LINEAR PROGRAMMING: LETTING COMPUTERS MAKE OUR DECISIONS

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What are we trying to do with analytics?

Every day in the media a lot of tech-like terms are being used: analytics, big data, artificial intelligence.

However, it may not be clear how this methods and technologies may help people in their everyday life.

Are we trying to build robots that will destroy human kind? Are we building apps that read the human mind? Will we replace all human workforce with robots?

We are using data in to take better and more effective decisions


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The New York Times

NEWS ANALYSIS

The Age of Big Data

By Steve Lohr

What can we do with analytics?

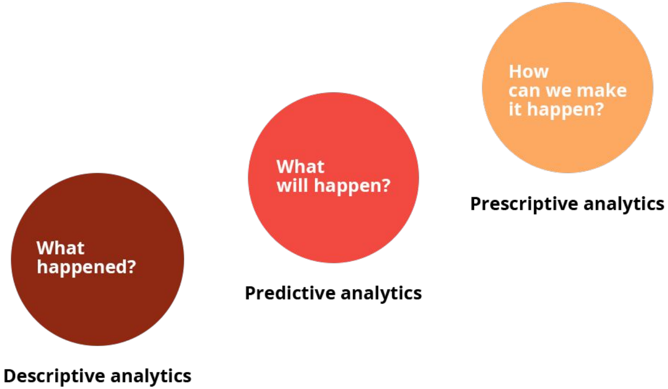
Different types of tasks can be tackled with analytics techniques. Let’s define them and give an example

Descriptive analytics □ How much did my sales grow in the last month?

Predictive analytics □ How are my sales going to be the next month?

Prescriptive analytics □ How many units should I produce in order to maximize profits?

Linear programming, Dynamic programming, etc.



LINEAR PROGRAMMING

A brief definition

“

Linear programming implies the abstraction of a problem or system that captures its behavior through mathematical relationships

Linear programming ≠ Coding

In this case programming means **planification**, not coding

We are trying to represent a problem or system with mathematical variables and relationships

We are trying to **optimize** something (e.g. minimize cost, maximize profits) making the best possible decision

This problems can be solved in a variety of interfaces:

- Complex problems are usually solved via code and specific libraries & APIs
- However, simpler problems could be solved with Excel's Solver or even pen and paper

Linear programming key concepts

Decision variables

What can be changed to optimize the objective function

E.G. Production quantities

Constraints

The conditions the solution has to meet

E.G. Don’t produce more than the factory’s capacity

Objective Function

The objective that has to be optimized

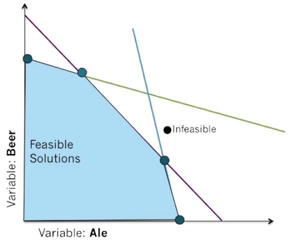
E.G. Profit formula

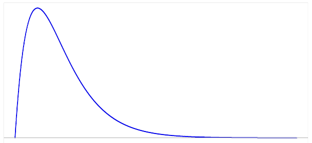
The constraints and objective función are linear □ Variables can´t be multiplied by another variable (e.g. x^2). They can be multiplied by a constant

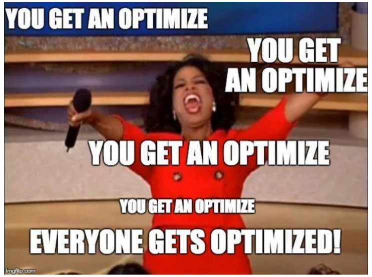
Linear programming pipeline

Define variables → Define constraints → Define obj function → Implement & Optimize! → Analyze the solution

x_i
Production of product i







What production mix maximizes profits

Cartoon equivalents to linear programming


MY MOM SAYS THIS IS TOO ABSTRACT
CAN WE SEE AN EXAMPLE?
imgflip.com

MIXED INTEGER[1] LINEAR PROGRAMMING

A practical example

[1]This is a Mixed Integer Linear Problem as it also has binary variables

The setup

After the Coronavirus pandemic, the Argentine Football Association (AFA) is organizing a small 4-team tournament between:

- Aldosivi (ALD)
- Godoy Cruz (GCM)
- Central Córdoba (CCO)
- Talleres (TAL)

The tournament will last two weeks and, after every match, the teams don't return to their home (they travel to the next match's home stadium)

The objective is to set up a schedule that minimizes the total distance traveled

Variable definition

Define variables → Define constraints → Define obj function → Implement & Optimize! → Analyze the solution

$$n \rightarrow Number\ of\ teams$$
$$m \rightarrow Number\ of\ matches\ per\ team = 2n - 2$$
$$r \rightarrow Number\ of\ matches\ per\ team\ in\ the\ first\ round = n - 1$$
$$N \rightarrow Set\ of\ teams. N = \{1,2,\dots,n\}$$
$$T \rightarrow Set\ of\ matchdays. T = \{1,2,\dots,m\}$$

$$x_{ijt} \rightarrow A\ binary\ variable\ that\ is\ equal\ to\ 1\ if\ team\ i\ plays\ at\ home\ against\ team\ j\ in\ match\ t$$
$$y_{ijkt} \rightarrow A\ binary\ variable\ that\ is\ equal\ to\ 1\ if\ team\ i\ travels\ from\ location\ j\ to\ k\ after\ match\ t$$
$$\overline{d_{jk}} \rightarrow Distance\ between\ teams\ j\ and\ k\ (This\ is\ a\ constant\ - not\ a\ decision\ variable)$$

