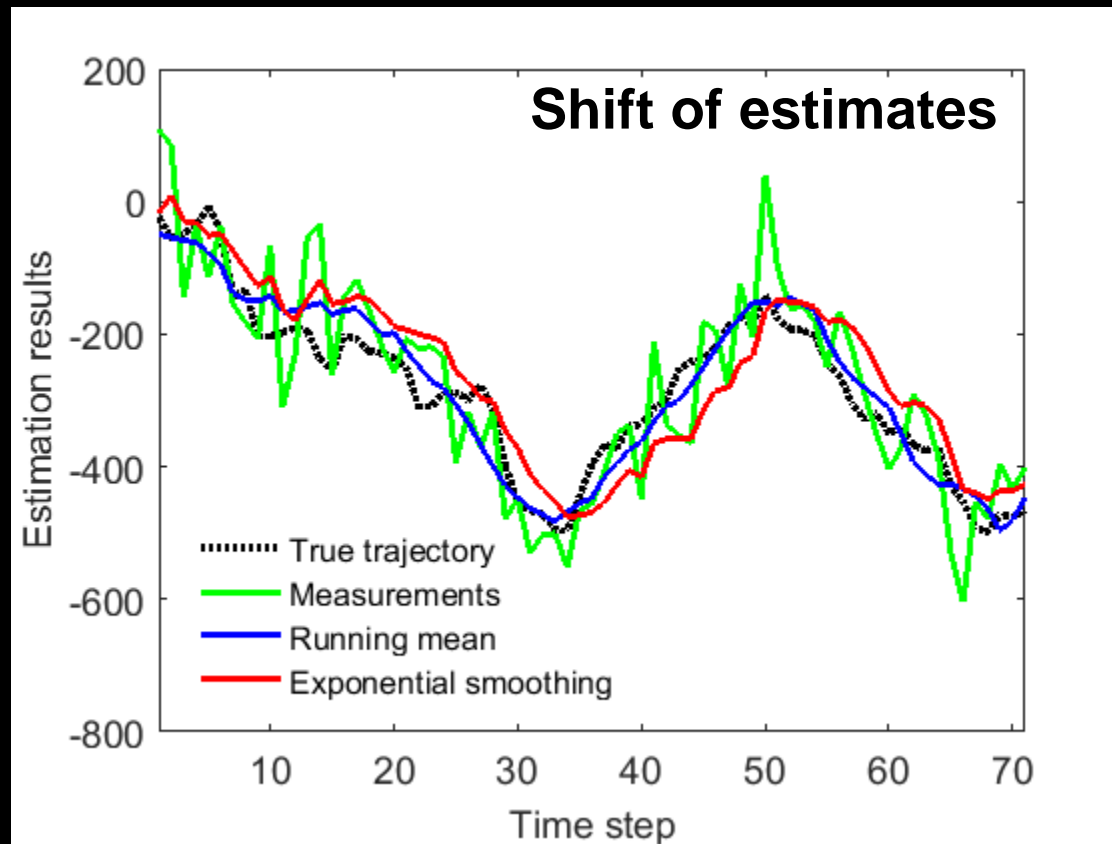


## “Experimental Data Processing”

### Laboratory work 2 Short discussion

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Term 1B, October 2017  
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# Shift of estimates in exponential smoothing

