The CIE 1997 Colour Appearance Model: CIECAM97s

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The Structure of the CIE 1997 Colour Appearance Model (CIECAM97s)

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Abstract: The components comprising the CIE 1997 Colour Appearance Model, CIECAM97s, are described, and the steps needed to implement it in both forward and reverse modes are listed. A worked example is also given. © 1998 John Wiley & Sons, Inc. Col Res Appl, 23, 138–146, 1998

Key words: colour appearance; CIECAM97s model; adaptation

INTRODUCTION

At the CIE Expert Symposium 96 on Colour Standards for Image Technology, held in Vienna in March 1996,[1] it was agreed that the authors should examine existing colour appearance models, try to combine the best features of these models into a high-performance model for general use, and test its performance against available experimental data. It was also agreed that this model should be available in a comprehensive version, and in a related simple version for use in limited conditions. At its meeting held in Kyoto in May 1997, CIE Technical Committee TC 1-34 agreed to adopt as the simple version the model CIECAM97s described in this article; the way in which this version might be extended to provide a comprehensive version, CAM97c, is also described, but this version has not yet been considered by the CIE. It is important to note that these models are empirical. The inclusion of the year 97 in the designations is intended to indicate the interim nature of the models, pending the availability of better models, which are expected to emerge in the future.

The CIECAM97s model depends on the work of many different investigators, and acknowledgments are made to them in the text. One of these investigators, Y. Nayatani, has also played a major role in advancing the general philosophy of model building, and this is gratefully acknowledged by the authors. The CIECAM97s model incorporates features from various prior models including: that by Nayatani and his collaborators, K. Hashimoto, H. Sobagaki, K. Takahama, and T. Yano; the RLAB model by R. S. Berns and M. D. Fairchild; the LLAB model by M. R. Luo, M. C. Lo, and W. G. Kuo; and that by Hunt and his collaborators, M. R. Pointer and M. R. Luo; references to these models are made in the text where relevant.

STRUCTURES OF COLOUR APPEARANCE MODELS

Colour appearance models usually comprise three stages: a chromatic adaptation transform, a dynamic response function, and a colour space for representing the correlates of the percepts. A simple chromatic adaptation transform can be provided by a multiplicative normalization of X, Y, Z tristimulus values to make them constant for whites in different conditions; this was proposed by Evans,[2] and is incorporated in the CIELAB system.[3] A more fundamental approach is to carry out the normalization on cone responses, as first proposed by Von Kries,[4] and most recent models use modified Von Kries transforms; however, the Bradford transform,[5] which is used in the LLAB model,[6] depends on a set of spectral sensitivities that are sharpened, compared to cone responses, and have some small negative lobes (as shown in Fig. 1); this transform also incorporates a power function in its blue channel. The dynamic response functions used are cube-root in the RLAB[7] and LLAB[6] models, hyperbolic in the Hunt model,[8,9] and logarithmic in the Nayatani model[10]; these different functions diverge most for very light and for very dark colours. The colour spaces used in these models are similar in that they all provide approximate correlates of redness–greenness and yellowness–blueness, ratios of which are used to derive angular correlates

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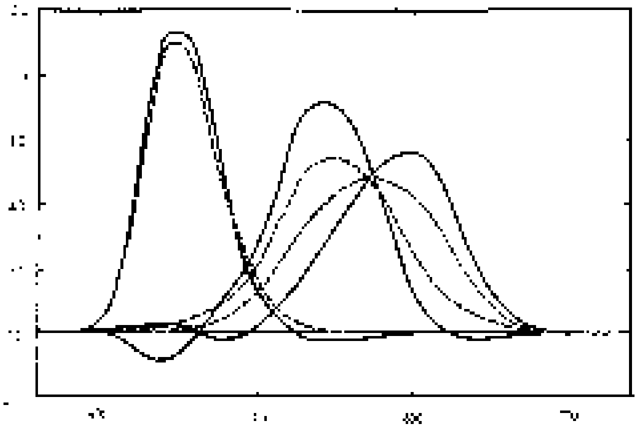

FIG. 1. Full lines: spectral sensitivities used in the Bradford chromatic adaptation transform. Broken lines: spectral sensitivities for cones, similar to those found by Estévez. Both sets of curves are linear transformations of the $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ functions.

of hue, and square-roots of the sum of the squares of which are used to derive correlates of chroma, while nonlinear functions of an achromatic signal are used to derive correlates of lightness; but only some of the models also provide correlates of saturation, colourfulness, and brightness. The differences in the chromatic adaptation transforms, in the dynamic response functions, and in the colour spaces, all contribute to the differences between the predictions made by the models.

