EUROPEAN CENTRE FOR RESEARCH AND ADVANCED TRAINING IN SCIENTIFIC COMPUTING

Large Eddy Simulation: Practical issues

Filtering, modelling, and errors

wall-modeling

turbulence injection

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Large eddy simulations: Practical issues

LES: A brief reminder

NS equation and turbulence
LES, filtering and models
Numerics, errors and LES
Implicit LES

Wall treatment

Wall-bounded turbulence
The logarithmic law-of-the-wall
Wall-modeled LES: analytical, TBLE
Detached-Eddy Simulations
Log-layer mismatch and applications

Turbulence injection

Synthetic methods (Fourier / POD)
Precursor method
Wall-bounded turbulent injection

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NS equations and turbulence

Compressible NS equations:

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0}$$

$$\boxed{\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}}$$

$$\boxed{\frac{\partial \rho E}{\partial t} + \frac{\partial \rho E u_j}{\partial x_j} = \frac{\partial u_i \tau_{ij}}{\partial x_j} - \frac{\partial q_j}{\partial x_j}}$$

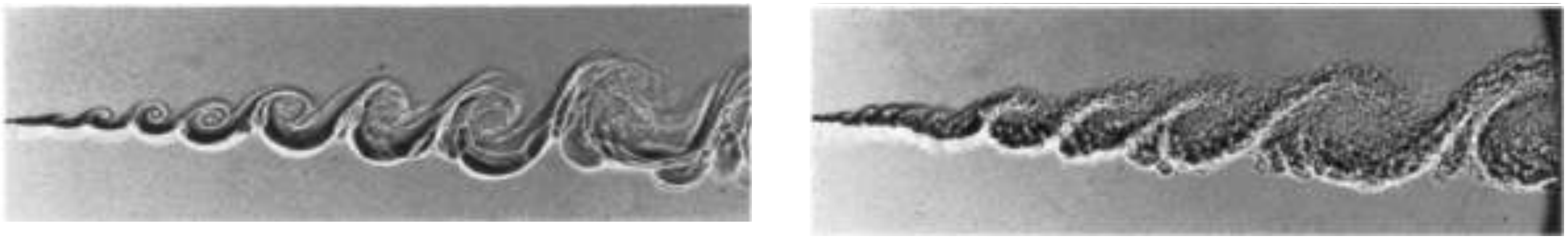
Viscous stress tensor

$$\tau_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}$$

Heat flux

$$q_j = -\lambda \frac{\partial T}{\partial x_j}$$

Navier Stokes equations: **Non linearity** leads to **turbulence**!



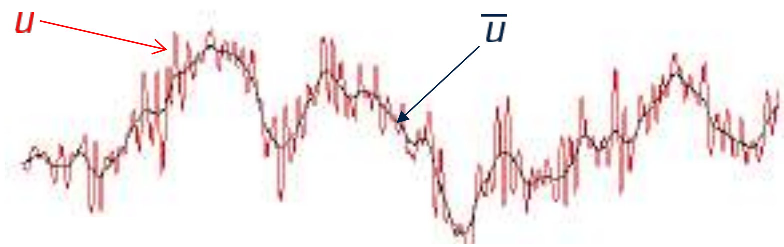
Determinist: For 1 initial condition => 1 unique solution

Hardly-predictible: An initial perturbation amplifies on a **large range** of scale.

Reynolds decomposition:

$$u = \overline{u} + u'$$

RANS decomposition



For **compressible** flows:

Favre averaging (mass-weighted averaging)

$$\tilde{u} = \frac{\overline{\rho u}}{\bar{\rho}}$$

Fluctuation:

$$u'' = u - \tilde{u}$$

Accounts for coupled density / velocity fluctuations

Favre **averaged** NS equations:

$$\boxed{\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\widetilde{u}_i}{\partial x_i} = 0}$$

$$\boxed{\frac{\partial \bar{\rho}\widetilde{u}_i}{\partial t} + \frac{\partial \bar{\rho}\widetilde{u}_i\widetilde{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j} - \frac{\partial R_{ij}}{\partial x_j}}$$

Turbulent stress tensor:

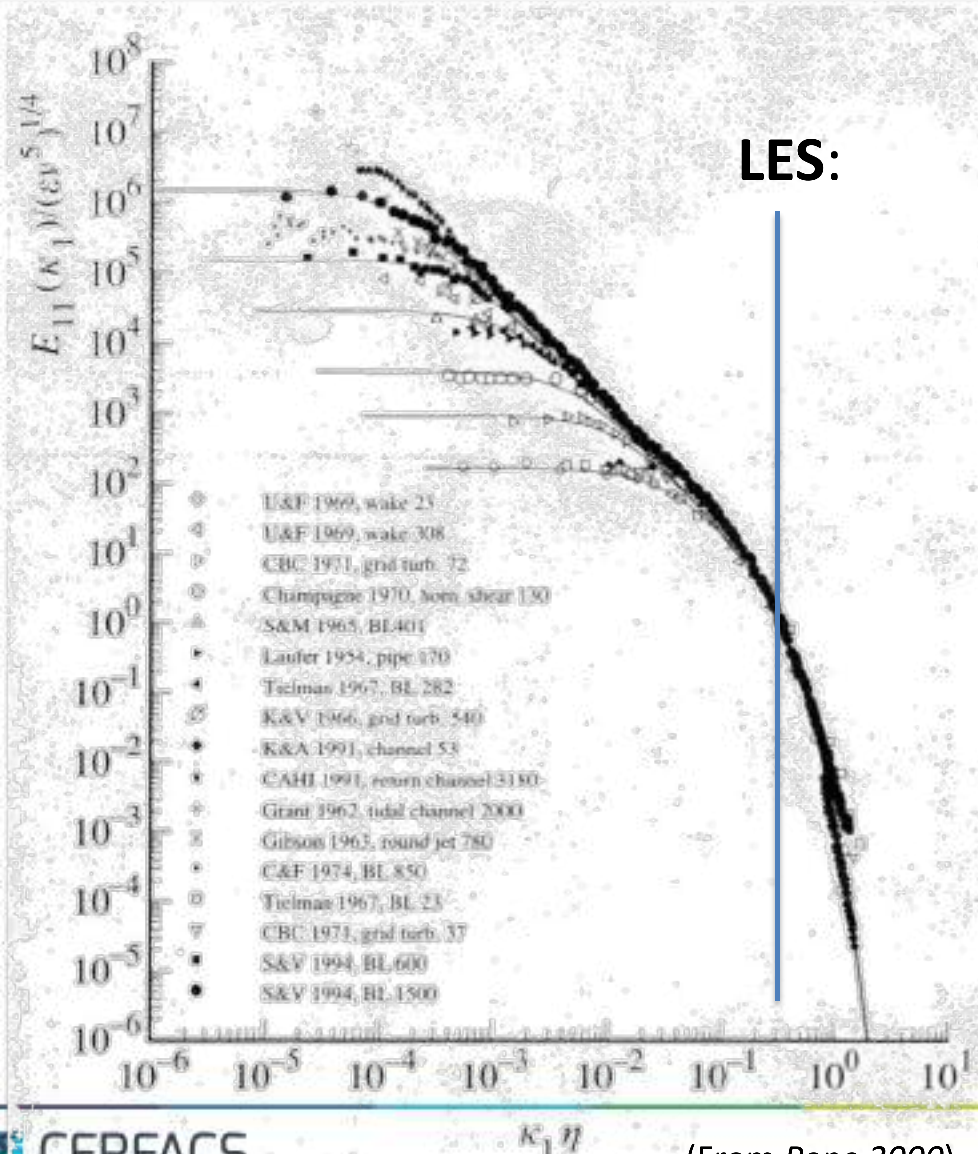
$$R_{ij} = -\overline{\rho u_i'' u_j''}$$

$$\boxed{\frac{\partial \bar{\rho}\tilde{E}}{\partial t} + \frac{\partial \bar{\rho}\tilde{u}_j\tilde{E}}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(\overline{\rho u_j'' E''}\right) + \frac{\partial \overline{\tau_{ij} u_i}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial \bar{T}}{\partial x_j}\right)}$$

$$\tilde{E} = \frac{\tilde{u}_i\tilde{u}_i}{2} + c_v\tilde{T} + \frac{1}{2}\widetilde{u_i'' u_i''}$$

RANS => Averaging. Consideration of **mean + fluctuation**

Turbulence cascade

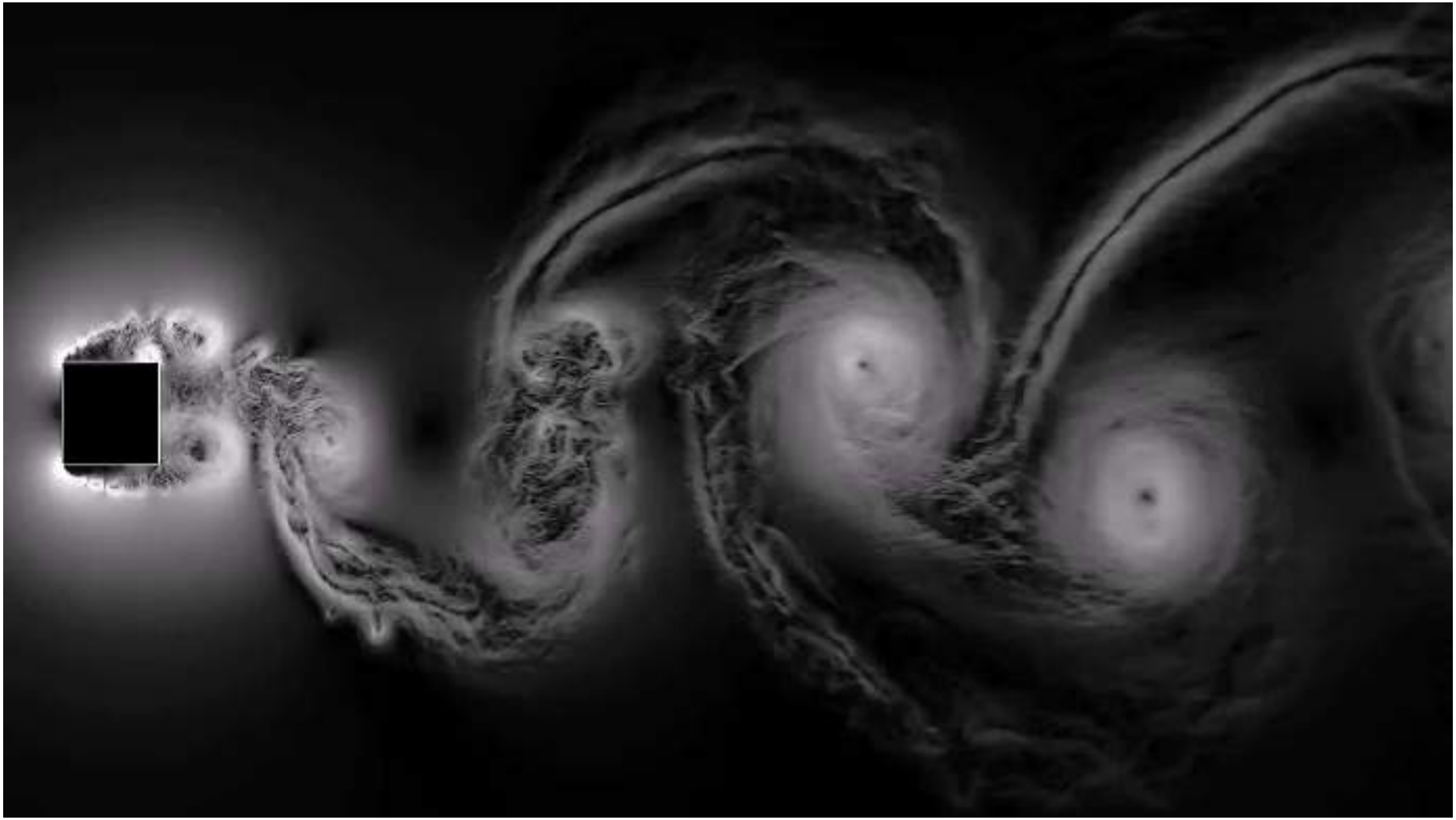


(From *Pope*,*2000*)

Compilation of numerous experiments (pipe, grids, jets, wake, BL…)

<u>**LES**</u>: Let's **filter** the high frequencies

Turbulence, auto-similarity



Trias et al., C&F, 2015

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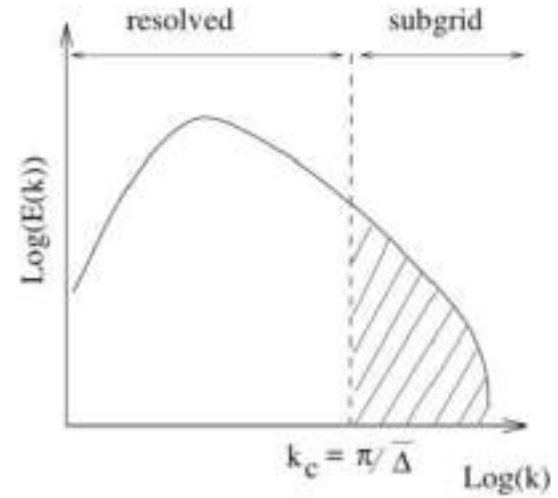
The LES concepts

<u>4 conceptual steps in LES:</u>

filtered modeled

- Define a **filtering operation :** $Q = \bar{Q} + Q'$
- **Filter NS equations** => residual stress tensor
- Define a **Closure** to **model** the residual stress tensor
- Filtered equations are **numerically** solved.

$\Rightarrow$ Provide an approximation of the large scales motion, for 1 realization of turbulent flow.



Filtering and **modeling** are **independent of the numerical method**, in particular of the grid.
Numerical method is supposed to provide an accurate **solution** to the **filtered equations.**

LES: about **filtering**

Filtering operation: convolution (Leonard, 1974)

$$\bar{Q}(\mathrm{x}) = \int Q(\mathrm{x}^*) F(\mathrm{x} - \mathrm{x}^*)\, \mathrm{dx}^*$$

with

$$\int_{-\infty}^{\infty} F(\mathbf{x}) \mathrm{d}\mathbf{x} = 1$$

(normalization)

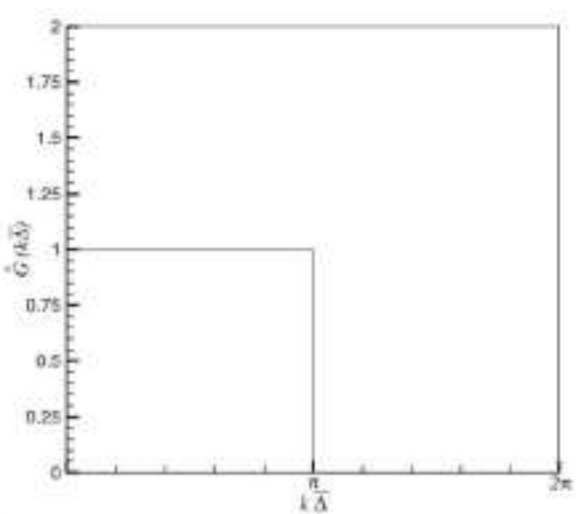
Filtered quantity

filter

<u>Filters exempla</u>

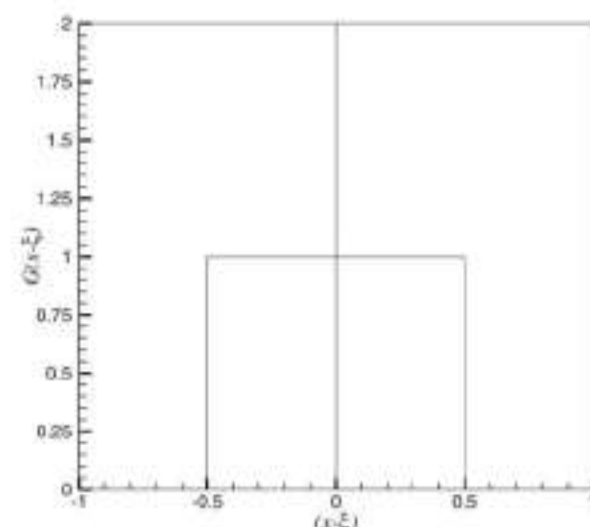
Cut-off filter (spectral space)

$$\bar{F}(k) = \begin{cases} 1 & \text{if } k \leq \pi/\Delta \\ 0 & \text{otherwise} \end{cases}$$



Box filter (physical space)

$$F(\mathrm{x}) = F(x_1, x_2, x_3) = \begin{cases} 1/\Delta^3 & \text{if } |x_i| \leq \Delta/2,\ i = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$



LES: about filtering

Similar to Favre **averaging**, one can define Favre **filtering:**

$$\bar{\rho}\tilde{Q}(\mathrm{x}) = \int \rho Q(\mathrm{x}^*)F(\mathrm{x}-\mathrm{x}^*)\mathrm{dx}^*$$

Averaging ≠ Filtering !

- Reynolds decomposition: $u = \overline{u} + u'$ average fluctuation $\overline{u'} = 0$

- Filtering: $Q = \bar{Q} + Q'$ resolved modeled $\overline{Q'} \neq 0$

See Veynante – Vervisch, PECS 2002

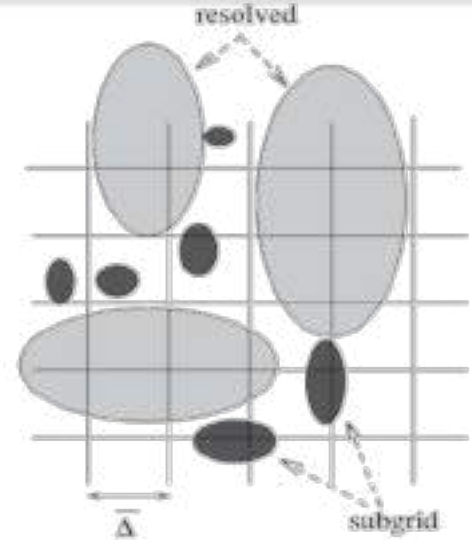
In practice, the **filter width** is usually given by the **local mesh size**. $\Delta = \sqrt[3]{\Delta x \Delta y \Delta z}$

=> **Numerical** and **modeling errors** can hardly be separated.

NS filtered equations

Similar to Favre **averaged** equations, one can define Favre **filtered** NS equations:

$$\begin{cases} \dfrac{\partial \overline{\rho}}{\partial t} + \dfrac{\partial}{\partial x_i}(\overline{\rho}\tilde{u}_i) = 0 \\ \dfrac{\partial \overline{\rho}\tilde{u}_i}{\partial t} + \dfrac{\partial}{\partial x_i}(\overline{\rho}\tilde{u}_i\tilde{u}_j) = -\dfrac{\partial \overline{p}}{\partial x_i} + \dfrac{\partial}{\partial x_j}\tilde{\sigma}_{ij} - \dfrac{\partial}{\partial x_j}\tau_{ij} \end{cases}$$



<u>Viscous stress tensor</u>

$$\tilde{\sigma}_{ij} = \overline{\mu}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial \tilde{u}_k}{\partial x_k}\right)$$

<u>**Subgrid** (residual) **stress tensor**</u>

$$\tau_{ij} = \rho\left(\widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j\right)$$

$\Rightarrow$ Need to be **modeled**

<u>Eddy viscosity assumption:</u>

$$\tau_{ij} = -2\nu_t S_{ij}$$

with $S_{ij} = \dfrac{1}{2}\left(\dfrac{\partial \tilde{u}_i}{\partial x_j} + \dfrac{\partial \tilde{u}_j}{\partial x_i}\right)$

<u>**Turbulent viscosity**</u>, provided by a SGS model

Reminder:

- Residual stress tensor: $\tau_{ij} = \rho \left(\widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j \right)$
- Assumption: $\tau_{ij} = -2 \nu_t S_{ij}$

Smagorinsky model (*1963*):

$$\nu_t = (C_s \Delta)^2 \sqrt{2 \widetilde{S_{ij}} \widetilde{S_{ij}}} = (C_s \Delta)^2 |S|$$

$C_s \in [0.1, \; 0.18]$

Shear flows, wall-bounded

HIT

Filter size (in practise, grid size)

A few LES models

Dynamic Smagorinsky model (*Germano, 1992*):

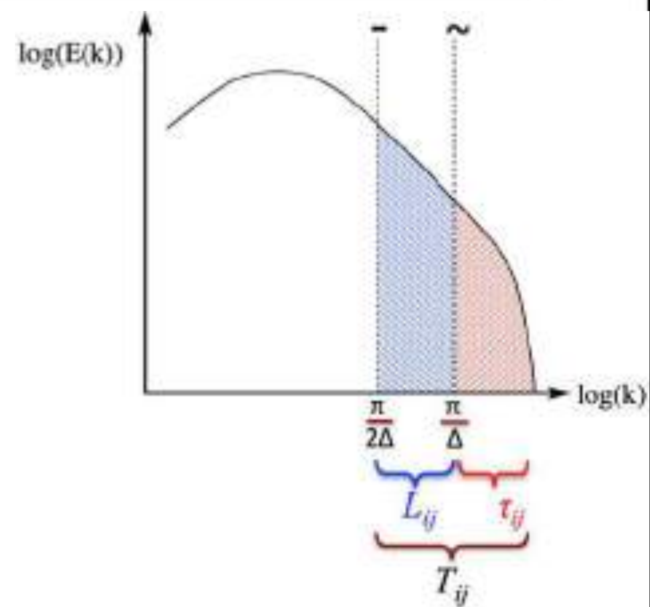
- Introduction of a **2nd filter** (test filter)
- Based on test filtered equations, **dynamic determination** of C_s

- **Grid** filter $\widetilde{\Delta}$
- **Test** filter $\widehat{\Delta} = 2\widetilde{\Delta}$

Filter NS

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

$$T_{ij} = \widehat{\widetilde{u_i u_j}} - \widehat{\tilde{u}_i \tilde{u}_j}$$



Germano identity :

$$\widehat{\tilde{u}_i \tilde{u}_j} - \hat{\tilde{u}}_i \hat{\tilde{u}}_j = T_{ij} - \widehat{\tau_{ij}}$$

Can be computed on the grid

Must be modeled

- Smagorinsky for τ_{ij} and T_{ij} :

$$T_{ij} = 2C_s \widehat{\Delta}^2 |\widehat{\tilde{S}}| \widehat{\tilde{S}_{ij}}$$

$$\tau_{ij} = 2C_s \widetilde{\Delta}^2 |\tilde{S}| \widetilde{S_{ij}}$$

=> Substitute into Germano identity

A few LES models

Dynamic Smagorinsky model (*Germano, 1992*):

- Substitute into Germano identity

$$\widehat{\widetilde{u_i}\widetilde{u_j}} - \widehat{\widetilde{u_i}}\,\widehat{\widetilde{u_j}} = 2C_s\left(\widehat{\Delta}^2|\widehat{\widetilde{S}}|\widehat{\widetilde{S}}_{ij} - \widehat{\widetilde{\Delta}^2|\widetilde{S}|\widetilde{S}_{ij}}\right)$$

Can be computed on the grid

- Finally $$C_s = \frac{1}{2}\frac{\widehat{\widetilde{u_i}\widetilde{u_j}} - \widehat{\widetilde{u_i}}\,\widehat{\widetilde{u_j}}}{\widehat{\Delta}^2|\widehat{\widetilde{S}}|\widehat{\widetilde{S}}_{ij} - \widehat{\widetilde{\Delta}^2|\widetilde{S}|\widetilde{S}_{ij}}}$$

Formulation later improved (Lilly 1992)

Main ideas:

- Introduction of a **test filter**
- Germano identity
- **Relying on resolved scales to model unresolved scales**

WALE model (*Nicoud, Ducros, 1999*):

Wall Adapting Local Eddy Viscosity

- Intended for **wall bounded flows** (*without law-of-the-wall…).*
- Turbulent viscosity must vanish according to $\nu_t \propto (y^+)^3$ close to a wall.

$$\nu_t = (C_\omega \Delta)^2 \frac{(s_{ij}^d s_{ij}^d)^{3/2}}{(\widetilde{S}_{ij}\widetilde{S}_{ij})^{5/2} + (s_{ij}^d s_{ij}^d)^{5/4}}$$

$$s_{ij}^d = \frac{1}{2}(\widetilde{g}_{ij}^2 - \widetilde{g}_{ij}^2) - \frac{1}{3}\widetilde{g}_{kk}^2 \delta_{ij}$$

$$\widetilde{g}_{ij} = \partial \widetilde{u}_i / \partial x_j$$

$$C_w = 0,4929$$

Sigma model (*Nicoud et al., 2011*):

- Intended for **wall bounded flows** (*without wall-law…).*
- Turbulent viscosity must vanish according to $\nu_t \propto (y^+)^3$ close to a wall.
- **WALE improvement for pure shear or solid rotation cases**

$$\nu_t = (C_\sigma \Delta)^2 \frac{\sigma_3(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)}{\sigma_1^2}$$

$C_\sigma = 1.5$

$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$

Singular values of the velocity gradient tensor

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Numerics, errors and LES

<u>Reminder:</u>

- Incompressible momentum equation $$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$
- Filtered quantity: $$\bar{f}(\vec{x},t) = \int f(\vec{y},t)\bar{G}_{\Delta x}(\vec{x}-\vec{y})d\vec{y}$$

<u>Filtered momentum equation:</u>

- $$\overline{\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j}} = 0$$
- $$\frac{\partial \bar{u}_i}{\partial t} + \overline{\frac{\partial u_i u_j}{\partial x_j}} + \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \overline{\tau_{ij}}}{\partial x_j} = 0$$

For non-uniform grids:

- $$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \overline{u_i u_j}}{\partial x_j} + \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \overline{\tau_{ij}}}{\partial x_j} = \mathbf{m}$$
- $$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} + \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \overline{\tau_{ij}}}{\partial x_j} = \mathbf{m} + \frac{\partial \bar{\tau}_{ij}{}^t}{\partial x_j}$$

Commutation error

$$m = \overline{\frac{\partial u_i u_j}{\partial x_j}} - \frac{\partial \overline{u_i u_j}}{\partial x_j}$$ [1,2]

Modeling induces resolution error

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[1] Ghosal AIAA 1999
[2] Van der Bos, Geurts, POF 2005

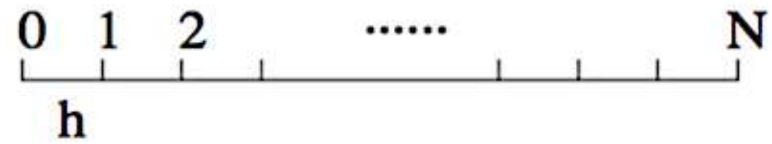
Numerics, errors and LES

Filtered quantity: $$\bar{f}(\vec{x},t) = \int f(\vec{y},t)\bar{G}_{\Delta x}(\vec{x}-\vec{y})d\vec{y}$$

Commutation and resolution errors

- On **regular** meshes, **commutation** with temporal and spatial derivatives
- **For variable** Δx => **variable width** for the filter.
- We **no longer have commutation** with spatial derivative. Induces "**Commutation error**".
- **Subgrid** scale **modelling** induces **resolution error**.

Discretization (Truncation) error



Transport equation:

$$\frac{\partial U}{\partial t} + \nabla \cdot F(U) = 0$$

Discretization

Equation satisfied by the **numerical solution** ("Modified Equation")

$$\frac{\partial U^N}{\partial t} + \nabla \cdot F(U^N) = \epsilon(U^N)$$

Discretization error

=> Application on Burgers equation…

Ex: truncation error and Burgers equation

Burgers Equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

Finite difference method:

1st order derivative $\quad \frac{\partial u}{\partial x} \approx \mathcal{D}_h^- u(x) \equiv \frac{u(x) - u(x-h)}{h}$

2nd order derivative $\quad \frac{\partial^2 u}{\partial x^2} \approx \mathcal{D}_h^2 u(x) \equiv \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$

Numerically solved equation:

$$\boxed{\frac{\partial u}{\partial t} + u\mathcal{D}_h^- u = \nu\mathcal{D}_h^2 u}$$

Equivalent to (**modified equation**):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = (\nu + \nu_{num})\frac{\partial^2 u}{\partial x^2} + \mathcal{O}(h^2)$$

$$\nu_{num} = \frac{1}{2}uh$$

- **1-order accurate** numerical scheme
- **Non-linear** error
- New **source terme** is the equation **actually solved**

Numerics, errors and LES

Finally: **<u>Incompressible momentum equation:</u>**

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial}{\partial x_j}\left(\tau_{ij}^t + \tau_{ij}^h\right) + m$$

Explicit SGS model contribution

Truncation error contribution

Commutation error for non-uniform grids

- **A good LES requires:** $$\frac{\partial}{\partial x_j}(\tau_{ij}^h) + m << \frac{\partial}{\partial x_j}(\tau_{ij}^t)$$

=> Need for accurate numerical scheme, and uniform grids

- <u>Ghosal 1996:</u> **“Models” and “errors” are comparable in typical LES…**

=> Investigation of high-order schemes, + “non-conventional” SGS modeling

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Implicit LES

- Assimilate the **discretization** to a **low-pass filter**.
- A SGS model is no longer needed: "**No-model" LES**.
- Scheme **must be accurate**: $\tau^h{}_{ij}$ (truncation error) must be small.

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial}{\partial x_i}\left(\tau_{ij}^t + \tau_{ij}^h\right) + m$$

~~**Explicit SGS model** contribution~~

Truncation error contribution

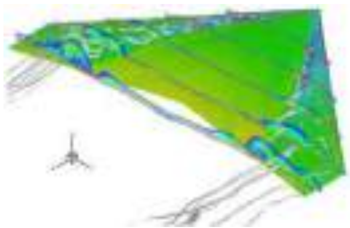
~~**Commutation error** for non-uniform grids~~

- **No all implicit SGS will work** !
  Truncation error must satisfies **SGS properties**
- No SGS model is used=> **modeling** and **numerics** are **inseparably coupled**.

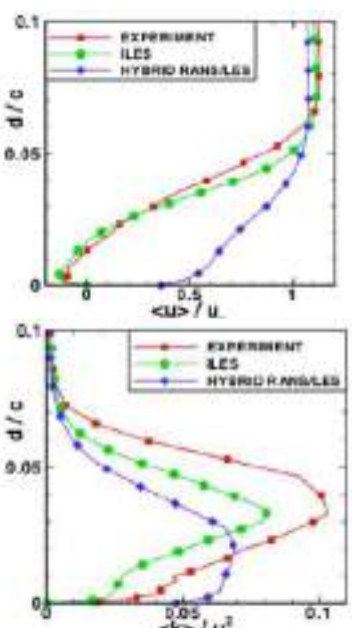
Implicit LES

ILES has advocates and detractors…

Despite controversies, **ILES** has proven to be **reliable** when used with adequate **high-order numerical schemes**.

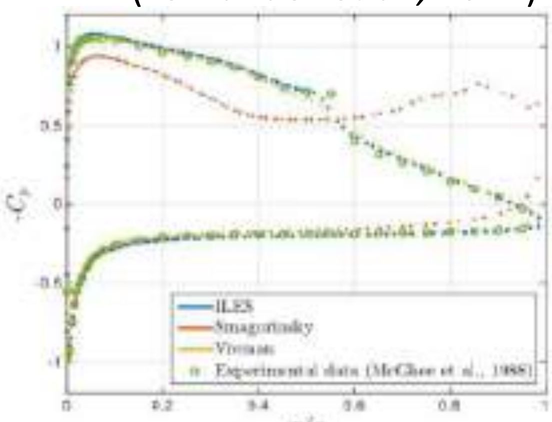


Drikakis et al., 2009



Transitional flow

(*Fernandez et al., 2017*)



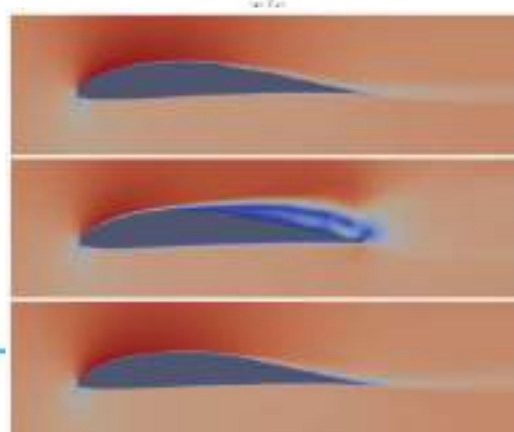
ILES:

High-Order

Discontinuous

Galerkin

scheme

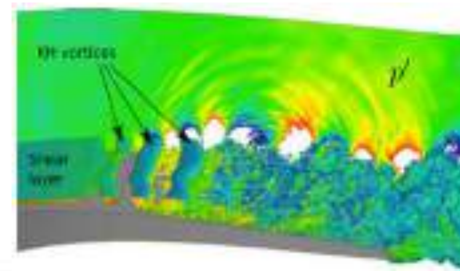


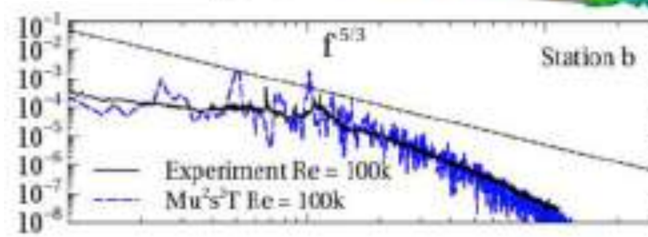
ILES

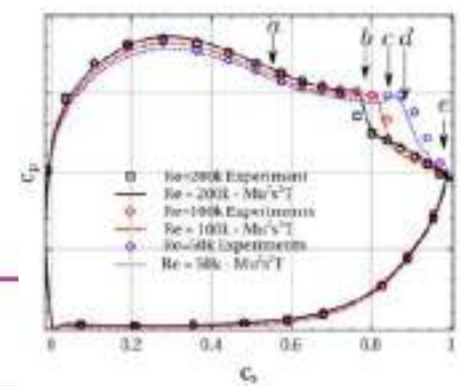
Smago

Vreman

Bolinches Gisbert et al. 2019







ILES:

High-Order Flux

Reconstruction

scheme

LES: about modeling

Explicit LES approach:

A **model** is introduced to account for the filtred subgrid scales:

Among others: (Dynamic) Smagorinsky
WALE/Sigma
Vreman, lagrangian averaging…

(See Sagaut (2001))

Implicit LES:

- **No SGS model**. The **numerical method** is chosen such that numerical error and resolution error cancel each other.

