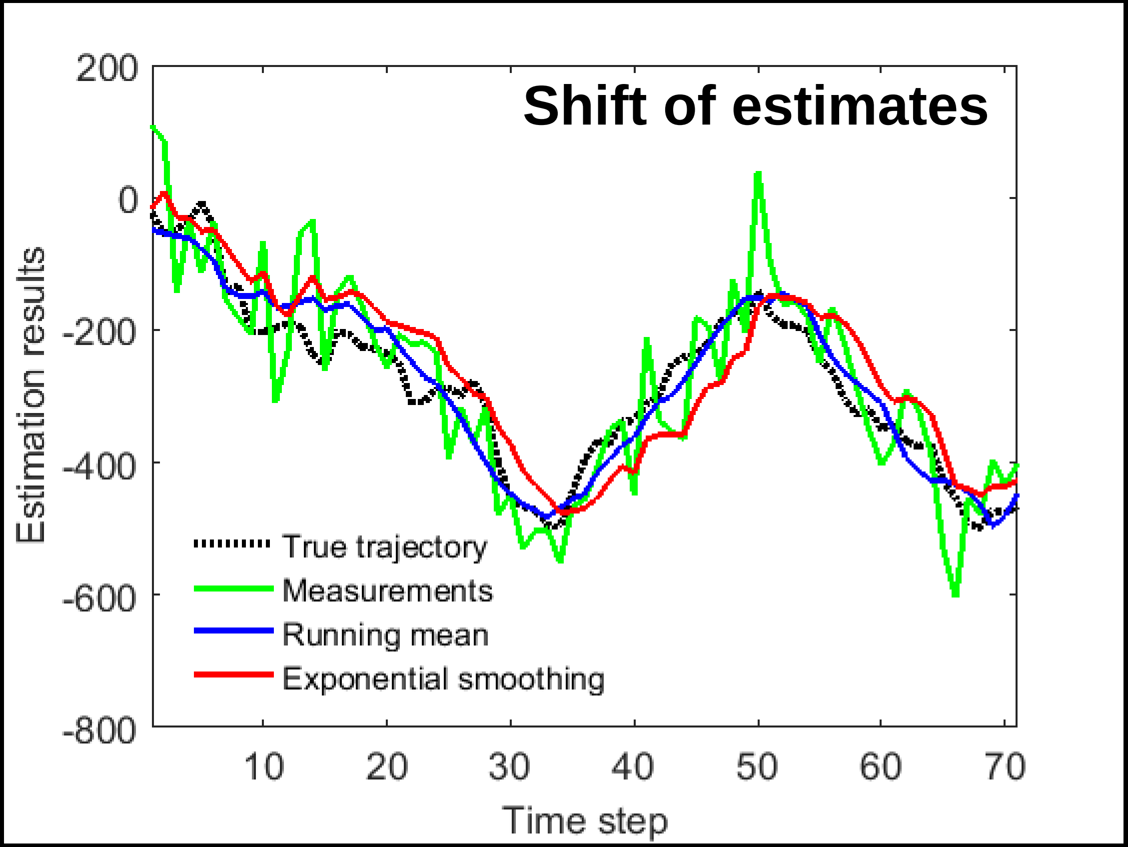
Skoltech
Skolkovo Institute of Science and Technology

“Experimental Data Processing”

