

#### **School of Mathematics**

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Student Signature _	Ma	Date _	16 March 2020	
Student Name	Nico Septianus	Student Number	201380903	

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# **PART 1: Regression**

1.) Using the GLM function in R on medal\_data, we obtained both 2008 and 2012 prediction such as,

#### **Medal Count in 2008**

```
call:
glm(formula = Medal2008 ~ GDP + Population, data = medal_data)
Deviance Residuals:
          1Q Median
   Min
                                30
                                         Max
                            0.842
-27.154
          -4.856
                   -1.702
                                      51.037
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 5.613e+00 1.506e+00 3.728 0.000395 ***
GDP 7.613e-03 7.353e-04 10.354 1.29e-15 ***
Population 8.435e-09 7.220e-09 1.168 0.246750
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for gaussian family taken to be 133.1455)
    Null deviance: 29595.1 on 70 degrees of freedom
Residual deviance: 9053.9 on 68 degrees of freedom
AIC: 553.72
Number of Fisher Scoring iterations: 2
```

## **Medal Count in 2012**

```
glm(formula = Medal2012 ~ GDP + Population, data = medal_data)
Deviance Residuals:
           1Q Median
                             3Q
   Min
                                      Max
-20.568
        -5.961
                 -2.462 3.932
                                   60.121
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.076e+00 1.500e+00 4.051 0.000133 ***
           7.564e-03 7.325e-04 10.326 1.45e-15 ***
Population 5.247e-09 7.193e-09 0.729 0.468225
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for gaussian family taken to be 132.1562)
   Null deviance: 28402.8 on 70 degrees of freedom
Residual deviance: 8986.6 on 68 degrees of freedom
AIC: 553.19
Number of Fisher Scoring iterations: 2
```

2.) Next to check the consistency for each variable (Population and GDP) corresponding to the prediction. We perform 95% confidence interval test for each variable in each year.

#### 2008 Data:

- GDP

```
"GDP 2008: 0.00614591191989105 0.00908035801420896"
```

The interval does not contain regression parameter 0 in it, which can be conclude that GDP is consistent towards the prediction in 2008.

- Population

```
"Population 2008: -5.97148075440265e-09 2.28411294839574e-08"
```

The interval contains regression parameter 0 in it, which means population inconsistent towards the prediction in 2008. This could be due to error or inefficiency in data points.

#### 2012 Data:

- GDP

```
"GDP 2012: 0.00610231906043588 0.00902584306021206"
```

Similar to 2008, GDP is consistent towards the prediction in 2012. This can be seen from the interval which does not contain regression parameter 0.

- Population

```
"Population 2012: -9.10593444565584e-09 1.95994344112297e-08"
```

Similar to 2008, the variable population is inconsistent towards the prediction in 2012. We can see from the interval contains regression parameter 0.

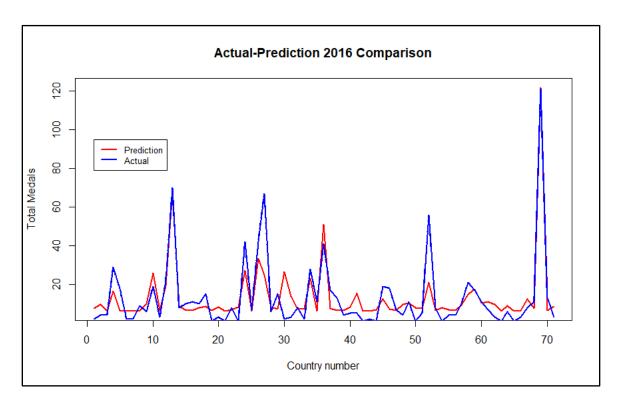
From comparison in 2008 and 2012, can be concluded that GDP plays big part on predicting medal obtained for every country. However, population in a country does not always refer to medal obtained for the countries.

3.) For prediction 2016 medal from 2012, we use the predict function on the linear regression of 2012 medal count. Below there are countries number, countries name, prediction results for 2016 and the actual data from medal\_data in 2016 respectively.