A "good LES":

- Filter and grid are sufficiently fine to **resolve 80% of the energy** (Pope, 2000)

A few conclusion

- **Averaging ≠ Filtering**

- “**Real life**” :
  modeling + spatio-temporal discretization ➔ **errors**
  modeling (resolution), commutation, truncation errors

- **Numerics** and **modeling** can be **inseparably coupled**

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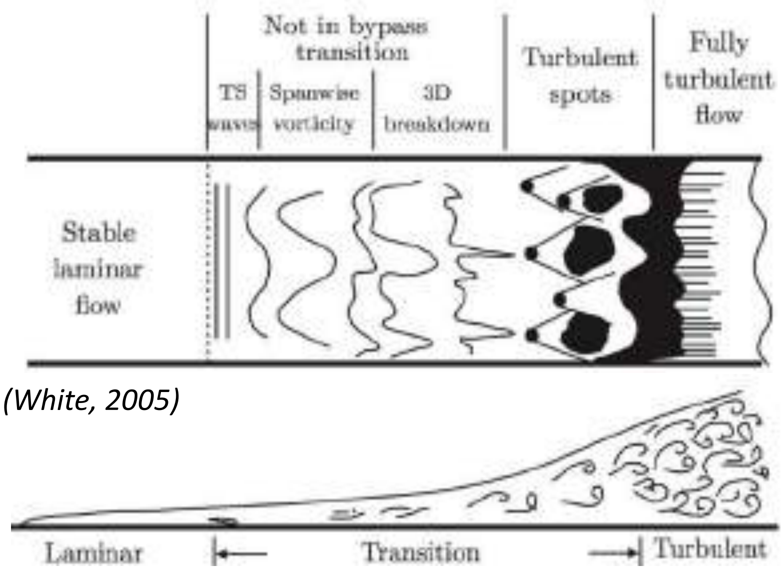
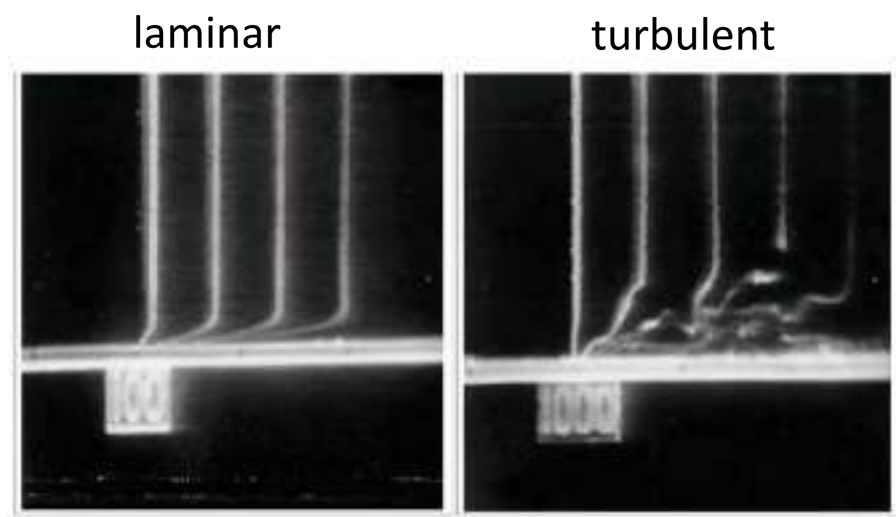
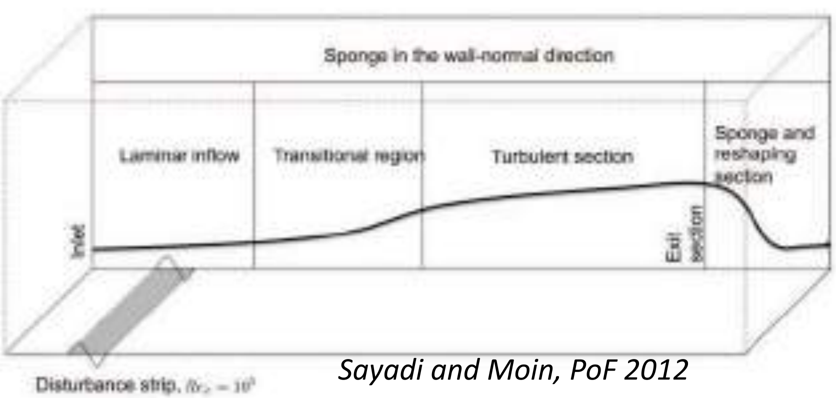
Wall bounded flows: Development of a boundary layer
Convection + diffusion of vorticity



(White, 2005)



University of Iowa



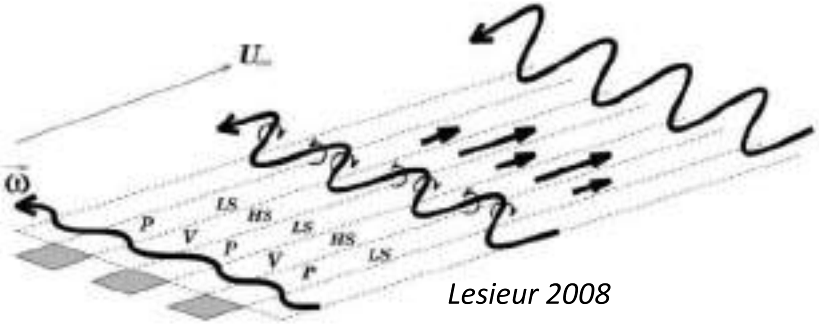
Sayadi and Moin, PoF 2012

3 main regimes:

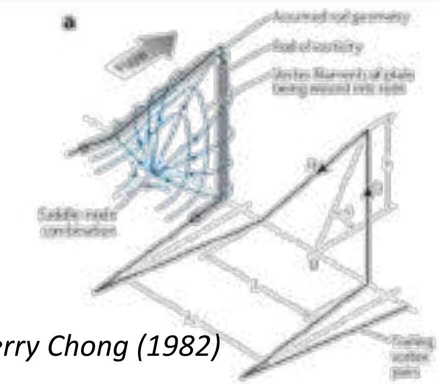
- **Laminar** boundary layer
- **Transition** laminar→ turbulence
- **Turbulent** boundary layer

(Natural) transition to turbulence

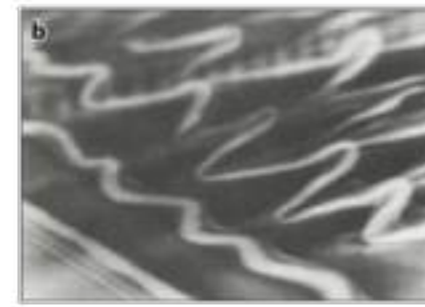
Vorticity evolution



Lesieur 2008

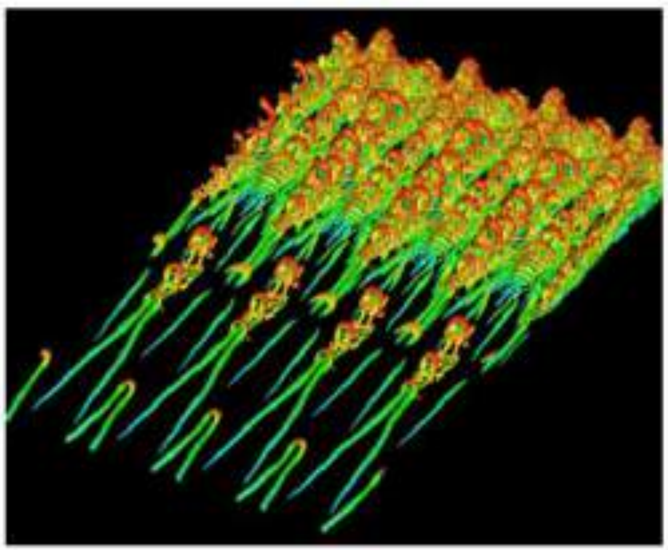


Perry Chong (1982)

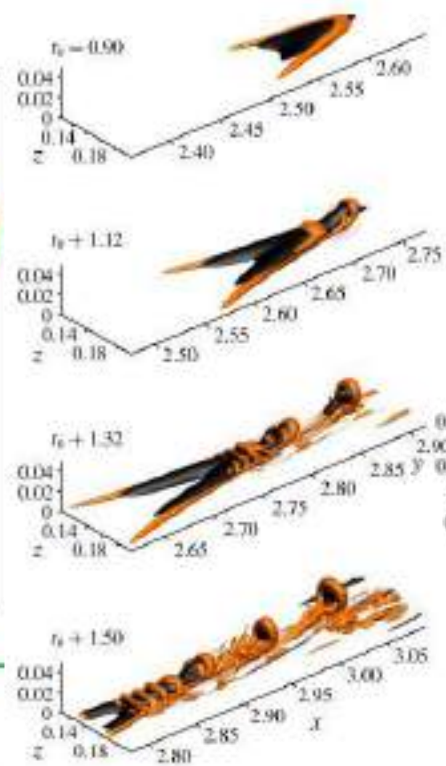


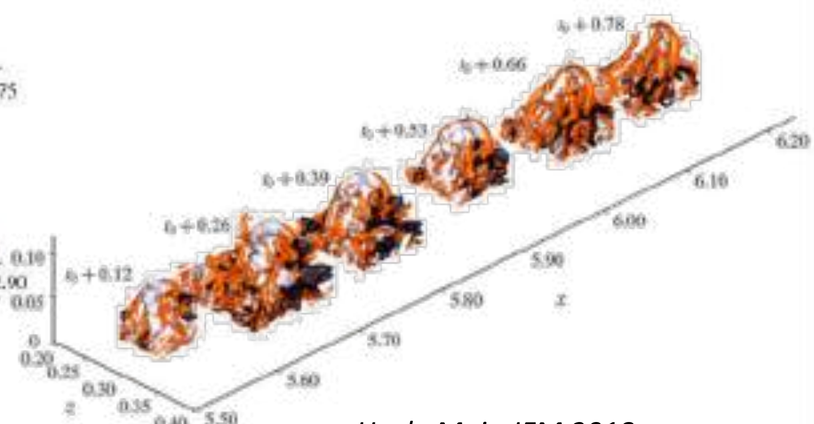
Perry et al. (1981)

Hairpin formation



Sayadi 2012

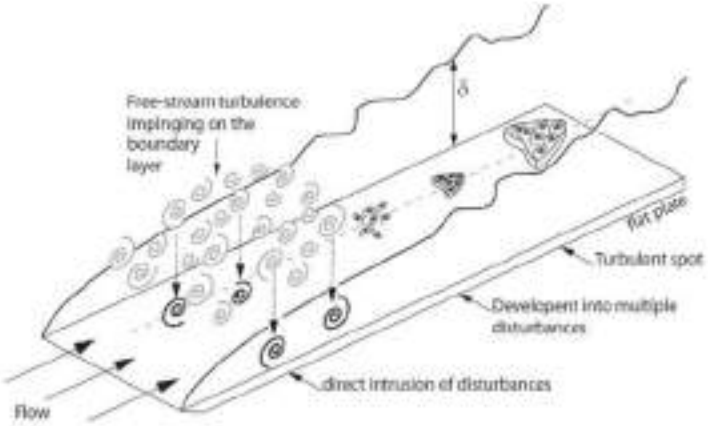




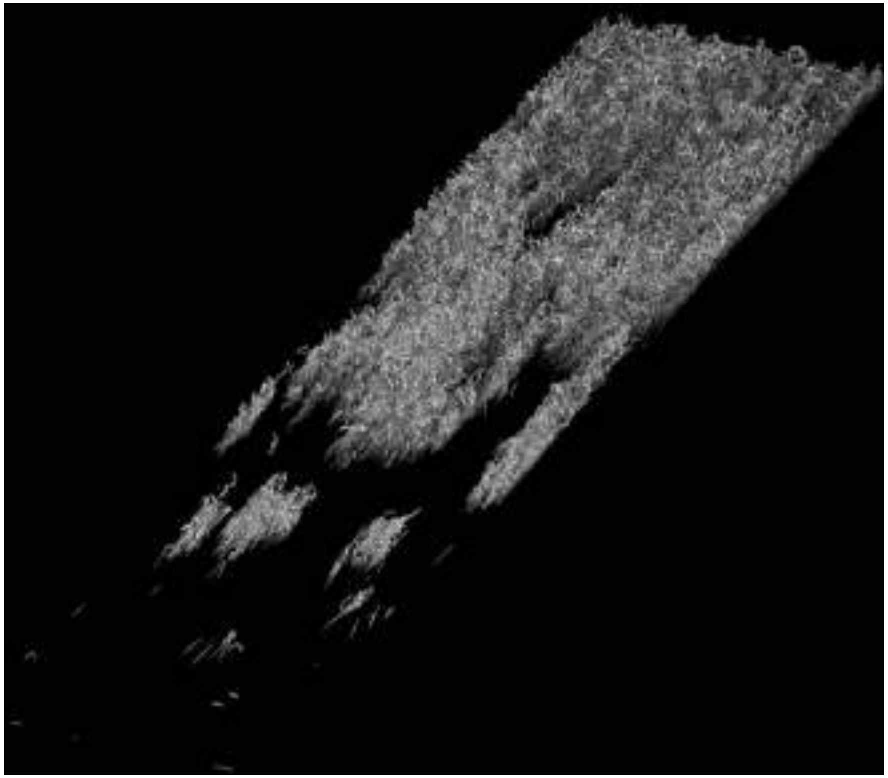
Hack, Moin JFM 2018

(Bypass) transition to turbulence

- Bypass transition: Non-natural transition to turbulence
- Transition **under freestream turbulence, noise**

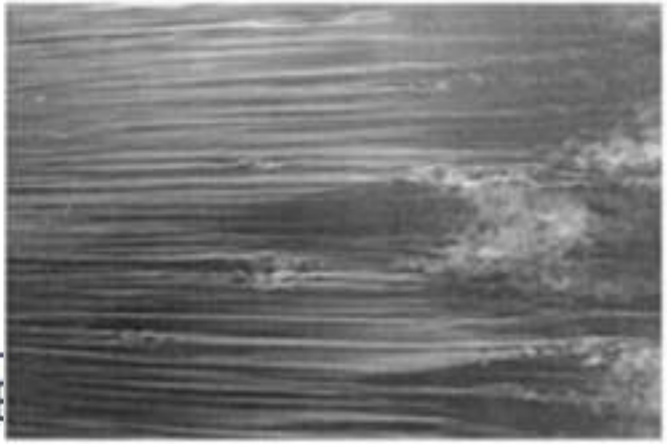


Ghasemi et al., IJHMT 2014



Wu et al., POF2014, PNAS 2017

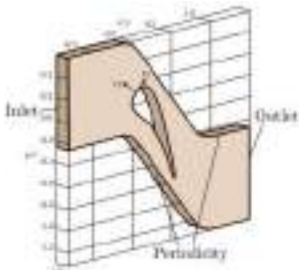
Turbulent spot



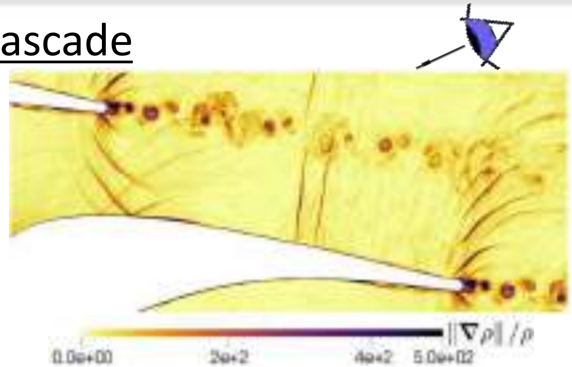
Matsubara Alfredsson 2001

Bypass transition

LS89 VKI turbine cascade

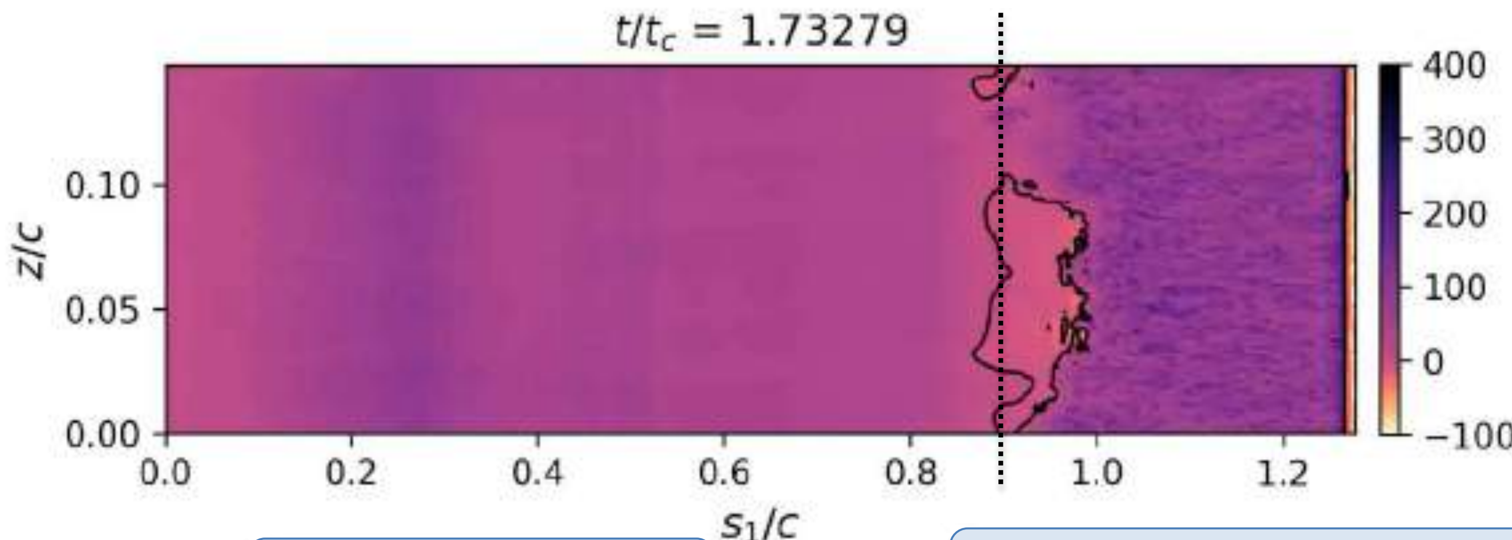


Dupuy et al., PoF2020



$\Rightarrow$ **Critical** phenomena **for** **heat transfer** prediction

Shock



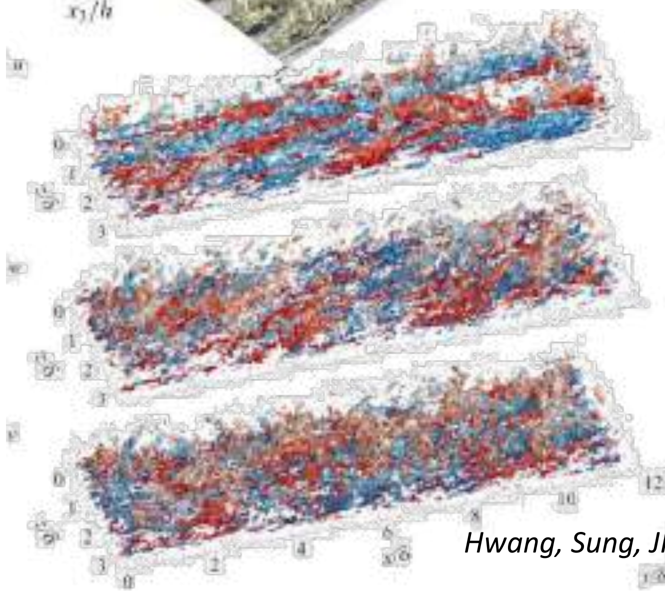
- Streaks
- Bypass transition
- Turbulent spots

- Bypass transition
- Turbulent BL
- Intermittent relaminarisation

Turbulent BL

Coherent structures in a **turbulent** BL:

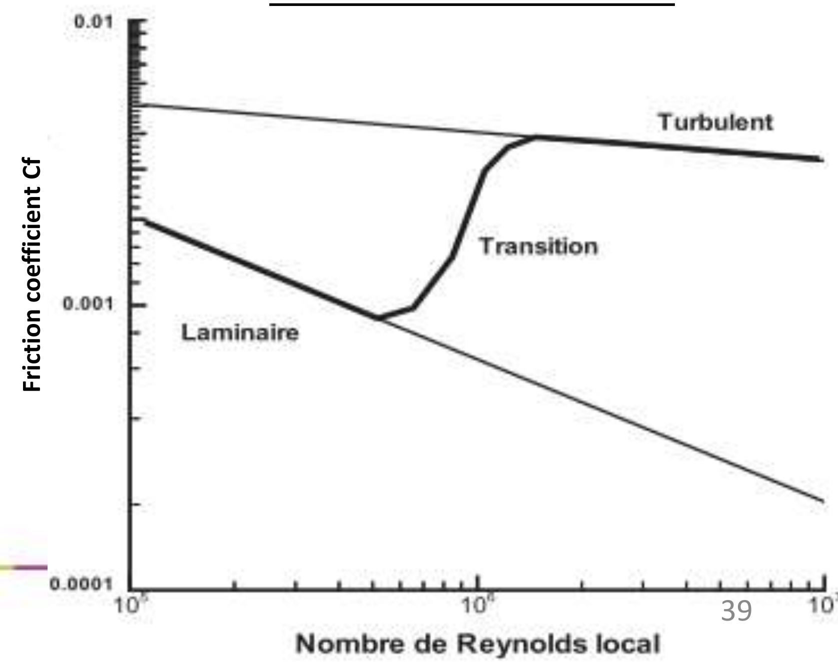
Jimenez, JFM 2018



Hwang, Sung, JFM 218

Falco 1974

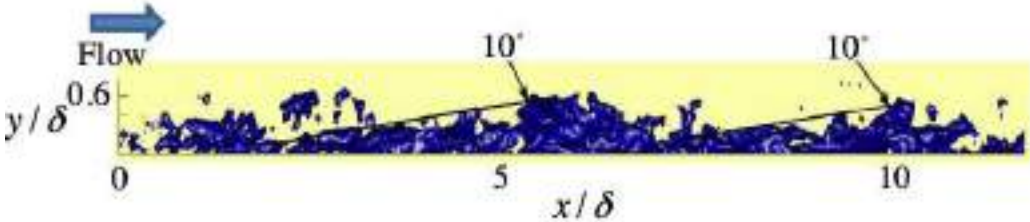
Friction coefficient



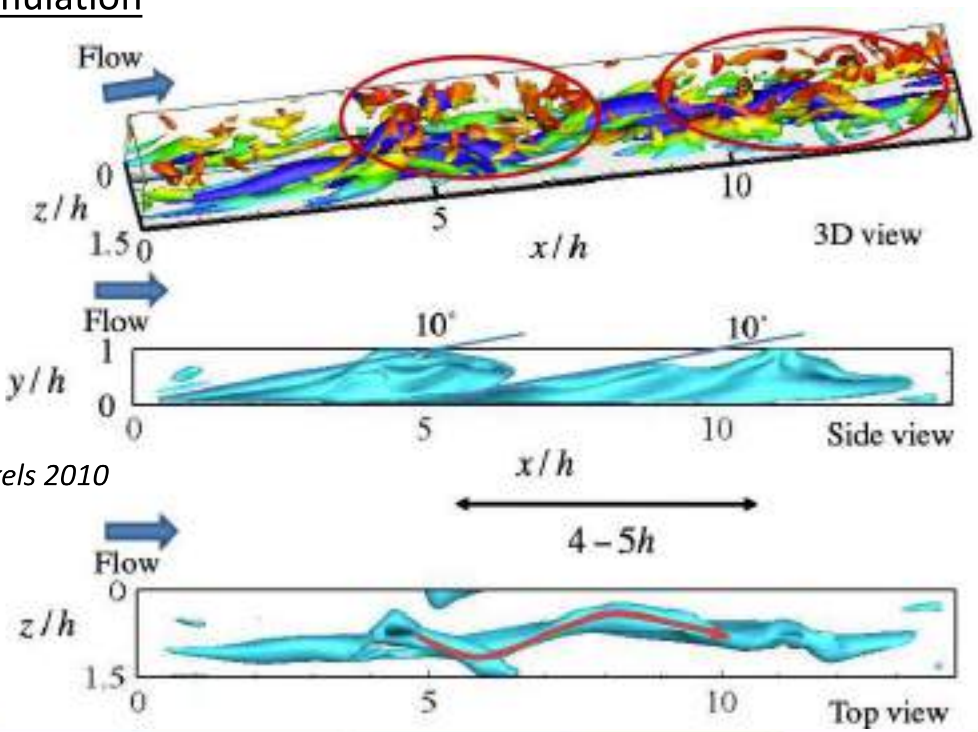
Once BL becomes turbulent: **Friction** significantly **increases**!!!

Coherence in turbulent BL

PIV measurement



Simulation



Nickels 2010

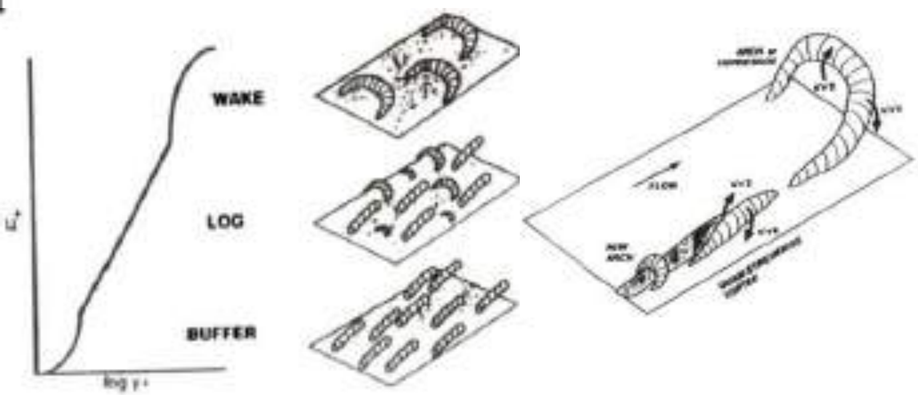
Coherence in turbulent boundary layer:

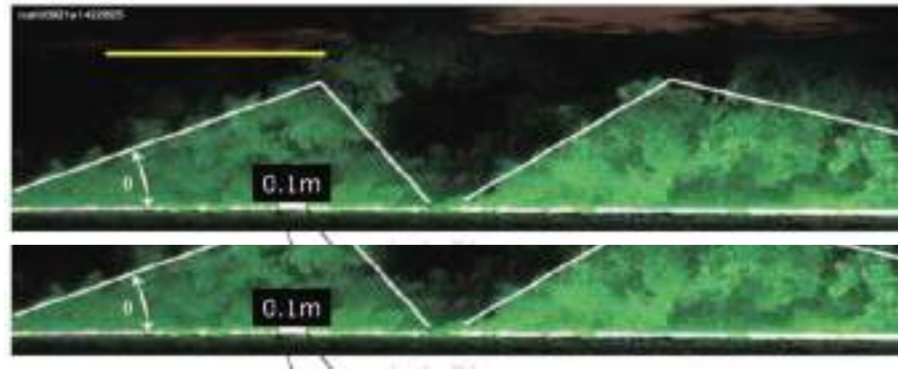
$\Rightarrow$ **Streaks**

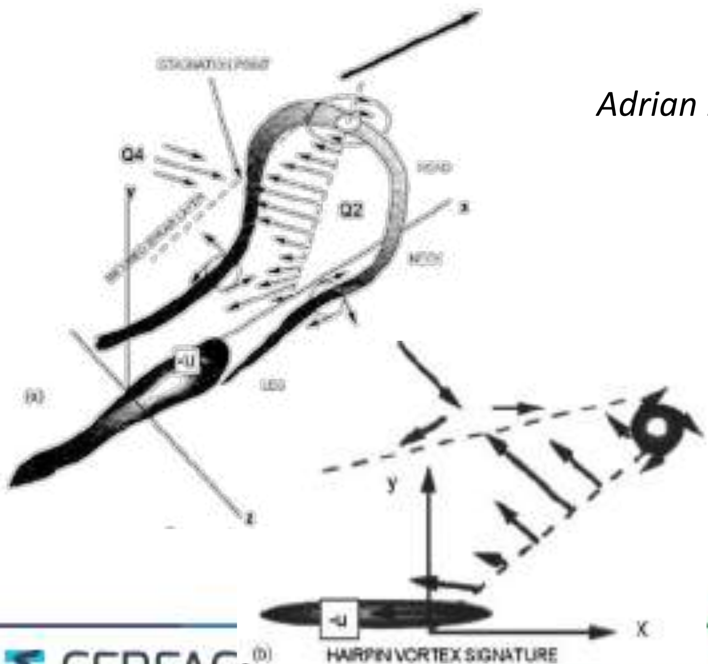
Elongated, anisotropic structures

Streaks and hairpins packets

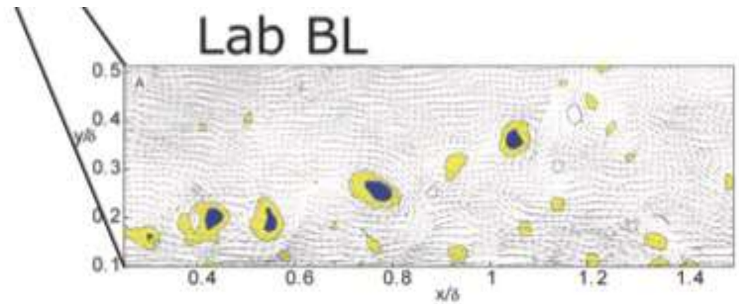
Large Scale Motions (LSM)

