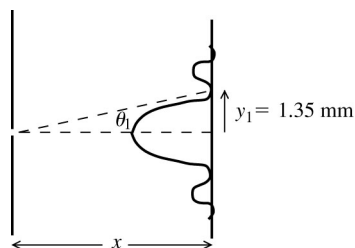


DIFFRACTION

- 36.1. IDENTIFY:** Use $y = x \tan \theta$ to calculate the angular position θ of the first minimum. The minima are located by $\sin \theta = \frac{m\lambda}{a}$, $m = \pm 1, \pm 2, \dots$. First minimum means $m = 1$ and $\sin \theta_1 = \lambda/a$ and $\lambda = a \sin \theta_1$. Use this equation to calculate λ .
- SET UP:** The central maximum is sketched in Figure 36.1.



EXECUTE: $y_1 = x \tan \theta_1$.

$$\begin{aligned}\tan \theta_1 &= \frac{y_1}{x} \\ \frac{1.35 \times 10^{-3} \text{ m}}{2.00 \text{ m}} &= 0.675 \times 10^{-3}. \\ \theta_1 &= 0.675 \times 10^{-3} \text{ rad}.\end{aligned}$$

Figure 36.1

$$\lambda = a \sin \theta_1 = (0.750 \times 10^{-3} \text{ m}) \sin(0.675 \times 10^{-3} \text{ rad}) = 506 \text{ nm}.$$

EVALUATE: θ_1 is small so the approximation used to obtain $y_m = x \frac{m\lambda}{a}$ is valid and this equation could have been used.

- 36.2. IDENTIFY:** The angle is small, so $y_m = x \frac{m\lambda}{a}$.

SET UP: $y_1 = 8.65 \text{ mm}$.

$$\text{EXECUTE: } y_1 = \frac{x\lambda}{a} \Rightarrow a = \frac{x\lambda}{y_1} = \frac{(0.600 \text{ m})(5.46 \times 10^{-7} \text{ m})}{8.65 \times 10^{-3} \text{ m}} = 3.79 \times 10^{-5} \text{ m} = 37.9 \text{ } \mu\text{m}.$$

EVALUATE: The diffraction pattern is observed at a distance of 60.0 cm from the slit.

- 36.3. IDENTIFY:** The dark fringes are located at angles θ that satisfy $\sin \theta = \frac{m\lambda}{a}$, $m = \pm 1, \pm 2, \dots$

SET UP: The largest value of $|\sin \theta|$ is 1.00.

EXECUTE: (a) Solve for m that corresponds to $\sin \theta = 1$: $m = \frac{a}{\lambda} = \frac{0.0666 \times 10^{-3} \text{ m}}{585 \times 10^{-9} \text{ m}} = 113.8$. The largest value m can have is 113. $m = \pm 1, \pm 2, \dots, \pm 113$ gives 226 dark fringes.

(b) For $m = \pm 113$, $\sin \theta = \pm 113 \left(\frac{585 \times 10^{-9} \text{ m}}{0.0666 \times 10^{-3} \text{ m}} \right) = \pm 0.9926$ and $\theta = \pm 83.0^\circ$.

EVALUATE: When the slit width a is decreased, there are fewer dark fringes. When $a < \lambda$ there are no dark fringes and the central maximum completely fills the screen.

- 36.4. IDENTIFY and SET UP:** λ/a is very small, so the approximate expression $y_m = x \frac{m\lambda}{a}$ is accurate. The distance between the two dark fringes on either side of the central maximum is $2y_1$.

EXECUTE: $y_1 = \frac{\lambda x}{a} = \frac{(633 \times 10^{-9} \text{ m})(3.50 \text{ m})}{0.750 \times 10^{-3} \text{ m}} = 2.95 \times 10^{-3} \text{ m} = 2.95 \text{ mm}$. $2y_1 = 5.90 \text{ mm}$.

EVALUATE: When a is decreased, the width $2y_1$ of the central maximum increases.

- 36.5. IDENTIFY:** The minima are located by $\sin \theta = \frac{m\lambda}{a}$.

SET UP: $a = 12.0 \text{ cm}$. $x = 8.00 \text{ m}$.

EXECUTE: The angle to the first minimum is $\theta = \arcsin\left(\frac{\lambda}{a}\right) = \arcsin\left(\frac{9.00 \text{ cm}}{12.00 \text{ cm}}\right) = 48.6^\circ$.

So the distance from the central maximum to the first minimum is just

$$y_1 = x \tan \theta = (8.00 \text{ m}) \tan(48.6^\circ) = \pm(9.07 \text{ m}).$$

EVALUATE: $2\lambda/a$ is greater than 1, so only the $m = 1$ minimum is heard.

- 36.6. IDENTIFY:** The angle that locates the first diffraction minimum on one side of the central maximum is given by $\sin \theta = \frac{\lambda}{a}$. The time between crests is the period T . $f = \frac{1}{T}$ and $\lambda = \frac{v}{f}$.

SET UP: The time between crests is the period, so $T = 1.0 \text{ h}$.

EXECUTE: (a) $f = \frac{1}{T} = \frac{1}{1.0 \text{ h}} = 1.0 \text{ h}^{-1}$. $\lambda = \frac{v}{f} = \frac{800 \text{ km/h}}{1.0 \text{ h}^{-1}} = 800 \text{ km}$.

(b) Africa-Antarctica: $\sin \theta = \frac{800 \text{ km}}{4500 \text{ km}}$ and $\theta = 10.2^\circ$.

Australia-Antarctica: $\sin \theta = \frac{800 \text{ km}}{3700 \text{ km}}$ and $\theta = 12.5^\circ$.

EVALUATE: Diffraction effects are observed when the wavelength is about the same order of magnitude as the dimensions of the opening through which the wave passes.

- 36.7. IDENTIFY:** We can model the hole in the concrete barrier as a single slit that will produce a single-slit diffraction pattern of the water waves on the shore.

SET UP: For single-slit diffraction, the angles at which destructive interference occurs are given by $\sin \theta_m = m\lambda/a$, where $m = 1, 2, 3, \dots$

EXECUTE: (a) The frequency of the water waves is $f = 75.0 \text{ min}^{-1} = 1.25 \text{ s}^{-1} = 1.25 \text{ Hz}$, so their wavelength is $\lambda = v/f = (15.0 \text{ cm/s})/(1.25 \text{ Hz}) = 12.0 \text{ cm}$.

At the first point for which destructive interference occurs, we have

$$\tan \theta = (0.613 \text{ m})/(3.20 \text{ m}) \Rightarrow \theta = 10.84^\circ. a \sin \theta = \lambda \text{ and}$$

$$a = \lambda / \sin \theta = (12.0 \text{ cm})/(\sin 10.84^\circ) = 63.8 \text{ cm}.$$

(b) Find the angles at which destructive interference occurs.

$$\sin \theta_2 = 2\lambda/a = 2(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_2 = \pm 22.1^\circ.$$

$$\sin \theta_3 = 3\lambda/a = 3(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_3 = \pm 34.3^\circ.$$

$$\sin \theta_4 = 4\lambda/a = 4(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_4 = \pm 48.8^\circ.$$

$$\sin \theta_5 = 5\lambda/a = 5(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_5 = \pm 70.1^\circ.$$

EVALUATE: These are large angles, so we cannot use the approximation that $\theta_m \approx m\lambda/a$.

- 36.8. IDENTIFY:** The angle is small, so $y_m = x \frac{m\lambda}{a}$ applies.

SET UP: The width of the central maximum is $2y_1$, so $y_1 = 3.00 \text{ mm}$.

EXECUTE: (a) $y_1 = \frac{x\lambda}{a} \Rightarrow a = \frac{x\lambda}{y_1} = \frac{(2.50 \text{ m})(5.00 \times 10^{-7} \text{ m})}{3.00 \times 10^{-3} \text{ m}} = 4.17 \times 10^{-4} \text{ m}.$

(b) $a = \frac{x\lambda}{y_1} = \frac{(2.50 \text{ m})(5.00 \times 10^{-5} \text{ m})}{3.00 \times 10^{-3} \text{ m}} = 4.17 \times 10^{-2} \text{ m} = 4.2 \text{ cm}.$

(c) $a = \frac{x\lambda}{y_1} = \frac{(2.50 \text{ m})(5.00 \times 10^{-10} \text{ m})}{3.00 \times 10^{-3} \text{ m}} = 4.17 \times 10^{-7} \text{ m}.$

EVALUATE: The ratio a/λ stays constant, so a is smaller when λ is smaller.

36.9. IDENTIFY and SET UP: $v = f\lambda$ gives λ . The person hears no sound at angles corresponding to diffraction minima. The diffraction minima are located by $\sin \theta = m\lambda/a$, $m = \pm 1, \pm 2, \dots$. Solve for θ .

EXECUTE: $\lambda = v/f = (344 \text{ m/s})/(1250 \text{ Hz}) = 0.2752 \text{ m}$; $a = 1.00 \text{ m}$. $m = \pm 1$, $\theta = \pm 16.0^\circ$; $m = \pm 2$, $\theta = \pm 33.4^\circ$; $m = \pm 3$, $\theta = \pm 55.6^\circ$; no solution for larger m .

EVALUATE: $\lambda/a = 0.28$ so for the large wavelength sound waves diffraction by the doorway is a large effect. Diffraction would not be observable for visible light because its wavelength is much smaller and $\lambda/a \ll 1$.

36.10. IDENTIFY: Compare E_y to the expression $E_y = E_{\max} \sin(kx - \omega t)$ and determine k , and from that

calculate λ . $f = c/\lambda$. The dark bands are located by $\sin \theta = \frac{m\lambda}{a}$.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$. The first dark band corresponds to $m = 1$.

EXECUTE: (a) $E = E_{\max} \sin(kx - \omega t)$. $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.40 \times 10^7 \text{ m}^{-1}} = 4.488 \times 10^{-7} \text{ m}.$

$f\lambda = c \Rightarrow f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{4.488 \times 10^{-7} \text{ m}} = 6.68 \times 10^{14} \text{ Hz}.$

(b) $a \sin \theta = \lambda$. $a = \frac{\lambda}{\sin \theta} = \frac{4.488 \times 10^{-7} \text{ m}}{\sin 28.6^\circ} = 9.38 \times 10^{-7} \text{ m}.$

(c) $a \sin \theta = m\lambda (m = 1, 2, 3, \dots)$. $\sin \theta_2 = \pm 2 \frac{\lambda}{a} = \pm 2 \frac{4.488 \times 10^{-7} \text{ m}}{9.38 \times 10^{-7} \text{ m}}$ so $\theta_2 = \pm 73.2^\circ$.

EVALUATE: For $m = 3$, $\frac{m\lambda}{a}$ is greater than 1 so only the first and second dark bands appear.

36.11. IDENTIFY: Calculate the angular positions of the minima and use $y = x \tan \theta$ to calculate the distance on the screen between them.

(a) **SET UP:** The central bright fringe is shown in Figure 36.11a.

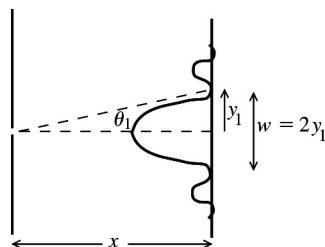


Figure 36.11a

EXECUTE: The first minimum is located by

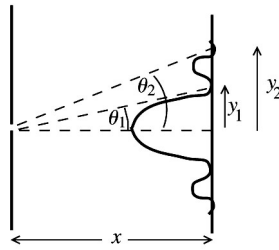
$\sin \theta_1 = \frac{\lambda}{a} = \frac{633 \times 10^{-9} \text{ m}}{0.350 \times 10^{-3} \text{ m}} = 1.809 \times 10^{-3}.$

$\theta_1 = 1.809 \times 10^{-3} \text{ rad}.$

$y_1 = x \tan \theta_1 = (3.00 \text{ m}) \tan(1.809 \times 10^{-3} \text{ rad}) = 5.427 \times 10^{-3} \text{ m}.$

$w = 2y_1 = 2(5.427 \times 10^{-3} \text{ m}) = 1.09 \times 10^{-2} \text{ m} = 10.9 \text{ mm}.$

(b) **SET UP:** The first bright fringe on one side of the central maximum is shown in Figure 36.11b.



