

**The title of my thesis project**

**Candidate:** Nicola Dal Lago

**Advisor:** Prof. Luca Schenato

**Advisor:** Prof. George Nikolakopoulos

**Advisor:** ...

Master in Control Engineering  
Department of Information Engineering  
2016



# Abstract

This is my abstract



# Contents

<b>Contents</b>	<b>v</b>
<b>1 Introduction</b>	<b>1</b>
<b>2 Design and model</b>	<b>3</b>
2.1 Mechanical design . . . . .	3
2.2 Mathematical model . . . . .	4
2.2.1 Quaternion math . . . . .	4
2.2.2 Quadrotor modelling . . . . .	8
2.2.3 Adding the rotating platform . . . . .	10
<b>3 System Identification</b>	<b>11</b>
<b>Bibliography</b>	<b>13</b>



# 1

## Introduction

In these last years, a growing interest has been shown in robotics, In fact, several industries (automotive, medical, manufacturing, space, etc.), require robots to replace men in dangerous, repetitive or onerous situations. A wide area of this research is dedicated to Unmanned Aerial Vehicle (UAV) and especially the one of having the capability of Vertical TakeOff and Landing (VTOL) [1]. This kind of vehicle can be use in a variety of different scenario, do to the reasonable price, small dimensions and large sensors capability. In particular, nowadays intensive research as been accomplish in the area of enviroment monitoring and exploration, accomplish with different strategies and sensors.



Figure 1.1: T-Hawk, a US-made UAV, commonly used to search for roadside bombs in Iraq, made its debut when it photographed the Fukushima nuclear plant from above, providing a detailed look at the interior damage.

Many type of UAVs have been developed over the last years, in particular the quadrotor type [2], a quadrotor type UAV consists of two pairs of counter rotating rotors and propellers. The aim of this thesis is to contribute to the develop of the so called *Prometheus project*, a fully autonomus vertical tekeoff and landing vehicle, able to perform indoor enviroment exploration and mapping. To do this, we inspired from the film *Prometheus*, where drones are able to map an indoor cave. Of course, do to technology and budjet limitations, the vehicle will not have the same performance, but will have in theory the same capabilities. As previously said, this thesis is only a part

of the project, that has been divided in three main parts:

- mechanical design and building of the UAV [3];
- mathematical model, system identification and control;
- usage of the sensors, mapping and navigation algorithms.

This thesis will focus on the second point, but briefly introductions will give also in the other two points, in particular in the mechanical design, necessary for develop a mathematical model.

Figure 1.2: Frame of the prometheus movie, where the drone perform the exploration and mapping of the cave.



Description of the varius chapters.....



# 2

## Design and model

In this chapter we will focus in the description of the mechanical model of the UAV and the sensor system and, from these, a mathematical model will derive, necessary for build and simulate a control law, and to perform system identification.

### 2.1 Mechanical design

The overall objective of the Prometheus project is navigate and mapping, for these we mean to obtain a 3D reconstruction of a indoor physical environment, using a 360 degrees *Lidar* laser scanner, which, coupled to a standard UAV, will explore in a autonomus way. Lidar is a surveying technology that measures distance by illuminating a target with a laser light. Lidar is an acronym of Light Detection And Ranging, (sometimes Light Imaging, Detection, And Ranging).



Figure 2.1: Lidar laser scanner, able to perform a 360 degrees mapping.

Lidar is popularly used as a technology to make high-resolution maps, with appli-

cations in geodesy, geomatics, archaeology, geography, geology, geomorphology, seismology, forestry, atmospheric physics and so on. What is known as Lidar is sometimes simply referred to as laser scanning or 3D scanning, with terrestrial, airborne and mobile applications <sup>1</sup>. The specific Lidar laser scanner used in this project is report in figure 2.1, where is possible to see the rotating structure moved by a motor attach in the bottom of the frame. However, this sensor is only able to perform 2D mapping and, attach to a drone, make it practically impossible to perform a complete 3D mapping. To solve this problem, several approaches could be adopted, such as use a more complicated and more expensive sensor, that can 3D map, or just by simply use more than one Lidar. However, the solution adopted in this project is again inspired from the movie Prometheus where the sensors are also rotating around the UAV. In such a way, the Lidar has three degrees of freedom in the movement and 3D mapping can be perform. This solution comport, of course, the usage of only one laser scanner, but require a rotating structure that can move the sensor.

