Intro to 41525 ”Finite Element Methods”

Ole Sigmund and Kim Lau Nielsen

Department of Civil and Mechanical Engineering, Solid Mechanics

Preparatory reading:

Course Notes - Day 1

Chapter 1 in Cook et al.

FEM = Finite Element Methods

- Started in the 50’s – mechanical engineering
- Introduced in Denmark in the 60’s
- First taught at DTU around 1980
- Today: everyday tool in the industry

FEA - Popular softwares

- Abaqus
- Ansys
- COMSOL
- Nastran
- Loads of others …

FEA vs. FEM (Analysis vs. Methods)

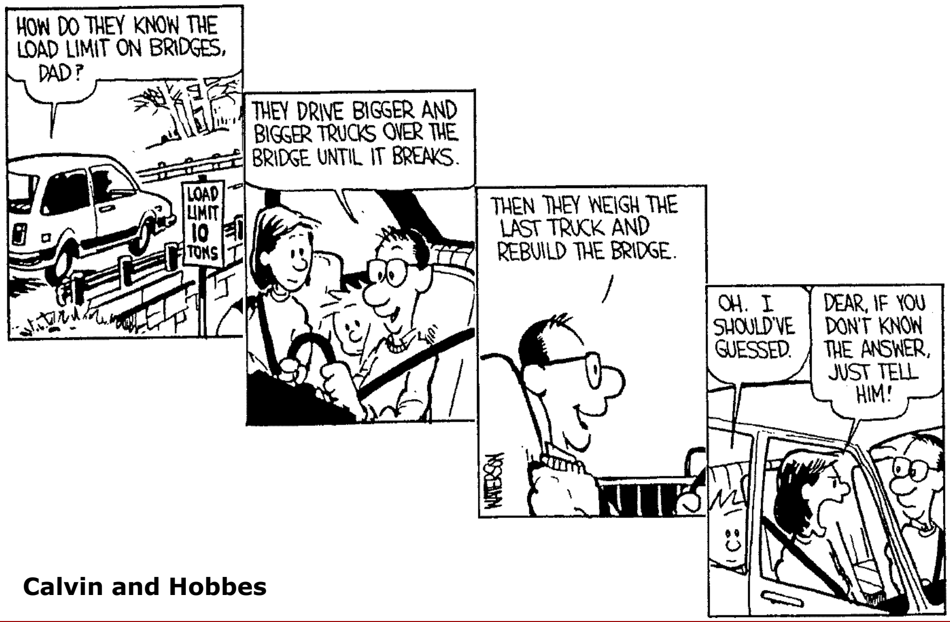
FEA (41812)

- Applied FEA
- Commercial FE codes
- Complex geometries and models
- Linear analysis

FEM (41525)

- Programming the FEM
- Own code development
- Simpler (planar) geometries
- Non-linear analysis

Why do we need FEM?



Calvin and Hobbes

FEM – a smart way to solve PDE's

PDE's Partial Differential Equations

BC's Boundary Conditions

Volume equi.: $\sigma_{ji,j} + \Phi_i = 0$ in Ω PDE

Constitutive eq.: $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$

Strain definition: $\varepsilon_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j})$

Prescribed displ.: $u_i = u_i^*$ on Γ_u

Surface tractions: $\sigma_{ij}n_i = F_j$ on Γ_F

BC's

Generally no analytical solutions – what to do?

Finite elements

Dividing the domain into finite elements

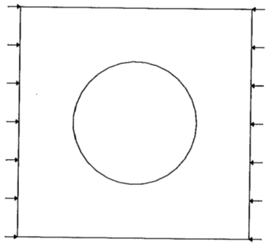
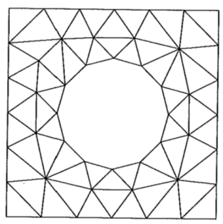
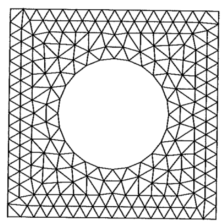


