A Coalitional Game Theoretical Approach for Electricity Distribution in Smart Grid

Simone Dalla Valle†, Nicola Gulmini‡

***Abstract*—Smart grid revolutionizes the old-fashioned power grids and transforms them into intelligent electric power distribution networks. This radical transformation is made possible by the usage of network communications, that allows interaction among the entities for monitoring, signaling or even collaborating purposes. In this context, game theoretical approaches constitute a powerful framework for intelligent decision making problems. In this paper, we analyse micro-grid distribution networks in which electrical power management relies on the communication and the cooperation among the entities. In this scenario, micro-grids are allowed to team up and the interactions among them are indeed modelled using coalitional game theory. We implement both exact and heuristic algorithms for coalition formation and for ordering sellers and buyers. Our experimental evaluations show that cooperation leads to better outcomes in terms of power loss and this approach may be applied in real contexts using approximation algorithms that guarantee feasible computation times.**

***Index Terms*—Smart Grid, Game Theory, Cooperative Game Theory, Coalition Formation.**

I. Introduction

Electrical energy distribution is increasingly becoming harder due to the growth of urban centers and high population density in small geographical areas. In the last few decades, researchers deeply investigated this problem from different points of view, such as power electronics, urban planning and social phenomena. One of the most promising ways for optimizing the distribution and reducing the costs is due to Information and Communications Technologies. In this complex scenario, we distinct between traditional electricity grids and smart grids. The former refers to electrical grid in which there are not almost storage capabilities and they have hierarchical structures. Instead, the latter are electricity networks in which both electricity and digital communications data may flow across the grid and that allow to develop self-healing capabilities and to enable end-users to become active participants. Data communication is made possible by the usage of many power sensors and smart meters placed between the electricity provider and customers. Thanks to these technologies, power grids is able to improve their reliability and efficiency. Moreover, electrical operators may successfully manage the peak load and the changes in the electricity supply by setting flexible price plans. Consequently, end-user may control their demand and make the best consumption according to the price plans [1], [2].

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One of the key aspects of smart grid is the accurate demand response (DR) management. Demand response is defined as the reduction in demand designed to reduce electricity peaks and to enhance reliability.

In this paper, we investigate how a game theoretical approach can be applied in smart grid context, mostly focusing on coalitional games. Agents are interested in forming coalition because they desire to increase their profit and to decrease the power loss. Choosing a game theoretical approach, we assume that players act independently and rationally, since each of them aims to maximize their profits [3].

In Section II we briefly recap which are the most useful game theoretical concepts in context of smart grid. Then, Section IV collects the main results available in literature, both using cooperative and non-cooperative approaches. After that, we present the most interesting results obtained in Section III. Finally, a summary is issued in Section VII.

II. Background to Game Theory

Shortly, game theory is the study of mathematical models of strategic decision-making. Therefore, game theory studies conflicts and cooperation among intelligent and rational players. Everyone chooses strategies and the choice affects the strategies of the other players. Every player receive a reward, that depends on the strategy chosen by all the players, and each of them aims to maximize his/her profit (definition of rationality) [4]. More precisely, a game includes a set of players N and each of whom plays a strategy $s_i \in S_i$, where S_i is the set of possible strategies among which player i can choose. The set of all the combination of moves is $S = S_1 \times S_2 \times \ldots \times S_n$. Players want to maximize their utility u_i, where the utility function is defined as $u_i(s) : S \to \mathbb{R}$.

There exist two main types of strategy, called pure strategy and mixed strategy. In the first case, a player chooses an action for sure, while in the second one, he/she selects a probability distribution over the set of actions.

Given two pure strategies available to player i, called A and B, we state that B *strictly dominates* A, if choosing B always gives a better outcome than choosing A, regardless of the choices of the other players. More precisely, $u_i(A, s_{-i}) > u_i(B, s_{-i}) \; \forall s_{-i} \in S_{-i}$, where $s_{-i} = (s_j)_{j \neq i} = (s_1, s_2, ..., s_{i-1}, s_{i+1}, ..., s_n)$ for brevity.

Nash equilibrium is a set of strategies, one for each of the n players of a game, such that each player's choice is his/her best response to the choices of the $n-1$ other players [5]. In other words, any player does not benefit by varying his/her strategy keeping fixed the others. Nash's Existence

Theorem proves that, if mixed strategies are allowed, there exists at least one Nash equilibrium for every game with a finite number of players. However, finding the Nash equilibria of a game is a complex task. In fact, many algorithms have been proposed, but none known to run in polynomial time [6]. One of the most common methods used to find them is called *iterated elimination of strictly dominated strategies*. The algorithm proceeds as follows: we eliminate a strictly dominated strategy from the strategy space, since a rational player would never play this strategy. So, we have obtained a smaller game and we repeat the procedure applying the first step to the reduced game until no dominated strategy is found for any player. When the algorithm stops, if there is only one strategy for each player, the remaining strategies will form the unique Nash equilibrium [7]. This algorithm does not give any guarantee of finding the Nash equilibrium and so other algorithms must be applied in that circumstances.