EXECUTE: $w = y_2 - y_1$.

$$y_1 = 5.427 \times 10^{-3} \text{ m (part (a))}.$$

$$\sin \theta_2 = \frac{2\lambda}{a} = 3.618 \times 10^{-3}.$$

$$\theta_2 = 3.618 \times 10^{-3} \text{ rad.}$$

$$y_2 = x \tan \theta_2 = 1.085 \times 10^{-2} \text{ m.}$$

Figure 36.11b

$$w = y_2 - y_1 = 1.085 \times 10^{-2} \text{ m} - 5.427 \times 10^{-3} \text{ m} = 5.4 \text{ mm.}$$

EVALUATE: The central bright fringe is twice as wide as the other bright fringes.

36.12. IDENTIFY: The space between the skyscrapers behaves like a single slit and diffracts the radio waves.

SET UP: Cancellation of the waves occurs when $a \sin \theta = m\lambda$, $m = 1, 2, 3, \dots$, and the intensity of the

waves is given by $I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2$, where $\beta/2 = \frac{\pi a \sin \theta}{\lambda}$.

EXECUTE: (a) First find the wavelength of the waves:

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s}) / (88.9 \text{ MHz}) = 3.375 \text{ m.}$$

For no signal, $a \sin \theta = m\lambda$.

$$m = 1: \sin \theta_1 = (1)(3.375 \text{ m}) / (15.0 \text{ m}) \Rightarrow \theta_1 = \pm 13.0^\circ.$$

$$m = 2: \sin \theta_2 = (2)(3.375 \text{ m}) / (15.0 \text{ m}) \Rightarrow \theta_2 = \pm 26.7^\circ.$$

$$m = 3: \sin \theta_3 = (3)(3.375 \text{ m}) / (15.0 \text{ m}) \Rightarrow \theta_3 = \pm 42.4^\circ.$$

$$m = 4: \sin \theta_4 = (4)(3.375 \text{ m}) / (15.0 \text{ m}) \Rightarrow \theta_4 = \pm 64.1^\circ.$$

$$(b) I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2, \text{ where } \beta/2 = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi(15.0 \text{ m}) \sin(5.00^\circ)}{3.375 \text{ m}} = 1.217 \text{ rad.}$$

$$I = (3.50 \text{ W/m}^2) \left[\frac{\sin(1.217 \text{ rad})}{1.217 \text{ rad}} \right]^2 = 2.08 \text{ W/m}^2.$$

EVALUATE: The wavelength of the radio waves is very long compared to that of visible light, but it is still considerably shorter than the distance between the buildings.

36.13. IDENTIFY: The minima are located by $\sin \theta = \frac{m\lambda}{a}$. For part (b) apply $I = I_0 \left\{ \frac{\sin[\pi a(\sin \theta)/\lambda]}{\pi a(\sin \theta)/\lambda} \right\}^2$.

SET UP: For the first minimum, $m = 1$. The intensity at $\theta = 0$ is I_0 .

EXECUTE: (a) $\sin \theta = \frac{m\lambda}{a} = \sin 90.0^\circ = 1 = \frac{m\lambda}{a} = \frac{\lambda}{a}$. Thus $a = \lambda = 580 \text{ nm} = 5.80 \times 10^{-4} \text{ mm}$.

(b) Using $I = I_0 \left\{ \frac{\sin[\pi a(\sin \theta)/\lambda]}{\pi a(\sin \theta)/\lambda} \right\}^2$ gives

$$\frac{I}{I_0} = \left\{ \frac{\sin[\pi a(\sin \theta)/\lambda]}{\pi a(\sin \theta)/\lambda} \right\}^2 = \left\{ \frac{\sin[\pi(\sin \pi/4)]}{\pi(\sin \pi/4)} \right\}^2 = 0.128.$$

EVALUATE: If $a = \lambda/2$, for example, then at $\theta = 45^\circ$, $\frac{I}{I_0} = \left\{ \frac{\sin[(\pi/2)(\sin \pi/4)]}{(\pi/2)(\sin \pi/4)} \right\}^2 = 0.65$. As a/λ decreases, the screen becomes more uniformly illuminated.

36.14. IDENTIFY: $I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2$. $\beta = \frac{2\pi}{\lambda} a \sin \theta$.

SET UP: The angle θ is small, so $\sin \theta \approx \tan \theta \approx y/x$.

EXECUTE: $\beta = \frac{2\pi a}{\lambda} \sin \theta \approx \frac{2\pi a}{\lambda} \frac{y}{x} = \frac{2\pi(4.50 \times 10^{-4} \text{ m})}{(6.20 \times 10^{-7} \text{ m})(3.00 \text{ m})} y = (1520 \text{ m}^{-1})y.$

(a) $y = 1.00 \times 10^{-3} \text{ m}: \frac{\beta}{2} = \frac{(1520 \text{ m}^{-1})(1.00 \times 10^{-3} \text{ m})}{2} = 0.760.$

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(0.760)}{0.760} \right)^2 = 0.822 I_0.$$

(b) $y = 3.00 \times 10^{-3} \text{ m}: \frac{\beta}{2} = \frac{(1520 \text{ m}^{-1})(3.00 \times 10^{-3} \text{ m})}{2} = 2.28.$

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(2.28)}{2.28} \right)^2 = 0.111 I_0.$$

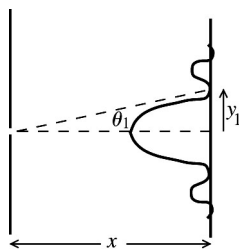
(c) $y = 5.00 \times 10^{-3} \text{ m}: \frac{\beta}{2} = \frac{(1520 \text{ m}^{-1})(5.00 \times 10^{-3} \text{ m})}{2} = 3.80.$

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(3.80)}{3.80} \right)^2 = 0.0259 I_0.$$

EVALUATE: The first minimum occurs at $y_1 = \frac{\lambda x}{a} = \frac{(6.20 \times 10^{-7} \text{ m})(3.00 \text{ m})}{4.50 \times 10^{-4} \text{ m}} = 4.1 \text{ mm}.$ The distances in parts (a) and (b) are within the central maximum. $y = 5.00 \text{ mm}$ is within the first secondary maximum.

- 36.15. (a) IDENTIFY:** Use $\sin \theta = m \frac{\lambda}{a}$ with $m=1$ to locate the angular position of the first minimum and then use $y = x \tan \theta$ to find its distance from the center of the screen.

SET UP and EXECUTE: The diffraction pattern is sketched in Figure 36.15.



$$\sin \theta_1 = \frac{\lambda}{a} = \frac{540 \times 10^{-9} \text{ m}}{0.240 \times 10^{-3} \text{ m}} = 2.25 \times 10^{-3}.$$

$$\theta_1 = 2.25 \times 10^{-3} \text{ rad}.$$

Figure 36.15

$$y_1 = x \tan \theta_1 = (3.00 \text{ m}) \tan(2.25 \times 10^{-3} \text{ rad}) = 6.75 \times 10^{-3} \text{ m} = 6.75 \text{ mm}.$$

- (b) **IDENTIFY and SET UP:** Use $I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2$ and $\beta/2 = \frac{\pi a \sin \theta}{\lambda}$ to calculate the intensity at this point.

EXECUTE: Midway between the center of the central maximum and the first minimum implies

$$y = \frac{1}{2}(6.75 \text{ mm}) = 3.375 \times 10^{-3} \text{ m}.$$

$$\tan \theta = \frac{y}{x} = \frac{3.375 \times 10^{-3} \text{ m}}{3.00 \text{ m}} = 1.125 \times 10^{-3}; \theta = 1.125 \times 10^{-3} \text{ rad}.$$

The phase angle β at this point on the screen is

$$\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta = \frac{2\pi}{540 \times 10^{-9} \text{ m}} (0.240 \times 10^{-3} \text{ m}) \sin(1.125 \times 10^{-3} \text{ rad}) = \pi.$$

Then $I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2 = (6.00 \times 10^{-6} \text{ W/m}^2) \left(\frac{\sin \pi/2}{\pi/2} \right)^2$.

$$I = \left(\frac{4}{\pi^2} \right) (6.00 \times 10^{-6} \text{ W/m}^2) = 2.43 \times 10^{-6} \text{ W/m}^2.$$

EVALUATE: The intensity at this point midway between the center of the central maximum and the first minimum is less than half the maximum intensity. Compare this result to the corresponding one for the two-slit pattern, Exercise 35.20.

- 36.16. IDENTIFY:** The intensity on the screen varies as the light spreads out (diffracts) after passing through the single slit.

SET UP: $I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$ where $\beta = \frac{2\pi}{\lambda} a \sin \theta$.

EXECUTE: $\beta = \frac{2\pi}{\lambda} a \sin \theta = \left(\frac{2\pi}{592 \times 10^{-9} \text{ m}} \right) (0.0290 \times 10^{-3} \text{ m}) (\sin 1.20^\circ) = 6.44589 \text{ rad}$.

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 = (4.00 \times 10^{-5} \text{ W/m}^2) \left[\frac{\sin(6.44589 \text{ rad})}{6.44589 \text{ rad}} \right]^2 = 2.54 \times 10^{-8} \text{ W/m}^2.$$

EVALUATE: The intensity is less than 1/1500 of the intensity of the light at the center of the central maximum.

- 36.17. IDENTIFY and SET UP:** Use $\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta$ to calculate λ and use $I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2$ to calculate I .

$$\theta = 3.25^\circ, \quad \beta = 56.0 \text{ rad}, \quad a = 0.105 \times 10^{-3} \text{ m}.$$

EXECUTE: (a) $\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta$ so

$$\lambda = \frac{2\pi a \sin \theta}{\beta} = \frac{2\pi (0.105 \times 10^{-3} \text{ m}) \sin 3.25^\circ}{56.0 \text{ rad}} = 668 \text{ nm}.$$

(b) $I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2 = I_0 \left(\frac{4}{\beta^2} \right) (\sin(\beta/2))^2 = I_0 \frac{4}{(56.0 \text{ rad})^2} [\sin(28.0 \text{ rad})]^2 = 9.36 \times 10^{-5} I_0$.

EVALUATE: At the first minimum $\beta = 2\pi$ rad and at the point considered in the problem $\beta = 17.8\pi$ rad, so the point is well outside the central maximum. Since β is close to $m\pi$ with $m = 18$, this point is near one of the minima. The intensity here is much less than I_0 .

- 36.18. IDENTIFY:** The intensity at the screen is due to a combination of single-slit diffraction and double-slit interference.

SET UP: $I = I_0 \left(\cos^2 \frac{\phi}{2} \right) \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$, where $\phi = \frac{2\pi d}{\lambda} \sin \theta$ and $\beta = \frac{2\pi}{\lambda} a \sin \theta$.

EXECUTE: $\tan \theta = \frac{9.00 \times 10^{-4} \text{ m}}{0.750 \text{ m}} = 1.200 \times 10^{-3}$. θ is small, so $\sin \theta \approx \tan \theta$.

$$\phi = \frac{2\pi d}{\lambda} \sin \theta = \frac{2\pi (0.640 \times 10^{-3} \text{ m})}{568 \times 10^{-9} \text{ m}} (1.200 \times 10^{-3}) = 8.4956 \text{ rad}.$$

$$\beta = \frac{2\pi a}{\lambda} \sin \theta = \frac{2\pi (0.434 \times 10^{-3} \text{ m})}{568 \times 10^{-9} \text{ m}} (1.200 \times 10^{-3}) = 5.7611 \text{ rad}.$$

$$I = (5.00 \times 10^{-4} \text{ W/m}^2) (\cos 4.2478 \text{ rad})^2 \left[\frac{\sin 2.8805 \text{ rad}}{2.8805} \right]^2 = 8.06 \times 10^{-7} \text{ W/m}^2.$$

EVALUATE: The intensity is decreased by a factor of almost a thousand, so it would be difficult to see the light at the screen.

