Rescheduling the NBA regular season via Integer Programming

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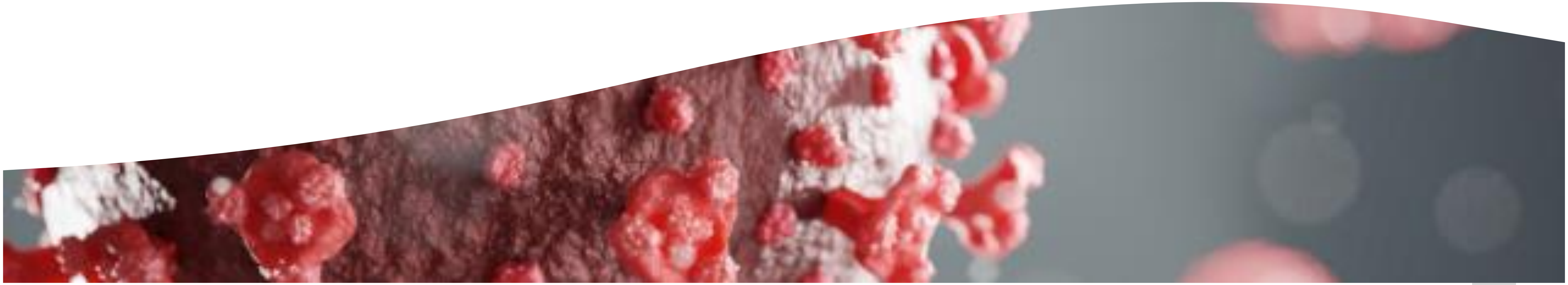
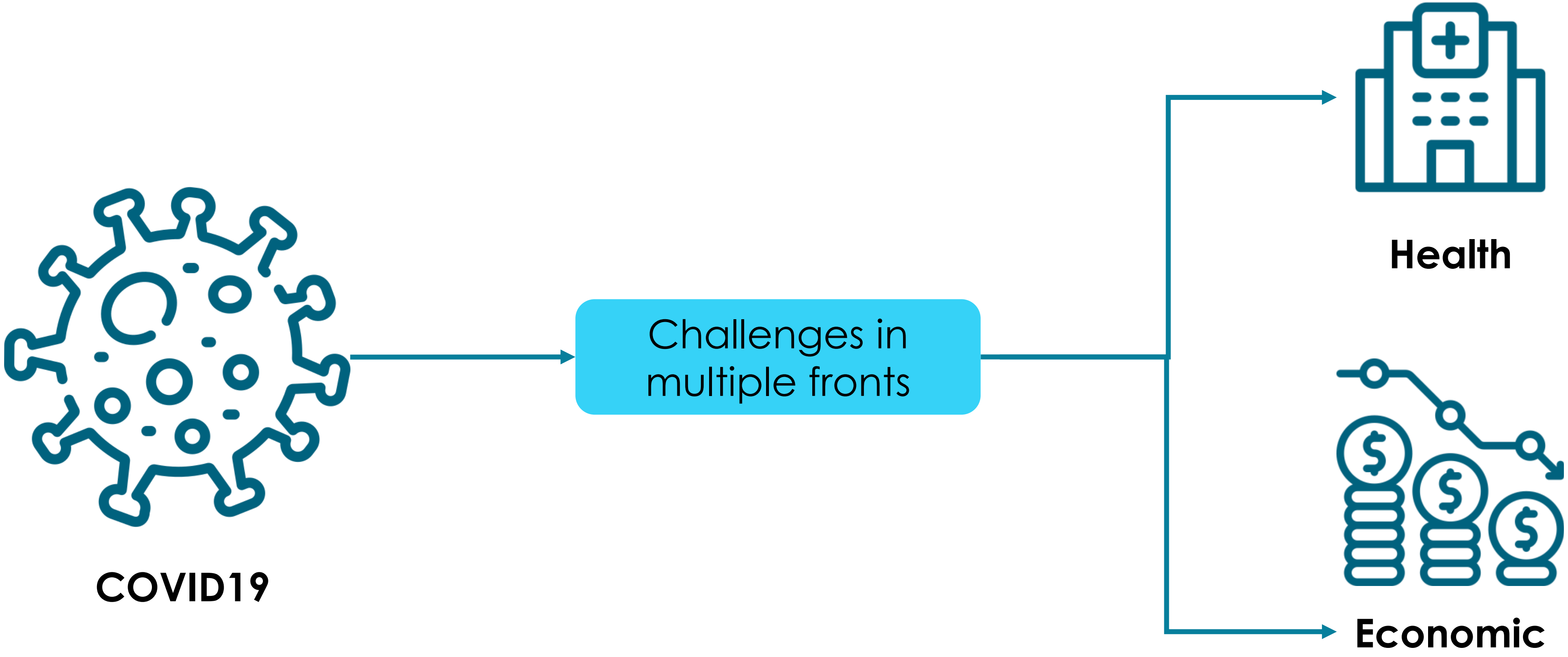
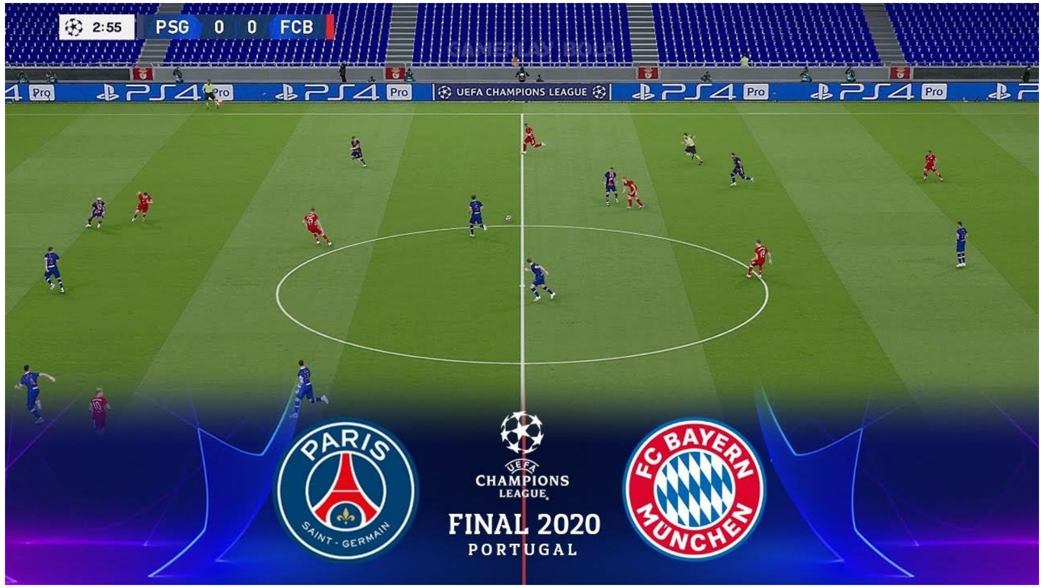
The challenges that the pandemic brought



