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Influence of probabilistic abstention on spatial cooperative games

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"Controversies in Game Theory"

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Contents

Individual Contributions

Who did what....

Introduction

Here we write the introduction.

1 Theoretical background

1.1 Game theory

1.2 Games

1.3 Spatial games

1.4 Abstention

2 Implementation

The game has been implemented in Python, the code is publicly available on GitHub [1]. The spatial environment has been implemented as a 50×50 2D grid. A player is present at every point of grid. Every player is characterized by a strategy (collaborate / defect) and by an abstention probability $0 \leq \alpha \leq 1$. The game is defined by the five parameters T, R, P, S, L and by a noise factor $0 \leq \eta \leq 1$. The first five parameters determine the game played: prisoner dilemma (PD), snowdrift (SD), stag hunt (SH) or harmony game (HG).

The game is initialized assigning to each player a random strategy, with a $50-50$ probability, and an abstention probability α. The latter can be assigned in three different ways:

- No abstention: $\alpha = 0$ for all players. The game is reduced to the standard prisoner dilemma (PD) or another standard game
- Optional abstention: $\alpha = 0$ or 1 with a $50 - 50$ probability. The game is reduced to the optional prisoner dilemma (OPD) or the equivalent for another game
- Probabilistic abstention: $\alpha \in [0, 1]$. The players have a finite probability of absteining, uniformly distributed between 0 and 1. We call this situation prisoner dilemma with probabilistic abstention (PDPA) or the equivalent for other games.

At each iteration of the game, each player plays against their four nearest neighbours. Each player earns an amount equal to the sum of the payoff of the four games they played. The payoff of a single match between two players is determined by the strategy of the two players and by their abstention probability. Each player joins the game with a probability $1 - \alpha$. If both players join the game, the game is played and payoffs are distributed according to the players strategies. If one of the players abstains, the game is not played, and both players receive a payoff L.

After every player has played, the imitation process begins. The imitation process is influenced by the noise parameter η. Each player changes their strategy to the opposite strategy with a probability η and imitates their best performing neighbours with a probability $1 - \eta$. If the imitation option is chosen, the player looks at their four nearest neighbours payoffs, and imitates the strategy of the one who performed the best. If more players share the same optimal outcome, a random one is chosen. In addition to imitating the best-performing player's strategy, the abstention probability is imitated as well.

The process is iterated for a fixed number of times t (usually $t = 200$). At each iteration, the players play the game and imitate their neighbours' strategies and abstention probabilities. At the end of the cycle, some statistics are extracted from the final configuration. Taking inspiration from Cardinot *et al.* [**cardinot2018**] we evaluate the mean abstention probability $\langle\alpha\rangle$ and the mean the effective cooperation frequency $\varepsilon = (1 - \alpha) \cdot (1 - s)$, where $s = 0$ for a cooperator and $s = 1$ for a defector. In addition to these two quantities, we also analyse the correlation between being a cooperator and having a low abstention frequency α. To do so, when playing the version of the game with probabilistic abstention, we plot the number cooperators and defectors for each value of α and compare the results.

3 Results

First of all we start by comparing the results we derive with the ones obtained by [**cardinot2018**] for the dependence of effective cooperation frequency on the temptation payoff the with the ones we obtain:

