CHAPTER 3

INTERACTION OF ELECTROMAGNETIC RADIATION WITH MATTER IN MEDICAL IMAGING

Historically, the foundation of medical imaging was laid when Wilhelm Conrad Roentgen invented X rays and described their diagnostic capabilities in 1895 (1). Roentgen received the first Nobel Prize for his discovery of X rays in 1901. X rays were first used in three-dimensional (3-D) medical imaging using computed tomography (CT) methods in 1972, independently by Godfrey Hounsfield and Allen Cormack, who shared the Nobel Prize for medicine in 1979 (2–4). Today, X rays are the primary and most widely used radiation source for radiological imaging in X-ray radiography, X-ray mammography, and X-ray CT. After the discovery of natural radioactivity by Antonio Henri Becquerel in 1896, the first use of radioactive tracers in medical imaging through a rectilinear scanner was reported by Cassen et al. in 1951 (5). This was followed by the development of a gamma-ray pinhole camera for in vivo studies by Anger in 1952 (6). The clinical use of positron emission tomography (PET) was demonstrated by G. Brownell in 1953 (7). Later, visual light-based imaging methods including fluoroscopy and endoscopy became useful in biomedical and clinical sciences (8, 9). All of these modalities deal with the radiation emerging from the electromagnetic (EM) spectrum as discussed in Chapter 1. Recently, other modalities including magnetic resonance imaging (MRI) and ultrasound have been applied to many medical applications (10–14). It is apparent that medical imaging modalities dealing with radiation sources related to the EM spectrum are most widely used in diagnostic and therapeutic radiological imaging. In this chapter, the basic principles of interaction of EM with matter are briefly discussed.

3.1. ELECTROMAGNETIC RADIATION

In general, radiation is a form of energy that can be propagated through space or matter. It travels in vacuum with a speed of 2.998×10^8 m/s. In other media, the propagation of EM radiation would depend on the transport characteristics of the

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matter. The behavior of EM radiation can be described by a wave or by particle-like bundles of energy called quanta or photons. The wave nature of EM radiation explains reflection, refraction, diffraction, and polarization. However, the physics of EM radiation is often better understood in terms of particles. Short EM waves, such as X rays, may react with matter as if they were treated as particles rather than waves.

EM radiation can be broken down into two mutually dependent perpendicular components. The first carries the electrical properties and is therefore called the electrical field. The other component exhibits the magnetic properties of the radiation and is therefore known as the magnetic field. Both fields propagate in space with the same frequency, speed, and phase. Thus, the combination of the two fields justifies the name "electromagnetic" radiation. The EM radiation wave is characterized by its wavelength λ, frequency ν, and speed c, which is a constant. This provides an inverse relationship between the wavelength and frequency with the following equation:

$$c = \lambda\nu. \tag{3.1}$$

The particle-like quantum or photon is described by a specific amount of energy in EM radiation. The energy, E, of a quantum or photon is proportional to the frequency and is given by

$$E = h\nu \tag{3.2}$$

where h is Planck's constant (6.62×10^{-34} J/s or 4.13×10^{-18} keV/s).

It is apparent from above that the energy of a photon can also be given by

$$E = hc/\lambda = 1.24/\lambda \text{ keV} \tag{3.3}$$

where c (3×10^{8} m/s) is the velocity of light by which a photon can travel in space, and λ is in nanometer (nm).

The particle nature of EM radiation, particularly at short wavelengths, is important in diagnostic radiology. EM radiation usually travels in a straight line within a medium but sometimes can change direction due to scattering events. With respect to energy, the interaction of a photon with matter may lead to penetration, scattering, or photoelectric absorption. While the scattering causes a partial loss of energy and a change of the direction, the photoelectric absorption leads to a complete loss of energy and removal of the photon from the radiation beam.

3.2. ELECTROMAGNETIC RADIATION FOR IMAGE FORMATION

The interaction of photons with matter leads to the formation of an image that carries the information about the absorption properties of the medium. The human body has several types of matter such as bone, soft tissue, and fluid. When a beam of short-wavelength EM radiation such as X ray is transmitted through the body, the X-ray photons usually travel in a straight line and are attenuated depending on the density and atomic properties of the matter in the medium. Thus, the X rays when passed through the body carry the attenuation information that is used to form an image. Since different body matters have different absorption coefficients, an X-ray image reflects an anatomical map of the spatial distribution of the absorption coefficients of the various body matters.