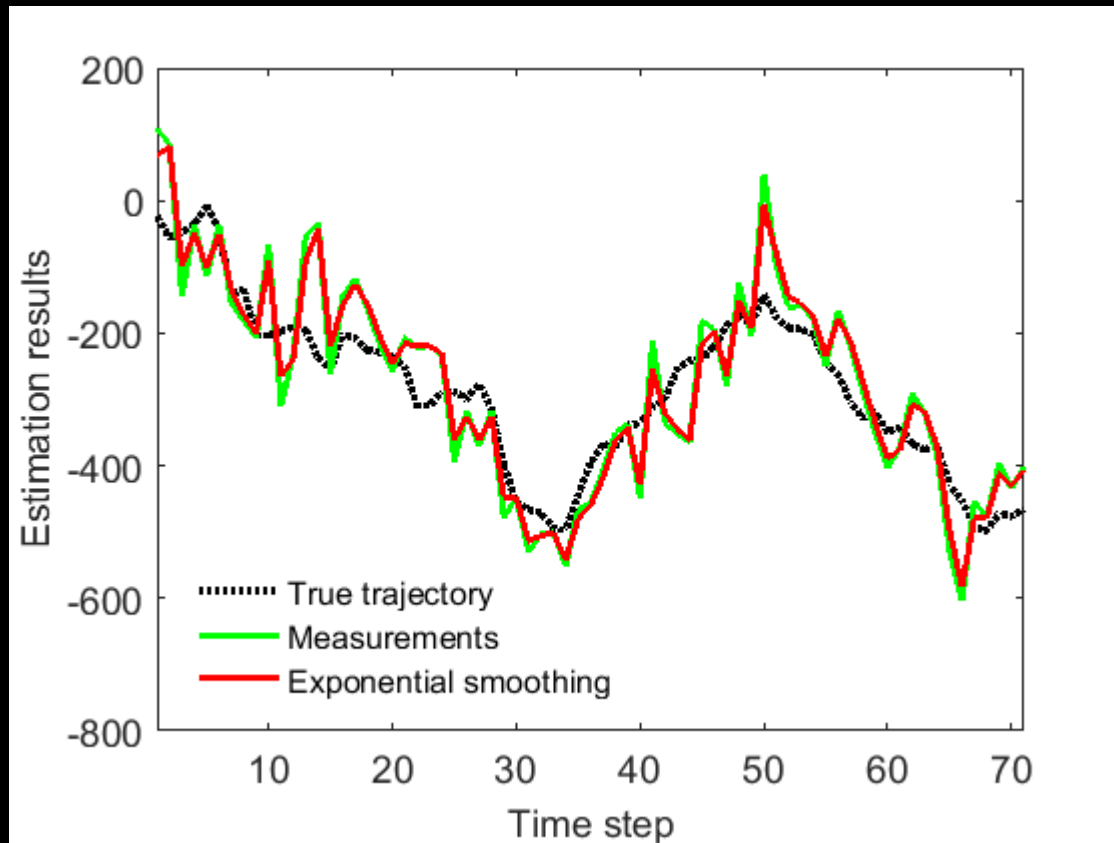


Optimal  
smoothing  
constant  
 $\alpha = 0.25$

Equal component  
of estimation error  
related with  
measurement errors

Size of  
running mean  
window  
 $M = 7$

# No filtration of measurement errors

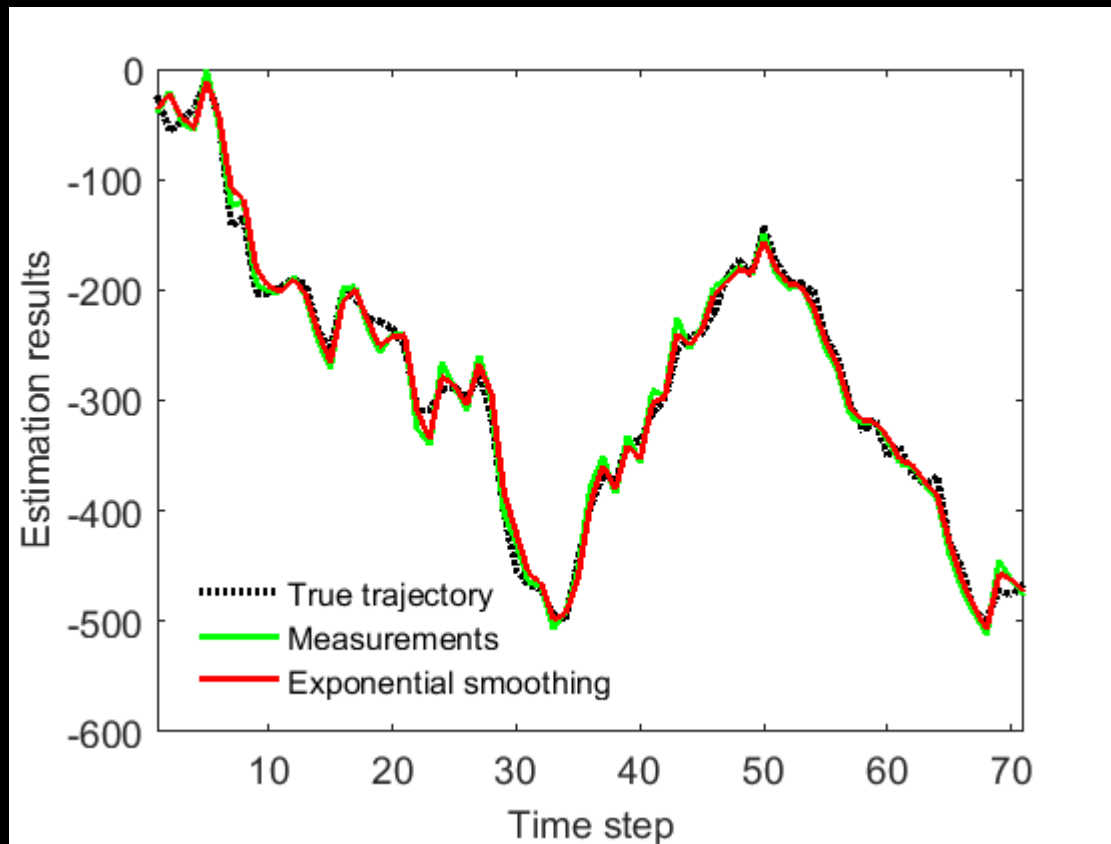


**Non-optimal  
smoothing  
constant  
 $\alpha = 0.8$**



**No shift  
of estimates,  
but no filtration of  
