

EVALUATE: The total energy $K + U$ is positive.

13.19. IDENTIFY: Mechanical energy is conserved. At the escape speed, the object has no kinetic energy when it is very far away from the planet.

SET UP: Call m the mass of the object, M the mass of the planet, and r its radius. $K_1 + U_1 = K_2 + U_2$, $K = \frac{1}{2}mv^2$, $U = -GmM/r$, $g = GM/r^2$.

EXECUTE: Energy conservation gives $\frac{1}{2}mv^2 - GmM/r = 0 + 0$. $M = rv^2/2G$. Putting this into $g = GM/r^2$

gives $g = \frac{G\left(\frac{rv^2}{2G}\right)}{r^2} = \frac{v^2}{2r}$. Putting in the numbers gives

$g = (7.65 \times 10^3 \text{ m/s})^2/[2(3.24 \times 10^6 \text{ m})] = 9.03 \text{ m/s}^2$.

EVALUATE: This result is not very different from g on earth, so it is physically reasonable for a planet.

13.20. IDENTIFY: Newton's second law and his law of gravitation both apply to the satellite.

SET UP: $T = \frac{2\pi r}{v}$. r and v are also related by applying $\Sigma \vec{F} = m\vec{a}$ to the motion of the satellite. The satellite

has $a_{\text{rad}} = v^2/R$, and the only force on the satellite is the gravitational force, $F_g = G\frac{m_E m}{r^2}$.

$m_E = 5.97 \times 10^{24}$ kg.

EXECUTE: (a) The free-body diagram of the satellite is shown in Figure 13.20. $F_g = ma_{\text{rad}}$ gives

$G\frac{m_E m}{r^2} = m\frac{v^2}{r}$, which simplifies to $G\frac{m_E}{r} = v^2$. Solving for r gives

$r = \frac{Gm_E}{v^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{(6200 \text{ m/s})^2} = 1.04 \times 10^7 \text{ m}.$

$T = \frac{2\pi r}{v} = \frac{2\pi(1.04 \times 10^7 \text{ m})}{6200 \text{ m/s}} = 1.05 \times 10^4 \text{ s} = 176 \text{ min} = 2.93 \text{ h}.$

(b) $a_{\text{rad}} = \frac{v^2}{r} = \frac{(6200 \text{ m/s})^2}{1.04 \times 10^7 \text{ m}} = 3.70 \text{ m/s}^2.$

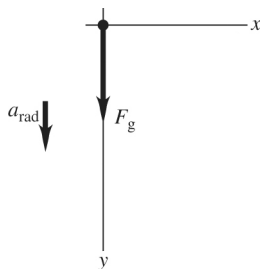
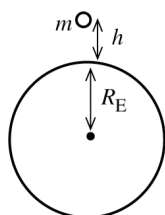


Figure 13.20

EVALUATE: The acceleration of the satellite is 38% of the acceleration due to gravity at the earth's surface.

- 13.21. IDENTIFY:** Apply Newton's second law to the motion of the satellite and obtain an equation that relates the orbital speed v to the orbital radius r .

SET UP: The distances are shown in Figure 13.21a.

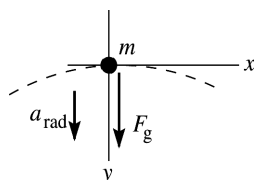


The radius of the orbit is $r = h + R_E$.

$$r = 8.90 \times 10^5 \text{ m} + 6.37 \times 10^6 \text{ m} = 7.26 \times 10^6 \text{ m}.$$

Figure 13.21a

The free-body diagram for the satellite is given in Figure 13.21b.



(a) EXECUTE: $\Sigma F_y = ma_y$

$$F_g = ma_{\text{rad}}$$

$$G \frac{mm_E}{r^2} = m \frac{v^2}{r}$$

Figure 13.21b

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{7.26 \times 10^6 \text{ m}}} = 7.408 \times 10^3 \text{ m/s} \text{ which rounds to } 7410 \text{ m/s}.$$

$$\text{(b) } T = \frac{2\pi r}{v} = \frac{2\pi(7.26 \times 10^6 \text{ m})}{7.408 \times 10^3 \text{ m/s}} = 6158 \text{ s} = 1.71 \text{ h}.$$

EVALUATE: Note that $r = h + R_E$ is the radius of the orbit, measured from the center of the earth. For this satellite r is greater than for the satellite in Example 13.6, so its orbital speed is less.

- 13.22. IDENTIFY:** The time to complete one orbit is the period T , given by $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$. The speed v of the

satellite is given by $v = \frac{2\pi r}{T}$.

SET UP: If h is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_E$.

$$R_E = 6.37 \times 10^6 \text{ m} \text{ and } m_E = 5.97 \times 10^{24} \text{ kg}.$$

$$\text{EXECUTE: (a) } T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}} = \frac{2\pi(7.05 \times 10^5 \text{ m} + 6.37 \times 10^6 \text{ m})^{3/2}}{\sqrt{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}} = 5.93 \times 10^3 \text{ s} = 98.8 \text{ min}$$

$$\text{(b) } v = \frac{2\pi(7.05 \times 10^5 \text{ m} + 6.37 \times 10^6 \text{ m})}{5.93 \times 10^3 \text{ s}} = 7.50 \times 10^3 \text{ m/s} = 7.50 \text{ km/s}.$$

EVALUATE: The satellite in Example 13.6 is at a lower altitude and therefore has a smaller orbit radius than the satellite in this problem. Therefore, the satellite in this problem has a larger period and a smaller orbital speed. But a large percentage change in h corresponds to a small percentage change in r and the values of T and v for the two satellites do not differ very much.

- 13.23. IDENTIFY:** We know orbital data (speed and orbital radius) for one satellite and want to use it to find the orbital speed of another satellite having a known orbital radius. Newton's second law and the law of universal gravitation apply to both satellites.

SET UP: For circular motion, $F_{\text{net}} = ma = mv^2/r$, which in this case is $G \frac{mm_p}{r^2} = m \frac{v^2}{r}$.

EXECUTE: Using $G \frac{mm_p}{r^2} = m \frac{v^2}{r}$, we get $Gm_p = rv^2 = \text{constant}$. $r_1 v_1^2 = r_2 v_2^2$.

$$v_2 = v_1 \sqrt{\frac{r_1}{r_2}} = (4800 \text{ m/s}) \sqrt{\frac{7.00 \times 10^7 \text{ m}}{3.00 \times 10^7 \text{ m}}} = 7330 \text{ m/s}.$$

EVALUATE: The more distant satellite moves slower than the closer satellite, which is reasonable since the planet's gravity decreases with distance. The masses of the satellites do not affect their orbits.

- 13.24. IDENTIFY:** We can calculate the orbital period T from the number of revolutions per day. Then the period and the orbit radius are related by $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$.

SET UP: $m_E = 5.97 \times 10^{24} \text{ kg}$ and $R_E = 6.37 \times 10^6 \text{ m}$. The height h of the orbit above the surface of the earth is related to the orbit radius r by $r = h + R_E$. 1 day = $8.64 \times 10^4 \text{ s}$.

EXECUTE: The satellite moves 15.65 revolutions in $8.64 \times 10^4 \text{ s}$, so the time for 1.00 revolution is

$$T = \frac{8.64 \times 10^4 \text{ s}}{15.65} = 5.52 \times 10^3 \text{ s}. \quad T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}} \text{ gives}$$

$$r = \left(\frac{Gm_E T^2}{4\pi^2} \right)^{1/3} = \left(\frac{[6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2][5.97 \times 10^{24} \text{ kg}][5.52 \times 10^3 \text{ s}]^2}{4\pi^2} \right)^{1/3}. \quad r = 6.75 \times 10^6 \text{ m} \text{ and}$$

$$h = r - R_E = 3.8 \times 10^5 \text{ m} = 380 \text{ km}.$$

EVALUATE: The period of this satellite is slightly larger than the period for the satellite in Example 13.6 and the altitude of this satellite is therefore somewhat greater.

- 13.25. IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the motion of the baseball. $v = \frac{2\pi r}{T}$.

SET UP: $r_D = 6 \times 10^3 \text{ m}$.

EXECUTE: (a) $F_g = ma_{\text{rad}}$ gives $G \frac{m_D m}{r_D^2} = m \frac{v^2}{r_D}$.

$$v = \sqrt{\frac{Gm_D}{r_D}} = \sqrt{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.5 \times 10^{15} \text{ kg})}{6 \times 10^3 \text{ m}}} = 4.07 \text{ m/s} \text{ which rounds to } 4.1 \text{ m/s}.$$

4.1 m/s = 9.1 mph, which is easy to achieve.

$$\text{(b)} \quad T = \frac{2\pi r}{v} = \frac{2\pi(6 \times 10^3 \text{ m})}{4.07 \text{ m/s}} = 9263 \text{ s} = 154.4 \text{ min} = 2.6 \text{ h}. \text{ The game would last a very long time indeed!}$$

EVALUATE: The speed v is relative to the center of Deimos. The baseball would already have some speed before we throw it because of the rotational motion of Deimos.

- 13.26. IDENTIFY:** $T = \frac{2\pi r}{v}$ and $F_g = ma_{\text{rad}}$.

SET UP: The sun has mass $m_S = 1.99 \times 10^{30} \text{ kg}$. The radius of Mercury's orbit is $5.79 \times 10^{10} \text{ m}$, so the radius of Vulcan's orbit is $3.86 \times 10^{10} \text{ m}$.

EXECUTE: $F_g = ma_{\text{rad}}$ gives $G \frac{m_S m}{r^2} = m \frac{v^2}{r}$ and $v^2 = \frac{Gm_S}{r}$.

$$T = 2\pi r \sqrt{\frac{r}{Gm_S}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_S}} = \frac{2\pi(3.86 \times 10^{10} \text{ m})^{3/2}}{\sqrt{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}} = 4.13 \times 10^6 \text{ s} = 47.8 \text{ days}$$

EVALUATE: The orbital period of Mercury is 88.0 d, so we could calculate T for Vulcan as

$$T = (88.0 \text{ d})(2/3)^{3/2} = 47.9 \text{ days}.$$

- 13.27. IDENTIFY:** The orbital speed is given by $v = \sqrt{Gm/r}$, where m is the mass of the star. The orbital period is given by $T = \frac{2\pi r}{v}$.

SET UP: The sun has mass $m_S = 1.99 \times 10^{30} \text{ kg}$. The orbit radius of the earth is $1.50 \times 10^{11} \text{ m}$.

EXECUTE: (a) $v = \sqrt{Gm/r}$.

$$v = \sqrt{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.85 \times 1.99 \times 10^{30} \text{ kg})/((1.50 \times 10^{11} \text{ m})(0.11))} = 8.27 \times 10^4 \text{ m/s}.$$

(b) $2\pi r/v = 1.25 \times 10^6 \text{ s} = 14.5 \text{ days}$ (about two weeks).

EVALUATE: The orbital period is less than the 88-day orbital period of Mercury; this planet is orbiting very close to its star, compared to the orbital radius of Mercury.

- 13.28. IDENTIFY:** The period of each satellite is given by $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_p}}$. Set up a ratio involving T and r .

SET UP: $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_p}}$ gives $\frac{T}{r^{3/2}} = \frac{2\pi}{\sqrt{Gm_p}} = \text{constant}$, so $\frac{T_1}{r_1^{3/2}} = \frac{T_2}{r_2^{3/2}}$.

EXECUTE: $T_2 = T_1 \left(\frac{r_2}{r_1} \right)^{3/2} = (6.39 \text{ days}) \left(\frac{48,000 \text{ km}}{19,600 \text{ km}} \right)^{3/2} = 24.5 \text{ days}$. For the other satellite,

$$T_2 = (6.39 \text{ days}) \left(\frac{64,000 \text{ km}}{19,600 \text{ km}} \right)^{3/2} = 37.7 \text{ days}.$$

EVALUATE: T increases when r increases.

- 13.29. IDENTIFY:** Kepler's third law applies.

SET UP: $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_s}}$, $d_{\min} = a(1 - e)$, $d_{\max} = a(1 + e)$.

EXECUTE: (a) Kepler's third law gives

$$T = \frac{2\pi a^{3/2}}{\sqrt{Gm_s}} = \frac{2\pi (5.91 \times 10^{12} \text{ m})^{3/2}}{\sqrt{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}} = 7.84 \times 10^9 \text{ s} [(1 \text{ y})/(3.156 \times 10^7 \text{ s})] = 248 \text{ y}.$$

(b) $d_{\min} = a(1 - e) = (5.91 \times 10^{12} \text{ m})(1 - 0.249) = 4.44 \times 10^{12} \text{ m}$; $d_{\max} = a(1 + e) = 7.38 \times 10^{12} \text{ m}$.

EVALUATE: $d_{\max} = 1.66d_{\min}$, which is *much* greater than for the earth's orbit since the earth moves in a much more circular orbit than Pluto.

- 13.30. IDENTIFY:** $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$, where m_{star} is the mass of the star. $v = \frac{2\pi r}{T}$.

SET UP: $3.09 \text{ days} = 2.67 \times 10^5 \text{ s}$. The orbit radius of Mercury is $5.79 \times 10^{10} \text{ m}$. The mass of our sun is $1.99 \times 10^{30} \text{ kg}$.

