Report: Laboratory work 6

Analysis of accuracy decrease of filtration in conditions of correlated biased state and measurement noise

Team #1: Viktor Liviniuk, Alina Liviniuk

The objective of this laboratory work is to analyze the sensitivity of estimation results obtained by a Kalman filter that doesn't take into account bias of state and measurement noise.

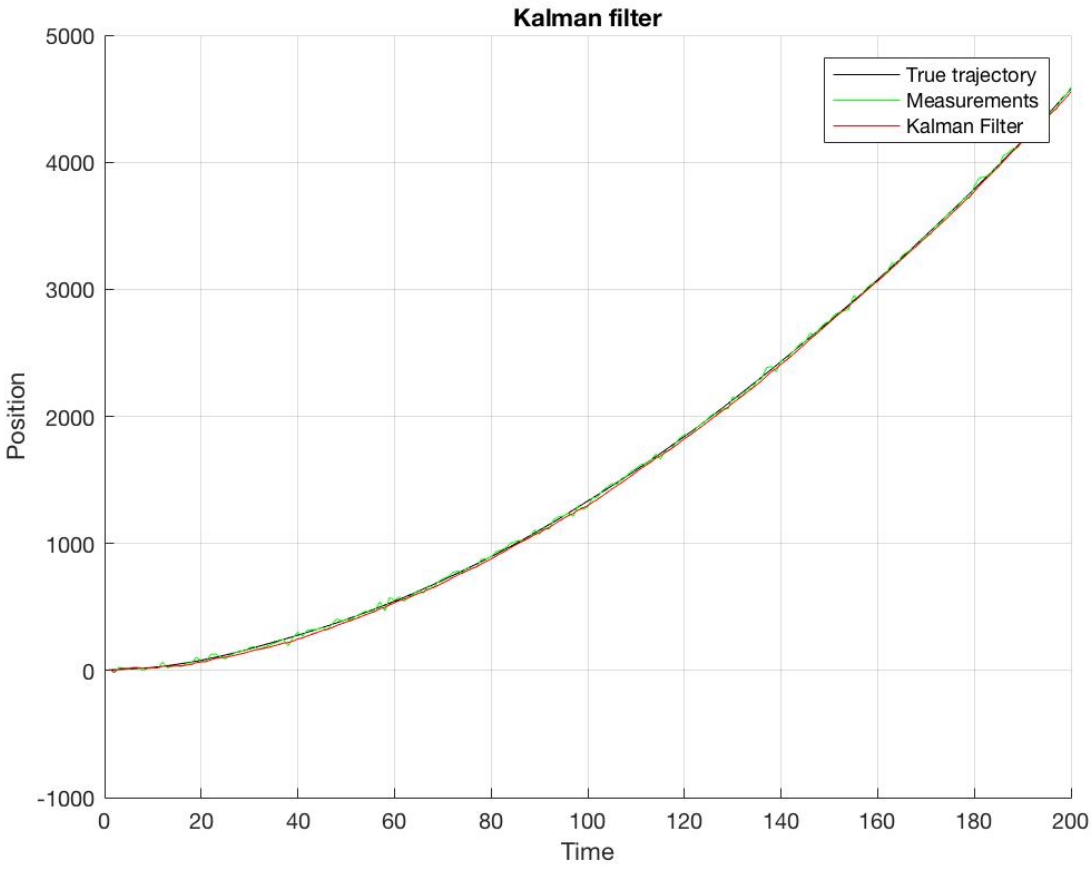
Divergence of Kalman filter when bias of state noise (acceleration) is neglected in assimilation algorithm. Development of optimal Kalman filter that takes into account bias of state noise (acceleration).

