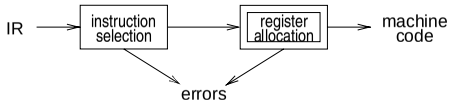
Register allocation



Register allocation:

- have value in a register when used
- limited resources
- changes instruction choices
- can move loads and stores
- optimal allocation is difficult
  $\Rightarrow$ NP-complete for $k \geq 1$ registers

Register allocation by simplification

Assume K registers

1. *Build* interference graph G: for each program point
   (a) compute set of temporaries simultaneously live
   (b) add edge to graph for each pair in set
2. *Simplify*: Color graph using a simple heuristic
   (a) suppose G has node m with degree $< K$
   (b) if $G' = G - \{m\}$ can be colored then so can G, since nodes adjacent to m have at most $K - 1$ colors
   (c) each such simplification will reduce degree of remaining nodes leading to more opportunity for simplification
   (d) leads to recursive coloring algorithm

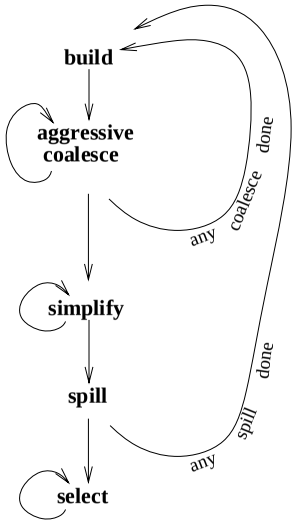
Register allocation by simplification (cont.)

3. *Spill*: suppose $\nexists m$ of degree $< K$
   (a) target some node (temporary) for spilling (optimistically, spilling node will allow coloring of remaining nodes)
   (b) remove and continue simplifying
4. *Select*: assign colors to nodes
   (a) start with empty graph
   (b) must be a color for non-spill nodes (basis for removal)
   (c) if adding spill node and no color available (neighbors already K-colored) then mark as an *actual spill*
   (d) repeat select
5. *Start over*: if select has no actual spills then finished, otherwise
   (a) rewrite code: fetch spills at use, store at definition
   (b) recalculate liveness and repeat

Coalescing

- Can delete a *move* instruction when source s and destination d do not interfere:
  - *coalesce* them into a new node whose edges are the union of those of s and d
- In principle, any pair of non-interfering nodes can be coalesced
  - unfortunately, the union is more constrained and new graph may no longer be K-colorable
  - overly aggressive

Simplification with aggressive coalescing



Conservative coalescing

Apply tests for coalescing that preserve colorability.

Suppose a and b are candidates for coalescing into node ab.

Briggs: coalesce only if ab has $< K$ neighbors of *significant* degree $\geq K$

- *simplify* first removes all insignificant-degree neighbors
- ab will then be adjacent to $< K$ neighbors
- *simplify* can then remove ab

George: coalesce only if all significant-degree neighbors of a already interfere with b

- *simplify* removes all insignificant-degree neighbors of a
- remaining significant-degree neighbors of a already interfere with b; coalescing does not increase degree of any node

Iterated register coalescing

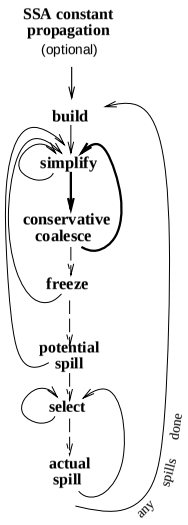
Interleave simplification with coalescing to eliminate most moves while guaranteeing not to introduce spills:

1. *Build* interference graph G and distinguish move-related from non-move-related nodes
2. *Simplify*: remove non-move-related nodes of low degree one at a time
3. *Coalesce*: conservatively coalesce move-related nodes
   - remove associated move instruction
   - if resulting node is non-move-related it can now be simplified
   - repeat simplify and coalesce until only significant-degree or uncoalesced moves

Iterated register coalescing (cont.)