Plate with a Hole

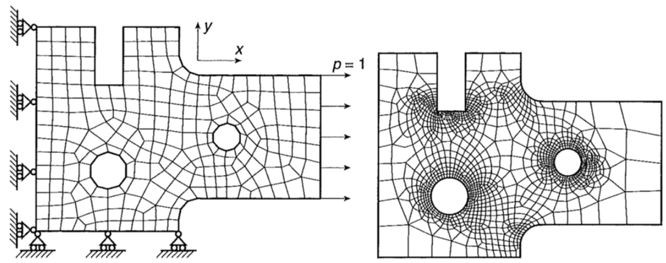
Triangular Finite Element



Finite Element Model



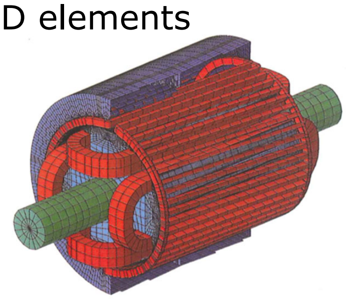
Refined Finite Element Model

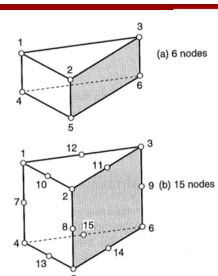


Finer mesh -> better solution !