Figure here RENDER

In figure [] is possible to see clearly the platform, made of two lightweight rings, and the cart that provide the circular movement of the sensor. An important choice was also the selection of the UAV, that has to guarantee to flight also with the weight of the mechanical structure, sensor and all the eletronics needed to fly and control the movement of the cart.

## 2.2 Mathematical model

Is pretty much clear from the previous section that this UAV is different from almost every other vheicle that is possible to buy, this of course require a complete and detailed study to characterized the mathematical model. To characterized the model, is before necessary to provide some definitions, that are also valid for standars commercial quadrotors.

A quadrotor helicopter is made of a central frame and four propellers that are attach to the frame with respectively four arms. Moreover, the propellers' rotation direction must be opposite in pairs, like illustrate in figure 2.2.

Furthermore, is necessary to define two frames, the world fixed frame and the body frame attach to the vehicle.

In figure 2.3 is possible to see the two frames, the word frame, in black, is fixed to a point and can't be move, the body frame, in blue, instead is attach to the quadrotor and can move with three degrees of freedom. In this, we are interesting in know the translation and rotation of the body frame in respect to the world frame. For represent the translation, a three dimension vector  $\mathbf{x}$  is enough, that actually indicate the position of the quadrotor in the space. Instead, for the rotation, we used quaternions [4], that will be introduce in the following section.

### 2.2.1 Quaternion math

A quaternion is a hyper complex number of rank 4, wich can be represented as follow

$$\mathbf{q} = [q_0 \ q_1 \ q_2 \ q_3]^T \quad (2.1)$$

---

<sup>1</sup><https://en.wikipedia.org/wiki/Lidar>

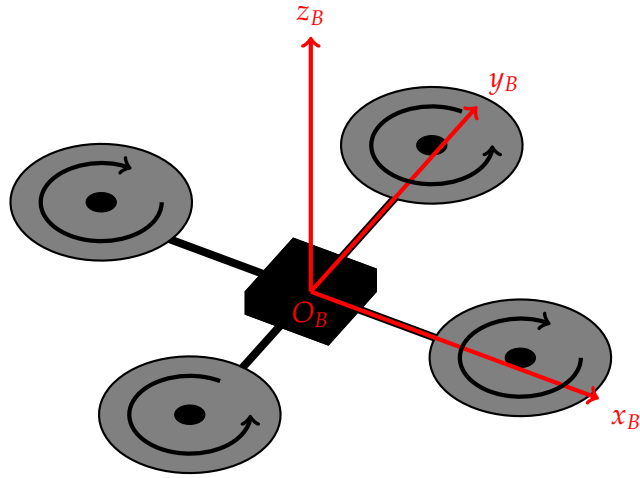


Figure 2.2: Sketch of a standard quadrotor with its body frame attach.

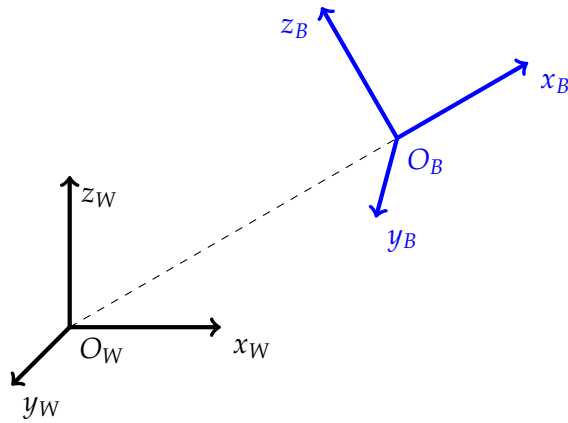


Figure 2.3: Illustration of the world and body frames.