In the CIECAM97s model, the chromatic adaptation transform is the Bradford transform (devised by Lam and Rigg[5]), in which is incorporated a factor, D (similar to those suggested by Fairchild[7] and by Nayatani[11]) to regulate the extent to which the chromatic adaptation occurs (as used in the LLAB model[7]). A matrix then converts the variables to cone responses (following the work of Estévez[12]), which are used as the basis of the rest of the model; the matrix is normalized so that the cone responses are equal for the Equi-energy Stimulus, S_E. The cone responses are multiplied by a luminance-level adaptation factor,[8] and then modified by a dynamic response function that is hyperbolic (as introduced by Seim and Valberg,[13] and as found in physiological studies of rhesus monkeys by Valeton and Van Norren[14]). The colour space used is similar to that used in the Hunt model,[8] but with predictors for lightness and brightness that avoid the possibility of negative values occurring (as achieved by Nayatani[10] and Fairchild[6] in the Nayatani and RLAB models, respectively). Allowance for the effects of different surrounds is based on the work by Bartleson and Breneman.[15]

CHROMATIC ADAPTATION TRANSFORM

The Bradford chromatic adaptation transform uses as input the ratios X/Y, Y/Y, Z/Y, instead of X, Y, Z, and this means that a series of colours of the same chromaticity have the same input into the transform, and, therefore, the same output from it, apart from differences associated with the luminance factors.

The transform converts the tristimulus values, X, Y, Z, of test colours in a set of test conditions to tristimulus values, X_c, Y_c, Z_c, of corresponding colours, which define stimuli that have the same appearance in a set of reference conditions. Because it is necessary to model reduced levels of chromatic adaptation, the reference conditions must include a reference chromaticity relative to which test conditions can be considered more or less chromatic; this reference chromaticity is chosen (as in the current RLAB model[7]) to be the same as that of the Equi-energy Stimulus, S_E, because, as has been found by Hurvich and Jameson,[16] such stimuli appear achromatic to the dark-adapted eye. This reference chromaticity is assigned to a reference white. The Y value ascribed to this reference white, Y_{wr}, does not affect the corresponding colours produced by the transform; this is because the reference-white parameters used in the transform, R_{wr}, G_{wr}, B_{wr}, are derived from the ratios of its tristimulus values, and these are equal to the ratios of its chromaticity coordinates. However, it is convenient to put $Y_{wr} = 100$, so that the reference white is the perfect diffuser. But, when using the model to obtain the correlate of lightness, this is evaluated relative to the corresponding colour (in the reference condition) of the adopted test white; this is because, if evaluated relative to the perfect diffuser, the adopted white would usually have a lower lightness and this would imply that it was not a true white. (In most applications in the surface colour industries, Y_w, the Y value of the adopted white, is set to 100; this results in Y_{wc}, the Y value of its corresponding colour, also being equal to 100.)

The factor, D, in the chromatic adaptation transform, is normally derived as a function of the luminance of the adapting field, L_A, which results in D increasing (and, hence, the degree of chromatic adaptation increasing) as L_A increases. But, if the colour of the illuminant is completely discounted (complete chromatic adaptation), D is set equal to 1.0; if there is partial discounting, D can be set half way between its variable value and 1.0. If there is no chromatic adaptation, D is set equal to zero.

The nonlinear function in the B channel of the transform means that special procedures have to be used when the B signal happens to be negative.

DYNAMIC RESPONSE FUNCTION

The hyperbolic nature of the dynamic response function results in the response gradually reducing to a noise level of 1.0 at very low stimulus intensities, and gradually rising to a maximum of 41 at very high stimulus intensities, with a transition in between that approximates a square-root function. The ratio of maximum to minimum response, 41:1, was chosen to accord approximately with the ratio of the maximum to minimum useful numbers of nerve impulses per second, this being estimated at about 400 to about 10.

The luminance-level adaptation factor, F_L, is proportional to the cube-root of the luminance of the adapting field, L_A, down to about cone threshold ($5L_A$ equal to about 1 cd/m^2) and this provides partial luminance-level adaptation over this range (full luminance-level adaptation would correspond to F_L being constant); below cone threshold, F_L is proportional to L_A, and this then provides no luminance-level adaptation, as is appropriate.

COLOUR SPACE

The colour space includes an eccentricity factor to allow for the fact that perceived achromatic colours are not at the center of contours of low saturation (as in the Hunt[8,9] and Nayatani[10] models).

The formula for predicting chroma is constructed to allow for the fact that, although dark backgrounds make most colours appear of higher chroma, in the case of light colours the opposite effect occurs so that they appear of lower chroma (as reported by Hunt[8]).