36.19. (a) IDENTIFY and SET UP: The interference fringes (maxima) are located by $d \sin \theta = m\lambda$, with $m = 0, \pm 1, \pm 2, \dots$. The intensity I in the diffraction pattern is given by $I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2$, with

$\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta$. We want $m = \pm 3$ in the first equation to give θ that makes $I = 0$ in the second equation.

EXECUTE: $d \sin \theta = m\lambda$ gives $\beta = \left(\frac{2\pi}{\lambda} \right) a \left(\frac{3\lambda}{d} \right) = 2\pi(3a/d)$.

$I = 0$ says $\frac{\sin \beta/2}{\beta/2} = 0$ so $\beta = 2\pi$ and then $2\pi = 2\pi(3a/d)$ and $(d/a) = 3$.

(b) IDENTIFY and SET UP: Fringes $m = 0, \pm 1, \pm 2$ are within the central diffraction maximum and the $m = \pm 3$ fringes coincide with the first diffraction minimum. Find the value of m for the fringes that coincide with the second diffraction minimum.

EXECUTE: Second minimum implies $\beta = 4\pi$.

$$\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta = \left(\frac{2\pi}{\lambda} \right) a \left(\frac{m\lambda}{d} \right) = 2\pi m(a/d) = 2\pi(m/3).$$

Then $\beta = 4\pi$ says $4\pi = 2\pi(m/3)$ and $m = 6$. Therefore the $m = \pm 4$ and $m = \pm 5$ fringes are contained within the first diffraction maximum on one side of the central maximum; two fringes.

EVALUATE: The central maximum is twice as wide as the other maxima so it contains more fringes.

36.20. IDENTIFY: The net intensity is the *product* of the factor due to single-slit diffraction and the factor due to double slit interference.

SET UP: The double-slit factor is $I_{DS} = I_0 \left(\cos^2 \frac{\phi}{2} \right)$ and the single-slit factor is $I_{SS} = \left(\frac{\sin \beta/2}{\beta/2} \right)^2$.

EXECUTE: (a) $d \sin \theta = m\lambda \Rightarrow \sin \theta = m\lambda/d$.

$\sin \theta_1 = \lambda/d$, $\sin \theta_2 = 2\lambda/d$, $\sin \theta_3 = 3\lambda/d$, $\sin \theta_4 = 4\lambda/d$.

(b) At the interference bright fringes, $\cos^2 \phi/2 = 1$ and $\beta/2 = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi(d/3) \sin \theta}{\lambda}$.

At θ_1 , $\sin \theta_1 = \lambda/d$, so $\beta/2 = \frac{\pi(d/3)(\lambda/d)}{\lambda} = \pi/3$. The intensity is therefore

$$I_1 = I_0 \left(\cos^2 \frac{\phi}{2} \right) \left(\frac{\sin \beta/2}{\beta/2} \right)^2 = I_0 (1) \left(\frac{\sin \pi/3}{\pi/3} \right)^2 = 0.684 I_0.$$

At θ_2 , $\sin \theta_2 = 2\lambda/d$, so $\beta/2 = \frac{\pi(d/3)(2\lambda/d)}{\lambda} = 2\pi/3$. Using the same procedure as for θ_1 , we have

$$I_2 = I_0 (1) \left(\frac{\sin 2\pi/3}{2\pi/3} \right)^2 = 0.171 I_0.$$

At θ_3 , we get $\beta/2 = \pi$, which gives $I_3 = 0$ since $\sin \pi = 0$.

At θ_4 , $\sin \theta_4 = 4\lambda/d$, so $\beta/2 = 4\pi/3$, which gives $I_4 = I_0 \left(\frac{\sin 4\pi/3}{4\pi/3} \right)^2 = 0.0427 I_0$.

(c) Since $d = 3a$, every third interference maximum is missing.

(d) In Figure 36.12c in the textbook, every fourth interference maximum at the sides is missing because $d = 4a$.

EVALUATE: The result in this problem is different from that in Figure 36.12c in the textbook because in this case $d = 3a$, so every third interference maximum at the sides is missing. Also the “envelope” of the intensity function decreases more rapidly here than in Figure 36.12c in the text because the first diffraction minimum is reached sooner, and the decrease in intensity from one interference maximum to the next is faster for $a = d/3$ than for $a = d/4$.

- 36.21. (a) IDENTIFY and SET UP:** If the slits are very narrow then the central maximum of the diffraction pattern for each slit completely fills the screen and the intensity distribution is given solely by the two-slit interference. The maxima are given by $d \sin \theta = m\lambda$ so $\sin \theta = m\lambda/d$. Solve for θ .

EXECUTE: 1st order maximum: $m = 1$, so $\sin \theta = \frac{\lambda}{d} = \frac{580 \times 10^{-9} \text{ m}}{0.530 \times 10^{-3} \text{ m}} = 1.094 \times 10^{-3}$; $\theta = 0.0627^\circ$.

2nd order maximum: $m = 2$, so $\sin \theta = \frac{2\lambda}{d} = 2.188 \times 10^{-3}$; $\theta = 0.125^\circ$.

- (b) IDENTIFY and SET UP:** The intensity is given by Eq. (36.12): $I = I_0 \cos^2(\phi/2) \left(\frac{\sin \beta/2}{\beta/2} \right)^2$. Calculate

ϕ and β at each θ from part (a).

EXECUTE: $\phi = \left(\frac{2\pi d}{\lambda} \right) \sin \theta = \left(\frac{2\pi d}{\lambda} \right) \left(\frac{m\lambda}{d} \right) = 2\pi m$, so $\cos^2(\phi/2) = \cos^2(m\pi) = 1$.

(Since the angular positions in part (a) correspond to interference maxima.)

$\beta = \left(\frac{2\pi a}{\lambda} \right) \sin \theta = \left(\frac{2\pi a}{\lambda} \right) \left(\frac{m\lambda}{d} \right) = 2\pi m(a/d) = m2\pi \left(\frac{0.320 \text{ mm}}{0.530 \text{ mm}} \right) = m(3.794 \text{ rad})$.

1st order maximum: $m = 1$, so $I = I_0(1) \left(\frac{\sin(3.794/2) \text{ rad}}{(3.794/2) \text{ rad}} \right)^2 = 0.249 I_0$.

2nd order maximum: $m = 2$, so $I = I_0(1) \left(\frac{\sin 3.794 \text{ rad}}{3.794 \text{ rad}} \right)^2 = 0.0256 I_0$.

EVALUATE: The first diffraction minimum is at an angle θ given by $\sin \theta = \lambda/a$ so $\theta = 0.104^\circ$. The first order fringe is within the central maximum and the second order fringe is inside the first diffraction maximum on one side of the central maximum. The intensity here at this second fringe is much less than I_0 .

- 36.22. IDENTIFY:** The diffraction minima are located by $\sin \theta = \frac{m_d \lambda}{a}$ and the two-slit interference maxima are

located by $\sin \theta = \frac{m_i \lambda}{d}$. The third bright band is missing because the first order single-slit minimum occurs at the same angle as the third order double-slit maximum.

SET UP: The pattern is sketched in Figure 36.22. $\tan \theta = \frac{3 \text{ cm}}{90 \text{ cm}}$, so $\theta = 1.91^\circ$.

EXECUTE: Single-slit dark spot: $a \sin \theta = \lambda$ and $a = \frac{\lambda}{\sin \theta} = \frac{500 \text{ nm}}{\sin 1.91^\circ} = 1.50 \times 10^4 \text{ nm} = 15.0 \mu\text{m}$ (width).

Double-slit bright fringe: $d \sin \theta = 3\lambda$ and $d = \frac{3\lambda}{\sin \theta} = \frac{3(500 \text{ nm})}{\sin 1.91^\circ} = 4.50 \times 10^4 \text{ nm} = 45.0 \mu\text{m}$ (separation).

EVALUATE: Note that $d/a = 3.0$.

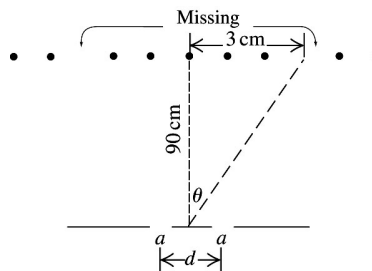


Figure 36.22

- 36.23. IDENTIFY:** Knowing the wavelength of the light and the location of the first interference maxima, we can calculate the line density of the grating.

SET UP: The line density in lines/cm is $1/d$, with d in cm. The bright spots are located by $d \sin \theta = m\lambda$, $m = 0, \pm 1, \pm 2, \dots$

EXECUTE: (a) $d = \frac{m\lambda}{\sin \theta} = \frac{(1)(632.8 \times 10^{-9} \text{ m})}{\sin 17.8^\circ} = 2.07 \times 10^{-6} \text{ m} = 2.07 \times 10^{-4} \text{ cm}$. $\frac{1}{d} = 4830 \text{ lines/cm}$.

(b) $\sin \theta = \frac{m\lambda}{d} = m \left(\frac{632.8 \times 10^{-9} \text{ m}}{2.07 \times 10^{-6} \text{ m}} \right) = m(0.3057)$. For $m = \pm 2$, $\theta = \pm 37.7^\circ$. For $m = \pm 3$, $\theta = \pm 66.5^\circ$.

EVALUATE: The angles are large, so they are not equally spaced; $37.7^\circ \neq 2(17.8^\circ)$ and $66.5^\circ \neq 3(17.8^\circ)$.

36.24. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: The order corresponds to the values of m .

EXECUTE: First-order: $d \sin \theta_1 = \lambda$. Fourth-order: $d \sin \theta_4 = 4\lambda$.

$$\frac{d \sin \theta_4}{d \sin \theta_1} = \frac{4\lambda}{\lambda}, \sin \theta_4 = 4 \sin \theta_1 = 4 \sin 11.3^\circ \text{ and } \theta_4 = 51.6^\circ.$$

EVALUATE: We did not have to solve for d .

36.25. IDENTIFY and SET UP: The bright bands are at angles θ given by $d \sin \theta = m\lambda$. Solve for d and then solve for θ for the specified order.

EXECUTE: (a) $\theta = 78.4^\circ$ for $m = 3$ and $\lambda = 681 \text{ nm}$, so $d = m\lambda / \sin \theta = 2.086 \times 10^{-4} \text{ cm}$.

The number of slits per cm is $1/d = 4790 \text{ slits/cm}$.

(b) 1st order: $m = 1$, so $\sin \theta = \lambda/d = (681 \times 10^{-9} \text{ m}) / (2.086 \times 10^{-6} \text{ m})$ and $\theta = 19.1^\circ$.

2nd order: $m = 2$, so $\sin \theta = 2\lambda/d$ and $\theta = 40.8^\circ$.

(c) For $m = 4$, $\sin \theta = 4\lambda/d$ is greater than 1.00, so there is no 4th-order bright band.

EVALUATE: The angular position of the bright bands for a particular wavelength increases as the order increases.

36.26. IDENTIFY: The bright spots are located by $d \sin \theta = m\lambda$.

SET UP: Third-order means $m = 3$ and second-order means $m = 2$.

EXECUTE: $\frac{m\lambda}{\sin \theta} = d = \text{constant}$, so $\frac{m_r \lambda_r}{\sin \theta_r} = \frac{m_v \lambda_v}{\sin \theta_v}$.

$$\sin \theta_v = \sin \theta_r \left(\frac{m_v}{m_r} \right) \left(\frac{\lambda_v}{\lambda_r} \right) = (\sin 65.0^\circ) \left(\frac{2}{3} \right) \left(\frac{400 \text{ nm}}{700 \text{ nm}} \right) = 0.345 \text{ and } \theta_v = 20.2^\circ.$$

EVALUATE: The third-order line for a particular λ occurs at a larger angle than the second-order line. In a given order, the line for violet light (400 nm) occurs at a smaller angle than the line for red light (700 nm).

36.27. IDENTIFY and SET UP: Calculate d for the grating. Use $d \sin \theta = m\lambda$ to calculate θ for the longest

wavelength in the visible spectrum (750 nm) and verify that θ is small. Then use $y_m = x \frac{m\lambda}{a}$ to relate the linear separation of lines on the screen to the difference in wavelength.

