Analysis of Fresnel diffraction - Numerical integration

Date: 12/02/24; Student Nr: 2173375

Introduction

In this exercise, Fresnel diffraction of light through an aperture in 1D and 2D was simulated via numerical integration techniques, and the resulting diffraction patterns analysed. Scipy's quad as well as dblquad integration methods were employed in parts a) and b), and the diffraction investigated under varying parameters such as screen distance and aperture width. In the last part the Monte Carlo technique was explored for 2D diffraction as an alternative integration method.

Part a – 1D diffraction

Method

In this part, scipy's quad integration method is used to evaluate the real and imaginary parts of the one-dimensional Fresnel integral. This is done by looping over the screen coordinates, each of them being used to integrate over the aperture and then obtain a value for the intensity at that screen coordinate.

To analyse how the diffraction pattern changes with different values of screen distance (z) and aperture width (ap), the code sweeps over a range of z- and ap-values (holding the other value constant), and the width of the central peak as well as its intensity are plotted.

Finally, the accuracy of the integration is analysed by performing it for all screen coordinates at different values of epsabs, epsrel and limit (parameters of the quad function), and plotting the RMSE for each of the integration runs as a function of these values.

Results

Default parameters:

Wavelength, λ (m)	Screen distance, z (m)	Aperture width, a (m)	E-field of the incident light in the aperture, E0 (V/m)	Number of data points N for the integral
1*10^(-6)	0.02	0.00002	0.01	200

With the given values, at fix λ and aperture width, near-field effects (Fresnel diffraction) occur approximately at or below a screen distance of 0.00002 m.

With the given values, a fix λ and screen distance, near-field effects (Fresnel diffraction) occur approximately at or above an aperture width of 0.0002 m.

<u>1D- diffraction patterns</u>

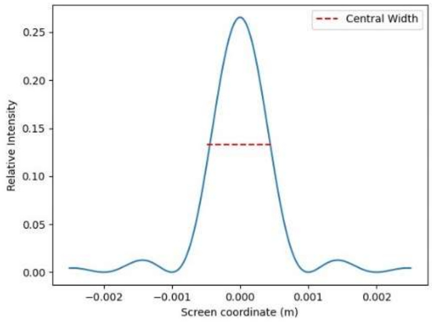
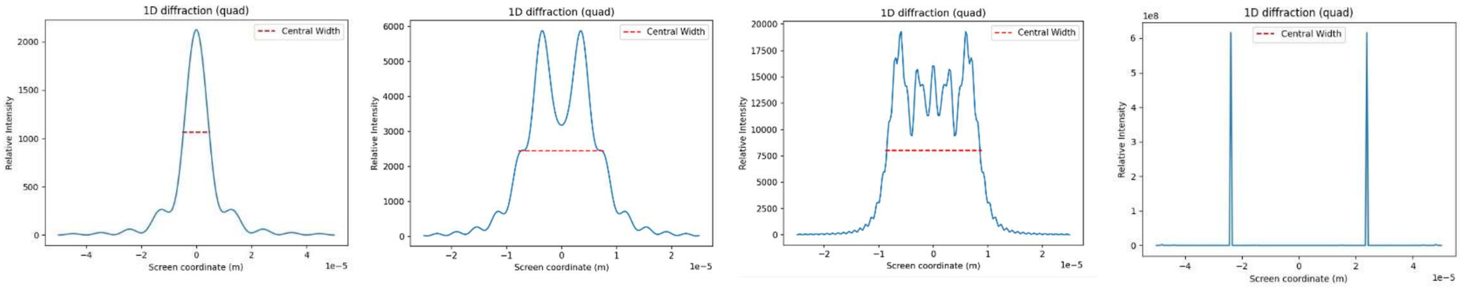
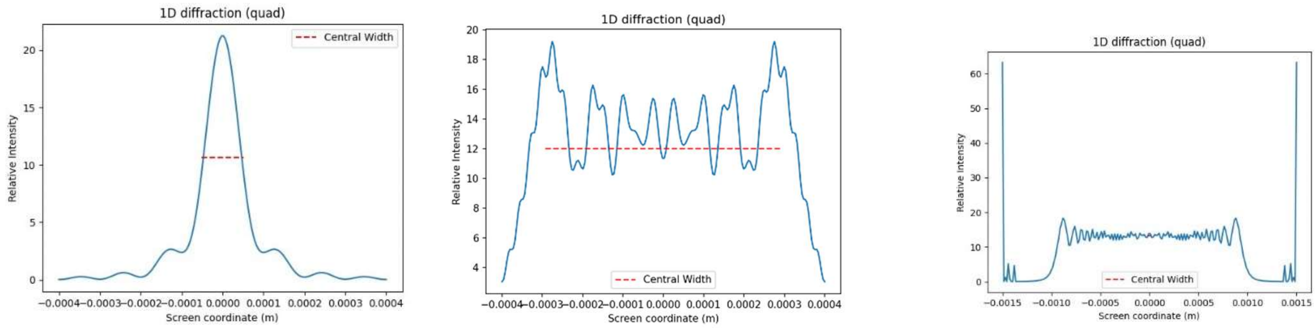