EXECUTE: (a) $T = 2.67 \times 10^5 \text{ s}$. $r = (5.79 \times 10^{10} \text{ m})/9 = 6.43 \times 10^9 \text{ m}$. $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$ gives

$$m_{\text{star}} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2 (6.43 \times 10^9 \text{ m})^3}{(2.67 \times 10^5 \text{ s})^2 (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 2.21 \times 10^{30} \text{ kg}. \quad \frac{m_{\text{star}}}{m_{\text{sun}}} = 1.11, \text{ so } m_{\text{star}} = 1.11 m_{\text{sun}}.$$

(b) $v = \frac{2\pi r}{T} = \frac{2\pi (6.43 \times 10^9 \text{ m})}{2.67 \times 10^5 \text{ s}} = 1.51 \times 10^5 \text{ m/s} = 151 \text{ km/s}$

EVALUATE: The orbital period of Mercury is 88.0 d. The period for this planet is much less primarily because the orbit radius is much less and also because the mass of the star is greater than the mass of our sun.

- 13.31. IDENTIFY:** Knowing the orbital radius and orbital period of a satellite, we can calculate the mass of the object about which it is revolving.

SET UP: The radius of the orbit is $r = 10.5 \times 10^9$ m and its period is $T = 6.3$ days $= 5.443 \times 10^5$ s. The

mass of the sun is $m_S = 1.99 \times 10^{30}$ kg. The orbital period is given by $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_{HD}}}$.

EXECUTE: Solving $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_{HD}}}$ for the mass of the star gives

$$m_{HD} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2 (10.5 \times 10^9 \text{ m})^3}{(5.443 \times 10^5 \text{ s})^2 (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 2.3 \times 10^{30} \text{ kg, which is } m_{HD} = 1.2 m_S.$$

EVALUATE: The mass of the star is only 20% greater than that of our sun, yet the orbital period of the planet is much shorter than that of the earth, so the planet must be much closer to the star than the earth is.

- 13.32. IDENTIFY:** Section 13.6 states that for a point mass outside a spherical shell the gravitational force is the same as if all the mass of the shell were concentrated at its center. It also states that for a point inside a spherical shell the force is zero.

SET UP: For $r = 5.01$ m the point mass is outside the shell and for $r = 4.99$ m and $r = 2.72$ m the point mass is inside the shell.

EXECUTE: (a) (i) $F_g = \frac{Gm_1 m_2}{r^2} = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(1000.0 \text{ kg})(2.00 \text{ kg})}{(5.01 \text{ m})^2} = 5.31 \times 10^{-9} \text{ N}.$

(ii) $F_g = 0.$ (iii) $F_g = 0.$

(b) For $r < 5.00$ m the force is zero and for $r > 5.00$ m the force is proportional to $1/r^2$. The graph of F_g versus r is sketched in Figure 13.32.

EVALUATE: Inside the shell the gravitational potential energy is constant and the force on a point mass inside the shell is zero.

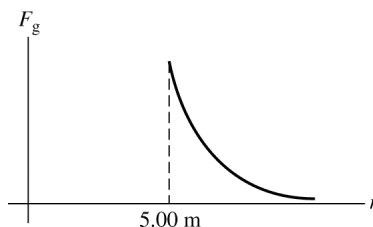


Figure 13.32

- 13.33. IDENTIFY:** Section 13.6 states that for a point mass outside a uniform sphere the gravitational force is the same as if all the mass of the sphere were concentrated at its center. It also states that for a point mass a distance r from the center of a uniform sphere, where r is less than the radius of the sphere, the gravitational force on the point mass is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The density of the sphere is $\rho = \frac{M}{\frac{4}{3}\pi R^3}$, where M is the mass of the sphere and R is its radius.

The mass inside a volume of radius $r < R$ is $M_r = \rho V_r = \left(\frac{M}{\frac{4}{3}\pi R^3} \right) \left(\frac{4}{3}\pi r^3 \right) = M \left(\frac{r}{R} \right)^3$. $r = 5.01$ m is

outside the sphere and $r = 2.50$ m is inside the sphere. $F_g = \frac{Gm_1 m_2}{r^2}$.

EXECUTE: (a) (i) $F_g = \frac{GMm}{r^2} = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(1000.0 \text{ kg})(2.00 \text{ kg})}{(5.01 \text{ m})^2} = 5.31 \times 10^{-9} \text{ N}.$

$$(ii) F_g = \frac{GM'm}{r^2}. \quad M' = M \left(\frac{r}{R} \right)^3 = (1000.0 \text{ kg}) \left(\frac{2.50 \text{ m}}{5.00 \text{ m}} \right)^3 = 125 \text{ kg}.$$

$$F_g = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(125 \text{ kg})(2.00 \text{ kg})}{(2.50 \text{ m})^2} = 2.67 \times 10^{-9} \text{ N}.$$

$$(b) F_g = \frac{GM(r/R)^3 m}{r^2} = \left(\frac{GMm}{R^3} \right) r \text{ for } r < R \text{ and } F_g = \frac{GMm}{r^2} \text{ for } r > R. \text{ The graph of } F_g \text{ versus } r \text{ is}$$

sketched in Figure 13.33.

EVALUATE: At points outside the sphere the force on a point mass is the same as for a shell of the same mass and radius. For $r < R$ the force is different in the two cases of uniform sphere versus hollow shell.

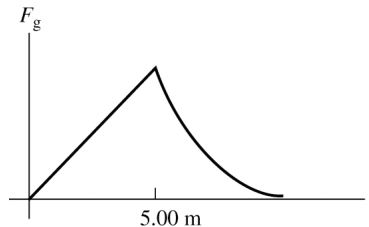


Figure 13.33

- 13.34. IDENTIFY:** The gravitational potential energy of a pair of point masses is $U = -G \frac{m_1 m_2}{r}$. Divide the rod into infinitesimal pieces and integrate to find U .

SET UP: Divide the rod into differential masses dm at position l , measured from the right end of the rod. $dm = dl(M/L)$.

$$\text{EXECUTE: (a) } U = -\frac{Gm \, dm}{l+x} = -\frac{GmM}{L} \frac{dl}{l+x}.$$

$$\text{Integrating, } U = -\frac{GmM}{L} \int_0^L \frac{dl}{l+x} = -\frac{GmM}{L} \ln \left(1 + \frac{L}{x} \right). \text{ For } x \gg L, \text{ the natural logarithm is } \sim (L/x), \text{ and } U \rightarrow -GmM/x.$$

$$(b) \text{ The } x\text{-component of the gravitational force on the sphere is } F_x = -\frac{\partial U}{\partial x} = \frac{GmM}{L} \frac{(-L/x^2)}{(1+(L/x))} = -\frac{GmM}{x^2 + Lx},$$

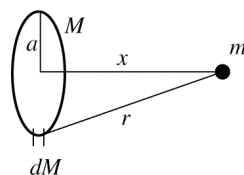
with the minus sign indicating an attractive force. As

$x \gg L$, the denominator in the above expression approaches x^2 , and $F_x \rightarrow -GmM/x^2$, as expected.

EVALUATE: When x is much larger than L the rod can be treated as a point mass, and our results for U and F_x do reduce to the correct expression when $x \gg L$.

- 13.35. IDENTIFY:** Find the potential due to a small segment of the ring and integrate over the entire ring to find the total U .

(a) SET UP:



Divide the ring up into small segments dM , as indicated in Figure 13.35.

Figure 13.35

EXECUTE: The gravitational potential energy of dM and m is $dU = -GmdM/r$.

The total gravitational potential energy of the ring and particle is $U = \int dU = -Gm \int dM/r$.

But $r = \sqrt{x^2 + a^2}$ is the same for all segments of the ring, so

$$U = -\frac{Gm}{r} \int dM = -\frac{GmM}{r} = -\frac{GmM}{\sqrt{x^2 + a^2}}.$$

(b) EVALUATE: When $x \gg a$, $\sqrt{x^2 + a^2} \rightarrow \sqrt{x^2} = x$ and $U = -GmM/x$. This is the gravitational potential energy of two point masses separated by a distance x . This is the expected result.

(c) IDENTIFY and SET UP: Use $F_x = -dU/dx$ with $U(x)$ from part (a) to calculate F_x .

EXECUTE: $F_x = -\frac{dU}{dx} = -\frac{d}{dx} \left(-\frac{GmM}{\sqrt{x^2 + a^2}} \right)$

$$F_x = +GmM \frac{d}{dx} (x^2 + a^2)^{-1/2} = GmM \left(-\frac{1}{2} (2x) (x^2 + a^2)^{-3/2} \right)$$

$F_x = -GmMx/(x^2 + a^2)^{3/2}$; the minus sign means the force is attractive.

EVALUATE: (d) For $x \gg a$, $(x^2 + a^2)^{3/2} \rightarrow (x^2)^{3/2} = x^3$

Then $F_x = -GmMx/x^3 = -GmM/x^2$. This is the force between two point masses separated by a distance x and is the expected result.

(e) For $x = 0$, $U = -GmM/a$. Each small segment of the ring is the same distance from the center and the potential is the same as that due to a point charge of mass M located at a distance a .

For $x = 0$, $F_x = 0$. When the particle is at the center of the ring, symmetrically placed segments of the ring exert equal and opposite forces and the total force exerted by the ring is zero.

- 13.36. IDENTIFY:** At the north pole, Sneezy has no circular motion and therefore no acceleration. But at the equator he has acceleration toward the center of the earth due to the earth's rotation.

SET UP: The earth has mass $m_E = 5.97 \times 10^{24}$ kg, radius $R_E = 6.37 \times 10^6$ m and rotational period

$T = 24$ hr $= 8.64 \times 10^4$ s. Use coordinates for which the $+y$ direction is toward the center of the earth. The free-body diagram for Sneezy at the equator is given in Figure 13.36. The radial acceleration due to

Sneezy's circular motion at the equator is $a_{\text{rad}} = \frac{4\pi^2 R}{T^2}$, and Newton's second law applies to Sneezy.

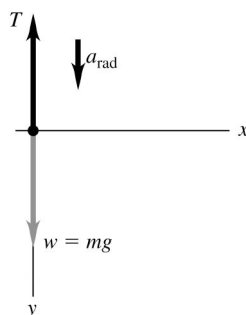


Figure 13.36

EXECUTE: At the north pole Sneezy has $a = 0$ and $T = w = 395.0$ N (the gravitational force exerted by the earth). Sneezy has mass $m = w/g = 40.31$ kg. At the equator Sneezy is traveling in a circular path and

has radial acceleration $a_{\text{rad}} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 (6.37 \times 10^6 \text{ m})}{(8.64 \times 10^4 \text{ s})^2} = 0.03369 \text{ m/s}^2$. Newton's second law

$\Sigma F_y = ma_y$ gives $w - T = ma_{\text{rad}}$. Solving for T gives

$$T = w - ma_{\text{rad}} = m(g - a_{\text{rad}}) = (40.31 \text{ kg})(9.80 \text{ m/s}^2 - 0.03369 \text{ m/s}^2) = 394 \text{ N}.$$

EVALUATE: At the equator Sneezy has an inward acceleration and the outward tension is less than the true weight, since there is a net inward force.

13.37. IDENTIFY and SET UP: At the north pole, $F_g = w_0 = mg_0$, where g_0 is given by $g = G \frac{m_E}{r^2}$ applied to

Neptune. At the equator, the apparent weight is given by $w = w_0 - mv^2/R$. The orbital speed v is obtained from the rotational period using $v = 2\pi R/T$.

EXECUTE: (a) $g_0 = Gm/R^2 = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.02 \times 10^{26} \text{ kg})/(2.46 \times 10^7 \text{ m})^2 = 11.25 \text{ m/s}^2$.

This agrees with the value of g given in the problem.

$F = w_0 = mg_0 = (3.00 \text{ kg})(11.25 \text{ m/s}^2) = 33.74 \text{ N}$, which rounds to 33.7 N. This is the true weight of the object.

(b) We have $w = w_0 - mv^2/R$

$$T = \frac{2\pi r}{v} \text{ gives } v = \frac{2\pi r}{T} = \frac{2\pi(2.46 \times 10^7 \text{ m})}{(16 \text{ h})(3600 \text{ s/1 h})} = 2.683 \times 10^3 \text{ m/s}$$

$$v^2/R = (2.683 \times 10^3 \text{ m/s})^2 / (2.46 \times 10^7 \text{ m}) = 0.2927 \text{ m/s}^2$$

Then $w = 33.74 \text{ N} - (3.00 \text{ kg})(0.2927 \text{ m/s}^2) = 32.9 \text{ N}$.

EVALUATE: The apparent weight is less than the true weight. This effect is larger on Neptune than on earth.

13.38. IDENTIFY: The radius of a black hole and its mass are related by $R_S = \frac{2GM}{c^2}$.

SET UP: $R_S = 0.50 \times 10^{-15} \text{ m}$, $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ and $c = 3.00 \times 10^8 \text{ m/s}$.

$$\text{EXECUTE: } M = \frac{c^2 R_S}{2G} = \frac{(3.00 \times 10^8 \text{ m/s})^2 (0.50 \times 10^{-15} \text{ m})}{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 3.4 \times 10^{11} \text{ kg}$$

EVALUATE: The average density of the black hole would be

$$\rho = \frac{M}{\frac{4}{3}\pi R_S^3} = \frac{3.4 \times 10^{11} \text{ kg}}{\frac{4}{3}\pi (0.50 \times 10^{-15} \text{ m})^3} = 6.49 \times 10^{56} \text{ kg/m}^3. \text{ We can combine } \rho = \frac{M}{\frac{4}{3}\pi R_S^3} \text{ and } R_S = \frac{2GM}{c^2} \text{ to}$$

give $\rho = \frac{3c^6}{32\pi G^3 M^2}$. The average density of a black hole increases when its mass decreases. The average

density of this mini black hole is much greater than the average density of the much more massive black hole in Example 13.11.

