Assignment 1: Finite Element Methods (41525)

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Report requirements

Formal rules

- This first assignment consists of four subassignments in total (one pr. course day).
- Each subassignment must be answered on one separate page including figures and discussions.
- Apart from the 4 pages discussed above, a cover page must include the following:
  - Group number and code.
  - Student's names and student's numbers.
  - A statement on who is responsible for what parts of the assignment and codes[1].
- One (in total) additional and separate page may be added containing answers to *-exercises related to the subassignments. Recall that one can still achieve top grades without solving any *-exercises!
- Use minimum 12 pt, single line spacing, 2cm margin left and right, 3cm margin top and bottom on your A4 paper. (NB! This document is formatted according to these rules)
- The report must be uploaded to DTU Learn as one separate pdf-file before the deadline (Tuesday September 27th at 10pm) and handed to the teacher in hard copy with signatures before the following lecture at the latest.
- One separate pdf-file containing source codes for one representative Matlab code pr. subassignment must be uploaded to DTU Learn as well.
- Failure to adhere to above rules will count negatively in the evaluation.

Recommendations:

- Use the feedback sheet (see the sample outline next page) as work guidance and check-list.
- The one page answers to each subassignment may contain plots, graphs, equations and text.
- All results must be accompanied by short and precise explanations, discussions and critical interpretations. Note that these discussions and interpretations also count in the evaluation.
- Create simple, clear and readable illustrations and graphs.

September 2022, Ole Sigmund and Kim L. Nielsen

[1] From the DTU rules: *"It must be clearly specified for which sections each student has the (main) responsibility. A group project is not deemed to be individualized if the students merely state that they have contributed equally to all sections of the report or the like. If a group project does not comply with these requirements for individualization or other formal requirements, the report may be rejected and no assessment given."*

Sample evaluation sheet

Tentative evaluation scheme for Assignment 1

Group code:

	ok	NA	Unacceptable	Just acceptable	Average	Good	Excellent	Remarks
Report specific issues								
Layout and format								
Matlab codes								
Cover page								
Page limit								
Competition 1								
Clarity and preciseness								
Language								
Readability of graphs								
Code organization								
Subassignment 1								
Ex. 1								
Ex. 2								
Ex. 3								
Ex. 4								
Ex. 5								
Ex. 6								
Subassignment 2								
Ex. 1								
Ex. 2								
Ex. 3								
Ex. 4								
Ex. 5								
Subassignment 3								
Ex. 1								
Ex. 2								
Ex. 3								
Ex. 4								
Subassignment 4								
Ex. 1								
Ex. 2								
Ex. 3								

Exercise 1 - Linear truss analysis

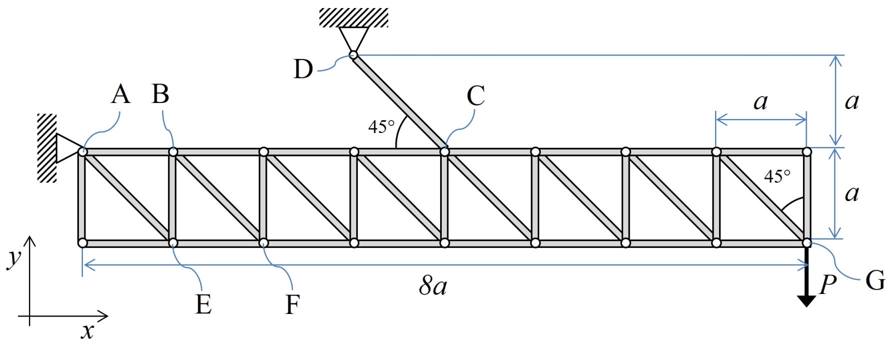

Figure 1: Truss beam structure.

Consider the truss beam structure shown in Figure 1. Dimensions and material properties of the truss structure are the following: all bars have areas $A = 0.0002\text{m}^2$, $P = 15\text{kN}$ and the Young's modulus is 70GPa. The length of each horizontal and vertical bar is $a = 0.5\text{m}$.

Perform hand calculations to compute (include short extract of hand-calculations/sketches):

1. Reaction forces.

2. Bar forces and stresses in the bars connecting nodes A and B as well as the bar connecting E and F.

Create an input file and use your linear Matlab code for the following:

3. Plot the deformed structure with tension bars in blue, compression bars in red and bars with less than 10^{-5} times the maximal numerical element stress value with green. Interpret and discuss the force (i.e. color) distribution.

4. Compute reaction forces and compare with the analytical solution.

5. Compute bar forces and stresses in the elements connecting nodes A and B as well as the element connecting nodes E and F and compare with the analytical solution as well as the yield stress for Aluminum of 350MPa.

