Unit Plan for 9th Grade Math Big Ideas Chapter 12 Congruent Triangles

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Unit Goal:

Students should be able to effectively recognize defining characteristics of different triangles and classify them. They should learn that triangles require certain tests to determine if they are congruent or not to another triangle. Students should be able to explain notation related to triangles and angles. They should also describe the connection between certain angles and certain triangle types. Students should recognize when certain algorithms and relationships should be used to aid in solving a problem with triangles.

Unit Objectives:

1. Students should make connections to previously learned material about shape transformations and similarity. (Construct a Concept)
2. Students should understand what congruence means and how that relates to side lengths of triangles. (Construct a Concept)
3. Students should use inductive reasoning to distinguish examples of a congruent pair of triangles from a non-congruent pair of triangles. (Construct a Concept)
4. Students construct the concept that triangles need to have certain requirements to be congruent (such as SAS, ASA, AAS, or SSS congruencies). (Construct a Concept)
5. When a student sees different types of triangles, they should be able to say their names. (Simple Knowledge)
6. When a student is given an isosceles triangle, they should know that there are two congruent angles called base angles. (Simple Knowledge)
7. Student should know that the different tick marks relate to congruency of side lengths. (Simple Knowledge)
8. Students should be able to explain the notation for measures of an angles using their own words. (Comprehension and Communication)
9. Students should be able to communicate the difference between angle measures and angles themselves. (Comprehension and Communication)
10. They should be able to effectively communicate about how the angles relate to each and how they are different than each other using correct mathematical vocabulary. (Comprehension and Communication)

11. Students should also be able to communicate effectively about the differences between supplementary angles and complementary angles. (Comprehension and Communication)
12. Students should be able to accurately determine if congruent triangle rules can be applied to solve a specific scenario. (Application)
13. Students should determine the "when" and "how" to use the appropriate concepts, algorithms, and relationships developed in this unit to complete problems about triangles and angles. (Application)

Construct a Concept Lesson

Construct a Concept Learning Level

Lesson Objectives:

1. Students should make connections to previously learned material about shape transformations and similarity.
2. Students should understand what congruence means and how that relates to side lengths of triangles.
3. Students should use inductive reasoning to distinguish examples of a congruent pair of triangles from a non-congruent pair of triangles.
4. Students construct the concept that triangles need to have certain requirements to be congruent (such as SAS, ASA, AAS, or SSS congruencies).

1. Sorting and Categorizing

Ask the students to create their own triangles on Geogebra, print the triangle, and label the triangle as they did in the example problem (from previous lesson). Ask students to make as many different "types" of triangles as they can think of. Then ask the students to make predictions about if each group member's triangle is congruent to the one that they made. Ask the students, "will all the other students have the exact same Triangle A as you did? Why or why not?"

After students print out their triangles and make their predictions, have them post all Triangle As in one side of the board, Triangle Bs on another part of the board and so on. The Triangle type will be determine based on if the student thinks that it looks similar (in at least two or three defining characteristics) to the one pasted at the top of the certain board.

2. **<u>Reflecting and Explaining:</u>**

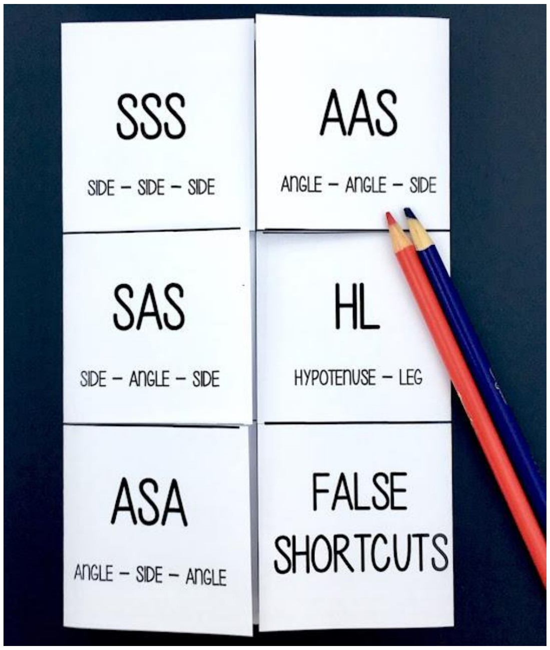
Then, instruct students to grab their predictions and do an individual gallery walk to assess their predictions. They need to answer the questions:

a. Were their predictions correct? Why or why not?
b. If the triangles are congruent, what parameters made them all be the same?
c. How could you guarantee that the triangles are congruent without using a ruler or protractor?

