Vision and Image Processing: Camera Models, Shading, Photometric Stereo

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Plan for today

- Before we start, Linear Algebra Again.
- Motivation and a Bit of History.
- Camera Models, the Pinhole Camera Model.
- Notions of projection and projective geometry.
- Beyond the Pinhole Camera Model.
- And Before: Orthographic Projection Camera Model.
- A few words on Wednesday.

Outline

Matrices /linear mappings as geometric transformations

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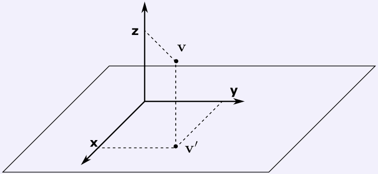
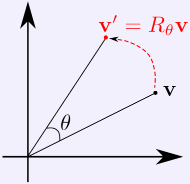
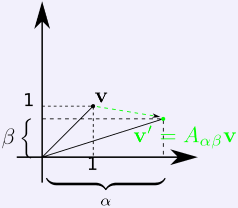
Projection on $x - y$ plane

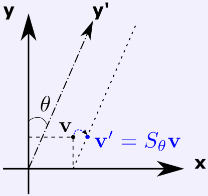




Rotation of angle θ



anisotropic scaling



shear

- projection $\mathbb{R}^3 \to \mathbb{R}^2$

$$F\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix},\quad F = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix}$$

- Rotation of angle θ from $\mathbb{R}^2 \to \mathbb{R}^2$:

$$R_\theta\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x\cos\theta - y\sin\theta\\x\sin\theta + y\cos\theta\end{bmatrix},\quad R_\theta = \begin{bmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{bmatrix}$$

- Scaling by a factor α in x and β in y:

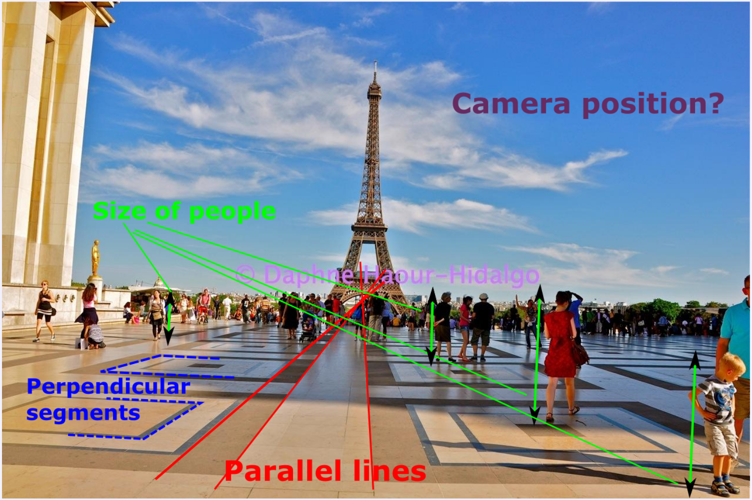
$$A_{\alpha\beta}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\alpha x\\\beta y\end{bmatrix},\quad A_{\alpha\beta} = \begin{bmatrix}\alpha & 0\\0 & \beta\end{bmatrix}$$

- Shear of the y-axis with angle θ:

$$S_\theta\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x + \sin\theta y\\y\end{bmatrix},\quad S_\theta = \begin{bmatrix}1 & \sin\theta\\0 & 1\end{bmatrix}$$

Outline

Motivation


Camera position?
Size of people
© Daphne Haour-Hidalgo
Perpendicular segments
Parallel lines

Questions

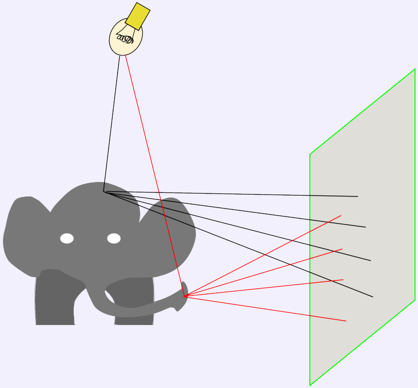
Previous picture raises some questions about:

- Lines?
- Parallelism?
- Angles / orthogonality?
- Sizes?
- Camera position / Horizon?