INPUT DATA

The input data to the model includes the luminance, L_A, of the test adapting field in cd/m^2. L_A can usually be taken as $L_w/5$, where L_w is the luminance in cd/m^2 of the perfect diffuser in the test illuminant; this is why $5L_A$ is often used in the model instead of L_A (if the adopted white is a Lambertian diffuser, L_w may be calculated as EY_w/π, where E is the test illuminance in lux). Also required are the chromaticities and luminance factors of the following: the sample in the test conditions; the adopted white in the test conditions; the background in the test conditions; and the reference white in the reference conditions. In addition, it is necessary to know whether the surround conditions are average, dim, dark, or ''cut-sheet''; by *average* is meant that the surround luminance is similar to the average luminance of the colours in the viewing field, as is typically the case when surface colours are viewed; by *dim* is meant that the surround luminance is appreciably less than the average luminance of the viewing field, as is typically the case when viewing television; by *dark* is meant that the surround luminance is very low compared to the average luminance of the viewing field, as is typically the case when viewing film projected in a darkened room; by ''*cut-sheet*'' is meant the conditions typical for viewing cut-sheet film on light boxes. If the samples subtend more than 4° at the eye, this also needs to be known. (These details, concerning the surround and the angular subtense, determine the values of the parameters, F, c, F_{LL}, and N_c, used in the model.) It is also necessary to know if the colour of the illuminant is being completely or partially discounted (in which case D is set at 1.0 or at an intermediate value), or if there is no chromatic adaptation (in which case D is set at zero).

The simple version of the model assumes that the conditions are such that the amount of cone pigment bleaching is negligible, that there is no significant contribution from the rods, and that there is no low-luminance tritanopia. For adapting luminances in the normal photopic range, L_A equal to values between about 2 and about 2000 cd/m^2, these assumptions are likely to be valid. The simple version also assumes that there is no significant Helson–Judd effect, and this is probably valid for most typical ''white'' illuminants, but would not be true for highly chromatic illuminants. The simple version also makes no allowance for the Helmholtz–Kohlrausch effect, although this will be significant for colours of high chroma, except in the case of yellows.

OUTPUT DATA

The model gives predictions for hue, h and H; lightness, J; brightness, Q; saturation, s; chroma, C; and colourfulness, M. The two predictors for hue are hue-angle, h, and hue quadrature, H; from hue quadrature the percentages of the unique hue components, the hue composition, can be readily determined, and this is useful for visualizing the perceived hue. However, hue composition has a very nonuniform relationship with hue difference, because of the considerably larger number of perceived hue-difference increments between blue and red than between red and yellow, or between yellow and green, or between green and blue; hue-angle represents perceived hue-differences more uniformly, but is less useful for visualizing the perceived hue.

DETAILS OF THE MODEL

The steps required to use the simple version of the model (CIECAM97s) are given in Appendix A, and in reverse mode in Appendix B; a worked example is given in Appendix C.

REVERSING THE MODEL

For some practical applications it is necessary to be able to operate the model in reverse. The steps required to do this for the simple version are given in Appendix B. It is to facilitate reversal that, in the formula for saturation, s, the term $R'_a + G'_a + B'_a$ used in the Hunt94 model has been replaced by $R'_a + G'_a + (21/20)B'_a$.

The Bradford transform does not reverse exactly by simple means because of the exponent in the B channel; but the procedure given in Appendix B is accurate enough for all practical purposes.[17]

The comprehensive version of the model, details of which are given in Appendix D, is difficult to reverse because of the inclusion of the rod response; methods of dealing with this situation are described in Appendix E.[18]

VISUAL AREAS IN THE OBSERVING FIELD

For related colours, five different visual fields are recognized in the model:

- Colour element considered: typically a uniform patch of about 2° angular subtense.
- Proximal field: the immediate environment of the colour element considered, extending typically for about 2° from the edge of the colour element considered in all or most directions.
- Background: the environment of the colour element considered, extending typically for about 10° from the edge of the proximal field in all, or most directions. When the proximal field is the same colour as the background, the latter is regarded as extending from the edge of the colour element considered.
- Surround: the field outside the background.
- Adapting field: the total environment of the colour element considered, including the proximal field, the background, and the surround, and extending to the limit of vision in all directions.

The visual patterns of scenes viewed in practice are almost infinitely variable, but the phenomenon of colour constancy indicates that the effects of this variety on colour appearance are, to some extent, limited. The regime of fields described above is an attempt to simplify the situation sufficiently to make it feasible for modeling, while making it possible to include the most important factors that affect colour appearance. The proximal field is not used in the simple version of the model; it is included to facilitate future incorporation in the comprehensive version of the effects of simultaneous contrast, or of assimilation for very small stimuli.

If the colour considered has an angular subtense of more than 4°, the colorimetric measures used in the model must be for the CIE 1964 Supplementary Colorimetric Observer, and the symbols representing the output data of the model must all carry a subscript 10. Otherwise, the CIE 1931 Standard Colorimetric Observer is used.

