

$$R = \{0,1\}$$

$$p(1) = 0.8, \quad p(0) = 0.2$$

1. What is the probability that a word contains exactly four 1's and two 0's?

Let X be the number of 1's in a word of 6 digits. Then the probability of getting exactly four 1's out of 6 is:

$$P(X = 4) = \text{Binomial}(4, p) = \binom{6}{4} p^4 (1 - p)^2 = \frac{6!}{4! 2!} * 0.8^4 * 0.2^2 = \mathbf{0.24576}$$

2. What is the probability that a word contains at least four 1's?

$$\begin{aligned} P(X \geq 4) &= P(X = 4) + P(X = 5) + P(X = 6) = 0.24576 + \text{Bin}(5, p) + \text{Bin}(6, p) = \\ &= 0.24576 + \binom{6}{5} p^5 (1 - p) + \binom{6}{6} p^6 = 0.24576 + 6 * 0.8^5 * 0.2 + 0.8^6 = \mathbf{0.90112} \end{aligned}$$

3. Assume that the first digit is $X_1 = 1$. What is the probability that the sum of the first two digits is 2?

Let Y_n be the sum of the first n digits. Then:

$$P_{X_1=1}(Y_2 = 2) = P(X_2 = 1) = \mathbf{p = 0.8}$$