13.39. IDENTIFY: The orbital speed for an object a distance r from an object of mass M is $v = \sqrt{\frac{GM}{r}}$. The

mass M of a black hole and its Schwarzschild radius R_S are related by $R_S = \frac{2GM}{c^2}$.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$. $1 \text{ ly} = 9.461 \times 10^{15} \text{ m}$.

EXECUTE:

$$\text{(a) } M = \frac{rv^2}{G} = \frac{(7.5 \text{ ly})(9.461 \times 10^{15} \text{ m/ly})(200 \times 10^3 \text{ m/s})^2}{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 4.3 \times 10^{37} \text{ kg} = 2.1 \times 10^7 M_S.$$

(b) No, the object has a mass very much greater than 50 solar masses.

$$\text{(c) } R_S = \frac{2GM}{c^2} = \frac{2v^2 r}{c^2} = 6.32 \times 10^{10} \text{ m, which does fit.}$$

EVALUATE: The Schwarzschild radius of a black hole is approximately the same as the radius of Mercury's orbit around the sun.

- 13.40. IDENTIFY:** The clumps orbit the black hole. Their speed, orbit radius and orbital period are related by $v = \frac{2\pi r}{T}$. Their orbit radius and period are related to the mass M of the black hole by $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$. The radius of the black hole's event horizon is related to the mass of the black hole by $R_S = \frac{2GM}{c^2}$.

SET UP: $v = 3.00 \times 10^7$ m/s. $T = 27$ h $= 9.72 \times 10^4$ s. $c = 3.00 \times 10^8$ m/s.

EXECUTE: (a) $r = \frac{vT}{2\pi} = \frac{(3.00 \times 10^7 \text{ m/s})(9.72 \times 10^4 \text{ s})}{2\pi} = 4.64 \times 10^{11}$ m.

(b) $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$ gives $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (4.64 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(9.72 \times 10^4 \text{ s})^2} = 6.26 \times 10^{36}$ kg.
 $= 3.15 \times 10^6 M_S$, where M_S is the mass of our sun

(c) $R_S = \frac{2GM}{c^2} = \frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(6.26 \times 10^{36} \text{ kg})}{(3.00 \times 10^8 \text{ m/s})^2} = 9.28 \times 10^9$ m

EVALUATE: The black hole has a mass that is about 3×10^6 solar masses.

- 13.41. IDENTIFY:** $g_n = G \frac{m_n}{R_n^2}$, where the subscript n refers to the neutron star. $w = mg$.

SET UP: $R_n = 10.0 \times 10^3$ m. $m_n = 1.99 \times 10^{30}$ kg. Your mass is $m = \frac{w}{g} = \frac{675 \text{ N}}{9.80 \text{ m/s}^2} = 68.9$ kg.

EXECUTE: $g_n = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{1.99 \times 10^{30} \text{ kg}}{(10.0 \times 10^3 \text{ m})^2} = 1.33 \times 10^{12} \text{ m/s}^2$

Your weight on the neutron star would be $w_n = mg_n = (68.9 \text{ kg})(1.33 \times 10^{12} \text{ m/s}^2) = 9.16 \times 10^{13}$ N.

EVALUATE: Since R_n is much less than the radius of the sun, the gravitational force exerted by the neutron star on an object at its surface is immense.

- 13.42. IDENTIFY:** Apply $F_g = Gm_1m_2/r^2$ to calculate the magnitude of each gravitational force. Each force is attractive.

SET UP: The forces on one of the masses are sketched in Figure 13.42. The figure shows that the vector sum of the three forces is toward the center of the square.

EXECUTE: $F_{\text{on A}} = 2F_B \cos 45^\circ + F_D = 2 \frac{Gm_A m_B \cos 45^\circ}{r_{AB}^2} + \frac{Gm_A m_D}{r_{AD}^2}$.

$F_{\text{on A}} = \frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(8.00 \text{ kg})^2 \cos 45^\circ}{(2.00 \text{ m})^2} + \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(8.00 \text{ kg})^2}{2(2.00 \text{ m})^2} = 2.04 \times 10^{-9}$ N

toward the center of the square.

EVALUATE: We have assumed each mass can be treated as a uniform sphere. This is reasonable since an 8.00-kg sphere could easily be small enough to fit inside a square that is 2.00 m on each side.

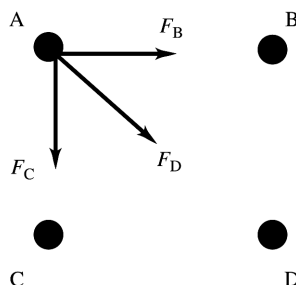


Figure 13.42

13.43. IDENTIFY: Use $F_g = Gm_1m_2/r^2$ to find each gravitational force. Each force is attractive. In part (b) apply conservation of energy.

SET UP: For a pair of masses m_1 and m_2 with separation r , $U = -G\frac{m_1m_2}{r}$.

EXECUTE: (a) From symmetry, the net gravitational force will be in the direction 45° from the x -axis (bisecting the x - and y -axes), with magnitude

$$F = (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.0150 \text{ kg}) \left[\frac{(2.0 \text{ kg})}{(2(0.50 \text{ m})^2)} + 2 \frac{(1.0 \text{ kg})}{(0.50 \text{ m})^2} \sin 45^\circ \right] = 9.67 \times 10^{-12} \text{ N}$$

(b) The initial displacement is so large that the initial potential energy may be taken to be zero. From the work-energy theorem, $\frac{1}{2}mv^2 = Gm \left[\frac{(2.0 \text{ kg})}{\sqrt{2}(0.50 \text{ m})} + 2 \frac{(1.0 \text{ kg})}{(0.50 \text{ m})} \right]$. Canceling the factor of m and solving

for v , and using the numerical values gives $v = 3.02 \times 10^{-5} \text{ m/s}$.

EVALUATE: The result in part (b) is independent of the mass of the particle. It would take the particle a long time to reach point P .

13.44. IDENTIFY: Use $g = G\frac{m_E}{r^2}$ to calculate g for Europa. The acceleration of a particle moving in a circular path is $a_{\text{rad}} = r\omega^2$.

SET UP: In $a_{\text{rad}} = r\omega^2$, ω must be in rad/s. For Europa, $R = 1.560 \times 10^6 \text{ m}$.

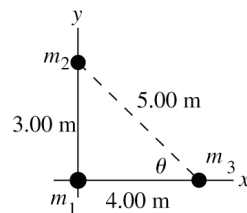
EXECUTE: $g = \frac{Gm}{R^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(4.80 \times 10^{22} \text{ kg})}{(1.560 \times 10^6 \text{ m})^2} = 1.316 \text{ m/s}^2$. $g = a_{\text{rad}}$ gives

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{1.316 \text{ m/s}^2}{4.25 \text{ m}}} = (0.5565 \text{ rad/s}) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 5.31 \text{ rpm}.$$

EVALUATE: The radius of Europa is about one-fourth that of the earth and its mass is about one-hundredth that of earth, so g on Europa is much less than g on earth. The lander would have some spatial extent so different points on it would be different distances from the rotation axis and a_{rad} would have different values. For the ω we calculated, $a_{\text{rad}} = g$ at a point that is precisely 4.25 m from the rotation axis.

13.45. IDENTIFY: Use $F_g = Gm_1m_2/r^2$ to calculate each gravitational force and add the forces as vectors.

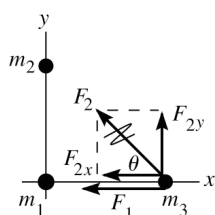
(a) SET UP: The locations of the masses are sketched in Figure 13.45a.



Section 13.6 proves that any two spherically symmetric masses interact as though they were point masses with all the mass concentrated at their centers.

Figure 13.45a

The force diagram for m_3 is given in Figure 13.45b (next page).



$$\cos \theta = 0.800$$

$$\sin \theta = 0.600$$

Figure 13.45b

EXECUTE: $F_1 = G \frac{m_1 m_3}{r_{13}^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(50.0 \text{ kg})(0.500 \text{ kg})}{(4.00 \text{ m})^2} = 1.043 \times 10^{-10} \text{ N}$

$$F_2 = G \frac{m_2 m_3}{r_{23}^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(80.0 \text{ kg})(0.500 \text{ kg})}{(5.00 \text{ m})^2} = 1.068 \times 10^{-10} \text{ N}$$

$$F_{1x} = -1.043 \times 10^{-10} \text{ N}, \quad F_{1y} = 0$$

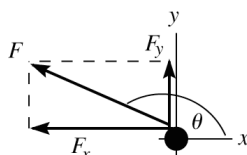
$$F_{2x} = -F_2 \cos \theta = -(1.068 \times 10^{-10} \text{ N})(0.800) = -8.544 \times 10^{-11} \text{ N}$$

$$F_{2y} = +F_2 \sin \theta = +(1.068 \times 10^{-10} \text{ N})(0.600) = +6.408 \times 10^{-11} \text{ N}$$

$$F_x = F_{1x} + F_{2x} = -1.043 \times 10^{-10} \text{ N} - 8.544 \times 10^{-11} \text{ N} = -1.897 \times 10^{-10} \text{ N}$$

$$F_y = F_{1y} + F_{2y} = 0 + 6.408 \times 10^{-11} \text{ N} = +6.408 \times 10^{-11} \text{ N}$$

F and its components are sketched in Figure 13.45c.



$$F = \sqrt{F_x^2 + F_y^2}$$

$$F = \sqrt{(-1.897 \times 10^{-10} \text{ N})^2 + (+6.408 \times 10^{-11} \text{ N})^2}$$

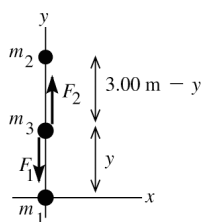
$$F = 2.00 \times 10^{-10} \text{ N}$$

$$\tan \theta = \frac{F_y}{F_x} = \frac{+6.408 \times 10^{-11} \text{ N}}{-1.897 \times 10^{-10} \text{ N}}; \quad \theta = 161^\circ$$

Figure 13.45c

EVALUATE: Both spheres attract the third sphere and the net force is in the second quadrant.

(b) SET UP: For the net force to be zero the forces from the two spheres must be equal in magnitude and opposite in direction. For the forces on it to be opposite in direction the third sphere must be on the y -axis and between the other two spheres. The forces on the third sphere are shown in Figure 13.45d.



EXECUTE: $F_{\text{net}} = 0$ if $F_1 = F_2$

$$G \frac{m_1 m_3}{y^2} = G \frac{m_2 m_3}{(3.00 \text{ m} - y)^2}$$

$$\frac{50.0}{y^2} = \frac{80.0}{(3.00 \text{ m} - y)^2}$$

$$\sqrt{80.0} y = \sqrt{50.0} (3.00 \text{ m} - y)$$

$$(\sqrt{80.0} + \sqrt{50.0}) y = (3.00 \text{ m}) \sqrt{50.0} \quad \text{and} \quad y = 1.32 \text{ m.}$$

Thus the sphere would have to be placed at the point $x = 0$, $y = 1.32 \text{ m}$.

Figure 13.45d

EVALUATE: For the forces to have the same magnitude the third sphere must be closer to the sphere that has smaller mass.

- 13.46. IDENTIFY:** The gravitational pulls of Titan and Saturn on the *Huygens* probe should be in opposite directions and of equal magnitudes to cancel.

SET UP: The mass of Saturn is $m_S = 5.68 \times 10^{26}$ kg. When the probe is a distance d from the center of Titan it is a distance $1.22 \times 10^9 \text{ m} - d$ from the center of Saturn. The magnitude of the gravitational force is given by $F_{\text{grav}} = GmM/r^2$.

EXECUTE: Equal gravity forces means the two gravitational pulls on the probe must balance, so

$$G \frac{mm_T}{d^2} = G \frac{mm_S}{(1.22 \times 10^9 \text{ m} - d)^2}. \text{ Simplifying, this becomes } d = \sqrt{\frac{m_T}{m_S}} (1.22 \times 10^9 \text{ m} - d). \text{ Using the masses}$$

$$\text{from the text and solving for } d \text{ we get } d = \sqrt{\frac{1.35 \times 10^{23} \text{ kg}}{5.68 \times 10^{26} \text{ kg}}} (1.22 \times 10^9 \text{ m} - d) = (0.0154)(1.22 \times 10^9 \text{ m} - d),$$

$$\text{so } d = 1.85 \times 10^7 \text{ m} = 1.85 \times 10^4 \text{ km}.$$

EVALUATE: For the forces to balance, the probe must be much closer to Titan than to Saturn since Titan's mass is much smaller than that of Saturn.

- 13.47. IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the motion of the two spheres.

SET UP: Denote the 50.0-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2.

EXECUTE: (a) Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1 v_1 = m_2 v_2$.