Another form of EM radiation commonly used in nuclear medicine is gamma-ray emission obtained through the radioactive decay of a nucleus. Through a selective administration of radioactive pharmaceuticals, the body matter becomes a source of radioactive decay with emission of gamma rays or other particles such as positrons that eventually lead to the emission of gamma rays. A gamma-ray image shows a spatial distribution of the activity of the source emission. Since body matters, depending on the metabolism, absorb different quantities of radioactive material, the gamma-ray image shows a metabolic or functional map along with some information about anatomical regions.

Since the purpose of gamma-ray imaging is to provide a distribution of the radioactive tracer in the object, the attenuation caused by the object medium is undesirable. In X-ray imaging, the attenuation caused by the object medium is the imaging parameter. Another major difference in these two imaging modalities is radiation dose. The amount and duration of radioactivity caused by the administration of radioactive pharmaceuticals in the body needs to be kept at low levels for the safety of the patient. This is another reason why attenuation and scattering cause a significant problem in nuclear medicine imaging. The gamma-ray photons are approximately in the same energy range as X-ray photons, yet low energy photons are not desirable because they are more likely to be absorbed within the body. Any gamma-ray photon absorbed in the body causes a loss of signal and therefore reduces the signal-to-noise ratio in the image, leading to a poor image quality. Also, any scattering event within the body will degrade the localization of the signal and therefore will cause a poor image with unreliable information.

Due to these differences in imaging, the image reconstruction, processing, and interpretation require different strategies for X-ray and nuclear medicine imaging modalities. These issues are discussed in the subsequent chapters.

3.3. RADIATION INTERACTION WITH MATTER

The photons that are transmitted through a medium either penetrate, scatter, or are removed from the beam due to complete loss of energy. The removal of photons from the radiation beam is called absorption. If a photon is not absorbed, it may continue traveling in the same direction or be scattered in another direction with or without any loss of energy. An X-ray radiation beam for body imaging is selected with an appropriate energy level to provide a reasonable absorption and negligible or very low scattering.

With EM radiation, there are four different types of interaction of photons with matter: (1) coherent or Rayleigh scattering, (2) photoelectric absorption, (3) Compton scattering, and (4) pair production.

3.3.1 Coherent or Rayleigh Scattering

This is an elastic collision of the photon with the matter that causes a slight change in the direction of the photon travel with no loss of energy. This type of elastic scattering occurs with the low-energy photons in the range of a few kilo-electron-volts (keV). It is not a major form of interaction in diagnostic radiology, as photons with energy levels of 20 keV or higher are used, but is often used in X-ray diffraction studies.

3.3.2 Photoelectric Absorption

At lower energy levels, the photon interaction with matter is dominated by photoelectric absorption, which is a major factor in attenuation. The amount of attenuation is described by the attenuation coefficient. In a general model of an atom, electrons are organized in shells or orbits around the nucleus based on their energy levels. During photoelectric absorption, a photon loses its energy by interacting with a tightly bound electron in the body matter, which is subsequently ejected from the atom due to the increased kinetic energy. The ejected electron is dissipated in the matter, leaving a vacancy that is filled by another electron falling from the next shell. The energy dissipation associated with this event is accompanied by the emission of a fluorescent radiation. The photoelectric absorption increases significantly with the increase in atomic number, in other words, the density of the material. It should be noted that low-energy photons are usually absorbed by M and L shells of the atomic structure, while the high-energy photons are absorbed in the inner K-shell. The increase in the energy level associated with the K-shell is characterized by the K absorption edge. Figure 3.1 shows the component of mass attenuation coefficients of water and lead.

