Suppose that X and Y are independent, uniformly distributed in $\{0,1,2\}$. Let S = X + Y and W = X · Y.

$$X \sim Uniform(3)$$
$$Y \sim Uniform(3)$$
$$S = X + Y$$
$$S \sim \{(0,1/9),(1,2/9),(2,1/3),(3,2/9),(4,1/9)\}$$
$$W = X * Y$$
$$W \sim \{(0,5/9),(1,1/9),(2,2/9),(4,1/9)\}$$

i. Compute the entropy of the random variables S and W.

$$H(S) = -\sum_{i=1}^{5} P(S=s_i)\log P(S=s_i) = 2*\frac{1}{9}\log_2 9 + 2*\frac{2}{9}\log_2\frac{9}{2} + \frac{1}{3}\log_2 3 = 2.197$$

$$H(W) = -\sum_{i=1}^{5} P(W=w_i)\log P(W=w_i) = \frac{5}{9}*\log_2\frac{9}{5} + 2*\frac{1}{9}*\log_2 9 + \frac{2}{9}*\log_2\frac{9}{2} = 1.658$$

ii. What is the mutual information $I(S,W)$?

Joint probabilities:

		S				
		0	**1**	**2**	**3**	**4**
W	**0**	1/9	2/9	2/9	0	0
	1	0	0	1/9	0	0
	2	0	0	0	2/9	0
	4	0	0	0	0	1/9

$$I(S,W) = \sum_{j=1}^{5}\sum_{k=1}^{4} p_{jk}\log_2\left(\frac{p_{jk}}{p_j q_k}\right) =$$
$$= \frac{1}{9}*\log 2(\frac{1}{9}*9*\frac{9}{5}) + \frac{2}{9}*\log_2(\frac{2}{9}*\frac{9}{2}*\frac{9}{5}) + \frac{2}{9}\log_2(\frac{2}{9}*3*\frac{9}{5}) +$$
$$+\frac{1}{9}*\log_2(\frac{1}{9}*3*9) + \frac{2}{9}*\log_2(\frac{2}{9}*\frac{9}{2}*\frac{9}{2}) + \frac{1}{9}*\log_2(\frac{1}{9}*9*9) =$$

$$= \frac{1}{3}*\log_2\frac{9}{5} + \frac{2}{9}*\log_2\frac{6}{5} + \frac{1}{9}*\log_2 3 + \frac{2}{9}*\log_2\frac{9}{2} + \frac{1}{9}*\log_2 9 = 1.352$$