The multiple challenges coming from the COVID19 Pandemic



The sports world didn't escape this phenomenon



UEFA Champions League Final

NBA Bubble

Rescheduling matches pre and post COVID

Pre-COVID	COVID
• Suspended matches are **rare** • Suspensions are usually **not related** to each other	• Suspended matches are **more common** due to the health situation • If multiple players become sick, **consecutive games** might be suspended

34%

Probability of a match being suspended in the NBA 2020-21 season, given than the previous match was suspended

Thinking about a systematic solution

Existing approaches for rescheduling

Rescheduling necessities

Literature

Yi et al (2020) present proactive and reactive strategies for time-constrained schedules

Practice

Leagues like NBA, NHL and MLB have rescheduled games, but haven't disclosed their methodological approach

We propose a two-stage approach to do this

We try to find a new date for each *disruption*

Definition: Each game in the executed timetable that is played before or after its scheduled round is considered a **disruption**. In our setup, each suspended game will therefore translate into a disruption that needs to be rescheduled in the remaining of the season schedule

1

Pre Processing

We carry out a pre-processing exercise to find potential possible dates for each disruption

2

Linear Optimization

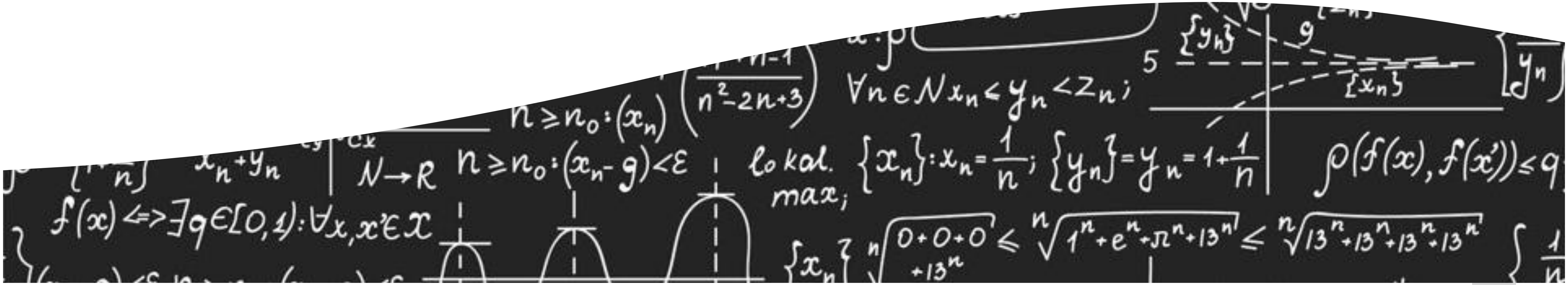
We build a linear optimization model to find the best possible date for each disruption and, if we can't, we reschedule it after the end of the schedule's original date

