

Alessandro Azzani, Francesco Gargiulo  
Lea Haas & Nicolò Montalti

# Influence of probabilistic abstention on spatial cooperative games

Project for the course  
“Controversies in Game Theory”

June 12, 2023

Contents

Individual contributions	ii
Introduction	1
1 Theoretical background	1
1.1 Game theory . . . . .	1
1.2 Games . . . . .	1
1.3 Spatial games . . . . .	1
1.4 Abstention . . . . .	1
2 Implementation	1
3 Results	2
4 Conclusions	2

## Individual Contributions

Who did what....

# Introduction

Here we write the introduction.

## 1 Theoretical background

### 1.1 Game theory

### 1.2 Games

### 1.3 Spatial games

### 1.4 Abstention

## 2 Implementation

The game has been implemented in Python, the code is publicly available on GitHub [1]. The spatial environment has been implemented as a  $50 \times 50$  2D grid. A player is present at every point of grid. Every player is characterized by a strategy (collaborate / defect) and by an abstention probability  $0 \leq \alpha \leq 1$ . The game is defined by the five parameters  $T, R, P, S, L$  and by a noise factor  $0 \leq \eta \leq 1$ . The first five parameters determine the game played: prisoner dilemma (PD), snowdrift (SD), stag hunt (SH) or harmony game (HG).

The game is initialized assigning to each player a random strategy, with a 50–50 probability, and an abstention probability  $\alpha$ . The latter can be assigned in three different ways:

- No abstention:  $\alpha = 0$  for all players. The game is reduced to the standard prisoner dilemma (PD) or another standard game
- Optional abstention:  $\alpha = 0$  or 1 with a 50 – 50 probability. The game is reduced to the optional prisoner dilemma (OPD) or the equivalent for another game
- Probabilistic abstention:  $\alpha \in [0, 1]$ . The players have a finite probability of abstaining, uniformly distributed between 0 and 1. We call this situation prisoner dilemma with probabilistic abstention (PDPA) or the equivalent for other games.

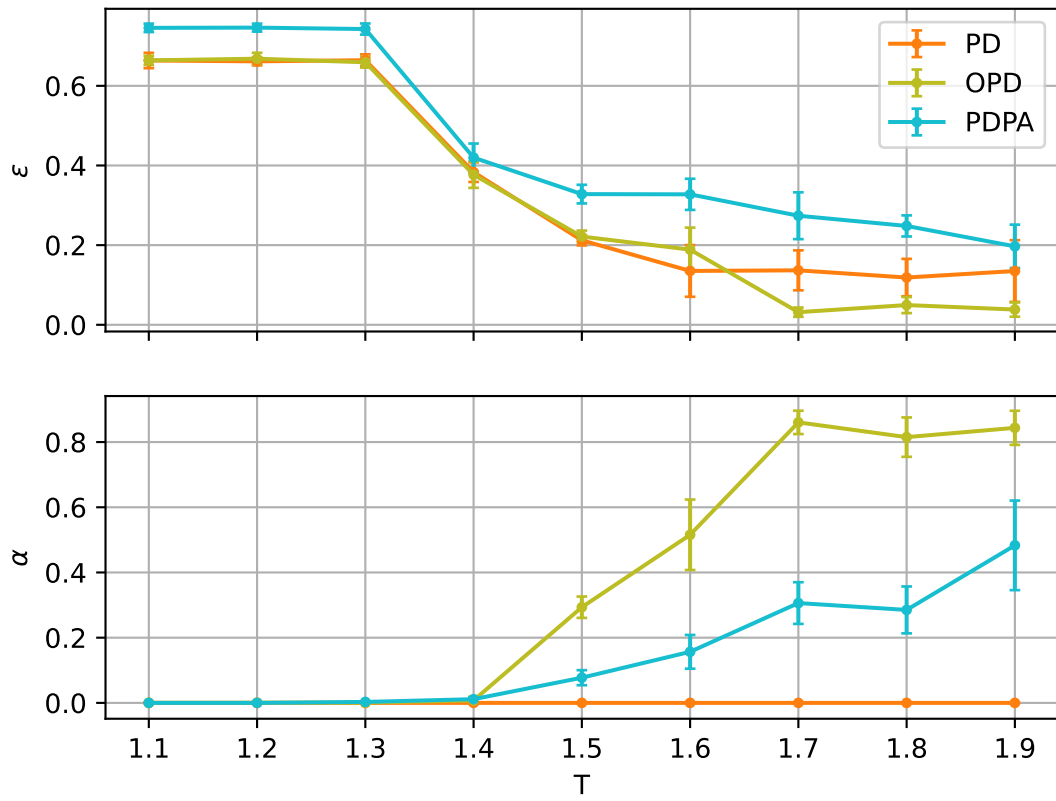
At each iteration of the game, each player plays against their four nearest neighbours. Each player earns an amount equal to the sum of the payoff of the four games they played. The payoff of a single match between two players is determined by the strategy of the two players and by their abstention probability. Each player joins the game with a probability  $1 - \alpha$ . If both players join the game, the game is played and payoffs are distributed according to the players strategies. If one of the players abstains, the game is not played, and both players receive a payoff  $L$ .