EXECUTE: (a) $d = \left(\frac{1}{900} \right) \text{ cm} = 1.111 \times 10^{-5} \text{ m}$.

For $\lambda = 750 \text{ nm}$, $\lambda/d = 6.8 \times 10^{-2}$. The first-order lines are located at $\sin \theta = \lambda/d$; $\sin \theta$ is small enough for $\sin \theta \approx \theta$ to be an excellent approximation.

(b) $y = x\lambda/d$, where $x = 2.50 \text{ m}$.

The distance on the screen between first-order bright bands for two different wavelengths is

$$\Delta y = x(\Delta\lambda)/d, \text{ so } \Delta\lambda = d(\Delta y)/x = (1.111 \times 10^{-5} \text{ m})(3.00 \times 10^{-3} \text{ m})/(2.50 \text{ m}) = 13.3 \text{ nm}.$$

EVALUATE: The smaller d is (greater number of lines per cm) the smaller the $\Delta\lambda$ that can be measured.

36.28. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: $350 \text{ slits/mm} \Rightarrow d = \frac{1}{3.50 \times 10^5 \text{ m}^{-1}} = 2.857 \times 10^{-6} \text{ m}$. The visible spectrum is between

approximately 380 nm and 750 nm.

EXECUTE: (a) $m = 1$: $\theta_{380} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{3.80 \times 10^{-7} \text{ m}}{2.857 \times 10^{-6} \text{ m}}\right) = 6.643^\circ$.

$$\theta_{750} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{7.50 \times 10^{-7} \text{ m}}{2.857 \times 10^{-6} \text{ m}}\right) = 15.2185^\circ. \quad \Delta\theta_1 = 15.2185^\circ - 6.643^\circ = 8.58^\circ.$$

(b) $m = 3$: $\theta_{380} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(3.80 \times 10^{-7} \text{ m})}{2.857 \times 10^{-6} \text{ m}}\right) = 23.516^\circ$.

$$\theta_{750} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(7.50 \times 10^{-7} \text{ m})}{2.857 \times 10^{-6} \text{ m}}\right) = 51.952^\circ. \quad \Delta\theta_1 = 51.952^\circ - 23.516^\circ = 28.4^\circ.$$

EVALUATE: $\Delta\theta$ is larger in third order.

36.29. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: 5000 slits/cm $\Rightarrow d = \frac{1}{5.00 \times 10^5 \text{ m}^{-1}} = 2.00 \times 10^{-6} \text{ m}$.

EXECUTE: (a) $\lambda = \frac{d \sin \theta}{m} = \frac{(2.00 \times 10^{-6} \text{ m}) \sin 13.5^\circ}{1} = 4.67 \times 10^{-7} \text{ m}$.

(b) $m = 2$: $\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{2(4.67 \times 10^{-7} \text{ m})}{2.00 \times 10^{-6} \text{ m}}\right) = 27.8^\circ$.

EVALUATE: Since the angles are fairly small, the second-order deviation is approximately twice the first-order deviation.

36.30. IDENTIFY: The grooves in a CD and DVD form a reflection diffraction grating. Constructive interference when $d \sin \theta = m\lambda$.

SET UP: The maxima are located by $d \sin \theta = m\lambda$, where $d = 1.60 \times 10^{-6} \text{ m}$ for a CD and $d = 0.740 \times 10^{-6} \text{ m}$ for a DVD.

EXECUTE: (a) For a CD we have: $\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(6.328 \times 10^{-7} \text{ m})}{1.60 \times 10^{-6} \text{ m}}\right) = \arcsin(0.396m)$. For

$m = 1$ we have $\theta_1 = 23.3^\circ$. For $m = 2$ we have $\theta_2 = 52.3^\circ$. There are no other maxima.

(b) For a DVD we have: $\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(6.328 \times 10^{-7} \text{ m})}{0.740 \times 10^{-6} \text{ m}}\right) = \arcsin(0.855m)$. For $m = 1$ we

have $\theta_1 = 58.8^\circ$. There are no other maxima.

EVALUATE: The reflective surface produces the same interference pattern as a grating with slit separation d .

36.31. IDENTIFY: The resolving power depends on the line density and the width of the grating.

SET UP: The resolving power is given by $R = Nm = \lambda/\Delta\lambda$.

EXECUTE: (a) $R = Nm = (5000 \text{ lines/cm})(3.50 \text{ cm})(1) = 17,500$

(b) The resolving power needed to resolve the sodium doublet is

$$R = \lambda/\Delta\lambda = (589 \text{ nm})/(589.59 \text{ nm} - 589.00 \text{ nm}) = 998$$

so this grating can easily resolve the doublet.

(c) (i) $R = \lambda/\Delta\lambda$. Since $R = 17,500$ when $m = 1$, $R = 2 \times 17,500 = 35,000$ for $m = 2$. Therefore

$$\Delta\lambda = \lambda/R = (587.8 \text{ nm})/35,000 = 0.0168 \text{ nm}.$$

$$\lambda_{\min} = \lambda + \Delta\lambda = 587.8002 \text{ nm} + 0.0168 \text{ nm} = 587.8170 \text{ nm}.$$

(ii) $\lambda_{\max} = \lambda - \Delta\lambda = 587.8002 \text{ nm} - 0.0168 \text{ nm} = 587.7834 \text{ nm}$

EVALUATE: (iii) Therefore the range of unresolvable wavelengths is $587.7834 \text{ nm} < \lambda < 587.8170 \text{ nm}$.

36.32. IDENTIFY: The resolution is described by $R = \frac{\lambda}{\Delta\lambda} = Nm$. Maxima are located by $d \sin \theta = m\lambda$.

SET UP: For 500 slits/mm, $d = (500 \text{ slits/mm})^{-1} = (500,000 \text{ slits/m})^{-1}$.

EXECUTE: (a) $N = \frac{\lambda}{m\Delta\lambda} = \frac{6.5645 \times 10^{-7} \text{ m}}{2(6.5645 \times 10^{-7} \text{ m} - 6.5627 \times 10^{-7} \text{ m})} = 1820 \text{ slits.}$

(b) $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) \Rightarrow \theta_1 = \sin^{-1}((2)(6.5645 \times 10^{-7} \text{ m})(500,000 \text{ m}^{-1})) = 41.0297^\circ$ and

$\theta_2 = \sin^{-1}((2)(6.5627 \times 10^{-7} \text{ m})(500,000 \text{ m}^{-1})) = 41.0160^\circ. \Delta\theta = 0.0137^\circ.$

EVALUATE: $d \cos \theta d\theta = \lambda/N$, so for 1820 slits the angular interval $\Delta\theta$ between each of these maxima

and the first adjacent minimum is $\Delta\theta = \frac{\lambda}{Nd \cos \theta} = \frac{6.56 \times 10^{-7} \text{ m}}{(1820)(2.0 \times 10^{-6} \text{ m}) \cos 41^\circ} = 0.0137^\circ.$ This is the

same as the angular separation of the maxima for the two wavelengths and 1820 slits is just sufficient to resolve these two wavelengths in second order.

36.33. IDENTIFY and SET UP: $\frac{\lambda}{\Delta\lambda} = Nm.$

EXECUTE: $N = \frac{\lambda}{m\Delta\lambda} = \frac{587.8002 \text{ nm}}{(587.9782 \text{ nm} - 587.8002 \text{ nm})} = \frac{587.8002}{0.178} = 3302 \text{ slits.}$

$\frac{N}{1.20 \text{ cm}} = \frac{3302}{1.20 \text{ cm}} = 2752 \frac{\text{slits}}{\text{cm}}.$

EVALUATE: A smaller number of slits would be needed to resolve these two lines in higher order.

36.34. IDENTIFY: The maxima are given by $2d \sin \theta = m\lambda$, $m = 1, 2, \dots$

SET UP: $d = 3.50 \times 10^{-10} \text{ m.}$

EXECUTE: (a) Using $m = 1$ gives

$\lambda = \frac{2d \sin \theta}{m} = 2(3.50 \times 10^{-10} \text{ m}) \sin 22.0^\circ = 2.62 \times 10^{-10} \text{ m} = 0.262 \text{ nm} = 262 \text{ pm.}$ This is an x ray.

(b) $\sin \theta = m \left(\frac{\lambda}{2d} \right) = m \left(\frac{2.62 \times 10^{-10} \text{ m}}{2(3.50 \times 10^{-10} \text{ m})} \right) = 0.3743m.$ $m = 2: \theta = 48.5^\circ.$ The equation doesn't have any solutions for $m > 2.$

EVALUATE: In this problem $\lambda/d = 0.75.$

36.35. IDENTIFY and SET UP: The maxima occur at angles θ given by $2d \sin \theta = m\lambda$, where d is the spacing between adjacent atomic planes. Solve for $d.$

EXECUTE: Second order says $m = 2.$

$d = \frac{m\lambda}{2 \sin \theta} = \frac{2(0.0850 \times 10^{-9} \text{ m})}{2 \sin 21.5^\circ} = 2.32 \times 10^{-10} \text{ m} = 0.232 \text{ nm.}$

EVALUATE: Our result is similar to d calculated in Example 36.5.

36.36. IDENTIFY: The crystal behaves like a diffraction grating.

SET UP: The maxima are at angles θ given by $2d \sin \theta = m\lambda$, where $d = 0.440 \text{ nm.}$

EXECUTE: $m = 1. \lambda = \frac{2d \sin \theta}{1} = 2(0.440 \text{ nm}) \sin 39.4^\circ = 0.559 \text{ nm.}$

EVALUATE: The result is a reasonable x-ray wavelength.

36.37. IDENTIFY and SET UP: The angular size of the first dark ring is given by $\sin \theta_1 = 1.22\lambda/D.$ Calculate θ_1 , and then the diameter of the ring on the screen is $2(4.5 \text{ m}) \tan \theta_1.$

EXECUTE: $\sin \theta_1 = 1.22 \left(\frac{620 \times 10^{-9} \text{ m}}{7.4 \times 10^{-6} \text{ m}} \right) = 0.1022; \theta_1 = 0.1024 \text{ rad.}$

The radius of the Airy disk (central bright spot) is $r = (4.5 \text{ m}) \tan \theta_1 = 0.462 \text{ m.}$ The diameter is $2r = 0.92 \text{ m} = 92 \text{ cm.}$

EVALUATE: $\lambda/D = 0.084.$ For this small D the central diffraction maximum is broad.

36.38. IDENTIFY and SET UP: For the first dark ring, $\sin \theta_1 = 1.22 \frac{\lambda}{D}$ and for the second dark ring

$\sin \theta_2 = 2.23 \frac{\lambda}{D}$. If y is the distance from the center to the ring, we can use the small-angle approximation

because $x \ll y$, where x is the distance from the aperture to the screen. Therefore $\sin \theta \approx \tan \theta = y/x$. The difference in radii of the first two dark rings is 1.65 mm.

EXECUTE: The radius of the first dark ring is $y_1 = x \tan \theta_1 = x(1.22 \lambda/D)$. The radius of the second ring is $y_2 = x \tan \theta_2 = x(2.23 \lambda/D)$. Dividing these two equations gives $y_2/y_1 = 2.23/1.22 = 1.8279$, so $y_2 = 1.8279 y_1$. We are told that the difference in radii is 1.65 mm, so $y_2 - y_1 = 1.65$ mm. Substituting for y_2 gives $1.8279 y_1 - y_1 = 1.65$ mm, so $y_1 = 1.9931$ mm. Now find D using $y_1 = x(1.22 \lambda/D)$.

$1.9931 \times 10^{-3} \text{ m} = (1.20 \text{ m})(1.22)(490 \text{ nm})/D$, so $D = 3.60 \times 10^5 \text{ nm} = 3.60 \times 10^{-4} \text{ m} = 0.360 \text{ mm}$.

EVALUATE: This is about the diameter of a good-sized pinhole.