Fig.1



Figs. 2-5 (left to right)



Figs. 6-8 (left to right)

Fig.1: With default vals: z = 0.02m, a = 0.00002m. Fresnel nr = 0.005
Figs. 2-5: Decrease of z with other values fixed.
Fig.2: z = 0.0002m. Fresnel nr= 0.5
Fig.3: z = 0.00007m. Fresnel nr = 1.43
Fig.4: z = 0.00002m. Fresnel nr = 5
Fig.5: z = 0.000002m. Fresnel nr = 50

Figs. 6-8: Increase of a with other values fixed.
Fig.6: a = 0.0002m. Fresnel nr = 0.5
Fig.7: a = 0.0008m. Fresnel nr = 8
Fig.8: a = 0.002m. Fresnel nr = 50

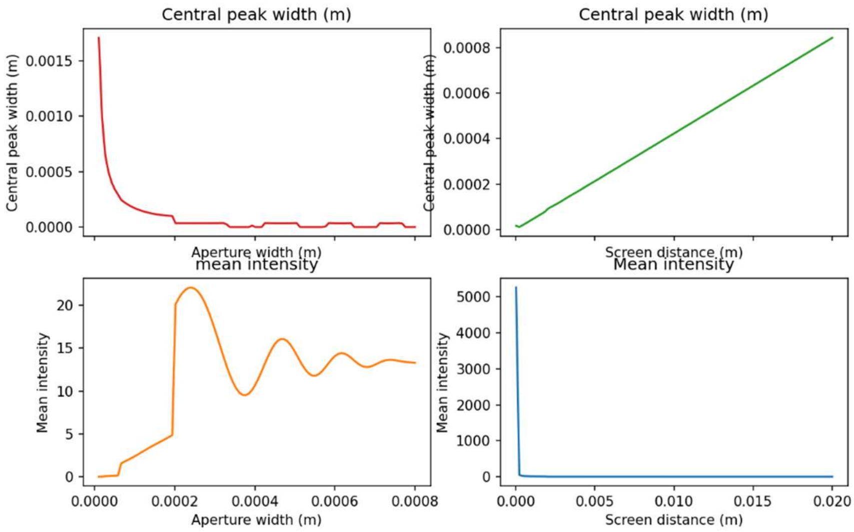


Fig.9: Central peak width and Mean intensity of the diffraction pattern plotted against aperture width and screen distance