4. *Freeze*: if unable to simplify or coalesce
   (a) look for move-related node of low-degree
   (b) freeze its associated moves (give up on coalescing)
   (c) now treat as non-move-related; resume iteration of simplify and coalesce
5. *Spill*: if no low-degree nodes
   (a) select candidate for spilling
   (b) remove to stack and continue simplifying
6. *Select*: pop stack assigning colors (with actual spills)
7. *Start over*: if select has no actual spills then finished, otherwise
   (a) rewrite code: fetch spills before use, store after def
   (b) recalculate liveness and repeat

Iterated register coalescing



Spilling

- Spills require repeating *build* and *simplify* on the whole program
- To avoid increasing number of spills in future rounds of *build* can simply discard coalescences
- Alternatively, preserve coalescences from before first *potential* spill, discard those after that point
- Move-related spilled temporaries can be aggressively coalesced, since (unlike registers) there is no limit on the number of stack-frame locations

Precolored nodes

Precolored nodes correspond to machine registers (e.g., stack pointer, arguments, return address, return value)

- *select* and *coalesce* can give an ordinary temporary the same color as a precolored register, if they don't interfere
- e.g., argument registers can be reused inside procedures for a temporary
- *simplify*, *freeze* and *spill* cannot be performed on them
- also, precolored nodes interfere with other precolored nodes

So, treat precolored nodes as having infinite degree

This also avoids needing to store large adjacency lists for precolored nodes; coalescing can use the George criterion

Temporary copies of machine registers

Since precolored nodes don't spill, their live ranges must be kept short:

1. use *move* instructions
2. move callee-save registers to fresh temporaries on procedure entry, and back on exit, spilling between as necessary
3. *register pressure* will spill the fresh temporaries as necessary, otherwise they can be coalesced with their precolored counterpart and the moves deleted

Caller-save and callee-save registers

Variables whose live ranges span calls should go to callee-save registers, otherwise to caller-save

This is easy for graph coloring allocation with spilling

- calls interfere with caller-save registers
- a cross-call variable interferes with all precolored caller-save registers, as well as with the fresh temporaries created for callee-save copies, forcing a spill
- choose nodes with high degree but few uses, to spill the fresh callee-save temporary instead of the cross-call variable
- this makes the original callee-save register available for coloring the cross-call variable

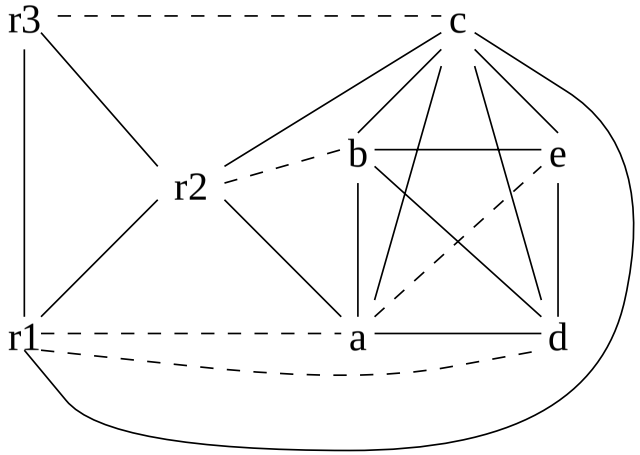
Example

```
enter:
  c := r3
  a := r1
  b := r2
  d := 0
  e := a
loop:
  d := d + b
  e := e - 1
  if e > 0 goto loop
  r1 := d
  r3 := c
  return [ r1, r3 live out ]
```

- Temporaries are `a`, `b`, `c`, `d`, `e`
- Assume target machine with $K = 3$ registers: `r1`, `r2` (caller-save/argument/result), `r3` (callee-save)
- The code generator has already made arrangements to save `r3` explicitly by copying into temporary `a` and back

Example (cont.)

Interference graph:



Example (cont.)