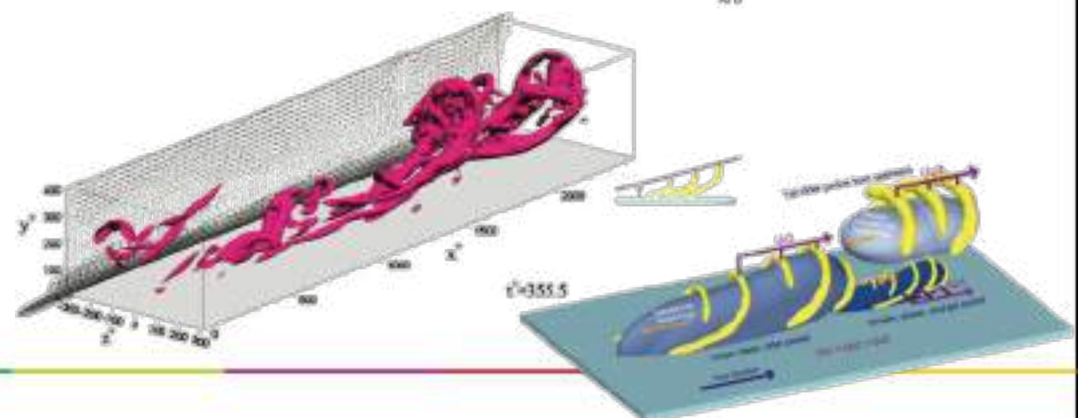






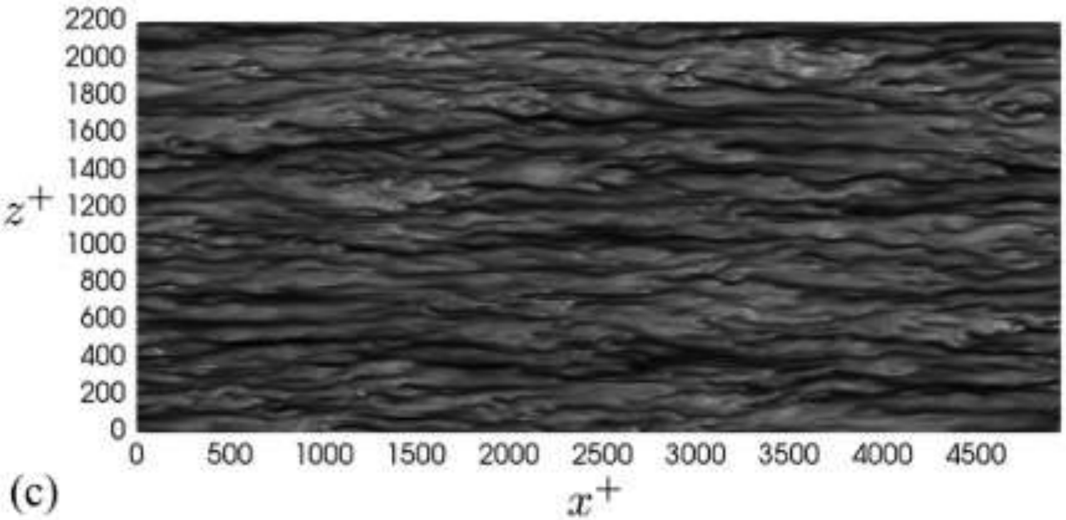
Adrian POF 2007



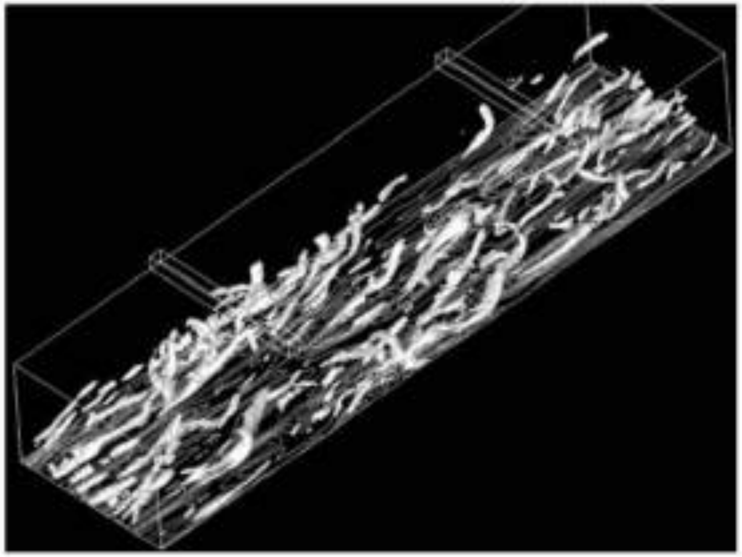


Close to the wall: “Streaks”

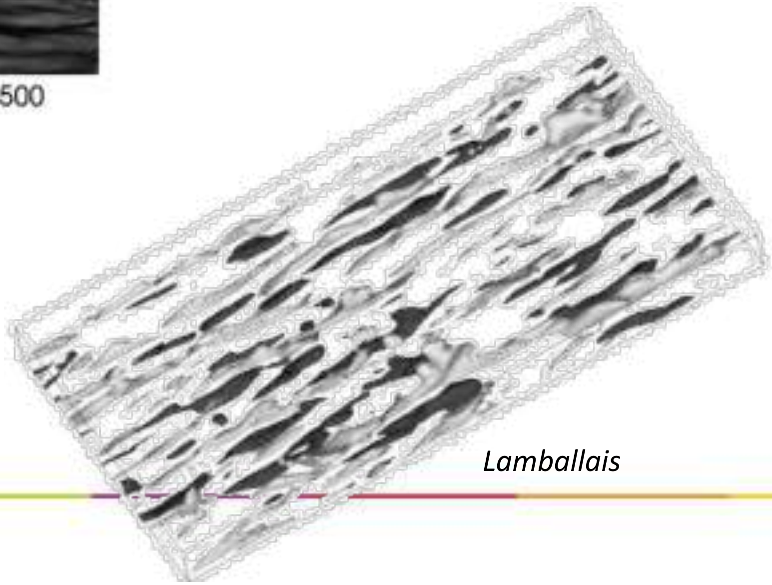
Anisotropic structures: “**Streaks**”



Rezaeiravesh Liefvendahl, PoF2018



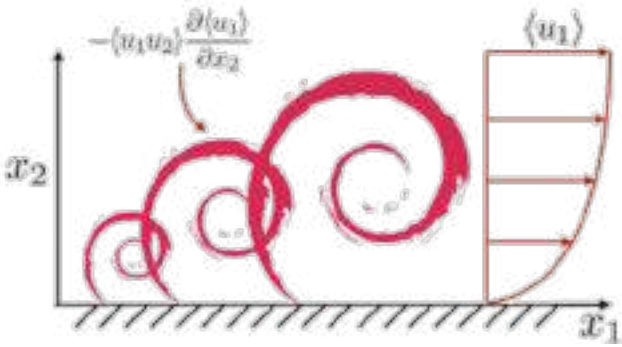
Dubief



Lamballais

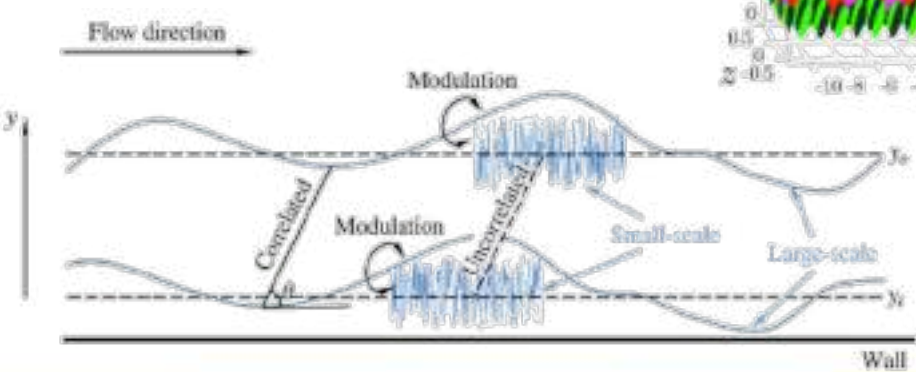
Wall-attached eddies

Townsend 1976: Structural model: attached eddies to the wall

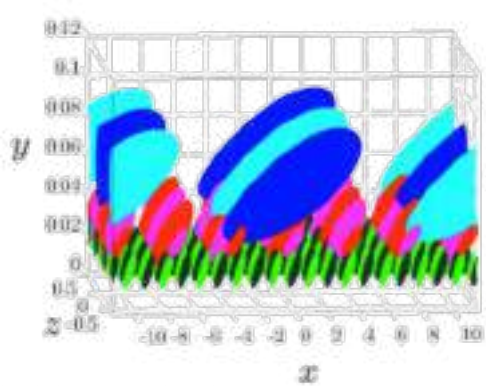


Lozano Duran, Bae 2019

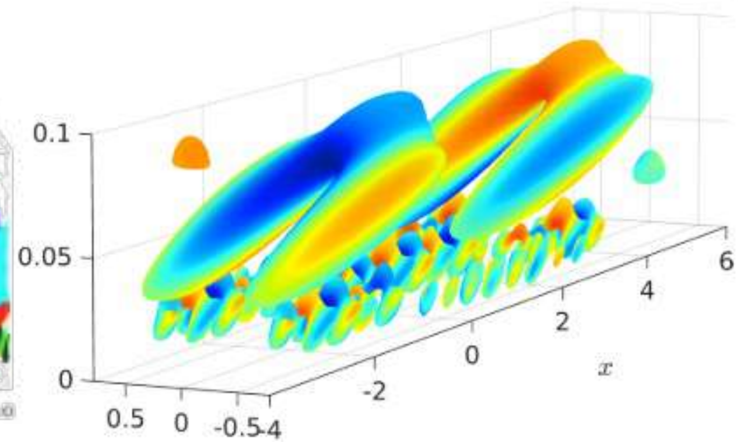
$$u^* \sim \sqrt{-\langle u_1u_2\rangle}$$
$$t^* \sim \frac{\partial\langle u_1\rangle}{\partial x_2}^{-1}$$
$$l^* \sim u^* \cdot t^*$$



Howland, Yang, JFM 2018



Self similar modes in turbulent BL:



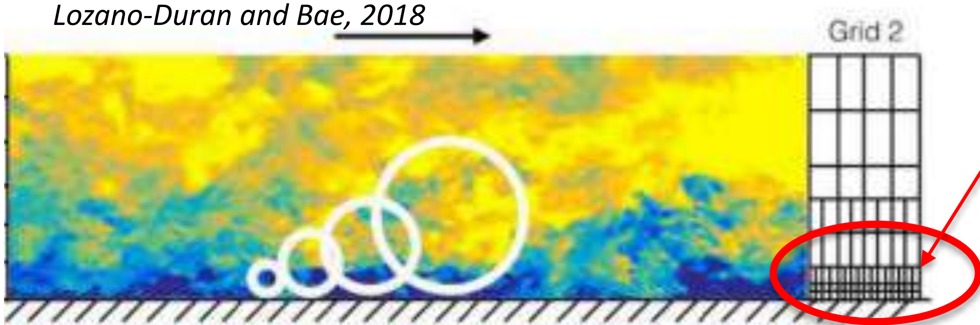
Sharma et al., 2017

=> Order in chaos…

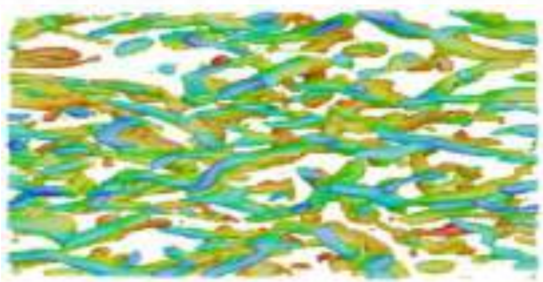
See book by Bergé, Pomeau, Vidal…

Turbulent BL and numerical simulations

Lozano-Duran and Bae, 2018



A fine grid is mandatory to resolve all structures!



"Sreaks": Anisotropic wall turbulent structures:
=> Spanwise and streamwise **constraints**:

$\Delta y_1^+ < 2$

$\Delta x^+ \sim 50 - 150$

$\Delta z^+ \sim 15 - 40$

Chapman (1979), Choi & Moin (2012):

Outer Layer (LES)

- Spatial resolution : $N_x N_y N_z \sim \mathrm{Re}^{0.4}$
- Total cost : $N_x N_y N_z \times N_t \sim \boxed{\mathrm{Re}^{0.6}}$

Inner Layer (LES)

- Stretched, nested grids : $N_x N_y N_z \propto \mathrm{Re}^{1.8}$
- Total cost : $N_x N_y N_z \times N_t \sim \boxed{\mathrm{Re}^{2.3} \text{ to } \mathrm{Re}^{3.6}}$

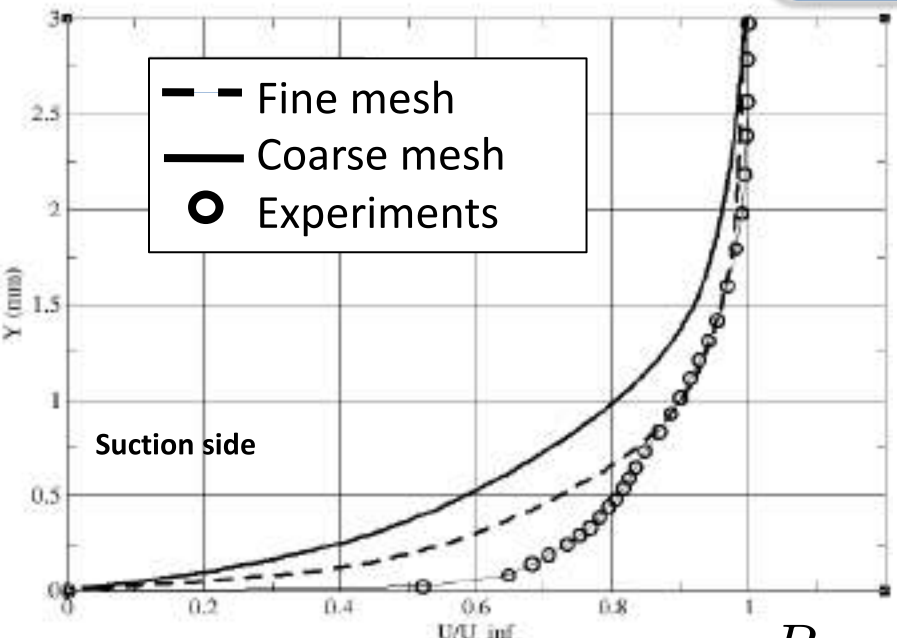
Resolving the inner layer is CPU demanding!!

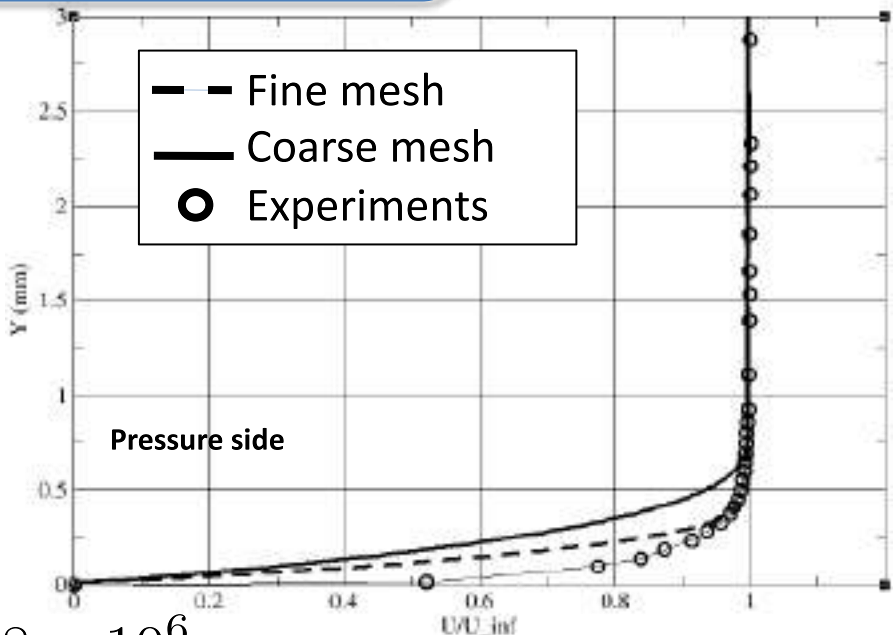
Wall resolved simulations

Even with such constraints, friction prediction remains a challenge at High Reynolds

"Fine mesh": $$\begin{aligned} \Delta y^+ &\simeq 2 \\ \Delta x^+ &\simeq 100 \\ \Delta z^+ &\simeq 50 \end{aligned}$$

Leonard et al., 2010

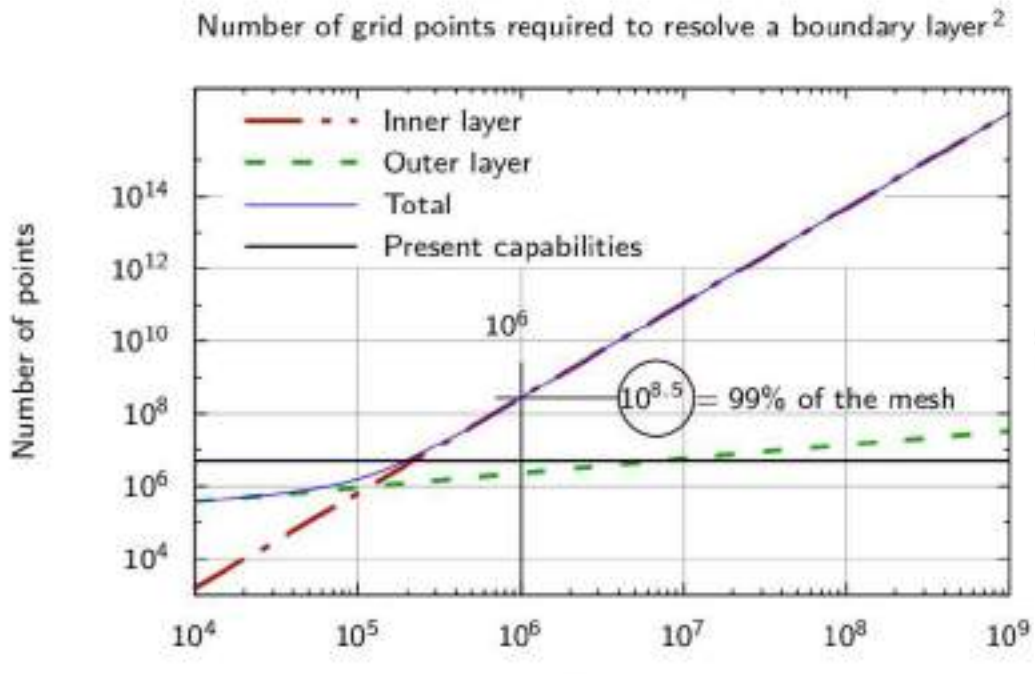




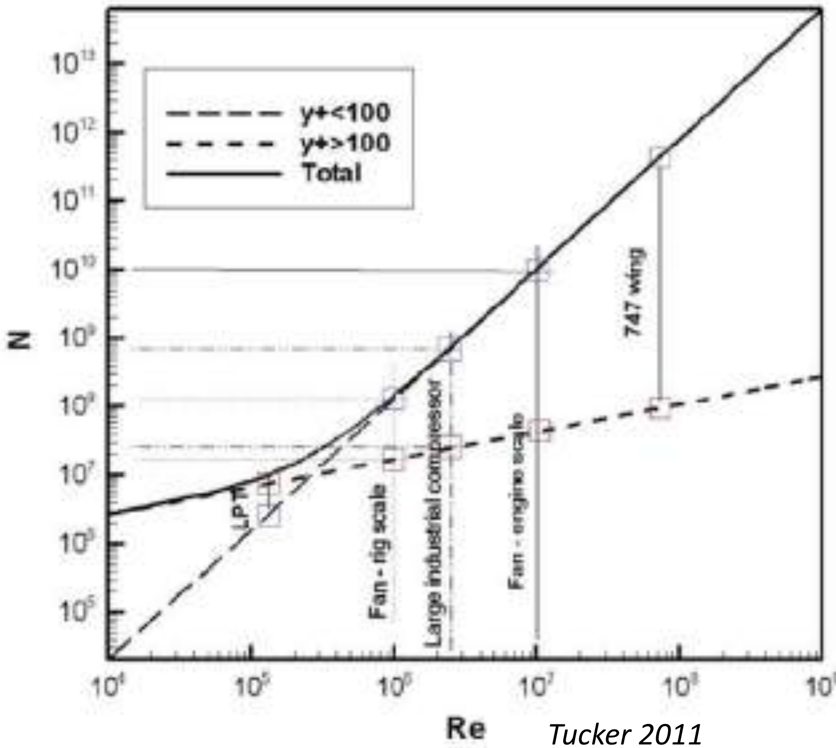
$$R_e = 2.8 \times 10^6$$

A need for wall modeling

In any case, **resolving the entire boundary layer is still intractable** for real applications.



Piomelli et al (2002)



Tucker 2011

- Full airplane wall-resolved simulation not expecting by 2100.
- **Modeling** the boundary layer is **mandatory** for real applications

Large eddy simulations: Practical issues

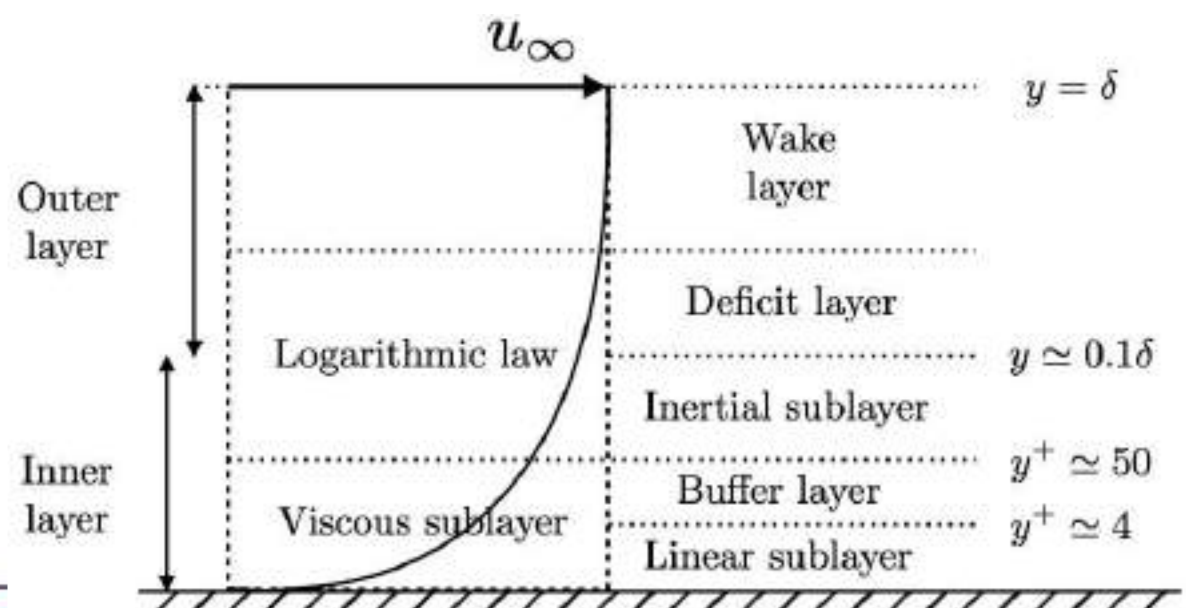
Turbulent boundary layer

- Let's consider **an incompressible 2D** flow**, zero pressure gradient, steady mean** flow.

  $\Rightarrow$ Momentum equation becomes:

$$\underbrace{\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y}}_{\text{inertial terms}} = \underbrace{\nu\frac{\partial^2 \overline{u}}{\partial y^2}}_{\text{viscous stress}} + \underbrace{\frac{\partial(-\overline{u'v'})}{\partial y}}_{\text{Reynolds shear stress}}$$

- BL is divided in regions, according to the **total shear stress** $\tau = \mu\frac{\partial \overline{u}}{\partial y} - \rho\overline{u'v'}$



(Very) Large Scale Motion (VLSM)

streaks

Velocity profile in a turbulent BL

$$\underbrace{\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}}_{\text{inertial terms}} = \underbrace{\nu\frac{\partial^2 \bar{u}}{\partial y^2}}_{\text{viscous stress}} + \underbrace{\frac{\partial(-\overline{u'v'})}{\partial y}}_{\text{Reynolds shear stress}}$$

In the limit of **infinite Reynolds**: $Re = \frac{UL}{\nu} = \frac{T_{vis}}{T_{in}}$

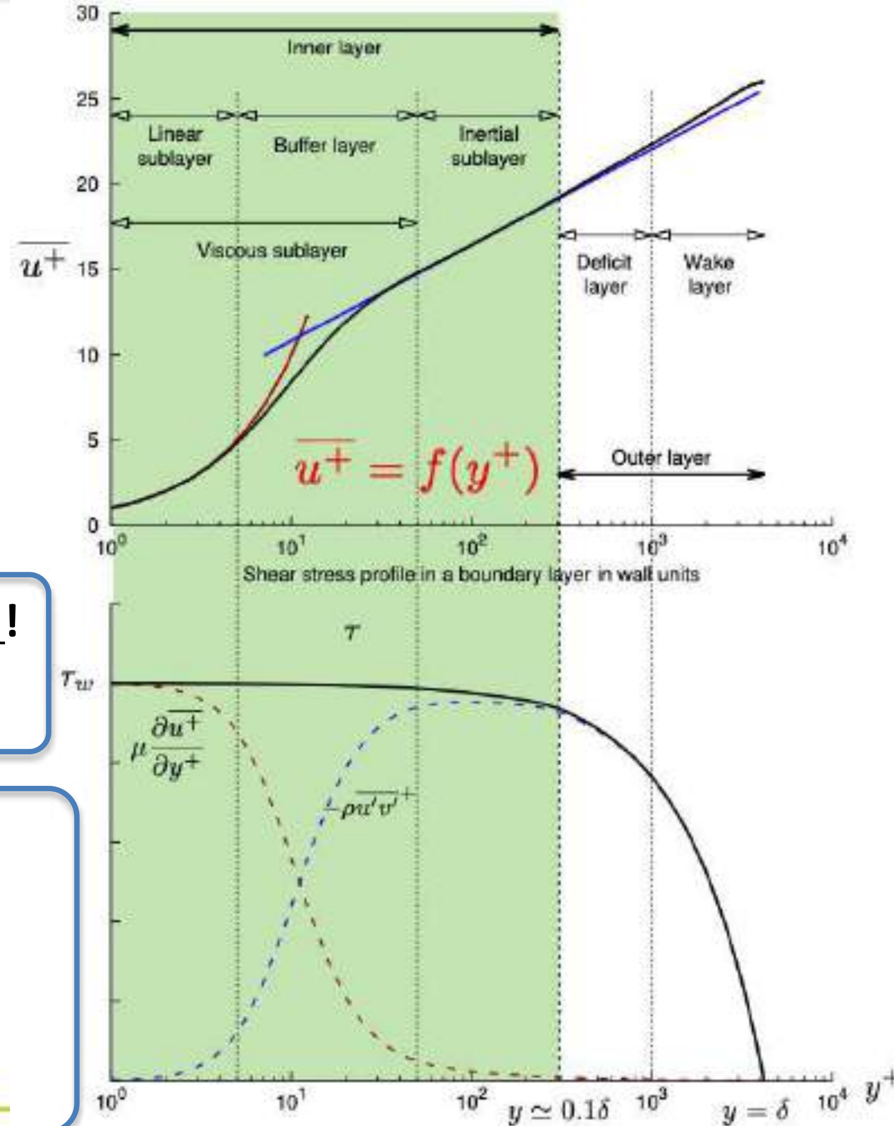
$$0 = \frac{\partial}{\partial y}\left[-\rho\overline{u'v'} + \mu\frac{\partial \bar{u}}{\partial y}\right] = \frac{\partial \tau}{\partial y}$$

=> In the inner layer, **total shear stress = constant !**

$$\tau = \text{cst} = \tau_w$$

Let's define:

- $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$
- $\overline{u^+} = \frac{\bar{u}}{u_\tau} = f(y^+)$
- $y^+ = \frac{y u_\tau}{\nu}$



$$0 = \underbrace{\nu \frac{\partial^2 \bar{u}}{\partial y^2}}_{\text{viscous stress}} + \underbrace{\frac{\partial \left(-\overline{u'v'}\right)}{\partial y}}_{\text{Reynolds shear stress}}$$

Velocity profile in a turbulent BL

- Very near the wall:

  Reynolds stress negligible: $\mu \frac{\partial^2 \overline{u}}{\partial y^2} = 0$

- Integration:

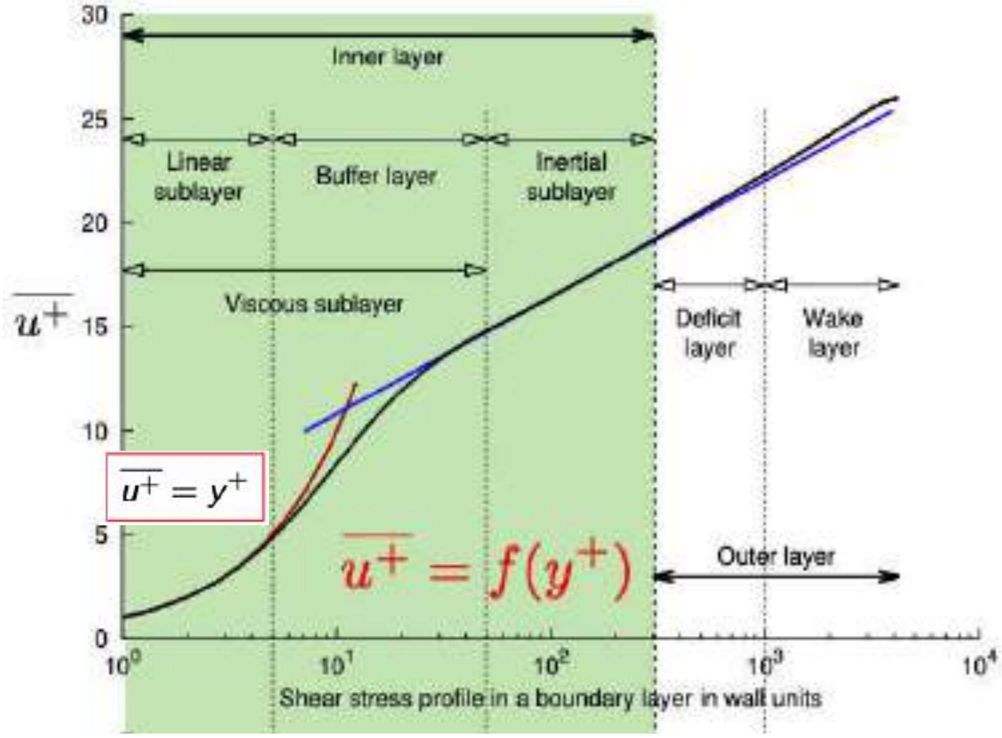
  $$u = \text{cst} \times y = \frac{\tau_w}{\mu} y$$

- In inner variables:

  $$\boxed{\overline{u^+} = y^+}$$

  Linear layer

  $y^+ \sim 3\text{–}5$



$$\overline{u^+} = \frac{\overline{u}}{u_\tau} \qquad u_\tau = \sqrt{\frac{\overline{\tau_w}}{\rho}}$$

$$y^+ = \frac{y u_\tau}{\nu}$$

$$0 = \underbrace{\nu \frac{\partial^2 \bar{u}}{\partial y^2}}_{\text{viscous stress}} + \underbrace{\frac{\partial \left(-\overline{u'v'}\right)}{\partial y}}_{\text{Reynolds shear stress}}$$

Velocity profile in a turbulent BL

- Away from the viscous region: Reynolds stress dominates : $\tau = \cancel{\mu \frac{\partial \overline{u}}{\partial y}} - \rho \overline{u'v'}$
- Boussinesq hypothesis:

$$-\overline{u'v'} = \nu_t \frac{\partial \overline{u}}{\partial y}$$

- Mixing length hypothesis:

$$\nu_t = \kappa u_\tau y$$

Von Karman "constant", $\kappa \simeq 0.41$

$$\mu_t \frac{\partial \overline{u}}{\partial y} = \tau_w$$

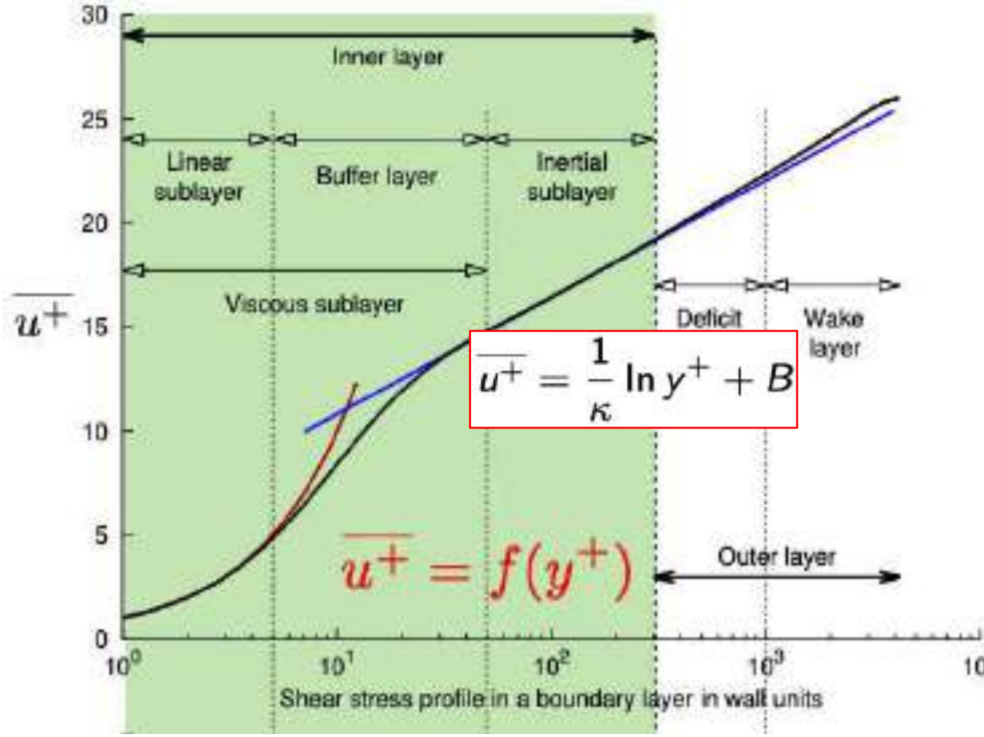
$$\rho \kappa u_\tau y \frac{\partial \overline{u}}{\partial y} = \tau_w = \rho u_\tau^2$$

$$\kappa \frac{\partial \overline{u}}{u_\tau} = \frac{\partial y}{y}$$

$$\mathrm{d}u^+ = \frac{1}{\kappa} \frac{\mathrm{d}y^+}{y^+}$$

$$\boxed{u^+ = \frac{1}{\kappa} ln(y^+) + B}$$

Logaritmic layer



$$\overline{u^+} = \frac{\overline{u}}{u_\tau} \qquad y^+ = \frac{y u_\tau}{\nu} \qquad u_\tau = \sqrt{\frac{\overline{\tau_w}}{\rho}}$$

$$0 = \underbrace{\nu \frac{\partial^2 \bar{u}}{\partial y^2}}_{\text{viscous stress}} + \underbrace{\frac{\partial\left(-\overline{u'v'}\right)}{\partial y}}_{\text{Reynolds shear stress}}$$