COMPREHENSIVE VERSION OF THE MODEL, CAM97C

In Appendix D an outline is given of the way in which a comprehensive version of the model, CAM97C, could be developed from the simple version, CIECAM97s. Factors are included for cone pigment bleaching, for the Helson–Judd effect, for the rod contribution to the achromatic signal, for low-luminance tritanopia, and for the Helmholtz–Kohlrausch effect. Also required, but not yet implemented, are means for incorporating the effects of the proximal field on simultaneous contrast, and on assimilation (for stimuli of very small angular subtense).

The allowance for the Helmholtz–Kohlrausch effect is based on the experimental results published by Fairchild and Pirrotta.[19–21]

The rod contribution is calculated in the same way as in the Hunt94 model[8]; instead of using the scotopic luminance, L_{AS}, of the adapting field, $L_{AS}/2.26$ is used; because, for the Equi-energy Stimulus, S_E, scotopic luminances divided by 2.26 are equal to photopic luminances. If L_{AS} is not known, it can be approximated by using

$$L_{AS}/2.26 = L_A(T/4000 - 0.4)^{1/3},$$

where T is the correlated colour temperature of the illuminant. If the scotopic luminance of the sample relative to that of the adopted white, S/S_w, is not known, the equivalent photopic values, Y/Y_w, can be used instead.

The comprehensive version described above has not yet been considered by the CIE.

MODEL FOR UNRELATED COLOURS

Unrelated colours are those that are seen in isolation from other colours. Their distinguishing feature is that they are incapable of possessing the attribute of greyness, so that they have brightnesses but no lightnesses, and colourfulnesses and saturations, but no chromas. Light sources are frequently perceived as unrelated colours.

There is at present no agreed model for unrelated colours. Pending the availability of anything better, use can be made of the model for unrelated colours associated with the Hunt94 model.[22]

APPENDIX A: STEPS FOR USING THE CIE 1997 SIMPLE COLOUR APPEARANCE MODEL, CIECAM97s

Starting data:			
Sample in test conditions	x	y	Y
Adopted white in test conditions:	x_w	y_w	Y_w
Background in test conditions:	x_b	y_b	Y_b
Reference white in reference conditions:	$x_{wr} = \frac{1}{3}$	$y_{wr} = \frac{1}{3}$	$Y_{wr} = 100$
Luminance of test adapting field (cd/m^2)	L_A†		

† L_A is normally taken as $\frac{1}{5}$ of the luminance of the adopted test white.

Surround parameters:	F	c	F_{LL}	N_c
Average (with sample over 4°)	1.0	0.69	0	1.0
Average	1.0	0.69	1.0	1.0
Dim	0.9	0.59	1.0	1.1
Dark	0.9	0.525	1.0	0.8
Cut-sheet	0.9	0.41	1.0	0.8

Background parameters:

$$N_{bb} = N_{cb} = 0.725(1/n)^{0.2}$$

$$z = 1 + F_{LL}n^{1/2},$$

where $n = Y_b/Y_w$.

If the test background chromaticity is different from that of the adopted test white, then, in the above expressions, the Y tristimulus values of the corresponding colours in the reference conditions, Y_{bc} and Y_{wc}, have to be used instead of Y_b and Y_w.

Step 1. For the sample, calculate:

$X = xY/y$, Y, $Z = (1 - x - y)Y/y$ and X/Y, Y/Y, Z/Y

and

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = M_{BFD} \begin{pmatrix} X/Y \\ Y/Y \\ Z/Y \end{pmatrix},$$

$$\text{where } M_{BFD} = \begin{pmatrix} 0.8951 & 0.2664 & -0.1614 \\ -0.7502 & 1.7135 & 0.0367 \\ 0.0389 & -0.0685 & 1.0296 \end{pmatrix}.$$

Similarly, from x_w, y_w, Y_w calculate R_w, G_w, B_w
from x_b, y_b, Y_b calculate R_b, G_b, B_b
from x_{wr}, y_{wr}, Y_{wr} calculate R_{wr}, G_{wr}, B_{wr}.

Step 2. Calculate the degree of chromatic adaptation, D:

$$D = F - F/[1 + 2(L_A^{1/4}) + (L_A^2/300)].$$

But if the chromatic adaptation is complete (illuminant colour completely discounted), put $D = 1.0$; or if there is partial discounting put $D = \frac{1}{2}\{1 + F - F/[1 + 2(L_A^{1/4}) + (L_A^2/300)]\}$; or if there is no chromatic adaptation, put $D = 0$.

Step 3. From R, G, B calculate the corresponding tristimulus values R_c, G_c, B_c for the sample under the reference conditions:

$$R_c = [D(R_{wr}/R_w) + 1 - D]R$$

$$G_c = [D(G_{wr}/G_w) + 1 - D]G$$

$$B_c = [D(B_{wr}/B_w^p) + 1 - D]|B|^p$$

(when B is negative, B_c must be made negative),

where $p = (B_w/B_{wr})^{0.0834}$.