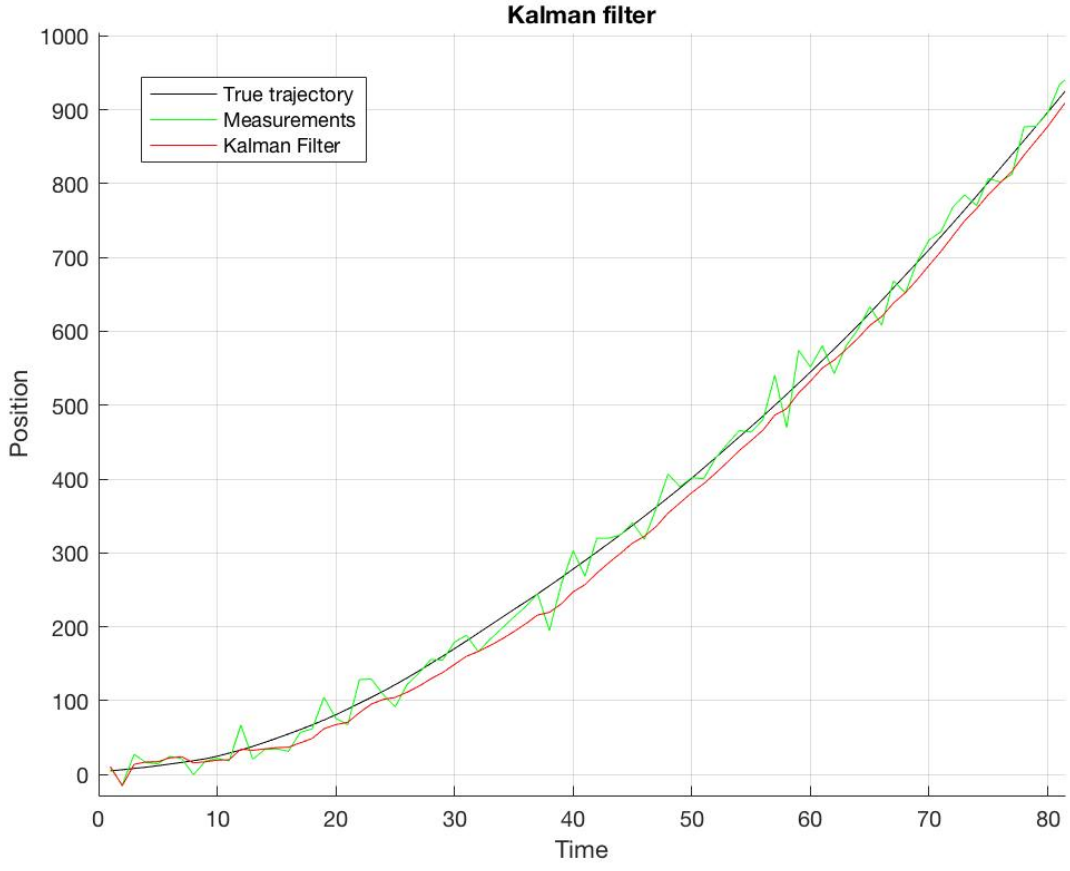
A trajectory X_i of an object motion disturbed by normally distributed BIASED random acceleration was generated. Measurements of the trajectory were also generated.

$$a_{i-1}^{biased} = a_{i-1} + q$$

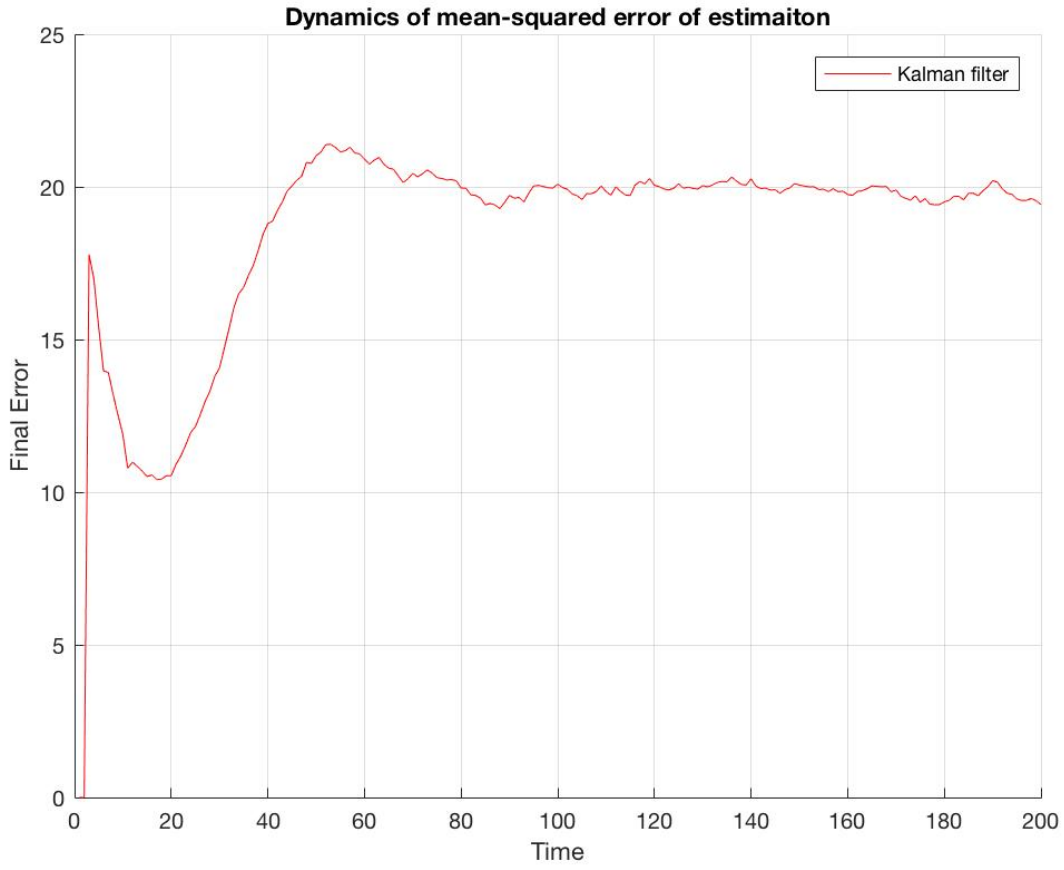
$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}^{biased}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}^{biased}T$$

Also, estimates of state vector $X = | \, x \; V \, |$ by Kalman filter in assumption of unbiased acceleration ($q = 0$) where made:


Kalman filter
True trajectory
Measurements
Kalman Filter
Position
Time
5000
4000
3000
2000
1000
0
-1000
0
20
40
60
80
100
120
140
160
180
200