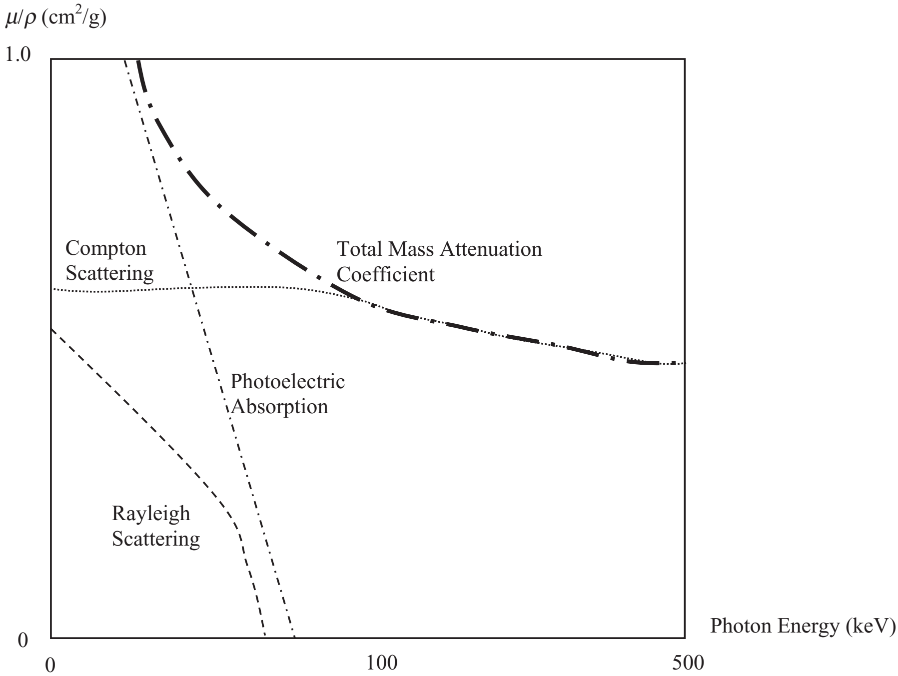

Figure 3.1 The mass attenuation coefficients of water under the 511 keV energy range.

3.3.3 Compton Scattering

Compton scattering is caused by an inelastic collision of a photon with an outer-shell electron with a negligible binding energy. After the collision, the photon with reduced energy is deflected, while the electron with an increased energy is ejected from the atom. With the conservation of energy, the loss of photon energy is equal to the gain in energy for the electron. While the photoelectric effect dominates in materials with high atomic numbers, Compton scattering is more significant in materials with lower atomic numbers. Also, Compton scattering dominates with high-energy photons. Since the higher energy photons would cause a larger deflection angle, a higher energy radiation is not desirable in radiological imaging. In diagnostic radiology and nuclear medicine, Compton scattering is the most problematic interaction of photons with the body matter. The deflections in scattering events cause uncertainties in photon localization as it becomes difficult to keep the desired radiation transmission path. The scattering events not only cause degrading image artifacts such as blur but also lead to a lower signal-to-noise ratio.

3.3.4 Pair Production

Pair production occurs when a high-energy photon on the order of 1 MeV interacts near the nucleus of an atom in a manner similar to the positron emission in a radioactive decay. The transfer of high energy from a photon to the nucleus causes ionization of the atom with a negatron (negatively charged) and a positron (positively charged) electron, each of 511 keV. The positron after coming to rest can interact with an electron, resulting in an annihilation creating two photons of 511 keV, each moving in the opposite direction. Since the high-energy photons are not used in diagnostic radiology, pair production does not play a significant role in radiological imaging.

In diagnostic radiology, the total attenuation coefficient, μ, is the sum of attenuation coefficients caused by Ralyeigh scattering, photoelectric absorption, and Compton scattering. Since Rayleigh scattering is negligible in diagnostic radiological imaging with X rays at energy levels of 20 keV or higher, the effective attenuation coefficient is the sum of photoelectric absorption and Compton scattering.

$$\begin{aligned}\mu &= \mu_{Rayl} + \mu_{Photo} + \mu_{Comp} \\ \mu_{Rayl} &= \rho Z^2 E^{-1} \\ \mu_{Photo} &= \rho Z^3 E^{-3} \\ \mu_{Comp} &= \rho\left(\frac{N_{av} Z}{A_m}\right) E^{-1}\end{aligned} \tag{3.4}$$

where μ_{Ray}, μ_{Photo}, and μ_{Com} are, respectively, the attenuation coefficients caused by Rayleigh scattering, photoelectric absorption, and Compton scattering and ρ, Z, E, N_{av}, and A_m are, respectively, the density, atomic number of the material, energy of the photon, Avogadro's number (6.023×10^{23}), and atomic mass.

3.4. LINEAR ATTENUATION COEFFICIENT

Due to absorption, a radiation beam experiences attenuation when transmitted through a medium. A linear attenuation coefficient, μ, of a medium can be represented as

$$N_o = N_{in} e^{-\mu t} \tag{3.5}$$

where N_{in} is the total number of photons entering into the medium of thickness t with a linear attenuation coefficient μ, and N_o is the total number of photons that come out of the medium.