3. **<u>Generalizing:</u>**

Teacher says "You and your teammates will compare notes and come up with rules for each type of congruent triangle group. Be ready to share your answers with the class. Remember what you learned from the example from last class about classifying triangles by similar characteristics with angles and side lengths."

Next generalize the class's thoughts to fit the format SAS, ASA, AAS, HL, or SSS. Then discuss how certain orders of angles and sides don't tell us any information about congruence (such as SSA).

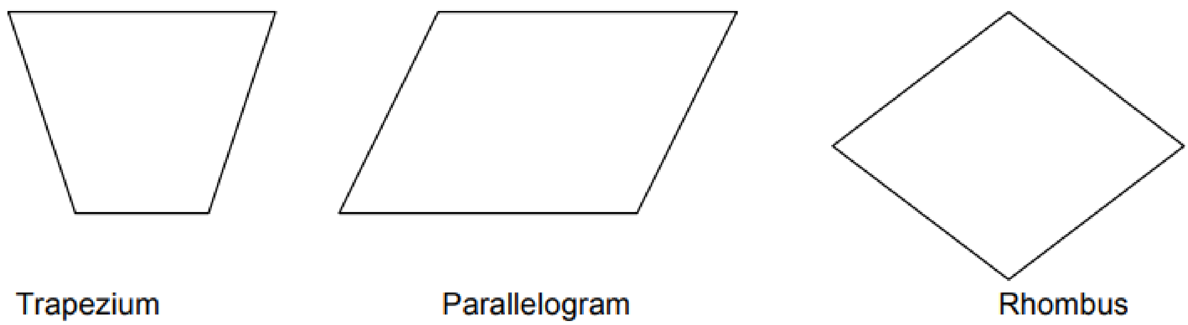


4. **<u>Verifying and Refining:</u>**

As a class, discuss their findings and fill out a paper as shown above (they will create that in class) with all of the Triangle Congruence Conjectures. Have students share their findings and add any information for them that they could not come up with on their own.

Miniexperiment

Which of the quadrilaterals below will make two congruent triangles when divided along the diagonal? Explain congruence without measuring the angles and sides.



Rubric:

Problem Type	Possible Points	Requirement for Points	Total
Trapezium	+2, +1, +0	Student should say that they are not congruent. Then student should explain that when cut in half along the diagonal, the Trapezium will make two different types of triangles (one is acute and one is obtuse.) You are only given	/2

		one side and one angle, and that’s not enough information to determine congruence.	
Parallelogram	+2, +1, +0	Student should say that they are congruent. Then student should explain that when cut in half along the diagonal, the Parallelogram will have two triangles inside that are congruent due to SAS or ASA.	/2
Rhombus	+2, +1, +0	Student should say that they are congruent. Then they shoul explain that when cut in half along the diagonal, the Rhombus will have two triangles inside that are congruent due to SAS or ASA.	/2
Total			/6

Simple Knowledge Lesson

Big Ideas Chapter 12 Section 4 Equilateral and Isosceles Triangles:

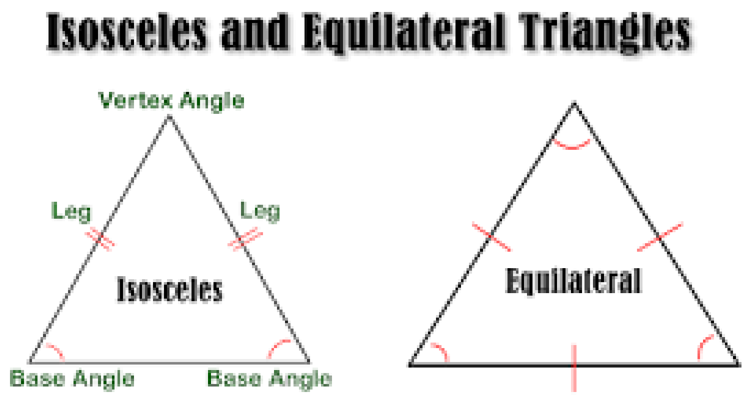
Simple Knowledge Learning Level

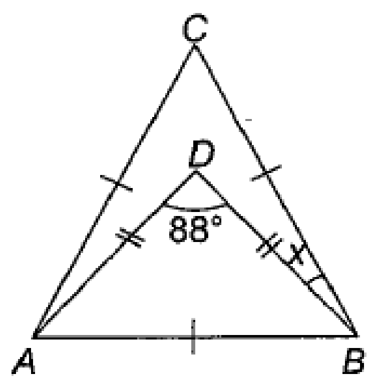
Lesson Objectives:

1. When a student sees different types of triangles, they should be able to say their names.
2. When a student is given an isosceles triangle, they should know that there are two congruent angles called base angles.
3. Student should know that the different tick marks relate to congruency of side lengths.

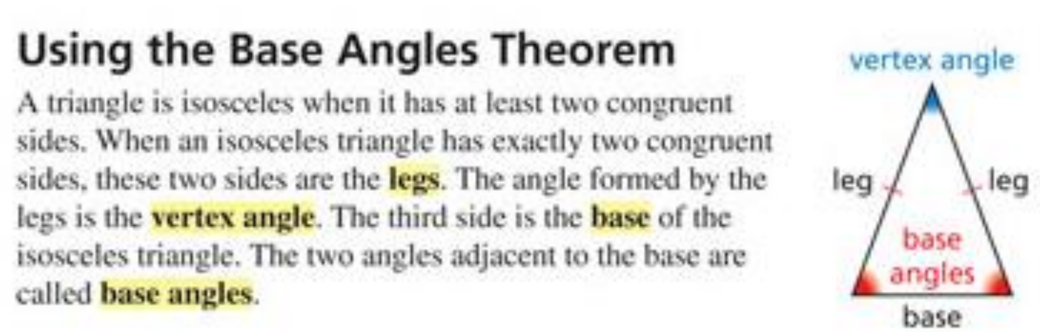
1. Exposition:

Today we will discuss the properties of two triangles: Equilateral and Isosceles Triangles. The types of triangles we will be dealing with today are defined by their side lengths. Look at the following pictures to see the defining characteristics of each triangle.





Define leg, vertex angle, and base angle by reading the following explanation:

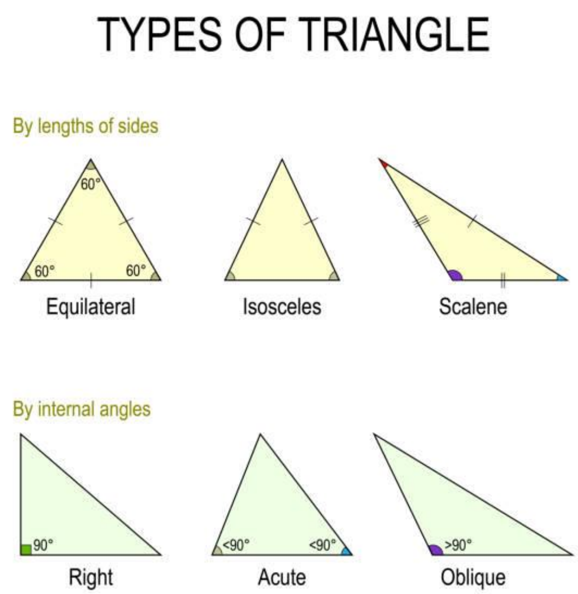


Using the Base Angles Theorem

A triangle is isosceles when it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.

