L14: Pose SLAM and Point Cloud registration

* Summary of L13 $\sqrt{SAM}$

Cholesky $\quad A^T A = \Lambda = R^T R$, R upper triangular

solve: $\quad Ry = A^T b$ } backsubstitution

$\quad R^T \delta = y$ } forward substitution

QR

$$Q^T A = \begin{bmatrix} R \\ 0 \end{bmatrix} \quad , \quad \text{solve} \quad R\delta = d$$

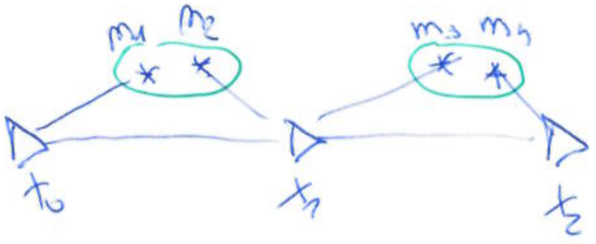
Schur : Λ_m is diagonal $\Rightarrow$ easy to invert.

Eliminate all dependencies from landmarks and then Cholesky.

Ordering of the graph for variable elimination (colperm, coland)

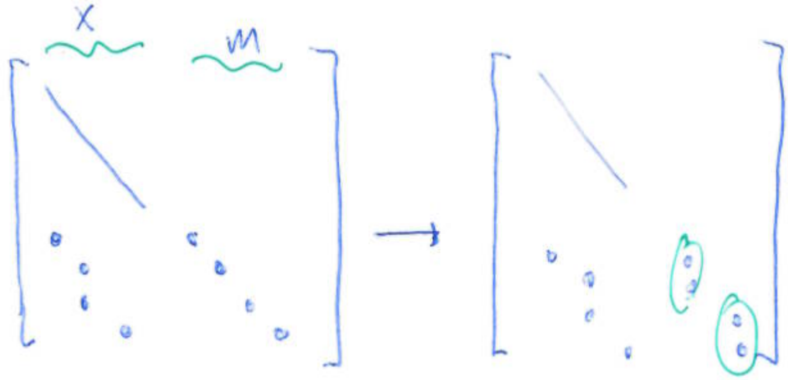
Incremental update of R (iSAM)

* Data Association in $\sqrt{SAM}$



Given the full trajectory, DA allows for easy correspondences over time $c_{0:t}$, while EKF "must" filter to only the last correspondences c_t

$A =$



Grouping landmarks, undoing wrong correspondences, etc. implies a change on A.

Grouping landmarks by a likelihood test → greedy but tractable.

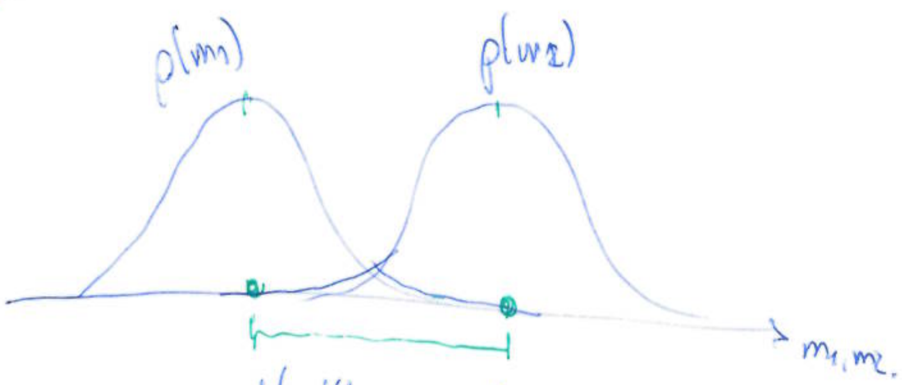
$$\Delta_{j,k} = \begin{bmatrix} m_k - m_j \\ m_j - m_k \end{bmatrix}$$

$$\|\Delta_{j,k}\|^2_{\Sigma_{j,k}} < \chi^2_{d,\alpha} \qquad \left(\begin{array}{l} d = 4 \\ \alpha = \text{confidence interval} \end{array} \right)$$

Other alternatives might work, like conditioning the joint Gaussian. Open problem.

We need the posterior covariance Σ_t, and from there marginalize everything except j, k landmarks.

Ex. 1d



if they are the same landmark Δ_m should be small.

* Covariance in $\sqrt{SAM}$

$\Sigma = \Lambda^{-1}$ (Λ is sparse but inverting is not efficient)

idea: No need to invert Λ, we have R.

