Laboratory work 8
Tracking and forecasting in conditions of measurement gaps

Performance – Thursday, October 12, 2017
Due to submit a performance report – Monday, October 16, 2017

The objective of this laboratory work is to develop the estimation and tracking algorithm in conditions of measurement gaps, that is of prime importance for many practical control and forecasting problems. This will bring about a deeper understanding of main difficulties of practical Kalman filter implementation and skills to overcome these difficulties to get optimal assimilation output.

This laboratory work is performed in the class by students as in teams of 2-4 on October 12, 2017 and the team will submit one document reporting about the performance till Monday, October 16, 2017. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

Here is the recommended procedure:

1. Generate a true trajectory X_i of an object motion disturbed by normally distributed unbiased random acceleration a_i with variance $\sigma_a^2 = 0.2^2$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}T$$

In general measurements of coordinate x_i are performed every second with variance of measurement noise $\sigma_\eta^2 = 20^2$. Observation interval is 200 seconds. However, there are measurement gaps.

Probability of measurement gaps is
a) $P = 0.2$;

Hint 1: Create measurements with gaps
To create the measurements with gaps, create the random value ξ that is **randomly** distributed at every step i. If $\xi_i \le P$, then $z(i) = NaN$ (gap), if $\xi_i > P$, then $z_i = X_i + \eta_i$.

Hint 2: Kalman filter in condition of gaps
Kalamn filter is a recurrent algorithm consisting of two procedures, extrapolation and filtration. We do filtration when we have the available measurements. When the measurements are absent, to not interrupt the recurrent process, at filtration step we should indicate that the filtered estimate is equal to extrapolated estimate $X_{i,i} = X_{i,i-1}$ and filtration error covariance matrix is equal to extrapolation error covariance matrix $P_{i,i} = P_{i,i-1}$. We do this as we don't have any information to improve our estimates at filtration step (no measurements).

2. Develop Kalman filter to track moving object under this conditions.

3. Determine filtered and extrapolated errors of estimation (1 step and 7 steps ahead) over 500 runs of filter. Compare them with true estimation errors.

4. Analyze the decrease of estimation accuracy in conditions of measurement gaps.
   Compare results when of probability of measurement gaps is
   a) $P = 0.3$;
   b) $P = 0.5$;

5. Make general conclusions.

Performance report

1. Performance report should contain all the items listed

2. The code should be commented. It should include:
   - Title of the laboratory work, for example
     % Converting a physical distance to a grid distance using least-square method

   - The names of a team, indication of Skoltech, and date, for example,
     %Tatiana Podladchikova, Skoltech, 2017
     Main procedures also should be commented, for example
     %13-month running mean
     …here comes the code

3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.