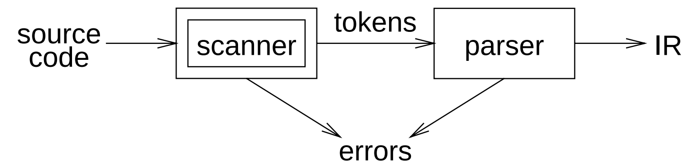
CS502: Scanning (Chapter 2)

Front end: Scanner



- maps characters into *tokens* – the basic unit of syntax
  `x = x + y;`
  becomes
  <id, `x`> `=` <id, `x`> `+` <id, `y`> `;`
- pattern for a *token* is a *lexeme*
- typical tokens: *number*, *id*, `+`, `-`, `*`, `/`, `do`, `end`
- eliminates white space (*tabs, blanks, comments*)
- key issue is speed
  ⇒ use specialized recognizer (as opposed to `lex`)

Specifying patterns

A scanner must recognize the units of syntax
Some parts are easy:

white space

```
<ws>  ::=  <ws> ' '
        |  <ws> '\t'
        |  ' '
        |  '\t'
```

keywords and operators
specified as literal patterns: `do`, `end`

comments
opening and closing delimiters: `/*` ··· `*/`

Specifying patterns

A scanner must recognize the units of syntax
Other parts are much harder:

identifiers
alphabetic followed by k alphanumerics (_, $, &, ...)

numbers
integers: 0 or digit from 1-9 followed by digits from 0-9
decimals: integer `'.'` digits from 0-9
reals: (integer or decimal) `'E'` (+ or -) digits from 0-9
complex: `'('` real `','` real `')'`

We need a powerful notation to specify these patterns

Operations on languages

Operation	Definition
union of L and M written $L \cup M$	$L \cup M = \{s \mid s \in L \text{ or } s \in M\}$
concatenation of L and M written LM	$LM = \{st \mid s \in L \text{ and } t \in M\}$
Kleene closure of L written L^*	$L^* = \bigcup_{i=0}^{\infty} L^i$
positive closure of L written L^+	$L^+ = \bigcup_{i=1}^{\infty} L^i$

Regular expressions

Patterns are often specified as *regular languages*

Notations used to describe a regular language (or a regular set) include both *regular expressions* and *regular grammars*

Regular expressions (*over an alphabet* Σ):

1. ε is a RE denoting the set $\{\varepsilon\}$
2. if $a \in \Sigma$, then a is a RE denoting $\{a\}$
3. if r and s are REs, denoting $L(r)$ and $L(s)$, then:
   - (r) is a RE denoting $L(r)$
   - $(r) \mid (s)$ is a RE denoting $L(r) \bigcup L(s)$
   - $(r)(s)$ is a RE denoting $L(r)L(s)$
   - $(r)^*$ is a RE denoting $L(r)^*$

Precedence for operators means extra parentheses go away: assume *closure* over *concatenation* over *alternation*.

Examples

identifier

letter $\rightarrow (a \mid b \mid c \mid \ldots \mid z \mid A \mid B \mid C \mid \ldots \mid Z)$

digit $\rightarrow (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)$

id $\rightarrow$ *letter* (*letter* | *digit*)*

numbers

integer $\rightarrow (+ \mid - \mid \varepsilon)\ (0 \mid (1 \mid 2 \mid 3 \mid \ldots \mid 9)$ *digit*$^*)$

decimal $\rightarrow$ *integer* `.` (*digit*)*

real $\rightarrow$ (*integer* | *decimal*) `E` $(+ \mid -)$ *digit**

complex $\rightarrow$ `'('` *real* `,` *real* `')'`

Numbers can get much more complicated

Most tokens can be described with REs

We can use REs to build scanners automatically

Algebraic properties of REs

Axiom	Description
$r \mid s = s \mid r$	$\mid$ is commutative
$r \mid (s \mid t) = (r \mid s) \mid t$	$\mid$ is associative
$(rs)t = r(st)$	concatenation is associative
$r(s \mid t) = rs \mid rt$ $(s \mid t)r = sr \mid tr$	concatenation distributes over $\mid$
$\varepsilon r = r$ $r\varepsilon = r$	ε is the identity for concatenation
$r^* = (r \mid \varepsilon)^*$	relation between * and ε
$r^{**} = r^*$	* is idempotent