After every player has played, the imitation process begins. The imitation process is influenced by the noise parameter  $\eta$ . Each player changes their strategy to the opposite strategy with a probability  $\eta$  and imitates their best performing neighbours with a probability  $1 - \eta$ . If the imitation option is chosen, the player looks at their four nearest neighbours payoffs, and imitates the strategy of the one who performed the best. If more players share the same optimal outcome, a random one is chosen. In addition to imitating the best-performing player's strategy, the abstention probability is imitated as well.

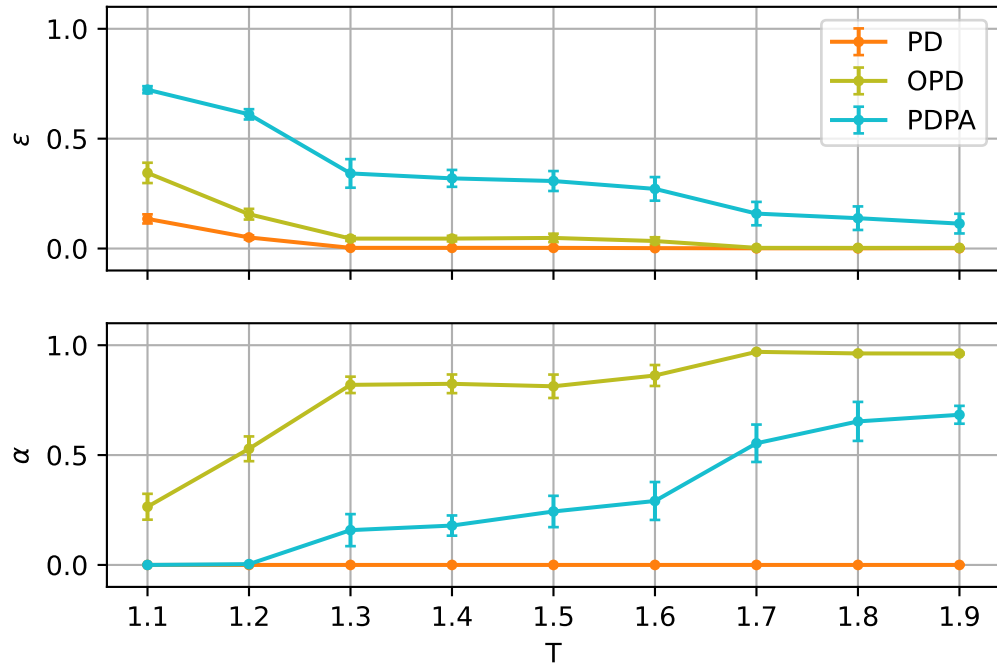
The process is iterated for a fixed number of times  $t$  (usually  $t = 200$ ). At each iteration, the players play the game and imitate their neighbours' strategies and abstention probabilities. At the end of the cycle, some statistics are extracted from the final configuration. In particular, we evaluate the mean abstention probability  $\langle \alpha \rangle$  and the mean the effective cooperation frequency  $\varepsilon = (1 - \alpha) \cdot (1 - s)$ , where  $s = 0$  for a cooperator and  $s = 1$  for a defector. In addition to these two quantities, we also analyse the correlation between being a cooperator and having a low abstention frequency  $\alpha$ . To do so, when playing the version of the game with probabilistic abstention, we plot the number cooperators and defectors for each value of  $\alpha$  and compare the results.