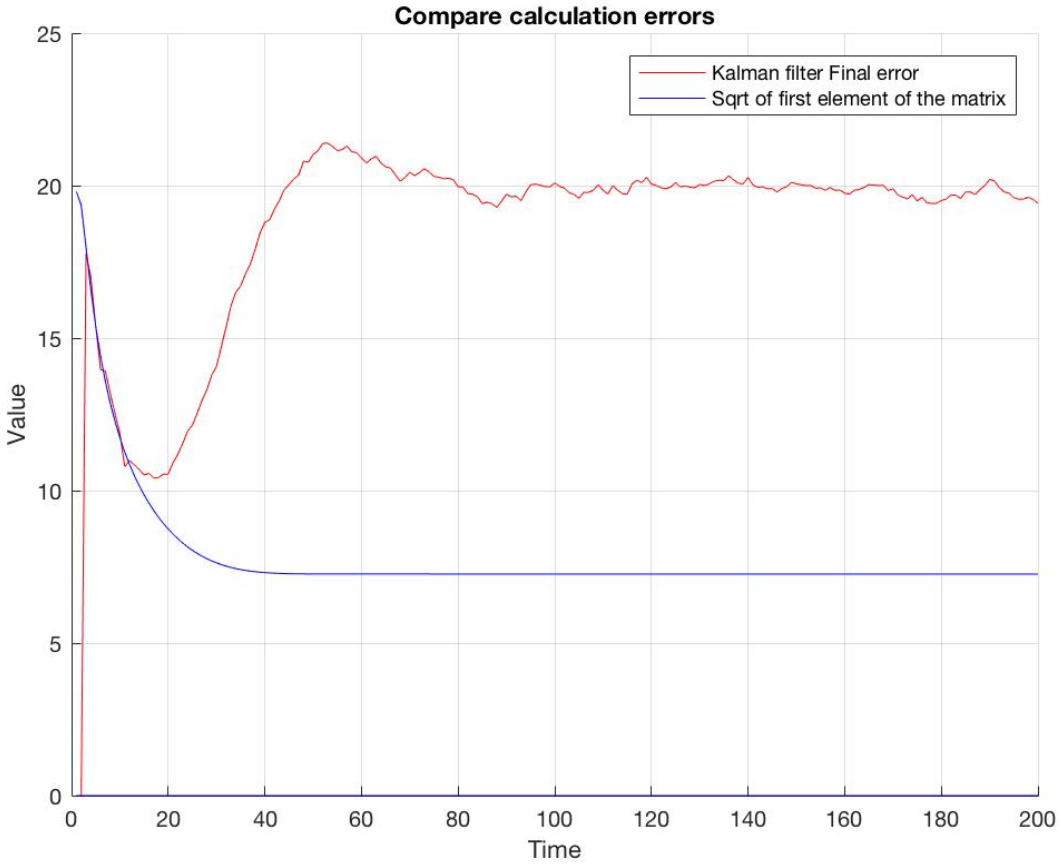

Kalman filter
True trajectory
Measurements
Kalman Filter
Position
Time
1000
900
800
700
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80

After 500 runs of filter and dynamics of mean-squared error of estimation over observation interval was estimated:



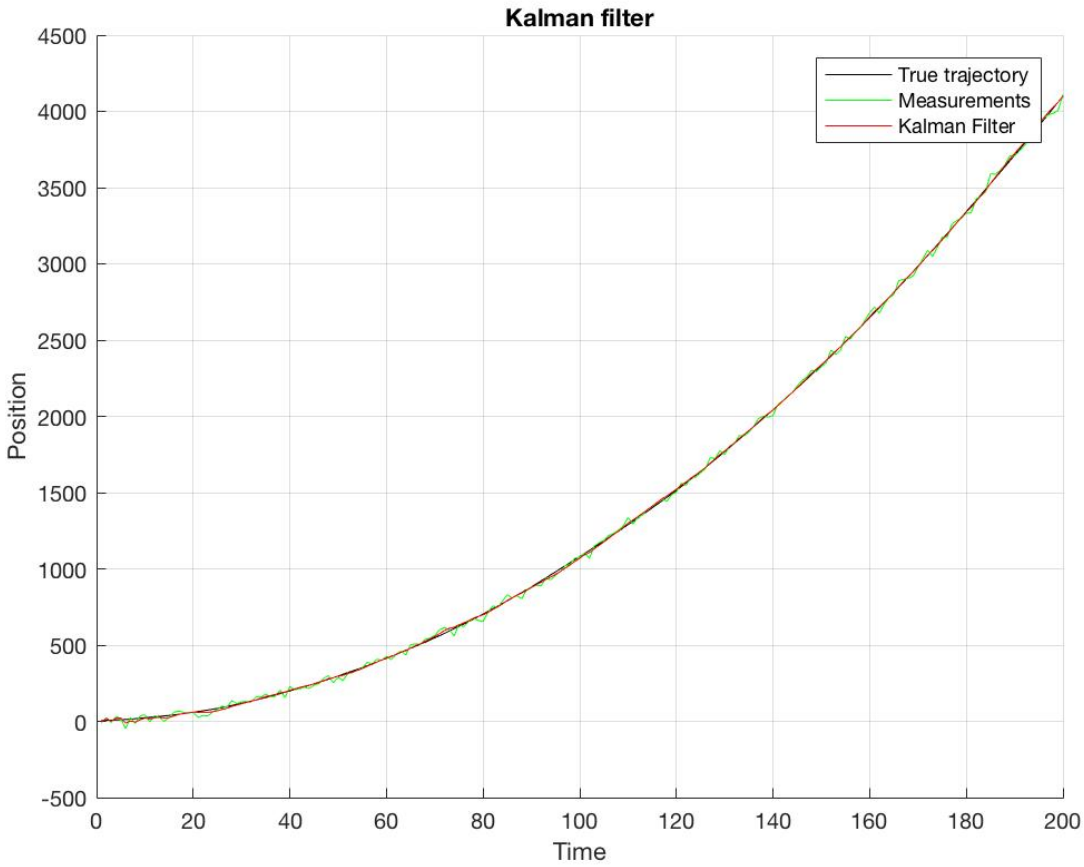
The true error is high – it saturates at about 20.