Velocity profile in a turbulent BL

- <u>Very near the wall</u>:

  Reynolds stress negligible:

  $\boxed{\overline{u^+} = y^+}$ **Linear layer**

  $y^+ \sim 3\text{–}5$

- <u>Away from the viscous region:</u>

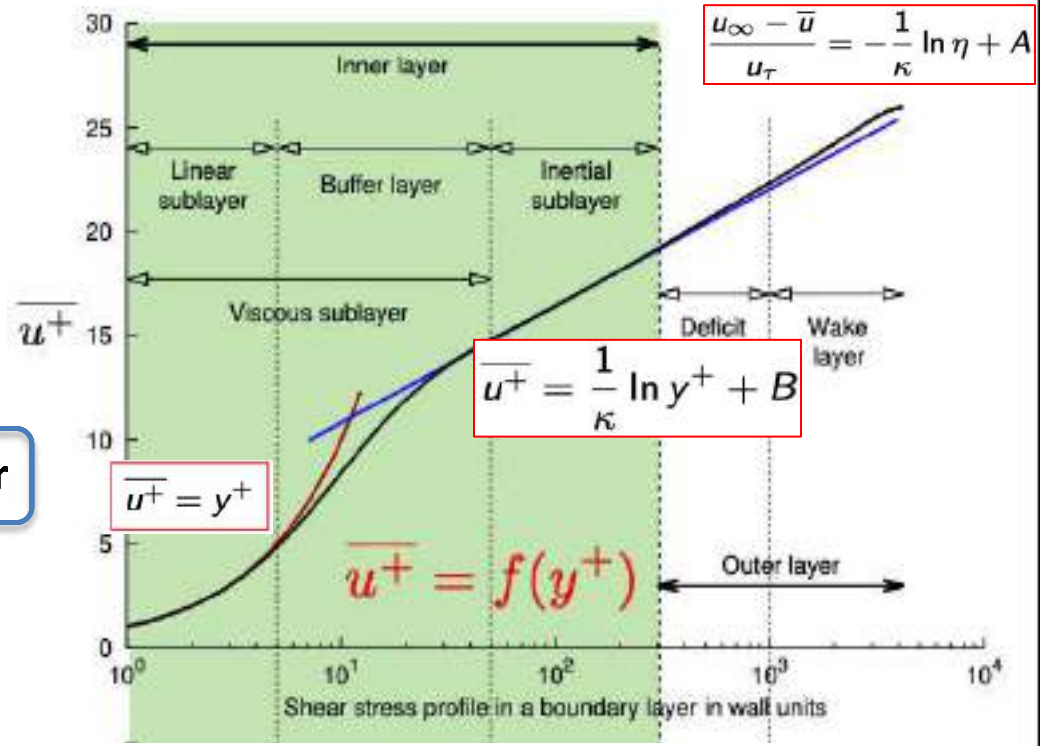
  Reynolds stress dominates:

  $\boxed{\overline{u^+} = \frac{1}{\kappa} \ln y^+ + B}$ **Logaritmic layer**

- <u>We can also demonstrate</u>:

  Close to $y \simeq \delta$

  $\boxed{\frac{u_\infty - \overline{u}}{u_\tau} = -\frac{1}{\kappa} \ln \eta + A}$ **Wake layer** $\eta = y/\delta$

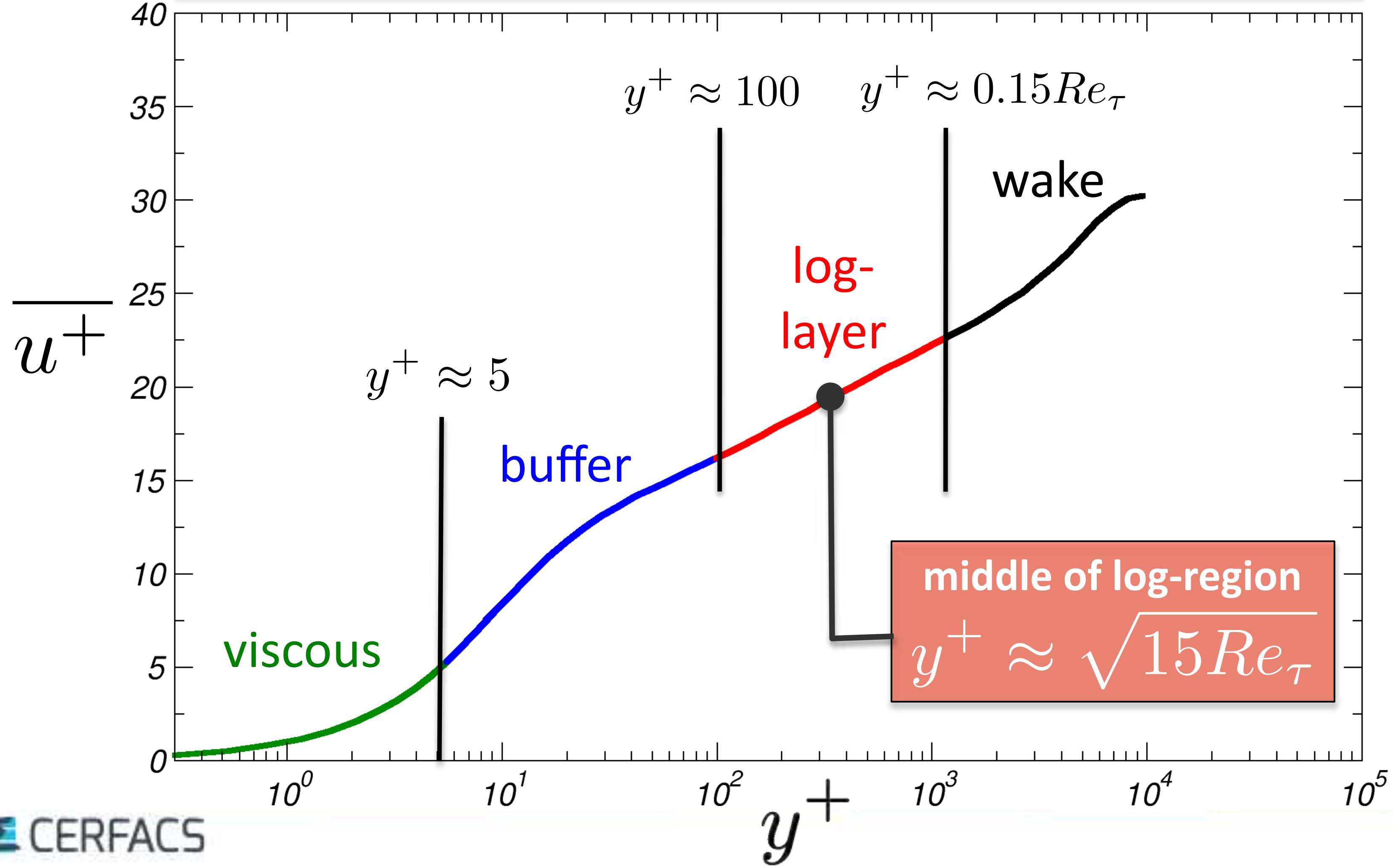


$$\overline{u^+} = \frac{\overline{u}}{u_\tau} \qquad u_\tau = \sqrt{\frac{\overline{\tau_w}}{\rho}}$$

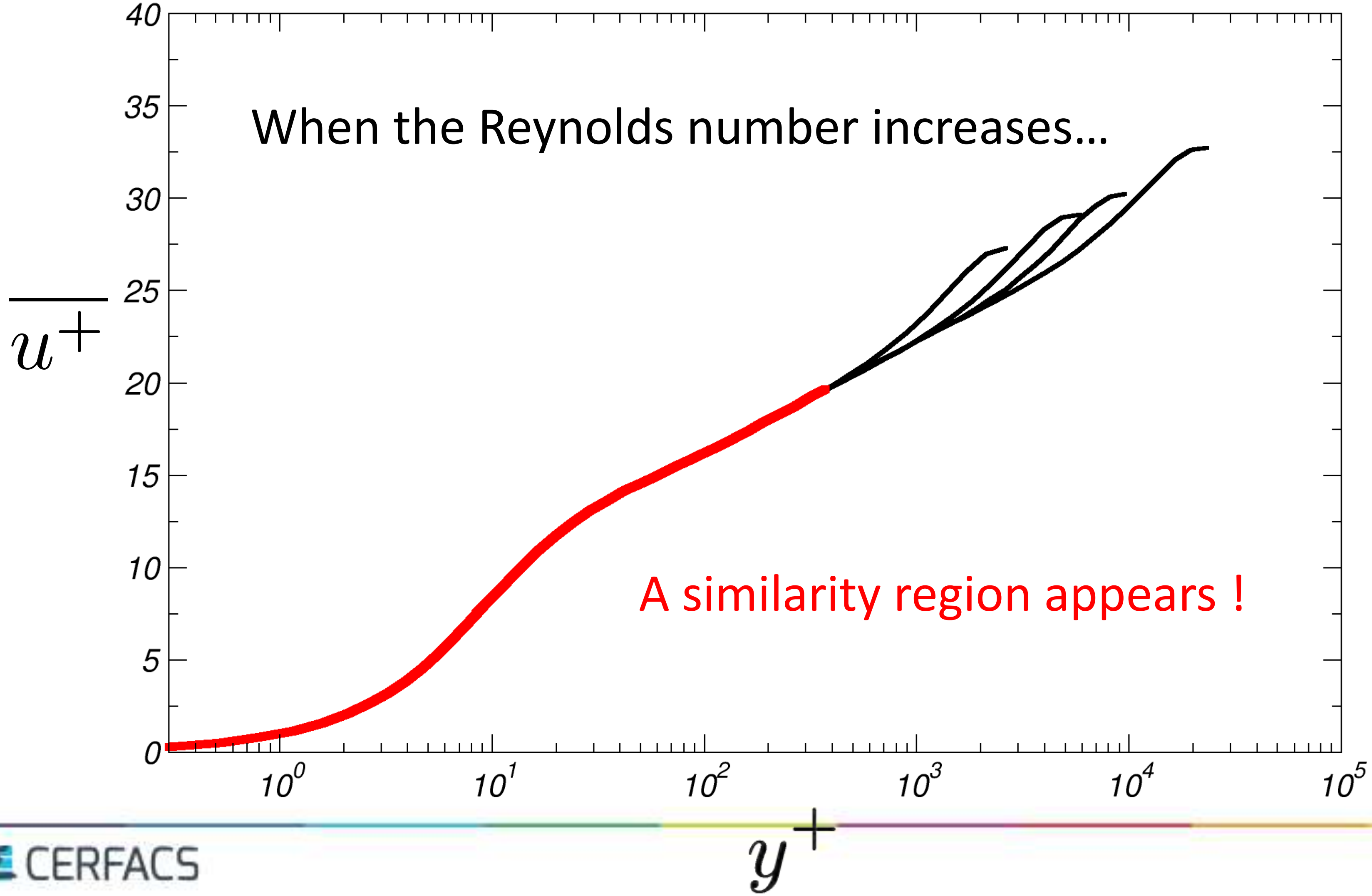
$$y^+ = \frac{y u_\tau}{\nu}$$

Velocity profile in a turbulent BL

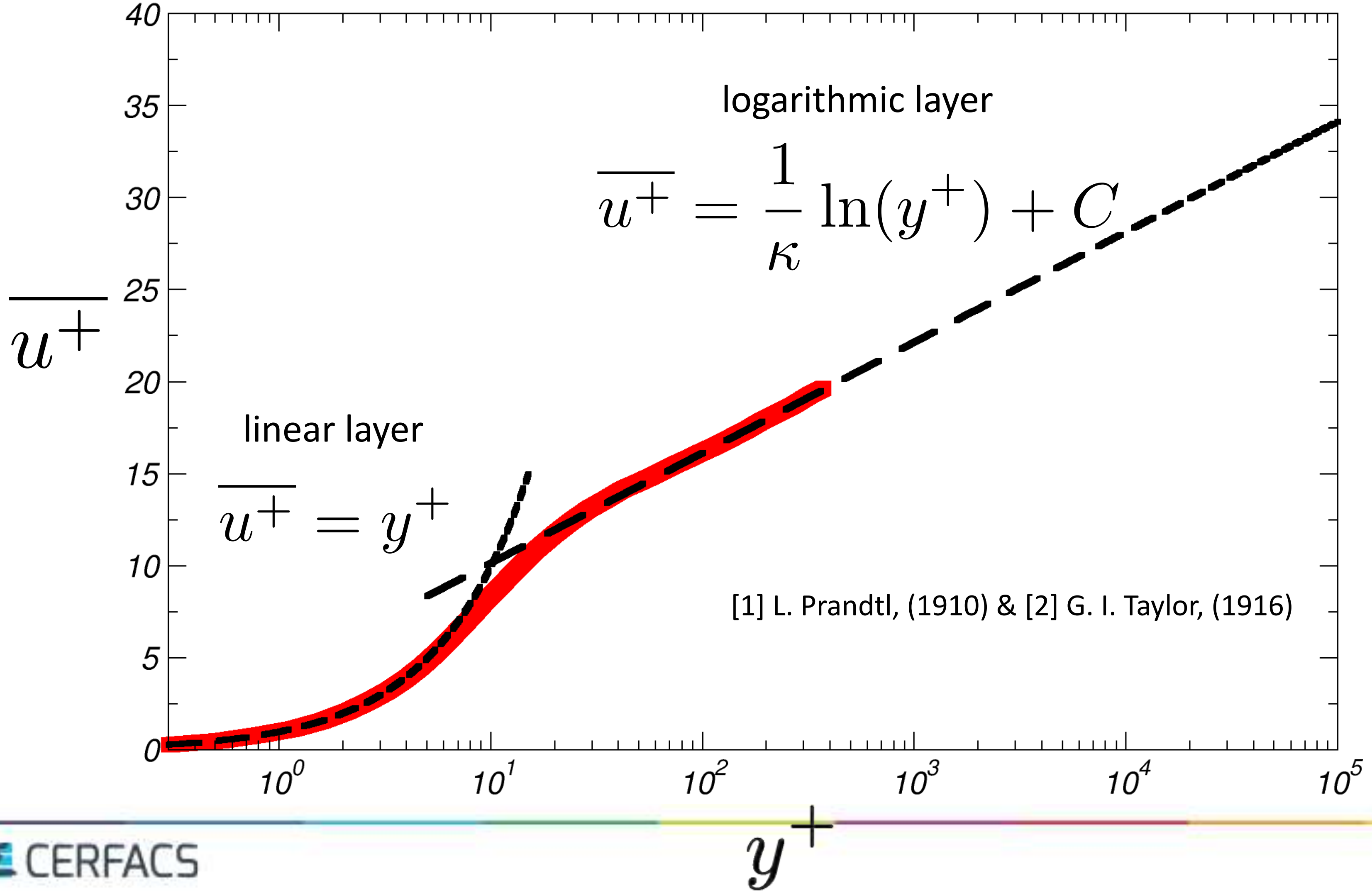
scaling $\overline{u^+} = \overline{u}/u_\tau$ with $u_\tau = \sqrt{\overline{\tau_{w,x}}/\rho_w}$



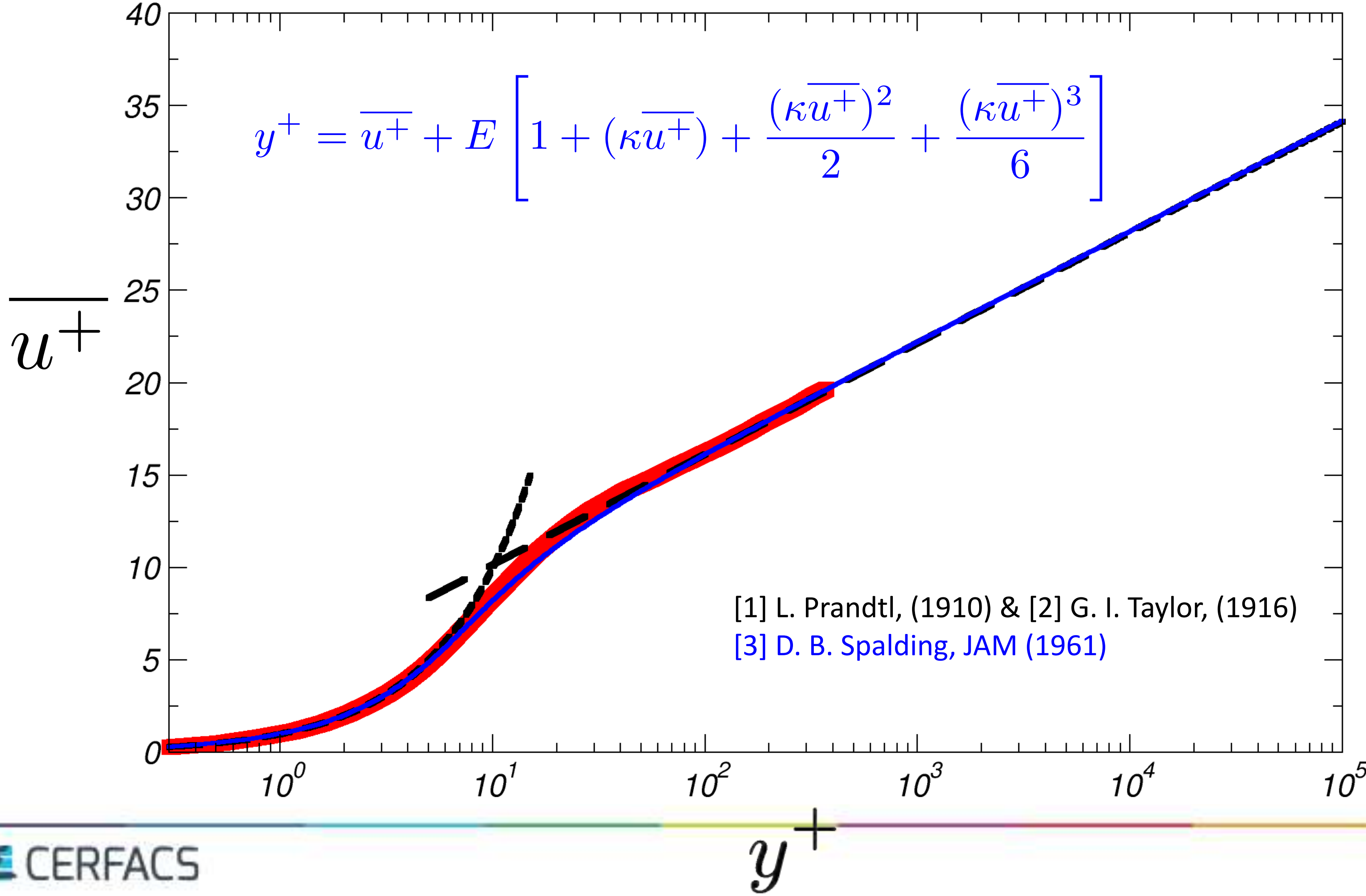
Velocity profile in a turbulent BL



2-layer approach [1-2]



Spalding's Law [3]

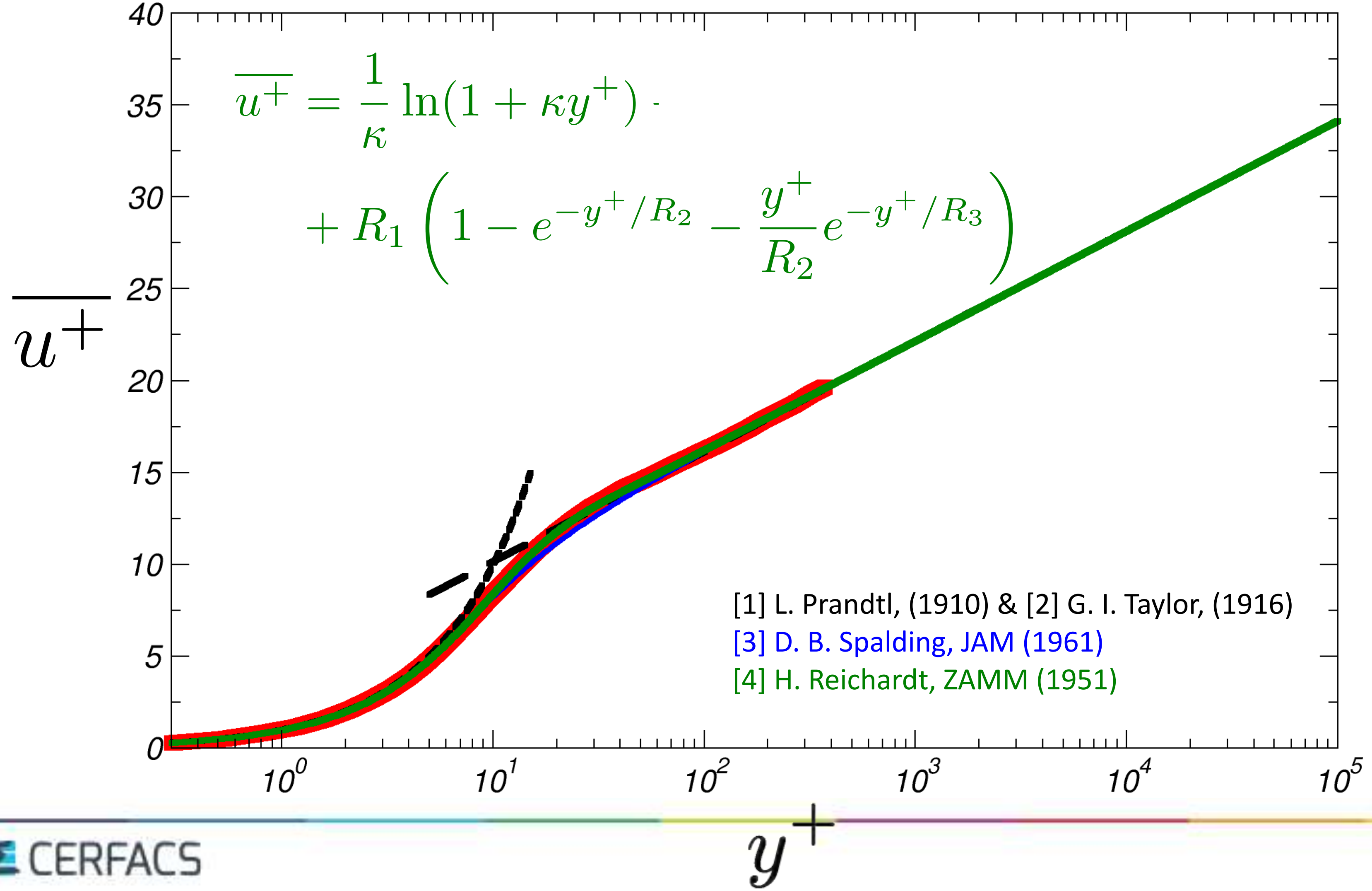


$$y^+ = \overline{u^+} + E\left[1 + (\kappa\overline{u^+}) + \frac{(\kappa\overline{u^+})^2}{2} + \frac{(\kappa\overline{u^+})^3}{6}\right]$$

[1] L. Prandtl, (1910) & [2] G. I. Taylor, (1916)

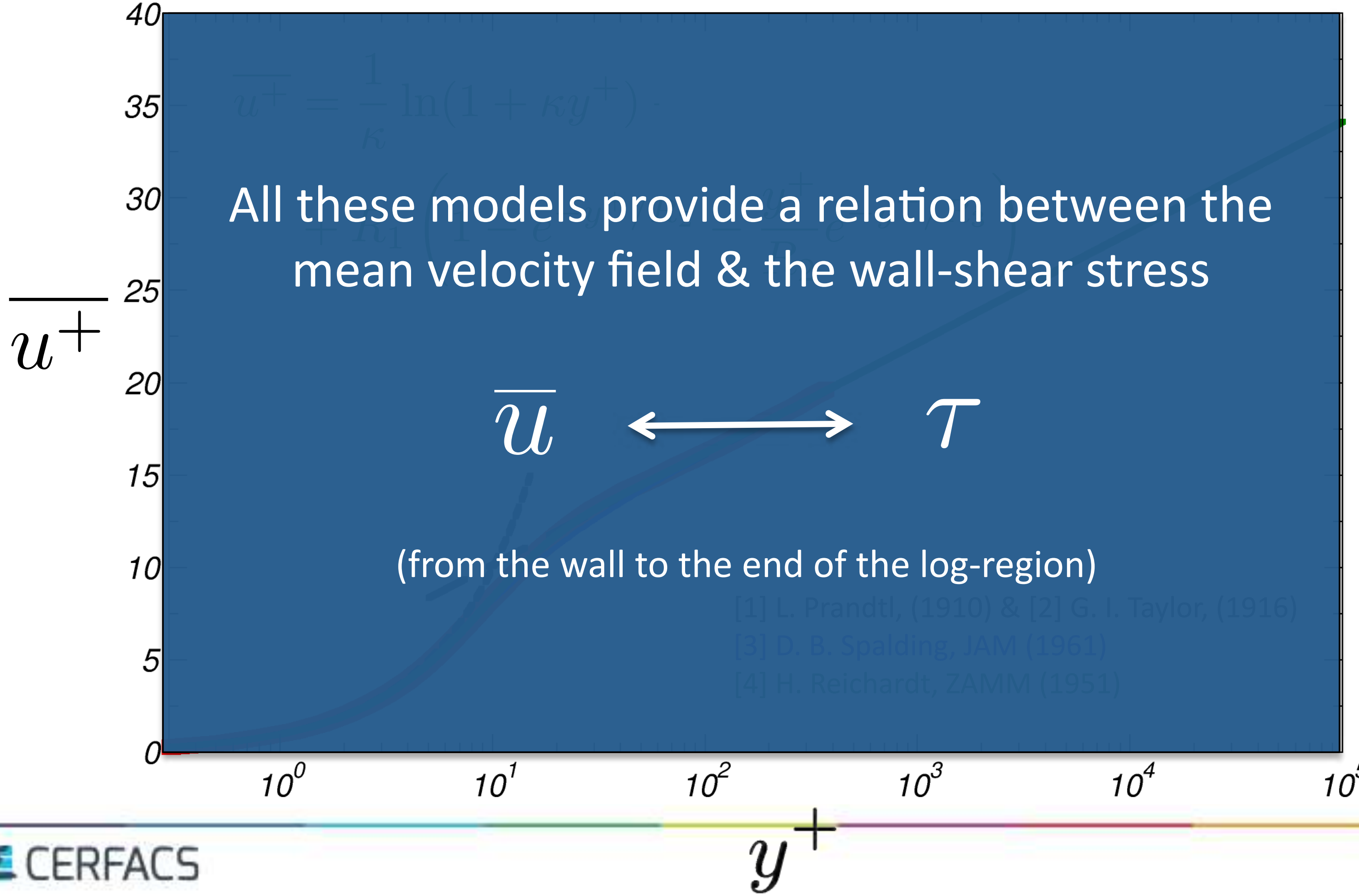
[3] D. B. Spalding, JAM (1961)

Reichardt's Law [4]



The wall-law

Let’s model this velocity profile…



All these models provide a relation between the mean velocity field & the wall-shear stress

$$\overline{u} \longleftrightarrow \tau$$

(from the wall to the end of the log-region)

The wall-law

$\overline{u^+} = f(y^+)$ is called the "**wall-law**", or "**law-of-the-wall**"

The **simplest** wall-law is the **2-layer law**:

- $\overline{u^+} = y^+$ For low y+
- $\overline{u^+} = \frac{1}{\kappa} \ln y^+ + B$ For y+>10

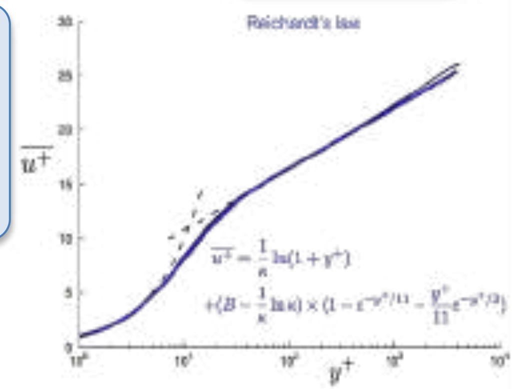
$$\overline{u^+} = \frac{\overline{u}}{u_\tau}$$

$$y^+ = \frac{y u_\tau}{\nu}$$

$$u_\tau = \sqrt{\frac{\overline{\tau_w}}{\rho}}$$

Provide a **relation** between **mean velocity** and mean wall **shear stress**:

$$\boxed{\overline{u} \longleftrightarrow \tau_w}$$



Relies on **assumptions**!