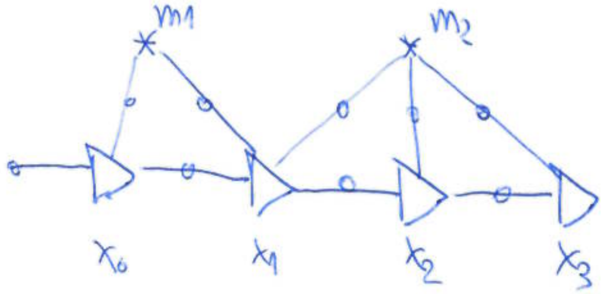
$$\Lambda = A^T A = R^T R = \Sigma^{-1}$$

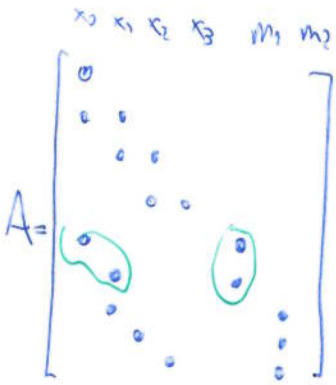
$$\Rightarrow R^T R \cdot \Sigma = I$$

$$\begin{cases} R^T \cdot Y = I \\ R \cdot \Sigma = Y \end{cases}$$

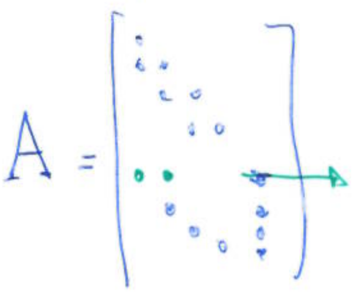
2 back substitutions, now of a matrix (set of vectors)

* Landmark elimination

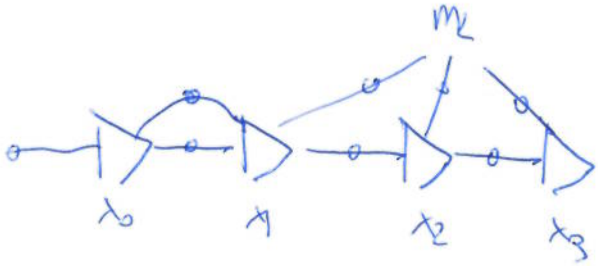




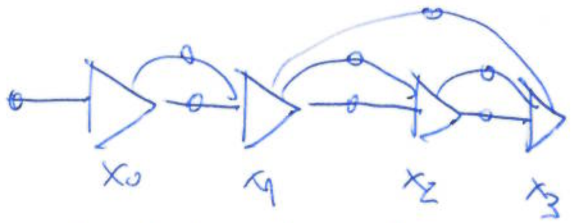
Eliminate m1, will add more factors to substitute the previous factors to m1.



new "observation" relating x_0, x_1 through the removed m1



eliminate m_2



$$\Lambda = \begin{bmatrix} \circ & \circ & & \\ \circ & \circ & \circ & \circ \\ & \circ & \circ & \circ \\ & \circ & \circ & \circ \end{bmatrix} = A^T A.$$

all landmarks eliminated, but new (equivalent) factors have appeared to express the same relations

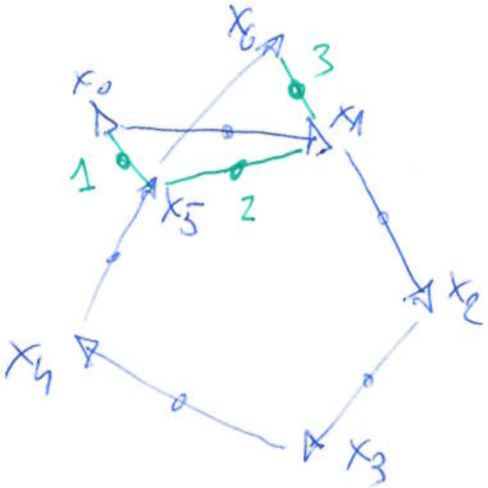
* Relation to the Schur complement.

$$\Lambda = A^T A = \begin{bmatrix} \Lambda_x & \Lambda_{xm} \\ \Lambda_{mx} & \Lambda_m \end{bmatrix} \qquad \delta = \begin{bmatrix} \delta_x \\ \hline \delta_m \end{bmatrix}, \qquad A^T b = \begin{bmatrix} b_x \\ \hline b_m \end{bmatrix}$$

$$\left(\Lambda_x - \Lambda_{xm} \Lambda_m^{-1} \Lambda_{mx}\right) \delta_x = b_x - \Lambda_{xm} \Lambda_m^{-1} b_m$$

the Schur complement is equivalent to eliminate (marginalize) all landmarks in the information matrix. These new factors are a new way to express the marginalization.