In this context, we distinct between competitive games and cooperative games. In the first case, players act selfishly and independently and any type of coordination is not allowed. While, in cooperative games, players might do not compete against the others in order to earn, but they can agree on strategies selection for achieving more benefits. In this type of game, they are able to communicate and coordinate attempting at reaching a consensus.

A. Cooperative games

Cooperative games study the interactions among group of agents, called *coalitions*, that are the actors of this kind of games, also called *coalitional games* for this reason. In cooperative game theory binding agreements are allowed, in contrast to non-cooperative game theory. Here, the focus is not on strategies, but on payoffs and coalitions. Coalitional game theoretical approach may be better to model complex scenarios than a competitive one, from the computational point of view.

Formally, if N is still the set of $n \in \mathbb{N}$ players, a coalition[1] is a subset $S \subseteq N$, where the coalition $S = N$ is called *grand coalition*. Given N, let us define the *characteristic function* $v : 2^N \to \mathbb{R}$ that assigns a value $v(S)$ at each coalition $S \subseteq N$ (2^N denotes the *power set* of N) and that satisfies $v(\varnothing) = 0$. A coalitional game is defined as a pair (N, v). The *payoff distribution* (also called *solution concept*) for S is a vector of real numbers $(x_i)_{i \in S}$ and represents the allocation of each player. We write $x(S) = \sum_{i \in S} x_i$. A payoff distribution $x \in \mathbb{R}^n$ is an *imputation* if

- x is *individually rational*, i.e.[2] $x_i \geq v(i)$ for all $i \in N$;
- x is *efficient*, i.e. $x(N) = v(N)$.

An imputation is a distribution of the worth $v(N)$ of the grand coalition N, which gives a payoff x_i to each player i. x_i is at least as much as he/she can obtain when he/she operates alone. The set of imputations is denoted by $I(v)$. We now present other definitions that will be useful in the following: a coalitional game (N, v) is *additive*, if $v(S \cup T) = v(S) + v(T)$ for all $S, T \subseteq N$ such that $S \cap T = \varnothing$. For an additive coalitional game $I(v) = (v(1), \dots, v(n))$. A coalitional game (N, v) is called *super-additive* if $v(S \cup T) \geq v(S) + v(T)$ for all $S, T \subseteq N$ such that $S \cap T = \varnothing$. An imputation is *coalitionally rational* if $x(S) \geq v(S)$ for all $S \in 2^N$. The *core* of a coalitional game (N, v) is the set of all coalitionally rational imputations:

$$C = \{x \in \mathbb{R}^n | x \in I(v) \wedge x(S) \geq v(S) \ \forall S \subseteq N\}$$

and may be empty or very large. Another important concept is the *dominance*. Let (N, v) be a coalitional game and $y, z \in I(v)$. We say that "y dominates z via a coalition S", denoted by $y \succ_S z$, if

- $y_i > z_i$ for all $i \in S$;
- $y(S) \leq v(S)$, i.e. the payoff is reachable for the coalizion S.

Finally, the following definition is also important in coalitional game theory. The set $V \subseteq I(v)$ is a *stable set* (also called *von Neumann-Morgenstern solution*), if the following conditions are satisfied:

- *internal stability*: $x, y \in V \Rightarrow \nexists S \subseteq N$ such that $y \not\succ_S x$, no payoff vector in the stable set is dominated by another vector in the set;
- *external stability*: $x \in V \Rightarrow \exists y \notin V$ and $S \subseteq N$ such that $x \succ_S y$, all payoff vectors outside the set are dominated by at least one vector in the set.

The stable set, when it exists, is generally not easy to compute, but it has interesting properties. For instance, it is typically not unique. From these simple definitions is clear that the coalitional game theory has to handle the computational complexity of its problems, allowing us to design new methods to obtain good solutions, in terms of time and memory.

III. Game Theory in Smart Grid: Applications

Using game theoretical approaches we are able to solve a large variety of problems related to smart grid [8], since it is a powerful mechanism to achieve optimization goals in many different contexts. For instance, demand-side management and micro-grids controlling may be solved using non-cooperative games or sellers may adapt and optimize pricing strategies according to the nature of the grid using dynamic games [9].

Moreover, thanks to the growth of information and communication technologies, nodes may interact each others in different ways allowing cooperation among them. In this context, cooperative game theoretical approaches model the interactions, trying to enhance the efficiency and the robustness of the network.

We now present some well-known game theory applications in smart grid, that make use of non-cooperative schemes.

[1] We abuse the notation using, from now, S for coalitions and no more for the strategies.