- **Stationary** flow
- No pressure gradient
- Attached flow
- Flat plate
- 2D
- Incompressible
- Fully turbulent

Large eddy simulations: Practical issues

LES: A brief reminder

NS equation and turbulence
LES, filtering and models
Numerics, errors and LES
Implicit LES

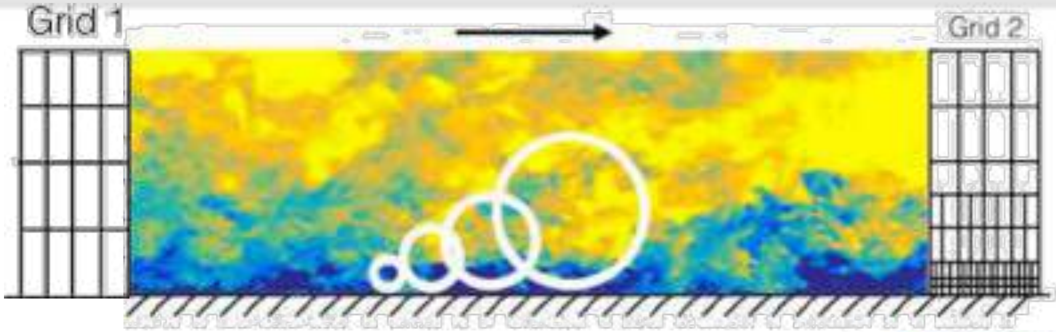
Wall treatment

Wall-bounded turbulence
The logarithmic law-of-the-wall
Wall-modeled LES: analytical, TBLE
Detached-Eddy Simulations
Log-layer mismatch and applications

Turbulence injection

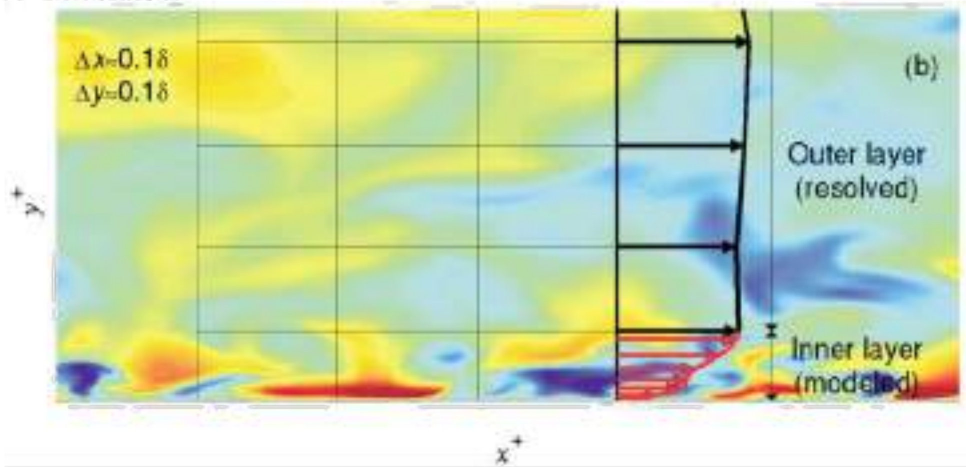
Synthetic methods (Fourier / POD)
Precursor method
Wall-bounded turbulent injection

Wall-**modeled** LES: principle



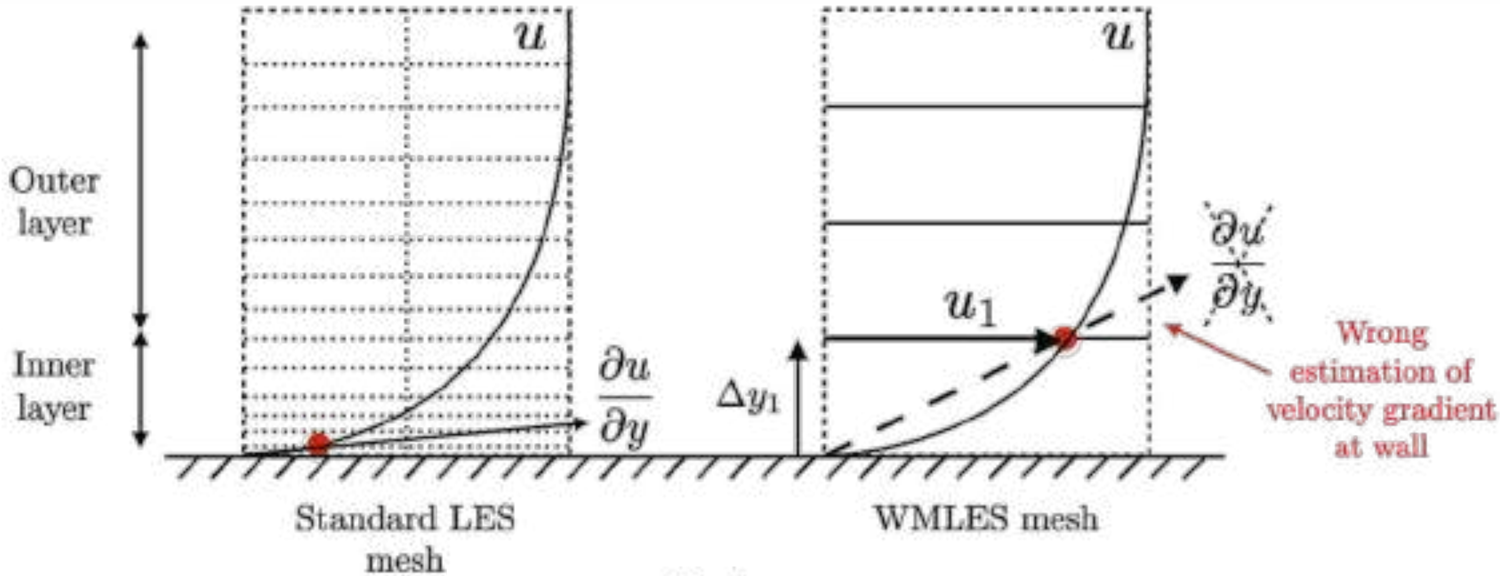
Bae et al. 2018

Piomelli Balaras 2002



- Mesh dedicated to the **outer layer.**
- **Inner layer** not resolved in WMLES => a **model** is needed.
- **SGS model** and **numerical method** likely to induce **non-physical structures**…

Wall-modeled LES: principle



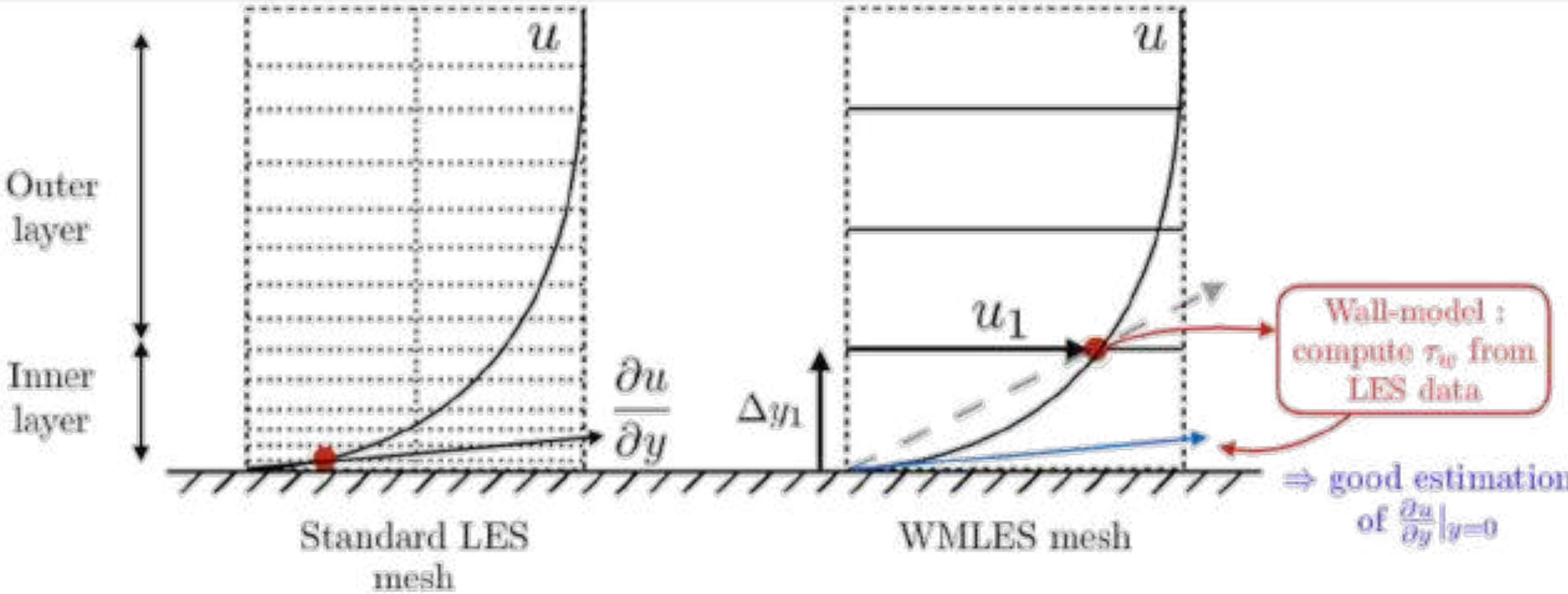
- We need to compute $\tau_w = \mu_w \left.\frac{\partial u}{\partial y}\right|_{y=0}$

- For wall-**resolved** LES: (fine mesh) $\boxed{\tau_w \simeq \mu_w \frac{u_1}{\Delta y_1}}$

- For wall-**modeled** LES: (Coarse mesh) $\cancel{\tau_w \simeq \mu_w \frac{u_1}{\Delta y_1}}$

Using finite-difference, τ_w is **overestimated** on **coarse** meshes.
=> **A model is needed**.

Analytical Wall-modeled LES



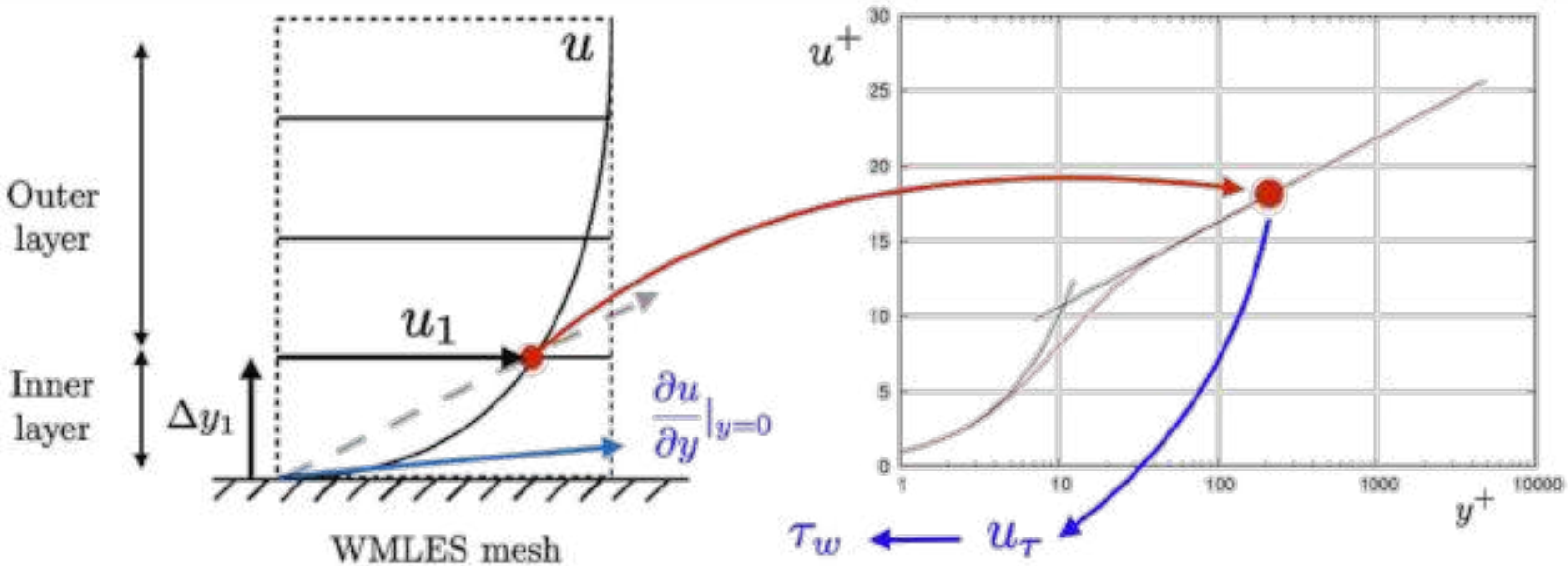
Wall modeling: provide an estimate of the wall –flux τ_w using the LES velocity at the first grid node.

Analytical law: $\overline{u^+} = f(y^+)$ (based on the log-law) provides τ_w .

Cost of an analytical law is negligible compared to a LES iteration.

Assumptions: stationary 2D flow, no pressure gradient, attached BL.

Analytical Wall-modeled LES



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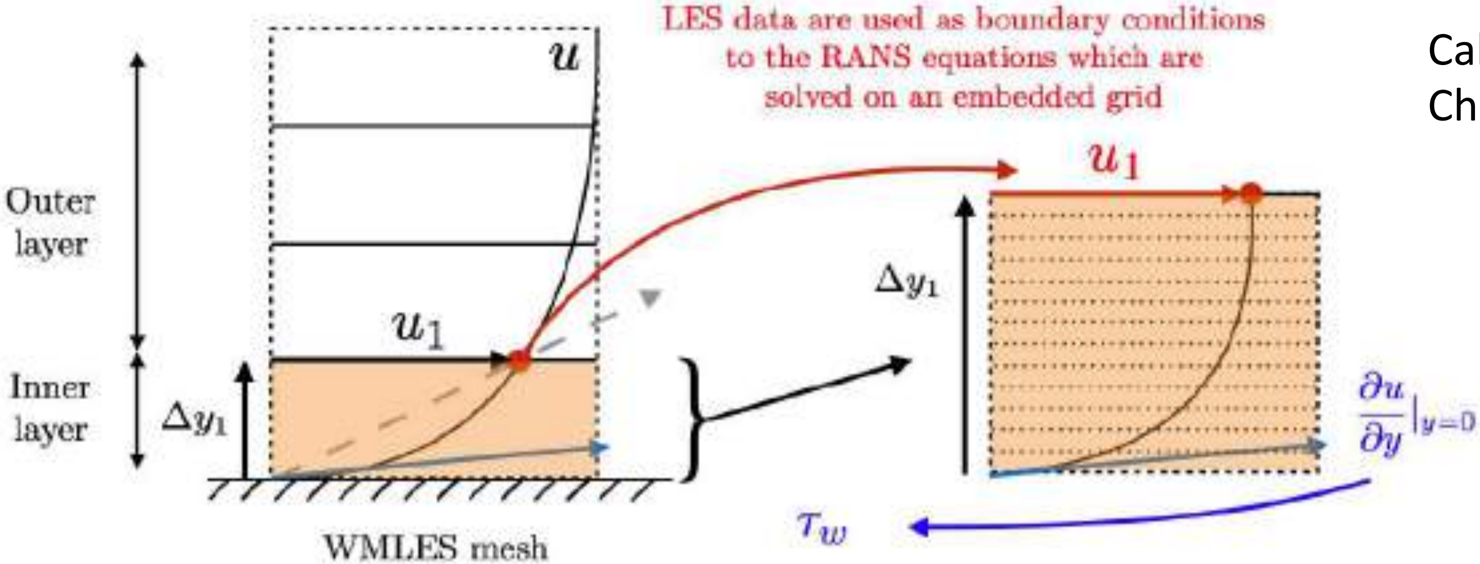
“Numerical” wall modeling: TBLE



Cabot & Moin,
Chen et al.

Instead using analytical law, a **set of differential equations** is solved on an embedded grid (=> **numerical** model).

Thin Boundary Layer Equations:

$$\cancel{\rho\frac{\partial u}{\partial t}} + \cancel{\rho v_i \frac{\partial u}{\partial x_i}} = -\cancel{\frac{\partial p}{\partial x}} + \frac{\partial}{\partial y}[(\mu + \mu^t)\frac{\partial u}{\partial y}]$$

$$\frac{\partial}{\partial y}[(\mu + \mu^t)\frac{\partial u}{\partial y}] = 0$$

2nd order **ordinary** differential equation

(Ex: Thomas Algorithm)

“Numerical” wall modeling: TBLE

$$\cancel{\frac{\partial \widetilde{u}_i}{\partial t}} + \cancel{\frac{\partial (\widetilde{u}_i \widetilde{u}_j)}{\partial x_j}} + \cancel{\frac{\partial P_m}{\partial x_i}} = \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \widetilde{u}_i}{\partial y}\right]$$

(Cabot and Moin 2000)

Let’s assume $\boxed{\nu_t = \kappa y u_\tau (1 - e^{-y^+/A})^2}$ (Mixing length with Van Driest damping)

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

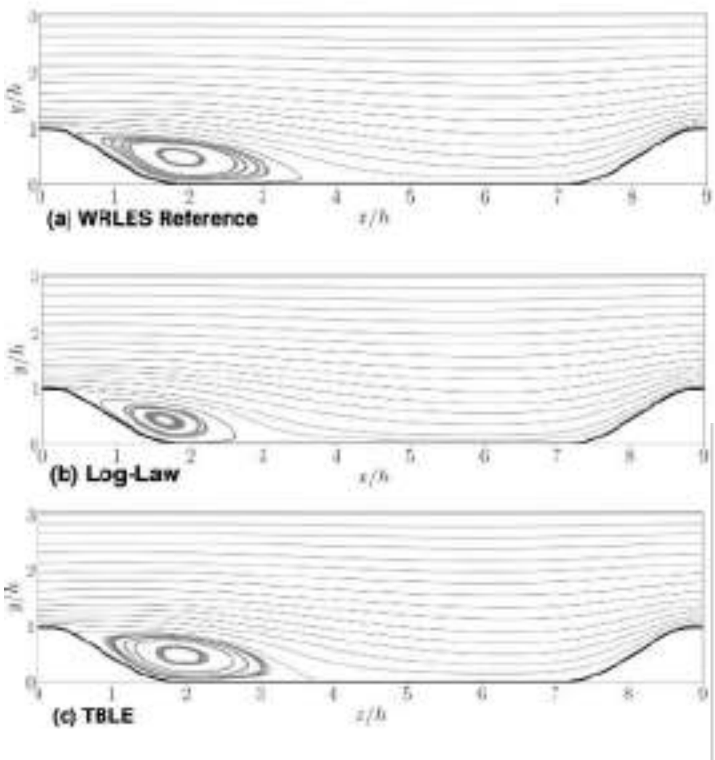
=> **Provides τ_w**

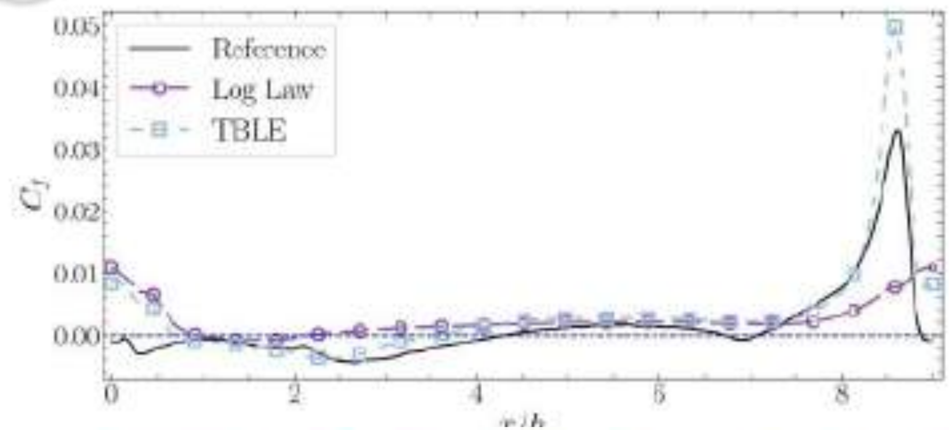
- Computational 10 to 20% more expensive than analytical law (but rather affordable regarding the LES iteration).
- Allow to account for complex effects (Pressure gradient, non-equilibrium, convective terms).

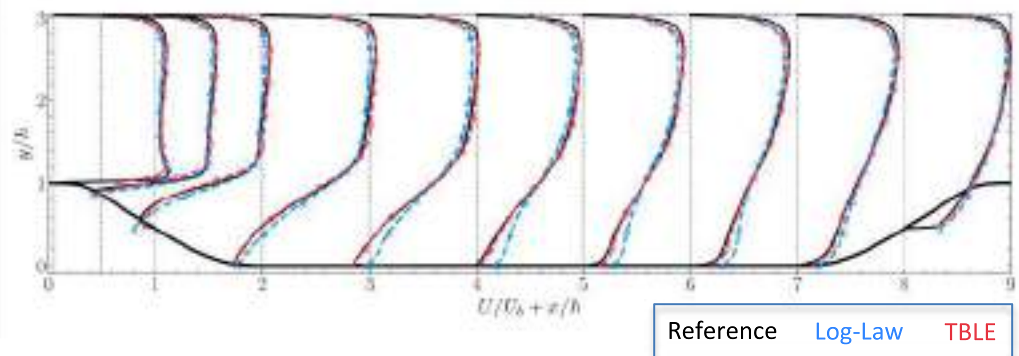
TBLE: Pressure gradient effect

- TBLE and **Pressure gradient :** $\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x} = \frac{\partial}{\partial y}\left([\bar{\nu}+\nu_t]\frac{\partial \bar{u}}{\partial y}\right)$

PhD Cizeron, 2024







- **Temperature gradient** can also be accounted for:

$$\rho\bar{u}C_p\frac{\partial \overline{T}}{\partial x} + \frac{\partial}{\partial y}\left[(\lambda+\lambda_t)\frac{\partial \overline{T}}{\partial y}\right] = 0$$

PhD Gelain, 2021

Back to analytical WMLES… Complex physics: **Heat Flux**

- A logarithmic relation can also be derived for **thermal flux**:

$$\begin{cases} T^+ = \frac{P_{r_t}}{\kappa}\ln(y^+) + \beta(P_r) & \text{Provides } q_w \\ u^+ = \frac{1}{\kappa}\ln(y^+) + C_{vd} & \text{Provides } \tau_w \end{cases}$$

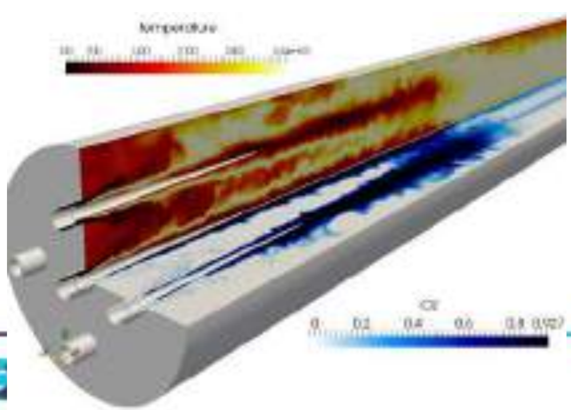
$$T^+ = \frac{\overline{T_w} - \overline{T}}{T_\tau}$$

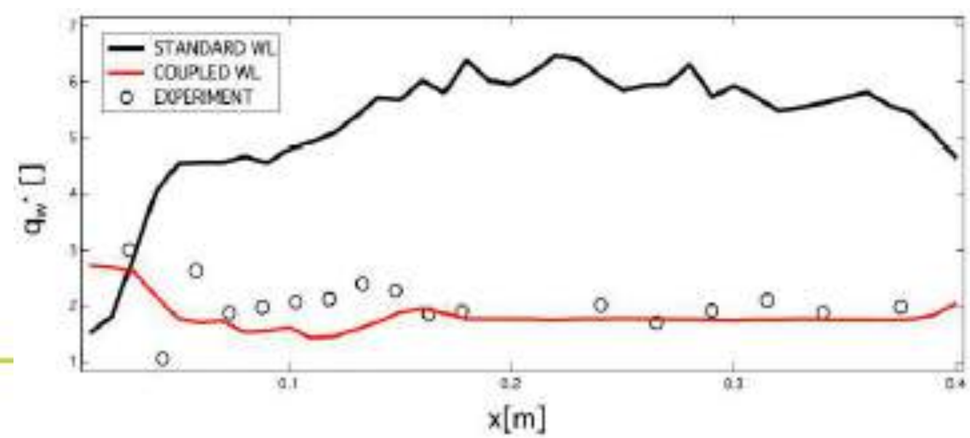
$$T_\tau = \frac{\overline{q_w}}{\overline{\rho_w}\, C_{p,w}\, u_\tau}$$

- **<u>Cabrit, 2009</u>**: **Coupled velocity-temperature** analytical wall model

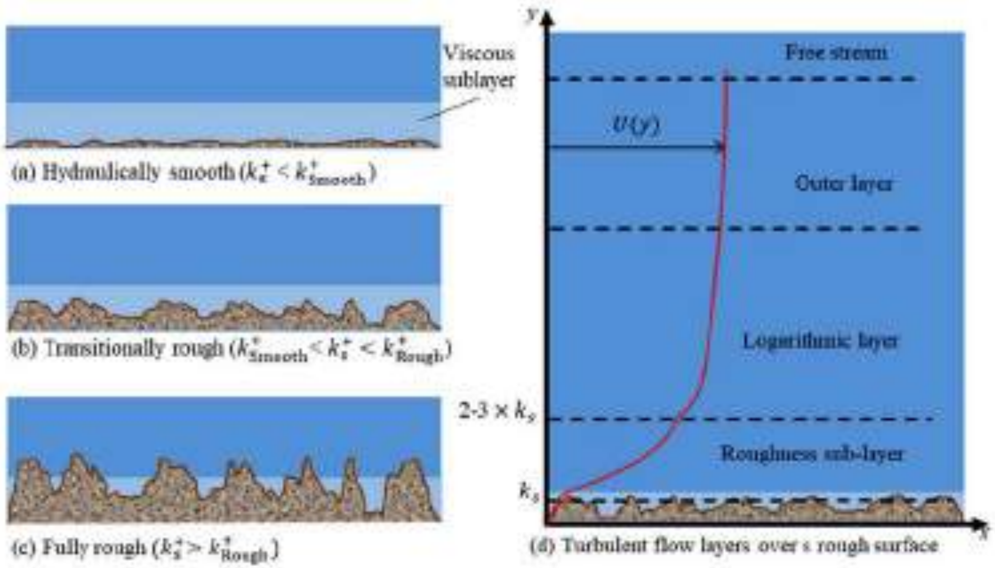
$$\begin{cases} \frac{2}{P_{r_t} B_q}\left(\sqrt{1 - K B_q} - \sqrt{\frac{T}{T_w}}\right) = \frac{1}{\kappa}\ln y^+ + C_{vd} \\ T^+ = P_{r_t} u^+ + K \end{cases}$$

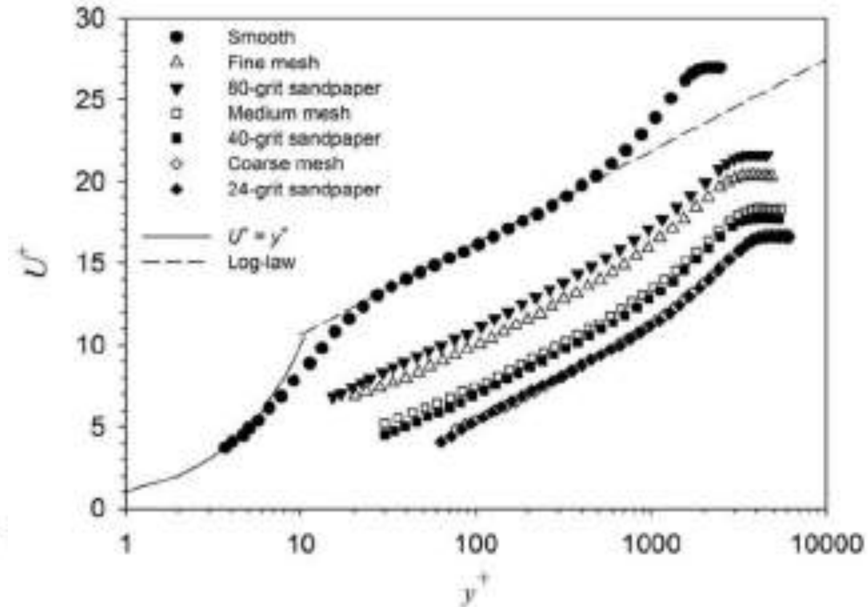
Strong improvement for **large temperature gradients** (rocket engines) :





Back to analytical WMLES… Complex physics: **Roughness**





Flack et al. POF 2007

Roughness modifies the mean velocity profile

Roughness can be accounted for through:

$$u^+ = \frac{1}{\kappa} ln \left(\frac{y^+}{y_0} \right) + B$$

Reviews in :

Flack et al POF 2014
Kadivar 2021

A complex topic !

Large eddy simulations: Practical issues

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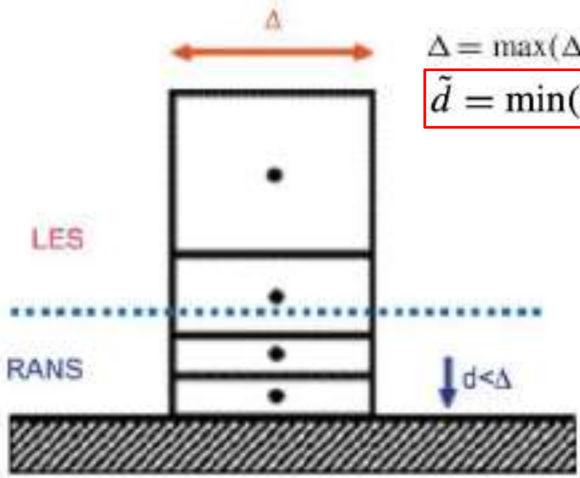
Detached-Eddy Simulation

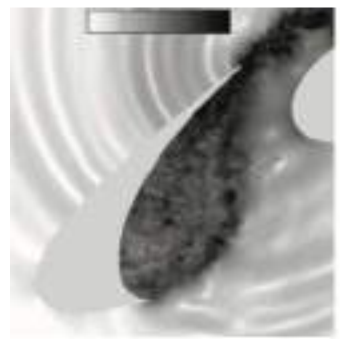
Originally proposed by Spalart (1997): Intended for “detached” flow:

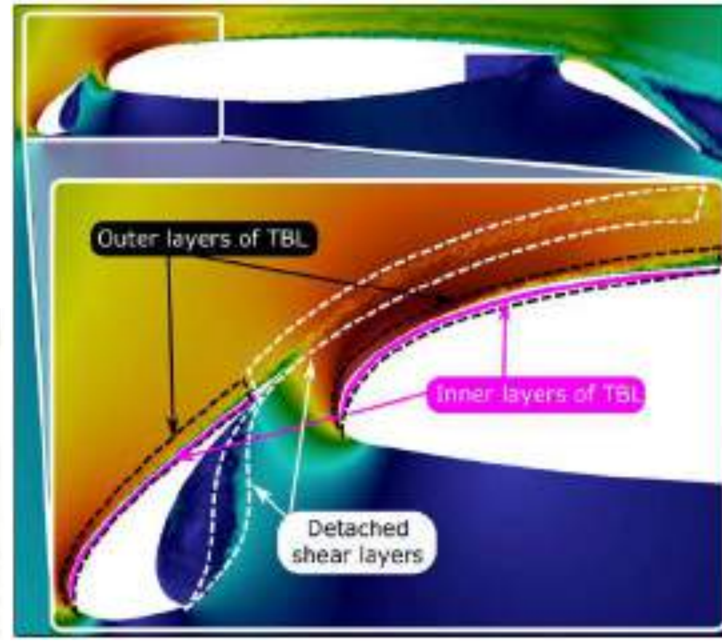
=> LES for **detached** flow, **RANS** for **attached BL.**



$$\Delta = \max(\Delta_x, \Delta_y, \Delta_z)$$

$$\boxed{\tilde{d} = \min(d, C_{\mathrm{DES}}\Delta)}$$





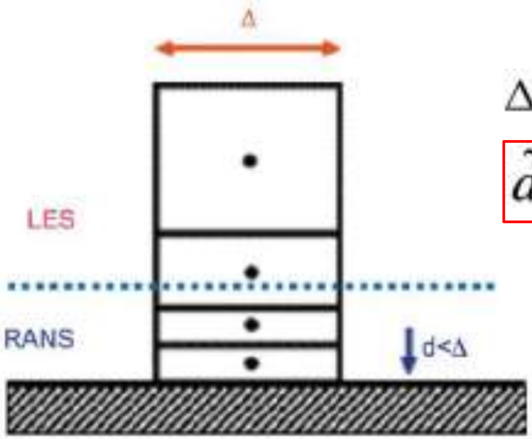
<u>Concept:</u>

Switch **from LES SGS** model to **RANS turbulence model** depending on the wall distance.

- **Close** to the wall: **RANS** modeling
- **Away** from the wall: **LES** modeling

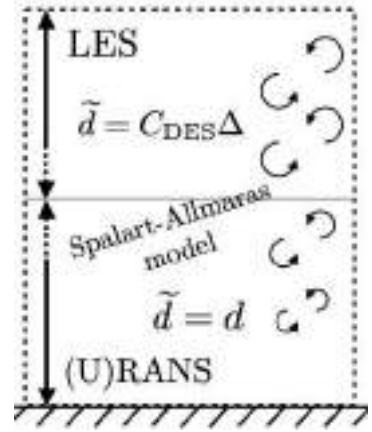
Uses a single grid.