The quaternion units from  $q_1$  to  $q_3$  are called the vector part of the quaternion, while  $q_0$  is the scalar part [5]. Multiplication of two quaternions  $\mathbf{p}$  and  $\mathbf{q}$ , is benignly performed by the Kronecker product, denoted as  $\otimes$ . If  $\mathbf{p}$  represents one rotation and  $\mathbf{q}$  represents another rotation, then  $\mathbf{p} \otimes \mathbf{q}$  represents the combined rotation.

$$\mathbf{p} \otimes \mathbf{q} = \begin{bmatrix} p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3 \\ p_0 q_1 + p_1 q_0 + p_2 q_3 - p_3 q_2 \\ p_0 q_2 - p_1 q_3 + p_2 q_0 + p_3 q_1 \\ p_0 q_3 + p_1 q_2 - p_2 q_1 + p_3 q_0 \end{bmatrix} \quad (2.2)$$

$$= Q(\mathbf{p})\mathbf{q} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (2.3)$$

$$= \bar{Q}(\mathbf{q})\mathbf{p} = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & q_3 & -q_2 \\ q_2 & -q_3 & q_0 & q_1 \\ q_3 & q_2 & -q_1 & q_0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} \quad (2.4)$$

The norm of a quaternion is define as

$$\|\mathbf{q}\| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} \quad (2.5)$$

If the norm of the quaternion is equal to 1, then the quaternion is called unit quaternion. The complex conjugate of a quaternion has the same definition as normal complex numbers.

$$\mathbf{q}^* = [q_0 \quad -q_1 \quad -q_2 \quad -q_3]^T \quad (2.6)$$

The inverse of a quaternion is define as a normal inverse of a complex number.

$$\mathbf{q}^{-1} = \frac{\mathbf{q}^*}{\|\mathbf{q}\|^2} \quad (2.7)$$

The time derivative of the unit quaternion is the vector of quaternion rates [6]. It requires some alebraic manipulation but is important to notice that the quaternion rates,  $\dot{\mathbf{q}}$ , are related to the angular velocity  $\boldsymbol{\omega} = [\omega_x \quad \omega_y \quad \omega_z]^T$ . It can be represented in two way:

- as in equation (2.8) in case that the angular velocity is in the world frame

$$\dot{\mathbf{q}}_w(\mathbf{q}, w) = \frac{1}{2}\mathbf{q} \otimes \begin{bmatrix} 0 \\ \boldsymbol{\omega} \end{bmatrix} = \frac{1}{2}Q(\mathbf{q}) \begin{bmatrix} 0 \\ \boldsymbol{\omega} \end{bmatrix} \quad (2.8)$$

- as in equation (2.9) if the angular velocity vector is in the body frame of reference.

$$\dot{\mathbf{q}}_{w'}(\mathbf{q}, w') = \frac{1}{2} \begin{bmatrix} 0 \\ \boldsymbol{\omega}' \end{bmatrix} \otimes \mathbf{q} = \frac{1}{2}\bar{Q}(\mathbf{q}) \begin{bmatrix} 0 \\ \boldsymbol{\omega}' \end{bmatrix} \quad (2.9)$$

A unit quaternion can be used also as a rotation operator, however the transformation requires both the quaternion and its conjugate, as show in equation (2.10). This rotates the vector  $\mathbf{v}$  from the world frame to the body frame represented by  $\mathbf{q}$ .

$$\omega = \mathbf{q} \otimes \begin{bmatrix} 0 \\ \mathbf{v} \end{bmatrix} \otimes \mathbf{q}^* \quad (2.10)$$

