

Write intro

### Construction of an $N^3\text{LO}$ DIS scheme

During my Master Thesis, under the supervision of Dr. Marco Bonvini and in collaboration with another Master student, I worked on the development of a so-called variable flavor number scheme (VFNS) for deep-inelastic-scattering (DIS) predictions at next-to-next-to-next-to-leading order ( $N^3\text{LO}$ ) in perturbation theory. This is needed to correctly consider the heavy quarks mass effects when computing theory predictions electron-proton scattering, whose understanding is crucial in PDFs fit. Different proposal exist in the literature and they are all equivalent at all order in perturbation theory but differ at any given order in the way the different ingredients are combined. What is crucial to discuss is that all of these schemes consider the heavy quark PDF of  $\mathcal{O}(\alpha_s^0)$ . Instead, our construction takes into account the fact that the heavy quark PDFs are generated perturbatively and therefore have to be considered of  $\mathcal{O}(\alpha_s)$  (this is true in the case in which intrinsic components are neglected, while in the case in which they are not the scheme will have to be modified disentangling the intrinsic part from the one perturbatively generated).

Currently, the ingredients needed for such a construction are completely known up to NNLO in perturbation theory, while at  $N^3\text{LO}$  there are still some missing informations. Even though I was also involved in the construction of the scheme, the bulk of the work I did during my Master thesis was to construct an approximation of the unknown terms of the  $N^3\text{LO}$  partonic cross section for DIS (the so-called coefficient functions) by combining some known limits. In this way it was possible to construct a VFNS at approximate  $N^3\text{LO}$ .

Ho approfondito bla bla bla

This result is important since it allows a better understanding of DIS theoretical predictions on two levels. First of all our VFNS will allow a more precise treatment of the heavy quark PDF. Moreover, the inclusion of the  $N^3\text{LO}$  corrections, even if in an approximate form, allows a better description of the DIS data and therefore it will increase the accuracy of the PDFs fits, that are fundamental for every theory prediction in high energy physics.

As a result of this work two codes were written to produce the results. Both of them are now public and open source. The first one, <https://github.com/niclaurenti/adani>, is a C++ code which computes the approximation of the heavy quark coefficient functions at  $N^3\text{LO}$ . The reason why the C++ language was chosen is that in the  $N^3\text{LO}$  massive coefficient function there are some terms that are exactly known but only in numerical form since they are obtained through the integration of the lowest order coefficient functions and splitting functions. Therefore, a compiled code was more suitable to handle the heavy integrals that are required to be computed.

Then a Python code, [https://github.com/andreab1997/DIS\\_TP](https://github.com/andreab1997/DIS_TP), whose aim is to combine the different ingredients to actually compute the theory predictions with our scheme.

Regarding this work, a publication is in preparation.

## PDFs fit

During my Ph.D. I continued with study of phenomenology in high energy physics under the supervision of Prof. Stefano Forte. In particular I focused on the topic of PDFs fits: PDFs are objects that link the hadronic cross sections, that are what is actually observed experimentally, to partonic cross sections, that are what is computed theoretically. Being non-perturbative objects, the PDFs cannot be computed in perturbation theory but they must be fitted from data. Their extraction is a fundamental step in the computation of theory predictions in hadronic collisions, like the ones happening at the LHC, since every observable depends on the PDFs. I delved into this subject joining the NNPDF collaboration, where I had the opportunity to study both the theoretical and the computational aspects.

My main project has been the inclusion of Quantum Electrodynamics (QED) effects in PDFs fit. The final aim was to produce the QED fit of the NNPDF4.0 series that replaced the old NNPDF3.1QED PDF set. In order to achieve it, different things had to be implemented in different codes.

The first step is the implementation of QED corrections to the PDF evolution equations, i.e. the DGLAP equations. They are a system of integro-differential equations, whose solution is highly non-trivial. **expand a bit** The solution of DGLAP equations, in presence of QED effects, has been implemented in the public code EKO (<https://github.com/NNPDF/eko>), a code written in Python that computes the solution of the DGLAP equation with the technique of Mellin transform. Before I joined the NNPDF collaboration, it could handle only the pure QCD case, but I extended it to consider also QED corrections. This passes through various steps: find a PDFs basis that maximally decouple the system, write the DGLAP equations in this basis, compute the Mellin transform of the known QED corrections of the splitting functions (i.e. the  $\mathcal{O}(\alpha)$ ,  $\mathcal{O}(\alpha\alpha_s)$  and  $\mathcal{O}(\alpha^2)$ ) and analytically continue them in the complex Mellin space and in the end find a suitable numerical solution of the system in Mellin space.

The second step has been the extension of the NNPDF fitting code (<https://github.com/NNPDF/nnpdf>) to consider the presence of a photon PDF. Indeed, once we consider QED effects, that means that we allow photons emissions from the quarks inside the proton, we have to consider also a photon PDF. This is obtained through the LuxQED method [1, 2]: with this approach the photon PDF is linked through a perturbative calculation to the DIS structure functions. Since the structure functions themselves depend on the quarks and gluon PDFs, the LuxQED method links the photon PDF to the “QCD” PDFs. I interfaced the FiatLux code, a public code written in C++ that implements the LuxQED formula, with the NNPDF code. In this way the fitting code is able to include the photon contribution in photon initiated processes, DGLAP evolution and momentum sum rules.

A publication for this work in preparation and preliminary results were presented in [3].

I was also involved in the approximate N<sup>3</sup>LO PDF determination with the NNPDF4.0 methodology. Indeed, almost all the ingredients needed for a PDF fit are exactly known up to NNLO, while at N<sup>3</sup>LO only partial informations are available (this is precisely true only for DIS predictions, as I mentioned in the past section, but not for all the proton-proton collision processes whose data are included in the fit). The aim of this project was to include the N<sup>3</sup>LO corrections, using the exact results when available and an approximate one for the missing terms. I was mainly involved in the implementation of the approximate N<sup>3</sup>LO massive DIS coefficient functions in the NNPDF code. In order to do it, I interfaced the code I discussed about in the past section with the NNPDF code. This involves some technical problems like the interface of a C++ code with a Python one, or the distribution of a compiled package for all possible operative systems. **Parte un po' inutile**

## Side projects

During my Ph.D. I also had the opportunity to work on different side projects.  
[4]

## References

- [1] Aneesh Manohar, Paolo Nason, Gavin P. Salam, and Giulia Zanderighi. How bright is the proton? A precise determination of the photon parton distribution function. *Phys. Rev. Lett.*, 117(24):242002, 2016.
- [2] Aneesh V. Manohar, Paolo Nason, Gavin P. Salam, and Giulia Zanderighi. The Photon Content of the Proton. *JHEP*, 12:046, 2017.
- [3] Niccolò Laurenti. Inclusion of qed corrections in pdfs fits. *Nuclear and Particle Physics Proceedings*, 2023.
- [4] Niccolò Laurenti, Tanjona R. Rabemananjara, and Roy Stegeman. Approximating missing higher-orders in transverse momentum distributions using resummations, 2022.