II.1.3 Bayes optimal classification and probabilistic classification

For the given problem, we have thy hypothesis class H={h0(x)=0, h1(x)=1} because there it is only one element in the input space and possible values are 0 or 1. The Bayes optimal classifier is the hypothesis for which we have the minimal risk: min({1 – ¼, 1 – ¾}) = ¼ which corresponds to the risk of hypothesis h1(x).

The risk of the classifier is the sum of the risks for each classifier.

p(y = 0, h(x) = 1) = p(y = 0 | h(x) = 1)p(h(x) = 1) = 0.25 · 0.75 = 0.1875

p(y = 1, h(x) = 0) = p(y = 1 | h(x) = 0)p(h(x) = 0) = 0.75 · 0.25 = 0.1875

Rp(h) = 0.1875 + 0.1875 = 0.375

As a conclusion, the risk of this classifier is worse than using the Bayes optimal classifier.