A Historical Perspective on Linear Equations

Nicola Baird

Lesson Plan:

I would explain that around 4000 years ago, the people of Babylon knew how to solve a simple 2X2 system of linear equations with two unknowns. Around 200 BC, the Chinese published that "Nine Chapters of the Mathematical Art," they displayed the ability to solve a 3X3 system of equations. The simple equation of ax+b=0 is an ancient question worked on by people from all walks of life. The main progress in linear algebra did not come about until the late 17^{th} century.

Systems of linear equations arose in Europe when René Descartes introduced coordinates in geometry in 1637. In this new geometry (now called Cartesian geometry), lines and planes are represented by linear equations, and computing their intersections amounts to solving systems of linear equations.

To best teach this material, I would use a lot of pictures to show why it was so important that Descartes used geometry to show how to solve systems of linear equations. For example:

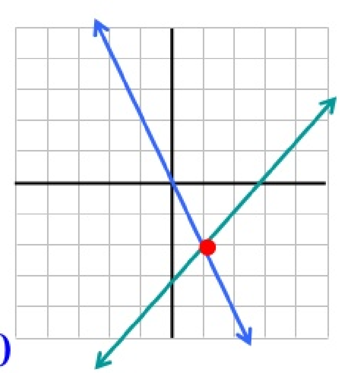
Solving Systems of Linear Equations by Graphing

$$\begin{cases} x - y = 3 \\ 2x + y = 0 \end{cases} \qquad \begin{cases} y = x - 3 \\ y = -2x \end{cases}$$

$$Solution: (1, -2)$$

$$1 - (-2) = 3 \qquad 2(1) + (-2) = 0$$

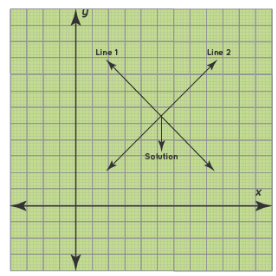
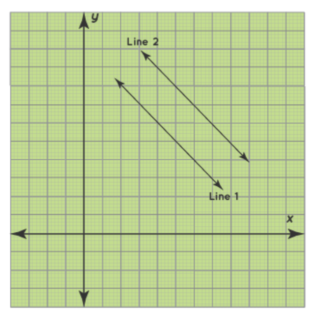
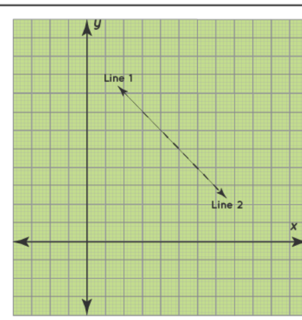
$$3 = 3 \qquad 0 = 0$$



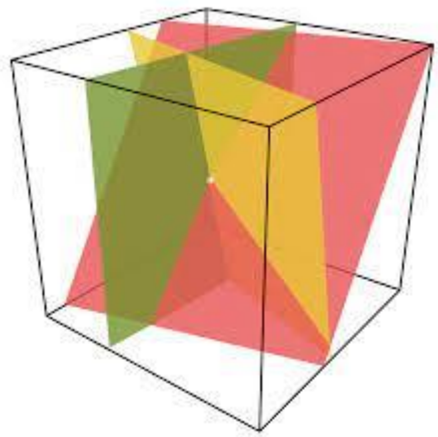
This picture would help me engage in a discussion with students about why principles of geometry would help find a solution to a pair of linear equations. We could discuss how this intersection point relates to finding a solution that works for both equations. We could then use the following picture to explain how geometry relates to equations that have no solution, or those that are all real solutions.

Types of Solutions and their Graphs

cuemath
THE MATH EXPERT

Type of solution	Conditions	Graphical Representation
Unique Solution (Consistent and Independent)	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	 Unique Solution (Consistent and independent)
No Solution (Inconsistent and Independent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	 No Solution (Inconsistent and independent)
Infinite Number of Solutions (Consistent and Dependent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	 Infinite number of solutions (consistent and dependent)

We could then use this logic to expand to how to solve systems of linear equations with more than two equations. This could lead us into an interesting discussion about planar geometry in 3d.



We could develop this logic further, saying that the mathematicians realized that they needed to find a different way to mathematically represent linear equations because they couldn't realistically do math in 4d, 5d, 6d, etc. Planar geometry with the current understanding and notation could only take them so far.

This would be a prime time to review matrices and vector spaces to show how solving systems of linear equations developed over time. While mathematicians before the 19th century preferred systems of linear equations to be expressed algebraically or with matrices, after the 19th century, vector spaces were preferred. It's fun to show how mathematics notation and understanding of linear equations has changed throughout history.