Assignment 2: Finite Element Methods (41525)

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Report requirements

Formal rules

- This second assignment consists of four subassignments in total (one pr. course days 6-9).
- The 4 subassignments must be answered on at most 5 pages including figures and discussions. Also include subassignment numbers in answers, c.f. 3.1, 3.2, etc.
- Apart from the 5 pages discussed above, a cover page must include the following:
  - Group number and code.
  - Student's names and student's numbers.
  - A statement on who is responsible for what parts of the assignment and codes[1].
- One (in total) additional and separate page may be added containing answers to *-exercises related to the subassignments. Recall that one can still achieve top grades without solving any *-exercises! This page may also be used to brag about special programming or smart file handling details that you have implemented in connection with the exercises.
- Use minimum 12 pt, single line spacing, 2cm margin left and right, 3cm margin top and bottom on your A4 paper. (NB! This document is formatted according to these rules)
- The final report must be uploaded to DTU Learn as one separate pdf-file before the deadline (Tuesday November 8th at 10pm) and handed to the teacher in hard copy with signatures before the following Wednesday lecture at the latest.
- One separate pdf-file containing your source codes for the fea.f90 and plane42.f90 (isoparametric formulation) modules must be uploaded as well.
- Failure to adhere to above rules will count negatively in the evaluation.

Recommendations:

- Use the feedback sheet (see the sample outline next page) as work guidance and check-list.
- The answers to each subassignment may contain plots, graphs, equations and text.
- All results must be accompanied by short and precise explanations, discussions and critical interpretations. Note that these discussions and interpretations also count in the evaluation.
- Create simple, clear and readable illustrations and graphs.

October 2022, Ole Sigmund and Kim L. Nielsen

[1] From the DTU rules: *"It must be clearly specified for which sections each student has the (main) responsibility. A group project is not deemed to be individualized if the students merely state that they have contributed equally to all sections of the report or the like. If a group project does not comply with these requirements for individualization or other formal requirements, the report may be rejected and no assessment given."*

Sample evaluation sheet

Evaluation of final report

Group code:

	ok	NA	Unacceptable	Just acceptable	Average	Good	Excellent	Remarks
Report specific issues								
Layout and format								
Matlab codes								
Cover page								
Page limit								
Competition 1								
Clarity and preciseness								
Language								
Readability of graphs								
Code organization								
Subassignment 1								
Ex. 1								
Ex. 2								
Ex. 3								
Ex. 4								
Subassignment 2								
Ex. 1								
Ex. 2								
Ex. 3								
Ex. 4								
Ex. 5								
Subassignment 3								
Ex. 1								
Ex. 2								
Ex. 3								
Ex. 4								
Subassignment 4								
Ex. 1								
Ex. 2								
Ex. 3								

Exercise 1 - Equation solving

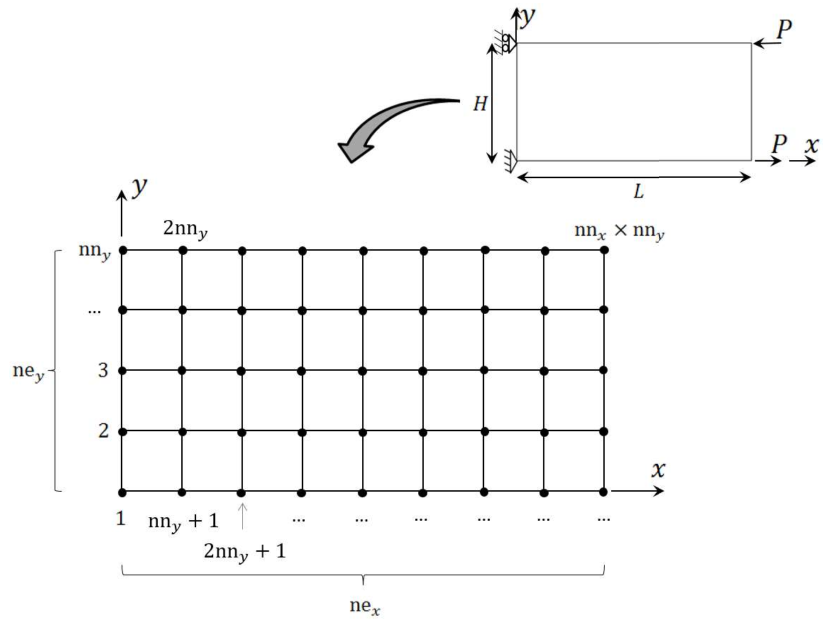

Figure 1: Schematic of 2D domain discretized by 2000 elements. The number of elements in the $x-$ and $y-$directions are ne_x and ne_y, respectively. The node numbering starts at the bottom left corner (at $[x, y] = [0, 0]$), continues along the y-axis before making a shift along the x-axis. The node numbering ends at the top right corner (at $[x, y] = [L, H]$).