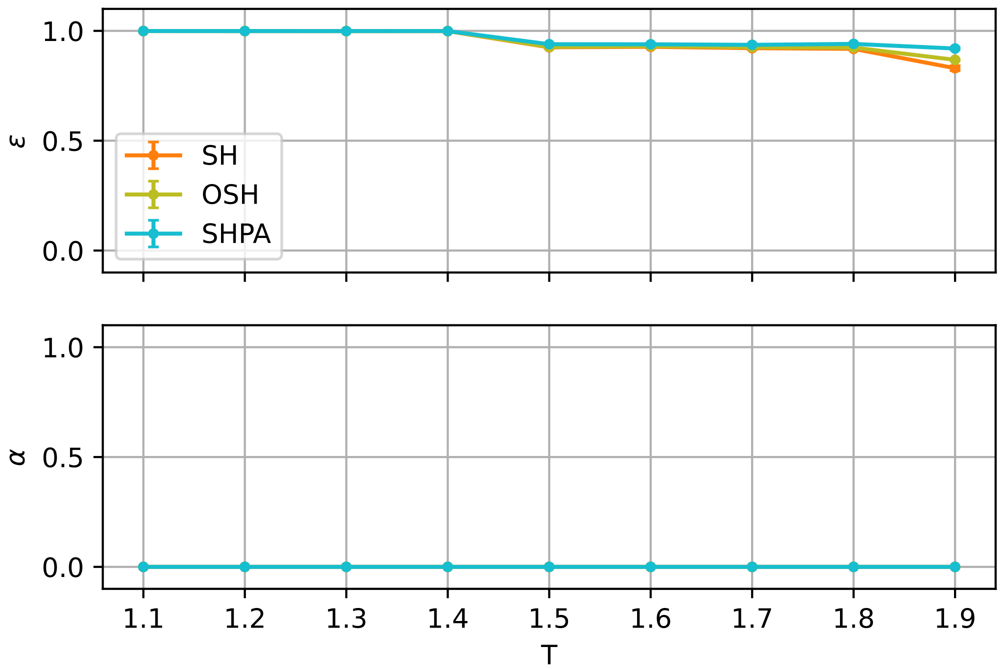
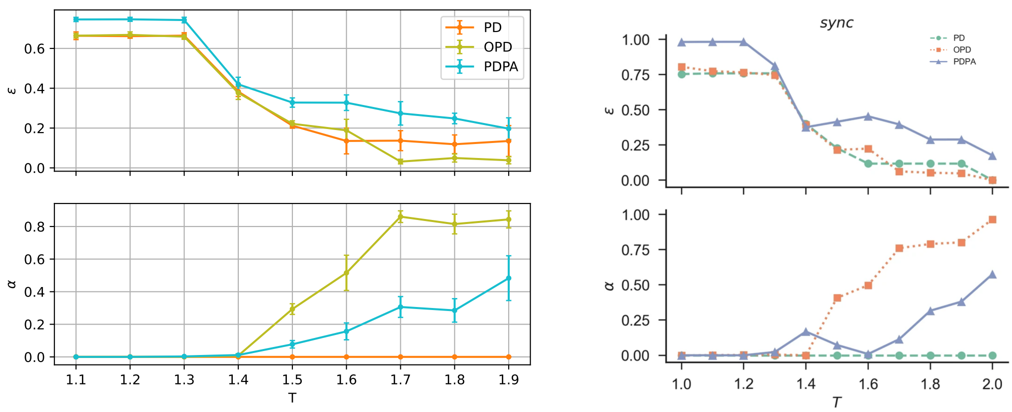


Figure 2: R = 1 S = 0 P = 0 L = 0.4 noise = 0.001 t = 200



(a) **(b)**

Figure 1: (a) shows the dependence we obtained for α and ϵ on T where we set R = 1 S = 0 P = 0.3 L = 0.4 noise = 0.001 t = 200. The results are obtained after averaging 10 runs (b) correspond to the results obtained from Cardinot et Al [**cardinot2018**].

Once we have made sure that the results are compatible with the ones derived we proceed to analyze the other possible games. We start by looking at snowdrift