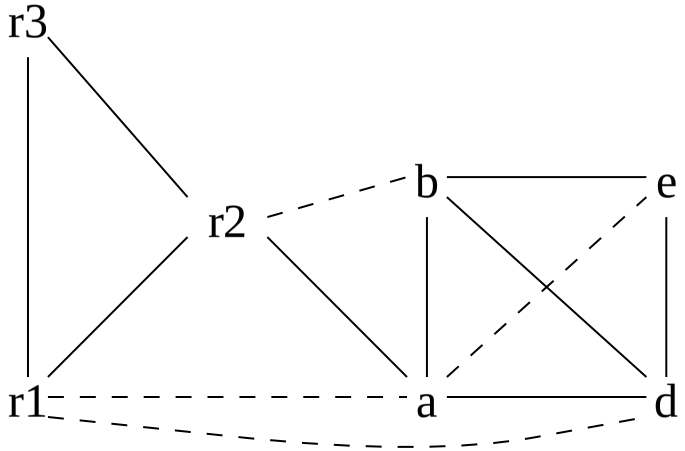
- No opportunity for *simplify* or *freeze* (all non-precolored nodes have significant degree $\geq K$)
- Any *coalesce* will produce a new node adjacent to $\geq K$ significant-degree nodes
- Must *spill* based on priorities:

Node	uses + defs outside loop		uses + defs inside loop		degree		priority
`a`	(2	$+10\times$	0)	/	4	=	0.50
`b`	(1	$+10\times$	1)	/	4	=	2.75
`c`	(2	$+10\times$	0)	/	6	=	0.33
`d`	(2	$+10\times$	2)	/	4	=	5.50
`e`	(1	$+10\times$	3)	/	3	=	10.30

- Node `c` has lowest priority so spill it

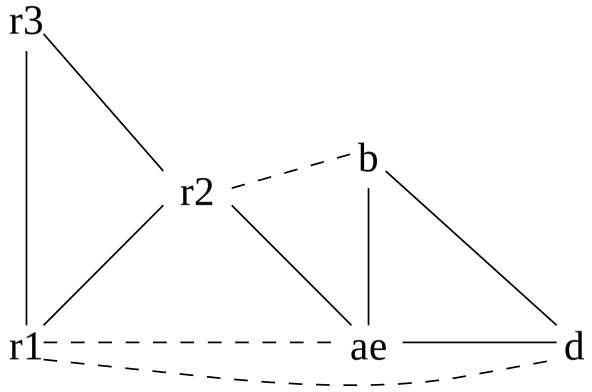
Example (cont.)

Interference graph with `c` removed:



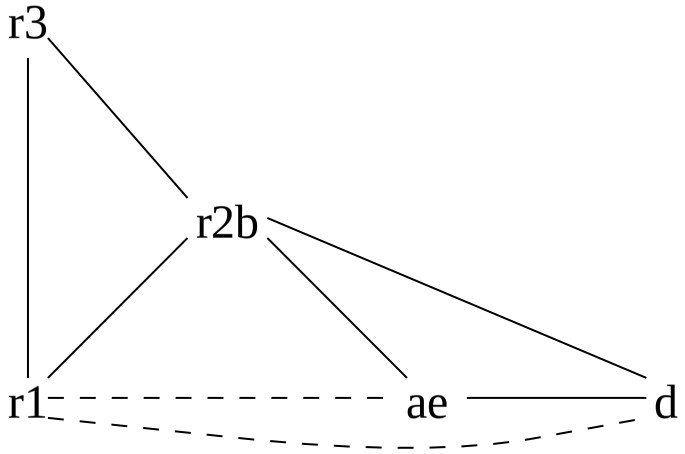
Example (cont.)

