Question 2

Optimization Problem:

true position

$d_{T_i} = \left\| \begin{bmatrix} x_T \\ y_T \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\|$

$n_i \sim \mathcal{N}(0, \sigma_i^2)$

MAP estimate of object position: $r_i = d_{T_i} + n_i$

$$\underset{\theta}{\arg\max}\{P(\theta|R)\} \quad \text{where } R \in \{r_1, \dots, r_k\}, \quad \theta = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\underset{\theta}{\arg\max}\{P(R|\theta)\,P(\theta)\}$$

Where: $P(\theta) = (2\pi\sigma_x\sigma_y)^{-1} e^{-\frac{1}{2}\theta^T\Sigma^{-1}\theta}$ where $\Sigma = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}$

and $P(R|\theta) \sim \mathcal{N}(d_{T_i}, \sigma_i^2) = \prod_{i=1}^{k} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(r_i - d_{T_i})^2}{2\sigma_i^2}}$

$$\underset{\theta}{\arg\max}\{\ln P(R|\theta) + \ln P(\theta)\}$$

$$\underset{\theta}{\arg\max}\left\{\sum_{i=1}^{k}\left[-\frac{1}{2}\ln(2\pi\sigma_i)^2 - \frac{(r_i - \overbrace{d_{T_i}}^{\|\theta_T - \theta_i\|})^2}{2\sigma_i^2}\right] - \ln(2\pi\sigma_x\sigma_y) - \frac{\theta^T\Sigma^{-1}\theta}{2}\right\}$$

Removing terms that won't impact map estimate:

$$\underset{\theta}{\arg\max}\left\{-\sum_{i=1}^{k}\frac{(r_i - d_{T_i})^2}{2\sigma_i^2} - \frac{\theta^T\Sigma^{-1}\theta}{2}\right\}$$