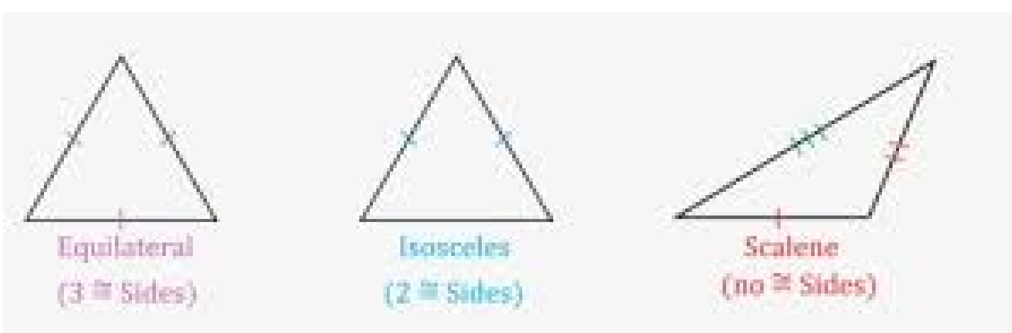
2. Explication:

When you see a triangle, you should begin to determine its characteristics to decide how to classify it. You should determine the information you are given. Specifically see if you are given the lengths, angles, or congruencies with another triangle. Look for relationships of how the angles and sides work together. To determine if a triangle is an isosceles or equilateral triangle, look at the specific lengths of the sides. The diagram below shows the names of triangles based on lengths and based on angles.



3. Mnemonics:

You can remember the distinctions between these two types of triangles by lining them up in a row starting with the most number of congruent side lengths going to the least number of congruent side lengths. This is demonstrated bellow:



You can see that they are ordered from left to right and each word’s first letter is in the order of the alphabet. “E” comes before “I” which comes before “S”.

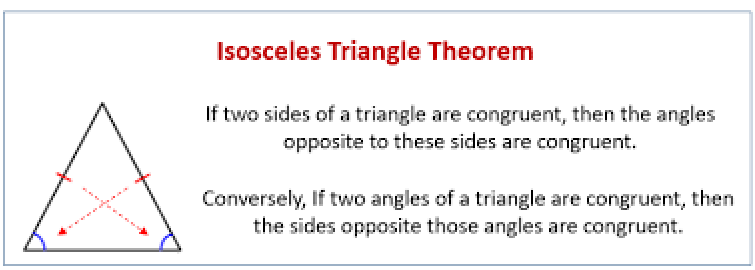
Also recall the definition of the word “equal”. It simply means equivalent. The equilateral triangle has three sides lengths that are all equivalent.

To remember the terminology “leg”, just think about a human. You have two legs that are probably the same length. Otherwise you would be leaning to one side when you walk. Thinking about how your two legs are the same length can help you remember that the legs of an isosceles triangle are the same length.

Each angle corresponds to a side length that is directly opposite it. This means that you can put your finger on an angle and point to the side length.

4. Monitoring and Feedback:

Use the following picture to explain what base angles are and what makes them different than any other set of angles. Then explain another defining characteristic of isosceles triangles and how they differ from equilateral triangles. Define a vertex angle and a what legs of a triangle are. Discuss which type of triangle they relate to.



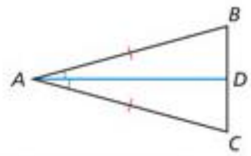
5. overlearning

Use the principles we have discussed to explain verbally why the Base Angles Theorem and its converse make sense.

PROOF **Base Angles Theorem**

Given $\overline{AB} \cong \overline{AC}$

Prove $\angle B \cong \angle C$



Plan for Proof
a. Draw $\overline{AD}$ so that it bisects $\angle CAB$.
b. Use the SAS Congruence Theorem to show that $\triangle ADB \cong \triangle ADC$.
c. Use properties of congruent triangles to show that $\angle B \cong \angle C$.