How do we find potential candidate dates for each disruption

Each potential date *t* must follow these conditions

1. *t* occurs after the disruption's date in the original schedule
2. There are no planned games for the teams involved in the disruption on date *t*
3. There are no scheduling rules violations if the corresponding match is scheduled on *t*
4. Both teams travel a reasonable distance if the corresponding match is scheduled on *t*

Mathematical Model



Approach and variables used

Approach

We will try to insert disrupted matches onto the existing schedule, without changing the date of non-disrupted matches

Variables used

General

Set of teams: $\mathrm{S} = \{1, \dots, m\}$

Set of original rounds: $\mathrm{T} = \{1, \dots, r\}$

Match between j and k: $(j,\ k)$

Scheduled game: Match between j and k on round t: $\alpha =< (j,\ k),\ t >$

Schedule Rules

Scheduled games of team i: R_i

Disrupted games of team i: R_i^{dis}

Total disrupted games: $R^{dis} = \cup_{i \in S} R_i^{dis}$

Potential candidate variables dates for match $\alpha = T_\alpha^{free}$

Maximum number of games that a team can play within every window of $t_2 - t_1$ days: $MG_{t_1.t_2}$

Number of non-disrupted games a team is playing between dates t_2 and t_1: $k_{t_1.t_2}^i$

Mathematical Model

$$\max \sum_{\alpha \in R^{dis}} \sum_{t \in T_\alpha^{free}} x_{\alpha t} \qquad (1)$$

st

$$\sum_{t \in T_\alpha^{free}} x_{\alpha t} \leq 1 \qquad \forall \alpha \in R^{dis} \qquad (2)$$

$$\sum_{\alpha \in R_i^{dis}} x_{\alpha t} + k_{t_1,t_2}^{i} \leq MG_{t_1,t_2} \qquad \forall t_1, t_2 \in T, 1 \leq t_2 - \mathrm{t}_1 \leq 7, i \in S \qquad (3)$$

$$x_{\alpha \mathrm{t}} \in \{0; 1\} \qquad \forall \alpha \in R^{dis}, t \in T_\alpha^{free} \qquad (4)$$

Mathematical Model

Maximize the number of matches rescheduled on original dates

$$\max \sum_{\alpha \in R^{dis}} \sum_{t \in T_\alpha^{free}} x_{\alpha t} \qquad (1)$$

Objective Function MaxGames

st

$$\sum_{t \in T_\alpha^{free}} x_{\alpha t} \leq 1 \qquad \forall \alpha \in R^{dis} \qquad (2)$$

$$\sum_{\alpha \in R_i^{dis}} x_{\alpha t} + k_{t_1,t_2}^i \leq MG_{t_1,t_2} \qquad \forall t_1, t_2 \in T, 1 \leq t_2 - \mathrm{t}_1 \leq 7, i \in S \qquad (3)$$

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Mathematical Model

$$\max \sum_{\alpha \in R^{dis}} \sum_{t \in T_\alpha^{free}} x_{\alpha t} \qquad (1)$$

st

Each match should be scheduled not more than once

$$\sum_{t \in T_\alpha^{free}} x_{\alpha t} \leq 1 \qquad \forall \alpha \in R^{dis} \qquad (2)$$

$$\sum_{\alpha \in R_i^{dis}} x_{\alpha t} + k_{t_1,t_2}^i \leq MG_{t_1,t_2} \qquad \forall t_1, t_2 \in T, 1 \leq t_2 - \mathrm{t}_1 \leq 7, i \in S \qquad (3)$$

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st

$$\sum_{t \in T_\alpha^{free}} x_{\alpha t} \le 1 \qquad \forall \alpha \in R^{dis} \qquad (2)$$

