- $f_1(x) = c_1 e^{-x}$ on $(0, +\infty)$

$$\text{if } c_1 \geq 0: \;\; f_1(x) \geq 0 \;\; \forall\, x \in (0, +\infty)$$

We impose that the integral of $f_1(x)$ on $(0, +\infty)$ is equal to 1:

$$\int_0^{+\infty} c_1 e^{-x} dx = [-c_1 e^{-x}]_0^{+\infty} = c_1 = 1$$

For $c_1 = 1$, $f_1(x)$ is a probability density function.

- $f_2(x) = c_2 e^x$

$$\text{if } c_2 \geq 0: \;\; f_2(x) \geq 0 \;\; \forall\, x \in (0, +\infty)$$

We impose that the integral of $f_2(x)$ on $(0, +\infty)$ is equal to 1:

$$\int_0^{+\infty} c_2 e^x dx = [c_2 e^x]_0^{+\infty} = +\infty \neq 1$$

$$\implies \nexists\, c_2 \;\; s.t. \;\; f_2(x) \;\; is \;\; a \;\; p.d.f.$$

- $f_3(x) = c_3(x-1)$ on $(2, 3)$

$$\text{if } c_3 \geq 0: \;\; f_1(x) \geq 0 \;\; \forall\, x \in (2, 3)$$

We impose the integral of $f_2(x)$ on $(2, 3)$ equal to 1:

$$\int_2^3 c_3(x-1)dx = c_3 * \left[\frac{x^2}{2} - x\right]_2^3 = \frac{3}{2}c_3 = 1$$

For $c_3 = \frac{2}{3}$, $f_3(x)$ is a p.d.f.

- $f_4(x) = c_4(x-1)$ on $(0, 2)$
Since $f_4(0) = -c_3$ and $f_4(2) = c_3$:

$$\nexists\, c_4 \;\; s.t. \;\; f_4(x) \geq 0 \;\; \forall\, x \in (0, 2)$$

$$\implies \nexists\, c_4 \;\; s.t. \;\; f_4(x) \;\; is \;\; a \;\; p.d.f.$$

- $f_5(x) = c_5(x-1)^2$ on $(0, 2)$

$$\text{if } c_5 \geq 0: \;\; f_5(x) \geq 0 \quad \forall\, x \in (0, 2)$$

We impose that the integral of $f_5(x)$ on $(0, 2)$ is equal to 1:

$$\int_0^2 c_5(x-1)^2 dx = c_5 * \int_0^2 (x^2 - 2x + 1)dx = c_5 * \left[\frac{x^3}{3} - x^2 + x\right]_0^2 =$$

$$= \frac{2}{3}c_5 = 1$$

For $c_5 = \frac{3}{2}$, $f_5(x)$ is a p.d.f.