What happens when you take a picture (or Daphné Haour-Hidalgo in the previous case :-))

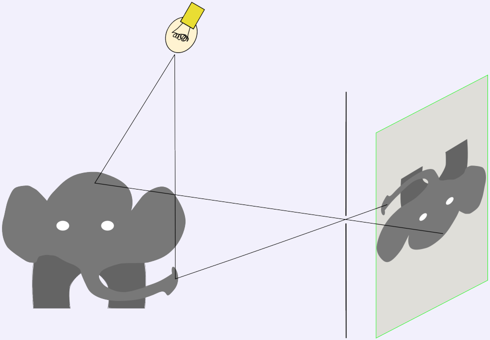
Outline

Getting an Image – I



Many rays emanating from the same position touch the image sensitive array at many location: big blur!

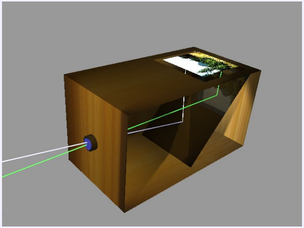
Getting an Image – II



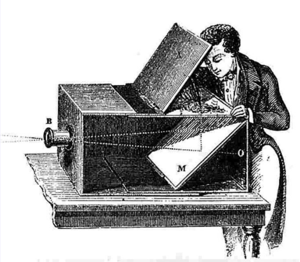
Filtering the rays via a pinhole: get an (inverted) image. Principle of the Camera Obscura (dark room).

Outline

Camera Obscura



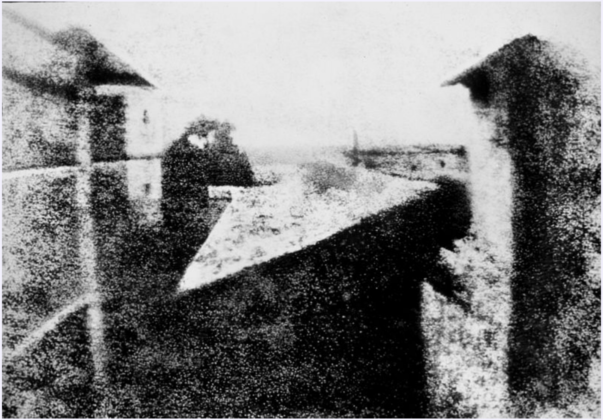
Principle of Camera Obscura



18th Century Camera Obscura

- Known from old chines writings
- Mentioned by Aristotle
- Plaque with photosensitive material: Photographic camera!

The Very First Photography, 1826



J. Nicéphore Nièpce, View from the window at Le Gras, Saint Loup de Varennes, France – Now at University of Texas at Austin.

The Pioneers

J. Nicéphore Nièpce

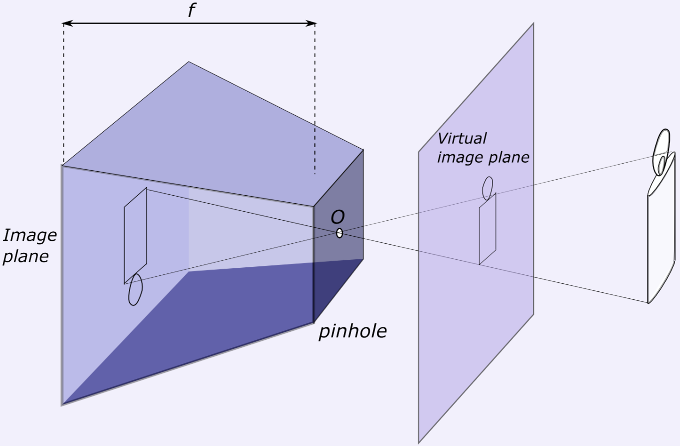
Louis Daguerre

Henri F. Talbot

Now...

