An urn contains four balls numbered from 1 to 4.

$$X \sim Uniform(1/4)$$

$$H(X) = - * \log_2(1/4) = 2 \ bits$$

1. If we add a new ball with the number 4, does the entropy associated with the extraction of a ball from the urn increase or decrease?

$$X \sim \{(1, 1/5), (2, 1/5), (3, 1/5), (4, 2/5)\}$$

$$H(X) = - \sum_{i=1}^{4} p_i \log p_i = 3 * \frac{1}{5} * \log_2 5 + \frac{2}{5} \log_2(5/2) = \log_2 5 = 1.922 \ bits$$

The entropy decreases. This is expected since the uniform distribution has maximum entropy.

3. And would it have happened if the added ball had the number 5?

$$X \sim Uniform(1/5)$$

$$H(X) = - \log_2(1/5) = 2.32 \ bits$$

The entropy increases when one ball is added.