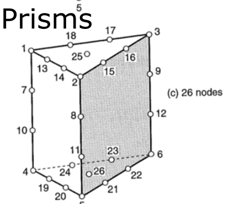
Element types

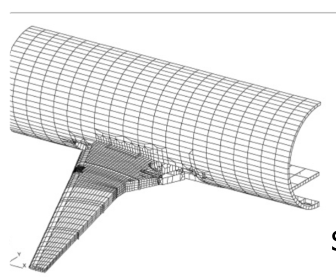
3D elements



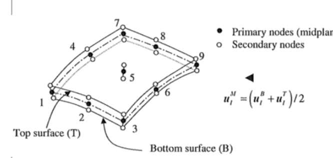


Prisms

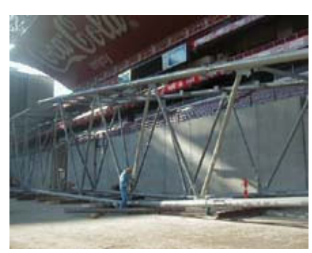




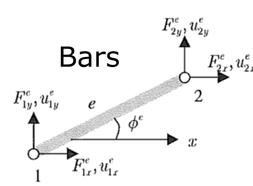
Shells

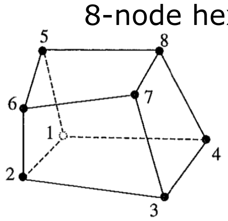


Bars

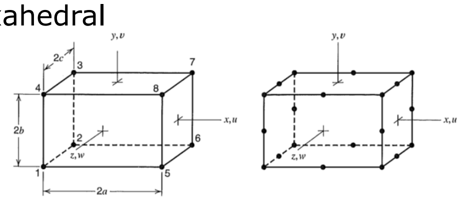


8-node hexahedral

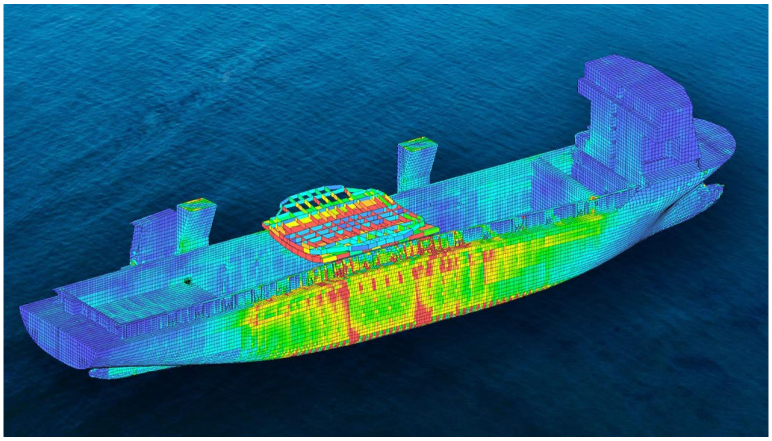




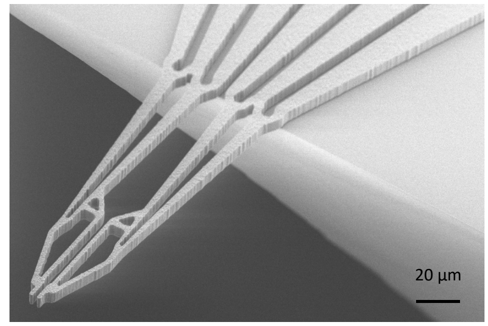
Beams

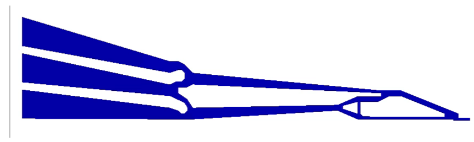