Similarly, from R_w, G_w, B_w calculate R_{wc}, G_{wc}, B_{wc}
from R_b, G_b, B_b calculate R_{bc}, G_{bc}, B_{bc}.

Step 4. Calculate $F_L = 0.2k^4(5L_A) + 0.1(1 - k^4)^2(5L_A)^{1/3}$, where $k = 1/(5L_A + 1)$.

Step 5. Calculate

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = M_H M_{BFD}^{-1} \begin{pmatrix} R_cY \\ G_cY \\ B_cY \end{pmatrix}$$

$$\begin{pmatrix} R'_w \\ G'_w \\ B'_w \end{pmatrix} = M_H M_{BFD}^{-1} \begin{pmatrix} R_{wc}Y_w \\ G_{wc}Y_w \\ B_{wc}Y_w \end{pmatrix},$$

$$\text{where } M_{BFD}^{-1} = \begin{pmatrix} 0.98699 & -0.14705 & 0.15996 \\ 0.43231 & 0.51836 & 0.04929 \\ -0.00853 & 0.04004 & 0.96849 \end{pmatrix}$$

$$\text{and } M_H = \begin{pmatrix} 0.38971 & 0.68898 & -0.07868 \\ -0.22981 & 1.18340 & 0.04641 \\ 0.00000 & 0.00000 & 1.00000 \end{pmatrix}.$$

Step 6. Calculate

$$Y_{bc} = (0.43231R_{bc} + 0.51836G_{bc} + 0.04929B_{bc})Y_b,$$

$$Y_{wc} = (0.43231R_{wc} + 0.51836G_{wc} + 0.04929B_{wc})Y_w,$$

$$n = Y_{bc}/Y_{wc}, \quad N_{bb} = 0.725(1/n)^{0.2}, \quad N_{cb} = 0.725(1/n)^{0.2}.$$

(If $x_b = x_w$ and $y_b = y_w$, then $R_b = R_w$ and, hence, $R_{bc} = R_{wc}$, and similarly for G and B, so that $Y_{bc}/Y_{wc} = Y_b/Y_w$).

Step 7. Calculate

$$R'_a = 40(F_LR'/100)^{0.73}/[(F_LR'/100)^{0.73} + 2] + 1$$

$$G'_a = 40(F_LG'/100)^{0.73}/[(F_LG'/100)^{0.73} + 2] + 1$$

$$B'_a = 40(F_LB'/100)^{0.73}/[(F_LB'/100)^{0.73} + 2] + 1.$$

Similarly, from R'_w, G'_w, B'_w calculate $R'_{aw}, G'_{aw}, B'_{aw}$.

If R' is less than 0 use: $R'_a = -40(-F_LR'/100)^{0.73}/[(-F_LR'/100)^{0.73} + 2] + 1$ and similarly for R'_{aw}, and for the G and B equations.

Step 8. Calculate:

Redness–Greenness $a = R'_a - 12G'_a/11 + B'_a/11$
Yellowness–Blueness $b = (1/9)(R'_a + G'_a - 2B'_a)$
Hue angle $h = \arctan(b/a)$.

Step 9. Using the following unique hue data:

	Red	Yellow	Green	Blue
h	20.14	90.00	164.25	237.53
e	0.8	0.7	1.0	1.2

$$\text{calculate } e = e_1 + (e_2 - e_1)(h - h_1)/(h_2 - h_1),$$

where e_1 and h_1 are the values of e and h, respectively, for the unique hues having the nearest lower value of h; and e_2 and h_2 are the values of e and h, respectively, for the unique hues having the nearest higher value of h.

Calculate the hue quadrature:

$$H = H_1 + 100[(h - h_1)/e_1]/[(h - h_1)/e_1 + (h_2 - h)/e_2],$$

where H_1 is 0, 100, 200, or 300 according to whether red, yellow, green, or blue, respectively, is the hue having the nearest lower value of h.

Calculate the Hue Composition, H_C,
where H_P is the part of H after its hundreds digit, if:

$H = H_P$, the Hue Composition is

$$H_P \text{ Yellow}, 100 - H_P \text{ Red},$$

$H = 100 + H_P$, the Hue Composition is

$$H_P \text{ Green}, 100 - H_P \text{ Yellow},$$

$H = 200 + H_P$, the Hue Composition is

$$H_P \text{ Blue}, 100 - H_P \text{ Green},$$

$H = 300 + H_P$, the Hue Composition is

$$H_P \text{ Red}, 100 - H_P \text{ Blue}.$$

Step 10. Calculate

$$A = [2R'_a + G'_a + (1/20)B'_a - 2.05]N_{bb}$$

$$A_w = [2R'_{aw} + G'_{aw} + (1/20)B'_{aw} - 2.05]N_{bb}.$$

Step 11. Calculate

$$J = 100(A/A_w)^{cz}, \text{ where } z = 1 + F_{LL}n^{1/2}.$$

Step 12. Calculate

$$Q = (1.24/c)(J/100)^{0.67}(A_w + 3)^{0.9}.$$

Step 13. Calculate

$$s = [50(a^2 + b^2)^{1/2}100e(10/13)N_cN_{cb}]/[R'_a + G'_a + (21/20)B'_a],$$

$$C = 2.44s^{0.69}(J/100)^{0.67n}(1.64 - 0.29^n),$$

$$M = CF_L^{0.15}.$$

APPENDIX B: STEPS FOR USING THE CIECAM97s MODEL IN REVERSE MODE

Starting data: Q or J, M or C, H or h. Also required:

Q_w, A_w, and Y_{bc}/Y_{wc}, obtained by using the model forward with the adopted test white and test background. (If $x_b = x_w$ and $y_b = y_w$, then $Y_{bc}/Y_{wc} = Y_b/Y_w$.)

Surround parameters used: F, c, F_{LL}, and N_c.
Luminance-level parameters used: L_A, D.
Unique hue data:

	Red	Yellow	Green	Blue
h	20.14	90.00	164.25	237.53
e	0.8	0.7	1.0	1.2

Step 1. From Q obtain J:

$$J = 100(Qc/1.24)^{1/0.67}/(A_w + 3)^{0.9/0.67}.$$

Step 2. From J obtain A:

$$A = (J/100)^{1/cz}A_w, \text{ where } z = 1 + F_{LL}(Y_{bc}/Y_{wc})^{1/2}.$$

Step 3. Using H, determine h_1, h_2, e_1, e_2, where e_1 and h_1 are the values of e and h, respectively, for the unique hues having the nearest lower value of h; and e_2 and h_2 are the values of e and h, respectively, for the unique hues having the nearest higher value of h.

Step 4. Calculate h:

$$h = [(H - H_1)(h_1/e_1 - h_2/e_2) - 100h_1/e_1]/[(H - H_1)(1/e_1 - 1/e_2) - 100/e_1],$$

where H_1 is 0, 100, 200, or 300 according to whether red, yellow, green, or blue, respectively, is the hue having the nearest lower value of h.

Step 5. Calculate $e = e_1 + (e_2 - e_1)(h - h_1)/(h_2 - h_1)$, where e_1 and h_1 are the values of e and h, respectively, for the unique hues having the nearest lower value of h; and e_2 and h_2 are the values of e and h, respectively, for the unique hues having the nearest higher value of h.

Step 6. Calculate C:

$$C = M/F_L^{0.15}.$$

Step 7. Calculate s:

$$s = C^{1/0.69}/[2.44(J/100)^{0.67n}(1.64 - 0.29^n)]^{1/0.69},$$

where $n = Y_{bc}/Y_{wc}$.

Step 8. Calculate a and b:

$$a = s(A/N_{bb} + 2.05)/\{[1 + (\tan h)^2]^{1/2}[50\,000eN_cN_{cb}/13] + s[(11/23) + (108/23)(\tan h)]\}.$$

In calculating $[1 + (\tan h)^2]^{1/2}$, the result is to be taken as: positive if h is equal to or greater than 0 and less than 90; negative if h is equal to or greater than 90 and less than 180; negative if h is equal to or greater than 180 and less than 270; positive if h is equal to or greater than 270 and less than 360.

$$b = a(\tan h).$$

Step 9. Calculate

$$R'_a = (20/61)(A/N_{bb} + 2.05) + (41/61)(11/23)a + (288/61)(1/23)b,$$

$$G'_a = (20/61)(A/N_{bb} + 2.05) - (81/61)(11/23)a - (261/61)(1/23)b,$$

$$B'_a = (20/61)(A/N_{bb} + 2.05) - (20/61)(11/23)a - (20/61)(315/23)b.$$

Step 10. Calculate

$$F_LR' = 100[(2R'_a - 2)/(41 - R'_a)]^{1/0.73},$$

$$F_L G' = 100[(2G'_a - 2)/(41 - G'_a)]^{1/0.73},$$

$$F_L B' = 100[(2B'_a - 2)/(41 - B'_a)]^{1/0.73}.$$

If $R'_a - 1$ is less than 0, use

$$F_L R' = -100[(2 - 2R'_a)/(39 + R'_a)]^{1/0.73},$$

and similarly for the G and B equations.
Divide each by F_L to obtain R', G', B'.

