Pstat 115 HW3

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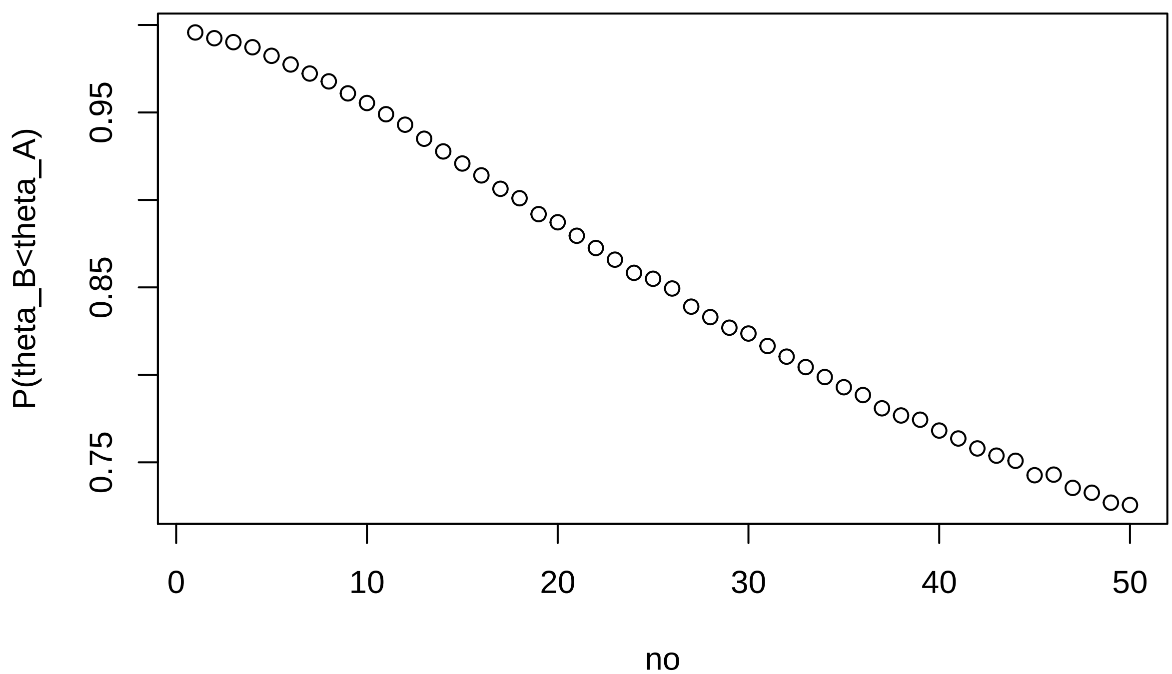
2018/10/27

1.

(a) By conjugation, we know that the posterior distributions are $\theta_A|y_A \sim gamma(120 + \sum y_A, 10 + 10)$ and $\theta_B|y_B \sim gamma(12 + \sum y_A, 1 + 13)$.

```
for(no in 1:50)
{
  gamma_A=rgamma(samples,120+sum(y_A),10+length(y_A))
  gamma_B=rgamma(samples,12*no+sum(y_B),no+length(y_B))
  diff_theta[no]=mean(gamma_A>gamma_B)
}
plot(diff_theta,xlab="no",ylab="P(theta_B<theta_A)")
```

