

$$p(k) = \begin{cases} 1/8 & k = -1 \\ 1/4 & k = 0 \\ 1/2 & k = 1 \\ 1/8 & k = 2 \end{cases}$$

a. Compute the expectation of X and of X^2 .

$$\mathbf{E}[X] = \sum_{x \in R} xp(x) = -\frac{1}{8} + \frac{1}{2} + \frac{1}{4} = \frac{5}{8}$$

$$\mathbf{E}[X^2] = \sum_{x \in R} x^2 p(x) = \frac{1 * (-1)^2}{8} + \frac{4 * 1^2}{8} + \frac{1 * 2^2}{8} = \frac{9}{8}$$

a. Compute the variance of X and of X^2 .

$$V(X) = \mathbf{E}[X^2] - (E[X])^2 = \frac{9}{8} - \left(\frac{5}{8}\right)^2 = \frac{72}{64} - \frac{25}{64} = \frac{47}{64}$$

$$\mathbf{E}[X^4] = \sum_{x \in R} x^4 p(x) = \frac{1 * (-1)^4}{8} + \frac{4 * 1^4}{8} + \frac{1 * 2^4}{8} = \frac{21}{8}$$

$$V(X^2) = \mathbf{E}[(X^2)^2] - (E[X^2])^2 = \frac{21}{8} - \left(\frac{9}{8}\right)^2 = \frac{168}{64} - \frac{81}{64} = \frac{87}{64}$$