

II.1.3 Bayes optimal classification and probabilistic classification

For the given problem, we have the hypothesis class $H = \{h_0(x)=0, h_1(x)=1\}$ because there is only one element in the input space and possible values are 0 or 1. The Bayes optimal classifier is the hypothesis for which we have the minimal risk: $\min(\{1 - \frac{1}{4}, 1 - \frac{3}{4}\}) = \frac{1}{4}$ which corresponds to the risk of hypothesis $h_1(x)$.

The risk of the classifier is the sum of the risks for each classifier.

$$p(y = 0, h(x) = 1) = p(y = 0 \mid h(x) = 1)p(h(x) = 1) = 0.25 \cdot 0.75 = 0.1875$$

$$p(y = 1, h(x) = 0) = p(y = 1 \mid h(x) = 0)p(h(x) = 0) = 0.75 \cdot 0.25 = 0.1875$$

$$R_p(h) = 0.1875 + 0.1875 = 0.375$$

As a conclusion, the risk of this classifier is worse than using the Bayes optimal classifier.