Let us consider $I_{in}(E)$ to be the intensity of input radiation of energy, E, and $I_d(x, y)$ to be the intensity of the beam detected at a plane perpendicular to the direction of the input beam. Considering no loss of photons due to the detector, $I_d(x, y)$ can be represented by

$$I_d(x, y) = \int I_{in}(x, y, E) e^{-\int \mu(x,y,z,E)dz} d(E) \tag{3.6}$$

where $\mu(x, y, z, E)$ is the linear attenuation coefficient at each region of the object in the field of view of radiation.

If we assume that the medium is homogeneous with a uniform linear attenuation coefficient μ_0 throughout the volume of thickness, t, and the radiation beam is obtained from a monochromatic energy source, $I_d(x, y)$ can be represented by

$$I_d(x, y) = I_{in}(x, y) e^{-\mu_0 t}. \tag{3.7}$$

The linear attenuation coefficient of a material is dependent on the energy of the photon and the atomic number of the elements in the material. Due to the dependency on the mass of the material, often a mass attenuation coefficient (μ/ρ) of a material is considered more useful and is defined by a ratio of the linear attenuation coefficient (μ) and the density (ρ) of the material. Figure 3.2 shows X-ray mass attenuation coefficients for compact bone and fat found in human body.

3.5. RADIATION DETECTION

Spectrometric detectors are used for detection of EM radiation, particularly for X-ray and gamma-ray imaging modalities. There are two basic principles used in spectrometric detectors: ionization and scintillation. Ionization-based detectors create a specific number of ion pairs depending on the energy of the EM radiation. In scintillation detectors, charged particles interact with the scintillation material to emit optical photons. Photomultiplier tubes (PMTs) are used to amplify the light intensity and to convert the optical photons into voltage proportional to the energy of charged particles. Semiconductor detectors have been developed for improving the detection performance of small sized detectors with minimal power requirements.

3.5.1 Ionized Chambers and Proportional Counters

The family of ionized detectors includes ionization chambers and proportional counters. The quantum or photon particle interacts with the sensitive volume of the

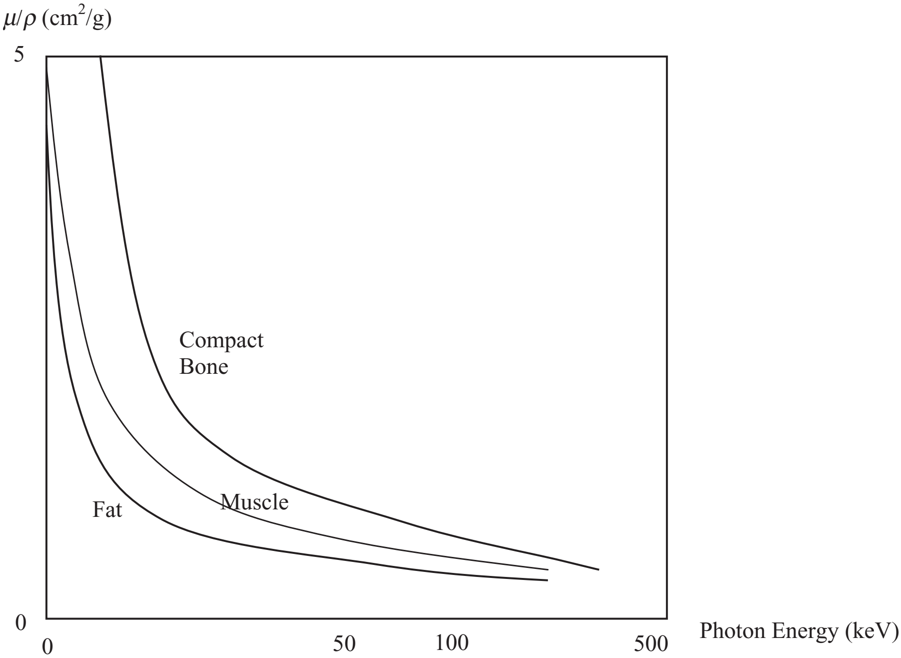

Figure 3.2 The mass attenuation coefficients of compact bone, muscle, and fat.

