Rescheduling the NBA regular season via Integer Programming

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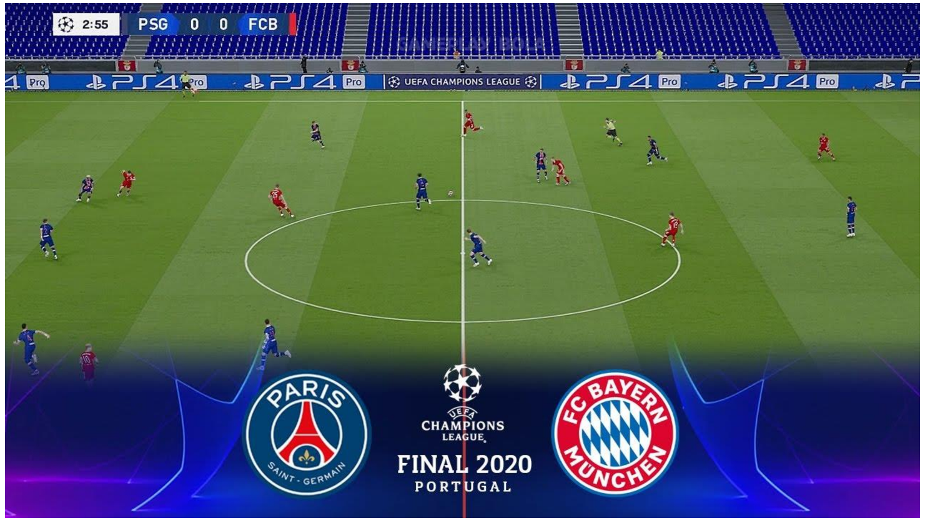
Impact in sport leagues around the world



UEFA Champions League Final (2020)

NBA Bubble (2019 - 2020)

Rescheduling matches pre and post COVID

Pre-COVID	COVID
• Suspended matches are **rare** • Suspensions are usually **not related** to each other	• Suspended matches are **more common** due to the health situation • If multiple players become sick, **consecutive games** might be suspended

34%

Probability of a match being suspended in the NBA 2020-21 season, given than the previous match was suspended

Rescheduling in sports

Rescheduling necessities

Literature

Two types of responses

- **Proactive**
- **Reactive**

Yi et al (EJOR, 2020) present proactive and reactive strategies for time-constrained schedules

Yi et al (ANOR, 2022) present proactive and reactive strategies for time-relaxed schedules

Practice

Leagues like NBA, NHL and MLB have rescheduled games, but haven't disclosed their methodological approach

The 2020 - 2021 NBA Season

The NBA applied the following contingency actions during the season:

- **Proactive scheduling strategy:**
    1. Schedule only the first half of the season (All-Star Game);

    Q: Is there a way to see a breakdown of the schedule for 2020-21?
    Yes, the 2020-21 regular season opponent matrix for each NBA team is available here. Full team-by-team schedules for the first half of 2020-21 are here.

    1. Postpone games if a teams has insufficient players to the second half;
    2. Generate a schedule for the second half + suspended games
    3. Ad-hoc reschedule for games suspended during the second half.

72

Games per team

31

Suspended games

3%

Games suspended

Our approach: a reactive strategy

Definition

Each game in the executed timetable that is played before or after its scheduled round is considered a **disruption**. In our setup, each suspended game will therefore translate into a disruption that needs to be rescheduled in the remaining of the season schedule

Research Question

Can we systematically generate an ***adjusted fixture*** by rescheduling postponed games within the planned schedule?

Approach: a reactive strategy

- Consider as input the **planned schedule** for the entire season.
- Insert the disrupted matches onto the existing schedule, maintaining the original scheduled dates for non-disrupted matches
- We build a linear optimization model to find the best possible date for each disruption and, if we can't, we reschedule it after the end of the schedule's original date

Notation and Definitions

<u>General</u>

Set of teams: $S = \{1, \ldots, m\}$

Set of original rounds: $T = \{1, \ldots, r\}$

Match between j and k: $(j,\ k)$

Scheduled game: Match between j and k on round t: $\alpha =< (j,\ k),\ t >$

<u>Schedule Rules</u>

Scheduled games of team i: R_i

Disrupted games of team i: R_i^{dis}

Total disrupted games: $R^{dis} = \cup_{i \in S} R_i^{dis}$

Potential candidate variables dates for match $\alpha = T_\alpha^{free}$

Maximum number of games that a team can play within every window of $t_2 - t_1$ days: $MG_{t_1.t_2}$

