TECHNICAL UNIVERSITY OF DENMARK

41525 FINITE ELEMENT METHODS

Assignment 1

TEAM 26 - LION

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1 Exercise 1

1. Solving the system of equilibrium equations $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_A = 0$ the analytical support reactions could be found: $R_{A,x} = 2P$, $R_{A,y} = -P$, $R_{D,x} = -2P$, $R_{D,y} = 2P$, with $P = 15$ [kN] the external applied load.

2. By applying Ritter's method to the bars AB, BE, and EF, it could be seen that bars AB and EF are both in a compression state with a load $P = -15$ [kN]. The respective stresses can be found as $\sigma_{AB} = \sigma_{EF} = -P/A = -15$ [kN]/2 [cm^2] $= -75$ [MPa].

3. Figure 1 shows the structure in undeformed and deformed configurations.

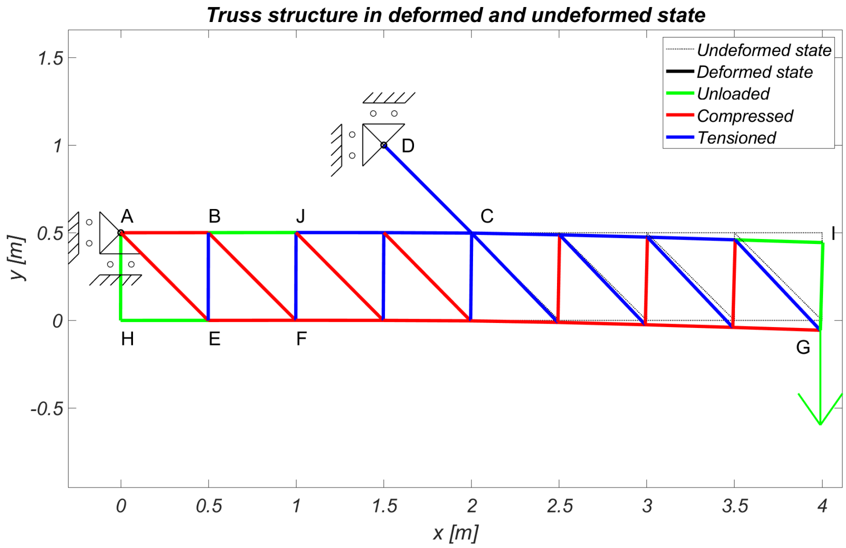


Figure 1: Truss structure under study in undeformed and deformed configurations

The equilibrium on nodes H and I determines that the bars connected to them are unloaded. Furthermore, bar BJ is not loaded, as Ritter's method confirms. The bar CD is in tension, as it has to provide the support reaction in D. Bars on the lower side are compressed because the structure tends to be displaced leftwards by the load, but this movement is constrained by the support reaction in A.

4. The application of the FEA code gives $R_{A,x} = 30$ [kN], $R_{A,y} = -15$ [kN], $R_{D,x} = -30$ [kN], and $R_{D,y} = 30$ [kN]. As the results from point 1.1 show, the reactions coincide.

5. Forces and stresses on bars AB and EF result to be equal and corresponding to $F_{AB} = F_{EF} = -P = -15$ [kN] and $\sigma_{AB} = \sigma_{EF} = -75$ [MPa], coinciding with the results of point 1.2.
   The bar stresses determine a safety factor against plasticity of $\nu = \sigma_{y,Al}/\sigma_{AB} = 4.67$, where $\sigma_{y,Al} = 350$ [MPa].

6. The resulting displacement vector of node G is $(x_G, y_G) = (-0.010, -0.056)$ [m]. Considering that the load is situated at the furthest side of the structure, the displacements' direction is reasonable. Furthermore, considering that the applied load P is quite high and that rotation around D is allowed, the obtained y displacement is a consistent result.

2 Exercise 2

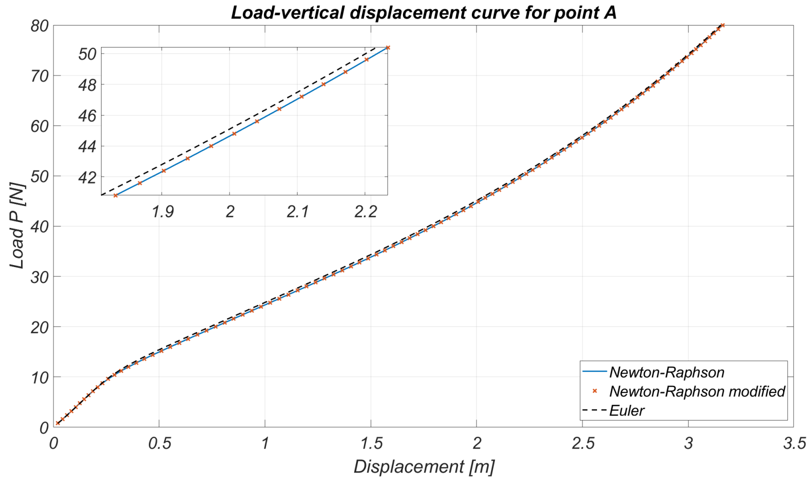


Figure 2: Load-Displacement curves for 100 load increments

1. Figure 2 shows the behaviour of the 3 methods. As the magnification highlights, Newton Raphson (NR) and Modified Newton-Raphson (MNR) are superimposed, whereas Euler method is clearly shifted from the others.

