

$$P(head) = P(tail) = 0.5$$

Then the probability p_n

$$p_n = (P(tail))^{n-1} * P(head) = 0.5^n$$

So the probability that the game ends on the n^{th} flip is $p_n = 0.5^n$

The probability that the winner is the first player is:

$$\begin{aligned} P(\text{first player wins}) &= 0.5 + 0.5^3 + 0.5^5 + \dots = \sum_{i=0}^{\frac{n-1}{2}} 0.5^{2i+1} = \\ &= 0.5 \sum_{i=0}^{\frac{n-1}{2}} 0.25^i \end{aligned}$$

If $n \rightarrow \infty$ the series

$$0.5 \sum_{i=0}^{\infty} 0.25^i$$

converges to

$$\frac{1}{2} * \frac{1}{1 - \frac{1}{4}} = \frac{1}{2} * \frac{4}{3} = \frac{2}{3}$$

Therefore, the probability that the winner is the first player is $\frac{2}{3}$