L08 Localization

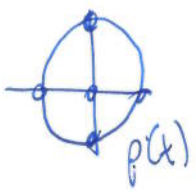
* Summary from L07

EKF : non-linear state estimation

$$x_t = g(x_{t-1}, u_t) \simeq g(\mu_{t-1}, u_t) + G_t \cdot \Delta x_{t-1}$$
$$z_t = h(x_t) \simeq h(\mu_t) + H_t \cdot \Delta x_t$$
Extended KF.

Unscented transformation

A set of sigma points transformed by a function $g(\cdot)$



$$\chi^{[i]} = \mu \pm (\gamma \sqrt{\Sigma_x})_i \qquad 2n+1 \text{ points}$$

$$\gamma = \sqrt{n+\lambda}$$ Sigma points lie on the isocontour of radius γ

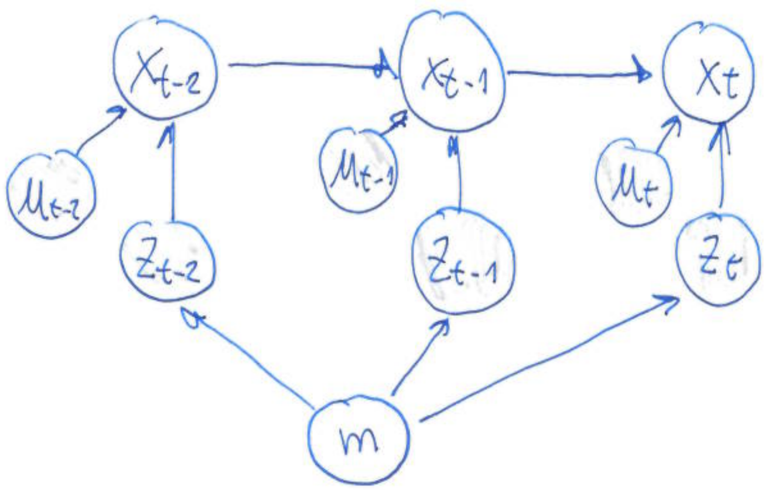
Apply Unscented transformation to the KF $\Rightarrow$ UKF

Canonical form $\mathcal{N}^{-1}(\xi, \Lambda)$

↳ if we derive from Bayes filter the update and pred.:

$\Rightarrow$ Information Filter (IF) $\xrightarrow[\text{linearize}]{}$ Extended IF.