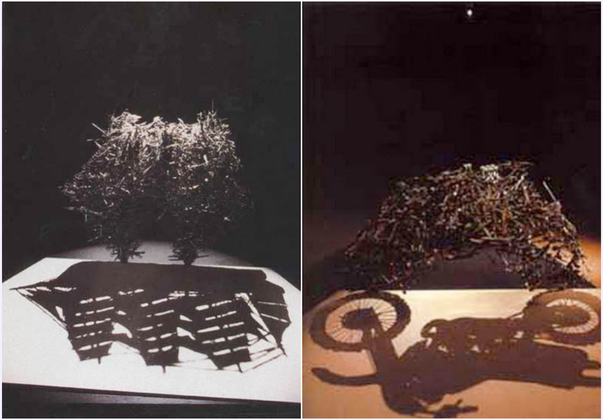
Outline

The Pinhole Camera Model



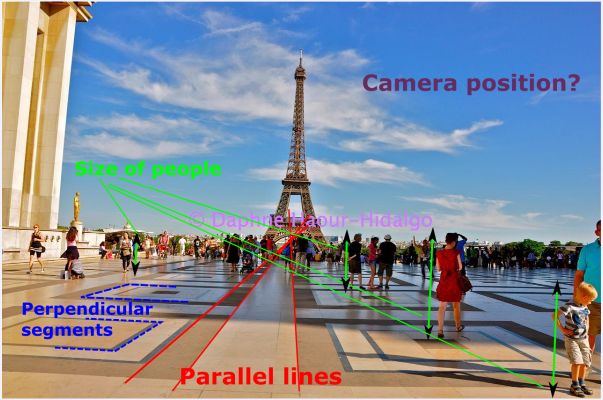
- f is the focal length,
- O is the camera center.

Projection is Tricky!



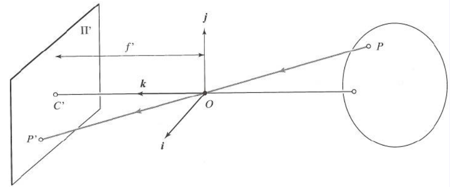
Some illusions from Shigeo Fukuda.

Lost in Projection and Preserved By Projection



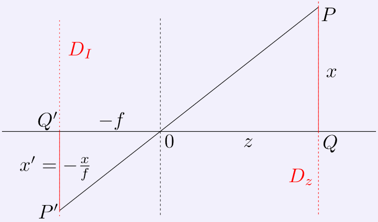
More about than with Søren after New Year.

Projection Equations



- $P(x,y,z)$, $P'(x',y',z')$. P' in the image plane $\implies$ $z' = f$. Thales a.k.a Similar Triangles Theorem:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} f\dfrac{x}{z} \\ \\ f\dfrac{y}{z} \end{bmatrix}, \quad D_I // D_z$$



Homogeneous Coordinates

- Mapping $Pr : \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \end{bmatrix}$ non linear but...
- Use Homogeneous Coordinates

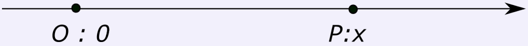
$$(a : b : c) = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{bmatrix} \frac{a}{c} \\ \frac{b}{c} \end{bmatrix}$$

- With them, $\begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \end{bmatrix} = \begin{pmatrix} fx \\ fy \\ z \end{pmatrix}$, (Obs: [] and ()),

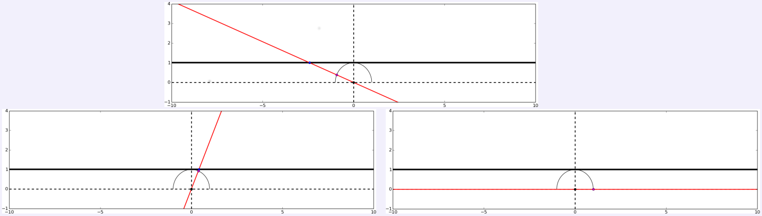
$$Pr = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Pr \begin{pmatrix} \lambda a \\ \lambda b \\ \lambda c \end{pmatrix} = \begin{pmatrix} f\lambda a \\ f\lambda b \\ \lambda c \end{pmatrix} = \begin{bmatrix} \dfrac{fa}{c} \\ \\ \dfrac{fb}{c} \end{bmatrix}$$

Projective Line

- In 1D: standard coordinate = 1 number.



- Change of coordinates: $x \sim \begin{pmatrix} x \\ 1 \end{pmatrix}$, $\begin{pmatrix} x \\ w \end{pmatrix} \sim x/w$
- "Tame" infinity! Point $P_\infty = (x, 0)^T \sim x/0 = \pm\infty \sim (\pm\infty, 1)^\top$.



Negative coordinate, positive coordinate, ∞ coordinate.