Respect scheduling rules

$$\sum_{\alpha \in R_i^{dis}} x_{\alpha t} + k_{t_1,t_2}^{i} \le MG_{t_1,t_2} \qquad \forall t_1, t_2 \in T, 1 \le t_2 - \mathrm{t}_1 \le 7, i \in S \qquad (3)$$

$$x_{\alpha \mathrm{t}} \in \{0; 1\} \qquad \forall \alpha \in R^{dis}, t \in T_\alpha^{free} \qquad (4)$$

Mathematical Model

$$\max \sum_{\alpha \in R^{dis}} \sum_{t \in T_\alpha^{free}} x_{\alpha t} \quad (1)$$

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Variable domain

$$x_{\alpha t} \in \{0; 1\} \qquad \forall \alpha \in R^{dis}, t \in T_\alpha^{free} \quad (4)$$

Another way to distribute matches

As part of the continuation of our work, we define a new objective function

Objective #2: Minimize the day difference between the original and new date (MinDiff)

$d_{\alpha t}$: The number of days between the original date of match α and date t

$$\min \sum_{\alpha \in R^{dis}} \sum_{t \in T_{\alpha}^{free}} d_{\alpha t} * x_{\alpha t} \tag{5}$$

If we use this objective function, constraint (2) must be set to equality

Preliminary Results

How we set up the analysis

- In order to get a better understanding of the performance of our solution we try to replicate the rescheduling process for the suspended games of the 2020-21 NBA season

Rescheduling methodology

Monthly

Dec	Jan	Feb	Mar	Apr	May

At the end of each month, we reschedule the disruptions that happened there

Post All-Star

Dec-Jan-Feb	Mar	Apr	May

We wait until the All-Star break to reschedule first set of matches, then we do a monthly approach

Monthly* and Post All Star*

We repeat each approach, without considering distance constraints and using NBA's reschedules if they happened within the month of the original game

Main Results

metric / method	NBA exec	*monthly*		*monthly**		*Post All-Star*		*Post All-Star**	
		MaxG	MinD	MaxG	MinD	MaxG	MinD	MaxG	MinD
distance	-0.2%	0.9%	1.0%	1.5%	0.8%	1.3%	0.8%	1.3%	0.4%
breaks	0.6%	-0.2%	-0.1%	-0.3%	0.2%	-0.5%	0.2%	-0.4%	0.4%
# dates added	-	11	7	8	4	8	5	6	3
games after	-	14	8	9	3	15	8	10	3