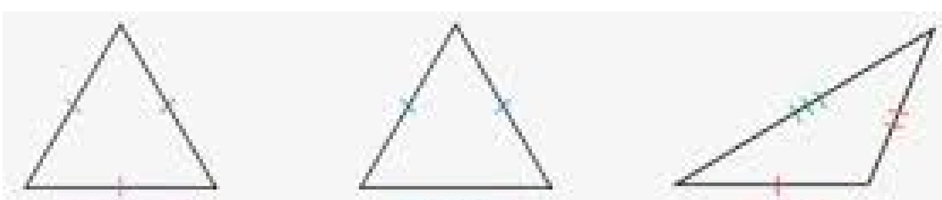
Plan in Action

	STATEMENTS	REASONS
a.	**1.** Draw $\overline{AD}$, the angle bisector of $\angle CAB$.	**1.** Construction of angle bisector
	2. $\angle CAD \cong \angle BAD$	**2.** Definition of angle bisector
	3. $\overline{AB} \cong \overline{AC}$	**3.** Given
	4. $\overline{DA} \cong \overline{DA}$	**4.** Reflexive Property of Congruence
b.	**5.** $\triangle ADB \cong \triangle ADC$	**5.** SAS Congruence Theorem
c.	**6.** $\angle B \cong \angle C$	**6.** Corresponding parts of congruent triangles are congruent.

We have already shown the proof for the Base Angles Theorem. If have additional time, prove the converse of the Base Angle Theorem. Use the following diagram to discuss how to do this proof as a class.

Miniexperiment:

1. Using the picture below, name the three different types of triangles.
2. What do the tick marks on each of the triangles mean in contect to that triangle?
3. What are the two identical angles in an isoceles triangle called?



Miniexperiment	Possible Points	Explanation of Point Distribution	Totals
#1	+3, +2,+1, +0	Student correctly names the triangles from left to right as Equilateral, Isosceles, and Scalene.	/ 3

#2	+2, +0	Student explains that the tick marks represent congruency. Student gets second point if they explain what that means about each of the side lengths in context for each triangle.	/2
#3	+1, +0	Student puts “base angles”	/ 1
Totals		NA	/ 6

<u>Comprehension and Communication Lesson</u>

This lesson is essential for this unit because it helps students communicate about how angles make up different types of triangles. This will help them make and solidify important connections about what makes triangles similar or different.

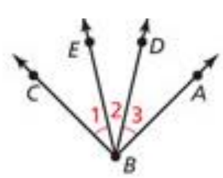
Comprehension and Communication Learning Level

Lesson Objectives:

1. Students should be able to explain the notation for measures of an angles using their own words.
2. Students should be able to communicate the difference between angle measures and angles themselves.
3. They should be able to effectively communicate about how the angles relate to each and how they are different than each other using correct mathematical vocabulary.
4. Students should also be able to communicate effectively about the differences between supplementary angles and complementary angles.

Given $m\angle 1 = m\angle 3$

Prove $m\angle EBA = m\angle CBD$



<u>Vocabulary:</u>

Acute angle-an angle that measures less than 90°

Obtuse angle-an angle that measure greater than 90°, but less than 180°

Complementary angle-two angles whose measure have a sum of 90°

Right angle-an angle that measures 90°

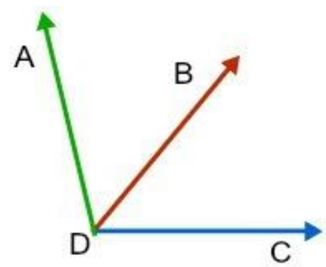
Opposite angles- the angles opposite each other that are formed when two lines intersect

Supplementary angle-two angles whose measures have a sum of 180°

Straight angle-an angle that measures 180°

Ray-a part of a line starting at a particular point and extending infinitely in one direction

Adjacent angles-two angles that have a common side and a common vertex (corner point) but do not overlap in any way.



Angle ADB and Angle BDC
share a common vertex of D
and have a common side of DB.