36.39. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: $\theta = \frac{W}{h}$, where $W = 28 \text{ km}$ and $h = 1200 \text{ km}$. θ is small, so $\sin \theta \approx \theta$.

EXECUTE: $D = \frac{1.22 \lambda}{\sin \theta} = 1.22 \lambda \frac{h}{W} = 1.22(0.036 \text{ m}) \frac{1.2 \times 10^6 \text{ m}}{2.8 \times 10^4 \text{ m}} = 1.88 \text{ m}$

EVALUATE: D must be significantly larger than the wavelength, so a much larger diameter is needed for microwaves than for visible wavelengths.

36.40. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: $\theta = (1/60)^\circ$

EXECUTE: $D = \frac{1.22 \lambda}{\sin \theta} = \frac{1.22(5.5 \times 10^{-7} \text{ m})}{\sin(1/60)^\circ} = 2.31 \times 10^{-3} \text{ m} = 2.3 \text{ mm}$

EVALUATE: The larger the diameter the smaller the angle that can be resolved.

36.41. IDENTIFY: The diameter D of the mirror determines the resolution.

SET UP: The resolving power is $\theta_{\text{res}} = 1.22 \frac{\lambda}{D}$.

EXECUTE: The same θ_{res} means that $\frac{\lambda_1}{D_1} = \frac{\lambda_2}{D_2}$. $D_2 = D_1 \frac{\lambda_2}{\lambda_1} = (8000 \times 10^3 \text{ m}) \left(\frac{550 \times 10^{-9} \text{ m}}{2.0 \times 10^{-2} \text{ m}} \right) = 220 \text{ m}$.

EVALUATE: The Hubble telescope has an aperture of 2.4 m, so this would have to be an *enormous* optical telescope!

36.42. IDENTIFY: Use Rayleigh's criterion to find the resolution.

SET UP: The resolving power is $\theta_{\text{res}} = 1.22 \frac{\lambda}{D}$. The diameter of Jupiter is $1.40 \times 10^8 \text{ m}$. The angular size

of an object is $\theta = \frac{y}{s}$. 1 light-year = $9.461 \times 10^{15} \text{ m}$.

EXECUTE: $\theta = \frac{y}{s} = \frac{1.40 \times 10^8 \text{ m}}{4.3(9.461 \times 10^{15} \text{ m})} = 3.44 \times 10^{-9} \text{ rad}$. Setting $\theta_{\text{res}} = \theta$ gives

$D = \frac{1.22 \lambda}{\theta} = 1.22 \left(\frac{550 \times 10^{-9} \text{ m}}{3.44 \times 10^{-9} \text{ rad}} \right) = 195 \text{ m}$.

EVALUATE: This is larger than any present-day ground-based telescopes, but putting very large telescopes into orbit may be a possibility in the future.

36.43. IDENTIFY and SET UP: Let y be the separation between the two points being resolved and let s be their

distance from the telescope. Then the limit of resolution corresponds to $1.22 \frac{\lambda}{D} = \frac{y}{s}$.

EXECUTE: (a) Let the two points being resolved be the opposite edges of the crater, so y is the diameter of the crater. For the moon, $s = 3.8 \times 10^8$ m. $y = 1.22\lambda s/D$.

Hubble: $D = 2.4$ m and $\lambda = 380$ nm gives the maximum resolution, so $y = 73$ m.

Arecibo: $D = 305$ m and $\lambda = 0.75$ m; $y = 1.1 \times 10^6$ m.

(b) $s = \frac{yD}{1.22\lambda}$. Let $y \approx 0.30$ m (the size of a license plate).

$$s = (0.30 \text{ m})(2.4 \text{ m}) / [(1.22)(380 \times 10^{-9} \text{ m})] = 1600 \text{ km}.$$

EVALUATE: D/λ is much larger for the optical telescope and it has a much larger resolution even though the diameter of the radio telescope is much larger.

36.44. IDENTIFY: Rayleigh's criterion limits the angular resolution.

SET UP: Rayleigh's criterion is $\sin \theta \approx \theta = 1.22\lambda/D$. Call $R = 11.5$ m = the distance from the bear to the lens.

EXECUTE: (a) Using Rayleigh's criterion

$$\sin \theta \approx \theta = 1.22\lambda/D = (1.22)(550 \text{ nm})/(135/4 \text{ mm}) = 1.99 \times 10^{-5} \text{ rad}.$$

On the bear this angle subtends a distance x . Using $\theta = x/R$ gives

$$x = R\theta = (11.5 \text{ m})(1.99 \times 10^{-5} \text{ rad}) = 2.29 \times 10^{-4} \text{ m} = 0.23 \text{ mm}.$$

(b) At $f/22$, D is $4/22$ times as large as at $f/4$. Since θ is proportional to $1/D$, and x is proportional to θ , x is $1/(4/22) = 22/4$ times as large as it was at $f/4$. $x = (0.229 \text{ mm})(22/4) = 1.3 \text{ mm}$.

EVALUATE: A wide-angle lens, such as one having a focal length of 28 mm, would have a much smaller opening at $f/2$ and hence would have an even less resolving ability.

36.45. IDENTIFY and SET UP: Resolved by Rayleigh's criterion means angular separation θ of the objects equals $1.22\lambda/D$. The angular separation θ of the objects is their linear separation divided by their distance from the telescope.

EXECUTE: $\theta = \frac{250 \times 10^3 \text{ m}}{5.93 \times 10^{11} \text{ m}}$, where 5.93×10^{11} m is the distance from the earth to Jupiter. Thus

$$\theta = 4.216 \times 10^{-7} \text{ rad}.$$

$$\text{Then } \theta = 1.22 \frac{\lambda}{D} \text{ and } D = \frac{1.22\lambda}{\theta} = \frac{1.22(500 \times 10^{-9} \text{ m})}{4.216 \times 10^{-7} \text{ rad}} = 1.45 \text{ m}.$$

EVALUATE: This is a very large telescope mirror. The greater the angular resolution the greater the diameter the lens or mirror must be.

36.46. IDENTIFY: Single-slit diffraction occurs and creates a pattern on the screen.

SET UP: The first maximum occurs when $a \sin \theta_1 = \lambda$. The distance between the first dark fringes on either side of the central maximum is equal to twice the distance of the screen from the slit. Call the distance from the slit to the screen x .

EXECUTE: $\tan \theta_1 = x/x = 1$, so $\theta_1 = 45^\circ$. For the first dark fringe, we have $a \sin \theta_1 = \lambda$, so

$$a/\lambda = 1/\sin \theta_1 = 1/\sin(45^\circ) = \sqrt{2} = 1.414.$$

EVALUATE: We certainly cannot use the small-angle approximation in this case!

36.47. IDENTIFY: We can apply the equation for single-slit diffraction to the hair, with the thickness of the hair replacing the thickness of the slit.

SET UP: The dark fringes are located by $\sin \theta = m \frac{\lambda}{a}$. The first dark fringes are for $m = \pm 1$. $y = x \tan \theta$ is

the distance from the center of the screen, where $x = 125$ cm is the distance from the hair to the screen. The distance y from the center to one minimum is 2.61 cm.

$$\text{EXECUTE: } \tan \theta = \frac{y}{x} = \frac{2.61 \text{ cm}}{125 \text{ cm}} = 0.02088 \text{ so } \theta = 1.20^\circ. \quad a = \frac{\lambda}{\sin \theta} = \frac{632.8 \times 10^{-9} \text{ m}}{\sin 1.20^\circ} = 30.2 \mu\text{m}.$$

EVALUATE: Although the thickness of human hairs can vary considerably, $30 \mu\text{m}$ is a reasonable thickness.

- 36.48. IDENTIFY:** The two holes behave like double slits and cause the sound waves to interfere after they pass through the holes. The motion of the speakers causes a Doppler shift in the wavelength of the sound.

SET UP: The wavelength of the sound that strikes the wall is $\lambda = \lambda_0 - v_s T_s$, and destructive interference first occurs where $\sin \theta = \lambda/2$.

EXECUTE: (a) First find the wavelength of the sound that strikes the openings in the wall.

$\lambda = \lambda_0 - v_s T_s = v/f_s - v_s/f_s = (v - v_s)/f_s = (344 \text{ m/s} - 80.0 \text{ m/s})/(960 \text{ Hz}) = 0.275 \text{ m}$. Destructive interference first occurs where $d \sin \theta = \lambda/2$, which gives $d = \lambda/(2 \sin \theta) = (0.275 \text{ m})/(2 \sin 11.4^\circ) = 0.636 \text{ m}$.

(b) $\lambda = v/f = (344 \text{ m/s})/(960 \text{ Hz}) = 0.3583 \text{ m}$.

$\sin \theta = \lambda/2d = (0.3583 \text{ m})/[2(0.696 \text{ m})] \rightarrow \theta = \pm 14.9^\circ$.

EVALUATE: The moving source produces sound of shorter wavelength than the stationary source, so the angles at which destructive interference occurs are smaller for the moving source than for the stationary source.

- 36.49. IDENTIFY:** In the single-slit diffraction pattern, the intensity is a maximum at the center and zero at the dark spots. At other points, it depends on the angle at which one is observing the light.

SET UP: Dark fringes occur when $\sin \theta_m = m\lambda/a$, where $m = 1, 2, 3, \dots$, and the intensity is given by

$$I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2, \text{ where } \beta/2 = \frac{\pi a \sin \theta}{\lambda}.$$

EXECUTE: (a) At the maximum possible angle, $\theta = 90^\circ$, so

$$m_{\max} = (a \sin 90^\circ)/\lambda = (0.0250 \text{ mm})/(632.8 \text{ nm}) = 39.5.$$

Since m must be an integer and $\sin \theta$ must be ≤ 1 , $m_{\max} = 39$. The total number of dark fringes is 39 on each side of the central maximum for a total of 78.

(b) The farthest dark fringe is for $m = 39$, giving

$$\sin \theta_{39} = (39)(632.8 \text{ nm})/(0.0250 \text{ mm}) \Rightarrow \theta_{39} = \pm 80.8^\circ.$$

(c) The next closer dark fringe occurs at $\sin \theta_{38} = (38)(632.8 \text{ nm})/(0.0250 \text{ mm}) \Rightarrow \theta_{38} = 74.1^\circ$.

The angle midway these two extreme fringes is $(80.8^\circ + 74.1^\circ)/2 = 77.45^\circ$, and the intensity at this angle is

$$I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2, \text{ where } \beta/2 = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi(0.0250 \text{ mm}) \sin(77.45^\circ)}{632.8 \text{ nm}} = 121.15 \text{ rad, which gives}$$

$$I = (8.50 \text{ W/m}^2) \left[\frac{\sin(121.15 \text{ rad})}{121.15 \text{ rad}} \right]^2 = 5.55 \times 10^{-4} \text{ W/m}^2.$$

EVALUATE: At the angle in part (c), the intensity is so low that the light would be barely perceptible.

- 36.50. IDENTIFY:** $d = \frac{1}{N}$, so the bright fringes are located by $\frac{1}{N} \sin \theta = \lambda$.

SET UP: Red: $\frac{1}{N} \sin \theta_R = 750 \text{ nm}$. Violet: $\frac{1}{N} \sin \theta_V = 380 \text{ nm}$.

EXECUTE: (a) $\frac{\sin \theta_R}{\sin \theta_V} = \frac{75}{38}$. $\theta_R - \theta_V = 27.0^\circ$, so $\theta_R = \theta_V + 27.0^\circ$. $\frac{\sin(\theta_V + 27.0^\circ)}{\sin \theta_V} = \frac{75}{38}$. Using the

trigonometry identity from Appendix B for the sin of the sum of two angles gives

$$\frac{\sin \theta_V \cos 27.0^\circ + \cos \theta_V \sin 27.0^\circ}{\sin \theta_V} = \frac{75}{38} \rightarrow \cos 27.0^\circ + \cot \theta_V \sin 27.0^\circ = 75/38.$$

$$\tan \theta_V = 0.4193 \Rightarrow \theta_V = 22.75^\circ \text{ and } \theta_R = \theta_V + 27.0^\circ = 22.75^\circ + 27.0^\circ = 49.75^\circ.$$

Using $\frac{1}{N} \sin \theta_R = 750 \text{ nm}$ gives

$$N = \frac{\sin \theta_R}{750 \text{ nm}} = \frac{\sin 49.75^\circ}{750 \times 10^{-9} \text{ m}} = 1.02 \times 10^6 \text{ lines/m} = 1.02 \times 10^4 \text{ lines/cm}.$$

(b) The spectrum begins at 22.7° and ends at 49.7° .