A

B

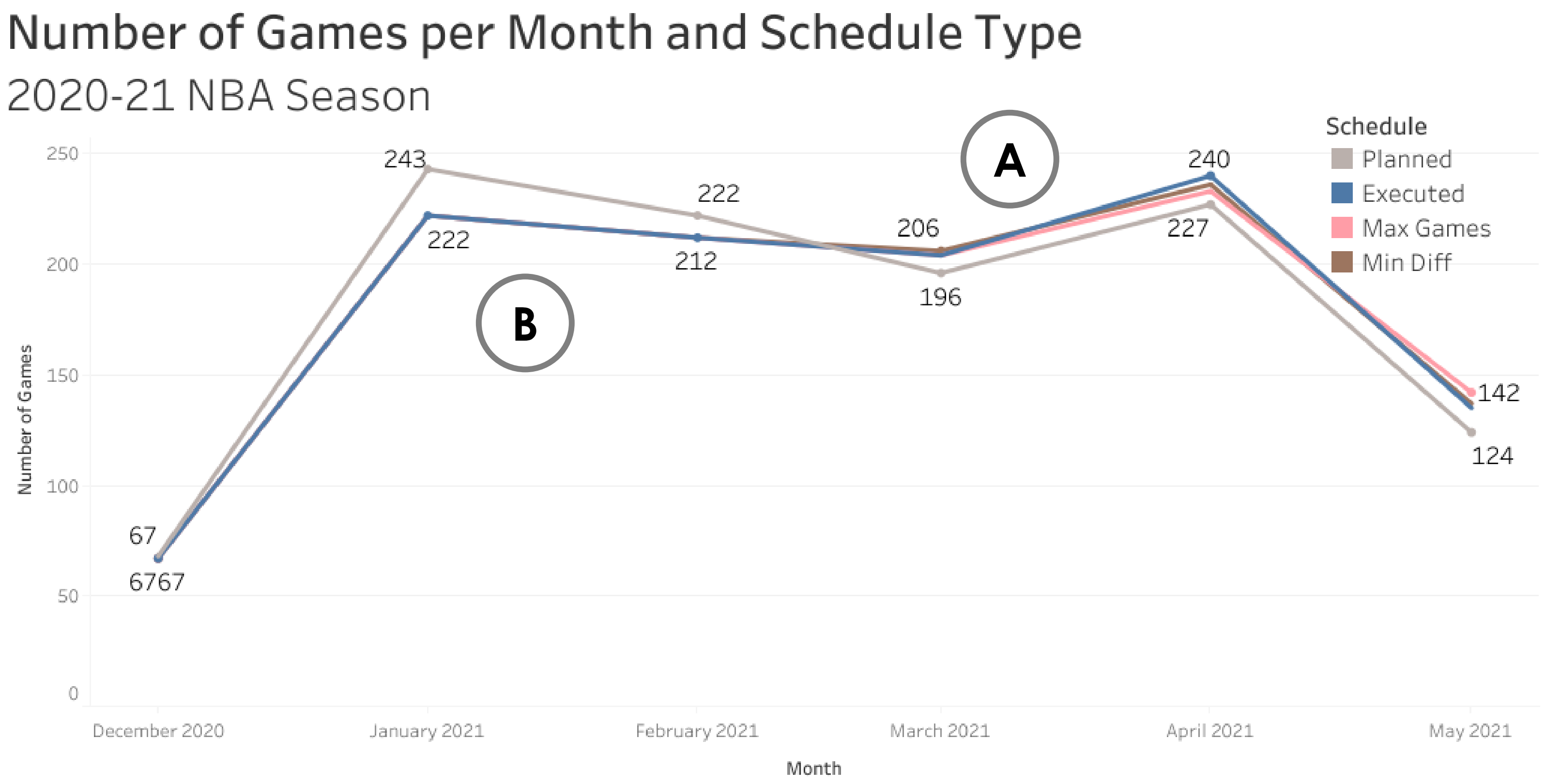
C

A All differences are small (around 1%)

B There is usually an improvement in breaks and distance worsening

C The distance constraint has effects on the additional number of dates needed

Game Calendar



A Number of games per month is similar for all strategies

B It appears the NBA left a less concentrated second half to have a greater buffer

Bigger Instances Analysis

We also created bigger instances to see how our approach would behave

- **15 more games:** ~50% increase in the problem size
- **25 more games:** ~80% increase in the problem size
- **15 more March games:** Similar to a second COVID wave

instance / metric	distance	breaks	# dates added	games after
15 more games	3.3%	-0.3%	9	15
25 more games	5.8%	-0.2%	9	20
15 more games in March	1.8%	0.5%	9	17

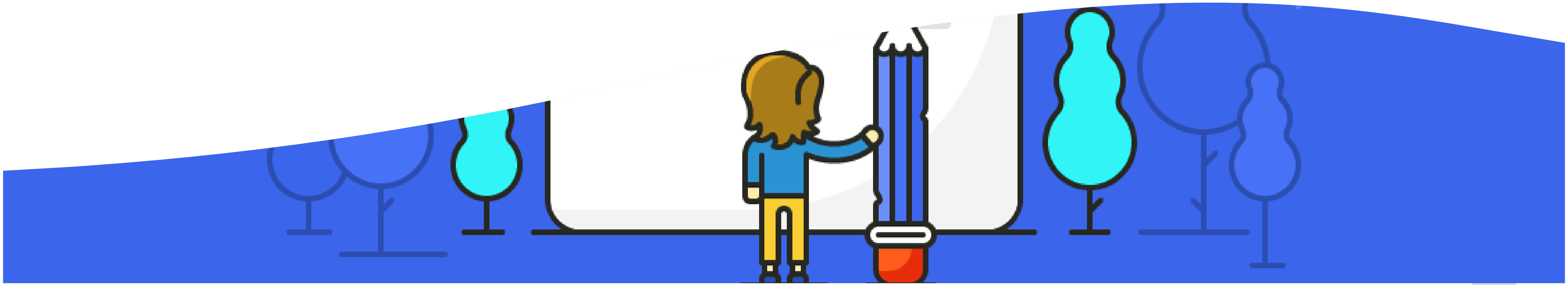
A

Table 2: Difference against the planned NBA schedule, *monthly** strategy and MIND objective.

A

Effects on relevant KPIs is not negligible

Conclusions and future work



Conclusions and future work

Conclusions

- We identified the problem of rescheduling in time-relaxed systems
- We propose a two-stage systematic approach to deal with multiple dependent reschedules
- Our first stage involves identify candidate dates for each disruption
- Our second stage involves finding the best candidate with a MIP
- Initial results show we are obtaining similar results than the ones produced by the NBA
- We are able to quantify the effect on relevant KPIs of bigger instances

Future work

- Consider introduce small local modifications to the non-disrupted matches

THANK YOU