Only possibility is to *coalesce* `a` and `e`: `ae` will have $< K$ significant-degree neighbors (after coalescing `d` will be low-degree, though high-degree before)



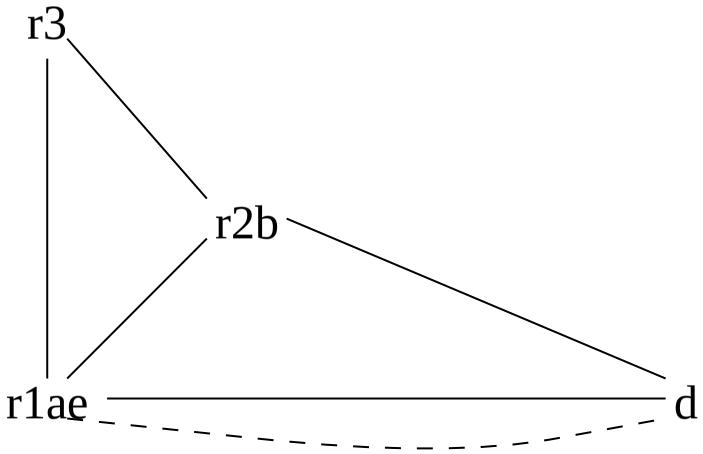
Example (cont.)

Can now *coalesce* `b` with `r2` (or coalesce `ae` and `r1`):



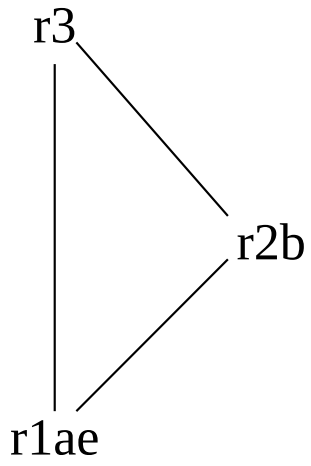
Example (cont.)

Coalescing `ae` and `r1` (could also coalesce `d` with `r1`):



Example (cont.)

Cannot *coalesce* `r1ae` with `d` because the move is *constrained*: the nodes interfere. Must *simplify* `d`:



Example (cont.)

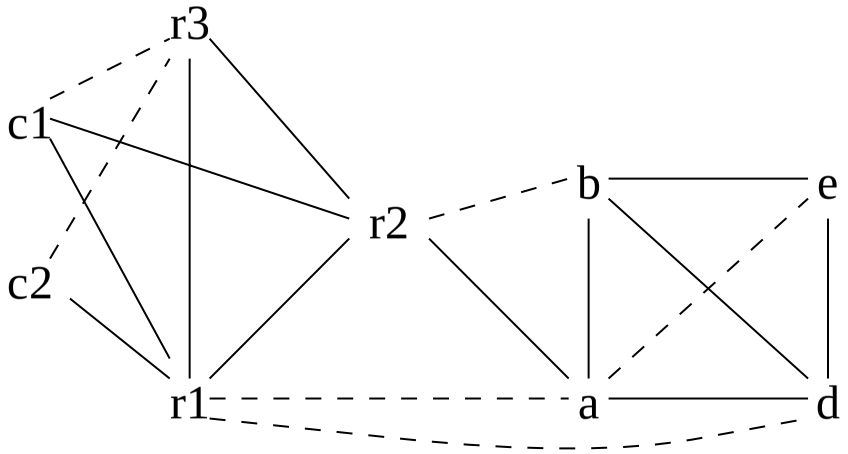
- Graph now has only precolored nodes, so pop nodes from stack coloring along the way
  - `d` $\equiv$ `r3`
  - `a`, `b`, `e` have colors by coalescing
  - `c` must spill since no color can be found for it
- Introduce new temporaries `c1` and `c2` for each use/def, add loads before each use and stores after each def

Example (cont.)

```
enter:
  c1 := r3
  M[c_loc] := c1
  a := r1
  b := r2
  d := 0
  e := a
loop:
  d := d + b
  e := e - 1
  if e > 0 goto loop
  r1 := d
  c2 := M[c_loc]
  r3 := c2
  return [ r1, r3 live out ]
```