[2] We write $v(i)$ instead of $v(\{i\})$ for simplicity: $i \in S$ is the player belonging to a coalition S, $\{i\} \subseteq N$ is the singleton coalition consisting of the only player i. It should be clear from the context which is the meaning of the notation.

A. *Energy consumption regulation through appliances' scheduling*

As just said, game theory can be used for demand-side management. We now focus on a non-cooperative approach for modelling the relations between an energy supplier and the consumers. Possible extensions may consider multiple energy sources, but in this case we omit this chance. Standard demand-side managements control the appliances of each consumer or incentives voluntary shifts in energy consumption by pricing energy based on time slots [9]. On the contrary, the authors of [10] provide an alternative scheme that optimizes the properties of the aggregate load of consumers, instead of focusing on individual energy consumption. Users are independent decision makers and their choices may effect the strategies of the others. The aim is allowing smart meters, installed in users' home, to choose the best appliances' scheduling in order to minimize the total cost to the utility company and, afterwards, minimize the prices for every individual user. The authors also present an algorithm to find the Nash equilibrium. Their simulations shows that the energy costs may be reduced up to 18% with respect to existing solutions [10].

In this scenario, future works might investigate the optimization of the trade-off between waiting time and billing charges, as suggested in [9].

B. *Demand-side management with storage device*

Let now consider an extension of the previous problem, in which houses have *energy storage capacity*. This perspective opens to new opportunities and demand-side management drastically changes. In fact, consumers may choose to store energy during off-peak hours and then directly utilize this energy during peak hours if needed. Storage capacities could enhance the energy efficiency and give the users more flexibility, but introduce several challenges at the same time. For example, storage devices may decide to be charged simultaneously from the electrical grid, causing overload in demand and potentially breaking down the system. The authors in [11] present a management technique that leads storage devices to converge to worthwhile and effective behaviour. They develop storage strategies that are able to adapt to time slots pricing and market conditions. Their experiments shows that it is possible to reach more robust solution, in term of reliability, and savings of up to 13% on average in electrical bill.

Once again, this result demonstrates the power of game theory-based strategy for electrical distribution management.

We decide to deal with cooperative game theory, since it is promising and partially unexplored scope. For this reason, in the following sections we are going to present a coalitional game theoretical approach for cooperative micro-grid distribution networks [12].

IV. Literature Review

In this section we provide a short recap of the most interesting work in which a coalitional game theoretical approach is used to tackle a smart grid related problem.

In [13] a detailed model of micro-grids distribution networks is provided and it is studied from a coalitional game theoretical point of view. In particular, an optimal coalition formation mechanism, called *hierarchical coalition formation* mechanism, is presented. The optimality of the mechanism is guaranteed by a *cooperative equilibrium*. The presented method is scalable, and it shows that, even with a quite low number of micro-grids, they have an incentive to form coalitions, that legitimizes us to perform simple simulations to test our algorithms.

In [14] a different model is presented, and a more theoretic analysis is done to show how sharing resources among consumers could be beneficial for everyone, and to quantifying this benefit. Also interesting extensions of the model are provided, but there is not enough attention to the computational aspects of the problem, although a case study is presented.

A more simple and realistic model is provided in [12] and it is used also in our work. Here, a complete analysis of a strategy that can minimize the power loss is provided. The most important contribution of this work is that it fixes a simple but efficient approach. The optimality of the presented results is a *Pareto-optimality*. Other contributions of this paper are appropriately expanded later in the section V.

Finally, in [15] the authors discuss the methods proposed by the works in [13], [12] and others. In [12] the algorithm is not scalable because of its NP-hardness, so they propose an heuristic algorithm, based on a similar model, to obtain computational preferable performances, sacrificing the optimality. They use a quadratic programming-based method, and consider the scenario in which not all the micro-grids can communicate each other (i.e. the line does not connect all the micro-grids). Contrary, in our work, since we use a low number of micro-grids, is still reasonable to think that all of them are connected by the distribution line, otherwise the algorithms' performances becomes less remarkable.

As suggested in [16], we focus on developing algorithms for coalition formation in order to decrease the time of processing and to increase the scalability of the model. Before introducing the algorithms and the results, it is necessary to provide a formal description of the adopted model.

V. Model

Our model is the same as [12], we briefly recap its main characteristics. Let N a set of n micro-grids that forms a distribution network, at the center of which there is a substation: the macro-station. At a certain time, a micro-grid $i \in N$ generates an amount of power G_i to satisfy a consumers' demand D_i. Let $Q_i = G_i - D_i$ be

- the surplus of power if $Q_i > 0$. In this case the micro-grid is a *seller*;
- the need if $Q_i < 0$. In this case the micro-grid is called *buyer*.

Otherwise, $Q_i = 0$, the micro-grid is able to meet its demand. For the complexity of the scenario, the demand is typically considered random.