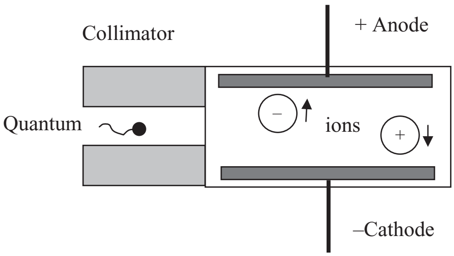

Figure 3.3 A schematic diagram of an ionization chamber-based detector.

detector material and produces a specific number of ion pairs, which is proportional to the energy of the quantum.

An ionized chamber-based detector is comprised of a hermetically sealed vessel filled with gas such as N_2 or Ar_2. A high electric field is used across the electrodes in the presence of a stimulating gas to improve ionization performance. An incoming quantum causes ionization in the gas medium (Fig. 3.3). The positive ions are collected at the cathode and negative ions are collected at the anode to charge an interelectrode capacitance to produce voltage proportional to the energy of the quantum. The electrodes in the ionization chambers are designed in different shapes to maximize the probability of uniform distribution of ions along the surface of the electrodes. Grids are used to improve the movements of charged particles for better efficiency. Since the origin or the traveling path of a scattered quantum from source to the detector cannot be determined with accuracy, the scattered radiations create error in image reconstruction and subsequent quantitative analysis. To avoid the errors in measurements due to scattered events that lower the signal-to-noise ratio, these scattered events must be rejected by the detector. One efficient way of achieving this is to use a collimator with a high density material such as lead. Lead absorbs the scattered quantum that approaches the detector area at an angle.

Let us assume that a quantum of energy, E, produces N number of ions. Let us also assume q to be the charge of an ion. The ionization would ideally lead to a voltage V such that

$$V = Nq/C \tag{3.8}$$

where C is the interelectrode capacitance.

If e_{ion} is the net energy required to create an ion, then

$$N = E/e_{ion}. \tag{3.9}$$

In proportional counters, additional grids are used with an increased voltage on the electrode in the ionization chamber. The proportional counters are usually of cylindrical form because such a shape permits a high potential gradient near the collecting electrode that is placed at the center. This creates a region of impact ionization of gas molecules. The impact ionization improves the process of ionization and increases the number of electrons reaching the collecting electrode. As a result, the proportional counters have a gas multiplication factor to produce better voltage transformation. However, linearity of proportional counters may be affected in the presence of impurities. In practice, gas multiplication factors can provide a gain up to 10,000. Thus, they are more efficient than simple pulse ionization chambers.

3.5.2 Semiconductor Detectors

Recently, emphasis has been given to semiconductor detectors because of their compactness, low power requirements, and potential for high sensitivity and specificity. In ionized detectors, an electric field is established with the help of electrodes. In semiconductor detectors, the particle energy is transformed into electric pulses at the junction region of the semiconductor material. On the surface of the detector a thin layer of p-type material is created over the n-type of material that serves as the base. With a p-n junction, the holes in p-type of material diffuse into n-type material and electrons in n-type material diffuse into p-type material to create a barrier potential across the junction to stop any further movement. The depletion layer across which the barrier potential exists can be modified by applying an external voltage to the p and n regions. The thickness of the depletion layer and, hence, the barrier voltage can be increased by applying positive voltage to the n region and negative voltage to the p region. This creates a high potential gradient in the depletion layer, which serves as an ionization chamber. A quantum interacting with the surface of the detector will create electron-hole pairs in the depletion layer, resulting in the movement of electrons toward positive electrodes and holes toward the negative electrodes to produce a voltage. The voltage that is produced through charging a capacitor is proportional to the energy of the input quantum. The thickness of the depletion layer is analogous to the diameter of an ionization chamber. The voltage produced in the semiconductor detectors can be modeled in terms of number of electron-hole pairs generated, the mobility of the holes, and depletion layer field. Let us assume that there are N numbers of electron-hole pairs generated by a quantum when the voltage across the *p-n* junction is $V_{p\text{-}n}$, and the mobility of holes in the depletion layer is μ_p. The voltage produced by an incoming quantum of energy E can then be modeled as:

$$v(t) = \frac{Nq}{C}(1 - e^{-t/t_0})e^{-t/\tau}$$

$$\text{with } t_0 = \frac{w^2}{4\mu_p V_{p\text{-}n}}. \tag{3.10}$$

where C is the capacitance across the *p-n* junction with τ time constant, q is the charge of a hole, and w is the width of depletion layer.

3.5.3 Advantages of Semiconductor Detectors

It should be noted that for accurate measurement, the thickness of the depletion layer must be much larger than the range of measured particles. This can create a problem in fabricating semiconductor detectors for specific energy ranges. Nevertheless, there are several advantages for using semiconductor detectors. These are listed below.