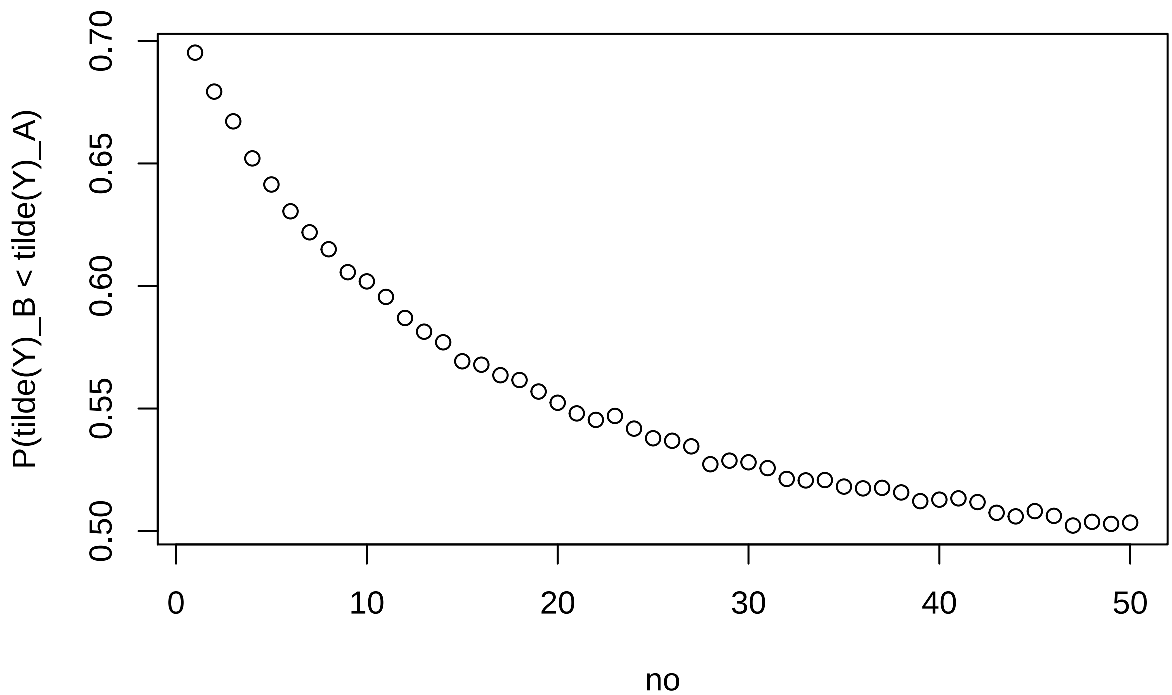


From the plot, we see that as n_0 increase,the probability of the event decrease. This shows that the event is sensitve to the prior distribution on θ_B since there is a linear relationship.

(b)

```
for(no in 1:50)
{
  gamma_A=rgamma(samples,120+sum(y_A),10+length(y_A))
  gamma_B=rgamma(samples,12*no+sum(y_B),no+length(y_B))
```

```
  y_tilda_a=rpois(samples,gamma_A)
  y_tilda_b=rpois(samples,gamma_B)
  diff_y[no]=mean(y_tilda_a>y_tilda_b)
}
plot(diff_y,xlab="no",ylab="P(tilde(Y)_B < tilde(Y)_A)")
```



We see that as n_0 increases, the probability of the event $\tilde{Y}_B < \tilde{Y}_A$ decreases. In addition, this event is more sensitive to n_0 then the event in part (a). We see that as $n_0 > 45$, the $P(\tilde{Y}_B < \tilde{Y}_A) \approx 0.5$.

(c) Calculating the probability of the event $\theta_B < \theta_A$ which the expection of mice B of the rate of tumor is lower than the expection of mice A of the rate of tumor. On the other hand, calculating the probability of the event $\tilde{Y}_B < \tilde{Y}_A$ which the rate of tumor of random mice B is lower than the rate of tumor of mice A.

2.