In the non-cooperative case, micro-grids exchange power, according to their needs, with the macro-station, with a medium voltage U_0. The power transfer implies a power loss

$$P_{i0}^{\text{loss}} = R_{i0}\left(\frac{P_i(Q_i)}{U_0}\right)^2 + \beta P_i(Q_i) \tag{1}$$

where R_{i0} is the resistance of the distribution line between the micro-grid i and the macro-station, β is the fraction of power lost at the transformer of the macro-station, and $P_i(Q_i)$ represents the power flowing between the micro-grid i and the macro-station given by

$$P_i(Q_i) = \begin{cases} Q_i & \text{if } Q_i > 0 \\ L_i^* & \text{if } Q_i < 0 \\ 0 & \text{if } Q_i = 0. \end{cases} \tag{2}$$

where L_i^* is the total amount of power that needs to be generated by the macro-station to ensure that the micro-grid i receives $P_i^{\text{required}} = -Q_i > 0$ and it is given by the following

$$L_i^* = \begin{cases} \frac{(1-\beta)-\sqrt{(\beta-1)^2+4\frac{Q_i R_{i0}}{U_0^2}}}{2R_{i0}/U_0^2} & \text{if } (\beta-1)^2 + \frac{4Q_i R_{i0}}{U_0^2} > 0 \\ \frac{(1-\beta)U_0^2}{2R_{i0}} & \text{otherwise.} \end{cases} \tag{3}$$

We also make the hypothesis that the macro-station can generate enough power to satisfy all the buyers of the network.

We now define the *non-cooperative utility function* of the micro-grid i as the power loss due to the power transfer

$$u(i) = -w_i P_{i0}^{\text{loss}} \tag{4}$$

where w_i is the price paid by the micro-grid per unit of power loss (the minus sign turns the problem to a maximization problem).

To maximize their payoff, the micro-grids have to reduce as much as possible the power loss due to the transfer. When a coalition $S \subseteq N$ of micro-grids is built, its members transfer power to each other, according to their surplus and their needs. This is convenient because the transfer can take place between micro-grids that are closer to each other than they are with the the macro-station. Indeed, the number of transfers with the macro-station is reduced and so the number of passages through the transformer. For these reasons the micro-grids have an incentive to form coalitions.

A. Cooperative micro-grids

We have already provided a formal definition of cooperative games in section II-A. To model our scenario with coalitional game theory it is necessary to define the characteristic function v.

Let us suppose, for simplicity, that every micro-grid has $Q_i \neq 0$. In a coalition S, the micro-grids can be divided into two disjoint sets: the set of buyers $S_b = \{i \in S : Q_i < 0\} \subset S$ and the set of sellers $S_s = \{i \in S : Q_i > 0\} \subset S$. It holds that $S_b \cup S_s = S$ and $S_b \cap S_s = \varnothing$. We now provide the power exchange process among sellers and buyers of the same coalition, shown in [12]. Let us focus on a fixed coalition S. Given a buyer $b \in S_b$ and an ordering of the sellers μ, i.e. a permutation of S_s (that can be defined in many different ways), the power exchange follows the algorithm 1.

Algorithm 1 Power exchange among a buyer $b \in S_b$ and the sellers of the same coalition S

```
S_s ← {s_μ(1), ..., s_μ(||S_s||)} with some ordering μ
while Q_b ≠ 0 or S_s ≠ ∅ do
    s ← the first available seller of the set S_s
    if s can ensure a received power of Q_b to b then
        Q_b ← 0
        Q_s ← Q_s − (|Q_b|+P_sb^loss)
        return
    else
        b acquire as much power as possible from s
        Q_b ← Q_b + (Q_s − P_sb^loss)
        Q_s ← 0
        S_s ← S_s/{s}
while Q_b ≠ 0 do
    perform power exchanges with the macro-station
while Q_s ≠ 0 do
    perform power exchanges with the macro-station
```

Given an ordering also for the set of buyers $S_b = \{b_{\sigma(1)}, \ldots, b_{\sigma(\|S_b\|)}\}$, the power exchange for the coalition S follows the algorithm 2.