EVALUATE: As N is increased, the angular range of the visible spectrum increases.

36.51. IDENTIFY and SET UP: $\sin \theta = \lambda/a$ locates the first dark band. In the liquid the wavelength changes and this changes the angular position of the first diffraction minimum.

EXECUTE: $\sin \theta_{\text{air}} = \frac{\lambda_{\text{air}}}{a}$; $\sin \theta_{\text{liquid}} = \frac{\lambda_{\text{liquid}}}{a}$. $\lambda_{\text{liquid}} = \lambda_{\text{air}} \left(\frac{\sin \theta_{\text{liquid}}}{\sin \theta_{\text{air}}} \right) = \lambda_{\text{air}} \frac{\sin 21.6^\circ}{\sin 38.2^\circ} = 0.5953 \lambda_{\text{air}}$.

$$\lambda_{\text{liquid}} = \lambda_{\text{air}}/n, \text{ so } n = \lambda_{\text{air}}/\lambda_{\text{liquid}} = \frac{\lambda_{\text{air}}}{0.5953 \lambda_{\text{air}}} = 1.68.$$

EVALUATE: Light travels faster in air and n must be > 1.00 . The smaller λ in the liquid reduces θ that located the first dark band.

36.52. IDENTIFY: The wavelength of the light is smaller under water than it is in air, which will affect the resolving power of the lens, by Rayleigh's criterion.

SET UP: The wavelength under water is $\lambda = \lambda_0/n$, and for small angles Rayleigh's criterion is $\theta = 1.22\lambda/D$.

EXECUTE: (a) In air the wavelength is $\lambda_0 = c/f = (3.00 \times 10^8 \text{ m/s})/(6.00 \times 10^{14} \text{ Hz}) = 5.00 \times 10^{-7} \text{ m}$. In water the wavelength is $\lambda = \lambda_0/n = (5.00 \times 10^{-7} \text{ m})/1.33 = 3.76 \times 10^{-7} \text{ m}$. With the lens open all the way, we have $D = f/2.8 = (35.0 \text{ mm})/2.80 = (0.0350 \text{ m})/2.80$. In the water, we have

$$\sin \theta \approx \theta = 1.22\lambda/D = (1.22)(3.76 \times 10^{-7} \text{ m})/[(0.0350 \text{ m})/2.80] = 3.67 \times 10^{-5} \text{ rad}.$$

Calling w the width of the resolvable detail, we have

$$\theta = w/x \rightarrow w = x\theta = (2750 \text{ mm})(3.67 \times 10^{-5} \text{ rad}) = 0.101 \text{ mm}.$$

(b) $\theta = 1.22\lambda/D = (1.22)(5.00 \times 10^{-7} \text{ m})/[(0.0350 \text{ m})/2.80] = 4.88 \times 10^{-5} \text{ rad}.$

$$w = x\theta = (2750 \text{ mm})(4.88 \times 10^{-5} \text{ rad}) = 0.134 \text{ mm}.$$

EVALUATE: Due to the reduced wavelength underwater, the resolution of the lens is better under water than in air.

36.53. IDENTIFY: $I = I_0 \left(\frac{\sin \gamma}{\gamma} \right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

SET UP: $\frac{d \sin \gamma}{d \gamma} = \cos \gamma$. $\frac{d}{d \gamma} \left(\frac{1}{\gamma} \right) = -\frac{1}{\gamma^2}$.

EXECUTE: (a) $\frac{dI}{d\gamma} = I_0 \frac{d}{d\gamma} \left(\frac{\sin \gamma}{\gamma} \right)^2 = 2 \left(\frac{\sin \gamma}{\gamma} \right) \left(\frac{\cos \gamma}{\gamma} - \frac{\sin \gamma}{\gamma^2} \right) = 0$. $\frac{\cos \gamma}{\gamma} - \frac{\sin \gamma}{\gamma^2} \Rightarrow \gamma \cos \gamma = \sin \gamma \Rightarrow \gamma = \tan \gamma$.

(b) The graph in Figure 36.53 is a plot of $f(\gamma) = \gamma - \tan \gamma$. When $f(\gamma)$ equals zero, there is an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the two smallest values of γ are $\gamma = 4.49 \text{ rad}$ and 7.73 rad .

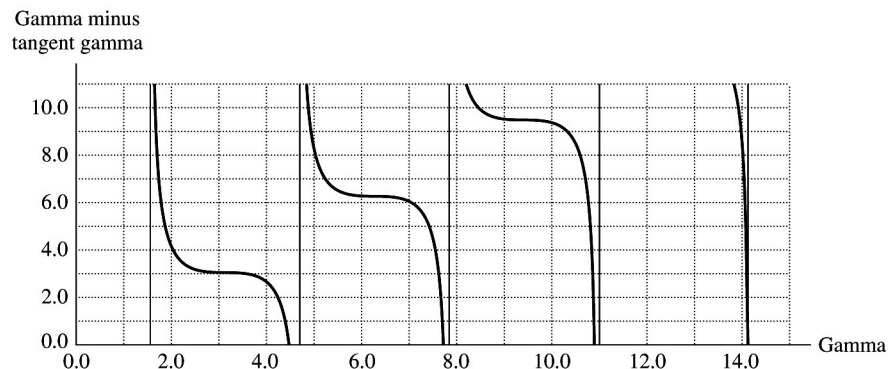


Figure 36.53

(c) The minima occur when $\gamma = \pm m\pi$, where $m = 1, 2, 3, \dots$, so the first three positive ones are $\gamma = \pi, 2\pi$, and 3π . From part (b), the first maximum is at $\gamma = 4.49$ rad. Midway between the first and second minima is $\gamma = \pi + \pi/2 = 3\pi/2 = 4.71$ rad. Clearly $4.49 \text{ rad} \neq 4.71 \text{ rad}$, so this maximum is not midway between the adjacent minima. The second maximum is at $\gamma = 7.73$ rad. Midway between the second and third minima is $\gamma = 2\pi + \pi/2 = 5\pi/2 = 7.85$ rad. Since $7.73 \neq 7.85$, the second maximum is not midway between the adjacent minima.

(d) Using $a = 12\lambda$, we have $\gamma = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi(12\lambda) \sin \theta}{\lambda} = 12\pi \sin \theta$. Using this we get the following:

First minimum: $\gamma = \pi$, so $\pi = 12\pi \sin \theta \rightarrow \theta = 4.78^\circ$.

First maximum: $\gamma = 2\pi$, so $2\pi = 12\pi \sin \theta \rightarrow \theta = 6.84^\circ$.

Second minimum: $\gamma = 3\pi$, so $3\pi = 12\pi \sin \theta \rightarrow \theta = 9.59^\circ$.

Midway between these minima is $\theta = (9.59^\circ + 4.78^\circ)/2 = 7.19^\circ$. The first maximum is at $\theta = 6.84^\circ$, and since $6.84^\circ \neq 7.19^\circ$, the first maximum is *not* midway between the adjacent minima.

EVALUATE: $\gamma = 0$ is the central maximum. The three values of γ we found are the locations of the first two secondary maxima. The first three minima are at $\gamma = 3.14$ rad, 6.28 rad, and 9.42 rad. The maxima are between adjacent minima, but not precisely midway between them.

- 36.54. (a) IDENTIFY and SET UP:** The angular position of the first minimum is given by $a \sin \theta = m\lambda$, with $m = 1$. The distance of the minimum from the center of the pattern is given by $y = x \tan \theta$.

EXECUTE: $\sin \theta = \frac{\lambda}{a} = \frac{540 \times 10^{-9} \text{ m}}{0.360 \times 10^{-3} \text{ m}} = 1.50 \times 10^{-3}$; $\theta = 1.50 \times 10^{-3} \text{ rad}$.

$y_1 = x \tan \theta = (1.20 \text{ m}) \tan(1.50 \times 10^{-3} \text{ rad}) = 1.80 \times 10^{-3} \text{ m} = 1.80 \text{ mm}$.

(Note that θ is small enough for $\theta \approx \sin \theta \approx \tan \theta$, and $y_m = x \frac{m\lambda}{a}$ applies.)

(b) **IDENTIFY and SET UP:** Find the phase angle β where $I = I_0/2$. Then use $\beta = \left(\frac{2\pi}{\lambda}\right)a \sin \theta$ to solve for θ and $y = x \tan \theta$ to find the distance.

EXECUTE: $I = I_0 \left(\frac{\sin \beta/2}{\beta/2}\right)^2$ gives that $I = \frac{1}{2} I_0$ when $\beta = 2.78 \text{ rad}$.

$\beta = \left(\frac{2\pi}{\lambda}\right)a \sin \theta$, so $\sin \theta = \frac{\beta\lambda}{2\pi a}$.

$y = x \tan \theta \approx x \sin \theta \approx \frac{\beta\lambda x}{2\pi a} = \frac{(2.78 \text{ rad})(540 \times 10^{-9} \text{ m})(1.20 \text{ m})}{2\pi(0.360 \times 10^{-3} \text{ m})} = 7.96 \times 10^{-4} \text{ m} = 0.796 \text{ mm}$.

EVALUATE: The point where $I = I_0/2$ is not midway between the center of the central maximum and the first minimum; see Exercise 36.15.

- 36.55. IDENTIFY:** Heating the plate causes it to expand, which widens the slit. The increased slit width changes the angles at which destructive interference occurs.

SET UP: First minimum is at angle θ given by $\tan \theta = \frac{(2.75 \times 10^{-3}/2)}{0.620}$. Therefore, θ is small and the

equation $y_m = x \frac{m\lambda}{a}$ is accurate. The width of the central maximum is $w = \frac{2x\lambda}{a}$. The change in slit width is $\Delta a = a\alpha\Delta T$.

EXECUTE: $dw = 2x\lambda \left(-\frac{da}{a^2}\right) = -\frac{2x\lambda}{a^2} da = -\frac{w}{a} da$. Therefore, $\Delta w = -\frac{w}{a} \Delta a$. The equation for thermal

expansion says $\Delta a = a\alpha\Delta T$, so $\Delta w = -w\alpha\Delta T = -(2.75 \text{ mm})(2.4 \times 10^{-5} \text{ K}^{-1})(500 \text{ K}) = -0.033 \text{ mm}$. When the temperature of the plate increases, the width of the slit increases and the width of the central maximum decreases.

EVALUATE: The fractional change in the width of the central maximum is $\frac{0.033 \text{ mm}}{2.75 \text{ mm}} = 1.2\%$. This is small, but observable.

- 36.56. IDENTIFY:** The wavelength of the helium spectral line from the receding galaxy will be different from the spectral line on earth due to the Doppler shift in the light from the galaxy.

SET UP: $d \sin \theta = m\lambda$. $\sin \theta_{\text{lab}} = \frac{2\lambda_{\text{lab}}}{d}$. $\sin \theta_{\text{galaxy}} = \frac{2\lambda_{\text{galaxy}}}{d}$. $\sin \theta_{\text{galaxy}} = \sin \theta_{\text{lab}} \left(\frac{\lambda_{\text{lab}}}{\lambda_{\text{galaxy}}} \right)$. The Doppler

formula says $f_R = \sqrt{\frac{c-v}{c+v}} f_S$. Using $f = \frac{c}{\lambda}$, we have $\frac{1}{\lambda_R} = \frac{1}{\lambda_S} \sqrt{\frac{c-v}{c+v}}$. Since the lab is the receiver R and

the galaxy is the source S, this becomes $\lambda_{\text{lab}} = \lambda_{\text{galaxy}} \sqrt{\frac{c+v}{c-v}}$.