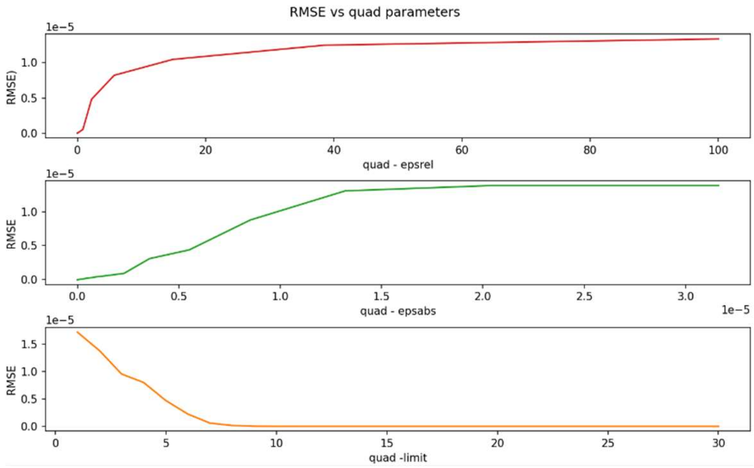


Fig.10: RMSE of integration plotted against the epsrel, epsabs and limit parameters of scipy's quad function.

Theory and Discussion

According to the Huygens-Fresnel principle, each point on a wavefront acts as a secondary source of waves. In the case of coherent light passing through a single narrow slit, the secondary wavelets emanating from different parts of the slit interfere, leading to a diffraction pattern on the screen which results from their angle-dependent constructive and destructive interference.

The diffraction can be described with following Fresnel integral:

$$E(x,y,z) = \frac{kE_0}{2\pi z} \int_{y'_1}^{y'_2} \int_{x'_1}^{x'_2} \exp\left\{\frac{ik}{2z}[(x-x')^2 + (y-y')^2\,]\right\} dx'dy'$$

Where E_0 is the electric field of the light in the aperture, z and y are the screen coordinates and x' and y' are the aperture coordinates. The integration is performed over aperture.

The electric field of the diffracted light can be related to the intensity via:

$$I(x, y, z) = \varepsilon_0 c |E(x, y, z)|^2$$

Whether near-field or far-field effects occur can be characterized by the Fresnel number:

$$N_F = \frac{(a/2)^2}{\lambda z}$$

If N_F is approximately 1or larger than 1, near-field effects become visible whereas far-field effects occur when N_F << 1.

When the screen distance is large enough compared with the size of the aperture, far-field effects can be observed. Hereby the diffraction pattern approximates that produced by Fraunhofer diffraction, described by a sinc function: a central maximum (the most intense, as all of the light is in phase at that point) surrounded by alternating, less intense peaks and throughs.

The default parameters for λ, z, and the aperture width produce such a far-field diffraction pattern, as seen in figure 1.

However decreasing z or increasing the aperture width (while keeping all other values constant) eventually leads to near-field effects. The transition from far-field to near-field patterns is visualised in figures 2-5, where z is decreased from 0.02m to 0.000002m. Increasing the aperture width from 0.00002m to 0.002m leads to similar patterns at similar Fresnel-number values (figs 6-8).

The diffraction pattern in the near field case is a lot more complicated. This is because in the Fresnel regime, the outgoing wavefronts are not approximately planar anymore (as in Fraunhofer diffraction), but are instead spherical, and edge diffraction can't be neglected. This leads to the formation of subpeaks within the central maximum.

The width of the central maximum decreases exponentially with aperture size and is seen to increase linearly with screen distance. As screen distance increases, the mean intensity drops very rapidly. However it does not vary a lot with aperture size – large variations would be due to the overshoots at the edges.

The overshoot at the edges of the near-field patterns is due to the Gibbs phenomenon, which occurs whenever there are jump discontinuities in integral transforms (such as Fourier or here, in the theory of diffraction). At very small z the overshoot becomes very large (figs 5 and 8), which affects the accuracy of the integration.

The Fresnel integral also only accounts for the first two terms of the expanded path length equation – at very small z the higher order terms cannot be neglected anymore, and a complete integral like the Ryleigh-Sommerfeld integral may be better.