Unit quaternion can be use also to represents rotation matrixes. Consider a vector  $\mathbf{z}$  in the world frame. If  $\mathbf{v}'$  is the same vector in the body coordinates, the the following relations hold

$$\begin{bmatrix} 0 \\ \mathbf{v}' \end{bmatrix} = \mathbf{q} \cdot \begin{bmatrix} 0 \\ \mathbf{v} \end{bmatrix} \cdot \mathbf{q}^* \quad (2.11)$$

$$= \bar{Q}(\mathbf{q})^T Q(\mathbf{q}) \begin{bmatrix} 0 \\ \mathbf{v} \end{bmatrix} \quad (2.12)$$

$$= \begin{bmatrix} 1 & \mathbf{0}^T \\ \mathbf{0} & R_{\mathbf{q}}(\mathbf{q}) \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{v} \end{bmatrix} \quad (2.13)$$

where

$$R_{\mathbf{q}}(\mathbf{q}) = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2q_1q_2 + 2q_0q_3 & 2q_1q_3 - 2q_0q_2 \\ 2q_1q_2 - 2q_0q_3 & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2q_2q_3 + 2q_0q_1 \\ 2q_1q_2 + 2q_0q_3 & 2q_2q_3 - 2q_0q_1 & q_0^2 - q_1^2 - q_2^2 - q_3^2 \end{bmatrix} \quad (2.14)$$

That is,

$$\mathbf{v}' = R_{\mathbf{q}}(\mathbf{q})\mathbf{v} \quad (2.15)$$

$$\mathbf{v} = R_{\mathbf{q}}(\mathbf{q})^T \mathbf{v}' \quad (2.16)$$

Just as with rotation matrices, sequences of rotations are represented by products of quaternions. That is, for unit quaternions  $\mathbf{q}$  and  $\mathbf{p}$ , it holds that

$$R_{\mathbf{q}}(\mathbf{q} \cdot \mathbf{p}) = R_{\mathbf{q}}(\mathbf{q})R_{\mathbf{q}}(\mathbf{p}) \quad (2.17)$$

Finally, for representing quaternion rotations in a more intuitive manner, the conversion from Euler angles (roll  $\phi$ , pith  $\theta$  and yaw  $\psi$ ) to quaternion and viceversa can be performed by utilizing the following two equations respectively.

$$q = \begin{bmatrix} \cos(\phi/2) \cos(\theta/2) \cos(\psi/2) + \sin(\phi/2) \sin(\theta/2) \sin(\psi/2) \\ \sin(\phi/2) \cos(\theta/2) \cos(\psi/2) - \cos(\phi/2) \sin(\theta/2) \sin(\psi/2) \\ \cos(\phi/2) \sin(\theta/2) \cos(\psi/2) + \sin(\phi/2) \cos(\theta/2) \sin(\psi/2) \\ \cos(\phi/2) \cos(\theta/2) \sin(\psi/2) - \sin(\phi/2) \sin(\theta/2) \cos(\psi/2) \end{bmatrix} \quad (2.18)$$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \text{atan2}(2(q_0q_1 + q_2q_3), q_0^2 - q_1^2 - q_2^2 + q_3^2) \\ \text{asin}(2(q_0q_2 - q_3q_1)) \\ \text{atan2}(2(q_0q_3 + q_1q_2), q_0^2 + q_1^2 - q_2^2 - q_3^2) \end{bmatrix} \quad (2.19)$$

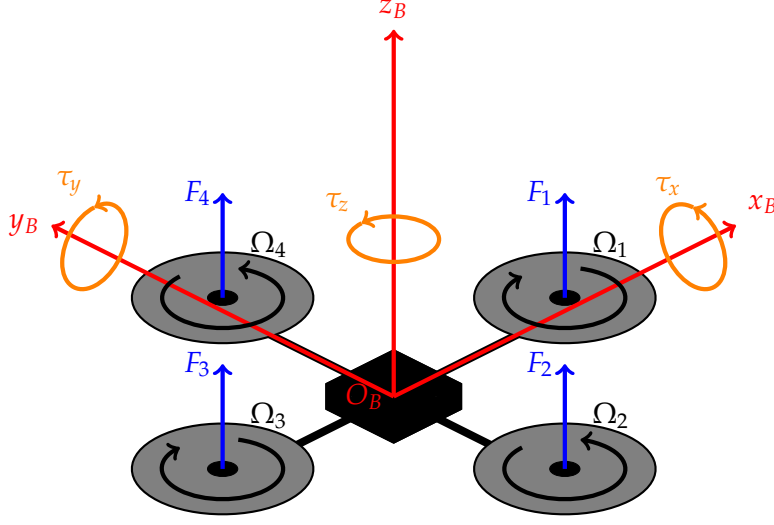


Figure 2.4: Sketch of a standard quadrotor.