EXECUTE: $\sin \theta_{\text{galaxy}} = \sin \theta_{\text{lab}} \sqrt{\frac{c+v}{c-v}} = \sin(18.9^\circ) \sqrt{\frac{2.998 \times 10^8 \text{ m/s} + 2.65 \times 10^7 \text{ m/s}}{2.998 \times 10^8 \text{ m/s} - 2.65 \times 10^7 \text{ m/s}}}$ which gives

$$\theta_{\text{galaxy}} = 20.7^\circ.$$

EVALUATE: The galaxy is moving away, so the wavelength of its light will be lengthened, which means that the angle should be increased compared to the angle from light on earth, as we have found.

- 36.57. IDENTIFY and SET UP:** The condition for an intensity maximum is $d \sin \theta = m\lambda$, $m = 0, \pm 1, \pm 2, \dots$. Third order means $m = 3$. The longest observable wavelength is the one that gives $\theta = 90^\circ$ and hence $\sin \theta = 1$.

EXECUTE: 9200 lines/cm so 9.2×10^5 lines/m and $d = \frac{1}{9.2 \times 10^5} \text{ m} = 1.087 \times 10^{-6} \text{ m}$.

$$\lambda = \frac{d \sin \theta}{m} = \frac{(1.087 \times 10^{-6} \text{ m})(1)}{3} = 3.6 \times 10^{-7} \text{ m} = 360 \text{ nm}.$$

EVALUATE: The longest wavelength that can be obtained decreases as the order increases.

- 36.58. IDENTIFY:** Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: θ is small, so $\sin \theta \approx \frac{\Delta x}{R}$, where Δx is the size of the details and R is the distance to the earth.

So $\frac{\Delta x}{R} \approx 1.22 \frac{\lambda}{D}$. $1 \text{ ly} = 9.41 \times 10^{15} \text{ m}$ and $10 \mu\text{m} = 1.0 \times 10^{-5} \text{ m}$.

EXECUTE: $\Delta x = 1.22 \frac{\lambda R}{D} = \frac{(1.22)(1.0 \times 10^{-5} \text{ m})(59 \text{ ly})(9.41 \times 10^{15} \text{ m/ly})}{6.0 \times 10^6 \text{ m}} = 1.1 \times 10^6 \text{ m} = 1100 \text{ km}.$

$$\frac{\Delta x}{D_{\text{planet}}} = \frac{1100 \text{ km}}{1.4 \times 10^5 \text{ km}} = 7.9 \times 10^{-3}. \text{ So } \Delta x \text{ is less than 1\% of the diameter of the planet.}$$

EVALUATE: The very large diameter of the array allows it to resolve planet-sized detail at great distances.

- 36.59. IDENTIFY:** The maxima are given by $d \sin \theta = m\lambda$. We need $\sin \theta = \frac{m\lambda}{d} \leq 1$ in order for all the visible wavelengths to be seen.

SET UP: For 650 slits/mm $\Rightarrow d = \frac{1}{6.50 \times 10^5 \text{ m}^{-1}} = 1.53 \times 10^{-6} \text{ m}$. The visible spectrum is approximately between 380 nm and 750 nm.

EXECUTE: $\lambda_1 = 3.8 \times 10^{-7} \text{ m}$: $m = 1$: $\frac{\lambda_1}{d} = 0.247$; $m = 2$: $\frac{2\lambda_1}{d} = 0.494$; $m = 3$: $\frac{3\lambda_1}{d} = 0.741$.

$\lambda_2 = 7.5 \times 10^{-7} \text{ m}$: $m = 1$: $\frac{\lambda_2}{d} = 0.4875$; $m = 2$: $\frac{2\lambda_2}{d} = 0.975$; $m = 3$: $\frac{3\lambda_2}{d} = 1.46$. So, the third order does not contain the violet end of the spectrum, and therefore only the first- and second-order diffraction patterns contain all colors of the spectrum.

EVALUATE: θ for each maximum is larger for longer wavelengths.

36.60. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: θ is small, so $\sin \theta \approx \frac{\Delta x}{R}$, where Δx is the size of the detail and $R = 7.2 \times 10^8$ ly.

1 ly = 9.41×10^{12} km. $\lambda = c/f$.

EXECUTE: $\sin \theta = 1.22 \frac{\lambda}{D} \approx \frac{\Delta x}{R} \Rightarrow \Delta x = \frac{1.22 \lambda R}{D} = \frac{(1.22)cR}{Df} = \frac{(1.22)(3.00 \times 10^5 \text{ km/s})(7.2 \times 10^8 \text{ ly})}{(77.000 \times 10^3 \text{ km})(1.665 \times 10^9 \text{ Hz})} = 2.06 \text{ ly}.$
 $(9.41 \times 10^{12} \text{ km/ly})(2.06 \text{ ly}) = 1.94 \times 10^{13} \text{ km}.$

EVALUATE: $\lambda = 18 \text{ cm}$. λ/D is very small, so $\frac{\Delta x}{R}$ is very small. Still, R is very large and Δx is many orders of magnitude larger than the diameter of the sun.

36.61. IDENTIFY: The liquid reduces the wavelength of the light (compared to its value in air), and the scratch causes light passing through it to undergo single-slit diffraction.

SET UP: $\sin \theta = \frac{\lambda}{a}$, where λ is the wavelength in the liquid. $n = \frac{\lambda_{\text{air}}}{\lambda}$.

EXECUTE: $\tan \theta = \frac{(22.4/2) \text{ cm}}{30.0 \text{ cm}}$ and $\theta = 20.47^\circ$.

$\lambda = a \sin \theta = (1.25 \times 10^{-6} \text{ m}) \sin 20.47^\circ = 4.372 \times 10^{-7} \text{ m} = 437.2 \text{ nm}.$ $n = \frac{\lambda_{\text{air}}}{\lambda} = \frac{612 \text{ nm}}{437.2 \text{ nm}} = 1.40.$

EVALUATE: $n > 1$, as it must be, and $n = 1.40$ is reasonable for many transparent films.

36.62. IDENTIFY: The resolution of the eye is limited because light diffracts as it passes through the pupil. The size of the pupil determines the resolution.

SET UP: The smallest angular separation that can be resolved is $\theta_{\text{res}} = 1.22 \frac{\lambda}{D}$. The angular size of the object is its height divided by its distance from the eye.

EXECUTE: (a) The angular size of the object is $\theta = \frac{50 \times 10^{-6} \text{ m}}{25 \times 10^{-2} \text{ m}} = 2.0 \times 10^{-4} \text{ rad}.$

$\theta_{\text{res}} = 1.22 \frac{\lambda}{D} = 1.22 \left(\frac{550 \times 10^{-9} \text{ m}}{2.0 \times 10^{-3} \text{ m}} \right) = 3.4 \times 10^{-4} \text{ rad}.$ $\theta < \theta_{\text{res}}$ so the object cannot be resolved.

(b) $\theta_{\text{res}} = \frac{y}{s}$ and $y = s \theta_{\text{res}} = (25 \text{ cm})(3.4 \times 10^{-4} \text{ rad}) = 8.5 \times 10^{-3} \text{ cm} = 85 \mu\text{m}.$

(c) $\theta = \theta_{\text{res}} = 3.4 \times 10^{-4} \text{ rad} = 0.019^\circ = 1.1 \text{ min}.$ This is very close to the experimental value of 1 min.

(d) Diffraction is more important.

EVALUATE: We could not see any clearer if our retinal cells were much smaller than they are now because diffraction is more important in limiting the resolution of our vision.

36.63. IDENTIFY: A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.

SET UP: Single-slit diffraction dark fringes occur when $a \sin \theta = m\lambda$, and double-slit interference bright fringes occur when $d \sin \theta = m'\lambda$.

EXECUTE: (a) The angle at which the first bright fringe occurs is given by $\tan \theta_1 = (1.53 \text{ mm})/(2500 \text{ mm}) \Rightarrow \theta_1 = 0.03507^\circ.$ $d \sin \theta_1 = \lambda$ and

$d = \lambda/(\sin \theta_1) = (632.8 \text{ nm})/\sin(0.03507^\circ) = 0.00103 \text{ m} = 1.03 \text{ mm}.$

(b) The 7th double-slit interference bright fringe is just cancelled by the 1st diffraction dark fringe, so $\sin \theta_{\text{diff}} = \lambda/a$ and $\sin \theta_{\text{interf}} = 7\lambda/d$.

The angles are equal, so $\lambda/a = 7\lambda/d \rightarrow a = d/7 = (1.03 \text{ mm})/7 = 0.148 \text{ mm}.$

EVALUATE: We can generalize that if $d = na$, where n is a positive integer, then every n^{th} double-slit bright fringe will be missing in the pattern.

36.64. IDENTIFY: Light from the slit produces a diffraction pattern on the screen.

SET UP: The first dark fringe occurs when $a \sin \theta_1 = \lambda$, and $\tan \theta_1 = (w/2)/R = w/2R$, where R is the distance from the slit to the screen.

EXECUTE: (a) If θ_1 is small, we can use the small-angle approximation $\sin \theta_1 \approx \tan \theta_1 = w/2R$. In that case, $aw = 2R\lambda$, which is constant. But if θ_1 is large, we cannot use $\sin \theta_1 \approx \tan \theta_1$, and wa is not constant.

Figure 36.64 shows the graph of aw versus a for the data in the problem, and its curvature for small a shows that wa is not constant for small values of a .

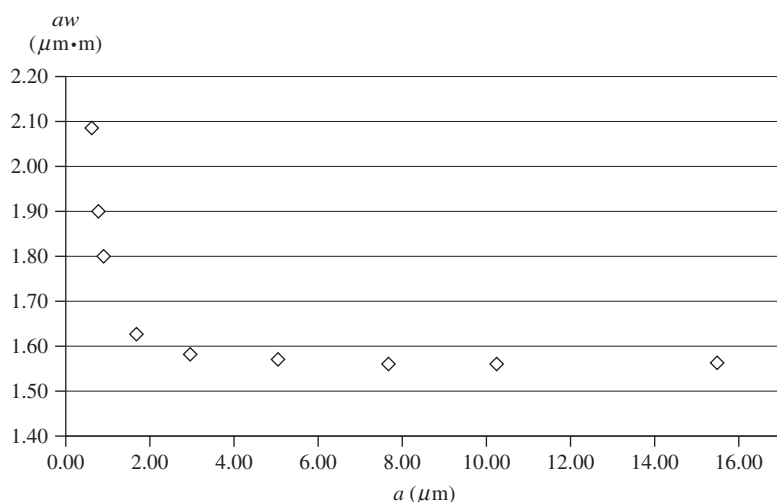


Figure 36.64

(b) We see that our graph flattens out for large values of a and $wa \rightarrow 1.56 \times 10^{-6} \text{ m}^2$. This means we can apply the small-angle approximation. In that case, $aw = 2R\lambda$, so $\lambda = aw/2R = (1.56 \times 10^{-6} \text{ m}^2)/[2(1.50 \text{ m})] = 5.20 \times 10^{-7} \text{ m} = 520 \text{ nm}$.

(c) (i) Use $a \sin \theta_1 = \lambda$ with $a = 0.78 \mu\text{m} = 780 \text{ nm}$.

$$(780 \text{ nm}) \sin \theta_1 = 520 \text{ nm} \rightarrow \theta_1 = 42^\circ.$$

(ii) Now use $a = 15.60 \mu\text{m} = 15,600 \text{ nm}$.

$$(15,600 \text{ nm}) \sin \theta_1 = 520 \text{ nm} \rightarrow \theta_1 = 1.91^\circ.$$

EVALUATE: In part (c), we could use the small-angle approximation in (ii) but not in (i).

36.65. IDENTIFY: The hole produces a diffraction pattern on the screen.

SET UP: The first dark ring occurs when $D \sin \theta_1 = 1.22 \lambda$, and $\tan \theta_1 = r/x$.

EXECUTE: (a) Solving for D gives $D = (1.22 \lambda) / (\sin \theta_1)$. First use $\tan \theta_1 = r/x$ to find θ_1 , and then calculate D .