The accuracy of the integration varies with the epsabs, epsrel and limit parameters of scipy's quad function (fig 10)

Epsabs determines the maximum absolute error that is allowed – the integration process continues until the estimated error falls below this threshold. An increasing epsabs value thus entails an increasing RMSE value, however the error plateaus at a certain point and does not increase further.

Epsrel sets the maximum allowable relative error in the integration, and thus controls the relative accuracy. The RMSE thus also decreases with a decreasing epsrel.

Limit specifies the maximum number of subdivisions allowed in the integration process – quad divides the integration interval into smaller subintervals, adapting their number until the convergence criteria are met. The RMSE of integration thus decreases with increasing number of subintervals, though reaches a plateau of how small it can get.

Part b) and c) – 2D diffraction

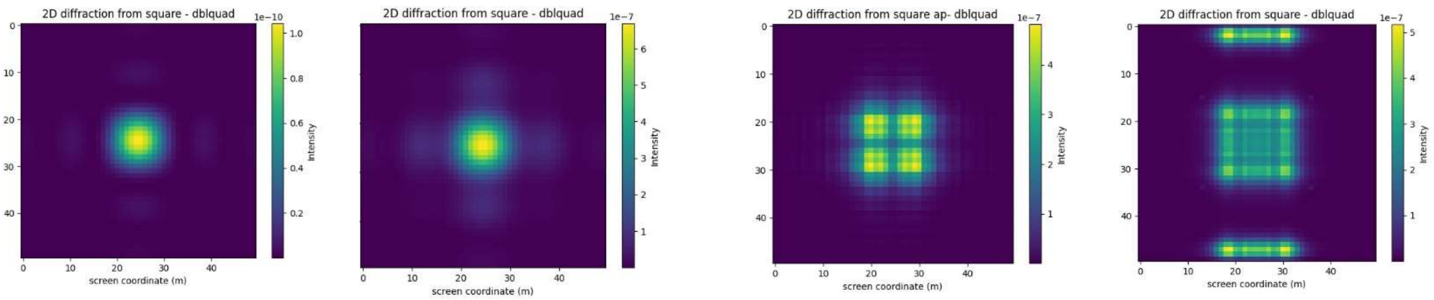
Method

For part b) and c) the 2d- Fresnel integral is evaluated with scipy’s dblquad integration method. To do this the code loops through all screen x-coordinates for each y-coordinate, performing the integration over the aperture for each of those x-y pairs, and subsequently obtaining a 2d-intensity array.

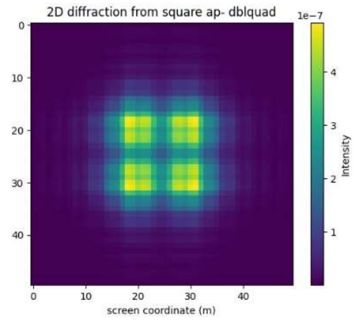
In part b) the intensity diffraction pattern is plotted for a square and a rectangular aperture.

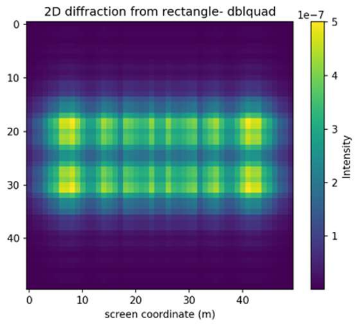
In part c) a circular aperture is used by defining the x-aperture integration limits as being functions of the y-aperture coordinates.

Results

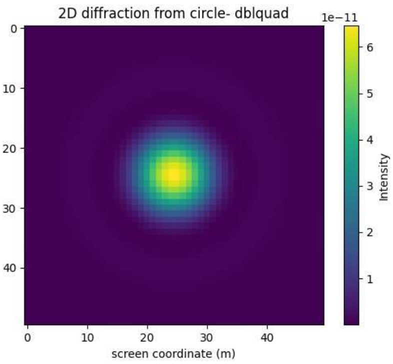


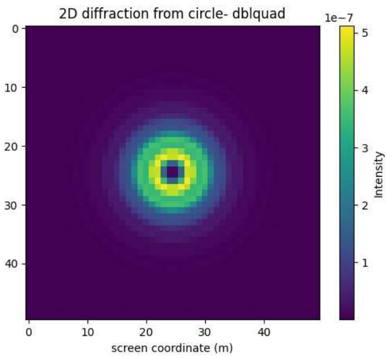
Figs. 1-4 (left to right)

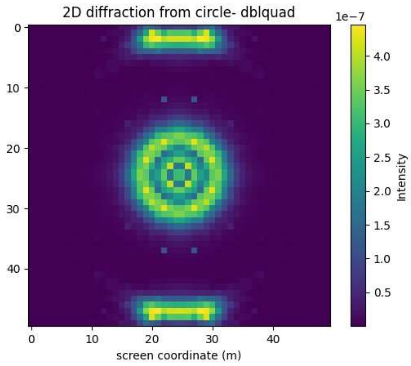




Figs. 5 and 6 (left to right)







Figs. 7-9 (left to right)

Figs. 1-4: Square aperture. Decrease of z with other values fixed.

- Fig.1: default vals: z = 0.02m, a = 0.00002m. Fresnel nr = 0.005
- Fig.2: z = 0.0002m. Fresnel nr= 0.5
- Fig.3: z = 0.00005m. Fresnel nr = 2
- Fig.4: z = 0.00002m. Fresnel nr = 5

Fig.5: Square aperture. a = 0.0004. Fresnel nr = 2
Fig.6: Rectangular aperture. a = 0.0004. Fresnel nr = 2

Figs. 7-9: Circular aperture. Decrease of z with other values fixed.

- Fig.7: default vals: z = 0.02m, a = 0.00002m. Fresnel nr = 0.005
- Fig.8: z = 0.00005m. Fresnel nr = 2
- Fig.9: z = 0.00002m. Fresnel nr= 5

With N = 50

Discussion

The diffraction patterns from square, rectangular and circular apertures follow the same rules as for 1D diffraction.

In the far-field (figs 1, 2 and 7) the constructive and destructive interference of the wavelets can be seen in the alternating bright and dark intensity values (radially away from the origin) and a bright central maximum. The shape of the aperture in the square case however only really becomes discernible in the near-field, with more complicated diffraction occurring within the aperture shape. In the rectangular case, the rectangular shape also gets translated into the diffraction pattern (fig 6).

Increasing the aperture width again leads to near field effects as well (fig 5), with similar patterns being produced at the same Fresnel numbers.

In the near-field pattern of diffraction through a circular aperture, the constructive and destructive interference effects are even more pronounced, as seen by the alternating bright and dark rings (fig 8).

Part d) – 2D Monte Carlo integration

Method

In this part the Monte Carlo integration technique was applied to get a 2D diffraction pattern through a circular aperture.

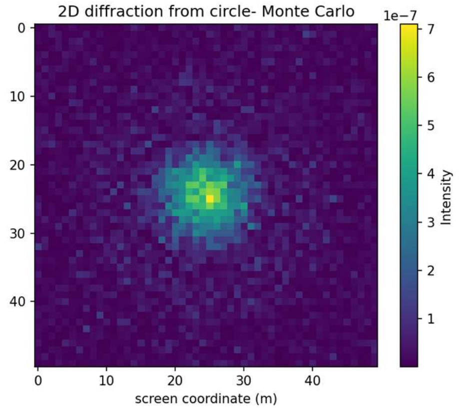
It works by generating a certain number of random aperture coordinates within a square of width 2*radius at each 2D screen coordinate. For each randomly generated aperture coordinate pair, it is checked whether they lie within the radius, and only if they do so are the real and imaginary parts of the Fresnel integration kernel calculated.