Laboratory work 2
Short discussion

Tatiana Podladchikova
Term 1B, October 2018
t.podladchikova@skoltech.ru

Shift of estimates in exponential smoothing

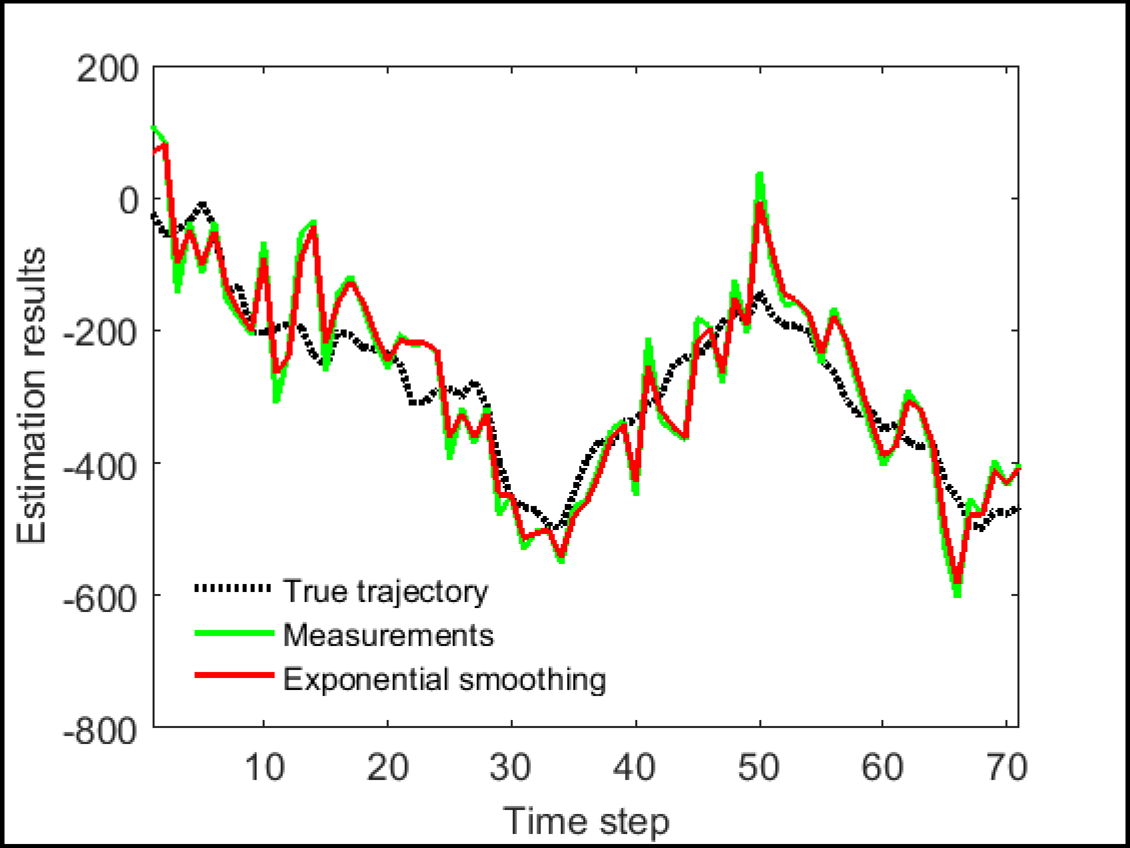


Optimal smoothing constant $\alpha = 0.25$

Equal component of estimation error related with measurement errors

Size of running mean window $M = 7$