Number of non-disrupted games a team is playing between dates t_2 and t_1: $k_{t_1.t_2}^i$

Decision variables

$x_{\alpha t} = 1$ iff disrupted game $\boldsymbol{\alpha}$ is rescheduled into round t

Mathematical Model (MinD)

Minimize total day difference with the original schedule

$$\min \sum_{\alpha \in R^{\text{dis}}} \sum_{t \in T_\alpha^{\text{free}}} d_{\alpha t} x_{\alpha t} \qquad (1)$$

Games with feasible dates are rescheduled

$$\text{s.t.} \sum_{t \in T_\alpha^{\text{free}}} x_{\alpha t} = 1 \qquad \forall \alpha \in R^{\text{dis}} \qquad (2)$$

$$\sum_{\alpha \in R_i^{\text{dis}}} \sum_{\substack{t_1 \leq t \leq t_2, \\ t \in T_\alpha^{\text{free}}}} x_{\alpha t} + k_{t_1,t_2}^i \leq MG_{d_{t_1,t_2}} \qquad \forall t_1, t_2 \in T, 1 \leq t_2 - t_1 \leq 7, i \in S \qquad (3)$$

$$x_{\alpha t} \in \{0,1\} \qquad i \in R^{\text{dis}}, t \in T_\alpha^{\text{free}} \qquad (4)$$

Satisfy maximum number of games in $[t_1, t_2]$

Preliminary experiments

- Rescheduled dates for postponed games are not considered as part of the input schedule (except for some special configurations)

- We consider the following metrics:
  - Classical from scheduling: distance, breaks.
  - New: # of day added and # of games played after the end of the season in the **planned schedule**.

- Models and algorithms implemented in Python + CPLEX.

Rescheduling Strategies

- In order to get a better understanding of the performance of our solution we try to replicate the rescheduling process for the suspended games of the 2020-21 NBA season

Rescheduling methodology

Post All-Star

Dec-Jan-Feb	Mar	Apr	May

Wait until the All-Star break to reschedule first set of matches, then follow the monthly approach

At the end of each month, we reschedule the disruptions that happened there.
We consider the rest of the sesason.

Post All Star*

Use the NBA’s reschedules if they happened within the month of the original game

Main Results

Number of additional days needed after the end of the season

Metric	NBA exec	Post All Star	Post All Star*
Distance	-0.2%	0.4%	0.4%
Breaks	0.6%	0.2%	0.7%
# Dates Added	-	6	3
Games After	-	7	3
Exec. Time		8.85 min	9.29 min

A

Number of games scheduled after the end of the season

B

Key Takeouts

A All differences are small (around 1%)

B The NBA's proactive approach has effects on the additional number of dates needed

Further Disruption Scenarios

We consider instances stressing the the number and timing of the disruptions:

- **15 more games:** ~50% increase in the problem size
- **25 more games:** ~80% increase in the problem size
- **15 more March games:** Similar to a second COVID wave

Instance / Metric	Distance	Breaks	# dates added	Games after	Exec. Time
15 more games	3.8%	-0.7%	13	21	10.41 min
25 more games	5.0%	-0.3%	13	26	12.91 min
15 more games in March	1.6%	0.2%	10	22	10.06 min

A

Difference against the planned NBA schedule, post All-Star strategy

A Effects on relevant KPIs is not negligible

How can we improve the existing solution?

Given the current result, specially the effect on relevant KPIs of bigger instances, we might consider that a different approach is necessary:

Example

Day	1	2	3	4	5	6
Match						

If we wanted to schedule a disruption, we would have to do it on day 6

Day	1	2	3	4	5	6
Match						

In this sense, we can think about matches **swaps and switches!**

If we could move the game in day 3 to day 2, we could complete the schedule in 5 days

How can we adapt the existing model?