Examples

Let $\Sigma = \{a, b\}$

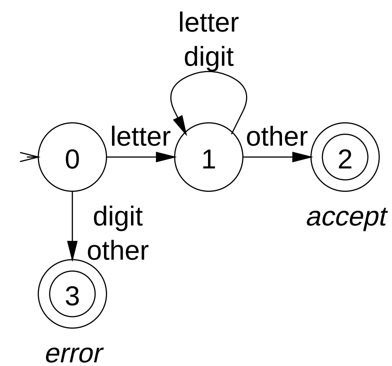
1. $a|b$ denotes $\{a, b\}$
2. $(a|b)(a|b)$ denotes $\{aa, ab, ba, bb\}$
   i.e., $(a|b)(a|b) = aa|ab|ba|bb$
3. a^* denotes $\{\varepsilon, a, aa, aaa, \ldots\}$
4. $(a|b)^*$ denotes the set of all strings of a's and b's (including ε)
   i.e., $(a|b)^* = (a^*b^*)^*$
5. $a|a^*b$ denotes $\{a, b, ab, aab, aaab, aaaab, \ldots\}$

Recognizers

From a regular expression we can construct a

deterministic finite automaton (DFA)

Recognizer for *identifier*:



Code for the recognizer

```
char ← next_char();
state ← 0;          /* code for state 0 */
done ← false;
token_value ← ""   /* empty string */
while( not done ) {
   class ← char_class[char];
   state ← next_state[class,state];
   switch(state) {
      case 1:     /* building an id */
         token_value ← token_value + char;
         char ← next_char();
         break;
```



```
      case 2:     /* accept state */
         token_type = identifier;
         done = true;
         break;
      case 3:     /* error */
         token_type = error;
         done = true;
         break;
   }
}
return token_type;
```

Tables for the recognizer

Two tables control the recognizer

`char_class`:

	$a-z$	$A-Z$	$0-9$	other
value	letter	letter	digit	other

`next_state`:

class	0	1	2	3
letter	1	1	—	—
digit	3	1	—	—
other	3	2	—	—

To change languages, we can just change tables

Automatic construction

Scanner generators automatically construct code from RE-like descriptions

- construct a DFA
- use state minimization techniques
- emit code for the scanner (table driven or direct code)

A key issue in automation is an interface to the parser

`lex` is a scanner generator supplied with UNIX

- emits C code for scanner
- provides macro definitions for each token (used in the parser)

Grammars for regular languages

Can we place a restriction on the *form* of a grammar to ensure that it describes a regular language?

Provable fact:

For any RE r, $\exists$ a grammar g such that $L(r) = L(g)$

Grammars that generate regular sets are called *regular grammars*:

They have productions in one of 2 forms:

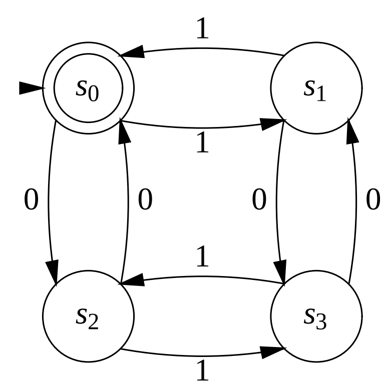
1. $A \to aA$
2. $A \to a$

where A is any non-terminal and a is any terminal symbol

These are also called *type 3* grammars (Chomsky)

More regular languages

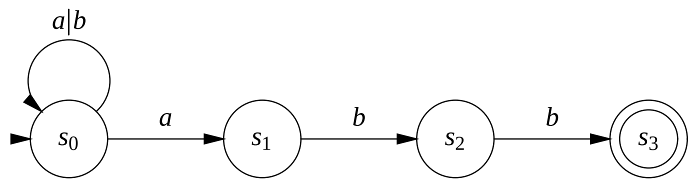
Example: the set of strings containing an even number of zeros and an even number of ones



The RE is $(00 \mid 11)^*((01 \mid 10)(00 \mid 11)^*(01 \mid 10)(00 \mid 11)^*)^*$

More regular expressions

What about the RE $(a \mid b)^*abb$?



