

## **Laboratory work 2**

### **Comparison of exponential and running mean for random walk model**

Performance –Wednesday, October 4, 2017

Due to submit a performance report – 10 p.m. Friday, October 6, 2017

The objective of this laboratory work is to compare the errors of exponential and running mean to choose the most effective quasi-optimal estimation method in conditions of uncertainty. Additional important outcome of this exercise is the solution of identification problem of noise statistics that is crucial for reliable estimation.

This laboratory work is performed in the class by students as in teams of 2-4 on October 4, 2017 and the team will submit one document reporting about the performance till 10 p.m. Friday, October 6, 2017. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

This laboratory work consists of two parts:

- I. Determination of optimal smoothing constant in exponential mean.
- II. Comparison of methodical errors of exponential and running mean.

***Here is the recommended procedure for part I:***

***Determination of optimal smoothing constant in exponential mean***

1. Please conduct a simulation experiment. First generate a true trajectory, then generate measurements of this true trajectory.

1.1. Generate a true trajectory  $X_i$  using the random walk model

$$X_i = X_{i-1} + w_i \quad (1)$$

$w_i$  –normally distributed random noise with zero mathematical expectation and variance  $\sigma_w^2$ .

Group 1:  $\sigma_w^2 = 10$ ;

Group 2:  $\sigma_w^2 = 12$ ;

Group 3:  $\sigma_w^2 = 15$ ;

Group 4:  $\sigma_w^2 = 10$ ;

Group 5:  $\sigma_w^2 = 16$ ;

Group 6:  $\sigma_w^2 = 8$ ;

Size of trajectory is

1) 3000 points

2) 300 points

To generate true trajectory use initial condition  $X_1 = 10$ .

1.2. Generate measurements  $z_i$  of the process  $X_i$

$$z_i = X_i + \eta_i \quad (2)$$

$\eta_i$  – normally distributed random noise with zero mathematical expectation and variance  $\sigma_\eta^2$ .

Group 1:  $\sigma_\eta^2 = 15$ ;

Group 2:  $\sigma_\eta^2 = 12$ ;

Group 3:  $\sigma_\eta^2 = 10$ ;

Group 4:  $\sigma_\eta^2 = 10$ ;

Group 5:  $\sigma_\eta^2 = 8$ ;

Group 6:  $\sigma_\eta^2 = 16$ ;

2. Identify  $\sigma_w^2$  and  $\sigma_\eta^2$  using identification method presented on slide 55 (Topic\_2\_Quasi-optimal approximation under uncertainty.pdf). Perform identification for different size of trajectory (3000 and 300). Compare estimation results with true values of  $\sigma_w^2$  and  $\sigma_\eta^2$ . Compare the accuracy of estimation.

3. Determine optimal smoothing coefficient in exponential smoothing

$$\alpha = \frac{-\chi + \sqrt{\chi^2 + 4\chi}}{2} \quad (3)$$

$$\chi = \frac{\sigma_w^2}{\sigma_\eta^2}$$

***Here is the recommended procedure for part II:***

***Comparison of methodical errors of exponential and running mean.***

1. Generate a true trajectory  $X_i$  using the random walk model (1).  
Size of trajectory is 300 points.  
Initial condition  $X_1 = 10$ .  
Variance of noise  $w_i$ ,  $\sigma_w^2 = 28^2$
2. Generate measurements  $z_i$  of the process  $X_i$  using equation (2)  
Variance of noise measurement noise  $\eta_i$ ,  $\sigma_\eta^2 = 97^2$
3. Determine optimal smoothing coefficient  $\alpha$  using equation (3).  
(There is no need to identify it again, just use equation for  $\alpha$  from part I).
4. The component of full error that is related to measurement errors is determined as  
(from slide 37, Topic\_2\_Quasi-optimal approximation under uncertainty.pdf)

Running mean (RM):

$$\sigma_{RM}^2 = \frac{\sigma_\eta^2}{M} \quad (4)$$

Exponential smoothing (ES):

$$\sigma_{ES}^2 = \sigma_\eta^2 \frac{\alpha}{2 - \alpha} \quad (5)$$

Determine the window size  $M$  (use round values) that provides equality of  $\sigma_{RM}^2$  and  $\sigma_{ES}^2$  using determined smoothing constant  $\alpha$

$$\sigma_{ES}^2 = \sigma_{RM}^2$$

5. Apply running mean using determined window size  $M$  and exponential mean (see, for instance, page 30, Topic\_2\_Quasi-optimal approximation under uncertainty.pdf) using determined smoothing constant  $\alpha$  to measurements  $z_i$ . Plot true trajectory  $X_i$ , measurements  $z_i$ , running and exponential mean.
6. Make visual comparison of results. Make conclusions which methods give greater methodical error in conditions of equal errors conditioned by measurement errors for this particular generated trajectory.

### ***Performance report***

1. Performance report should contain all the items listed
2. The code should be commented. It should include:
  - Title of the laboratory work, for example  
    % Converting a physical distance to a grid distance using least-square method
  - The names of a team, indication of Skoltech, and date, for example,  
    % Tatiana Podladchikova, Skoltech, 2017  
    Main procedures also should be commented, for example  
    % 13-month running mean  
    ...here comes the code
3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.