From huge to tiny

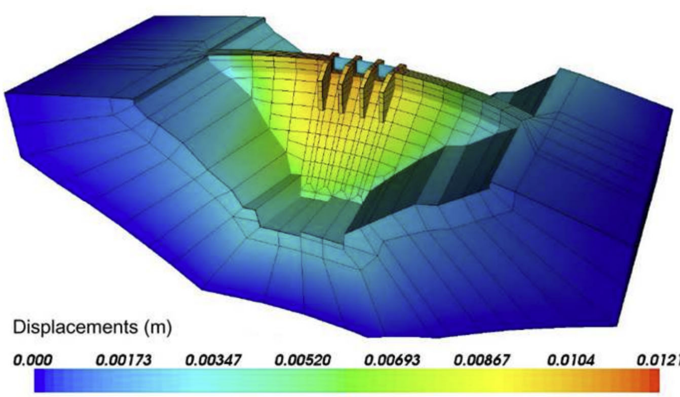


DNV

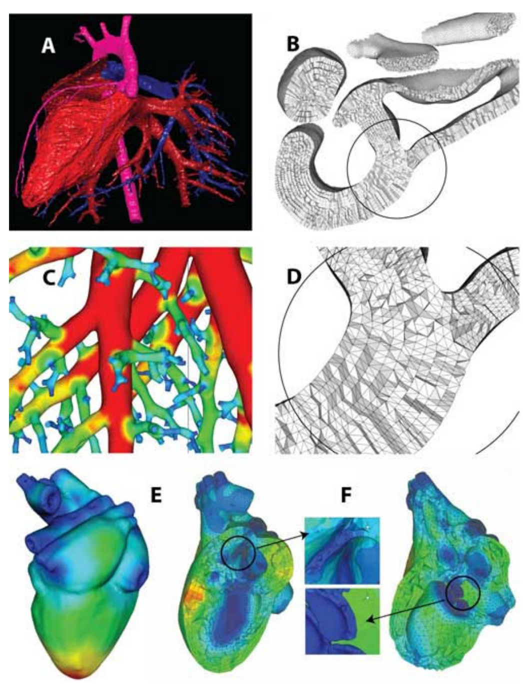




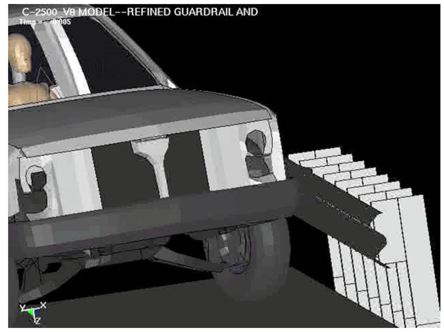
From hard to soft

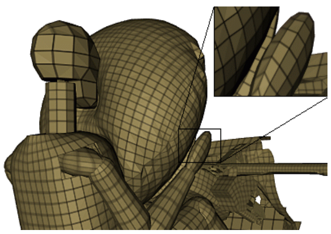


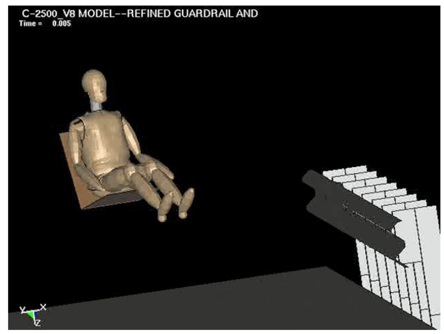
CoBa-group



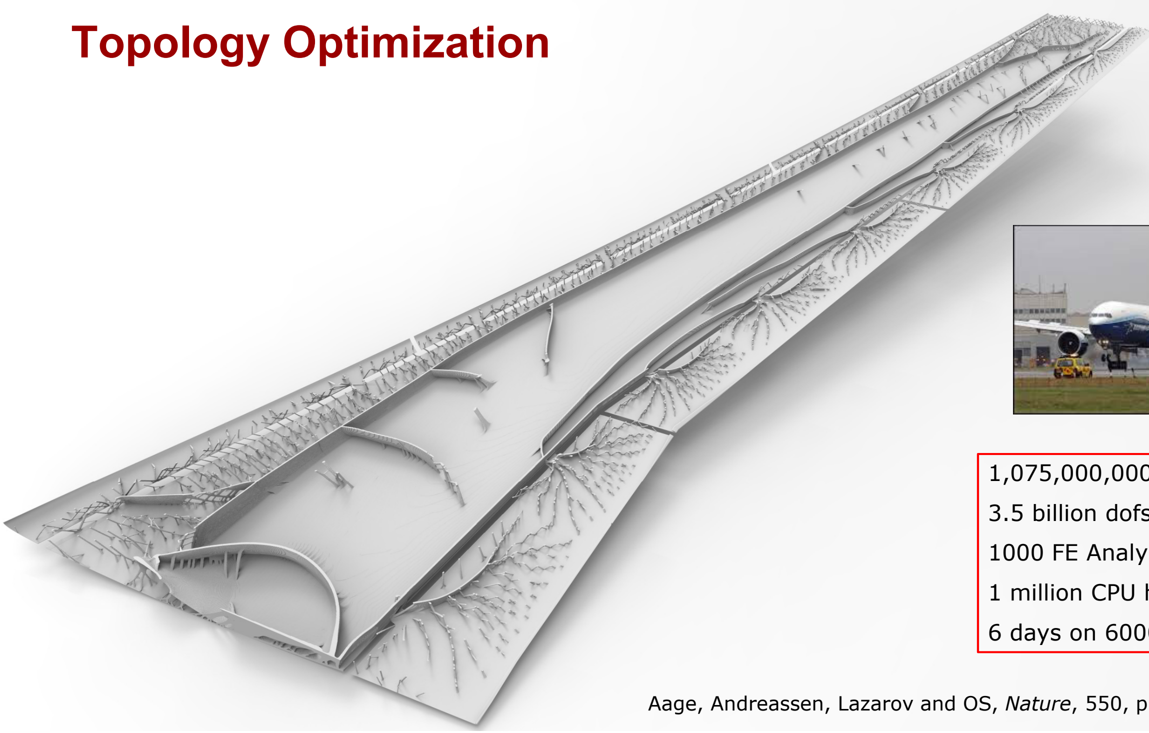
Time dependence







Topology Optimization



1,075,000,000 elements

3.5 billion dofs

1000 FE Analyses

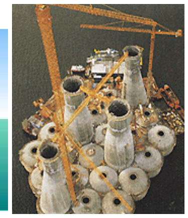
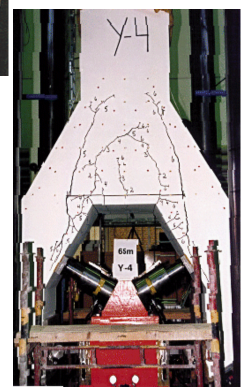
1 million CPU hours