Detached-Eddy Simulation



$$\Delta = \max(\Delta_x, \Delta_y, \Delta_z)$$

$$\boxed{\tilde{d} = \min(d, C_{\text{DES}}\Delta)}$$



- **<u>Close to the wall</u>**: $\boxed{d_w < C_{\text{DES}}\Delta}$ => **URANS**

<u>Spalart-Allmaras model:</u>

$$\nu_t = \tilde{\nu} f_{v1}$$

$$\frac{\partial \tilde{\nu}}{\partial t} + u_j \frac{\partial \tilde{\nu}}{\partial x_j} = C_{b1}[1 - f_{t2}]\tilde{S}\tilde{\nu} + \frac{1}{\sigma}\{\nabla \cdot [(\nu + \tilde{\nu})\nabla\tilde{\nu}] + C_{b2}|\nabla\tilde{\nu}|^2\} - \left[C_{w1} f_w - \frac{C_{b1}}{\kappa^2} f_{t2}\right]\left(\frac{\tilde{\nu}}{d}\right)^2 + f_{t1}\Delta U^2$$

- **<u>Away from the wall:</u>** $\boxed{d_w > C_{\text{DES}}\Delta}$ => **LES**

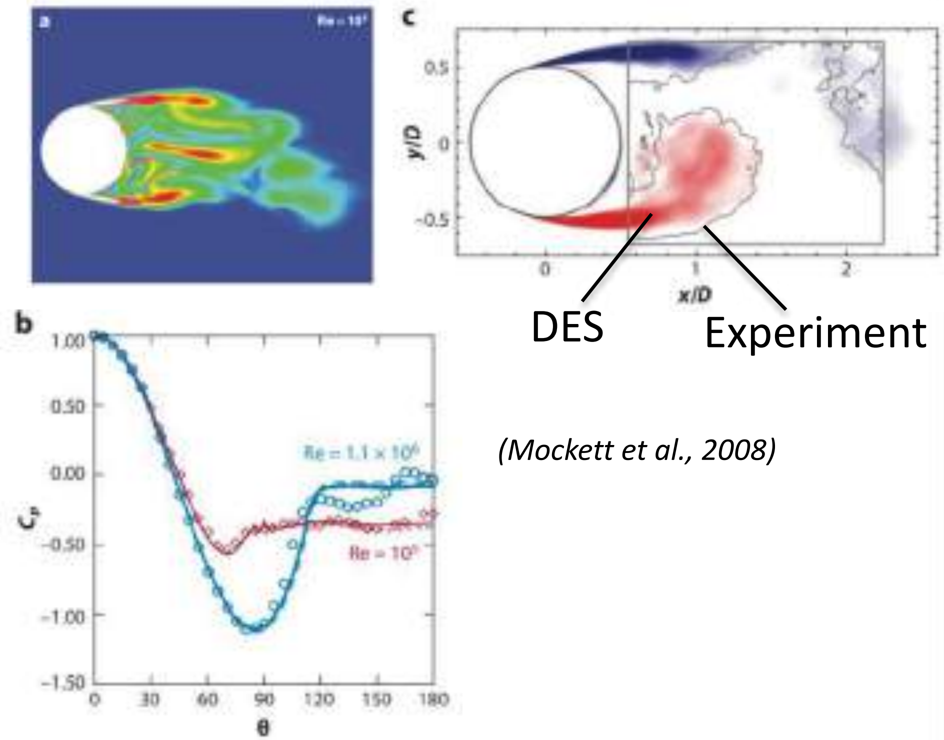
Smagorinsky model: $\nu_t = (C_s\Delta)^2 \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}$

Detached-Eddy Simulation

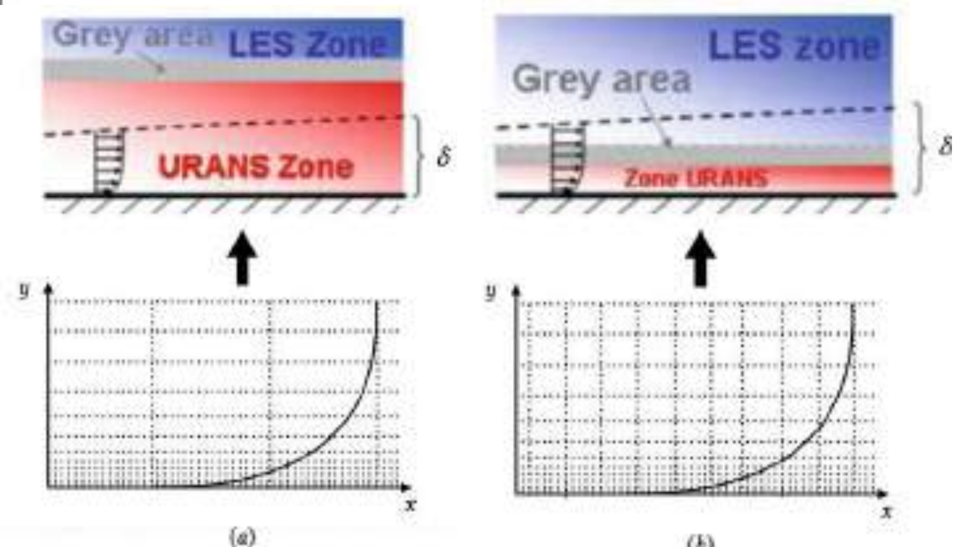
- Very efficient for massively separated flows

instantaneous

Phase averaging



(Mockett et al., 2008)



RANS / LES transition depends on the mesh resolution.
$\Rightarrow$ Significant solution dependency

Not so efficient for attached turbulent BL.

DES: other approaches

Delayed Detached Eddy Simulations (*Spalart, 2006*)

Add a constraint on *d*, to enforce URANS/LES transition out of the boundary layer.
Enforce BL resolution with URANS, reduces grid dependency.

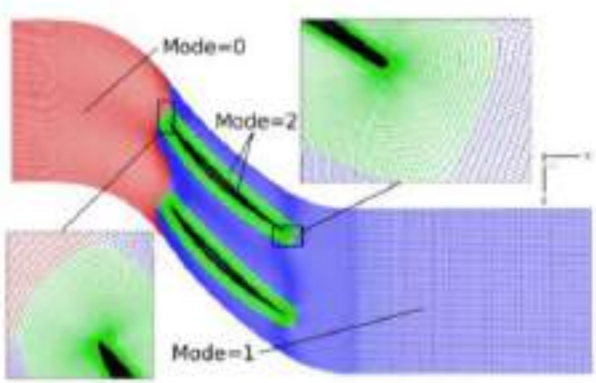
However, solution can depends on initial solution (Fröhlich and Terzi, 2008)

Improved Detached Eddy Simulation (*Shur et al., 2008*)

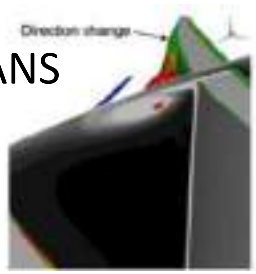
Define Δ as a function of grid size + wall distance: $\Delta = f(h_x, h_y, h_z, d_w)$

Zonal Detached Eddy Simulation (*Deck et al., 2005, 2011*)

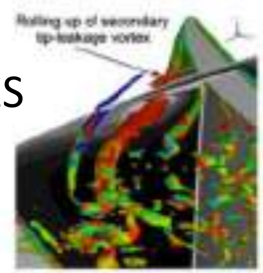
Explicit definition of URANS / LES transition

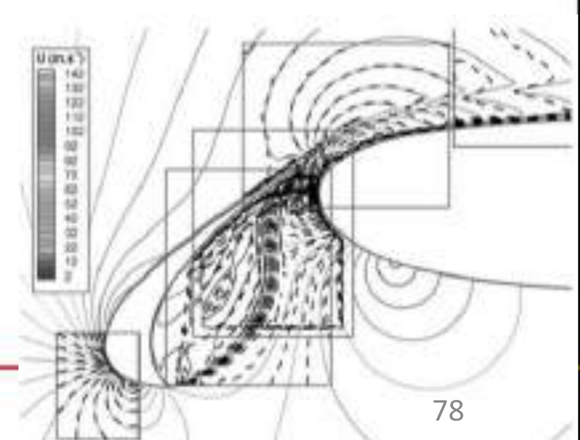


URANS



ZDES





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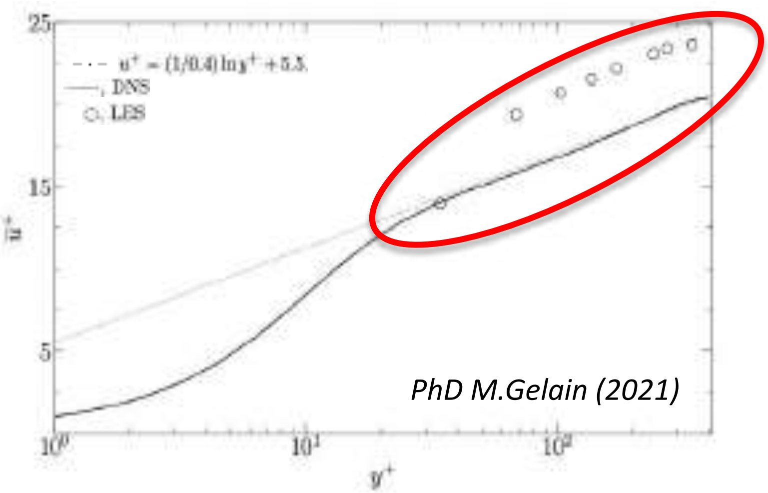
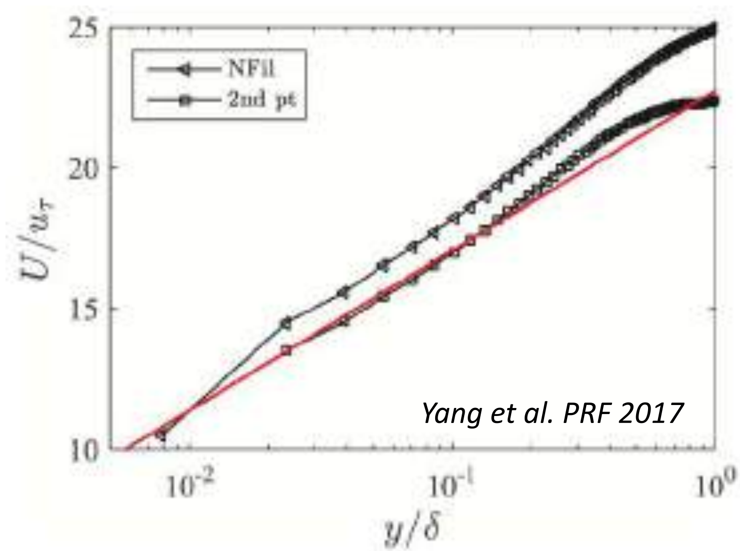
Log-layer mismatch

- SGS models: WALE, Sigma, … developed in a **Wall-resolved** context
- Wall-models: First intended for RANS context, **without SGS** models

When coupling both...: « **Log-layer mismatch** » !





⇒ Correction of SGS viscosity

⇒ Matching point at 2nd, 3rd off-wall node

⇒ Temporal filtering

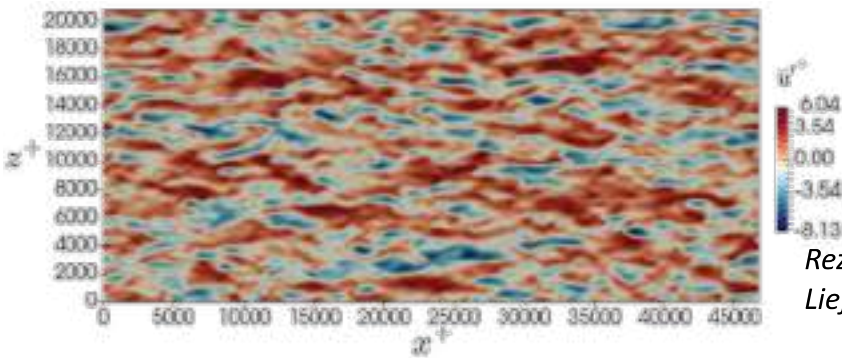
⇒ Stochastic forcing in 1st cell

Still a current topic of interest…

Impact of modeling

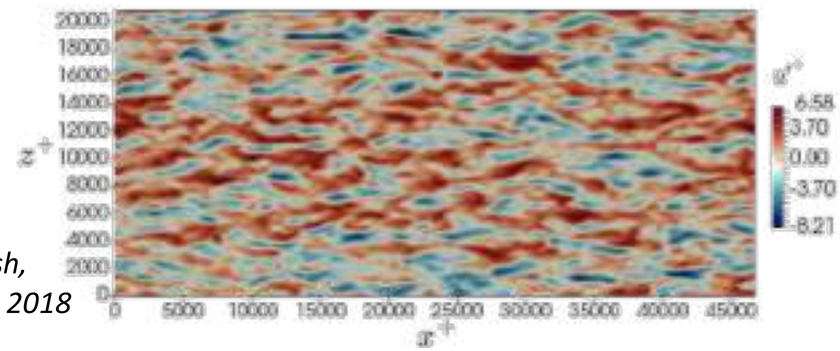
Impact of SGS modeling: flat plate, using an analytical Spalding wall-model

No SGS

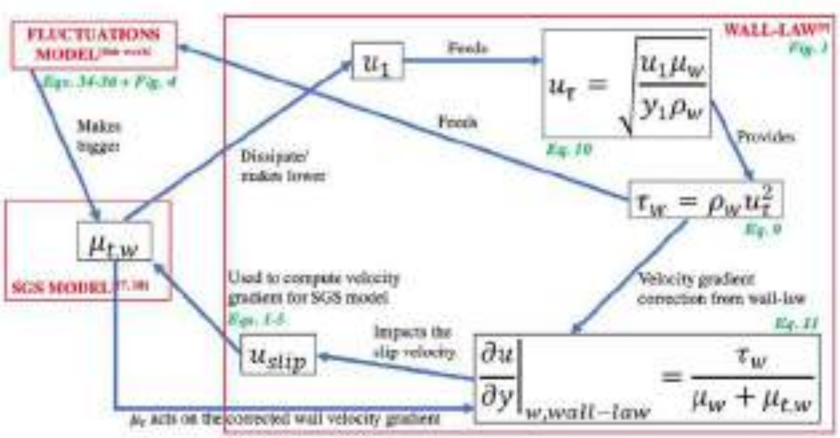


Rezaeiravesh, Liefvendahl 2018

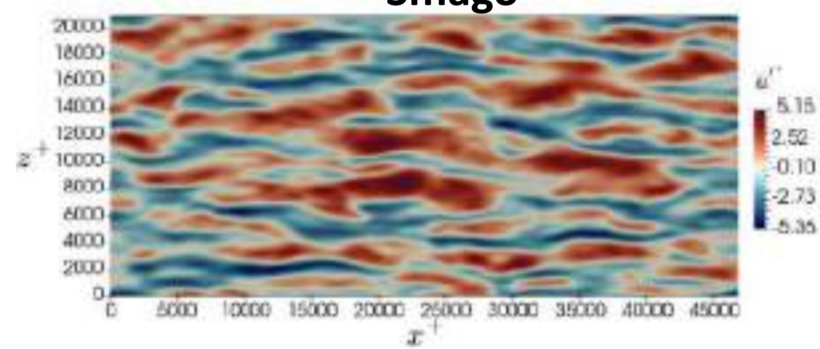
WALE



Wall law – SGS coupling



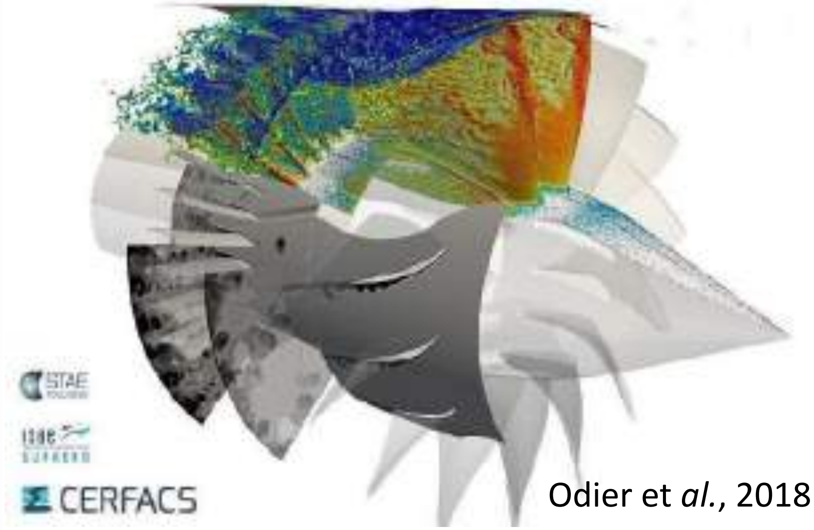
Smago



- Wall-modeling and SGS modeling can be **tightly coupled**.
- Result can be highly sensitive to the SGS model.

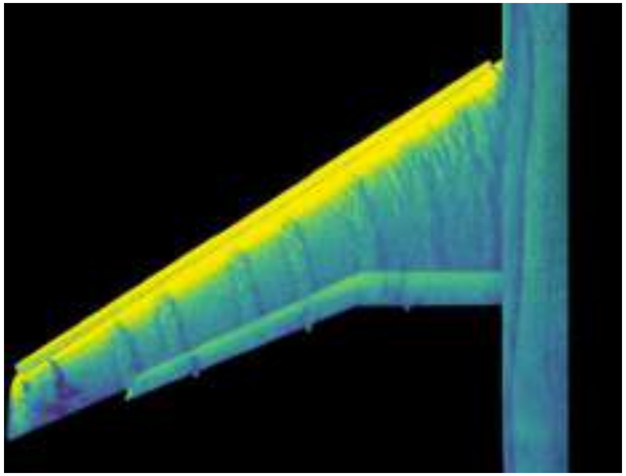
Applications: **analytical WMLES**

Turbomachinery Aerodynamics



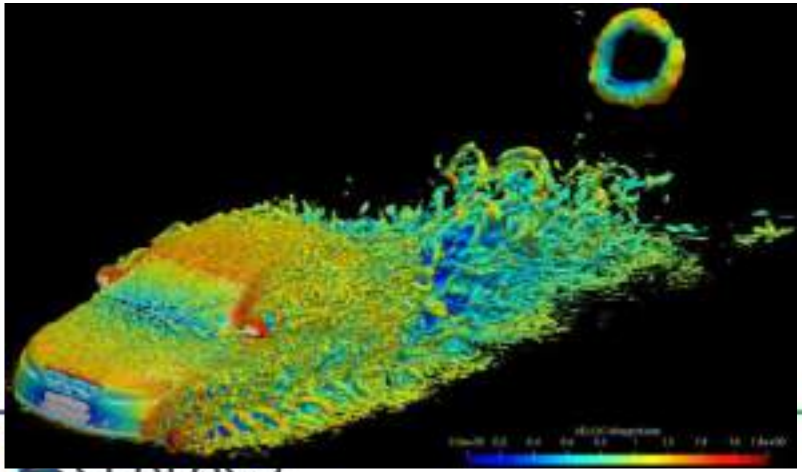
Odier et *al.*, 2018

Aeronautical flows at stall conditions



Lehmkuhl et *al.*, 2018

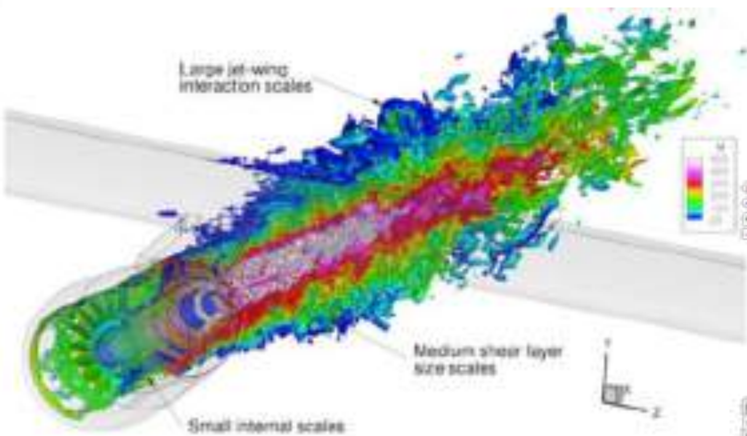
Automotive aerodynamics

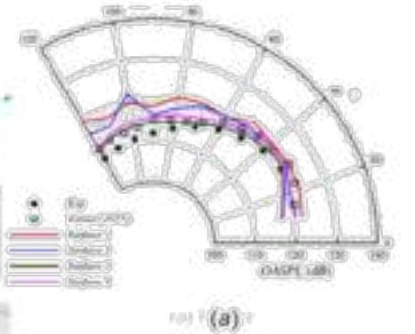


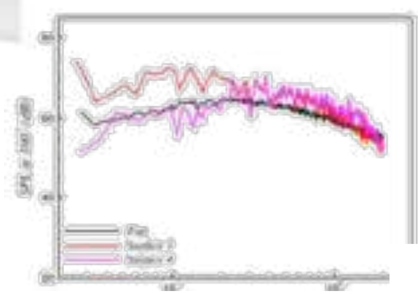
Lehmkuhl et *al.*, 2018

Applications: Hybrid RANS/LES

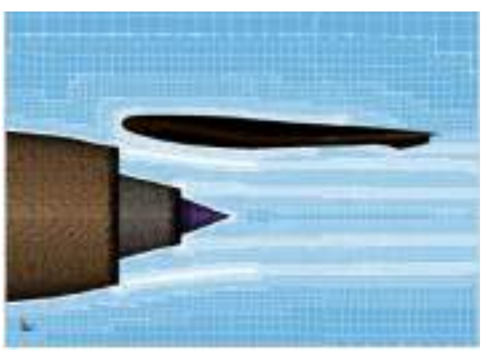
Aeroacoustic of an installed engine



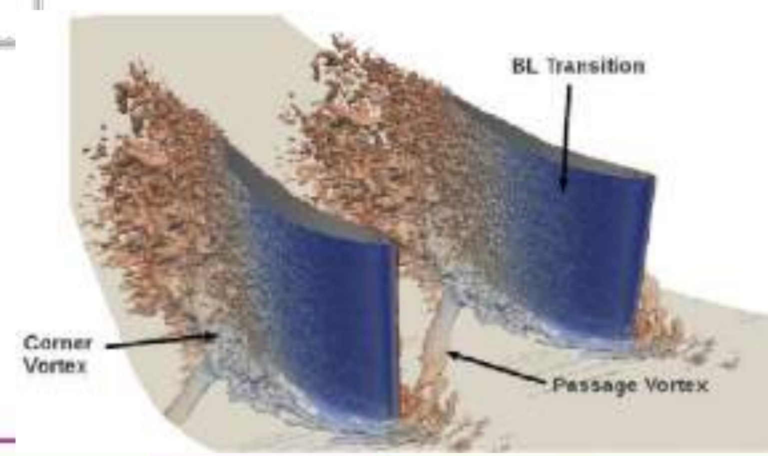




(Tyacke et al., 2017)

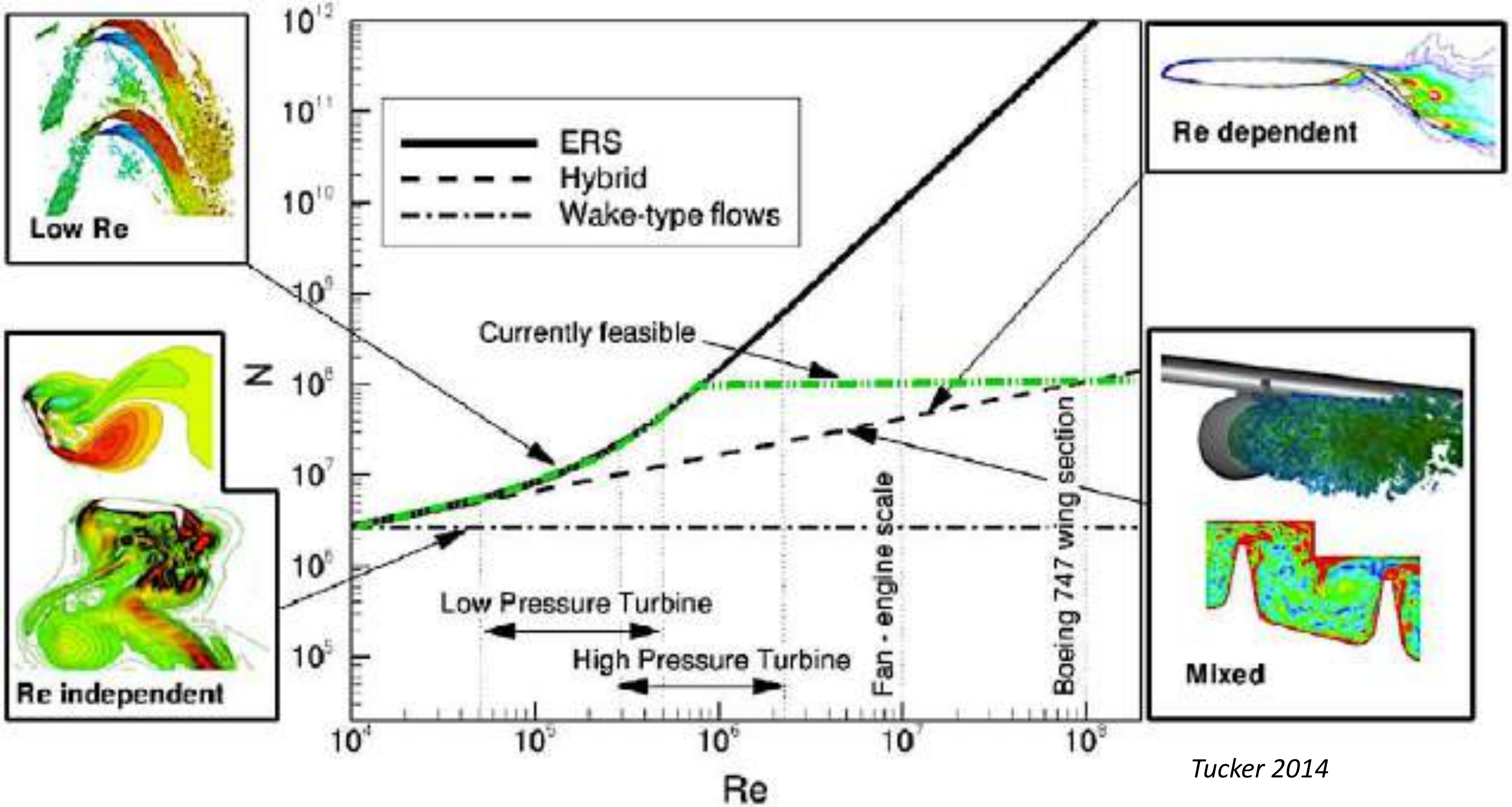


Aerodynamics



Tucker, 2016

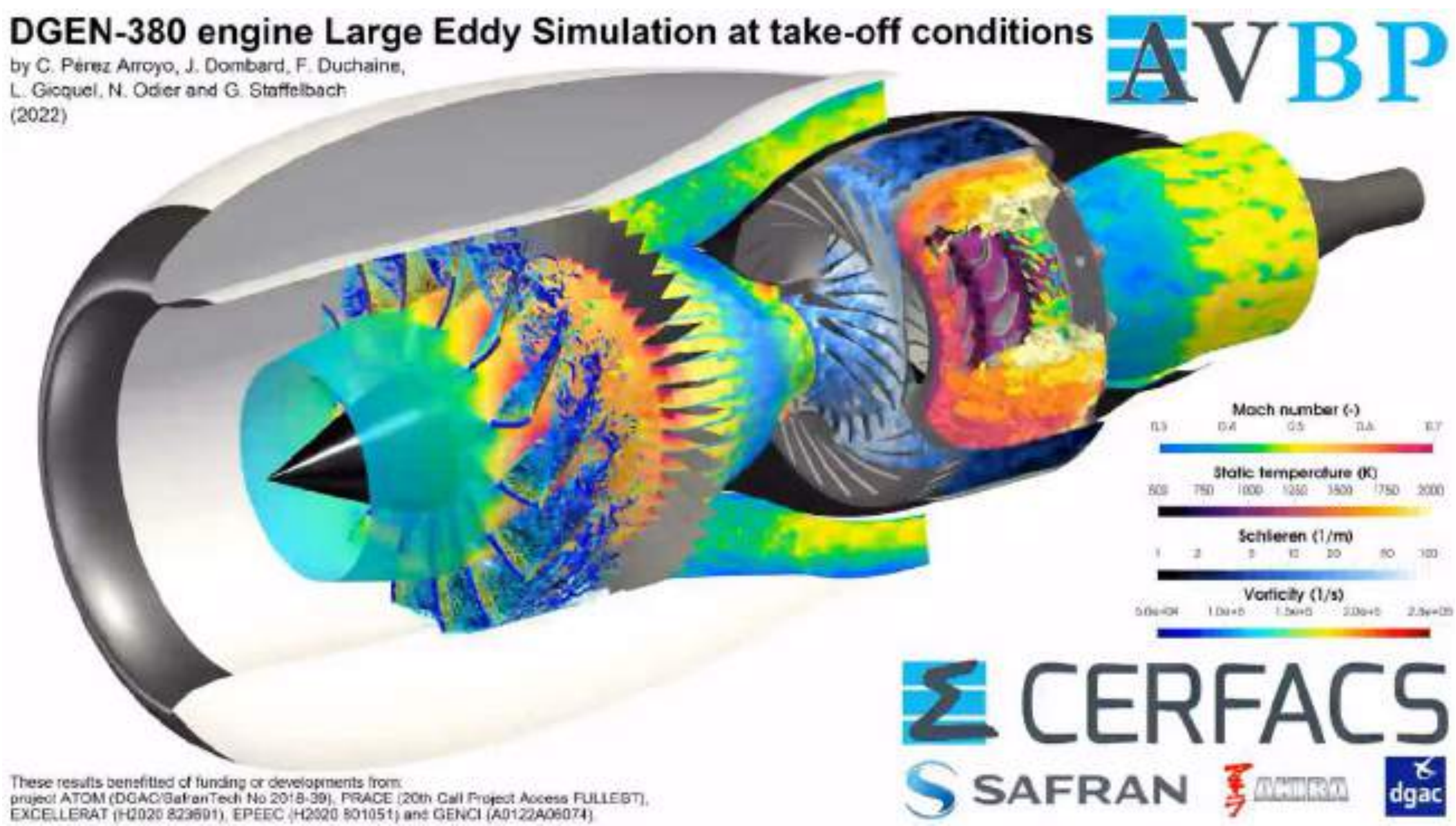
Wall-modeling and real applications



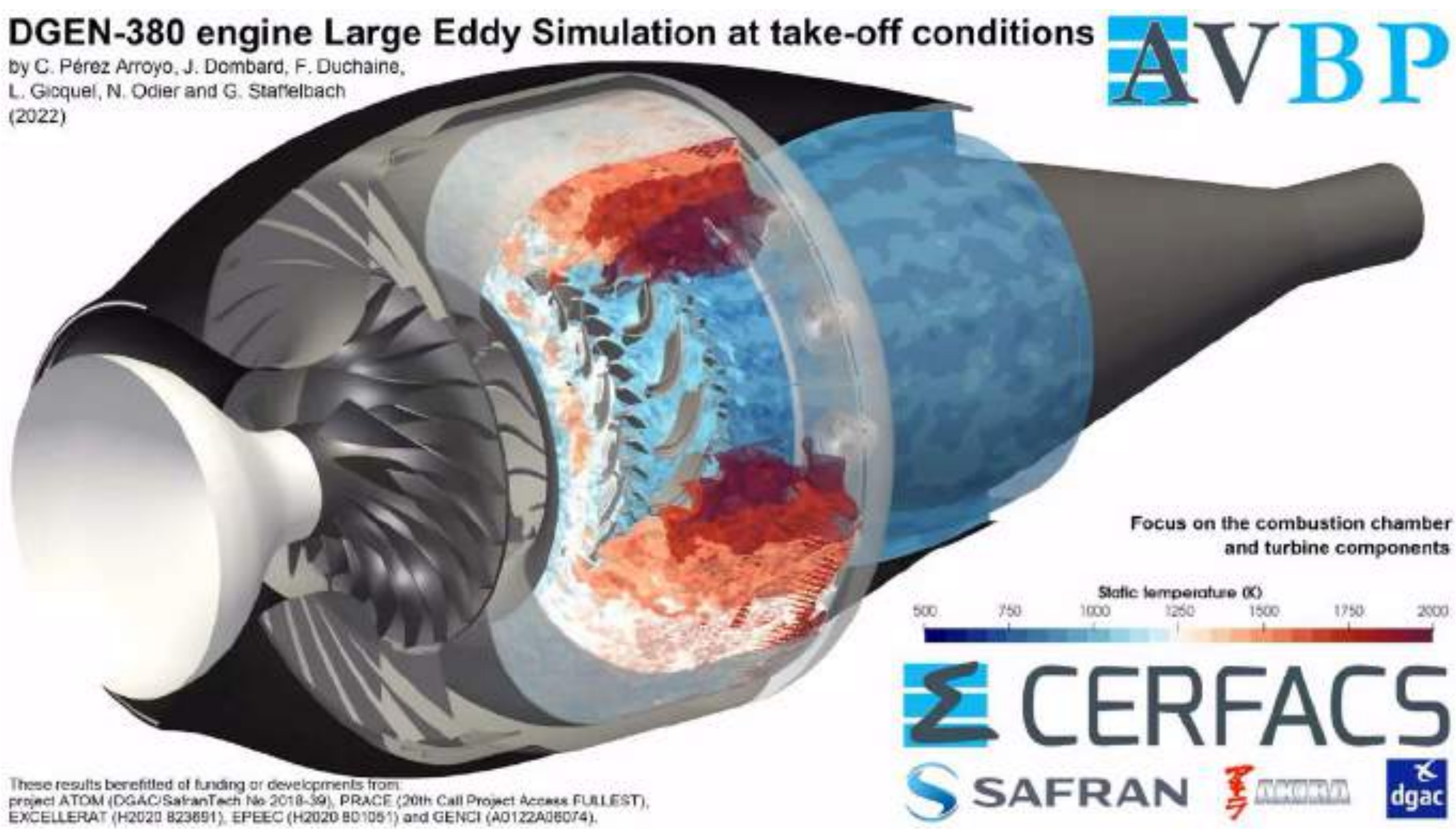
Tucker 2014

- Eddy-resolved simulation forbids many real applications
- Hybrids / wall-modeled LES are currently computationally affordable

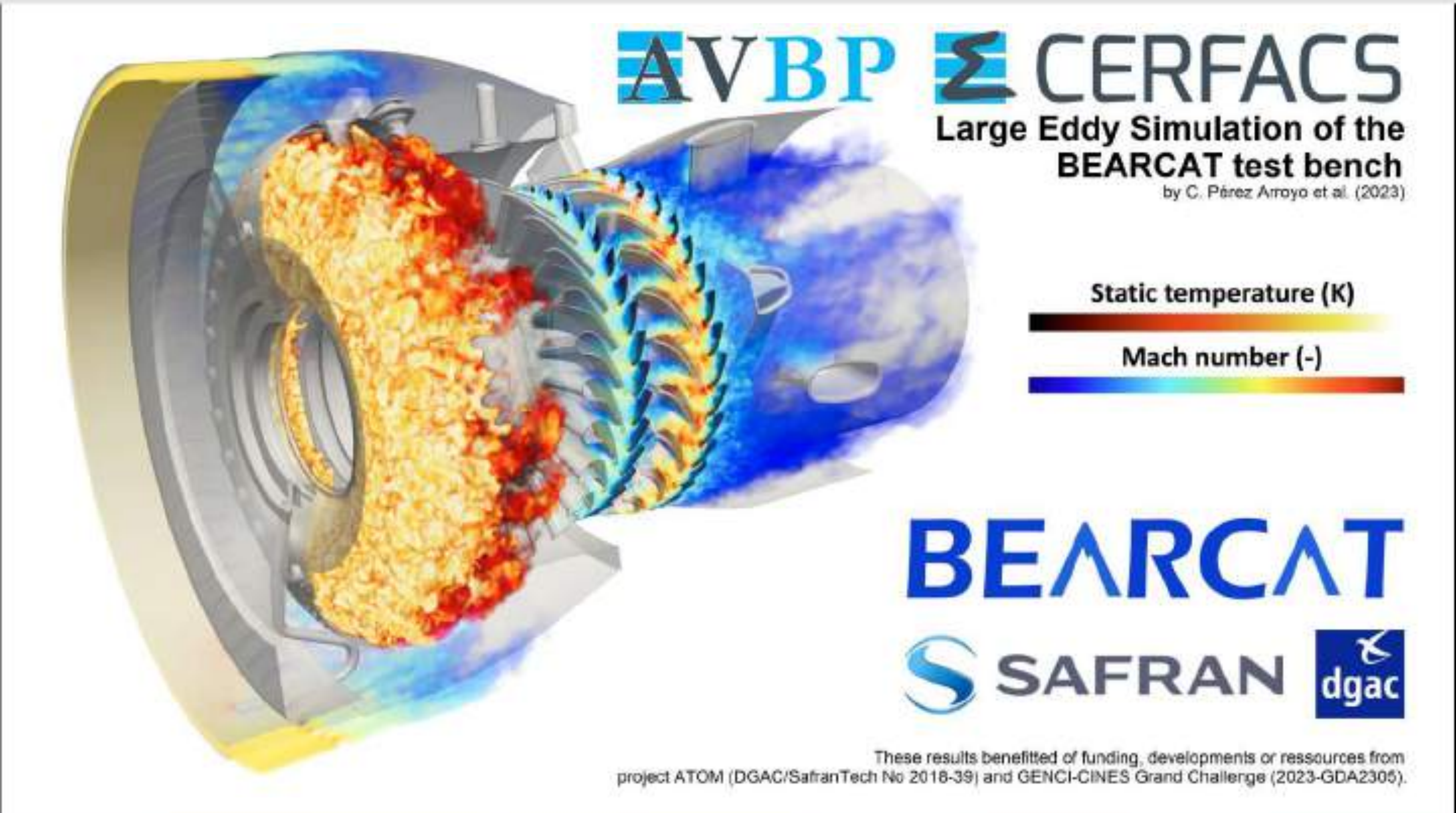
Real application



Real application



Real application



A few conclusions

Wall-bounded turbulence:

- A **complex physics**: transition to turbulence, streaks, large-scale structures…
- **Self-similar physics.**
- A **universal** velocity profile: the **logarithmic law-of-the-wall.**

- **(U)RANS** approaches are **not efficient** for complex physics predictions.
- **Large-eddy simulation**: a **wall-modeling** is mandatory for real applications.