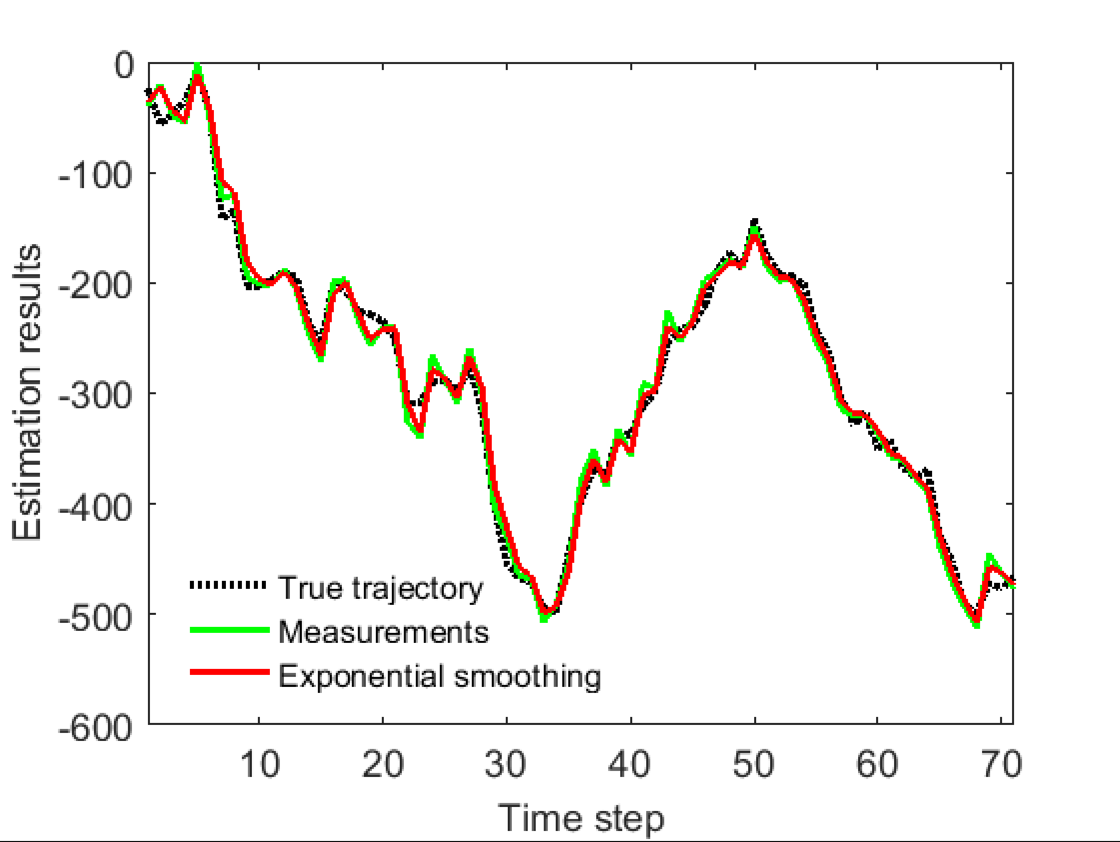
No filtration of measurement errors



Non-optimal smoothing constant $\alpha = 0.8$

No shift of estimates, but no filtration of measurement errors

Smoothing in conditions of small measurement errors

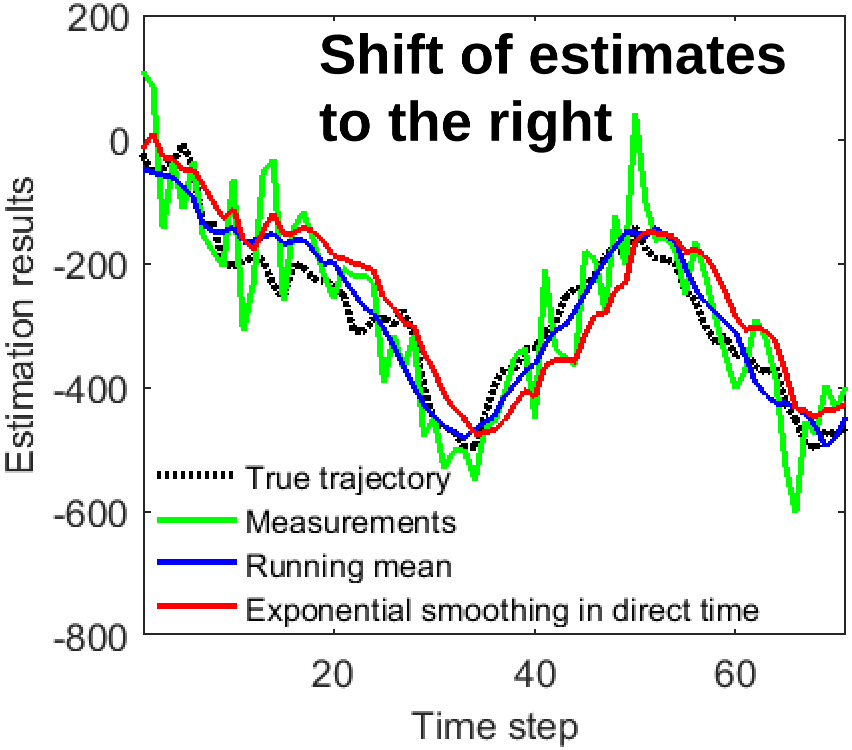


Optimal smoothing constant $\alpha = 0.81$