Congruent angle- an angle that has the same measure as another angle

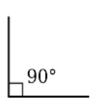
Vertex-is the common endpoint of the rays forming the angle

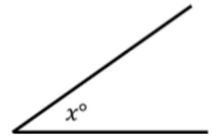
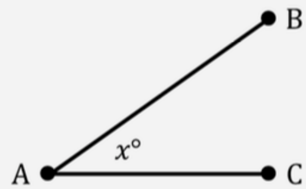
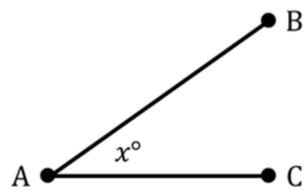
Technical Expressions:

Measure of an angle is notated as:

$m\angle 1$

A right angle is notated as:



Degree	$x°$ *x degrees*		A degree is the measurement of an angle
Angle	$\angle BAC$ *Angle BAC* *A is the vertex*		An angle is formed by two lines, segments, or rays that have the same endpoint, which is called the vertex
Angle Measure	$m\angle BAC$ *Measure of angle BAC* *A is the vertex* $m = x°$		The measure of an angle can be acute (< 90°), right (= 90°), or obtuse (> 90°). An angle measuring 180° is a straight line.

<u>Concepts:</u>

Angle Names: Angles are named by selecting a point from either ray/line segment and then putting the vertex point label and then the other ray/line segment. So for the angle above, angle BAC where you select a point, vertex, point.

Congruent Angles: What is needed for an angle to be congruent to another one?

<u>Relationships:</u>

How is the measure of an angle different than the geometric angle itself?

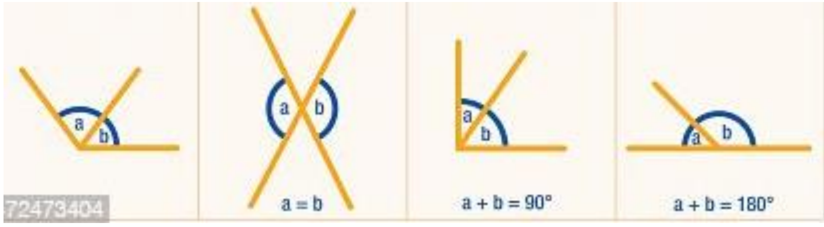
What are the differences between acute, right, obtuse, and straight line angles? (how will you remember them)?

What geometric parts make up an angle?

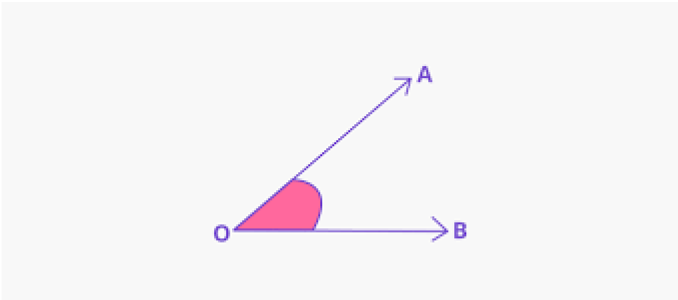
What is the difference between supplementary and complimentary angles?

Miniexperiment:

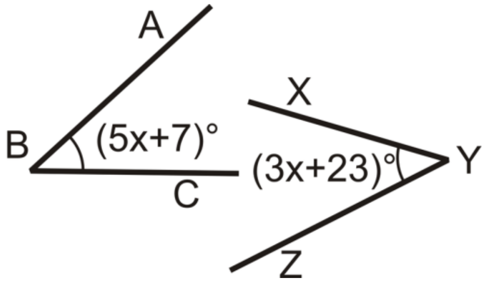
1. What are the differences between obtuse, acute, right, and straight-line angles?
2. Draw a picture to illustrate adjacent angles?
3. Name the following pictures: complimentary, supplementary, opposite, or adjacent angles. Then explain how you used the definitions to make those decisions.



4. What two names could you give the following angle?

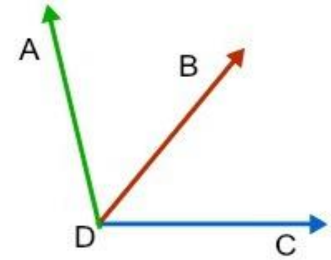


5. Please solve for x in the angles below. Why do you think it's helpful to know that the two angles are congruent?



Miniexperiment Rubric

Prompt Number	Points Possible	Distribution of Points	Totals
1	+5, +4 +3, +2, +1, +0	Student should explain the definitions for each of the 4 angle types to demonstrate how they are different from each other. +1 point for no irrelevant information.	