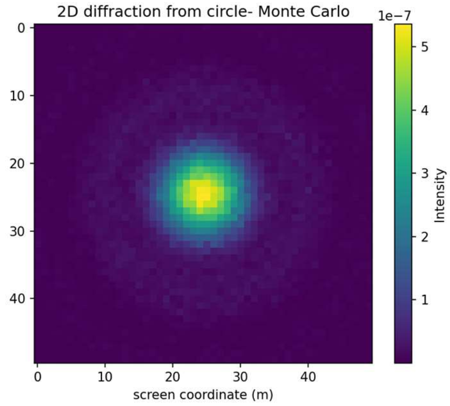
The average values for the real and imaginary kernels for the samples at each screen point are then obtained. These are multiplied by the area of the square to obtain the final integrated value (E).

This integration is performed for every point on the screen, resulting in a diffraction pattern.

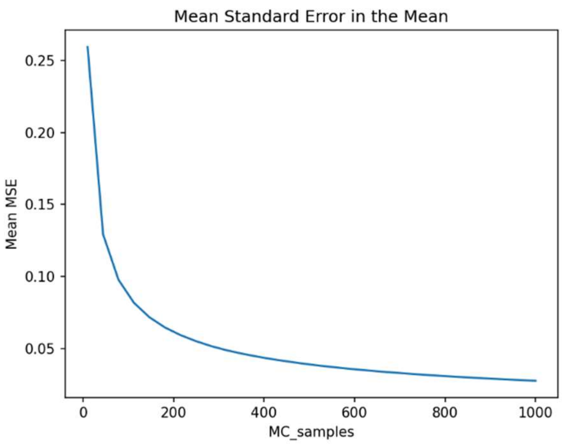
Finally, the mean standard error in the mean of the integration of all screen points, and the mean absolute error relative to scipy’s dblquad function is calculated and analysed.

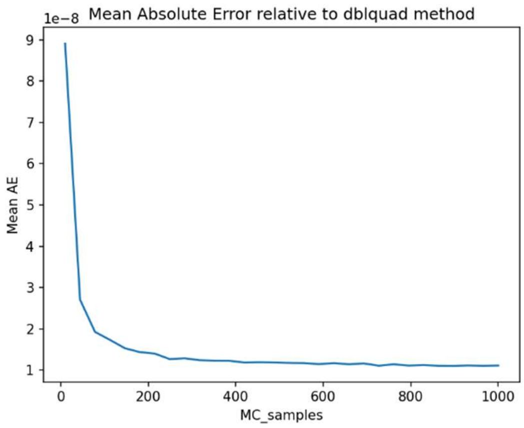
Results





Figs.: 1 and 2 (left to right)





Figs.: 3 and 4 (left to right)

Fig. 1 and 2: Monte Carlo diffraction pattern at 50 and 1000 samples (left to right)
Figs 3 and 4: Mean Standard error in the mean and Mean absolute error for each complete screen integration vs the number of samples the integration was performed at.

Discussion

Figures 2 displays the diffraction pattern that is expected when a circular aperture is used (as was shown before) and thus the Monte Carlo integration method is proven to work.

The accuracy of the integration increases with the number of Monte Carlo samples generated at each screen point. This can be seen in figures 1 and 2, where the pattern becomes less “pixeled”. In figure 3, the mean value of the standard error in the mean of all screen points is plotted against number of samples N, and it can be seen that decreases as 1/sqrt(N). This is also the case when the mean absolute error relative to scipy’s dblquad function is calculated (fig 4).

Conclusion

In conclusion, the quad, dblquad and Monte Carlo integration methods were successfully employed to simulate the diffraction of light through different aperture shapes, sizes, and different screen distances.

Though more complete integrals rather than the Fresnel integral may be better for very close screen distances, and the results are inaccurate in some cases, evaluating the Fresnel integral with these methods allowed for a valuable visualisation of the transition from far-field effects to near-field effects, capturing the complex phenomena that arises in the near field without having to exert much computational power.