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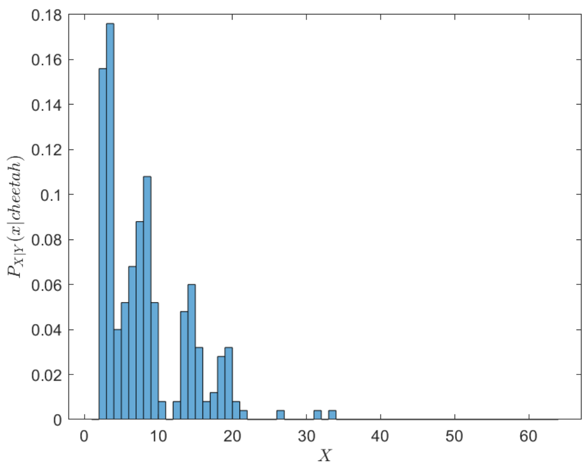
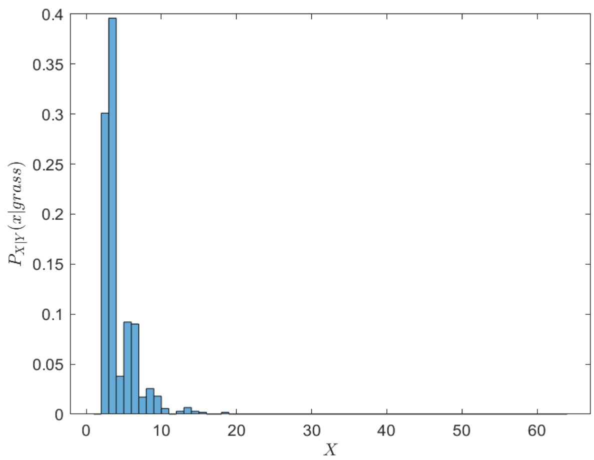
a. To perform Bayesian analysis, we calculate the prior probability of two features, cheetah (foreground) and grass (background). From the training data *TrainingSamplesDCT_8.mat*, we have two samples, *TrainsampleDCT_BG* and *TrainsampleDCT_FG,* representing foreground and background, respectively. In each row of the sample, a 64-dimensional vector is transformed from 8x8 matrix after computing the discrete cosine transform. Originally, for each vector, feature belongs to the foreground would be placed in the foreground sample, while feature belongs to the background would be placed in the background sample. Therefore, in the training sample, the more rows in the set represent more probability of having such feature in the testing sample.

$$P_Y(cheetah) = \frac{row\ size\ of\ cheetah}{total\ row\ size} \approx 0.19$$

$$P_Y(grass) = 1 - P_Y(cheetah) = \frac{row\ size\ of\ grass}{total\ row\ size} \approx 0.81$$

b. From the training data *TrainingSamplesDCT_8.mat*, we take the histogram of X versus $P_{X|Y}(x|cheetah)$ and X versus $P_{X|Y}(x|grass)$, where X is the index of the DCT coefficient with 2nd greatest energy and is the integer value between 2 and 64.





c. We compute an array of feature X from the image *cheetah.bmp* by using 8x8 block. We also use padding technique on the array converted from image to have a dimension of the array of feature X equivalent to the size of image. From lecture, we know the MAP rule has formula,

$$g^*(x) = \arg\max_i P_{Y|X}(i \mid x) = \arg\max_i P_{X|Y}(x \mid i) P_Y(i)$$

, where g*(x) is the maximum a-posteriori probability rule. Since we only have two category and x is an integer between 2 and 64, below is the list of posterior probability with optimal decision.

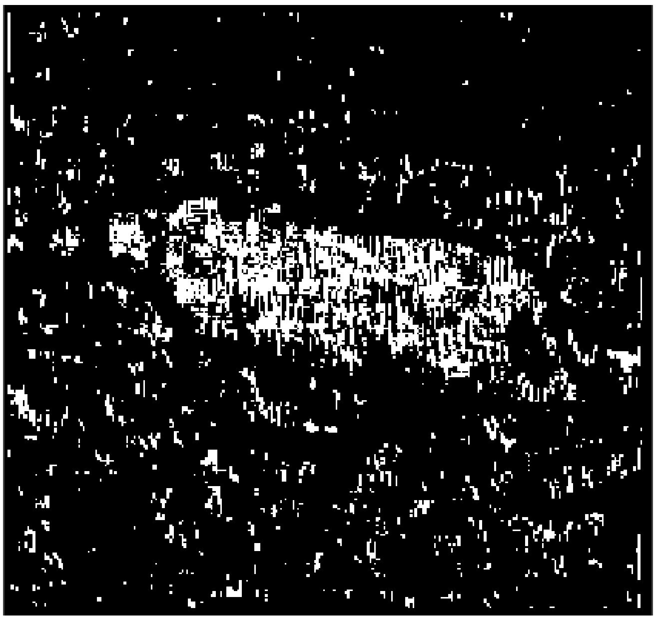
x	2	3	4	5	6	7	8	9	10
P(x|0)P(0)	0.2433	0.32	0.0307	0.0744	0.0729	0.0138	0.0207	0.0146	0.0046
P(x|1)P(1)	0.0299	0.0338	0.0077	0.01	0.0131	0.0169	0.0207	0.01	0.0015
Decision	*x*=0	*x*=0	*x*=0	*x*=0	*x*=0	*x*=1	*x*=0	*x*=0	*x*=0
x	12	13	14	15	16	17	18	19	20
P(x|0)P(0)	0.0023	0.0054	0.0023	0.0015	0	0	0.0015	0	0
P(x|1)P(1)	0.0015	0.0092	0.0115	0.0061	0.0015	0.0023	0.0054	0.0061	0.0015
Decision	x=0	*x*=1	*x*=1	*x*=1	*x*=1	*x*=1	*x*=1	*x*=1	*x*=1

x	21	26	31	33
P(x|0)P(0)	0	0	0	0
P(x|1)P(1)	0.0008	0.0008	0.0008	0.0008
Decision	*x*=1	*x*=1	*x*=1	*x*=1

Using this classification table, we can generate an image from the array of feature *X.* Notice that if the value of X is not in the classification table, the default decision will classify it as background as it indicates both *P(x|0)P(0)* and *P(x|1)P(1)* have 0 probability.

d. Given the truth image that separate background and foreground, we can calculate the probability of error. Formula for probability of error is

$$Error = \sum_{i=0}^{1} P_Y(i) \frac{amount\ \ of\ \ incorrect\ \ pixel\ \ of\ \ i}{size\ \ of\ \ i\ \ of\ \ truth} = 0.1722$$



Appendix

```
load('TrainingSamplesDCT_8.mat');

%A
pri_f=size(TrainsampleDCT_FG,1)...
    /(size(TrainsampleDCT_BG,1)+size(TrainsampleDCT_FG,1));
pri_b=1-pri_f;

%B
%take absolute vlalues
bg=abs(TrainsampleDCT_BG);
fg=abs(TrainsampleDCT_FG);
%find 2nd largest
sortb= sort(bg,2, 'descend');
sortf= sort(fg,2,'descend');
%disp(size(sortb,1)
for i=1:size(bg,1)
    for j=1:size(bg,2)
        if(sortb(i,2)==bg(i,j))
            b_ind(i)=j;
        end
    end
end
for i=1:size(fg,1)
    for j=1:size(fg,2)
        if(sortf(i,2)==fg(i,j))
            f_ind(i)=j;
        end
    end
end
b_hist=histogram(b_ind,1:size(bg,2),'Normalization','pdf');
xlabel({'$X$'},'Interpreter','latex');
ylabel({'$P_{X|Y}(x|grass)$'},'Interpreter','latex');
figure();
f_hist=histogram(f_ind,1:size(fg,2),'Normalization','pdf');
xlabel({'$X$'},'Interpreter','latex');
ylabel({'$P_{X|Y}(x|cheetah)$'},'Interpreter','latex');
```

```
%C
zig=load('Zig-Zag Pattern.txt')+1;
cheetah= im2double(imread('cheetah.bmp'));
%clear cheetah_p
cheetah_p=padarray(cheetah,[4,4],0,'pre');
cheetah_p=padarray(cheetah_p,[3,3],0,'post');
for i=1:size(cheetah_p,1)-7
    for j=1:size(cheetah_p,2)-7
        cheetah_dct=abs(dct2(cheetah_p(i:i+7, j:j+7)));
        for k=1:64
            [z1,z2]=find(zig==k);
            X_dct(k)=cheetah_dct(z1,z2);
        end
        sortX=sort(X_dct, 'Descend');
        for m=1:64 %find feature X
            if(X_dct(m)==sortX(2))
                X(i,j)=m;
            end
        end
    end
end

%Using MAP
clear b_prob    f_prob
b_prob=histcounts(b_ind,1:65)/size(TrainsampleDCT_BG,1);
f_prob=histcounts(f_ind,1:65)/size(TrainsampleDCT_FG,1);

for i=1:size(cheetah,1)
    for j=1:size(cheetah,2)
        %disp(X(i,j))
        if(b_prob(X(i,j))*pri_b>=f_prob(X(i,j))*pri_f)
            final(i,j)=0;
        else
            final(i,j)=1;
        end
    end
end

figure();
```

```
imshow(final)

for i=2:64
	post(1,i)=b_prob(i)*pri_b;
	post(2,i)=f_prob(i)*pri_f;
end

%D, calcuate error and Bayes Error (Risk)
truth=imread('cheetah_mask.bmp');
truth=im2double (truth);
err=0;

for i=1:size(truth,1)
	for j=1:size(truth,2)
		if (final(i,j)~= truth(i,j))
			err=err+1;
		end
	end
end
```