			/ 5
2	+3, +2, +1, +0	Angle ADB and Angle BDC share a common vertex of D and have a common side of DB. Students should draw something like the picture above. They should also say that angles ADB and BDC have a common line segment DB. +1 point for no erroneous information.	/ 3
3	+4, +3, +2, +1, +0	Students get 1 point for each picture they correctly identified and explained their reasoning for. From left to right: Adjacent; opposite; complimentary; supplementary	/ 4
4	+2, +1, +0	Angle AOB and Angle BOA.	/ 2
5	+3, +2, +1, +0	Students say x=8. Students explains that since the angles are congruent they are able to set the two angle equations equal to each	

		other. +1 for no erroneous information.	/ 3
Totals:	NA	NA	/ 17

Application Lesson Plan

Application Learning Level

Lesson Objectives:

1. Students should be able to accurately determine if congruent triangle rules can be applied to solve a specific scenario.
2. Students should determine the “when” and “how” to use the appropriate concepts, algorithms, and relationships developed in this unit to complete problems about triangles and angles.

Initial Problem Confrontation and Analysis:

Consider the following two scenarios and determine which of them requires using triangle congruence properties.

1. You are about to go hang-gliding, and your nerves start acting up; you want to make sure that the hang glider will fly straight. Given that both halves of the glider use the same amount of material (same area) what other properties need to be met in order for the sides to be equal?
2. If you draw a line to connect the tips of the hang glider, you create a new triangle. You want to know what the area of the new triangle is. Assume triangle QRM (M for middle of the hang glider) and triangle RSM are the same. The length from Q to R is 7 feet, the angle QRS is 120 degrees and the middle bar is 3 feet. What is the distance from Q to S?

Lead the class in a discussion about what each question was asking, and which one would require the use of congruence properties. (Problem 1 requires congruence and Problem 2 doesn't).

Subsequent Problem Confrontation and Analysis:

In this step, we are actually finding the answers for the two questions and giving new problems for the students to determine the objective is relevant.

Students would discuss and find the answer for problem 1. The discussion would follow the steps listed in the solution in the picture below.

Previous
congruent figures
corresponding parts
construction
perpendicular lines

Using Congruent Triangles to Solve Problems

EXAMPLE 1 **Using Congruent Triangles**

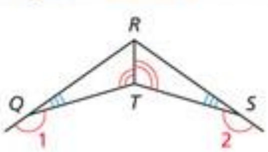
Explain how you can use the given information to prove that the hang glider parts are congruent.

Given $\angle 1 \cong \angle 2$, $\angle RTQ \cong \angle RTS$

Prove $\overline{QT} \cong \overline{ST}$

SOLUTION

Mark given and deduced information.



If you can show that $\triangle QRT \cong \triangle SRT$, then you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then mark the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$ by the Reflexive Property of Congruence. Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle QRT \cong \triangle SRT$.

▶ Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

For problem 2, the students would calculate the distance using the given lengths of the two triangles.

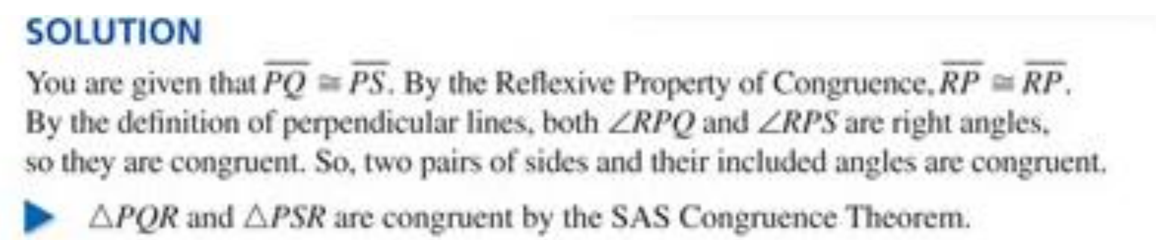
ADDITIONAL PROBLEMS:

1. You are making a canvas sign to hang on the triangular portion of a barn wall. You think that you can use two triangular sheets of canvas. You know that the length RP and the length QS are perpendicular, and that the length PQ is the equivalent to the length PS. How would you know for sure if these two triangles are congruent or not?
2. How long is length PR? You are given that the length QR is 15 and that the length QP is 10.