1. Semiconductor detectors can be fabricated with smaller e (minimum energy needed to create an electron-hole pair), which is ~3.6 eV for silicon. This leads to better detector resolution.
2. Due to the strong electric potential gradient across the depletion layer, the e value becomes independent of the mass and charge of the particle.
3. Semiconductor detectors provide a small charge collection time (<10 ns).
4. Semiconductor detectors cause very small recombination losses due to fast charge collection.

3.5.4 Scintillation Detectors

Scintillation detectors use a scintillation phosphor and a PMT coupled together through an optical contact. The charged particles interact with the scintillation material to excite molecules. The excited molecules emit optical photons during the relaxation process to return to the ground state. The intensity of each scintillation event (amount of light) is proportional to the energy lost by the charged particle or quantum in the phosphor. Thus, scintillation absorbs the energy of ionized radiation quantum and converts it into small flash of light usually in the visible spectrum. Scintillation materials are also called phosphors, or fluorescent materials. For medical imaging applications, a good scintillation material should have a high efficiency of conversion of incident radiation energy into scintillation photons with a linear response over the required energy range, high-energy resolution, good spectral response to match with spectral sensitivity of the connected PMT, a short rise time for fast timing applications, and a fast decay time to reduce detector dead time for faster sampling of dynamic events. While efficiency is defined as the percentage of incident particles that are detected by the detector, energy resolution is measured as the ratio of the full width at half maximum (FWHM) of a given energy peak to the peak position. A good scintillation detector must have a strong stopping power over the required energy range. Stopping power, based on the absorption length, is the ability to stop the quantum in as little material as possible to keep the overall size of the scintillation detector as small as possible. The density of the material is also an important factor in determining the size of the detector to meet spatial resolution requirements of the imaging application.

There are several types of scintillation materials including organic crystals, organic liquids, inorganic crystals, plastics, gaseous scintillators, and glasses. Organic crystals for scintillation, such as anthracene ($C_{14}H_{14}$), stibene ($C_{14}H_{14}$), and naphthalene ($C_{14}H_{14}$), have the advantage of very fast decay time but suffer from anisotropic response, resulting in poor energy resolution. They are hard to develop in the shape and sizes required for medical imaging applications. Organic liquids are also not very practical to use in medical imaging applications, as they provide low efficiency and require sealed chambers. Plastic scintillation materials such as polyvinyltoluene and polystyrene provide high light output and can be molded into any desirable shape. However, they have not been used in the clinical environment because of their low density (approximately 1.1 to 1.2 g/cm^3). Because of their low density, plastic scintillators have poor stopping power and are therefore not used for medical imaging applications. Inorganic crystal materials such as sodium iodide doped with thallium (Na(TI)) are most commonly used because of their high luminescence efficiency, excellent spectral response, and economic price. Other inorganic alkali halide crystals suitable for medical imaging applications include bismuth germanate (BGO), cesium iodide activated by thallium (CsI(TI)), lutetium oxyorthosilicate (LSO), and yttrium oxyorthosilicate activated by cerium (YSO(Ce)). Table 3.1 provides the decay time and light yield for popular scintillation materials used in producing detectors in medical imaging.

PMTs are used to amplify the optical photon intensity or light produced by scintillation materials to provide measurable electric voltage proportional to the energy of the incident quantum. A typical PMT consists of a photocathode, focusing

TABLE 3.1 Characteristic Properties of Commonly Used Scintillation Materials

Material	Density g/cm^3	Refraction index	Decay time ns	Emission wavelength nm	Light yield quanta/keV
NaI(TI)	3.67	1.85	230	410	38
BGO	7.13	2.15	300	480	8.2
LSO	7.44	1.82	40	420	15–27
YSO(Ce)	4.54	1.8	35	420	10–24
CsI(TI)	4.51	1.79	1100	540	38

electrodes, a number of dynodes that work as electron multiplier, and an anode (electron collector electrode) sealed in a vacuum tube. The light photons that are emitted by absorption of an incident quantum in the scintillation material now strike the photocathode of the PMT. As these light photons strike the photocathode, photoelectrons are emitted. The number of photoelectrons emitted by the photocathode, N_c, can be given as

$$N_c = kg_0 wE/E_{ph} \tag{3.11}$$

where g_0 is the coefficient to account for losses at the photocathode, w is quantum efficiency, and E_{ph} is the average energy of photon.