### 2.2.2 Quadrotor modelling

We consider first a standard quadrotor, without a rotating platform, like in figure 2.4. In figure 2.4 are also impres the force vectors  $F_i$  generate from each motor-propeller, the torques vectors  $\tau_x$ ,  $\tau_y$  and  $\tau_z$  about the three axis and the propeller's speed  $\Omega_i$ . Now, for modeling the rigid body of a multirotor, the standard Newton-Euler kinematics equations can be utilized [7].

$$\begin{bmatrix} \mathbf{F} \\ \boldsymbol{\tau} \end{bmatrix} = \begin{bmatrix} m \cdot I_{3 \times 3} & \mathbf{0} \\ \mathbf{0}^T & I_{cm} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_B \\ \dot{\boldsymbol{\omega}}_B \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\omega}_B \times I_{cm} \cdot \boldsymbol{\omega}_B \end{bmatrix} \quad (2.20)$$

Where  $\mathbf{F} = [F_x \ F_y \ F_z]^T$  is the vector of the total force,  $\boldsymbol{\tau} = [\tau_x \ \tau_y \ \tau_z]^T$  is the total torque,  $m$  is the mass of the quadrtotor,  $I_{cm}$  is the matrix of inertia related to the center of mass,  $\ddot{\mathbf{x}}_B$  is the acceleration of the quadrotor center of mass related to the body frame and  $\boldsymbol{\omega}_B = [\omega_x \ \omega_y \ \omega_z]^T$  is the rotational rates in the body frame.

Before deriving the torque relationship, the motor models from the input signal to the thrust force are needed. In specific, the four input signals are the speed of the propellers  $u_i$ , map between 0 (zero speed) and 1 (full speed). Then, the thrust for each propeller can be simply derive as follow

$$F_i(t) = A_{F,i} \Omega_i^2 = A_{F,i} \Omega_{max,i}^2 u_i(t)^2 \quad (2.21)$$

where  $A_{F,i} \in \mathbb{R}_+$  are the thrust constants of the motor-propeller cobination,  $\Omega_{max,i} \in \mathbb{R}_+$  are the maximum rotational soeed of the motors and  $u_i(t)$  are the motor signals. What is missing in equation (2.21) is the model of the DC motors and in particular, a map between the input signal  $u_i(t)$  and the control signal  $u_{in,i}(t)$ . To keep the model simple but still accurate <sup>2</sup>, the motor has been modeled like a delay, like in equation

<sup>2</sup><http://pi19404.github.io/pyVision/2015/04/10/25/>

(2.22).

$$u_i(t) \approx \frac{1}{\tau_i s + 1} u_{in,i}(t) \quad (2.22)$$

This approach is very common [8], since all the parameters of a motor are not provide from datasheet, especially from cheap motors that is possible to find quiet often in a commercial quadrotor. Furthermore, to represent the direction of the thrust from a motor it should be considered that

$$\mathbf{F}_i(t) = A_{F,i} \Omega_{max,i}^2 u_i(t) \mathbf{n}_i \quad (2.23)$$

$$\mathbf{n}_i = R_i \cdot [0 \ 0 \ 1]^T \quad (2.24)$$

Where, in this case,  $\mathbf{F}_i(t)$  is the force vector for each propeller and  $R_i$  is the rotational matrix encoding the direction of the thrust and torque vector. Then the torque representation is given by

$$\boldsymbol{\tau}_i(t) = -\text{sgn}(\Omega_i) B_{F,i} \Omega_{max,i}^2 u_i(t)^2 \mathbf{n}_i \quad (2.25)$$

where  $B_{F,i} \in \mathbb{R}_+$  is the torque constant.

Now, by defing the vector  $\mathbf{l}_i = [l_{x,i} \ l_{y,i} \ l_{z,i}]^T$  the distance between the center of mass and the position where the propeller  $i$  is attach, combining equations (2.23), (2.24) and (2.25) is possible to obtain equation (2.26) as in the work [9].

$$\begin{bmatrix} \mathbf{F}_{total} \\ \boldsymbol{\tau}_{total} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^4 \mathbf{F}_i(u_i^2) \\ \sum_{i=1}^4 \mathbf{l}_i \times \mathbf{F}_i(u_i^2) + \boldsymbol{\tau}_i(u_i^2) \end{bmatrix} \quad (2.26)$$

This combined with the Newton-Euler kinematics of equation (2.20) gives the final model, from control signal to accelerationa and angular acceleration, as depicted in equations (2.27) and (2.28).

$$\begin{aligned} \begin{bmatrix} \ddot{\mathbf{x}}_B \\ \dot{\boldsymbol{\omega}}_B \end{bmatrix} &= \begin{bmatrix} \dots & \frac{A_{F,i} \Omega_{max,i}^2 \mathbf{n}_i}{m} & \dots \\ \dots & I_{cm}^{-1} [\mathbf{l}_i \times A_{F,i} \Omega_{max,i}^2 \mathbf{n}_i - \text{sgn}(\Omega_i) B_{F,i} \Omega_{max,i}^2 \mathbf{n}_i] & \dots \end{bmatrix} \begin{bmatrix} \vdots \\ u_i^2 \\ \vdots \end{bmatrix} + \\ &+ \begin{bmatrix} \mathbf{0} \\ I_{cm}^{-1} (\boldsymbol{\omega}_B \times I_{cm} \boldsymbol{\omega}_B) \end{bmatrix} \end{aligned} \quad (2.27)$$

$$u_i = \frac{1}{\tau_i s + 1} u_{in,1} \quad (2.28)$$

From the model (2.27) the linear and angular accelerations are given, is then necessary to convert those to the world frame and integrate to obtain the position and orientation of the quadrotor with the respect to the world frame. ....

### 2.2.3 Adding the rotating platform

Till now, all the model was design for a standard quadrotor vehicle, what we want to do in this section is to add the model of the rotating platform, necessary to deduce a controller and simulate this.

The movement of the platform, introduce a time variant center of mass gravity, that is simply modelled by time variant vectors  $\mathbf{l}_i(t)$ , that identify the displacement of the center of the propeller with the respect of the CoG. If we know precisely the position of the CoG of the quadrotor (without the moving cart) and the position of the CoG of the cart, the result position can be compute.

