Laboratory work 13
Joint assimilation of navigation data coming from different sources

Performance – Thursday October 19, 2017
Due to submit a performance report – Monday October 23, 2017

The objective of this laboratory work is to develop a navigation filter by assimilating data coming from different sources. Important outcome of this exercise is getting skill to incorporate all available measurement information into assimilation algorithm and develop a tracking filter for nonlinear models.

This laboratory work is performed in the class by students as in teams of 2-4 on October 19, 2017 and the team will submit one document reporting about the performance till October 23, 2017. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

1. ***Here is the recommended procedure:***
   Generate a true trajectory X_i of an object motion disturbed by normally distributed random acceleration

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2}$$
$$V_i^x = V_{i-1}^x + a_{i-1}^x T$$
$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2}$$
$$V_i^y = V_{i-1}^y + a_{i-1}^y T$$

   Initial conditions to generate trajectory
   (a) Size of trajectory is $N = 500$ points.

   (b) $T = 2$ seconds – time step.

   (c) Initial coordinates
   $x_0 = 1000;\ y_0 = 1000$

   (a) Initial components of velocity V
   $V_x = 100; V_y = 100;$

   (b) Variance of noise a_i, $\sigma_a^2 = 0.3^2$ for both a_i^x, a_i^y

2. Generate also true values of range D and azimuth β

$$D_i = \sqrt{x_i^2 + y_i^2}$$
$$\beta_i = arctg\left(\frac{x}{y}\right)$$

3. Generate measurements of D^m and β^m of range D and azimuth β provided by first observer that arrive every 4 seconds.

$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$
$$i = 1,3,5,\dots,N-1 - \textbf{odd} \text{ time steps}$$

   Variances of measurement noises η_i^D, η_i^β are given by

$\sigma_D^2 = 50^2; \sigma_\beta^2 = 0.004^2$

4. Generate more accurate measurements of azimuth β^m provided by second observer that arrive between measurement times of the first observer.

$$\beta_i^m = \beta_i + \eta_i^\beta$$

$i = 4,6,8,\ldots,N$ – **even** time steps

Variance of measurement noise η_i^β in this case is given by

$$\sigma_{\beta_{add}}^2 = 0.001^2$$

5. Initial conditions for Extended Kalman filter algorithm

Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} x_3^m \\ \dfrac{x_3^m - x_1^m}{2T} \\ y_3^m \\ \dfrac{y_3^m - y_1^m}{2T} \end{vmatrix}$$

$$x_1^m = D_1^m sin\beta_1^m$$
$$x_3^m = D_3^m sin\beta_3^m$$
$$y_1^m = D_1^m cos\beta_1^m$$
$$y_3^m = D_3^m cos\beta_3^m$$

Initial filtration error covariance matrix $P_{0,0}$

First use great initial filtration error covariance matrix

$$P_{0,0} = \begin{vmatrix} 10^4 & 0 & 0 & 0 \\ 0 & 10^4 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 10^4 \end{vmatrix}$$

6. Develop Kalman filter algorithm to estimate state vector X_i (extrapolation and filtration). **Start algorithm from time step = 4.**

6.1. At every filtration step depending on observer, measurement vector z_i and observation function $h(X_i)$ have different form.
Consult charts, pages 3 – 6, Lab12_Brief_explanations.pdf.

6.2. The form of measurement noise covariance matrix R also varies:

1) Observer 1, odd time steps

$$R = \begin{vmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\beta^2 \end{vmatrix}$$

2) Observer 2, even time steps

$$R = \sigma_{\beta_{add}}^2$$

6.3. Using extrapolated and filtered estimates at every extrapolation and filtration step you will need to calculate

(a) range D

(b) azimuth β

6.4. At every filtration step in the algorithm you should linearize measurement equation by determining

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}}$$

Consult charts, pages 3 – 6, Lab12_Brief_explanations.pdf.

7. Run Kalman filter algorithm over $M = 500$ runs.
   Calculate true estimation errors of
   (a) Errors of extrapolation and filtration estimates of range D
   (b) Errors of extrapolation and filtration estimates of azimuth β
   Please plot these errors on two different plots for the analysis.

8. Compare estimation results with measurement errors of D and β.

9. Analyze again estimation errors of range D and azimuth β.
   Please make conclusions why the accuracy of estimation varies for odd and even time steps for both D and β.

Performance report

1. Performance report should contain all the items listed

2. The code should be commented. It should include:
   - Title of the laboratory work, for example
     % Converting a physical distance to a grid distance using least-square method
   - The names of a team, indication of Skoltech, and date, for example,
     %Tatiana Podladchikova, Skoltech, 2017
     Main procedures also should be commented, for example
     %13-month running mean
     …here comes the code

3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.