Advanced topics in Machine Learning

Assignment 0

Mariuta Nicolae (rqt629) September 6, 2015

# Question 1

**1.** Sample space S = {(red, red), (red, orange) ,(red, blue) (orange, red), (orange,orange), (orange,blue), (blue, red), (blue, orange)}

**2**. P{(red, red)} =

P{(red,orange) } =

P{(red,blue)}=

P{(orange,red)}=

P{(orange,orange)}=

P{(orange,blue)}=

P{(blue,red)}=

P{(blue,orange)}=

**3.** Possible values for X are: 0,1,2

**4.** P{X=0} = I summed the probabilities of all draws that do not contain orange ball.

**5.** P{X=1} =

P{X=2} =

# Question 2

**1.** If the expectation of a random variable is I can then expand the expectation of sums to:

If I consider only the first element of the sum, then:

So the inner sum here is (the event: “X=x” is the same as the event “X=x and Y takes any value ”).

In the same way we obtain that

By combining these results, we obtain that as required.

2.

Because X and Y are independent, then:

**3.** I take the case of tossing two fair dices. X is the number of dices greater or equal with 3 and Y is the maximum number being shown. For these random variables, I have the following tables with all possible values for X and Y for each roll dice:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 2 | 2 | 2 | 2 |
| 4 | 1 | 1 | 2 | 2 | 2 | 2 |
| 5 | 1 | 1 | 2 | 2 | 2 | 2 |
| 6 | 1 | 1 | 2 | 2 | 2 | 2 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Y | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 2 | 3 | 4 | 5 | 6 |
| 3 | 3 | 3 | 3 | 4 | 5 | 6 |
| 4 | 4 | 4 | 4 | 4 | 5 | 6 |
| 5 | 5 | 5 | 5 | 5 | 5 | 6 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

The joint probability distribution table is the following:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| X/Y | 1 | 2 | 3 | 4 | 5 | 6 | sum |
| 0 | 1/36 | 3/36 | 0 | 0 | 0 | 0 | 4/36 |
| 1 | 0 | 0 | 4/36 | 4/36 | 4/36 | 4/36 | 16/36 |
| 2 | 0 | 0 | 1/36 | 3/36 | 5/36 | 7/36 | 16/36 |
| sum | 1/36 | 3/36 | 5/36 | 7/36 | 9/36 | 11/36 | 1 |

The expectations for each variable:

Since then, the two random variables are dependent.

**4.**

In this case, it is like the E[X] in interior of the first expectation is a function (we can say f(x)).

In this case, I can write like the expectation of a function:

But in our case, f(x) is expectation of X so we can expand and write like:

And the sum of all probabilities for a random variable is 1 so we can conclude that:

It is certainly not the optimal way to write that proof but I hope I got the right idea.

**5.**

I will start by just opening the parenthesis:

I have written the proof as for the discrete case but I should have written for the continuous case by using integrals instead of sums.

# Question 3

**1.** I will define the random variable S as suggested in the hint:

By doing this, we can bound the probability by using the Markov’s inequality by using the formula:

Since there is equal chance for each to take value 0 or 1, then the expectation of is 0.5 and then . As a conclusion, we can bound the inequality:

**2.** I am not sure if I understood correctly the Hoeffding’s inequality but since we are interested in the upper limit of the sum, I only have to calculate the upper bound of the equality:

Since we are interested in the probability that S to be smaller or equal than 9 and I have previously found that E[X] = 5, then I think it should be 4.

**3.** Since there are 10 possible combinations of X1,X2…X10 that have sum 9 and 1 combination that has sum 10 out of combinations in total, then the probability of the event is:

**4.** By looking at the results obtained previously, I can say that Hoeffding’s inequality provides a more accurate bounding of the probabilities.

# Question 4

1. I assign random variable if the passenger shows up and 0 otherwise. In this case, we have the probabilities:

and

I will again assign random variable S for the total number of passengers and the expectation of S for 100 passengers is:

By using the Markov’s inequality, I obtain the bound:

By using the Hoeffding’s inequality, I obtain: