

CSCI2824 Assignment 7

Due date: Monday April 18 in class

Submission Format: both, hard hard copy in class and online on moodle

Your answers should be clear and well-organized, and written in full sentences in proper English when asked to provide explanations. Please type your answers, or write VERY neatly. Also, be advised that **late submissions will not be accepted**. Do not forget to staple your hard-copy sheets if you have multiple pages.

Problem 1. (32 pts)

1a. Write out the values of F1(1) through F1(6) for the following function. What does this function do (can you express what's going on here in a brief, informative way)?

Define F1(N)

```
IF N = 1
  RETURN 2
ELSE RETURN [F1(N-1)]2
```

1b Write out the values of F2(1) through F2(6) for the following function. What does this one do?:

Define F2(N)

```
IF N = 1
  RETURN 2
ELSE RETURN 2[F2(N-1)]
```

1c. Write out the values of F3(0) through F3(8) for the following function. What does this function do? Can you express the value that it seems to be approaching?

Define F3(N)

```
IF N = 0
  RETURN 1
ELSE RETURN SQRT(1 + F3(N-1))
```

1d. Write out the values of F4(0) through F4(4) for the following function. Also write out a column showing $(2^{(N+1)} * F4(N))$ for values of N going from 0 to 4. What number do you seem to be approaching?

Define F4(N)

```
IF N = 0
  RETURN SQRT(2)
ELSE
  LET PART = (0.5 * (F4(N-1)))2
  RETURN
    SQRT(PART + (1 - SQRT(1 - PART))2)
```

Problem 2. (32 pts)

2a. Here is a difference table for a function $S(N)$. Your job is to fill in the table, and then write out the closed polynomial expression corresponding to $S(N)$. The third level difference row is assumed to be a constant of 1.

N	0	1	2	3	4	5
$S(N)$	2					
Δ	0					
Δ^2	3					
Δ^3	1	1	1	1	1	1

2b. Write out a closed form expression for the following recurrence:

$$T(0) = 2$$
$$T(N) = 5 * T(N-1)$$

2c. Write out a closed form expression for the following recurrence:

$$T(0) = 2$$
$$T(N) = T(N-1) + 5 * N$$

2d. You have an ample supply of pennies, nickels, dimes, and quarters. How many distinct ways are there of making change for 45 cents?

Problem 3. (18 points)

3a. Suppose you have a drawer with B black socks and W white socks inside it (both B and W are greater than 0). Now, you draw out a sock at random and place it on the bed. Then you draw out a second sock at random and place it next to the first. Write an expression for the probability that both socks are black.

3b. What is the smallest possible number of socks in the drawer for which the probability in 3a is 0.5?

3c. Suppose we know that there are an even number of white socks. Now what is the smallest number of socks in the drawer for which the probability in 3a is 0.5?

Problem 4. (18 points)

In your tennis club there are two other members. One of them is a very good player. His name is Roger (think Federer) and you have a 0.1 probability of winning a match against him. The other player is Joe (think average). He is a decent tennis player and you have a 0.4 probability of winning a match against him.

In a tennis tournament, you have two choices for the sequence of matches:

Roger, Joe, Roger
or
Joe, Roger, Joe

To get a prize in the tournament you need to win at least two matches in a row among your set of three matches. (Note: this does not mean winning two out of three matches; you have to win at least two consecutive matches.)

4a. Which of these two schedules (RJR or JRJ) gives you the better chance at a prize?

4b. What is the expected number of wins for you in the sequence RJR? In the sequence JRJ?