Algorithm 2 Power exchange among buyers and sellers of S

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S_s ← {s_μ(1), ..., s_μ(||S_s||)} with some ordering μ
S_b ← {b_σ(1), ..., b_σ(||S_b||)} with some ordering σ
for i ← 1 to ||S_b|| do
    perform the algorithm 1 among b_σ(i) and S_s
return
```

It is clear that the order in which sellers and buyers act matters. While the ordering μ can be chosen in order to minimize the loss, minimizing the distance between the buyer and the selected seller, or in other ways; the ordering σ is not trivial to select. Let $\mathcal{I}_S$ be the set of orderings over S_b. Then, given $\sigma \in \mathcal{I}_S$, the total loss over the distribution lines due to the power transfer among the sellers and buyers of S is

$$u(S,\sigma) = -\left(\sum_{i\in S_s, j\in S_b} P_{ij}^{\text{loss}} + \sum_{i\in S_s} P_{i0}^{\text{loss}} + \sum_{j\in S_b} P_{j0}^{\text{loss}}\right), \tag{5}$$

where:

- P_{i0}^{loss} and P_{j0}^{loss} are given by the equations 1 and 2;
- P_{ij}^{loss} is the power loss in the distribution lines during the power transfer inside the coalition between a seller i and a buyer j, which is also given by the equations 1 and 2. We can make similar considerations as we done before about the case that the power that can reach a buyer is not enough for that buyer. In this case the seller send as much power as he can, or the power that minimizes the loss along the line (equation 3). Moreover, $\beta = 0$ for the absence of the transformer and the voltage of the power

line is U_1, which is typically lower than the medium voltage U_0.

Now it is possible to conclude the definition of the coalitional game, providing the characteristic function

$$v(S) = \max_{\sigma \in \mathcal{I}_S} u(S, \sigma). \tag{6}$$

VI. Experimental Evaluations

For our experiments, we fixed:

- $w_i = 1$ for each $i \in N$ (the price per unit of power);
- resistance of the distribution line: $0.2\ \Omega/km$;
- $\beta = 0.02$, i.e. the 2% of the power is lost due to the transformer of the macro-station;
- $U_0 = 70000\ V$ and $U_1 = 22000\ V$;
- a variable number of micro-grids;
- a random distribution of the micro-grids in a 30×30 kilometers square.
- the position of the macro-station is $(0, 0)$;
- $Q_i \sim \mathcal{N}(0, \mathcal{U}(10, 316))\ MW$ for each $i \in N$.

In this section we provide several methods to perform the coalition formation and the sellers-buyers power transfer, and then we compare their performances.

A. Coalition formation

In the previous section we fixed a coalition and studied how sellers and buyers exchange power among themselves. Now we focus on the coalition formation stage. Given the set N of n micro-grids, the number of different possible non-empty coalitions is B_n. The number B_n is the *Bell number*, and it holds $B_n \in O((n/\log(n+1))^n)$, as proved in [17]. Hence, the exhaustive search of the optimal coalition division is difficult to perform, and we focus on other approaches that can be easier to compute and still reach good results.

1) K-means clustering coalition formation: given the distance-based nature of the problem, we can use a clustering technique. The basic idea is to try to minimize the power loss minimizing the distances between micro-grids of the same coalition.

Typically, clustering algorithms, in particular the most common center-based clustering algorithms, require the number k of cluster to compute. In our scenario there is no reason to think that a given number of coalitions is preferable over another number, so the only way to obtain the best one is to run the clustering algorithm multiple times: one for each $k = 1, \ldots, n$, and then choose the one that yields the lowest power loss.

We use k-means as the clustering algorithm, and analyze its performances in terms of average power loss as the number of coalitions to form increases. We report the results in Fig. 1 and Fig. 2, comparing them with the average power loss obtained with the non-cooperative strategy, in which there is not any coalition.

In the first simulation (Fig. 1), given a coalition, the stage of negotiation among its sellers and buyers is performed with a random ordering of the buyers and reiterated many times, saving the optimal ordering. Given a buyer, the first seller to be chosen is the nearest one. When k reaches the total number of micro-grids n, the cooperative loss approaches the non-cooperative one.