State s_0 has multiple transitions on a!
$\Rightarrow$ *nondeterministic finite automaton*

	a	b
s_0	$\{s_0, s_1\}$	$\{s_0\}$
s_1	–	$\{s_2\}$
s_2	–	$\{s_3\}$

Finite automata

A *non-deterministic finite automaton* (NFA) consists of:

1. a set of *states* $S = \{s_0, \ldots, s_n\}$
2. a set of input symbols Σ (the alphabet)
3. a transition function mapping state-symbol pairs to sets of states
4. a distinguished *start state* s_0
5. a set of distinguished *accepting* or *final* states F

A *Deterministic Finite Automaton* (DFA) is a special case:

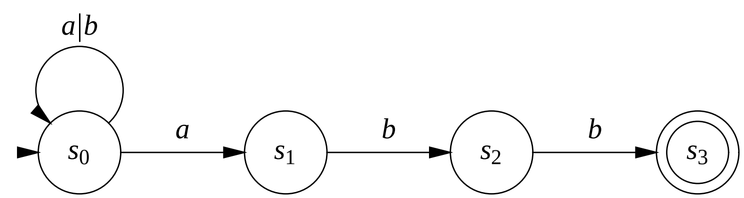
1. no state has a ε-transition, and
2. for each state s and input symbol a, $\exists$ at most one edge labelled a leaving s

A DFA *accepts* x iff. $\exists$ a *unique* path through the transition graph from s_0 to a final state such that the edges spell x.

DFAs and NFAs are equivalent

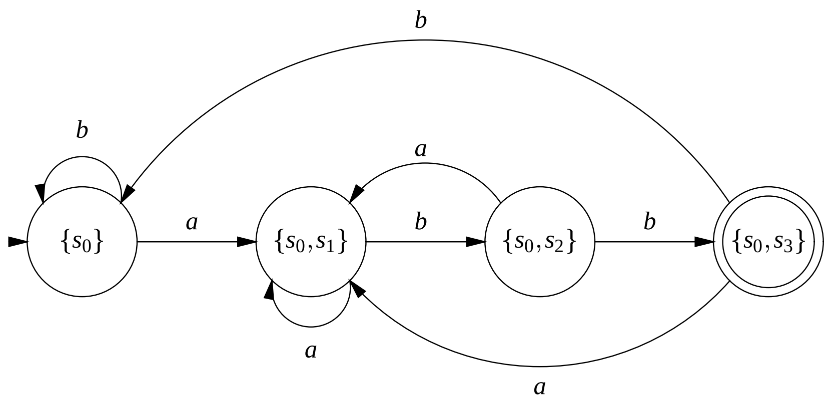
1. DFAs are clearly a subset of NFAs
2. Any NFA can be converted into a DFA, by simulating sets of simultaneous states:
   - each DFA state corresponds to a set of NFA states
   - possible exponential blowup

NFA to DFA via subset construction: Example 1



	a	b
$\{s_0\}$	$\{s_0, s_1\}$	$\{s_0\}$
$\{s_0, s_1\}$	$\{s_0, s_1\}$	$\{s_0, s_2\}$
$\{s_0, s_2\}$	$\{s_0, s_1\}$	$\{s_0, s_3\}$
$\{s_0, s_3\}$	$\{s_0, s_1\}$	$\{s_0\}$

Example 1 (continued)