					_		54 455455	
1		Prediction		3		Japan		41
1	Algeria	7.697937	2	3		Kazakhstan	7.572239	17
2	Argentina		4	3		Kenya	6.532974	13
3	Armenia	6.170773	4	3		Lithuania	6.416057	4
4	Australia	16.572245	29	4	_	Malaysia	8.332637	5
5	Azerbaijan	6.603459	18	4		Mexico	15.404428	5
6	Bahamas	6.136872	2	4		Moldova	6.147716	1
7	Bahrain	6.248223	2	4	_	Mongolia		2
8	Belarus	6.542817	9	4	•	Morocco	7.004824	1
9	Belgium	10.002805	6	4		Netherlands	12.489418	19
10	Brazil	25.819025	19	4		New Zealand	7.087823	18
11	Bulgaria	6.519486	3	4		North Korea	6.368698	7
12	Canada	19.390152	22	4		Norway	9.776986	4
13	China	68.348721	70	4		Poland	10.169817	11
14	Colombia	8.828562	8	5	0	Portugal	7.928127	1
15	Croatia	6.581570	10	5		Romania	7.535952	5
16	Cuba	6.595043	11	5		Russian Federation	20.878996	56
17	Czech Republic	7.759147	10	5	3	Serbia	6.454139	8
18	Denmark	8.621790	15	5	4	Singapore	7.916400	1
19	Dominican Republic	6.545939	1	5	5	slovakia	6.830738	4
20	Egypt	8.242126	3	5	6	Slovenia	6.461612	4
21	Estonia	6.250779	1	5	7	South Africa	9.429469	10
22	Ethiopia	6.758360	8	5	8	South Korea	14.774385	21
23	Finland	8.117037	1	5	9	Spain	17.595080	17
24	France	27.394391	42	6	0	Sweden	10.196346	11
25	Georgia	6.208237	7	6	1	Switzerland	10.925493	7
26	Germany	33.513444	42	6	2	Taiwan	9.722857	3
27	Great Britain	24.795509	67	6	3	Tajikistan	6.165369	1
28	Greece	8.392310	6	6	4	Thailand	9.034171	6
29	Hungary	7.187558	15	6	5	Trinidad and Tobago	6.253046	1
30	India	26.568161	2	6	6	Tunisia	6.478984	3
31	Indonesia	13.728427	3	6	7	Turkey	12.315867	8
32	Iran	7.130986	8	6	8	Ukraine	7.565541	11
33	Ireland	7.743689	2	6	9	United States	121.892569	121
34	Italy	22.996238	28	7	0	Uzbekistan	6.572002	13
35	Jamaica	6.204280	11	7	1.	Venezuela	8.612422	3

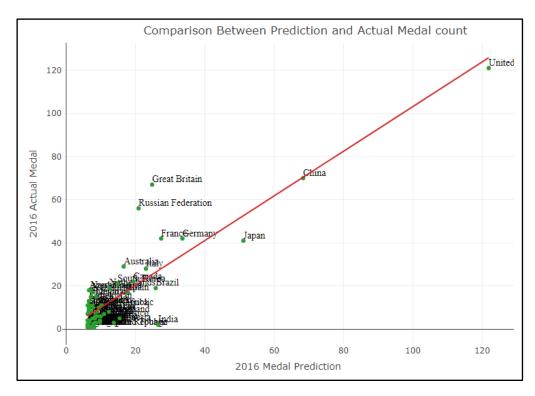
The country number will be used to track for finding the trend and outliers. Therefore, for question 4 need to refer to these tables.

