MathDM PW2 - ILP Network Design

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1 Problem statement

The French telecommunication company Orange is the unique owner and handler of the telecommunication network in the figure below.

The costs on the links are proportional to the distances d(i, j) between the nodes, expressed in units of 10km. Because of anti-trust regulations, Orange must delegate to another two companies (i.e., SFR and Bouygues) two subnetworks, each having at least two nodes (with Orange handling the third part). Orange therefore needs to design a backbone network to connect the three subnetworks. Transforming an existing link into a backbone link costs c = 25 e/km.

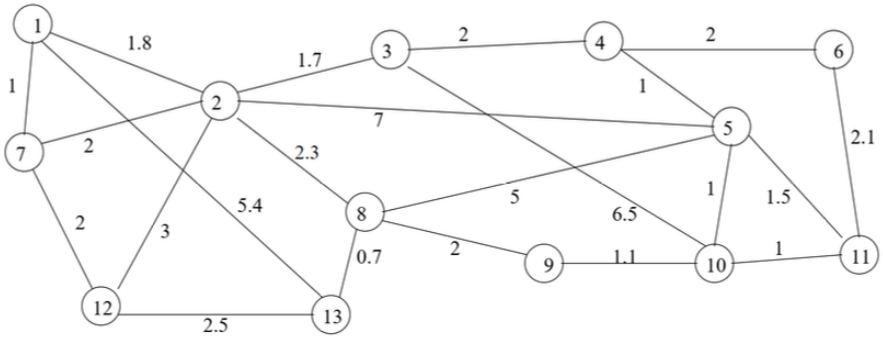

Figure 1: The topology of the telecom network with given link distances in units of [10km].

2 Solutions

2.1 MIP model formulation

Formulate a mathematical (integer linear programming) model to minimize the cost of implementing a backbone connecting the three sub-networks :

Indices: $i, j \in V$, $h, k \in K = \{1, 2, 3\}$.

Parameters: for each $i, j \in E$, d_{ij} is the edge weight (distance between i and j).
c: backbone updating cost.
m: minimum cardinality of the subnetworks.

Variables: for each $i \in V, h \in K$, let $x_{ih} = 1$ if vertex i is in Vh, and 0 otherwise.

Objective Function :

$$min \frac{1}{2} \sum_{h \neq k \in K} \sum_{ij \in E} c d_{ij} x_{ij} x_{jk},$$

S.t. constraints :

$$\begin{array}{ll}
\forall \{i\} \in V & \sum_{k \in K} x_{ik} = 1 \\
\forall \{h\} \in K & \sum_{i \in V} x_{ik} \geq m \\
\forall i \in V \ and \ h \in K & x_{ih} \in \{1, 2, 3\} \\
\forall \{i, j\} \in E, h \neq k \in K & w_{ij}^{hk} \geq x_{ih} + x_{jk} - 1 \quad \big(if \ x_{ih} = x_{jk} = 1, w_{ij}^{hk} = 1\big) \\
\forall \{i, j\} \in E, h \neq k \in K & w_{ij}^{hk} \leq x_{ih} \quad \big(if \ x_{ih} = 0, w_{ij}^{hk} = 0\big) \\
\forall \{i, j\} \in E, h \neq k \in K & w_{ij}^{hk} \leq x_{jk} \quad \big(if \ x_{jk} = 0, w_{ij}^{hk} = 0\big)
\end{array}$$

2.2 Model implementation and solution

Implement the ILP model in either CPLEX or Python with Pyomo. Solve the model with the given dataset.