Additional variables

- Non Disrupted Games of Team i: R_i^{ND}
- Set of rounds post-tournament: $TP = \{r+1, \dots r+181\}$
- Maximum days of difference between match date and original date for non disruptions: d
- Original date for non disrupted match $\bar{\alpha}$: $\tau_{\bar{\alpha}}$
- Potential candidate dates for non disrupted match $\bar{\alpha}$: $T_{\bar{\alpha}}^{free} = \{\tau_{\bar{\alpha}} - d, \dots \tau_{\bar{\alpha}} + d\}$
- Tours of team i: $TO_i = \{TO_{i1}, TO_i\}$
- Tour n of team i: $TO_{i1} = \{\overline{\alpha_{n1}, \alpha_{n2}, \dots}\}$
- Maximum non disruptions whose date can be modified within a tour: td
- Variable that indicates that non disrupted match $\bar{\alpha}$ is scheduled on round t: $\overline{x_{\alpha t}}$

New mathematical model

$$\min \sum_{\alpha \in R^{DIS}} \sum_{t \in T_\alpha^{free}} d_{\alpha t} * x_{\alpha t} + \sum_{\alpha \in R^{DIS}} \sum_{t \in TP} 100 * d_{\alpha t} * x_{\alpha t} \tag{5}$$

$$s.t \sum_{t \in T_\alpha^{free}} x_{\alpha t} + \sum_{t \in TP} x_{\alpha t} = 1 \qquad \forall\, \alpha \in R^{DIS} \tag{6}$$

$$\sum_{t \in T_{\bar{\alpha}}^{free}} \overline{x_{\alpha t}} = 1 \qquad \forall\, \bar{\alpha} \in R^{ND} \tag{7}$$

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$$\sum_{\bar{\alpha} \in TO_{in}} \sum_{\substack{t \in T_{\bar{\alpha}}^{free} \\ t \neq \tau_{\bar{\alpha}}}} \overline{x_{\alpha t}} \le td \qquad \forall\, n \in TO_i, i \in S \tag{9}$$

$$x_{\alpha t}, \overline{x_{\alpha t}} \in \{0; 1\} \tag{10}$$

New mathematical model

Minimize Day Difference Between Original and Final Date

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All disruptions should be scheduled in existing or extra dates

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All non disruptions should be scheduled in possible dates

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Schedule rules must be followed

$$\sum_{\alpha \in R_i^{DIS}} \sum_{\substack{t \in T_{\bar{\alpha}}^{free} \\ t_1 \le t \le t_2}} x_{\alpha t} + \sum_{\bar{\alpha} \in R_i^{ND}} \sum_{\substack{t \in T_{\bar{\alpha}}^{free} \\ t_1 \le t \le t_2}} \overline{x_{\alpha t}} \le MG_{d_{t_1 t2}} \qquad \forall\, t_1, t_2 \in T \cup TP, 1 \le t_2 - t_1 \le 7, i \in S \qquad (8)$$

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No more tan *td* non disruptions can be changed per tour

$$\sum_{\bar{\alpha} \in TO_{in}} \sum_{\substack{t \in T_{\bar{\alpha}}^{free} \\ t \ne \tau_{\bar{\alpha}}}} \overline{x_{\alpha t}} \le td \qquad \forall\, n \in TO_i, i \in S \qquad (9)$$