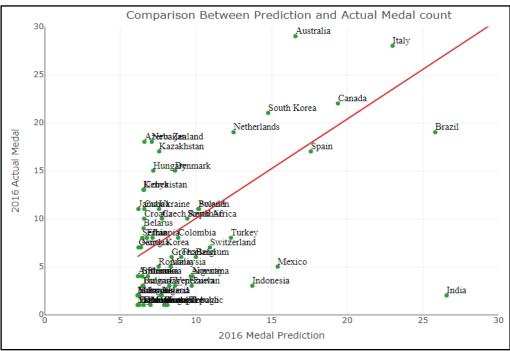
- 4.) To compare between both predicted and actual medal data in 2016. We plot 3 different graphs. There will be line graph comparison, scatter plot with transformation axes and boxplot to find the outliers.
  - Line graph



From the graphs we can see the comparison for predicted and actual medal count. The x-axis refers to country number and y-axis is total medal obtained. We can see more or less the predicted graph (red) follows the actual data (blue). For instance, the country number 69 (USA) reached the peak at 121 medals is predicted same with the actual data. Another good prediction come from country number 12 (China) which receive roughly 68 medals. Some other low receiving medals countries also show some good prediction such as country number 14 (Columbia), 57 (South Africa), 59 (Spain) and more.

## Scatter Plot





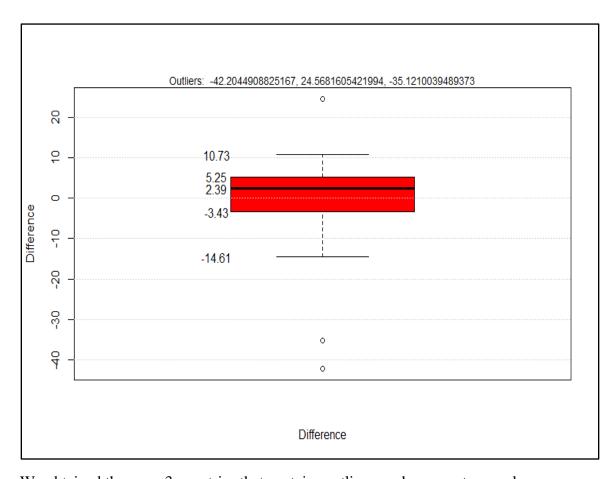
The red line is the linear regression in 2016 where x = y where x is predicted medal data and y is actual data. As we can see the linear regression go in same way with the previous line chart which China and USA have the majority medals and they fit nicely with the regression.

# Boxplot

For boxplot we use the difference between the predicted data and actual data or we can say the error in the regression line from each predicted value. This in order to find the outlier from trend (huge difference). First, we make data frame of the difference (predicted – actual) for each country,

_	Country	Difference <sup>‡</sup>	^	Country	Difference			
1	Algeria	5.6979373	25	Georgia	-0.7917633			
2	Argentina	5.6600810	26	Germany	-8.4865558			
3	Armenia	2.1707729	27	Great Britain	-42,2044909			
4	Australia	-12.4277552	28	Greece	2.3923104		Portugal	6.928126
5	Azerbaijan	-11.3965415	29	Hungary	-7.8124415	51		2.535951
6	Bahamas	4.1368719	30	India	24.5681605	52	Russian Federation	-35.1210
7	Bahrain	4.2482230	31	Indonesia	10.7284275	53	Serbia	-1.54586
8	Belarus	-2,4571829	32	Iran	-0.8690137	54	Singapore	6.916399
9	Belgium	4.0028050	33	Ireland	5.7436890	55	Slovakia	2.830738
	_		34		-5.0037617	56	Slovenia	2.461612
10	Brazil	6.8190248		Italy		57	South Africa	-0.57053
11	Bulgaria	3.5194861	35	Jamaica	-4.7957204	58	South Korea	-6.22561
12	Canada	-2.6098481	36	Japan	10.1254379	59	Spain	0.595080
13	China	-1.6512793	37	Kazakhstan	-9.4277608	60	Sweden	-0.80365
14	Colombia	0.8285623	38	Kenya	-6.4670260	61	Switzerland	3,925492
15	Croatia	-3.4184296	39	Lithuania	2.4160571	62	Taiwan	6,722856
16	Cuba	-4.4049566	40	Malaysia	3.3326367			
17	Czech Republic	-2,2408534	41	Mexico	10.4044280	63	Tajikistan	5.165369
	•		42	Moldova	5.1477165	64	Thailand	3.034171
18		-6.3782098	43	Mongolia	4.1551999	65	Trinidad and Tobago	5,253046
19	Dominican Republic	5.5459390	44	Morocco	6.0048243	66	Tunisia	3.478983
20	Egypt	5.2421261	45	Netherlands	-6.5105821	67	Turkey	4.315867
21	Estonia	5.2507787	46	New Zealand	-10.9121769	68	Ukraine	-3.43445
22	Ethiopia	-1.2416397	47	North Korea	-0.6313021	69	United States	0.892568
23	Finland	7.1170365	48	Norway	5.7769863	70	Uzbekistan	-6.42799
24	France	-14.6056091	49	Poland	-0.8301831	71	Venezuela	5.612422

