	$Y=0$	$Y=1$
$X=0$	0	1/8
$X=1$	1/8	3/4

$$P(X=0)=P(0,0)+P(0,1)=\frac{1}{8}\quad P(X=1)=P(1,0)+P(1,1)=\frac{7}{8}$$

$$P(Y=0)=P(0,0)+P(0,1)=\frac{1}{8}\quad P(Y=1)=P(1,0)+P(1,1)=\frac{7}{8}$$

i. Compute the entropy $H(X)$ and the conditional entropy $H_Y(X)$, and check that $H(X)\geq H_Y(X)$.

$$H(X)=-\sum_{i=1}^{2}p_i\log p_i=\frac{1}{8}\log_2 8+\frac{7}{8}\log_2\frac{8}{7}=0.544$$

$$P_{Y=0}(X=0)=\frac{P(Y=0,X=0)}{P(Y=0)}=0\quad P_{Y=0}(X=1)=\frac{P(Y=0,X=1)}{P(Y=0)}=1$$

$$P_{Y=1}(X=0)=\frac{P(Y=1,X=0)}{P(Y=1)}=\frac{1}{7}\quad P_{Y=1}(X=1)=\frac{P(Y=1,X=1)}{P(Y=1)}=\frac{6}{7}$$

$$H_0(X)=-\sum_{i=1}^{2}P_0(X=x_i)\log P_0(X=x_i)=0+1*\log_2 1=0$$

$$H_1(X)=-\sum_{i=1}^{2}P_1(X=x_i)\log P_1(X=x_i)=\frac{1}{7}*\log_2 7+\frac{6}{7}\log_2\frac{7}{6}=0.592$$

$$H_Y(X)=\mathbb{E}[H.(X)]=\frac{1}{8}*0+\frac{7}{8}*0.592=0.518$$

Since $H(X)=0.544$ and $H_Y(X)=0.518$,

$$H(X)\geq H_Y(X)$$

ii. Show that under the conditional probability $P_{Y=1}$, the entropy of X is higher than with respect to the a-priori probability P.

As calculated above:

$$P_{Y=1}(X=0)=\frac{P(Y=1,X=0)}{P(Y=1)}=\frac{1}{7}\quad P_{Y=1}(X=1)=\frac{P(Y=1,X=1)}{P(Y=1)}=\frac{6}{7}$$

Therefore:

$$H_1(X) = -\sum_{i=1}^{2} P_1(X = x_i) \log P_1(X = x_i) = \frac{1}{7} * \log_2 7 + \frac{6}{7} \log_2 \frac{7}{6} = 0.592$$

$$H_{Y=1}(X) > H(X)$$

iii. Compute the joint information I(X, Y).

$$I(X,Y) = \sum_{j=1}^{2} \sum_{k=1}^{2} p_{jk} \log_2 \left(\frac{p_{jk}}{p_j q_k} \right) =$$

$$= 0 + \frac{1}{8} * \log_2 (\frac{1}{8} * 8 * \frac{8}{7}) + \frac{1}{8} * \log_2 (\frac{1}{8} * \frac{8}{7} * 8) + \frac{3}{4} * \log_2 (\frac{3}{4} * \frac{8}{7} * \frac{8}{7}) =$$

$$= \frac{1}{4} * \log_2 \frac{8}{7} + \frac{3}{4} \log_2 \frac{48}{49} = 0.0258$$