2. It is noted that the number of increments has two main effects. For the MNR method there is a threshold of about 30 increments under which the method does not properly work. For the Euler method, instead, the increase of the number of increments particularly influences the norm of the residuals, which initially is very large, but then starts decreasing up to stabilizing to a constant value. Therefore, with this method it is disadvantageous to use too much increments, because accuracy does not increase, while the computational cost does.

3. Given n load increments, Euler and MNR methods require n factorizations (i.e. 1 per each load increment), whereas the NR method requires at most $n \cdot i$ factorizations, where i is the number of iterations needed to the convergence of the norm of the residuals at each load increment. For the given problem, the order of magnitude of the norm of the final residuals is 10^0 for Euler, 10^{-12} for NR and 10^{-7} for MNR. The huge difference between Euler and the other two can be justified by the nature of the method: as Euler does not take any residual into account when proceeding, the others make them converging before doing a new step.

4. The final residual for Euler method is a bit larger than before. Instead, for the NR method the number of required factorizations is larger (estimated from 710 to 736). The MNR requires a higher number of iterations (estimated from 991 to 1034). This happens because with the new set of constants $E(\varepsilon)$ is more non-linear, implying that residuals of NR and MNR methods require more iterations to converge.

5. If the problem is small, NR method can be used, since the factorization is applied to a small [K] matrix. Instead, if the problem is large MNR is the best option, as it is accurate and requires the minimum number of factorizations. Its only downside is that it requires a larger number of increments in order to properly work. Euler's method can be applied in both cases but lacks in precision if steps are too small.

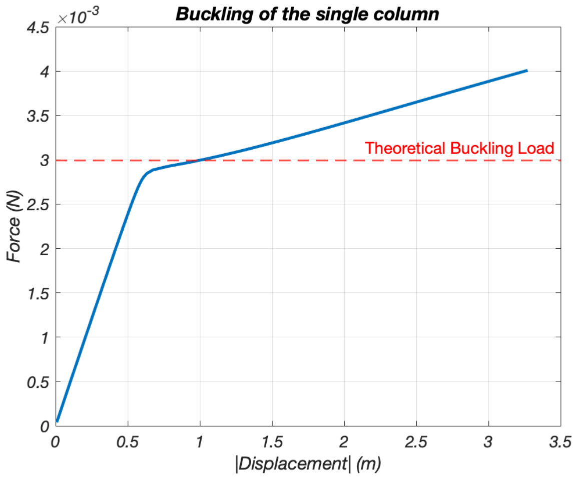
3 Exercise 3

1. A normal load of $P = 5 \cdot 10^{-5}$ [N] is applied in the middle of a roller-pin supported beam and the corresponding displacement of $\delta = 0.275$ [m] is measured. From beam theory: $EI = \frac{PH^3}{48 \cdot \delta} = \frac{5 \cdot 10^{-5} 80^3}{48 \cdot 0.275} = 1.94$ [N m^2]. Considering the same support configuration, the critical eulerian load for buckling is $P_{cr} = \frac{\pi^2 \cdot EI}{H^2} = 2.99 \cdot 10^{-3}$ [N].

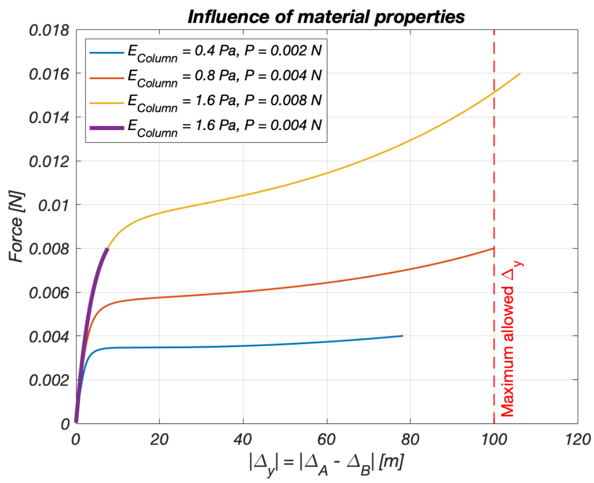
2. Theoretical and computed results are compatible. From the force-displacement plot of the column alone (Figure 3a), a lost in linearity could be seen from a load of $2.8 \cdot 10^{-3}$ [N] and a displacement of 0.61 [m].

3. As can be seen in Figure 3b for $E_C = 0.8$ [Pa] and $P = 0.004$ [N], the structure becomes non linear for a load larger than 0.004 [N]. The buckling of the whole structure happens at a higher load than the single column alone: the constraints provided by the beams to the column are less compliant to the rotation than in the case 3.1, decreasing its effective length and so increasing its critical load.

