$$X \sim Uniform(0,1)$$

$$Y = X^2$$

$$f_X(X) = \begin{cases} 1 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

$$F_X(X) = \int_0^x f(t)dt = \begin{cases} x & 0 < x < 1 \\ 1 & x \geq 1 \\ 0 & x \leq 0 \end{cases}$$

$$F_Y(Y) = F_Y(X^2) = P(X^2 \leq y) = P(X \leq \sqrt{y}) = \begin{cases} \sqrt{y} & 0 < y < 1 \\ 1 & y \geq 1 \\ 0 & y \leq 0 \end{cases}$$

$$f_Y(Y) = F'_Y(Y) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 < y < 1 \\ 0 & otherwise \end{cases}$$

$$\mathbb{E}[Y] = \int_0^1 y f_y(Y)dy = \int_0^1 \frac{y}{2\sqrt{y}} = \int_0^1 \frac{\sqrt{y}}{2} = \left[\frac{y^{3/2}}{3}\right]_0^1 = \frac{1}{3}$$