$$N = 8000 \quad q = 5$$

$$Prizes: \quad 1*2000, \ 2*750, \ 5*100$$

$$X: ''net \ gain \ from \ purchasing \ a \ randomly \ selected \ ticket''$$

1. Construct the probability distribution function of X.

$$R_X = \{-5, 95, 745, 1995\}$$

$$P(X = 1995) = \frac{1}{8000}$$

$$P(X = 735) = \frac{2}{8000}$$

$$P(X = 95) = \frac{5}{8000}$$

$$P(X = -5) = 1 - P(X \neq -5) = 1 - \frac{1+2+5}{8000} = \frac{7992}{8000}$$

Then PDF of X is:

$$p_X(k) = \begin{cases} 7992/8000 & k = -5 \\ 5/8000 & k = 95 \\ 2/8000 & k = 745 \\ 1/8000 & k = 1995 \end{cases}$$

2. Compute the expected value of X and interpret its meaning.

$$\mu = \sum_{x \in R} x p_X(x) = \frac{-5*7992}{8000} + \frac{95*5}{8000} + \frac{745*2}{8000} + \frac{1995}{8000} = -\frac{36000}{8000} = -\frac{9}{2} = -4.5$$

This result means that the expected net gain when purchasing a ticket is negative because either the probability of not winning is too high, or the win is too low to justify the potential loss.

3. Compute the standard deviation of X.

$$\mathbf{E}[X^2] = \sum_{x \in R} x^2 p_X(x) = \frac{(-5)^2*7992}{8000} + \frac{95^2*5}{8000} + \frac{745^2*2}{8000} + \frac{1995^2}{8000} = 666.875$$

$$V(X) = \mathbf{E}[X^2] - \mu^2 = 666.875 - 20.25 = 646.625$$

Then the standard deviation of X is:

$$\sigma_X = \sqrt{V(X)} = \sqrt{646.625} = 25.429$$