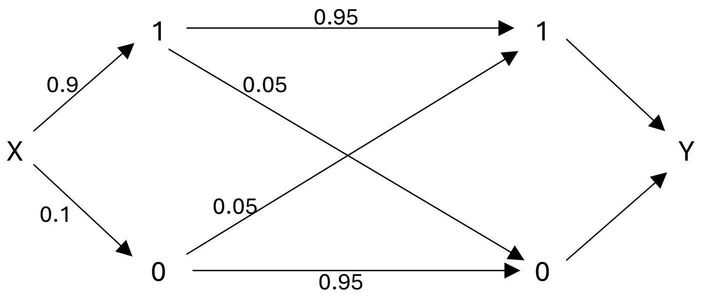
$$R_X = \{0,1\}$$

$$p_X(1) = p = 0.9, \quad p_X(0) = 1 - p = 0.1$$

$$\epsilon = 0.05$$



1. Show that *Y* is a Bernoulli distribution with parameter q.

$$\boldsymbol{R_Y} = \{\mathbf{0}, \mathbf{1}\}$$

$$\boldsymbol{p_Y}(\mathbf{1}) = P(Y = 1) = P_{X=1}(Y = 1) * P(X = 1) + P_{X=0}(Y = 1) * P(X = 0\,) =$$

$$= (1 - \epsilon) * p_X(1) + \epsilon * p_X(0) =$$

$$= p - p\epsilon + \epsilon(1 - p) = p - \mathrm{p}\epsilon + \epsilon - \mathrm{p}\epsilon = \boldsymbol{p} + \boldsymbol{\epsilon} - \mathbf{2}\boldsymbol{p\epsilon} = \mathbf{q}$$

$$\boldsymbol{p_Y}(\mathbf{0}) = P(Y = 0) = P_{X=0}(Y = 0) * P(X = 0) + P_{X=1}(Y = 0) * P(X = 1\,) =$$

$$= (1 - \epsilon) * p_X(0) + \epsilon * p_X(1) =$$

$$= (1 - \epsilon)(1 - p) + p\epsilon = 1 - p - \epsilon + p\epsilon + p\epsilon = \mathbf{1} - (\boldsymbol{p} + \boldsymbol{\epsilon} - \mathbf{2}\boldsymbol{p\epsilon}) = \mathbf{1} - \boldsymbol{q}$$

Since $\boldsymbol{R_Y} = \{\mathbf{0}, \mathbf{1}\}$, $\boldsymbol{p_Y}(\mathbf{1}) = \boldsymbol{q}$ and $\boldsymbol{p_Y}(\mathbf{0}) = \mathbf{1} - \boldsymbol{q}$, Y is a Bernoulli distribution with parameter q.

2. Determine the parameter q.

$$\boldsymbol{q} = p + \epsilon - 2p\epsilon = 0.9 + 0.05 - 2 * 0.9 * 0.05 = \mathbf{0.86}$$

$$\mathbf{1} - \boldsymbol{q} = 1 - 0.86 = \mathbf{0.14}$$

3. Compute the joint probability distribution function of *(X,Y)*.

$$R_X = \{x_1 = 0, x_2 = 1\}, \quad R_Y = \{y_1 = 0, y_2 = 1\}$$

$$p_{ij} = P\big(X = x_i,\ Y = y_j\big) \quad for \quad 1 \le i \le 2, \quad 1 \le j \le 2$$

$$\boldsymbol{p_{00}} = P(X = 0, Y = 0) = P(X = 0) * P_{X=0}(Y = 0) = 0.1 * 0.95 = \mathbf{0.095}$$

$$\boldsymbol{p_{11}} = P(X = 1, Y = 1) = P(X = 1) * P_{X=1}(Y = 1) = 0.9 * 0.95 = \mathbf{0.855}$$

$$\boldsymbol{p_{01}} = P(X = 0, Y = 1) = P(X = 0) * P_{X=0}(Y = 1) = 0.1 * 0.05 = \mathbf{0.005}$$

$$\boldsymbol{p_{10}} = P(X = 1, Y = 0) = P(X = 1) * P_{X=1}(Y = 0) = 0.9 * 0.05 = \mathbf{0.045}$$