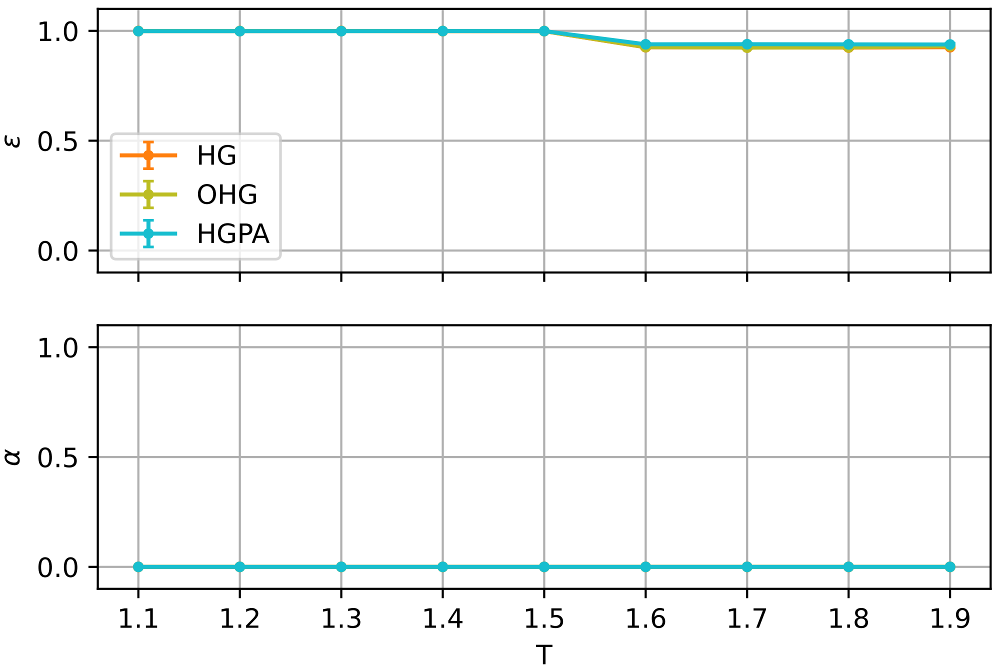


Figure 3: R = 2 S = 0.3 P = 0 L = 0.4 noise = 0.001 t = 200

4 Conclusions

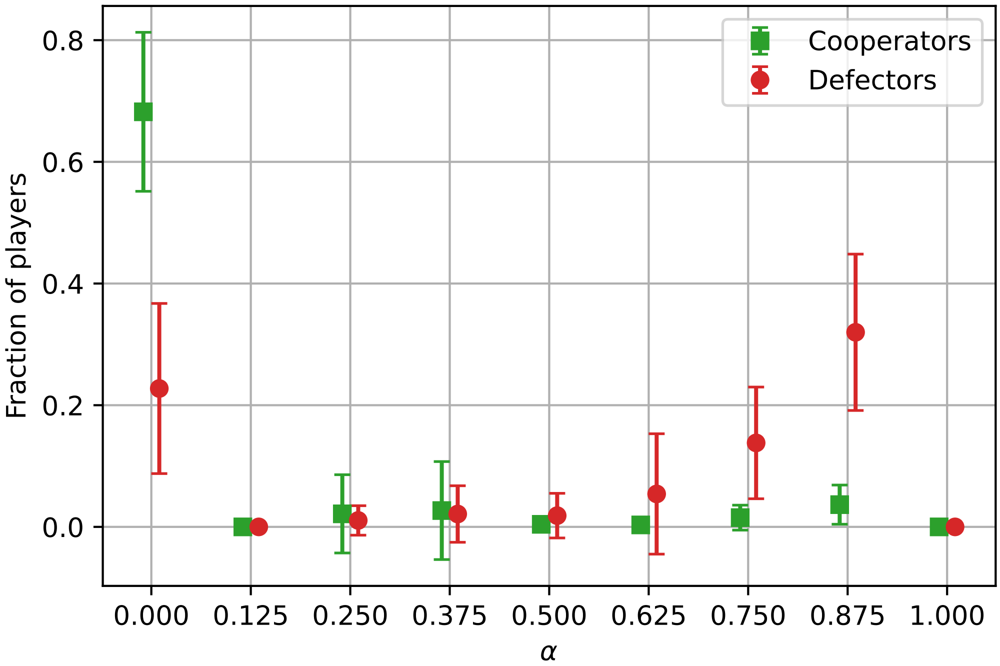


Figure 4: Correlation between abstention and cooperation. For each value of α, the relative number of cooperators is compared to the relative number of defectors. The numbers are normalized dividing by the total number of players. The marks of cooperators (defectors) are slightly shifted to the left (right) for better readability. Parameters: $T = 1.4, R = 1, S = 0, P = 0.3, L = 0.4$

References

[1] Nicoló Montalti, Alessandro Azzani, and Francesco Gargiulo. *Controversies in Game Theory*. June 4, 2023. URL: https://github.com/nicmontalti/GameTheory.