Sujet de Travaux Dirigés / Pratiques - TP MRF - IMA203
Only one paper document per 2 people.
BUT one jupyter notebook on e-campus for each student.

NAME :

NAME :

Introduction to Markov Random Fields for image processing

Objective of the session :

The aim of this session is to program the Gibbs sampler algorithm and study it in the binary case. This program will then be used to do image classification in a bayesian framework (next practical work).
You have to fill by hand-writing the printed version of the practical work (this document) for 2 students and upload the filled jupyter notebook on e-campus for each of you. This report should be given on the 1st of december during the course. The filled notebook should be also uplodaded on e-campus for the 24th of november.

1 Ising model

In this section we consider a binary Markov random field (taking values in $E = 0, 1$. The neighborhood is defined in 4-connexity and the potential of a clique of order 2 is defined by : $V_c(0, 1) = V_c(1, 0) = +\beta$ and $V_c(1, 1) = V_c(0, 0) = 0$ (the potential for singleton clique is 0).

— Draw in the grid the imaU generated with the notebook (fill in black the pixels with value 0) :

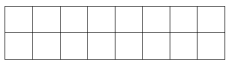

FIGURE 1 – Image generated by the notebook

— Q1 For the Ising model defined above, and the imaU generated in the previous cell, give the formula of the global energy and give its value as a function of β for the generated imaU :

— Draw in the grid the local configuration generated with the notebook (fill in black the pixels with value 0) :

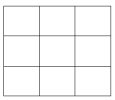

FIGURE 2 – Configuration ImaVois of the local neighborhood (the pixel s to be considered is in the center of the 3x3 window).

— Q2 Write the general form of the local conditional probability in a pixel s. For the neighborhood configuration ImaVois generated with the notebook and represented in figure 2, compute the 2 local conditional energies (for the value 0 and for the value 1 of the central pixel), then the local conditional probabilities (as a function of β). What is the most probable class ? (NB : do the calculation for an 8-neighborhood).

Program the Gibbs sampler on the notebook.

— Q3 Run the program several times. Do you still get the same image ? Comment on this.

— Q4 Vary β from 0.5 to 20. Comment on the results.

— Q5 Which image minimizes the overall energy for this model ?

— Q6 Change β and give it a negative value. Describe the result and justify it.

We now work in 8-neighborhood, but still with cliques of order 2 (non-isotropic this time).

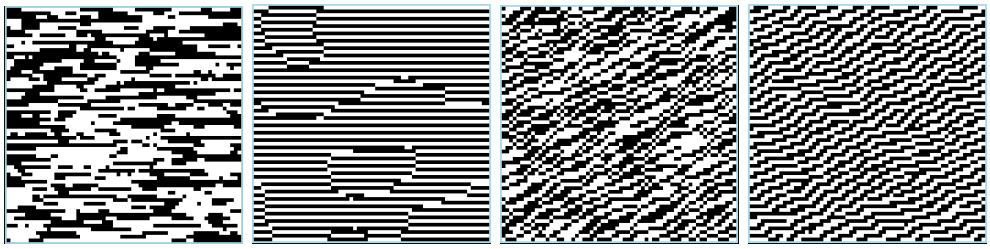

FIGURE 3 – Image A, B, C, D (de gauche à droite)

For each of these images, propose the clique potentials that allow us to obtain these realizations. Initially all clique potentials are zero.

— Image A : there is only one clique potential of order 2 which is -1.
— Image B : in addition to the previous one, there is a clique potential of order 2 which is 1. Indicate which one.
— Image C : in addition to the 2 previous ones, there is a clique potential of order 2 which is -1. Indicate which one.
— Image D : in addition to the 3 previous ones, there is a second order clique potential which is +1. Indicate which one.

— Q8 Propose the clique potentials that allow us to obtain these realizations

Potential	horiz. $V_c(0,1)$ $V_c(1,0)$	horiz. $V_c(0,0)$ $V_c(1,1)$	vertical $V_c(0,1)$ $V_c(1,0)$	vertical $V_c(0,0)$ $V_c(1,1)$	diagonal $(+\frac{\pi}{4})$ $V_c(0,1)$ $V_c(1,0)$	diagonal $(+\frac{\pi}{4})$ $V_c(0,0)$ $V_c(1,1)$	diagonal $(+\frac{3\pi}{4})$ $V_c(0,1)$ $V_c(1,0)$	diagonal $(+\frac{3\pi}{4})$ $V_c(0,0)$ $V_c(1,1)$
Image A								
Image B								
Image C								
Image D								

Modify your program to obtain these results (you can copy and paste the previous cells).

Q9 Modify your program to define an Ising model with diagonal attractive potentials only (the other potentials are zero). It means that this model encourages similar labels (either 0 or 1) for neighboring pixels in the diagonal directions ($\pi/4$ or $3\pi/4$ directions). Describe and comment on the result.