Small measurement errors - small estimation problems

Shift of estimates in exponential smoothing

→ **Smoothing in direct time**



← **Smoothing in backward time**

Shift of estimates by exponential smoothing in backward time?

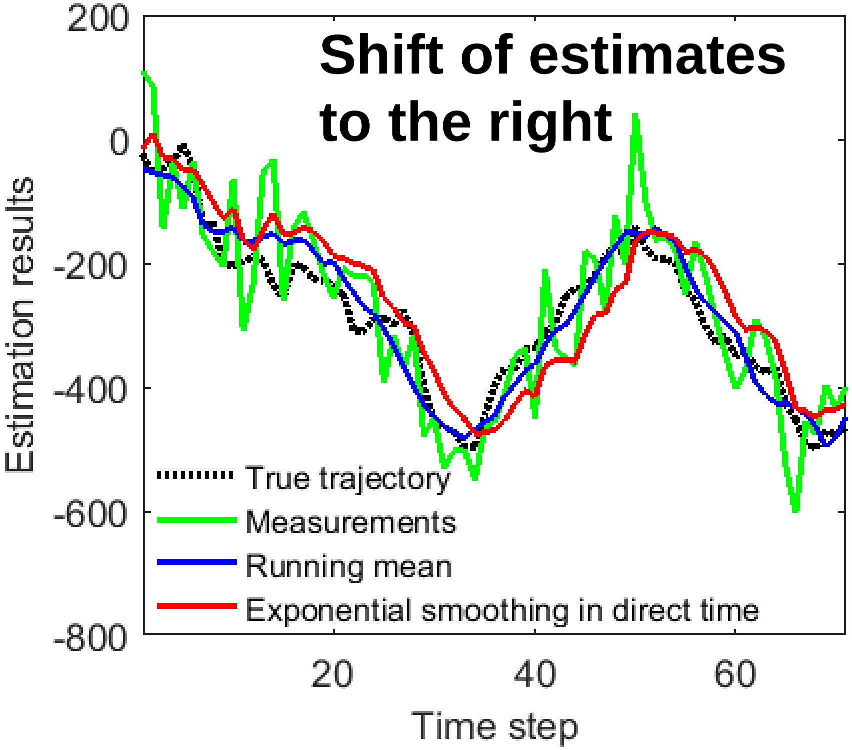
Optimal smoothing constant $\alpha = 0.25$