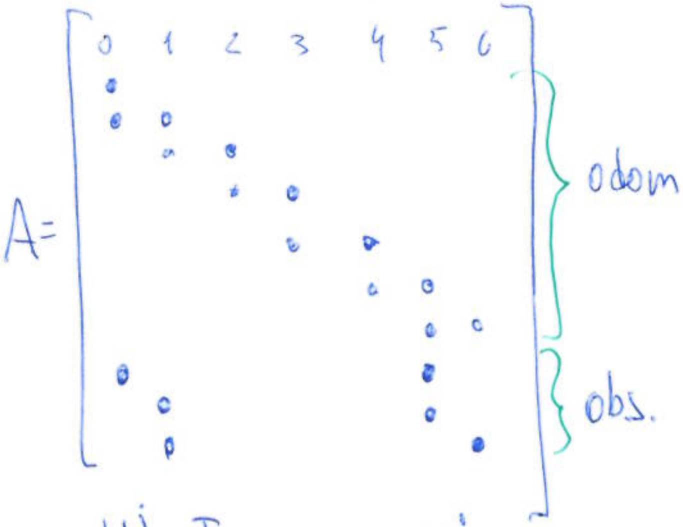
* Pose SLAM : only poses are estimated



factors (observations) :
- Odometry
- Relative pose observations or loop closure.

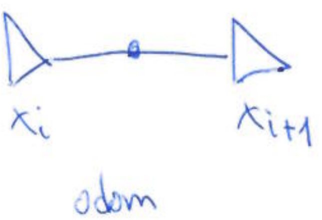
$$h(i,j) = x_j - x_i$$

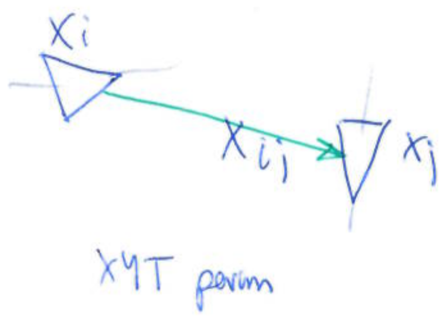
"from pose i we observe j"



$H_k^j = I$ $\qquad$ $H_k^i = -I$

how to generate observations between poses??





$h(i,j) = x_j - x_i$

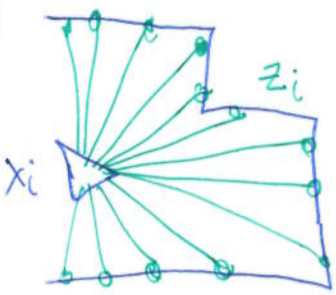
1) Pose SLAM from landmark marginalization (not practical)

2) Virtual observation between poses: In general a Registration problem.

* Point Cloud Registration

PC here can be 3D or 2D, set of points

Ex:



$z_i = \{ p_1, p_2, \dots, p_K \}$

points from sensor (range-finder) observed at pose x_i.

Problem: find the transformation from a pair of poses x_i, x_j, such that (homogeneous coord)

$$\boxed{{}^jT_i \, z_i = z_j}$$

for all points in z.

Called registration, alignment, scanmatching (2D)

Now, this definition considers 2 unrealistic things:

1 – point to point correspondences is correct.

2 – we sample the same points from x_i and x_j

* SVD methods (Arun '87)

If we have enough points, we can assume true the previous considerations

then, we define a cost function to minimize.

$$J(z_i, z_j) = \sum_k^K \| z_j^k - {}^jT_i \, z_i^k \|^2$$

the solution is found by decoupling translation and Rotation:

$$ {}^{i}t_i + E\{z_i\} = E\{z_j\} \quad \text{(same centroids translated)} $$

and then apply an SVD solution of a cross term

$$ \Sigma_{z_i, z_j}. $$

Only 2 points in 2D
3 points in 3D.

Drawbacks:

sensitive to outliers
requires known correspondences

* RANSAC (Fischer '81)

↳ Random sample consensus (General alg. for param. estimation).

We reject/eliminate outliers by sampling a subset of observation correspondences z_j', z_i' minimal for solving:

- SVD-registration $(z_j', z_i') = {}^{j}T_i'$ solve the registrat.
- $J(z_i, z_j, {}^{j}T_i')$ test the hypothesis.

Create a consensus set

* Iterative closest point (ICP)

given two point clouds, z_i, z_j with no a priori correspondences.

1) find closest points

$$d(p_i, z_j) = \min_{p_{z_j} \in z_j} \| p_{z_j} - p_i \| \quad , \quad p_i \in z_i \quad (p_i \in {}^iT_i z_i)$$

Brute force search of a single point vs all the other points. (Kd-tree)

2) Align the pointcloud z_j and $\{p_i\}$ with correspondences.

3) Do while convergence..

Ex:

Of course there will be outliers but iteratively we get rid of them.
SVD techniques are not the most convinient. ⇒
Gradient based techniques work best.

$${}^jT_i := {}^jT_i^0 \oplus \nabla_T J(z_i, z_j)$$

(next Prob Rob Ch 9)