

# Analyzing the Robustness of the Chilean Optical Network

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## ABSTRACT

Chile's propensity for natural disasters makes it necessary to have a resilient Internet network that ensures national connectivity even during crises. In 2015 the Chilean government decided to address this issue, asking our team for a study of the Chilean optical network's robustness and a proposal for improving it. In this work we present our findings and graph-based approach we took to derive a constructed solution and its cost estimate, along with recommendations on how to improve these algorithms, and comparing our cost with the one calculated using an optimal model. We conclude that Chile's Internet network is not resilient and we share how we found a cost-effective routes that need to be built in order to guarantee a robust network. Our results and proposals are currently being used by the Chilean government to analyze project feasibility and prioritize route construction.

## CCS CONCEPTS

• **Computer systems organization** → *Redundancy*; • **Networks** → Network reliability.

## KEYWORDS

network robustness, optical networks

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## 1 INTRODUCTION

Chile is a country highly susceptible to natural disasters: earthquakes, tsunamis, volcanic eruptions, and landslides are just some examples of them. A shared characteristic of such disasters is that they cause damage to large areas. These disasters not only directly impact the population, but also communication networks.

By design the Internet is a resilient network, prepared to correct partial failures and to continue operations while there are alternative paths available. The Internet should be available and operational 100% of the time, especially during great disasters as it allows people to organize and communicate with their loved ones, making it easier for them to stay calm. However in practice that is not always the case. A clear example is the 2010 earthquake in Chile, during which national connectivity failed and most of the Web requests made afterwards went to servers outside the country, saturating the international link [17].

In 2015, the National Secretary of Telecommunications (SUBTEL) asked us to study the *robustness* of the Internet's national infrastructure. For this work, SUBTEL defined robustness as the system's ability to resist disruptions to normal operations, whether these disruptions are random or deliberate. Considering that Chile is administratively divided in 16 Regions, each Region is divided into Provinces, each Province is divided into Communes (there are 346 Communes in the Country), and a city can include a set of Communes and/or a Commune can include a set of towns. SUBTEL requested:

- To check if every Commune's center is connected to every other Commune's center by at least two *disjoint* paths. Where disjoint paths is a pair of paths that not share edges (but could share a node).
- To check if the Commune's center in which the administrative office for Regional Government is located is connected by disjoint paths of optical fiber with at least three other Commune's centers.
- In case of having a non-robust infrastructure, to calculate the investment that would be needed in order to have a robust infrastructure.

The system must be quick enough to:

- support at least daily updates on fiber, road, and/or communications information (made by an *on-the-road* engineer),
- changer restrictions and parameters,
- answer online queries such as “how much will cost to build a network infrastructure for only the 90% of the country's population?”, “give me the keys fibers of the network”, or “what happens now if we link this two cities by a optical fiber”.

Given the NP-complete nature of this type of problem and the dynamic quality of the data we were collecting, we reduced the problem's dimensions by using a simplified version of it, reducing the problem to “obtaining the minimal cost of connecting all Communes to any two other Communes”, for which we first obtained all

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disjoint shortest paths between Communes ordered by cost using Dijkstra's algorithm [10].

Then, in order to check the loss generated by the dimension reduction, we compared the cost of building new infrastructure produced by the latest collected data with the theoretical optimal cost obtained by the Suurballe and Tarjan algorithm [22].

This article presents the process and result of the study commanded by SUBTEL and the a comparison with the optimal value, including some insights appeared by our simplifications. This article does not aim to introduce new algorithms for calculating the robustness infrastructure problem, but using well known algorithms in order to solve a real problem, comparing the engineering and the academic solutions. The rest of the paper is organized as follows: Related Work is presented in Section 2, followed by the work ordered by SUBTEL in Section 3. The process of finding the optimal value is presented in Section 4, followed by the Discussion (Section 5) and Conclusions (Section 6).