measurement errors**

# Smoothing in conditions of small measurement errors



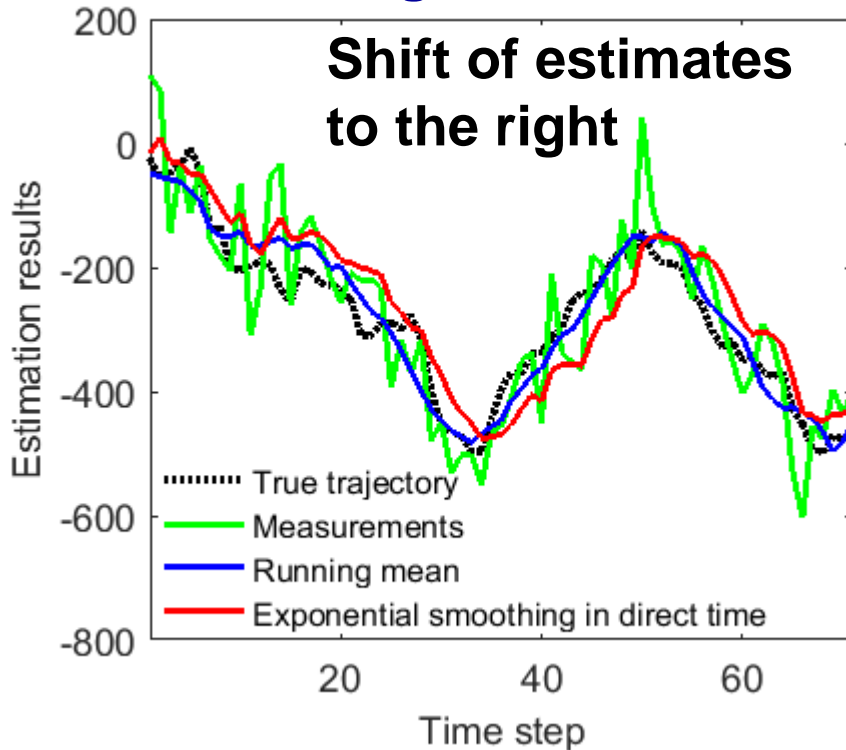
**Optimal  
smoothing  
constant**  
 $\alpha = 0.81$

**Small measurement errors -  
small estimation problems**

# Shift of estimates in exponential smoothing

→ Smoothing in direct time

← Smoothing in backward time



Shift of estimates  
by exponential  
smoothing in  
backward time?

