	X=-2	X=0	X=2	Row sum
Y=1	0	2a	a	3a
Y=2	2a	0	2a	4a
Y=4	a	2a	0	3a
Col sum	3a	4a	3a	

$$S = X + Y \quad Z = X - Y$$

$$R_S = \{-1, 0, 1, 2, 3, 4, 6\}$$

$$R_Z = \{-6, -4, -3, -2, -1, 0, 1\}$$

1. Find the value of a and the marginal probability density function of X

The sum of the probabilities of all possible outcomes must be equal to 1:

$$0 + 2a + a + 2a + 0 + 2a + a + 2a + 0 = 1$$

$$10a = 1 \quad a = \frac{1}{10}$$

We can get the PDF of X by the sum of the columns:

$$p_X(x) = \begin{cases} 3/10 & x = -2 \\ 4/10 & x = 0 \\ 3/10 & x = -2 \end{cases}$$

2. Are X and Y independent?

$$P(X = -2, Y = 1) = 0, \quad P(X = -2) * P(Y = 1) = 0.3 * 0.3 = 0.09$$

Since $P(X = -2, Y = 1) \neq P(X = -2) * P(Y = 1)$, X and Y are not independent.

3. Compute the covariance of S and Z

$$\mathbb{E}[X] = -2 * 0.3 + 0 * 0.4 + 2 * 0.3 = 0$$

$$\mathbb{E}[Y] = 1 * 0.3 + 2 * 0.4 + 4 * 0.3 = 2.3$$

$$Cov(S, Z) = Cov(X + Y, X - Y) = \mathbb{E}[(X + Y)(X - Y)] + \mathbb{E}[X + Y] * \mathbb{E}[X - Y]$$

	X=-2	X=0	X=2
Y=1	0	0.2	0.1
Y=2	0.2	0	0.2
Y=4	0.1	0.2	0

X+Y	-2	0	2
1	-1	1	3
2	0	2	4
4	2	4	6

X-Y	-2	0	2
1	-3	-1	1
2	-4	-2	0
4	-6	-4	-2

$$\mathbb{E}[S] = \mathbb{E}[X+Y] = 0.2*1+0.1*3+0.2*4+0.1*2+0.2*4 = 2.3$$

$$\mathbb{E}[Z] = \mathbb{E}[X-Y] = 0.2*(-1)+0.1*1+0.2*(-4)+0.1*(-6)+0.2*(-4) = -2.3$$

$$Cov(S,Z) = \mathbb{E}[(S-\mu_S)(Z-\mu_Z)] =$$

$$= 0.2*(1-2.3)(-1+2.3)+0.1*(3-2.3)(1+2.3)+0.2*(0-2.3)(-4+2.3)+$$

$$+0.2*(4-2.3)(0+2.3)+0.1*(2-2.3)(-6+2.3)+0.2*(4-2.3)(-4+2.3) =$$

$$= 0.99$$

The covariance of S and Z is 0.99.

4. Are S and Z independent?

No, they are dependent since their covariance is not zero.