Equal component of estimation error related with measurement errors

Size of running mean window $M = 7$

Shift of estimates in exponential smoothing

→ **Smoothing in direct time**



← **Smoothing in backward time**

Optimal smoothing constant $\alpha = 0.25$

Equal component of estimation error related with measurement errors

Size of running mean window $M = 7$

Forward – backward exponential smoothing

→ ❶ **Forward exponential smoothing**

$$X_i^f = X_{i-1}^f + \alpha\left(z_i - X_{i-1}^f\right), i = 2, \ldots, N$$

X_i^f → Estimates of forward exponential smoothing

z_i → Input - measurements

Forward – backward exponential smoothing

→ ① **Forward exponential smoothing**

$$X_i^f = X_{i-1}^f + \alpha\left(z_i - X_{i-1}^f\right), i = 2, \dots, N$$

X_i^f → Estimates of forward exponential smoothing

z_i → Input - measurements

← ② **Backward exponential smoothing**

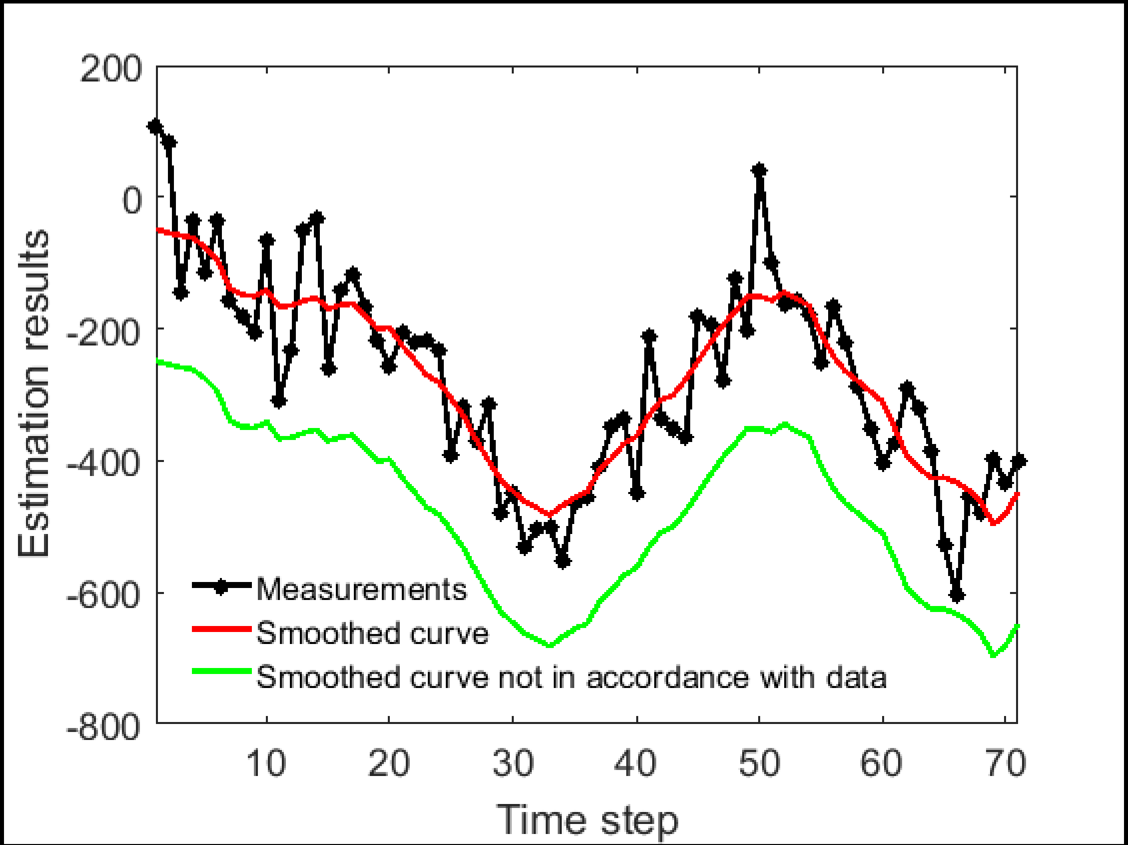
$$X_i^b = X_{i+1}^b + \alpha\left(X_i^f - X_{i+1}^b\right), \mathrm{i} = \mathrm{N} - 1, \dots, 1$$

$$X_N^b = X_N^f$$

X_i^b → Estimates of backward exponential smoothing

X_i^f → Input – forward estimates

How to verify the effectiveness of smoothing when true trajectory is unknown?



Requirement of estimation to be close to measurements

1 **Deviation indicator**

$$I_d = \sum_{i=1}^{N} \left(z_i - \hat{X}_i\right)^2$$

z_i - measurements
$\hat{X}_i$ - estimation

How to verify the effectiveness of smoothing when true trajectory is unknown?