(b) (i) From the work-energy theorem in the form $K_i + U_i = K_f + U_f$, with the initial kinetic energy

$$K_i = 0 \text{ and } U = -G \frac{m_1 m_2}{r}, \quad G m_1 m_2 \left[\frac{1}{r_f} - \frac{1}{r_i} \right] = \frac{1}{2} (m_1 v_1^2 + m_2 v_2^2). \text{ Using the conservation of momentum}$$

$$\text{relation } m_1 v_1 = m_2 v_2 \text{ to eliminate } v_2 \text{ in favor of } v_1 \text{ and simplifying yields } v_1^2 = \frac{2Gm_2^2}{m_1 + m_2} \left[\frac{1}{r_f} - \frac{1}{r_i} \right], \text{ with a}$$

$$\text{similar expression for } v_2. \text{ Substitution of numerical values gives } v_1 = 1.49 \times 10^{-5} \text{ m/s, } v_2 = 7.46 \times 10^{-6} \text{ m/s.}$$

(ii) The magnitude of the relative velocity is the sum of the speeds, $2.24 \times 10^{-5} \text{ m/s}$.

(c) The distance the centers of the spheres travel (x_1 and x_2) is proportional to their acceleration, and

$$\frac{x_1}{x_2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}, \text{ or } x_1 = 2x_2. \text{ When the spheres finally make contact, their centers will be a distance of}$$

$2r$ apart, or $x_1 + x_2 + 2r = 40 \text{ m}$, or $2x_2 + x_2 + 2r = 40 \text{ m}$. Thus, $x_2 = 40/3 \text{ m} - 2r/3$, and $x_1 = 80/3 \text{ m} - 4r/3$. The point of contact of the surfaces is $80/3 \text{ m} - r/3 = 26.6 \text{ m}$ from the initial position of the center of the 50.0-kg sphere.

EVALUATE: The result $x_1/x_2 = 2$ can also be obtained from the conservation of momentum result that

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}, \text{ at every point in the motion.}$$

- 13.48. IDENTIFY:** The gravity force for each pair of objects is given by $F_g = Gm_1 m_2 / r^2$. The work done is $W = -\Delta U$.

SET UP: The simplest way to approach this problem is to find the force between the spacecraft and the center of mass of the earth-moon system, which is $4.67 \times 10^6 \text{ m}$ from the center of the earth. The distance from the spacecraft to the center of mass of the earth-moon system is $3.82 \times 10^8 \text{ m}$ (Figure 13.48, next page). $m_E = 5.97 \times 10^{24} \text{ kg}$, $m_M = 7.35 \times 10^{22} \text{ kg}$.

EXECUTE: (a) Using the law of gravitation, the force on the spacecraft is 3.4 N, an angle of 0.61° from the earth-spacecraft line.

(b) $U = -G \frac{m_A m_B}{r}$. $U_2 = 0$ and $r_1 = 3.84 \times 10^8$ m for the spacecraft and the earth, and the spacecraft and the moon.

$$W = U_2 - U_1 = + \frac{GMm}{r_1} = + \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg} + 7.35 \times 10^{22} \text{ kg})(1250 \text{ kg})}{3.84 \times 10^8 \text{ m}}.$$

$$W = 1.31 \times 10^9 \text{ J}.$$

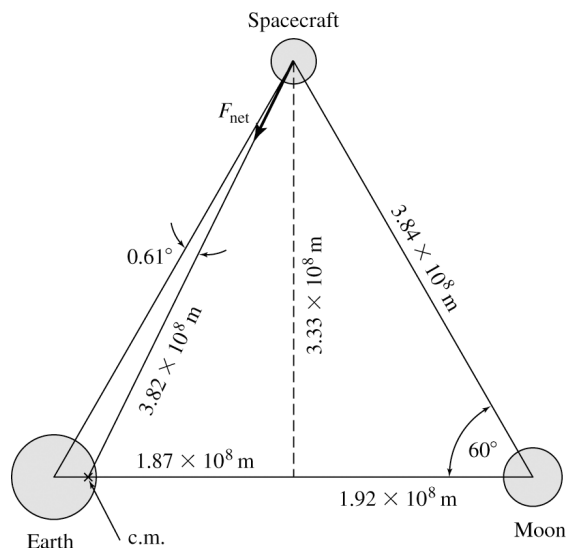


Figure 13.48

EVALUATE: The work done by the attractive gravity forces is negative. The work you do is positive.

- 13.49. **IDENTIFY and SET UP:** (a) To stay above the same point on the surface of the earth the orbital period of the satellite must equal the orbital period of the earth:

$T = 1 \text{ d}(24 \text{ h}/1 \text{ d})(3600 \text{ s}/1 \text{ h}) = 8.64 \times 10^4 \text{ s}$. The equation $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives the relation between the orbit

radius and the period.

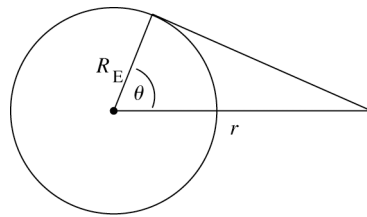
EXECUTE: $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives $T^2 = \frac{4\pi^2 r^3}{Gm_E}$. Solving for r gives

$$r = \left(\frac{T^2 G m_E}{4\pi^2} \right)^{1/3} = \left(\frac{(8.64 \times 10^4 \text{ s})^2 (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) (5.97 \times 10^{24} \text{ kg})}{4\pi^2} \right)^{1/3} = 4.23 \times 10^7 \text{ m}$$

This is the radius of the orbit; it is related to the height h above the earth's surface and the radius R_E of the earth by $r = h + R_E$. Thus $h = r - R_E = 4.23 \times 10^7 \text{ m} - 6.37 \times 10^6 \text{ m} = 3.59 \times 10^7 \text{ m}$.

EVALUATE: The orbital speed of the geosynchronous satellite is $2\pi r/T = 3080 \text{ m/s}$. The altitude is much larger and the speed is much less than for the satellite in Example 13.6.

(b) Consider Figure 13.49.



$$\cos \theta = \frac{R_E}{r} = \frac{6.37 \times 10^6 \text{ m}}{4.23 \times 10^7 \text{ m}}$$

$$\theta = 81.3^\circ$$

Figure 13.49

A line from the satellite is tangent to a point on the earth that is at an angle of 81.3° above the equator. The sketch shows that points at higher latitudes are blocked by the earth from viewing the satellite.

- 13.50. IDENTIFY:** The information about Europa allows us to evaluate g at the surface of Europa. Since there is no atmosphere, $p_0 = 0$ at the surface. The pressure at depth h is $p = \rho gh$. The inward force on the window is $F_\perp = pA$.

SET UP: $g = \frac{Gm}{R^2}$, where $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. $R = 1.560 \times 10^6 \text{ m}$. Assume the ocean water has density $\rho = 1.00 \times 10^3 \text{ kg/m}^3$.

EXECUTE: $g = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(4.80 \times 10^{22} \text{ kg})}{(1.560 \times 10^6 \text{ m})^2} = 1.3156 \text{ m/s}^2$. The maximum pressure at the

window is $p = \frac{8750 \text{ N}}{(0.250 \text{ m})^2} = 1.40 \times 10^5 \text{ Pa}$. $p = \rho gh$ so for h we get

$$h = \frac{1.40 \times 10^5 \text{ Pa}}{(1.00 \times 10^3 \text{ kg/m}^3)(1.3156 \text{ m/s}^2)} = 106 \text{ m}.$$

EVALUATE: 8750 N is the inward force exerted by the surrounding water. This will also be the net force on the window if the pressure inside the submarine is essentially zero.

- 13.51. IDENTIFY:** From Example 13.5, the escape speed is $v = \sqrt{\frac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ .

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$.

EXECUTE: In terms of the density ρ , the ratio M/R is $(4\pi/3)\rho R^2$, and so the escape speed is

$$v = \sqrt{(8\pi/3)(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(2500 \text{ kg/m}^3)(150 \times 10^3 \text{ m})^2} = 177 \text{ m/s}.$$

EVALUATE: This is much less than the escape speed for the earth, 11,200 m/s.

- 13.52. IDENTIFY:** Apply $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ to relate the orbital period T and M_p , the planet's mass, and then use

$w = \frac{Gm_E m}{r^2}$ applied to the planet to calculate the astronaut's weight.

SET UP: The radius of the orbit of the lander is $5.75 \times 10^5 \text{ m} + 4.80 \times 10^6 \text{ m}$.

EXECUTE: From $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$, we get $T^2 = \frac{4\pi^2 r^3}{GM_p}$ and

$$M_p = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (5.75 \times 10^5 \text{ m} + 4.80 \times 10^6 \text{ m})^3}{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.8 \times 10^3 \text{ s})^2} = 2.731 \times 10^{24} \text{ kg},$$

or about half the earth's mass. Now we can find the astronaut's weight on the surface from $w = \frac{Gm_E m}{r^2}$.

(The landing on the north pole removes any need to account for centripetal acceleration.)

$$w = \frac{GM_p m_a}{r_p^2} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(2.731 \times 10^{24} \text{ kg})(85.6 \text{ kg})}{(4.80 \times 10^6 \text{ m})^2} = 677 \text{ N}.$$

EVALUATE: At the surface of the earth the weight of the astronaut would be 839 N.

13.53. IDENTIFY: Apply the law of gravitation to the astronaut at the north pole to calculate the mass of planet.

Then apply $\Sigma \vec{F} = m\vec{a}$ to the astronaut, with $a_{\text{rad}} = \frac{4\pi^2 R}{T^2}$, toward the center of the planet, to calculate the

period T . Apply $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ to the satellite in order to calculate its orbital period.

SET UP: Get radius of X: $\frac{1}{4}(2\pi R) = 18,850 \text{ km}$ and $R = 1.20 \times 10^7 \text{ m}$. Astronaut mass:

$$m = \frac{w}{g} = \frac{943 \text{ N}}{9.80 \text{ m/s}^2} = 96.2 \text{ kg}.$$

EXECUTE: $\frac{GmM_X}{R^2} = w$, where $w = 915.0 \text{ N}$.

$$M_X = \frac{mg_X R^2}{Gm} = \frac{(915 \text{ N})(1.20 \times 10^7 \text{ m})^2}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(96.2 \text{ kg})} = 2.05 \times 10^{25} \text{ kg}$$

Apply Newton's second law to the astronaut on a scale at the equator of X. $F_{\text{grav}} - F_{\text{scale}} = ma_{\text{rad}}$, so

$$F_{\text{grav}} - F_{\text{scale}} = \frac{4\pi^2 mR}{T^2}. \quad 915.0 \text{ N} - 850.0 \text{ N} = \frac{4\pi^2 (96.2 \text{ kg})(1.20 \times 10^7 \text{ m})}{T^2} \quad \text{and}$$

$$T = 2.65 \times 10^4 \text{ s} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 7.36 \text{ h}.$$

$$\text{(b) For the satellite, } T = \sqrt{\frac{4\pi^2 r^3}{Gm_X}} = \sqrt{\frac{4\pi^2 (1.20 \times 10^7 \text{ m} + 2.0 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(2.05 \times 10^{25} \text{ kg})}} = 8.90 \times 10^3 \text{ s} = 2.47 \text{ hours}.$$

EVALUATE: The acceleration of gravity at the surface of the planet is $g_X = \frac{915.0 \text{ N}}{96.2 \text{ kg}} = 9.51 \text{ m/s}^2$, similar

to the value on earth. The radius of the planet is about twice that of earth. The planet rotates more rapidly than earth and the length of a day is about one-third what it is on earth.

13.54. IDENTIFY and SET UP: The observed period allows you to calculate the angular velocity of the satellite relative to you. You know your angular velocity as you rotate with the earth, so you can find the angular velocity of the satellite in a space-fixed reference frame. $v = r\omega$ gives the orbital speed of the satellite and Newton's second law relates this to the orbit radius of the satellite.

EXECUTE: (a) The satellite is revolving west to east, in the same direction the earth is rotating. If the angular speed of the satellite is ω_s and the angular speed of the earth is ω_E , the angular speed ω_{rel} of the satellite relative to you is $\omega_{\text{rel}} = \omega_s - \omega_E$.

$$\omega_{\text{rel}} = (1 \text{ rev})/(12 \text{ h}) = \left(\frac{1}{12}\right) \text{ rev/h}$$

$$\omega_E = \left(\frac{1}{24}\right) \text{ rev/h}$$

$$\omega_s = \omega_{\text{rel}} + \omega_E = \left(\frac{1}{8}\right) \text{ rev/h} = 2.18 \times 10^{-4} \text{ rad/s}$$

$$\Sigma \vec{F} = m\vec{a} \text{ says } G \frac{mm_E}{r^2} = m \frac{v^2}{r}$$

$$v^2 = \frac{Gm_E}{r} \text{ and with } v = r\omega \text{ this gives } r^3 = \frac{Gm_E}{\omega^2}; \quad r = 2.03 \times 10^7 \text{ m}$$

This is the radius of the satellite's orbit. Its height h above the surface of the earth is

$$h = r - R_E = 1.39 \times 10^7 \text{ m.}$$

(b) Now the satellite is revolving opposite to the rotation of the earth. If west to east is positive, then

$$\omega_{\text{rel}} = \left(-\frac{1}{12}\right) \text{ rev/h}$$

$$\omega_s = \omega_{\text{rel}} + \omega_E = \left(-\frac{1}{24}\right) \text{ rev/h} = -7.27 \times 10^{-5} \text{ rad/s}$$

$$r^3 = \frac{Gm_E}{\omega^2} \text{ gives } r = 4.22 \times 10^7 \text{ m and } h = 3.59 \times 10^7 \text{ m.}$$

EVALUATE: In part (a) the satellite is revolving faster than the earth's rotation and in part (b) it is revolving slower. Slower v and ω means larger orbit radius r .