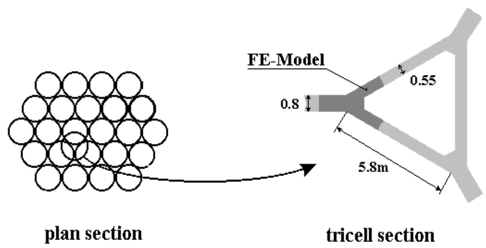
6 days on 6000 processors

Aage, Andreassen, Lazarov and OS, *Nature*, 550, pp. 84-86, **2017**

Be careful! The Sleipner A offshore platform

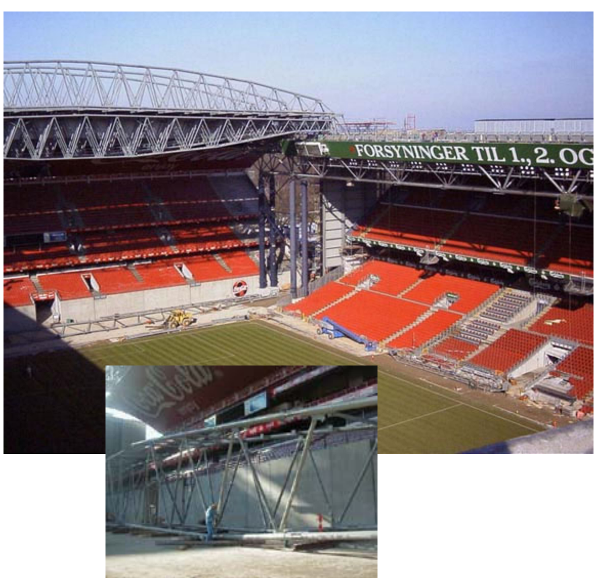


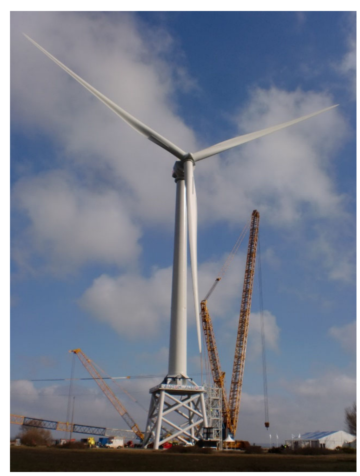





Stress underestimated by 47%, failure postpredicted to 62m, sank at 65m. 57kT, 700mil.$.

Truss structures



Alstom Turbine with LM-73.5

Practicalities at start of course: FEM-Heavy (41525)

Lectures Wednesdays at 8am in room 072, Building 421. Duration: around 2 hours. Smaller follow-up lectures may be given during the day.

Computer exercises 003, 010, 014, 018 in building 421.

Teachers Ole Sigmund (OS), Department of Mechanical Engineering, Solid Mechanics, Building 404, room 134, e-mail: sigmund@mek.dtu.dk

Kim Lau Nielsen (KLN), Department of Mechanical Engineering, Solid Mechanics, Building 404, room 118, e-mail: kin@mek.dtu.dk

Teaching Assistants Christoffer Fyllgraf Christensen, email: chrify@dtu.dk
Yafeng Wang, email: vafwa@dtu.dk
Peter Dørffler Ladegaard, email: pdlj@dtu.dk
Federico Ferrari, email: feferr@dtu.dk

Office hours Office hours will be announced before the assignment periods. Often email requests are more efficient but please do not send same email separately to several TA's.

Apologies Don't miss classes! In case of Force Majeure please send an e-mail to the teachers.

Literature Cook, Malkus and Plesha, "Concepts and Applications of Finite Element Analysis", 4th edition, 2002, John Wiley and Sons. Buy it in the bookstore (Building 101) - see www.polyteknisk.dk/home/dtu/detailed_view/9780471356059. Handouts during the course.

Technical problems If you have technical problems in the M-Data-bar please contact MEK-support (icon on the desktop).

Homework Course notes have one chapter pr. course week. Each chapter also refers to recommended reading in the Cook book. Further info and material may be found on DTU Learn.

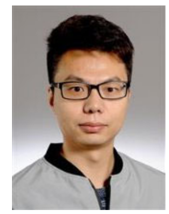
Assignments and examination The course evaluation consists of the evaluation of three individualized group reports and a grade can only be awarded if the individual student's contribution to the project can be ascertained. It must be clearly specified for which sections each student has the (main) responsibility. A group project is not deemed to be individualized if the students merely state that they have contributed equally to all sections of the report or the like. Each group will consist of maximum two students. The reports (respectively 4, 5 and a maximum of 15 pages) must demonstrate and discuss solutions of the assignments. The final grade is based on the overall performance the individual student has demonstrated in the three reports.

Assignments 1) Full assignment published on DTU Learn 21/9, 1pm. To be uploaded to DTU Learn by 27/9 at 10pm (maximum 4 pages + cover page).
2) Full assignment published on DTU Learn 2/11, 1pm. To be uploaded to DTU Learn by 8/11 at 10pm (maximum 5 pages + cover page).
3) Full assignment published in class 9/11. To be uploaded to DTU Learn by 5/12 at 1pm and delivered in a paper version to the teachers (maximum 15 pages + cover page).