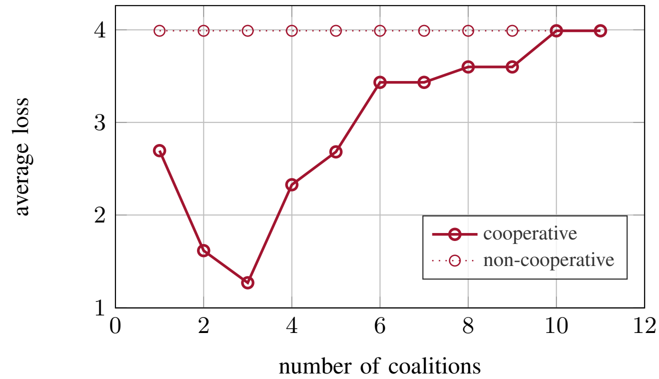

Figure 1: The power loss per micro-grid as the number of coalitions k, given at k-means algorithm, increases. The non-cooperative loss is independent of the number of coalitions. The negotiation stage is randomly performed.

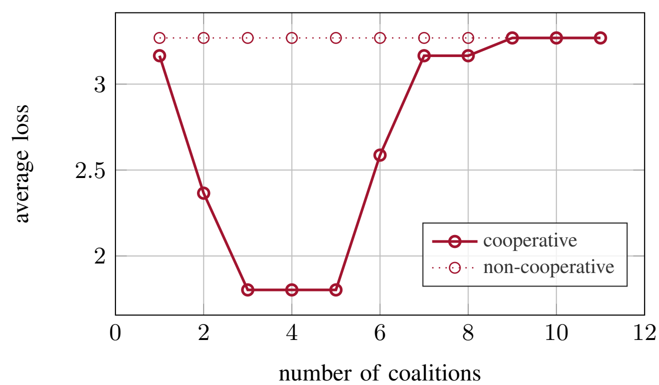

Figure 2: In this simulation the setting is the same as the Fig. 1, but the negotiation stage is performed with an exhaustive search.

In the second simulation (Fig. 2), the configuration is the same as the previous one, but the negotiation stage among sellers and buyers of a coalition is performed with an exhaustive search of the ordering of the buyers, producing all the permutations of the list and keeping the optimal one. Given a buyer, the first seller to be chosen is the nearest one, as in the previous simulation.

Both the simulations start with 1 coalition (because no coalition does not make sense) and finish with $n = 11$ singleton coalitions.

The cooperative average power loss reaches the non-cooperative one before $k = n$, because when $k \approx n$ many coalitions are singleton, and the non-singleton ones have only sellers or only buyers, so all the micro-grids exchange power only with the macro-station, as in the non-cooperative case. Indeed, a weakness of this algorithm is that it does not check if there are at least one seller and one buyer in a coalition, but it groups the micro-grids looking only at the distances among them.

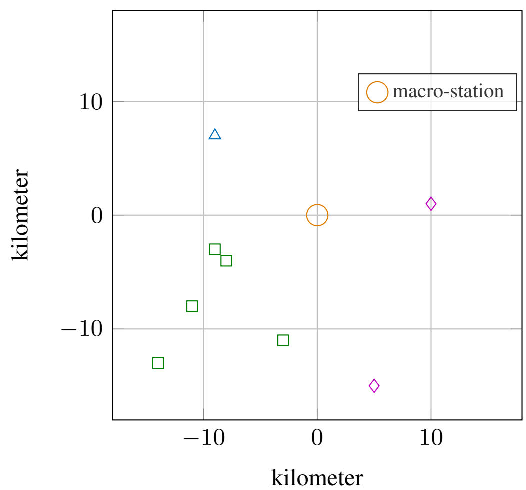

Figure 3: Positions of 8 micro-grids in a 30×30 kilometers square, where the macro-station is located at the center, and micro-grids of the same coalition share the marker.

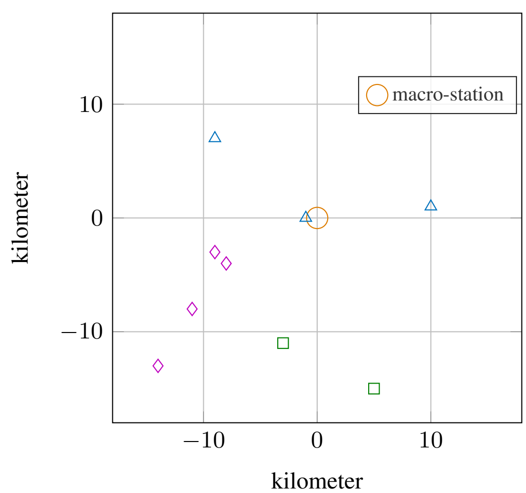

Figure 4: Positions of 9 micro-grids in a 30×30 kilometers square, with the macro-station in the center. Here the coalitions are much different from the Fig. 3, despite there is only one new micro-grid.

This problem leads to another consideration: when the number of micro-grids changes, the coalitions can significantly vary. Fig. 3 and Fig. 4 present an example of this phenomenon. Changing from 8 to 9 the number of micro-grids distributed in a 30×30 kilometers square, the optimal number of coalitions remains 3 (that is generally not true), but the shape of the coalitions is much different.

The last simulations, whose results are shown in Fig. 5, provide a comparison between the cooperative average power loss and the non-cooperative one on varying the number of micro-grids. The cooperative average loss is computed with a k-means coalition formation, and the ordering of the buyers of the coalition negotiation stage is random, as in Fig. 1. The trend is so variable because at each simulation the value of Q_i changes, and the loss with it.

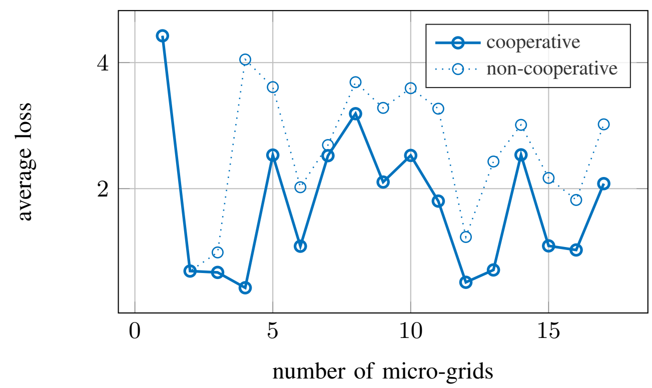

Figure 5: Comparison between the average power loss achieved by the cooperative strategy (k-means coalition formation and random buyers ordering) and the non-cooperative one. Given the number of micro-grids, the cooperative average power loss taken is the best among the several possible coalition configurations considered by the algorithm, as shown in Fig. 1 and Fig. 2.

The instability of this technique may not be an issue, but in a realistic scenario, the dynamic of the power surplus or need of a micro-grid has to be taken into account. For this reason, a possible improvement of this method is to modify the k-means clustering in a way that permits it to consider which micro-grid is a seller, and which one is a buyer.

Moreover, as the number of micro-grids n increases, try all $k = 1, \dots, n$, running k-means n times becomes infeasible. To handle this issue, k-means could only use $k \in [2, n/3]$, that seems reasonable from our simulations. However, there are not theoretical guarantees that suggest an optimal number of coalitions lower than $n/3$.

2) Random coalitions: since the previous method does not provide directly a number of coalitions, we try with this technique. Let random$(\cdot)$ be a primitive that removes and returns a random element from a given set. The random coalition formation strategy is performed by the algorithm 3.