Constraint definition

Define variables → **Define constraints** → Define obj function → Implement & Optimize! → Analyze the solution

$$x_{iit} = 0 \quad \forall\, i \in N, \forall\, t \in T \quad (1)$$

Teams can't play against themselves

$$\sum_{j,j\neq i} x_{ijt} + \sum_{j,j\neq i} x_{jit} = 1 \quad \forall\, i \in N, \forall\, t \in T \quad (2)$$

Teams play against one team per matchday

$$\sum_{t,t\leq r} x_{ijt} + \sum_{t,t\leq r} x_{jit} = 1 \quad \forall\, i,j \in N, i \neq j \quad (3)$$

Teams play once against each rival in the first round

$$\sum_{t} x_{ijt} + \sum_{t} x_{jit} = 2 \quad \forall\, i,j \in N, i \neq j \quad (4)$$

Teams play twice against each rival in the tournament

$$\sum_{t} x_{ijt} = 1 \quad \forall\, i,j \in N, i \neq j \quad (5)$$

Teams can't play in their stadium against the same team twice

$$\left.\begin{aligned}
y_{ijkt} &\geq x_{jit} + x_{kit+1} - 1 \quad \forall\, i,j,k \in N, \forall\, t \in T, t < m \quad (6)\\
y_{iikt} &\geq x_{ijt} + x_{kit+1} - 1 \quad \forall\, i,j,k \in N, \forall\, t \in T, t < m \quad (7)\\
y_{ijit} &\geq x_{jit} + x_{ikt+1} - 1 \quad \forall\, i,j,k \in N, \forall\, t \in T, t < m \quad (8)\\
y_{iiit} &\geq x_{ijt} + x_{ikt+1} - 1 \quad \forall\, i,j,k \in N\ \forall\, t \in T, t < m \quad (9)
\end{aligned}\right\}$$

These constraints relate the x and y variables

Objective function definition

Define variables → Define constraints → **Define obj function** → Implement & Optimize! → Analyze the solution

$$\min \sum_i \sum_j \sum_k \sum_t \overline{d_{jk}} * y_{ijkt} \rightarrow \text{Total distance}$$

Solution Analysis

Define variables → Define constraints → Define obj function → Implement & Optimize! → **Analyze the solution**

Generated Schedule

Matchday	ALD	GCM	CCO	TAL
1	@CCO	TAL	ALD	@GCM
2	@TAL	CCO	@GCM	ALD
3	GCM	@ALD	TAL	@CCO
4	@GCM	ALD	@TAL	CCO
5	TAL	@CCO	GCM	@ALD
6	CCO	@TAL	@ALD	GCM

Total Distance
15.604 km

Advantages of Linear Programming

Using a model that is an abstraction of reality allows the following things:

- Adapting this model for other applications (e.g vehicle routing)
- If new constraints appear or other objective function should be used, the model can be easily adapted

So, let's change the model!

Changing the model

Instead of minimizing total distance, AFA wants to maximize the tournament's interest in the last matches of the tournament. In order to do this, they have created a metric p_{ijt} that evaluates the interest generated by a match played by teams i and j, played in matchday t, which uses the position of each team in last year's tournament (the position of team i is defined by the constant pos_i).

$$p_{ijt} = \frac{t}{1 + |pos_i - pos_j|}$$

So, how does the formulation of the problem change?

- Constraints (6) to (9) disappear as we are no longer considering distance
- The objective function is changed. The new objective function is equal to:

$$\max \sum_i \sum_j \sum_t x_{ijt} * p_{ijt}$$

Changing the model - results

New Generated Schedule

Matchday	ALD	GCM	CCO	TAL
1	@TAL	@CCO	GCM	ALD
2	@CCO	@TAL	ALD	GCM
3	GCM	@ALD	TAL	@CCO
4	TAL	CCO	@GCM	@ALD
5	CCO	TAL	@ALD	@GCM
6	@GCM	ALD	@TAL	CCO

Objective Function	Total Distance	Total Interest
Distance Minimization	15.604 km	4.93
Interest Maximization	19.392 km	5.41

RESOURCES

What is Operations Research?

Operations Research and Optimization: Improving Decisions from Data

An Application of the Traveling Tournament Problem: The Argentine Volleyball League

Scheduling the Chilean Soccer League by Integer Programming

Using Sports Scheduling to Teach Integer Programming

OR Tools Introduction for Python

Thanks!

Any questions?

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