Next, we plot the boxplot from these data to find the outliers from trend and we can refer back to the line chart and see which country number that have the huge difference or outliers.



We obtained there are 3 countries that contains outliers, such as country number

- 27 (Great Britain = -42,2),
- 30 (India = 24,56),
- 52 (Russian Federation = -35,12)

This shows overall the model between actual and predicted data are quite close with only 3 outliers in it.

## **PART 2: Model Selection**

- 1.) First, we name each model for question 2 and 3.
  - Model 1 (Population only)

```
glm(formula = Medal2012 ~ Population, data = medal_data)
Deviance Residuals:
                              3Q
  Min 1Q Median
                                     Max
-54.311
       -8.421 -5.507 1.786 81.059
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.032e+01 2.295e+00 4.498 2.7e-05 ***
Population 4.026e-08 1.009e-08 3.990 0.000162 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for gaussian family taken to be 334.4558)
   Null deviance: 28403 on 70 degrees of freedom
Residual deviance: 23077 on 69 degrees of freedom
AIC: 618.15
Number of Fisher Scoring iterations: 2
```

# • Model 2 (GDP only)

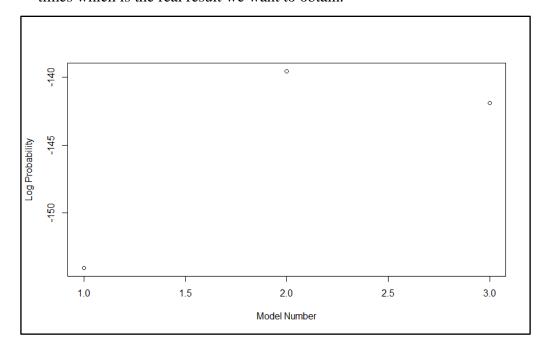
```
glm(formula = Medal2012 ~ GDP, data = medal_data)
Deviance Residuals:
        1Q Median
                                     Max
        -6.025 -2.590 3.885 60.244
-20.211
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.2359593 1.4788174 4.217 7.39e-05 ***
          0.0078160 0.0006438 12.140 < 2e-16 ***
GDP
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
(Dispersion parameter for gaussian family taken to be 131.2601)
   Null deviance: 28402.8 on 70 degrees of freedom
Residual deviance: 9056.9 on 69 degrees of freedom
AIC: 551.74
Number of Fisher Scoring iterations: 2
```

• Model 3 (Population + GDP)