Consider the 2D plane stress domain loaded by a concentrated forces of magnitude, P, at the top-right corner point ($x = L$ and $y = H$) and at the bottom-right corner point ($x = L$ and $y = 0$). The domain is fixed (so that $u_x = u_y = 0$) at points $[x, y] = [0, 0]$ and constrained in the x_1-direction at point $[x, y] = [0, H]$ (so that $u_x = 0$). The domain size and shape stay fixed throughout the following investigation of the employed solvers.

The total number of elements (ne) that discretizes the domain equals 2000, such that $\text{ne} = \text{ne}_x \times \text{ne}_y = 2000$, with ne_x and ne_y being the number of elements in the $x-$ and $y-$direction, respectively. The nodes are numbered as illustrated in Fig. 1. Thus, the node numbering starts at the bottom left corner (at $[x, y] = [0, 0]$) and runs along the $y-$axis before making a shift along the $x-$axis. The node numbering ends at the top right corner (at $[x, y] = [L, H]$).

For $\text{ne}_x = [2000, 1000, 500, 250, 125, 16, 8, 4, 2, 1]$ (in total 10 different meshes), your are asked to:

1. Create the input files (e.g. by using *Flextract*) and calculate the bandwidth (*bw*) as well as the number of equations (neqn) for all 10 meshes. Explain the numbers and make a prediction of the expected CPU time according to the discussion in Chapter 6 in the course notes and the Cook book.

2. Time the solution and compare results for the *full* and *banded* solver. Discuss the differences - if any - and compare with your theoretical predictions.

Employ the *bandfem* and *renum* programs (available on DTU Learn) to optimize the bandwidth for all 10 meshes.

3. Compare the optimized bandwidths to those of the original meshes. Do you see a change in bandwidth for all meshes - why/why not?

4. Compare calculation time for the 10 different meshes, before and after optimizing the bandwidth, and discuss the results.

Exercise 2 - Virtual Work Principle

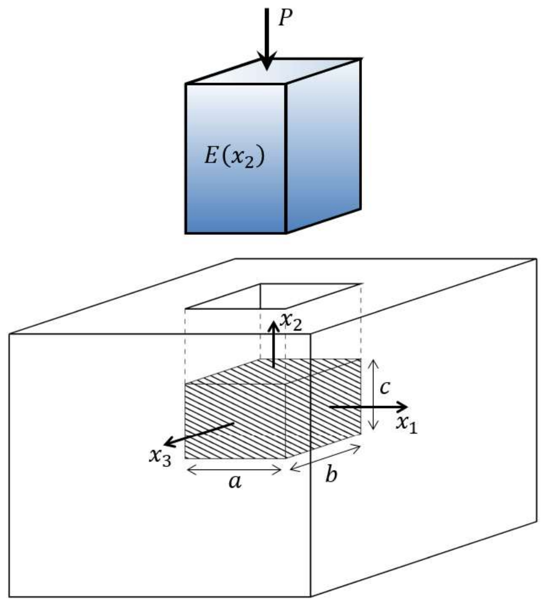

Figure 2: Schematic of a compression test of a linear elastic material constrained by a channel such that the specimen (in grey) only deforms in the x_2-direction. The specimen has height c, and in-plane dimensions a and b in the x_1- and x_3-direction respectively. The material is functionally graded such that the Young's modulus is denoted, $E(x_2)$, the shear modulus is $G(x_2)$, and Poisson ratio is $\nu(x_2)$.

Consider compression testing of a block of linear elastic material with height, c, and in-plane dimensions a and b in the x_1 and x_3-direction, respectively (see Fig. 2). The block is placed in the bottom of the channel and loaded by a pressure, $p = P/(ab)$, at the top boundary. The channel die and punch is assumed rigid such that the specimen only deformed in the x_2-direction, thus, $u_2(x_2) \neq 0$ and $u_1(x_2) = u_3(x_2) = 0$. No volume and friction forces exist. Based on linear elasticity you are asked to:

1. Determine the stress and strain state of the material. Write the respective vectors (e.g $\{\varepsilon\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{13}, \varepsilon_{23}\}^T$ and $\{\sigma\} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}\}^T$).

2. Determine the strain energy density, U. Write the expression in terms of the given parameters (**Hint:** recall that $U = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}$).

From Eq. (7.25) in Chapter 7 in the course notes, the principle of virtual work for the domain considered can by written as:

$$\int_V \delta\varepsilon_{ij}\sigma_{ij}\mathrm{d}V = \int_{S_F} \delta u_i F_i \mathrm{d}S + \int_V \delta u_i \Phi_i \mathrm{d}V \tag{1}$$

You are asked to:

3. Show that the weak form of the equilibrium equations in Eq. (1) reduces to 1D for the compression problem in Fig. 2. Write the simplified version of Eq. (1). (**Hint:** Notice that the variation in the displacement field is in the x_2-direction.)

4. Derive the discrete finite element equation based on the 1D variational problem accounting for the material gradient in the x_2-direction, i.e. $E = E(x_2)$ and $\nu = \nu(x_2)$.

5. Based on the derivations in question 4, develop a 1D element (not iso-parametric) and derive the element stiffness matrix.