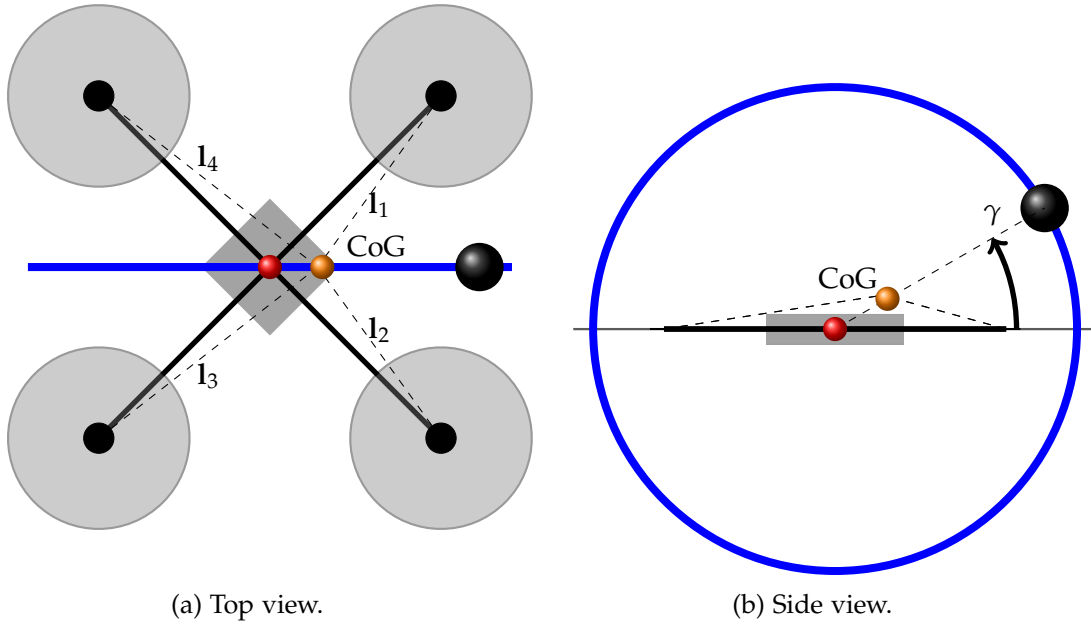


Figure 2.5: Quadrotor with the rotating platform in blue, in red the CoG of the quadrotor and in orange the resulting CoG.

In figure 2.5 is illustrate how the resulting CoG change with the position of the cart, is possible to see also the four  $\mathbf{l}_i(t)$  vectors in black dashed line. Then the position of the CoG is

$$\mathbf{p} = \frac{1}{m} \cdot (m_{quad}\mathbf{p}_{quad} + m_{cart}\mathbf{p}_{cart}) \quad (2.29)$$

where  $m = m_{quad} + m_{cart}$  is the sum of the mass of the quadrotor plus the mass of the moving cart, then the total mass,  $\mathbf{p}_{quad}$  is the position of the center of gravity of the quadrotor without the cart with the respect to the origin of the body frame (we assume that the quadrotor frame is not symmetrical) and  $\mathbf{p}_{cart}$  is the position of the CoG of the cart with the respect to the body frame. Then the vectors  $\mathbf{l}_i$  are just the distance between the center of the propeller  $i$  and  $\mathbf{p}$ .

Another different in using the rotating platform is that the moment of inertia  $I_{cm}$  is not constant, but depend from the position  $\gamma$  of the cart, like in figure 2.5b. This problem can be solved by using the detailed CAD model of the entire vehicle, provide in [3]. From this is possible to deduce the inertia for various position, and then create a simple piecewise model.



# 3

## System Identification

In this chapter, is going to be address an important part of this project. Since the model of the previous chapter is depending from many parameters, is necessary to identificate them, to be able to design an appropriate controller. A Kalman Filter approach will be used, based from the work [\[9\]](#).



## Bibliography

- [1] **P. Pounds, R. Mahony and P. Corke.** Modelling and control of a large quadrotor robot. *Control Engineering Practice* 18, 2010. [1](#)
- [2] **Claudia Mary, Luminita Cristiana Totu and Simon Konge Koldbæk.** Modelling and Control of Autonomous Quad-Rotor. *Aalborg Universitet*, June 2010. [1](#)
- [3] **Carlos Navarro Leoncio.** Design of the Prometheus UAV. *Master thesis project, Luleå university of technology*, 2016. [2](#), [10](#)
- [4] **Joan Solà.** Quaternion kinematics for the error-state KF. *February 2, 2016*. [4](#)
- [5] **Emil Fresk and George Nikolakopoulos.** Full Quaternion Based Attitude Control for a Quadrotor. *2013 European Control Conference (ECC)*, July. [5](#)
- [6] **James Diebel.** Representing Attitude: Euler Angles, Unit Quaternions, and Rotation Vectors. *Stanford University*, 2006. [6](#)
- [7] **Tommaso Bresciani.** Modelling, Identification and Control of a Quadrotor Helicopter. *Department of Automatic Control, Lund University*, October 2008. [8](#)
- [8] **Emil Fresk and George Nikolakopoulos.** Experimental Model Derivation and Control of a Variable Pitch Propeller Quadrotor. *IEEE Multi-Conference on System and Control*, 2014. [9](#)
- [9] **Emil Fresk, R. James Cotton and George Nikolakopoulos.** Generalized Model Identification for Multirotors: Towards applications in Auto Tunning. 2016. [9](#), [11](#)
- [10] **Taeyoung Lee, Melvin Leok and N. Harris McClamroch.** Nonlinear Robust Tracking Control of a Quadrotor UAV on  $SE(3)$ . *2012 American Control Conference, Fairmont Queen Elizabeth, Montréal, Canada, June 27 - June 29, 2012*.