

X c.r.v.

$$f(x) = ax^{-a-1} \mathbb{1}_{(1,+\infty)}(x), \quad a > 2$$

Compute the mean of X and its median.

$$\begin{aligned} \mathbb{E}(X) &= \int_{-\infty}^{+\infty} xf(x)dx = \int_1^{+\infty} ax^{-a}dx = \\ &= \left[\frac{a}{1-a} * \frac{1}{x^{a-1}} \right]_1^{+\infty} = -\frac{a}{1-a} = \frac{a}{a-1} \end{aligned}$$

The mean is $\frac{a}{a-1}$.

To compute the median, we solve the equation $F(m) = 0.5$.

$$F(x) = \int_1^x f(t)dt = \frac{at^{-a}}{-a} = \left[-\frac{1}{t^a} \right]_1^x = -\frac{1}{x^a} + 1 = 1 - \frac{1}{x^a}$$

$$1 - \frac{1}{m^a} = \frac{1}{2}$$

$$m^{-a} = \frac{1}{2}$$

$$m = \left(\frac{1}{2} \right)^{-\frac{1}{a}} = 2^{\frac{1}{a}}$$

$$m = \sqrt[a]{2}$$

The median is $m = \sqrt[a]{2}$.