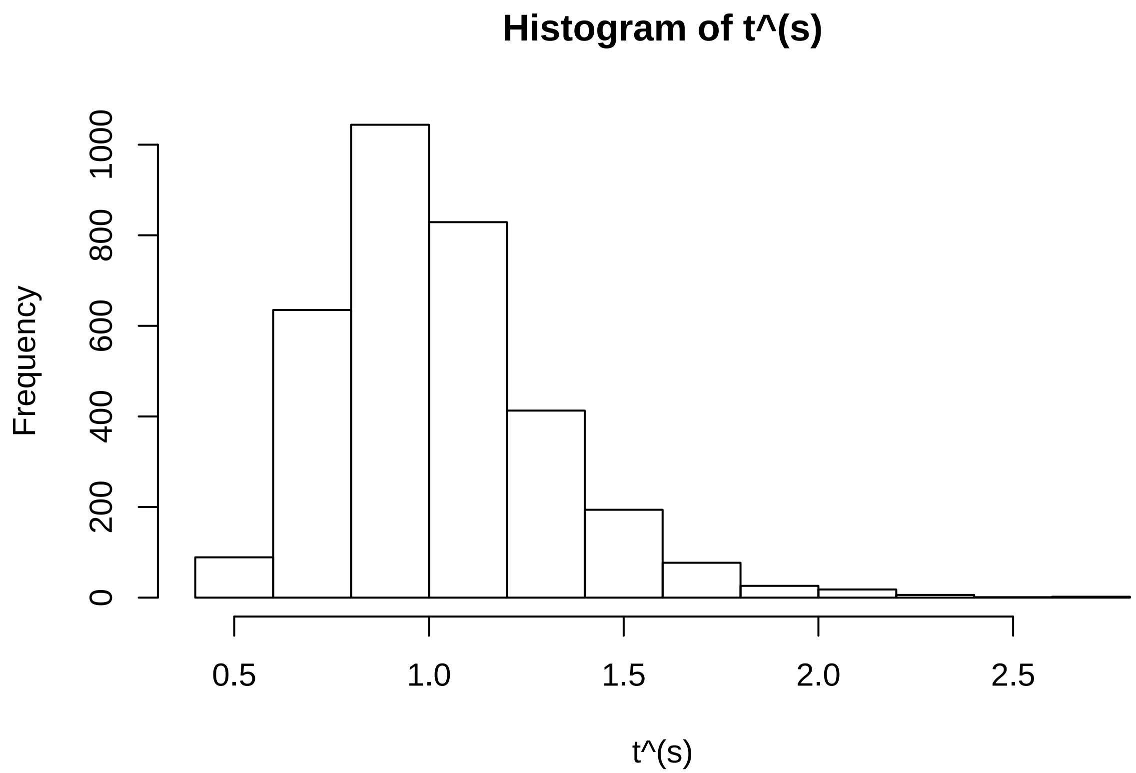
(a) If Poisson model was reasonable, then $t^{(s)}=1 $ because Poisson model has the same mean and vairance.

(b)

```
set.seed(1)
gamma_A=rgamma(samples,120+sum(y_A),10+length(y_A))
poisson_A=matrix(sapply(gamma_A,rpois,n=1),ncol=30)
```

```
## Warning in matrix(sapply(gamma_A, rpois, n = 1), ncol = 30): data length
## [100000] is not a sub-multiple or multiple of the number of rows [3334]
```

```
t_A=rowMeans(poisson_A)/apply(poisson_A,1,var)
hist(t_A,xlab="t^(s)",main="Histogram of t^(s)")
```




```
cat("In this historgram, 95% confidence interval is between (", quantile(t_A,0.025),",",quantile(t_A,0.9
```

```
## In this historgram, 95% confidence interval is between ( 0.5947732 , 1.72192 ).
```

```
cat("We see that 1 is in this confidence interval. In addition, ")
```

```
## We see that 1 is in this confidence interval. In addition,
```

```
cat("The mean of t^(s) is",mean(t_A), "which is close to the theoretical calculation for Poisson distrib
```

```
## The mean of t^(s) is 1.0213 which is close to the theoretical calculation for Poisson distribution
```

```
cat("It shows that this model is reasonable to use Poisson distribution.")
```