There are not simulation in which the number of coalitions changes, because here we are not looking for the optimal number of coalitions, so we can directly execute the simulations as done in Fig. 5 for the previous technique.

The results are shown in Fig. 6. They are quite similar to the ones obtained with the previous method, but in this case each simulation requires an unique run of the algorithm, while with k-means the number of runs for a fixed number of micro-grids n depends on n, and then the best result is taken. With similar results but less computational time required, the random coalition formation strategy is preferable.

The comparison among these two strategies, fixing the same negotiation method, is shown in Fig. 7.

It could be interesting to further increase the number of micro-grids to observe if the performances become significantly different or stay similar.

Contrary to the clustering-based coalition formation method, here the nature of seller or buyer of a micro-grid

Algorithm 3 Random coalition formation

mgs $\leftarrow N$
$j \leftarrow 0$
$S_j \leftarrow \{\}$
while mgs $\neq \varnothing$ **do**
  $i \leftarrow$ random(mgs)
  $S_j \leftarrow S_j \cup \{i\}$
  if i is a buyer **then**
    if there are free sellers **then**
      $s \leftarrow$ random($\{x \in$ mgs $: x$ is a seller$\}$)
      $S_j \leftarrow S_j \cup \{s\}$
    else
      close the coalition S_j
      $j \leftarrow j + 1$
      $S_j \leftarrow \{\}$
  else
    do the same with $\{x \in$ mgs $: x$ is a buyer$\}$
  if $S_j \neq \varnothing \wedge$ random($\{0, 1\}$) $= 1$ **then**
    close the coalition S_j
    $j \leftarrow j + 1$
    $S_j \leftarrow \{\}$
return

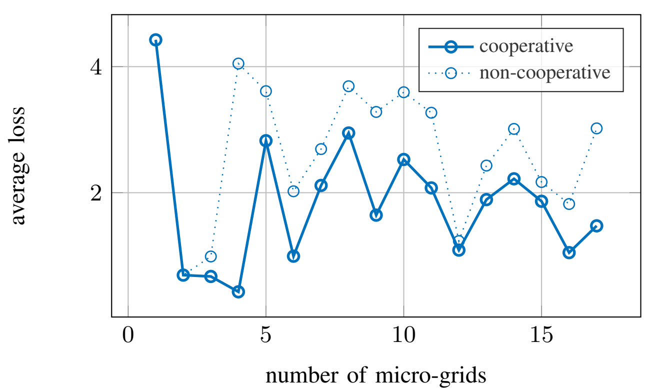

Figure 6: In this plot several simulations are performed, increasing the number of micro-grids randomly distributed in a 30×30 kilometers square. The cooperative average loss is computed with a random coalition formation, and the ordering of the buyers of the coalition's negotiation stage is random, as in Fig. 1.

is taken into account, in order to obtain meaningful coalitions (because a coalition with only sellers or only buyers is basically useless and counter-productive from a computational point of view), but the distance between the micro-grids are not considered. This induces a possible straightforward improvement for the algorithm, in which the probability to take a particular micro-grid to enter the coalition is proportional to its closeness to the coalition (with a proper definition of distance between a micro-grid and an already formed coalition).

Also a genetic approach could be interesting.

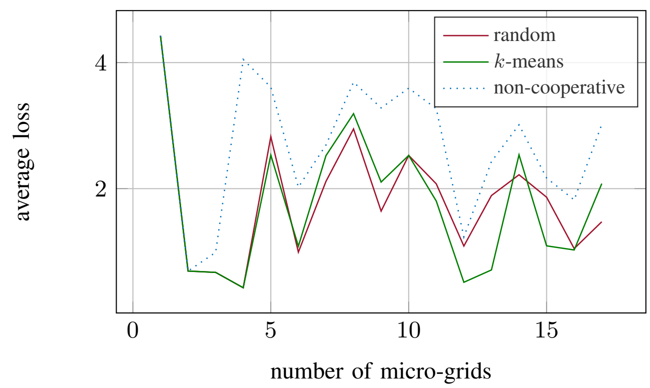

Figure 7: The curves are taken from Fig. 5 and Fig. 6, where markers are removed to better visualize the performances. The two strategies both perform better than the non-cooperative case, but quite similarly. The random one requires no number of coalitions and computes much faster, while k-means requires n runs to figure out which is the optimal average power loss, related to the number of coalitions. Fixing the number of micro-grids, the inherent randomness of these methods does not allow further meaningful comparisons, except by repeating the simulations several times, changing the seed.

B. Ordering of sellers and buyers

We now focus on the negotiation stage. Fixed the coalition formation, this stage can be performed in different manners. Given a coalition, the realization of the power exchange among the micro-grids of the coalition is given by a division of the micro-grids in sellers and buyers, and by a negotiation among them, that requires orderings.

Let us suppose that a coalition S has non-empty sets of sellers $S_s \neq \varnothing$ and buyers $S_b \neq \varnothing$. In this case, the power exchange algorithms described in 1 and 2 can be used. As indicated in the definition of the characteristic function (6), the minimization has to be done looking for the optimal ordering of the buyers σ.

