

“Experimental Data Processing”

Laboratory work 7

Analysis of accuracy decrease of filtration in conditions of correlated biased state and measurement noise

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Moving object which trajectory is disturbed by random acceleration

Motion
model

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}T$$

State equation

Motion
model

$$\begin{aligned}x_i &= x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2} \\V_i &= V_{i-1} + a_{i-1}T\end{aligned}$$

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

State
vector

It contains full
information about the
state of system at time i

State
equation

$$X_i = \Phi X_{i-1} + Ga_{i-1}$$

$$\Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

Transition
matrix

$$G = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}$$

Input
matrix

Measurement equation

Measurements
of coordinate x_i
with error η_i

$$z_i = x_i + \eta_i$$

Measurement
equation

$$z_i = HX_i + \eta_i$$

$$X_i = \begin{bmatrix} x_i \\ v_i \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Observation
matrix

Prediction procedure in Kalman filter

① Prediction (extrapolation)

Prediction of state vector at time i using $i - 1$ measurements

$$X_{i,i-1} = \Phi_{i,i-1} X_{i-1,i-1}$$

Prediction error covariance matrix

$$P_{i,i-1} = \Phi_{i,i-1} P_{i-1,i-1} \Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T]$$

$X_{i,i-1}$

First subscript i
denotes time on which
the prediction is made

Second subscript $i - 1$
represents the number of
measurements to get $X_{i,i-1}$

Filtration procedure in Kalman filter

② Filtration

Adjustment of predicted estimate

Improved estimate by incorporating a new measurement

$$X_{i,i} = X_{i,i-1} + K_i(z_i - HX_{i,i-1})$$

Residual

Filter gain, weight of residual

$$K_i = P_{i,i-1}H_i^T(H_iP_{i,i-1}H_i^T + R_i)^{-1}$$

Filtration error covariance matrix

$$P_{i,i} = (I - K_iH_i)P_{i,i-1}$$

$$P_{i,i} = E[(X_i - X_{i,i})(X_i - X_{i,i})^T]$$

**Standard Kalman filter
provides optimal estimate**



**State noise and measurement noise
are uncorrelated and unbiased**



**In practice these assumptions
are often not true**

**Analysis and modifications
of Kalman filter**

Biased state noise



**How to take bias of acceleration
into account in Kalman filter
algorithm?**

Prediction procedure in Kalman filter

① Prediction (extrapolation)

Prediction of state vector at time i using $i - 1$ measurements

$$X_{i,i-1} = \Phi_{i,i-1} X_{i-1,i-1}$$

Prediction error covariance matrix

$$P_{i,i-1} = \Phi_{i,i-1} P_{i-1,i-1} \Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T]$$

$X_{i,i-1}$

First subscript i
denotes time on which
the prediction is made

Second subscript $i - 1$
represents the number of
measurements to get $X_{i,i-1}$

Prediction procedure in Kalman filter taking into account bias of state noise

① Prediction (extrapolation)

Prediction of state vector at time i using $i - 1$ measurements

$$X_{i,i-1} = \Phi_{i,i-1}X_{i-1,i-1} + Gq$$

Prediction error covariance matrix

$$P_{i,i-1} = \Phi_{i,i-1}P_{i-1,i-1}\Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T]$$

$X_{i,i-1}$

First subscript i
denotes time on which
the prediction is made

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Correlated state noise

In practice correlated noise is often presented as a Gauss-Markov first-order process

Random acceleration

$$a_i = e^{-\lambda T} a_{i-1} + \zeta_i$$

ζ_i

Uncorrelated noise with variance

$$\sigma_{\zeta}^2 = \sigma_a^2 (1 - e^{-2\lambda T})$$

λ

Value that is inverse to correlation interval

$$\lambda = 1000$$

a_i - uncorrelated noise

$$\lambda = 0.1$$

a_i - correlated noise

T

Time interval between measurements

σ_a^2

Variance of acceleration