## 2 RELATED WORK

In a complex system, when failures are not permanent but rather are disruptions to the normal operation of the system components, the term disruption tolerance applies [21]. Sources of random failures in networked systems include component overload, loss of energy or telecommunication links, erroneous configuration of components, failures of electronic devices, and human error such as accidents and misconfigurations.

Random failures are not the only source of disruptions on a network, deliberate attacks can also cause disruptions to the normal operation of the system components [8]. In those cases the term vulnerability is often used, and the term survivability is used to denote resilience against deliberated attacks (cf. [21]). The usual goal of this type of attacker is to cause the greatest possible disruption to the system [11, 24]. The attack may be a single attack or a sequence of attacks [23]. Here connectivity and load balancing in networked systems that undergo eliminations of elements has been widely studied [5, 6, 19]. The dynamics of spreading failures caused by the elimination of one or more elements of the network percolation is known as percolation, which depending on the nature of the system and the failure may or not lead to an abrupt failure of the whole system [16].

To modify a system in order to increase its robustness is a task of structural optimization where some objective functions that measure the resilience are optimized by adding, removing, or modifying the vertices and the edges, while a set of constraints (such as location, capacity, cost) are respected [5].

When multiple threats exist against a system, the vulnerability of a system as a whole can be quantified in terms of the individual impacts and the occurrence probabilities of the threats [20]. This can be extended to robustness against random failures when the impacts and probabilities can be estimated. The possible events may then be ranked from the most severe (the highest product of impact and likelihood) to the least severe. In both cases, the impact of the damage is not the only measure of interest, but also the speed at which the system is able to recover from is relevant [15].

Strategies and methodologies for controlling the "resilience cycle" (referring to the state changes between operational, broken,

and recovered) [1, 20] emphasize how all systems are prone to failures and that a thorough understanding of the normal operation of a system provides a necessary foundation for identifying possible problems. The basic strategies involve identifying threats, detecting undesirable events, and having formulated response plans for any events that may compromise system functionality. Yet another type of threat are natural hazards that compromise network components in a specific geographic region and possibly with varying degrees of severity [1, 12].

For systems that contain smart components that are able to alter their configuration or behavior, this lets the system adapt and thus dynamically recover from adverse events. Such an adaptation requires identifying possible threats, generating possible actions for each and estimating their expected impact, selecting and executing an action, and then evaluating its effect in order to determine if adequate recovery has been achieved [7].

An alternative to adaptive or structurally optimized systems is to simulate the behavior of a system of interest, identify a potential problem, recreate it in a simulation, and explore possible counter actions to overcome it [18]. In a similar mindset, the study of complex networks has given rise to a field of research that proposes and analyzes generators that artificially produce network topologies with some desired characteristics, such as those of the Internet [13].

For a survey of general structural characteristics for graphs, including those that are not intended to capture resilience-related characteristics, we refer the reader to the work of Costa et al. [9], for a survey on resilience for the specific context of communication networks, to the work of Sterbenz et al. [21], and for a survey on frameworks for the study of robustness using multilayer networks please refer to the work of Bachmann et al. [3].

## 3 PART I: SIMPLE MODEL

The problem of studying the robustness of Chilean Internet infrastructure is not new [2, 14]. However, at the time, these studies did not count with all real world data presented on this study. In this paper, thanks to the support of SUBTEL and the national Ministry of Transport and Telecommunications (MTT), we have access to the following data: the map of optical fibers, the map of under-sea optical fibers, the map of roads, ship routes, and the map of the electrical grid. All of this data was used for two main purposes: analysis of network robustness, and to make suggestions about the addition of new edges to improve the network's robustness. Further information can be found on the project's website <http://www.niclabs.cl/yafun/>.

### 3.1 Bad news: the network is not robust

Part of our work was to check in-situ if the data provided by MTT and SUBTEL was accurate. To do this we traveled the country from South to North during 2 years annotating any differences with the theoretical data we had, that is the main reason we need the dimension reduction on our ILP problem, data can be updated daily or at most weekly<sup>1</sup>.

