

1 What is the PDF of X?

$$X \in R = \{1, 2, 3, 4\}$$

$$p = 0.2$$

$$X \sim \text{Geo}(p) \text{ for } X = 1, 2, 3$$

$$p_X(1) = p = 0.2$$

$$p_X(2) = (1 - p)p = 0.8 * 0.2 = 0.16$$

$$p_X(3) = (1 - p)^2 p = 0.8^2 * 0.2 = 0.128$$

$$p_X(4) = 1 - (p_X(1) + p_X(2) + p_X(3)) = 0.512$$

Then PDF of X is:

$$p_X(k) = \begin{cases} 0.2 & k = 1 \\ 0.16 & k = 2 \\ 0.128 & k = 3 \\ 0.512 & k = 4 \end{cases}$$

2 What is the probability that Hugo successfully completes his collection?

$$P(\text{completes}) = p_X(1) + p_X(2) + p_X(3) + (1 - p)^3 p = 0.488 + 0.8^3 * 0.2 = 0.5904$$

The probability that Hugo completes his collection is 0.5904.

3 Given that Hugo completes the collection, what is the probability that he only buys one pack?

$$\begin{aligned} P_{\text{completes}}(X = 1) &= \frac{P_{X=1}(\text{completes}) * P(X = 1)}{P(\text{completes})} = \\ &= \frac{1 * 0.2}{0.5904} = 0.3388 \end{aligned}$$