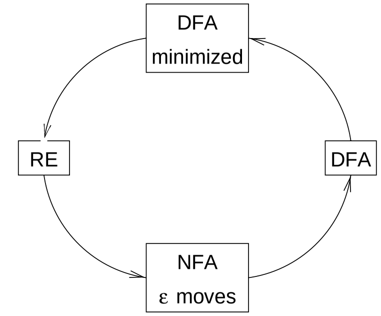
Constructing a DFA from a regular expression

RE →NFA w/ε moves
- build NFA for each term;
- connect them with ε moves

NFA w/ε moves to DFA
- construct the simulation;
- the "subset" construction

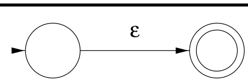
DFA → minimized DFA
- merge compatible states

DFA → RE
- construct $R_{ij}^{k} = R_{ik}^{k-1}(R_{kk}^{k-1})^{*}R_{kj}^{k-1} \bigcup R_{ij}^{k-1}$

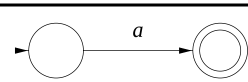


RE to NFA

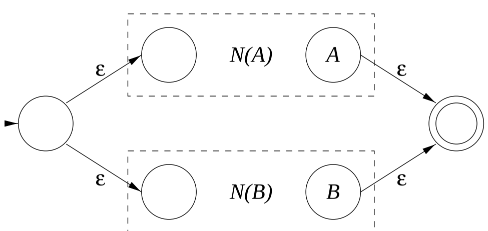
$N(\varepsilon)$



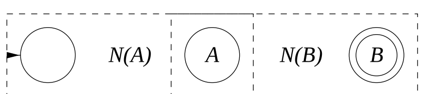
$N(a)$



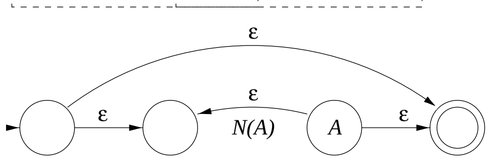
$N(A|B)$



$N(AB)$

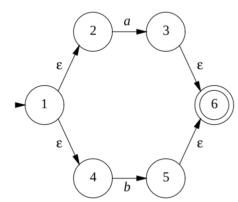


$N(A^*)$

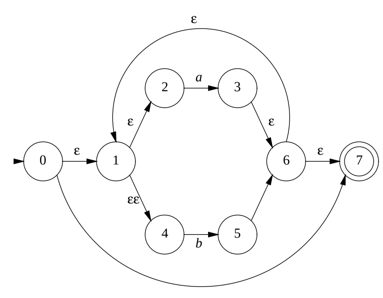


RE to NFA: Example $(a \mid b)^*abb$

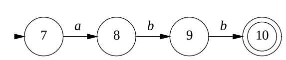
$a|b$



$(a|b)^*$



abb



NFA to DFA: the subset construction

Input: NFA N
Output: A DFA D with states *Dstates* and transitions *Dtrans* such that $L(D) = L(N)$
Method: Let s be a state in N and T be a set of states, and using the following operations:

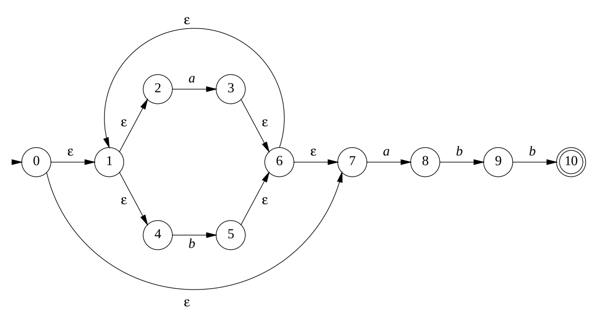
Operation	Definition
ε-*closure*(s)	set of NFA states reachable from NFA state s on ε-transitions alone
ε-*closure*(T)	set of NFA states reachable from some NFA state s in T on ε-transitions alone
move(T, a)	set of NFA states to which there is a transition on input symbol a from some NFA state s in T

NFA to DFA: the subset construction (cont.)

