L15 Occupancy Grid Mapping (ProbRob Ch9)

* Summary from L14

Variable elimination = marginalization (Schur)
(Graph) $(\Lambda = A^T A)$

DA in SAM: more options than filtering.

$R^T R \cdot \Sigma = I$ Efficient covariance recovery.

Pose SLAM: only robot poses are estimated.

Loop closure observations: $h(i,j) = x_j - x_i$ Linear obs. function

$H_k^j = I$ $H_k^i = -I$

Point cloud registration.

- SVD methods
- RANSAC
- ICP

* Why do we want maps?

Pose SLAM does not require landmarks. (dimensionality reduced)
How to calculate correctly loop closure observations?

scan - scan

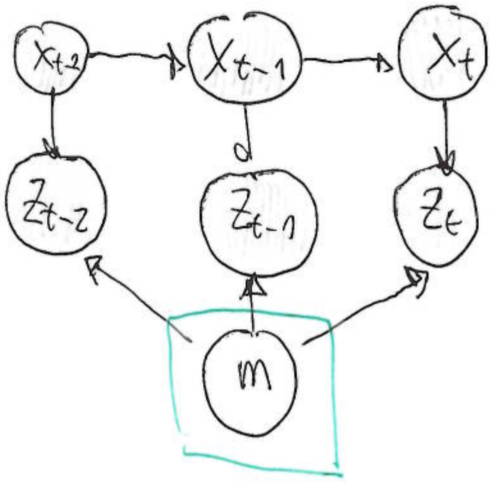
$\boxed{\begin{array}{l}\text{scan - map}\\ \text{map - map.}\end{array}}$

Several point clouds can be group for a "mini" map while doing SLAM
↳ "less noisy.

Planning purposes : Map contains information regarding free traversable space.

After solving SLAM, poses are known.

$p(m \mid Z, x)$



Inference of map

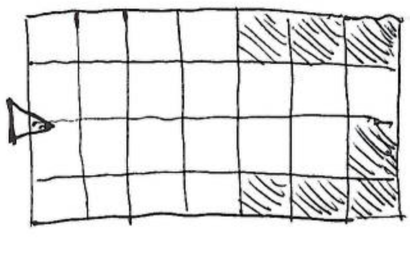
Explicit map: - lines
- objects
- surfaces

Implicit : grid based, voxels, ..
(more general, but more memory).

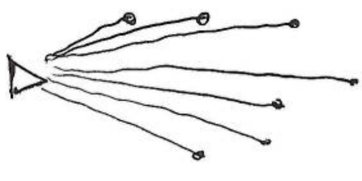
* Occupancy Grid as a Map.

~~Explicit~~ grid.

Ex:



Landmarks



Abstraction of the map

$$m_i = \begin{cases} 0 & \text{free} \\ 1 & \text{occupied} \end{cases}$$ Binary state

$m = \{ m_i \}_{6\times5}$ set of cells = map

↳ the combination of possible maps is huge

Ex: 7x4 grid are $2^{28} \sim 268M$

a map of 100×100 are $2^{10.000} \rightarrow$ Intractable.

We need some approximations:

1) $p(m \mid Z, X) = \prod_i p(m_i \mid Z, X)$ Cells are independent.

2) Inverse model $p(m \mid z_t)$

Too large space

we have used the likelihood function $p(z_t \mid m)$

where it was easier to describe distribution as causes (Bayes)

* Cells as random variables

cells are Binary random vars.

$$m_i = \begin{cases} 0 & \text{free} \\ 1 & \text{occupied} \end{cases}$$

$$p(m_i) = \begin{cases} 0 & \text{free} \\ 1 & \text{occ.} \\ 0.5 & \text{no knowledge} \end{cases}$$

Log odds (ProbRob p. 94)

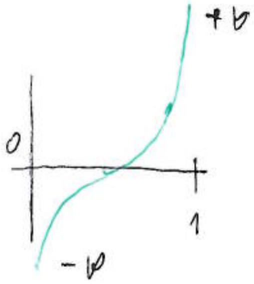
$$\frac{p(x)}{p(\bar{x})} = \frac{p(x)}{1 - p(x)}$$

An alternative way of using pdf's when the state is binary.

$$\Rightarrow \quad l(x) = \log \frac{p(x)}{1-p(x)} \qquad \text{(log odds)}$$

r.v x , $x \in [0, 1]$ (Binary)

r.v l , $l \in (-\infty, \infty)$



$$p(x) = 1 - \frac{1}{1 + \exp(l(x))} \qquad (\exists \text{ inverse})$$

Binary Bayes filter for cells

$$bel(m) = p(m|Z,X) = \prod p(m_i|Z,X)$$

1 cell:

$$p(m_i|z_{1:t},X) = \frac{p(z_t|m_i, z_{1:t-1}, X)\, p(m_i|z_{1:t-1}, x_{0:t})}{p(z_t|z_{1:t-1}, X)}$$

