NONLINEAR VIBRATIONS OF AEROSPACE STRUCTURES

2016-2017 PROJECT: FROM MEASUREMENTS TO UNDERSTANDING

Nonlinearities are more and more frequently encountered in real-world mechanical and aerospace structures. The objective of this project is to have you acquainted with state-of-the-art techniques in the area of nonlinear vibrations. To progress in this direction, you will carry out a detailed analysis, from measurements to understanding, of an academic nonlinear mechanical system. The result of your study is to be documented in a formal report that you will defend orally.

Besides obtaining accurate results, grading will be based on the quality of the writing and of the programming, on the fundamental understanding you will have gained about nonlinear vibrations and on the links that you will establish with the course material.

The system under consideration is a two-degree-of-freedom (DOF) system with mass, damping and stiffness matrices equal to:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathrm{C} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}, K = \begin{bmatrix} 2e4 & -1e4 \\ -1e4 & 2e4 \end{bmatrix}$$

The nonlinearities, whose mathematical forms and coefficients are unknown to you, can be located anywhere in the system.

1. MEASURE

You will be able to simulate experiments on the system of interest. Select adequate force types and set the experiment parameters, and send the generated force signals to jp.noel@ulg.ac.be. You will receive in return the displacements, velocities and accelerations of the two masses of the system. You will be given 5 opportunities to simulate experiments, considering up to 3 different force signals per experiment.

2. IDENTIFY

2a. Detection step: bring compelling evidence that your structure behaves nonlinearly.

2b. Characterization step: locate your nonlinearity(ies). Once located, visualize it(them) using the acceleration surface method (ASM) and select the best mathematical form to model it(them).

2c. Parameter estimation step: identify the nonlinear coefficients using the restoring force surface (RFS) method (see appendix A).

2d. Validation step: validate your results by comparing the response of the identified model to a sine-sweep response that will be provided as a target. You can use NI2D for your simulations.

3. SIMULATE AND UNDERSTAND

3a. Calculate the nonlinear normal modes (NNMs) of the system by combining a shooting method with sequential continuation. Discuss.

3b. Calculate the nonlinear frequency responses (NLFRs) of the system by combining a shooting method with sequential continuation. Discuss.

3c. Superpose the NNM backbone to the NLFRs. Discuss.

3d. Validate your calculations by comparing the NLFRs at a forcing level for which the nonlinearity(ies) is(are) significantly activated to a sine-sweep response obtained at the same forcing level.

INSTRUCTIONS

1. The project should be carried out by groups of two students.
2. The report should not exceed 25 pages. Electronic and paper copies will be provided to Jean-Philippe Noël and Gaëtan Kerschen.
3. You will have 15 minutes during the oral defense to summarize your findings. There will be time for questions after the presentation.
4. Both the report and the presentation (slides) are due February 15, 2017. The defense will take place at the end of February.

ADVICES

1. Useful Matlab functions for the project: ode 45 and fminunc.
2. An oral presentation should be used to convey a different message/perspective compared to a written document.
3. To copy the work of others will be considered as plagiarism and will result in the disciplinary action administered by the university.

USEFUL REFERENCES

M. Peeters, R. Viguié, G. Sérandour, G. Kerschen, J.C. Golinval, Nonlinear normal modes, Part II: Toward a practical computation using numerical continuation, Mechanical Systems and Signal Processing 23 (2009), 195-216.

APPENDIX A : RESTORING FORCE SURFACE METHOD FOR A 1DOF SYSTEM

The restoring force surface method is based on Newton's second law:

$$m\ddot{x}(t) + f(x(t), \dot{x}(t)) = p(t), \tag{1}$$

where $p(t)$ is the applied force and $f(x, \dot{x})$ is the restoring force, i.e., a non-linear function of the displacement and velocity. The time histories of the displacement and its derivatives, and of the applied force are assumed to be measured. In practice, the data must be sampled simultaneously at regular intervals. From equation (1), it is possible to find the restoring force defined as $f_i = p_i - m\ddot{x}_i$ where subscript i refers to the ith sampled value. Thus, for each sampling instant a triplet $(x_i, \dot{x}_i, f_i)$ is found, i.e., the value of the restoring force is known for each point in the phase plane $(x_i, \dot{x}_i)$.

It is important to describe the system by a mathematical model. The usual way is to fit to the data a model of the form:

$$f(x, \dot{x}) = \sum_{i=0}^{m} \sum_{j=0}^{n} \alpha_{ij} x^i \dot{x}^j. \tag{2}$$

Least-squares parameter estimation can be used to obtain the values of the coefficients α_{ij}. To have a measure of the error between the measured value x_i and the predicted value $\hat{x}_i$, the mean-square error (MSE) indicator is defined as

$$MSE(x) = \frac{100}{N\sigma_x^2} \sum_i (x_i - \hat{x}_i)^2, \tag{3}$$

where N is the total number of samples and σ_x^2 the variance of the measured input. Experience shows that an MSE value of less than 5% indicates good agreement while a value of less than 1% reflects an excellent fit. To determine which terms are significant and