$$P(H) = P_F(H) * P(F) + P_U(H) * P(U) = \frac{1}{2} * \frac{2}{3} + p * \frac{1}{3} = \frac{p+1}{3}$$

$$\boldsymbol{P_H(F)} = \frac{P_F(H) * P(F)}{P(H)} = \frac{\frac{1}{3}}{\frac{p+1}{3}} = \frac{\boldsymbol{1}}{\boldsymbol{p+1}}$$

The probability that I picked a fair coin given that the outcome is head is $\dfrac{1}{p+1}$