The primary emission of photoelectrons from the photocathode is focused on a series of dynodes with increasing higher voltage levels that stimulate a secondary emission due to a high potential gradient, working as positive feedback to provide a multiplication factor to N_c. The multiple dynodes with increasing voltage levels also accelerate the movement of electrons to provide an amplification factor for the transformation of voltage. Thus, the repetition of secondary emission through multiple dynodes creates electronic multiplication such that photoelectrons are increased up to one million times or more with 10–12 dynodes, making the PMT the most sensitive to the energy of the incident quantum.

In a scintillation detector system, a quantum of energy E produces a time-dependent process of photon emission that decays in a manner similar to an exponential radioactive decay with a time constant t_0. Let us assume that N_0 is the number of photons at time $t = 0$, the total number of photons N generated by a quantum can be expressed as

$$N = N_0 e^{-t/t_0}. \tag{3.12}$$

The decay of photons emission, dN/dt, can then be expressed as

$$\frac{dN}{dt} = \frac{N_0}{t_0} e^{-t/t_0}. \tag{3.13}$$

Although the PMT provides a multiplication factor through secondary emission, the voltage produced by the PMT is a time-dependent function due to the photon emission decay process. A voltage pulse $v(t)$ yielded by the PMT can be modeled as

$$v(t) = k\frac{N_0 q}{C}(1 - e^{-t/t_0})e^{-t/\tau} \tag{3.14}$$

where C is the interelectrode capacitance of the PMT with a time constant τ, k is the multiplication factor, and it is assumed that $\tau \gg t_0$.

Photomultiplier tubes provide excellent amplification factor with low noise as the signal is amplified within the vacuum tube. Figure 3.4 shows a schematic diagram of a scintillation detector showing a collimator to reject scattering events, scintillation phosphor or crystal area, and PMT with multiple dynodes. The spectral response of the scintillation material must be matched with spectral sensitivity of the PMT to provide maximum possible efficiency. Figure 3.5 shows typical quantum efficiency with respect to spectral sensitivity of PMTs manufactured by Hamamatsu. It can be noted that the typical quantum efficiency of PMT is usually within the

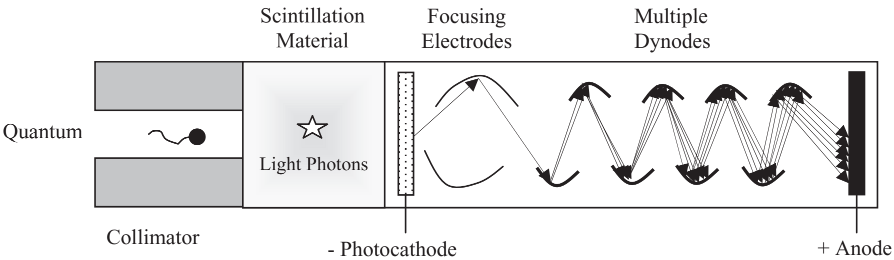

Figure 3.4 A schematic diagram of photomultiplier tube.

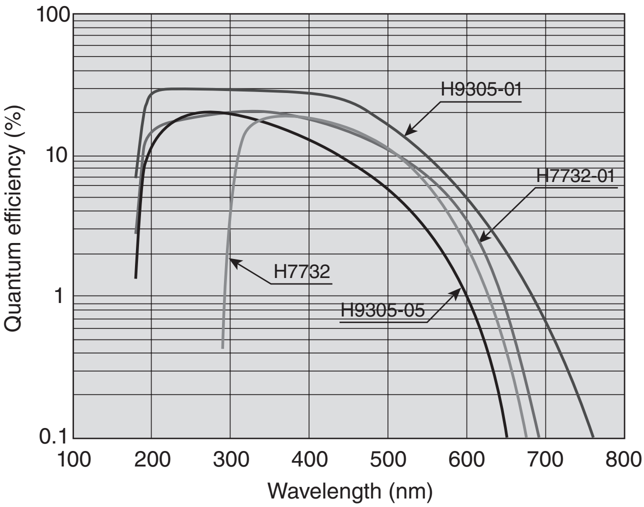

Figure 3.5 Typical spectral sensitivity and quantum efficiency of some of the Hamamatsu photomultiplier tubes (from http://sales.hamamatsu.com).