- Usual line + P_∞ = Projective Line $\mathbb{P}^1$ and P_∞ = Point at Infinity

1D homography

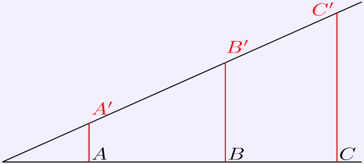
- Homography: Isomorphic (1-to-1) transformation of projective line onto itself.
- Written as 2x2 matrix H with $\det(H) \neq 0$.

$$H = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix} \sim \frac{ax+by}{cx+dy}$$

- Linear mapping on line: multiplication by scalar a. Can be written

$$\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ 1 \end{pmatrix} = \begin{pmatrix} ax \\ 1 \end{pmatrix} \sim ax.$$

  Special type of homography
- Linear mappings conserve distance ratios:



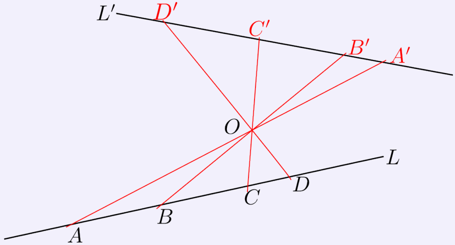
$$\frac{A'B'}{A'C'} = \frac{AB}{AC}$$

Cross-Ratio

- Cross-Ratio of 4 points A, B, C and D:

$$(A;B;C;D) = \frac{AC \cdot BD}{BC \cdot AD}$$

- Geometric Representation of Homography



- O would be camera optical center, Line L in 3-space and line L' in camera plane. Transformation from L to L': 1D homography.
- Homographies conserve Cross-Ratio

$$(A;B;C;D) = (A';B';C';D') = \frac{A'C' \cdot B'D'}{B'C' \cdot A'D'}$$

Projective Geometry

- Invented by artists, Late middle age.
- First formalization by Desargues (1591–1661).
- Idea: complete the line, plan, space with "things" at infinity and make infinity "close".

Homogeneous Coordinates.

- Add an extra coordinate!
- in Plan $[x, y] \sim (x, y, 1)$, $(x, y, z) \sim [x/z, y/z]$, $z \neq 0$.
- Planar Line equation, Projective planar line equation

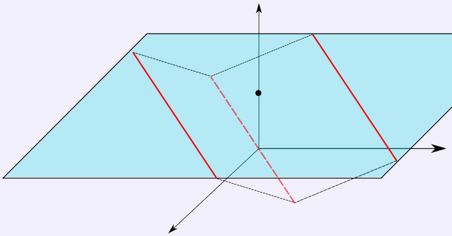
$$\text{Euclidean: } ax + by + c = 0, \quad \text{Projective: } ax + by + cz = 0.$$

- Parallel lines in Euclidean Plan:

$$\begin{cases} L_1: & ax + by + c = 0 \\ L_2: & ax + by + d = 0 \end{cases}, \quad L_1 \cap L_2 = \emptyset$$

Parallel lines in Projective Space?

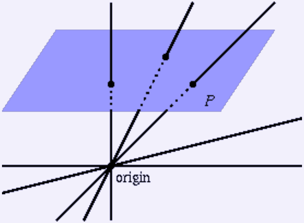
$$\begin{cases} L_1: & ax+by+cz=0 \\ L_2: & ax+by+dz=0 \end{cases}, \quad L_1 \cap L_2 = \begin{cases} ax+by=0 \\ \boxed{z=0} \end{cases}$$

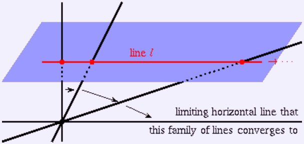


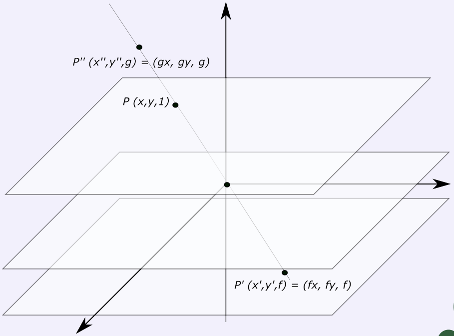
$z = 0$ means “in the extra stuff at infinity”!