- 13.55. IDENTIFY:** The free-fall time of the rock will give us the acceleration due to gravity at the surface of the planet. Applying Newton's second law and the law of universal gravitation will give us the mass of the planet since we know its radius.

SET UP: For constant acceleration, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. At the surface of the planet, Newton's second

law gives $m_{\text{rock}}g = \frac{Gm_{\text{rock}}m_p}{R_p^2}$.

EXECUTE: First find $a_y = g$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. $a_y = \frac{2(y - y_0)}{t^2} = \frac{2(1.90 \text{ m})}{(0.480 \text{ s})^2} = 16.49 \text{ m/s}^2 = g$.

$$g = 16.49 \text{ m/s}^2. \quad m_p = \frac{gR_p^2}{G} = \frac{(16.49 \text{ m/s}^2)(8.60 \times 10^7 \text{ m})^2}{6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 1.83 \times 10^{27} \text{ kg.}$$

EVALUATE: The planet's mass is over 100 times that of the earth, which is reasonable since it is larger (in size) than the earth yet has a greater acceleration due to gravity at its surface.

- 13.56. IDENTIFY:** Use the measurements of the motion of the rock to calculate g_M , the value of g on Mongo.

Then use this to calculate the mass of Mongo. For the ship, $F_g = ma_{\text{rad}}$ and $T = \frac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward.

$$g_M = G \frac{m_M}{R_M^2}. \text{ The radius of Mongo is } R_M = \frac{c}{2\pi} = \frac{2.00 \times 10^8 \text{ m}}{2\pi} = 3.18 \times 10^7 \text{ m. The ship moves in an orbit}$$

of radius $r = 3.18 \times 10^7 \text{ m} + 3.00 \times 10^7 \text{ m} = 6.18 \times 10^7 \text{ m}$.

EXECUTE: (a) $v_{0y} = +12.0 \text{ m/s}$, $v_y = -12.0 \text{ m/s}$, $a_y = -g_M$ and $t = 4.80 \text{ s}$. $v_y = v_{0y} + a_y t$ gives

$$-g_M = \frac{v_y - v_{0y}}{t} = \frac{-12.0 \text{ m/s} - 12.0 \text{ m/s}}{4.80 \text{ s}} \text{ and } g_M = 5.00 \text{ m/s}^2.$$

$$m_M = \frac{g_M R_M^2}{G} = \frac{(5.00 \text{ m/s}^2)(3.18 \times 10^7 \text{ m})^2}{6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 7.577 \times 10^{25} \text{ kg which rounds to } 7.58 \times 10^{25} \text{ kg.}$$

(b) $F_g = ma_{\text{rad}}$ gives $G \frac{m_M m}{r^2} = m \frac{v^2}{r}$ and $v^2 = \frac{Gm_M}{r}$.

$$T = \frac{2\pi r}{v} = 2\pi r \sqrt{\frac{r}{Gm_M}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_M}} = \frac{2\pi(6.18 \times 10^7 \text{ m})^{3/2}}{\sqrt{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.577 \times 10^{25} \text{ kg})}}$$

$$T = 4.293 \times 10^4 \text{ s} = 11.9 \text{ h.}$$

EVALUATE: $R_M = 5.0R_E$ and $m_M = 12.7m_E$, so $g_M = \frac{12.7}{(5.0)^2} g_E = 0.508g_E$, which agrees with the value calculated in part (a).

- 13.57. IDENTIFY:** Use the orbital speed and altitude to find the mass of the planet. Use this mass and the planet's radius to find g at the surface. Use projectile motion to find the horizontal range x , where $x = \frac{v_0^2 \sin(2\alpha)}{g}$.

SET UP: For an object in a circular orbit, $v = \sqrt{GM/r}$. $g = GM/R^2$. Call r the orbital radius and R the radius of the planet.

EXECUTE: $v = \sqrt{GM/r}$ gives $M = rv^2/G$. Using this to find g gives
 $g = GM/R^2 = G(rv^2/G)/R^2 = v^2 r/R^2 = (4900 \text{ m/s})^2 (4.48 \times 10^6 \text{ m} + 6.30 \times 10^5 \text{ m}) / (4.48 \times 10^6 \text{ m})^2 = 6.113 \text{ m/s}^2$. Now use this acceleration to find the horizontal range.

$$x = \frac{v_0^2 \sin(2\alpha)}{g} = (12.6 \text{ m/s})^2 \sin[2(30.8^\circ)] / (6.113 \text{ m/s}^2) = 22.8 \text{ m}.$$

EVALUATE: On this planet, $g = 0.624g_E$, so the range is about 1.6 times what it would be on earth.

- 13.58. IDENTIFY:** The 0.100 kg sphere has gravitational potential energy due to the other two spheres. Its mechanical energy is conserved.

SET UP: From energy conservation, $K_1 + U_1 = K_2 + U_2$, where $K = \frac{1}{2}mv^2$, and $U = -GmM/r$.

EXECUTE: Using $K_1 + U_1 = K_2 + U_2$, we have $K_1 = 0$, $m_A = 5.00 \text{ kg}$, $m_B = 10.0 \text{ kg}$ and $m = 0.100 \text{ kg}$.

$$U_1 = -\frac{Gmm_A}{r_{A1}} - \frac{Gmm_B}{r_{B1}} = -(6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.100 \text{ kg}) \left(\frac{5.00 \text{ kg}}{0.400 \text{ m}} + \frac{10.0 \text{ kg}}{0.600 \text{ m}} \right)$$

$$U_1 = -1.9466 \times 10^{-10} \text{ J}.$$

$$U_2 = -\frac{Gmm_A}{r_{A2}} - \frac{Gmm_B}{r_{B2}} = -(6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.100 \text{ kg}) \left(\frac{5.00 \text{ kg}}{0.800 \text{ m}} + \frac{10.0 \text{ kg}}{0.200 \text{ m}} \right)$$

$$U_2 = -3.7541 \times 10^{-10} \text{ J}. \quad K_2 = U_1 - U_2 = -1.9466 \times 10^{-10} \text{ J} - (-3.7541 \times 10^{-10} \text{ J}) = 1.8075 \times 10^{-10} \text{ J}.$$

$$\frac{1}{2}mv^2 = K_2 \quad \text{and} \quad v = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(1.8075 \times 10^{-10} \text{ J})}{0.100 \text{ kg}}} = 6.01 \times 10^{-5} \text{ m/s}.$$

EVALUATE: The kinetic energy gained by the sphere is equal to the loss in its potential energy.

- 13.59. IDENTIFY and SET UP:** First use the radius of the orbit to find the initial orbital speed, from $v = \sqrt{GM/r}$ applied to the moon.

EXECUTE: $v = \sqrt{Gm/r}$ and $r = R_M + h = 1.74 \times 10^6 \text{ m} + 50.0 \times 10^3 \text{ m} = 1.79 \times 10^6 \text{ m}$

$$\text{Thus } v = \sqrt{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22} \text{ kg})}{1.79 \times 10^6 \text{ m}}} = 1.655 \times 10^3 \text{ m/s}$$

After the speed decreases by 20.0 m/s it becomes $1.655 \times 10^3 \text{ m/s} - 20.0 \text{ m/s} = 1.635 \times 10^3 \text{ m/s}$.

IDENTIFY and SET UP: Use conservation of energy to find the speed when the spacecraft reaches the lunar surface.

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

Gravity is the only force that does work so $W_{\text{other}} = 0$ and $K_2 = K_1 + U_1 - U_2$

EXECUTE: $U_1 = -Gm_m m/r$; $U_2 = -Gm_m m/R_m$

$$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + Gmm_m(1/R_m - 1/r)$$

And the mass m divides out to give $v_2 = \sqrt{v_1^2 + 2Gm_m(1/R_m - 1/r)}$

$$v_2 = 1.682 \times 10^3 \text{ m/s} (1 \text{ km}/1000 \text{ m})(3600 \text{ s}/1 \text{ h}) = 6060 \text{ km/h}$$

EVALUATE: After the thruster fires the spacecraft is moving too slowly to be in a stable orbit; the gravitational force is larger than what is needed to maintain a circular orbit. The spacecraft gains energy as it is accelerated toward the surface.

- 13.60. IDENTIFY:** In part (a) use the expression for the escape speed that is derived in Example 13.5,

$$v = \sqrt{\frac{2GM}{R}}. \quad \text{In part (b) apply conservation of energy.}$$

SET UP: $R = 4.5 \times 10^3$ m. In part (b) let point 1 be at the surface of the comet.

EXECUTE: (a) The escape speed is $v = \sqrt{\frac{2GM}{R}}$ so

$$M = \frac{Rv^2}{2G} = \frac{(4.5 \times 10^3 \text{ m})(1.0 \text{ m/s})^2}{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 3.37 \times 10^{13} \text{ kg}.$$

(b) (i) $K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0.100K_1$, $U_1 = -\frac{GMm}{R}$; $U_2 = -\frac{GMm}{r}$. $K_1 + U_1 = K_2 + U_2$ gives

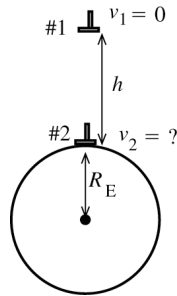
$$\frac{1}{2}mv_1^2 - \frac{GMm}{R} = (0.100)\left(\frac{1}{2}mv_1^2\right) - \frac{GMm}{r}. \text{ Solving for } r \text{ gives}$$

$$\frac{1}{r} = \frac{1}{R} - \frac{0.450v_1^2}{GM} = \frac{1}{4.5 \times 10^3 \text{ m}} - \frac{0.450(1.0 \text{ m/s})^2}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(3.37 \times 10^{13} \text{ kg})} \text{ and } r = 45 \text{ km. (ii) The debris}$$

never loses all of its initial kinetic energy, but $K_2 \rightarrow 0$ as $r \rightarrow \infty$. The farther the debris are from the comet's center, the smaller is their kinetic energy.

EVALUATE: The debris will have lost 90.0% of their initial kinetic energy when they are at a distance from the comet's center of about ten times the radius of the comet.

- 13.61. IDENTIFY and SET UP:** Apply conservation of energy. We must use $U = -\frac{GMm}{r}$ for the gravitational potential energy since h is not small compared to R_E .



As indicated in Figure 13.61, take point 1 to be where the hammer is released and point 2 to be just above the surface of the earth, so $r_1 = R_E + h$ and $r_2 = R_E$.

Figure 13.61

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Only gravity does work, so $W_{\text{other}} = 0$.

$$K_1 = 0, \quad K_2 = \frac{1}{2}mv_2^2$$

$$U_1 = -G\frac{mm_E}{r_1} = -\frac{Gmm_E}{h + R_E}, \quad U_2 = -G\frac{mm_E}{r_2} = -\frac{Gmm_E}{R_E}$$

$$\text{Thus, } -G\frac{mm_E}{h + R_E} = \frac{1}{2}mv_2^2 - G\frac{mm_E}{R_E}$$

$$v_2^2 = 2Gm_E \left(\frac{1}{R_E} - \frac{1}{R_E + h} \right) = \frac{2Gm_E}{R_E(R_E + h)} (R_E + h - R_E) = \frac{2Gm_E h}{R_E(R_E + h)}$$

$$v_2 = \sqrt{\frac{2Gm_E h}{R_E(R_E + h)}}$$

EVALUATE: If $h \rightarrow \infty$, $v_2 \rightarrow \sqrt{2Gm_E/R_E}$, which equals the escape speed. In this limit this event is the reverse of an object being projected upward from the surface with the escape speed. If $h \ll R_E$, then

$$v_2 = \sqrt{2Gm_E h/R_E^2} = \sqrt{2gh}, \text{ the same result if } mgh \text{ is used for } U.$$

- 13.62. IDENTIFY:** In orbit the total mechanical energy of the satellite is $E = -\frac{Gm_E m}{2R_E}$. $U = -G\frac{m_E m}{r}$.

$$W = E_2 - E_1.$$

SET UP: $U \rightarrow 0$ as $r \rightarrow \infty$.

EXECUTE: (a) The energy the satellite has as it sits on the surface of the Earth is $E_1 = \frac{-GmM_E}{R_E}$. The energy it has when it is in orbit at a radius $R \approx R_E$ is $E_2 = \frac{-GmM_E}{2R_E}$. The work needed to put it in orbit is

$$\text{the difference between these: } W = E_2 - E_1 = \frac{GmM_E}{2R_E}.$$

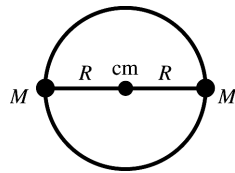
(b) The total energy of the satellite far away from the earth is zero, so the additional work needed is

$$0 - \left(\frac{-GmM_E}{2R_E} \right) = \frac{GmM_E}{2R_E}.$$

EVALUATE: (c) The work needed to put the satellite into orbit was the same as the work needed to put the satellite from orbit to the edge of the universe.