* What is localization position (pose) estimation



Pose inferred from observed data

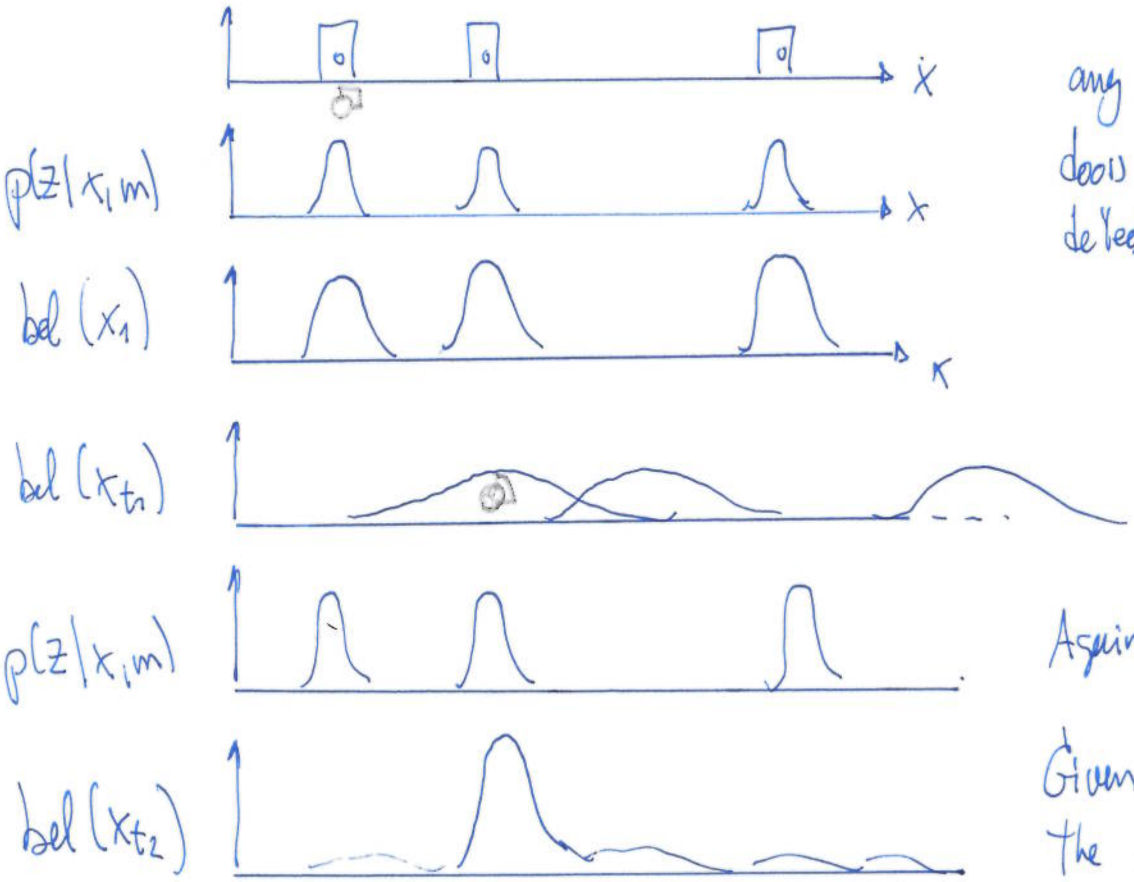
$$bel(x_t) = p(x_t \mid U, Z, m)$$

* **Markov localization** : directly uses Bayes filter.

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}, m)\, bel(x_{t-1})\, dx_{t-1}$$

$$bel(x_t) = \eta \cdot p(z_t \mid x_t, m)\, \overline{bel}(x_t)$$

Ex: 1d 3 doors problem.



any of the three doors could have been detected

Only propagation

Again a door is detected

Given the previous bel the robot is better localiz

* <u>Localization problems</u> (taxonomy)

- local , (position tracking) VS global - x_0 unknown
  given x_0 - kidnapped problem

x_t can change at any time.

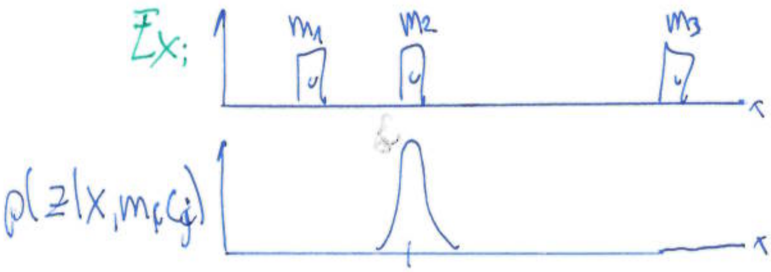
- Static vs Dynamic (moving furniture, doors, snow, ...
- Passive vs Active (exploration, belief planning)
- Single-robot vs Multi-robot

* EKF localization

but Gaussian are unimodal! $p(z|x)$ (on 3 doors we used a multi-mod

we need to solve the data association problem landmark - observa

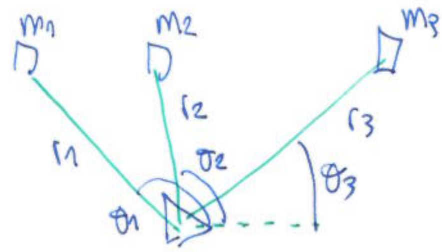
Ex:



We will assume Known correspondences

$c_j = j$ (from landmark m_j)

Ex: 2D



3 observations of each landmark

$$p(z|x,m,c) = \prod_{j=1}^{3} p(z_j|x, m, c_j)$$

Algorithm: EKF localization with Known correspondences

Inputs: $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m$ (Prob Rob 204)