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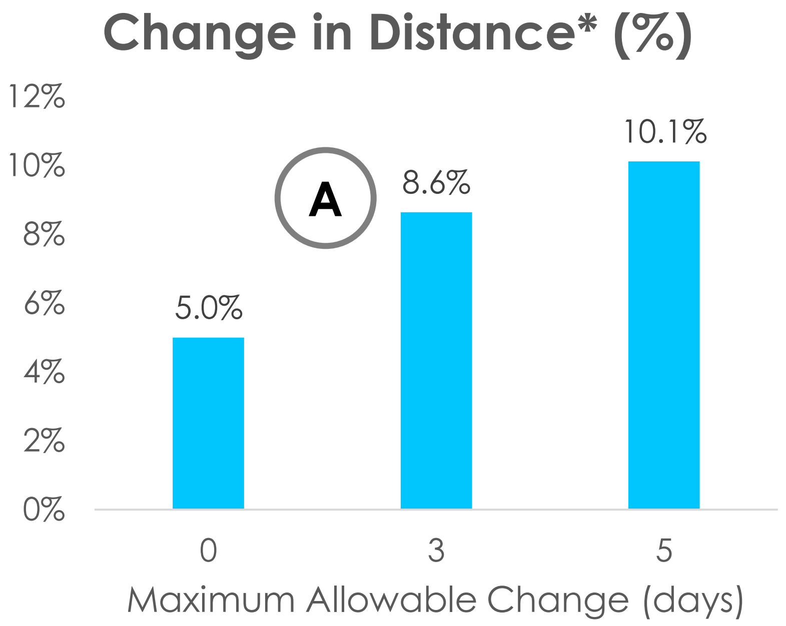
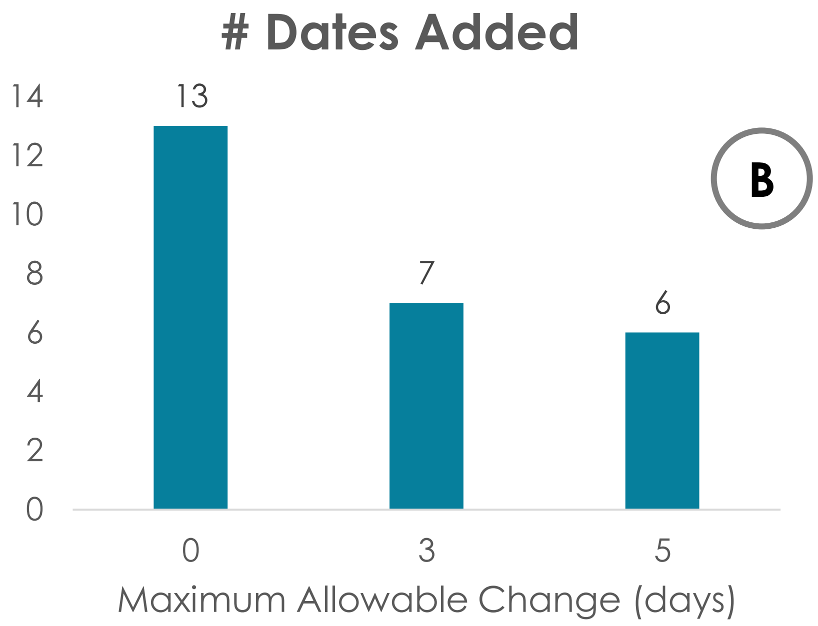
Variable domain

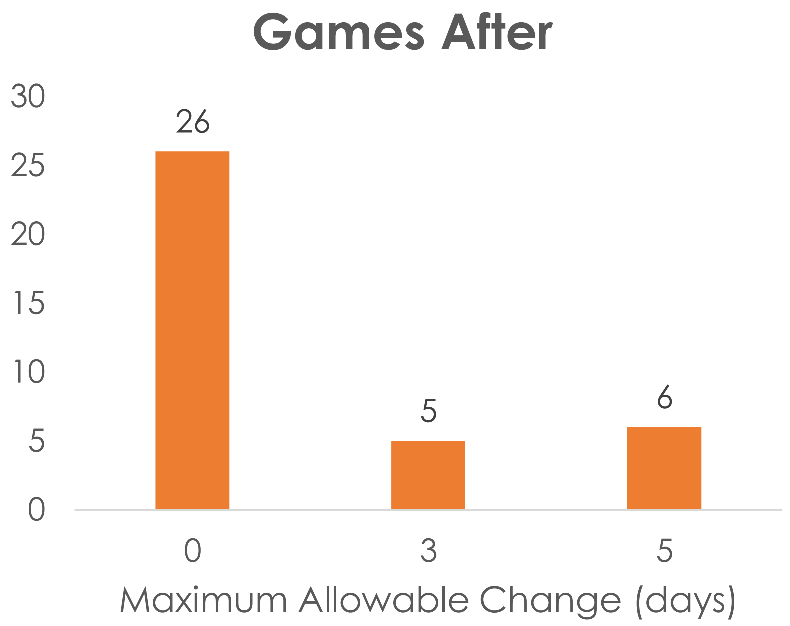
$$x_{\alpha t}, \overline{x_{\alpha t}} \in \{0; 1\} \quad (10)$$

Effects on relevant KPIs

To get an understanding on the effect of this possible changes, let's analyze the results with a Post All Star scheduling strategy with our biggest instance: original disruptions + 25 new ones:







A When we increase the potential adjustment, logically, total distance increases

B However, the number of additional dates needed to complete the tournament decreases