### 3 Results

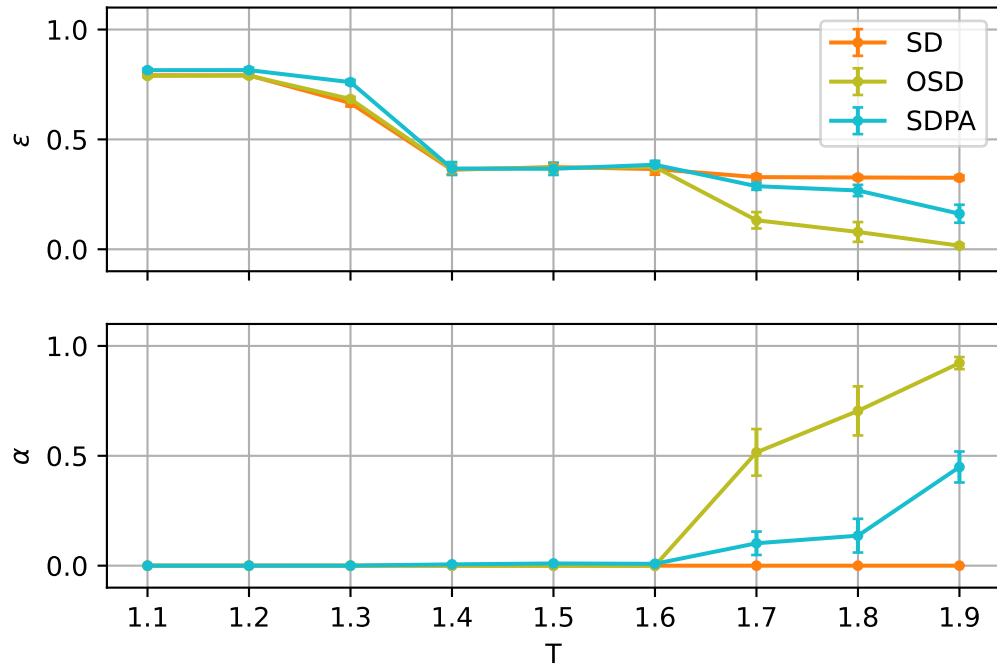
## 4 Conclusions



**Figure 1:**  $R = 1$   $S = 0$   $P = 0.3$   $L = 0.4$  noise = 0.001  $t = 200$



**Figure 2:**  $R = 1$   $S = 0$   $P = 0.3$   $L = 0.4$  noise = 0.001  $t = 200$



**Figure 3:** (same as paper)  $R = 1$   $S = 0.3$   $P = 0$   $L = 0.4$  noise = 0.001  $t = 200$

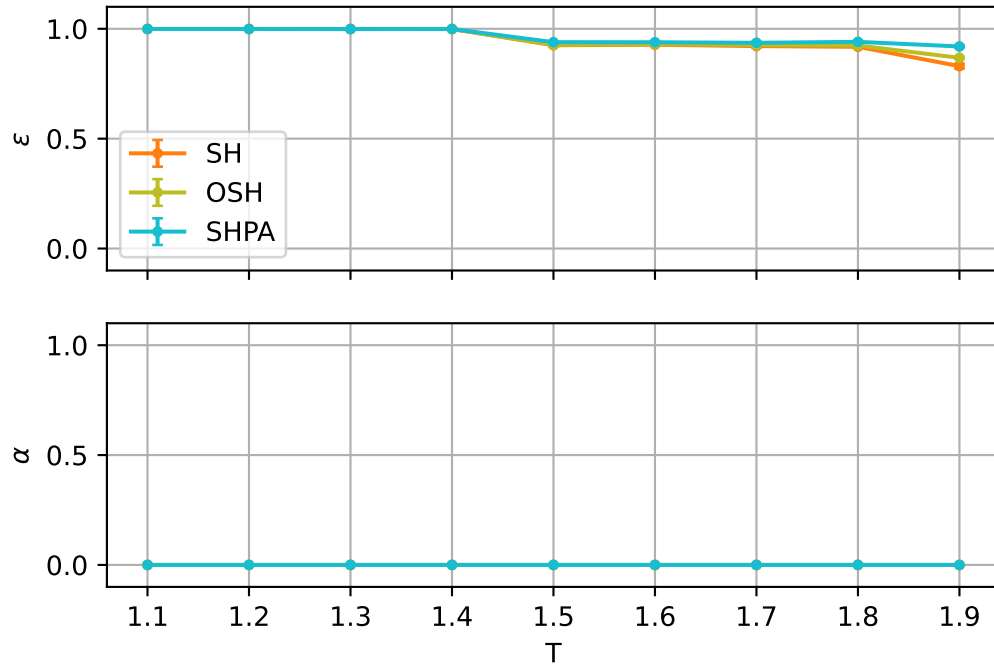


Figure 4:  $R = 1$   $S = 0$   $P = 0$   $L = 0.4$  noise = 0.001  $t = 200$

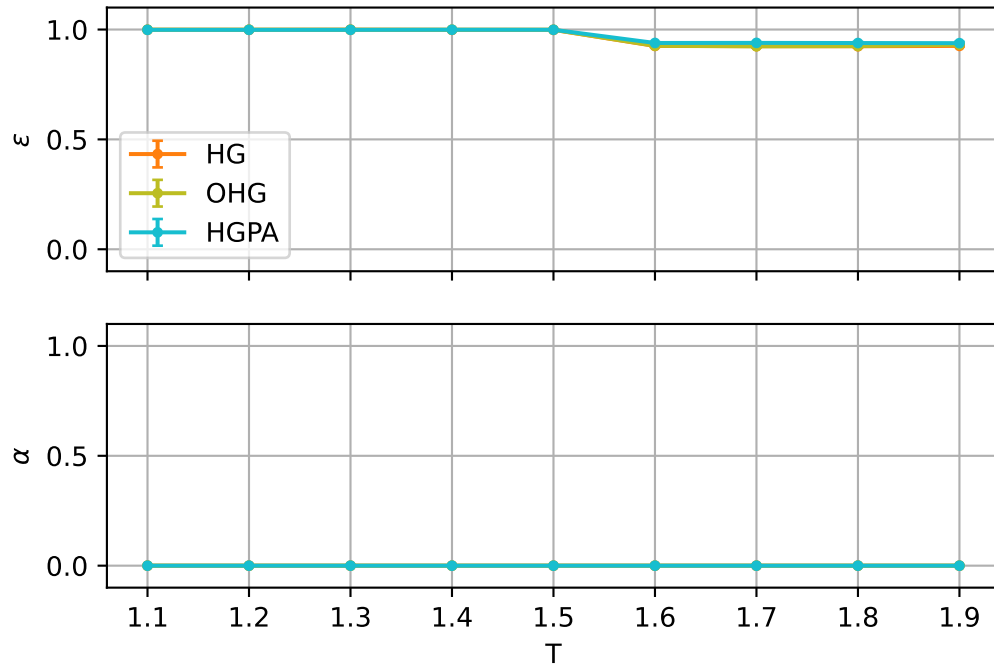
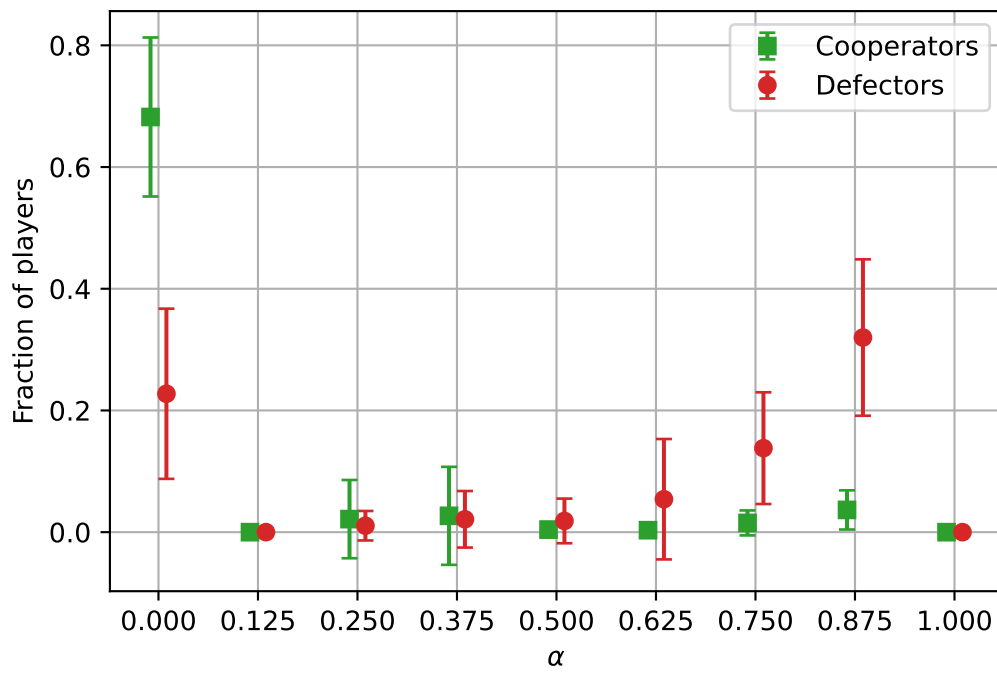


Figure 5:  $R = 2$   $S = 0.3$   $P = 0$   $L = 0.4$  noise = 0.001  $t = 200$





**Figure 6:** Correlation between abstention and cooperation. For each value of  $\alpha$ , the relative number of cooperators is compared to the relative number of defectors. The numbers are normalized dividing by the total number of players. The marks of cooperators (defectors) are slightly shifted to the left (right) for better readability. Parameters:  $T = 1.4$ ,  $R = 1$ ,  $S = 0$ ,  $P = 0.3$ ,  $L = 0.4$

---

## References

- [1] Nicolás Montalti, Alessandro Azzani, and Francesco Gargiulo. *Controversies in Game Theory*. June 4, 2023. URL: <https://github.com/nicmontalti/GameTheory>.