First set: $\tan \theta_1 = r/x = (5.6 \text{ cm})/(100 \text{ cm}) \rightarrow \theta_1 = 3.205^\circ$.

$$D = (1.22)(562 \text{ nm})/[\sin(3.205^\circ)] = 1.226 \times 10^4 \text{ nm}.$$

Second set: $\tan \theta_1 = r/x = (8.5 \text{ cm})/(150 \text{ cm}) \rightarrow \theta_1 = 3.2433^\circ$.

$$D = (1.22)(562 \text{ nm})/[\sin(3.2433^\circ)] = 1.212 \times 10^4 \text{ nm}.$$

Third set: $\tan \theta_1 = r/x = (11.6 \text{ cm})/(200 \text{ cm}) \rightarrow \theta_1 = 3.3194^\circ$.

$$D = (1.22)(562 \text{ nm})/[\sin(3.3194^\circ)] = 1.184 \times 10^4 \text{ nm}.$$

Fourth set: $\tan \theta_1 = r/x = (14.1 \text{ cm})/(250 \text{ cm}) \rightarrow \theta_1 = 3.2281^\circ$.

$$D = (1.22)(562 \text{ nm})/[\sin(3.2281^\circ)] = 1.218 \times 10^4 \text{ nm}.$$

Taking the average gives

$$D_{\text{av}} = [(1.226 + 1.212 + 1.184 + 1.218) \times 10^4 \text{ nm}]/4 = 1.21 \times 10^4 \text{ nm} = 12.1 \mu\text{m}.$$

$$(b) \sin \theta_2 = 2.23 \lambda/D = (2.23)(562 \text{ nm})/(1.21 \times 10^4 \text{ nm}) \rightarrow \theta_2 = 5.945^\circ.$$

$$r_2 = x_2 \tan \theta_2 = (1.00 \text{ m}) \tan(5.945^\circ) = 0.104 \text{ m} = 10.4 \text{ cm}.$$

$$\sin \theta_3 = 3.24 \lambda/D = (3.24)(562 \text{ nm})/(1.21 \times 10^4 \text{ nm}) \rightarrow \theta_3 = 8.6551^\circ.$$

$$r_3 = x_3 \tan \theta_3 = (1.00 \text{ m}) \tan(8.6551^\circ) = 0.152 \text{ m} = 15.2 \text{ cm}.$$

EVALUATE: This is a very small hole, but its diameter is still over 20 times the wavelength of the light passing through it.

36.66. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) Each source can be thought of as a traveling wave evaluated at $x = R$ with a maximum amplitude of E_0 . However, each successive source will pick up an extra phase from its respective

pathlength to point P . $\phi = 2\pi \left(\frac{d \sin \theta}{\lambda} \right)$ which is just 2π , the maximum phase, scaled by whatever fraction

the path difference, $d \sin \theta$, is of the wavelength, λ . By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

(b) $e^{i(kR - \omega t + n\phi)} = \cos(kR - \omega t + n\phi) + i \sin(kR - \omega t + n\phi)$. The real part is just $\cos(kR - \omega t + n\phi)$. So,

$$\text{Re} \left[\sum_{n=0}^{N-1} E_0 e^{i(kR - \omega t + n\phi)} \right] = \sum_{n=0}^{N-1} E_0 \cos(kR - \omega t + n\phi). \text{ (Note: Re means "the real part of..."). But this is just}$$

$$E_0 \cos(kR - \omega t) + E_0 \cos(kR - \omega t + \phi) + E_0 \cos(kR - \omega t + 2\phi) + \cdots + E_0 \cos(kR - \omega t + (N-1)\phi).$$

$$(c) \sum_{n=0}^{N-1} E_0 e^{i(kR - \omega t + n\phi)} = E_0 \sum_{n=0}^{N-1} e^{-i\omega t} e^{+ikR} e^{in\phi} = E_0 e^{i(kR - \omega t)} \sum_{n=0}^{N-1} e^{in\phi}. \quad \sum_{n=0}^{\infty} e^{in\phi} = \sum_{n=0}^{N-1} (e^{i\phi})^n. \text{ But recall}$$

$$\sum_{n=0}^{N-1} x^n = \frac{x^N - 1}{x - 1}. \text{ Putting everything together:}$$

$$\sum_{n=0}^{N-1} E_0 e^{i(kR - \omega t + n\phi)} = E_0 e^{i(kR - \omega t + (N-1)\phi/2)} \frac{(e^{iN\phi/2} - e^{-iN\phi/2})}{(e^{i\phi/2} - e^{-i\phi/2})}$$

$$= E_0 [\cos(kR - \omega t + (N-1)\phi/2) + i \sin(kR - \omega t + (N-1)\phi/2)] \left[\frac{\cos N\phi/2 + \sin N\phi/2 - \cos N\phi/2 + i \sin N\phi/2}{\cos \phi/2 + i \sin \phi/2 - \cos \phi/2 + i \sin \phi/2} \right].$$

$$\text{Taking only the real part gives } \Rightarrow E_0 \cos(kR - \omega t + (N-1)\phi/2) \frac{\sin(N\phi/2)}{\sin \phi/2} = E.$$

$$(d) I = |E|_{\text{av}}^2 = I_0 \frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}. \text{ (The } \cos^2 \text{ term goes to } \frac{1}{2} \text{ in the time average and is included in the}$$

$$\text{definition of } I_0.) \quad I_0 \propto \frac{E_0^2}{2}.$$

$$\text{EVALUATE: (e) } N = 2. \quad I = I_0 \frac{\sin^2(2\phi/2)}{\sin^2 \phi/2} = \frac{I_0 (2 \sin \phi/2 \cos \phi/2)^2}{\sin^2 \phi/2} = 4 I_0 \cos^2 \frac{\phi}{2}. \text{ Looking at Eq. (35.9),}$$

$$I'_0 \propto 2 E_0^2 \text{ but for us } I_0 \propto \frac{E_0^2}{2} = \frac{I'_0}{4}.$$

36.67. IDENTIFY and SET UP: From Problem 36.66, $I = I_0 \frac{\sin^2(N\phi/2)}{\sin^2 \phi/2}$. Use this result to obtain each result specified in the problem.

$$\text{EXECUTE: (a) } \lim_{\phi \rightarrow 0} I \rightarrow \frac{0}{0}. \text{ Use l'Hôpital's rule: } \lim_{\phi \rightarrow 0} \frac{\sin(N\phi/2)}{\sin \phi/2} = \lim_{\phi \rightarrow 0} \left(\frac{N/2}{1/2} \right) \frac{\cos(N\phi/2)}{\cos(\phi/2)} = N. \text{ So}$$

$$\lim_{\phi \rightarrow 0} I = N^2 I_0.$$

(b) The location of the first minimum is when the numerator first goes to zero at $\frac{N}{2}\phi_{\min} = \pi$ or $\phi_{\min} = \frac{2\pi}{N}$.

The width of the central maximum goes like $2\phi_{\min}$, so it is proportional to $\frac{1}{N}$.

(c) Whenever $\frac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity.

That is, I is a minimum wherever $\phi = \frac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero

as well, which occurs when $\frac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from

part(a), there is a maximum. That is, if $\frac{n}{N}$ is an integer, there will be a maximum.

(d) From part (c), if $\frac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where

$\frac{n}{N}$ is not an integer for fixed N and integer n .) For example, $n=0$ will be a maximum, but $n=1, 2, \dots, N-1$ will be minima with another maximum at $n=N$.

(e) Between maxima $\frac{\phi}{2}$ is a half-integer multiple of π (i.e., $\frac{\pi}{2}, \frac{3\pi}{2}$ etc.) and if N is odd then

$$\frac{\sin^2(N\phi/2)}{\sin^2\phi/2} \rightarrow 1, \text{ so } I \rightarrow I_0.$$

EVALUATE: These results show that the principal maxima become sharper as the number of slits is increased.

36.68. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) From the segment dy' , the fraction of the amplitude of E_0 that gets through is

$$E_0 \left(\frac{dy'}{a} \right) \Rightarrow dE = E_0 \left(\frac{dy'}{a} \right) \sin(kx - \omega t).$$

(b) The path difference between each little piece is

$$y' \sin \theta \Rightarrow kx = k(D - y' \sin \theta) \Rightarrow dE = \frac{E_0 dy'}{a} \sin(k(D - y' \sin \theta) - \omega t). \text{ This can be rewritten as}$$

$$dE = \frac{E_0 dy'}{a} (\sin(kD - \omega t) \cos(ky' \sin \theta) + \sin(ky' \sin \theta) \cos(kD - \omega t)).$$

(c) So the total amplitude is given by the integral over the slit of the above.

$$\Rightarrow E = \int_{-a/2}^{a/2} dE = \frac{E_0}{a} \int_{-a/2}^{a/2} dy' (\sin(kD - \omega t) \cos(ky' \sin \theta) + \sin(ky' \sin \theta) \cos(kD - \omega t)).$$

But the second term integrates to zero, so we have:

$$\begin{aligned} E &= \frac{E_0}{a} \sin(kD - \omega t) \int_{-a/2}^{a/2} dy' (\cos(ky' \sin \theta)) = E_0 \sin(kD - \omega t) \left[\left(\frac{\sin(ky' \sin \theta)}{ka \sin \theta/2} \right) \right]_{-a/2}^{a/2} \\ &\Rightarrow E = E_0 \sin(kD - \omega t) \left(\frac{\sin(ka(\sin \theta)/2)}{ka(\sin \theta)/2} \right) = E_0 \sin(kD - \omega t) \left(\frac{\sin(\pi a(\sin \theta)/\lambda)}{\pi a(\sin \theta)/\lambda} \right). \end{aligned}$$

$$\text{At } \theta = 0, \frac{\sin[\dots]}{[\dots]} = 1 \Rightarrow E = E_0 \sin(kD - \omega t).$$

(d) Since $I \propto E^2 \Rightarrow I = I_0 \left(\frac{\sin(ka(\sin \theta)/2)}{ka(\sin \theta)/2} \right)^2 = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2$, where we have used

$$I_0 = E_0^2 \sin^2(kx - \omega t).$$

EVALUATE: The same result for $I(\theta)$ is obtained as was obtained using phasors.

36.69. IDENTIFY and SET UP: For an interference maxima, $2d \sin \theta = m\lambda_0/n$. The microspheres are in water, so we use $n=1.3$. Do a rough calculation to estimate the distance between microspheres.

EXECUTE: Use the numbers in the problem and $n=1.3$. Solving $2d \sin \theta = m\lambda_0/n$ for d gives

$$d = \frac{m\lambda_0}{2n \sin \theta} = \frac{(1)(650 \text{ nm})}{2(1.3) \sin 39^\circ} \approx 400 \text{ nm.}$$

This distance is much greater than the spacing of atoms in a crystal

and is comparable to the wavelength of visible light. Therefore choice (d) is correct.

EVALUATE: The atoms in a crystal are tightly bound by the electric force between them and are therefore very close together.

36.70. IDENTIFY and SET UP: For an interference maxima, $2d \sin \theta = m\lambda_0/n$. The microspheres are in water, so we use $n=1.33$.

EXECUTE: $d = \frac{m\lambda_0}{2n \sin \theta} = \frac{(1)(650 \text{ nm})}{2(1.33) \sin 39^\circ} \approx 390 \text{ nm.}$ Choice (a) is the correct one.

EVALUATE: We are assuming that the microspheres do not affect the index of refraction of the water.

36.71. IDENTIFY and SET UP: For an interference maxima, $2d \sin \theta = m\lambda_0/n$, so $\sin \theta \propto \frac{1}{d}$. If d is smaller, $\sin \theta$, and hence θ , is larger.

EXECUTE: At the top $\theta_{\text{top}} = 37^\circ$, and at the bottom $\theta_{\text{bottom}} = 41^\circ$. Since $\theta_{\text{top}} < \theta_{\text{bottom}}$, it follows that $d_{\text{top}} > d_{\text{bottom}}$. Therefore the microspheres are closer together at the bottom than at the top, which makes choice (a) the correct one.

EVALUATE: Gravity tends to pull the spheres downward so they bunch up at the bottom, making them closer together.