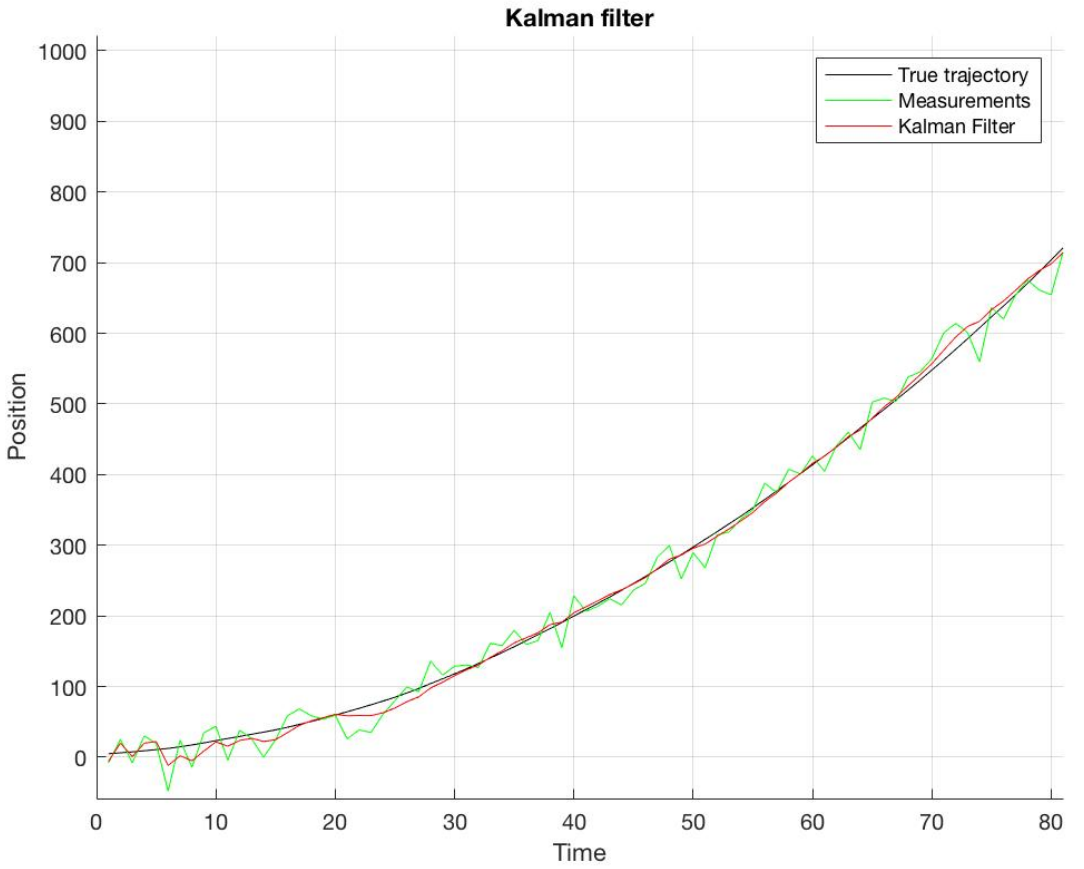
To compare true estimation error with errors of estimation, provided by Kalman filter algorithm, the following plot was made:



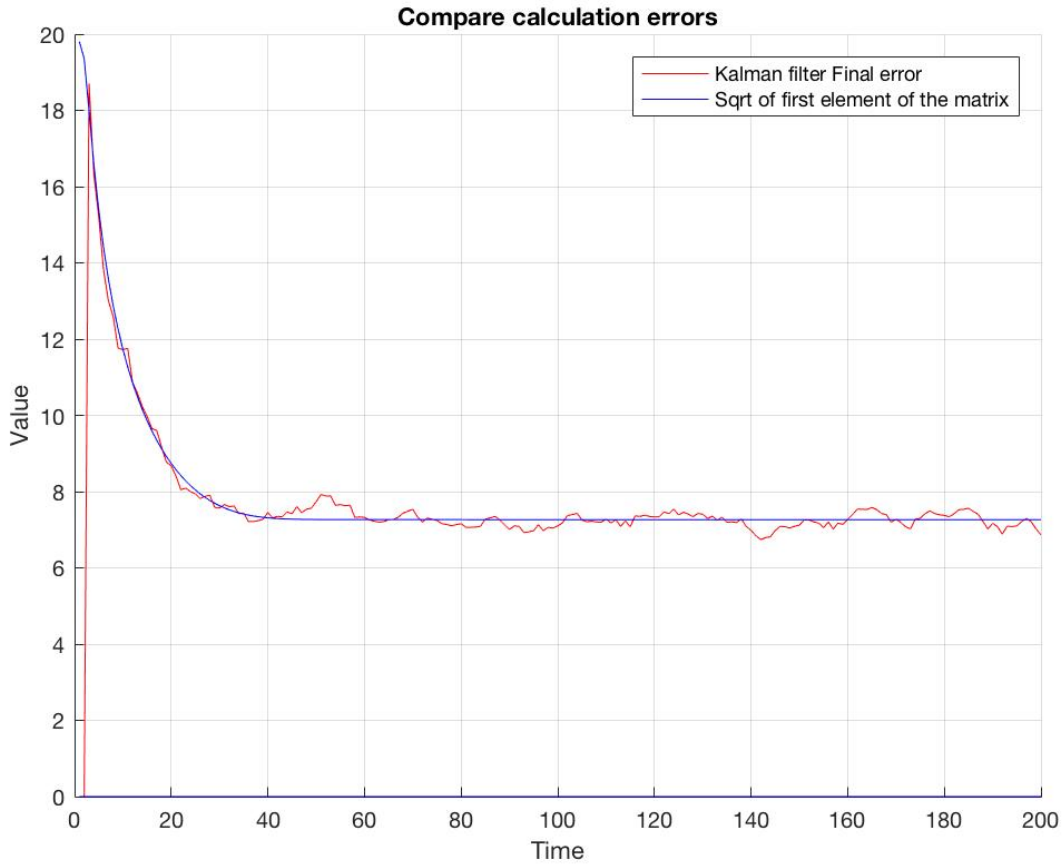
The true error corresponds to calculation errors of estimation only at first steps, then increases and becomes 3 times greater. The filter is non-optimal.

Kalman filter that takes into account bias of acceleration was constructed:


Kalman filter
True trajectory
Measurements
Kalman Filter
Position
Time
4500
4000
3500
3000
2500
2000
1500
1000
500
0
-500
0
20
40
60
80
100
120
140
160
180
200


Kalman filter
True trajectory
Measurements
Kalman Filter
Position
Time
1000
900
800
700
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80

In optimal conditions, true error corresponds to calculation errors of estimation:



Conclusion

Kalman filter should be used in optimal conditions. When Kalman filter that doesn't take into account bias of state and measurement noise, error of estimation greatly increases, because of the method's sensitivity.