

Consider a ternary source, associated to the symbols R, G, B; we consider two probability density distributions:

Source	R	G	B
Q_1	1/6	1/2	1/3
Q_2	(1-p)/2	p	(1-p)/2

i. What is the entropy of the source Q_1 ?

$$H(Q_1) = - \sum_{i=1}^3 p_i \log p_i = \frac{1}{6} \log_2 6 + \frac{1}{2} \log_2 2 + \frac{1}{3} \log_2 3 = 1.459$$

ii. What is the value of p that maximises the entropy of the source Q_2 ?

The principle of maximum entropy states that the entropy is maximised if there is a uniform distribution, therefore $H(Q_2)$ is maximum if:

$$\frac{1-p}{2} = p$$

$$1-p = 2p$$

$$p = \frac{1}{3}$$

In this case, the entropy of Q_2 is:

$$H(Q_2) = - \sum_{i=1}^3 p_i \log p_i = \log_2 3 = 1.585$$