- Either **analytical model** (based on log-law), **numerical model** (TBLE) to deal with turbulent boundary layer.
- Other **hybrid RANS/LES** models: Detached Eddy Simulation
- An accurate wall model / SGS model coupling is still a research topic…

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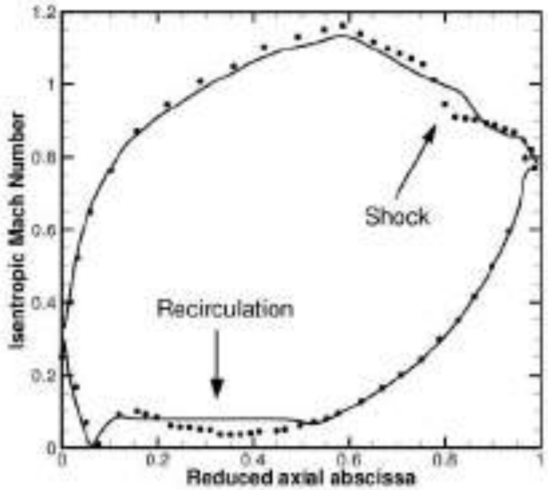
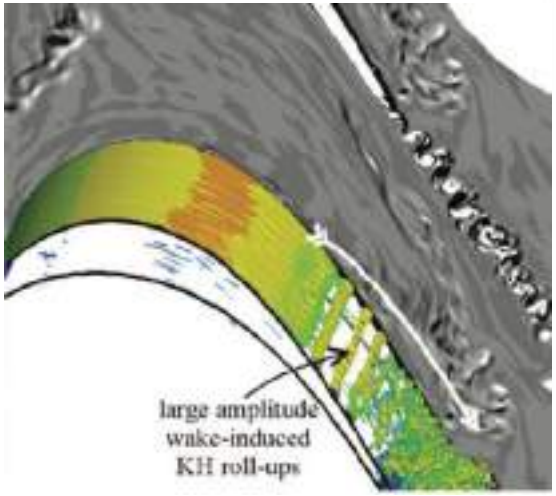
Context

Why do we care about turbulence injection in LES ?

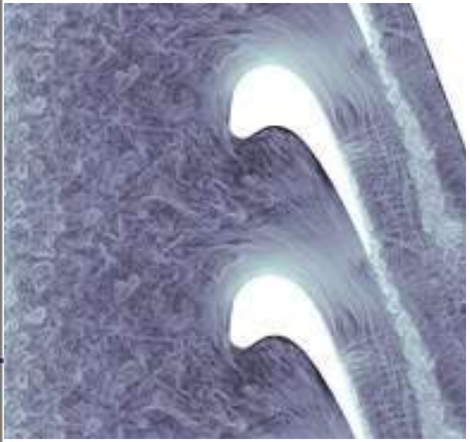


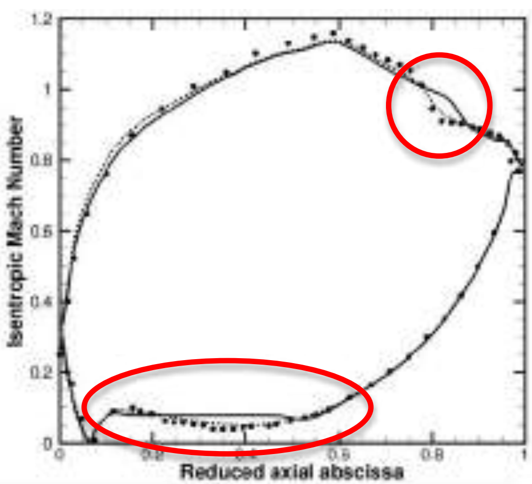
Laminar inlet



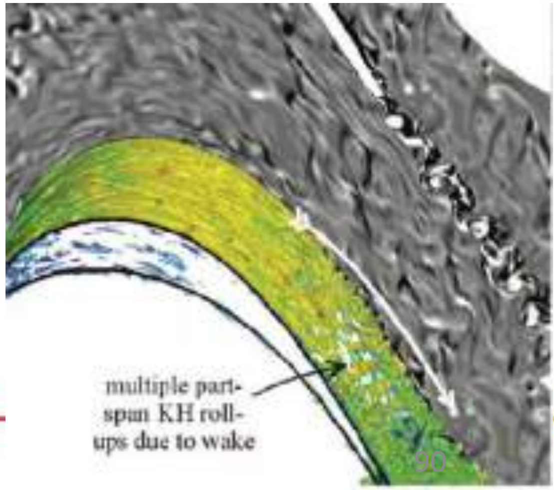
Harnieh et al., 2017

Cui et al. 2015





Turbulent inlet



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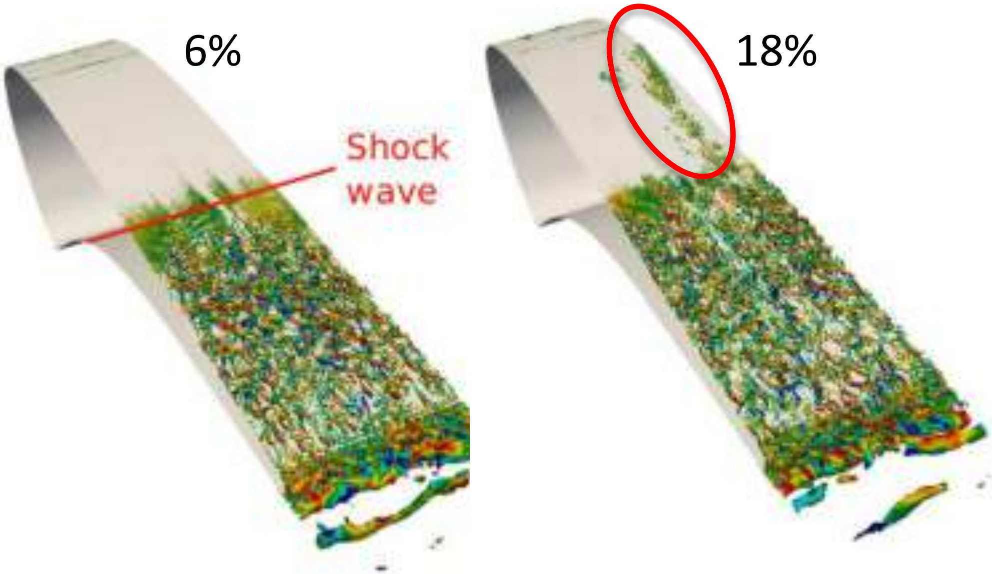
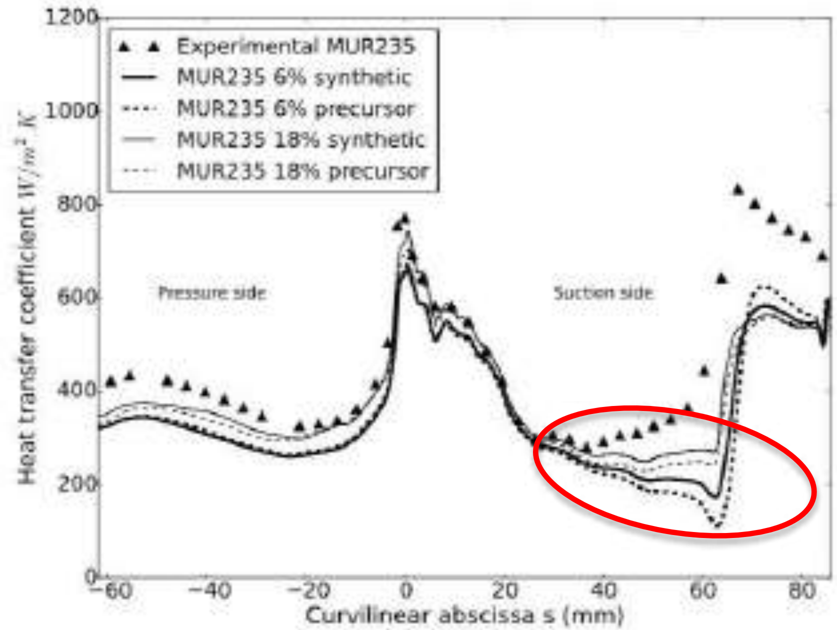




Inlet turbulence level may modify the boundary layer transition process:

⇒ Significant influence on the **average pressure distribution**

⇒ Significant impact on **resulting losses**

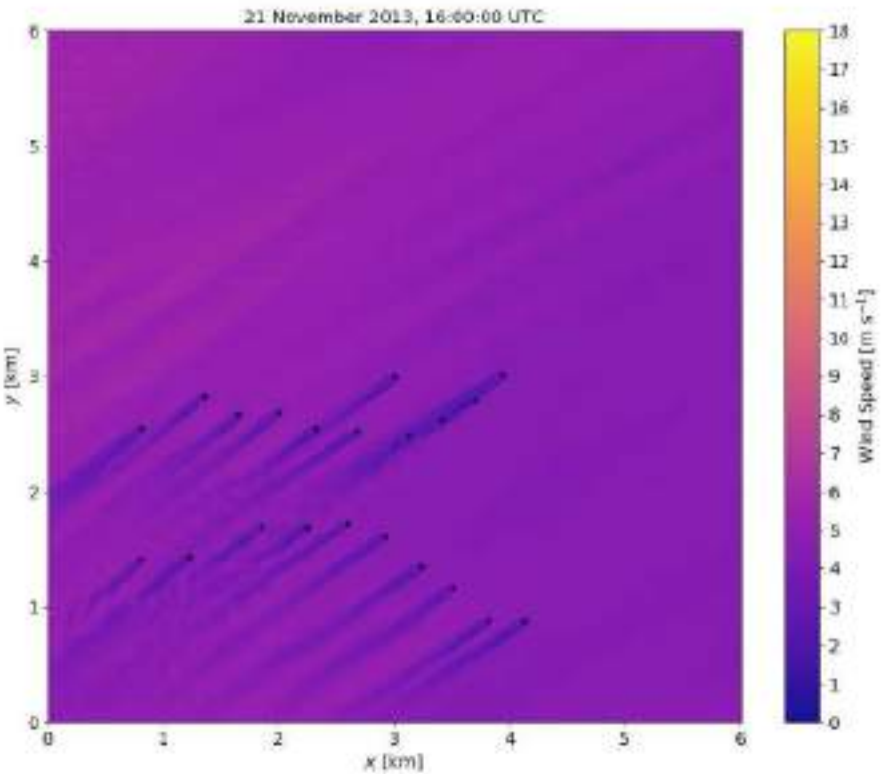
⇒ Significant impact on **heat transfer**

=> Importance of inlet boundary condition !

Context

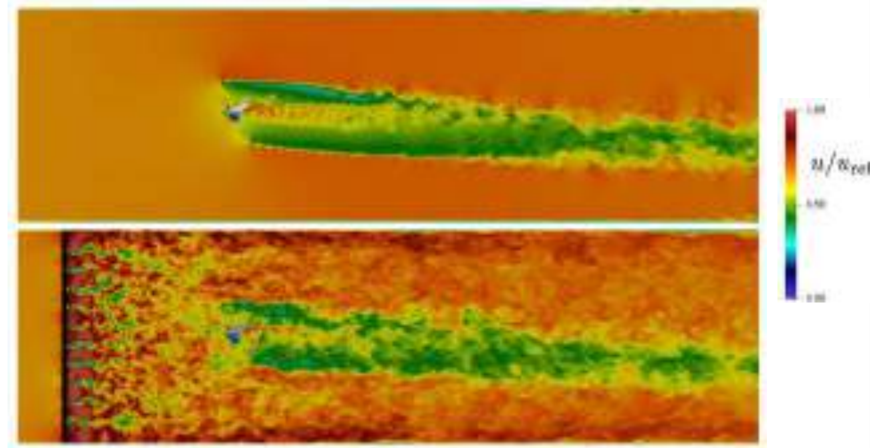
Offshore wind farm

Radar observation

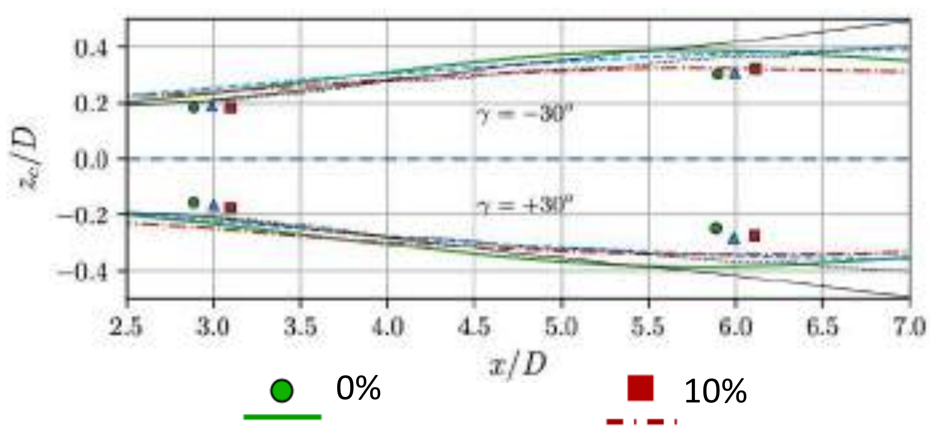


Arthur et al., 2020

Standalone wind turbine



Houtin-Mongrolle et al., 2020



Turbulence modifies wake development

Large eddy simulations: Practical issues

LES: A brief reminder

- NS equation and turbulence
- LES, filtering and models
- Numerics, errors and LES
- Implicit LES

Wall treatment

- Wall-bounded turbulence
- Wall modeling: The “wall-law”, TBLE
- Detached-Eddy Simulations

Turbulence injection

- Synthetic methods (Fourier / POD)
- Precursor method
- Wall-bounded turbulent injection

Turbulence injection

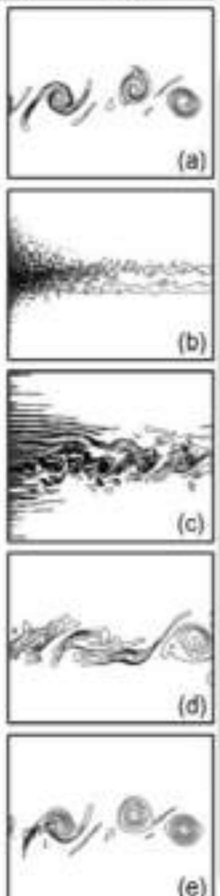
<u>Basic principle:</u> Generation of instantaneous realizations statistically equivalent to the freestream flow.

$$\mathbf{u}(\mathbf{x_0}, t) = U_0(x_0) + \boxed{\mathbf{u}'(x_0, t)}$$

???

Reference simulation

<u>Imposed</u> $\mathbf{u}'(x_0, t)$:



Druault et al., 2004

Exact solution

<u>White noise</u> with same single point statistics as in ref.

<u>White noise</u> + same two-points temporal statistics

<u>White noise</u> + same two-points spatial statistics

Reconstruction POD

<u>White noise:</u>

- **Lacks** spatial and temporal **coherence** of turbulence

=> **Other approaches are needed**

Synthetic methods: Fourier techniques

<u>Objective</u>: Build velocity fluctuations that behave similarly to turbulent fluctuations.

Kraichnan (1970): Fluctuations can be seen as a **sum of random Fourier modes**:

$$\underline{u}'(\underline{r},t) = \sum_{n=1}^{N} \underline{\hat{u}}^n(\underline{k}^n) e^{(j\underline{k}^n \underline{r} + j\omega^n t)}$$

$\underline{\hat{u}}^n.\underline{k}^n = 0 \quad \forall n$ (continuity equation)

$$\underline{\hat{u}}^n(\underline{k}^n) = \underline{\hat{v}}^n(\underline{k}^n) + j\underline{\hat{w}}^n(\underline{k}^n)$$

Recast as:

$$\underline{u}'(\underline{r},t) = \sum_{n=1}^{N} [\underline{\hat{v}}^n(\underline{k}^n) cos(\underline{k}^n.\underline{r} + \omega^n t) + \underline{\hat{w}}^n(\underline{k}^n) sin(\underline{k}^n.\underline{r} + \omega^n t)]$$

Pulsation

Wave vector

Taken from isotropic **Gaussian distributions** so shaped that **E(k)** is reached in the limit N→ ∞

Velocity fluctuations are finally **added to the mean flow** imposed at the inlet (“Frozen turbulence hypothesis”, Taylor 1938)

Synthetic methods: Fourier techniques

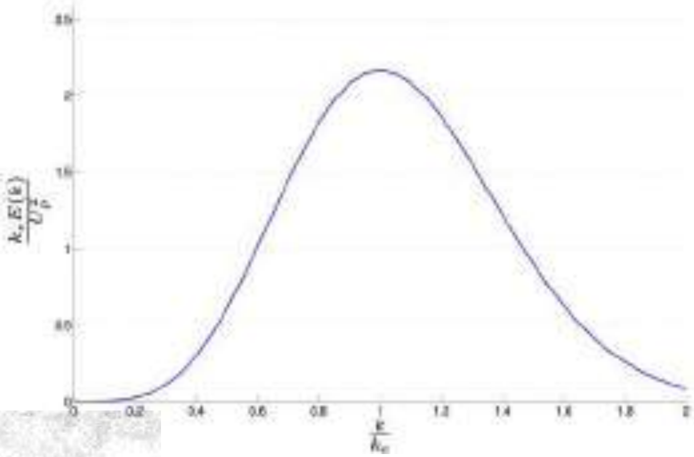
Spectra example:

Passot-Pouquet

$$E(k) = A\left(\frac{k}{k_e}\right)^4 exp\left(-2\left(\frac{k}{k_e}\right)^2\right)$$

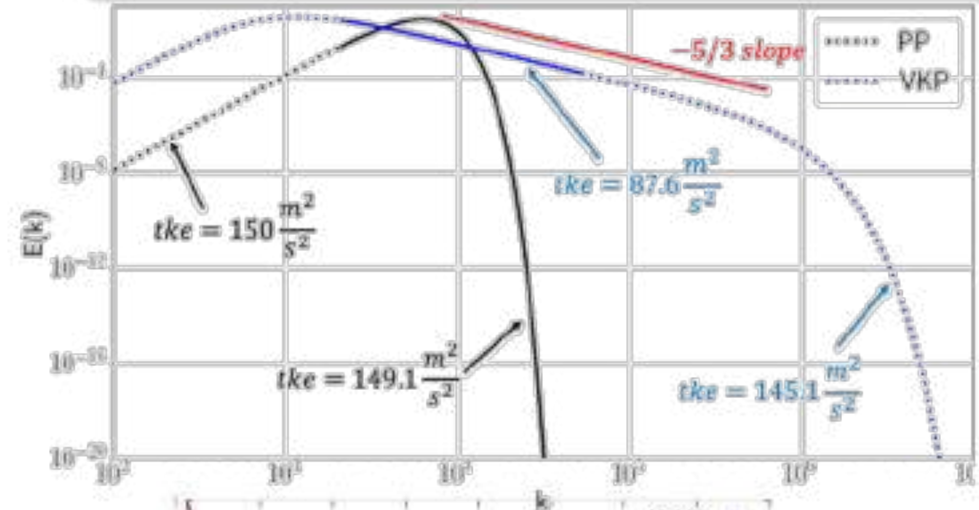
with $A = \frac{16U_p^2}{k_e}\sqrt{\frac{2}{\pi}}$

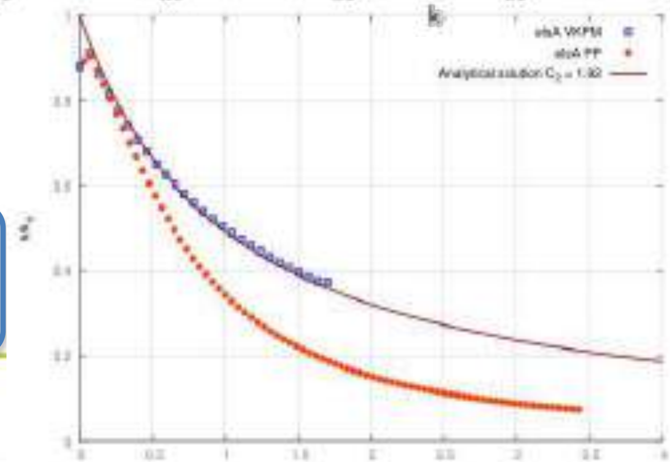
Integration gives : $k \equiv \frac{1}{2}\overline{u_i'u_i'} = \frac{3}{2}U_p^2$



Von Karman-Pao

$$E(k) = \alpha\frac{u'^2}{k_e}\frac{(k/k_e)^4}{[1+(k/ke)^2]^{17/6}}exp\left[-2\left(\frac{k}{k_{Kol}}\right)^2\right]$$



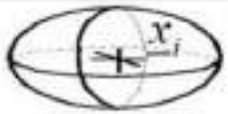


Turbulence decay in a turbulent channel

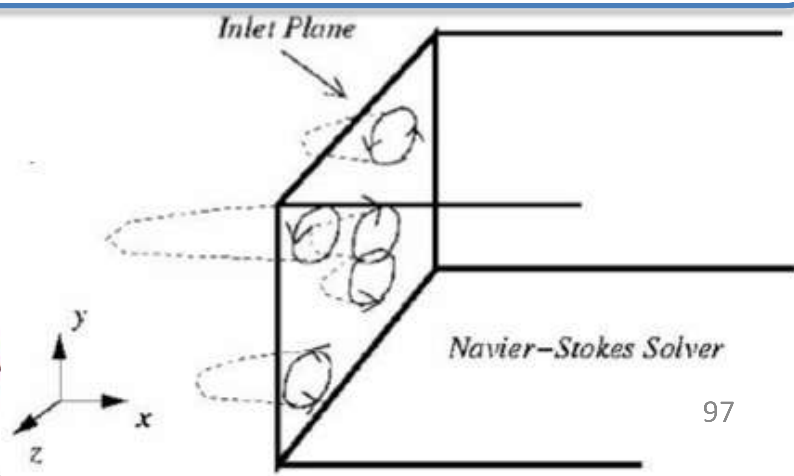
CERFACS

Synthetic Eddy Method

(Jarrin et al. 2006, 2009)

- Turbulent flow is a **superposition of coherent eddies** 

- Each eddy is described by a **shape function** $f_\sigma(x)$, on compact support $[-\sigma,\sigma]$
  $[-r_x;r_x]\times[-r_y;r_y]\times[-r_z;r_z]$
- Each eddy has a **random position** (y_i, z_i) on the inlet plane.
- Each eddy is convected through the inlet plane until it is not active anymore ($r_x < x_i$)
- Then, a new one is regenerated at $x_i = -r_x$



Synthetic Eddy Method

Gaussian

Contribution of the turbulent spot *i* to the velocity field is $\boxed{u^{(i)}(x) = \varepsilon_i f_\sigma(x - x_i)}$

Random step (+1 or -1)

Contribution of N eddies: $\boxed{u(x) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \varepsilon_i f_\sigma(x - x_i)}$

Unsteady velocity field: $\boxed{u_j'(\underline{x}, t) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \varepsilon_{ij} f_j(\underline{x} - \underline{x}_i(t))}$

Final velocity field: $\boxed{u_i = \bar{u}_i + a_{ij} u_j'}$

$\bar{u}_i$ Mean velocity field

a_{ij} Deduced from prescribed Reynolds Stress Tensor R_{ij}

Mean velocity and R_{ij} must be known in advance (RANS or experiment)

$$\begin{pmatrix} \sqrt{R_{11}} & 0 & 0 \\ R_{21}/a_{11} & \sqrt{R_{22} - a_{21}^2} & 0 \\ R_{31}/a_{11} & (R_{32} - a_{21}a_{31})/a_{22} & \sqrt{R_{33} - a_{31}^2 - a_{32}^2} \end{pmatrix}$$

(Cholesky decomposition of Reynolds Stress Tensor)

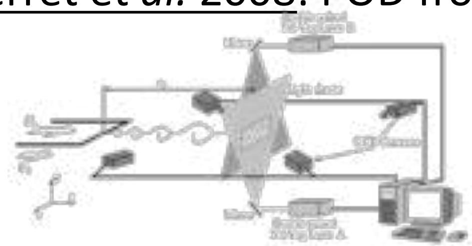
<u>Proper Orthogonal Decomposition:</u>

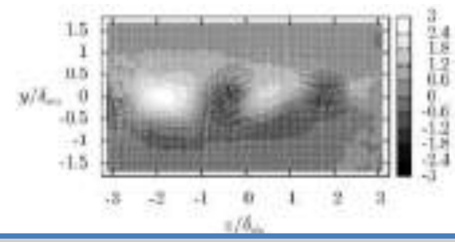
Decomposes an ensemble of unsteady snapshots into **spatial** and **temporal eigenvectors**:

$$\tilde{u}(X,t) = \sum_n a_n(t)\Phi^{(n)}(X)$$

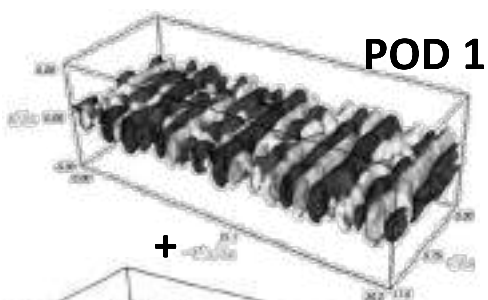
=> Allows to **reconstruct the most energetic structures** from reference

<u>Perret et *al.* 2008</u>: POD from experiment + synthetic Fourier method



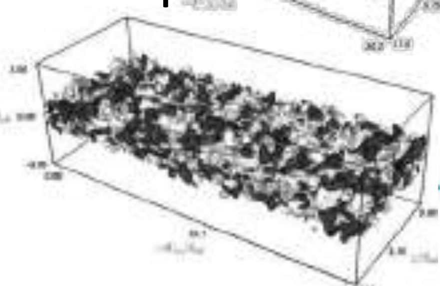


POD mode from PIV



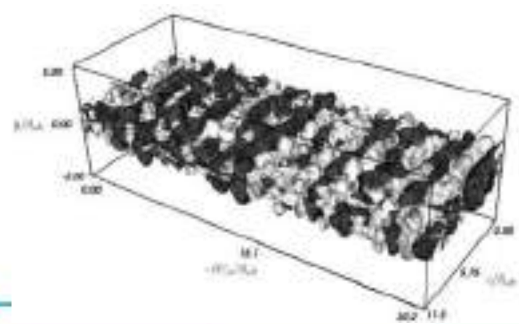
POD 12 modes

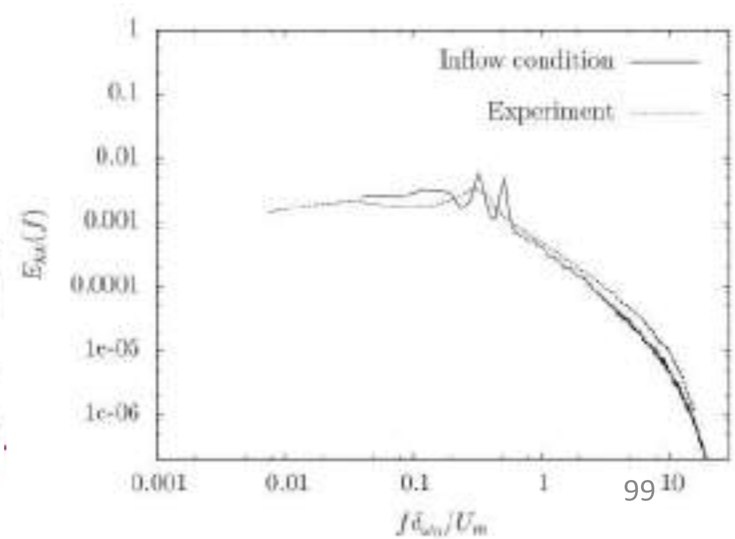
+

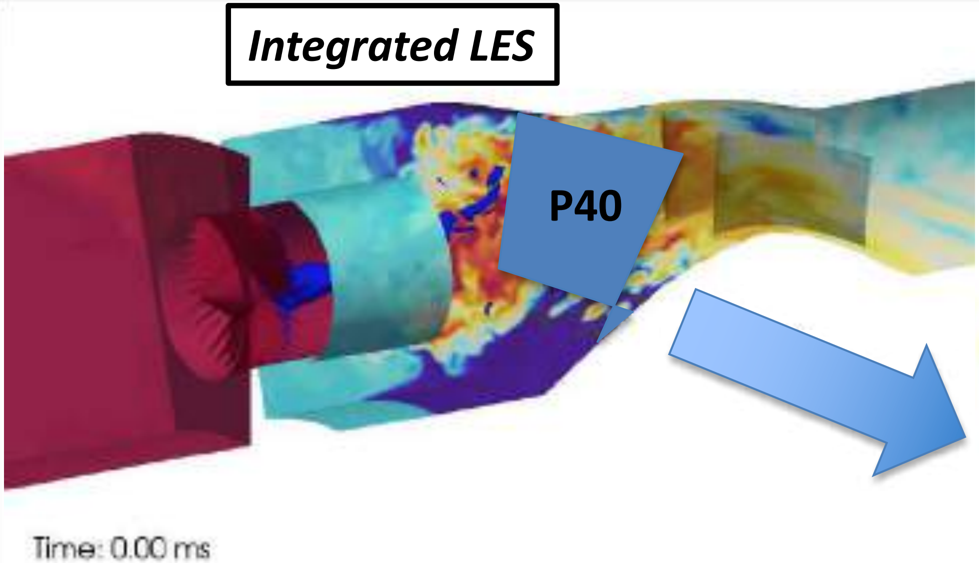


Fourier

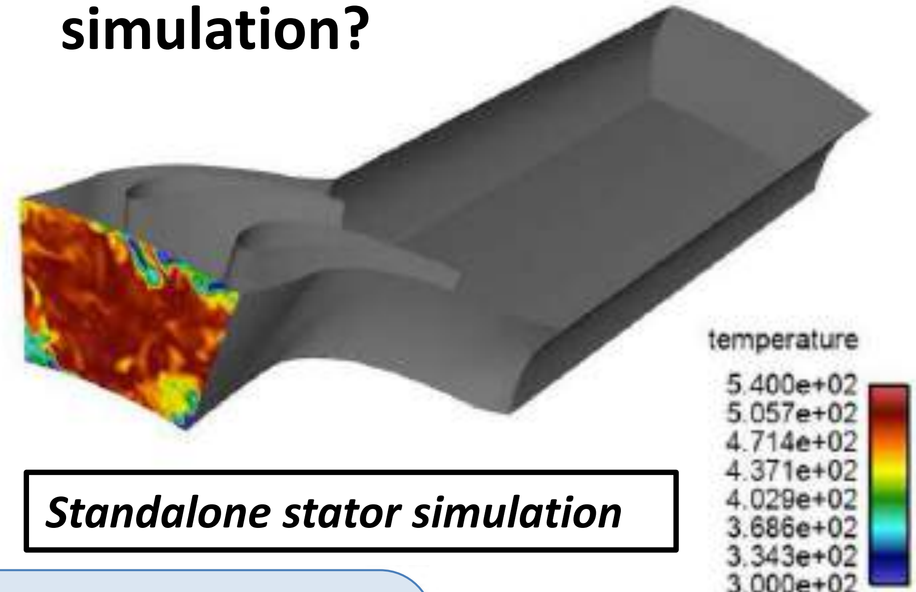
=







What to impose on isolated simulation?