- 13.63. IDENTIFY:** Use $F_g = Gm_1m_2/r^2$ to calculate F_g . Apply Newton's second law to circular motion of each star to find the orbital speed and period. Apply the conservation of energy to calculate the energy input (work) required to separate the two stars to infinity.

(a) **SET UP:** The cm is midway between the two stars since they have equal masses. Let R be the orbit radius for each star, as sketched in Figure 13.63.



The two stars are separated by a distance $2R$, so

$$F_g = GM^2/(2R)^2 = GM^2/4R^2$$

Figure 13.63

(b) **EXECUTE:** $F_g = ma_{\text{rad}}$

$$GM^2/4R^2 = M(v^2/R) \text{ so } v = \sqrt{GM/4R}$$

$$\text{And } T = 2\pi R/v = 2\pi R\sqrt{4R/GM} = 4\pi\sqrt{R^3/GM}$$

(c) **SET UP:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the system of the two stars. Separate to infinity implies $K_2 = 0$ and $U_2 = 0$.

$$\text{EXECUTE: } K_1 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = 2\left(\frac{1}{2}M\right)(GM/4R) = GM^2/4R$$

$$U_1 = -GM^2/2R$$

Thus the energy required is $W_{\text{other}} = -(K_1 + U_1) = -(GM^2/4R - GM^2/2R) = GM^2/4R$.

EVALUATE: The closer the stars are and the greater their mass, the larger their orbital speed, the shorter their orbital period and the greater the energy required to separate them.

- 13.64. IDENTIFY:** In the center of mass coordinate system, $r_{\text{cm}} = 0$. Apply $\vec{F} = m\vec{a}$ to each star, where F is the gravitational force of one star on the other and $a = a_{\text{rad}} = \frac{4\pi^2 R}{T^2}$.

SET UP: $v = \frac{2\pi R}{T}$ allows R to be calculated from v and T .

EXECUTE: (a) The radii R_1 and R_2 are measured with respect to the center of mass, and so $M_1R_1 = M_2R_2$, and $R_1/R_2 = M_2/M_1$.

(b) The forces on each star are equal in magnitude, so the product of the mass and the radial accelerations are equal: $\frac{4\pi^2 M_1 R_1}{T_1^2} = \frac{4\pi^2 M_2 R_2}{T_2^2}$. From the result of part (a), the numerators of these expressions are

equal, and so the denominators are equal, and the periods are the same. To find the period in the symmetric form desired, there are many possible routes. An elegant method, using a bit of hindsight, is to use the above expressions to relate the periods to the force $F_g = \frac{GM_1 M_2}{(R_1 + R_2)^2}$, so that equivalent expressions for the

period are $M_2 T^2 = \frac{4\pi^2 R_1 (R_1 + R_2)^2}{G}$ and $M_1 T^2 = \frac{4\pi^2 R_2 (R_1 + R_2)^2}{G}$. Adding the expressions gives

$$(M_1 + M_2)T^2 = \frac{4\pi^2 (R_1 + R_2)^3}{G} \quad \text{or} \quad T = \frac{2\pi (R_1 + R_2)^{3/2}}{\sqrt{G(M_1 + M_2)}}.$$

(c) First we must find the radii of each orbit given the speed and period data. In a circular orbit,

$$v = \frac{2\pi R}{T}, \quad \text{or} \quad R = \frac{vT}{2\pi}. \quad \text{Thus} \quad R_\alpha = \frac{(36 \times 10^3 \text{ m/s})(137 \text{ d})(86,400 \text{ s/d})}{2\pi} = 6.78 \times 10^{10} \text{ m} \quad \text{and}$$

$$R_\beta = \frac{(12 \times 10^3 \text{ m/s})(137 \text{ d})(86,400 \text{ s/d})}{2\pi} = 2.26 \times 10^{10} \text{ m}. \quad \text{Now find the sum of the masses.}$$

$$(M_\alpha + M_\beta) = \frac{4\pi^2 (R_\alpha + R_\beta)^3}{T^2 G}. \quad \text{Inserting the values of } T \text{ and the radii gives}$$

$$(M_\alpha + M_\beta) = \frac{4\pi^2 (6.78 \times 10^{10} \text{ m} + 2.26 \times 10^{10} \text{ m})^3}{[(137 \text{ d})(86,400 \text{ s/d})]^2 (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 3.12 \times 10^{30} \text{ kg}. \quad \text{Since}$$

$$M_\beta = M_\alpha R_\alpha / R_\beta = 3M_\alpha, \quad 4M_\alpha = 3.12 \times 10^{30} \text{ kg}, \quad \text{or} \quad M_\alpha = 7.80 \times 10^{29} \text{ kg}, \quad \text{and} \quad M_\beta = 2.34 \times 10^{30} \text{ kg}.$$

(d) Let α refer to the star and β refer to the black hole. Use the relationships derived in parts (a) and (b):

$$R_\beta = (M_\alpha / M_\beta) R_\alpha = (0.67/3.8) R_\alpha = (0.176) R_\alpha, \quad R_\alpha + R_\beta = \sqrt[3]{\frac{(M_\alpha + M_\beta) T^2 G}{4\pi^2}}. \quad \text{For Monocerotis,}$$

inserting the values for M and T gives $R_\alpha = 1.9 \times 10^9 \text{ m}$, $v_\alpha = 4.4 \times 10^2 \text{ km/s}$ and for the black hole

$$R_\beta = 34 \times 10^8 \text{ m}, \quad v_\beta = 77 \text{ km/s}.$$

EVALUATE: Since T is the same, v is smaller when R is smaller.

13.65. IDENTIFY and SET UP: Use conservation of energy, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. The gravity force exerted by the sun is the only force that does work on the comet, so $W_{\text{other}} = 0$.

$$\text{EXECUTE:} \quad K_1 = \frac{1}{2} m v_1^2, \quad v_1 = 2.0 \times 10^4 \text{ m/s}$$

$$U_1 = -G m_S m / r_1, \quad \text{where} \quad r_1 = 2.5 \times 10^{11} \text{ m}$$

$$K_2 = \frac{1}{2} m v_2^2$$

$$U_2 = -G m_S m / r_2, \quad r_2 = 5.0 \times 10^{10} \text{ m}$$

$$\frac{1}{2} m v_1^2 - G m_S m / r_1 = \frac{1}{2} m v_2^2 - G m_S m / r_2$$

$$v_2^2 = v_1^2 + 2G m_S \left(\frac{1}{r_2} - \frac{1}{r_1} \right) = v_1^2 + 2G m_S \left(\frac{r_1 - r_2}{r_1 r_2} \right)$$

$$v_2 = 6.8 \times 10^4 \text{ m/s}$$

EVALUATE: The comet has greater speed when it is closer to the sun.

- 13.66. IDENTIFY:** $g = \frac{GM}{R^2}$, where M and R are the mass and radius of the planet.

SET UP: Let m_U and R_U be the mass and radius of Uranus and let g_U be the acceleration due to gravity at its poles. The orbit radius of Miranda is $r = h + R_U$, where $h = 1.04 \times 10^8$ m is the altitude of Miranda above the surface of Uranus.

EXECUTE: (a) From the value of g at the poles,

$$m_U = \frac{g_U R_U^2}{G} = \frac{(9.0 \text{ m/s}^2)(2.5360 \times 10^7 \text{ m})^2}{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 8.674 \times 10^{25} \text{ kg} \text{ which rounds to } 8.7 \times 10^{25} \text{ kg}.$$

$$\text{(b)} \quad Gm_U/r^2 = g_U(R_U/r)^2 = 0.35 \text{ m/s}^2.$$

$$\text{(c)} \quad Gm_M/R_M^2 = 0.079 \text{ m/s}^2.$$

EVALUATE: (d) No. Both the object and Miranda are in orbit together around Uranus, due to the gravitational force of Uranus. The object has additional force toward Miranda.

- 13.67. (a) IDENTIFY and SET UP:** Use $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, applied to the satellites orbiting the earth rather than the sun.

EXECUTE: Find the value of a for the elliptical orbit:

$2a = r_a + r_p = R_E + h_a + R_E + h_p$, where h_a and h_p are the heights at apogee and perigee, respectively.

$$a = R_E + (h_a + h_p)/2$$

$$a = 6.37 \times 10^6 \text{ m} + (400 \times 10^3 \text{ m} + 4000 \times 10^3 \text{ m})/2 = 8.57 \times 10^6 \text{ m}$$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}} = \frac{2\pi(8.57 \times 10^6 \text{ m})^{3/2}}{\sqrt{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}} = 7.90 \times 10^3 \text{ s}$$

(b) Conservation of angular momentum gives $r_a v_a = r_p v_p$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{6.37 \times 10^6 \text{ m} + 4.00 \times 10^6 \text{ m}}{6.37 \times 10^6 \text{ m} + 4.00 \times 10^5 \text{ m}} = 1.53.$$

(c) Conservation of energy applied to apogee and perigee gives $K_a + U_a = K_p + U_p$

$$\frac{1}{2}mv_a^2 - Gm_E m/r_a = \frac{1}{2}mv_p^2 - Gm_E m/r_p$$

$$v_p^2 - v_a^2 = 2Gm_E(1/r_p - 1/r_a) = 2Gm_E(r_a - r_p)/r_a r_p$$

$$\text{But } v_p = 1.532v_a, \text{ so } 1.347v_a^2 = 2Gm_E(r_a - r_p)/r_a r_p$$

$$v_a = 5.51 \times 10^3 \text{ m/s}, \quad v_p = 8.43 \times 10^3 \text{ m/s}$$

(d) Need v so that $E = 0$, where $E = K + U$.

$$\text{at perigee: } \frac{1}{2}mv_p^2 - Gm_E m/r_p = 0$$

$$v_p = \sqrt{2Gm_E/r_p} = \sqrt{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})/(6.77 \times 10^6 \text{ m})} = 1.085 \times 10^4 \text{ m/s}$$

This means an increase of $1.085 \times 10^4 \text{ m/s} - 8.43 \times 10^3 \text{ m/s} = 2.42 \times 10^3 \text{ m/s}$.

$$\text{at apogee: } v_a = \sqrt{2Gm_E/r_a} = \sqrt{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})/(1.037 \times 10^7 \text{ m})} = 8.763 \times 10^3 \text{ m/s}$$

This means an increase of $8.763 \times 10^3 \text{ m/s} - 5.51 \times 10^3 \text{ m/s} = 3.25 \times 10^3 \text{ m/s}$.

EVALUATE: Perigee is more efficient. At this point r is smaller so v is larger and the satellite has more kinetic energy and more total energy.

- 13.68. IDENTIFY:** The engines do work on the rocket and change its kinetic energy and gravitational potential energy.

SET UP: Call M the mass of the earth and m the mass of the rocket. $U_g = -GMm/r$, $K = \frac{1}{2}mMG/r$ for a circular orbit, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

EXECUTE: (a) For a circular orbit, using $K = \frac{1}{2} mMG/r$ gives the difference in kinetic energy:

$$K_2 - K_1 = \frac{1}{2} mMG(1/r_2 - 1/r_1). \text{ Using the given numbers } m = 5000 \text{ kg, } M = 5.97 \times 10^{24} \text{ kg,}$$

$r_2 = 8.80 \times 10^6 \text{ m}$, and $r_1 = 7.20 \times 10^6 \text{ m}$, we get $K_2 - K_1 = -2.52 \times 10^{10} \text{ J}$. The minus sign tells us that the kinetic energy decreases.

(b) $U_2 - U_1 = -GmM/r_2 - (-GmM/r_1) = GmM(1/r_1 - 1/r_2) = -2(K_2 - K_1) = +5.03 \times 10^{10} \text{ J}$. The plus sign means that the energy increases.

(c) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$$W_{\text{other}} = (K_2 - K_1) + (U_2 - U_1) = -2.51 \times 10^{10} \text{ J} + 5.03 \times 10^{10} \text{ J} = +2.51 \times 10^{10} \text{ J}.$$

EVALUATE: In part (b), the potential energy increases because it becomes less negative. The work is positive because the total energy increases.

13.69. IDENTIFY and SET UP: Apply conservation of energy, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, and solve for W_{other} .

Only $r = h + R_E$ is given, so use $v = \sqrt{GM/r}$ to relate r and v .

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = -Gm_M m/r_1$, where m_M is the mass of Mars and $r_1 = R_M + h$, where R_M is the radius of Mars and $h = 2000 \times 10^3 \text{ m}$.

$$U_1 = -(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(6.42 \times 10^{23} \text{ kg})(5000 \text{ kg})}{3.39 \times 10^6 \text{ m} + 2000 \times 10^3 \text{ m}} = -3.97230 \times 10^{10} \text{ J}$$

$U_2 = -Gm_M m/r_2$, where r_2 is the new orbit radius.

$$U_2 = -(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(6.42 \times 10^{23} \text{ kg})(5000 \text{ kg})}{3.39 \times 10^6 \text{ m} + 4000 \times 10^3 \text{ m}} = -2.89725 \times 10^{10} \text{ J}$$

For a circular orbit $v = \sqrt{Gm_M/r}$, with the mass of Mars rather than the mass of the earth.

