

Three fair dices are thrown. Let X denote the number of dice that land with the same number of dots. Describe the probability distribution function of X .

$$R = \{1,2,3\}$$

$$\begin{aligned} \text{All different: } (X = 1) &= \{(1,2,3), (1,2,4), \dots\} = \\ &= \{(x, y, z) : 1 \leq x, y, z \leq 6, x \neq y \neq z\} \end{aligned}$$

$$\begin{aligned} \text{Two equal: } (X = 2) &= \{(1,2,2), \dots, (1,2,1), \dots, (1,1,2)\} = \\ &= \{(x, y, y), (y, x, y), (y, y, x) : 1 \leq x, y \leq 6, x \neq y\} \end{aligned}$$

$$\begin{aligned} \text{All equal: } (X = 3) &= \{(1,1,1), (2,2,2), \dots\} = \\ &= \{(x, x, x) : 1 \leq x \leq 6\} \end{aligned}$$

$$\#(X = 1) = 6 * 5 * 4 = 120$$

$$\#(X = 2) = 6 * 5 * 1 + 6 * 5 * 1 + 6 * 1 * 5 = 30 * 3 = 90$$

$$\#(X = 3) = 6 * 1 * 1 = 6$$

There are $6^3 = 216$ total possible combinations. Therefore, the probability distribution function of X is:

$$p_X(1) = P(X = 1) = \frac{\#(X = 1)}{TOT} = \frac{120}{216} = \frac{5}{9}$$

$$p_X(2) = P(X = 2) = \frac{\#(X = 2)}{TOT} = \frac{90}{216} = \frac{5}{12}$$

$$p_X(3) = P(X = 3) = \frac{\#(X = 3)}{TOT} = \frac{6}{216} = \frac{1}{36}$$