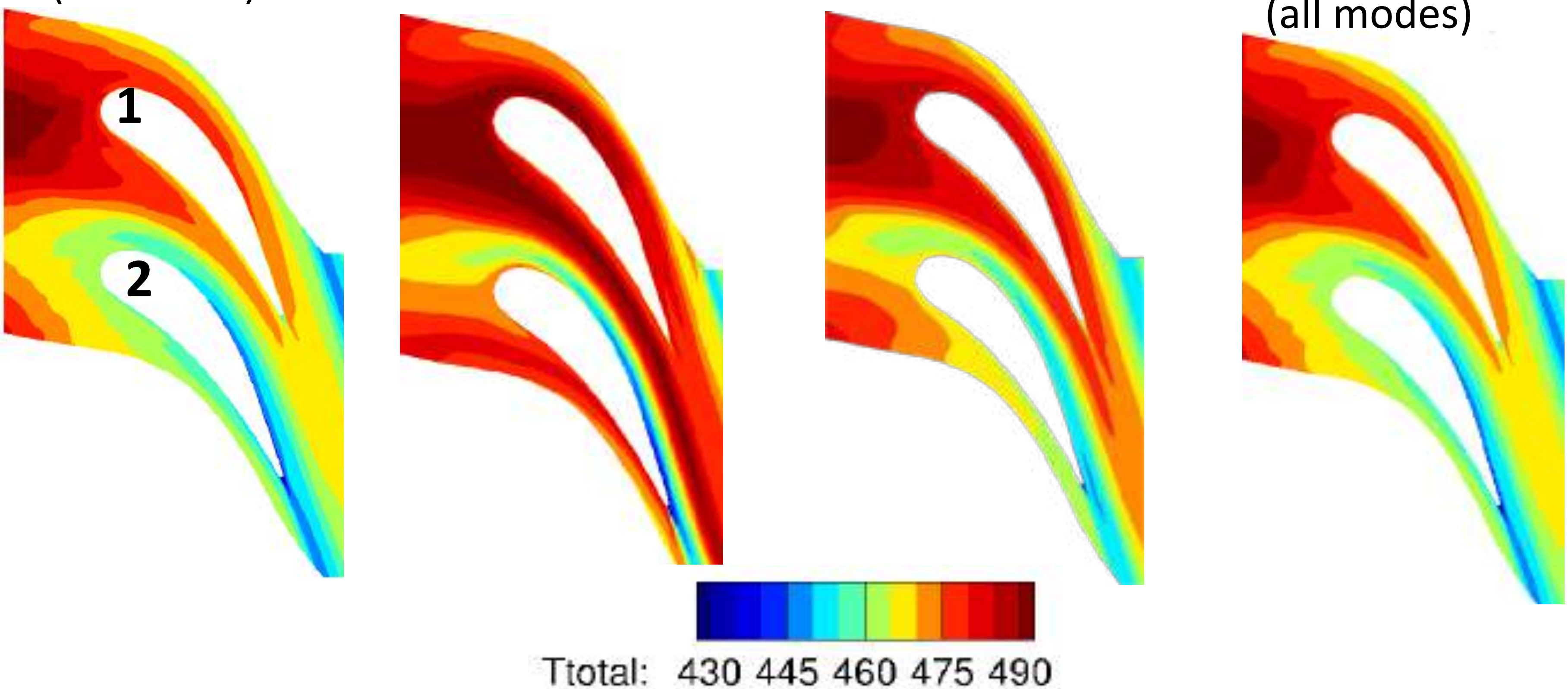
- Extract P40 fully unsteady flow fields
- Recast the data into POD modes.
- Reconstruct several inflow conditions:
  - **- Mean 2D profile**
  - **- Mean 2D profile + POD modes corresponding to the PVC**
  - **- Fully unsteady flow fields**

[1]: Thomas et al., *ETMM12*, 2018

POD method

Numeric / Numeric

Integrated (reference) | **Constant 2D map** | **2D + PVC modes** | **Fully unsteady** (all modes)



- Reference and Full modes simulation are very similar
- PVC has a strong impact on flow field

Synthetic methods

Pros:

- Can be used as a standalone boundary condition in the main computational domain
- Relatively easy to implement
- Affordable computational cost

Cons:

- Injected turbulence **doesn’t respect NS Equations**.
- An **adaptation distance** is mandatory to reach the target turbulence characteristics.

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The logarithmic law-of-the-wall

Wall-modeled LES: analytical, TBLE

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Log-layer mismatch and applications

Turbulence injection

Synthetic methods (Fourier / POD)

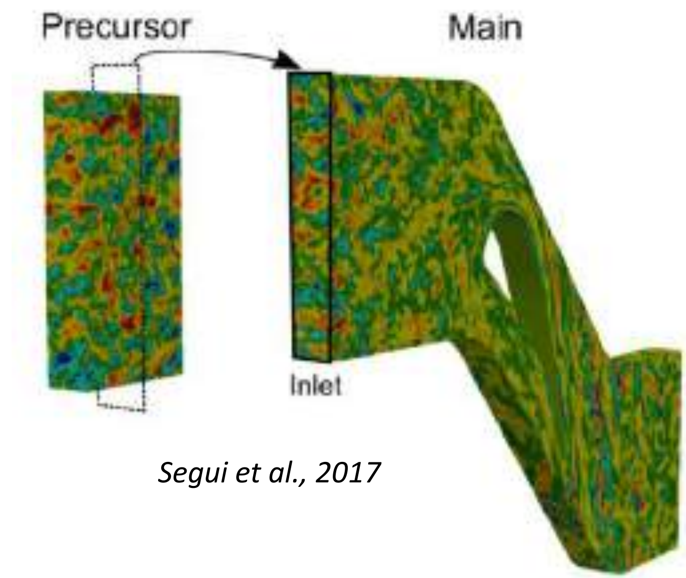
Precursor method

Wall-bounded turbulent injection

Precursor simulation

Turbulence is computed in a **separated domain**, then imposed to the main domain inlet

A **precursor** periodic **HIT** simulation is performed.



Segui et al., 2017

Taylor hypothesis (1938): "Frozen turbulence"

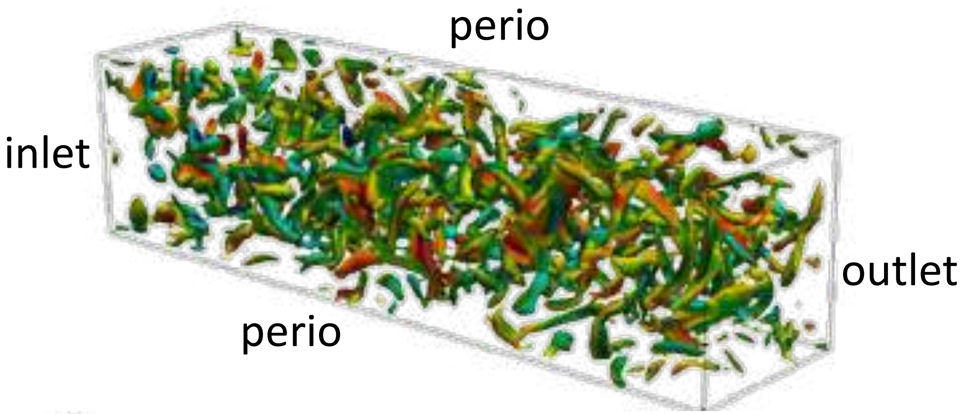
- Turbulence of the advected field and turbulence in the fixed HIT are comparable

$\Rightarrow$ The extraction plane is advected at the main domain inlet bulk velocity.

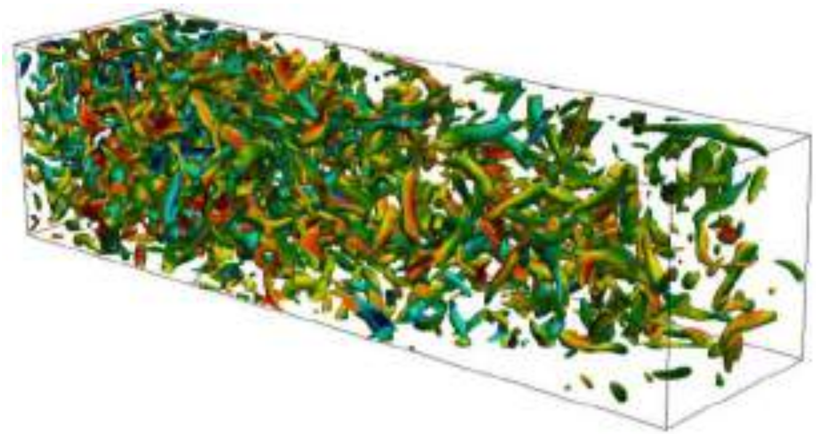
$\Rightarrow$ Extracted turbulence imposed through the NSCBC condition.

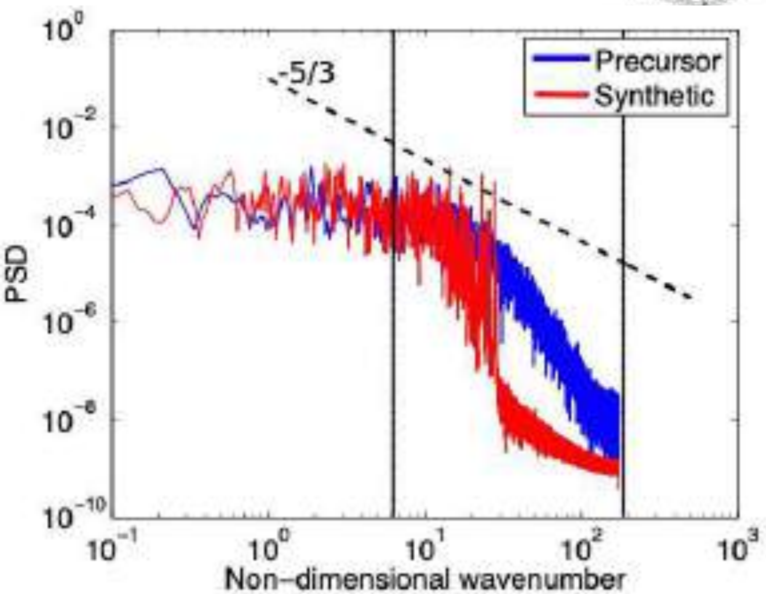
Assessment: Turbulent channel

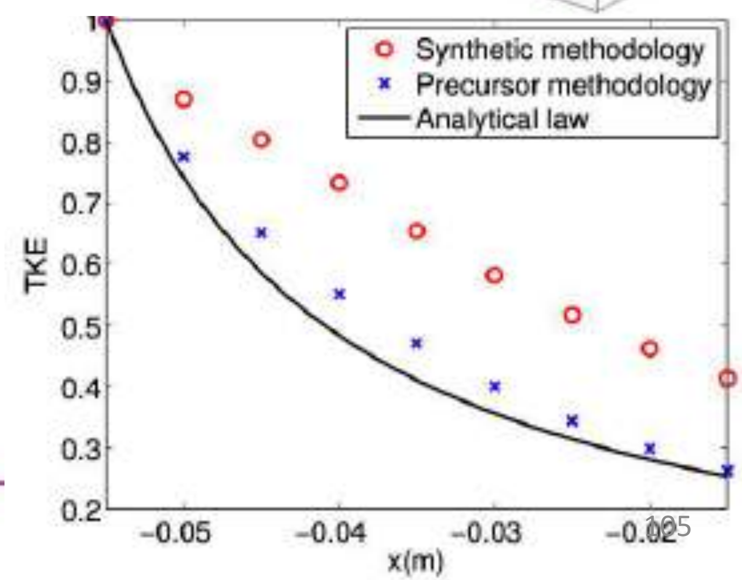
Synthetic Fourier Method (Kraichnan)



Precursor simulation







Pros:

- Almost completely eliminates the errors encountered with synthetic methods
- Provides excellent results.

Cons:

- Implementation is not straightforward
- One-way coupling: no information from main simulation to precursor (e.g. acoustic wave)
- Computational cost

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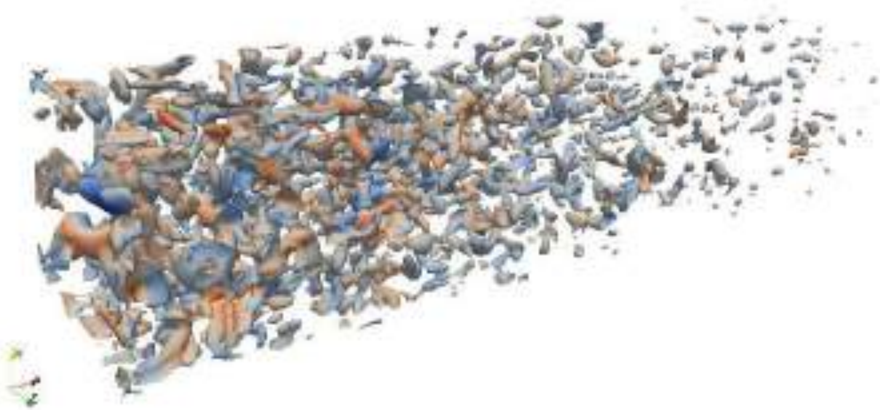
Precursor method

Wall-bounded turbulent injection

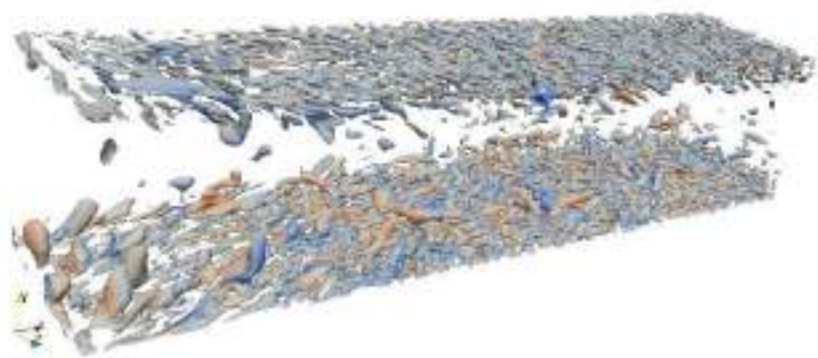
Wall-bounded turbulence injection

Turbulence injection:

Homogeneous, isotropic



Non-homogeneous, anisotropic

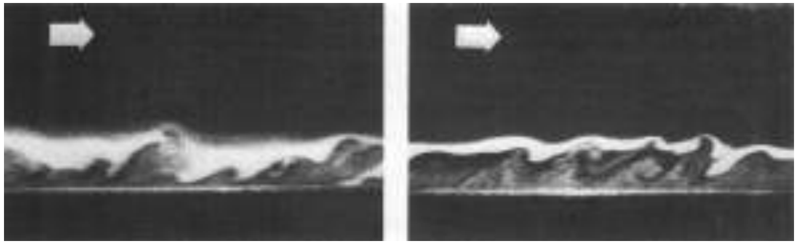


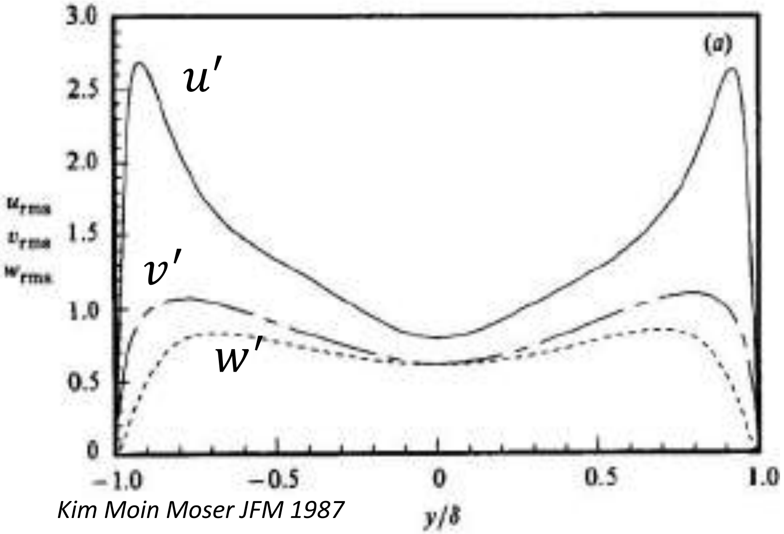
Wall-Bounded flows

Boudin, 2021

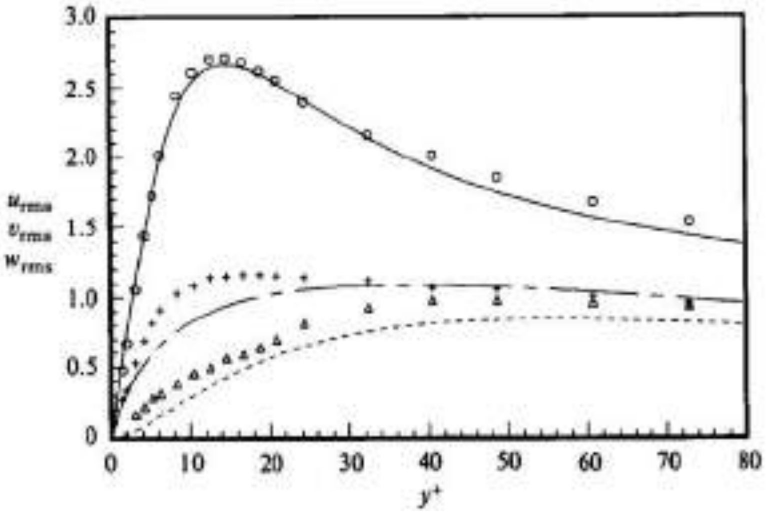
Reminder: Statistics in a BL

Head Bandyopadhyay JFM 1981





Kim Moin Moser JFM 1987



Non-uniformity of **statistics** (Urms, Vrms, Wrms) in a turbulent boundary layer.

=> **Turbulent BL requires dedicated turbulence injection** !

Smirnov /Celik transformation

For a anisotropic velocity correlation tensor r_{ij} :

=> Let find a **transformation tensor a_{ij}** which **diagonalizes r_{ij}**: $r_{ij} \equiv \overline{u_i' u_j'}$

$$a_{mi} a_{nj} r_{ij} = \delta_{mn} c_{(n)}^2$$
$$a_{ik} a_{kj} = \delta_{ij}$$

$$a_{ij} = \begin{pmatrix} \sqrt{R_{11}} & 0 & 0 \\ R_{21}/a_{11} & \sqrt{R_{22} - a_{21}^2} & 0 \\ R_{31}/a_{11} & (R_{32} - a_{21}a_{31})/a_{22} & \sqrt{R_{33} - a_{31}^2 - a_{32}^2} \end{pmatrix}$$

c_n corresponds to the fluctuations (u', v', w') to inject, to satisfy with r_{ij}

$$c_n = \{c_1, c_2, c_3\} \Rightarrow (u', v', w')$$

Smirnov Methodology:

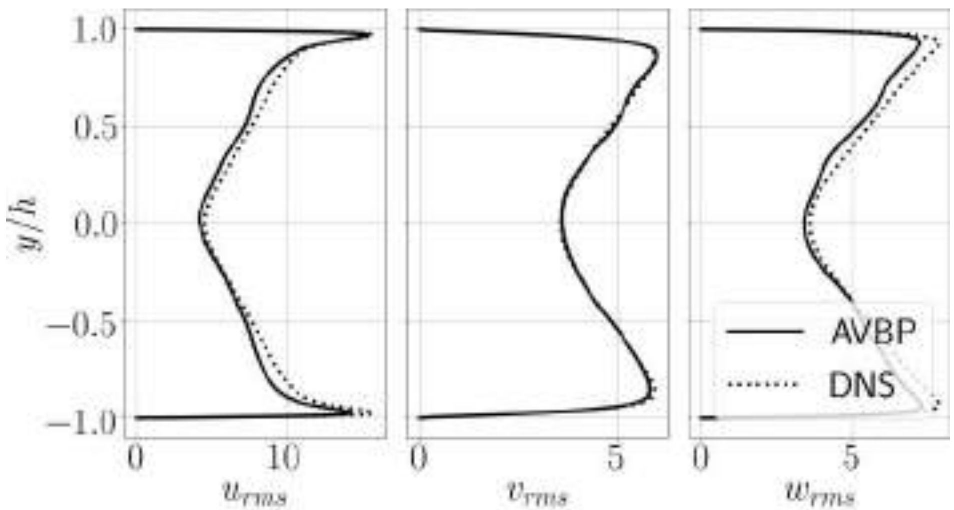
- Generate an **isotropic** fluctuation through **Kraichnan** method => $v_i'(\vec{x}, t)$
- Apply **scaling** and **orthogonal transformation** to this isotropic flow fiels

$$w_i = c_{(i)} v_{(i)}$$
$$u_i = a_{ik} w_k$$

Smirnov, Shi, Celik, 2001

Synthetic method for **wall-bounded flows**

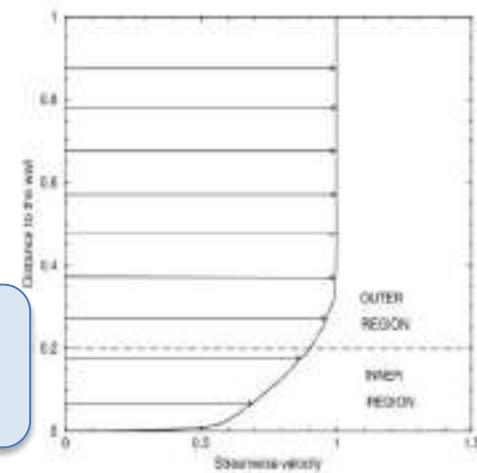
Application: Imposition of r_{ij} extracted from a DNS *(Hoyas / Jimenez 2008.)*



Synthetic method for **wall-bounded flows**

Yao 2002–Sandham 2003

Fluctuations in the **inner** and **outer** part of the BL have **different characteristic scales**
=> Specific disturbances are introduced in each part.



Inner fluctuations: Streaks, with energy max at $y^+_{p,j}$

$$\hat{u}^{\text{inner}} = c_{1,0}\, y^+ \mathrm{e}^{-y^+/y^+_{\mathrm{p},0}} \sin(\omega_0 t)\cos(\beta_0 z + \phi_0)$$

Outer fluctuations:

$$\hat{u}^{\text{outer}} = \sum_{j=1}^{3} c_{1,j}\, y/y_{\mathrm{p},j}\, \mathrm{e}^{-y/y_{\mathrm{p},j}} \sin(\omega_j t)\cos(\beta_j z + \phi_j)$$

ϕ_j Phase shift

ω_j Forcing frequency

β_j Spanwise wave number

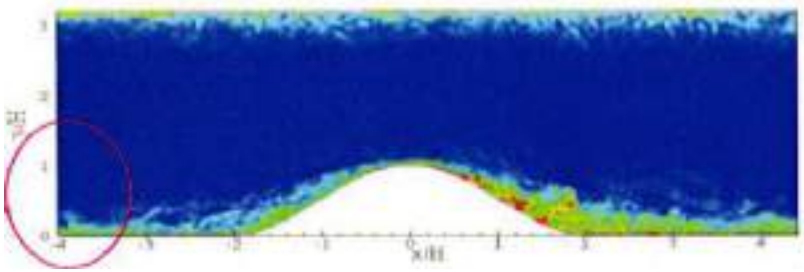
To be adapted depending on the boundary layer

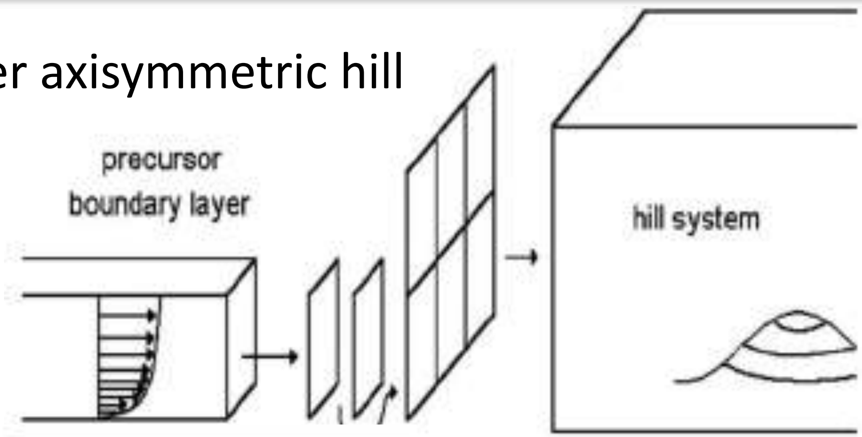
<u>Issue</u>: How can we create **representative turbulence to impose a boundary layer** of given size δ ?

<u>Precursor simulation:</u>

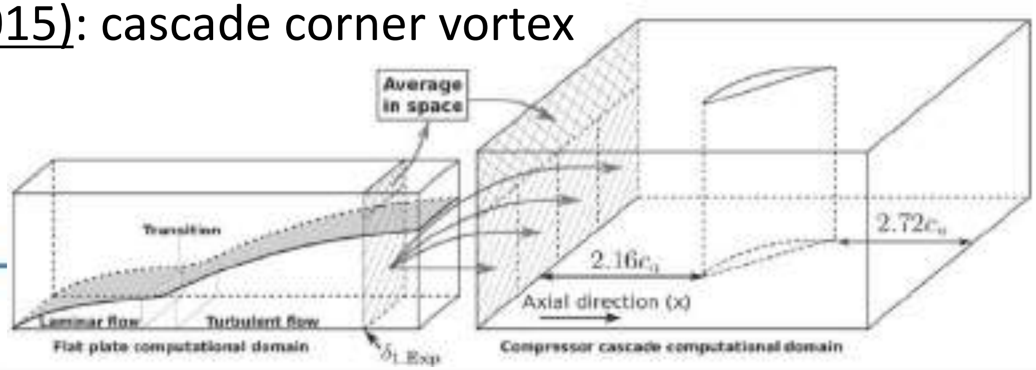
Simulation of a turbulent boundary layer over a flat plate, until the desired BL thickness.

<u>Castagna et al., (2014)</u>: turbulent flow over axisymmetric hill

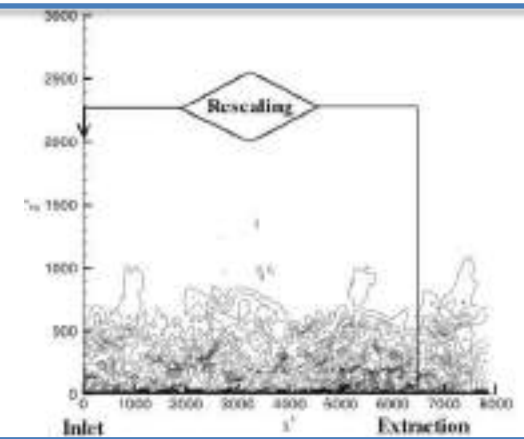




<u>Xie et al. (2015)</u>: cascade corner vortex



<u>Recycling method</u>: *Lund, 1998*



- Information imposed at the inlet based on the information extracted from a plane downstream
- No longer need a precursor simulation

<u>Issue:</u>

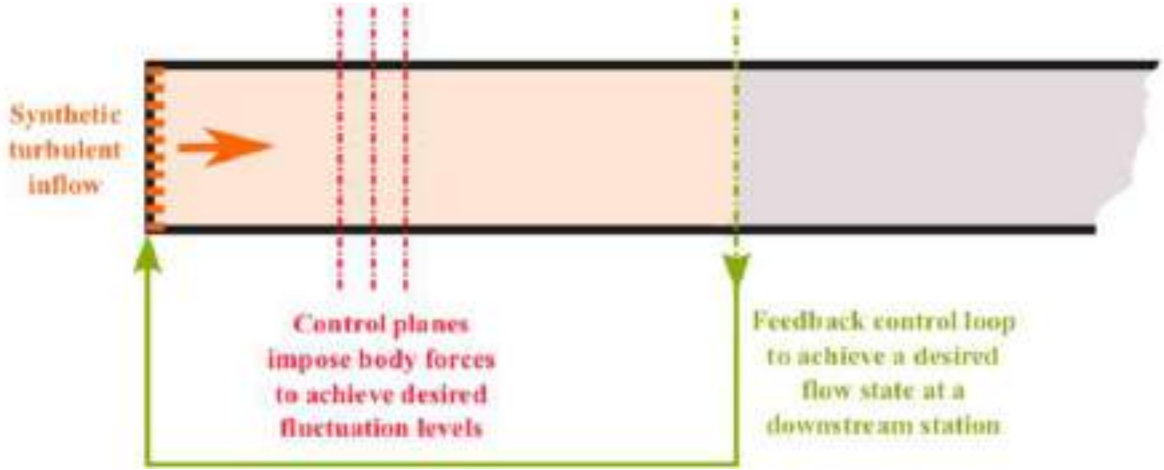
- The boundary layer thickness spatially increases…

=> The **flow extracted** downstream must be **rescaled** before being used at the inlet plane.

Efficient, but must be used with care:

- Only valid for fully turbulent self-similar boundary layers
- Implementation / parallelization not straightforward
- Extraction plane must be far enough, to avoid spurious coupling

Suggested improvement *(Spille-Kohoff and Kaltenbach, 2001)*:



Active control planes, where the wall-normal velocity fluctuations are amplified or damped to **match a target Reynolds shear stress**.

A few conclusion

- Accounting for turbulence injection is mandatory for real flows.

- Random / **white noise is not accurate** since it does not account for turbulence coherence.

- **Synthetic methods** (Fourier/POD) are much more reliable. However, still lack of physics.

- **Precursor simulations** provide excellent results. However, computational cost significantly increases.

- Specific **care** must be taken for **wall-bounded flows**.

General conclusions

- LES must faces **“real-life” issues** when dealing with real flows: several **errors** are induced.
- Numerical scheme must be accurate enough (2^{nd} order minimum).
- **Numerical** / **modeling** errors can be highly **coupled**.

- **LES** is efficient for massively **separated industrial flows**, where RANS is not.
- For attached boundary layers, **modeling is mandatory** to reduce the computational cost.
- Validity of such models for real flows is still an **open question**…

- **Turbulence injection** is mandatory for numerous flows. There is still work to do for an accurate coupling with NSCBC.

- LES is **still an expert** approach… A use in the industry is currently beginning.