Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Maths/TP/TSM
SF TSM

Trinity Term 2016

MA1212 — Linear Algebra II

Friday, May 20 Sports Centre 9:30 – 11:30

Dr. Paschalis Karageorgis

Instructions to Candidates:

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.

You may not start this examination until you are instructed to do so by the Invigilator.

1. Find a matrix A that has v_1 as an eigenvector with eigenvalue $\lambda_1 = 4$ and v_2 as an eigenvector with eigenvalue $\lambda_2 = 5$ when

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

2. Find the Jordan form and a Jordan basis for the matrix

$$A = \begin{bmatrix} 4 & -7 & 5 \\ 1 & -3 & 4 \\ 1 & -6 & 7 \end{bmatrix}.$$

3. The following matrix A has a triple eigenvalue. Find the minimal polynomial of A and use it to determine the inverse of A.

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & -2 & 2 \\ 3 & -6 & 5 \end{bmatrix}.$$

4. Let Q be the quadratic form on $\mathbb{R}^3$ which is defined by the formula

$$Q(x, y, z) = 2x^2 + (a+4)y^2 + (a+4)z^2 + 2axy + 6axz.$$

Find the values of the real parameter a for which the form is positive definite.

5. Suppose that A is an invertible $n \times n$ real matrix. Show that there exists a positive definite symmetric matrix P such that $P^2 = A^t A$.