- Exercise: P with coordinates $(x, y, 1)$. $Q(x', y', w)$ point on the line through $O(0, 0, 0)$ and P. Compute x' and y'. Considering $[x', y', w]$ as homogeneous 2D coordinates, what are the corresponding 2D coordinates?

Projective Plane and Camera







$z = 1$ and $z = -f$

Homography in 2D

- Isomorphism of the projective plane
- Maps lines to lines.
- Can be represented by a (non-unique) 3x3 invertible matrix (Non zero determinant).

$$H \sim \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \sim \begin{pmatrix} \lambda a & \lambda b & \lambda c \\ \lambda d & \lambda e & \lambda f \\ \lambda g & \lambda h & \lambda i \end{pmatrix}$$

- Standard vector $[x, y] \sim (x, y, 1)^T$. If $i \neq 0$ in H: divide all terms by i: $a \leftarrow a/i$, $b \leftarrow b/i \dots$

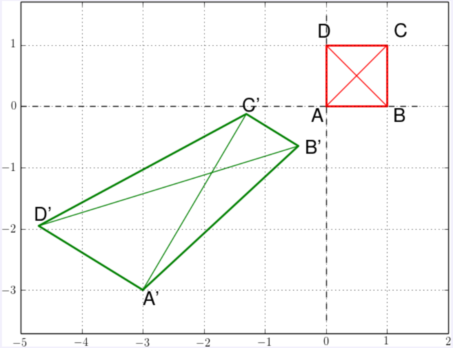
$$\begin{pmatrix} \lambda a & \lambda b & \lambda c \\ \lambda d & \lambda e & \lambda f \\ \lambda g & \lambda h & \lambda 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

- No guaranty that $z' \neq 0$: Can send a point to infinity.

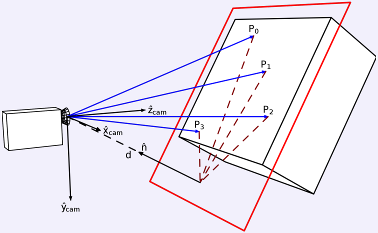
Example of 2D Homography

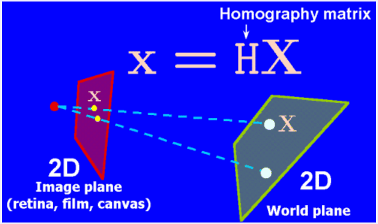
Matrix of homography is

$$H = \begin{pmatrix} 2\cos\theta & -\sin\theta & -\frac{3}{2} \\ \sin\theta & \cos\theta & -\frac{3}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}, \quad \theta = 1.02.$$



Homographies in Computer Vision





Homogeneous coordinates, 3D

- From 3D point coordinate to 3D Homogeneous coordinate

$$(x, y, z) \Longrightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- From 3D homogeneous coordinates to 3D coordinates

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Longrightarrow (x/w, y/w, z/w)$$

Homographies

Matrix known *up to a scalar*.

- Contains Linear mappings and translations for points not at ∞.
- $2D$ linear mapping as homography

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad H_A = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Translation by vector $v = [v_x, v_y]$ as homography:

$$H_v = \begin{pmatrix} 1 & 0 & v_x \\ 0 & 1 & v_y \\ 0 & 0 & 1 \end{pmatrix}$$

Why Are They Useful

- Projection to image plane in standard coordinates:

$$P : (x, y, z) \mapsto P' : (f\frac{x}{z}, f\frac{y}{z})$$

- In homogeneous coordinates:

Why Are They Useful

- Projection to image plane in standard coordinates:

$$P : (x, y, z) \mapsto P' : (f\frac{x}{z}, f\frac{y}{z})$$

- In homogeneous coordinates:

$$P : \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \mapsto P' : \begin{bmatrix} fx \\ fy \\ z \end{bmatrix}$$

- Matrix notation

$$\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- M is the Camera Matrix

World,Camera and Image Coordinates

In the previous slides, Many coordinate systems are implicitly known:

- 3D World Coordinates: Coordinate system of the 3D world.
- Camera Coordinates: 3D coordinate system attached to the camera.
- Image Coordinates: 2D Coordinate system attached to the image plane.

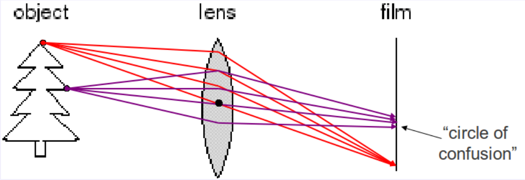
Not that simple in practice! Søren After New Year!