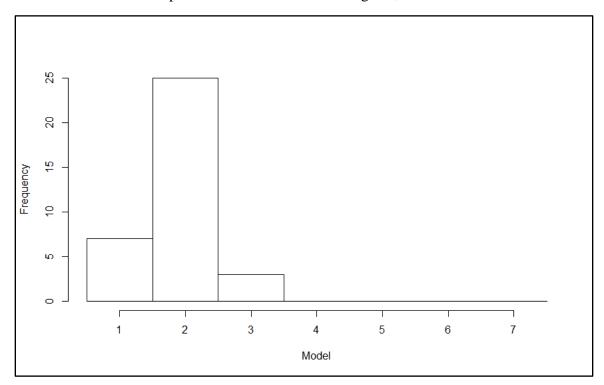
```
call:
glm(formula = Medal2012 ~ GDP + Population, data = medal_data)
Deviance Residuals:
                  Median
   Min
             1Q
                               3Q
                                       Max
-20.568
         -5.961
                  -2.462
                            3.932
                                    60.121
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.076e+00 1.500e+00 4.051 0.000133 ***
           7.564e-03 7.325e-04 10.326 1.45e-15 ***
Population 5.247e-09 7.193e-09
                                  0.729 0.468225
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for gaussian family taken to be 132.1562)
   Null deviance: 28402.8 on 70 degrees of freedom
Residual deviance: 8986.6 on 68 degrees of freedom
AIC: 553.19
Number of Fisher Scoring iterations: 2
```

From 3 different models we can see if model 2 (GDP only) has the lowest AIC with 551,74. Followed by model 3 (AIC = 553,19) and lastly model 1 remain with the highest AIC (618,15). Thus, we choose model 2.

2.) To ensure the AIC result we use cross-validation method to check if model 2 is really feasible model. Here we are going to plot in 2 different way which is just the training set in scatter plot and histogram which contain the iteration of 35 times which is the real result we want to obtain.



Above we have reliable result through the log-likelihood probability. We got model 2 with the highest log probability. Nevertheless, another test of 35 iteration is still needed. To make it different we put the iteration result in histogram,



Similarly, to the AIC and the scatter graph, we have model 2 as the highest frequency. This is good sign since we can conclude that the model 2 is the preferable model compare to model 1 and 3. This could be because model 3 (GDP + population) might overfit with more variable and model 1 (population only) is weak to explain the medal prediction.

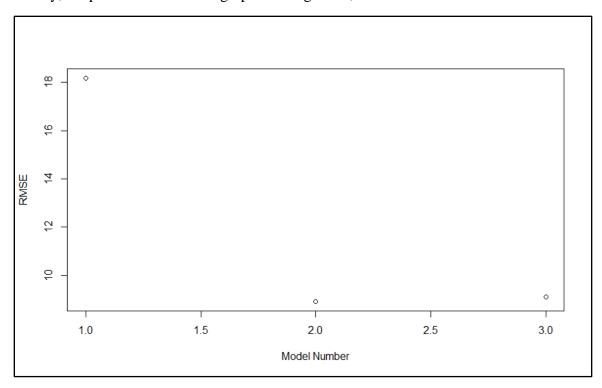
3.) Similar to part 1, we use the predict function to predict each model corresponding to each variable.