Step 11. Calculate

$$\begin{pmatrix} R_cY \\ G_cY \\ B_cY \end{pmatrix} = M_{BFD} M_H^{-1} \begin{pmatrix} R' \\ G' \\ B' \end{pmatrix},$$

$$\text{where } M_H^{-1} = \begin{pmatrix} 1.91019 & -1.11214 & 0.20195 \\ 0.37095 & 0.62905 & 0.00000 \\ 0.00000 & 0.00000 & 1.00000 \end{pmatrix}.$$

Step 12. Calculate

$$Y_c = 0.43231 R_cY + 0.51836 G_cY + 0.04929 B_cY$$

and, hence,

$$(Y/Y_c)R_c,\ (Y/Y_c)G_c,\ (Y/Y_c)B_c.$$

Step 13. Calculate

$$(Y/Y_c)R = (Y/Y_c)R_c/[D(R_{wr}/R_w) + 1 - D],$$

$$(Y/Y_c)G = (Y/Y_c)G_c/[D(G_{wr}/G_w) + 1 - D],$$

$$(Y/Y_c)^{1/p}B = [|(Y/Y_c)B_c|]^{1/p}/[D(B_{wr}/B_w^p) + 1 - D]^{1/p}.$$

If $(Y/Y_c)B_c$ is negative, $(Y/Y_c)^{1/p}B$ is negative.

Step 14. Calculate

$$Y' = 0.43231YR + 0.51836YG + 0.04929(Y/Y_c)^{1/p}BY_c$$

and

$$(Y'/Y_c)^{(1/p-1)}.$$

Step 15. Calculate

$$\begin{pmatrix} X''/Y_c \\ Y''/Y_c \\ Z''/Y_c \end{pmatrix} = M_{BFD}^{-1} \begin{pmatrix} (Y/Y_c)R \\ (Y/Y_c)G \\ (Y/Y_c)^{1/p}B/(Y'/Y_c)^{(1/p-1)} \end{pmatrix}.$$

Step 16. Multiply each by Y_c to obtain X'', Y'', Z'' equal to X, Y, Z, to a very close approximation.

Note: Y' differs from Y because, instead of YB, $(Y/Y_c)^{1/p}BY_c$ is used: but this is multiplied by 0.04929 so that the difference is small. The term $(Y/Y_c)^{1/p}B/(Y'/Y_c)^{(1/p-1)} = (Y/Y_c)B(Y/Y')^{(1/p-1)}$; because Y and Y' are similar, and p is not usually very different from 1.0 (for Illuminant A, $p = 0.914$), this term is approximately equal to $(Y/Y_c)B$, which is what is required to give the correct values of X/Y_c, Y/Y_c, and Z/Y_c.

APPENDIX C: WORKED EXAMPLE FOR THE CIE 1997 SIMPLE COLOUR APPEARANCE MODEL, CIECAM97s

The CIECAM97s model gives the following results for a sample in Standard Illuminant A (S_A) at four different levels of adapting luminance, L_A.

Starting data:

Sample in test conditions	$x = 0.3618$		$y = 0.4483$	$Y = 23.93$
Adopted white in test conditions:	$x_w = 0.4476$		$y_w = 0.4074$	$Y_w = 90.0$
Background in test conditions:	$x_b = 0.4476$		$y_b = 0.4074$	$Y_b = 18.0$
Reference white in reference conditions:	$x_{wr} = \frac{1}{3}$		$y_{wr} = \frac{1}{3}$	$Y_{wr} = 100$
Luminance of test adapting field (cd/m^2), L_A	2000	200	20	2
Surround: average (small sample):	$F = 1.0$	$c = 0.69$	$F_{LL} = 1.0$	$N_c = 1.0$
D factor dependent on L_A				

Predictions for the adopted white:

L_A	2000	200	20	2
Hue angle, h	41.8	57.4	58.8	59.5
Hue Quad., H	28.3	50.0	52.0	53.0
Hue Comp., H_C	28Y72R	50Y50R	52Y48R	53Y47R
Lightness, J	100.0	100.0	100.0	100.0
Brightness, Q	70.1	52.7	37.9	26.8
Saturation, s	0.0	0.5	12.6	25.9
Chroma, C	0.1	1.3	12.1	19.8
Colourfulness, M	0.1	1.3	10.8	15.7

Predictions for the sample:

L_A	2000	200	20	2
Hue angle, h	190.2	190.0	183.5	175.7
Hue Quad., H	239.7	239.4	229.9	218.2
Hue Comp., H_C	40B60G	39B61G	30B70G	18B82G
Lightness, J	53.0	48.2	45.2	44.2
Brightness, Q	45.8	32.3	22.3	15.5
Saturation, s	120.0	125.9	114.0	96.5
Chroma, C	52.4	53.5	49.5	44.0
Colourfulness, M	58.8	53.5	44.1	34.9

The saturation, chroma, and colourfulness of the white increase as the adapting luminance falls, because the degree of chromatic adaptation decreases; compared to the chroma, the colourfulness increases less, because it is reduced by the falling adapting luminance.

APPENDIX D: STEPS FOR USING THE 1997 COMPREHENSIVE COLOUR APPEARANCE MODEL, CAM97c

The steps required for the comprehensive version of the model, CAM97c, are the same as those for the simple version, CIECAM97s, given in Appendix A, except as indicated below.