We fixed, as done in [12], the ordering of the sellers by choosing "the first available seller" as the closest to the considered buyer, but also this ordering is not trivial to find. Ideally, the optimal ordering (i.e. the one that minimizes the loss) is the best among all the possible permutations. In this sense, an exhaustive search on both the sellers and the buyers can be done, but it would take a lot of computational time. Hence, in our simulation, we used the proposed algorithms 1 and 2 to find the ordering of the sellers, and we now investigate a couple of strategies to find a good ordering of the buyers.

1) Exhaustive search: the exhaustive search among all possible permutations of the buyers also would take a lot of time and it is feasible only if we are dealing with quite small coalitions. Despite the coalitions are indeed small in our simulations, the comparison between this technique and the following one must take into account this fact.

The exhaustive search give us the guarantee that the optimal ordering of the buyers is found.

2) Random ordering: using a random ordering of the buyers is not a good technique, so we reiterate the negotiation and the power exchange many times ($T = 15$ in all our simulations) and then we take the best ordering found. The small size of our coalitions allows us to find almost always the optimal ordering, so no remarkable comparison between this and the previous technique can be done in term of the quality of the results.

The only observed difference is the computational time: the exhaustive search execution time is greater than the random ordering time and the latter can be controlled deciding how many iterations of the power exchange perform by changing T. For these reasons, we use this random strategy in all our simulations.

In Fig. 1 the ordering of the buyers is found with the $T = 15$ iterations random technique and, in Fig. 2, the exhaustive search is performed. Note that no comparisons can be done between the two plots because they come from different simulations, that means different power values. In these simulations the assumption that each coalition has non-empty sets of sellers and buyers is false, because the k-means-based coalition formation method is used. Moreover, the small size of the coalitions does not allow us to appreciate the ordering search and the differences between the proposed techniques. For these reasons, the time to compute the optimal ordering is unexpectedly low, and the contribution of these techniques is not fully valuable.

Future works may compare these techniques with larger coalitions, also introducing other strategies.

VII. Conclusion

In this work we have presented the smart grid: intelligent electric power distribution networks, in which the "intelligence" comes from the ability to communicate, that allows the entire network to reach results of general interest. In multi-entity interaction scenarios, game theory finds fertile ground. In particular, after a short introduction to (cooperative) game theory, we have focused on the power exchange among micro-grids, grouped in coalitions. Once the model and the basic idea are defined, we have compared some heuristic algorithms to perform the power exchange, forming coalitions and minimizing the average power loss by minimizing the expensive exchanges with the macro-station.

We expect that cooperative game theory may give further interesting contributions to this field. One possible direction could be to consider more episodes in which micro-grids change their power surplus or need, testing the robustness of the proposed algorithms. In addition to the other improvements suggested along the discussion of our results, a more realistic scenario would allow an in-depth study of the probability distribution of power surplus/need, depending on the daily time slot and then introducing machine learning technique to support the algorithms, based on real-world data.

Another interesting aspect of this scenario, that we have not mentioned, is the amount of data that the micro-grids can handle: it is taken for granted that the micro-grids follow the algorithms, but at a certain time the realization of the power transfer has to take place based on a efficient communication between micro-grids, that has to tackle issues of security, maintenance and environment origin [2], [8].

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