Filter synthesis

Roland Badeau,
roland.badeau@telecom-paris.fr

TSIA201

Part I

Linear phase FIR filters

Linear phase FIR filters

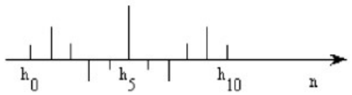
- Impulse response : $h(n) \in \mathbb{R}$, $n = 0 \dots N-1$
- Frequency response : $H(e^{i2\pi\nu}) = e^{i2\pi(\beta - \alpha\nu)} H_R(\nu)$
- Constant and equal group and phase delays (if $\beta = 0$)
- Advantages : always causal and stable, preserve the waveform of a narrow-band signal (if $\beta = 0$)
- Drawback : high computational complexity

Characterization

- 1 - periodicity of $H(e^{2i\pi\nu}) \Rightarrow \alpha = p/2$, $p \in \mathbb{Z}$ and H_R is 2-periodic
- Hermitian symmetry $\Rightarrow d = e^{2i\pi\beta} = 1$ or i and H_R is even or odd
- As $H_R(\nu)$ is 2-periodic, we can define $G(e^{2i\pi\nu}) = d\,H_R(2\nu)$ where $g(n)$ is real, even or odd
- We can thus write $H(z^2) = z^{-p} G(z)$ (filter of length $2N-1$)
- We choose $p = N-1$ for a causal filter
- 4 possibilities depending on d and the parity of N :
  - $d = 1$, N even or odd : $h(n) = h(N-1-n)$
  - $d = i$, N even or odd : $h(n) = -h(N-1-n)$

Types of filters

- Type 1 : N odd, symmetric ($d = 1$)

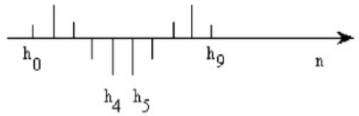


$$H(e^{i2\pi\nu}) = e^{-i2\pi\nu\frac{N-1}{2}} \sum_{n=0}^{\frac{N-1}{2}} a_n \cos(2\pi\nu n)$$

- Use : low-pass, high-pass, band-pass

Types of filters

- Type 2 : N even, symmetric ($d = 1$)

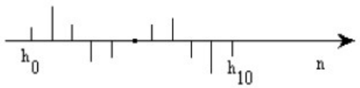


$$H(e^{i2\pi\nu}) = e^{-i2\pi\nu\frac{N-1}{2}} \sum_{n=1}^{\frac{N-1}{2}} b_n \cos\left(2\pi\nu\left(n - \tfrac{1}{2}\right)\right))$$

- Property : $H(-1) = 0$ ($\nu = \frac{1}{2}$)
- Use : low-pass, band-pass

Types of filters

- Type 3 : N odd, antisymmetric ($d = i$)

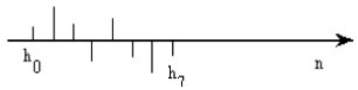


$$H(e^{i2\pi\nu}) = i\, e^{-i2\pi\nu\frac{N-1}{2}} \sum_{n=1}^{\frac{N-1}{2}} c_n \sin(2\pi\nu n)$$

- Property : $H(1) = H(-1) = 0$ ($\nu = 0$ or $\frac{1}{2}$)
- Use : differentiator ($H(f) = i2\pi f$), Hilbert transform ($H(f) = -i\,\mathrm{sign}(f)$) in band-pass form

Types of filters

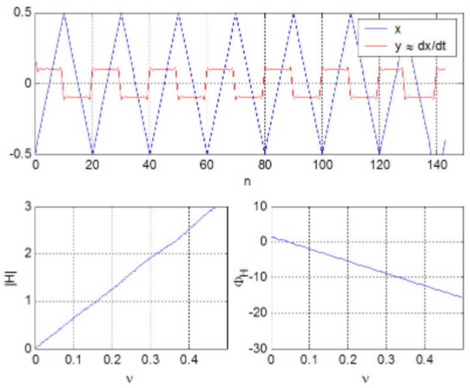
- Type 4 : N even, antisymmetric ($d = i$)



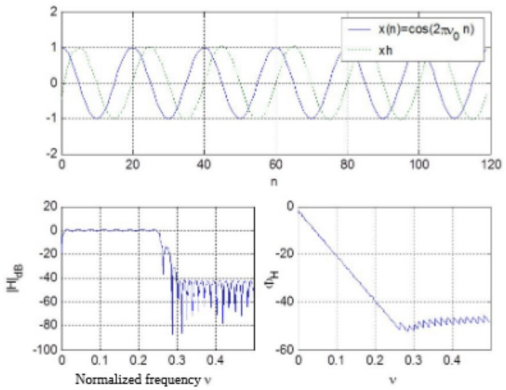
$$H(e^{i2\pi\nu}) = i\, e^{-i2\pi\nu\frac{N-1}{2}} \sum_{n=1}^{\frac{N-1}{2}} d_n \sin\left(2\pi\nu\left(n - \tfrac{1}{2}\right)\right))$$

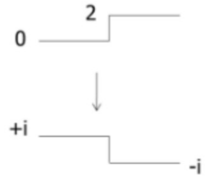
- Property : $H(1) = 0$ ($\nu = 0$)
- Use : differentiator ($H(f) = i2\pi f$), Hilbert transform ($H(f) = -i\,\mathrm{sign}(f)$) in high-pass form

Differentiators



Hilbert transform

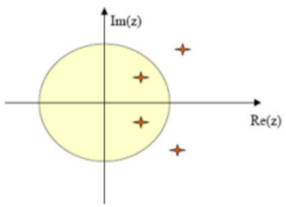




Position of the zeros

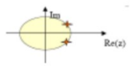
- Complex zero outside the unit circle :

$$(1-\rho e^{i\theta}z^{-1})(1-\rho e^{-i\theta}z^{-1})(1-\frac{1}{\rho}e^{i\theta}z^{-1})(1-\frac{1}{\rho}e^{-i\theta}z^{-1})$$

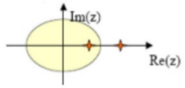


Position of the zeros

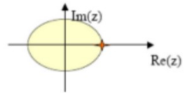
- Zero on the unit circle



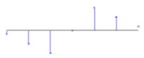
- Real zero



- Real zero on the unit circle



Summary

Type I N odd symmetric		-	Low-pass High-pass Band-pass
Type II N even symmetric		$H(-1) = 0$	Low-pass, Band-pass
Type III N odd antisym.		$H(1) =$ $H(-1) = 0$	Differentiator, Hilbert Transform, Band-pass
Type IV N even antisym.		$H(1) = 0$	Differentiator, Hilbert Transform, High-pass

Part II

FIR filters: iterative methods

Iterative methods

- Advantages
  - Optimal design
  - Flexible method
  - Constant amplitude ripples
  - Minimum order for a given template
- Drawbacks
  - Computationally demanding synthesis
  - Not suitable for real-time processing
  - Not suitable for long filters (numerical stability issues)

Filter template