1 Deviation indicator

$$I_d = \sum_{i=1}^{N} \left(z_i - \hat{X}_i\right)^2$$

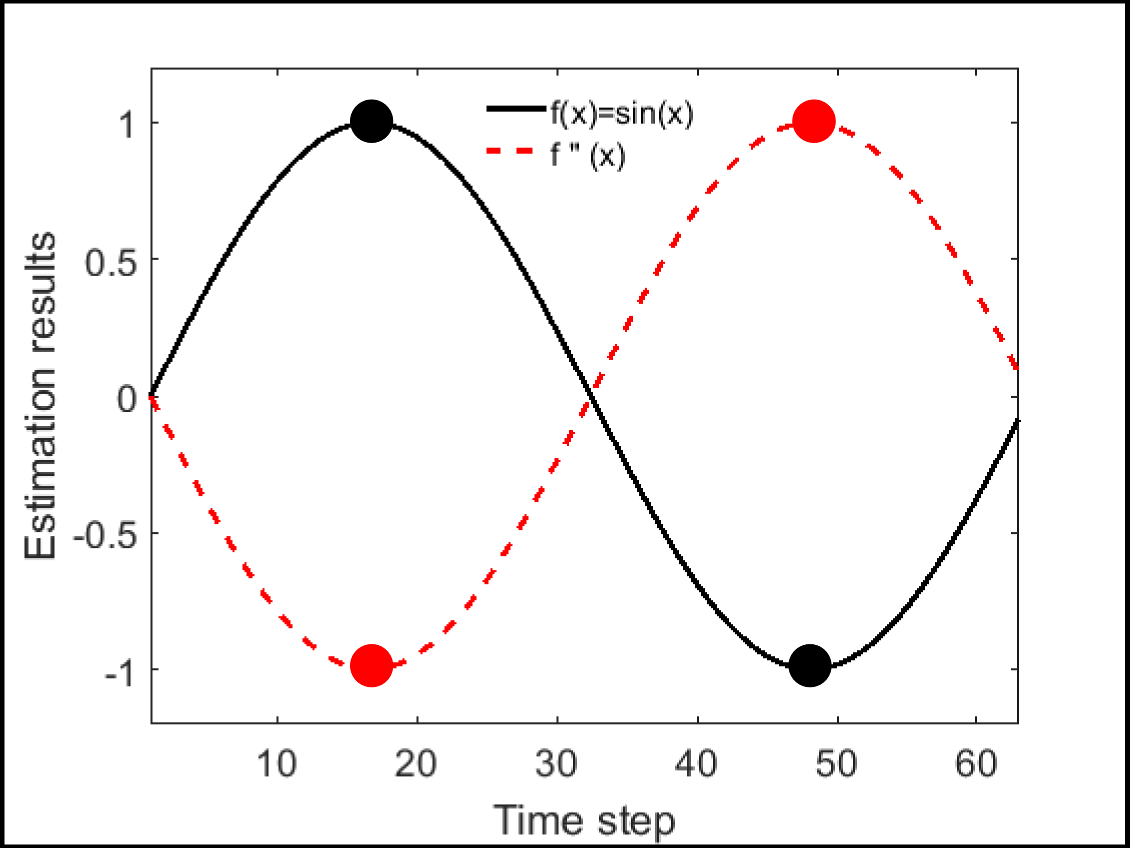
$$I_d = 0$$

No filtration of measurement noise

$$\hat{X}_i = z_i$$

Not enough to use only deviation indicator. Additional criterion is needed

How to verify the effectiveness of smoothing when true trajectory is unknown?



Absolute value of second derivative is maximal at points of the greatest "variability" of curve

Maximal rate of change of the process

② Variability indicator

$$I_v = \sum_{i=1}^{N-2} \left(\widehat{X}_{j+2} - 2\widehat{X}_{j+1} + \widehat{X}_j\right)^2$$

$\widehat{X}_i$ - estimation