Christoffer


Peter


Federico


Yafeng

Tentative course schedule

Week	Date	Lecturer	Topic
1	31/8	OS	Matlab truss FE-analysis
2	7/9	KLN	Matlab truss FE-analysis (material non-linearity)
3	14/9	KLN	Matlab truss FE-analysis (geometrical non-linearity)
4	21/9	OS	Matlab truss FE-analysis (Topology Optimization) Publishing of first full report assignment on DTU Learn (1pm)
	27/9		First report (upload no later than 10pm)
5	28/9	KLN	Continuum type FE-analysis
6	5/10	KLN OS	Continuum type FE-analysis (Equation solving/stress computations/distributed forces) Feedback to first report
7	12/10	OS	Continuum type FE-analysis (energy principles).
	19/10		FALL BREAK
8	26/10	OS	Continuum type FE-analysis (Isoparametric formulation)
9	2/11	KLN	Dynamics (eigenvalue analysis) Introduction of final projects Publishing of second full report assignment on DTU Learn (1pm)
	8/11		Second report (upload no later then 10pm)
10	9/11	OS	Topology Optimization. Industry talk (to be announced). Final assignment.
11	16/11	KLN	Alternative methods and High Performance Computing. Final assignment. Feedback to second report
12	23/11	OS	Error estimates + Mesh-generation. Final assignment.
13	30/11	KLN	Course evaluation. Final assignment.
	5/12		Hand in third report on final assignment before 1pm

Project	Description
1	Plate modelling
2	Geometrical non-linearity
3	Contact analysis
4	Transient problems
5	Plasticity
6	Incompressibility/Stokes flow
7	Special elements
8	Acoustic structure interaction
9	Your own idea ...

Assignments

Report 1: 4 pages*
Assignment handed out Day 2/4
Upload report to Learn before Day 5

Report 2: 5 pages*
Assignment handed out Day 7/9
Upload report to Learn before Day 10

Report 3: Max. 15 pages
Assignment handed out Day 10
Upload report to Learn first day of exam period (noon December 5th)

Group work

- Form groups of **2** (1 and 3 only in absolute ”emergencies”)
- Preferably same available time / similar ambitions in group
- Preferably programming and elasticity experience in all groups
- Groups may be reformed after first or second report

(get accept from teachers and organize on or before Day 5 or 10)

DTU rules:

The course evaluation consists of the evaluation of three individualized group reports and *a grade can only be awarded if the individual student’s contribution to the project can be ascertained. It must be clearly specified for which sections each student has the (main) responsibility. A group project is not deemed to be individualized if the students merely state that they have contributed equally to all sections of the report or the like.*

Recommendations

Each Day/Chapter has a number of exercises at the end

Use the exercises for the following purposes:
- as guideline for your work and code implementation
- as means for checking that your code works and produces reasonable results
- as a trigger for learning to interpret and evaluate the numerical results
- as a checklist for what kind of questions you can expect to see in the assignments

It is highly recommended to prepare you codes such that:
- they easily can solve for alternating structures, loads and boundary conditions
- it is easy to generate plots and graphs for visualization and interpretation of the results

Set up a template for your assignment reports (Word, Latex, etc.) and prepare front page, etc.
Exercises marked with a * are voluntary exercises.

Teaching/learning goals

A student who fully satisfies the course goals, will be able to:
- Use the principle of virtual work to set up finite element equations
- Implement the finite element method for truss and continuum problems
- Apply the finite element method to the solution of static and dynamic mechanical problems
- Solve material and geometrically non-linear problems by use of explicit and implicit solution methods
- Optimize simple mechanical structures based on finite element calculations
- Implement iso-parametric finite elements based on numerical integration
- Verify correctness of calculations based on test problems and comparisons with analytical solutions
- Evaluate the quality of a finite element model when the exact solution is unknown
- Evaluate the quality of finite element calculations based on convergence analyses
- Present results of finite element analyses in a transparent, easily accessible and efficient form
- Set up continuous and discrete forms of the principle of virtual work for arbitrary partial differential equations
- Plan a larger finite element programming project and in an independent way suggest and implement test examples.