6. Compute horizontal and vertical displacements in node G of the structure. Do the displacements seem reasonable considering material selection, geometry and loads?

Exercise 2 - Material non-linearity

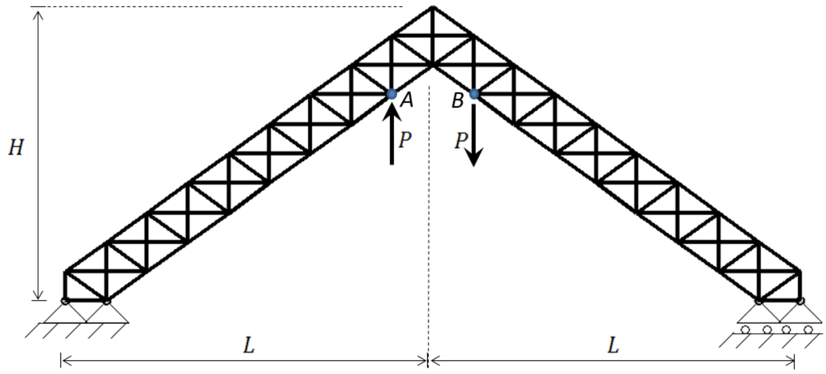

Figure 2: Larger truss structure (”TrussExercise2.m”), with $L = 50$, $H = 40$, and $P = 80$.

The rubber-like material from Day 2 is considered and the stress-strain relation is given by the Signorini model

$$\sigma(\varepsilon) = c_1(\lambda - \lambda^{-2}) + c_2(1 - \lambda^{-3}) + c_3(1 - 3\lambda + \lambda^3 - 2\lambda^{-3} + 3\lambda^{-2}) \tag{1}$$

where $\lambda = 1 + c_4\varepsilon$ is the stretch. In the following you are asked to use the material constants $c_1 = 1.2, c_2 = 5, c_3 = 0.2$, and $c_4 = 50$.

Consider the structure shown in Figure 2, by loading the mesh file "TrussExercise2.m" into your incremental code that takes non-linear material behavior into account. Consider here the Pure Euler, the Newton-Raphson, and the Modified Newton-Raphson algorithms.

1. Use the three different algorithms to plot and compare the load-displacement curve (in the vertical direction) for point A (use nincr= 100 and $P = P_{final} = 80$).
2. Discuss the effect of using more/less load increments in relation to the study above.
3. In relation to task 1, compare the number of factorizations for the system of equations and the norms of final residuals for the three algorithms (Pure Euler, Newton-Raphson, and Modified Newton-Raphson). Discuss the differences. (For the NR-algorithms use $||R|| < 10^{-8}P_{final}$ as convergence criterion).
4. In relation to the results obtained above: do the performances of the algorithms change when changing $c_1 = 1.2$ to $c_1 = 0$, $c_2 = 5$ to $c_2 = 1$, and $c_4 = 50$ to $c_4 = 200$? Why/Why not?
5. Which method do you prefer for small and large problems?