(Markov) $$= \frac{\boxed{p(z_t|m_i, x_t)}\, p(m_i|z_{1:t-1}, x_{0:t-1})}{p(z_t|z_{1:t-1}, X)} \qquad (1)$$ x_t pose is not required.

even for 1 cell this h.H is harder to compute than the inverse model.

$$p(z_t|m_i, x_t) = \frac{\overbrace{p(m_i|z_t, x_t)}\, p(z_t|x_t)}{p(m|x_t)}$$ (this we can calculate.)

substituting in (1)

$$p(m_i \mid z_{1:t}, X) = \frac{p(m_i \mid z_t, x_t)\,\cancel{p(z_t \mid x_t)}}{\underbrace{p(m_i \mid x_t)}_{p(m_i) \text{ for simplification}}} \cdot \frac{p(m_i \mid z_{1:t-1}, x_{0:t-1})}{\cancel{p(z_t \mid z_{1:t-1}, X)}}$$

Markov.

Equivalently for the negated $\bar{m}_i$:

$$p(\bar{m}_i \mid z_{1:t}, X) = \frac{p(\bar{m}_i \mid z_t, x_t)\,\cancel{p(z_t \mid x_t)}}{p(\bar{m}_i)} \cdot \frac{p(\bar{m}_i \mid z_{1:t-1}, x_{0:t-1})}{\cancel{p(z_t \mid z_{1:t-1}, X)}}$$

We are interested in the log odds (quotient):

$$\frac{p(m_i \mid Z, X)}{p(\bar{m}_i \mid Z, X)} = \frac{p(m_i \mid z_t, x_t)\,p(m_i \mid z_{1:t-1}, x_{0:t-1})}{p(m_i)} \cdot \frac{p(\bar{m}_i)}{p(\bar{m}_i \mid z_t, x_t)\,p(\bar{m}_i \mid z_{1:t-1}, X}$$

$$= \underbrace{\frac{p(\bar{m}_i)}{p(m_i)}}_{\text{Prior}} \cdot \underbrace{\frac{p(m_i \mid z_t, x_t)}{p(\bar{m}_i \mid z_t, x_t)}}_{\text{current update}} \cdot \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{0:t-1})}{p(\bar{m}_i \mid z_{1:t-1}, x_{0:t-1})}}_{\text{recursive term.}}$$

$$l_{t,i} = l(m_i \mid Z, X) = \log\left(\frac{p(m_i \mid Z, X)}{p(\bar{m}_i \mid Z, X)}\right) = \dots$$

$$= \underbrace{\log \frac{p(\bar{m}_i)}{p(m_i)}}_{l_0 = -\log \frac{p(\bar{m}_i)}{p(m_i)}} + \underbrace{\log \frac{p(m_i \mid z_t, x_t)}{p(\bar{m}_i \mid z_t, x_t)}}_{\text{Inverse sensor model}} + \underbrace{\log \frac{p(m_i \mid z_{1:t-1}, x_{0:t-1})}{p(\bar{m}_i \mid z_{1:t-1}, x_{0:t-1})}}_{l_{t-1,i}}$$

$$\boxed{= l_{t-1,i} + \text{inv\_sensor\_model}(m_i, z_t, x_t) - l_0}$$

$$p(m_i \mid Z, X) = 1 - \frac{1}{1 + \exp(l_{t,i})}$$

Alg: Occuppancy Grid Mapping

OGM($\{l_{t-1,i}\}$, x_t, z_t):

for all cells m_i:

if $m_i \subset$ 'perceptual field of z_t':

$l_{t,i} = l_{t-1,i} + \text{inv\_sensor\_model}(m_i, z_t, x_t) - l_0$

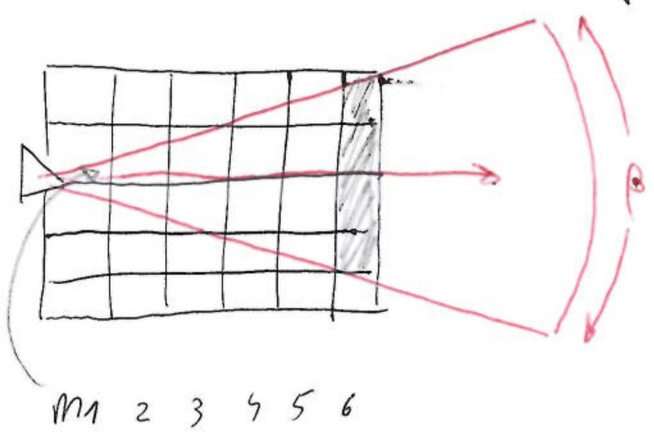
else

$l_{t,i} = l_{t-1,i}$

return $\{l_{t,i}\}$

Ex:

perceptual field



(inv. model)

$m_1 \to l_{free}$

$l_{t-1,1:6} = 0$

$l_{t,1} = 0 + l_{free} - l_0 = -100$

$l_{t,6} = 0 + l_{occ} - l_0 = 100$

(more in ProbRob 288)

the inverse models returns 'occuppied' or 'free' for each cell by raytracing up to the observation. For undetermined results returns l_0.