Alice proposes to Bob the following bet. Alice tosses a fair coin n times and computes the number of heads X.
Bob tosses the coin $n+1$ times and obtains Y heads.
Bob wins the bet if $Y > X$. Is the bet fair?
Bonus version: compute the answer for a general coin.

$$X = \#heads\ \ in\ \ Alice's\ \ n\ \ trials$$

$$Y = \#heads\ \ in\ \ Bob's\ \ n+1\ \ trials$$

Let's also define Z as:

$$Z = Y - X$$

Then, if p is the probability of obtaining head:

$$P(Y > X) = P(Z > 0)$$

$$Y \sim Bin(n+1, p)$$

$$X \sim Bin(n, p)$$

Since $Z = Y - X$:

$$Z \sim Bin(n+1-n, p) = Bin(1, p) = Ber(p)$$

$$Z \sim Ber(p)$$

1 Case $p = \frac{1}{2}$

$$P(Y > X) = P(Z > 0) = P(Z = 1) = \frac{1}{2}$$

The bet is fair.

2 Case $p > \frac{1}{2}$

$$P(Y > X) = P(Z > 0) = P(Z = 1) > \frac{1}{2}$$

The bet is unfair, Bob has more chance of winning

3 Case $p < \frac{1}{2}$

$$P(Y > X) = P(Z > 0) = P(Z = 1) < \frac{1}{2}$$

The bet is unfair, Alice has more chance of winning