20–30% range. There are multianode position-sensitive enclosures that are available today in a square-shaped configuration of 8 × 8 PMTs. These position-sensitive PMT blocks are used in modern nuclear medicine as well as CT single photon emission computed tomography (SPECT) and CT–PET dual modality imaging cameras.

3.6. DETECTOR SUBSYSTEM OUTPUT VOLTAGE PULSE

As described above, the primary detector, such as a scintillation phosphor, crystal, or semiconductor material, is interfaced to a PMT and/or gain amplifier that provides a multiplication factor and produces a voltage pulse in an electronic circuit. The design of the electronic circuit may vary but it usually carries an output circuit

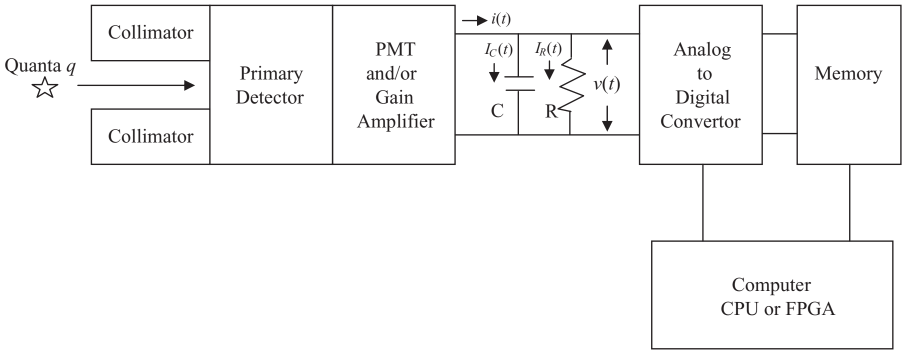

Figure 3.6 A schematic diagram of detector subsystem with output pulse shaping and storage.

segment with a capacitor connected with a load resistor in parallel to shape the output voltage pulse as shown in Figure 3.6. The output voltage pulse is then sampled by an analog-to-digital convertor and the values can be stored in a memory unit using a computer or field programmable gate arrays (FPGA) board.

As shown in Figure 3.6, the current $i(t)$ from the PMT or gain amplifier circuit is split into a capacitor current $i_C(t)$ and a load resistor current $i_R(t)$, producing an output voltage pulse $v(t)$. The output voltage pulse can be shaped using a proper time constant τ, determined by the values of capacitor C and load resistance R. It can be seen from Figure 3.6 that

$$i(t) = i_R(t) + i_C(t)$$

and

$$i_R(t)R = \frac{1}{C}\int i_C(t)dt \tag{3.15}$$

$$i_C(t) = RC\frac{di_R(t)}{dt}.$$

The relationship between the capacitor and resistor currents is given by

$$\frac{di_R(t)}{dt} + \frac{i_R(t)}{RC} = \frac{i(t)}{RC}$$

$$\text{with } i_R(t) = \frac{1}{\tau}e^{-t/\tau}\int_0^t i(t')e^{-t'/\tau}dt' \quad \text{and } \tau = RC. \tag{3.16}$$

The output voltage pulse $v(t)$ can then be expressed as

$$v(t) = i_R(t)R = \frac{1}{C}e^{-t/\tau}\int_0^t i(t')e^{-t'/\tau}dt'. \tag{3.17}$$

It should be noted that the final shape of the output voltage pulse also depends on the detector, PMT, and gain amplifier responses with their respective time decay constants, as they are critical in shaping the current pulse $i(t)$.

3.7. EXERCISES

3.1. Find the wavelength of a quantum with 511 keV.

3.2. What is the linear attenuation coefficient? Why is it important in medical imaging?

3.3. What is Rayleigh scattering? What is its significance in medical imaging?

3.4. Define Compton scattering and photoelectric absorption and describe their differences.

3.5. What happens when photons higher than 1 MeV interact with matter? Describe how this process is different from Compton scattering.

3.6. Why is a monochromatic energy source preferred in medical imaging? Explain your answer.

3.7. Describe the operation of semiconductor detectors and their advantages over the ionized chambers.

3.8. Describe the gain function of a photomultiplier tube on the secondary emission.

3.9. Does a PMT provide better signal-to-noise ratio than a semiconductor amplifier? Explain your answer.

3.10. Why are the photomultiplier tubes extensively used in nuclear medicine imaging?

3.11. Why is it important to have a short decay time for a detector?

3.8. REFERENCES

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