* Against original planned schedule

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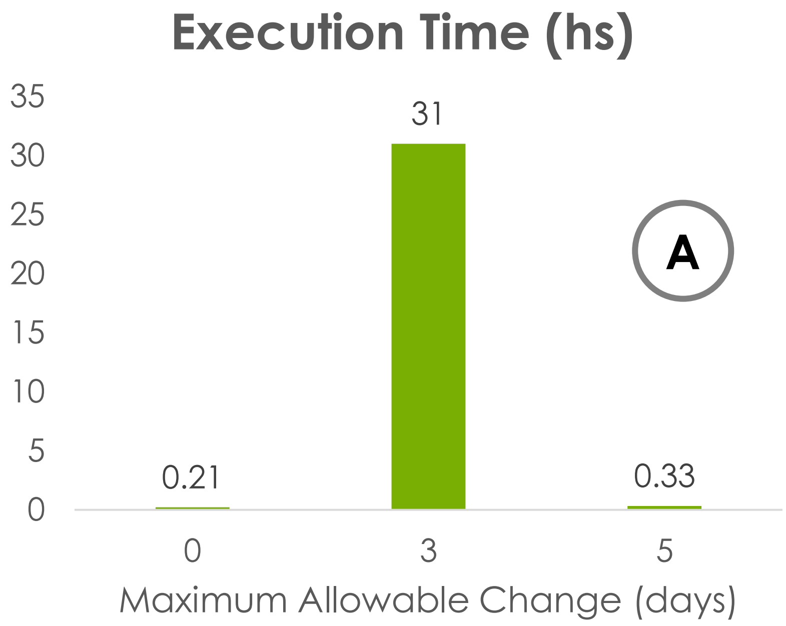

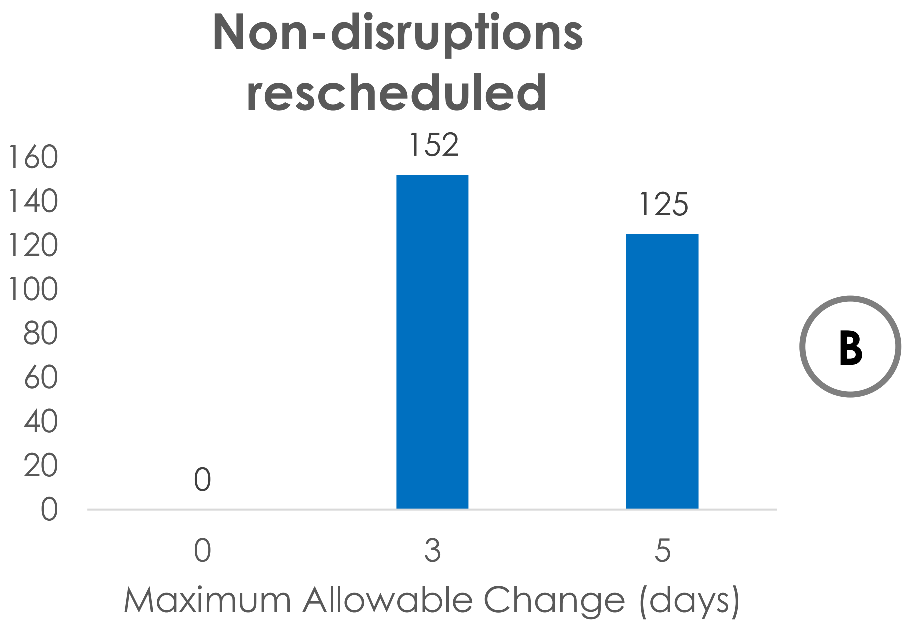

A The possibility to reschedule more games increases complexity, although relationship is not clear

B As we are not limiting the number of total reschedules, this results in approximately 15% of total games being changed

* Against original planned schedule

A shortcoming of this approach

Example: Original Schedule

	Home Tour			Away Tour		
Match	**1**	**2**	**3**	**4**	**5**	**6**
Rival	A	B	C	@D	@E	@F

Changed Schedule

Day	**1**	**2**	**3**	**4**	**5**	**6**
Match	A	@D	C	@E	B	@F

Our current approach might change the tour order, **increasing the total distance travelled**

Summary

- We framed the problem of rescheduling in time-relaxed tournaments
- We evaluate different rescheduling strategies building on MIP models
- Initial results show we are obtaining similar results than the ones produced by the NBA
- More stressed disrupted scenarios have a considerable impact on relevant KPIs
- Schedule modifications may help reduce the impact of multiple disruptions on more stressed scenarios

Next steps

- Limit potential changes in order to find a good balance between additional dates needed to complete a schedule and total distance travelled
- Elaborate new model that enumerates potential tour changes and finds a schedule that generates the minimum distance.

THANK YOU