Laboratory work 6
Analysis of accuracy decrease of filtration in conditions of correlated biased state and measurement noise

Performance –Tuesday, October 10, 2017
Due to submit a performance report – Thursday, October 12, 2017

The objective of this laboratory work is to analyze the sensitivity of estimation results obtained by a Kalman filter that doesn't take into account bias of state and measurement noise. This will bring about a deeper understanding of main difficulties of practical Kalman filter implementation and skills to overcome these difficulties to get optimal assimilation output.

The first part of this laboratory work is performed in the class by students as in teams of 2-4 on Tuesday, October 10 and the team will submit one document reporting about the performance till Thursday, October 12, 2017. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

This laboratory work consists of one part:

I. Divergence of Kalman filter when bias of state noise (acceleration) is neglected in assimilation algorithm. Development of optimal Kalman filter that takes into account bias of state noise (acceleration).

Here is the recommended procedure for part I:

1. Generate a true trajectory X_i of an object motion disturbed by normally distributed **BIASED** random acceleration. Bias (mathematical expectation) of random noise $q = 0.2$

$$a_{i-1}^{biased} = a_{i-1} + q$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}^{biased}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}^{biased}T$$

   Size of trajectory is 200 points.
   Initial conditions: $x_1 = 5; V_1 = 1; T = 1$
   Variance of noise a_i, $\sigma_a^2 = 0.2^2$

2. Generate measurements z_i of the coordinate x_i

$$z_i = x_i + \eta_i$$

   η_i –normally distributed random noise with zero mathematical expectation and variance $\sigma_\eta^2 = 20^2$.

3. Obtain estimates of state vector $X = \begin{vmatrix} x \\ V \end{vmatrix}$ by Kalman filter in assumption of unbiased acceleration ($q = 0$). Please use already developed code or detailed recommendations how to develop Kalman filter algorithm in Lab 5. In this conditions it will be non-optimal filter.

4. Plot results including true trajectory, measurements, filtered estimates of state vector X_i.

5. Make $M = 500$ runs of filter and estimate dynamics of mean-squared error of estimation over observation interval. Please calculate this error only for filtered estimate of coordinate $x_{i,i}$.

Hint how to do:
Calculate squared deviation of true coordinate x_i from its estimation $\hat{x}_{i,i}$ for every run over the whole observation interval N=200.

$$Error^{Run}(i) = \left(x_i - \hat{x}_{i,i}\right)^2$$

Run – number of run;

$i = 3, \ldots, N$ - observation interval

(please start error calculation from step $i = 3$);

$Run = 1, \ldots, M$ - number of runs;

Find average value of $Error^{Run}(i)$ over M runs for every step i and calculate its square root

$$Final\_Error(i) = \sqrt{\frac{1}{M-1}\sum_{Run=1}^{M} Error^{Run}(i)}$$

This final error is true error of estimation as you used true trajectory to get this error.

6. Compare true estimation error obtained in item 5 with errors of estimation $P_{i.i}$ provided by Kalman filter algorithm

   Hint how to do:
   Make a plot of two curves:
   a) Final error (true estimation error) obtained over $M = 500$ runs according to item 5.
   b) Filtration error covariance matrix $P_{i,i}$ (calculation error provided by Kalman filter) Please use square root of the first diagonal element of $P_{i,i}$ that corresponds to standard deviation of estimation error of coordinate x_i. It doesn't depend on runs, for every run it is the same, as it depends only on model parameters $\Phi, \mathrm{H}, \mathrm{Q}, \mathrm{R}$

   Verify if calculation errors of estimation correspond to true estimation errors.
   As bias of acceleration is neglected in Kalman filter algorithm, than true estimation errors should be significantly greater than calculation errors of estimation $P_{i,i}$ as filter is non-optimal.

7. Develop optimal Kalman filter algorithm that takes into account bias of acceleration (state noise).

   Hint
   Adjust equation to get the predicted (extrapolated) estimate by introducing correction Gq.

8. Calculate again true estimation errors and errors of estimation $P_{i.i}$ provided by optimal Kalman filter that takes into account bias of acceleration (state noise). Compare these errors in optimal conditions.

9. Conclusions.

Performance report

1. Performance report should contain all the items listed

2. The code should be commented. It should include:
   - Title of the laboratory work, for example
     % Converting a physical distance to a grid distance using least-square method
   - The names of a team, indication of Skoltech, and date, for example,
     %Tatiana Podladchikova, Skoltech, 2017
     Main procedures also should be commented, for example
     %13-month running mean
     …here comes the code

3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.