(205) 1: $G_t = \dfrac{\partial g(x_{t-1}, u_t)}{\partial x_{t-1}}$, $V_t = \dfrac{\partial g(x_{t-1}, u_t)}{\partial u_t}$, $M_t^{arc} = \begin{bmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 \cdot v_t^2 + \alpha_4 w \end{bmatrix}$

2: $\bar{\mu}_t = g(\mu_{t-1}, u_t)$ (I) (arc circular model ↑)

3: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + \underbrace{V_t M_t V_t^T}_{R_t}$ (II)

8.4

4: $Q_t = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}$ Range and bearing observation cov. noise. Known correspondences $\Rightarrow$ $\sigma_s^2 = 0$ (eliminated)

5: for $\{ i \mid z_t^i = [r_t^i, \phi_t^i]^T \}$:

6: $$\hat{z}_t^i = \begin{bmatrix} \underbrace{\sqrt{(m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \mu_{t,y})^2}}_{=\sqrt{q}} \\ \mathrm{atan2}(m_{j,y} - \bar{\mu}_{t,y}, m_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{bmatrix} \quad (L06)$$

7: $$H_t^i = \frac{\partial h(x_t)}{\partial x_t}\bigg|_{\bar{\mu}_t} = \begin{bmatrix} -\frac{(m_{j,x} - \bar{\mu}_{t,x})}{\sqrt{q}} & -\frac{(m_{j,y} - \mu_{t,y})}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \bar{\mu}_{t,y}}{q} & -\frac{(m_{j,x} - \bar{\mu}_{t,x})}{q} & -1 \end{bmatrix}_{2\times 3}$$

8: $S_t^i = H_t^i \bar{\Sigma}_t (H_t^i)^T + Q_t$

9: $K_t^i = \bar{\Sigma}_t (H_t^i)^T (S_t^i)^{-1}$

10: $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ innovation vector for z_t^i

11: $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$

12: endfor

13: $\mu_t = \bar{\mu}_t$ $\left(\mu_t = \bar{\mu}_t + \sum_i K_t^i (z_t^i - \hat{z}^i)\right)$

14: $\Sigma_t = \bar{\Sigma}_t$

15: $p_{z_t} = \prod_i \eta \cdot \exp\left\{ -\frac{1}{2} (z_t^i - \hat{z}_t^i)^T (S_t^i)^{-1} (z_t^i - \hat{z}_t^i) \right\}$

(8.5)

Q: Why are we updating the prediction belief $\overline{bel}(x_t)$ I times? Because we assume $\{z_t^i\}$ are independent.

$$bel(x_t) = p(z|x, m, c) \cdot \overline{bel}(x_t)$$
$$= \prod p(z^i|x, m, c)\, \overline{bel}(x_t)$$
$$= p(z^1|x, m, c) \cdot$$
$$p(z^2|x, m, c)$$
$$\vdots$$
$$p(z^I|x, m, c)\, \overline{bel}(x_t)$$

summation of $K_t^i(z^i - \hat{z}^i)$

recussive conditiony

$\prod^{I-1} p()N()$

conditioning a joint Gaussian.

* UKF localization (ProbRob 221)

Exactly the same idea than EKF-loc but sigma-points

we will assume 1 observation at a time t (for simplicity)

$M_t^{arc} \neq M_t^{odom}$ (ProbRob 134), $Q_t = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}$

$$\mu_{t-1}^a = [\ \underbrace{\mu_{t-1}^T}_{},\ \underbrace{[0,0]}_{\varepsilon_{u_t}},\ \underbrace{[0,0]}_{\delta_{z_t}\ (\eta_{z_t})}\ ]^T$$ Augmented state

$$\Sigma_{t-1}^a = \underbrace{\begin{bmatrix} \Sigma_{t-1} & 0 & 0 \\ 0 & M_t & 0 \\ 0 & 0 & Q_t \end{bmatrix}}_{7\times 7} \begin{matrix} \}3 \\ \}2\ (\text{arc model}),\ 3\ (\text{odom model}) \\ \}2 \end{matrix}$$

$$6: \chi^a_{t-1} = \{ \mu^a_{t-1} ,\ \mu^a_{t-1} \pm \gamma \sqrt{\Sigma^a_{t-1}}\Big|_i \} \qquad 2L+1 = 15 \text{ sigma point}$$

