Laboratory work 2
Comparison of the exponential and running mean for random walk model

Performance –Thursday, October 4, 2018
Due to submit a performance report – Monday, October 8, 2018, 23:59 p.m.

The objective of this laboratory work is to compare the errors of exponential and running mean to choose the most effective quasi-optimal estimation method in conditions of uncertainty. Additional important outcome of this exercise is the solution of identification problem of noise statistics that is crucial for reliable estimation.

This laboratory work is performed in the class by students as in teams of 3 on October 4, 2018 and the team will submit one document reporting about the performance till Monday, October 8, 2018. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

This laboratory work consists of two parts:

I. Determination of optimal smoothing constant in exponential mean.
II. Comparison of methodical errors of exponential and running mean.

***Here is the recommended procedure for part I*:**
Determination of optimal smoothing constant in exponential mean

1. Please conduct a simulation experiment. First generate a true trajectory, then generate measurements of this true trajectory.

   1.1.Generate a true trajectory X_i using the random walk model

   $$X_i = X_{i-1} + w_i \quad (1)$$

   w_i –normally distributed random noise with zero mathematical expectation and variance σ_w^2.
   Group 1: $\sigma_w^2 = 8$;
   Group 2: $\sigma_w^2 = 9$;
   Group 3: $\sigma_w^2 = 10$;
   Group 4: $\sigma_w^2 = 11$;
   Group 5: $\sigma_w^2 = 12$;
   Group 6: $\sigma_w^2 = 13$;
   Group 7: $\sigma_w^2 = 14$;
   Group 8: $\sigma_w^2 = 15$;
   Group 9: $\sigma_w^2 = 16$;
   Group 10: $\sigma_w^2 = 17$;
   Group 11: $\sigma_w^2 = 18$;
   Group 12: $\sigma_w^2 = 19$;
   Group 13: $\sigma_w^2 = 20$;

   Size of trajectory is
   1) 3000 points
   2) 300 points

   To generate true trajectory use initial condition $X_1 = 10$.

   1.2.Generate measurements z_i of the process X_i

$$z_i = X_i + \eta_i \quad (2)$$

η_i – normally distributed random noise with zero mathematical expectation and variance σ_η^2.

Group 1: $\sigma_\eta^2 = 16$;
Group 2: $\sigma_\eta^2 = 12$;
Group 3: $\sigma_\eta^2 = 13$;
Group 4: $\sigma_\eta^2 = 10$;
Group 5: $\sigma_\eta^2 = 9$;
Group 6: $\sigma_\eta^2 = 8$;
Group 7: $\sigma_\eta^2 = 10$;
Group 8: $\sigma_\eta^2 = 9$;
Group 9: $\sigma_\eta^2 = 11$;
Group 10: $\sigma_\eta^2 = 10$;
Group 11: $\sigma_\eta^2 = 12$;
Group 12: $\sigma_\eta^2 = 10$;
Group 13: $\sigma_\eta^2 = 15$;

2. Identify σ_w^2 and σ_η^2 using identification method presented on slide 55 (Topic_2_Quasi-optimal approximation under uncertainty.pdf). Perform identification for different size of trajectory (3000 and 300). Compare estimation results with true values of σ_w^2 and σ_η^2. Compare the accuracy of estimation.

3. Determine optimal smoothing coefficient in exponential smoothing

   $$\alpha = \frac{-\chi + \sqrt{\chi^2 + 4\chi}}{2} \qquad (3)$$

   $$\chi = \frac{\sigma_w^2}{\sigma_\eta^2}$$

4. Perform exponential smoothing with the determined smoothing coefficient. Plot results. For the comparison add measurements, true values of process and exponentially smoothed data.

***Here is the recommended procedure for part II*:**
Comparison of methodical errors of exponential and running mean.

1. Generate a true trajectory X_i using the random walk model (1).
   Size of trajectory is 300 points.
   Initial condition $X_1 = 10$.
   Variance of noise w_i, $\sigma_w^2 = 28^2$

2. Generate measurements z_i of the process X_i using equation (2)
   Variance of noise measurement noise η_i, $\sigma_\eta^2 = 97^2$

3. Determine optimal smoothing coefficient α using equation (3).
   (There is no need to identify it again, just use equation for α from part I).

4. The component of full error that is related to measurement errors is determined as
   (from slide 37, Topic_2_Quasi-optimal approximation under uncertainty.pdf)

Running mean (RM):

$$\sigma_{RM}^2 = \frac{\sigma_\eta^2}{M} \quad (4)$$

Exponential smoothing (ES):

$$\sigma_{\mathrm{ES}}^2 = \sigma_\eta^2 \frac{\alpha}{2-\alpha} \quad (5)$$

Determine the window size M (use round values) that provides equality of σ_{RM}^2 and σ_{ES}^2 using determined smoothing constant α

$$\sigma_{\mathrm{ES}}^2 = \sigma_{\mathrm{RM}}^2$$

5. Apply running mean using determined window size M and exponential mean (see, for instance, page 30, Topic_2_Quasi-optimal approximation under uncertainty.pdf) using determined smoothing constant α to measurements z_i. Plot true trajectory X_i, measurements z_i, running and exponential mean.

6. Make visual comparison of results. Make conclusions which methods give greater methodical error in conditions of equal errors conditioned by measurement errors for this particular generated trajectory.

7. Prepare performance report and submit to Canvas:
   Performance report should include 2 documents:
   1) A report (PDF) with performance of all the items listed above
   2) Code
      The code should be commented. It should include:
      -Title of the laboratory work;
      -The names of a team, indication of Skoltech, and date;
      -Main procedures also should be commented, for example
      %13-month running mean