Step 7. Calculate

$$R'_a = B_R\{40(F_L R'/100)^{0.73}/[(F_L R'/100)^{0.73} + 2] + R'_D\} + 1,$$

$$G'_a = B_G\{40(F_L G'/100)^{0.73}/[(F_L G'/100)^{0.73} + 2] + G'_D\} + 1,$$

$$B'_a = B_B\{40(F_L B'/100)^{0.73}/[(F_L B'/100)^{0.73} + 2] + B'_D\} + 1,$$

where B_R, B_G, B_B are cone bleach factors:

$$B_R = 10^7/[10^7 + 5L_A(R'_w/100)]$$

$$B_G = 10^7/[10^7 + 5L_A(G'_w/100)]$$

$$B_B = 10^7/[10^7 + 5L_A(B'_w/100)].$$

If no cone bleaching occurs, these factors are all equal to 1.

R'_D, G'_D, B'_D are Helson–Judd effect factors:

$$R'_D = k_D(R'/R'_w - Y_{bc}/Y_{wc})(R'/R'_w - 1/3),$$

$$G'_D = k_D(G'/G'_w - Y_{bc}/Y_{wc})(G'/G'_w - 1/3),$$

$$B'_D = k_D(B'/B'_w - Y_{bc}/Y_{wc})(B'/B'_w - 1/3),$$

where k_D is a constant, the value of which is adjusted to provide the appropriate amount of Helson–Judd effect. If there is no Helson–Judd effect, or if the colour of the illuminant is discounted, $R'_D = G'_D = B'_D = 0$.

Step 10. Calculate the rod contribution, A_S:

$$A_S = B_S(3.05)(F_{LS}S/S_w)^{0.73}/[(F_{LS}S/S_w)^{0.73} + 2] + 0.3,$$

where $B_S = 0.5/\{1 + 0.3[(5L_{AS}/2.26)(S/S_w)]^{0.3}\} + 0.5/\{1 + 5[5L_{AS}/2.26]\}$

and $F_{LS} = 3800\,j^2 5L_{AS}/2.26 + 0.2(1 - j^2)^4(5L_{AS}/2.26)^{1/6}$

and $j = 0.00001/(5L_{AS}/2.26 + 0.00001)$.

If the scotopic luminance of the sample relative to that of the adopted white, S/S_w, is not known, the equivalent photopic value, Y/Y_w, can be used instead. If the scotopic luminance, L_{AS}, of the adapting field is not known, it can be approximated by using

$$L_{AS}/2.26 = L_A(T/4000 - 0.4)^{1/3},$$

where T is the correlated colour temperature of the illuminant.

Calculate

$$A = [2R'_a + G'_a + (1/20)B'_a + A_S - 2.31]N_{bb}.$$

Calculate A_w similarly.

Step 13. F_t is a low-luminance tritanopia factor:

$$F_t = L_A/(L_A + 0.1)$$

$$b_t = bF_t$$

$$s = [50(a^2 + b_t^2)^{1/2}100e(10/13)N_c N_{cb}]/[R'_a + G'_a + (21/20)B'_a].$$

Step 14. Helmholtz–Kohlrausch effect added:

$$J_{HK} = J + (|100 - J|)(C/300)(\sin|((h - 90)/2)|),$$

$$Q_{HK} = (1.24/c)(J_{HK}/100)^{0.67}(A_w + 3)^{0.9}.$$

APPENDIX E: METHODS OF REVERSING THE COMPREHENSIVE VERSION OF THE MODEL, CAM97c

Reversing the comprehensive version of the model, CAM97c, is complicated by the presence of the rod contribution in the achromatic signal.

The value of A_S depends on S/S_w; this is not usually known, and Y/Y_w is usually used in the model instead as an approximation. But, when using the model in reverse, if Y/Y_w is not known, some extra steps are necessary. There are two alternative methods of dealing with this situation. The first method is to use J to calculate $(S/S_w)_J$, an approximate value for S/S_w. The following formula for $(S/S_w)_J$ can be used for photopic conditions for colours of luminance factor not less than 3%:

$$(S/S_w)_J = \{J - [\log(5L_A)]^2/4\}^{1.8}/\{100 - [\log(5L_A)]^2/4\}^{1.8}.$$

$(S/S_w)_J$ can then be used in the reverse model to calculate X, Y, and Z. Using $(S/S_w)_J$ instead of Y/Y_w in the forward model results in H and s being unchanged, C and M being changed very slightly, and Q and J being changed slightly. For many applications these changes are negligible. If more precise results are required, the resulting value of Y/Y_w can be used as S/S_w to derive a new set of values of X, Y, and Z; this procedure can then be iterated until stable values of Y/Y_w are obtained. The second method is to require that S/S_w be equal to Y/Y_w from the outset, and to use methods of successive numerical approximation to complete the calculation.

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