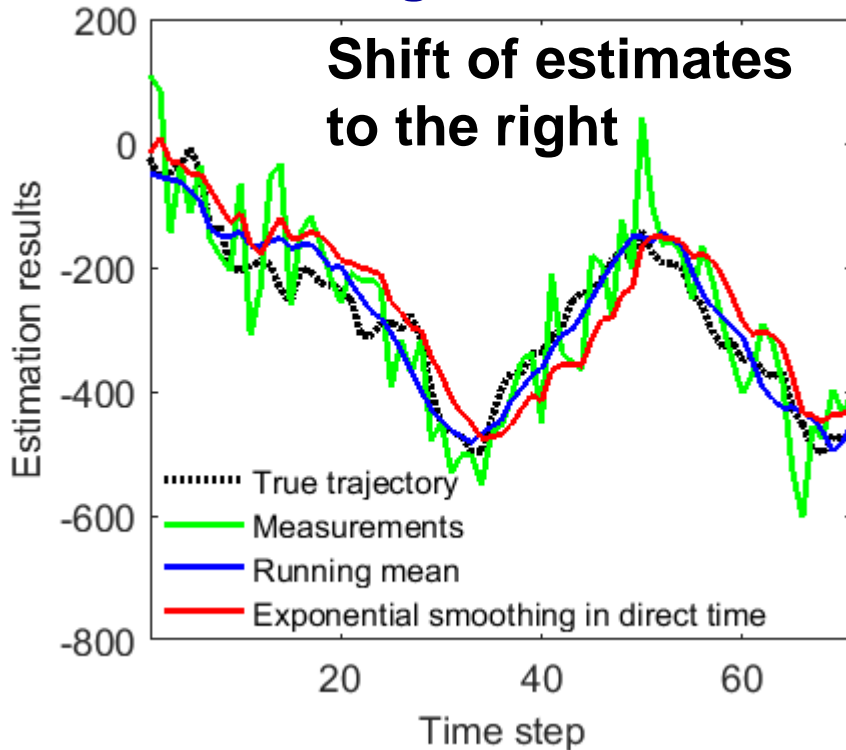
Optimal  
smoothing  
constant  
 $\alpha = 0.25$