4. Figure 3b shows the force $F = 2 \cdot P$ plotted as a function of the relative displacement of points A ad B. It is to note that the stiffness of the beams remains constant, while the one of the column and the applied load are scaled as proposed.



(a) Force - Displacement of the column

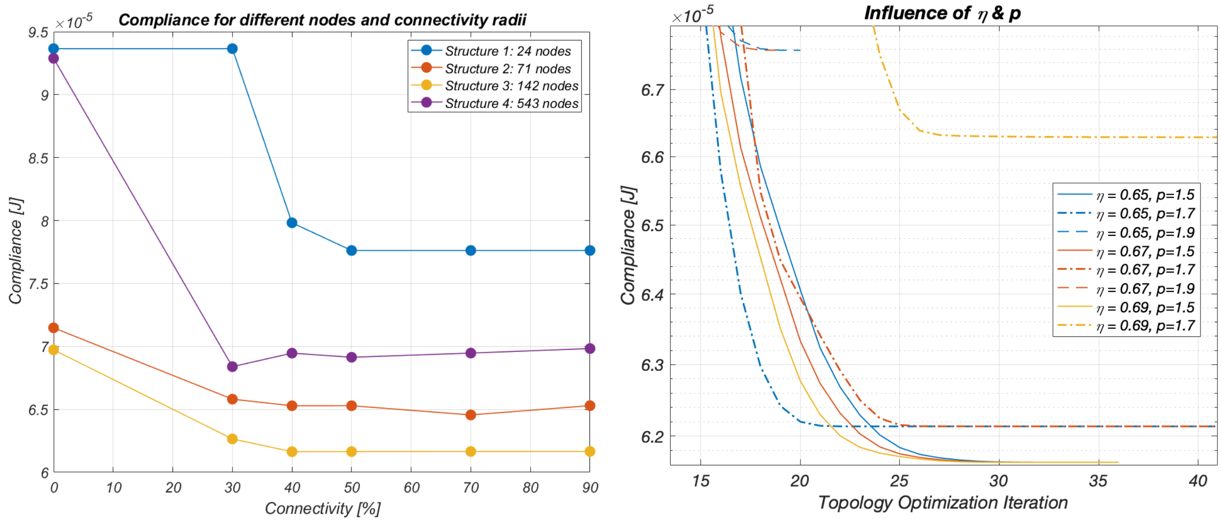


(b) Force - Displacement for different E_C/E_B

Figure 3: Force displacement curves for column alone (a) and for the structure (b)

The full load-displacement is always obtained when both Young's module and load are proportionally scaled. If only the former is changed, then the structure does not reach the buckling condition and only linear behaviour is visible (as can be seen for $E_C = 1.6$ [Pa] and $P = 0.004$ [N]).
When $E_C = 1.6$ [Pa] and $P = 0.008$ [N], the maximum Δ_y predicted from the model is higher than the initial distance between A and B. After $\Delta_y = 100$, the structure reaches a non physical configuration, but the model is not able to detect it, hence it proceeds to the maximum load.
The curvature when instability occurs and also the non-linear behaviour are not in similarity when the load and E_C are scaled. Since E_B remains constant, the beams provide the same bending stiffness and less deflection is allowed when loads are low.

4 Exercise 4



(a) Effects of connectivity and node number on the compliance

(b) Convergence plot for different penalization factors p and η

Figure 4: Compliance (a) and convergence (b) plots

1. For the given available area, 20 different structures have been tested using 5 connectivity percentages of specific radii for 4 node configurations. Figure 4a shows that the influence of the two parameters is not monotonic. Increasing the connectivity up to 40% the compliance decreases, while after that it stabilizes to a constant value. Increasing the number of nodes up to structure 3, the compliance improves, while for the fourth it is higher. For all the structures $\eta = 0.67$ and p=1.5 are used.

2. Structure 3 with 40% connectivity is tested as case study for all the combinations of $p = \{1.5, 1.7, 1.9\}$ and $\eta = \{0.65, 0.67, 0.69\}$. Results are reported in Figure 4b, and they highlight that the combination η=0.67 and p=1.5 is the best trade off between low compliance and fast convergence.
   For the tested η values, for p $\geq$ 1.8 the obtained compliance is too high or the method does not converge (case of η=0.69 and p=1.9 not visible in the plot). For $p < 1.3$ the method does not converge. Instead, for p $\in [1.4, 1.7]$ the results only differ for a small amount in compliance and number of iterations (except for the case η=0.69 and p=1.7). For p in this range, depending on the examined structure, the best compromise has to be chosen.

3. From the results of 4.1 the impossibility to break the record could not be demonstrated, although the obtained best value of C=$6.16 \cdot 10^{-5}$[J] is not too far from it. More freedom in the position of the nodes and constraints would be necessary in order to break the record. This would also allow to use bars with any length and angulations, instead of connecting only points of a fixed dimension grid. Indeed, as Figure 4a shows, the only increase of the number of nodes and connections is not sufficient to provide better results. It is supposed that the strategy to find the best solution is increasing the number of tested structures leaving also the possibility to move the nodes' position. Nevertheless, this would also require a higher computational effort.