Laboratory work 11
Noise statistics identification to construct tracking filter of a moving object

Performance - Tuesday, October 17, 2017
Due to submit a performance report – October 19, 2017

The objective of this laboratory work is to develop a tracking filter of a moving object on the basis of noise statistics identification which specification is crucial for optimal assimilation output. This will bring a deeper understanding of main difficulties of practical Kalman filter implementation and skills to overcome these difficulties.

This laboratory work is performed in the class by students as in teams of 2-4 on October 17 2017 and the team will submit one document reporting about the performance till October 19, 2017. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

Here is the recommended procedure:
Part I. Noise statistics identification

1. Generate a true trajectory X_i of an object motion disturbed by normally distributed **BIASED** random acceleration. Bias (mathematical expectation) of random noise $\boldsymbol{q = 6}$

$$a_{i-1}^{biased} = a_{i-1} + q$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}^{biased}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}^{biased}T$$

   Size of trajectory is **500 000** points.
   Initial conditions: $x_1 = 5; V_1 = 1; T = 1$
   Standard deviation of a_i, $\sigma_a = 3$

2. Generate measurements z_i of the coordinate x_i

$$z_i = x_i + \eta_i$$

   η_i –normally distributed random noise with zero mathematical expectation
   and standard deviation of $\sigma_\eta = 10$.

3. Identify bias q, standard deviation σ_a, and standard deviation σ_η using measurements z_i.
   Consult page 45,
   Topic_5_Part_I_Model construction at state space under uncertainty.pdf

Part II. Tracking filter of a moving object on the basis of noise statistics identification and sensitivity analysis of assimilation output to choice of noise statistics

1. Generate again a true trajectory X_i and measurements z_i of a moving object.
   The size of trajectory in this case is **200 points**.
   Use the same noise statistics and initial conditions as in part I, items 1,2.
   Bias (mathematical expectation) of random noise $q = 6$
   Variance of noise a_i, $\sigma_a = 3$
   Variance of noise η_i, $\sigma_\eta = 10$
   Initial conditions: $x_1 = 5; V_1 = 1; T = 1$

2. Obtain estimates of state vector $X = \begin{vmatrix} x \\ V \end{vmatrix}$ by Kalman filter and use identification results of q, σ_a, σ_η obtained in part I where it is needed in the estimation algorithm. Note that bias and covariance matrix of state noise Ga_i depends also on input matrix G.

   Use initial conditions
   Initial filtered estimate $X_0 = \begin{vmatrix} 2 \\ 0 \end{vmatrix}$
   Initial filtration error covariance matrix

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 \\ 0 & 10^{10} \end{vmatrix}$$

3. Make $M = 500$ runs of filter and and compare true estimation error of coordinate x_i with errors of estimation $P_{i.i}$ provided by Kalman filter algorithm. Plot results

4. Repeat item 3, but change the values of noise statistics and analyze sensitivity of Kalman filter output to these changes
   (a) Instead of identified bias q of noise a_i use $q = 0$,
   σ_a, σ_η– identified values in part I
   (b) Instead of identified standard deviation σ_a use value 10 times greater $\sigma_a = 10\sigma_a$,
   q and σ_η– identified values in part I
   (c) Instead of identified standard deviation σ_a use value 10 times less $\sigma_a = \sigma_a/10$
   q and σ_η– identified values in part I
   (d) Instead of identified standard deviation σ_η use value 10 times greater $\sigma_\eta = 10\sigma_\eta$,
   q and σ_a– identified values in part I
   (e) Instead of identified standard deviation σ_η use value 10 times less $\sigma_\eta = \sigma_\eta/10$,
   q and σ_a– identified values in part I

Performance report

1. Performance report should contain all the items listed

2. The code should be commented. It should include:
   - Title of the laboratory work, for example
     % Converting a physical distance to a grid distance using least-square method
   - The names of a team, indication of Skoltech, and date, for example,
     %Tatiana Podladchikova, Skoltech, 2017
     Main procedures also should be commented, for example
     %13-month running mean
     …here comes the code

3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.