Using this gives $K = \frac{1}{2} mv^2 = \frac{1}{2} m(Gm_M/r) = \frac{1}{2} Gm_M m/r$, so $K = -\frac{1}{2} U$.

$$K_1 = -\frac{1}{2} U_1 = +1.98615 \times 10^{10} \text{ J} \text{ and } K_2 = -\frac{1}{2} U_2 = +1.44863 \times 10^{10} \text{ J}$$

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$$W_{\text{other}} = (K_2 - K_1) + (U_2 - U_1)$$

$$W_{\text{other}} = (1.44863 \times 10^{10} \text{ J} - 1.98615 \times 10^{10} \text{ J}) + (+3.97230 \times 10^{10} \text{ J} - 2.89725 \times 10^{10} \text{ J})$$

$$W_{\text{other}} = 5.38 \times 10^9 \text{ J}.$$

EVALUATE: When the orbit radius increases the kinetic energy decreases and the gravitational potential energy increases. $K = -U/2$ so $E = K + U = -U/2$ and the total energy also increases (becomes less negative). Positive work must be done to increase the total energy of the satellite.

13.70. IDENTIFY: The engines do work on the rocket and change its kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2} mMG/r = -\frac{1}{2} U_g$ for a circular orbit; $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ and $U_g = -2K$. Combining these two equations gives

$K_1 - 2K_1 + W_{\text{other}} = K_2 - 2K_2$, so $K_2 = K_1 - W_{\text{other}}$. This gives $\frac{1}{2} mv_2^2 = \frac{1}{2} mv_1^2 - W_{\text{other}}$. Solving for v_2 gives

$$v_2 = \sqrt{v_1^2 - \frac{2W_{\text{other}}}{m}} = \sqrt{(9640 \text{ m/s})^2 - \frac{2(-7.50 \times 10^9 \text{ J})}{848 \text{ kg}}} = 10,500 \text{ m/s, which is greater than } v_1, \text{ so the}$$

speed increases.

EVALUATE: The work is negative, yet the speed increases. The potential energy decreases (becomes more negative), so the total energy decreases. Due to the friction, the satellite will go to a lower orbit (closer to the earth), so it must have a greater speed to remain in orbit.

- 13.71. IDENTIFY:** Integrate $dm = \rho dV$ to find the mass of the planet. Outside the planet, the planet behaves like a point mass, so at the surface $g = GM/R^2$.

SET UP: A thin spherical shell with thickness dr has volume $dV = 4\pi r^2 dr$. The earth has radius $R_E = 6.37 \times 10^6$ m.

EXECUTE: Get M : $M = \int dm = \int \rho dV = \int \rho 4\pi r^2 dr$. The density is $\rho = \rho_0 - br$, where

$$\rho_0 = 15.0 \times 10^3 \text{ kg/m}^3 \text{ at the center and at the surface, } \rho_s = 2.0 \times 10^3 \text{ kg/m}^3, \text{ so } b = \frac{\rho_0 - \rho_s}{R}.$$

$$M = \int_0^R (\rho_0 - br) 4\pi r^2 dr = \frac{4\pi}{3} \rho_0 R^3 - \pi b R^4 = \frac{4}{3} \pi R^3 \rho_0 - \pi R^4 \left(\frac{\rho_0 - \rho_s}{R} \right) = \pi R^3 \left(\frac{1}{3} \rho_0 + \rho_s \right) \text{ and}$$

$$M = 5.71 \times 10^{24} \text{ kg. Then } g = \frac{GM}{R^2} = \frac{G\pi R^3 (\frac{1}{3}\rho_0 + \rho_s)}{R^2} = \pi R G \left(\frac{1}{3}\rho_0 + \rho_s \right).$$

$$g = \pi (6.37 \times 10^6 \text{ m}) (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \left(\frac{15.0 \times 10^3 \text{ kg/m}^3}{3} + 2.0 \times 10^3 \text{ kg/m}^3 \right).$$

$$g = 9.34 \text{ m/s}^2.$$

EVALUATE: The average density of the planet is

$$\rho_{\text{av}} = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3(5.71 \times 10^{24} \text{ kg})}{4\pi (6.37 \times 10^6 \text{ m})^3} = 5.27 \times 10^3 \text{ kg/m}^3. \text{ Note that this is not } (\rho_0 + \rho_s)/2.$$

- 13.72. IDENTIFY and SET UP:** Use $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_s}}$ to calculate a . $T = 30,000 \text{ y} (3.156 \times 10^7 \text{ s/1 y}) = 9.468 \times 10^{11} \text{ s}$

$$\text{EXECUTE: } T = \frac{2\pi a^{3/2}}{\sqrt{Gm_s}}, \text{ which gives } T^2 = \frac{4\pi^2 a^3}{Gm_s}, \text{ so } a = \left(\frac{Gm_s T^2}{4\pi^2} \right)^{1/3} = 1.4 \times 10^{14} \text{ m.}$$

EVALUATE: The average orbit radius of Pluto is 5.9×10^{12} m (Appendix F); the semi-major axis for this comet is larger by a factor of 24. Converting to meters gives $4.3 \text{ light years} = 4.3(9.461 \times 10^{15} \text{ m}) = 4.1 \times 10^{16} \text{ m}$. The distance of Alpha Centauri is larger by a factor of 300. The orbit of the comet extends well past Pluto but is well within the distance to Alpha Centauri.

- 13.73. IDENTIFY:** The direct calculation of the force that the sphere exerts on the ring is slightly more involved than the calculation of the force that the ring exerts on the sphere. These forces are equal in magnitude but opposite in direction, so it will suffice to do the latter calculation. By symmetry, the force on the sphere will be along the axis of the ring in Figure E13.35 in the textbook, toward the ring.

SET UP: Divide the ring into infinitesimal elements with mass dM .

EXECUTE: Each mass element dM of the ring exerts a force of magnitude $\frac{(Gm)dM}{a^2 + x^2}$ on the sphere,

$$\text{and the } x\text{-component of this force is } \frac{GmdM}{a^2 + x^2} \frac{x}{\sqrt{a^2 + x^2}} = \frac{GmdMx}{(a^2 + x^2)^{3/2}}.$$

Therefore, the force on the sphere is $GmMx/(a^2 + x^2)^{3/2}$, in the $-x$ -direction. The sphere attracts the ring with a force of the same magnitude.

EVALUATE: As $x \gg a$ the denominator approaches x^3 and $F \rightarrow \frac{GMm}{x^2}$, as expected.

- 13.74. IDENTIFY and SET UP:** Use $F_g = Gm_1m_2/r^2$ to calculate the force between the point mass and a small segment of the semicircle.

EXECUTE: The radius of the semicircle is $R = L/\pi$.

Divide the semicircle up into small segments of length $R d\theta$, as shown in Figure 13.74.

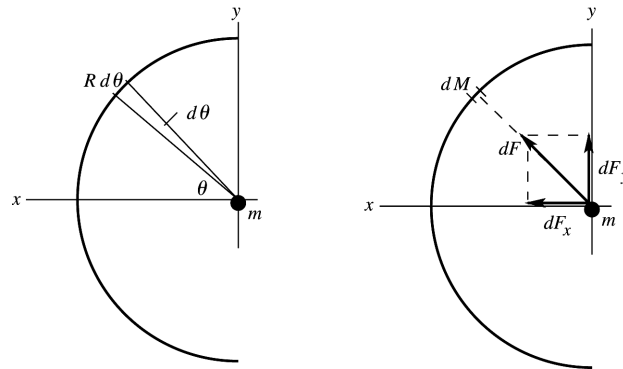


Figure 13.74

$$dM = (M/L)R d\theta = (M/\pi) d\theta$$

$d\vec{F}$ is the gravity force on m exerted by dM .

$\int dF_y = 0$; the y -components from the upper half of the semicircle cancel the y -components from the lower half.

The x -components are all in the $+x$ -direction and all add.

$$dF = G \frac{mdM}{R^2}$$

$$dF_x = G \frac{mdM}{R^2} \cos \theta = \frac{Gm\pi M}{L^2} \cos \theta d\theta$$

$$F_x = \int_{-\pi/2}^{\pi/2} dF_x = \frac{Gm\pi M}{L^2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = \frac{Gm\pi M}{L^2} (2)$$

$$F = \frac{2\pi GmM}{L^2}$$

EVALUATE: If the semicircle were replaced by a point mass M at $x = R$, the gravity force would be $GmM/R^2 = \pi^2 GmM/L^2$. This is $\pi/2$ times larger than the force exerted by the semicircular wire. For the semicircle it is the x -components that add, and the sum is less than if the force magnitudes were added.

13.75. IDENTIFY: Compare F_E to Hooke's law.

SET UP: The earth has mass $m_E = 5.97 \times 10^{24}$ kg and radius $R_E = 6.37 \times 10^6$ m.

EXECUTE: (a) For $F_x = -kx$, $U = \frac{1}{2}kx^2$. The force here is in the same form, so by analogy

$U(r) = \frac{Gm_E m}{2R_E^3} r^2$. This is also given by the integral of F_g from 0 to r with respect to distance.

(b) From part (a), the initial gravitational potential energy is $\frac{Gm_E m}{2R_E}$. Equating initial potential energy and final kinetic energy (initial kinetic energy and final potential energy are both zero) gives

$$v^2 = \frac{Gm_E}{R_E}, \text{ so } v = 7.91 \times 10^3 \text{ m/s.}$$

EVALUATE: When $r = 0$, $U(r) = 0$, as specified in the problem.

13.76. IDENTIFY: Kepler's third law applies to the planets.

SET UP: $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$

EXECUTE: Squaring $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ gives $T^2 = \left(\frac{4\pi^2}{Gm_S}\right)a^3$. If we graph T^2 versus a^3 , the equation is of the

slope- y -intercept form of a straight line, $y = mx + b$. In this case, the slope is $(4\pi^2/Gm_S)$, and the y -intercept is zero. Take logs of both sides of the equation in Kepler's third law, giving

$\log T = \log \left[\left(\frac{2\pi}{\sqrt{Gm_S}} \right) a^{3/2} \right]$, which can be written as $\log T = \frac{3}{2} \log a + \log \left(\frac{2\pi}{\sqrt{Gm_S}} \right)$. Therefore graphing

$\log T$ versus $\log a$ should yield a straight line. Figure 13.76 shows this graph.

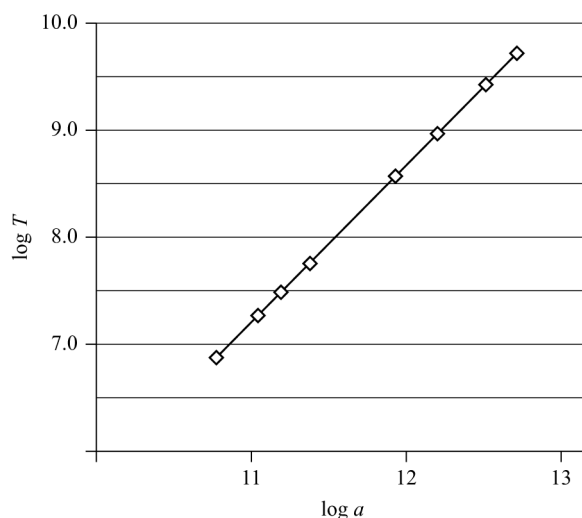


Figure 13.76

(b) In the slope- y -intercept form $y = mx + b$, the slope is $3/2$. The best-fit equation of the graph in the figure is $y = 1.4986x - 9.2476$. Since 1.4986 rounds to 1.50, which is equal to $3/2$, our graph has the expected slope.

(c) The y -intercept is $\log \left(\frac{2\pi}{\sqrt{Gm_S}} \right) = -9.2476$. Solving for m_S , we get $\frac{2\pi}{\sqrt{Gm_S}} = 10^{-9.2476}$. Squaring and

solving for m_S gives $m_S = 4\pi^2 [10^{2(9.2476)}]/G = 1.85 \times 10^{30}$ kg. From Appendix F, $m_S = 1.99 \times 10^{30}$ kg. Our result agrees to within about 7% with the value in Appendix F.

(d) Solving Kepler's third law for a gives $a = \left(\frac{T\sqrt{Gm_S}}{2\pi} \right)^{2/3}$. Expressing T in seconds, we get

$T = 1325.4$ d $(24 \text{ h/d})(3600 \text{ s/h}) = 1.145 \times 10^8$ s. Putting in $G = 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ and $m_S = 1.99 \times 10^{30}$ kg, we get $a = 3.53 \times 10^{11} \text{ m} = 353 \times 10^6 \text{ km}$. From the table with the problem, we see that Vesta's orbit lies between that of Mars and Jupiter.

EVALUATE: The asteroid belt lies between Mars and Jupiter, which is why Vesta is usually considered an asteroid.

13.77. IDENTIFY and SET UP: At the surface of a planet, $g = \frac{GM}{R^2}$, and average density is $\rho = m/V$, where

$V = \frac{4}{3} \pi R^3$ for a sphere.

EXECUTE: We have expressions for g and M : $g = \frac{GM}{R^2}$ and $M = \rho V = \rho \left(\frac{4}{3} \pi R^3 \right)$. Combining them we

get $g = \frac{G\rho \left(\frac{4}{3} \pi R^3 \right)}{R^2} = \frac{4\pi G\rho R}{3}$. Using $R = D/2$ gives $g = \frac{2\pi G\rho D}{3}$.