	Country	Population_model	GDP_model	PopGDP_model	Actual
1	Algeria	11.81632	7.710687	7.697937	2
2	Argentina	11.93780	9.721830	9.660081	4
3	Armenia	10.45411	6.316074	6.170773	4
4	Australia	11.24378	16.957674	16.572245	29
5	Azerbaijan	10.68936	6.731495	6.603459	18
6	Bahamas	10.33675	6.296846	6.136872	2
7	Bahrain	10.37222	6.407130	6.248223	2
8	Belarus	10.70346	6.666935	6.542817	9
9	Belgium	10.76345	10.234092	10.002805	6
10	Brazil	18.06845	25. 593526	25.819025	19
11	Bulgaria	10.61904	6.654195	6.519486	3
12	Canada	11.72256	19.804975	19.390152	22
13	China	64.57287	63.278115	68.348721	70
14	Colombia	12.19380	8.828145	8.828562	8
15	Croatia	10.49527	6.735013	6.581570	10
16	Cuba	10.77513	6.711252	6.595043	11
17	Czech Republic	10.74545	7.918125	7.759147	10
18	Denmark	10.54720	8.836196	8.621790	15
19	Dominican Republic	10.70014	6.670609	6.545939	1
20	Egypt	13.62126	8.029972	8.242126	3
21	Estonia	10.37557	6.409319	6.250779	1
22	Ethiopia	13.71765	6.483806	6.758360	8
23	Finland	10.54022	8.315570	8.117037	1
24	France	12.95379	27.910040	27.394391	42
25	Georgia	10.50246	6.348276	6.208237	7
					42
26	Germany	13.61739	34.143557	33.513444	
27	Great Britain	12.82945	25.241335	24.795509	67
28	Greece	10.75687	8.570841	8.392310	6
29	Hungary	10.72362	7.330438	7.187558	15
30	India	60.31055	20.679823	26.568161	2
31	Indonesia	19.89101	12.854806	13.728427	3
32	Iran	13.39505	6.912280	7.130986	8
33	Ireland	10.50725	7.934226	7.743689	2
34	Italy	12.76964	23.390187	22.996238	28
35	Jamaica	10.43145	6.353747	6.204280	11
36	Japan	15.46227	52.093769	51.125438	41
37	Kazakhstan	10.99565	7.691304	7.572239	17
38	Kenya	11.87712	6.498734	6.532974	13
39	Lithuania	10.45106	6. 569938	6.416057	4
40		11.46337	8.414052	8.332637	5
	Malaysia				5
41	Mexico	14.84568	15.265973	15.404428	
42	Moldova	10.46583	6.290671	6.147716	1 2
43	Mongolia	10.43270	6.302864	6.155200	
44	Morocco	11.63218	7.019282		
45	Nother lands			7.004824	1
46	Netherlands	10.99620	12.772191	12.489418	1 19
	New Zealand				1
47		10.99620	12.772191	12.489418	1 19
47 48	New Zealand	10.99620 10.50098	12.772191 7.257358	12.489418 7.087823	1 19 18
	New Zealand North Korea	10.99620 10.50098 11.29096	12.772191 7.257358 6.407912	12.489418 7.087823 6.368698	1 19 18 7
48	New Zealand North Korea Norway	10.99620 10.50098 11.29096 10.52406 11.87273	12.772191 7.257358 6.407912 10.032986	12.489418 7.087823 6.368698 9.776986 10.169817	1 19 18 7 4
48 49 50	New Zealand North Korea Norway Poland Portugal	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127	1 19 18 7 4 11
48 49 50 51	New Zealand North Korea Norway Poland Portugal Romania	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952	1 19 18 7 4 11 1 5
48 49 50 51 52	New Zealand North Korea Norway Poland Portugal Romania Russian Federation	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996	1 19 18 7 4 11 1 5
48 49 50 51 52 53	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139	1 19 18 7 4 11 1 5 56 8
48 49 50 51 52 53 54	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400	1 19 18 7 4 11 5 56 8
48 49 50 51 52 53 54 55	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738	1 19 18 7 4 11 5 56 8 1 4
48 49 50 51 52 53 54 55 56	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612	1 19 18 7 4 11 5 56 8 1 4 4
48 49 50 51 52 53 54 55 56	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469	1 19 18 7 4 11 5 56 8 1 4 4 4
48 49 50 51 52 53 54 55 56 57 58	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385	1 19 18 7 4 11 5 56 8 1 4 4 10 21
48 49 50 51 52 53 54 55 56 57 58 59	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469	1 19 18 7 4 11 5 56 8 1 4 4 4
48 49 50 51 52 53 54 55 56 57 58	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385	1 19 18 7 4 11 5 56 8 1 4 4 10 21
48 49 50 51 52 53 54 55 56 57 58 59	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17
48 49 50 51 52 53 54 55 56 57 58 59 60	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia South Africa South Korea Spain Sweden	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17
48 49 50 51 52 53 54 55 56 57 58 60 61 62	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999 11.204218 9.878228	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 7
48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Taiwan	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.44199 11.204218 9.878228 6.286920	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 11 7 3 1