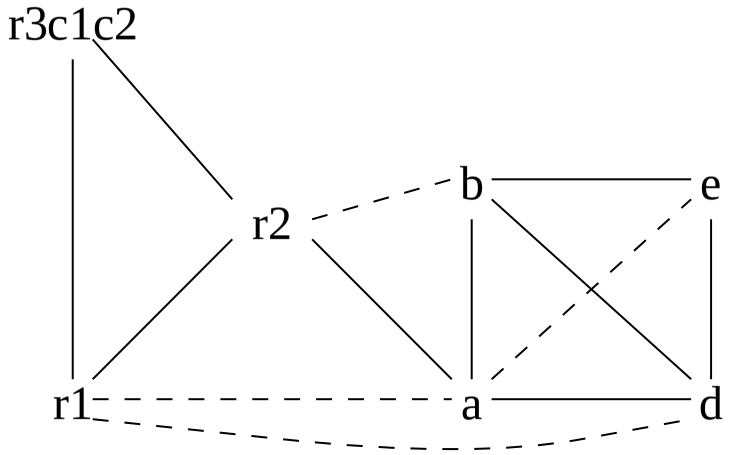

Example (cont.)

New interference graph:



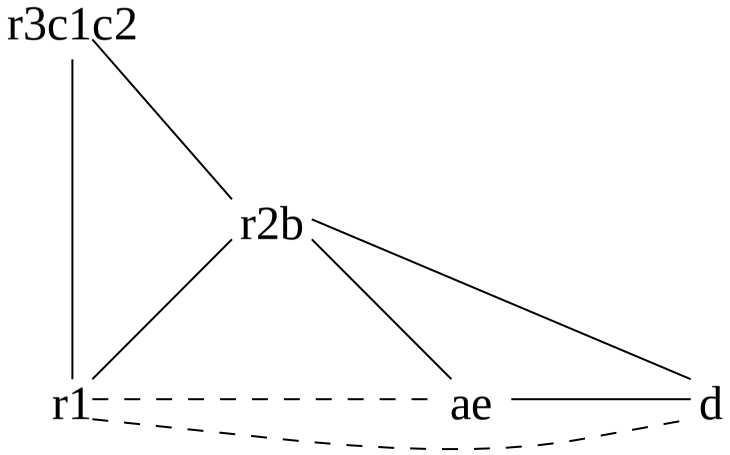
Example (cont.)

Coalesce `c1` with `r3`, then `c2` with `r3`:



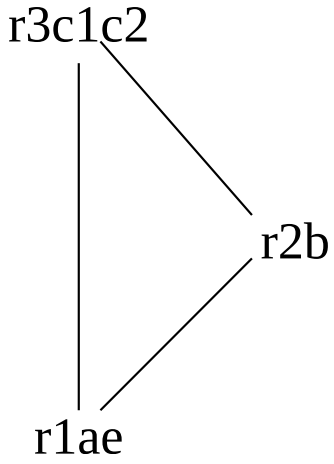
Example (cont.)

As before, *coalesce* `a` with `e`, then `b` with `r2`:



Example (cont.)

As before, *coalesce* `ae` with `r1` and *simplify* `d`:



Example (cont.)

Pop `d` from stack: select `r3`. All other nodes were coalesced or precolored. So, the coloring is:

- `a` ≡ `r1`
- `b` ≡ `r2`
- `c` ≡ `r3`
- `d` ≡ `r3`
- `e` ≡ `r1`

Example (cont.)

Rewrite the program with this assignment:

```
enter:
  r3 := r3
  M[c_loc] := r3
  r1 := r1
  r2 := r2
  r3 := 0
  r1 := r1
loop:
  r2 := r3 + r2
  r1 := r1 - 1
  if r1 > 0 goto loop
  r1 := r3
  r3 := M[c_loc]
  r3 := r3
  return [ r1, r3 live out ]
```


Example (cont.)

- Delete moves with source and destination the same (coalesced):

```
enter:
  M[c_loc] := r3
  r3 := 0
loop:
  r2 := r3 + r2
  r1 := r1 - 1
  if r1 > 0 goto loop
  r1 := r3
  r3 := M[c_loc]
  return [ r1, r3 live out ]
```

- One uncoalesced move remains