<sup>1</sup>a side project is to build a web service where all Communes could independently update their communication, electrical and road networks. Thus, data updating could easily be reduced from days to hours.

According to our field study, only 89.3% of the Communes have some kind of optical fiber connectivity. In many cases, the lines could not be verified on their entirety from the public road, thus we can ensure fiber optic connectivity in 263 (76.6%) Communes. To assess the resilience of the network following the SUBTEL guidelines we must only consider the connectivity of nodes that have 2 or more disjoint paths between them. Using the SUBTEL guideline, there were only 59 communes at the largest connected component of our network, meaning that only 17.1% of the network belongs to this component. Giving the high degree of network disconnection we did not perform any other robustness studies over the data, and instead focused on a proposal to improve the country connectivity.

### 3.2 Proposal

Based on the guidelines presented in Section 1, the following assumptions were made for the cases in which complete information was unavailable:

- (1) For the connection between Communes, we considered as central node the coordinates corresponding to the Commune's central park. All the road lines documented were connected from the beginning of the road to the central park coordinates, assuming the same kind of connection. In cases where there were more than one direct routes between two Communes, we only considered the one with the lowest construction cost.
- (2) In the case of islands near the continent, the route used to connect them to the continent was the ferry route.
- (3) We assume that the optic fiber infrastructure is enough to ensure connectivity, as there is a lack of information about the logical network.
- (4) Our proposal considered only the investment cost necessary for the construction of missing lines in order to obtain a robust network (CAPEX), and does not correspond to the total cost of ownership (TCO).

We broke down the data provided by MTT into sets of interconnected coordinates, which were converted to nodes and arcs of a graph. With the transformation of the geometric objects we obtained a graph  $G$ , composed of a set of all  $V$  ( $|V| = 77,991$ ) and a set of arcs  $E \in \{V \times V\}$  ( $|E| = 78,209$ ). Here, 346 nodes in  $V$  correspond to Communes.

For each edge in our graph we calculated its length as the orthodromic<sup>2</sup> path between the coordinates of both points according to the Formula 1, with  $R = 6,378.137$  kms the Earth's ratio. This was preferred to the euclidean distance since Chile is an extremely long country and thus the distances between 2 points within the country are indeed affected by the Earth curvature.

$$d = R \times 2 \arcsin \sqrt{\sin^2\left(\frac{|x_2 - x_1|}{2}\right) + \cos x_1 \cos x_2 \sin^2\left(\frac{|y_2 - y_1|}{2}\right)} \quad (1)$$

In order to minimize the cost of building a robust network, it was necessary to value the construction per kilometer of optical fiber for each of the existing types of roads, that is: already existing optical fiber ( $C_f = 0$ ), electricity supply or copper ( $C_p$ ), roads without cables ( $C_n$ ), and submarine connection in the case of the islands

<sup>2</sup>the shortest distance between two points on the surface of a sphere

( $C_s$ ). With these values we calculated, for each arc  $e_{i,j}$ , a weight  $w_{e_{i,j}}$  equivalent to the construction cost of an optical fiber along that path  $l_{e_{i,j}}$ .

Figure 1 shows a segment of the map of the Araucanía Region, in which the road documented between Nueva Toltén (A) and Teodoro Schmidt (C) can be seen. This road has two different paths, a first path between Toltén and the junction to Carahue ( $e_{A,B}$ ) with a length of  $l_{e_{A,B}} = 10.70$  kms. with a construction cost of  $C_f$  per kilometer; and a second section from the junction to Teodoro Schmidt ( $e_{B,C}$ ) without any type of road or lines, with a length  $l_{e_{B,C}} = 13.48$  kms. and construction cost of  $C_n$  per kilometer. Thus, the cost from Nueva Toltén to Teodoro Schmidt will be of  $w_{e_{A,B}} + w_{e_{B,C}}$ , which is equivalent to  $l_{A,B}C_f + l_{B,C}C_n$ .

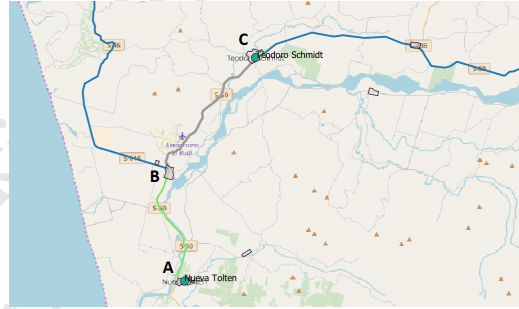


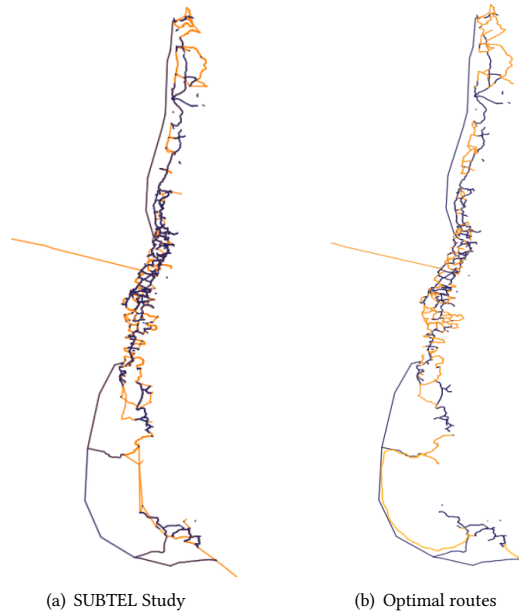
Figure 1: map with an example of construction costs.

The cost values informed by the MTT were (all values in USD):

- $C_f = \$0$
- $C_n = \$25,000$  per kilometer (includes installing the cable poles and the wiring itself)
- $C_p = C_n - \$8,000$  per kilometer already wired.
- $C_s = \$35,000$  per kilometer.
- $C_{ferry} = C_n$ .
- $C_a$  (no road at all) =  $1.41C_p$  per linear distance between Communes.

Once we defined the construction cost for every path, we proceed to build a matrix running the Dijkstra algorithm to get the lowest cost that a connection between Communes can have. The resulting matrix (named  $A$ ) corresponds to the adjacency matrix of a normalized graph, in which each element  $A_{i,j}$  corresponds to the lowest value of directly connecting the commune  $i$  with the commune  $j$  (if there is already fiber, the cost is \$0). Resulting matrix  $A$  can be graphically seen in figure 2 where the existing paths (black) and proposed paths (orange) are shown.

Given matrix  $A$  as previously described, let  $B$  be the matrix with the necessary paths to have a robust optical network that covers 100% of Communes. The following optimization problem is



**Figure 2: Map of the two proposals, in blue the actual fiber and in orange what has to be built.**

formulated:

$$\begin{aligned}
 C &= \min \sum_{i=1}^{N-1} \sum_{j=i+1}^N (B_{i,j} \times A_{i,j}) \\
 \text{s.t.} \\
 B_{i,j} &\in \{0, 1\} \\
 \sum_{j=1}^N B_{i,j} &\geq 2 \quad \forall i
 \end{aligned}$$