→ Prediction step

$$7: \bar{\chi}^{x[i]}_t = g(\chi^{x[i]}_{t-1},\ u_t + \chi^{u[i]}_{t-1}) \quad , i = 0, \dots, 2L \quad , \quad \begin{bmatrix} \chi^{x[i]}_{t-1} \\ \chi^{u[i]}_{t-1} \\ \chi^{z[i]}_{t-1} \end{bmatrix} = \chi^{a[i]}_{t-1}$$

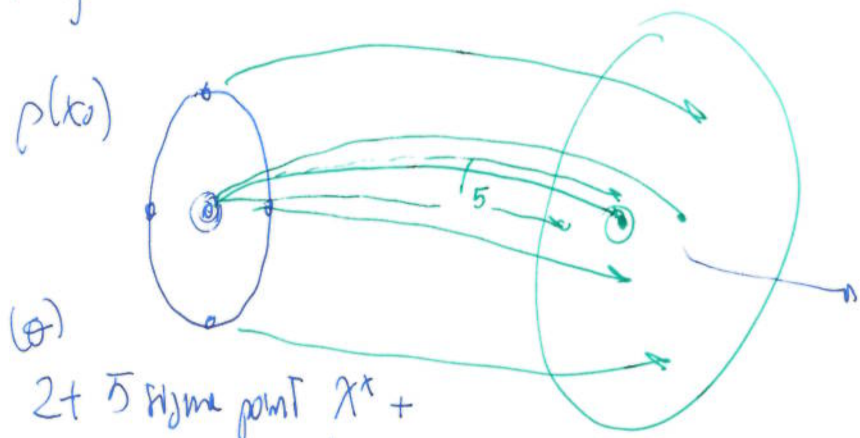
subvectors of $\chi^{a[i]}_{t-1}$

$$8: \bar{\mu}_t = \sum_{i=0}^{2L} w^{[i]}_m \bar{\chi}^{x[i]}_t \qquad (\text{EKF } \mathrm{I})$$

$$9: \bar{\Sigma}_t = \sum_{i=0}^{2L} w^{[i]}_c (\bar{\chi}^{x[i]}_t - \bar{\mu}_t)(\bar{\chi}^{x[i]}_t - \bar{\mu}_t)^T \qquad \mathrm{II}$$

Note: No R in (9) since we are already including it on sigma-points

Ex: prediction.



15 sigma point projected

↳ go to the mean resulting in the same sigma point (χ^z)

2+5 sigma point χ^x +
+ 2 (at mean) for χ^u
+ 2 at mean for χ^z

* Predict expected observations $\hat{z}_t$

10: $\bar{Z}_t = h(\bar{\mathcal{X}}_t^x) + \mathcal{X}_t^z$

11: $\hat{z}_t = \sum_{i=0}^{2L} w_m^{[i]} \bar{Z}_t^{[i]}$

12: $S_t = \sum_{i=0}^{2L} w_c^{[i]} (\bar{Z}_t^{[i]} - \hat{z}_t)(\bar{Z}_t^{[i]} - \hat{z}_t)^T$

13: $\Sigma_t^{x,z} = \sum_{i=0}^{2L} w_c^{[i]} (\bar{\mathcal{X}}_t^{x[i]} - \bar{\mu}_t)(\bar{Z}_t^{[i]} - \hat{z}_t)^T$

Note: All previous sigma points are transformed into observation space $h(x)$ and we add ("noise") sigma points in 3.

* Correction step

$K_t = \Sigma_t^{x,z} S_t^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - \hat{z}_t)$

$\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T$

* Final remarks

Initial (and gentle) introduction to Markov localization given known landmark-obs correspondences.

When not known, other approaches such as, maximum likelihood MHT, full Bayes, etc, ...

Next lecture: Ch 8 ProbRob