Outline

Shrinking the aperture

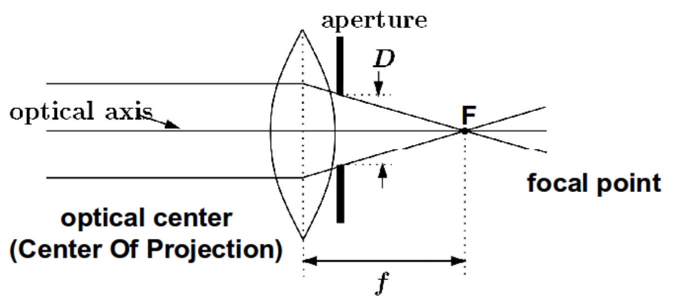
Less light in, diffraction.

Adding Lens



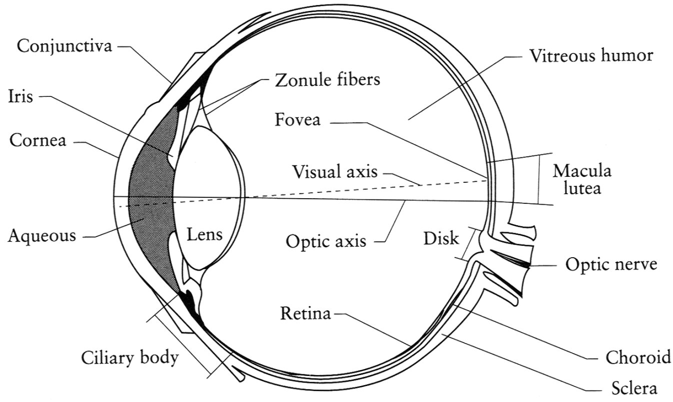
- Specific distance for which objects are in focus
- Changing shape of lens changes the focus distance.

Focal Length, Aperture, Depth of Field

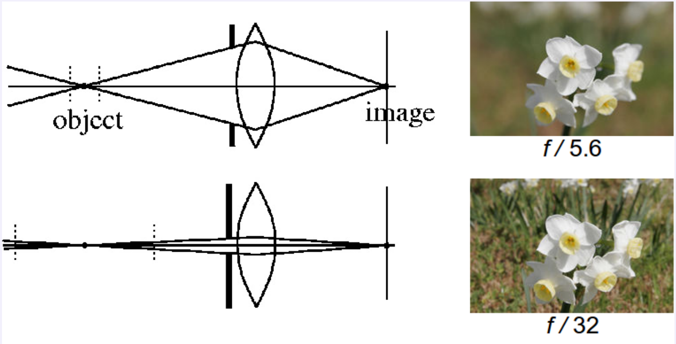


- Lens focuses parallel rays into a single point.
- Aperture restricts range of rays.

The Eye is a Camera with Lens



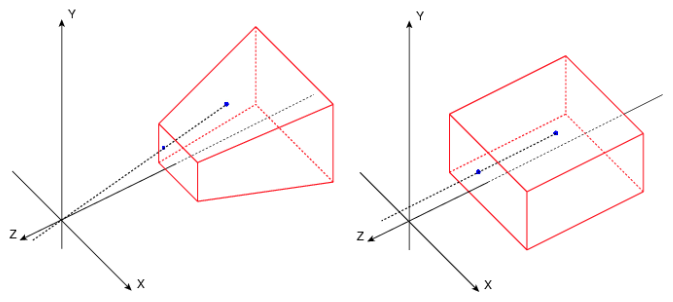
Depth of Field



Controlled by aperture size and focal length.

Outline

Orthographic Camera



- Non physical: "Camera at infinity", parallel beams.
- Projection Matrix:

$$\Pi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- Useful for "far objects". In Some Transmission Imaging too, e.g. CT Scanning, Transmission Electron Microscopy...

What It Preserves

- Parallel lines.
- Size.
- Other?
- Matrix in homogeneous coordinates:

$$\Pi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Observe: null determinant!
- Utility: Distance preserving drawings, and Wednesday!

Outline

Wednesday

- Image and Light.
- Light / Shading / Reflectance.
- Photometric Stereo.