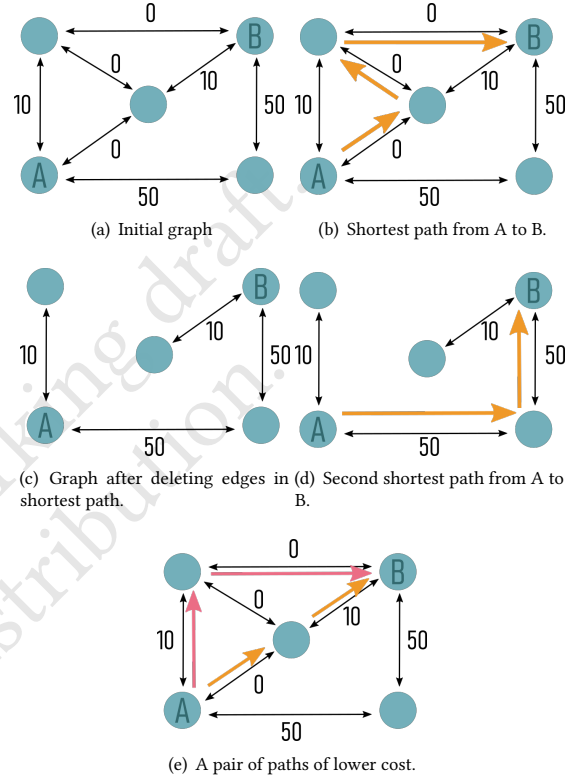
Here  $C$  denotes the cost of building a robust network, which is in this case, following SUBTEL requirements, \$447M (millions of US dollars).

#### 4 COMPARING THE STUDY AGAINST THE OPTIMAL VALUE

Once the experience under SUBTEL requirements finished, we aimed to know how far we were of the optimal value not including the simplifications, that is, using fiber, roads, and power maps as they are (the full graph). In this case, we took the original graph  $G$ , composed of a set of all cities and towns ( $|V| = 77,991$ , instead of the 346 of previous section) and a the road, ferry, telephone and electrical power networks that connect them  $E \in \{V \times V\}$  ( $|E| = 78,209$ ). Also, given the nature of the infrastructure of the country new drawbacks appeared.

First, our Dijkstra variation strategy chooses the two shortest paths that connect each commune with other two communes, forcing that at least one of them passes for a third commune. Of course, this does not always guarantee that the total cost of these paths is optimal. An example of this behavior can be seen in Figure 3,

where the Dijkstra variation would output a total cost of 100, even though we can see that another routing would cost 20. Furthermore, this variation may fail to find two paths connecting the vertices even when they are available as shown in Figure 4. Here, the first part of the algorithm disconnects the graph as it was explained by Suurballe and Tarjan in [22]. Therefore, the Suurballe and Tarjan algorithm was used to find the pair of disjoint paths among cities.

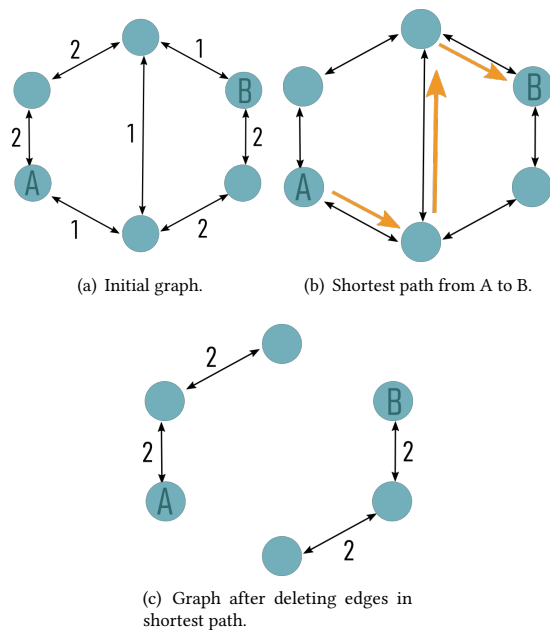


**Figure 3: Given the graph in (a), we want to find the pair of shortest paths from A to B. We begin by calculating the path of lowest cost and deleting its edges from the graph, (b) and (c). Then, we run Dijkstra's algorithm again to find the second shortest path from A to B, as shown in (d). The sum of the resulting paths' costs is 100, while in (e) we show two disjoint paths with a total cost of 20.**

Secondly, the requirement that each commune has to be connected to at least two other communes did not ensure a single connected component. The same can be observed for the requirement of having two disjoint paths between communes. Indeed, if three communes form a triangle with each commune being connected to the other two, but these three communes are isolated from the rest of the network we can generate several connected components that meet the requirements and do not form a single connected network. To ensure the existence of a single connected component we checked the resulting network once the minimum cost was obtained by solving the optimization problem.