add state $T = \varepsilon$-*closure*(s_0) unmarked to *Dstates*
while $\exists$ unmarked state T in *Dstates*
  mark T
  for each input symbol a
    $U = \varepsilon$-*closure*(*move*(T, a))
    if $U \notin$ *Dstates* **then** add U to *Dstates* unmarked
    Dtrans$[T, a] = U$
  endfor
endwhile

ε-*closure*(s_0) is the start state of D
A state of D is final if it contains at least one final state in N

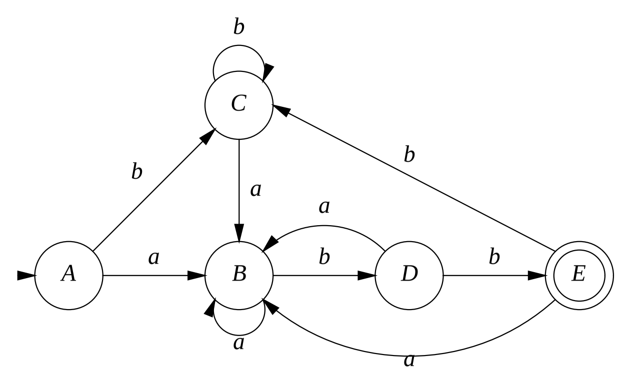
NFA to DFA: Example 2



$A = \{0,1,2,4,7\}$
$B = \{1,2,3,4,6,7,8\}$
$C = \{1,2,4,5,6,7\}$
$D = \{1,2,4,5,6,7,9\}$
$E = \{1,2,4,5,6,7,10\}$

	a	b
A	B	C
B	B	D
C	B	C
D	B	E
E	B	C

Example 2 (continued)



DFA to RE: the basic idea

For a DFA M, with start state s_1,

- let R^k_{ij} be the set of all strings that take M from state s_i to state s_j without going through a state s_l where $l > k$
- "through s_k" means both entering and leaving s_k

Then, $\mathcal{L}(M) = \bigcup_{s_j \in F(M)} R^n_{1j}$

More formally:

1. $R^k_{ij} = R^{k-1}_{ik}(R^{k-1}_{kk})^* R^{k-1}_{kj} \cup R^{k-1}_{ij}$
2. if $i \neq j$, $R^0_{ij} = \{a | \delta(s_i, a) = s_j\}$
3. if $i = j$, $R^0_{ij} = \{a | \delta(s_i, a) = s_j\} \cup \{\varepsilon\}$

See pp 33-34 in Hopcroft & Ullman: *Introduction to Automata Theory, Languages, and Computation*

Limits of regular languages

Not all languages are regular

One cannot construct DFAs to recognize these languages:

- $L = \{p^k q^k\}$
- $L = \{wcw^r \mid w \in \Sigma^*\}$

Note: neither of these is a regular expression!
(DFAs cannot count!)

But, this is a little subtle. One can construct DFAs for:

- alternating 0's and 1's
  $(\varepsilon \mid 1)(01)^*(\varepsilon \mid 0)$
- sets of pairs of 0's and 1's
  $(01 \mid 10)^+$

So what is hard?

Language features that can cause problems:

reserved words
PL/I had no reserved words
`if then then then = else; else else = then;`

significant blanks
FORTRAN and Algol68 ignore blanks
`do 10 i = 1,25`
`do 10 i = 1.25`

string constants
special characters in strings
`newline`, `tab`, `quote`, `comment delimiter`

finite closures
some languages limit identifier lengths
adds states to count length
FORTRAN 66 $\rightarrow$ 6 characters

These can be swept under the rug in the language design

How bad can it get?

```
1           INTEGERFUNCTIONA
2           PARAMETER(A=6,B=2)
3           IMPLICIT CHARACTER*(A-B)(A-B)
4           INTEGER FORMAT(10),IF(10),DO9E1
5     100   FORMAT(4H)=(3)
6     200   FORMAT(4 )=(3)
7           DO9E1=1
8           DO9E1=1,2
9              IF(X)=1
10             IF(X)H=1
11             IF(X)300,200
12    300      CONTINUE
13          END
      C     this is a comment
          $ FILE(1)
14          END
```