48 49 50 51 52 53 54 55 56 57 58 60 61 62 63 64	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Tajikistan Thailand	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916 12.95900	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999 11.204218 9.878228 6.286920 8.937569	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369 9.034171	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 11 7 3 1 6
48 49 50 51 52 53 54 55 56 57 58 60 61 62 63 64 65	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Taiwan Tajikistan Thailand	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916 12.95900 10.37556	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999 11.204218 9.878228 6.286920 8.937569 6.411664	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369 9.034171 6.253046	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 11 7 3 16 6
48 49 50 51 52 53 54 55 56 57 58 60 61 62 63 64 65 66	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Taiwan Tajikistan Thailand Trinidad and Tobago Tunisia	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916 12.95900 10.37556 10.75228	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.58793 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999 11.204218 9.878228 6.286920 8.937569 6.411664 6.594402	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369 9.034171 6.253046 6.478984	1 19 18 7 4 11 5 56 8 1 4 4 4 10 21 17 7 3 1 6 1 3
48 49 50 51 52 53 54 55 56 60 61 62 63 64 65 66 67	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Taiwan Tajikistan Thailand Trinidad and Tobago Tunkey	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916 12.95900 10.37556 10.37556 10.75228 13.33124	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999 11.204218 9.878228 6.286920 8.937569 6.411664 6.594402 12.278453	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369 9.034171 6.253046 6.478984 12.315867	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 7 11 6 6 1 3 8
48 49 50 51 52 53 54 55 56 67 68	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Taiwan Tajikistan Thailand Trinidad and Tobago Turnisia Turkey Ukraine	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916 12.95900 10.37556 10.75228 13.33124 12.16036	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999 11.204218 9.878228 6.286920 8.937569 6.411664 6.594402 12.278453 7.527558	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369 9.034171 6.253046 6.478984 12.315867 7.565541	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 7 11 6 1 3 8 11
48 49 50 51 52 53 54 55 56 67 68 66 67 68	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Taiwan Tajikistan Thailand Trinidad and Tobago Tunisia Turkey Ukraine	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916 12.95900 10.37556 10.75228 13.33124 12.16036 22.94067	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441899 11.204218 9.878228 6.286920 8.937569 6.411664 6.594402 12.278453 7.527558 124.211089	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369 9.034171 6.253046 6.478984 12.315867 7.565541 121.892569	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 11 7 3 1 6 11 3 8 11 121
48 49 50 51 52 53 54 55 56 67 68	New Zealand North Korea Norway Poland Portugal Romania Russian Federation Serbia Singapore Slovakia Slovenia South Africa South Korea Spain Sweden Switzerland Taiwan Tajikistan Thailand Trinidad and Tobago Turnisia Turkey Ukraine	10.99620 10.50098 11.29096 10.52406 11.87273 10.74776 11.08926 16.08260 10.60922 10.53123 10.54176 10.40535 12.35936 12.27856 12.18258 10.70464 10.63939 11.25801 10.62916 12.95900 10.37556 10.75228 13.33124 12.16036	12.772191 7.257358 6.407912 10.032986 10.257306 8.092422 7.641203 20.756342 6.587993 8.109461 6.986220 6.623165 9.426775 14.960601 17.888172 10.441999 11.204218 9.878228 6.286920 8.937569 6.411664 6.594402 12.278453 7.527558	12.489418 7.087823 6.368698 9.776986 10.169817 7.928127 7.535952 20.878996 6.454139 7.916400 6.830738 6.461612 9.429469 14.774385 17.595080 10.196346 10.925493 9.722857 6.165369 9.034171 6.253046 6.478984 12.315867 7.565541	1 19 18 7 4 11 5 56 8 1 4 4 10 21 17 7 11 6 1 3 8 11

To check which model predicts the best we use root mean squared error method (RMSE), by using rmse() function we obtained such as,

_	model1_Population	model2_GDP	model3_GDPPopulation
1	18.18077	8.908145	9.112593

Finally, we plot it on the scatter graph which give us,



The x-axis refers to which model we use and y-axis is the RMSE value. The smallest RMSE value the better fit model with the real data. This means again model 2 (8,9) has the best prediction followed by model 3 (9,11) and model 1 (18,18) respectively.