(a) A graph of g versus D is shown in Figure 13.77. As this graph shows, the densities vary considerably and show no apparent pattern.

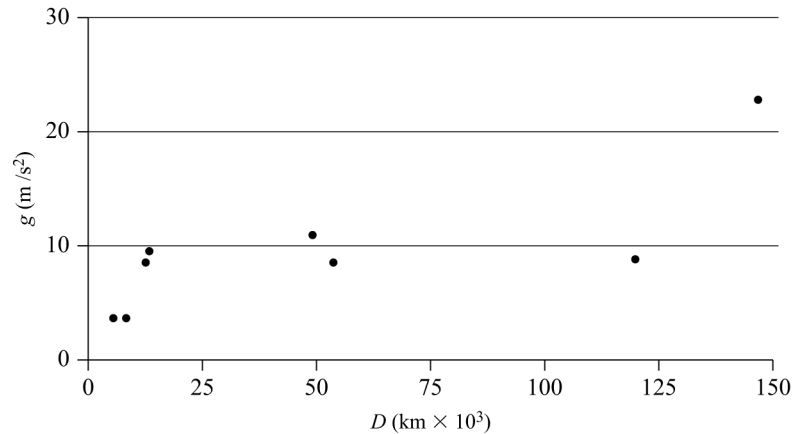


Figure 13.77

(b) Using the equation we just derived, $g = \frac{2\pi G\rho D}{3}$, we solve for ρ and use the values from the table given in the problem. For example, for Mercury we have

$$\rho = \frac{3g}{2\pi DG} = \frac{3(3.7 \text{ m/s}^2)}{2\pi(4.879 \times 10^6 \text{ m})(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 5400 \text{ kg/m}^3.$$

Continuing the calculations and putting the results in order of decreasing density, we get the following results.

Earth: 5500 kg/m³
 Mercury: 5400 kg/m³
 Venus: 5300 kg/m³
 Mars: 3900 kg/m³
 Neptune: 1600 kg/m³
 Uranus: 1200 kg/m³
 Jupiter: 1200 kg/m³
 Saturn: 534 kg/m³

(c) For several reasons, it is reasonable that the other planets would be denser toward their centers. Gravity is stronger at close distances, so it would compress matter near the center. In addition, during the formation of planets, heavy elements would tend to sink toward the center and displace light elements, much as a rock sinks in water. This variation in density would have no effect on our analysis however, since the planets are still spherically symmetric.

(d) $g = \frac{2\pi G\rho D}{3} = \frac{2\pi(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.20536 \times 10^8 \text{ m})(5500 \text{ kg/m}^3)}{3} = 93 \text{ m/s}^2.$

EVALUATE: Saturn is less dense than water, so it would float if we could throw it into our ocean (which of course is impossible since it is much larger than the earth). This low density is the reason that g at its “surface” is less than g at the earth’s surface, even though the mass of Saturn is much greater than that of the earth. Also note in our results in (b) that the inner four planets are much denser than the outer four (the gas giants), with the earth being the densest of all.

13.78. IDENTIFY and SET UP: $L = MvR$, $v = 2\pi R/T$.

EXECUTE: (a) Combining the two equations gives $L = mvR = m(2\pi R/T)R = 2\pi mR^2/T$.

(b) We use our formula with the quantities in Appendix F. For example, for Mercury we have

$L = 2\pi(3.30 \times 10^{23} \text{ kg})(5.79 \times 10^{10} \text{ m})^2/[(88.0 \text{ d})(24 \text{ h/d})(3600 \text{ s/h})] = 9.14 \times 10^{38} \text{ kg} \cdot \text{m}^2/\text{s}$. Similar calculations give the following results.

Mercury: $9.14 \times 10^{38} \text{ kg} \cdot \text{m}^2/\text{s}$

Venus: $1.84 \times 10^{40} \text{ kg} \cdot \text{m}^2/\text{s}$

Earth: $2.67 \times 10^{40} \text{ kg} \cdot \text{m}^2/\text{s}$

Mars: $3.53 \times 10^{39} \text{ kg} \cdot \text{m}^2/\text{s}$

Jupiter: $1.93 \times 10^{43} \text{ kg} \cdot \text{m}^2/\text{s}$

Saturn: $7.86 \times 10^{42} \text{ kg} \cdot \text{m}^2/\text{s}$

Uranus: $1.73 \times 10^{42} \text{ kg} \cdot \text{m}^2/\text{s}$

Neptune: $2.50 \times 10^{42} \text{ kg} \cdot \text{m}^2/\text{s}$

The total angular momentum is the sum of all of these since the planets all move in the same direction around the sun. $L_{\text{tot}} = 3.13 \times 10^{43} \text{ kg} \cdot \text{m}^2/\text{s}$.

(c) Treating the sun as a uniform solid sphere, we have

$$L = I\omega = \left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right) = \frac{4\pi MR^2}{5T} = \frac{4\pi(1.99 \times 10^{30} \text{ kg})(6.96 \times 10^8 \text{ m})^2}{5(24.6 \text{ d})(24 \text{ h/d})(3600 \text{ s/h})} = 1.14 \times 10^{42} \text{ kg} \cdot \text{m}^2/\text{s}.$$

(d) $L_S/L_P = (1.14 \times 10^{42} \text{ kg} \cdot \text{m}^2/\text{s})/(3.14 \times 10^{43} \text{ kg} \cdot \text{m}^2/\text{s}) = 0.0363$. The angular momentum of the sun is only 3.63% of the angular momentum of the planets. From Appendix F, the total mass of the planets is $m_P = 2.669 \times 10^{27} \text{ kg}$, so $m_S/m_P = (1.99 \times 10^{30} \text{ kg})/(2.669 \times 10^{27} \text{ kg}) = 746$, so the sun is 746 times as massive as the planets combined.

EVALUATE: The sun contains nearly all the mass in the solar system, yet it has only 3.6% of the angular momentum. There appears to be no progressive pattern in the angular momentum of the planets as we go from the inner planets to the outer ones.

13.79. IDENTIFY: Apply $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ to the transfer orbit.

SET UP: The orbit radius for earth is $r_E = 1.50 \times 10^{11} \text{ m}$ and for Mars it is $r_M = 2.28 \times 10^{11} \text{ m}$. From

Figure 13.18 in the textbook, $a = \frac{1}{2}(r_E + r_M)$.

EXECUTE: (a) To get from the circular orbit of the earth to the transfer orbit, the spacecraft's energy must increase, and the rockets are fired in the direction opposite that of the motion, that is, in the direction that increases the speed. Once at the orbit of Mars, the energy needs to be increased again, and so the rockets need to be fired in the direction opposite that of the motion. From Figure 13.18 in the textbook, the semimajor axis of the transfer orbit is the arithmetic average of the orbit radii of the earth and Mars, and so from $E = -Gm_S m/2r$, the energy of the spacecraft while in the transfer orbit is intermediate between the energies of the circular orbits. Returning from Mars to the earth, the procedure is reversed, and the rockets are fired against the direction of motion.

(b) The time will be half the period as given in $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, with the semimajor axis equal to

$$a = \frac{1}{2}(r_E + r_M) = 1.89 \times 10^{11} \text{ m} \text{ so}$$

$$t = \frac{T}{2} = \frac{\pi (1.89 \times 10^{11} \text{ m})^{3/2}}{\sqrt{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}} = 2.24 \times 10^7 \text{ s} = 259 \text{ days, which is more than } 8\frac{1}{2} \text{ months.}$$

(c) During this time, Mars will pass through an angle of $(360^\circ) \frac{(2.24 \times 10^7 \text{ s})}{(687 \text{ d})(86,400 \text{ s/d})} = 135.9^\circ$, and the spacecraft passes through an angle of 180° , so the angle between the earth-sun line and the Mars-sun line must be 44.1° .

EVALUATE: The period T for the transfer orbit is 526 days, the average of the orbital periods for earth and Mars.

13.80. IDENTIFY: Apply $\Sigma \vec{F} = m\vec{a}$ to each ear.

SET UP: Denote the orbit radius as r and the distance from this radius to either ear as δ . Each ear, of mass m , can be modeled as subject to two forces, the gravitational force from the black hole and the tension force (actually the force from the body tissues), denoted by F .

EXECUTE: The force equation for either ear is $\frac{GMm}{(r+\delta)^2} - F = m\omega^2(r+\delta)$, where δ can be of either sign.

Replace the product $m\omega^2$ with the value for $\delta = 0$, $m\omega^2 = GMm/r^3$, and solve for F :

$$F = (GMm) \left[\frac{r+\delta}{r^3} - \frac{1}{(r+\delta)^2} \right] = \frac{GMm}{r^3} \left[r+\delta - r(1+(\delta/r)^{-2}) \right].$$

Using the binomial theorem to expand the term in square brackets in powers of δ/r ,

$$F \approx \frac{GMm}{r^3} [r+\delta - r(1-2(\delta/r))] = \frac{GMm}{r^3} (3\delta) = 2.1 \text{ kN}.$$

This tension is much larger than that which could be sustained by human tissue, and the astronaut is in trouble.

(b) The center of gravity is not the center of mass. The gravity force on the two ears is not the same.

EVALUATE: The tension between her ears is proportional to their separation.

13.81. IDENTIFY: As suggested in the problem, divide the disk into rings of radius r and thickness dr .

SET UP: Each ring has an area $dA = 2\pi r dr$ and mass $dM = \frac{M}{\pi a^2} dA = \frac{2M}{a^2} r dr$.

EXECUTE: The magnitude of the force that this small ring exerts on the mass m is then

$$(Gm dM)(x/(r^2 + x^2)^{3/2}). \text{ The contribution } dF \text{ to the force is } dF = \frac{2GMmx}{a^2} \frac{rdr}{(x^2 + r^2)^{3/2}}.$$

The total force F is then the integral over the range of r ;

$$F = \int dF = \frac{2GMmx}{a^2} \int_0^a \frac{r}{(x^2 + r^2)^{3/2}} dr.$$

The integral (either by looking in a table or making the substitution $u = r^2 + a^2$) is

$$\int_0^a \frac{r}{(x^2 + r^2)^{3/2}} dr = \left[\frac{1}{x} - \frac{1}{\sqrt{a^2 + x^2}} \right] = \frac{1}{x} \left[1 - \frac{x}{\sqrt{a^2 + x^2}} \right].$$

Substitution yields the result $F = \frac{2GMm}{a^2} \left[1 - \frac{x}{\sqrt{a^2 + x^2}} \right]$. The force on m is directed toward the center of

the ring. The second term in brackets can be written as

$$\frac{1}{\sqrt{1+(a/x)^2}} = (1+(a/x)^2)^{-1/2} \approx 1 - \frac{1}{2} \left(\frac{a}{x} \right)^2$$

if $x \gg a$, where the binomial expansion has been used. Substitution of this into the above form gives

$$F \approx \frac{GMm}{x^2}, \text{ as it should.}$$

EVALUATE: As $x \rightarrow 0$, the force approaches a constant.

13.82. IDENTIFY and SET UP: Use $\rho = m/V$ to calculate the density of the planet and then use the table given in the problem to estimate its composition.

EXECUTE: Using $\rho = m/V$ gives

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{7.9m_E}{\frac{4}{3}\pi(2.3R_E)^3} = \frac{7.9}{(2.3)^3} \frac{m_E}{\frac{4}{3}\pi(R_E)^3} = 0.65\rho_E.$$

From the table, this density is in the range 0.4–0.9 times the density of the earth, so the planet probably has an iron core with a rock mantle and some lighter elements, which is choice (c).

EVALUATE: A method such as this gives only an estimation of the composition of a planet.

13.83. IDENTIFY and SET UP: Use $g = GM/R^2$.

EXECUTE: $g = GM/R^2 = G(7.9m_E)/(2.3R_E)^2 = [(7.9)/(2.3)^2](Gm_E/R_E^2) = 1.5g_E$, which is choice (c).

EVALUATE: Even though this planet has 7.9 times the mass of the earth, g at its surface is only $1.5g_E$ because the planet is 2.3 times the radius of the earth, which makes the surface farther away from its center than is the case with the earth.

13.84. IDENTIFY and SET UP: Apply Newton's second law and the law of universal gravitation to the planet, calling m the mass of the planet, M the mass of the star, r the orbital radius, and T the time for one orbit.

$\Sigma F = ma$, $F_g = Gm_1m_2/r^2$, $a_{\text{rad}} = mv^2/r$, $v = 2\pi r/T$.

EXECUTE: $\frac{GmM}{r^2} = \frac{mv^2}{r} = \frac{m(2\pi r/T)^2}{r} = \frac{4\pi^2 mr}{T^2}$. Now solve for r , which gives

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{G(0.70M_{\text{sun}})\left(\frac{9.5}{365}T_{\text{earth}}\right)^2}{4\pi^2} = (0.70)\left(\frac{9.5}{365}\right)^2\left(\frac{GM_{\text{sun}}T_{\text{earth}}^2}{4\pi^2}\right) = 0.000474r_{\text{earth}}^3$$

$r = (0.000474)^{1/3}r_{\text{earth}} = 0.078r_{\text{earth}}$, which is choice (b).

EVALUATE: The planet takes only 9.5 days for one orbit, yet the star has 70% the mass of our sun, so the planet must be very close to the star compared to the earth. And this is, in fact, what we have found, since r for this planet is 7.8% the distance of the earth from the sun.