Solutions:

Students will discuss how the first prompt is asking them to compare triangles and to decide if they are congruent. They will discuss how the second prompt is asking them to find the side length of only one of the triangles, and therefore does not require them to use any of the triangle equivalence properties.

1.



SOLUTION

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property of Congruence, $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two pairs of sides and their included angles are congruent.

▶ $\triangle PQR$ and $\triangle PSR$ are congruent by the SAS Congruence Theorem.

2. Students would use the Pythagorean theorem to compute the requested length.

Rule Articulation:

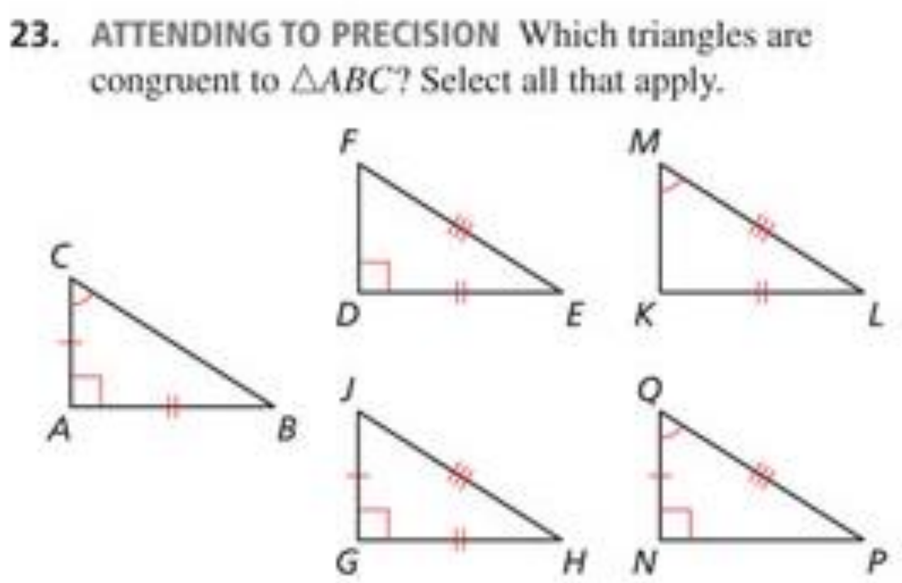
Look to see which steps the problem will require. Then see if you need to determine if two triangles are congruent. Then see if enough information is given. You will need to recall which information you will need by thinking about the different rules for congruence. You know that you can use ASA, SAS, SSS, and AAS. See if you have enough information given to declare congruence.

Extension Into Subsequent Lessons:

Previous units have discussed when to use the Pythagorean theorem and how to set up the equation to solve. Future units will discuss using trig to determine the length of a side of a hang glider. We will continue to develop usages for all of the properties discussed in these example prompts.

Miniexperiment:

Answer the following prompt and explain why each of the triangles either are or are note congruent to the given triangle.



a) Triangle FDE
b) Triangle LMK
c) Triangle JGH
d) Triangle QNP

Rubric for Miniexperiment:

Possible Points	Explanation	Total Points Allotted

+2 +1 +0	Student gets 2 points if they selected C and D as being congruent to the given triangle.	/ 2
+2 +1 +0	Students will get two points if they explain how they knew that C and D were congruent to the original triangle. C because of SAS and D because of ASA.	/ 2
+1 +0	Students will get 1 point if they explain that there is not enough information given in A or B to determine congruence.	/ 1
Total	NA	/ 5