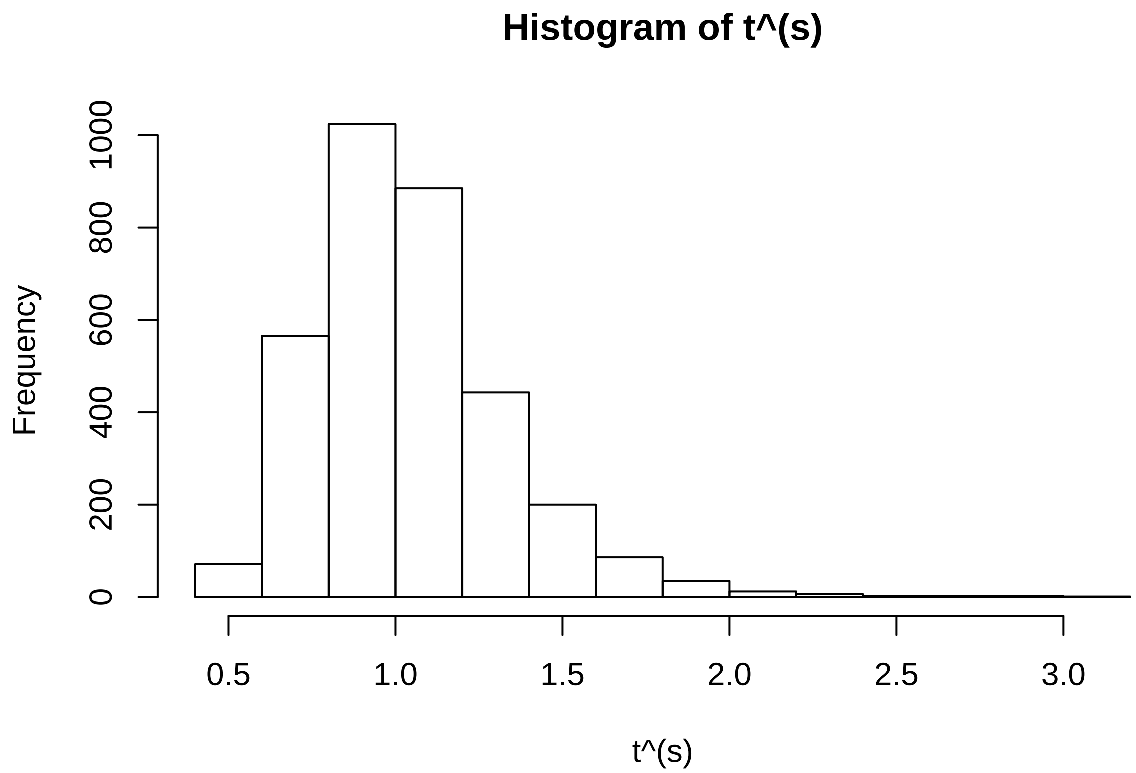
```
## It shows that this model is reasonable to use Poisson distribution.
```

(c)

```
gamma_B=rgamma(samples,156+sum(y_B),13+length(y_B))
poisson_B=matrix(sapply(gamma_B,rpois,n=1),ncol=30)
```

```
## Warning in matrix(sapply(gamma_B, rpois, n = 1), ncol = 30): data length
## [100000] is not a sub-multiple or multiple of the number of rows [3334]
```

```
t_B=rowMeans(poisson_B)/apply(poisson_B,1,var)
hist(t_B,xlab="t^(s)",main="Histogram of t^(s)")
```




```
cat("In this historgram, 95% confidence interval is between (", quantile(t_B,0.025),",",quantile(t_B,0.9
```

```
## In this historgram, 95% confidence interval is between ( 0.6165605 , 1.722591 ).
```

```
cat("We see that 1 is in this confidence interval. In addition, ")
```

```
## We see that 1 is in this confidence interval. In addition,
```

```
cat("The mean of t^(s) is",mean(t_B), "which is close to the theoretical calculation for Poisson distrib
```

```
## The mean of t^(s) is 1.043685 which is close to the theoretical calculation for Poisson distribution
```

```
cat("It shows that this model is reasonable to use Poisson distribution.")
```

```
## It shows that this model is reasonable to use Poisson distribution.
```