Optimal cost found by Suurballe and Tarjan algorithm was \$363 millions of dollars. The optimal network map is shown in Figure





**Figure 4: It is not possible to use the Dijkstra method on the graph in (a) to find the pair of shortest paths between vertex A and B. Deleting the edges that form the path found in the first run of Dijkstra's algorithm, pictured in (b), disconnects A and B. This makes it impossible to find a second shortest path in (c).**

2(b). Therefore, the solution presented to SUBTEL was is approximately 23% more expensive than the optimal solution. However, in the hardware assigned for the processing (a 24 cores server) the value is obtained within hours, while the simplified solution produces results in only minutes. Also, some insights appeared while comparing both solutions, which are discussed in the following Section.

## 5 DISCUSSION

A problem similar to presented on Figures 3 and 4 arose within communes located in the northern and southern area of Chile. This phenomenon can be explained because northern and southern communes cover large extensions with low population density, thus with less alternatives for paths, and even that only the double-path connection requirement was applied, the disjoint routes between communes increased the cost in infrastructure building.

In the North, the simplified solution build four new links joining Pozo Almonte with San Pedro de Atacama through Ollagüe which add around \$20 millions to the budget, that decision increased the network robustness with a new link through the countrys center instead of using near located infrastructure edges (routes and power grid network use to be near each other).

In the South, the optimal solution produces an isolated cluster-like connected component given that the new link follows the ferry path, which is almost the same that the path followed by the Austral National Fiber. Simplified solution forces the isolated Communes

to be connected by five links located through the Andes with a building cost of \$36.5 millions. These five links and their building costs are:

- (1) Villa O'higgins to Puerto Natales \$12.82 millions,
- (2) Cerro Castillo to Villa O'higgins \$10.99 millions,
- (3) Rio Verde to Cerro Castillo \$5.89 millions,
- (4) Puerto Williams to Pampa Guanaco \$4.39 millions, and
- (5) Punta Arenas to Rio Verde \$2.41 millions.

Therefore, even though our strategy is more expensive in terms of money, building infrastructure where there is no road at all aiming to join two communes by a third one, is more robust in terms of optical links because they are more geographically separated.

Finally, the simplified solution that connects low-density and isolated populations (extreme north and south of the country), increased the budget from the optimal value in almost a 16%, and building them it is a state-level decision that has to be taken by SUBTEL and MTT. When both extremes are out of the discussion (e.g., asking the cost of an at least two-connected Internet infrastructure for the 90% of the country population) the simplified and optimal solution produced similar costs.

## 6 CONCLUSION

In this work we presented our findings on the study of Chilean national optical infrastructure. We aimed to obtain a full national infrastructure study, a proposal to extend and improve the network, and an estimation of the cost of this improvement. In this paper we have shown that Chile's Internet network is not robust, and we show a way to find cost-effective routes to be added that improve the network robustness using a Dijkstra + ILP approach. We also compared the cost obtained through our approach with the optimal cost obtained using the Suurballe and Tarjan algorithm. The proposal presented in this paper is currently being used by the government to analyze the project feasibility, and to prioritize route construction. All information can be found on the project's website <http://www.niclabs.cl/yafun/>.

As future work we would like to study novel algorithms to find the optimal recommendation giving all constraints proposed by SUBTEL, explore different metrics and methods to assess the robustness of the system, and to improve the algorithms performance through parallel programming following the recommendations of Banerjee, Ghosh and Reddy [4].

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