Exercise 3 - Geometrical non-linearity

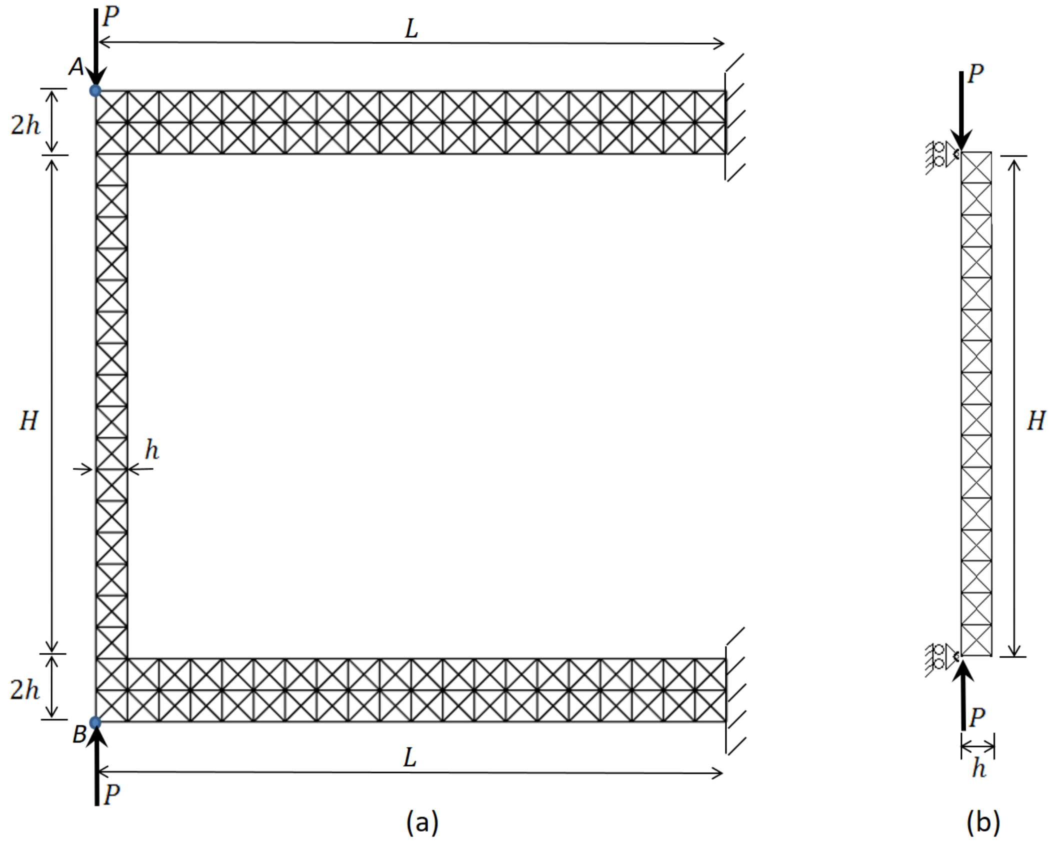

Figure 3: a) Column supported cantiliver beam structure ("TrussExercise3.m"), with $L = 100$, $h = 5$, $H + 4h = 100$, $A_{Column} = 0.2$, $A_{Beam} = 0.1$, $P = 0.004$, $E_{Column} = 0.8$, $E_{Beam} = 0.1$. The column has height H and is located between the to support beams each of height $2h$. Use $nincr = 200$, $i_{max} = 1000$, and $\varepsilon = 10^{-9}$. b) Support column corresponding to the column in the big structure.

Consider the structure displayed in Figure 3a by loading the mesh file "TrussExercise3.m" into your incremental (non-modified) Newton-Raphson code based on geometrically non-linear truss analysis. Here, considering linear elastic materials.

1. Determine the effective bending stiffness, EI, for the support column shown in Figure 3b (**Hint**: expose the slender column to a small transverse load and exploit your knowledge from simple beam theory. See also Exercise 3.3 in the course notes).

2. Compare the buckling load of the support column to an analytical solution. Discuss your results.

3. Plot the force, $F = 2P$, applied to the structure in Figure 3a as a function of the displacement: $\Delta_y = \Delta_y^{(A)} - \Delta_y^{(B)}$, recorded at the point A and B. At which load does the structure become unstable and how does this compare to the critical load for the support column alone, supported as seen in Figure 3b? Explain the results.

4. Demonstrate the influence of material properties on the load-deflection curve. For example, repeat task 3 for three different ratios of Young's moduli; E_{Column}/E_{Beam} (Use $E_{Column} = 0.4$, $E_{Column} = 0.8$, and $E_{Column} = 1.6$). (**Hint**: it may be a good idea to scale the applied loading accordingly). Do you obtain the full load-displacement curve in all cases? Explain the response of the structures.

Exercise 4 - Topology optimization

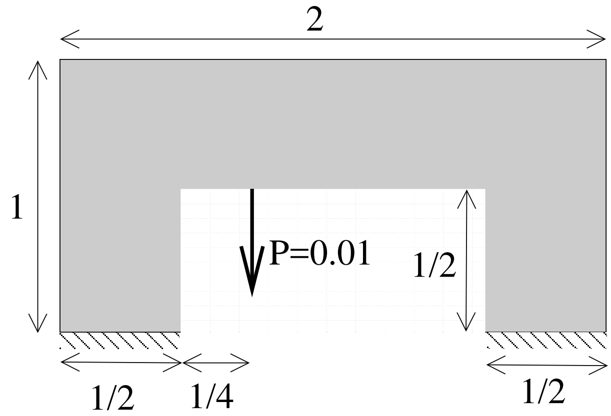

Figure 4: The competition truss topology optimization problem. The limit on total volume is $V^* = 6$.

Topology optimize the competition problem from Day 1 sketched in Figure 4, and investigate the following:

1. Run the optimization for different numbers of nodes and connectivities. Compare the resulting compliances. What is the tendency?

2. Run the optimization for different penalization factors, p. Discuss the results.

3. Can you break the record from the competition ($C = 5.94 \cdot 10^{-5}$)? Discuss why / why not - depending on your results.

Hints: Initialize the starting values of the design variables such that the volume constraint is satisfied (i.e. $\{\rho\}^T\{v\} = V^*$). Use a very small lower bound, i.e. $\rho_{min} = 10^{-6}$. Plot only bars where $\rho^e > 10\rho_{min}$.