Equal component  
of estimation error  
related with  
measurement errors

Size of  
running mean  
window  
 $M = 7$

# Shift of estimates in exponential smoothing

→ Smoothing in direct time



← Smoothing in backward time



Optimal  
smoothing  
constant  
 $\alpha = 0.25$

Equal component  
of estimation error  
related with  
measurement errors

Size of  
running mean  
window  
 $M = 7$

# Forward – backward exponential smoothing



**① Forward exponential smoothing**



$$X_i^f = X_{i-1}^f + \alpha (z_i - X_{i-1}^f), i = 2, \dots, N$$



# Forward – backward exponential smoothing

## → ① Forward exponential smoothing

$$X_i^f = X_{i-1}^f + \alpha (z_i - X_{i-1}^f), i = 2, \dots, N$$



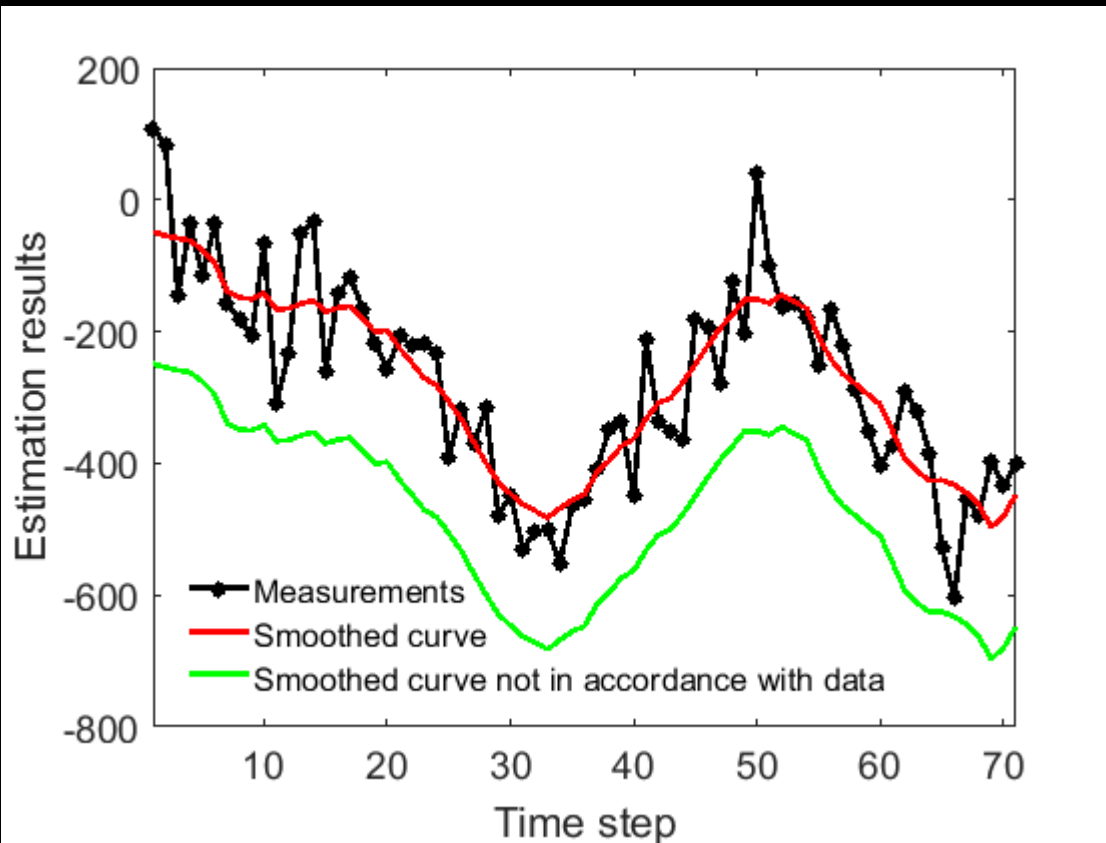
## ← ② Backward exponential smoothing

$$X_i^b = X_{i+1}^b + \alpha (X_i^f - X_{i+1}^b), i = N - 1, \dots, 1$$





# How to verify the effectiveness of smoothing when true trajectory is unknown?



**Requirement  
of estimation  
to be close to  
measurements**

1

**Deviation  
indicator**



$$I_d = \sum_{i=1}^N (z_i - \hat{X}_i)^2$$

$z_i$  - measurements  
 $\hat{X}_i$  - estimation

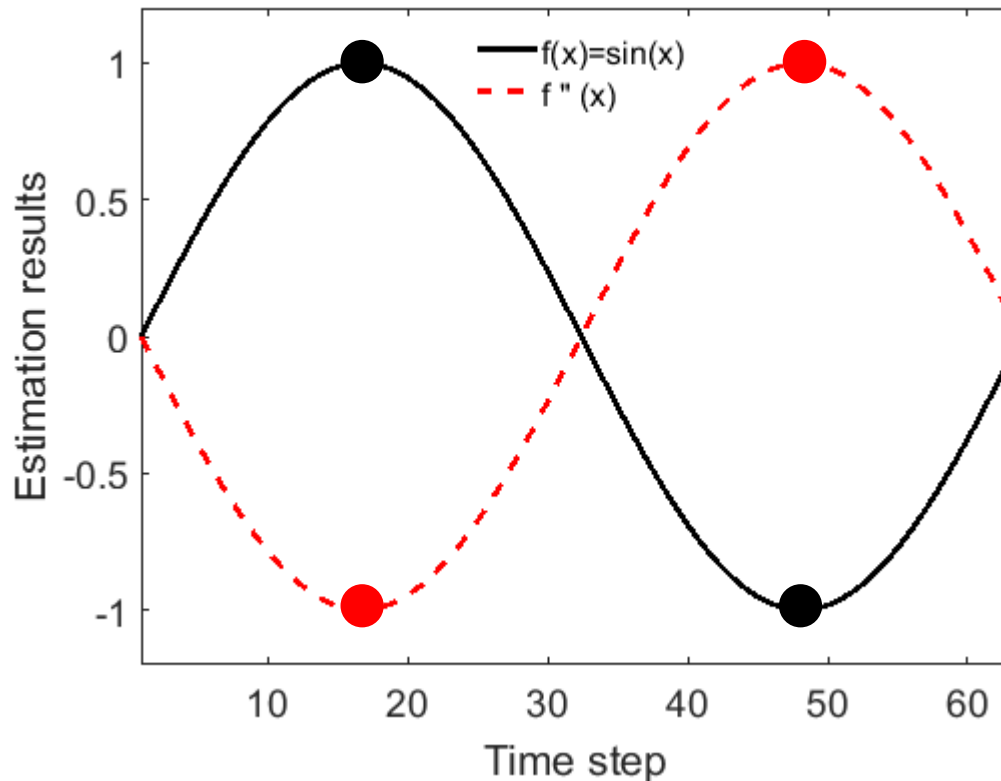
# How to verify the effectiveness of smoothing when true trajectory is unknown?

1 Deviation indicator  $\Rightarrow I_d = \sum_{i=1}^N (z_i - \hat{X}_i)^2$

$I_d = 0 \Rightarrow$  No filtration of measurement noise  
 $\hat{X}_i = z_i$

Not enough to use only deviation indicator.  
Additional criterion is needed

# How to verify the effectiveness of smoothing when true trajectory is unknown?



**Absolute value of second derivative is maximal at points of the greatest “variability” of curve**

**Maximal rate of change of the process**

**② Variability indicator**

$$I_v = \sum_{i=1}^{N-2} (\hat{X}_{j+2} - 2\hat{X}_{j+1} + \hat{X}_j)^2$$

$\hat{X}_i$  - estimation